Design of a Composite Morphing Wing

Wan Luqman Hakim Wan A Hamid

Supervised by Lorenzo Iannucci and Paul Robinson

This thesis is submitted for the degree of
Doctor of Philosophy of Imperial College London
April 2019
Abstract

Morphing aircraft components can increase the possibility of optimising the performance of an aircraft at various flight conditions. A morphing aircraft wing can change the wing shape to modify the lift and drag distribution on the wing surface, allowing the lift-to-drag ratio to be tailored to the desired performance. A camber morphing and a trailing edge morphing wing changes the aerodynamic lift by altering the camber and by deflecting the wing trailing edge, potentially reducing the aerodynamic drag by eliminating the gaps; which exist between the main wing and the control surfaces of a conventional wing.

Among the technology used to achieve camber morphing and trailing edge morphing, were mechanical and smart actuations, such as piezoelectrics and shape memory alloys (SMAs). Compliant structures, cellular structures, shape memory polymers, and multi-stable structures were exploited to improve the flexibility of the aerofoil sections or wings. SMA wires were one of the smart actuators which had been extensively utilised to morph various aerofoils/wings, mainly due to the high actuation force and compatibility, which reduce the volume and weight of the actuators and the complexity of moving mechanical components.

In this research, a user defined material model (UMAT) was developed within the explicit LS-DYNA FE code, for NiTi shape memory alloy (SMA) wires, and used for actuation of the composite morphing wing. The Tanaka SMA constitutive model was implemented in MATLAB and FORTRAN codes for the SMA-actuation of various structures.

The UMAT was used to simulate actuations of various complex morphing structures, including several aluminium and composite aerofoils with corrugated sections, and a pre-curved corrugated plate. Actuations of the two aluminium aerofoils, with corrugated sections in the lower surface and the middle cantilever section, by a 0.5mm-diameter SMA wire with a maximum recoverable strain or a pre-strain of 1.6%, resulted in trailing edge (TE) deflections of 7.8 mm and 65.9 mm, respectively. Actuation of the carbon fibre (CF) composite aerofoil, with the corrugated section as a middle cantilever section, and with 8 layers of CF in ±45° directions, produced a TE deflection of 52.0 mm.

To demonstrate the SMA-actuated morphing concept, a composite 3D-printing technology was explored to manufacture a carbon fibre (CF) composite structure, consisted of a flat vertical front plate, a corrugated section, and a rear trailing edge (TE) section. Due to the nature of 3D-
printing, two layers of CF were 3D-printed along the circumference of the corrugation and the TE section, and the minimum thickness of the structure was 3 mm. Experimentally, actuation of the CF composite corrugated structure by a NiTi SMA wire with a diameter of 0.2 mm and a pre-strain of 4.77%, and with a diameter of 0.5 mm and a pre-strain of 1.68%, aligned in the chordwise direction, resulted in 1.1 mm and 6.0 mm TE deflections, respectively. Cyclic tests (10 and 30 cycles) of the actuation of the CF composite corrugated structure showed the TE deflection converged after few cycles.

A 1.25m-span composite morphing wing was finally designed and manufactured, consisted of a CF composite D-nose spar which resisted the main aerodynamic loading, and rear sections which were made of rigid and flexible foams. CF composite spar flanges, spar web, front and rear ribs, were 3D-printed, and were assembled with a CF composite skin which was autoclave-manufactured, to form the CF composite D-nose spar. Sections of rigid and flexible foams were CNC-machined and were attached to the front CF composite D-nose spar, 3D-printed long rear ribs, trailing edge sections and the morphed corrugated structure, to form a complete composite morphing wing.
Declaration of Originality

The work presented hereafter is based on research carried out by the author at the Department of Aeronautics of Imperial College London. It is all the author’s own work except where otherwise acknowledged. None of the present work has been submitted elsewhere for another degree or qualification.

Wan Luqman Hakim Wan A Hamid
Copyright Declaration

The copyright of the thesis rests with the author. Unless otherwise indicated, its contents are licensed under a Creative Commons Attribution-Non Commercial 4.0 International License (CC BY -NC).

Under this licence, you may copy and redistribute the material in any medium or format. You may also create and redistribute modified versions of the work. This is on the condition that: you credit the author and do not use it, or any derivative works, for a commercial purpose.

When reusing or sharing this work, ensure you make the licence terms clear to others by naming the licence and linking to the licence text. Where a work has been adapted, you should indicate that the work has been changed and describe those changes.

Please seek permission from the copyright holder for uses of this work that are not included in the licence or permitted under UK Copyright Law.
Acknowledgements

First and foremost, I would like to express my utmost gratitude to my supervisors, Professor Lorenzo Iannucci and Professor Paul Robinson, for their patience, continuous support and valuable guidance throughout the PhD study.

I am also thankful to Dr. Matthew Santer, for his valuable advice and expertise in the development of the project.

I dedicate this work to my late father, who passed away during the first year of the PhD study, for his moral support, long struggle and great dedication to provide his children with the highest possible education. I am grateful to my wife, Aisyah Zahraa’ and my daughter, Wan Ayleen Alisyaa, for their love, sacrifice and companion during these years. I am also indebted to my siblings, especially Wan Noor Khadijah Syuhada’ and Wan Mohd Qusyairi Hakim for their assistance in supporting the family during my absence, after the departure of our father.

I am also thankful to my colleagues, technical and administration staff in the Department of Aeronautics, for their friendship and support during these years.

The work presented in this thesis was funded by International Islamic University Malaysia (IIUM) and Ministry of Education (MoE) Malaysia. Their sponsorship and financial support are hereby greatly acknowledged.
# Table of Contents

Abstract........................................................................................................................................... 1
Declaration of Originality .................................................................................................................. 3
Copyright Declaration ....................................................................................................................... 4
Acknowledgements ............................................................................................................................ 5
Table of Contents .............................................................................................................................. 6
List of Figures ..................................................................................................................................... 10
List of Tables ..................................................................................................................................... 23

## Chapter 1 Introduction .................................................................................................................. 25
  1.1 Research Problem and Research Statement........................................................................ 26
  1.2 Motivation................................................................................................................................. 28
  1.3 Research Objectives and Novelties ....................................................................................... 29
  1.4 Research Methodology ........................................................................................................... 30
  1.5 Thesis Outline ......................................................................................................................... 32

## Chapter 2 Literature Review ........................................................................................................ 34
  2.1 Overview ................................................................................................................................. 34
  2.2 Morphing Aerofoils/Wings by Mechanical Actuation .......................................................... 37
  2.3 Morphing Aerofoils/Wings by Smart Actuation ................................................................. 48
    2.3.1 Piezoelectric/Piezoceramic ............................................................................................ 48
    2.3.2 Micro-Fibre Composite (MFC) .................................................................................... 49
    2.3.3 Electro-Active Polymer (EAP) ..................................................................................... 50
    2.3.4 Shape Memory Polymer (SMP) ................................................................................... 51
    2.3.5 Shape Memory Alloy (SMA) ....................................................................................... 53
  2.4 Camber Morphing by Compliant Mechanism, Multistable Structures, and Other Structures ......................................................................................................................... 68
    2.4.1 Compliant Structures .................................................................................................... 68
    2.4.2 Multistable Structures .................................................................................................. 74
    2.4.3 Kagome Truss Structures .............................................................................................. 80
  2.5 Shape Memory Alloy (SMA) - Thermomechanical Behaviour and Physical Properties ................................................................................................................................. 85
  2.6 SMA Constitutive Models ....................................................................................................... 90
Chapter 5 SMA-Actuated Morphing Aerofoils

5.1 Finite Element Analysis (FEA) of SMA-Actuated Morphing Aerofoils

5.1.1 SMA-Actuated Aluminium Morphing Aerofoils

5.1.1.1 Mesh Convergence Study

5.1.1.2 Parameter Study

5.1.2 SMA-Actuated CF Composite Morphing Aerofoils

5.2 Finite Element Analysis of SMA-Actuated Pre-Curved Corrugated Plate

5.3 Experiments on SMA-Actuated 3D-Printed Carbon Fibre (CF) Composite Corrugation

5.3.1 Microstructure of Carbon Fibres and Onyx Filaments

5.3.2 Tensile Tests on 3D-Printed Carbon Fibre (CF) Composites

5.3.2.1 Carbon Fibre (CF) Composite Specimens Preparation

5.3.2.2 Tensile Tests Procedure

5.3.2.3 Tensile Tests Results

5.3.2.4 Microstructure of 3D-Printed Unidirectional Carbon Fibre Composite After Tensile Tests

5.3.3 3D-Printed CF Composite Corrugated Structure

5.3.3.1 Microstructure of 3D-Printed Carbon Fibre (CF) Composite Corrugated Structure

5.3.4 3-Point Bending Tests on 3D-Printed Carbon Fibre (CF) Composites

5.3.4.1 Analytical Solution of Flexural Stiffness of 3D-Printed CF Composite

5.3.5 Experimental Tests of SMA-Actuated CF Composite Corrugated Structure

5.3.5.1 Test A – SMA Diameter 0.2 mm, 4.77% Pre-Strain

5.3.5.2 Test B – SMA Diameter 0.5 mm, 1.68% Pre-Strain

5.3.6 Control Surface Area of the Composite Morphing Wing

5.3.7 Static Load on Carbon Fibre (CF) Composite Corrugated Structure

5.3.7.1 Point Loading on Three Locations

5.3.7.2 Tip Loading

5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation
5.4.1 Virtual 3-Point Bending Tests on 3D-Printed Carbon Fibre (CF) Composites
5.4.2 FEA Simulation of SMA-Actuated Carbon Fibre Composite Corrugated Structure
5.4.3 Predictive Virtual Testing of SMA-Actuated Carbon Fibre Composite Corrugated Structure
5.5 Chapter Summary and Conclusions

Chapter 6 SMA-Actuated Composite Morphing Wing

6.1 Aerodynamic Loads
6.2 Design of CF Composite D-Nose Spar
   6.2.1 CF Composite Spar Caps/Flanges and Cell Walls
   6.2.2 Buckling of CF Composite D-Nose Spar
6.3 3D-Printed Carbon Fibre (CF) Composite D-Nose Spar
   6.3.1 CF Composite Spar Flanges
   6.3.2 CF Composite Spar Web
   6.3.3 CF Composite Front and Rear Ribs
   6.3.4 CF Composite D-Nose Skin
   6.3.5 Manufacturing Difficulties, Constraints and Iterations
6.4 Assembly of CF Composite D-Nose Spar
6.5 Foam Sections And 3D-Printed Trailing Edge Sections
6.6 FE Simulations of Composite Morphing Wing
6.7 Chapter Summary and Conclusions

Chapter 7 Conclusions and Recommendations

7.1 Conclusions
7.2 Recommendations for Future Research

Bibliography

Appendix

Appendix A: Experimental Tests on Shape Memory Alloy (SMA) Wires
Appendix B: SMA Transformation Equations
Appendix C: UMAT, Fortran and Matlab Codes
Appendix D: Experimental Tests on 3D-Printed CF Composite
Appendix E: FEA Simulation of Virtual 3-Point Bending Tests on CF Composite Beam – Two Beams FEA Model
Appendix F: Composite Morphing Wing
## List of Figures

**Figure 1.1.** (a) Wings with conventional control surfaces, and (b) several possible concepts of the proposed morphing wings. ........................................................................ 27

**Figure 1.2.** Morphing wing inspiration from the nature of flying birds. ................................................................. 28

**Figure 1.3.** A full envelope aircraft performance that can be achieved by a morphing wing (Gilbert, 1981). ......................................................................................................................... 29

**Figure 1.4.** Research Methodology .......................................................................................................................... 31

**Figure 2.1** Several types of morphing wings: (a) a span morphing, (b) a twist morphing, (c) a sweep morphing, (d) a camber morphing and (e) a trailing edge morphing. ........ 36

**Figure 2.2.** Mission Adaptive Wing (MAW) (a) trailing edge mechanism concept and (b) the deflected shapes (Gilbert, 1981). ............................................................................................................. 38

**Figure 2.3.** An aluminium flexible rib model of the formvariable flap structure (Monner, 2001). .................................................. 39

**Figure 2.4.** Articulated cell concept (Iannucci et al., 2009). ..................................................................................... 40

**Figure 2.5.** (a) Actuation system in between morphing skin panels (MSP), and the achieved control surface deflection (Thill et al., 2010). ............................................................. 41

**Figure 2.6.** Active winglet of a flying wing (Gatto et al., 2010). .............................................................. 42

**Figure 2.7.** A Rotating Ribs (RR) concept with linear slide connections (left) and a ‘proof-of-concept’ trailing edge model without upper skin (right) (Ricci et al., 2006, Miller et al., 2010). ......................................................................................................................... 42

**Figure 2.8.** Hingeless trailing edge control surface (HCS) actuated by eccentuators and ultrasonic motor (Wang et al., 2001). ................................................................................................................. 43

**Figure 2.9.** A modified eccentuator concept (Iannucci et al., 2009). .......................................................... 43

**Figure 2.10.** A seamless aeroelastic wing (SAW) with a torque tube actuation mechanism (Perera et al., 2010). ............................................................................................................................. 44

**Figure 2.11.** A morphing leading edge section actuated by an eccentric beam actuation mechanism (EBAM), at a neutral position (left) and at a fully-deployed position (right) (Morishima et al., 2010). ................................................................. 44

**Figure 2.12.** (a) A Pneumatic Artificial Muscle (PAM) actuator, (b) top view of the FishBAC aerofoil section and (c) cross-sectional view and trailing edge deflection of the FishBAC aerofoil section (Woods et al., 2014). ................................................................................................................. 45

**Figure 2.13.** A Pneumatic Muscle Fibre (PMF) actuator as an active morphing lower skin for a camber morphing structure (Feng et al., 2015). .............................................................. 46

**Figure 2.14.** Deployable/inflatable wing using on-board compressed air tank (Jacob and Smith, 2009). ................................................................................................................................. 47
Figure 2.15. A variform concept for an unmanned aerial vehicle wing: (a) an initial bulky NACA 23015 profile (outer line) morphed into a sleeker FX 60-126 profile (inner section), and (b) the possible fuel bladder configurations (Gano and Renaud, 2002)..........................................................47

Figure 2.16. A controllable camber rotor aerofoil prototype with a compliant mechanism and piezostack actuators (Gandhi et al., 2008). .........................................................48

Figure 2.17. Numerical results in term of pressure distribution for (a) the flapped and (b) the twisted wings (Donadon and Iannucci, 2014). .............................................................49

Figure 2.18. (a) A SSMA concept and (b) an experimental construction of the SSMA concept (Pankonien et al., 2013)............................................................................50

Figure 2.19. (a) Principle of actuation of the electro-active polymer (EAP) dimple actuator, (b) dimple topology at a maximum deflection for a 5 mm radius dimple, and (c) out-of-plane deflection versus applied voltage for the 5 mm, 10 mm and 15 mm radius dimples (left to right) (Dearing et al., 2010)..............................................................51

Figure 2.20. A shape memory polymer (SMP) concept for a morphing wing: (a) activation and actuation of the SMP morphing skin, (b) active top surface area of the SMP skin, and (c) active bottom surface area of the SMP skin (James et al., 2009). 52

Figure 2.21. Shape memory wire concept (James et al., 2009). ..........................................................53

Figure 2.22. SMA-actuated underwater vehicle (Webb et al., 2000)......................................................53

Figure 2.23 SMA-actuated biomimetic hydrofoil (Rediniotis et al., 2002). ..............................................54

Figure 2.24. SMA-actuated trailing edge tab of a helicopter rotor blade (Epps and Chopra, 2001). ......................................................................................................................55

Figure 2.25. SAMPSON (a) adaptive lip (Pitt et al., 2001) and (b) adaptive internal wall (Pitt et al., 2002). ..................................................................................................................55

Figure 2.26. An offset cantilever beam (Banerjee et al., 2008). ...............................................................56

Figure 2.27. (a) Multistable structures, and brainstormed morphing concepts based on (b) several multistable structures and (c) embedded shape memory wires (or other active materials) for flight control, and (d) a SMA-actuated flexible curved composite plate (Iannucci and Fontanazza, 2008)..................................................................................................58

Figure 2.28. Morphing laminar wing with flexible extrados (Georges et al., 2009).........................59

Figure 2.29. Morphing flap of a balsa wing (Kang et al., 2012). ..........................................................59

Figure 2.30. A 3D-printed poly-lactid acid (PLA) morphing wing rib with two different positions of SMA wires (AB and CB) (Almeida et al., 2015). ..........................60

Figure 2.31. A morphing wing with SMA springs fitted in holes across the stainless steel ribs to deform top and bottom composite skins (Dong et al., 2008). .........................61

Figure 2.32. SMA torque tube actuator assembly for Test 2 of phase 1 DARPA Smart Wing Program (Jardine et al., 1999). ......................................................................................62

Figure 2.33. (a) SMA torque tube assembly and (b) torsional test setup (Prahld and Chopra, 2001). ...........................................................................................................................63

Figure 2.34. (a) SMA training method and (b) a cured SMA elastomer block (Talling et al., 2007). .............................................................................................................................64
Figure 2.35. (a) Static chevrons on the core and bypass nozzles in the NASA Langley Research Centre Low-Speed Aeroacoustic Wind Tunnel and (b) SMAHC adaptive chevron (Turner et al., 2008) ........................................65

Figure 2.36. Bio-inspired robotic morphing wing (Basaeri et al., 2014) ........................................66

Figure 2.37. Antagonistic Flexural Cell (AFC) concept (Elzey et al., 2003) ....................................67

Figure 2.38. Hingeless Control Surface (HCS) (Wang et al., 2001) .................................................67

Figure 2.39. Belt-rib concept (Campanile and Sachau, 2000) ..........................................................69

Figure 2.40. Tendon actuated compliant cellular trusses (Ramrkahyani et al., 2004) .......................70

Figure 2.41. (a) Chord-wise cross sectional view of an aerofoil member, and (b) cross sectional aerofoil shape changes achieved through morphing (Iannucci, 2012). ..........................................................70

Figure 2.42. Morphing aerofoil with a composite chiral structure (Airoldi et al., 2012) ............71

Figure 2.43. (a) Experimental setup of a corrugated aerofoil; maximum (b) downward and (c) upward deflections of a bench-top model (Ruangjirakit, 2013) .................................72

Figure 2.44. A morphing wing concept consisted of compliant belt ribs (Previtali et al., 2011, 2014). ........................................................................................................73

Figure 2.45. A compliance morphing wing (Molinari et al., 2016) ................................................73

Figure 2.46. Compliant multistable structures (Santer and Pellegrino, 2008) ...............................75

Figure 2.47. A compliant adaptive leading edge (Santer and Pellegrino, 2009) .........................76

Figure 2.48. (a) Rotor blade section with a bistable flap, (b) lay-up of the bistable laminates, and (c) load-displacement tests setup of the bistable flap of the rotor blade section (Daynes et al., 2009). ..........................................................77

Figure 2.49. (i) Two configurations of bistable element in the initial concept of a multistable plate structure made of Nylon6 from injection moulding, (ii) a prototype model of the multistable plate structure, and (iii) sequential actuations of the multistable plate structure prototype (Santer and Pellegrino, 2011) ..........................78

Figure 2.50. (a) Stacking sequence of tow-steered laminates; Optimum fiber orientation of laminates with (b) identical top two plies for a maximum deployed flap-angle (top) and a maximum flap-angle increment (bottom), and with independent top two plies for (c) a maximum deployed flap-angle and (d) a maximum flap-angle increment; Deformed flap (state 2) with (e) identical case and (f) independent case (Panesar and Weaver, 2012). ................................................................................79

Figure 2.51. (a) An aerofoil structure with an embedded variable stiffness multi-stable element, (b) a tailored lay-out of the multi-stable composites, (c) experimental tests on the compliant rib in a stiff mode (left) and a compliant mode (right) (Arrieta et al., 2014). .................................................................80

Figure 2.52. A sandwich plate actuated by a Kagome planar truss (Wicks and Hutchinson, 2004). ..............................................................................................................................81

Figure 2.53. A morphing UAV wing made of polystyrene foam and ABS sliders (Bolinches et al., 2011) ..................................................................................................................82

Figure 2.54. Stress-strain behavior of shape memory alloy (SMA): (a) Shape Memory Effect (SME) and (b) Pseudoelasticity (PE) (Nishimura, 1997). ..................................................86
Figure 2.55. Phase transformation associated with the shape memory effect (SME) (Chopra, 2002). .......................................................... 87

Figure 2.56. (a) Transformation lines ($M_s$ and $A_s$) of martensitic transformation (MT) in cycles $N =$ 1 and 100, and (b) schematic of starting and end lines of MT and R-phase transformation (RPT) in the 1st cycle and Nth cycle (Tobushi et al., 1996). ......................................................... 88

Figure 2.57. (a) An airship and its landing gear consisted of SMA wires (Alipour et al., 2015, taken from previously published paper, Dayananda et al., 2007), and (b) previously simplified experimental model (left) and the author simplified FE model (right) of the actuated landing gear. .......................................................... 97

Figure 2.58. An adaptive strip actuated by SMA wires (Solomou et al., 2014). ......................... 97

Figure 2.59. A finite element model of a conical shell structure with a SMA strip (Roh et al., 2006). ........................................... 99

Figure 2.60. SMA sandwich beam, (a) experiment and (b) FEA simulation results (Okabe et al., 2011). ...................................................... 100

Figure 2.61. Potential applications of morphing aerofoils. .................................................... 104

Figure 3.1. A Differential Scanning Calorimeter (DSC) machine .............................................. 109

Figure 3.2. The prepared SMA wires with (a) 0.1 mm, (b) 0.2 mm and (c) 0.5 mm diameters. ........................................................................ 109

Figure 3.3. (a) A weight scale and (b) a Tzero press. ............................................................... 109

Figure 3.4. SMA transformation temperatures obtained from Differential Scanning Calorimetry (DSC) tests on NiTi shape memory alloy wires with (a) 0.1 mm, (b) 0.2 mm and (c) 0.5 mm diameter. .......................................................... 112

Figure 3.5. Phase transformation temperatures of shape memory alloy (SMA) wires obtained from Differential Scanning Calorimetry (DSC) tests. ....................................................... 113

Figure 3.6. Shape memory alloy (SMA) wires with diameter of (a) 0.1 mm, (b) 0.2 mm and (c) 0.5 mm. ......................................................... 114

Figure 3.7. (a) Experimental setup of the tensile tests on shape memory alloy (SMA) wires, (b) a small cord and yarn grip (INSTRON 2714-031) at a zero gap position, and (c) a clamped 0.2mm-diameter SMA wire on the large cord and yarn grip (INSTRON 2714-040). .......................................................... 115

Figure 3.8. Tensile stress-strain behaviour of NiTi SMA wires with a diameter of 0.1 mm and gauge lengths of (a) 250 mm and (b) 300 mm. ......................................................... 116

Figure 3.9. Tensile stress-strain behaviour of NiTi SMA wires with a diameter of 0.2 mm and a gauge length of 250 mm. ......................................................... 118

Figure 3.10. Tensile stress-strain behaviour of NiTi SMA wires with a diameter of 0.5 mm and gauge length of (a) 200 mm and (b) 250 mm. ......................................................... 119

Figure 3.11. (a) Load-displacement and (b) stress-strain results of load-controlled tensile tests on NiTi SMA wires, with a diameter of 0.1 mm, a gauge length of 200 mm, and a maximum applied tensile load of 10N. ......................................................... 121
Figure 3.1. Captured images of a NiTi SMA wire with a diameter of 0.1 mm and a gauge length of 200 mm, subjected to a tensile load at a loading rate of 10 N/min (left: at the initial position, middle: at the maximum load of 10N, and right: at the final unloading position). .......................................................... 122

Figure 3.12. Captured images of a NiTi SMA wire with a diameter of 0.2 mm and a gauge length of 300 mm, subjected to a tensile load at a loading rate of 10 N/min (left: at the initial position, middle: at the maximum load of 30N, and right: at the final unloading position) .......................................................... 124

Figure 3.13. Captured images of a NiTi SMA wire with a diameter of 0.2 mm, a gauge length of 300 mm, subjected to a tensile load at a loading rate of 10 N/min (left: at the initial position, middle: at the maximum load of 30N, and right: at the final unloading position) .......................................................... 125

Figure 3.14. (a) Load-displacement and (b) stress-strain results based on strain measurement of dotted points on the SMA wires and (c) stress-strain results based on strain measurement of circular tabs, of load-controlled tensile test results on NiTi SMA wires with a diameter of 0.2 mm, a gauge length of 300 mm, and a maximum applied tensile load of 30N. ........................................................................ 127

Figure 3.15. Captured images of a NiTi SMA wire with a diameter of 0.5 mm and a gauge length of 200 mm, subjected to a tensile load at a loading rate of 20 N/min (left: at the initial position, middle: at the maximum load of 200N, and right: at the final unloading position). ........................................................................ 128

Figure 3.16. (a) Load-displacement and (b) stress-strain results of load-controlled tensile tests on NiTi SMA wires with a diameter of 0.5 mm, a gauge length of 200 mm, and a maximum applied tensile load of 200N. .......................................................... 129

Figure 3.17. SMA stress-strain behaviour from the supplier datasheet (SAES Getters, 2008). ........................................................................ 130

Figure 3.18. NiTi SMA wires with a diameter of (a) 0.2 mm and (b) 0.5 mm clamped on the tension fixture on the RSA-G2 machine ........................................................................ 131

Figure 3.19. Frequency sweep of the DMTA tensile tests ........................................................................ 132

Figure 3.20. Results of DMTA frequency sweep tensile tests on NiTi SMA wires with a diameter of (a) 0.2 mm and (b) 0.5 mm ........................................................................ 133

Figure 3.21. Amplitude/strain sweep of the DMTA tensile tests ........................................................................ 134

Figure 3.22. Results of DMTA amplitude sweep tensile tests on NiTi SMA wires with a diameter of (a) 0.2 mm and (b) 0.5 mm ........................................................................ 135

Figure 3.23. Transition temperatures of SMA under different loads (Butera, 2008) ........................................................................ 136

Figure 4.1. A SMA wire - linear spring structure ........................................................................ 137

Figure 4.2. UMAT structure for one element of the shape memory alloy (SMA) material model ........................................................................ 138

Figure 4.3. A SMA wire - linear spring structure ........................................................................ 139

Figure 4.4. SMA stress and strain during the actuation of the SMA wire - linear spring structure ........................................................................ 140

Figure 4.5. Global damping energy and global internal energy ........................................................................ 141

Figure 4.6. Internal energy of the SMA wire and the linear spring ........................................................................ 142

Figure 4.7. Force-displacement curve of the SMA wire – linear spring model ........................................................................ 143

Figure 4.8. Strain energy integrated from the force-displacement curve ........................................................................ 144
Figure 4.9. Finite element model of a SMA-actuated cantilever beam. .................................................. 149

Figure 4.10. SMA stress and strain of the SMA-actuated cantilever beam. ........................................... 150

Figure 4.11. Tip deflection of the SMA-actuated cantilever beam .......................................................... 151

Figure 4.12. SMA stress and strain extracted from the mesh sensitivity study, for the SMA wire with 1, 2, 5, 10 and 100 beam elements. .......................................................... 151

Figure 4.13. Variables of interest in the key parameter study and the associated normalised stiffness ........................................................................................................................................ 152

Figure 4.14. SMA stress and strain for six normalised stiffness in the key parameter study. ........................................................................................................................................ 153

Figure 4.15. Relationship between SMA recovery and residual stresses, SMA recovery and residual strains, and normalised stiffness ........................................................................................................................................ 154

Figure 5.1. SMA-actuated aluminium morphing aerofoil A, with a corrugated section as a lower skin. ........................................................................................................................................ 158

Figure 5.2. SMA-actuated aluminium morphing aerofoil B, with a corrugated section as a middle cantilever beam. ........................................................................................................................................ 159

Figure 5.3. FEA results of the SMA-actuated aluminium morphing aerofoil A: (a) vertical deflection and (b) von Mises stress ........................................................................................................................................ 160

Figure 5.4. FEA results of the SMA-actuated aluminium morphing aerofoil B: (a) vertical deflection and (b) von Mises stress ........................................................................................................................................ 161

Figure 5.5. SMA-actuated composite morphing wing with several possible configurations and positions of the aerofoil sections, which consist of SMA-actuated corrugated sections: (a) one corrugated section at the wing tip and (b) seven corrugated sections along the span of the flexible wing section. ........................................................................................................................................ 163

Figure 5.6. Mesh sensitivity study: the corrugated section with shell element size of (i) 5 mm, (ii) 2.5 mm, (iii) 1.25 mm, (iv) 1 mm, (v) 0.625 mm, and (vi) 0.5 mm. ........................................................................................................................................ 164

Figure 5.7. Results of mesh convergence study: tip deflection of aluminium morphing aerofoil B ........................................................................................................................................ 165

Figure 5.8. Aluminium morphing aerofoil with a middle corrugated section, actuated by 12 SMA wires on the lower part and 11 SMA wires on the upper part of the corrugated section: von Mises stress (top) and trailing edge deflection (bottom). ........................................................................................................................................ 167

Figure 5.9. Aluminium morphing aerofoil with a cantilever corrugated section, actuated by a single long SMA wire between the D-nose spar and the trailing edge section: von Mises stress (top) and trailing edge deflection (bottom). ........................................................................................................................................ 168

Figure 5.10. Aluminium morphing aerofoil with a cantilever plate, actuated by a single long SMA wire between the D-nose spar and the trailing edge: von Mises stress (top) and trailing edge deflection (bottom). ........................................................................................................................................ 170

Figure 5.11. Aluminium morphing aerofoil with a middle corrugated section, and upper and lower skins, actuated by 12 SMA wires: von Mises stress (top) and trailing edge deflection (bottom). ........................................................................................................................................ 171
Figure 5.12. Aluminium morphing aerofoil with corrugated upper and lower skins, actuated by 12 SMA wires on the outer part of the lower skin and 11 SMA wires on the inner part of the upper skin: von Mises stress (top) and trailing edge deflection (bottom). .......................................................................................................................... 172

Figure 5.13. Trailing edge deflections of composite morphing aerofoil sections: corrugated plate with (a) a symmetrical lay-up and (b) an anti-symmetrical lay-up........ 175

Figure 5.14. Pre-curved corrugated aluminium plates actuated by one SMA wire per cell: (a) isometric view and (b) side view. ................................................................. 177

Figure 5.15. Pre-curved corrugated aluminium plates actuated by two SMA wires per cell: (a) isometric view and (b) side view. ................................................................. 178

Figure 5.16. Pre-curved corrugated aluminium plates actuated by 2 SMA wires per cell, which were arranged in parallel: (a) isometric view and (b) side view............. 180

Figure 5.17. Pre-curved corrugated aluminium plates actuated by 2 SMA wires per cell, which were arranged in ‘V’ configuration: (a) isometric view and (b) side view. .................................................................................................................................. 181

Figure 5.18. A carbon fibre (CF) filament, an onyx filament and a 3D-printed half-hexagonal CF composite specimens for microstructure analysis. ....................... 183

Figure 5.19. Microstructure images of a single carbon fibre filament (left) and a single onyx filament (right). ................................................................................................. 184

Figure 5.20. 3D-printed carbon fibre composite specimens with (a) unidirectional [0°]₈ and (b) specially orthotropic (45/-45/-45/45/45/-45)° lay-ups......................... 187

Figure 5.21. Tensile stress-strain behaviour of (a) unidirectional [0°]₈ and (b) specially orthotropic (45/-45/-45/45/45/-45)° 3D-printed carbon fibre composites. ................................................................................................................................. 188

Figure 5.22. Results of tensile tests on (a) unidirectional [0°]₈ and (b) specially orthotropic (45/-45/-45/45/45/-45)° 3D-printed carbon fibre composites. ............... 189

Figure 5.23. Cross-section view of the 3D-printed unidirectional (UD) carbon fibre (CF) composite.................................................................................................................. 190

Figure 5.24. A comparison between 3D-printed unidirectional (UD) carbon fibre (CF) composites (a) without and (b) with debonding................................................... 191

Figure 5.25. Magnified images (a) 20× and (b) 50× of the 3D-printed unidirectional (UD) carbon fibre (CF) specimen ................................................................. 192

Figure 5.26. A model of 3D-printed CF composite corrugated structure with a rigid trailing edge section. ........................................................................................................ 193

Figure 5.27. 3D-printing position 1, and the carbon fibre and onyx distributions............. 194

Figure 5.28. 3D-printing position 2, and the carbon fibre and onyx distributions............. 194

Figure 5.29. Design iterations of the 3D-printed carbon fibre (CF) composite corrugated structure. ................................................................................................................. 195

Figure 5.30. 3D-printed onyx washers .................................................................................. 195

Figure 5.31. Microstructures of a small section (one half-hexagonal cell) of the 3D-printed carbon fibre composite corrugated structure. .................................................. 197
Figure 5.32. 3D-printed (a) CF composite corrugated structure and (b) CF composite specimens for 3-point bending tests. ................................................................. 198

Figure 5.33. Force-displacement curve of the 3D-printed carbon fibre composites under a 3-point bending load. ................................................................. 199

Figure 5.34. Stress-strain curve of 3D-printed carbon fibre composites under 3-point bending load. ................................................................. 200

Figure 5.35. Conditions of the 3D-printed carbon fibre composites at 0.6% and 0.8% flexural strain. ................................................................. 202

Figure 5.36. 3D-printed CF composites under a 3-point bending load: initial state and states at failure. ................................................................. 203

Figure 5.37. A comparison between the unidirectional carbon fibre composite specimens (a) with a lay-up following the ASTM standard and (b) as a representative of the CF composite corrugated structure. ................................................................. 205

Figure 5.38. Experimental setup of the SMA-actuated composite morphing corrugated structure. ................................................................. 206

Figure 5.39. Temperature measurement devices: (a) Ti400 FLUKE thermal imager, (b) K-type thermocouple with COMARK thermometer, and (c) NI 9211 thermocouple module. ................................................................. 207

Figure 5.40. Initial (top) and final (bottom) configurations of the SMA-actuated CF composite corrugated structure with a SMA wire having a diameter of 0.2 mm and a pre-strain of 4.77% (10-cycles test). ................................................................. 209

Figure 5.41. Trailing edge deflection and SMA wire temperature for the 10-cycles test – SMA wire with 0.2 mm diameter and 4.77% pre-strain. ................................................................. 209

Figure 5.42. Initial (top) and final (bottom) configurations of the SMA-actuated CF composite corrugated structure with a SMA wire having a diameter of 0.2 mm and a pre-strain of 4.77% (30-cycles test). ................................................................. 209

Figure 5.43. Trailing edge deflection and SMA wire temperature for the 30-cycles test – SMA wire with 0.2 mm diameter and 4.77% pre-strain. ................................................................. 210

Figure 5.44. Initial (top) and final (bottom) configurations of the SMA-actuated CF composite corrugated structure with a SMA wire having a diameter of 0.5 mm and a pre-strain of 1.68% (10-cycles test). ................................................................. 211

Figure 5.45. Trailing edge deflection and SMA wire temperature for the 10-cycles test – SMA wire with 0.5 mm diameter and 1.68% pre-strain. ................................................................. 211

Figure 5.46. Initial (top) and final (bottom) configurations of the SMA-actuated CF composite corrugated structure with a SMA wire having a diameter of 0.5 mm and a pre-strain of 1.68% (30-cycles test). ................................................................. 212

Figure 5.47. Trailing edge deflection and SMA wire temperature for the 30-cycles test – SMA wire with 0.5 mm diameter and 1.68% pre-strain. ................................................................. 212

Figure 5.48. Comparison of trailing edge deflections of the SMA-actuated CF composite corrugated structure, between test A (0.2 mm SMA diameter and 4.77% pre-strain) and test B (0.5 mm SMA diameter and 1.68% pre-strain), for the (a) 10-cycles test and (b) 30-cycles test. ................................................................. 213

Figure 5.49. Smart composite morphing wing. ................................................................. 215
Figure 5.50. Pressure distribution on a NACA 0012 aerofoil at Re $1.141 \times 10^6$, solved in XFOIL. .................................................................216

Figure 5.51. (a) Re-plotted chordwise pressure distribution on a NACA 0012 aerofoil, and (b) the pressure distribution that was used and excluded in the calculation of aerodynamic load acting on the CF composite corrugated structure. ............................217

Figure 5.52. Trailing edge deflection of the CF composite corrugated structure subjected to downward vertical point loadings at three locations: (a) experimental result and (b) FEA simulation result. ............................................................218

Figure 5.53. Mild steel cylinders which were in-house manufactured and were used in the static test of the CF composite corrugated structure. .................................................................218

Figure 5.54. Trailing edge deflection of the CF composite corrugated structure subjected to a tip loading: (a) experimental result and (b) FEA simulation result. .................219

Figure 5.55. Five integration points across the thickness of CF composite beam. ..............220

Figure 5.56. FEA model of 3-point bending test on the CF composite beam. ..................221

Figure 5.57. Input curve of applied displacement vs. time. ...........................................222

Figure 5.58. FEA simulation result of the applied load on the middle section/nodes of the CF composite beam under a 3-point bending load. ........................................223

Figure 5.59. FEA simulation result of the centre-displacement of the CF composite beam under a 3-point bending load. .................................................................224

Figure 5.60. FEA simulation result of the flexural stress vs. flexural strain of the CF composite beam. .................................................................224

Figure 5.61. Seven integration points across the thickness of the CF composite beam. .....225

Figure 5.62. Twelve integration points across the thickness of the CF composite beam. ....225

Figure 5.63. FEA simulation results of the applied load on the middle section/nodes of the CF composite beam, with 5, 7 and 12 integration points, subjected to a 3-point bending load. ...........................................................................227

Figure 5.64. FEA simulation results of the flexural stress vs. flexural strain of the CF composite beam, with 5, 7 and 12 integration points (IP), subjected to a 3-point bending load. ...........................................................................227

Figure 5.65. Comparison of the FEA simulation results of the flexural stress vs. flexural strain of the CF composite beam, with the experimental results. ......................227

Figure 5.66. FEA result of deformation (vertical deflection) of the CF composite beam under 3-point bending load. ...........................................................................228

Figure 5.67. CF composite beam model with two beams having (a) 6 integration points (IPs), (b) 8 IPs, and (c) 10 IPs. .........................................................................................229

Figure 5.68. Input of applied displacement vs. time for the CF composite modelled with two beams. .........................................................................................229

Figure 5.69. FEA simulation results of the flexural stress vs. flexural strain of the CF composite beam, modelled with 2 beams and with 6, 8 and 10 integration points (IPs), and subjected to a 3-point bending load. ..................................................230
Figure 5.70. (a) FE model and (b–c) FEA result of deformation (vertical deflection) of the CF composite beam, which was modelled as two beams, and was subjected to a 3-point bending load. ................................................................. 231

Figure 5.71. FE model of SMA-actuated 3D-printed CF composite corrugated structure... 232

Figure 5.72. An example of a cellular honeycomb structure to form smooth upper and lower aerofoil surfaces.......................................................................................................................... 233

Figure 5.73. Deformation and trailing edge deflection of SMA-actuated 3D-printed CF composite corrugated structure: (a) FE simulation and (b) experimental test results.......................................................... 233

Figure 5.74. Result of predictive virtual test on SMA-actuated CF composite corrugated structure......................................................................................................................... 234

Figure 5.75. Illustration of deformation and trailing edge deflection of the SMA-actuated CF composite corrugated structure from the predictive virtual tests......... 235

Figure 6.1. Spanwise lift distribution of the composite morphing wing at sea level flight. .239

Figure 6.2. Eight sections of the carbon fibre composite D-nose spar. ......................... 240

Figure 6.3. A section (S2) of the FE model of the CF composite D-nose spar. ................ 242

Figure 6.4. Quarter chord line or nodes along the semi-span of the CF composite D-nose spar. ........................................................................................................................................... 244

Figure 6.5. Applied aerodynamic lift loading (FOS of 6.75) on the quarter chord of the CF composite D-nose spar. ........................................................................................................ 245

Figure 6.6. FEA result of deformation of the first-iterated CF composite D-nose spar under an applied aerodynamic lift load at FOS of 6.75. .................................................. 247

Figure 6.7. Buckling of the CF composite D-nose skin of sections S1 and S2 of the first-iterated CF composite D-nose spar under an applied aerodynamic lift load at FOS of 6.75. ....................................................................................... 248

Figure 6.8. Buckling of the top CF composite spar flange and the CF composite D-nose skin of sections S4–S6 of the first-iterated CF composite D-nose spar under an applied aerodynamic lift load at FOS of 6.75. ............................................. 248

Figure 6.9. Final design of carbon fibre (CF) composite D-nose spar............................ 250

Figure 6.10. Deformation (vertical displacement) of the CF composite D-nose spar under an applied aerodynamic lift load at FOS of 6.75...................................................... 252

Figure 6.11. Result of tip deflection of the CF composite D-nose spar under an applied aerodynamic lift load at FOS of 6.75. ................................................................. 252

Figure 6.12. Applied aerodynamic lift loading (FOS of 1) on the quarter chord of the CF composite D-nose spar........................................................................................................ 253

Figure 6.13. Deformation (vertical displacement) of the CF composite D-nose spar under an applied aerodynamic lift load at FOS of 1.............................................................. 253

Figure 6.14. Result of tip deflection of the CF composite D-nose spar under an applied aerodynamic lift load at FOS of 1. ................................................................. 253

19
Figure 6.15. Initial full-length composite spar flange of one section of the CF composite D-nose spar of the morphing wing. ................................................................. 255

Figure 6.16. An initial 3D-printed CF composite flange supported by ‘brim’ feature. ....... 255

Figure 6.17. 3D-printed carbon fibre (CF) composite spar flange with flat slots for composite spar web and ribs. ................................................................. 256

Figure 6.18. 3D-printed CF composite top flanges of section S8 with the scarf joint, with steps across the flange thickness. ......................................................... 257

Figure 6.19. 3D-printed bottom flanges of section S5 with the scarf joint, having steps across the flange width: (a) two printed parts and (b) the combined two parts through the scarf joint. ................................................................. 257

Figure 6.20. Carbon fibre (CF) composite strips for additional reinforcement of the scarf joints on the composite spar flanges................................................. 258

Figure 6.21. Evolution of the design and manufacturing iterations of the 3D-printed CF composite spar flanges................................................................. 259

Figure 6.22. Initial design of 3D-printed CF composite spar web.................................. 260

Figure 6.23. Four of the 3D-printed CF composite spar webs after several design iterations. ................................................................. 261

Figure 6.24. Evolution of the design and manufacturing iterations of the 3D-printed CF composite spar web. ................................................................. 261

Figure 6.25. Initial design of CF composites front and rear ribs, fitted into slots on top and bottom flanges. ................................................................. 263

Figure 6.26. 3D-printed CF composites front and rear ribs, (a) top view and (b) isometric view. ................................................................. 263

Figure 6.27. Evolution of the design and manufacturing iterations of the 3D-printed CF composite front and rear ribs. ................................................................. 265

Figure 6.28. A 3D-printed mould for fabrication of D-nose skin.................................. 266

Figure 6.29. 300mm-long male (top) and female (bottom) moulds on the virtual Markforged 3D-printing bed................................................................. 267

Figure 6.30. A 1.5m-long solid wood female mould.................................................. 269

Figure 6.31. Manufacturing of the CF composite D-nose skin: (a) consolidation of four plies of the CF composite prepregs, and (b) the final twelve plies of the CF composite prepregs on the female mould inside the vacuum bag before curing process in the autoclave. ................................................................. 270

Figure 6.32. Curing cycle for honeycomb and monolithic component of Hexcel 8552 CF composite prepregs (Hexcel Corporation, 2016).......................................... 270

Figure 6.33. A 1.5m-long male mould made of a solid Mahogany wood, for manufacturing of the CF composite D-nose skin of the CF composite D-nose spar.............. 271

Figure 6.34. Autoclave cure cycle for monolithic component (<15mm thick) of Hexcel M21 CF composite prepregs (Hexcel Corporation, 2015)........................................ 272

Figure 6.35. The second trial of the CF composite D-nose skin, (a) before and (b) after autoclave curing................................................................. 273
Figure 6.36. The third trial of the CF composite D-nose skin. ........................................... 275
Figure 6.37. The fourth trial of the CF composite D-nose skin. ........................................... 276
Figure 6.38. The fourth fabricated CF composite D-nose skin ........................................... 277
Figure 6.39. An autoclave-cured braided CF composite tube ........................................... 277
Figure 6.40. Initial design of the spar flanges and spar web, and tolerance for the slots. .... 278
Figure 6.41. Detached carbon fibres of the top flange of the CF composite D-nose spar. .... 279
Figure 6.42. 3D-printed CF composite spar flanges covered with blue protective tapes. .... 281
Figure 6.43. 3D-printed CF composite spar flanges on the vacuum table. ......................... 281
Figure 6.44. The assembled 3D-printed CF composite spar flanges. ................................ 282
Figure 6.45. Sequence of assembly of the 3D-printed CF composite spar flanges. .......... 283
Figure 6.46. Sequence of assembly of the 3D-printed CF composite spar web. .......... 284
Figure 6.47. Assembly of the CF composite spar flanges and spar web. .................... 285
Figure 6.48. Squeezed-out solidified acrylic adhesive at the slots on (a) the CF composite spar flanges and (b) the CF composite spar web, for the CF composite front and rear ribs. ............................................................ 286
Figure 6.49. Assembly of 3D-printed CF composite front ribs. ........................................ 287
Figure 6.50. The assembled 3D-printed CF composite rear ribs. ...................................... 288
Figure 6.51. Foam sections machined by a CNC machine .................................................. 289
Figure 6.52. (a) Damaged trailing edge of the foam sections and (b) a rigid 3D-printed CF composite trailing edge section ................................................................. 290
Figure 6.53. SMA-actuated composite morphing wing at a cruise flight condition........ 292
Figure 6.54. Optimum SMA-actuated composite morphing wing at a cruise flight condition ................................................................. 293

Figure A1. Differential Scanning Calorimetry (DSC) tests on shape memory alloy wires with 0.1 mm diameter .......................................................... 307
Figure A2. Differential Scanning Calorimetry (DSC) tests on shape memory alloy wires with 0.2 mm diameter .......................................................... 308
Figure A3. Differential Scanning Calorimetry (DSC) tests on shape memory alloy wires with 0.5 mm diameter .......................................................... 309
Figure A4. Captured images of NiTi SMA wires with a diameter of 0.1 mm and a gauge length of 200 mm, subjected to a tensile load at a loading rate of (a) 0.5 N/min and (b) 5 N/min, at the start of tests (left), at the maximum tensile load of 10N (middle), and at the end of tests (right) .............................................. 310
Figure A5. Captured images of NiTi SMA wires with a diameter of 0.2 mm and a gauge length of 300 mm, subjected to a tensile load at a loading rate of (a) 1 N/min and (b) 5 N/min, at the start of tests (left), at the maximum tensile load of 30N (middle), and at the end of tests (right) .............................................. 311
Figure A6. Captured images of NiTi SMA wires with a diameter of 0.5 mm and a gauge length of 200 mm, subjected to a tensile load at a loading rate of 5 N/min, at the
start of tests (left), at the maximum tensile load of 200N (middle), and at the end of tests (right)………………………………………………………………………………312

**Figure D1.** The developed temperature measurement system of the SMA wire, using the NI 9211 thermocouple module and LabVIEW from National Instruments: (a) block diagram and (b) panel view………………………………………………………………………………331

**Figure F1.** Idealised booms of the D-nose spar. .................................................................338

**Figure F2.** Shear flow in the cell walls of the D-nose spar..................................................342

**Figure F3.** The burned CF composite prepregs and wood female mould on the first trial of manufacture of the CF composite D-nose skin.................................................................348

**Figure F4.** (a) The Nabertherm furnace and (b) the wood with the applied adhesive spray and with the applied filler before (top) and after (bottom) heated in the furnace for 3 hours......................................................................................348

**Figure F5.** Assembly of 3D-printed CF composite rear ribs.............................................. 349
List of Tables

Table 2.1. Morphing concepts deformed by mechanical actuation............................................. 82
Table 2.2. Morphing concepts with smart materials/actuations................................................ 83
Table 2.3. Morphing concepts utilising compliant structures and mechanism-based structures, which could be actuated by mechanical or smart actuators ............................................... 84
Table 2.4. Comparison of shape memory alloy (SMA) with other smart materials - advantages and disadvantages (Chopra and Sirohi, 2014) ................................................................. 85
Table 3.1. Transformation temperatures of NiTi SMA wires obtained from Differential Scanning Calorimetry (DSC) tests ...................................................................................... 113
Table 3.2. SMA material properties obtained from tensile tests on SMA wires with a diameter of 0.1 mm, and gauge length of 250 mm and 300 mm ........................................ 117
Table 3.3. SMA material properties obtained from tensile tests on SMA wires with a diameter of 0.2 mm, and a gauge length of 250 mm .............................................................. 118
Table 3.4. SMA material properties obtained from tensile tests on SMA wires with a diameter of 0.5 mm, and gauge length of 200 mm and 250 mm ................................................. 119
Table 3.5. Summary of load-controlled tensile tests on NiTi SMA wires ................................ 128
Table 3.6. RSA-G2 specifications (TA Instruments, 2014) .......................................................... 129
Table 3.7. Thermomechanical properties of NiTi SMA wires obtained from experimental tests. ........................................................................................................................................ 133
Table 4.1. NiTi SMA material properties (Butera, 2008, SAES, 2009) and additional parameters for the UMAT ............................................................................................................. 141
Table 5.1. Summary and results of mesh sensitivity study on the morphing aerofoil .............. 165
Table 5.2. Material properties of composites, polymers, and metals (Matthews and Rawlings, 1999). ............................................................................................................................... 174
Table 5.3. Material properties of carbon fibre composites for the SMA-actuated composite aerofoils, compared to the previous aluminium material ................................................. 174
Table 5.4. FEA simulations results of the SMA-actuated pre-curved corrugated plates (PCCP) ........................................................................................................................................ 181
Table 5.5. Polishing stages of the carbon fibre composites for microstructure analysis ...... 183
Table 5.6. Results of microstructure analysis on carbon fibre (CF) and onyx filaments ......... 185
Table 5.7. Geometry of 3D-printed carbon fibre composite specimens ................................. 187
Table 5.8. Mechanical properties of 3D-printed carbon fibre composites ............................ 189
Table 5.9. Results of microstructure analysis on 3D-printed carbon fibre composites ...... 192
Table 5.10. Dimension of the 3D-printed composite specimens for 3-point bending test ......... 199
Table 5.11. Flexural properties of the 3D-printed carbon fibre composites ............................ 201
Table 5.12. Experimental trailing edge deflections of the SMA-actuated CF composite corrugated structure ................................................................. 212

Table 5.13. Comparison of trailing edge deflection of the carbon fibre (CF) composite corrugated structure when subjected to three points loading and a tip loading. 219

Table 5.14. Details of the integration points used in the FEA simulation of CF composite beam with 5 integration points across the thickness, subjected to 3-point bending load .................................................................................. 220

Table 5.15. Details of the integration points used in the FEA simulation of CF composite beam with 7 integration points across the thickness, subjected to 3-point bending load .................................................................................. 225

Table 5.16. Details of the integration points used in the FEA simulation of CF composite beam with 12 integration points across the thickness, subjected to a 3-point bending load .................................................................................. 226

Table 6.1. Applied aerodynamic lift load (FOS of 6.75) on 8 sections of the CF composite D-nose spar .......................................................... 245

Table 6.2. The initial design of the CF composite D-nose spar based on analytical solutions .................................................................................. 246

Table 6.3. The first-iterated design of the CF composite D-nose spar for the start of the FEA simulation ........................................................................ 246

Table 6.4. Final composite lay-up of the CF composite D-nose spar ......................................................................................................................... 249

Table 6.5. Thickness and number of carbon fibre (CF) layers of the top and bottom composite spar flanges .............................................................. 254

Table 6.6. Dimensions of the 3D-printed carbon fibre (CF) composite strips for reinforcement of the composite spar web .................................. 262

Table 6.7. Dimensions of the 3D-printed carbon fibre composite front and rear ribs .... 264

Table C1. Important keywords and parameters of Finite Element Model (FEM) of the shape memory alloy wires ................................................................. 315

Table E1. Details of the integration points used in the FEA simulation of the CF composite beam, which was modelled as two beams, with 6 integration points (IPs) across the thickness (3 IPs per beam), subjected to 3-point bending load ........ 332

Table E2. Details of the integration points used in the FEA simulation of the CF composite beam, which was modelled as two beams, with 8 integration points (IPs) across the thickness (4 IPs per beam), subjected to 3-point bending load ........... 332

Table E3. Details of the integration points used in the FEA simulation of the CF composite beam, which was modelled as two beams, with 10 integration points (IPs) across the thickness (5 IPs per beam), subjected to 3-point bending load ........... 332

Table F1. Design and sizing of the spar caps/flanges of the CF composite D-nose spar .... 341

Table F2. Design and sizing of the spar web and D-nose skin of the CF composite D-nose spar ....................................................................................................... 346
Morphing of aircraft wings has the potential of changing the wing geometry between different configurations. Wing span morphing can change the span or the aspect ratio of a wing, which can increase the aerodynamic lift and reduce the induced drag. A wing sweep morphing can change the wing sweep angle, to alter the wing configuration between cruise to another configuration, such as high speed flight. A wing camber morphing changes the wing aerofoil shape from one aerofoil configuration to another, and a leading/trailing edge morphing deforms the leading or trailing edge of the aerofoil to alter the aerodynamic lift generated by the aerofoil. Leading/trailing edge morphing can potentially replace conventional control surfaces and high lift devices, which have gaps between the control surfaces and the main wing/elevator/rudder structures. With the use of morphing technologies, the gaps or the hinge line may be removed, and consequently the aerodynamic drag due to the flow separation along the hinge line can be potentially reduced. Over a long flight range, this indirectly improves the fuel economy, increases the range and endurance, and potentially increases the payload (passengers or luggage), which gives potential profit to the airline operators. In addition to the improvement to aircraft performance, morphing wing technologies, with a combination of smart materials such as a shape memory alloy (SMA), offer further advantages, including a reduction in moving mechanical parts. These additional advantages consequently could reduce the maintenance cost and reduce the operating cost of an aircraft. Exploitation of additive manufacturing process, such as the emerging 3D-printing technology, to manufacture substructures of the morphing wings offers additional benefit, which is a reduction in the manufacturing cost, as the internal structure of the morphing aerofoil and the trailing edge section, for instance, could be manufactured as a single structure.
1.1 Research Problems and Research Statement

Aerodynamic efficiency and flight performances of aircrafts are subject to continuous studies, since the invention of the flying machine. To improve the aerodynamic efficiency and the flight performance, one of the approaches which has been continuously investigated is the morphing of an aircraft wing. For instance, a span morphing wing can increase the aspect ratio of the wing and hence can significantly improve the flight range and endurance. A sweep morphing wing can transform the aircraft wing from a normal wing which is optimised for a cruise flight, to a swept wing which is optimised for a high speed flight.

A camber morphing wing, or a leading/trailing edge morphing, which is one of several types of wing morphing, can potentially eliminate the gap between the main wing section and the traditional control surfaces, and can produce a smooth camber and aerodynamic surface changes, while providing a sufficient trailing edge deflection. This may delay flow separation along the wing chord and improve the aerodynamic efficiency in term of drag reduction. Consequently, the flight performances, such as flight endurance, flight range, power and fuel consumptions, can be improved.

A combination of the camber morphing concept with a high strength and lightweight composite structure, and a smart material, such as a shape memory alloy (SMA), can offer additional benefits, such as a significant reduction in the aircraft weight and system complexity, and therefore can deliver further improvement in the above-mentioned flight performances. In addition, with the lower fuel consumption, the emission of carbon dioxide (CO₂) and the air pollution can be reduced, to tackle the critical issue of global warming.

To achieve these ultimate goals, a high aspect ratio composite morphing wing, which consists of a rigid and stiff composite D-nose spar, a flexible rear wing section, and SMA-actuated corrugated sections, is proposed. The cross-section, or the aerofoil section, of the morphing wing, is illustrated in Fig. 1.1, with a comparison to the conservative wings with conventional control surfaces. In general, the morphing aerofoil consists of two sections; a rigid and stiff D-nose section and a rear flexible morphing section. The front D-nose section should be adequately stiff to resist the resultant aerodynamic loading, which acts at the quarter-chord of the aerofoil/wing, and the rear morphing section should be highly flexible to allow morphing, but sufficiently stiff to resist the aerodynamic loading. The research focuses on utilising a corrugated structure to achieve the flexibility, and utilising SMA wires as a smart actuator, within the morphing section.
1.1 Research Problems and Research Statement


(b) Several possible concepts of the proposed morphing wings

Figure 1.1. (a) Wings with conventional control surfaces, and (b) several possible concepts of the proposed morphing wings.
1.2 Motivation

Morphing of aircraft wings is inspired by flying birds, as illustrated in Fig. 1.2. Unlike bird wings which can be deformed to various forms and shapes during flight, existing wings of aircrafts have a fixed geometry which are usually optimised for cruise flight. With the morphing technology, one aircraft wing can be deformed to several shapes or dimensions by changing the length of the wing span, the dihedral and sweep angles, or the leading and trailing edges deflection angles.

Morphing of aircraft wings offers the possibilities of changing the wing shapes to multiple wing configurations, so that they give the optimum aerodynamic performance at various flight conditions. Fig 1.3 shows an example of the optimum shapes of a fighter aircraft wing to achieve optimum performance at different flight regimes within a complete flight envelope (Gilbert, 1981). The aerodynamic performance can be optimised for various flight altitudes,
flight speeds, and types of manoeuvre, by changing the sweep angle, and the leading and trailing edges deflection angles.

A further important benefit of morphing aircraft wings is the delay or elimination of flow separation, due to gaps between the main wing and conventional control surfaces (flaps and ailerons), or between the horizontal tail and the conventional elevator, or between the vertical tail and the conventional rudder, when the control surfaces are deployed. This may reduce the drag along the wing chord and hence improve the aircraft aerodynamic performance.

![Diagram](image)

**Figure 1.3.** A full envelope aircraft performance that can be achieved by a morphing wing (Gilbert, 1981).

### 1.3 Research Objectives and Novelties

The key objectives of the research are:

1. Review various morphing concepts, focusing on camber morphing concepts of aerofoils/wings, using mechanical and smart actuations.
2. Investigate the thermo-mechanical behaviour and material properties of shape memory alloy (SMA).
3. Develop and validate a user defined material (UMAT) model for SMA wires, implement into the explicit finite element software, LS-DYNA.
4. Design and simulate the actuation of several metallic and composite morphing aerofoils, using the new SMA user routine in LS-DYNA.
5. Design and manufacture a composite morphing wing based on the morphing aerofoil which produces a sufficiently large trailing edge deflection and an acceptable aerofoil deformation.
The novelties of the research on the design of a composite morphing wing are presented in prepared manuscripts and in a series of oral presentations of conferences.

Publications:
1. Finite element modelling of NiTi shape memory wires for morphing aerofoils (in preparation for submission to *Smart Materials and Structures*).
2. Flexural behaviour of 3D-printed carbon fibre composites: experimental and virtual tests (in preparation for submission to *Composite Structures*).

Oral presentations:
1. Finite element modelling of a NiTi shape memory alloy (SMA) wire, 8th ECCOMAS Thematic Conference on Smart Structures and Materials, SMART 2017, Madrid, Spain, 5th - 8th June 2017.

1.4 Research Methodology

As illustrated in Fig. 1.4, the composite morphing wing research starts with a detailed review of various morphing aerofoils/wings, which are morphed by mechanical and smart actuations. The review focuses on morphing concepts which involve small shape changes, such as a camber morphing, and a leading/trailing edge morphing, of aerofoils and wings. Based on previous studies, shape memory alloys (SMAs) have a high potential as smart actuators due to several advantages, including a large deformation or a high recovery strain, and a high blocking force. In addition, the SMAs have a high energy density, capable of operating in harsh environment, and have an excellent cyclic behaviour (Buehler and Wiley, 1961).

Due to the promising advantages and the excellent material properties of the SMAs, a more detailed review is conducted on the thermo-mechanical behaviour and material properties of the SMAs, including several SMA constitutive models. This is followed by a deeper understanding of the thermomechanical behaviour of the SMAs, which is used to develop a user defined material (UMAT) model in LS-DYNA. The development of the UMAT for the SMA wires focuses on the shape memory effect (SME) behaviour, and will be utilised in the composite morphing wing research. The newly developed UMAT of the SMA wires is then validated for several fundamental structures.

The explicit LS-DYNA is selected as the analysis can be used for a fast/dynamic environment, such as during actuation of the morphing wing. The second reason is to
significantly save computational cost, in term of the memory requirement to store the matrices (stiffness matrix inversion) at every time step, which is large for implicit simulations.

After validation of the UMAT, several FEA simulation studies are conducted on the key internal structure of the morphing wing. A mesh sensitivity study and a parameter study are conducted to find the suitable corrugated structure with the right positions of the SMA wires, and to investigate their effect on the deformation of the morphing aerofoils. The material of the aerofoil is varied to investigate the effect of the material variation (i.e. aluminium and carbon fibre composite with different fibre orientations) on the deformation of the selected morphing aerofoil.
Following final design of the SMA-actuated corrugated structure, which is appropriate for the composite morphing wing, a series of experimental fabrications and tests are conducted, using 3D-printing. Several experimental designs of the composite corrugated structure are investigated, before the final design. Carbon fibre (CF) composite is selected as the main material for the smart composite morphing wing. The main reasons are high strength-to-weight ratio or specific strength (strength-to-density ratio), and high stiffness-to-weight ratio or high specific modulus (stiffness-to-density ratio). Another crucial reason is that the stiffness and strength of the CF composite can be tailored in different directions, to effectively carry the external aerodynamic loads.

Finally, a composite morphing wing, with the internal SMA-actuated composite corrugated structure, is designed, fabricated and assembled. In general, the composite morphing wing consists of a load-bearing structural part and a morphing flexible part. The load-bearing structural part consists of a composite D-nose spar and a rigid rear wing section close to the wing root. The morphing flexible part consists of SMA-actuated composite corrugated sections/aerfoils and flexible foam sections. The experimental tests on the composite morphing wing with aerodynamic loads (represented by static loads/weights) will confirm the finite element LS-DYNA simulations of the SMA-actuated composite morphing wing using the new UMAT.

1.5 Thesis Outline

Chapter 1 introduces the research, defines the research problems and statement, and explains the motivation behind the research. The research objectives, novelties and methodology are outlined.

Chapter 2 summarises the past literature on morphing aerofoils and wings, which are actuated by mechanical and smart actuations. Various approaches to improve the flexibility of the morphing aerofoils/wings are reviewed. The theory behind the working principle of the shape memory alloy (SMA), as a smart actuator is presented. The literature review is crucial to select a suitable structure and a potential actuator for the composite morphing aerofoil/wing.

Chapter 3 reports the results of experimental tests on Nickel-Titanium (NiTi) SMA wires, including Differential Scanning Calorimetry (DSC) test, displacement-controlled and load-controlled tensile tests, and Dynamic Mechanical Thermal Analysis (DMTA) test. The experimental tests are carried out to obtain the transformation temperatures and mechanical
properties of the SMA wires. The transformation temperatures are important because the SMA stress-strain behaviour is affected by the SMA temperature.

Chapter 4 describes the approach used to develop a user defined material model (UMAT) of the SMA wires, for actuation of the morphing aerofoils/wings. Validations of the UMAT, and results of a mesh sensitivity study and a key parameters study to investigate the effect of several important parameters on the stress-strain behaviour of the SMA wires, are presented. The validation of the UMAT and its application to the mentioned studies are carried out to ensure that the UMAT is working correctly, before being applied to a more complex actuated structures such as morphing aerofoils.

Chapter 5 presents the applicability of the developed UMAT of the SMA wires to actuate more complex structures, including aluminium and composite aerofoils, and a pre-curved corrugated structure. SMA-actuation of several aerofoils with different arrangement of corrugation and SMA wires are investigated and evaluated to make a final selection of an aerofoil that can produce a sufficiently large trailing edge deflection and a reasonably smooth aerofoil deformation. SMA-actuation of the pre-curved corrugated structure is investigated to explore the possibility of resolving the permanent residual strain of the SMA wires at the end of cooling stage, which usually occurs for SMA-actuated structures. To demonstrate SMA-actuation of a composite corrugated structure with a trailing edge section, which represents an internal structure of an aerofoil, manufacturing of a carbon fibre (CF) composite corrugated structure using a 3D-printing technology, and test results of its actuation, are presented. To correctly model SMA-actuation of the 3D-printed CF composite corrugated structure in a FE software LS-DYNA, microstructural analysis, as well as experimental and virtual 3-point bending tests on the 3D-printed CF composite specimens, are conducted and discussed to understand the flexural behaviour of the SMA-actuated CF composite corrugated structure.

Chapter 6 describes the experimental design, manufacturing and assembly of the composite morphing wing. The 3D-printing technologies are explored and exploited to manufacture the substructures of the composite morphing wing. The objective of the composite wing manufacture and assembly is to conduct a bench-top static test on the wing, and to evaluate and compare the result of the wing deformation to the predicted results of finite element simulations on the composite D-nose spar and the composite corrugated structure.

Chapter 7 concludes the research on the design of a composite morphing wing, and recommends several ideas for future research on morphing structures.
Chapter 2

Literature Review

2.1 Overview

Morphology generally means the study or classification of a shape, a form, an external structure or an arrangement (Oxford English Dictionary, 2007). Morphing, in an engineering field, refers to a continuous shape change where one entity deforms upon actuation, without movement of discrete parts relative to each other (Thill et al., 2008). Shape morphing of an aircraft refers to geometrical changes, which can be categorised into an aerofoil adjustment, an out-of-plane transformation and a planform alteration (Barbarino et al., 2011). Shape morphing can also refer to a strong shape variation to optimise the aircraft functionality and performance at different flight missions (Baier and Datashvili, 2011).

Throughout the research of the design of a composite morphing wing, the past work on various morphing structures, especially morphing aerofoils and wings, were consistently reviewed. As illustrated in Fig. 2.1, morphing aircraft wing could be classified to a span morphing (wing span variation to change the wing surface area) (Gamboa et al., 2009, Murugan et al., 2012, Vocke et al., 2012, Ajaj et al., 2014, Woods and Friswell, 2015), a sweep morphing (sweep angle variation) (Basaeri et al., 2014), a twist morphing (wing twist mainly for roll control) (Jardine et al., 1999, Iannucci and Fontanazza, 2008, Donadon et al., 2009), a camber morphing (alteration of a camber line, hence the aerofoil section) (Ruangjirakit, 2013, Molinari et al., 2016), and a leading edge or a trailing edge morphing (movement of high lift devices or control surfaces, e.g. flaps and ailerons, without mechanical hinges) (Iannucci and Fontanazza, 2008, James et al., 2009, Ameduri et al., 2011).

The camber morphing, the leading edge (LE) and trailing edge (TE) morphing had been extensively investigated in the last few decades due to its potential of maintaining maximum aerodynamic efficiency (lift-to-drag ratio) at multiple flight missions, by tuning the aerofoil camber to its optimum configurations during flight. Another potential benefit was a significant drag reduction, due to elimination of gaps between the control surfaces and the main wing/elevator/rudder. Therefore, morphing structures based on changes of an aerofoil shape,
such as morphing aerofoils, morphing LE/TE sections and morphing ailerons, were the main focus of the review. Other morphing structures that could be potentially used within an aerofoil, such as multistable structures and Kagome truss plate structures, were also included in the review. From the previous research, the morphing structures were actuated by various mechanical actuators, such as motors, gearboxes and eccentuators, and by many smart actuators such as piezoelectrics/piezoceramics, micro-fibre composites (MFC), an electro-active polymer (EAP), and shape memory alloys (SMAs) in various forms.
2.1 Overview

Zero-poisson honeycomb wing (Vocke et al., 2012)

Span morphing with a compliant spar (Ajaj et al., 2014)

Adaptive aspect ratio morphing wing (Woods and Friswell, 2015)

(a) Span morphing

DARPA Smart Wing (Jardine et al., 1999)

(b) Twist morphing

Bio-inspired robotic morphing wing (Basaeri et al., 2014)

(c) Sweep morphing

Compliant morphing wing (Previtali et al., 2011)

(d) Camber morphing

Morphing flap (Ameduri et al., 2011)

(e) Trailing edge or flap morphing

Figure 2.1. Several types of morphing wings: (a) a span morphing, (b) a twist morphing, (c) a sweep morphing, (d) a camber morphing and (e) a trailing edge morphing.
2.2 Morphing Aerofoils/Wings by Mechanical Actuation

Many past and recent research on morphing structures still used mechanical actuators, such as motors/gearboxes and hydraulic systems (Fig. 2.2) for the Mission Adaptive Wing (MAW) (Gilbert, 1981), huge and bulky cross-bars (Fig. 2.5) which were extended and retracted by a torque wrench to actuate a sandwiched morphing skin panel (Thill et al., 2010), high-torque Hitec HSR-5995TG robot servos combined with a belt-drive system to actuate active winglets (Fig. 2.6) of a flying wing (Gatto et al., 2010), servo-actuators to morph a trailing edge (Fig. 2.7) based on a rotating rib concept (Ricci et al., 2006, Miller et al., 2010), eccentuators to morph a leading edge section (Morishima et al., 2010) as shown in Fig. 2.11 and a trailing edge section of a Seamless Aeroelastic Wing (SAW) (Perera et al., 2010) as shown in Fig. 2.10, electro-mechanic linear actuators to actuate compliant belt-ribs of a morphing wing (Previtali et al., 2014), and a Pneumatic Artificial Muscle (PAM) actuator (Fig. 2.12a) to morph a Fish Bone Active Camber (FishBAC) aerofoil (Woods et al., 2014).

Gilbert (1981) utilised a hydraulic system with a gearbox to morph a Mission Adaptive Wing (MAW) of the F-111A fighter aircraft. As shown in Fig. 2.2b, the MAW consisted of a middle rigid spar, and deformable leading edge and trailing edge sections. All components of the leading edge section were made of an aluminium material, while the skins were made of a standard aircraft grade woven cloth fiberglass composite. The second figure of Fig. 2.2b (from top) was the wing configuration at a neutral position, while the first and third figures were the wing configurations for roll manoeuvre, and the last figure was the wing configuration for approach take-off and landing. The internal view of the trailing edge section, which consisted of various mechanical components, is shown in Fig. 2.2a. Each of the hydraulics produced 2.4 in (61 mm) stroke and 14,000 lb (62,275 N) load. The achieved maximum deflection angles of the leading and trailing edge sections were 30° and 25°, respectively.

Monner (2001) investigated a formvariable flap structure with a reference to Airbus A340-300 wing and flap geometries. The flap was located at 90% of the wing chord, with the flap chord length of 1680 mm. The flexible region of the flap was 50% of the flap chord, which was 840 mm. The target maximum trailing edge deflections were ±185 mm, corresponded to the requirement of ±15° maximum camber variations of the A340-300 aircraft. The aluminum model of the flexible rib, deformed by a spindle actuator is shown in Fig. 2.3. The figure shows the maximum upper deflection, the neutral position and the maximum lower deflection of the flexible rib (top to bottom, respectively).
2.3 Morphing Aerofoils/Wings by Mechanical Actuation

In a research of a morphing wing design, Iannucci et al. (2009) proposed an articulated cell concept, which aerofoil consisted of a rigid central box that was sufficiently stiff to resist a bending moment, a deformable intermediate box, a leading edge and an aileron sections, as shown in Fig. 2.4. The central rigid box of the articulated cells concept was designed to be adequately stiff to support the torsional and bending moments generated by the aerodynamic lift and drag forces. The other three cells, which were the leading edge section, the intermediate box, and the aileron were designed to achieve -13° to 9°, -3° to 0°, and -9° to 11° rotations, respectively. They were allowed to rotate about the rotation axes shown in the top figure. The approach of using fewer sections, if compared to the previous formvariable concept (Monner, 2001), was meant to reduce mechanical components/joints and friction between the boxes/sections. The top, middle and bottom figures show the aerofoil at a neutral position, a maximum downward deflection, and a maximum upward deflection, respectively, upon

Figure 2.2. Mission Adaptive Wing (MAW); (a) the trailing edge mechanism and (b) the deflected shapes (Gilbert, 1981).
deflections of individual boxes/sections. The advantages of this concept were low activation energy and high bandwidth (10 Hz) requirements to perform a roll control.

Thill et al. (2010) experimentally investigated a morphing trailing edge section, which skin panels were made of a composite corrugated sandwich structure. The wing had a chord of 0.8 m, a span of 1.52 m, and a morphing trailing edge section located at 32% aft of the chord. The composite corrugated sandwich skin panels were fabricated from an E-glass/977-2 epoxy tape and a foam core. As shown in Fig. 2.5a, they used two huge cross bars (one for each wing) with a torque wrench to deform the trailing edge. Control surface deflections of up to 12° were achieved at low speeds wind tunnel air flows (20 m/s and 30 m/s), as depicted in Fig. 2.5b. Compared to the wing position at zero angle of attack (AOA), the maximum lift coefficient, $C_{L_{\text{max}}}$, was increased by 8%, while the drag coefficient, $C_d$, was decreased by 7%.

Figure 2.3. An aluminium flexible rib model of the formvariable flap structure (Monner, 2001).
In an experimental research on an active winglet of a flying wing, Gatto et al. (2010) utilised two digital Hitec HSR-5995TG high-torque robot servos with a belt-drive system on each wing tip, to produce a winglet rotation (Fig. 2.6). The flying wing had a Zagi 12 wing section, with a total wingspan of 1.54 m (with winglets), root and tip chords of 326 mm and 185 mm, respectively, and a leading edge sweep angle of 30 deg. In the wind tunnel test with a Reynolds number of $5.53 \times 10^5$ (flow speed of 30 m/s), a maximum dihedral deflection of $\pm 75$ degree was achieved. The concept was proved to be able to adjust and tailor the aerodynamic loading towards the main wing tip area (1.2 m span), which could be useful in enhancing a secondary performance/efficiency, such as in a gust load alleviation.

A Smart Aircraft Morphing Technologies (SMorph) project which combined three collaborative research partners (the UK, Portugal and Italy), attempted on several morphing concepts such as Rotating Ribs (RR), adaptive wing tip devices, rotating spars, multi-stable composites and telescopic wing (Miller et al., 2010). The Rotating Ribs concept (Fig. 2.7) utilised linear slides instead of traditional rivets for rib-skin connection, allowing a smooth deflection of a trailing edge section. This connection allowed the upper and lower skins to follow contour of the deflected ribs. As reported in an initial work of Ricci et al. (2006) for an Active Aeroelastic Aircraft Structures (3AS) project, a static test on the RR resulted in a camber
angle change of $\pm 5.5^\circ$. Servo-actuators which were mounted between the two spars, were used to rotate the four ribs along the 1.4m-span trailing edge section.

![Picture of actuation system](image)

**Figure 2.5.** (a) Actuation system in between morphing skin panels (MSP), and the achieved control surface deflection (Thill *et al.*, 2010).

Eccentuators with ultrasonic motors, as shown in Fig 2.8a (Wang *et al.*, 2001), were used to morph a hingeless trailing edge control surface (HCS) of a 9.2ft-span (2.8m-span) unmanned combat air vehicle (UCAV) wing. The flexible HCS consisted of a center laminate, a honeycomb core, silicone skins, and an aluminum tip (Fig. 2.8b). With 10-segments control surface, several shapes of trailing edge deflections with maximum deflections of up to 20° were obtained in less than 0.2 sec. Fig. 2.8c shows the maximum 13° cosine and 12° ramp shapes that were achieved.
2.3 Morphing Aerofoils/Wings by Mechanical Actuation

Iannucci et al. (2009) proposed a modified eccentuator concept, or an eccentric torque-tube disk mechanism (ETDM) concept, in a morphing wing design for a missile flight control, as shown in Fig. 2.9. The concept consisted of a series of torque tube disks, mounted on circular rods, and skins with stringers and a sliding trailing edge mechanism. The sliding trailing edge was proposed to minimise the actuation power. The actuation load was supposed to be transferred from the torque tube disks to the skins through the stringers. Several ETDM was proposed to be placed along the wing span to obtain a desirable morphing shape. The ETDM could be actuated by linear or rotary piezo motors, which were not back driven, to reduce power consumption. The concept, however, had not been taken forward to the next stage, because of the identified disadvantage, which was the friction between the torque tube disks and the stringers on the skins.

Figure 2.6. Active winglet of a flying wing (Gatto et al., 2010).

Figure 2.7. A Rotating Ribs (RR) concept with linear slide connections (left) and a ‘proof-of-concept’ trailing edge model without upper skin (right) (Ricci et al., 2006, Miller et al., 2010).
2.3 Morphing Aerofoils/Wings by Mechanical Actuation

![Diagram showing morphing aerofoils/wings by mechanical actuation](image)

**Figure 2.8.** Hingeless trailing edge control surface (HCS) actuated by eccentruators and ultrasonic motors (Wang et al., 2001).

![Diagram showing a modified eccenator concept](image)

**Figure 2.9.** A modified eccentuator concept (Iannucci et al., 2009).

Perera et al. (2010) exploited a torque tube actuation mechanism (TTAM) to deform a flexible trailing edge section of a seamless aeroelastic wing (SAW). The 300mm-chord, 600mm-span wing consisted of front and rear wood spars, a rigid foam between the spars, and a glass/polyester composite skin, as shown in Fig. 2.10. The composite skin was made of three layers of 0/90 glass/polyester fabric. Two sets of TTAM were used with two RS4549 digital servo motors, which each of them produced a maximum torque of 0.085 Nm at a supplied voltage of 6V. No experimental results of tip deflection were reported. However, to achieve a 60 mm tip deflection which corresponded to a 10° conventional flap deflection, the predicted actuation force was 200 N, using the PATRAN Finite Element Analysis (FEA) software.

Morishima et al. (2010) introduced an eccentric beam actuation mechanism (EBAM) to morph a leading edge (LE) section of a 39.65m-span, 5.15m-mean-chord, civil aircraft wing. The morphed LE section had a 0.86 m chord length, a 1 m span and a 0.44 m height (Fig. 2.11). The upper and lower skins of the section was reinforced by metallic I-shaped stringers. Twelve
percent of the LE section near the spar, was assumed to be rigid, and the target maximum vertical and horizontal deflections were 6% and 1.2% of the 4 m wing chord, respectively. The skin was made of a 12 layers, 3mm-thick, glass fibre/epoxy composite laminate, while the front spar was made of a 16 layers, 4mm-thick, carbon fibre/epoxy composite laminate. The required actuation force to achieve the target deflection was 11400 N, predicted by the commercial finite element software, PATRAN/NASTRAN. Because the normal and shear stresses exceeded the material allowable strength, a LE section with an aluminium skin was then simulated and proposed.

A seamless aeroelastic wing (SAW) with a torque tube actuation mechanism (Perera et al., 2010).

A morphing leading edge section actuated by an eccentric beam actuation mechanism (EBAM), at a neutral position (left) and at a fully-deployed position (right) (Morishima et al., 2010).

A Pneumatic Artificial Muscle (PAM) actuator, as shown in Fig. 2.12a, was used to actuate a 500mm-span, 300mm-chord FishBAC aerofoil section (Woods et al., 2014). The PAM actuator consisted of an airtight inflatable bladder, surrounded by an expandable braided sleeve. When pressurised air was pumped into the bladder, the expansion of the bladder exerted axial tensile force in the stiff filaments of the braided sleeve, and produced a final required axial force/displacement. The PAM actuator was reported to have a large specific output work of about 4000 J/kg, if compared to other mechanical actuators, such as electromechanical actuators (300 J/kg) and pneumatic pistons (1200 J/kg). Inside a hollow leading-edge spar of the FishBAC aerofoil section, the PAM actuator was arranged in the spanwise direction along with a spiral pulley, a bevel gear and a tendon pulley, as shown in Fig. 2.12b. The rigid trailing edge section was connected to the tendon pulley by a pair of antagonistic tendons. In the research, the kinematics of the spiral pulley was studied to achieve the target trailing edge
deflection of 49 mm (Fig. 2.12c), or 16% of the chord, and a tendon pulley rotation of 50 degrees.

Feng et al. (2015) utilised a pneumatic muscle fibre (PMF) actuator to deform a 250mm-chord, 50mm-span Clark Y aerofoil. The pneumatic muscle fibres, with a length of 125 mm, were embedded in a silicone rubber, and the embedded structure was used as the lower skin of
the aerofoil. The embedded lower skin had a dimension of $150 \times 50 \times 6.6$ mm (Fig. 2.13a). The upper skin of the aerofoil was made of a carbon fibre composite plate. Similar to the PAM actuator, the PMF actuator also utilised air pressure to produce an output force and a contraction of the lower skin. Fig. 2.13b shows the aerofoil deformation at supplied air pressure of 0.25 MPa, 0.3 MPa and 0.35 MPa. At the maximum air pressure, the output force was about 73.59 N and the contraction of the lower surface was about 9.7%. The achieved trailing edge deflection was about 53 mm.

![A Pneumatic Muscle Fibre (PMF) actuator as an active morphing lower skin for a camber morphing structure](Feng et al., 2015).

Instead of utilising the pressurised air for actuation, it was also used to deploy a trailing edge section of an aerofoil or to deploy the entire wing of small air vehicles, as shown in Fig. 2.14 (Jacob and Smith, 2009). Fig. 2.14a shows a deployable wing consisted of a carbon fibre composite rigid spar, a rigid leading edge section, and an inflatable trailing edge section. The
hollow space on the spar was used for placement of air hoses and servo wires, and for storage space to store the deflated inflatable trailing edge section. This concept, however, might be limited to small, low altitude air vehicles.

![Deployable/inflatable wing using on-board compressed air tank](image)

**Figure 2.14.** Deployable/inflatable wing using on-board compressed air tank (Jacob and Smith, 2009).

![Variform concept for an unmanned aerial vehicle wing](image)

**Figure 2.15.** A variform concept for an unmanned aerial vehicle wing: (a) an initial bulky NACA 23015 profile (outer line) morphed into a sleeker FX 60-126 profile (inner section), and (b) the possible fuel bladder configurations (Gano and Renaud, 2002).

Gano and Renaud (2002) investigated a variform concept for a morphing wing of unmanned aerial vehicles (UAVs), to improve range and endurance of the UAVs. The variform wing concept morphed the wing shape from bulky profiles into sleeker profiles, for instance from a NACA 23015 profile into a FX 60-126 profile. As shown in Fig. 2.15a, the outer blue line is the bulky NACA 23015 profile, morphed into the FX 60-126 sleeker shape represented
by the inner solid section. The concept utilised fuel consumption for the shape changing, where the change occurred slowly when the fuel was consumed throughout the flight. The possible balloon-like fuel bladder configurations are shown in Fig. 2.15b. Based on combination of both computational fluid dynamics (CFD) and analytical aerodynamic equations, the range was improved by 22.3% further and the endurance was improved by 22.0% longer.

2.3 Morphing Aerofoils/Wings by Smart Actuation

2.3.1 Piezoelectric/Piezoceramic

Gandhi et al. (2008) utilised piezoelectric stack actuator to actively control camber of a 560mm-chord NACA 0012 aerofoil, to replace trailing edge flaps used for reduction in helicopter vibration. The optimised aerofoil consisted of a fixed D-spar, a compliant mechanism, and a rear trailing edge section. The aerofoil prototype with the piezostack actuators is shown in Fig. 2.16. The prototype was fabricated from a 0.5in-thick T-6061 aluminium plate. The top piezostack actuators contracted, while the bottom piezostack actuators expanded, deforming the aerofoil. The piezostack actuators, which were made of PIC 151 PZT ceramic material, required a maximum of ±1000V supplied voltage to achieve only 0.15% free strain. The actuators displacement was measured by an optoNCDT 1800 laser measurement system, while the tip deflection was measured by a dial indicator. A static tip deflection of 3.65 mm was achieved at the maximum supplied voltage.

![Figure 2.16. A controllable camber rotor aerofoil prototype with a compliant mechanism and piezostack actuators (Gandhi et al., 2008).](image)

Donadon et al. (2009), and Donadon and Iannucci (2014), investigated aerodynamic performances of two morphing wing configurations of Unmanned Aerial Vehicles (UAVs), which were flapped and twisted wings, aimed to change the wing shape for flight control. The wings had a NACA 0012 aerofoil section with a chord length of 270 mm and a span of 1400
mm. They were fixed at 3° angle of attack, under air flow of 6.86×10^5 Reynolds number. Both of the hinging line of the flapped wing and the twisting line of the twisted wing were positioned at 70% aft of the chord. The maximum flap deflection and twisting angle were 10°. The flexible trailing edge sections were made of an elastomeric skin, and were clamped to the remaining front part of the wing. A Vortex Lattice Method (VLM) was implemented in MATLAB and ABAQUS software to predict the associated aerodynamic loads such as pitching moment, rolling moment, pressure distribution, lift distribution, and aerodynamic energy. Results of pressure distribution on both wings are depicted in Fig. 2.17. Compared to the twisted wing, the flapped wing generated higher maximum lift coefficient (\( C_L \)) (approximately 1.4 compared to 0.9), pitching moment, aerodynamic energy, and elastic energy density (6.0 J/kg compared to 3.6 J/kg). The latest, however, showed the advantage of the twisted wing because of lower actuation energy. The predicted elastic energy density of both wings was compared with the reviewed elastic energy density of ten smart materials, and the Single Crystal PZN-PT material was suggested as the potential candidate for actuation of the smart morphing wing structures.

![Figure 2.17. Numerical results in term of pressure distribution for (a) the flapped and (b) the twisted wings (Donadon and Iannucci, 2014).](image)

### 2.3.2 Micro-Fibre Composite (MFC)

Pankonien et al. (2013) integrated a shape memory alloy (SMA) and a piezo-composite (micro-fibre composite - MFC) actuators for a Synergistic Smart Morphing Aileron (SSMA). The purpose of the combination was to overcome individual limitation of both smart materials. The SSMA concept and its experimental setup are illustrated in Fig. 2.18. Performance of the SSMA was measured in term of response amplitude (maximum amplitude of motion) and
2.3 Morphing Aerofoils/Wings by Smart Actuation

relative time constant (RTC) (a measure of actuator response speed) of the square wave response function (SWRF). The SSMA was first actuated by SMA wires, and then by MFC. Results of the SMA-actuated aileron showed a decrease in the performance (the actuation amplitude decreased and the RTC increased) as the frequency of input square waves increased. The MFC-actuated aileron also showed a similar tendency but at larger frequencies (one order-of-magnitude). The performance of a combination of both the MFC and the SMA was evaluated based on actuation of a combination of flexure box aileron and hinge. Consequently, overall actuation range and bandwidth were improved.

![Image](image_url)

**Figure 2.18.** (a) A SSMA concept and (b) an experimental construction of the SSMA concept (Pankonien et al., 2013).

### 2.3.3 Electro-Active Polymer (EAP)

Dearing *et al.* (2010) designed and characterised an electro-active polymer (EAP) dimple actuator, for turbulent flow control to reduce skin-friction drag. The circular EAP dimple actuator consisted of a 100 μm MED4930 silicone rubber film, as an elastomer, sandwiched between graphite electrodes. On application of electric field, the elastomer film was compressed and caused an in-plane expansion. Exploiting a clamp boundary condition, the in-plane expansion was constrained and caused the elastomer to buckle, forming a bump. This is shown in Fig. 2.19(a). The radius of the investigated dimple actuator were 5 mm, 10 mm and 15 mm. They found that the critical buckling voltage and out-of-plane displacement increased with the radius size, as shown in Fig. 2.19(c). The critical buckling voltages were 2.0 kV, 3.0 kV, and 3.5 kV (corresponded to 20 MV/m, 25 MV/m and 30 MV/m) for the dimple radius of 5 mm, 10 mm and 15 mm, respectively. Another research on morphing aerofoil utilising film actuator, called dielectric barrier discharge (DBD) plasma actuator (Poggie *et al.*, 2010), is worth mentioned.
2.3 Morphing Aerofoils/Wings by Smart Actuation

![Diagram of Morphing Aerofoils/Wings by Smart Actuation](image)

Figure 2.19. (a) Principle of actuation of the electro-active polymer (EAP) dimple actuator, (b) dimple topology at a maximum deflection for a 5 mm radius dimple, and (c) out-of-plane deflection versus applied voltage for the 5 mm, 10 mm and 15 mm radius dimples (left to right) (Dearing et al., 2010).

2.3.4 Shape Memory Polymer (SMP)

James et al. (2009) developed a shape memory polymer (SMP) concept for a morphing wing skin. The morphing wing with the SMP concept consisted of a structural closed-section spar, a D-nose skin, a compliant deformable trailing edge section made of a carbon fibre laminate (two layers of carbon cloth laid at ±45° direction) with a VeriflexE2-100 SMP matrix, and an actuator, as shown in Fig. 2.20. The SMP skin was temperature-activated, where the stiffness reduced to almost zero as the temperature increased. The trailing edge section was actuated when the SMP skin was in the low stiffness state. After actuation, the temperature was decreased to regain the structural stiffness. The active area for the SMP is shown in Fig. 2.20b. With additional active area on the lower skin as shown in Fig. 2.20c, the ABAQUS predicted trailing edge deflection was amplified by a factor of five, which was 22.06 mm, for an actuation force of 100 N. Experimental tensile tests on the small SMP specimens (25×150×0.6 mm) resulted in 82 N extension force, to extend the specimens by 8%, which corresponded to a 15° flap deflection angle. This extension force was considered too high for the change of the structural shape. Consequently, alternative fibre orientations (e.g. continuous fibres in a zigzag direction, short fibres in a random mat, and a woven cloth with a shallow ±75° angle) and fabric types were suggested to achieve a lower extension/actuation force. In addition to the SMP concept, they also foreseen the possibility of a multistable structures concept, an electroactive elastomer concept, an electroactive or shape memory polymer foams concept, and
a SMA wire concept for the morphing wing. The SMA wire concept, consisted of flexible plates with NiTi SMA elements embedded in the surface is shown in Fig. 2.21.

Figure 2.20. A shape memory polymer (SMP) concept for a morphing wing: (a) activation and actuation of the SMP morphing skin, (b) active top surface area of the SMP skin, and (c) active bottom surface area of the SMP skin (James et al., 2009).
2.3 Morphing Aerofoils/Wings by Smart Actuation

2.3.5 Shape Memory Alloy (SMA)

Shape memory alloy (SMA) material is commercially available in various forms, such as wires, springs, tubes, strips and thin sheets (S.A.E.S Group, 2009). The SMA in these forms were utilised to morph various aerofoil sections, wings and aerospace structures.

Webb et al. (2000) conducted laboratory tests on SMA wires, for application to a biomimetic underwater vehicle. The proposed concept of the biomimetic underwater vehicle consisted of hinged skeletal structures, with actuated and relaxed SMA wires, as shown in Fig. 2.22. Two years later, Rediniotis et al. (2002) experimented a biomimetic hydrofoil made of almost similar hinged skeletal structures, but with slightly different arrangement of SMA wires, which were arranged alternately with springs, as shown in Fig. 2.23a. The hydrofoil had a NACA 0009 aerofoil section with a 762 mm chord length. It consisted of seven 6.4 mm-thick aluminium plates or so-called ribs which were connected by six sets of two hinges, segmented aluminium skins, SMA wires for actuation and restoring springs. In air actuation, deflection angle of each segment was 8°, which resulted in a total maximum static deflection angle of 50° from the head to the tail as shown in Fig. 2.23b. Even though the deflection angle was high, the bi-directional movement of the hydrofoil in water generated about 0.4 lb thrust and about 0.2 lb drag, for the tail-only actuation. The resultant thrust was therefore small, which was believed due to the too flexible tail, and the conclusion was that the tail had to be redesigned to generate more thrust to propel the underwater vehicle.
2.3 Morphing Aerofoils/Wings by Smart Actuation

In an experimental work of in-flight tracking of helicopter rotor blades, Epps and Chopra (2001) arranged 3 sets of SMA-SMA wire actuators in parallel, to deform a hinged trailing edge tab of a helicopter rotor blade, as shown in Fig. 2.24. The rotor blade used a NACA 0012 aerofoil section, with 12 in chord and span, while the trailing edge tab had a 2.4 in chord and a 4 in span. Nitinol SMA wires with a diameter of 0.015 in and an initial pre-strain of 3.158% were used for the actuation. The wire clamp and the hinge tube were fabricated with fiberglass to electrically insulate the SMA wires. Upon SMA actuation, a trailing edge tab deflection of 19° was achieved. The experimental results of strain vs. temperature and tab deflection vs. temperature matched the analytical results which were predicted using the Brinson SMA constitutive model. Both experimental and analytical results showed a permanent residual strain upon completion of the cooling stage of the SMA wires.

In a Smart Aircraft and Marine Project System Demonstration (SAMPSON) program, Pitt et al. (2001) utilised 40 Nickel-Titanium SMA wires to antagonistically actuate an adaptive lip of an engine inlet of an F-15 Strike Eagle fighter aircraft (Fig. 2.25a). Under static loading corresponded to aerodynamic loading at Mach number of 0.75, adaptive lip angles from -7° to 10° were achieved upon actuation. In another SAMPSON project, Pitt et al. (2002) developed a SMA-powered adaptive internal wall for engine inlet of an F-15 fighter aircraft, as shown in Fig. 2.25b. The adaptive internal wall was called “Smart Flex Skin”, consisted of elastomeric
or rubber material, 80 composite structural rods, and 10 NiTi SMA wires. A smooth 3” bump was formed upon SMA actuation, forming a compression ramp on the upstream side and an expansion ramp on the downstream side.

Figure 2.24. SMA-actuated trailing edge tab of a helicopter rotor blade (Epps and Chopra, 2001).

Figure 2.25. SAMPSON (a) adaptive lip (Pitt et al., 2001) and (b) adaptive internal wall (Pitt et al., 2002).

Banerjee et al. (2008) investigated an optimum discrete location of a Flexinol SMA wire for actuation of cantilever beams with different flexural stiffness. The 0.125mm-diameter SMA wire was positioned parallel to the length of the cantilever beam with an offset, as shown in
2.3 Morphing Aerofoils/Wings by Smart Actuation

Fig. 2.26. One end of the SMA wire was mechanically fixed to the fixed end of the beam, and another end of the SMA wire was mechanically fixed to the free end of the beam. The 100mm-length acrylic beam tip was deflected when the SMA wire was heated resistively. The flexural rigidity of the four cantilever beams ranged from 3,060 Nmm$^2$ to 47,800 Nmm$^2$. They found that an optimum offset existed, and it increased or shifted away from the neutral axis, with an increase in the beam flexural stiffness.

Iannucci and Fontanazza (2008) discussed morphing design methodologies, which included specification of morphing objectives, approached strategies, system requirement and sub-system design. For flight control objective, they identified piezoelectric (PZT) material, multistable structures and embedded active material, as possible smart materials to actuate a compliant trailing edge. The combination of multistable structures with piezoelectric patches bonded on the surface, as shown in Fig. 2.27a, deformed the multistable structures from one stable state to another. However, the approach was not ideal for flight control, as the structures could usually hold only two stable configurations (Santer and Pellegrino, 2008, Santer and Pellegrino, 2011, Panesar and Weaver, 2012, and Arrietta et al., 2014). Even though it was possible to employ additional multistable units (Fig. 2.27b) to achieve all required configurations within the flight control envelope, this would only increase the system complexity and would raise other issues such as interface deformation and smoothness of the adaptive surface.

Alternatively, a more preferred morphing concept for flight control was a shape memory wire concept, where NiTi SMA wires (or other active materials) were embedded in the surface, as shown in Fig. 2.27c. An ABAQUS flexible plate model, which approximated a cambered wing with NiTi SMA wires embedded in the surface, is shown in Fig. 2.27d (top). The cambered wing had a small curvature of 2% maximum camber, and a thickness-to-chord ratio of 3.4%. The expansion and contraction of the SMA wires were modelled as equivalent...
temperature changes in the wires. For the curved composite plate which was fixed on the left edge (Fig. 2.27d top), with the NiTi SMA wire embedded along and aside the principal diagonal such that the trailing edge was connected to the SMA wires, a wing twist was achieved. Upon actuation or shrinkage of the NiTi SMA wires, about $5.272 \text{ mm}$ maximum out-of-plane displacement was achieved, as shown in blue colour at the bottom right corner of the curved composite plate in Fig. 2.27d (bottom). However, the composite fibre orientation and the material properties of the SMA wires, especially the size and the maximum recoverable strain, were not revealed.

Apart from multistable structures with piezoelectric patches, and embedded shape memory wires concepts, an electroactive elastomer (DEA) concept was also investigated for flight control of the morphing wing. This concept utilised an in-plane expansion of the dielectric component, squeezed between two compliant electrodes upon voltage supply, for actuation.

Georges et al. (2009) and Coutu et al. (2009) employed two groups of NiTi SMA wire actuators, with maximum of six SMA wires per actuator group, to morph four-ply carbon/kevlar epoxy matrix composite flexible extrados of a 500mm-chord, 990.6mm-span morphing laminar wing. The morphing laminar wing is shown in Fig. 2.28. Each actuator group consisted of SMA wires, a bias gas spring, and a slider with a crank mechanism. Prior to the experimental work, XFOIL numerical studies revealed that the laminar-to-turbulent flow transition point shifted backward towards the trailing edge, and caused a significant drag reduction, upon morphing of the flexible extrados (Coutu et al., 2009). Experimentally, the flexible extrados was manufactured by a resin transfer moulding (RTM) technology, with unidirectional carbon plies and woven hybrid carbon/kevlar plies (Georges et al., 2009). Force-displacement characteristics of the SMA wires were used to determine the working envelope of the two groups of the SMA actuators. Under laboratory condition or without aerodynamic load, a maximum vertical extrados displacement of $7.7 \text{ mm}$ was achieved.
Figure 2.27. (a) Multistable structures, and brainstormed morphing concepts based on (b) several multistable structures and (c) embedded shape memory wires (or other active materials) for flight control, and (d) a SMA-actuated flexible curved composite plate (Iannucci and Fontanazza, 2008).
Kang et al. (2012) and Rim et al. (2014) utilised 6 SMA wires which were arranged in parallel to morph a flap of a 275mm-chord, 366mm-span bench-top balsa wing, as shown in Fig. 2.29. The wing spars, ribs and stringers were made of balsa wood, while the wing skin was manufactured from a PVC plate. The flap of the wing with a Clark Y aerofoil shape was actuated by the SMA wires through resistive heating by a DC power supply. The flap deflection increased nonlinearly with an increase of electric current (1.5A - 3.3A), due to nonlinear thermomechanical characteristics of the SMA wires. A maximum flap deflection of 21° was observed at 3.3A supplied current. No wind tunnel test was conducted. Aerodynamic analyses were performed numerically at 3.84 × 10^5 Reynolds number using a commercial GAMBIT/FLUENT software. Three models (0°, 6.4°, and 15.8° deflection angles) were analysed, and the highest lift-to-drag ratio (15.22) was reported at the 6.4° deflection angle. Coefficient of pressure distributions on the upper surface changed rapidly at 70% of the chord length due to a rapid change in the wing shape. This led to a flow separation and hence aerodynamic losses. Therefore, a smoother change of the wing shape was recommended to suppress the flow separation and improve the aerodynamic performance.

Almeida et al. (2015) experimentally applied a single trained NiTi SMA wire to deform a wing rib made of 3D-printed poly-lactid acid (PLA) material. The rib had a NACA 0012 aerofoil section with a chord length of 250 mm, and a trailing edge section located at 50% of
2.3 Morphing Aerofoils/Wings by Smart Actuation

the chord. The SMA wires had a diameter of 0.4 mm and exhibited a two-way shape memory effect (TWSME) behaviour. They were heated by the Joule effect, by means of supplying electrical current through aluminium tubes, which acted as stringers and connection structures for the SMA wires. The position of the SMA wire was varied to achieve several trailing edge deflections. The results showed that by varying the positions of the SMA wires (AB and CB) as shown in Fig. 2.30, with maximum recoverable strains of only 1.06% and 1.05%, resulted in trailing edge deflections of 11.6 mm (5.3°) and 14.0 mm (6.4°), respectively. The aerodynamic analyses were carried out using XFOIL software by simply sketching the final configurations of the morphing rib and the hinged-flap aerofoil, and then importing them into the software for the analyses. At small angle of attack (< 1.5°), these resulted in an improvement of maximum lift-to-drag ratio by 10.26% and 13.26%, respectively, when compared to the hinged-flap aerofoil.

Figure 2.30. A 3D-printed poly-lactid acid (PLA) morphing wing rib with two different positions of SMA wires (AB and CB) (Almeida et al., 2015).

On top of the SMA wires, SMA springs, tubes and strips were also utilised to actuate various morphing structures. Dong et al. (2008) utilised SMA springs to deform upper and lower skins of a 350mm-chord bench top composite aerofoil. As shown in Fig. 2.31, the aerofoil consisted of front and rear rigid composite sections, connected by three stainless steel thick plates called ribs, and top and bottom composite skins. The front and rear parts were manufactured from T700 carbon fibre composite prepreg with a [0°/±45°/90°] lay-up. The composite skins (the top skin was not shown in the figure) were manufactured from similar
material with a symmetrical \([90^\circ/0^\circ/90^\circ/0^\circ/90^\circ/0^\circ/90^\circ/0^\circ/90^\circ]\) lay-up. Ten SMA springs were fitted in holes along the chord of each rib to deform the skins. Five springs actuated the top skin, while the other five deformed the bottom skin. Nylon sleeves were used to isolate the SMA springs from other metallic parts, avoiding possibilities of short circuit and power damage. They were electrically connected in series to ensure equal amount of supplied current. To reduce strain of the composite skins during morphing, a slidable (simply supported) boundary condition was used between the front leading edge section and the skins, and a fixed boundary condition was used between the skins and the rear trailing edge section. Micro laser emitters were used to measure the skin deformation. Only results of the skin deformation were analysed, which closely matched the simulated and targeted deformation. Slight differences in the deformation of the upper and lower skins were observed from the aerofoil root to tip, indicating a non-uniform deformation.

In a Smart Wing Program initiated by Defense Advanced Research Projects Agency (DARPA), Jardine et al. (1999) utilised a SMA torque tube to morph a military-sized aircraft wing, as shown in Fig. 2.32. They improved the tube design and performance, which include a new torque loading path (single-spanwise instead of dual torque tubes) that produced a direct applied torque from wing root to tip, and a new blocking torque capability of 3500±100 in-lb. As a result, variable spanwise twist improved from 1.25° to 5°, and aerodynamic performances (lift and rolling moment) were improved by 8-12%, as similarly reported by Kudva et al. (1999). A maximum power consumption of 200W was required to twist the wing and 20W was
required to maintain it. Several challenges were identified such as a non-uniform spanwise twist (concentrated towards wing tips) because of stiffer inboard panels, and a requirement of cautious manufacturing process of the SMA torque tube to avoid buckling.

![Image of SMA torque tube actuator assembly](image)

**Figure 2.32.** SMA torque tube actuator assembly for Test 2 of phase 1 DARPA Smart Wing Program (Jardine et al., 1999).

Prahlad and Chopra (2001) experimented a SMA torque tube to twist a tiltrotor blade. The SMA torque tube assembly and torsional test setup are shown in Fig. 2.33. They identified an optimum tube outer radius of 10.2-15.2 mm and an optimum tube thickness ratio (inner/outer radius) of 0.65-0.8. The modelling of the SMA torque tube actuator was based on Brinson SMA constitutive model, and the analytical stress-strain behaviour and the tube torque-angle relationship matched the experimental test results. However, the comparison was only for the SMA torque tube tested at a room temperature, but not during actuation cycle where SMA temperature changed. The SMA torque tubes were pre-strained by 12 degrees, and upon actuation, they recovered about 10.5° under free recovery and about 9.5° when the tube acted against a torsional restoring spring. For both cases, residual strains (tube angles) were observed at the end of the cooling cycle. They used thermoelectric modules to release excessive heat through the blade surface, to dramatically decrease the cooling cycle time and overall actuation time. The cooling time was reduced from 9 minutes to 5 minutes, to decrease the tube temperature from 95°C to 35°C.

In a morphing wing technologies research, Fontanazza et al. (2006) proposed four morphing concepts for flight control, which included a deformed compliant wing, a twisted wing, multistable composites, and variable stiffness materials. They attempted to develop a smart composite material consisting of NiTi shape memory alloy (SMA) as a precipitate within a low modulus, high yield strain, compliant Titanium alloy (Gum metal – Ti-23Nb-0.7Ta-2Zr-1.2O and Ti-23Nb-0.7Ta-2Zr-0.3O) matrix. Preliminary in situ X-ray synchrotron tensile tests
were conducted on the Gum metal, and no stress-induced phase transformation was observed. The Gum metal had only 2% yield strain, which might be insufficient to accommodate the high strain recovery of the SMA. The conclusion was that the deformation behaviour of the Gum metal was not completely understood, and should be subjected to a more detailed research and understanding, before the smart composite material could be produced.

In addition to X-ray diffraction experiments on 90% cold worked Ti-36Nb-2Ta-3Zr-0.3O (wt. %) alloy (Gum metal), Talling et al. (2007) investigated a morphing conceptual design by embedding NiTi SMA hollow tubes within an Adiprene L 900/Caytur 30 elastomer resin. The SMA hollow tubes were trained at 500 °C to remember a curved shape, as shown in Fig. 2.34a. A cured laminate of the SMA hollow tubes with the elastomer resin, where the SMA tubes were in a straight shape, is shown in Fig. 2.34b. The SMA hollow tubes were heated internally with SMA wires. They found that the performance was poor if compared to the predicted actuator stroke that was supplied by the manufacturer. They concluded that the training should
be optimum on straight hollow tubes, instead of curved tubes. The believed reason was that the SMA material near the neutral axis was not fully actuated, when they were trained as curved tubes. They also indicated that it was necessary to include a thermocouple for temperature monitoring, because of the possible development of ‘hot spots’ along the SMA tubes, which could generate a non-uniform actuation.

![Image](image1.png)

**Figure 2.34.** (a) SMA training method and (b) a cured SMA elastomer block (Talling *et al.*, 2007).

Turner *et al.* (2008) embedded 2 Nitinol SMA strips in a composite laminate of a scaled-down adaptive chevron (Fig. 2.35), with a final objective of reducing the noise of a jet engine. The SMA strips/ribbons had a cross-sectional dimension of $2.29 \times 0.15$ mm, were pre-strained (elongated) by 4% strain, and were heated by resistive heating. The composite laminate was fabricated from glass-epoxy (S2-glass / 3501-6 resin) unidirectional preimpregnated (prepreg) tape, with a nominal cured ply thickness of 0.1 mm. The SMA hybrid composite (SMAHC) laminate had a $-45^\circ/+45^\circ/90^\circ$/SMA$+/45^\circ$/SMA$-45^\circ$ lamination sequence, exploiting the neutral axis of the composite laminate. The target chevron tip deflection was $1.27$ mm, and the achieved static tip deflection was about $1.14$ mm ($1.52$ mm actuated – $0.38$ mm unactuated). After 1st thermal cycle, the SMAHC chevron did not return to its original undeflected shape,
and this additional permanent deformation decreased with an increase in thermal cycles and stabilised after the 56th cycle.

Calkins et al. (2008) comprehensively reviewed Boeing’s morphing aerostructures based on shape memory alloy (SMA) actuation, which include the F-15 smart inlet in the Smart Aircraft and Marine Project System Demonstration (SAMPSON) program, the controllable rotor blades in the Reconfigurable Rotor Blade (RRB) program, the morphing chevrons of a jet engine in the Variable Geometry Chevron (VGC) program, the deployable rotor blade aerodynamic device, and the variable area fan nozzle of a jet engine. These aerostructures were actuated by bundled SMA wires, SMA rotary actuators, SMA plates, an Active Hinge Pin Actuator (AHPA) which used a SMA torque tube as a structural hinge pin, and SMA flexure actuators, respectively. This indicated the reliability and maturity of the SMA to actuate various large-scale morphing aerostructures.

![Image](image1.jpg)

*Figure 2.35.* (a) Static chevrons on the core and bypass nozzles in the NASA Langley Research Centre Low-Speed Aerosacoustic Wind Tunnel and (b) SMAHC adaptive chevron (Turner et al., 2008).

In the early period of morphing research based on shape memory alloy (SMA) actuators, hinged structures seemed to attract much attention and were accepted by the research
Morphing Aerofoils/Wings by Smart Actuation

community (see for instance Fig. 2.22 (Webb et al., 2000), Fig. 2.23 (Rediniotis et al., 2002), Fig. 2.24 (Epps and Chopra, 2001), and Fig. 2.25 (Pitt et al., 2001)). These hinged structures are rarely investigated these days, but few research were still carried out recently, for instance an experimental study on a Synergistic Smart Morphing Aileron (SSMA) by Pankonien et al. (2013), as shown in the previous Fig. 2.18, and an experimental study by Basaeri et al. (2014) on sweep and gull/dihedral morphing of a bio-inspired robotic wing, as shown in Fig. 2.36. Another example was the design and experiment of hinged truss structures actuated by multiple NiTi SMA wires (Sofla et al., 2009). Sofla et al. (2008) investigated an antagonistic flexural cell (AFC) concept actuated by SMA ribbons, as shown in Fig. 2.37, which was a continuation of the previously introduced shape morphing panel concept (Elzey et al., 2003). The fact that the interest on this type of structures had declined probably because the hinged structures could be easily or more efficiently deformed by conventional motors, instead of using the expensive SMAs that generate high actuation force. As a simple analogy, a large hinged door could be pushed by a finger, and did not require a large applied force to be opened/closed.

Figure 2.36. Bio-inspired robotic morphing wing (Basaeri et al., 2014).

Instead of hinged structures, the SMAs were also investigated as potential actuators for hingeless structures since late 1990s. Antagonistic SMA wires were one of the investigated actuators for the hingeless control surface (HCS) concept of the Northrop Grumman unmanned combat aircraft vehicle (UCAV) (Wang et al., 2001). One of the concepts had two SMA wires from spar to the trailing edge section (Fig. 2.38a), while the other concept had a series of thin
film SMAs or SMA wires from the spar to the centre laminate (Fig. 2.38b). However, the concepts were not taken forward to the final design of the HCS because of complexity of the proposed concept and high requirement of the UCAV manoeuvre control, which was more than 75°/s.

Figure 2.37. Antagonistic Flexural Cell (AFC) concept (Elzey et al., 2003).

Figure 2.38. Hingeless Control Surface (HCS) (Wang et al., 2001).
2.4 Camber Morphing by Compliant Mechanism, Multistable Structures, and Other Structures

Alternative approaches to achieve camber morphing of aerofoils/wings were exploitation of compliant structures, mechanism-based structures such as bi-stable or multi-stable structures, and other form of structures such as a cellular truss and a Kagome planar truss.

2.4.1 Compliant structures

At the German Aerospace Centre or Deutsches Zentrum für Luft- und Raumfahrt e.V. (DLR), Campanile and Sachau (2000) proposed a belt-rib concept to replace a classical aerofoil section of outboard flap of Airbus A340, which comprised of spars, ribs, stringers and skins, as depicted in Fig. 2.39a. The belt-rib concept consisted of a closed shell belt, connected to spokes or in-plane stiffeners by solid state hinges (Fig. 2.39b). For a flap chord of 1.5 m and a span of 10.21 m, the target flap deflection was ±5° or 50 mm. The camber change was mainly at the trailing edge section, at 40% aft of the flap chord. In experimental bench-top tests, a 500mm-wide flap made of carbon fibre/epoxy composite and metallic hinges, was deflected mechanically by an eccentric cam driven by a handle, as shown in Fig. 2.39c. Other potential actuation methods were proposed, as shown in Fig. 2.39d, which included a truss configuration (top), rotary actuators or active hinges (middle) and an embedded active material in the closed shell belt (bottom).

Ramrkahyani et al. (2004, 2005) proposed a tendon-actuated cellular truss concept for a morphing aerofoil, as shown in Fig. 2.40a. The proposed trusses were made of an aircraft grade aluminium alloy, while the proposed tendons or cables were made of a high strength polymer, such as Spectra. The tendons worked in a way that different cables were active in different load conditions. The trusses and cables were connected through compliant joints, which acted as rotational springs. The possible compliant joints were cylindrical SMAs in pseudoelastic mode. The proposed skin was made of a lightweight polymer material, such as vectran. The bench-top demonstrator model of the cellular trusses is shown in Fig. 2.40b.

As a result of continuous research, a camber morphing concept was invented and patented by Iannucci (2012), which highlighted the potential of a morphing wing made of a honeycomb core and a polyurethane reinforced glass/carbon fibre composite skin (Fig. 2.41). The honeycomb core was potentially deformed and actuated by a piezoelectric element, a magnetostriective element, or SMA/SMP/monolithic single crystal piezo composites. Each potential element was proposed to be
activated using electrodes, a magnetic field generator, and temperature changes/electric charges, respectively. A polypropylene material was suggested for hinges of the skin-core attachment due to its flexibility. A significant advantage of this concept was that the shape changes of the honeycomb core did not require the shape changes of the skin, and hence reducing the complexity in designing the morphing skin.

Figure 2.39. Belt-rib concept (Campanile and Sachau, 2000).
Airoldi et al. (2012) investigated a morphing aerofoil with a carbon fibre composite chiral honeycomb core structure. The leading and trailing edge sections of the 1m-chord, 1m-span wing were considered as rigid bodies (Fig. 2.42a). The optimised hexa-chiral cell had a radius of 10.3 mm, a length between cells of 19.3 mm, and a thickness of 0.232 mm. In the finite
element analysis (FEA) simulation work, the skin was connected to the chiral honeycomb core by rigid beams and pinned beams. The optimised chiral wing section before and after 10 degrees camber change is shown in Fig. 2.42b.

Ruangjirakit (2013) introduced a composite corrugated structure for a morphing aerofoil section. The structure had a NACA 0014 aerofoil section with a chord length of 300 mm, a span/width of 80 mm, and a single spar located at 25% of the chord. The bench-top model with a total length of 225 mm (75% of the chord) was fabricated from a carbon fibre reinforced polyurethane material (for the corrugated lower surface), and a VeroBlack FullCure 870 material (for the smooth upper surface) using a 3-D printer (Objet Connex 350™). The experimental setup is depicted in Fig. 2.43(a). A pull force from a linear actuator deflected the trailing edge downward, while a push force deflected it upward. Results of maximum downward and upward defections (without aerodynamic load) were 4° and 6°, as illustrated in Fig. 2.43(b) and Fig. 2.43(c), respectively. The maximum required actuation force was reported as around 10 N push force and approximately 12 N pull force, for vertical displacements of 2 mm and -3 mm, respectively. At a low Reynolds number, a corrugated aerofoil was expected to perform better than a smooth aerofoil, because of a lower drag. This could be analogically compared to a golf ball with a dimple pattern, which performed better at a low Reynolds
number, as investigated by Terwagne et al. (2014) on smart morphable surfaces for aerodynamic drag control.

![Wing model](image1.png) ![Actuator](image2.png) ![Force gauge](image3.png)

**Figure 2.43.** (a) Experimental setup of a corrugated aerofoil; maximum (b) downward and (c) upward deflections of a bench-top model (Ruangjirakit, 2013).

Previtali et al. (2014) investigated a morphing wing concept consisted of compliant belt ribs with single and double corrugations, as shown in Fig. 2.44. Electro-mechanical linear actuators were proposed to actuate the morphing wing. A maximum aileron deflection of $25^\circ$ was aimed and the performance of the morphing wing was compared with a conventional wing. Comparatively, the morphing wing had a lower structural weight at a low speed but a heavier structural weight at a higher speed, compared to a conventional wing.

Molinari et al. (2016) fabricated and tested a morphing wing based on distributed compliance structures, aimed for flight control of a small unmanned aerial vehicle (UAV). The compliance morphing wing concept, which consisted of a front rigid D-nose wing box and a rear compliant section, is shown in Fig. 2.45a. The fabricated and assembled demonstrator of the $300\text{mm}$-chord compliance wing is shown in Fig. 2.45b. Majority of the wing components were fabricated from glass- and carbon-fibre reinforced plastic (GFRP and CFRP), as generally reported. A total of 18 piezoelectric MFCs were used for actuation. In each of six groups of MFC actuators along the wing span, two piezoelectric patches were used on the upper surface and one piezoelectric patch was used on the lower surface. The piezoelectric actuators were operated at a supplied voltage between $-500$ V and $1050$ V. At low Reynolds numbers ($185,500$, $371,000$ and $556,500$, corresponded to flight speeds of $10$, $20$ and $30$ m/s), the wind
tunnel tests resulted in average vertical displacement of the rearmost section of the wing of about 7.4 mm (without airflow), and reduced to about 3.9 mm at the maximum speed. In a subsequent flight test, a maximum roll rate of 17.6 deg/s was achieved at a flight speed of 21.55 m/s.

![Diagram of a morphing wing](image1)

Figure 2.44. A morphing wing concept consisted of compliant belt ribs (Previtali et al., 2011, 2014).

![Diagram of wing box](image2)

Figure 2.45. A compliance morphing wing (Molinari et al., 2016).
2.4 Camber Morphing by Compliant Mechanism, Multistable Structures and Other Structures

### 2.4.2 Multistable Structures

Santer and Pellegrino (2008) investigated compliant multistable structures, known as asymmetrically-bistable tetrahedron, which was made of linear and nonlinear springs. The linear springs used was slender rods made of carbon fibre reinforced polymer (CFRP), while the nonlinear springs used were tape springs made of triaxial-weave CFRP. The length of the rods and the tape springs were 100 mm and 80 mm, respectively. The angle between rods was 30°, and the radius of the tape spring was 8 mm. The changes from one stable shape to another were achieved by applied moment at the shorter edge of the tape springs, as shown in Fig. 2.46a. The ABAQUS finite element analysis (FEA) predictions of the three stable equilibrium configurations of the tetrahedron is shown in Fig. 2.46b. A good comparison between the FEA predictions and the experimental results of the physical model is shown in Fig. 2.46c. A truly bistable tetrahedron model was then introduced, which had only two stable states. Several truly bistable tetrahedrons were combined to form a multistable super-tetrahedron structure, as shown in Fig. 2.46d. The height difference between stowed (Fig. 2.46d-i) and fully deployed (Fig. 2.46d-v) configuration was about 4.1 cm.

Santer and Pellegrino (2009) designed and fabricated a compliant adaptive leading edge section of an aircraft wing, based on a load-path-based topology optimisation. The aircraft wing had a NACA-2421 aerofoil profile with a chord length of 1056 mm, envisaged to be incorporated in an unmanned aerial vehicle (UAV). The compliant adaptive leading edge section consisted of the first quarter chord, which was 264 mm, front of the wing section. It consisted of 5mm-thick rib and 1mm-thick skin, as shown in Fig. 2.47a. The optimisation study accounted for aerodynamic pressure loading, which was obtained from an inviscid flow condition at a flight speed of 134 m/s, at sea level, and at a 5 degrees angle of attack, using the XFOIL software. The pressure loading was applied on the skin of the leading edge section, which was modelled in a SAMCEF finite element software. With the objective of a vertical displacement corresponded to a 5 degrees rotation of the leading edge section about the quarter chord, the internal lattice structure was optimised for a minimum mass and a minimum shape change error. The optimised compliant adaptive leading edge section was fabricated from a high-strength aluminium material, with a Young’s modulus of 72 GPa, a Poisson’s ratio of 0.33, and a yield strength of 395 MPa. The optimum compliant adaptive leading edge section, after post processing by eliminating the highly compressive stressed member, and replacing the highly tensional stressed member with a member having localised hinges, is shown in Fig. 2.47b. The figure shows the demonstration model before actuation (left) and after actuation by
an imposed chordwise displacement of 4.8 mm (right), at a single-point internal actuation, by means of a mechanical screw.

**Figure 2.46.** Compliant multistable structures (Santer and Pellegrino, 2008).
2.4 Camber Morphing by Compliant Mechanism, Multistable Structures and Other Structures

Under an Intelligent Responsive Composite Structures (IRCS) program, Daynes et al. (2009) investigated a bistable composite aerofoil for hover and forward flight of a helicopter main rotor blade. The aerofoil had a cambered NACA 24016 aerofoil section, with a 680 mm chord length, a 100 mm span, and a 16% maximum thickness-to-chord ratio. The bistable composite was designed for the flap located at 15% rear of the chord, as shown in Fig. 2.48a. The bistable flap consisted of six bistable pre-stressed (1.1% pre-strained) buckled laminates, made of Hexcel 913 glass fibre reinforced plastic (GFRP) prepreg. The six bistable laminates had a [0/90/90/0] T lay-up, with the pre-stressing applied in the 0 degree direction (Fig. 2.48b), while the upper and lower external skins were not bistable and had a [90/0/90] T lay-up. Limited to load-displacement tests using an INSTRON machine, as shown in Fig. 2.48c, two stable states were achieved, at which the corresponded snap-through angles were 10.2° and -2.3°.

Santer and Pellegrino (2011) investigated a multistable plate structure which can be deformed to cylindrical curved shapes. The compliant plate was actuated by bistable snap-through truss plates. The prototype consisted of five bistable elements (Fig. 2.49ii), resulting in 32 stable configurations. It was fabricated from an acrylic-based polymer FullCure720 material, which had a Young’s modulus of 2870 MPa and a Poisson’s ratio of 0.3, using the PolyJet 3D printing rapid prototype technology. The snap-through truss plates had a dimension of 9.79 mm long, 9 mm wide and 0.5 mm thick, while the living hinges that connected the truss plates to the rest of the bistable element, had a dimension of 2 mm long and 0.2 mm thick. The
2.4 Camber Morphing by Compliant Mechanism, Multistable Structures and Other Structures

sequential actuation of the prototype is shown in Fig. 2.49(iii). A finite element software, SAMCEF, was used to determine the displacement or stroke required to shift between the two stable configurations of the bistable element, which was 1.73 mm.

![Figure 2.48](image)

\( \theta_1 = 0 \) deg
\( \theta_2 = 10 \) deg

Figure 2.48. (a) Rotor blade section with a bistable flap, (b) lay-up of the bistable laminates, and (c) load-displacement tests setup of the bistable flap of the rotor blade section (Daynes et al., 2009).

Panesar and Weaver (2012) investigated a morphing flap made of bistable laminates as an alternative to a plain flap. It has a dimension of 1400 x 400 x 1 mm. Utilizing symmetrical boundary condition along the mid-span, the length used in Matlab optimization and ABAQUS finite element model was reduced to 700 mm. Experimentally, the bistable laminates were manufactured using a tow-steering technique, with half of the actual dimension (700 x 200 x 0.5 mm). They were comprised of 4 plies (Fig. 2.50a) with bottom two plies had a fixed fibre orientation (0°) and top two plies had a variable fibre orientation in 0°/30°/60°/90° directions. Two cases were investigated: identical top two plies (Fig. 2.50b) and independent top two plies (Fig. 2.50c and d). An actuation load was applied on corners B and B-mid (unidentified exact value) of the laminates to attain the first stable state, and on the opposite direction to achieve the second stable state. An ant colony optimisation (ACO) technique was implemented to optimise the laminate configuration or fibre orientation. For both cases of identical and independent top two plies, optimum fibre orientations were obtained for each of the following objectives: a maximum absolute deflection in a stable state (deployed flap-angle), and a maximum relative deflection between the two stable states (flap-angle increment).
Figure 2.49. (i) Two configurations of bistable element in the initial concept of a multistable plate structure made of Nylon6 from injection moulding, (ii) a prototype model of the multistable plate structure, and (iii) sequential actuations of the multistable plate structure prototype (Santer and Pellegrino, 2011).
Figure 2.50. (a) Stacking sequence of tow-steered laminates; Optimum fiber orientation of laminates with (b) identical top two plies for a maximum deployed flap-angle (top) and a maximum flap-angle increment (bottom), and with independent top two plies for (c) a maximum deployed flap-angle and (d) a maximum flap-angle increment; Deformed flap (state 2) with (e) identical case and (f) independent case (Panesar and Weaver, 2012).

Arrieta et al. (2014) utilised variable stiffness multi-stable composites within a morphing rib to achieve passive load alleviation control. Instead of common morphing objectives to increase aerodynamic performance or to continuously change it to match different mission profiles, their aim was to reduce lift distribution when the structure was subjected to perturbation, e.g. a gust load. This ensured its structural integrity by avoiding an excessive bending stress or a fatigue failure. The 500mm-chord, 60mm-span NACA 0012 aerofoil section was made of glass fibre reinforced plastic (GFRP) skins and spars, and a foam-filled front section (Fig. 2.51a). The multi-stable composites were manufactured from carbon fibre reinforced plastic (CFRP) with a tailored lay-out shown in Fig. 2.51b. Experimental tests resulted in a trailing edge deflection of 19 mm at 6 N applied load (Fig. 2.51c). Aerodynamic analysis at $1.4 \times 10^5$ Reynolds number using a XFLR5 software resulted in about 10-20% reduction in lift coefficient, for a range of -5° to 10° angle of attack. This concept had an advantage of elimination of moving mechanical parts and active smart materials for actuation, which may lead to reduction in total weight and complexity. However, the major drawbacks of the concept were a permanent structure deformation, which was limited to two modes of deformation, and a poor deformed shape. Once the snap-through was triggered, the aerofoil shape could not return back to its original shape, even after decreasing the wind speed.
2.4 Camber Morphing by Compliant Mechanism, Multistable Structures and Other Structures

Figure 2.51. (a) An aerofoil structure with an embedded variable stiffness multi-stable element, (b) a tailored lay-out of the multi-stable composites, (c) experimental tests on the compliant rib in a stiff mode (left) and a compliant mode (right) (Arrieta et al., 2014).

2.4.3 Kagome Truss Structures

Wicks and Hutchinson (2004) analytically analysed deformation of a sandwich plate actuated by a Kagome planar truss. The sandwich plate consisted of one solid face sheet and one actuated Kagome face sheet, which were joined by a pyramidal/tetrahedral truss core, as shown in Fig. 2.52a. Actuation of a selected one column of the Kagome members (Fig. 2.52b - left) resulted in a plate deformation shown in Fig. 2.52c. Actuation of the selected two columns of the Kagome members (Fig. 2.52b - right) resulted in a plate deformation shown in Fig. 2.52d. These were achieved based on calculated normalised strain of 0.015. The advantages of this approach were good in-plane stiffness, low internal resistance to actuation, and achievability of wide variety of deformed shapes. The disadvantage was a failure of a single truss might require a replacement of the entire plate, because of the complexity of the internal trusses.

The morphing aerofoils/wings/structures which were covered in sections 2.2-2.4, were summarised in Table 2.1-2.3. The morphing aerofoils/wings/structures deformed by mechanical actuation are listed in Table 2.1. The aerofoils/wings/structures morphed by smart actuation are summarised in Table 2.2. Compliant structures which could be actuated by either mechanical or smart actuation are summarised in Table 2.3. Even though they were not directly comparable, because the aerofoils/wings/structures had various sizes and made of different materials, the tables should give a rough idea on the various selection of available actuators, the required power supply, and the achievable stroke or deformation or trailing edge deflection, of the morphing aerofoils/wing/structures.
From the review, a conventional method of measuring actuation force was by deforming the aerofoil section numerically or experimentally to a target deformation, and then measuring...
the force acted on the actuation point in the deformed configuration. For example, Bolinches et al. (2011) experimentally measured actuation forces of 17 N and 27 N to relatively displace a 1m-span 350mm-chord wing made of polystyrene foam and 3D-printed ABS sliders (Fig. 2.53), by 5 mm and 10 mm in the chord wise direction, respectively. Another example was the predicted trailing edge deflection of the SMP morphing concept in ABAQUS by James et al., (2009), which was 22.06 mm, for an actuation force of 100 N. This approach could be used for morphing wings with very few actuation points. The design of morphing wings was sometimes constrained by this limitation. The development of the UMAT for the SMA wires could possibly overcome this limitation, as the SMA wires could be modelled and placed at any sections/parts of the morphing structures.

**Figure 2.53.** A morphing UAV wing made of polystyrene foam and ABS sliders (Bolinches et al., 2011).

<table>
<thead>
<tr>
<th>Morphing Concepts</th>
<th>References</th>
<th>Actuators</th>
<th>Actuation Power/Stroke/Force</th>
<th>Achievement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formvariable Flap</td>
<td>Monner (2001)</td>
<td>Spindle drive</td>
<td>N/A</td>
<td>±15° maximum camber variations. ±185 mm TE deflections.</td>
</tr>
<tr>
<td>Rotating Ribs</td>
<td>Ricci et al. (2006)</td>
<td>Servo-actuators</td>
<td>N/A</td>
<td>±5° TE deflections.</td>
</tr>
<tr>
<td>Seamless Aeroelastic Wing (SAW)</td>
<td>Perera et al. (2010)</td>
<td>Eccentuators with servomotors and gears</td>
<td>Voltage: 6V. Torque: 0.085 Nm.</td>
<td>9.4-14.4 mm TE deflections.</td>
</tr>
<tr>
<td>Morphing Leading Edge</td>
<td>Morishima et al. (2010)</td>
<td>Eccentuators</td>
<td>Force: 11,400 lb.</td>
<td>6% and 1.2% chord (4 m) vertical and horizontal deflections, respectively.</td>
</tr>
<tr>
<td>Morphing FishBAC Aerofoil</td>
<td>Woods et al. (2014)</td>
<td>Pneumatic artificial muscle (PAM)</td>
<td>Specific output work: 4000 J/kg.</td>
<td>49 mm TE deflection (16% of the chord).</td>
</tr>
<tr>
<td>Active Morphing Skin</td>
<td>Feng et al. (2015)</td>
<td>Pneumatic muscle fibre (PMF)</td>
<td>0.35 MPa air pressure Force: max 73.59 N.</td>
<td>53 mm maximum TE deflection.</td>
</tr>
</tbody>
</table>

N/A – not available
### Table 2.2. Morphing concepts with smart materials/actuations.

<table>
<thead>
<tr>
<th>Morphing Concepts</th>
<th>References</th>
<th>Actuators</th>
<th>Actuator Size</th>
<th>Actuation Power/Stroke/Force</th>
<th>Achievement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape Memory Polymer (SMP) Concept</td>
<td>James et al. (2009)</td>
<td>Linear actuator</td>
<td>N/A</td>
<td>Force: 100 N</td>
<td>22.06 mm trailing edge deflection.</td>
</tr>
<tr>
<td>Dimple Actuators for Flow Control</td>
<td>Dearing et al. (2010)</td>
<td>Electro-active polymer (EAP)</td>
<td>Radius: 5 mm, 10 mm, 15 mm.</td>
<td>Voltage: 42-45 MV/m</td>
<td>280 μm, 480 μm, and 800 μm out-of-plane deflections.</td>
</tr>
<tr>
<td>Controllable Camber Rotor Airfoil</td>
<td>Gandhi et al. (2008)</td>
<td>Piezoelectric stack actuators</td>
<td>N/A</td>
<td>Voltage: ±1000V Stroke: 0.15%</td>
<td>3.65 mm trailing edge deflection at ±1000V supplied voltage.</td>
</tr>
<tr>
<td>DARPA Smart Wing</td>
<td>Jardine et al. (1999)</td>
<td>Shape memory alloy (SMA) torque tube</td>
<td>Diameter: 10.2 mm (outer), 7.1 mm (inner), Length: 152.4 mm</td>
<td>Power: 12W (4V, 3A). Stroke: 12° pre-strain</td>
<td>5° spanwise twist.</td>
</tr>
<tr>
<td>Variable Twist Tiltrotor Blade</td>
<td>Prahlad and Chopra (2001)</td>
<td></td>
<td>N/A</td>
<td></td>
<td>9.5°-10.5° twist.</td>
</tr>
<tr>
<td>Hingeless Control Surface (HCS)</td>
<td>Wang et al. (2001)</td>
<td></td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Biomimetic Hydrofoil</td>
<td>Rediniotis et al. (2002)</td>
<td></td>
<td>Diameter: 0.06 mm Length: 260 mm</td>
<td>Stroke: 3.5%</td>
<td>50° maximum deflection angle.</td>
</tr>
<tr>
<td>Embedded Shape Memory Wire Concept</td>
<td>Iannucci and Fontanazza (2008), James et al. (2009)</td>
<td>SMA wires</td>
<td>N/A</td>
<td>N/A</td>
<td>Wing twist with a maximum 5.27 mm out-of-plane displacement.</td>
</tr>
<tr>
<td>Morphing Laminar Wing with Flexible Extrados</td>
<td>Georges et al. (2009)</td>
<td></td>
<td>Cross-section area: 4.2 mm² Length: 1800 mm</td>
<td>Force: 100-180 N</td>
<td>7.7 mm maximum vertical displacement.</td>
</tr>
<tr>
<td>Morphing Wing Rib</td>
<td>Almeida et al. (2015)</td>
<td></td>
<td>Diameter: 2.058mm Length: 103.5 mm Length: 152.1 mm</td>
<td>Stroke: 1.05-1.06%</td>
<td>11.6 mm - 14 mm trailing edge deflections (5.3º-6.4º).</td>
</tr>
<tr>
<td>Antagonistic Flexural Cell (AFC)</td>
<td>Elzey et al. (2003)</td>
<td>SMA strips</td>
<td>Thickness: 0.25 mm</td>
<td>Stroke: 4% pre-strain</td>
<td>N/A</td>
</tr>
<tr>
<td>SMAHC Adaptive Chevron</td>
<td>Turner et al. (2008)</td>
<td></td>
<td>Cross-section: 2.29×0.15mm.</td>
<td>Power: 3.5W (1.6V, 2.25A). Stroke: 4% pre-strain</td>
<td>1.14 mm chevron tip deflection.</td>
</tr>
<tr>
<td>Changeable Aerofoil</td>
<td>Dong et al. (2008)</td>
<td>SMA springs</td>
<td>Spring outer diameter: 9 mm Cross-section diameter: 1.5 mm</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

N/A – not available
2.4 Camber Morphing by Compliant Mechanism, Multistable Structures and Other Structures

Table 2.3. Morphing concepts utilising compliant structures and mechanism-based structures, which could be actuated by mechanical or smart actuators.

<table>
<thead>
<tr>
<th>Morphing Concepts</th>
<th>References</th>
<th>Actuators</th>
<th>Achievement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belt Rib Concept</td>
<td>Campanile and Sachau (2000)</td>
<td>Eccentric cam driven by a handle</td>
<td>±5° or ±50 mm flap deflections.</td>
</tr>
<tr>
<td>Biomimetic Hydrofoil</td>
<td>Rediniotis et al. (2002)</td>
<td>SMA wires</td>
<td>50° maximum deflection angle.</td>
</tr>
<tr>
<td>Compliant Leading Edge</td>
<td>Santer and Pellegrino (2009)</td>
<td>Screw</td>
<td>5° leading edge tip deflection.</td>
</tr>
<tr>
<td>Aerofoil Member</td>
<td>Iannucci (2012)</td>
<td>Mechanical or smart actuators</td>
<td>N/A</td>
</tr>
<tr>
<td>Aerofoil with Composite Chiral Structure</td>
<td>Aeroldi et al. (2012)</td>
<td>Mechanical or smart actuators</td>
<td>10° camber change.</td>
</tr>
<tr>
<td>Composite Corrugated Aerofoil</td>
<td>Ruangjirakit (2013)</td>
<td>Linear mechanical actuator</td>
<td>-4° to 6° trailing edge deflections.</td>
</tr>
<tr>
<td>Distributed Compliance Morphing Wing</td>
<td>Molinari et al. (2016)</td>
<td>Piezoelectric Macro Fibre Composites (MPCs)</td>
<td>3.9 mm - 7.4 mm trailing edge deflections.</td>
</tr>
<tr>
<td>Compliant Multistable Structure</td>
<td>Santer and Pellegrino (2008)</td>
<td>N/A</td>
<td>41 mm height difference.</td>
</tr>
<tr>
<td>Multistable Plate Structure</td>
<td>Santer and Pellegrino (2011)</td>
<td>Bistable snap-through truss plates</td>
<td>N/A</td>
</tr>
<tr>
<td>Bistable Morphing Flap</td>
<td>Panesar and Weaver (2012)</td>
<td>N/A</td>
<td>6.6 mm tip deflection.</td>
</tr>
<tr>
<td>Compliant Trailing Edge with Multistable Composite</td>
<td>Arrietta et al. (2014)</td>
<td>Mechanical - INSTRON machine</td>
<td>19 mm trailing edge deflection.</td>
</tr>
<tr>
<td>Compliant Cellular Truss Concept</td>
<td>Ramrakahyani et al. (2004)</td>
<td>Tendons or cables made of high strength polymer (i.e Spectra)</td>
<td>N/A</td>
</tr>
<tr>
<td>Sandwich Plates</td>
<td>Wicks and Hutchinson (2004)</td>
<td>Kagome truss plate</td>
<td>N/A</td>
</tr>
</tbody>
</table>

N/A – not available

From the extensive review on the various morphing structures that had been explored over three decades, shape memory alloys (SMAs) were realised as a favourable smart actuator over other smart materials and mechanical actuators, mainly because of a higher energy density, a high actuation stroke, a reduced complexity, and a compact volume. Therefore, the physics and thermo-mechanical behaviour of the SMA were then deeply studied and reviewed, including some of earliest SMA constitutive models. The knowledge and understanding of the SMA behaviour and constitutive models were then applied to develop a FORTRAN code for modelling SMA actuation in an explicit finite element analysis (FEA) software, LS-DYNA, which would be comprehensively covered in Chapter 4. In the review of the SMA material, the past work on FEA simulations involving SMAs were also covered, which were limited mostly to implicit simulations, and some were constrained to simulations of oversimplified models.
2.5 Shape Memory Alloy (SMA) – Thermomechanical Behaviour and Physical Properties

Shape memory alloy (SMA) material was carefully selected as the most promising actuator for the morphing wing, after considering other potential smart materials such as piezoelectrics, shape memory polymers, and electrostrictives. The crucial reasons of the SMA selection were a large deformation capability and the unique shape memory effect (SME) properties. The large deformation (reported up to 8% strain) was important to produce sufficiently high deflection of the morphing aerofoil section. During actuation, the SME properties utilised heat transfer to recover its initial shape from a deformed shape, reducing complexity in mechanical rotors/parts which resulted in saving of material and maintenance costs. Comparison of the SMA with other smart materials is summarised in Table 2.4 (Chopra and Sirohi, 2014).

<table>
<thead>
<tr>
<th>Smart Materials</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape Memory Alloys (SMAs)</td>
<td>• Shape Memory Effect (SME).</td>
<td>• Slow response (&lt; 1Hz).</td>
</tr>
<tr>
<td></td>
<td>• Large recovery strain (up to 8%) and elongation prior to failure (up to 50%).</td>
<td>• Speed of transformation is limited by heat-transfer rate.</td>
</tr>
<tr>
<td></td>
<td>• High recovery stress (up to 800 MPa) and ultimate tensile stress (up to 1000 MPa).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• High corrosion resistance.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Easy workability and great damping capacity.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Long cycle life.</td>
<td></td>
</tr>
<tr>
<td>Piezoelectrics</td>
<td>• Higher bandwidth than SMAs.</td>
<td>• Leakage of charge with time – cannot be used for static response and measurements.</td>
</tr>
<tr>
<td></td>
<td>• More compact than magnetostrictives.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Bidirectional, unlike electrostrictives.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Insensitive to electromagnetic fields and radiation (applications in harsh environment).</td>
<td></td>
</tr>
<tr>
<td>Shape Memory Polymer (SMP)</td>
<td>• Easily tailored.</td>
<td>• Low strength, stiffness and recovery stress.</td>
</tr>
<tr>
<td></td>
<td>• 100% strain recoverable.</td>
<td>• Limited cycle life (200 cycles).</td>
</tr>
<tr>
<td></td>
<td>• Low cost, light-weight, non-toxic and bio-degradable.</td>
<td></td>
</tr>
<tr>
<td>Electrostrictives</td>
<td>• Electrostrictive effect is quadratic with electric field (unlike piezoelectric which is linear).</td>
<td>• Do not show spontaneous polarization (like piezoelectric), hence very low hysteresis effect even at high operating frequencies.</td>
</tr>
<tr>
<td></td>
<td>• Repeatable performance (contrary to piezoelectrics).</td>
<td>• Temperature sensitive (important limitation) – need to be maintained at ±10°C, restricted to underwater and vivo applications.</td>
</tr>
<tr>
<td></td>
<td>• Not initially poled, hence elongation takes place for both positive and negative voltages.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Electric capacitance 4-5 times as high as piezoelectric actuators.</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.4. Comparison of shape memory alloy (SMA) with other smart materials - advantages and disadvantages (Chopra and Sirohi, 2014).
Shape memory alloy (SMA) exhibited two important and unique thermomechanical properties, namely Shape Memory Effect (SME) and Pseudoelasticity (PE) (or sometimes referred to superelasticity - SE). They were associated with austenite and martensite phases transformations. Fig. 2.54a illustrates the stress-strain behaviour of the SMA, called the shape memory effect (SME), at a temperature lower than martensite finishing temperature \( T < M_f \). On mechanical loading or deformation, the SMA stress increased linearly with an increase in the SMA strain, followed by an almost plateau region and another linearly increased SMA stress. Upon unloading, the SMA stress decreased linearly to zero, and the SMA strain decreased to a certain value, called a recovery strain or a maximum recoverable strain \( \varepsilon_L \) at the maximum achievable by the SMA. The phase transformation associated with the SME, which was utilised for actuation, is depicted in Fig. 2.55. On cooling of the SMA, the SMA crystal structure transformed from an austenite phase to a martensite phase (1). At this twinned martensite phase, the SMA was deformed to set the recovery strain (2). The deformation of the SMA at a low temperature, below the martensite finishing temperature, resulted in the stress-strain behaviour shown in the previous Fig. 2.54a. Upon heating of the SMA, the deformed martensite phase (also referred as stress-induced martensite) transformed back to the initial/parent austenite phase, and recovered the strain (3). Schuerch (1968) theoretically predicted a maximum recoverable strain \( \varepsilon_L \) of up to 12\% for SMA having single perfectly oriented crystals, and experimentally observed up to 10\% maximum recoverable strain \( \varepsilon_L \).

The pseudoelastic (PE) or superelastic (SE) properties of the SMA is the SMA isothermal properties at a temperature higher than the austenite finishing temperature, \( T > A_f \), where the SMA is capable of recovering high recovery strain upon removal of applied loads, as shown by the SMA stress-strain behaviour in Fig. 2.54b. Shaw et al. (2007) experimentally achieved over 50\% strain recovery for their superelastic NiTi honeycomb subjected to an in-plane compression loading.

![Figure 2.54. Stress-strain behavior of shape memory alloy (SMA): (a) Shape Memory Effect (SME) and (b) Pseudoelasticity (PE) (Nishimura, 1997).](image-url)
The shape memory effect (SME) characteristic could also be associated with a R-phase (rhombohedral phase) transformation, depending on the manufacturing process of the SMA. Occasionally, intermediate rhombohedral phase (R-phase) transformation existed on cooling from an above austenite finishing temperature ($A_f$) to a below martensite finishing temperature ($M_f$), before subsequent transformation from the R-phase to the martensite phase. This was typical for SMA supplied by Nitinol suppliers in a trained state. From a differential scanning calorimeter (DSC) test on NiTi SMA ribbons supplied by Nitinol Devices and Components (Fremont, CA), Sofla et al. (2008) reported the phase transformation from the austenite phase to the R-phase, followed by the R-phase to the martensite phase transformation, on the cooling stage. They suggested that the NiTi SMA ribbons could be used as linear actuators for their antagonistic shape morphing structures, provided the R-phase transformation temperatures were above the room temperature. This depended on prior thermal history of the SMA, as stated by Schuerch (1968), who observed up to three exothermic transitions on cooling of the Nitinol SMA wires. Ren et al. (2001) discovered three different martensitic transformation paths from a parent B2 phase (austenite) to the final martensite B19' phase in Ti-Ni-based alloys. They were B2-to-B19’, B2-to-R-to-B19’ and B2-to-B19-to-B19’ transformation paths. They stated that these transformation paths depended on the prior manufacturing thermal history of the Ti-Ni SMA (either quenching process or aging process), the deformation of the Ti-Ni alloys, and the existence of additional elements in the Ti-Ni alloys (such as Fe and Cu).

The SME (main focus of this research, which would be utilised for actuation) associated with the R-phase transformation (RPT) showed an excellent cyclic properties and a fast
response in heating-cooling cycles, compared to the SME associated with martensitic transformation (MT). In the study of TiNi SMA subjected to cyclic loadings, Tobushi et al. (1996) concluded that the RPT stress and temperature of the transformation lines (Fig. 2.56b - starting and finishing lines with parentheses, ’()’) did not change, in contrast to the MT transformation lines, indicating a stable behavior (constant transformation temperatures under cyclic deformation. Additionally, a small temperature hysteresis allowed small temperature changes to complete the phase transformation, and hence provided a fast response (response to heating-cooling cycles) for actuation.

![Figure 2.56](image.png)

**Figure 2.56.** (a) Transformation lines (\(M_s\) and \(A_s\)) of martensitic transformation (MT) in cycles \(N = 1\) and 100, and (b) schematic of starting and end lines of MT and R-phase transformation (RPT) in the 1st cycle and \(N^{th}\) cycle (Tobushi et al., 1996).
2.5 Shape Memory Alloy (SMA) - Thermomechanical Behaviour and Physical Properties

In contrast, as clearly illustrated in Fig. 2.56a, the MT temperature increased and the MT stress decreased, as the number of cycles increased. Referring to Fig. 2.56b (starting and finishing lines without parentheses, ‘”‘), after N-th cycles, the shape memory alloy under a given constant stress did not complete the austenite phase transformation if heated to temperature G. Instead, the SMA needed to be heated to a higher temperature (H) to complete the transformation. In other words, the power consumption required to heat the SMA increased with number of cycles. Similar findings on the shift of MT stress and temperature were also reported by Tobushi et al. (1991).

Even though the shape memory effect (SME) associated with the R-phase transformation (RPT) showed an excellent cyclic properties and a fast response, unfortunately it showed a small recovery strain (that could be recovered upon heating) after unloading. Appropriate amount of recovery strain was important for applications that required sufficiently large deformation. The reported recovery strain for the SME due to the RPT was very small, which was less than 1% (Chopra and Sirohi, 2014). Miyazaki et al. (1988) also reported a small recovery strain, approximately 0.4%, in the study of the SME and the pseudoelasticity (PE) associated with the R-phase transition in Ti-50.5 at.% Ni. In contrast, the recovery strain for the SME due to the martensitic transformation (MT) was comparably large, approximately 4-6% (Tobushi et al., 1991, Tobushi et al., 1996).

On the heating-cooling techniques of shape memory alloy (SMA) to initiate the martensitic and the reverse transformation, several techniques had been used in the past. The most frequently used were resistive heating and environmental heating. Kang et al. (2012) and Rim et al. (2014) used resistive heating by applying direct current to actuate their SMA wire actuator to operate a morphing wing made of a PVC plate and a Balsa wood. Prahlad and Chopra (2001) applied both resistive and environmental heating in experimental characterisation of SMA wires, and compared the obtained thermo-mechanical behaviour with results from one-dimensional constitutive models. Another heating-cooling technique was heating using hot water and force-cooling using flow of cold air from an electric fan. This method was used in the study of TiNi SMA wire under variable stress and temperature (Tobushi et al., 1991), and the study of cyclic properties of TiNi SMA helical spring (Tobushi et al., 1992). In addition the above mentioned techniques, thermoelectric modules were also possible to be used to heat and cool the SMA, as proposed by Wang et al. (2001) for their hingeless control surface (HCS).

On the cyclic behaviour of the shape memory alloy (SMA), it had a remarkable cyclic properties, which was reported around $3 \times 10^6$ cycles in a heat engine (Banks, 1975), $25 \times 10^6$
cycles in a rotating beam test (Schuerch, 1968), 50×10^6 cycles (free recovery) in temperature monitoring devices (Buehler et al., 1976). In an experimental study on 0.7mm-diameter TiNi SMA wires, Tobushi et al. (2002) discovered that the martensite transformation stress decreased while the residual strain increased, with an increase in the number of cycles. These stress and residual strain inclined to reach a saturation point, which was believed due to the dislocations that accumulated around the defects in the material as the number of cycles increased. In a study of cyclic degradation of SMA ribbons for actuation of antagonistic flexural unit cell (AFC), Sofla et al. (2008) found that the degradation in the SMA recovery strain or in the actuation stroke stabilised after about 100 cycles. For a 5% pre-strained SMA ribbon, the SMA recovery strain decreased from about 2.8% on the first cycle to approximately 1.7% after 100th cycles.

Other significant material properties of the NiTi shape memory alloy (SMA) were paramagnetic, that was non-reactive to a magnetic field, excellent corrosion and oxidation resistance, high impact strength, good damping characteristic (Buehler et al., 1965), improvable strength, high strength to density ratio and high ductility (Rozner and Buehler, 1967).

2.6 SMA Constitutive Models

Since the discovery of the NiTi shape memory alloy (SMA) material in 1960s by Naval Ordnance Laboratory (Buehler and Wiley, 1961), hence the name “NitiNOL”, several SMA constitutive models were developed to provide additional understanding on the special metal behaviour. Among the famous and frequently applied SMA constitutive models were Tanaka model, Liang and Rogers model and Brinson model. The Tanaka model, one of the earliest SMA constitutive model, used exponential functions to express the value of martensite volume fraction during martensitic and reverse transformations (Tanaka et al., 1986, Tanaka et al., 1992). Instead of the exponential functions, Liang in his work (Liang, 1990) introduced cosine functions for the martensite volume fraction, which was later known as Liang and Rogers model. The main constitutive equation which related the SMA stress to the SMA strain, temperature and the martensite volume fraction, remained the same as the Tanaka model. Brinson and Lammering (1993) improved the Liang and Rogers cosine model by separating thermal-induced martensite and stress-induced martensite in the functions of martensite volume fraction, as well as in the main constitutive equation. There were several other SMA
constitutive equations, such as the one dimensional constitutive model developed by Shaw (2002). The choice of a suitable SMA constitutive equation depended on the intended applications and the type of SMA that was used for actuation of the structures. For applications that were actuated by SMA wires (or SMA strips in some cases), all first three of the above mentioned one dimensional SMA constitutive models should predict the SMA stress and strain satisfactorily, as long as the phase transformation temperatures were interpreted properly, as suggested by Epps and Chopra (2001).

### 2.6.1 One-Dimensional SMA Constitutive Models

Three SMA constitutive models had been studied to understand the thermo-mechanical behavior of a SMA wire under uniaxial deformation, namely the Tanaka model, the Liang and Rogers model, and the Brinson model. Modification of the Brinson model by Chung et al. (2007) to resolve a problem of the martensite volume fraction exceeding unity at a certain region, was also investigated. This subsection briefly describes the SMA constitutive models.

#### 2.6.1.1 Tanaka SMA Constitutive Model

The Tanaka SMA constitutive model expressed stress in the SMA material as a function of strain, temperature and martensite volume fraction. The model described the SMA uniaxial thermo-mechanical behaviour by the following constitutive equation (Tanaka et al., 1992).

\[
\dot{\sigma} = D \dot{\epsilon} + \Theta \dot{T} + \Omega \dot{\xi}
\]

(2.1)

\(D\), \(\Theta\), and \(\Omega\) were the elastic constant, the thermoelastic coefficient and the phase transformation coefficient, respectively. \(\dot{\sigma}, \dot{\epsilon}, \dot{T}, \text{and} \dot{\xi}\) are time derivatives of the SMA stress, the SMA strain, the SMA temperature, and the martensite volume fraction of the SMA, respectively. This constitutive equation was derived from energy balance and Clausius-Duhem inequality which could be referred from work of Tanaka et al. (1986) and Nishimura (1997). The SMA constitutive equation could also be written as

\[
\sigma - \sigma_0 = E(\xi)(\epsilon - \epsilon_0) + \Theta(T - T_0) + \Omega(\xi)(\xi - \xi_0)
\]

(2.2)

where the Young’s modulus, \(E\) and the phase transformation coefficient, \(\Omega\) were functions of the martensite volume fraction, \(\xi\).

\[
E(\xi) = E_A + \xi(E_M - E_A)
\]

(2.3)
2.6 SMA Constitutive Models

\[ \Omega(\xi) = -\varepsilon_L E(\xi) \]  

(2.4)

where \( \varepsilon_L \) is the maximum recoverable strain.

The martensite volume fraction, \( \xi \) indicated the percentage of volume of shape memory alloy that was in the martensite phase, which had a value between 0 and 1 (\( 0 \leq \xi \leq 1 \)). The Tanaka model used exponential functions to describe this variable on the cooling and heating stages of the SMA, as expressed by Eq. 2.5 and Eq. 2.6 respectively.

For martensitic transformation (austenite-to-martensite) \( (T > M_f \text{ and } C_m(T - M_s) < \sigma < C_m(T - M_f)) \), the martensite volume fraction was defined as

\[ \xi_{A\rightarrow M} = 1 - e^{a_M(M_s-T)+b_M}\sigma \]  

(2.5)

For reverse transformation (martensite-to-austenite) \( (T > A_s \text{ and } C_A(T - A_f) < \sigma < C_A(T - A_s)) \), the martensite volume fraction was defined as

\[ \xi_{M\rightarrow A} = e^{a_A(A_s-T)+b_A}\sigma \]  

(2.6)

where \( a_M, b_M, a_A \) and \( b_A \) were material constants, defined as

\[ a_M = \ln(0.01)/\left(M_s-M_f\right), b_M = \frac{a_M}{C_m} \]

\[ a_A = \ln(0.01)/\left(A_s-A_f\right), b_A = \frac{a_A}{C_A} \]

These two equations (Eq. 2.5 and Eq. 2.6) were integrated from their transformation kinetics (Eq. 2.7 and Eq. 2.8), respectively. Detail integration is presented in Appendix B.

\[ \frac{\dot{\xi}}{1-\xi} = b_M c_M \dot{T} - b_M \dot{\sigma} \]  

(2.7)

\[ \frac{-\dot{\xi}}{\dot{\xi}} = b_A c_A \dot{T} - b_A \dot{\sigma} \]  

(2.8)

2.6.1.2 Liang and Rogers SMA Constitutive Model

Liang and Rogers model used cosine function instead of the exponential function to describe the martensite volume fraction of the shape memory alloy (SMA) during phase transformations (Liang, 1990, and Chopra and Sirohi, 2014).
For austenite-to-martensite transformation (\(T > M_f\) and \(C_M(T - M_s) < \sigma < C_M(T - M_f)\)), the martensite volume fraction (MVF) was defined as

\[
\xi_{A\rightarrow M} = \frac{1 - \xi_A}{2} \cos[a_M(T - M_f) + b_M\sigma] + \frac{1 + \xi_A}{2} \tag{2.9}
\]

For martensite-to-austenite transformation (\(T > A_s\) and \(C_A(T - A_f) < \sigma < C_A(T - A_s)\)), the martensite volume fraction (MVF) was defined as

\[
\xi_{M\rightarrow A} = \frac{\xi_M}{2} \{\cos[a_A(T - A_s) + b_A\sigma] + 1\} \tag{2.10}
\]

where \(a_M, b_M, a_A\) and \(b_A\) were material constants, and were defined differently from the Tanaka model.

\[
a_M = \frac{\pi}{(M_s - M_f)}, b_M = -\frac{a_M}{C_M}
\]

\[
a_A = \frac{\pi}{(A_f - A_s)}, b_A = -\frac{a_A}{C_A}
\]

### 2.6.1.3 Brinson SMA Constitutive Model

The previous SMA constitutive models described only the martensitic transformation (Eq. 2.5 and Eq. 2.9) and reverse transformation (Eq. 2.6 and Eq. 2.10). They did not distinguish between thermal-induced and stress-induced martensite (Fig. 2.55). Transformation from the thermal-induced (twinned) martensite to the stress-induced (detwinned) martensite was responsible for the shape memory effect (SME) and isothermal SMA stress-strain behavior at temperatures below austenite starting temperature, \(A_s\). The Brinson SMA constitutive model took into account the difference between these two martensite variants. The constitutive equation of the Brinson model was defined as (Brinson and Lammering, 1993, Chopra and Sirohi, 2014)

\[
\sigma - \sigma_0 = E(\xi)\varepsilon - E(\xi_0)\varepsilon_0 + \Omega(\xi)\xi_s - \Omega(\xi_0)\xi_{s0} + \Theta(T - T_0) \tag{2.11}
\]

The model was developed by separating the martensite volume fraction (MVF) to the stress-induced (due to applied stress) martensite and the temperature-induced (due to temperature change) martensite.

\[
\xi = \xi_s + \xi_T \tag{2.12}
\]
For transformation to detwinned martensite, these components were defined as

(i) For $T > M_s$ and $(\sigma_s^{cr} + C_M(T - M_S)) < \sigma < (\sigma_f^{cr} + C_M(T - M_S))$

$$\xi_s = \frac{1 - \xi S_0}{2} \cos \left(\frac{\pi}{\sigma_s^{cr} - \sigma_f^{cr}} \left[\sigma - \sigma_f^{cr} - C_M(T - M_S)\right]\right) + \frac{1 + \xi S_0}{2}$$

(ii) For $T < M_s$ and $\sigma_s^{cr} < \sigma < \sigma_f^{cr}$

$$\xi_s = \frac{1 - \xi S_0}{2} \cos \left(\frac{\pi}{\sigma_s^{cr} - \sigma_f^{cr}} \left(\sigma - \sigma_f^{cr}\right)\right) + \frac{1 + \xi S_0}{2}$$

where,

$$\Delta_{\tau e} = \frac{1 - \xi T_0}{2} \cos \left[a_M(T - M_f)\right] + 1}$$

For transformation to austenite, the components were defined as

(iii) For $T > A_s$ and $C_A(T - A_f) < \sigma < C_A(T - A_s)$

$$\xi_s = \xi S_0 - \frac{\xi S_0}{\xi 0} (\xi_0 - \xi)$$

$$\xi T = \xi T_0 - \frac{\xi T_0}{\xi 0} (\xi_0 - \xi)$$

where,

$$\xi = \frac{\xi 0}{2} \cos \left[a_A(T - A_s - \frac{\sigma}{C_A})\right] + 1}$$

94
2.6 SMA Constitutive Models

The material constants $a_M, b_M, a_A$ and $b_A$ were the same with the ones defined by Liang and Rogers SMA constitutive model.

2.6.1.4 Improvement in the Brinson SMA Constitutive Model

The total martensite volume fraction (MVF), $\xi = \xi_S + \xi_T$, could exceed unity when applying the Brinson SMA constitutive model in a 1-D analysis of the shape memory alloy (SMA), particularly in the dual transformation region (within $M_f < T < M_s$ and $\sigma_s^{cr} < \sigma < \sigma_f^{cr}$). This problem was addressed by Chung et al. (2007), who proposed revised equations of the stress-induced and the temperature-induced MVF for this region.

For $T < M_s$ and $\sigma_s^{cr} < \sigma < \sigma_f^{cr}$

\[
\xi_S = \frac{1 - \xi_{S0}}{2} \cos \left[ \frac{\pi}{\sigma_s^{cr} - \sigma_f^{cr}} (\sigma - \sigma_f^{cr}) \right] + \frac{1 + \xi_{S0}}{2} \tag{2.19}
\]

\[
\xi_T = \Delta_T \xi - \frac{\Delta_T \xi}{1 - \xi_{S0}} (\xi_S - \xi_{S0}) \tag{2.20}
\]

where

if $M_f < T < M_s$ and $T < T_0$,

\[
\Delta_T \xi = \frac{1 - \xi_{S0} - \xi_{T0}}{2} \cos \left[ \frac{\pi}{M_s - M_f} (T - M_f) \right] + \frac{1 - \xi_{S0} + \xi_{T0}}{2}
\]

otherwise,

\[
\Delta_T \xi = \xi_{T0}
\]

This modified SMA constitutive model was proved by Chung et al. (2007) to satisfy the following conditions:

i. Total martensite volume fraction ($\xi = \xi_S + \xi_T$) must be smaller or equal to 1.

ii. $\xi_S$ (stress-induced martensite volume fraction) must be 1 when $\sigma = \sigma_f^{cr}$.

iii. $\xi$ (total martensite volume fraction) must be 1 when $T = M_f$.

iv. Continuity of the transformation kinetics.
2.7 Finite Element Analysis (FEA) of Morphing Structures Involving Shape Memory Alloys (SMAs)

2.7.1 Morphing Structures with SMAs

Since the discovery of SMA, numerous research have been conducted on modelling and finite element analysis (FEA) of the SMA. However, most papers are limited to the development of SMA constitutive models within the MATLAB code, which were further implemented as a UMAT in ABAQUS. Most research focused on the superelastic/pseudoelastic effect of the SMA. If the shape memory effect (SME) behaviour was investigated, only a simple constant load or a constant stress example was investigated, with very few successful and convincing results on application of the SMA to actuate real complex structures. For example, Alipour et al. (2015) developed a MATLAB code for the SMA, based on the Brinson SMA constitutive model, which was later implemented as a UMAT in ABAQUS. The research mostly investigated the pseudoelastic effect of the SMAs, and the shape memory effect (SME) behaviour was only for a SMA wire under a constant stress. On the application to more complex structures, such as a smart landing gear of an airship (Fig. 2.57a), only one result of load against displacement curve was presented. More importantly, the result was based on a rather simplified FE model that used only beams, as shown in Fig. 2.57b. This showed that a significant improvement was still required on the FEA models of the SMA, so that the SMA could be used to actuate real and more complex aerospace structures comprised of multiple nodes shell/solid elements, rather than limited only to beam elements.

A further example was the finite element (FE) work on SMA actuators by Solomou et al. (2014), which was based on the Lagoudas SMA constitutive model, and was implemented as a user element (UEL) subroutine in ABAQUS. The research focused mainly on a beam element of a single SMA wire with a fixed end and a free end boundary conditions, which was rather simpler than the previously described work by Alipour et al. (2015). On application of the UEL of the SMA wire to actuate a real structure, an adaptive strip made of solid elements, as shown in Fig. 2.58, was simulated. Even though the form of elements of the actuated structure was more complex (solid elements rather than beam elements), the adaptive strip itself was considered as a simple form of an actuated structure. Moreover, only the strip tip displacement was reported, without any images of the strip in the actuated state.
2.7 Finite Element Analysis of Morphing Structures Involving Shape Memory Alloys

![Diagram of an airship and its landing gear](image1)

**Figure 2.57.** (a) An airship and its landing gear consisted of SMA wires (Alipour et al., 2015, taken from previously published paper, Dayananda et al., 2007), and (b) previously simplified experimental model (left) and the author simplified FE model (right) of the actuated landing gear.

![Diagram of an adaptive strip](image2)

**Figure 2.58.** An adaptive strip actuated by SMA wires (Solomou et al., 2014).

Tabesh et al. (2012) developed shape memory alloy (SMA) actuators, in the forms of SMA wires and strips. They implemented the Lagoudas SMA constitutive model as a subroutine in ABAQUS. They investigated the effect of the latent heat on the performance of the SMA actuators. However, the research focused on the superelasticity or the pseudoelasticity properties of the SMA, where most of the presented results were on the pseudoelasticity properties. For the shape memory recovery, only a SMA strip under a constant stress of 100 MPa was studied. More importantly, no SMA-actuation of real structures was covered in the FEA simulations. This was another relevant example that showed the urgent need for further development of the SMA material model in the commercial FEA software, for applications to actual SMA-actuated structures.
Roh et al. (2006) simulated SMA wire and strip actuators based on the Lagoudas SMA constitutive model. The model was implemented into the ABAQUS finite element program through a user supplied subroutine, UMAT. A 3-D SMA model was developed using eight node brick elements, and two separate boundary conditions were applied, to treat the model as a wire and a cantilever beam. Even though the approach was slightly unusual, having rectangular cross section instead of circular cross section for a wire, the results showed reasonable stress-strain and strain-temperature behaviours, if compared to Lagoudas numerical results. The actuation of the cantilever elastic (aluminium) beam with a dimension of 80×10×2 mm by the SMA strip with a dimension of 80×10×1 mm, resulted in approximately 20 mm vertical tip deflection, which was about 25% of the beam length (80mm). This was corresponded to the SMA strip with an initial strain of 5%. The 0.5mm-thick SMA strip actuator was then applied to a variable-area fan nozzle (VAFN) with a length (L) of 150 mm, a shallow angle (Ø) of 30°, radius of (Ra) 100 mm and (Rb) 50 mm, and a thickness of 2 mm. A conical shell structure which resembled a section of the VAFN is depicted in Fig. 2.5. With initial SMA strains of 3% and 5%, tip deflections of about 30 mm and 38 mm were achieved, respectively. Once again, similar to the above-mentioned research on an adaptive strip (Solomou et al., 2014), only a graph of the tip deflections was presented, without any illustration of the structure in the deformed state. Another important outcome of the simulation study was that the deformed shape was not completely recovered at the end of the temperature cycle, because the recovery stress of the SMA strip did not reduce to zero.

Other approaches of simulating SMA-actuated structures in finite element analysis (FEA) program were by developing SMA models based on thermal expansion of the SMA, and by using an existing shape memory material model, which was only suitable for superelasticity/pseudoelasticity behaviour of the SMA, combined with appropriately applied boundary conditions. Turner et al. (2008) developed an FEA model of a SMA-actuated chevron, based on coefficient of thermal expansion of the SMA. The model was subjected to nonlinear static analyses in MSC.Nastran and ABAQUS. Only strain and temperature contours were reported, without any contours of the chevron tip deflection. However, a curve of tip deflection against temperature, showing a maximum tip deflection of approximately 1.5 mm, was presented. Barbarino et al. (2009) initially considered the Liang and Rogers SMA constitutive model for the development of a morphing wing trailing edge, but finally modelled the contraction of the SMA wires in MSC/Nastran with beam elements having fictitious negative thermal expansion coefficient. In experimental and numerical work of a lightweight
actuator structure, which consisted of a TiNi SMA honeycomb core and CFRP skins as shown in Fig. 2.60, Okabe et al. (2011) acknowledged the difficulties of modelling the shape memory effect (SME) of SMA with LS-DYNA. Consequently, they modelled the SMA sandwich structure with the existing superelastic material model for the SMA, which was MAT_030-MAT_SHAPE_MEMORY, with applied boundary conditions which were a completely fixed/constrained lower surface and a horizontal movement of the top skin of the sandwich structure. The recovery shear force of the honeycomb core was exploited to deform the structure. With an implicit solver and SMA maximum recovery strain of 3.4%, a tip deformation of 7.1 mm was obtained.

**Figure 2.59.** A finite element model of a conical shell structure with a SMA strip (Roh et al., 2006).

Another important constraint of the previously developed finite element model of SMA in the commercial FEA software was an implicit formulation (e.g. the actuated landing gear (Alipour et al., 2009), the variable-area fan nozzle (Roh et al., 2006) and the SMA sandwich structure (Okabe et al., 2011)). The UMAT researched by Alipour et al. (2015), which was based on the Brinson SMA constitutive model, was implemented as an implicit code in
2.7 Finite Element Analysis of Morphing Structures Involving Shape Memory Alloys

ABAQUS. The user element (UEL) subroutine developed by Solomou et al. (2014), even though not clearly stated, was also believed as implicitly implemented, because of significant numbers of matrix inversion in the formulation. The UMAT developed by Roh et al. (2006) for the VAFN was not clearly stated as implicitly or explicitly implemented, but it was believed as implicitly formulated based on the use of the Jacobian matrix and the loading histories. The SMA model which was based on coefficient of thermal expansion, by Turner et al. (2008) for the SMA-actuated chevron, was also believed as implicitly implemented because of the static analyses. The simulation of the SMA sandwich beam in LS-DYNA by Okabe et al. (2011) was clearly stated using an implicit solver.

It was important to note that even though various FEA studies had been conducted on the modelling the shape memory alloy (SMA) for application to SMA-actuated structures, the research should be continuously progressed because of several important reasons. One of the reasons was that there might be several limitations in the previous SMA model, such as it could only be applied on implicit simulations. Another reason was that some uncertainties in the FEA results of the SMA-actuated structures, such as the permanent deformation at the end of the cooling stage, could be verified/supported by current and future research using different SMA constitutive models. In addition, various previously investigated SMA-actuated structures had different boundary conditions, structural constrains, materials, loading types and directions,
SMA types and dimensions, and hence would be partially or completely different from the current research on the design of composite morphing wings.

2.7.2 Time Integration in Explicit LS-DYNA

For a brief overview on the explicit formulation used in LS-DYNA, which is extensively used in the current composite morphing wing research, the time integration used in the explicit formulation is covered in this subsection. In LS-DYNA, the equation of motion is solved by a time integration method. For a single degree of freedom (DOF) damped system with a linear behaviour, the equations of motion lead to a linear ordinary differential equation.

\[ m\ddot{u} + c\dot{u} + ku = p(t) \]  \hspace{1cm} (2.21)

where \( \ddot{u} \), \( \dot{u} \) and \( u \) are the acceleration, velocity and displacement, respectively. \( m \), \( c \), \( k \) and \( p(t) \) are the mass, damping coefficient, linear stiffness and external forces in the system, respectively. The displacement, \( u(t) \), for this linear ordinary differential equation can be solved analytically.

For non-linear problems, however, where the internal force varies as a nonlinear function of the displacement, the analytical solution is not available. The nonlinear ordinary differential equation is

\[ m\frac{d^2u}{dt^2} + c\frac{du}{dt} + f_{\text{int}}(u) = p(t) \]  \hspace{1cm} (2.22)

To solve this nonlinear ordinary differential equation, LS-DYNA uses the explicit central difference time integration method to integrate from acceleration to velocity, and then from velocity to displacement. The equations of motion (semi-discrete) at time \( n \) is

\[ Ma^n = P^n - F^n + H^n \]  \hspace{1cm} (2.23)

where \( M \), \( P^n \), \( F^n \) and \( H^n \) are the diagonal mass matrix, the external and body force loads, the stress divergence vector, and the hourglass resistance, respectively.

The central difference time integration is used, to advance to time \( n+1 \). Firstly the acceleration is calculated from the above equation.

\[ a^n = M^{-1}(P^n - F^n + H^n) \]  \hspace{1cm} (2.24)
Then, the obtained global nodal acceleration at time $n$ is used to calculate the global nodal velocity at time $(n + 1/2)$.

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = a\Delta t$$

$$v^{n+\frac{1}{2}} = v^{n-\frac{1}{2}} + a^n\Delta t^n$$ \hspace{1cm} (2.25)

Then, the resulted global nodal velocity at time $(n + 1/2)$ is used to calculate the global nodal displacement at time $(n + 1)$.

$$v = \frac{\Delta u}{\Delta t}$$

$$\Delta u = v\Delta t$$

$$u^{n+1} = u^n + v^{n+\frac{1}{2}}\Delta t^{n+\frac{1}{2}}$$ \hspace{1cm} (2.26)

where

$$\Delta t^{n+\frac{1}{2}} = \frac{\Delta t^n + \Delta t^{n+1}}{2}$$

The geometry of the finite element (FE) model is then updated by adding the solved displacement increments to the initial geometry.

$$x^{n+1} = x^0 + u^{n+1}$$ \hspace{1cm} (2.27)

For a Hughes-Liu beam, which is used in the development of the user defined material (UMAT) for the shape memory alloy (SMA) wire, the incremental displacement is used to calculate the incremental displacement gradient and the incremental strain.

$$G_{ij} = \frac{\partial \Delta u_i}{\partial y_j}$$ \hspace{1cm} (2.28)

$$\Delta \varepsilon_{ij} = \frac{1}{2}(G_{ij} + G_{ji})$$ \hspace{1cm} (2.29)

This incremental strain is passed to the UMAT as $\text{eps}(1)$ for the next time step. For the UMAT of the SMA, a total strain is used in the SMA constitutive model, instead of the incremental strain. Hence, the total strain is assigned to one of the history variables, to accumulate the incremental strain.
\[ \varepsilon_{n+1} = \varepsilon_n + \Delta \varepsilon_{ij} \] (2.30)

The SMA stress is then calculated based on the SMA total strain, and is assigned to the \( sig(1) \) variable.

For the Hughes-Liu beam element formulation (ELFORM = 1) in LS-DYNA, the time step changes with a change in the element length and the wave speed. The wave speed, which is a function of modulus of elasticity and density of the beam, changes during the phase transformation of the SMA because the modulus of elasticity changes. Therefore, the time step also changes. The time step and the wave speed are calculated using the following equations, respectively (LSTC, 2006).

\[
\Delta t_e = \frac{L}{c} 
\] (2.31)

\[
c = \sqrt{\frac{E}{\rho}} 
\] (2.32)

where \( L \), \( c \), \( E \), and \( \rho \) are the length, the wave speed, the modulus of elasticity and the density of the beam element, respectively.

### 2.8 Other Morphing Aerofoil Applications

The composite morphing aerofoil was selected as a structural unit to be extensively investigated and developed in this research because the morphing aerofoil could be used not only on aircraft structures, but also on other road vehicles such as high performance cars and racing cars, and also underwater vehicles, such as submarines. For examples, the morphing aerofoil could be used as a rear wing of a FORD GT high performance car (Fig. 2.61a), as front wings and a rear wing of a ‘F1 W09 EQ Power+’ Mercedes Formula One car (Fig. 2.61b), and as a rear control surface of submarines (Fig. 2.61c). The morphing aerofoil application on the high performance cars and Formula One cars was for increasing/controlling the downward force on the vehicles to increase the stability, while the application on the submarines was mainly for navigation.
2.8 Other Morphing Aerofoil Applications

(a) FORD GT rear wing (www.fordgt.com)

(b) Front and rear wings of the Mercedes Formula One car (www.mercedesamgf1.com)

(c) Tail control surface of a submarine (Kota and Hetrick, 2008).

Figure 2.61. Potential applications of morphing aerofoils.

2.9 Critical View and Research Plan

Previous research on morphing aerofoils/wings introduced too complex actuation mechanisms, either mechanically actuated (Gilbert, 1981, Woods et al., 2014) or smartly actuated (Coutu et al., 2009, Georges et al., 2009). The complex systems consisted of various internal mechanical parts, such as gearboxes and gears, which added unnecessary weight to the aerofoils/wings.
This should be avoided as the weight saving is one of the critical factors in building aerostructures.

Another critical issue that should be addressed from some of the previous research was that the design of the morphing wing did not take into account the structural design of the whole complete wing. For examples, Dong et al. (2008) used thick stainless steel plates as ribs of a composite morphing aerofoil, Bolinches et al. (2011) used only polystyrene foam for a 1m-span wing without any internal structure to carry the aerodynamic loads especially in the spanwise direction, and Almeida et al. (2015) used aluminium tubes as stringers to support the morphing rib. The morphing wings should be properly designed with thin-walled aluminium/composite ribs and spars with varied thickness from the wing root to the wing tip (thicker towards the wing root), to efficiently carry the aerodynamic loads (lift, drag and bending moments) acted on the wings, and to minimise the total weight of the wings, which was crucial in aeronautics applications.

To make the aerofoils more compliant or more easily deformable, the approaches included the use of hinges (Epps and Chopra, 2001, Pitt et al., 2001, Elzey et al., 2003, Pankonien et al., 2013, and Basaeri et al., 2014), the use of corrugated structures (Ruagjirakkit, 2013, Previtali et al., 2014, and Molinari et al., 2016), the use of cellular structures (Ramrkahyani, 2004, Iannucci, 2012, and Airoldi, 2012) and utilisation of boundary conditions (Ricci et al., 2006, Dong et al., 2008, Iannucci et al., 2009, Miller et al., 2010).

Out of several concepts of morphing aerofoils/wings, as listed in the previous Table 2.1 - 2.3, a corrugated structure was chosen as a potential internal structure for the current research on the composite morphing wing, by ruling out other possible morphing options. The hinged morphing concept was avoided in the current composite morphing wing research because of three main reasons. Firstly, the integrity of the structure was questionable. A failure of the SMA wires would cause the collapse of the whole aerofoil/wing structure as it was subjected to external aerodynamic loads. Secondly, this kind of hinged structure could be easily or more efficiently deformed by conventional motors because of a small actuation force requirement, and hence did not justify the use of SMA wires which produced a high actuation force. Thirdly, another set of SMA wires was required in the opposite direction/side of actuation, to return the structure back to its initial position. The multi-stable structure concept was ruled out because of limited mode of deformation (usually only two modes of deformations, which were the initial and final aerofoil configurations). The stable states in between the two extremes could not be easily achieved. They could possibly be achieved by utilising multiple multi-stable
structures/plates, but it would increase the system complexity and would hardly produce a smooth deformation of the morphing aerofoil/wing. The morphing concept with embedded actuators (e.g. SMA wires) were opted out as the structural stiffness of the morphing structure would reduce the performance (e.g. achievable stroke) of the actuators. The elimination of the possible morphing concepts narrowed down the selection to a corrugated structure and a cellular structure. The corrugated structure could represent a structural unit of the cellular structure, and therefore was selected to be focused on, in the research of the composite morphing wing.

After evaluating various mechanical and smart actuators, shape memory alloy (SMA) material was selected as the suitable actuator for the composite morphing wing, because of a high energy density, a large strain recovery and a high actuation force generated from the unique shape memory effect (SME) properties. Therefore, a deep review on the SMA thermo-mechanical properties and the SMA constitutive models was conducted and reported. Among the SMA constitutive models that had been reviewed, the Tanaka SMA constitutive model was chosen as a suitable SMA model for the development of the user defined material model in a commercial explicit finite element analysis software, LS-DYNA. The main reasons include:

i. The model was one of the earliest SMA constitutive model (pioneer) which covered the phase transformations from martensite to austenite and vice versa. There were other SMA models, which surfaced close to the time the Tanaka model was first introduced, such as by Perkins (1981) and Achenbach (1989), but they were either too general or were not widely established and laboriously/experimentally tested, if compared to the Tanaka model.

ii. Liang (1990), who developed the Liang and Rogers SMA constitutive model, in his thesis, acknowledged that the predicted results by the Tanaka SMA constitutive model were closer to the experimental results, if compared to his cosine model for the SMA. It should be noted again that the subsequent Brinson SMA constitutive model was an improved version of the Liang and Rogers SMA constitutive model.

iii. The Tanaka SMA constitutive model was used recently, such as in the development of a shape memory alloy (SMA) heat engine (Tobushi et al., 2010). As another example, Barbarino et al., (2009) considered the Liang and Rogers SMA constitutive model in the development of a morphing wing trailing edge, where the only difference between the Liang and Rogers model and the Tanaka model was the use of cosine functions and
exponential functions to describe the changes of the martensite volume fraction of the SMA, respectively.
Chapter 3
Experimental Tests: Characterisation of NiTi Shape Memory Alloy Wires

Experimental tests were conducted on Nickel-Titanium (NiTi) shape memory alloy (SMA) wires to measure their material and mechanical properties. The experimental tests were Differential Scanning Calorimetry (DSC) tests, tensile tests, and Dynamic Mechanical Thermal Analysis (DMTA) tests. The DSC tests were carried out to obtain the SMA transformation temperatures. The SMA transformation temperatures were crucial because the SMA stress-strain behaviour depended on the temperatures. The obtained SMA transformation temperatures were used as a guideline to heat the SMA wires above the austenite finishing temperature, before the tensile tests, to recover the SMA pre-strain, as the SMA wires were supplied in a trained state. A series of tensile tests was conducted at a room temperature to obtain the stress-strain behaviour of the SMA wires, from which the SMA elastic modulus in a martensite phase could be calculated. The DMTA tests were carried out at a room temperature to support the finding of elastic modulus from the tensile tests, as the obtained stress-strain behaviour of the SMA wires from the tensile tests did not exactly match the typical theoretical SMA stress-strain behaviour in the martensite phase. Other SMA material properties were obtained from manufacturer datasheet. The obtained elastic modulus in the martensite phase and the other material properties were required for the development of the user defined material model (UMAT) of the SMA wires. The NiTi SMA wires were supplied by a commercial SMA supplier, SAES (Societa Apparecchi Elettriche Scientifici – Electrical and Scientific Components Company).

3.1 Differential Scanning Calorimetry (DSC) Tests

A series of Differential Scanning Calorimetry (DSC) tests were conducted on the NiTi shape memory alloy (SMA) wires to obtain their phase transformation temperatures under no loading condition. The DSC tests were conducted using a Differential Scanning Calorimeter (DSC)
machine from TA Instrument (Fig. 3.1). The machine measured temperatures and heat flows associated with thermal transitions or phase transformations of the SMA material. The SMA wires had three different diameters, which were 0.1 mm, 0.2 mm and 0.5 mm. The SMA wires were cut to short pieces and were arranged in parallel in the pans, as shown in Fig. 3.2. The empty weight of the pans, and their weight with the SMA wires were measured using a scale with a 0.0001 g precision (Fig. 3.3a), to obtain the weight of the SMA wires. The pans filled with SMA wires were then enclosed by the lids and were pressed using a Tzero press (Fig. 3.3b) to ensure a good contact between the pan, lid and the SMA wires. Three specimens were prepared for each SMA diameter, to ensure the repeatability and reliability of the test results.

![Figure 3.1. A Differential Scanning Calorimeter (DSC) machine.](image)

![Figure 3.2. The prepared SMA wires with (a) 0.1 mm, (b) 0.2 mm and (c) 0.5 mm diameters.](image)

![Figure 3.3. (a) A weight scale and (b) a Tzero press.](image)

The specimens were placed on the DSC machine (TA Instrument). The temperature was increased from a room temperature to 150 °C, and then was decreased to -70 °C. The specific heat flow into the SMA wires during heating stage, and out of the SMA wires during cooling stage were observed and recorded. These tests were conducted to obtain the transformation temperatures of the SMA wires under no loads, as well as the specific heat absorption during
martensite-to-austenite phase transformation and specific heat dissipation during martensitic (austenite-to-martensite) phase transformation.

The results of the specific heat flow of the SMA wires with diameter of 0.1 mm, 0.2 mm and 0.5 mm are shown in Fig. 3.4a, Fig. 3.4b and Fig. 3.4c, respectively. The red curves were the specific heat flow into the SMA wires (endothermic) on heating, while the blue curves were the specific heat flow out of the SMA wires (exothermic) on cooling. Only one result was presented for each diameter of the SMA wires, and the other results were reported in Appendix A.

The results showed that only one peak existed on the heating stage, which indicated a phase transformation from a martensite phase to an austenite phase. The peak was highest in magnitude for the SMA wires with the smallest diameter (0.1 mm), and decreased as the SMA diameter increased to 0.2 mm and 0.5 mm. On the cooling stage, two peaks existed, which indicated two phase transformations. The crystal structure of the SMA wires first changed from an austenite phase to an intermediate Rhombohedral phase (R-phase), and then from the R-phase to the martensite phase. The magnitude of these two peaks were smallest for SMA wires with a diameter of 0.1 mm, and increased with the increase in the SMA wire diameter. The peaks, however, were almost similar for SMA wires with diameter of 0.2 mm and 0.5 mm.

The width of the phase transformation regions on the heating stage was smallest for the 0.1mm-diameter SMA wires, indicated a fast phase transformation, because of a small cross-sectional area of the SMA wires. However, on the cooling stage, the width was almost similar for all SMA diameters. Comparing the width of the phase transformation regions between the heating stage and the cooling stage, the width were narrow on the heating stage, but much wider on the cooling stage. This indicated that fast actuation could be achieved on heating, but it would take more time to cool down the SMA wires for the actuated structure to return back to the initial position.

From the plotted results, the transformation temperatures of the SMA wires, as well as their specific energy absorption and specific energy dissipation, were extracted. The transformation temperatures were obtained from intersection of the steepest slopes for each phase transformation, with the baseline curves, as depicted in Fig. 3.4. This step was repeated for all 3 tests of each SMA diameter, to ensure repeatability of the DSC tests as well as the procedure of obtaining the transformation temperatures. On the heating stage, the austenite starting and finishing temperatures ($A_s$ and $A_f$) were obtained. On the cooling stage, the R-phase
starting and finishing temperatures ($R_s$ and $R_f$), and the martensite starting and finishing temperatures ($M_s$ and $M_f$) were obtained. The phase transformation temperatures were summarised in Table 3.1, and illustrated in column chart in Fig. 3.5. A maximum tolerance/deviation of ±0.75 °C also indicated repeatability of the tests. From the charts, it was observed that the SMA wires with a smaller cross-section required smaller temperature changes to complete the martensite-to-austenite phase transformation on the heating stage. However, to complete the R-phase and the martensite phase transformations on the cooling stage, all of the SMA wires required approximately similar amount of reduction in temperatures, except for SMA wires with 0.1 mm diameter during the martensitic transformation. These transformation temperatures were expected to be higher when the SMA wires were subjected to applied loadings, e.g. under a constant stress or during actuation of a structure.

The two phases transformation during the cooling stage, from the austenite phase to the R-phase and then from the R-phase to the martensite phase, were typical for SMAs supplied by SMA suppliers. As a reference, Sofla et al. (2008) also reported similar trend of dual phase transformation from their DSC test on a NiTi SMA ribbon, which was supplied by Nitinol Devices and Components (Fremont, CA). It looks like the commercial NiTi SMAs which were already trained, would exhibit such dual phase transformations.
3.1 Differential Scanning Calorimetry (DSC Tests)

Figure 3.4. SMA transformation temperatures obtained from Differential Scanning Calorimetry (DSC) tests on NiTi shape memory alloy (SMA) wires with (a) 0.1 mm, (b) 0.2 mm and (c) 0.5 mm diameter.
Table 3.1. Transformation temperatures of NiTi SMA wires obtained from Differential Scanning Calorimetry (DSC) tests.

<table>
<thead>
<tr>
<th>Transformation Temperatures</th>
<th>SMA Diameter 0.1 mm</th>
<th>SMA Diameter 0.2 mm</th>
<th>SMA Diameter 0.5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test 1</td>
<td>Test 2</td>
<td>Test 3</td>
</tr>
<tr>
<td>Austenite starting temperature, $A_s$ (°C)</td>
<td>80.2</td>
<td>79.9</td>
<td>80.2</td>
</tr>
<tr>
<td>Austenite finishing temperature, $A_f$ (°C)</td>
<td>82.1</td>
<td>81.6</td>
<td>81.5</td>
</tr>
<tr>
<td>R-phase starting temperature, $R_s$ (°C)</td>
<td>73.0</td>
<td>73.0</td>
<td>73.2</td>
</tr>
<tr>
<td>R-phase finishing temperature, $R_f$ (°C)</td>
<td>47.6</td>
<td>47.5</td>
<td>47.0</td>
</tr>
<tr>
<td>Martensite starting temperature, $M_s$ (°C)</td>
<td>37.5</td>
<td>37.5</td>
<td>37.5</td>
</tr>
<tr>
<td>Martensite finishing temperature, $M_f$ (°C)</td>
<td>2.5</td>
<td>3.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Figure 3.5. Phase transformation temperatures of shape memory alloy (SMA) wires obtained from Differential Scanning Calorimetry (DSC) tests.
3.2 SMA Tensile Tests

3.2.1 Displacement-Controlled Tensile Tests on NiTi SMA Wires

A series of displacement-controlled tensile tests were conducted on SMA wires with a 0.1 mm diameter and three gauge length, which were 200 mm, 250 mm and 300 mm. The 0.1mm, 0.2mm and 0.5mm-diameter SMA wires are depicted in Fig. 3.6. A small cord and yarn grip (INSTRON part number: 2714-031), as shown in Fig. 3.7b, was used to firmly clamp both ends of the SMA wires. The gauge length at zero grip separation (Fig. 3.7b) of the small grip was 54 mm. Therefore, for the test gauge lengths of 200 mm, 250 mm and 300 mm, the vertical distances between the grips were set to 146 mm, 196 mm and 246 mm, respectively. The strain rate was kept constant at 2%/min (Tanaka et al., 1986, 1992), and hence the displacement rates of the INSTRON machine cross-head were set to 4 mm/min, 5 mm/min and 6 mm/min for the SMA gauge lengths of 200 mm, 250 mm and 300 mm, respectively.

![Figure 3.6](image)

*Figure 3.6. Shape memory alloy (SMA) wires with diameter of (a) 0.1 mm, (b) 0.2 mm and (c) 0.5 mm.*
The experimental setup of the SMA tensile tests is depicted in Fig. 3.7. The tensile loads on the SMA wires were measured by a load cell on the cross-head of the INSTRON machine, and the SMA strains were measured by an IMETRUM optical system. The SMA strains were measured by measuring the relative displacement of two points, which were marked on the SMA wires, and divided the change in the displacement with the initial distance of the two defined points. Five specimens or SMA wires were tested for each gauge length.

Figure 3.7. (a) Experimental setup of the tensile tests on shape memory alloy (SMA) wires, (b) a small cord and yarn grip (INSTRON 2714-031) at a zero gap position, and (c) a clamped 0.2mm-diameter SMA wire on the large cord and yarn grip (INSTRON 2714-040).
3.2 SMA Tensile Tests

The resulted stress-strain behaviour of the 0.1mm-diameter Nickel-Titanium (NiTi) shape memory alloy (SMA) wires are illustrated in Fig. 3.8. From observation of the stress-strain relationship, the NiTi SMA wires had a short linear-elastic regime, followed by a region of gradual increase in the SMA stress (instead of an almost constant plateau region), and then a second elastic region, before reduction of stiffness and ended with failure.

![Graph](image)

**Figure 3.8.** Tensile stress-strain behaviour of NiTi SMA wires with a diameter of 0.1 mm and gauge lengths of (a) 250 mm and (b) 300 mm.

The tensile tests on the SMA wires with a gauge length of 200 mm resulted in inaccurate tensile stress-strain curves, which had too large failure tensile strains (120% - 146%), and hence were not included in the reported results. For the SMA wire gauge length of 250 mm and 300 mm, the average ultimate tensile stress and the average failure strain, were found as approximately 1564 MPa and 14.1%, and 1526 MPa and 13.5%, respectively.
From the SMA tensile stress-strain curves, the SMA Young’s modulus was calculated from the slope of the first elastic region, using the following equation

\[ E = \frac{\Delta \sigma}{\Delta \varepsilon} \]

where \( E \), \( \Delta \sigma \), and \( \Delta \varepsilon \), are the modulus of elasticity, the difference in the tensile stress between the two selected strain points, and the difference between the two selected strain points, respectively. For the 250 mm and 300 mm gauge length, the average Young’s modulus was found as 31.9 GPa and 34.1 GPa. They are listed in Table 3.2, with the maximum tensile load, ultimate tensile strength and the failure tensile strain.

Table 3.2. SMA material properties obtained from tensile tests on SMA wires with a diameter of 0.1 mm, and gauge length of 250 mm and 300 mm.

<table>
<thead>
<tr>
<th>SMA gauge length (mm)</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMA wire specimens</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>31.9</td>
<td>34.1</td>
</tr>
<tr>
<td>Maximum tensile load (N)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>12.2</td>
<td>12.0</td>
</tr>
<tr>
<td>Ultimate tensile strength (MPa)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>1564</td>
<td>1526</td>
</tr>
<tr>
<td>Failure tensile strain (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>14.1</td>
<td>13.5</td>
</tr>
</tbody>
</table>

With similar experimental setup and procedures, the displacement-controlled tensile tests were repeated on SMA wires with a diameter of 0.2 mm diameter and a gauge length of 250 mm. Five SMA wires were tested, and the valid results of the SMA tensile stress plotted against the SMA tensile strain, are shown in Fig. 3.9. The average results of the Young’s modulus, the maximum tensile load, the ultimate tensile strength, and the failure tensile strain were found as approximately 30.1 GPa, 44.2 N, 1451 MPa, and 11.2%, respectively. They are listed in Table 3.3.
3.2 SMA Tensile Tests

Figure 3.9. Tensile stress-strain behaviour of NiTi SMA wires with a diameter of 0.2 mm and a gauge length of 250 mm.

Table 3.3. SMA material properties obtained from tensile tests on SMA wires with a diameter of 0.2 mm, and a gauge length of 250 mm.

<table>
<thead>
<tr>
<th>SMA gauge length (mm)</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMA wire specimens</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>29.1</td>
</tr>
<tr>
<td></td>
<td>27.5</td>
</tr>
<tr>
<td></td>
<td>36.4</td>
</tr>
<tr>
<td></td>
<td>22.3</td>
</tr>
<tr>
<td></td>
<td>35.0</td>
</tr>
<tr>
<td>Average</td>
<td>30.1</td>
</tr>
<tr>
<td>Maximum tensile load (N)</td>
<td>44.1</td>
</tr>
<tr>
<td></td>
<td>44.2</td>
</tr>
<tr>
<td></td>
<td>44.4</td>
</tr>
<tr>
<td></td>
<td>44.0</td>
</tr>
<tr>
<td></td>
<td>44.3</td>
</tr>
<tr>
<td>Average</td>
<td>44.2</td>
</tr>
<tr>
<td>Ultimate tensile strength (MPa)</td>
<td>1446</td>
</tr>
<tr>
<td></td>
<td>1451</td>
</tr>
<tr>
<td></td>
<td>1457</td>
</tr>
<tr>
<td></td>
<td>1444</td>
</tr>
<tr>
<td></td>
<td>1455</td>
</tr>
<tr>
<td>Average</td>
<td>1451</td>
</tr>
<tr>
<td>Failure tensile strain (%)</td>
<td>11.6</td>
</tr>
<tr>
<td></td>
<td>11.5</td>
</tr>
<tr>
<td></td>
<td>11.2</td>
</tr>
<tr>
<td></td>
<td>10.3</td>
</tr>
<tr>
<td></td>
<td>11.6</td>
</tr>
<tr>
<td>Average</td>
<td>11.2</td>
</tr>
</tbody>
</table>

The displacement-controlled tensile tests were finally repeated on SMA wires with a 0.5 mm diameter, with similar experimental setup and procedures. Like the 0.2 mm-diameter SMA wires, only two gauge lengths were studied, which were 200 mm and 250 mm. The SMA tensile stresses were plotted against the SMA tensile strains, as depicted in Fig. 3.10. For the 0.5 mm-diameter SMA wires with a gauge length of 200 mm, the average Young’s modulus, maximum tensile load, ultimate tensile strength, and failure tensile strain were found as 37.6 GPa, 269.2 N, 1457 MPa, and 10.7%, respectively. For the SMA wires with a gauge length of 250 mm, they were found as 36.9 GPa, 268.5 N, 1453 MPa, and 11.1%, respectively. They are tabulated in Table 3.4.
3.2 SMA Tensile Tests

(a) SMA wire diameter 0.5 mm, gauge length 200 mm.

(b) SMA wire diameter 0.5 mm, gauge length 250 mm.

Figure 3.10. Tensile stress-strain behaviour of NiTi SMA wires with diameter of 0.5 mm and gauge length of (a) 200 mm and (b) 250 mm.

Table 3.4. SMA material properties obtained from tensile tests on SMA wires with a diameter of 0.5 mm, and gauge length of 200 mm and 250 mm.

<table>
<thead>
<tr>
<th>SMA gauge length (mm)</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMA wire specimens</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>40.0</td>
<td>35.3</td>
</tr>
<tr>
<td>Average</td>
<td>37.6</td>
<td></td>
</tr>
<tr>
<td>Maximum tensile load (N)</td>
<td>270.2</td>
<td>268.3</td>
</tr>
<tr>
<td>Average</td>
<td>269.2</td>
<td></td>
</tr>
<tr>
<td>Ultimate tensile strength (MPa)</td>
<td>1463</td>
<td>1452</td>
</tr>
<tr>
<td>Average</td>
<td>1457</td>
<td></td>
</tr>
<tr>
<td>Failure tensile strain (%)</td>
<td>10.6</td>
<td>10.9</td>
</tr>
<tr>
<td>Average</td>
<td>10.7</td>
<td></td>
</tr>
</tbody>
</table>

In short, the displacement-controlled tensile tests on the NiTi SMA wires with several diameters and gauge lengths, resulted in a general stress-strain behaviour, consisted of a short
linear elastic region, followed by a region of gradually increasing stress, and another linear region, before reduction in stiffness and tensile failure. Even though the stress-strain behaviour did not exactly match the theoretical behaviour which should have a constant plateau region after the first elastic region, the obtained Young’s modulus, especially for the 0.5mm-diameter SMA wires, was almost close to the supplied data from the manufacturer datasheet, which was 40 GPa.

3.2.2 Load-Controlled Tensile Tests on NiTi SMA Wires

A series of load-controlled tensile tests were conducted on NiTi SMA wires with a diameter of 0.1 mm and a gauge length of 200 mm. The loading rates were 0.5 N/min, 5 N/min, and 10 N/min. A small cord and yard grip (INSTRON part number: 2714-031), as shown in Fig. 3.7b, was used for the end fixture of the SMA wires. The SMA wires were extended until the tensile load reached 10 N, and then the tensile load was decreased to zero Newton under the same loading rate. The resulted SMA tensile load as a function of the INSTRON cross-head displacement, and the SMA tensile stress as a function of the SMA tensile strain, for several loading rates, are plotted in Fig. 3.11a and Fig. 3.11b, respectively. The curves in green, red and black colours were the results for loading rates of 0.5 N/min, 5 N/min and 10 N/min, respectively. The tensile load/force and the displacement were obtained from the load cell and the cross-head displacement of the INSTRON machine, respectively. The SMA tensile stress was obtained by dividing the tensile force with the cross-sectional area of the SMA wire, while the SMA tensile strain was measured between two points on the tested SMA wire, by the contactless IMETRUM optical system.

At the maximum tensile load of 10 N, the maximum SMA tensile stress was about 1280 MPa, which almost reached the ultimate tensile stress, while the maximum SMA tensile strain was approximately 5.8%. On complete unloading, the recoverable strain was about 3.1%. Calculation of the elastic modulus of the NiTi SMA wires, within the range of 0.001 strain to 0.003 strain, resulted in 37.5 GPa – 41.6 GPa. The results of the experimental tensile tests showed that the load-displacement and the tensile stress-strain behaviour of the NiTi SMA wires at all loading rates (0.5 N/min, 5 N/min and 10 N/min), almost coincided with each other. Hence, it could be concluded that the stress-strain characteristic of the NiTi SMA wires was almost not affected, or was only slightly affected by the loading rates. The captured images of the NiTi SMA wire subjected to a tensile load, at a loading rate of 10 N/min are shown in Fig. 3.12. The left, centre and right images show the NiTi SMA wire at the beginning of the
experiment, at the maximum applied load of 10 N, and at the final unloading position, respectively. For the other two loading rates (0.5 N/min and 5 N/min), the images of the NiTi SMA wires are presented in Appendix A.

Figure 3.11. (a) Load-displacement and (b) stress-strain results of load-controlled tensile tests on NiTi SMA wires, with a diameter of 0.1 mm, a gauge length of 200 mm, and a maximum applied tensile load of 10N.
3.2 SMA Tensile Tests

Figure 3.12. Captured images of a NiTi SMA wire with a diameter of 0.1 mm and a gauge length of 200 mm, subjected to a tensile load at a loading rate of 10 N/min (left: at the initial position, middle: at the maximum load of 10N, and right: at the final unloading position).

The load-controlled tensile tests were repeated on the NiTi SMA wires with a diameter of 0.2 mm and a gauge length of 300 mm. The loading rates were 1 N/min, 5 N/min, and 10 N/min. A large cord and yard grip (INSTRON part number: 2714-040), as shown in the
previous Fig. 3.7c, was used for the end fixture of the NiTi SMA wires. The SMA wires were extended until the tensile load reached 30 N, and then the tensile load was decreased to zero Newton under a similar loading rate. Prior to this series of tests, tensile tests were conducted on 0.2mm-diameter and 200mm-gauge length NiTi SMA wires, but the obtained SMA stress-strain results were slightly inaccurate, with too large strain measurement (about 26% at a maximum load of 20 N). Hence, for the SMA wires with a diameter of 0.2 mm, the gauge length was increased or changed to 300 mm, and external strips with bullet circular marks were attached to the SMA wires as shown in Fig. 3.1. The figure showed the captured images of the NiTi SMA wire under a loading rate of 10 N/min, at the beginning of the experiment (left), at the maximum applied load of 30 N (centre), and at the final unloading position (right). The images of the NiTi SMA wires under the other two loading rates (1 N/min and 5 N/min), were presented in Appendix A. The captured images showed that both top and bottom marks (on the wire and on the circular dots) moved up when the machine upper crosshead moved up, and both marks moved down when the machine upper crosshead moved down. This suggested that the strain of the SMA wire must be measured by the IMETRUM optical strain system, and should not be simply calculated from the change in the crosshead displacement to the initial gauge length of the SMA wire.

The results of the SMA tensile load as a function of the crosshead displacement, for the NiTi SMA wires with a diameter of 0.2 mm and a gauge length of 300 mm, are shown in Fig. 3.14a. The SMA tensile stress-strain results which were obtained from the strain measurement of the dotted points on the SMA wires are shown in Fig. 3.14b. The SMA tensile stress-strain results which were obtained from the strain measurement of the bullet circular marks/tabs attached on the SMA wires are shown in Fig. 3.14c. At the maximum tensile force of 30 N, the maximum SMA tensile stress was about 960 MPa, while the maximum SMA tensile strains were approximately 4.4-4.8% and 4.0-4.15%, when the strains were measured between two dotted points on the SMA wires, and between two bullet circular marks/tabs attached to the SMA wires, respectively. On complete unloading, the recoverable strain was about 2.2-2.4%, and 1.95-2.0%, respectively. Both of the maximum SMA strains and recoverable SMA strains were slightly lower when the strains were measured based on the movement of the bullet circular tabs, if compared to when the strains were measured based on the movement of the dotted points on the SMA wires. The calculated Young’s modulus was therefore slightly higher for the SMA wires with the attached circular tabs (26.5-30.7 GPa), compared to the SMA wires...
with the dotted points on the SMA wires (24.0-27.3 GPa). In general, it could be said that the NiTi SMA behaviour was slightly affected by the loading rates.

**Figure 3.13.** Captured images of a NiTi SMA wire with a diameter of 0.2 mm and a gauge length of 300 mm, subjected to a tensile load at a loading rate of 10 N/min (left: at the initial position, middle: at the maximum load of 30N, and right: at the final unloading position).
3.2 SMA Tensile Tests

Figure 3.14. (a) Load-displacement and (b) stress-strain results based on strain measurement of dotted points on the SMA wires and (c) stress-strain results based on strain measurement of circular tabs, of load-controlled tensile tests on NiTi SMA wires with a diameter of 0.2 mm, a gauge length of 300 mm, and a maximum applied tensile load of 30N.
3.2 SMA Tensile Tests

The load-controlled tensile tests were then repeated on 0.5mm-diameter, 200mm-length NiTi SMA wires. Loading rates of 5 N/min and 20 N/min were used. Smaller loading rates were not possible, because the maximum applied load was 200 N, and it would take more than 2 hours and 15 minutes (the maximum limit of the operating time of the IMETRUM optical system) to complete a single test with one loading-unloading cycle. Similar to the previous 0.2mm-diameter SMA wires, a large cord and yard grip (INSTRON part number: 2714-040) was used for both end fixtures of the SMA wires, because the small cord and yarn grips (2714-031) had a maximum load capability of only 50N (INSTRON, 2013). For each loading rate, the SMA wires were extended until the tensile load reached 200 N, and were then unloaded to zero load under the same rate.

![Fig. 3.15](image)

**Figure 3.15.** Captured images of a NiTi SMA wire with a diameter of 0.5 mm and a gauge length of 200 mm, subjected to a tensile load at a loading rate of 20 N/min (left: at the initial position, middle: at the maximum load of 200N, and right: at the final unloading position).

The captured images of the 0.5mm-diameter NiTi SMA wire subjected to a tensile load, at a loading rate of 20N/min are shown in Fig. 3.15. The left, centre and right images showed the NiTi SMA wire at the beginning of the experiment, at the maximum applied load of 200 N, and at the final unloading position, respectively. The images of the NiTi SMA wires under 5
3.2 SMA Tensile Tests

N/min loading rate were presented in Appendix A. Fig. 3.16a shows the SMA tensile load as a function of the crosshead displacement of the INSTRON machine, and Fig. 3.16b shows the SMA tensile stress as a function of the SMA tensile strain. The maximum SMA tensile stresses were about 1020 MPa at the maximum tensile load of 200 N. At the maximum applied load, the maximum SMA tensile strains were approximately 4.3% and 3.8% for the loading rates of 5 N/min and 20 N/min, respectively. On complete unloading, the SMA recoverable strain were about 1.85% and 1.4%, respectively. The elastic modulus was calculated from 0.001 strain to 0.003 strain, and was found as 32.9 GPa – 34.0 GPa.

The results of the load-controlled tensile tests on the NiTi SMA wires with diameters of 0.1 mm, 0.2 mm, and 0.5 mm, gauge lengths of 200 mm and 300 mm, loading rates of 0.5 N/min, 1 N/min, 5 N/min, 10 N/min, and 20 N/min, and two cord and yarn grips are
summarised in Table 3.2. The Young’s modulus of the 0.1mm-diameter, 200mm-gauge length NiTi SMA wires were the closest to the supplied material properties data (40 GPa), which was provided by the manufacturer datasheet (SAES Getters, 2008). None of the SMA stress-strain results matched the theoretical stress-strain, which usually consisted of a linear elastic region followed by a plateau region up to about 8% strain, and then another linear region before failure. The SMA stress-strain behaviour provided by the supplier is illustrated in Fig. 3.17 (SAES Getters, 2008), which shows a typical theoretical SMA stress-strain behaviour. From further discussion with the SAES technical specialist, the typical theoretical SMA stress-strain behaviour might be obtained if the ingot shape memory alloy material, or the untrained SMA, was tested. Therefore, from the series of load-controlled tensile tests on the SMA wires, only the modulus of elasticity of the SMA in the martensite phase could be extracted, which was concluded as 40 GPa, to be used in the next stage of finite element modelling of the user defined material (UMAT) model of the SMA wires.

Table 3.5. Summary of load-controlled tensile tests on NiTi SMA wires.

<table>
<thead>
<tr>
<th>SMA Diameter (mm)</th>
<th>Cord and Yarn Grips</th>
<th>Loading Rate (N/min)</th>
<th>Maximum Tensile Load (N)</th>
<th>Maximum Tensile Stress (MPa)</th>
<th>Maximum Tensile Strain (%)</th>
<th>Recoverable Strain (%)</th>
<th>Elastic Modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>Small (2714-031)</td>
<td>0.5 10.0</td>
<td>10</td>
<td>1280</td>
<td>5.80</td>
<td>3.10</td>
<td>37.5 – 41.6</td>
</tr>
<tr>
<td>0.2</td>
<td>Large (2714-040)</td>
<td>1.0 5.0 10.0</td>
<td>30</td>
<td>960</td>
<td>4.80</td>
<td>2.40</td>
<td>24.0 – 27.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.0 20.0</td>
<td>200</td>
<td>1020</td>
<td>4.25</td>
<td>1.85</td>
<td>32.9 – 34.0</td>
</tr>
</tbody>
</table>

Figure 3.17. SMA stress-strain behaviour from the supplier datasheet (SAES Getters, 2008).
3.3 Dynamic Mechanical Thermal Analysis (DMTA) Tests

A series of Dynamic Mechanical Thermal Analysis (DMTA) tests were carried out on the NiTi shape memory alloy (SMA) wires to support the mechanical properties obtained from the previous displacement-controlled and load-controlled tensile tests. The RSA-G2 machine from TA Instruments, with the specifications listed in Table 3.3 (TA Instruments, 2014), was used for the testing. The NiTi SMA wires with a diameter of 0.2 mm and 0.5 mm, and a gauge length of 20 mm were tested. The SMA wires were clamped with a constant torque of 90 cN.m applied on all bolts. The NiTi SMA wires which were fixed onto the tension clamp on the RSA-G2 machine are illustrated in Fig. 3.18. Two types of tensile tests were carried out, which were frequency sweep and amplitude sweep tensile tests.

Table 3.6. RSA-G2 specifications (TA Instruments, 2014)

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Force</td>
<td>0.0005 N</td>
</tr>
<tr>
<td>Maximum Force</td>
<td>35 N</td>
</tr>
<tr>
<td>Force Resolution</td>
<td>0.00001 N</td>
</tr>
<tr>
<td>Dynamic Displacement Range</td>
<td>±0.00005 to ± 1.5 mm</td>
</tr>
<tr>
<td>Displacement Resolution</td>
<td>1 Nanometer</td>
</tr>
<tr>
<td>Modulus Range</td>
<td>$10^3$ to $3 \times 10^{12}$ N</td>
</tr>
<tr>
<td>Modulus Precision</td>
<td>± 1%</td>
</tr>
<tr>
<td>Tan δ Sensitivity</td>
<td>0.0001</td>
</tr>
<tr>
<td>Tan δ Resolution</td>
<td>0.00001</td>
</tr>
<tr>
<td>Frequency Range</td>
<td>$2 \times 10^{-5}$ to 100 Hz</td>
</tr>
<tr>
<td>Temperature Control</td>
<td>Forced Convection Oven</td>
</tr>
<tr>
<td>Temperature Range</td>
<td>-150 to 600 °C</td>
</tr>
<tr>
<td>Heating Rate</td>
<td>0.1 to 60 °C/min</td>
</tr>
<tr>
<td>Cooling Rate</td>
<td>0.1 to 60 °C/min</td>
</tr>
<tr>
<td>Isothermal Stability</td>
<td>± 0.1 °C</td>
</tr>
</tbody>
</table>

Figure 3.18. NiTi SMA wires with a diameter of (a) 0.2 mm and (b) 0.5 mm, clamped on the tension fixture of the RSA-G2 machine.
3.3 Dynamic Mechanical Thermal Analysis (DMTA) Tests

3.3.1 Tensile Tests - Frequency Sweep

The frequency sweep tensile tests were carried out on the NiTi SMA wires, by applying a constant strain of 0.02% within the elastic region of the SMA stress-strain behaviour (sinusoidal wave of 0.02% strain, repetitively), and increasing the frequency from 0.01 Hz to 10 Hz. The applied strain with an increasing frequency is illustrated in Fig. 3.19. Ten measurements were recorded between 0.01 Hz and 0.1 Hz, followed by another 10 measurements between 0.1 Hz and 1 Hz, and final 10 measurements between 1 Hz and 10 Hz. The results of the DMTA frequency sweep tensile tests on the 0.2mm-diameter and 0.5mm-diameter NiTi SMA wires are shown in Fig. 3.20a and Fig. 3.20b, respectively.

![Figure 3.19. Frequency sweep of the DMTA tensile tests.](image)

![Figure 3.20. Results of DMTA frequency sweep tensile tests on NiTi SMA wires with a diameter of (a) 0.2 mm and (b) 0.5 mm.](image)
The results showed an almost constant storage modulus of approximately 35.5 GPa and 36.0 GPa, in average, for the NiTi SMA wires with a diameter of 0.2 mm and 0.5 mm respectively. The loss modulus were almost zero in both cases. Therefore, the average elastic modulus were 35.5 GPa and 36.0 GPa, respectively.

### 3.3.2 Tensile Tests – Amplitude Sweep

The tensile tests were repeated on the NiTi SMA wires, but with a constant frequency of 1 Hz and an increasing oscillation strain between 0.0005% strain and 0.5% strain. The increasing applied strain with a constant frequency is illustrated in Fig. 3.21. The results of the DMTA amplitude sweep tensile tests on the NiTi SMA wires with a diameter of 0.2 mm and 0.5 mm, are shown in Fig. 3.22a and Fig. 3.22b, respectively.

![Amplitude/strain sweep of the DMTA tensile tests.](image)

The results showed that at low strain (amplitude), the storage modulus remained almost constant at an average of 37.1 GPa and 36.6 GPa at oscillation strains below 0.02%, for the NiTi SMA wires with a diameter of 0.2 mm and 0.5 mm, respectively. For the 0.2mm-diameter SMA wire, the maximum storage modulus was about 39.5 GPa, and after 0.02% strain, it started to drop to the lowest of 25.6 GPa. For the 0.5mm-diameter SMA wire, the maximum storage modulus was about 38.2 GPa, and after 0.02% strain, it started to drop to the lowest of 30.9 GPa.

The trend of the results, where the storage modulus was constant at the beginning and then decreasing after a certain frequency, was a common trend because the strain sweep tests were actually meant to identify the linear viscoelastic region (LVR) of a material. However, the results of the constant storage modulus within the elastic region could be used for comparison with the results obtained from the previous frequency sweep tests.
3.3 Dynamic Mechanical Thermal Analysis (DMTA) Tests

Figure 3.22. Results of DMTA amplitude sweep tensile tests on NiTi SMA wires with a diameter of (a) 0.2 mm and (b) 0.5 mm.
3.4 Chapter Summary and Conclusions

In summary, this Chapter 4 of the experimental tests on the NiTi shape memory alloy (SMA) wires, reported the SMA phase transformation temperatures under no loading condition, and the elastic modulus of the SMA wires in the martensite phase. They are summarised in Table 3.7. The experimental SMA elastic modulus was slightly lower than the provided data from the supplier datasheet, except for SMA wires with a 0.1 mm diameter. Regardless of the differences, an elastic modulus of 40 GPa for the SMA wire in the martensite phase, which was provided by the supplier and was within the range of one of the tests, were used with the other provided SMA material properties, to develop a user defined material model (UMAT) of the SMA wire, which would be reported in the next Chapter 4.

<table>
<thead>
<tr>
<th>Experimental Tests</th>
<th>SMA Diameter (mm)</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSC Tests</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Austenite starting temperature, $A_s$ (°C)</td>
<td>80.05 ± 0.15</td>
<td>76.65 ± 0.15</td>
<td>79.40 ± 0.10</td>
<td></td>
</tr>
<tr>
<td>Austenite finishing temperature, $A_f$ (°C)</td>
<td>81.80 ± 0.30</td>
<td>79.40 ± 0.10</td>
<td>83.25 ± 0.25</td>
<td></td>
</tr>
<tr>
<td>R-phase starting temperature, $R_s$ (°C)</td>
<td>73.10 ± 0.10</td>
<td>73.50 ± 0.50</td>
<td>74.00</td>
<td></td>
</tr>
<tr>
<td>R-phase finishing temperature, $R_f$ (°C)</td>
<td>47.30 ± 0.30</td>
<td>47.25 ± 0.75</td>
<td>48.25 ± 0.75</td>
<td></td>
</tr>
<tr>
<td>Martensite starting temperature, $M_s$ (°C)</td>
<td>37.50</td>
<td>40.65 ± 0.15</td>
<td>39.15 ± 0.35</td>
<td></td>
</tr>
<tr>
<td>Martensite finishing temperature, $M_f$ (°C)</td>
<td>2.25 ± 0.75</td>
<td>20.40 ± 0.60</td>
<td>19.15 ± 0.35</td>
<td></td>
</tr>
<tr>
<td>SMA Tensile Tests</td>
<td>Young’s Modulus (GPa)</td>
<td>24.4 - 42.7</td>
<td>26.5 - 30.7</td>
<td>28.9 - 40.0</td>
</tr>
<tr>
<td></td>
<td>(Displacement-controlled tensile tests)</td>
<td>37.5 - 41.6</td>
<td>26.5 - 30.7</td>
<td>32.9 - 34.0</td>
</tr>
<tr>
<td></td>
<td>(Load-controlled tensile tests)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DMTA Tests</td>
<td>Young’s Modulus (GPa)</td>
<td>N/A</td>
<td>35.5</td>
<td>36.0</td>
</tr>
<tr>
<td></td>
<td>(Frequency sweep)</td>
<td>N/A</td>
<td>37.1</td>
<td>36.6</td>
</tr>
<tr>
<td></td>
<td>(Strain/amplitude sweep)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The main reason of the decision was lots of the expensive NiTi SMA wires were already wasted in the series of tensile tests, without any positive results in term of the SMA stress-strain behaviour, which should theoretically consisted of a linear elastic region, followed by a plateau region up to 8% strain, and ended with another linear elastic region (previous Fig. 3.18). It was deduced that if the right SMA stress-strain curve could not be obtained from the experimental tests at the low temperature (martensite phase), the SMA pseudoelastic stress-strain behaviour would not be obtained from another series of experimental tests at a high temperature (austenite phase).
The other material properties of the NiTi SMA wires were obtained from the manufacturer datasheet and the published work of the SAES manufacturer (Butera, 2008). The stress influenced coefficients at both martensite and austenite phase, for example, were taken as 8.2 MPa/°C. They were obtained from the slopes of the SMA stress vs temperature, which were plotted from transition temperature of the SMA wire under different loads, as depicted in Fig. 3.23. The martensite starting and finishing temperatures, and the austenite starting and finishing temperatures, were also obtained from the same plots, by projecting the transformation temperature lines further down to the temperature axis or zero stress. These material properties of the NiTi SMA wires, as listed in Table 4.1, would be used in the modelling and programming the user defined material (UMAT) model of the SMA wires in the next Chapter 4.

![Figure 3.23. Transition temperatures of SMA under different loads (Butera, 2008).](image-url)
Chapter 4

A User Defined Material Model of Shape Memory Alloy (SMA) Wires

To realise the potential of shape memory alloy (SMA) wires to actuate composite morphing aerofoils/wings, one of the earliest SMA constitutive model, the Tanaka model, was initially implemented in MATLAB, and then in the explicit LS-DYNA code as a User Defined Material (UMAT). The UMAT can be used as a design tool to predict the deformation of the SMA-actuated composite morphing aerofoils/wings. This chapter consists of a general description of a SMA material model in the LS-DYNA framework; a more detailed description on the development of the UMAT of the SMA; an extended version of the UMAT for multiple SMA wires. Validations of the UMAT, a mesh sensitivity study and a key parameters study are also presented.

4.1 SMA Material Model

The SMA wire was modelled using beam elements, having a circular cross section and one integration point, mimicking truss elements. The beam elements were used instead of a truss elements because the number of beam integration points must be defined in a “Database_Extend_Binary” keyword file, otherwise results of history variables would not be available in the output file. These integration points could not be assigned to truss (or cable) elements that used a resultant formulation. The circular cross section was assigned to the beam elements, which was appropriate for a wire, by choosing a tubular cross section type (CST = 1) in Section_Beam, with zero inner diameter and 0.5 mm outer diameter.

4.2 UMAT Structure

The user defined material (UMAT) model of the shape memory alloy (SMA) wire was developed for the main purpose of actuating composite morphing structures (e.g. morphing aerofoils and a morphing wing). The structure of the UMAT consisted of equations and
formulations which were developed for one complete heating-cooling cycle, based on Tanaka
SMA constitutive model. The first part of this section describes the developed mathematical
equations. The second part of this section shows how the mathematical equations were
implemented in LS-DYNA through a FORTRAN code, by using history variables, \( h_{sv}() \). The
third part of this section shows an improved FORTRAN code which was further developed for
multiple SMA wires with many elements, to significantly save computational time.

4.2.1 Tanaka SMA Constitutive Model

The Tanaka SMA constitutive model was initially studied and analytically developed
specifically for a simple fundamental structure, consisted of a SMA wire connected to a linear
spring in series, as shown in Fig. 4.1. This structure was chosen with a vision that the stiffness
of the linear spring could be replaced with a stiffness of the actual geometries of morphing
structures, in the direction of actuation.

![Figure 4.1. A SMA wire - linear spring structure.](image)

The SMA constitutive equation (Tanaka et al 1986, Tanaka et al 1992) is

\[
\sigma - \sigma_0 = E(\xi)(\varepsilon - \varepsilon_0) + \Theta(T - T_0) + \Omega(\xi)(\xi - \xi_0)
\]  

where \(\sigma, \varepsilon, T, \) and \(\xi,\) are the stress, strain, temperature and martensite volume fraction of the
SMA, respectively. The subscript ‘0’ symbolises the initial state of each variable. The variables
\(E, \Theta,\) and \(\Omega,\) are the Young’s modulus, thermoelastic coefficient and phase transformation
coefficient, respectively.

In this research, thermal expansion was neglected which is a common assumption for the
SMA, because of much smaller strain, \(10^{-6}\) to \(10^{-5}\) (Turner et al., 2008), due to the thermal
expansion, if compared to the SMA recovery strain (\(10^{-2}\)). The Young’s modulus and phase
transformation coefficient are in functions of the martensite volume fraction, as stated in Eq.
4.2 and Eq. 4.3, respectively.

\[
E(\xi) = E_A - \xi(E_A - E_M)
\]  

\[
\Omega(\xi) = -\varepsilon_L E(\xi)
\]  

where \(E_A\) and \(E_M\) are the Young’s modulus of the SMA at an austenite phase and a martensite
phase, respectively, while \(\varepsilon_L\) is the SMA maximum recoverable strain. With the assumption of
negligible thermal expansion and simplified expression of the phase transformation coefficient, the SMA constitutive equation was therefore simplified to

\[ \sigma - \sigma_0 = E(\xi)(\varepsilon - \varepsilon_0) - \varepsilon_L E(\xi)(\xi - \xi_0) \quad (4.4) \]

The martensite volume fractions, \( \xi \), for a forward martensitic transformation and a reverse transformation are in exponential forms (Tanaka et al. 1986, Tanaka et al. 1992). They were obtained from transformation kinetics, which were derived from the 1\(^{\text{st}}\) law (the conservation of energy principle) and the 2\(^{\text{nd}}\) law of thermodynamics. For a forward martensitic transformation (cooling stage), the martensite volume fraction was defined as

\[ \xi_{A \rightarrow M} = 1 - e^{a_M(M_s-T)+b_M\sigma} \quad (4.5) \]

For a reverse transformation (heating stage), it was defined as

\[ \xi_{M \rightarrow A} = e^{a_A(A_s-T)+b_A\sigma} \quad (4.6) \]

where \( A_s \) and \( M_s \) are the austenite and martensite starting temperatures, respectively. The material constants, \( a_a, b_a, a_m \) and \( b_m \) are defined in Eqs. 4.7-4.8, where \( A_f \) and \( M_f \) are the austenite and martensite finishing temperatures, respectively. \( C_A \) and \( C_M \) are the stress influence coefficient at the austenite and martensite phases, respectively, which are obtained from the supplier material datasheet (Butera, 2008, SAES, 2009).

\[ a_A = \frac{\ln(0.01)}{A_s - A_f}, b_A = \frac{\ln(0.01)}{C_A(A_s - A_f)} = \frac{a_A}{C_A} \quad (4.7) \]
\[ a_M = \frac{\ln(0.01)}{M_s - M_f}, b_M = \frac{\ln(0.01)}{C_M(M_s - M_f)} = \frac{a_M}{C_M} \quad (4.8) \]

The simplified SMA constitutive equation (Eq. 4.4) was solved for the SMA stress and strain by implementing Newton-Raphson iteration method for one complete heating-cooling cycle. In this research, since the SMA wires were used as actuators to morph composite morphing structures, the UMAT was initially developed for one actuation cycle. A stress function (Eq. 4.9) was obtained by setting Eq. 4.4 to zero.

\[ f(\sigma) = \sigma - \sigma_0 - E(\xi)(\varepsilon - \varepsilon_0) + \varepsilon_L E(\xi)(\xi - \xi_0) \quad (4.9) \]

For the reverse austenitic transformation (heating stage), with zero initial stress and strain \( (\sigma_0 = 0, \varepsilon_0 = 0) \), and initial martensite volume fraction of one (fully martensite, \( \xi_0 = 1 \)), the stress function and its derivative with respect to stress are

\[ f(\sigma) = \sigma - E(\xi)\varepsilon + \varepsilon_L E(\xi)(\xi - 1) \quad (4.10) \]
The partial differentiation of both martensite volume fraction and Young’s modulus in Eq. 4.11 are given by Eq. 4.12 and Eq. 4.13, respectively. These were obtained by differentiating Eq. 4.6 and Eq. 4.2 with respect to stress, respectively.

\[
\frac{\partial \xi}{\partial \sigma} = b_Ae^{a_A(A_s-T)} + b_A = b_A \xi_{M \rightarrow A} \\
\frac{\partial E(\xi)}{\partial \sigma} = (E_M - E_A) \frac{\partial \xi}{\partial \sigma} = (E_M - E_A)b_M \xi_{M \rightarrow A}
\]  

For the forward martensitic transformation (cooling stage), with zero initial martensite volume fraction (fully austenite, \( \xi_0 = 0 \)), the stress function and its derivative are

\[
f(\sigma) = \sigma - \sigma_0 - E(\xi)(\varepsilon - \varepsilon_0) + \varepsilon_L E(\xi)\xi
\]

\[
f'(\sigma) = \frac{\partial f(\sigma)}{\partial \sigma} = 1 - (\varepsilon - \varepsilon_0) \frac{\partial E(\xi)}{\partial \sigma} + \varepsilon_L \left( E(\xi) \frac{\partial \xi}{\partial \sigma} + \xi \frac{\partial E(\xi)}{\partial \sigma} \right)
\]

Here, the partial differentiation of both martensite volume fraction and Young’s modulus in Eq. 4.15 are given by Eq. 4.16 and Eq. 4.17, respectively. These were obtained by differentiating Eq. 4.5 and Eq. 4.2 with respect to stress, respectively.

\[
\frac{\partial \xi}{\partial \sigma} = -b_Me^{a_M(M_s-T)} + b_M = b_M (\xi_{A \rightarrow M} - 1)
\]

\[
\frac{\partial E(\xi)}{\partial \sigma} = (E_M - E_A) \frac{\partial \xi}{\partial \sigma} = (E_M - E_A)b_M (\xi_{A \rightarrow M} - 1)
\]

These equations (Eq. 4.10 - 4.17) were used to calculate the stress developed in the SMA wire iteratively for each time step, through the following relation

\[
\sigma_{n+1} = \sigma_n - \frac{f(\sigma_n)}{f'(\sigma_n)}
\]

In the differentiation from Eq. 4.10 to Eq. 4.11, and from Eq. 4.14 to Eq. 4.15, the SMA strain, \( \varepsilon \) was considered as a constant, because it was obtained from LS-DYNA each time the SMA stress was solved. In a prior analytical solution (solved in MATLAB for a validation purpose) of the SMA wire connected to a linear spring, however, the SMA strain was in function of the SMA stress, and had to be taken into account in the differentiation.

From Fig. 4.1, the axial force acting on the linear spring was

\[
F = k_{spring}x_{spring} = k(-\Delta l_{SMA})
\]
where $F$, $k$, $x$ and $l$ are the axial force, spring stiffness, change in length in the linear spring, and change in length in the SMA wire, respectively. Hence the change in length in the SMA wire was

$$
\Delta l_{SMA} = -\frac{F}{k_{spring}} \tag{4.20}
$$

The SMA strain was therefore

$$
\varepsilon = \frac{\Delta l_{SMA}}{l_{SMA}} = -\frac{F}{k_{spring}l_{SMA}} = -\frac{\sigma A}{kl} \tag{4.21}
$$

and the differentiation of the strain with respect to stress was

$$
\frac{\partial \varepsilon}{\partial \sigma} = -\frac{A}{kl} \tag{4.22}
$$

where $A$, $k$ and $l$ are the SMA wire cross-sectional area, spring stiffness and SMA wire length, respectively. Therefore, in the analytical solution (MATLAB), the stress functions and their derivatives for the reverse transformation (heating stage) and the forward martensitic transformation (cooling stage), were given by Eq. 4.23 - 4.24 and Eq. 4.25 - 4.26, respectively. The partial differentiation of both martensite volume fraction ($\frac{\partial k}{\partial \sigma}$) and Young’s modulus ($\frac{\partial E}{\partial \sigma}$) in Eq. 4.24 were defined by the previous Eq. 4.12 and Eq. 4.13, respectively, while these in Eq. 4.26 were defined by the previous Eq. 4.16 and Eq. 4.17, respectively.

$$
f(\sigma) = \sigma + E(\xi)\frac{\sigma A}{kl} + \varepsilon_L E(\xi)(\xi - 1) \tag{4.23}
$$

$$
f'(\sigma) = \frac{\partial f(\sigma)}{\partial \sigma} = 1 + A \frac{E(\xi) + \sigma \frac{\partial E(\xi)}{\partial \sigma}}{kl} + \varepsilon_L \left\{E(\xi) \frac{\partial \xi}{\partial \sigma} + \frac{\partial E(\xi)}{\partial \sigma}(\xi - 1)\right\} \tag{4.24}
$$

$$
f(\sigma) = \sigma - \sigma_o + E(\xi)\left(\frac{\sigma A}{kl} + \varepsilon_o\right) + \varepsilon_L E(\xi)\xi \tag{4.25}
$$

$$
f'(\sigma) = \frac{\partial f(\sigma)}{\partial \sigma} = 1 + A \frac{E(\xi) + \sigma \frac{\partial E(\xi)}{\partial \sigma}}{kl} + \varepsilon_o \frac{\partial E(\xi)}{\partial \sigma} + \varepsilon_L \left\{E(\xi) \frac{\partial \xi}{\partial \sigma} + \xi \frac{\partial E(\xi)}{\partial \sigma}\right\} \tag{4.26}
$$

Besides the main differences of the stress functions and their derivatives, between the user defined material (UMAT) model (Eqs. 4.10 - 4.11 and Eqs. 4.14 - 4.15) and the analytical solution (Eqs. 4.23 - 4.26), other differences are the input of the spring stiffness, $k$ and the SMA wire length, $l$, which are not required in the UMAT code, but have to be defined in the MATLAB analytical solution. This implies that the developed and generalised material model could be applied for SMA wires having any length, and the SMA wires could be connected to
any actuated structure having any stiffness. In other words, LS-DYNA user did not have to figure out and define the stiffness of the actuated structures in the actuation directions, and also the length of the SMA wires in the keyword input deck. This greatly simplified the finite element modelling stage.

In LS-DYNA, other material properties that changed during the phase transformation were the shear modulus and the bulk modulus of the SMA. Because the SMA Young’s modulus changed during the martensite-to-austenite phase transformation (heating stage) and the austenite-to-martensite phase transformation (cooling stage), the shear modulus and the bulk modulus of the SMA also changed, as they were in functions of the SMA Young’s modulus. Their relation with the SMA Young’s modulus are shown in Eq. (4.27) and Eq. (4.28), respectively. The specific addresses of the material constants for them were cm(17) and cm(16), respectively. In the Newton-Raphson iteration, they were assigned to the eleventh and twelve history variables, hsv(11) and hsv(12), respectively. After solved at every time step, the history variables were passed to the material constants, respectively. This was another important difference between the coded UMAT in LS-DYNA and the analytical solution which was solved in MATLAB for validation. Both of the shear and bulk modulus were not required in the MATLAB code.

\[
G(\xi) = \frac{E(\xi)}{2(1 + v)} \quad (4.27)
\]

\[
B(\xi) = \frac{E(\xi)}{3(1 - 2v)} \quad (4.28)
\]

The SMA material properties used in the equations are listed in Table 4.1 with other additional parameters. For a clear illustration, the structure of the UMAT is illustrated in Fig. 4.2. The above equations were implemented in a FORTRAN code for explicit FEA simulations using LS-DYNA, and in MATLAB for analytical solutions. This would be described in detail in the following section 4.2.2, 4.2.3, and in the MATLAB code section in Appendix C4.
Table 4.1. NiTi SMA material properties (Butera, 2008, SAE, 2009) and additional parameters for the UMAT.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass density, ( \rho )</td>
<td>( 7.89 \times 10^4 ) kg/mm(^3)</td>
</tr>
<tr>
<td>Poisson’s ratio, ( \nu )</td>
<td>0.3</td>
</tr>
<tr>
<td>SMA Young’s modulus at martensite phase, ( E_M )</td>
<td>40 GPa</td>
</tr>
<tr>
<td>SMA Young’s modulus at austenite phase, ( E_A )</td>
<td>75 GPa</td>
</tr>
<tr>
<td>Martensite starting temperature, ( M_s )</td>
<td>47 °C</td>
</tr>
<tr>
<td>Martensite finishing temperature, ( M_f )</td>
<td>43 °C</td>
</tr>
<tr>
<td>Austenite starting temperature, ( A_s )</td>
<td>60 °C</td>
</tr>
<tr>
<td>Austenite finishing temperature, ( A_f )</td>
<td>65 °C</td>
</tr>
<tr>
<td>Stress influenced coefficient at martensite phase, ( C_M )</td>
<td>( 8.2 \times 10^{-3} ) Gpa/°C</td>
</tr>
<tr>
<td>Stress influenced coefficient at austenite phase, ( C_A )</td>
<td>( 8.2 \times 10^{-3} ) Gpa/°C</td>
</tr>
<tr>
<td>Maximum recoverable strain, ( \varepsilon_L )</td>
<td>0.016</td>
</tr>
<tr>
<td>SMA wire diameter, ( \Omega )</td>
<td>0.1/0.2/0.4/0.5 mm</td>
</tr>
<tr>
<td>Maximum no. of iteration (convergence), ( n_{\text{max}} )</td>
<td>500</td>
</tr>
<tr>
<td>Bulk modulus, ( B )</td>
<td>33.33 GPa</td>
</tr>
<tr>
<td>Shear modulus, ( G )</td>
<td>15.38 GPa</td>
</tr>
</tbody>
</table>

The SMA material model was validated for two cases: (1) a SMA wire connected to a linear spring in series, where the spring stiffness may represent the stiffness of actuated geometrical structures, and (2) a SMA-actuated cantilever beam. For the second case, a SMA-actuated cantilever beam, LS-DYNA results were validated with analytical solution of the SMA-spring model, with the spring stiffness defined by Eq. 4.29.

\[
k = \frac{F}{\delta_{\text{tip}}} = \frac{3EI}{L^3}
\]  

(4.29)

where \( F, \delta_{\text{tip}}, E, I \) and \( L \) are the vertical force acting on the beam tip, tip deflection, Young’s modulus, second moment of area, and the length of the cantilever beam, respectively.

Once validated, several design parameters such as the spring stiffness, the SMA wire cross-sectional area and length, were varied to study their effects on thermomechanical behaviour of the SMA wire. A mesh sensitivity study and a constant load case were also presented and analysed.
4.2 UMAT Structure

User Input Parameters – Material Constants and History Variables

---

**SMA Material Properties**
\[ \rho, \nu, E_M, E_A, C_M, C_A, \varepsilon_L, G, B \]

**SMA Dimension**
\[ \phi \]

**SMA Transformation Temperatures**
\[ M_s, M_f, A_s, A_f \]

**Applied Thermal Load**
\[ LCID \]

---

**Heating stage (martensite-to-austenite)**

**Initial Conditions**
\[ \sigma_0 = 0, \varepsilon_0 = 0, \xi_0 = 1 \]

**Temperature Change**
\[ \Delta T (T_L \rightarrow T_H) \quad T \text{ above } A_s \]

**Martensite Volume Fraction (MVF) - Eq. 4.6**

**Young’s Modulus – Eq. 4.2**

**SMA Stress**
\[ \sigma = E(\xi)\varepsilon - \varepsilon_L E(\xi)(\xi - 1) \]

**Newton Raphson Iteration - Eqs. 4.10 - 4.11 & 4.18**

---

**Cooling stage (austenite-to-martensite)**

**Initial Conditions**
\[ \sigma_0 = \sigma, \varepsilon_0 = \varepsilon, \xi_0 = 0 \]

**Temperature Change**
\[ \Delta T (T_H \rightarrow T_L) \quad T \text{ below } (M_s + \sigma_0/C_A) \]

**Martensite Volume Fraction (MVF) - Eq. 4.5**

**Young’s Modulus – Eq. 4.2**

**SMA Stress**
\[ \sigma = \sigma_0 + E(\xi)(\varepsilon - \varepsilon_0) - \varepsilon_L E(\xi) \xi \]

**Newton Raphson Iteration - Eqs. 4.14 - 4.15 & 4.18**

---

Figure 4.2. UMAT structure for one element of the shape memory alloy (SMA) material model.
4.2.2 FORTRAN Code in LS-DYNA

The equations (Eq. 4.1 - Eq. 4.18) related to the constitutive behaviour of the shape memory alloy (SMA) wire, which were presented in the previous section 4.2.1, were implemented in a FORTRAN code in the explicit LS-DYNA. The subroutine called umat41 was systematically developed and documented in Appendix C2, strictly without any duplication of other UMATs from any previous researches.

4.2.3 Vectorised UMAT (FORTRAN Code in LS-DYNA)

The main programmed UMAT presented in the previous section 4.2.2 was sufficient to complete Finite Element Analysis (FEA) simulations involving a single SMA wire, such as a SMA wire - linear spring structure (section 4.3.1), a SMA-actuated cantilever beam (section 4.3.2), and simulations of the SMA wire - linear spring structure in the mesh sensitivity and key parameter studies (section 4.4 and 4.5, respectively). However, for more complex structures actuated by multiple SMA wires, such as morphing aerofoils, which would be presented in the next Section 5, a vectorised UMAT had to be developed to significantly speed up the simulations and save computational time. This allowed the processing of all SMA elements simultaneously at every time step. The code structure of the vectorised UMAT (umat41v) is shown in Appendix C3, where the previous fundamental code (umat41) was called within the vectorised UMAT code.
4.3 Validations of the SMA Material Model

The SMA material model implemented in LS-DYNA was validated by an analytical solution solved in MATLAB. The MATLAB code is documented in Appendix C4. Two cases of validations are presented: (1) a SMA wire - linear spring structure and (2) a SMA-actuated cantilever beam.

4.3.1 UMAT Validation: SMA Wire – Linear Spring Structure

The SMA material model was initially tested and validated for a simple fundamental structure, which was a SMA wire connected to a linear spring in series, as shown in Fig. 4.3. The SMA wire was connected to the linear spring at one end, and the other ends were fixed in all degree of freedoms. In the Finite Element Model (FEM), the SMA wire was modelled with a beam element as described in section 4.1, while the linear spring was modelled with a discrete element. The diameter and length of the SMA wire were 0.5 mm and 100 mm, respectively. Similar dimension of SMA wire was used in the next sections of this chapter, except in the section of key parameter study. The maximum recoverable strain of the SMA wire was 1.6%, and other SMA material properties were listed in Table 4.1 in section 4.2, obtained from the manufacturer datasheet (SAES, 2009). The linear spring stiffness was 3.5 N/mm. A thermal load (Load_Thermal_Load_Curve) of one complete heating-cooling cycle was applied on the SMA wire, and the resulted SMA thermomechanical behaviour was analysed.

![Figure 4.3. A SMA wire - linear spring structure.](image)

Fig. 4.4 graphically compares the SMA stress and strain, which were solved by the FEA simulation and the MATLAB analytical solution, and presented by the solid lines and the dotted lines, respectively. The red lines are the SMA stress and strain on the heating stage, while the blue lines are the ones on the subsequent cooling stage. When heated, as the temperature exceeded the austenite starting temperature ($A_s$) of 60 °C, the SMA stress increased because the SMA wire contracted (SMA strain decreased) and extended the linear spring. The SMA stress and strain remained at a constant magnitude when the phase transformation from a martensite phase to an austenite phase completed. This martensite-to-austenite phase...
transformation completed at a temperature higher than the austenite finishing temperature ($A_f$) of 65 °C listed in Table 4.2, because the SMA transformation temperature was influenced by the SMA stress. When the SMA wire was cooled and the temperature dropped below the transition temperature, the SMA stress started to decrease. The SMA strain, in contrast, started to increase because the stiffness of the SMA wire decreased, causing the elongation of the SMA wire by the linear spring. This temperature was higher than the martensite starting temperature ($M_s$) of 47 °C, because of the influence of the SMA stress on the SMA transformation temperature. The SMA stress and strain remained constant when the martensitic transformation completed. An excellent comparison between the results of the FEA simulation (solid lines) and analytical solution (dotted lines), was obtained.

![Graph showing SMA stress and strain during actuation](image)

**Figure 4.4.** SMA stress and strain during the actuation of the SMA wire - linear spring structure.

At the end of the heating stage, the maximum SMA recovery stress was 27.65 MPa and the maximum SMA recovery strain was 1.56%. The SMA strain did not reach the maximum recoverable strain of 1.6% because the spring stiffness resisted the recovery of the SMA wire. The magnitude of these maximum SMA recovery stress and strain were influenced by the stiffness of the actuated structure and also other parameters, which was investigated and would
be reported in the next section 4.5. At the end of the cooling stage, small amount of SMA stress and strain remained at constant magnitudes. This finding was generally consistent with results from other research on SMA-actuated structures (Epps and Chopra 2001, Roh et al. 2006). These SMA stress and strain, which were usually called SMA residual stress and strain, were expected to converge after cycles of actuation.

4.3.1.1 Energy Analysis

To briefly explain the remaining (residual) stress and strain at the end of the cooling stage, the global energy as well as the part energy of the SMA wire and the linear spring were analysed. The global damping energy and global internal energy are shown in Fig. 4.5. The global damping energy was required to damp the fluctuations in the SMA force at the end of heating and cooling stages. The negative global internal energy was consequently dissipated with equal magnitudes to balance the total global internal energy. The first energy step (3.95×10^{-5} J) was associated with the heating stage, and the second energy step (8.17×10^{-5} J) was associated with the subsequent cooling stage. These energy were extracted from ASCII_GLSTAT output file. This had connection with the difference in part internal energy of the SMA wire and the linear spring. They are shown in Fig. 4.6, which were extracted from ASCII_MATSUM output file. The SMA wire internal energy was negative because of work done by the SMA wire on the linear spring, while the linear spring internal energy was positive because of the potential energy stored in the spring when it was stretched/extended by the SMA wire. Based on the conservation of energy principle, the energy due to the work done by the SMA wire on the linear spring (black curve) should be equal to the energy stored by the linear spring (blue curve). However, the energy due to the work done by the SMA wire was higher in magnitude than the energy stored in the linear spring. It turned out that this increase in energy was due to the additional negative global internal energy (−3.95×10^{-5}J on heating and −8.17×10^{-5}J on cooling) shown in Fig. 4.5. For a clearer clarification, the part internal energy of the SMA wire on both heating and cooling stages, was calculated by Equations 4.30-4.31,

\[ E_{SMA_{Heating}} = -4.21 \times 10^{-3}J + (-3.95 \times 10^{-5}J) = -4.25 \times 10^{-3}J \] (4.30)
\[ E_{SMA_{Cooling}} = -1.66 \times 10^{-6}J + (-8.17 \times 10^{-5}J) = -8.33 \times 10^{-5}J \] (4.31)

which were equal to the SMA internal energy shown in Fig. 4.6.
4.3 Validations of the SMA Material Model

Strain energy of the SMA wire was obtained from the area under the curve of the force-displacement curve. From the force-displacement curve shown in Fig. 4.7, the force increased as the SMA wire was heated (as shown in red), and then the force decreased as the SMA wire was cooled (as shown in blue). The non-linear curves at the beginning of the heating and cooling stages were due to the overshoot and fluctuations of the forces, before they reached equilibrium once the forces were completely damped. Integration of this force-displacement curve produced the strain energy, as shown in Fig. 4.8, with respect to the resulted displacement. The red and blue lines were the strain energy during the heating and cooling stages, respectively. On the completion of the heating stage, the strain energy was 4.21×10⁻³ J, which was equal to the energy stored in the linear spring. This means 4.21×10⁻³ J was absorbed by the SMA wire to complete the martensite-to-austenite transformation. On the completion of the cooling stage, the strain energy was comparably small, approximately 6.11×10⁻⁶ J. This means 4.204×10⁻³ J (4.21×10⁻³ J - 6.11×10⁻⁶ J = 4.204×10⁻³ J) was dissipated by the SMA wire to complete the martensitic transformation. This dissipated energy was slightly lower than...
the absorbed energy during the heating stage, because the SMA strain was not fully recovered upon cooling, as shown in the previous Fig. 4.4.

Figure 4.7. Force-displacement curve of the SMA wire – linear spring model.

Figure 4.8. Strain energy integrated from the force-displacement curve.

To analyse the specific energy absorption and the specific energy dissipation on the heating and cooling stages, the following calculations are presented. For the SMA wire with a diameter of 0.5 mm, a length of 100 mm, and a volume of $7.89 \times 10^{-6}$ kg/mm$^3$, the volume of the SMA wire is

$$V_{SMA} = A \cdot l = \left(\frac{\pi}{4}D^2\right)(l) = 19.6349 \text{ mm}^3$$ \hspace{1cm} (4.32)

and the mass of the SMA wire is

$$m_{SMA} = \rho \cdot V = 0.1549 \text{ g}$$ \hspace{1cm} (4.33)

Hence the specific energy of the SMA wire on heating and cooling stages are

$$e_{SMA_{\text{Heating}}} = \frac{E_{SMA_{\text{heating}}}}{m_{SMA}} = 2.7179 \times 10^{-2} \text{ J/g}$$ \hspace{1cm} (4.34)

$$e_{SMA_{\text{Cooling}}} = \frac{E_{SMA_{\text{cooling}}}}{m_{SMA}} = -2.7140 \times 10^{-2} \text{ J/g}$$ \hspace{1cm} (4.35)
4.3 Validations of the SMA Material Model

4.3.2 UMAT Validation: SMA-Actuated Cantilever Beam

The previous section showed the interaction between the user defined material model (UMAT) of the SMA wire and a discrete element, which was the linear spring. To show the interaction between the UMAT of the SMA wire and shell elements, and to provide additional validation, a cantilever beam actuated by the SMA wire as illustrated in Fig. 4.9 (a), was simulated and validated. The cantilever beam was modelled with an elastic aluminium material having mass density of \(2.81 \times 10^{-6} \text{ kg/mm}^3\), Young’s modulus of 71.7 GPa, and Poisson’s ratio of 0.33. The cantilever beam dimension was \(225 \times 1 \times 10 \text{ mm}^3\) (length × width × height), and it was meshed with 90 3D continuum shell elements. It was fixed/cantilevered at one end and connected to a SMA wire at the other end, with the SMA wire aligned perpendicular to the beam length and parallel to the beam height. A thermal load of one complete heating-cooling cycle was applied on the SMA wire, and the resulted SMA stress and strain as well as the beam tip deflection were analysed.

As depicted in Fig. 4.10, the trend and shape of the SMA stress and strain were similar to the previous SMA wire - linear spring structure: (1) the SMA stress increased on heating and then decreased on subsequent cooling, (2) the SMA strain decreased on heating and then
increased on subsequent cooling, and (3) constant SMA stress and strain once the reverse and martensitic transformations completed. The differences are the magnitude of final SMA stress and strain at the end of the heating and cooling stages, as well as the transformation temperatures, $A_f$ and $M_s$, because these temperatures were influenced by the SMA stress. The maximum SMA stress at the end of the heating stage, was lower than the previous case, and consequently the transformation temperatures were slightly lower. Conversely, the SMA strain was higher in magnitude at the end of the heating stage, closer to the maximum recoverable strain of 1.6%. These were due to the lower stiffness of the cantilever beam in the actuation direction (1.5737 N/mm – refer Eq. 29) compared to the linear spring stiffness of 3.5 N/mm. The respective temperature-history tip deflection of the cantilever beam is illustrated in Fig. 4.11. The tip deflection decreased (downward deflection) on heating and then increased back (upward deflection) on cooling. The maximum tip deflection due to the actuation was 1.57 mm, which can also be observed in fringes in Fig. 4.9 (b). The results of the SMA stress, SMA strain and beam tip deflection obtained from the FEA simulation (solid lines) were surprisingly close to the analytical predictions (dotted lines). This provided additional validation for the user defined material model (UMAT) of the SMA wire.

![Graph showing stress and strain](image)

**Figure 4.10.** SMA stress and strain of the SMA-actuated cantilever beam.
4.3 Validations of the SMA Material Model

4.4 Mesh Sensitivity Study

To ensure the reliability and accuracy of the UMAT, a mesh sensitivity study was conducted on the SMA wire - linear spring structure (Fig. 4.3) described in section 4.3.2. Instead of one beam element used for the SMA wire in the previous section 4.3.2, four additional simulations were completed with the SMA wire having 2, 5, 10 and 100 beam elements. One complete heating-cooling cycle was applied, and the resulted SMA stress and SMA strain were compared, as shown in Fig. 4.12.

The results showed exactly similar temperature-history SMA stress and strain, regardless of the number of beam elements of the SMA wire. Therefore, after debugging the developed code to fix a small but very significant error which was discovered in the FEA simulations using multiple beam elements for the SMA wire, it was concluded that one beam element was adequate to accurately simulate the thermomechanical behaviour of the SMA wire. Even though this study might look simple and insignificant, it was a very important step in the development and implementation of the UMAT of the SMA wire in the explicit LS-DYNA.
4.4 Mesh Sensitivity Study

Figure 4.12. SMA stress and strain extracted from the mesh sensitivity study, for the SMA wire with 1, 2, 5, 10 and 100 beam elements.

4.5 Key Parameter Study

Apart from the mesh sensitivity study, a key parameter study was also conducted to further understand the behaviour of the SMA wire. The SMA wire - linear spring structure described in the previous section 4.3.2, was used in the FEA simulations. A total of twelve cases were simulated with the SMA wire length, the SMA wire diameter and the spring stiffness treated as variables of interest. Two SMA wire length ($l$ in mm), four SMA wire diameter ($D$ in mm) and three spring stiffness ($k$ in N/mm), as illustrated in Fig. 4.13, were used in the FEA simulations. Combinations of the three variables resulted in six normalised stiffness ($\pi kl/4A$, in N/mm²), with two combinations for each normalised stiffness.

The results of the SMA stress and strain in function of temperature for the twelve cases are comparatively shown in Fig. 4.14. It was obviously observed that if the normalised stiffness was similar, even though the SMA length, the SMA diameter and the spring stiffness were varied, similar SMA stress and strain were obtained. The maximum SMA recovery stress at
the end of the heating stage, and the residual SMA stress at the end of the cooling stage, increased with increase in the normalised stiffness. Conversely, the maximum recovery strain at the end of the heating stage decreased as the normalised stiffness increased, because of the increase in the maximum recovery SMA stress. In other words, the strain recovery of the SMA wire decreased if the stiffness of the actuated structure was higher, or if the length of the SMA wire was longer, or if the diameter (cross-sectional area) of the SMA wire was smaller. Interestingly, the SMA residual strain at the end of the cooling stage had a different trend. It increased first but then decreased, with an increase in the normalised stiffness. This required further and deeper fundamental understanding of the SMA constitutive equation.

For a clearer picture, the maximum SMA recovery stress, the SMA residual stress, the maximum SMA recovery strain, and the SMA residual strain were plotted against the normalised stiffness, as depicted in Fig. 4.15. It was observed that there were nonlinear increase in the maximum SMA recovery stress, the SMA residual stress and the maximum SMA recovery strain, as the normalised stiffness increased. The SMA residual strain, however, decreased first but then increased, with an increase in the normalised stiffness.
Figure 4.14. SMA stress and strain for six normalised stiffness in the key parameter study.
Figure 4.15. Relationship between SMA recovery and residual stresses, SMA recovery and residual strains, and normalised stiffness.

4.6 Chapter Summary and Conclusions

In this Chapter 4, the development of a user defined material model (UMAT) of a SMA wire in an explicit LS-DYNA through a FORTRAN code, was described. To solve actuation of any complicated structures in the SMA wire axial direction, the approach was by solving a fundamental structure consisted of a SMA wire connected to a linear spring in series. The FEA model of the fundamental SMA wire - linear spring structure was simulated in the explicit LS-DYNA, and the resulted thermomechanical behaviour (SMA stress and SMA strain in function of the SMA temperature) was compared with analytical results from a MATLAB code. An additional validation case was presented, in form of a cantilever beam actuated by a single SMA wire, which was aligned perpendicular to the beam axis. Comparison of the SMA thermomechanical behaviour and the tip deflection, between the FEA model and another MATLAB code (SMA wire - linear spring structure with the right spring stiffness), showed an extremely good agreement. After validations, the fundamental structure was extensively tested for a mesh
sensitivity study (mesh variation of the SMA beam elements) and a key parameters study (variation of the spring stiffness, the length and the diameter of the SMA wire). The UMAT of the SMA wire was further improved by developing a vectorised UMAT, which enabled multiple SMA wires to be processed at a single time, and hence significantly save the computational time. The improved vectorised UMAT was beneficial for actuation of a more complex structure with multiple SMA wires, which would be presented in the next Chapter 5.
Chapter 5
SMA-Actuated Morphing Aerofoils

After the user defined material (UMAT) model of the shape memory alloy (SMA) had been successfully validated and tested for several fundamental cases in the previous Chapter 4, which involved a single SMA wire, the UMAT was then further extended to structures that were actuated by multiple SMA wires. The structures were SMA-actuated aluminium and composite aerofoils, which are covered in this chapter. The programmed code for the extended UMAT, or a vectorised UMAT, was briefly reported in the previous section 4.2.3. This chapter shows the capability of the vectorised UMAT of the SMA to predict the thermomechanical behaviour of multiple SMA wires, in actuating several aluminium and composite morphing aerofoils. The main objectives of virtually testing several aerofoils with different configurations of corrugated structure and different positions of SMA wires are to determine the right configurations and SMA locations so that the actuation of the aerofoil produces a sufficiently large trailing edge deflection and a smooth aerofoil deformation. Design, manufacturing with a 3D-printing technology, and experimental tests of the composite morphing aerofoil (corrugated structure with a trailing edge section), are then presented.

5.1 Finite Element Analysis (FEA) of SMA-Actuated Morphing Aerofoils

5.1.1 SMA-Actuated Aluminium Morphing Aerofoils

From the literature review, corrugated structures had been recognised as one of promising structures for morphing aerofoil/wing applications (Thill et al., 2010, Ruangjirakit, 2013, Previtali et al., 2014, Molinari et al., 2016). A corrugated structure/surface was introduced as a part of morphing aerofoils in this research. To identify the suitable arrangement of SMA wires within the corrugated structure, seven finite element models (FEMs) of aluminium aerofoils were created and simulated. Two of the possible and promising aerofoil configurations are illustrated in Fig. 5.1 and Fig. 5.2, in isometric and side views. The aerofoils had a NACA 0012 aerofoil profile, with a 300 mm chord length and a 10 mm span/width. In the initial study, the aerofoils had a D-nose spar located at 25% front of the chord, and a 12-
5.1 Finite Element Analysis (FEA) of SMA-Actuated Morphing Aerofoils

cells corrugated section which was positioned as a lower skin (aerofoil A - Fig. 5.1) or as a cantilever beam (aerofoil B - Fig. 5.2). It was assumed that the D-nose spar carried all the external aerodynamic loads (i.e. lift and drag forces) acted on the wing, and bending moments resulted from the aerodynamic loads. For the aerofoils with a cantilever corrugated plate (aerofoil B - Fig. 5.2), it was assumed that the void was filled with a very flexible structure, such as a flexible foam or a flexible cellular core with smooth upper and lower skins.

![Isometric View](image)

**Figure 5.1.** SMA-actuated aluminium morphing aerofoil A, with a corrugated section as a lower skin.

The aerofoils were modelled with 3D shell elements, with a thickness of 1 mm for the D-nose spar and 0.5 mm for the corrugated section, the upper skin and the trailing edge section. They were modelled with an elastic aluminium material, with a density of $2.81 \times 10^{-6}$ kg/mm$^3$, a Young’s modulus of 71.7 GPa and a Poisson’s ratio of 0.33. The corrugated section had twelve half-hexagonal-shape cells with approximately 10 mm cell size and 5 mm side length. The D-nose spar was constrained in all degree-of-freedoms, while the side edges of the corrugated section and the trailing edge were applied with a symmetrical boundary condition. One SMA wire with a 0.5 mm diameter and approximately 10 mm length, was positioned in each cell of the corrugation. One complete heating-cooling thermal loading was applied on the
SMA wires, and the resulted trailing edge deflections as well as the effective stress (von Mises stress) were evaluated. The effective stress (von Mises) was evaluated to determine whether yielding could have occurred.

![Isometric View](image1)

![Side View](image2)

**Figure 5.2.** SMA-actuated aluminium morphing aerofoil B, with a corrugated section as a middle cantilever beam.

The FEA simulation results of the SMA-actuated aluminium aerofoils A and B are illustrated in Fig. 5.3 and Fig. 5.4, respectively. The figures show the fringe of vertical z-displacement (in mm), the maximum trailing edge deflection, and the effective stress (von Mises in GPa) of the aerofoils, upon completion of the heating stage of the SMA wires. A large trailing edge deflection was achieved by aerofoil B, approximately 65.9 mm (Fig. 5.4a), compared to about 7.8 mm for aerofoil A (Fig. 5.3a). The corresponded deflection angle was about 16° and 2°, respectively. This was as expected because there was additional upper skin for the aerofoil A, which provided additional flexural stiffness to the aerofoil, and hence resulted in a lower SMA recovery strain and a lower trailing edge deflection.
Finite Element Analysis (FEA) of SMA-Actuated Morphing Aerofoils

Figure 5.3. FEA results of the SMA-actuated aluminium morphing aerofoil A: (a) vertical deflection and (b) effective stress (von Mises).
5.1 Finite Element Analysis (FEA) of SMA-Actuated Morphing Aerofoils

Figure 5.4. FEA results of the SMA-actuated aluminium morphing aerofoil B: (a) vertical deflection and (b) effective stress (von Mises).
The effective stress (von Mises), on the other hand, was higher for aerofoil A, approximately 16.1 MPa (Fig. 5.3b), compared to 7.7 MPa for aerofoil B (Fig. 5.4b). As shown in the figures, the stress was highest at the edges of each corrugation, at the mid-width of the aerofoils, where the SMA wires were connected to the corrugation. This was shown in red colour of the stress contour. The stresses for both aerofoils were far below the aluminium yield strength of 270 MPa, which ensured no structural yielding of the material during actuation. These results supported the preliminary conclusion from fundamental tests in the previous Chapter 4, which was the stiffer the actuated structure, the higher the SMA stress, and therefore the lower the SMA recovery strain, which eventually resulted in a lower trailing edge deflection.

From the results, a smooth deformation of the aerofoils was observed. This smooth deformation was obvious especially for aerofoil B, as shown in Fig. 5.4. The smooth changes of aerofoil shapes may lead to a smooth air flow distribution along the wing chord, if compared to conventional hinged control surfaces (flaps and ailerons) which had gaps between them and the front main wing section. Consequently, the morphing aerofoils were expected to generate a greater lift distribution and a lower drag, because of the possibility to eliminate the above-mentioned gaps that contributed to the flow separation. However, this prediction required further validation from aerodynamic studies such as incompressible fluid dynamic (ICFD) simulations.

The morphing aerofoils could be positioned along the wing span for a camber morphing wing or a trailing edge morphing wing, or they could be positioned at the wing tip for aileron morphing (Fig. 5.5a), or they could be positioned at several locations along the flexible section of a morphing wing to achieve a smooth wing tip deformation (Fig. 5.5b).

To check if the results of the trailing edge deflections and the effective stress (von Mises) of the morphing aerofoils were acceptable, a mesh convergence study was conducted on the aerofoil B, as presented in the next subsection 5.1.1.1.
Figure 5.5. SMA-actuated composite morphing wing with several possible configurations and positions of the aerofoil sections, which consist of SMA-actuated corrugated sections: (a) one corrugated section at the wing tip and (b) seven corrugated sections along the span of the flexible wing section.
5.1.1 Mesh Convergence Study

A mesh convergence study was conducted on the SMA-actuated aluminium aerofoil to ensure the reliability of the FEA simulation results. The previous aerofoil B which had a high trailing edge deflection upon SMA actuation, was selected as an aerofoil model in this study. As shown in Fig. 5.6, a total of six cases were simulated, with the number of the 3D continuum shell elements for one surface (5 mm × 10 mm flat surface) of the corrugated section ranged from two elements to 200 elements. The range of size of the shell elements for the corrugated section was from 5 mm to 0.5 mm, respectively. The number of elements, the element size and the results of trailing edge deflections are summarised in Table 5.1. The simulations were run on the High Performance Computer (HPC) of Imperial College London. The total CPU time used in the simulations was also reported, to analyse the computational time.

![Mesh sensitivity study: the corrugated section with shell element size of (i) 5 mm, (ii) 2.5 mm, (iii) 1.25 mm, (iv) 1 mm, (v) 0.625 mm, and (vi) 0.5 mm.](image)
5.1 Finite Element Analysis (FEA) of SMA-Actuated Morphing Aerofoils

As plotted in Fig. 5.7, the results of the mesh sensitivity study showed that the aerofoil tip deflection started to converge at mesh III (1.25 mm element size), and it remained almost constant as the number of shell element increased to mesh VI (0.5 mm element size). The previous results in Fig. 5.3 and Fig. 5.4 were based on mesh IV (1 mm element size), and hence the accuracy of the results was considered acceptable. Simulations of the other five aluminium aerofoils, which would be analysed in the next section 5.1.1.2, a parameter study section, were also based on mesh IV (1 mm element size).

Table 5.1. Summary and results of mesh sensitivity study on the morphing aerofoil.

<table>
<thead>
<tr>
<th>FEA Simulations</th>
<th>Side Length Elements</th>
<th>Width Elements</th>
<th>Total Shell Elements (one surface)</th>
<th>Element Size (mm)</th>
<th>Trailing Edge Deflection (mm)</th>
<th>Total CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>72.41</td>
<td>4H 47M</td>
</tr>
<tr>
<td>II</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>2.5</td>
<td>67.29</td>
<td>37H 12M</td>
</tr>
<tr>
<td>III</td>
<td>4</td>
<td>8</td>
<td>32</td>
<td>1.25</td>
<td>66.12</td>
<td>1055H 53M</td>
</tr>
<tr>
<td>IV</td>
<td>5</td>
<td>10</td>
<td>50</td>
<td>1</td>
<td>65.93</td>
<td>2325H 10M</td>
</tr>
<tr>
<td>V</td>
<td>8</td>
<td>16</td>
<td>128</td>
<td>0.625</td>
<td>65.76</td>
<td>7174H 37M</td>
</tr>
<tr>
<td>VI</td>
<td>10</td>
<td>20</td>
<td>200</td>
<td>0.5</td>
<td>65.71</td>
<td>9999H 52M</td>
</tr>
</tbody>
</table>

The time consumed (total CPU time) to complete the simulation of the aerofoil with mesh III (1056 hours) was less than half of the time consumed to complete the simulation of the
aerofoil with mesh IV (2325 hours), with almost similar results in the trailing edge deflection (only 0.2 mm difference). This showed a significant reduction in the computational time and a sufficient accuracy in the results if mesh III was used. Therefore, for larger aerofoils with a longer span/width (e.g. 31.25 mm), which were simulated after several design iterations and after considerations on 3D-printing tolerance and limitations, the mesh III (1.25 mm element size) was used to significantly save the computational time.

5.1.1.2 Parameter Study

In addition to the aluminium morphing aerofoils A and B, which were presented in the beginning of section 5.1.1, several other aluminium aerofoil configurations were also modelled, simulated and analysed. Firstly, for aerofoils with a corrugated section as a cantilever beam, the positions of the SMA wires were varied. As shown in Fig. 5.8, twelve SMA wires were positioned on the lower corrugation, and eleven SMA wires were positioned on the upper corrugation. Initially, this aerofoil configuration with SMA wires on both top and bottom corrugated surfaces, was modelled to achieve upward and downward trailing edge deflections, separately. Heating of the SMA wires on the bottom corrugated surface should deflect the trailing edge downward, while heating the SMA wires on the top corrugated surface should deflect the trailing edge upward. However, the developed UMAT of SMA wires could input one thermal load curve to activate the SMA wires. The UMAT could be improved and extended in the future to process multiple thermal load curves in a single simulation. As shown in Fig. 5.9, a single long SMA wire was positioned in between the D-nose spar and the rear spar of the trailing edge section. The aerofoil with this configuration/position of SMA wire was modelled to evaluate the achieved trailing edge deflection, if the fixing method of the SMA wire had to be simplified (in actual application, fixing a single SMA wire was considerably easier than fixing multiple SMA wires between corrugated surfaces). The trailing edge deflection should be compared with the trailing edge deflection of aerofoil B shown in the previous Fig. 5.4.

For all five aerofoils presented in this subsection, the diameter and maximum recoverable strain of the SMA wires were similar to the ones used for aerofoils A and B, which were 0.5 mm and 1.6%, respectively. The length of the SMA wires depended on the positions of the SMA wires, which took the distance between each corrugated cell or the distance between the D-nose spar and the trailing edge section. Other material properties of the SMA wires were as listed in the previous Table 4.1, in Chapter 4. An important assumption was made, which was the space between the D-nose spar and the trailing edge was filled with a very flexible structure,
such as a flexible foam or a flexible cellular core with upper and lower skins, to form a complete aerofoil.

Figure 5.8. Aluminium morphing aerofoil with a middle corrugated section, actuated by 12 SMA wires on the lower part and 11 SMA wires on the upper part of the corrugated section: effective stress (von Mises) (top) and trailing edge deflection (bottom).

The figures show the effective stress (von Mises) and trailing edge deflections on completion of the heating stage of the SMA wires. As shown by fringes in Fig. 5.8, only 5.32 mm trailing edge deflection was achieved when eleven additional SMA wires were positioned on the upper corrugation, in addition to the twelve SMA wires on the lower corrugation, which were initially designed for the previous aerofoil B. This was much lower than the trailing edge deflection of the aerofoil B with twelve SMA wires positioned only on the lower corrugation, which was about 65.9 mm (Fig. 5.4a). The reason was that when heated, the SMA wires on the lower corrugation tend to deform the trailing edge section downward, but the SMA wires on the upper corrugation inclined to deform the trailing edge section upward. Hence the downward deflection produced by the SMA wires on the lower corrugation was counter-deflected by the
upward deflection produced by the SMA wires on the upper corrugation. This aerofoil configuration was therefore not carried forward to the next stage of the morphing research.

As shown in Fig. 5.9, when the aerofoil with a cantilever corrugated section was actuated by a single long SMA wire, a trailing edge deflection of 25.6 mm was achieved. This was about 39% of the trailing edge deflection achieved by the previous aerofoil B, which had twelve short SMA wires on the lower corrugation (Fig. 5.4a). The length of the single SMA wire in the Fig. 5.9 was 185.41 mm, which was 65 mm longer than the total length of the twelve short SMA wires of the previous aerofoil B, which was 120 mm. This showed that by optimising the positions of the SMA wires on each cell of the corrugation as in the aerofoil B, a higher trailing edge deflection could be achieved with a shorter total length of the SMA wires. This eventually led to a significant power saving in resistive heating the SMA wires. Comparatively, the previous aerofoil B had a 0.55 mm trailing edge deflection per millimetre SMA wire, while the aerofoil with a single long SMA wire shown in Fig. 5.9 had a 0.14 mm trailing edge deflection.
per millimetre SMA wire. The later showed about four times lower in the performance, and therefore was filtered out of the morphing research. Observing the deformation of the aerofoil in the bottom figure of Fig. 5.9, the SMA wire seemed to be on the outside of the aerofoil when it was in the deflected position. This could be solved by shifting the locations of both ends of the SMA wire closer to the neutral axis, but a special slot on the imaginary foam section had to be designed to allow movement of the SMA wire.

Secondly, instead of a cantilever corrugated section actuated by a single long SMA wire shown if Fig. 5.9, the corrugated section was replaced with a cantilever flat plate, as shown in Fig. 5.10. The objective of the simulation of this aerofoil configuration was to provide a direct comparison of the types of the cantilever sections within the morphing aerofoil, between the flat cantilever section (Fig. 5.10) and the corrugated cantilever section (Fig. 5.9), and to evaluate their trailing edge deflection upon SMA actuation. Both of the aerofoils offered a simplification of the fixing method of the SMA wire, if compared to aerofoil B (Fig. 5.2). The length of the SMA wire was similar to the SMA wire used to actuate the previous aerofoil with a cantilever corrugated section. A similar assumption was made, which was the space between the D-nose spar and the trailing edge was filled with a very flexible structure to form a complete aerofoil. The structure was excluded from the model to simplify the design of the morphing aerofoil at this stage, by assuming it had a very low flexural stiffness and hence did not contribute to the flexibility and deformation of the aerofoil. Compared to the aerofoil with a cantilever corrugated section, the aerofoil with a flat cantilever plate had slightly higher trailing edge deflection, which was about 29.4 mm (15% higher). This was because the flat cantilever plate had a lower flexural stiffness compared to the cantilever corrugated section. Even though the trailing edge deflection was higher, it was still much lower than the previous aerofoil B. In term of trailing edge deflection per unit length of SMA wire, this aerofoil configuration produced about 0.16 mm trailing edge deflection per millimetre SMA wire, which was about 3.5 times lower than the previous aerofoil B. Therefore, this aerofoil configuration was also filtered out of the morphing research. Similar to the aerofoil with a cantilever corrugated section, the SMA wire appeared to be on the outside of the aerofoil when it was in the deflected position, which could be solved by shifting the locations of both ends of the SMA wire closer to the neutral axis, with a special slot on the imaginary foam section for movement of the SMA wire.
In addition to the above aerofoil configurations, an aluminium aerofoil with a middle cantilever corrugated section, a top skin and a bottom skin, as shown in Fig. 5.11, was also modelled, simulated and analysed. Compared to the original configuration of aerofoil B (Fig. 5.4), the top and bottom skins were added between the front spar and the trailing edge section to evaluate the aerofoil deformation upon SMA actuation. The objective of the simulation of this aerofoil was to explore the possibility of combining smooth top and bottom aerofoil skins with the SMA-actuated middle corrugated section, and to study the effect of the additional skins on the trailing edge deflection and the aerofoil deformation. The results show the effective stress (von Mises) and vertical deflection of the aerofoil on completion of the heating stage of the SMA wires. The maximum effective stress (von Mises) at the SMA connection points was 32.03 MPa, about four times higher than the maximum effective stress (von Mises) of the aerofoil B. This stress, however, was still much lower than the yield stress of the aluminium material.
It was observed that the top and bottom skins buckled, as they resisted the compressive force exerted by the trailing edge section, when the SMA wires contracted and deformed the corrugated section. This skin buckling could be advantageous in certain applications, such as in the variable inlet area of a jet engine (Pitt et al., 2002), but it was a disadvantage in view of the current morphing aerofoil. The maximum downward trailing edge deflection of the aerofoil was about 9.79 mm, which was about 15% of the trailing edge deflection achieved by the previous aerofoil B. This showed that by adding top and bottom aluminium skins, the trailing edge deflection was significantly reduced. Based on this finding, aluminium skins should be avoided in combination with the aerofoil with a centre cantilever corrugated section. The skins might be replaced with a low stiffness flexible foam which can provide smooth upper and lower aerodynamic surfaces.

Another aerofoil configuration was an aerofoil with a corrugated section as a bottom skin, such as in the aerofoil A previously introduced in the beginning of this chapter, and also as a...
top skin. The corrugated sections for both top and bottom skins had twelve cells, in such that they were mirrored about the horizontal symmetrical axis or the chord of the aerofoil. As shown in Fig. 5.12, eleven SMA wires were positioned on the lower part of the top corrugated skin (inside of the aerofoil), and twelve SMA wires were positioned on the lower part of the bottom corrugated skin (outside of the aerofoil). The objective of the simulation of the aerofoil was to investigate the possibility of increasing the trailing edge deflection when the SMA-actuated corrugated surface was introduced as both top and bottom skins (instead of only the bottom skin as in the previous aerofoil A).

Figure 5.12. Aluminium morphing aerofoil with corrugated upper and lower skins, actuated by 12 SMA wires on the outer part of the lower skin and 11 SMA wires on the inner part of the upper skin: effective stress (von Mises) (top) and trailing edge deflection (bottom).

Initially, based on the result of the previous aerofoil A in Fig. 5.3a, it was assumed that almost similar trailing edge deflection could be achieved if the smooth top skin was replaced with a SMA-actuated corrugated skin having SMA wires on the lower part (inside of the aerofoil), and the bottom skin was replaced with a smooth skin. Therefore, if an aerofoil had
both SMA-actuated lower and upper corrugated skins, it was expected that the total trailing edge deflection would be contributed by each of the lower and upper corrugated skins.

The effective stress (von Mises) and vertical deflection of the aerofoil upon completion of heating of the SMA wires are depicted in Fig. 5.12. A maximum tip deflection of about 0.71 mm was achieved. Surprisingly, this was about ten times lower if compared to the tip deflection of the previous aerofoil A (Fig. 5.3a). This showed that there was no advantage of the additional corrugated top skin with the SMA wires. The reason was that contraction of the top and bottom corrugated skins at the same time tend to pull the trailing edge section towards the D-nose spar, instead of deforming the aerofoil shape in a downward direction. These results showed how useful the UMAT of the SMA wires as a design tool to predict the deformation behaviour of the SMA-actuated morphing aerofoils. The simulation results helped the decision making process to exclude certain aerofoil configurations during the early stage of the morphing research, before finalising the morphing aerofoil configuration to be carried forward to the next stage of experimental manufacture and tests.

5.1.2 SMA-Actuated CF Composite Morphing Aerofoils

Convinced with the potential of integrating corrugated sections with SMA wires to produce sufficient morphing capability of the aluminium aerofoil, the model of the SMA-actuated aerofoil (aerofoil B) was extended to a more advanced material, which was a carbon fibre (CF) composite material. Compared to an aerofoil with isotropic aluminium material, an aerofoil with orthotropic CF composite material offered higher strength-to-weight ratio and greater stiffness-to-weight ratio. These superior properties were important for aerostructures, such as a morphing wing, to achieve the end goals, including fuel saving, greater endurance and flight range, and reduction in CO₂ emission.

For example, as shown in Table 5.2, the tensile strength of the Nylon66 + carbon fibre composite was about three times higher than the aluminium material, but about half of the mild steel material. However, because of a lower density, the specific strength was much higher than the aluminium and the mild steel, about six times and three times higher, respectively. With the similar reason, the specific modulus was almost comparable to the two metals, approximately 60% of their stiffness. Carbon (AS4 and HMS) epoxy composites were other examples with superior specific strength and modulus than the metallic materials.
5.1 Finite Element Analysis (FEA) of SMA-Actuated Morphing Aerofoils

Table 5.2. Material properties of composites, polymers, and metals (Matthews and Rawlings, 1999).

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (Mg/m³)</th>
<th>Young’s Modulus (GPa)</th>
<th>Tensile Strength (MPa)</th>
<th>Ductility (%)</th>
<th>Flexural Strength (MPa)</th>
<th>Specific Modulus [GPa/(Mg/m³)]</th>
<th>Specific Strength [MPa/(Mg/m³)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nylon66 + 40% carbon fibre</td>
<td>1.34</td>
<td>22</td>
<td>246</td>
<td>1.7</td>
<td>413</td>
<td>16</td>
<td>184</td>
</tr>
<tr>
<td>Carbon (AS4) epoxy</td>
<td>1.55</td>
<td>145</td>
<td>1480 (T) 1225 (C)</td>
<td>-</td>
<td>-</td>
<td>93</td>
<td>955 (T) 790 (C)</td>
</tr>
<tr>
<td>Carbon (HMS) epoxy</td>
<td>1.63</td>
<td>205</td>
<td>1275 (T) 1020 (C)</td>
<td>-</td>
<td>-</td>
<td>126</td>
<td>782 (T) 636 (C)</td>
</tr>
<tr>
<td>Epoxy</td>
<td>1.12</td>
<td>4</td>
<td>50</td>
<td>4</td>
<td>-</td>
<td>4</td>
<td>36</td>
</tr>
<tr>
<td>Nylon 6.6</td>
<td>1.14</td>
<td>2</td>
<td>70</td>
<td>-</td>
<td>-</td>
<td>18</td>
<td>61</td>
</tr>
<tr>
<td>Aluminium</td>
<td>2.70</td>
<td>69</td>
<td>77</td>
<td>47</td>
<td>-</td>
<td>26</td>
<td>29</td>
</tr>
<tr>
<td>Mild steel</td>
<td>7.86</td>
<td>210</td>
<td>460</td>
<td>35</td>
<td>-</td>
<td>27</td>
<td>59</td>
</tr>
</tbody>
</table>

The finite element (FE) models of the composite aerofoils used properties of carbon fibre reinforced plastic (CFRP) T300/914 composite. The material properties of a single ply of the T300/914 carbon fibre composite are listed in Table 5.3 (Laminate Analysis Program 4.1 (Anaglyph Ltd, 1995-2000), which sourced from Imperial College). The diameter and maximum recoverable strain of the SMA wires were similar to the ones used for the previous aerofoil B, which were 0.5 mm and 1.6%, respectively, while the other SMA material properties were listed in Table 4.1 in the previous Chapter 4. The corrugated section, in between the D-nose spar and the trailing edge section, had two different lay-ups: (a) a symmetrical lay-up, (90/45/-45/90/90/-45/45/90)°, and (b) an anti-symmetrical lay-up, (90/45/-45/90/90/45/-45/90)°. The reason behind the selection of the two lay-ups was to investigate the bending–twisting coupling of the composite aerofoils when they were bended/deformed upon SMA actuation. The D-nose spar and the trailing edge section had the anti-symmetrical layup, (90/45/-45/90/90/45/-45/90)°. Zero degree was in the chord wise direction. The thickness of all parts was 1 mm. A complete heating-cooling cycle was applied on the SMA wires, and deformations of the composite aerofoils were analysed.

Table 5.3. Material properties of carbon fibre composites for the SMA-actuated composite aerofoils, compared to the previous aluminium material.

<table>
<thead>
<tr>
<th>LS-DYNA Keywords</th>
<th>Parameters</th>
<th>Carbon Fibre Composites</th>
<th>Aluminium</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAT</td>
<td>RO (mass density, kg/mm³) E (Young’s modulus, GPa) G (Shear Modulus, GPa) PR (Poisson’s ratio)</td>
<td>MAT_022-Composite_Damage 1.53×10⁶ E₁₁ = 129, E₂₂ = 8.4 G₁₂ = 4.2 0.34 (ν₂₁ = 0.022)</td>
<td>MAT_001-Elastic 2.81×10⁶ 71.7 26.9 0.33</td>
</tr>
</tbody>
</table>
5.1 Finite Element Analysis (FEA) of SMA-Actuated Morphing Aerofoils

The trailing edge deflections of the composite aerofoils on completion of the heating stage of the SMA wires are shown in Fig. 5.13. The trailing edge of the composite aerofoils with corrugated section having symmetrical lay-up and anti-symmetrical lay-up were deflected by 52.5 mm and 52.0 mm, respectively. As expected, there was a small amount of twist for the composite aerofoil with the corrugated section having a symmetrical layup, while no twisting was observed from the composite aerofoil with the corrugated section having an anti-symmetrical lay-up. This was because the $D_{13}$ and $D_{23}$ components of the $D$ matrix for the symmetrical composite were non-zero, while they were zero for the anti-symmetrical composite. This condition was checked and confirmed in the composite lay-up analysis using the Laminate Analysis Program 4.1 (Anaglyph Ltd, 1995-2000). The results of trailing edge deflections were smaller than the previous aluminium aerofoil B, because of higher flexural stiffness of the composite aerofoils.
5.2 Finite Element Analysis of SMA-Actuated Pre-Curved Corrugated Plate

To demonstrate the applicability of the developed shape memory alloy (SMA) material model to actuate another complex structure, the model was further tested for actuation of a pre-curved corrugated plate (PCCP). This type of structure was studied for two main reasons: firstly, to explore a solution to a permanent deformation existed on the actuated aerofoil at the end of the cooling cycle, and secondly, the structure could potentially represent an internal structure of a winglet of a wing, which should perform as a normal winglet before SMA actuation and increased the wetted area of the wing upon SMA actuation.

The PCCP was modelled with 3D continuum shell elements, with a plate width of 10 mm, a circumference length of 225 mm, a thickness of 1 mm, and a total of 15 corrugated cells. The corrugated plate was modelled with an aluminium material, with material properties similar to the previous cantilever beam in section 4.3.2 and SMA-actuated aluminium aerofoils in section 5.1.1. The SMA wires had a diameter of 0.5 mm, with material properties listed in Table 4.1 in the previous Chapter 4. A Fixed boundary condition in all degree of freedoms was applied on one edge of the plates (the right edge of the presented figures – Fig. 5.14 to Fig. 5.17) while the other edge (the left end) was free. A symmetrical boundary condition was applied on the side edges of the plates. One complete heating-cooling cycle was applied on the SMA wires, and the vertical tip deflection of the free end was evaluated.

Two cases are presented for the actuation of the pre-curved corrugated plate. Firstly, each cell was actuated by one SMA wire and two SMA wires, to investigate the effect of increasing the number of SMA wires per cell. Secondly, the configurations or arrangements of the SMA wires in the cells were varied to study their effects on the tip deflection.

Fig. 5.14 and Fig. 5.15 show simulation results of the pre-curved corrugated aluminium plate actuated by one and two SMA wires per cell, respectively. The former was run on a computer with 4 CPU and 32 GB memory capacity, while the later was run on the High Performance Computer (HPC) of Imperial College London with 24 CPUs and 20 GB memory capacity. They were completed in 603 hours and 39 minutes (elapsed time), and 46 hours and 28 minutes (elapsed time), respectively. The simulation on HPC was completed about 13 times faster than the simulation on the normal computer, because of higher number of CPUs. The resulted vertical tip deflections due to the SMA wires actuation were 56.3 mm and 63.8 mm, respectively. It showed that doubling the number of the SMA wires (hence increase in power requirement) increased the tip deflection by only 13.3%.
5.2 Finite Element Analysis of SMA-Actuated Pre-Curved Corrugated Plate

Figure 5.14. Pre-curved corrugated aluminium plates actuated by one SMA wire per cell: (a) isometric view and (b) side view.
5.2 Finite Element Analysis of SMA-Actuated Pre-Curved Corrugated Plate

Figure 5.15. Pre-curved corrugated aluminium plates actuated by two SMA wires per cell: (a) isometric view and (b) side view.
In addition to the analysis of the number of SMA wires used for actuation of the PCCP, their configurations or arrangements were also varied. In this study, two SMA wires were arranged in parallel and in ‘V’ configurations within each cell, as depicted in Fig. 5.16 and Fig. 5.17, respectively. The PCCP with the SMA wires in ‘V’ configuration was intentionally studied to investigate the effect of arranging SMA wires in multiple directions/arrangements, as proposed by some of previous research on SMA-actuated structures (Barbarino et al., 2010). In this analysis, SMA wires with a diameter of 0.1 mm and a maximum recoverable strain of 3.9% were used. This was intentionally performed to allow the PCCP with the SMA wires arranged in parallel configuration to be compared with the previous PCCP actuated by SMA wires in a similar configuration, but having a larger diameter, and a smaller maximum recoverable strain (Fig. 5.15). The simulation results showed maximum vertical tip deflections of 38.1 mm and 35.7 mm, for the PCCP with SMA wires in the parallel and ‘V’ configurations, respectively. Therefore, the PCCP with SMA wires in the parallel configuration was preferable because a slightly larger tip deflection could be achieved. However, the maximum tip deflection (38.1 mm) for the parallel configuration was much smaller than the maximum tip deflection achieved by the previous PCCP actuated by SMA wires having 0.5 mm diameter and 1.6% maximum recoverable strain, which was shown in the previous Fig. 5.15 (63.8 mm). This showed that the influence of the SMA cross-sectional area on the tip deflection of the PCCP was greater than the influence of the SMA maximum recoverable strain. For comparison, the results of this study are reported in Table 5.4.

Initially, this specific study on the pre-curved corrugated plate (PCCP) was conducted to solve the permanent deformation of the morphing aerofoil trailing edge, which existed at the end of the cooling stage of the SMA wires, after one actuation cycle. However, the study was not pursued further, because the tip deflection was not sufficient for morphing aerofoil application. This can be observed from Fig. 5.15, where the tip deflection was just slightly below the horizontal axis, when the PCCP was actuated by two 0.5mm-diameter SMA wires per cell. The tip deflection can be increased by increasing the number of SMA wires along the 10mm-width/span of the PCCP, i.e. by using maximum 20 SMA wires which were arranged in parallel along the PCCP width/span. However, there would be some difficulties in the method of fixing many SMA wires on the narrow-width PCCP. Therefore, it is recommended for future research to explore the possibility of using the PCCP to solve the permanent deformation of morphing aerofoil, but SMA plates/strips should be used instead of the SMA wires, to simplify the method of fixing the SMAs.
Figure 5.16. Pre-curved corrugated aluminium plates actuated by 2 SMA wires per cell, which were arranged in parallel: (a) isometric view and (b) side view.
5.2 Finite Element Analysis of SMA-Actuated Pre-Curved Corrugated Plate

Figure 5.17. Pre-curved corrugated aluminium plates actuated by 2 SMA wires per cell, which were arranged in ‘V’ configuration: (a) isometric view and (b) side view.

Table 5.4. FEA simulations results of the SMA-actuated pre-curved corrugated plates (PCCP).

<table>
<thead>
<tr>
<th>Case Studies</th>
<th>No. and Arrangement of SMA Wires</th>
<th>SMA Diameter (mm)</th>
<th>SMA Maximum Recoverable Strain (%)</th>
<th>Results of Tip Deflection (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Single</td>
<td>0.5</td>
<td>1.6</td>
<td>56.3</td>
</tr>
<tr>
<td>2</td>
<td>Double, parallel</td>
<td>0.5</td>
<td>1.6</td>
<td>63.8</td>
</tr>
<tr>
<td>3</td>
<td>Double, parallel</td>
<td>0.1</td>
<td>3.9</td>
<td>38.1</td>
</tr>
<tr>
<td>4</td>
<td>Double, ‘V’ configuration</td>
<td>0.1</td>
<td>3.9</td>
<td>35.7</td>
</tr>
</tbody>
</table>
5.3 Experiments on SMA-Actuated 3D-Printed Carbon Fibre (CF) Composite Corrugation

To realise the potential of the SMA-actuated corrugated structure within a morphing aerofoil, a carbon fibre (CF) composite corrugated structure with a rigid trailing edge section was fabricated and tested. An additive manufacturing technique, a 3D-printing method, was selected to fabricate the CF composite corrugated structure, because of the following advantages:

1) The composite corrugated section and the trailing edge section could be fabricated as one single structure.

2) Dimensional accuracy and precision of the Markforged 3D-printer in manufacturing composite parts.

3) A kinking problem of conventional carbon fibre composite, if the composite corrugated section was manufactured with a traditional composite moulding technique, could be avoided.

Prior to the 3D-printing of the composite corrugated structure, microstructure of the 3D-printing materials, which were carbon fibre and onyx filaments, that were nylon-based, were studied and analysed. The onyx material comprised of chopped carbon fibre filament (less than 1 mm length) with a tough nylon matrix.

5.3.1 Microstructure of Carbon Fibres and Onyx Filaments

To understand the microstructure of the 3D-printed carbon fibre (CF) composites, which might affect the macro mechanical behaviour of the 3D-printed composite parts, three specimens were prepared for microstructure observation. The specimens were a single CF filament, a single onyx filament, and a 3D-printed half-hexagonal CF composite which represented a section of the CF composite corrugated structure.

The specimens were prepared by immersing them in polyester resin inside 20 mm\(^3\) epoxy tubs, as shown in Fig. 5.18a. The experiment procedures are as described below:

(i) The specimens were supported by support plastic rings (MetPrep Ltd, Ref. 113041) to keep the specimens standing straight inside the tubs.
(ii) Inside a fume cupboard, 60 ml of polyester resin was mixed with 36 drops of hardener in a paper cup (6 drops per 10 ml polyester resin). The mixture was stirred until the colour changed from a pink colour to a brown colour.

(iii) The mixture was then poured into the epoxy tubs (20 ml each), and the immersed specimens were left in the fume cupboard for 24 hours, before the next polishing stage.

The prepared specimens after 24 hours curing are shown in Fig. 5.18b.

![Figure 5.18. A carbon fibre (CF) filament, an onyx filament and a 3D-printed half-hexagonal CF composite specimens for microstructure analysis.](image)

Before the polishing stage, the bottom surfaces of the specimens were sanded using an abrasive paper to obtain flat surfaces, for placement of the specimens on the optical microscope. The top surfaces of the specimens were then polished using an automatic polisher (ATA SAPHIR 550, Germany). Four stages of the polishing steps, using different medium, are summarised in Table 5.5, with the polishing time, the applied force on the specimens and the rotational speed of the polisher head.

<table>
<thead>
<tr>
<th>Polishing Stage</th>
<th>Medium</th>
<th>Polishing Time (minutes)</th>
<th>Applied Force (N)</th>
<th>Rotational Speed (/minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1</td>
<td>water</td>
<td>2</td>
<td>25</td>
<td>150</td>
</tr>
<tr>
<td>Stage 2</td>
<td>9 μm</td>
<td>4</td>
<td>25</td>
<td>150</td>
</tr>
<tr>
<td>Stage 3</td>
<td>0.3 μm</td>
<td>4</td>
<td>25</td>
<td>100</td>
</tr>
<tr>
<td>Stage 4</td>
<td>0.06 μm</td>
<td>2</td>
<td>15</td>
<td>80</td>
</tr>
</tbody>
</table>

The microstructures of the carbon fibre (CF) and the onyx filaments were studied to understand the fibre distribution in a single CF filament and a single onyx filament. An optical
microscope (Olympus Zeiss AX10), with 1.25, 5, 20, 50 and 100 zooms capability, was used in the experiment. The captured images of the CF and onyx filaments are shown in Fig. 5.19.

The diameter of the single carbon fibre (CF) filament was measured as 393.66 \( \mu \text{m} \), or 0.39 mm. The diameter of the micro CF was in the range of 7.08 \( \mu \text{m} \) to 8.58 \( \mu \text{m} \), with an average diameter of 7.66 \( \mu \text{m} \). On the other hand, the single onyx filament had a diameter of 1753.06 \( \mu \text{m} \), or 1.75 mm. The micro CF in the single onyx filament had an average diameter of 9.05 \( \mu \text{m} \), with the range of 6.54 \( \mu \text{m} \) to 10.28 \( \mu \text{m} \).

**Figure 5.19.** Microstructure images of a single carbon fibre filament (left) and a single onyx filament (right).
On the pattern of micro CF distribution, the CF filament had groups of close-spaced fibres with a significantly wide distance between the groups. The onyx filament, in contrast, had a more even fibre distribution and almost even distance between the fibres. In term of the number of micro fibres, the CF filament was found to have a significantly greater number of micro fibres, compared to the onyx filament.

The fibre volume fractions were calculated for both of the carbon fibre (CF) and onyx filaments. For the CF filament, it was calculated from four squares in the 20× zoomed image, with a square area of 100 μm × 100 μm. The resulted fibre volume fraction of the CF filament ranged from 37.8% to 51.6%, with an average of 44.01%. For the onyx filament, the fibre volume fraction was calculated from five squares in the 20× zoomed image, with a square area of 100 μm × 100 μm. The average fibre volume fraction of the onyx filament was found to be 14.15%, ranging from 10.93% to 17.37%.

The results of the microstructure analysis of the CF and onyx filaments, including the filament diameter, the micro CF diameter, the pattern of CF distribution, and the calculated fibre volume fraction, are summarised in Table 5.6.

<table>
<thead>
<tr>
<th>Diameter of a single filament (μm)</th>
<th>CF Filament</th>
<th>Onyx Filament</th>
</tr>
</thead>
<tbody>
<tr>
<td>394</td>
<td>1753</td>
<td></td>
</tr>
<tr>
<td>Diameter of a single fibre (μm)</td>
<td>7.66</td>
<td>9.05</td>
</tr>
<tr>
<td>Fibre distribution</td>
<td>Closely spaced, wide distance between compact groups</td>
<td>Evenly spaced, almost even distance between smaller groups</td>
</tr>
<tr>
<td>Fibre volume fraction, V_f (%)</td>
<td>44</td>
<td>14</td>
</tr>
</tbody>
</table>

5.3.2 Tensile Tests on 3D-Printed Carbon Fibre (CF) Composites

A series of tensile tests was carried out on unidirectional and specially orthotropic [45/-45/-45/45/-45/45/-45]° 3D-printed carbon fibre (CF) composite specimens to characterise the 3D-printed CF composites.

5.3.2.1 Carbon Fibre (CF) Composite Specimens Preparation

Two sets of carbon fibre (CF) composites specimens were 3D-printed and tested: firstly unidirectional CF composites, [0°]s, and secondly specially orthotropic CF composites, with a [45/-45/-45/45/45/-45/-45]° lay-up. In practice, a balanced and symmetric lay-up was preferred, to ensure there would be no coupling between direct stress and shear strain, or the
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

direct and shear stiffnesses were uncoupled (a balanced lay-up, thus $A_{12} = A_{23} = 0$), and also no bending-membrane coupling, or the in-plane and flexural stiffness sub-matrices were uncoupled (a symmetric lay-up, hence $B$ matrix was zero) (Matthews and Rawlings, 1999, Katwijk, 1981). In addition to these two conditions, another desired behaviour of a composite lay-up was no bending-twisting coupling, or $D_{13} = D_{23} = 0$. All of these couplings could be avoided if the lay-up was a specially orthotropic laminates, which was the approach taken in this smart composite wing research. The load/moment-strains/twisting relationship for the specially orthotropic laminates, which consisted of the $A$, $B$ and $D$ matrices, is shown by Eq. 5.1 (Katwijk, 1981). The three conditions for the specially orthotropic composite, e.g. a $[45/-45/45/-45/45/45/-45]$ lay-up, were confirmed to be satisfied, by checking the $A$, $B$ and $D$ matrices in the ESDU 81047 program, and were also cross-checked in the Laminate Analysis Program 4.1 (Anaglyph Ltd, 1995-2000).

$$
\begin{bmatrix}
N_x \\
N_y \\
N_{xy} \\
M_x \\
M_y \\
M_{xy}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & 0 & 0 & 0 & 0 \\
A_{21} & A_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & A_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & D_{11} & D_{12} & 0 \\
0 & 0 & 0 & D_{21} & D_{22} & 0 \\
0 & 0 & 0 & 0 & 0 & D_{33}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_{xy} \\
-\frac{\partial^2 w}{\partial x^2} \\
-\frac{\partial^2 w}{\partial y^2} \\
-2\frac{\partial^2 w}{\partial x\partial y}
\end{bmatrix}
$$

(5.1)

Three CF composite specimens were 3D-printed and tested for each case. The 3D-printed CF composite specimens are shown in Fig. 5.20. The international standard test method for tensile properties of polymer matrix composite materials (ASTM D3039/D3039M-14) was used as a guideline in the tests. The length, width and thickness of the specimens were measured using a Vernier calliper with 0.01 mm accuracy. The measured geometry of the 3D-printed CF composite specimens and the ASTM recommended geometry are listed in Table 5.7. Note that the thickness of the specimens was 1 mm thicker than the ASTM recommended thickness, because the Markforged recommendation of using at least four onyx layers for each of the roof and floor of the 3D-printed CF composite parts, where each layer had a thickness of 0.125 mm. After 3D-printing, glass fibre tabs were bonded to both ends of the unidirectional CF composite specimens, using an Alcroyde 2011 adhesive. The fibre orientation of the glass fibre tabs was at 45° to the applied force direction or carbon fibre direction, and the length and thickness are included in the Table 5.7.
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

![Composite Corrugation](image)

Figure 5.20. 3D-printed carbon fibre composite specimens with (a) unidirectional [0°]ₘ and (b) specially orthotropic (45/-45/-45/45/45/45/45/-45)° lay-ups.

<table>
<thead>
<tr>
<th>Fiber layups</th>
<th>Unidirectional CF 8[0]°</th>
<th>Orthotropic CF [45/-45/-45/45/45/-45]°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimens</td>
<td>ASTM T01-1 T01-2 T01-3 Average</td>
<td>ASTM T02-1 T02-2 T02-3 Average</td>
</tr>
<tr>
<td>Length (mm)</td>
<td>250 250 250 250 250</td>
<td>250 250 250 250 250</td>
</tr>
<tr>
<td>Width (mm)</td>
<td>15.00 15.22 15.24 15.50 15.32</td>
<td>25 26.02 25.99 25.96 25.99</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>1.00 2.16 2.09 2.02 2.09</td>
<td>2.5 3.72 3.67 3.73 3.71</td>
</tr>
<tr>
<td>Tab length (mm)</td>
<td>56</td>
<td>N/A</td>
</tr>
<tr>
<td>Tab thickness (mm)</td>
<td>1.5</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 5.7. Geometry of 3D-printed carbon fibre composite specimens.

5.3.2.2 Tensile Tests Procedure

The 3D-printed CF composite specimens were tested on an INSTRON machine. A vertical displacement was applied on the specimens, with a stationary bottom fixture and a movable top fixture. The applied load was measured by a 50 kN load cell, while the strain was measured by an IMETRUM optical strain system. The strain was measured between two central points along the length of the specimens, with a gauge length of 100 mm. The frequency for the data collection on the INSTRON machine and the IMETRUM optical system was ensured to be equal, which was 10 Hz, to record 10 data points in a second. A standard head displacement rate of 2 mm/min was used in the tests. A preload of about 3 N was applied to all CF composite specimens.
5.3.2.3 Tensile Tests Results

Figure 5.21. Tensile stress-strain behaviour of (a) unidirectional [0°], and (b) specially orthotropic (45/-45/-45/-45/45/-45°) 3D-printed carbon fibre composites.

The tensile stress-strain results of the tensile tests on the unidirectional and specially orthotropic 3D-printed CF composites are shown in Fig. 5.21, and the conditions of the CF composite specimens after failure are depicted in Fig. 5.22. For the unidirectional composites, the results showed a linear stress-strain behaviour of the CF composites, until rupture or failure at a strain range of 1.27% to 1.34%. The tensile strain was directly obtained from the IMETRUM optical strain system, while the tensile stress was calculated by the applied force over the cross-sectional area of the CF composites. The actual cross-sectional area of the CF specimens (based on width and thickness in Table 5.7) was used in the calculation of the tensile stress. The area was about 32 mm² for the unidirectional CF composite specimens and about 96 mm² for the specially orthotropic CF composite specimens. The Young’s modulus, or the modulus of elasticity, was calculated by dividing the change in the tensile stress to the change...
in the tensile strain, between the two selected data points (at 0.001 and 0.003 strains). The calculated ultimate tensile strength and the Young’s modulus were about 50% less than the datasheet values. The reason of higher tensile strength and Young’s modulus in the datasheet was they were based on tensile tests on CF filament. The extracted mechanical properties of the 3D-printed CF composite are summarized in Table 5.8.

![Image](image.png)

**Figure 5.22.** Results of tensile tests on (a) unidirectional [0°]s and (b) specially orthotropic (45/-45/-45/45/45/-45/45)° 3D-printed carbon fibre composites.

**Table 5.8.** Mechanical properties of 3D-printed carbon fibre composites.

<table>
<thead>
<tr>
<th>Composite Material Properties</th>
<th>Specimen 1 (T01-1)</th>
<th>Specimen 2 (T01-2)</th>
<th>Specimen 3 (T01-3)</th>
<th>Average</th>
<th>Markforged Datasheet (2016)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate Tensile Load (kN)</td>
<td>12.4</td>
<td>11.5</td>
<td>12.2</td>
<td>12.0</td>
<td>N/A</td>
</tr>
<tr>
<td>Ultimate Tensile Strength (MPa)</td>
<td>377</td>
<td>363</td>
<td>389</td>
<td>376</td>
<td>700</td>
</tr>
<tr>
<td>Young’s Modulus (GPa)</td>
<td>26.8</td>
<td>26.2</td>
<td>28.4</td>
<td>27.1</td>
<td>54</td>
</tr>
<tr>
<td>Failure Strain (%)</td>
<td>1.34</td>
<td>1.27</td>
<td>1.30</td>
<td>1.30</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**5.3.2.4 Microstructure of 3D-Printed Unidirectional Carbon Fibre Composite after Tensile Tests**

After the tensile tests, the 3D-printed unidirectional (UD) carbon fibre (CF) composite specimens were cut, and the cross-section was observed under an optical microscope. The specimens were prepared in epoxy tubs and were polished before observation, with similar procedures described in Section 5.3.1. Fig. 5.23 shows the cross-section of the CF composite under a 5× zoom lense. It shows eight layers of CF, as numbered on the left side of the figure, and includes portions of the four layers of onyx on the top roof and bottom floor. The CF layers
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

consisted of groups of closed-spaced carbon fibres, as shown in white circles/dots, while the onyx layers consisted of widely-spaced but more evenly spaced carbon fibres. From the magnified microstructure of the CF, the average of total thickness of the CF layers was measured as 1199 μm, or 1.20 mm. In average, the width and thickness of a single strand of the 3D-printed carbon fibre were approximately 888 μm (0.89 mm) and 131 μm (0.13 mm), respectively.

![Cross-section view of the 3D-printed unidirectional (UD) carbon fibre (CF) composite.](image)

From the Fig. 5.23, voids with different sizes existed with the longest length of 172 μm. The locations of the voids seemed random, but they were most likely to form in between the adjacent 3D-printed CF layers. The figure shows the debonding between the CF layers and the bottom floor onyx layers, which resulted from the tensile test. A comparison between UD CF composite specimens with and without debonding is depicted in Fig. 5.24. From observation, one assumption could be made, that was the debonding between the carbon fibre layers and the top/bottom onyx layers, could occur because of the existence of voids near the boundary of the two different materials.

Fig. 5.25a shows a 20 times (20× lense) magnified image of the CF specimen in Fig. 5.24a. It shows three CF layers across the thickness and about one-and-half strands across the width. Fig. 5.25b shows a 50 times (50× lense) magnified image of the same specimen, from which the measured average radius of a single micro CF was approximately 3.72 μm. From the microstructure analysis of the 3D-printed CF composite specimens, the total thickness of 8 CF layers, the width and thickness of a single strand CF, and the diameter of a single micro CF were measured, averaged, and summarised in Table 5.9.
The fibre volume fractions of the CF composite specimens were also calculated and recorded in Table 5.9. The fibre volume fractions were calculated based on the 50 times (50× lens) magnified image, by dividing the total area of the micro carbon fibres with the image area. The total area of the micro carbon fibres was calculated by summing the total number of micro fibres, and then multiplying with cross-sectional area of a single micro carbon fibre, which was assumed as a circular cross-section with a diameter listed in the table.
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

![Magnified images (a) 20× and (b) 50× of the 3D-printed unidirectional (UD) carbon fibre (CF) specimen.](image)

**Figure 5.25.** Magnified images (a) 20× and (b) 50× of the 3D-printed unidirectional (UD) carbon fibre (CF) specimen.

<table>
<thead>
<tr>
<th>CF composite specimens</th>
<th>CF T01-1</th>
<th>CF T01-2</th>
<th>CF T01-3</th>
<th>AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total thickness CF (μm)</td>
<td>1199</td>
<td>1136</td>
<td>1229</td>
<td>1188</td>
</tr>
<tr>
<td>Width single strand CF (μm)</td>
<td>826</td>
<td>888</td>
<td>880</td>
<td>865</td>
</tr>
<tr>
<td>Thickness single strand CF (μm)</td>
<td>132</td>
<td>131</td>
<td>136</td>
<td>133</td>
</tr>
<tr>
<td>Diameter (μm)</td>
<td>7.74</td>
<td>7.44</td>
<td>7.16</td>
<td>7.45</td>
</tr>
<tr>
<td>Fibre volume fraction, (V_f) (%)</td>
<td>54.7</td>
<td>45.5</td>
<td>45.1</td>
<td>48.4</td>
</tr>
</tbody>
</table>

**Table 5.9.** Results of microstructure analysis on 3D-printed unidirectional carbon fibre composites.

5.3.3 3D-printed CF Composite Corrugated Structure

The carbon fibre (CF) composite corrugated structure with a rigid trailing edge section was 3D-printed using the Markforged 3D-printer, Mark Two. The 3D-printer was capable of printing carbon, fibreglass, or Kevlar composites, with nylon or onyx as a plastic material. The onyx material was made of chopped carbon fibre (less than 1 mm length) and nylon matrix. The CF and onyx materials were chosen as the printed materials for the composite corrugated...
structure with a rigid trailing edge section, because of higher flexural stiffness (Markforged, 2016).

Regardless of the above-mentioned advantages of the 3D-printing technique, it had critical limitations in fibre distribution, depending on the structural dimension and the design of the structure. For the design of the composite corrugated structure with a rigid trailing edge section, and with a dimension of 210×31.25×3 mm, as shown in Fig. 5.26, the CF layers could not be printed layer-by-layer uniformly in the ±45/90° directions, as initially investigated in the FEA simulations in the previous Section 5.1.2. This was because the 3D-printer printed the CF composite layers on the plane parallel to the printing bed. If the CF composite corrugated structure was printed in a position 1 shown in Fig. 5.27a, only the flat surface would have fibres (Fig. 5.27b) and the inclined walls would have no fibres at all but only filled with onyx triangular or honeycomb fills (Fig. 5.27c). The blue lines and the white lines on the composite corrugated structure represented the CF and onyx layers, respectively.

![Figure 5.26. A model of 3D-printed CF composite corrugated structure with a rigid trailing edge section.](image)

Therefore, to have uniform carbon fibres along the corrugation, the corrugated structure had to be printed in position 2, with the cross-section of the corrugation parallel to the printing bed (Fig. 5.28a). Two types of fibre fill were available on Markforged Mark Two 3D-printer, which were isotropic fibre fill and concentric fibre fill. The isotropic fibre fill produced a part that was relatively stiff in all directions, depending on the fibre directions (e.g. 0°/90°/±45°), while the concentric fibre fill was good for reinforcing the walls of the printed part. For the corrugated structure printed in position 2 as shown in Fig. 5.28a, the concentric fibre fill was chosen to obtain a uniform fibre distribution along the corrugation, as shown in Fig. 5.28b. For the particular printing position, the minimum thickness of the corrugated section was 3 mm. If the thickness was less than 3 mm, the printed layers would not be filled with fibres, but only filled with the onyx material. For the similar reason, the isotropic fibre fill was not chosen. The wall, roof and floor layers, which were printed with onyx material by default, were kept to a
minimum one layer, to save the total weight of the 3D-printed part, which was critical in aero structures.

![Figure 5.27](image1)

**Figure 5.27.** 3D-printing position 1, and the carbon fibre and onyx distributions.

![Figure 5.28](image2)

**Figure 5.28.** 3D-printing position 2, and the carbon fibre and onyx distributions.

The 3D-printed carbon fibre (CF) composite corrugated structure is illustrated in Fig. 5.29c. The CF composite corrugated structure had 10 half-hexagonal cells with a rear rigid trailing edge section. The flat vertical plate in front of the corrugated section was meant to be attached to the spar web of the composite D-nose spar. In the previous finite element analysis (FEA) simulations presented in Chapter 5.1, the total numbers of half-hexagonal cells were 12
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

cells. However, after several iterations, the maximum limit of 10 cells was finalised, because of the space required for mechanical screws/bolts and nuts, and 3D-printed washers, used to constrain the SMA wires. The 3D-printed onyx washers are shown in Fig. 5.30.

The earliest design of the CF composite corrugated structure is shown in Fig. 5.29a, with a smooth wavy corrugation shape instead of a half-hexagonal shape. The reason behind the smooth wavy corrugation shape was to keep the manufacturing options open with a possible traditional composite moulding technique, to avoid kinking problem of the CF composite in such case. Fig. 5.29b shows the second iteration of the CF composite corrugated structure, with 11 flat half-hexagonal cells. The curved surface was changed to flat surface to provide a smooth and stable surface for the mechanical screws and washers. The figure shows that no SMA wire could be fixed at the first and last cells, because the screws and washers could not be fitted onto the small spaces next to the cells. For this reason, the maximum number of half-hexagonal cells had to be limited to 10 cells, as depicted in Fig. 5.29c.

![Figure 5.29](image)

**Figure 5.29.** Design iterations of the 3D-printed carbon fibre (CF) composite corrugated structure.

![Figure 5.30](image)

**Figure 5.30.** 3D-printed onyx washers.
5.3.3.1 Microstructure of 3D-Printed Carbon Fibre (CF) Composite Corrugated Structure

A small section of the carbon fibre (CF) composite corrugated structure was 3D-printed. It was then carefully prepared and polished, and was observed under the optical microscope, to examine the distribution of the carbon fibres and onyx in the 3D-printed part. The printing position of the small section or one half-hexagonal cell was similar to the printing position used to print the CF composite corrugated structure, which was shown in the previous Fig. 5.28 and Fig. 5.29. The captured images at three different locations, using five lenses (1.25×, 5×, 20×, 50×, and 100× zoom), are illustrated in Fig. 5.31.

The images show that the average thickness of one layer of the 3D-printed carbon fibre was 1068.85 μm, or 1.07 mm, with a range of 944.49 μm to 1203.87 μm. In contrast, the 3D-printed onyx layers had an average width/thickness of 195.61 μm, or 0.20 mm, with a range of 132.88 μm to 248.90 μm. These showed that the 3D-printing process caused expansion to the CF (diameter/width from 0.39 mm before 3D-printing, to 1.07 mm after 3D-printing) and contraction to the onyx material (diameter/width from 1.75 mm before 3D-printing, to 0.20 mm after 3D-printing).

The observation of the results also revealed that the onyx layers had much shorter fibres compared to the carbon fibre layers. The average length of the short fibres in the onyx material was 53.42 μm, with a range of 44.08 μm to 67.25 μm. In contrast, the average length of the fibres in the 3D-printed carbon fibre layers was about 168.34 μm, with a range of 45.56 μm to 487.85 μm. This was surprisingly unexpected, as the initial assumption, before the beginning of the 3D-printing work of the CF composite corrugated structure and the CF composite D-nose spar, was that the carbon fibres were continuous along the printing directions.
Figure 5.31. Microstructures of a small section (one half-hexagonal cell) of the 3D-printed carbon fibre composite corrugated structure.
5.3.4 3-Point Bending Tests on 3D-Printed Carbon Fibre (CF) Composites

To determine the flexural properties of the carbon fibre (CF) composite corrugated structure, which were 3D-printed with the aerofoil cross-section parallel to the printing bed (Fig. 5.32a), CF composite specimens with similar thickness and fibre orientation (concentric fibre layers mode) (Fig. 5.32b), were 3D-printed and tested under a 3-point bending load. The blue and white lines in the figure were the CF layers and the onyx layers, respectively. The standard test method for flexural properties of polymer matrix composite materials (ASTM D7264/D7264M-15) was used as a reference. Procedure A of the ASTM standard was chosen, which was a 3-point loading with a centre loading in a simply supported beam. The radii of the loading nose and the supports were 5 mm, and the testing speed was set to 1.0 mm/min crosshead movement.

![Figure 5.32](image)

**Figure 5.32.** 3D-printed (a) CF composite corrugated structure and (b) CF composite specimens for 3-point bending tests.

The CF composite specimens had a similar thickness as the CF composite corrugated structure, which was 3 mm. The length and width of the CF composite specimens were 115.2 mm and 13 mm, respectively. With the standard support span-to-thickness ratio of 32:1, a support span of 96 mm was constantly used. The total length of the CF composite specimens was 20% longer than the support span, which was 115.2 mm. Theoretically, the support span must be long enough compared to the thickness, so that the failure of the CF composite beam would be caused by the resultant bending moment, and not by the shear force. The 3D-printed CF composite specimens were kept in a vacuum oven until the testing time. Before testing, the width, thickness and length of the specimens were measured with a Vernier calliper having 0.01 mm accuracy, and were recorded in Table 5.10.

The results of the applied force and centre-displacement of the 3D-printed carbon fibre (CF) composite specimens subjected to a 3-point bending load, are illustrated in Fig. 5.33. For
comparison, the load-displacement measured by the IMETRUM optical system and the INSTRON machine were plotted in the same figure. The curves on the left side of the y-axis are the applied force plotted against the centre-displacement measured by the IMETRUM optical system, while the curves on the right side of the y-axis are the applied force plotted against the crosshead movement of the INSTRON machine. The results show almost similar load-displacement characteristic between the two measurement systems, with the displacements measured by the IMETRUM optical system were slightly lower than the ones measured by the INSTRON machine, except for the third specimen (CF3). The maximum force, and the maximum vertical centre-displacement at failure, were about 207.3 N and 15.6 mm, respectively. Using the applied force and vertical centre-displacement measured by the IMETRUM optical system, the maximum flexural stress and the maximum flexural strain were calculated.

Table 5.10. Dimension of the 3D-printed composite specimens for 3-point bending test.

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Width (mm)</th>
<th>Thickness (mm)</th>
<th>Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.14</td>
<td>3.05</td>
<td>115.53</td>
</tr>
<tr>
<td>2</td>
<td>13.14</td>
<td>2.99</td>
<td>115.44</td>
</tr>
<tr>
<td>3</td>
<td>13.12</td>
<td>3.02</td>
<td>115.43</td>
</tr>
<tr>
<td>4</td>
<td>13.12</td>
<td>3.01</td>
<td>115.48</td>
</tr>
<tr>
<td>5</td>
<td>13.12</td>
<td>3.03</td>
<td>115.35</td>
</tr>
</tbody>
</table>

![Figure 5.33. Force-displacement curve of the 3D-printed carbon fibre composites under a 3-point bending load.](image)

The maximum flexural stress, which was the maximum stress at the outer surface occurred at mid-span of the composite beam, was calculated using the following equation

\[ \sigma_{\text{flexural}} = \frac{3PL}{2bh^2} \]  (5.2)

199
where \( \sigma, P, L, b, \) and \( h \), are the flexural stress at the outer surface at mid-span, the applied vertical load, the support span, the composite beam width and thickness, respectively.

The maximum flexural strain at the outer surface which occurred at mid-span of the composite beam, was calculated using the following equation

\[
\varepsilon_{\text{flexural}} = \frac{6\delta h}{L^2} \tag{5.3}
\]

where \( \varepsilon, \delta, L, \) and \( h \), are the maximum strain at the outer surface at mid-span, the mid-span/centre-beam deflection, the support span, and the composite beam thickness, respectively. The derivation of the flexural stress and the flexural strain equations are shown in Appendix D. The maximum flexural stress was plotted against the maximum flexural strain, as shown in Fig. 5.34. Out of the five tests, only four results were plotted and used in the calculation of the flexural properties of the 3D-printed CF composites. The data of specimen 3 (CF3) was omitted because of abnormality of the force-displacement and flexural stress-strain curves, compared to the other four specimens.

![Figure 5.34](image_url)

**Figure 5.34.** Stress-strain curve of 3D-printed carbon fibre composites under 3-point bending load.

Theoretically, the above formula for maximum flexural stress and flexural strain were strictly applied to materials for which the stress-strain relationship was linearly proportional up to the point of rupture and for which the strain or displacement was small (ASTM D7264/D7264M-15, 2015). The equations should be valid for data comparison up to the maximum fibre strain of 2%. The equations were also derived based on the assumption of homogenous beam theory, where the modulus was constant throughout the thickness and in all directions. This could cause discrepancies when the results were compared with an analytical or FEA numerical solution.
From the flexural stress-strain curves, the flexural chord modulus of elasticity was calculated within the suggested strain range of 0.002 (from 0.001 to 0.003 strain), using the following equation

$$E_{f}^{chord} = \frac{\Delta \sigma}{\Delta \varepsilon}$$

(5.4)

where $E_{f}^{chord}$, $\Delta \sigma$, and $\Delta \varepsilon$, are the flexural chord modulus of elasticity, the difference in the flexural stress between the two selected strains, and the difference between the two selected strains, respectively. The second flexural modulus was calculated using the same formula, with a similar strain range of 0.002, from 0.012 to 0.014. The calculated flexural properties of the 3D-printed CF composites were tabulated in Table 5.11. They were averaged and compared to the manufacturer data. In average, the flexural strength, the flexural strain at break, and the flexural modulus were 227 MPa, 2.98% and 22.8 GPa, respectively.

<table>
<thead>
<tr>
<th>Specimens</th>
<th>CF 1</th>
<th>CF 2</th>
<th>CF 4</th>
<th>CF 5</th>
<th>Average ( \bar{x} )</th>
<th>Markforged Datasheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexural Strength (MPa)</td>
<td>244</td>
<td>227</td>
<td>221</td>
<td>216</td>
<td>227.00</td>
<td>470</td>
</tr>
<tr>
<td>Flexural Strain at Break (%)</td>
<td>3.07</td>
<td>2.80</td>
<td>3.03</td>
<td>3.03</td>
<td>2.98</td>
<td>1.2</td>
</tr>
<tr>
<td>Flexural Modulus (GPa)</td>
<td>23.3</td>
<td>24.2</td>
<td>22.2</td>
<td>21.7</td>
<td>22.85</td>
<td>51</td>
</tr>
<tr>
<td>Second Flexural Modulus (GPa)</td>
<td>6.47</td>
<td>6.46</td>
<td>6.84</td>
<td>7.06</td>
<td>6.71</td>
<td>N/A</td>
</tr>
<tr>
<td>Carbon CFF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Onyx</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the flexural stress-strain diagram shown in Fig. 5.34, it is important to ensure that the composite wing operate within the linear elastic region, e.g. before the first drop of the flexural stress, or below 0.87% strain. The conditions of four specimens of the 3D-printed CF composites in this linear elastic region, for example at 0.6% and 0.8% strain, are illustrated in Fig. 5.35. The pictures show that no failure, such as delamination, visually occurred within this elastic region.

The conditions of the 3D-printed carbon fibre composites before and after the 3-point bending tests are shown in Fig. 5.36. Observations of the composite specimens showed that all specimens failed at the loading nose, with a clear delamination occurred for specimens 2 and 4 (CF2 and CF4).
Figure 5.35. Conditions of the 3D-printed carbon fibre composites at 0.6% and 0.8% flexural strain.
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

Final states at the end of tests

**Figure 5.36.** 3D-printed CF composites under a 3-point bending load: initial state and states at failure.
5.3.4.1 Analytical Solution of Flexural Stiffness of 3D-Printed CF Composite

To roughly check the validity of the flexural tests or 3-point bending tests (3PBTs), an analytical solution was developed to calculate the flexural stiffness of the 3D-printed carbon fibre (CF) composite beam. From the microstructure of the 3D-printed CF composite in Section 5.3.3.1, the individual thickness of the outermost onyx layers, carbon fibre layers and the middle weak bond layer, was identified as 0.25 mm, 1.075 mm, and 0.35 mm, respectively. The middle weak bond layer was assumed to be nylon, as it was the base material of both the CF and onyx materials. The flexural modulus of the carbon fibre, onyx, and nylon were 51 GPa, 2.9 GPa and 0.84 GPa, respectively.

\[
\Sigma E I = E_f \left( \frac{b t^3}{12} \right)
\]  

\[
2 \times 51 \times \left[ \frac{b \times 1.075^3}{12} + b \times 1.075 \times 0.7125^2 \right] + 2 \times 2.9
\times \left[ \frac{b \times 0.25^3}{12} + b \times 0.25 \times 1.375^2 \right] + 0.84 \times \left[ \frac{b \times 0.35^3}{12} \right] = E_f \left( \frac{b \times 3^3}{12} \right)
\]

\[
E_f = 30.66 \text{ GPa} \tag{5.6}
\]

The calculated flexural stiffness was found as 30.7 GPa, which was about 34% higher than the experimental results (see Table 5.11).

If the Young’s modulus of 41.0 GPa was used, which was obtained from tensile tests on 3D-printed CF composites with a similar fibre orientation as the specimens tested under 3-point bending tests (Fig. 5.32b), the flexural modulus was calculated as

\[
\Sigma E I = E_f \left( \frac{b t^3}{12} \right)
\]

\[
2 \times 41 \times \left[ \frac{b \times 1.075^3}{12} + b \times 1.075 \times 0.7125^2 \right] + 2 \times 2.9
\times \left[ \frac{b \times 0.25^3}{12} + b \times 0.25 \times 1.375^2 \right] + 0.84 \times \left[ \frac{b \times 0.35^3}{12} \right] = E_f \left( \frac{b \times 3^3}{12} \right)
\]

\[
55.9909 = E_f(2.25)
\]
The calculated flexural stiffness was found as 24.9 GPa, which was closer to the experimental results (see Table 5.1).

\[
E_f = 24.88 \text{ GPa} 
\]  \hspace{1cm} (5.7)

Figure 5.37. A comparison between the unidirectional carbon fibre composite specimens (a) with a lay-up following the ASTM standard and (b) as a representative of the CF composite corrugated structure.

The UD CF composite specimen with a similar fibre orientation or a similar type of fibre fill as the CF composite corrugated structure, was compared with the UD CF composite specimen in the earlier tensile test (following ASTM standard, presented in Section 5.3.2), as illustrated in Fig. 5.37. Fig. 5.37a shows the later and Fig. 5.37b shows the former. The difference was due to how the specimens were oriented on the printing bed, in addition to the
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

Variation of the thickness of the specimens and the types of the fibre fill. The thickness of the composite specimens in TEST 1 and TEST 2 was 2 mm and 3 mm, respectively. The 3 mm thickness of specimens in TEST 2 was referred to the width of Fig. 5.37b. The types of the fibre fill were *isotropic* fibre and *concentric* fibre, respectively. The total of CF strands along the width was 15 strands and 2 strands, respectively. The total of CF layers along the thickness/height was 8 layers and 118 layers, respectively. Therefore, the total of CF strands in the composites of TEST 1 and TEST 2 were 120 and 236, respectively.

5.3.5 Experimental Tests of SMA-Actuated CF Composite Corrugated Structure

The carbon fibre (CF) composite corrugated structure with a rigid trailing edge section, shown in Fig. 5.38, was actuated by a single NiTi shape memory alloy (SMA) wire which was positioned along the aerofoil chord. Several SMA wires with different diameters and pre-strains were used for the actuation, which were 0.2 mm diameter with a 4.77% pre-strain, and 0.5 mm diameter with a 1.68% pre-strain. The pre-strains of the SMA wires were achieved by using an INSTRON machine with a cord and yarn grips for the SMA wire extension, and an IMETRUM optical system for the strain measurement.

![Experimental setup of the SMA-actuated composite morphing corrugated structure.](image)

Figure 5.38. Experimental setup of the SMA-actuated composite morphing corrugated structure.

The CF composite corrugated structure with a rigid trailing edge section and 10 half-hexagonal corrugations, was clamped on the front flat plate, and a single SMA wire was attached to the structure using mechanical screws and nuts. Both ends of the SMA wire were crimped at the first and last screws, and the other nine screws were used for the wire alignment. Prior to the tests, other methods of attaching the SMA wire onto the CF composite corrugated structure were investigated, including fixing the SMA wire between top and bottom washers at all screws positions, and turning and fixing the SMA wire around the first and the last screws. The SMA wire was heated by resistive heating, by supplying an electrical current to the SMA wire using a power supply.
Measurement of the trailing edge deflection/displacement using the IMETRUM optical system required calibration of the system. The calibration was done by defining the height and the length of the rigid trailing edge section. The dimensions were measured by a Vernier calliper with a 0.01 mm precision, and they were defined in the optical system as 11.19 mm and 42.09 mm, respectively.

![Image of measurement devices](image1)

**Figure 5.39.** Temperature measurement devices: (a) Ti400 FLUKE thermal imager, (b) K-type thermocouple with COMARK thermometer, and (c) NI 9211 thermocouple module.

Several devices were commercially available to measure the temperature of the SMA wire, such as a thermal imager and a thermocouple. Thermal imager, such as a Ti400 FLUKE thermal imager (Fig. 5.39a), measured temperature based on emissivity of the observed material. Thermocouples, such as a K-type thermocouple with a COMARK N1001 thermometer (Fig. 5.39b) and a NI 9211 thermocouple module from National Instruments (Fig. 5.39c), measured temperature based on two different metals. The NI 9211 thermocouple module from National Instrument was used to measure the temperature of the SMA wires in the experiments of the morphing composite corrugated structure.

As shown in a block diagram in Appendix D, the NI 9211 device was used and a temperature measurement system was developed to measure the temperature of the SMA wires, due the following main reasons:

(i) Multiple thermocouples were used to measure and monitor the SMA temperatures at the same time. Four thermocouples were fitted into a single module, and a total of eight modules could be installed in a single DAQ deck, which make a total maximum of 32 thermocouples. This could be very useful in morphing applications that use multiple SMA wires for actuation.

(ii) The SMA temperatures were continuously monitored and recorded in time domain, which would be impossible with the other two methods/devices.

(iii) NI LabVIEW codes were programmable. The developed programme to measure and monitor the SMA temperatures could be coupled with a power input module to supply
current/voltage to the SMA wires. This would allow a better control of the composite morphing wing, as the SMA temperatures could be monitored and used as input to provide control feedback in form of supplied current/voltage, provided the relation between the SMA temperatures and the target actuated displacement was established. This could also prevent overheating of the SMA wires. This coupling aspect, however, was not able to be covered in the composite morphing wing research, and was strongly recommended for a future single complete research.

For each SMA wire, three types of tests were conducted. Firstly, a single heating-cooling cycle. Secondly, ten heating-cooling cycles. Thirdly, 30 heating-cooling cycles.

(i) In the first test, a single heating-cooling cycle, the electrical current was increased to the maximum by 0.1A increment on heating, and it was decreased to zero by 0.1A decrement on cooling. On each 0.1A increment/decrement, the supplied current was maintained for one minute before the next increment/decrement.

(ii) In the second test, ten heating-cooling cycles, the electrical current was increased to the maximum immediately, then the supplied current was maintained for five minutes, before the current was reduced to zero. Five minutes were also the waiting time on cooling, for the remaining 9 heating cooling cycles.

(iii) In the third test, 30 heating-cooling cycles, the electrical current was increased to the maximum immediately, and the waiting time was only one minute before immediate decrease to zero with the same one minute waiting time. Similar procedures were repeated for the remaining 29 cycles.

5.3.5.1 Test A – SMA Diameter 0.2 mm, 4.77% Pre-Strain

The first series of tests was the actuation of the CF composite corrugated structure using a shape memory alloy (SMA) wire with a diameter of 0.2 mm and a pre-strain of 4.77%. For the test with 10 heating-cooling cycles and five minutes waiting time, the results of the trailing edge (TE) deflection and SMA wire temperature are shown in Fig. 5.41, in a blue solid line and a red dotted line, respectively. The achieved TE deflection at the end of the heating stage (at a supplied current of 0.6A and a supplied voltage of 3.4V, or a supplied power of 2.04W) was approximately 1.10 mm, and the TE deflection at the end of cooling stage was about 0.08 mm due to the residual stress and residual strain of the SMA. This was expected from the previous extensive FEA simulations on SMA-actuated structures (spring, beam, pre-curved
corrugated plates, aluminium and composite aerofoils). The maximum and minimum temperatures at the end of the heating and cooling stages were about 64 °C and 26 °C, respectively.

**Figure 5.40.** Initial (top) and final (bottom) configurations of the SMA-actuated CF composite corrugated structure with a SMA wire having a diameter of 0.2 mm and a pre-strain of 4.77% (10-cycles test).

For the test with 30 heating-cooling cycles and one minute waiting time, Fig. 5.43 shows the results of the trailing edge (TE) deflection and the SMA wire temperature, in a blue solid line and a red dotted line, respectively. The maximum TE deflection converged at about **1.02 mm** after the 4th cycle. On the cooling stage, the TE deflection was about **0.18 mm**. The maximum and minimum SMA temperatures at the end of the heating and cooling stages were about 60 °C and 30 °C, respectively.

**Figure 5.42.** Initial (top) and final (bottom) configurations of the SMA-actuated CF composite corrugated structure with a SMA wire having a diameter of 0.2 mm and a pre-strain of 4.77% (30-cycles test).
The temperature profile (the red dotted line) of the 30 cycles test had a sharp shape, compared to the shape of the 10 cycles test, because the one minute waiting time in the 30 cycles test was a little short for the SMA temperature to converge. If the waiting time were five minutes, as in the 10 cycles test, it could be observed that the SMA temperature started to converge at the end of both heating and cooling stages. However, from the perspective of the achieved trailing edge (TE) deflection, increasing the waiting time from one minute to five minutes did not give a huge advantage, as the TE deflection only increased by 7.8% (from 1.02 mm to 1.10 mm).

5.3.5.2 Test B – SMA Diameter 0.5 mm, 1.68% Pre-Strain

The second series of tests was the actuation of the CF composite corrugated structure using a shape memory alloy (SMA) wire with a diameter of 0.5 mm and a pre-strain of 1.68%. For the 10-cycles test with five minutes waiting time, the trailing edge (TE) deflection and the SMA wire temperature are plotted in Fig. 5.45, in a blue solid line and a green dotted line, respectively. The temperatures of the mechanical screw and the CF composite corrugated structure were also plotted, in yellow and red dotted lines, respectively. TE deflections of about 6.0 mm and 0.3 mm were observed at the end of the heating stage (at a supplied power 5.75W – 2.5A, 2.3V) and the cooling stage, respectively. The highest and lowest SMA temperatures at the end of the heating and cooling stages were about 110 °C and 30 °C, respectively. The temperatures of the mechanical screw and the CF composite corrugated structure also increased on heating of the SMA wire, to the maximum of 50 °C and 40 °C, respectively. They both decreased to about 30 °C on cooling of the SMA wire. The cooling was achieved by natural
convection, without any external force-cooling devices, such as a fan or thermoelectric modules.

![Figure 5.44. Initial (top) and final (bottom) configurations of the SMA-actuated CF composite corrugated structure with a SMA wire having a diameter of 0.5 mm and a pre-strain of 1.68% (10-cycles test).](image)

For the 30-cycles test with one minute waiting time, the trailing edge (TE) deflection and the temperatures of the shape memory alloy (SMA) wire, the mechanical screw and the CF composite corrugated structure are plotted in Fig. 5.47, in a blue solid line, and green, yellow, and red dotted lines, respectively. The maximum and minimum TE deflections converged after the 5th cycle, at about 5.2 mm and 0.9 mm, at the end of the heating stage (at a supplied power 5.75W – 2.5A, 2.3V) and the cooling stage, respectively. The highest and lowest SMA temperatures at the end of the heating and cooling stages were about 98 °C and 39 °C, respectively. The temperatures of the mechanical screw and the CF composite corrugated structure also increased on the heating of the SMA wire, to the maximum of 41 °C and 36 °C, respectively. On cooling of the SMA wire, the temperatures decreased to the minimum of about 39 °C and 34 °C, respectively.

![Figure 5.45. Trailing edge deflection and SMA wire temperature for the 10-cycles test – SMA wire with 0.5 mm diameter and 1.68% pre-strain.](image)
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

Figure 5.46. Initial (top) and final (bottom) configurations of the SMA-actuated CF composite corrugated structure with a SMA wire having a diameter of 0.5 mm and a pre-strain of 1.68% (30-cycles test).

Figure 5.47. Trailing edge deflection and SMA wire temperature for the 30-cycles test – SMA wire with 0.5 mm diameter and 1.68% pre-strain.

The results of the trailing edge deflections of the SMA-actuated CF composite corrugated structure are summarised in Table 5.12, with the SMA dimension and pre-strain, and the supplied power.

Table 5.12. Experimental trailing edge deflections of the SMA-actuated CF composite corrugated structure.

<table>
<thead>
<tr>
<th>SMA Diameter (mm)</th>
<th>SMA Pre-strain (%)</th>
<th>Supplied Power (W)</th>
<th>Experimental Trailing Edge Deflections (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>10-Cycles Test</td>
</tr>
<tr>
<td>0.2</td>
<td>4.77</td>
<td>2.04W (0.6A, 3.4V)</td>
<td>1.10</td>
</tr>
<tr>
<td>0.5</td>
<td>1.68</td>
<td>5.75W (2.5A, 2.3V)</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Slight inconsistency of the results of the trailing edge deflections on few cycles (i.e. 15th and 19th cycle in Fig. 5.43, and 7th cycle in Fig. 5.45) was identified due to the time lagging, the speed of supplying/cutting the electrical power, and the overshoot of the electrical current. The time lagging was related to the very few seconds between turning on the knob of the power supply, starting the stopwatch, collecting and recording data, stopping the stopwatch, and turning off the knob of the power supply. Cumulative of the time lagging between these steps resulted in a total of a few seconds delay, which might increase the SMA temperature, and consequently increased the trailing edge deflections. Slight overshoot of the supplied electrical
current (i.e. 2.51A/2.52A instead of exactly 2.50A for the 0.5mm-diameter SMA wire) on the heating stage, was also a contributing factor to the inconsistency.

The effect of the time lagging on the results of the trailing edge (TE) deflections was more obvious when the TE deflection in test A (0.2 mm SMA diameter and 4.77% pre-strain) was compared with the TE deflection in test B (0.5 mm SMA diameter and 1.68% pre-strain), for both 10 cycles and 30 cycles tests. This is shown in Fig. 5.48a and Fig. 5.48b, respectively. The figures show that the total time taken by the SMA-actuated CF composite corrugated structure in test B for both 10-cycles and 30-cycles tests, exceeded the total time taken by the SMA-actuated CF composite corrugated structure in test A, even though the same waiting time was applied between cycles. This was due to the slightly more time taken to supply current to the larger diameter SMA wire, 2.5A, if compared to the smaller diameter SMA wire, 0.6A. It was important that the current knob was turned slowly when increasing the supplied current, to avoid oversupply of the current.

![Figure 5.48](image_url)

**Figure 5.48.** Comparison of trailing edge deflections of the SMA-actuated CF composite corrugated structure, between test A (0.2 mm SMA diameter and 4.77% pre-strain) and test B (0.5 mm SMA diameter and 1.68% pre-strain), for the (a) 10-cycles test and (b) 30-cycles test.
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

5.3.6 Control Surface Area of the Composite Morphing Wing

A typical aileron area of an aircraft wing covered about 50-90% or 50-100% of the wing span (Raymer, 2012). The additional 10% at the wing tips gave little control effectiveness because of vortex flow at the wing tips. From Fig. 6.3 in the reference (Raymer, 2012), for an aileron span-to-wing span ratio \( \frac{b_{aileron}}{b_{wing}} \) of 0.5, the proposed chord ratio \( \frac{c_{aileron}}{c_{wing}} \) was about 0.16. In the composite morphing wing research, for a wing with a 2.5 m span and 300 mm chord length, the ratios resulted in an aileron span of 1.25 m and an aileron chord of 48 mm, which resulted in a control surface area of 60,000 mm\(^2\). However, the proposed composite morphing wing had a morphing section located at 64% aft of the wing chord. This resulted in an aileron chord length of 192 mm. To maintain the same control surface area of 60,000 mm\(^2\), the aileron span was calculated as 312.5 mm. Therefore, the span of the morphing section was decided as one-fourth of half of the total wing span, which was 312.5 mm. The above descriptions are shown in the following calculations.

For a span ratio of

\[
\frac{b_{aileron}}{b_{wing}} = 0.5
\]

the proposed aileron chord ratio was

\[
\frac{c_{aileron}}{c_{wing}} = 0.16
\]

For the currently investigated composite morphing wing, with a total semi-span of 2.5 m and a chord length of 300 mm, the aileron span and chord were calculated as

\[
b_{aileron} = 0.5 \times b_{wing} = 0.5 \times 2.5 \text{ m} = 1.25 \text{ m}
\]
\[
c_{aileron} = 0.16 \times c_{wing} = 0.16 \times 300 \text{ mm} = 48 \text{ mm}
\]

The control surface area was then

\[
A_{control \ surface} = b_{aileron} \times c_{aileron} = 60,000 \text{ mm}^2
\]

However, the composite morphing wing was designed with a rigid, stiff and strong composite D-nose section, which covered 36% front of the wing chord, and a flexible morphing section which covered 64% aft of the wing chord. The chord length of the flexible morphing section was calculated as

\[
c_{aileron} = 0.64 \times c_{wing} = 192 \text{ mm}
\]
To maintain the above calculated control surface area with the new aileron (morphing part) chord, the new span of the flexible morphing section was calculated as

\[
b_{\text{ailer}} = \frac{A_{\text{control surface}}}{c_{\text{ailer}}} = \frac{60,000 \text{ mm}^2}{192 \text{ mm}} = 312.5 \text{ mm}
\]

Therefore, the final chord length and span of the morphing section were 192 mm and 312.5 mm, respectively, which span length was one-fourth of half of the total wing semi-span. This is illustrated in Fig. 5.49 in the next section, as the area of the morphing section was required to estimate the aerodynamic pressure load acting on it.

### 5.3.7 Static Load on Carbon Fibre (CF) Composite Corrugated Structure

The aerodynamic pressure loading acted on the morphing section (Fig. 5.49) of the composite wing was assumed to be taken by the carbon fibre (CF) composite corrugated structure. Realistically, some percentage of the aerodynamic load acted on the morphing section, was taken by the front composite D-nose spar, and some fraction of the aerodynamic load was taken by the rigid composite rear rib next to the morphing section. However, to simplify calculation and to consider extreme loading condition for the composite corrugated structure, it was assumed that all the aerodynamic pressure loading was taken by the CF composite corrugated structure.

![Smart composite morphing wing](image)

**Figure 5.49.** Smart composite morphing wing.

The pressure loading acted on the morphing section, was calculated using 2D XFOIL analysis, which was conducted on NACA 0012 aerofoil to obtain the chord wise pressure distribution. The aerofoil was set at 3° angle of attack (AOA), with the following flow conditions: air density \((\rho_\infty)\) of 1.225 kg/m³, pressure \((P_\infty)\) of \(1.0133 \times 10^5\) N/m², dynamic
viscosity (μ) of 17.89×10⁻⁶ kg/m.s, and cruise velocity (V∞) of 55.55 m/s or Mach number (M∞) of 0.1632. The Reynolds number for the 0.3m-chord morphing wing at these flow conditions was calculated as 1.141×10⁶. In XFOIL, the number of panels of the aerofoil was set to 240 panels, with a trailing edge/leading edge (TE/LE) panel density ratio of one. The XFOIL analysis was carried out with the above mentioned Reynolds number and Mach number.

The resulted pressure distribution on the NACA 0012 aerofoil is depicted in Fig. 5.50 and Fig. 5.51. Because 36% front of the composite morphing wing was made of the composite D-nose spar, and it was designed to be structurally stiff enough to resist the aerodynamic load, the pressure distribution acted on 36% front of the chord was excluded in the calculation of load acted on the composite corrugated structure.

![Figure 5.50. Pressure distribution on a NACA 0012 aerofoil at Re 1.141×10⁶, solved in XFOIL.](image)

Under the above mentioned flow conditions, at 36% chord, the pressure coefficients on the lower and upper surfaces of the aerofoil were -0.15 and -0.45, respectively. This resulted in a difference of pressure coefficient of 0.30. At 85% chord, the pressure coefficients on the lower and upper surfaces of the aerofoil were approximately 0 and -0.04, respectively, which resulted in a difference of 0.04 pressure coefficient. Pressure profile aft of the 85% chord overlapped, and hence was assumed to cancel out each other. From the differences in pressure coefficient, that were obtained at 36% chord and at 85% chord, which were 0.30 and 0.04 respectively, the pressures were calculated using Eq. 5.8, as 101,897 Pa and 101,406 Pa respectively.

\[
C_p = \frac{P - P_\infty}{\frac{1}{2} \rho_\infty V_\infty^2}
\] (5.8)
Calculation of the aerodynamic pressure acted on the morphing section, which consisted of 64% aft of the aerofoil chord and 312.5mm-length in the span wise direction, resulted in a total lift force of approximately 14.17 N. This lift force was assumed to be taken by the CF composite corrugated structure, and used in the static load tests on the structure. The load was applied on the CF composite corrugated structure in two different ways: firstly by applying loads at three different point, which decreased linearly from the front spar to the trailing edge section, and resulted in a total of 14.17 N, and secondly by applying the total load at only one point, which was the tip of the trailing edge section. They are shown in Fig. 5.52 and Fig. 5.54, respectively. Instead of upward vertical forces, which were closer or resembled the actual aerodynamic lift, the corrugated section was turned upside down and the vertical forces were applied in a downward direction.

![Graph](image1.png)

**Figure 5.51.** (a) Re-plotted chordwise pressure distribution on a NACA 0012 aerofoil, and (b) the pressure distribution that was used and excluded in the calculation of aerodynamic load acting on the CF composite corrugated structure.
5.3.7.1 Point Loading on Three Locations

This series of static load tests on the carbon fibre (CF) composite corrugated structure was conducted to observe the trailing edge (TE) deflection and deformation of the structure due to the applied static load, which represented the aerodynamic load acted on the morphing section and was entirely taken by the CF composite corrugated structure. The first test was the CF composite corrugated structure subjected to point loadings at three locations, as shown in Fig. 5.52. The loads were assumed to be linearly distributed, decreasing from the front to the rear of the aerofoil chord. The calculated loads were 8.11N, 4.98N and 1.08N, acted at the 2\textsuperscript{nd} cell, the 6\textsuperscript{th} cell and at the flat vertical wall of the rear trailing edge section, respectively. The loads were applied by hanging weights of stainless steel cylinders, which were in-house manufactured, as shown in Fig. 5.53. Experimentally, using an IMETRUM optical system, the measured trailing edge deflection was approximately 12.87 mm, compared to the predicted TE deflection of 11.50 mm obtained from the FE simulation.

(a) Experimental result of TE deflection

(b) FEA simulation result of TE deflection

Figure 5.52. Trailing edge deflection of the CF composite corrugated structure subjected to downward vertical point loadings at three locations: (a) experimental result and (b) FEA simulation result.

Figure 5.53. Mild steel cylinders which were in-house manufactured and were used in the static test of the CF composite corrugated structure.
5.3.7.2 Tip Loading

The second test consisted of a CF composite corrugated structure subjected to a point load at the tip of the trailing edge (TE) section, as shown in Fig. 5.54. This case was only studied for comparison with the LS-DYNA simulation result, which solved large deformation problems. It did not represent the aerodynamic loading acted on the aerofoil/wing under any flight condition. A total load of 14.17 N was applied at a single point, which was at the tip of the TE section. Experimentally, the applied static loading resulted in a 73.20 mm TE vertical deflection which was measured by the IMETRUM optical system, while numerically the TE deflection was 47.02 mm. The TE deflections of the CF composite corrugated structure subjected to three points loading and a tip loading are summarised and compared in Table 5.13.

![Figure 5.54](image)

(a) Experimental result of TE deflection

(b) FEA simulation result of TE deflection

**Figure 5.54.** Trailing edge deflection of the CF composite corrugated structure subjected to a tip loading: (a) experimental result and (b) FEA simulation result.

**Table 5.13.** Comparison of trailing edge deflection of the carbon fibre (CF) composite corrugated structure when subjected to three points loading and a tip loading.

<table>
<thead>
<tr>
<th>Types of Loading</th>
<th>Experiments</th>
<th>FEA Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-points loading</td>
<td>12.87 mm</td>
<td>11.50</td>
</tr>
<tr>
<td>Tip Loading</td>
<td>73.20 mm</td>
<td>47.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Forces and Their Units</th>
<th>3-points Loading</th>
<th>Tip Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force</td>
<td>14.17 N</td>
<td>14.17 N</td>
</tr>
<tr>
<td>Deflection</td>
<td>73.20 mm</td>
<td>47.02 mm</td>
</tr>
</tbody>
</table>
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

5.4.1 Virtual 3-Point Bending Tests on 3D-Printed Carbon Fibre (CF) Composites

A series of virtual 3-point bending tests on the carbon fibre (CF) composite were simulated implicitly in LS-DYNA. Based on the microstructure analysis of the CF composite corrugated structure, which was illustrated and presented in Fig. 5.31 in Section 5.3.3.1, the CF composite corrugated structure (as well as the CF composite specimens in the 3-point bending experimental test) had 5 composite layers. In the finite element analysis (FEA) model of the CF composite beam for the virtual 3-point bending tests, the 5 layers were modelled as 5 integration points ( IPs), as shown in Fig. 5.55. Each integration point represent each of the actual composite layers.

![Figure 5.55. Five integration points across the thickness of CF composite beam.](image)

The CF composite beam was modelled with shell elements, with a length, width and thickness of 115.2 mm, 13.0 mm and 3.0 mm, respectively. The user defined integration rule was used, with the details of the integration points listed in Table 5.14. The material, coordinate of integration point and weighting factor were assigned to each integration point. The coordinates of the IPs were obtained by normalising the shell thickness from -1 (for the beam lower surface) to 1 (for the beam top surface), with the neutral axis as 0 coordinate. The weighting factor for each IP was obtained by dividing the thickness of the IP/layer with the total thickness of the composite beam.

<table>
<thead>
<tr>
<th>Integration Points</th>
<th>Materials</th>
<th>Layer Thickness (mm)</th>
<th>Coordinate of IP, S (-1 to 1)</th>
<th>Weighting Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Onyx</td>
<td>0.250</td>
<td>-0.916667</td>
<td>0.083333</td>
</tr>
<tr>
<td>2</td>
<td>CF</td>
<td>1.075</td>
<td>-0.475</td>
<td>0.358333</td>
</tr>
<tr>
<td>3</td>
<td>Onyx</td>
<td>0.350</td>
<td>0.0</td>
<td>0.116667</td>
</tr>
<tr>
<td>4</td>
<td>CF</td>
<td>1.075</td>
<td>0.475</td>
<td>0.358333</td>
</tr>
<tr>
<td>5</td>
<td>Onyx</td>
<td>0.250</td>
<td>0.916667</td>
<td>0.083333</td>
</tr>
</tbody>
</table>
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

The composite beam was supported at two locations by rigid supports with cylindrical contact surfaces, as shown in Fig. 5.56. The supports had a radius of 5 mm, and the support span was 96 mm. The composite beam was modelled with an orthotropic elastic material, and the rigid supports were modelled with a rigid aluminium material. Similar to the standard experimental test procedure (ASTM D7264/D7264M-15, 2015), a vertical downward displacement was applied in the middle (as shown by the nodes along the middle line in Fig. 5.56) of the CF composite beam, with a speed of 1.0 mm/min. The direction of the applied displacement on the middle nodes was in the negative z-direction.

![Figure 5.56. FEA model of 3-point bending test on the CF composite beam.](image)

The input curve of the applied displacement against the time is shown in Fig. 5.57. It was crucial to note that, implicitly, the displacement must be applied in steps, and the structure which was the CF composite beam must reach equilibrium at each step. For the first 2 mm of applied displacement, the middle nodes were displaced linearly by 0.1 mm in 6 seconds (1 mm/min), and the allowed waiting time to reach equilibrium was 60 seconds at each step. After the 2 mm of applied displacement, the linear displacement on the middle nodes was 1 mm in 60 seconds (1 mm/min), with similar 60 seconds waiting time at each step. The slope of the curve or the speed of the applied displacement remained constant throughout the virtual testing. The number of steps of the applied displacement were higher at the first 2 mm, because this was the critical region at which the results would be compared with the theoretical solution and experimental results, which were based on a small deflection theory.

Prior to the FEA simulation of the virtual 3-point bending test on the CF composite beam, the virtual test was conducted on an isotropic beam with a Young’s modulus of 41 GPa in all directions. The virtual test on the isotropic beam was meant to check if the setup of the virtual 3-point bending test (3PBT) was modelled correctly. A similar FEA model, in term of the beam
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

dimension (length, width and total thickness), supports, support span, applied displacement, a total of 5 integration points as depicted in the previous Fig. 5.55, was simulated. The only difference was only one material, which was the isotropic material, was assigned to all of the 5 integration points across the beam thickness.

![Figure 5.57. Input curve of applied displacement vs. time.](image)

The results of the FEA simulations are illustrated in Fig. 5.58 to Fig. 5.60. To post-process the FEA simulation results in a similar approach the experimental results were processed (ASTM D7264/D7264M-15, 2015), the applied force on the middle section/nodes of the beam and the applied displacement were required. The FEA simulation result of the applied load, as shown in Fig. 5.58, was obtained from the nodes at the centre of the beam. The FEA simulation result of the vertical z-displacement of the nodes at the centre of the beam is shown in Fig. 5.59. Both of the results of the load and displacement were actually negatives, indicated that both of them were in the downward or negative z-direction. However, the absolute values of both were plotted for easy comparison with the experimental results, and for calculation of the flexural stress and flexural strain, later on.

From the FEA simulation results of the centre-load and centre-displacement in Fig. 5.58 and Fig. 5.59, the load and displacement at each step after reaching equilibrium, were extracted for calculation of the flexural stress and flexural strain. A total of 20 and 14 data points (load and displacement) were extracted before and after 2 mm applied displacement, respectively. For all 34 data points, the flexural stress and flexural strain were calculated by using the previous Eq. 5.2 and Eq. 5.3, respectively. The flexural stress was then plotted against the flexural strain, as shown in Fig. 5.60. From the plot of the flexural stress vs. flexural strain, the slope, which was the flexural chord modulus of elasticity, was calculated using the previous Eq. 5.4, within the range of 0.001 to 0.003 flexural strain. The calculated flexural modulus of
the CF composite beam was **20.54 GPa**, which was just slightly lower than the experimental results presented in the previous Section 5.3.4 (21.7-24.2 GPa). For convenience of analysis, the above-mentioned equations are re-stated below.

\[
\sigma_{\text{flexural}} = \frac{3PL}{2bh^2}
\]

\[
\varepsilon_{\text{flexural}} = \frac{6\delta h}{L^2}
\]

\[
E_{f}^{\text{chord}} = \frac{\Delta \sigma}{\Delta \varepsilon}
\]  
(5.9)

where \(\sigma, \varepsilon, P, \delta, L, b, h, E_{f}^{\text{chord}}, \Delta \sigma, \) and \(\Delta \varepsilon\), are the flexural stress at the outer surface at mid-span, the maximum strain at the outer surface at mid-span, the applied vertical load, the mid-span/centre-beam deflection, the support span, and the composite beam width and thickness, the flexural chord modulus of elasticity, the difference in the flexural stress between the two selected flexural strains, and the difference between the two selected flexural strains, respectively.

Figure 5.58. FEA simulation result of the applied load on the middle section/nodes of the CF composite beam under a 3-point bending load.

From the prior FEA simulation of the virtual 3-point bending test on the isotropic beam, the flexural modulus obtained by the above-described data processing was about 37.27 GPa, which was approximately 4 GPa lower than the input Young’s modulus of 41 GPa. For the isotropic beam, an alternative post-processing method, which was by directly extracting the stress in the x-direction and the lower integration point strain in the x-direction, of the lower surface of the two shell elements at the mid-span and the mid-width of the beam. In a simple word, the surface referred to the outermost lower surface of the mid-span of the beam, which
experienced the highest tensile stress. From the extracted stress and strain, the flexural modulus was calculated, which was found as 41.0 GPa, equal to the input Young’s modulus. The difference between the results of the flexural modulus obtained by the two post-processing method, could explain the FEA flexural modulus of the CF composite beam that was slightly lower the experimental results.

![Graph](image1.png)

**Figure 5.59.** FEA simulation result of the centre-displacement of the CF composite beam under a 3-point bending load.

![Graph](image2.png)

**Figure 5.60.** FEA simulation result of the flexural stress vs. flexural strain of the CF composite beam.

At this point, the deduction was that if the FEA model of the virtual 3-point bending test on the isotropic beam could be improved, in such that the flexural modulus obtained by the first method (based on the extracted vertical load and displacement) could be found closer to the flexural modulus obtained by the second method (based on the extracted stress and strain of the lower surface of the beam in the x-direction), then the flexural modulus of the CF composite beam could also be improved. Therefore, several additional FEA simulations were performed on the isotropic beam, by changing several parameters such as the shell element formulation (from ELFORM 2 to ELFORM 16), the boundary condition of the beam long
edges so that the beam only rotated on the x-z plane, and the number of shell elements which were quadrupled (one shell element was split into 4 shell elements). None of the trials improved the FEA simulation result.

The following approach was by increasing the number of integration points (IPs) across the thickness of the beam. They were increased from 5 integration points to 7 and 12 integration points, as illustrated in Fig. 5.61 and Fig. 5.62, respectively. The details of the integration points such as the assigned material, the thickness, the coordinate and the weighting factor of the integration points are listed in Table 5.15 and Table 5.16, respectively. This approach was somehow similar to the approach in the usual mesh convergence study, but here the resolution of the expected results were improved across the beam thickness, instead of across the beam length and width. The approach was tested for both the isotropic beam (only for 7 IPs) and the CF composite beam.

**Table 5.15.** Details of the integration points used in the FEA simulation of CF composite beam with 7 integration points across the thickness, subjected to 3-point bending load.

<table>
<thead>
<tr>
<th>Integration Points</th>
<th>Materials</th>
<th>Layer Thickness (mm)</th>
<th>Coordinate of IP, S (-1 to 1)</th>
<th>Weighting Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Onyx</td>
<td>0.2500</td>
<td>-0.916667</td>
<td>0.083333</td>
</tr>
<tr>
<td>2</td>
<td>CF</td>
<td>0.5375</td>
<td>-0.654167</td>
<td>0.179167</td>
</tr>
<tr>
<td>3</td>
<td>CF</td>
<td>0.5375</td>
<td>-0.295833</td>
<td>0.179167</td>
</tr>
<tr>
<td>4</td>
<td>Onyx</td>
<td>0.3500</td>
<td>0.0</td>
<td>0.116667</td>
</tr>
<tr>
<td>5</td>
<td>CF</td>
<td>0.5375</td>
<td>0.295833</td>
<td>0.179167</td>
</tr>
<tr>
<td>6</td>
<td>CF</td>
<td>0.5375</td>
<td>0.654167</td>
<td>0.179167</td>
</tr>
<tr>
<td>7</td>
<td>Onyx</td>
<td>0.2500</td>
<td>0.916667</td>
<td>0.083333</td>
</tr>
</tbody>
</table>

**Figure 5.61.** Seven integration points across the thickness of the CF composite beam.

**Figure 5.62.** Twelve integration points across the thickness of the CF composite beam.
Table 5.16. Details of the integration points used in the FEA simulation of CF composite beam with 12 integration points across the thickness, subjected to a 3-point bending load.

<table>
<thead>
<tr>
<th>Integration Points</th>
<th>Materials</th>
<th>Layer Thickness (mm)</th>
<th>Coordinate of IP, S (-1 to 1)</th>
<th>Weighting Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Onyx</td>
<td>0.1250</td>
<td>-0.958333</td>
<td>0.041667</td>
</tr>
<tr>
<td>2</td>
<td>Onyx</td>
<td>0.1250</td>
<td>-0.875</td>
<td>0.041667</td>
</tr>
<tr>
<td>3</td>
<td>CF</td>
<td>0.3583</td>
<td>-0.97889</td>
<td>0.119444</td>
</tr>
<tr>
<td>4</td>
<td>CF</td>
<td>0.3583</td>
<td>-0.475</td>
<td>0.119444</td>
</tr>
<tr>
<td>5</td>
<td>CF</td>
<td>0.3583</td>
<td>-0.253111</td>
<td>0.119444</td>
</tr>
<tr>
<td>6</td>
<td>Onyx</td>
<td>0.1750</td>
<td>-0.058333</td>
<td>0.058333</td>
</tr>
<tr>
<td>7</td>
<td>Onyx</td>
<td>0.1750</td>
<td>0.058333</td>
<td>0.058333</td>
</tr>
<tr>
<td>8</td>
<td>CF</td>
<td>0.3583</td>
<td>0.236111</td>
<td>0.119444</td>
</tr>
<tr>
<td>9</td>
<td>CF</td>
<td>0.3583</td>
<td>0.475</td>
<td>0.119444</td>
</tr>
<tr>
<td>10</td>
<td>CF</td>
<td>0.3583</td>
<td>0.713889</td>
<td>0.119444</td>
</tr>
<tr>
<td>11</td>
<td>Onyx</td>
<td>0.1250</td>
<td>0.875</td>
<td>0.041667</td>
</tr>
<tr>
<td>12</td>
<td>Onyx</td>
<td>0.1250</td>
<td>0.958333</td>
<td>0.041667</td>
</tr>
</tbody>
</table>

The results of the time-history applied force on the mid-span of the CF composite beam with 7 and 12 integration points (IPs) are plotted in Fig. 5.63, and were compared with the previous result of the CF composite beam with 5 integration points. It could be observed that there was a considerably high increase in the load at all steps, when the integration points were increased from 5 IPs to 7 IPs. When the integration points across the beam thickness were further increased from 7 IPs to 12 IPs, the load slightly increased at all steps. The results of the displacement of the mid-span of the CF composite beam were similar for all of the cases (5, 7 and 12 IPs), as plotted in the previous Fig. 5.59. From the load and displacement results, the flexural stress and the flexural strain at the mid-span of the CF composite beam were calculated for all equilibrium points, by using Eq. 5.9, and were plotted in Fig. 5.64 with the previous 5 IPs result. The results of the flexural stress vs. flexural strain were then compared with the previous experimental results, in Fig. 5.65. The comparison shows that FEA simulation results of the flexural modulus of the CF composite beam, which were the slopes of the curves, were within the range of the previous experimental results plotted as slightly transparent curves (CF1, CF2, CF4 and CF5). The results of the flexural modulus within a small strain range, were in a close agreement.
Figure 5.63. FEA simulation results of the applied load on the middle section/nodes of the CF composite beam, with 5, 7 and 12 integration points, subjected to a 3-point bending load.

Figure 5.64. FEA simulation results of the flexural stress vs. flexural strain of the CF composite beam, with 5, 7 and 12 integration points (IPs), subjected to a 3-point bending load.

Figure 5.65. Comparison of the FEA simulation results of the flexural stress vs. flexural strain of the CF composite beam, with the experimental results.

The result of the deformation of the CF composite beam with 12 integration points (IPs), when it was subjected to the 3-point bending load, is shown in Fig. 5.66. The figure shows the
deflection of the CF beam at a flexural strain of 0.0058 (refer Fig. 5.65), in isometric and front (without and with visible shell thickness) views.

![Isometric view](image1)

(a) Isometric view

![Front view](image2)

(b) Front view

![Front view – with visible shell thickness](image3)

(c) Front view – with visible shell thickness

Figure 5.66. FEA result of deformation (vertical deflection) of the CF composite beam under 3-point bending load.

To understand the drop in the flexural stiffness derived from the experimental results, a further series of FEA simulations were carried out on two beams of a similar CF composite material subjected to a 3-point bending load. It was deduced that after certain level of stress, which was about 120-140 MPa, the two layers of the CF layers across the thickness were de-bonded, and the CF composite specimen behaved like two individual beams. Therefore, the CF composite beam with the similar length (115.2 mm) and width (13 mm), was modelled as two beams, where the thickness of each of the beams (1.5 mm) was half of the previous total thickness (3 mm).

Similar to the previous FEA simulations, the number of integration points (IPs) across the thickness of the CF composite beams was increased from 3 IPs per beam (a total of 6 IPs for the entire CF composite beam), to 4 IPs per beam (a total of 8 IPs) and 5 IPs per beam (a total of 10 IPs), as illustrated in Fig. 5.67. The layer thickness, coordinates and weighting factor of the integration points for the three FE models are listed in Table E1, E2 and E3, respectively, in the Appendix D. For this series of virtual 3-point bending tests, a vertical displacement was applied on the mid-span nodes of the top CF composite beam, with a linear displacement and 16 discretised equilibrium points as shown in Fig. 5.68, instead of the previous applied
displacement in multiple steps (Fig. 5.57). Later, the number of the discretised points were increased to 160 points, but a similar result was obtained. The previous FEA simulation of a single beam with 7 IPs, which was subjected to the 3-point bending load, was also repeated with this alternative way of applying the vertical displacement, and as a result, a similar flexural stiffness of the CF composite beam was obtained.

![Image 1](image1.png)

**Figure 5.67.** CF composite beam model with two beams having (a) 6 integration points (IPs), (b) 8 IPs, and (c) 10 IPs.

![Image 2](image2.png)

**Figure 5.68.** Input of applied displacement vs. time for the CF composite modelled with two beams.

The results of the CF composite which was modelled as two beams, were post-processed in a similar way as the previous one beam FEA simulations. The resulted load and displacement
of the nodes on the mid-span of the beam were extracted, and from those values, the flexural stress and flexural strain were calculated. The flexural stress was plotted against the flexural strain in Fig. 5.69, for the three FE models (6 IPs, 8 IPs and 10 IPs), and the results were compared with the previous experimental and FEA simulation results. It could be observed that the slope of the curves, plotted in long dotted lines, converged when the total integration points (IPs) were increased from 6 IPs to 8 IPs and 10 IPs. It could also be observed that the slope of the converged FE model (10 IPs) was almost equal to the slope or the second flexural modulus of the previous experimental results (CF1-CF5). Therefore, the results supported the earlier hypotheses that the drop of the flexural stress after the first linear region was due to the delamination of the CF layers, and the CF composite plate acted/behaved as two individual beams after the delamination occurred. The FE model of the CF composite modelled as two beams is shown in Fig. 5.70a, and the result of deformation, at 0.0058 flexural strain, is depicted in Fig. 5.70b-d.

![Figure 5.69](image_url)

**Figure 5.69.** FEA simulation results of the flexural stress vs. flexural strain of the CF composite beam, modelled with 2 beams and with 6, 8 and 10 integration points (IPs), and subjected to a 3-point bending load.
Figure 5.70. (a) FE model and (b-c) FEA result of deformation (vertical deflection) of the CF composite beam, which was modelled as two beams, and was subjected to a 3-point bending load.
5.4.2 FEA Simulation of SMA-Actuated Carbon Fibre Composite Corrugated Structure

Based on the understanding of the 3D-printed CF composite microstructure and the technique to model it in the finite element (FE) software, LS-DYNA, a 3D-printed CF composite corrugated structure actuated by a NiTi shape memory alloy (SMA) wire was modelled and simulated. The NiTi SMA wire had a diameter of 0.5 mm and a recoverable strain of 1.68%. As illustrated in Fig. 5.71, the CF composite corrugated structure comprised of a flat vertical wall, a corrugated section with ten semi-hexagonal cells and a trailing edge section. The structure represented a morphing rear section of a NACA 0012 aerofoil/wing, located at 36% rear of the chord. The structure should be covered with a flexible structure, such as a cellular honeycomb structure shown in Fig. 5.72, or a flexible foam structure, to form smooth external upper and lower aerofoil surfaces. However, only the core structure which was the CF composite corrugated structure (with the front vertical plate and the rear trailing edge section) was virtually tested. The CF composite D-nose spar located at 36% front of the chord, on which the CF composite corrugated structure should be attached to, was also excluded in the FE model, as it should be sufficiently stiff and it should not contribute to the movement of the rear morphing section. The CF composite corrugated structure was modelled with shell elements and 12 integration points through the shell thickness, similar to one of the previous FE models of 3D-printed CF composite beam. A composite damage material model was used for the structure, instead of the previous orthotropic elastic material, because the UMAT of the SMA was developed for an explicit LS-DYNA. A temperature loading, consisted of a complete heating-cooling cycle, was applied on the NiTi SMA wire, and the deformation of the CF composite corrugated structure was evaluated.

Figure 5.71. FE model of SMA-actuated 3D-printed CF composite corrugated structure.
Figure 5.72. An example of a cellular honeycomb structure to form smooth upper and lower aerofoil surfaces.

The result of the FE simulation of the CF composite corrugated structure upon SMA-actuation is shown in Fig. 5.73a. On heating of the SMA wire, it recovered the pre-strain and deformed each of the semi-hexagonal cells of the CF composite corrugated structure. Consequently, a smooth camber morphing was achieved and the trailing edge was deflected downward by 7.3 mm. Compared to the experimental test result (Fig. 5.73b), which was previously reported in Section 5.3.5, the FE trailing edge deflection was about 1.3 mm higher than the experimental trailing edge deflection (6.0 mm). The reasons of the lower trailing edge deflection in the experimental test were due to the additional mechanical screws, nuts and washers used to fix and align the SMA wire, and the additional 3D-printed onyx washers to provide flat surfaces for the nuts, which increased the total stiffness of the CF composite corrugated structure.

Figure 5.73. Deformation and trailing edge deflection of SMA-actuated 3D-printed CF composite corrugated structure: (a) FE simulation and (b) experimental test results.
5.4 FEA Simulations of SMA-Actuated Carbon Fibre (CF) Composite Corrugation

5.4.3 Predictive Virtual Testing of SMA-Actuated Carbon Fibre Composite Corrugated Structure

As the finite element (FE) model of the SMA-actuated 3D-printed carbon fibre (CF) composite corrugated structure had been validated by the experimental test, the FE model was utilised in a series of FE simulations to find the optimum number of SMA wires that could produce the optimum trailing edge deflection. The number of SMA wires along the span of the CF composite corrugated structure was increased to 4, 8, 14, 27, 54, 81, 108 SMA wires. Prior to the SMA actuation, concentrated loads represented the aerodynamic lift load on the rear part of the aerofoil (a total of 14.17 N) were applied at three locations on the CF composite corrugated structure, as shown and described in the previous Section 5.3.7.1 (Fig. 5.52, but with the concentrated loads acted in the vertical upward direction).

The self-weight of the SMA wires was not included in the series of FE simulations. For justification, for one SMA wire with a diameter of 0.5 mm, a length of 150 mm, and a density of 7.89×10⁻⁶ kg/mm³, the calculated volume and mass of the SMA wire were 29.45 mm³ and 0.232 g, respectively. Therefore, the self-weight of a SMA wire was 2.2796×10⁻³ N. For 10, 50 and 100 SMA wires, the self-weight was 0.02 N, 0.11 N, and 0.23 N, respectively. For the maximum 108 SMA wires, the total self-weight was about 0.25 N. It was about 56 times lower than the applied aerodynamic load (14.17 N) in the opposite direction, and was therefore could be neglected.

The result of the predictive virtual test is shown in Fig. 5.74. The plot of the trailing edge deflection of the SMA-actuated CF composite corrugated structure against the number of SMA wires used for actuation shows that an optimum trailing edge deflection of approximately 67 mm was achieved with an optimum 54 SMA wires. The deformation and trailing edge deflection of the CF composite corrugated structure are illustrated in Fig. 5.75.

![Figure 5.74. Result of predictive virtual test on SMA-actuated CF composite corrugated structure.](image)
Figure 5.75. Illustration of deformation and trailing edge deflection of the SMA-actuated CF composite corrugated structure from the predictive virtual tests.
5.5 Chapter Summary and Conclusions

In this Chapter 5, the vectorised UMAT of the SMA wires was utilised to actuate more complex structures, including aluminium and CF composite aerofoils, and aluminium pre-curved corrugated plate. Various design of the morphing aerofoils with different arrangement of the SMA wires and different positions of the corrugated surfaces (e.g. as a middle cantilever structure, as a lower aerofoil surface, and as both upper and lower aerofoil surfaces) were investigated and simulated, and their effects on the aerofoil deformation and trailing edge deflection were evaluated. A final morphing aerofoil, comprised of a rigid D-nose section, a SMA-actuated corrugated section as a middle cantilever section, and a rear trailing edge section, was chosen. The CF composite corrugated section with the trailing edge section were manufactured using a composite 3D-printing technology. Experimental tests on the SMA-actuated CF composite corrugated structure showed a trailing edge deflection of 6.0 mm, at a supplied power of 5.75 Watt (2.5 A, 2.3 V).

CF composite beams with a similar cross-section as the CF composite corrugated structure, were 3D-printed and tested under a 3-point bending load. A new approach of finite element modelling a 3D-printed composite structure was explored by using a user defined integration through the shell thickness of the CF composite beam model, which was subjected to the 3-point bending load. The simulation results of the flexural stress vs. flexural strain matched the experimental results within the elastic region. After delamination occurred in the experimental tests, the CF composite beams behaved as two individual beams with reduced flexural stiffness, which was validated by another series of FEA simulations on the CF composite modelled as two beams. The FE modelling approach was then used in the simulation of a SMA-actuated 3D-printed CF composite corrugated structure, and a trailing edge deflection of 7.3 mm was achieved upon actuation, which was close to the experimental result. A predictive virtual test of the structure was then presented, with a predicted 54 SMA wires along the span to achieve an optimum trailing edge deflection of approximately 67 mm.

In a larger scale, several sections of the SMA-actuated CF composite corrugated structure with the trailing edge section could be located along a wing span to form a morphing wing trailing edge. However, it was finally decided to use the SMA-actuated composite corrugated structure with the trailing edge section, as a morphing part at the wing tip, in the final design of the composite morphing wing. The design and manufacturing of the composite morphing wing would be described in detail in the next Chapter 6.
Chapter 6
SMA-Actuated Composite Morphing Wing

The morphing structural research was applied to the design and manufacture of a composite wing, and integration of the SMA-actuated composite corrugated structure within the composite wing, to form a complete composite morphing wing. Initially, a semi-span of the composite wing was designed, with analytical knowledge of aerodynamics and aircraft structural analysis, combined with large deformation and composite buckling analysis in a finite element analysis (FEA) software, LS-DYNA. Half of the designed semi-span composite morphing wing was then manufactured with a recently emerging 3D-printing technology, and the 3D-printed composite wing parts were then assembled with other wing components, such as the foam and the SMA-actuated composite corrugated structure. The design, manufacturing and assembly of the composite morphing wing are described systematically in this Chapter 6.

6.1 Aerodynamic Loads

The composite morphing wing was designed for a typical medium-sized Unmanned Aircraft Vehicle (UAV). The composite morphing wing had a NACA 0012 aerofoil profile, with a constant chord of 300 mm and a wing span of 5 m. It was designed for a cruise flight at speed of 200 km/h (55.55 m/s), and at low angle of attack (AOA = 3°). The cruising altitude for the designed UAV composite wing was 4.5 km, at which the air density, \( \rho_{\infty} \) was 0.777 kg/m\(^3\) and the dynamic viscosity, \( \mu \) was 16.45×10\(^{-6}\) kg/(m.s). The Reynolds number (Abbott et al., 1959) for the incoming air flow was therefore,

\[
Re = \frac{\rho_{\infty} V_{\infty} c}{\mu} = 7.87 \times 10^5
\]  

(6.1)

where \( V_{\infty} \) and \( c \) were the air free stream velocity and the wing chord, respectively. The composite morphing wing had a high aspect ratio, calculated from the ratio of the squared of
the span to the surface area, or simply the ratio of the span to the chord for the rectangular wing (Abbott et al., 1959).

\[
AR = \frac{b^2}{S} = 16.67
\]  

(6.2)

where \(AR\), \(b\) and \(S\) were the aspect ratio, span and surface area of the composite morphing wing. One of the main advantages of the high aspect ratio wing was a less induced drag, if compared to a low aspect ratio wing at identical flight conditions. As a benchmark, the wing aspect ratio of the commercial aircrafts, Airbus A380 and Boeing B777 were 7.8 and 9.0, respectively. The aspect ratio of the designed composite wing was about twice of their aspect ratio.

To design a complete composite wing, the aerodynamic lift load acting on the wing at sea level was first calculated. The calculation was done for flight at sea level, because the air density was higher at sea level (1.225 kg/m\(^3\)), compared to the air density at 4.5 km altitude. Hence, the aerodynamic lift load was higher at sea level, which should be considered in the design of the composite morphing wing. The section lift coefficient and the section lift for the NACA 0012 aerofoil section, at a 3 degrees angle of attack (AOA) were simply obtained from the following equations (Abbott et al., 1959).

\[
c_l = 2\pi\alpha = 0.3290
\]  

(6.3)

\[
l = \frac{1}{2} \rho_\infty V_\infty^2 Sc_l = 186.55 \text{ N/m}
\]  

(6.4)

However, the section lift distribution along the span of a finite wing was not constant as calculated by Eq. 6.4, because it was affected by the trailing edge vortices. Therefore, the lift distribution along the wing span was calculated based on Prandtl lifting line theory (Flandro et al., 2012). The calculation is documented in Appendix F1.

The lift distribution along the wing span for the specified air density and velocity at the sea level, which were solved for \(n = 3\) and \(n = 7\) are shown in Fig. 6.1. The lift distribution curve with circular marks and diamond marks were for \(n = 3\), and \(n = 7\), respectively. The numbers 2 and 4 on the legend refer to the number of odd terms in the equations, respectively. The lift acted on the quarter chord of the wing, which location would be used as loading points in the FEA simulations of the composite morphing wing.
Figure 6.1. Spanwise lift distribution on the composite morphing wing at sea level flight.

The total lift acting on the full-span composite wing, for \( n = 3 \) and \( n = 7 \), were calculated using the following Eq. 6.5. The final form of the equation shows that the total lift acting on a finite wing was equal to a function of the wing span, density and velocity of the air flow, and only the first constant, \( A_1 \). The integration of the circulation along the wing span, \( \int_{-b/2}^{b/2} \Gamma(y) \), which resulted in the final form of Eq. 6.5, was derived and presented in Appendix F2. For the composite morphing wing, with a span of 5 m, \( A_1 \) of \( 5.1390 \times 10^{-3} \) and \( 5.9571 \times 10^{-3} \) for \( n = 3 \) and \( n = 7 \), respectively, air density of \( 1.225 \text{ kg/m}^3 \), and flight speed of \( 55.55 \text{ m/s} \), the resulted total lift were 763 N and 884 N, respectively.

\[
L = \int_{-b/2}^{b/2} \rho_\infty V_\infty \Gamma(y) dy = \rho_\infty V_\infty \left( \frac{\pi}{2} b^2 V_\infty A_1 \right) = \frac{\pi}{2} b^2 A_1 \rho_\infty V_\infty^2 \quad (6.5)
\]

\[
L_{n=3} = 762.86 \text{ N}
\]

\[
L_{n=7} = 884.30 \text{ N}
\]

In the early design stage of the composite morphing wing, a rough estimation of the payload that should be carried by the composite wing was 100 kg or 981 N. From the above total lift calculation, the total lift for \( n = 7 \) was closer to the initial designed payload. Therefore, the spanwise lift distribution curve of \( n = 7 \) in Fig. 6.1 was used to design and size the composite D-nose spar in the next section.
### 6.2 Design of CF Composite D-Nose Spar

The carbon fibre (CF) composite D-nose spar was designed based on the well-known idealised booms method, with assumptions that the spar caps/flanges resisted the direct stress due to the external bending moment, and the cell walls resisted the shear stress (Megson, 2017). The procedures of sizing the CF composite spar caps/flanges and the CF composite cell walls are described in the following two subsections, and then followed by another subsection analysing the buckling of the CF composite spar caps/flanges and the CF composite skins.

#### 6.2.1 CF Composite Spar Caps/Flanges and Cell Walls

As illustrated in Fig. 6.2, the CF composite D-nose spar was divided to 8 sections along the wing semi-span. For each section, the lift force acted on the section was estimated by calculating the area under the curve of the spanwise lift distribution, for \( n = 7 \), in the previous Fig. 6.1. Based on the lift force, the internal shear forces, the bending moments, the boom areas of the spar caps/flanges, the thickness of the CF composite spar caps/flanges, spar web and D-nose skin were calculated. The complete results of the calculations are documented in Appendix F3.
6.2.2 Buckling of CF Composite D-Nose Spar

The previously estimated sizes of the CF composite D-nose spar were only based on direct compressive stress and the associated compressive strength of the material, and the shear flow and the associated shear strength of the material. The analytical solutions which were based on the thin wall theory, however, had few limitations, such as

1) For isotropic material, such as aluminium.
2) Based on small deflection theory.
3) Stability or structural buckling was not included in the calculation.
4) Wing ribs were not included in the calculation.

Therefore, additional analytical software and finite element analysis (FEA) software were required to further refine the design of the CF composite D-nose spar. The ESDU (Engineering Science Data Unit) Composite Series tools (ESDU 81047) and the explicit LS-DYNA software were used, respectively. Initially, the ESDU 81047 program (Katwijk, 1981) was intensively explored. However, because of a certain limitation, that was an unavailable free edge boundary condition which should be applied on one edge of each analysed section of the composite spar flanges (Fig. 6.3), the composite buckling was then mainly analysed using the LS-DYNA software.

Additionally, a general manufacturing rule of minimum number of layers in each direction was also applied to several parts, where applicable. In general, the composite lay-ups for the different parts of the CF composite D-nose spar are as follow.

1) Top and bottom flanges have unidirectional CF composite \([0]^{\circ}\), in the spanwise direction, to effectively resist the bending moment and direct compressive and tensile stresses.

2) The spar web and D-nose skin were initially designed with specially orthotropic layers [i.e. \(45/-45/-45/45/45/45/-45\)^\circ], to effectively resist the shear load, but were then added with minimum of one CF layer in \(0^\circ\) and \(90^\circ\) directions.

3) All wing ribs have specially orthotropic 8 CF layers \([45/-45/-45/45/45/45/45/45\)^\circ], to effectively resist the shear load.

In LS-DYNA, the CF composite D-nose spar was modelled with the composite spar flanges, the composite spar webs, the composite D-nose skins, the front and rear composite
6.2 Design of CF Composite D-Nose Spar

ribs. Eight sections of the composite spar flanges, the composite spar webs and the composite D-nose skins were modelled for the semi-span of the composite D-nose spar, as shown in the previous Fig. 6.2. For each section, five front and rear composite ribs were modelled to maintain the aerofoil shape, and to attach/link the composite D-nose skins to the composite spar flanges and the composite spar webs. An illustration of one section (section S2) of the CF composite D-nose spar is shown in Fig. 6.3. Different colours of the composite parts indicated that they were modelled as different parts, so that they could be assigned with different sections having different wall thickness.

![Diagram of CF composite D-nose spar](image)

**Figure 6.3.** A section (section S2) of the FE model of the CF composite D-nose spar.

All of the composite parts were modelled with shell elements having Belytschko-Tsay element formulation. The CF composite parts were attached/connected to each other by merging their nodes at the connecting edges, assuming a perfect connection/bonding between them. The composite parts were assigned with a composite material. This material model was
selected for the CF composite D-nose spar, because the failure criteria could be activated by specifying the shear strength, the tensile strength and the compressive strength of the composite material. The activation of the failure criteria ensured that the CF composite D-nose spar or the composite wing would not fail under the applied aerodynamic load. The Young’s modulus of the CF composite in the fibre direction was 54 GPa, and the Young’s modulus in the other two transverse directions were 1.4 GPa (Markforged material datasheet, 2016). A minor Poisson’s ratio ($\nu_{21}$ or PRBA) of 0.014 was used. Two material cards, with globally orthotropic material axes (AOPT2), were defined and assigned to the composite parts. In one material card, the material axis for the composite spar flanges, the composite spar webs and the composite D-nose skin was defined by a $[0, 1, 0]$ vector, where the $0^\circ$ fibre direction was in the spanwise or the $y$-direction. In the other material card, the material axis for the composite front and rear ribs was defined by a $[0, 0, 1]$ vector, where the $0^\circ$ fibre direction was in the vertical direction of Fig. 6.3a, or the $z$-direction.

An integration rule was defined for every single part of the CF composite D-nose spar, by defining an equal spacing of integration points, so that the integration points were equally spaced through the shell thickness. By this method, the shell was divided into NIP (number of integration points, e.g. 12, for the spar web and the D-nose skin of section S8) layers of equal thickness. The number of integration points (NIP) in the integration card of every single composite part were ensured to match the NIP defined in the section card. A total of 33 integration cards and 33 section cards were defined for the entire CF composite D-nose spar (8 spar flanges, 8 spar webs, 8 D-nose skins and 9 ribs).

The CF composite D-nose spar was positioned at 3 degrees angle of attack (AOA), and a vertical load, representing the aerodynamic lift load, was applied on the quarter chord of the composite wing. The quarter chord line of the CF composite D-nose spar is shown in Fig. 6.4, in triangles/pyramids along the semi-span, and the applied lift force acted on the quarter chord is plotted in Fig. 6.5. As shown in Fig. 6.5, only one load curve was defined, but the lift load varied along the semi-span of the CF composite D-nose spar, by defining different load curve scale factor (SF) for every section (S1 to S8). The load curve scale factor (SF) of the 8 sections of the CF composite D-nose spar are listed in Table 6.1. The load increased linearly until it reached the maximum in 2 seconds, then it remained constant for 2 seconds to ensure the CF composite D-nose spar reached equilibrium, then the CF composite D-nose spar was unloaded to zero Newton in 2 seconds (under a similar loading/unloading rate), followed by a constant zero load in the final 2 seconds. Basically, the lift load on each section of the CF composite D-
nose spar was estimated from the area under the curve of the analytical spanwise lift distribution \((n = 7, \text{ or } n = 4 \text{ on the legend})\) shown in the previous Fig. 6.1. The load was applied using the LOAD_NODE_SET, where 8 sets of 20 nodes were defined for 8 sections of the CF composite D-nose spar.

For example, the section lift on the section S1 (inner most section at the wing root) was about 179.11 N/m. The section S1 had a span length of 312.5 mm and 20 nodes (as shown in Fig. 6.4b, every section had 20 nodes for the loading points) on which the vertical lift load was applied. Hence the load per node for factor of safety of 6.75 was

\[
L_{\text{per node}} = \frac{0.17911 \frac{kN}{m} \times 0.3125 \text{ m} \times 6.75}{20 \text{ nodes}} = 0.01889 \text{ kN/node}
\]

which was the input load curve shown in Fig. 6.5.
6.2 Design of CF Composite D-Nose Spar

Figure 6.5. Applied aerodynamic lift loading (FOS of 6.75) on the quarter chord of the CF composite D-nose spar.

Table 6.1. Applied aerodynamic lift load (FOS of 6.75) on 8 sections of the composite D-nose spar.

<table>
<thead>
<tr>
<th>Sections of CF composite D-nose spar</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load curve scale factor (SF)</td>
<td>1.000</td>
<td>0.999</td>
<td>0.996</td>
<td>0.993</td>
<td>0.995</td>
<td>1.021</td>
<td>1.078</td>
<td>1.108</td>
</tr>
<tr>
<td>Total load on each section (N)</td>
<td>377.8</td>
<td>377.4</td>
<td>376.3</td>
<td>375.1</td>
<td>375.9</td>
<td>385.7</td>
<td>407.3</td>
<td>418.6</td>
</tr>
<tr>
<td>Total load on semi-span of CF composite D-nose spar</td>
<td>[\sum L = 3094.1 , N \text{ (FOS 6.75)}]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The initial design or sizing of the CF composite D-nose spar which was based on the analytical solutions, is listed in Table 6.2. Only the thicknesses of the CF composite spar flanges, spar web, and D-nose skin were obtained from the calculations. The number of carbon fibre (CF) layers of the top and bottom composite spar flanges could be calculated based on a ply thickness of 0.125 mm. For the start of the finite element analysis (FEA) simulations, the design of the CF composite D-nose spar was iterated by

1) Assigning a specially orthotropic layers, consisted of 8 layers of 45° CF direction, or specifically \[45/-45/-45/45/-45/45/45/45\]°, for the composite spar web and the composite D-nose skin, to avoid the three well-known composite couplings, and to effectively resist the external shear load. A minimum two layers of each 0° and 90° directions were added to at least resist the in-plane tension/compression loading, and to obtain a symmetrical composite lay-up. This adjustment resulted in a **minimum 12 CF layers** or **1.5 mm thickness** for the CF composite spar web and D-nose skin. This affect the thickness of the spar web from section S4 to S8, and thickness of the D-nose skin for all sections (S1 to S8).

2) Increasing the thickness of the CF composite spar flanges of sections S6-S8 to 1.5 mm by adding more CF layers with the fibres aligned in the spanwise (0°) direction, because
the thickness of the D-nose skin changed to a minimum of 1.5 mm. This was to ensure a uniform thickness of each section in the chord wise direction (from the leading edge to the rear edge of the spar flanges).

3) Introducing front and rear composite ribs with specially orthotropic layers \([45/-45/-45/45/-45/45\] \(°\), to effectively resist the internal shear and torsional loads which resulted from the external aerodynamic loads, and to retain the aerofoil aerodynamic shape of the CF composite D-nose spar.

The design iteration of the CF composite D-nose spar, from the analytical solution to the starting point of the FEA simulation, was already described in general at the beginning of this subsection 6.2.3 (in the 3rd paragraph), but it was necessary to restated here with more details for further clarity. The above-mentioned changes in the design and sizes of the components of the CF composite D-nose spar are listed in Table 6.3.

### Table 6.2. The initial design of the CF composite D-nose spar based on analytical solutions.

<table>
<thead>
<tr>
<th>Sections</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spar flanges CF layers</td>
<td>72</td>
<td>55</td>
<td>41</td>
<td>28</td>
<td>18</td>
<td>10</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Spar flanges thickness (mm)</td>
<td>9</td>
<td>6.875</td>
<td>5.125</td>
<td>3.5</td>
<td>2.25</td>
<td>1.25</td>
<td>0.625</td>
<td>0.125</td>
</tr>
<tr>
<td>Spar web thickness (t_{12}) (mm)</td>
<td>2.0951</td>
<td>1.8339</td>
<td>1.5733</td>
<td>1.3135</td>
<td>1.0538</td>
<td>0.7910</td>
<td>0.5178</td>
<td>0.2288</td>
</tr>
<tr>
<td>Skin thickness (t_{21}) (mm)</td>
<td>0.2413</td>
<td>0.2112</td>
<td>0.1812</td>
<td>0.1513</td>
<td>0.1214</td>
<td>0.0911</td>
<td>0.0596</td>
<td>0.0264</td>
</tr>
</tbody>
</table>

### Table 6.3. The first-iterated design of the CF composite D-nose spar for the start of the FEA simulation.

<table>
<thead>
<tr>
<th>Sections</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spar flanges CF layers</td>
<td>72</td>
<td>55</td>
<td>41</td>
<td>28</td>
<td>18</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Spar flanges thickness (mm)</td>
<td>9.00</td>
<td>6.875</td>
<td>5.125</td>
<td>3.50</td>
<td>2.25</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Spar web CF layers</td>
<td>22</td>
<td>18</td>
<td>16</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Spar web thickness (t_{12}) (mm)</td>
<td>2.75</td>
<td>2.25</td>
<td>2.00</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Skin CF layers</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Skin thickness (t_{21}) (mm)</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Ribs CF layers</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ribs thickness</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.2 Design of CF Composite D-Nose Spar

The FEA simulation result of the first iterated CF composite D-nose spar, under aerodynamic lift load at factor of safety (FOS) of 6.75, is shown in Fig. 6.6. It could be observed that the CF composite D-nose skin of sections S1 and S2, as well as the top CF composite spar flanges and the CF composite D-nose skin of sections S4-S6, buckled due to the in-plane compressive loading, when the D-nose spar was deflected up by the applied aerodynamic lift load. Fig. 6.7 shows the zoomed area of sections S1 and S2 in a red box in Fig. 6.6, where buckling of the composite D-nose skin could be observed at two locations of the highly-compressed skin (exceeding about 400 GPa compressive stress in the y- or spanwise direction, as shown in fringes). The zoomed area of sections S4, S5 and S6 in a black box in Fig. 6.6, is shown in Fig. 6.8. The top composite spar flange and the composite D-nose skin buckled at several locations where the spanwise compressive stress was high, as shown by the blue fringe.

To solve the buckling of the CF composite D-nose spar, multiple iterations of the FEA simulations were carried out, by adding more carbon fibre layers in the 0° direction (spanwise direction) for the composite D-nose skin of sections S1 and S2, and for the composite spar flanges of sections S4-S6. Additional 6 CF layers (3 layers above the neutral axis and 3 layers below the neutral axis) and 4 CF layers (2 layers above the neutral axis and 2 layers below the neutral axis) were added on the sections S1 and S2 of the D-nose skin, respectively. In the iteration process, D-nose skin of section S3 was also affected (buckled), and hence additional 4 layers were added on the skin. For sections S4-S6, it was deduced that the thickness of the spar flange was more critical than the thickness of the D-nose skin, to avoid excessive deformation which led to buckling of the structure. The thickness of the composite spar flanges was increased by 1 mm, or by additional 8 CF layers, with the fibre direction was in the 0° or spanwise direction. The final iteration of the design of the composite D-nose spar, in term of
6.2 Design of CF Composite D-Nose Spar

thickness and number of layers is summarised in Table 6.4. The details of the CF composite lay-ups for the top and bottom flanges, the spar web, the D-nose skin, and the wing ribs of the CF composite D-nose spar are shown in Fig. 6.9.

Figure 6.7. Buckling of the CF composite D-nose skin of sections S1 and S2 of the first-iterated CF composite D-nose spar, under an applied aerodynamic lift load at FOS of 6.75.

Figure 6.8. Buckling of the top CF composite spar flange and the CF composite D-nose skin of sections S4-S6 of the first-iterated CF composite D-nose spar, under an applied aerodynamic lift load at FOS of 6.75.
Table 6.4. Final composite lay-up of the CF composite D-nose spar.

<table>
<thead>
<tr>
<th>Sections</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spar flanges CF layers</td>
<td>72</td>
<td>55</td>
<td>41</td>
<td>36</td>
<td>26</td>
<td>20</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Spar flanges thickness (mm)</td>
<td>9.00</td>
<td>6.875</td>
<td>5.125</td>
<td>4.50</td>
<td>3.25</td>
<td>2.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Spar web CF layers</td>
<td>22</td>
<td>18</td>
<td>16</td>
<td>16</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Spar web thickness t12 (mm)</td>
<td>2.75</td>
<td>2.25</td>
<td>2.00</td>
<td>2.00</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Skin CF layers</td>
<td>18</td>
<td>16</td>
<td>16</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Skin thickness t21 (mm)</td>
<td>2.25</td>
<td>2.00</td>
<td>2.00</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Ribs CF layers</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ribs thickness</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.2 Design of CF Composite D-Nose Spar

Figure 6.9. Final design of carbon fibre (CF) composite D-nose spar.
The final finite element analysis (FEA) simulation results of the CF composite D-nose spar subjected to the aerodynamic load, which was represented by the line load on the quarter chord, are illustrated in Fig. 6.10. Fig. 6.10a is the isometric view, while Fig. 6.10b is the rear view of the CF composite D-nose spar. The top figures show the initial state, while the bottom figures show the maximum deflected state (at 4000 milliseconds simulation time) of the CF composite D-nose spar. The figures show that the tip of the CF composite D-nose spar was deflected up to a maximum displacement of **565.1 mm**, which was about **22.6%** of the wing semi-span, or about **11.3%** of the full wing span. The time history of the tip deflection of the CF composite D-nose spar is plotted in Fig. 6.11. It could be observed that there were fluctuations at the maximum applied load (from 2 seconds to 4 seconds) and at the zero unloading (from 6 seconds to 8 seconds). The fluctuations could reach a complete equilibrium if the simulation time was extended for further few seconds, but it was avoided because of very expensive computational time. It was important to note that the deformation of the CF composite D-nose spar seemed high, because it was subjected to a lift load at a high factor of safety (FOS), which was 6.75.

In addition to the finite element analysis (FEA) simulation of the CF composite D-nose spar with a factor of safety (FOS) of 6.75, which represented an extreme flight condition such as a roll manoeuvre, another FEA simulation was carried out with an applied lift load at FOS of 1, which represented a normal flight condition. Here, the total lift load which was applied on the CF composite D-nose spar was approximately equal to the area under the curve of section lift vs. wing span (area for the wing semi-span, from 0 to 2.5 m, with \( n = 7 \)) shown in the previous Fig. 6.1. Similar to the previous simulation, only one load curve was defined, as shown in Fig. 6.12, and the load was varied along the semi-span of the CF composite D-nose spar by using different load curve scale factors (SF) for different sections (S1 to S8).

The FEA result of deformation of the CF composite D-nose spar under this loading condition is depicted in Fig. 6.13, in isometric and rear views. The initial state of the composite D-nose spar was similar to the initial state shown in the previous Fig. 6.10, and hence only the deformed state is shown in Fig. 6.13. The time history of the tip deflection of the CF composite D-nose spar is plotted in Fig. 6.14. As shown by the fringe levels, at 4000 milliseconds, the CF composite D-nose spar was deflected up to a maximum vertical or \( z \)-displacement of **85.5 mm**, which was about **3.4%** of the wing semi-span, or about **1.7%** of the full wing span.
6.2 Design of CF Composite D-Nose Spar

Figure 6.10. Deformation (vertical displacement) of the CF composite D-nose spar under an applied aerodynamic lift load at FOS of 6.75.

Figure 6.11. Result of tip deflection of the CF composite D-nose spar under an applied aerodynamic lift load at FOS of 6.75.
6.2 Design of CF Composite D-Nose Spar

Figure 6.12. Applied aerodynamic lift loading (FOS of 1) on the quarter chord of the CF composite D-nose spar.

Figure 6.13. Deformation (vertical displacement) of the CF composite D-nose spar under an applied aerodynamic lift load at FOS of 1.

Figure 6.14. Result of tip deflection of the CF composite D-nose spar under an applied aerodynamic lift load at FOS of 1.
The FEA simulations of the CF composite D-nose spar with a factor of safety (FOS) of 6.75 and FOS of 1 were run on the High Performance Computer (HPC) at Imperial College London, with 20 number of CPUs and Shared Memory Parallel (SMP). The total CPU time were 608 hours and 28 minutes, and 596 hours and 32 minutes, respectively.

6.3 3D-Printed Carbon Fibre (CF) Composite D-Nose Spar

The carbon fibre (CF) composite top and bottom flanges, spar web, front and rear ribs were 3D-printed using a Markforged 3D-printer. They were then assembled with an acrylic adhesive, which was suitable for the greasy nylon-based parts. The composite skin of the D-nose spar was manufactured by a vacuum moulding process to obtain a long single piece skin. The rear parts of the wing was made of several sections of rigid and flexible foams. The experimental design and manufacturing process of each component of the CF composite D-nose spar are described in this section.

6.3.1 CF Composite Spar Flanges

The initial design of the CF composite D-nose spar was shown in the previous Fig. 6.9, which semi-span was 2.5 m, a typical size for a medium size unmanned aerial vehicles (UAV) (Fahlstrom and Gleason, 2012). However, only half of the semi-span D-nose spar was fabricated using the Markforged 3D-printer, due to a time constraint. Hence the length of the fabricated CF composite D-nose spar of the morphing wing was 1.25 m. From the Fig. 6.9, this referred to the four sections close to the wing tip, which were sections S5 to S8. The top and bottom flanges had unidirectional carbon fibre (CF) lay-up, where the fibre direction was in the span wise direction, to efficiently resist the bending moment or bending stress resulted from the applied lift on the wing. The thickness and number of composite layers of the four sections are listed in Table 6.5. The actual thickness of the 3D-printed composite spar flanges was 0.25 mm thicker than the thickness listed in the table, because of the default minimum one layer of the plastic onyx material, for each of the roof and floor layers.

<table>
<thead>
<tr>
<th>Sections</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness (mm)</td>
<td>3.25</td>
<td>2.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>No. of CF layers</td>
<td>26</td>
<td>20</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>
Based on the previous Fig. 6.9, the spanwise length of each section of the sections S5-S8 was 312.5 mm. Initially, with the 3D-printing bed size of 320 mm × 140 mm, only four of the top CF composite flanges and four of the bottom CF composite flanges should be 3D-printed. As shown in Fig. 6.15, one section of the top flange could fit the printing bed. However, the requirement of the use of ‘brim’ features to support the thin triangular parts on the printing bed, which were designed as slots for the composite spar web and ribs, make it impossible, because the new full length of the flange with the ‘brim’ exceeded the size of the printing bed. Therefore, for each section of the sections S5-S8, two composite parts were printed, which make a total of eight parts for the top CF composite flange and eight parts for the bottom CF composite flange. A shorter version of the 3D-printed composite flange with the ‘brim’ is shown in Fig. 6.16. If the slots for the CF composite spar web and ribs had flat surfaces, instead of sharp triangles, as initially printed before series of iterations, the ‘brim’ was not required and the long section (312.5 mm) of the composite flanges could be printed. This is shown in Fig. 6.17.

![Figure 6.15. Initial full-length composite spar flange of one section of the composite D-nose spar of the morphing wing.](image1)

![Figure 6.16. An initial 3D-printed CF composite flange supported by ‘brim’ feature.](image2)
In addition to the ‘brim’ requirement, another reason for 3D-printing two CF composite parts for each section was a scarf joint introduced between the two parts, which increased the length of one part. The scarf joint was used in the latest design after several design iterations, as the scarf joint was known to be good for composites. Two types of scarf joints were explored for the 3D-printed composites; firstly a scarf joint with steps through the flange thickness, and secondly a scarf joint with steps through the flange width.

The scarf joint with steps through the flange thickness is shown in Fig. 6.18. This design was abandoned because of two factors related to the limitations of the Markforged 3D-printer. Firstly, the minimum one layer of onyx/plastic for the wall and roof of the printed parts, which reduced the number of carbon fibre (CF) layers at each step, and secondly, the slightly less accuracy in the thickness dimension of the 3D-printed composite parts, if compared to the accuracy in the length and width dimensions.

The scarf joint with steps through the flange width for section S5 is shown in Fig. 6.19. The Fig. 6.19a shows the two parts of the bottom composite flange of section S5, where one part shows the outer surface while the other part shows the inner surface of the composite flange. The visible long slots in the span wise direction were designed for the composite spar.
web insertion, and the shorter slots in the chord wise direction were designed for the placement of front and rear composite ribs. A total of 8 parts for each top and bottom composite flanges were 3D-printed, for all sections S5-S8 shown in the previous Fig. 6.9.

Figure 6.18. 3D-printed CF composite top flanges of section S8 with the scarf joint, having steps across the flange thickness.

Figure 6.19. 3D-printed bottom flanges of section S5 with the scarf joint, having steps across the flange width: (a) two printed parts and (b) the combined two parts through the scarf joint.
To reinforce the joints, carbon fibre (CF) strips with similar thickness and number of CF layers (as listed in the previous Table 6.5) of the CF composite spar flanges, were 3D-printed. The length and width of the CF strips were constant, 88.12 mm and 13.5 mm, respectively. Only the thickness of the CF composite strips varied along the span, depending on the thickness of the composite spar flanges. Two CF composite strips were 3D-printed for each joint, as shown in Fig. 6.20. A total of 32 CF composite strips were 3D-printed for the total of 16 joints along the span wise direction (8 joints on each top and bottom composite spar flanges).

The evolution of the design of the 3D-printed CF composite spar flanges is illustrated with details in Fig. 6.21. The design changed from one long single part of CF composite spar flanges (top and bottom flanges) for each section (sections S5-S8), with flat-surface slots for insertion of the CF composite spar web and front and rear CF composite ribs, and with a slot for attachment of the CF composite D-nose skin, to two parts of CF composite spar flanges for each section with triangle slots, scarf joints and CF composite reinforcement strips.
6.3 3D-Printed Carbon Fibre (CF) Composite D-Nose Spar

- One long single composite part for one section of sections S5-S8.
- Flat-surface slots for insertion of CF composite spar web and front and rear CF composite ribs.
- One long slots for the spar web, 3 slots for front ribs, and 3 slots for rear ribs.
- One long slot on the upper surface for the D-nose skin attachment.

- Flat-surface slots changed to triangle slots, with sharp edges on the printing bed.
- ‘Brim’ feature of the Markforged 3D-printer was required to anchor the thin section on the printing bed.
- With the ‘brim’, the previous long single part excessed the printing bed size, even if it was 3D-printed diagonally. Hence, two parts were 3D-printed for each section.

- A scarf joint, which was a good type of joints for composites, was introduced.
- Two methods had been tried, which were scarf joints through the thickness, and scarf joints through the width. The later was chosen.
- Flat edges at both ends of the length changed to stepped edges.

- For every scarf joint, two CF composite strips, with a similar thickness as the CF composite spar flanges, were 3D-printed to provide additional reinforcement on the scarf joints.

Figure 6.21. Evolution of the design and manufacturing iterations of the 3D-printed CF composite spar flanges.
6.3.2 CF Composite Spar Web

The carbon fibre (CF) composite spar web of the D-nose spar for all sections S5-S8, had twelve layers of carbon fibre composite, with the fibre directions of (0/45/-45/45/90/90/-45/45/45/-45/0)°. The 0° direction was in the span wise direction. The initial 3D-printed CF composite spar web before design iteration is shown in Fig. 6.22. It was one section of the sections S5-S8 shown in the previous Fig. 6.9. As shown in Fig. 6.22, the CF composite spar web almost fully occupied the printing bed.

![Figure 6.22. Initial design of 3D-printed CF composite spar web.](image)

After few design iterations, which included the use of triangle slots instead of flat-surface slots, for fitting of front and rear CF composite ribs, the use of a ‘brim’ feature to support the thin edges of the triangle slots on the printing bed, and the introduction of scarf joints between sections of the CF composite spar webs, it was decided to 3D-print two composite parts for each of the sections S5-S8, which resulted in a total of eight printed composite parts. Fig. 6.23 shows four parts of the 3D-printed composite spar webs, with scarf joints and slots for ribs placement. As shown in the figure, two CF composite strips were 3D-printed, with a similar CF orientation as the spar web, to reinforce the scarf joints along the span of the CF composite spar web. A total of 16 CF composite strips were 3D-printed for the total of 8 scarf joints on the CF composite spar web. The length and thickness of the CF composite strips for the spar web were constant at 88.12 mm and 1.75 mm, respectively, but the height varied along the sections, as listed in Table 6.6.

The evolution of the design of the CF composite spar web is illustrated in Fig. 6.24. The initial design was a long single CF composite spar web with a total of 6 flat-surfaced slots for supports of the front and rear CF composite ribs. With this design, only 4 parts were required to be 3D-printed for the 1.25m-span CF composite D-nose spar. However, because of the changes from flat-surfaced slots to triangle slots, the use of ‘brim’ feature to anchor the thin section on the printing bed, and the introduction of the scarf joints, the final design was two
CF composite parts of each section of the CF composite spar web sections (S5-S8), with CF composite strips for additional reinforcement.

**Figure 6.23.** Four of the 3D-printed CF composite spar webs after several design iterations.

- One long single CF composite spar web for one section of sections S5-S8.
- Flat-surface slots on both surfaces, for insertion of front and rear CF composite ribs.
- 3 slots for front ribs, and 3 slots for rear ribs.
- Flat-surface slots changed to triangle slots, with thin sharp edges on the printing bed.
- ‘Brim’ feature was required to anchor the thin section on the printing bed.
- With the ‘brim’, the previous long CF composite spar web exceeded the printing bed size, even if 3D-printed diagonally. Two parts were then 3D-printed for each section.
- A scarf joint was introduced.

**Figure 6.24.** Evolution of the design and manufacturing iterations of the 3D-printed CF composite spar web.
Table 6.6. Dimensions of the 3D-printed carbon fibre (CF) composite strips for reinforcement of the composite spar web.

<table>
<thead>
<tr>
<th>Sections of Composite D-Nose Spar</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, ( l ) (mm)</td>
<td>88.125</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height, ( h ) (mm)</td>
<td>20.40</td>
<td>22.37</td>
<td>25.00</td>
<td></td>
</tr>
<tr>
<td>Thickness, ( t ) (mm)</td>
<td>1.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CF Orientations</td>
<td>[0/45/-45/45/90/45/45/45/-45/0]°</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.3.3 CF Composite Front and Rear Ribs

The carbon fibre (CF) composites front and rear ribs were 3D-printed to support the top and bottom flanges, the spar web and the D-nose skin. The ribs were used mainly to avoid buckling of the flanges and the D-nose skin, by reducing the supported length in the span wise direction. Initially, the CF composite ribs were designed with eight layers of carbon fibre lay-ups in (45/-45/45/45/45/45/45/-45)° directions, as previously simulated FEA model of the CF composite D-nose spar, to effectively resist in-plane shear forces. To accommodate the compressive stress applied by the skins and flanges on the ribs, two carbon fibre layups in 90° were included. To account for bending stress on the ribs especially the front ribs, in the chord wise direction, two carbon fibre layups in 0° direction were included. The 0° direction was the chord wise direction. Consequently, the final lay-up of the front and rear CF composite ribs were (90/45/-45/-45/45/0/0/-45/45/90)°.

The 1.75mm-thick front and rear CF composite ribs, fitted into slots on the top and bottom CF composite flanges, is shown in Fig. 6.25. The figure shows the initial design of the long top and bottom CF composite flanges, which was one section of the sections S5-S8, with the front and rear CF composite ribs, and without the CF composite spar web. The design evolved with introduction of a larger surface area on the outer edges of the front CF composite ribs, for support and attachment of the CF composites D-nose skin, and shorter inner section (the part attached to the CF composite spar web, and the top and bottom CF composite flanges) to accommodate the thickness of CF strips reinforcement used at the scarf joints of the top and bottom flanges and the spar web. The final 3D-printed front and rear CF composite ribs are shown in Fig. 6.26. A total of 17 front CF composite ribs and 12 short version of the rear CF composite ribs were 3D-printed. It could be noticed that the length and height of the rear ribs and of the inner sections of the front ribs were different. The reasons were firstly each of the sections S5-S8 had different flanges thickness, and secondly the existence of additional CF strips reinforcement at the scarf joints locations.
6.3 3D-Printed Carbon Fibre (CF) Composite D-Nose Spar

Figure 6.25. Initial design of CF composites front and rear ribs, fitted into slots on top and bottom flanges.

Figure 6.26. 3D-printed carbon fibre composites front and rear ribs, (a) top view and (b) isometric view.

The dimensions of the front and rear ribs are listed in Table 6.7. The height of the CF composite ribs increased when moving outward spanwise from section S5 to section S8, because the thickness of the top and bottom CF composite spar flanges decreased. The height of the front and rear CF composite ribs for each section was similar, but the length of the front CF composite ribs was 0.2 mm longer than the length of CF composite rear ribs, because of
the gap allowance for the adhesive between the front CF composite ribs and the top and bottom CF composite flanges.

Table 6.7. Dimensions of the 3D-printed carbon fibre composite front and rear ribs.

<table>
<thead>
<tr>
<th>Sections of Composite D-Nose Spar</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Front Ribs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height, ( h ) (mm)</td>
<td>28.08</td>
<td>20.68</td>
<td>29.58</td>
<td>23.68</td>
</tr>
<tr>
<td>Length, ( l ) (mm)</td>
<td>15.175</td>
<td>17.125</td>
<td>15.175</td>
<td>17.125</td>
</tr>
<tr>
<td><strong>Rear Ribs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height, ( h ) (mm)</td>
<td>28.08</td>
<td>20.68</td>
<td>29.58</td>
<td>23.68</td>
</tr>
</tbody>
</table>

The evolution of design and manufacturing iterations of the front and rear CF composite ribs are shown in Fig. 6.27. The number of CF layers increased, but the size slightly decreased. A wider outer edge was introduced on the front CF composite rib, and the width increased through iterations.
- 8 CF layers.
- CF lay-up: 
  \[45/-45/-45/45/45/-45/-45/45\]^\circ.
- Height of the outer section of the CF composite front ribs matched the height of the CF composite spar flanges with a long narrow slot.

- 12 CF layers.
- CF lay-up: 
  \[90/45/-45/45/0/0/-45/45/45/90\]^\circ.
- A wider (5 mm) outer section of the front CF composite ribs, for contact surface with the CF composite D-nose skin.
- A smoother leading edge.
- Gaps on the corners of the inner section of the CF composite front and rear ribs, for the long spanwise slots on the CF composite spar flanges for placement of the CF composite spar web.

- The width of the outer section of the front CF composite ribs, for contact surface with the CF composite D-nose skin, was further increased to 10 mm.
- The height of the outer section of the front CF composite ribs was increased, so that it levelled with the height of the of CF composite spar flanges without a long narrow slot.
- Because of the introduction of the CF strips on the top and bottom flanges and the spar web, for additional reinforcement of the scarf joints, two different front and rear CF composite ribs were 3D-printed, for each section of the sections S5-S8. For the front CF composite ribs, one with flat surface on the vertical wall, and another with a flat surface on the horizontal walls. For the rear CF composite ribs, one with a taller height but shorter length, and another with a shorter height but longer length.

Figure 6.27. Evolution of the design and manufacturing iterations of the 3D-printed CF composite front and rear ribs.
6.3 3D-Printed Carbon Fibre (CF) Composite D-Nose Spar

6.3.4 CF Composite D-Nose Skin

The initial plan was to use a 3D-printed mould, which a small version is shown in Fig. 6.28. The figure shows the male and female mould which were 3D-printed with an onyx material (a nylon material with carbon black particles), on the Markforged 3D-printer. The time taken to 3D-print both the 30mm-long (spanwise) male and female moulds was 1 day and 7 hours. A longer version of the male and female moulds, which were 300 mm long, are shown in Fig. 6.29. If they were 3D-printed, the estimated printing time were 3 days 21 hours and 4 days 3 hours, respectively. A total of 5 parts of the 300mm-long male mould and 5 parts of the 300mm-long female mould were needed to form a 1.5m-long mould, and the total printing time was estimated as 40 days. Because of the extremely long printing time to 3D-print the 1.5m-long mould, in addition to the difficulty in combining several 3D-printed parts to form one single mould, the plan was aborted.

![Figure 6.28. A 3D-printed mould for fabrication of D-nose skin.](image)

If the mould was made of a single piece, 1.5m-long, solid wood or aluminium material, the time and material costs could be saved. Because of a shorter production time of the solid wood mould, a female wood mould was chosen for production, as shown in Fig. 6.30, instead of aluminium. Prior to the production of the long solid female wood mould, a thin male mould
of 36% front of the chord of the NACA 0012 aerofoil, with a thicker/offset profile than the original aerofoil profile to accommodate the CF composite skin thickness, was 3D-printed with an onyx material. The 3D-printed part was used to accurately draw the profile of the D-nose on the solid wood for the female mould production. A female mould was decided to be produced, instead of a male mould, because the outer surface of the CF composite D-nose skin was preferred to be smooth rather than the inner surface. With the female mould, the outer surface of the manufactured CF composite skin would take the smooth surface of the wood. The female mould was made of a solid gelatine wood. The mould was made of two parts and was then assembled. Six bolts and nuts, and five circular wood inserts were used to assemble and to precisely align the two parts, to form a single female mould. The aerofoil surface on the female mould was shifted outward of the actual aerofoil coordinates for the NACA 0012 aerofoil, by 1.5 mm, to accommodate the thickness of the carbon fibre (CF) composite skin.

The female mould was used to manufacture the first trial of the CF composite D-nose skin. Twelve layers of HexPly® 8552/IM7 unidirectional (UD) carbon prepregs (pre-impregnated) were cut to a dimension of 1350 mm × 270 mm, with fibre directions or a lay-up of [0/45/-45/-45/45/90/90/-45/45/45/-45/0]°, using the Blackman & White Genesis cutting machine. Prior to
the CF composite lay-up, the female mould was prepared with one layer of a blue release film with an adhesive spray to ensure the release film stick to the mould surface, and one layer of a brown peel ply. The first four CF composite plies were laid-up on a flat aluminium plate and were consolidated under vacuum for about half an hour, as shown in Fig. 6.31a. The centre-width of the flat lay-up was marked on the left and right edges. The consolidated first four plies were laid onto the female mould, with the centre-width mark aligned with the centre of the female mould (the leading edge). The centre of the four-plies was gently pressed against the centre of the female mould, by using a long circular aluminium rod. The four-plies were then consolidated on the female mould for couple of hours. The flat lay-up, consolidation, marking, lay-up onto the female mould, and another consolidation on the female mould processes were repeated with the following four CF composite plies, and the final four CF composite plies. The excess of the CF composite on the long edges of the female mould was trimmed, and a peel ply was laid-up before the final vacuum bagging. The final vacuum bagging of the CF composite prepregs, before moving into the autoclave, is shown in Fig. 6.31b.

With reference to Fig. 6.32, the cure cycle of the CF composite D-nose skin made of the HexPly® 8552/IM7 unidirectional (UD) carbon prepregs are listed below.

1) A full vacuum of 1 bar was applied.

2) 7 bar gauge autoclave pressure was applied.

3) The vacuum was reduced to a safety value of -0.2 bar once the autoclave pressure reached approximately 1 bar gauge.

4) Heat was applied at 3°C/min to 110°C ± 5°C.

5) The temperature was hold at 110°C ± 5°C for 60 minutes.

6) Heat was applied at 3°C/min to 180°C ± 5°C.

7) The temperature was hold at 180°C ± 5°C for 120 minutes.

8) The CF composite was cooled at 1°C/min to a room temperature.

9) The autoclave pressure was vented once the temperature of the CF composite reached 60°C.
Figure 6.30. A 1.5m-long solid wood female mould.
6.3 3D-Printed Carbon Fibre (CF) Composite D-Nose Spar

Figure 6.31. Manufacturing of the CF composite D-nose skin: (a) consolidation of four plies of the CF composite prepregs, and (b) the final twelve plies of the CF composite prepregs on the female mould, inside the vacuum bag, before curing process in the autoclave.

Figure 6.32. Curing cycle for honeycomb and monolithic component of Hexcel 8552 CF composite prepregs (Hexcel Corporation, 2016).

The outcome of the first trial of the manufacture of the CF composite D-nose skin was unfortunate, where the wood female mould and the CF composite prepregs caught fire and were completely burned inside the autoclave, as shown in Fig. F3 in Appendix F. The suspected
causes of fire were the filler used in the mould, the adhesive spray used to stick the blue release film onto the mould, a paper strip from the tacky tape which was accidentally left inside the vacuum bag, and an exothermic reaction of the resin. The first two suspected causes were rejected, after a test on two pieces of the similar wood applied with the filler and the adhesive spray, and then were put in a furnace (Fig. F4a in Appendix F) at 180°C for three hours. Neither of the wood pieces was burned or was on fire, during the test, as depicted in Fig. F4b in Appendix F. Hence, neglecting the very minor chance of accidentally leaving a paper inside the vacuum bag because it could be easily spotted through the transparent vacuum bag, the most probable cause of the fire was the last one, which was the exothermic reaction of the resin which might accumulate at certain location (most probably one end of the mould) along the span of the mould, and the accumulated heat from the exothermic reaction might start the fire. Therefore, it was extremely important to note that a male mould should be used instead of a female mould to avoid such incident.

A solid Mahogany wood was then used to produce a 1.5m-long male mould for the next fabrication of the CF composite D-nose skin. As shown in Fig. 6.33, the solid wood male mould had a shape of 36% front of the 300mm-chord NACA 0012 aerofoil, which make the mould height of 108 mm, with a total length of 1.5 m. Prior to plies lay-up, the mould was carefully and tightly covered with a red release film to prevent the carbon fibre plies sticking onto the mould after curing. The release film on the mould was ensured to be as smooth as possible to produce a CF composite D-nose skin with a good internal surface finish.

On the second trial of the manufacturing of the carbon fibre (CF) composite skin, eight layers of HexPly® M21/IM7 unidirectional (UD) carbon prepregs (pre-impregnated) were cut and were laid-up in [0/45/-45/90]°s directions. The M21/IM7 UD carbon prepregs were used,
because the previously used HexPly® 8552/IM7 UD carbon prepregs in the first trial were out of stock, and the new rolls were expected to arrive in 2-3 months. Eight layers were laid-up, instead of the previous 12 layers, because of the designed 1.5 mm thickness of the CF composite D-nose skin. With a cured ply thickness of 0.184 mm, 8 layers of the UD CF prepregs made a total thickness of 1.472 mm (the closest to 1.5 mm). With the maximum number of 8 layers and the above-mentioned sequence of lay-up (balanced and symmetric), the specially orthotropic CF composite skin could not be achieved, and only two of the three couplings could be avoided, which were the direct stress – shear strain coupling and the bending-membrane coupling. The third coupling, which was the bending-twisting coupling ($D_{13}$ and $D_{23} \neq 0$) could not be avoided with the above lay-up. With 8 layers of the CF lay-up, it was possible to achieve the specially orthotropic condition with a lay-up of [45/-45/-45/45/-45/45/-45/45]°. However, because of the critical requirement of at least two layers of CF (one layer above the neutral axis (NA) and another layer below the NA) in the spanwise or 0° direction, to prevent buckling of the composite D-nose spar, the option was opted out. After the 8 CF layers were laid-up, pieces or air breather were laid on the short (chordwise) and long (spanwise) edges of the CF layers, and continued to the vacuum hose, to ensure the air inside the CF prepregs was not trapped and was completely sucked out. No peel ply was used, and the vacuum bag was used as the final layer, because a shiny smooth surface was preferred and expected rather than a slightly rough surface if the peel ply was used. The CF composite skin was cured in the autoclave, with the temperature and pressure profile shown in Fig. 6.34. The CF composite D-nose skin before and after curing is shown in Fig. 6.35.

![Autoclave cure cycle for monolithic component (<15mm thick) of Hexcel M21 CF composite prepregs (Hexcel Corporation, 2015).](image)

**Figure 6.34.** Autoclave cure cycle for monolithic component (<15mm thick) of Hexcel M21 CF composite prepregs (Hexcel Corporation, 2015).
On the third trial, a high temperature silicone sheet (SILEX SILICONES LTD) was introduced as the outermost layer of the 8 plies CF composite lay-up, to obtain a CF composite D-nose skin with a smooth surface. The solid wood mould was prepared by covering the mould with a new red release film, as depicted in Fig. 6.36a. Eight layers of M21 unidirectional carbon fibre (CF) prepregs were cut using the Blackman & White Genesis cutting machine, with a dimension of 1400 mm × 250 mm and fibre directions of [0/45/-45/90]°s. The eight M21 carbon fibre plies were laid up on the male mould. Four first plies were laid up and were consolidated in a vacuum bag, followed by the remaining four layers and another consolidation in a vacuum bag. To obtain a smooth external surface finish, a high temperature silicone sheet (SILEX SILICONES LTD) was used as the final layer, on top of the carbon fibre layers.
breather pieces were placed along the edges of the layup, to allow air coming out of the pressed
carbon fibre plies when they were compressed under vacuum. The consolidated CF composite
prepregs were cured in the autoclave, with pressure and temperature profiles similar to the
profiles in the second trial. During the autoclave curing, a loss of vacuum occurred, and caused
a drop of temperature reading from one of the thermocouples. The temperature of the
operational thermocouple was let to increase to 180 °C, before adjustment was made on the
temperature-dropping thermocouple so that it increased to 180 °C as well. The result of the
autoclave-cured CF composite D-nose spar is shown in Fig. 6.36c and Fig. 6.36d. It could be
observed that the manufactured CF composite D-nose spar was over cured, most probably
because of the inconsistent temperature, and the top layer with fibre in 0° or span wise direction
was imperfect with de-bonding at multiple locations. Strangely, the 45° CF layer underneath
the 0° CF layer was perfectly cured and bonded to the rest of the CF layers. The suspected
reason was that there were possibly gaps between the silicone layer and the outermost CF layer,
when the silicone sheet wrinkle under the vacuum pressure.

An alternative approach to obtain a smooth outer surface of the CF composite D-nose skin,
was using a thin aluminium skin as the outermost layer, after layers of CF composites plies.
On the 4th trial of manufacturing the CF composite skin, 12 plies of HexPly® 8552/IM7
unidirectional (UD) carbon prepregs (pre-impregnated) were used, as the new batch of the
material had arrived. To save the material cost, the pre-pregs were cut to a dimension of 238
mm (chordwise circumference) × 100 mm (spanwise). The spanwise length was significantly
reduced to 100 mm, if compared to the previous three trials that had a length of 1400 mm. Prior
to the lay-up, the thin aluminium foil was heated to 200 °C for 3 hours, in a furnace
(Nabertherm), to ensure it did not burn under the applied temperature and to avoid the burning
incident of the solid wood mould. The aluminium foil was then prepared by applying three
layers of a liquid release film (ChemTrend Chemlease® 41-90 EZ). It was then cut to about a
similar size of the CF composite prepregs. The CF composite prepregs were laid-up on the
centre of the D-nose mould, as shown in Fig. 6.37a, with [0/45/-45/-45/45/90/90/-45/45/45/
-45/0]° fibre orientations, similar to the first trial. The aluminium skin was then laid-up as the
outermost layer, before the lay-up was covered with a brown peel ply, as shown in Fig. 6.37b.
Pieces of air breather were laid on top of the lay-up (Fig. 6.37c), before vacuum bagging (Fig.
6.37d). The vacuum was applied to consolidate the CF composite lay-up, as shown in Fig.
6.37e, and was left on for about an hour, before the autoclave curing. The autoclave curing
cycle of the short-width CF composite D-nose skin was similar the curing cycle used in the first trial, as shown in the previous Fig. 6.32.

![Images](a)(b)(c)(d)

**Figure 6.36.** The third trial of the CF composite D-nose skin.

The outcome of the 4th autoclave curing of the CF composite D-nose skin is shown in Fig. 6.38. A loss of vacuum occurred during the curing cycle, similar to the 3rd trial. However, the CF D-nose skin was properly cured. It was observed that the outer surface took the surface of the thin aluminium sheet and the peel ply, while the inner surface took the surface of the wood mould. Wrinkles were visible at certain locations on the aluminium sheet, in the spanwise
direction, which were suspected due to the dislocation of the outermost CF layer in the 0° (spanwise) direction, when the CF D-nose skin was cured under a high pressure (7 bar).

Figure 6.37. The fourth trial of the CF composite D-nose skin.
After the male wood mould had been into the autoclave for three times, and was heated up to 180 °C for a total duration of approximately 12 hours, the wood mould had a burning smell after the curing of the fourth CF composite D-nose skin. It was afraid that the mould would burn if it went into the autoclave again with another high-temperature curing CF prepregs. For this reason, ideally low temperature prepregs (lower than 120 °C curing temperature) should be used for the next iteration, however it was impossible because of a very long lead time for ordering of a new material. Alternatively, braided CF fabrics and low temperature epoxy resin films (MTM57-319) were chosen as the materials for the subsequent fabrication of the CF composite D-nose skin. After several iterations, the result of manufacture of a short (200 mm) rectangular CF composite tube, which was a trial before manufacturing a 1.35m-long CF composite D-nose skin, is shown in Fig. 6.39. The objectives of the manufacture of the trial braided CF composite tube were to ensure the manufacturing process was correct and to save the composite material, prior to the manufacturing of the CF composite D-nose skin.
6.3.5 Manufacturing Difficulties, Constraints and Iterations

Various manufacturing problems were encountered during the 3D-printing of the CF composite parts of the D-nose spar. Listed below are some of them and the necessary steps undertaken to tackle the problems.

1) Gap/tolerance of slots

When the width of the slots, for example the slot on the spar flange for connection to the spar web (Fig. 6.40), was modelled and printed as equal to the thickness of the spar web, the spar web could not be perfectly fitted into the slot because of slightly excessive thickness. Therefore, a width tolerance of 0.4 mm (0.2 mm on each side) was applied to the slots.

![Figure 6.40. Initial design of the spar flanges and spar web, and tolerance for the slots.](image)

2) Minimum number of layers for wall, roof and floor

The number of plastic (onyx) layers for the wall, top roof and bottom floor were kept to the minimum, which was one onyx layer, for all printed composite parts. This was the default minimum layer constrained by the Markforged printer. Hence, this constrain had to be taken into account in the design procedure especially during the assembly of multiple parts of the CF composite D-nose spar. However, the recommended number of layers by Markforged, for the top roof and bottom floor were four layers, and for the wall were two layers, in order to ensure the printed parts had a good surface finish and were watertight.
3) **Detached carbon fibre layers**

In one of the 3D-printing session, some of the fibre layers were suddenly detached or separated from the printed parts, as shown in Fig. 6.41. The figure shows the top flange of the 7th section (from wing root) of the composite D-nose spar, which had two detached carbon fibre rings. The detached carbon fibres are shown in the bottom right image. This occurred because the fibre or plastic nozzle accidentally hit the newly printed carbon fibre layers. This issue was solved by recalibrating the positions of the fibre and plastic nozzles, and performing a bed levelling utility.

![Detached carbon fibres](image)

**Figure 6.41.** Detached carbon fibres of the top flange of the CF composite D-nose spar.

4) **Non-uniform thickness**

Another manufacturing defect was non-uniform thickness of the carbon fibre layers, which occurred because of the defect of the first few layers of the onyx support. This defect of onyx support occurred because of poor surface condition of the printing bed. The problem was solved by monitoring the 3D-printing process for as long as possible, and by shifting the designed part upward to the printing bed area that had a better surface condition.
6.4 Assembly of CF Composite D-Nose Spar

In the Finite Element Analysis (FEA) simulations (Section 6.2), the connections between the CF composite spar flanges, spar webs, front and rear ribs, and D-nose skin were assumed to be perfectly bonded. The nodes at the connection points between the multiple parts were merged to achieve this perfect bonding condition. The main reasons of this approach were to significantly save computational time and to avoid complexity of the contact failure.

In the experimental bench top model assembly, the CF composite parts were assembled with an acrylic based adhesive. All eight pieces of the top spar flanges were assembled, followed by all eight pieces of bottom spar flanges and all eight pieces of spar webs. Then, the long assembled top spar flange, bottom spar flange and spar web were assembled to form a CF composite I-beam structure. Next, the front and rear ribs were assembled into the slots on the spar flanges and the spar web.

Prior to the assembly, the CF composite spar flanges, spar web, ribs, and reinforcement strips were stored in an oven at 40 °C overnight, to ‘dry-out’ the grease in the CF composites, because the matrix of the 3D-printed CF composites was nylon-based. On the assembly of the top CF composite spar flange, two parts were assembled at a time. The parts were prepared by covering the top surface of the parts using the blue adhesive tape, as shown in Fig. 6.42, for easy handling and to ensure the top surface would be clean from any acrylic adhesive. The CF strips to reinforce the scarf joints were also covered with the tapes to reduce the mess. The acrylic adhesive was applied on the side edges of the scarf joints, and in between the CF reinforcement strips and the bottom surface of the CF composite spar flanges. The adhesive application process was carried out in the fume cabinet, and the assembled parts were immediately but carefully moved onto a vacuum table that was covered with a brown peel ply, as shown in Fig. 6.43a. The assembled CF composite spar flanges with the CF composite reinforcement strips were supported along the length with thick aluminium plates covered with a blue release film, to ensure that the CF composite strips did not slip out when the whole assembly was under pressure when the vacuum was applied. The assembled CF composite spar flanges under the applied pressure is shown in Fig. 6.43b. The assembly was left for 24 hours to ensure a complete curing of the acrylic adhesive, and to achieve a maximum possible strength of the joints. The results of the assembly of the CF composite spar flanges are shown in Fig. 6.44. The assembly processes of the CF composite spar flanges were repeated for the rest of the scarf joints along the length of the spar flanges, until they were assembled to two (top and bottom) single 1.35m-long CF composite spar flanges, as depicted in Fig. 6.45.
6.4 Assembly of CF Composite D-Nose Spar

Figure 6.42. 3D-printed CF composite spar flanges covered with blue protective tapes.

Figure 6.43. 3D-printed CF composite spar flanges on the vacuum table.
A couple of important points had to be addressed. Firstly, the assembly of the 1.35m-long spar flanges was carried out on the vacuum table, to ensure the long assembled parts of the CF composite spar flanges remained flat during curing of the acrylic adhesive. The parts could possibly warp if the applied pressure was not uniform. Secondly, only two assemblies could be completed at one time, as the gel time of the acrylic adhesive was short. The acrylic adhesive started to solidify after 5-7 minutes, and hence the vacuum had to be applied on the parts as quickly as possible after the acrylic adhesive was applied on the CF composite spar flanges.

The assembly of the CF composite spar web was then carried out with similar methods and procedures, but with few alterations. The differences were two wider CF composite strips were bonded on each of the scarf joints to reinforce the joints, the CF orientation/lay-up of the CF composite strips were similar to the CF composite spar web in contrast to the unidirectional CF composite strips for the top and bottom flanges, and the spar web was supported by additional pieces of equal thickness of the CF composite strips, to ensure that the spar web did not bend under atmospheric pressure when the vacuum was applied. The sequence of the assembly process of the CF composite spar web is shown in Fig. 6.46.
Figure 6.45. Sequence of assembly of the 3D-printed CF composite spar flanges.
Figure 6.46. Sequence of assembly of the 3D-printed CF composite spar web.
The three long assembled structures, which were the CF composite top spar flange, bottom spar flange and spar web, were then assembled by inserting the spar web onto the long middle slots on the top and bottom spar flanges, with the acrylic adhesive applied between the edges of the spar web and surfaces of slots on the top and bottom spar flanges. Mixing nozzles (3M™ EPX mixing nozzles for 45ml cartridges) were required with the two part adhesive gun and the acrylic adhesive cartridge, instead of manually mixing the adhesive as previously done for parts of the CF composite spar flanges and spar web, because of the very long (1.35 m) spar web edges and slots on the top and bottom flanges, which required fast and even application of the adhesive. The CF composite spar flanges and spar web before and on the assembly process are shown in Fig. 6.47. G-clamps were used along the spar length/span, to apply pressure on the surfaces and edges with the applied adhesive. During the assembly, as the G-clamps were tightened, the CF composite ribs were slotted into the slots for the ribs on the top and bottom CF composite flanges and CF composite spar web, to ensure the top flange was parallel to the bottom flange. The G-clamps were supported on the table by the circular rods on the handle, to ensure their weight did not twist the CF composite spar flanges and spar web. The acrylic adhesive was let to cure for 24 hours, as usual.
The main reason of the assembly in stages was because of the properties of the acrylic adhesive for the nylon-based material, which required 24 hours to complete the curing process. After application of the acrylic adhesive on the joints, the part could not be touched and the curing time of 24 hours had to be allowed before subsequent joining process.

After the assembly of the CF composite spar flanges and spar web, the solidified acrylic adhesive which was squeezed out at the slots for the CF composite ribs, as shown in Fig. 6.48, was carefully removed as it prevented the front and rear CF composite ribs to be slotted properly on the next stage of the assembly. On the next assembly, the CF composite front ribs were bonded on the CF composite spar flanges and spar web, as depicted in Fig. 6.49. At one time, a maximum of three CF composite front ribs were bonded because of the short handling time of the acrylic adhesive, and the adhesive was left to cure for 24 hours before the next assembly of another three front ribs. The first three CF composite front ribs were bonded (Fig. 6.49a) at locations where the slots were on the top and bottom CF composite flanges, and the reinforcement strips were on the CF composite spar web (Fig. 6.48a). After 24 hours curing, the following three CF composite front ribs were bonded (Fig. 6.49b) at locations where the slots were on the CF composite spar web, and the reinforcement strips were on the top and bottom CF composite flanges (Fig. 6.48b). The assembly process was repeated until all 15 CF composite front ribs were assembled, as shown in Fig. 6.49c.

![Figure 6.48. Squeezed-out solidified acrylic adhesive at the slots on (a) the CF composite spar flanges and (b) the CF composite spar web, for the CF composite front and rear ribs.](image)

After the CF composite front ribs were assembled, the CF composite rear ribs were bonded onto the CF composite spar flanges and spar web, with similar sequences of process as the previous assembly of the CF composite front ribs. To support the CF composite D-nose spar and to ensure the top and bottom CF composite spar flanges remained flat and parallel during the assembly, circular cylinders were positioned at four locations along the span of the CF composite D-nose spar. The CF composite D-nose spar could not be simply placed on the table like the previous assembly, because of the assembled CF composite front ribs. Examples of the
assembled CF composite rear ribs are depicted in Fig. 6.50. The two long CF composite rear ribs were not yet bonded to the CF composite spar, as they were first to be bonded to the foam sections and rear trailing edge sections, before assembled onto the CF composite spar.

Figure 6.49. Assembly of 3D-printed CF composite front ribs.
Figure 6.50. The assembled 3D-printed CF composite rear ribs.
6.5 Foam Sections and 3D-Printed Trailing Edge Sections

The rear part of the 1.25m-span composite morphing wing, which was 64% rear of the wing chord (from 36% chord to 100% chord), was designed with four main sections. The four sections comprised of three sections of rigid foams and one section of flexible foam, which were fitted in between four 3D-printed CF composite rear ribs (long version) and a CF composite corrugated section. The foam sections were accurately machined with a CNC machine (AXIOM Precision), from stereolithography (STL) files of the foam sections, which were converted from finite element models in LS-DYNA.

![Foam sections machined by a CNC machine.](image)

For each foam section, a top section was machined followed by a bottom section. The top and bottom sections are shown in Fig. 6.51a. The CNC-machined foam sections had the NACA 0012 aerofoil shape (64% rear of the chord or 192 mm chordwise length), with additional thin
sections on both sides and additional wider section on the front side, which were slotted into the CF composite rear ribs (long version) and the assembled CF composite D-nose spar, respectively. The spanwise length of the three innermost sections of the rigid foams was 302.5 mm, while the outermost flexible foam section had a spanwise length of 276.25 mm.

Figure 6.52. (a) Damaged trailing edge of the foam sections and (b) a rigid 3D-printed CF composite trailing edge section.

The example of the foam sections simply supported by two long CF composite rear ribs are shown in Fig. 6.51b. The front part of the foam sections were carefully sanded with an abrasive paper, which one of the end results is shown in Fig. 6.51c, so that they fit into the slots between the CF composite spar flanges, spar web and short rear ribs. The top and bottom foam sections were then glued together to form the aerofoil shape. Because of the very thin foam material at the trailing edge, the cutter of the CNC machine destroyed most of the foam sections at the trailing edge, as shown in Fig. 6.52a. To solve this, three rear rigid trailing edge sections
were 3D-printed with carbon fibre and onyx materials. One of the 3D-printed CF composite trailing edge sections on the 3D-printing bed is depicted in Fig. 6.52b.

The rigid 3D-printed CF composite trailing edge (TE) sections had a length of 6.4 mm, with small slots on both sides. The slots were meant to be slotted onto the long CF composite rear ribs for better grip between the CF composite TE section and the two long rear ribs. The CF composite TE sections were 3D-printed with two concentric carbon fibre rings, triangular fill pattern, four roof and floor layers, and two wall layers. If a cross-section of the TE section, normal to the spanwise direction, was observed, the TE section was made of a sandwich structure consisted of top and bottom unidirectional CF in the spanwise direction, and a core of a triangular fill.

6.6 FE Simulations of Composite Morphing Wing

Due to time constraint, the experimental bench-top static test on the composite morphing wing could not be completed mainly because of the manufacturing of the CF composite D-nose skin, which required several additional time-consuming iterations to reach perfection. However, the expected experimental result should approximately matched the predicted result of the FE simulations of the composite morphing wing, which are presented in this section. The FE model of the SMA-actuated CF composite corrugated structure was integrated with the FE model of the CF composite D-nose spar. Only one section of the CF composite D-nose spar, which was section S8 (outermost section close to the wing tip), was included in the FE simulation to save computational time. They were subjected to the chordwise aerodynamic lift (which was represented by static point loads at three locations on the corrugation) and the spanwise aerodynamic lift (which was represented by a static line load on the quarter-chord of the wing, with only one-eighth of the total lift load, close to the wing tip). The composite morphing wing at the cruise flight condition, upon SMA-actuation (one SMA wire with a diameter of 0.5 mm) is shown in Fig. 6.53. The SMA-actuated composite morphing wing with the optimum trailing edge deflection (with optimum 54 SMA wires with a diameter of 0.5 mm) is shown in Fig. 6.54. The top, middle and bottom images are the composite morphing wing at the initial unactuated condition, on the applied aerodynamic loadings, and on SMA actuation, respectively. The fringes in the figures show the displacement in the vertical z-direction. With a single SMA wire along the corrugation, the achieved tip deflection was 7.4 mm, while with the optimum number of SMA wires, the achieved tip deflection was 15.1 mm.
Figure 6.53. SMA-actuated composite morphing wing at a cruise flight condition.
Figure 6.54. Optimum SMA-actuated composite morphing wing at a cruise flight condition.
The optimum tip deflection of the SMA-actuated composite morphing wing was significantly lower than the previously achieved tip deflection, when only the corrugated surface with the trailing edge section were actuated by series of SMA wires, as shown in the previous Fig. 5.75. The possible reason was the difference in the boundary condition of the fixed edge of the corrugated surface at the front of the wing chord. The edge was rigidly fixed in all degree of freedoms for the previous case in Chapter 5, while it was attached to the composite spar web of the composite D-nose spar in this Chapter 6. Because of the less fixed or simply supported boundary condition, the D-nose spar was also tend to deform when the SMA wires were activated, and hence reduced the tip deflection of the trailing edge section.

6.7 Chapter Summary and Conclusions

The composite morphing wing was designed and manufactured. The composite morphing wing comprised of a CF composite D-nose spar, foam sections, long CF composite rear ribs and trailing edge sections, and SMA-actuated CF composite corrugated section. The internal parts of the CF composite D-nose spar comprised of spar flanges and spar web, and front and rear ribs. They were manufactured using the composite 3D-printing technology. A composite moulding manufacturing process was used to manufacture the CF composite D-nose skin, and several iterations were attempted.

The experimental validation test of the composite morphing wing was not completed because of time constraint. The main reason was the difficulties in the manufacturing of the CF composite D-nose skin. Other internal parts of the CF composite D-nose spar were assembled, and they need to be assembled with the D-nose skin, before could be assembled with the other parts of the rear section of the composite morphing wing, such as the foam sections, the long CF composite rear ribs, the CF composite trailing edge sections, and the SMA-actuated CF composite corrugated section. Therefore, the experimental test on the composite morphing wing could not be completed to validate the wing (CF-composite D-nose spar) deformation which was predicted by the FE simulations. However, the expected result of the planned bench-top test on the SMA-actuated composite morphing wing should approximately match the FE simulation result presented in Section 6.6 at the end of this chapter.
Chapter 7
Conclusions and Recommendations

7.1 Conclusions

The conclusions on the research into the design of a composite morphing wing were:

1) Various morphing concepts, specifically aerofoils and wings camber morphing concepts were comprehensively reviewed. The review covered the morphing concepts based on mechanical actuation, such as motors and gearboxes, servos and eccentuators, and also based on smart actuation, such as shape memory alloys, piezoelectrics/piezoceramics, and electro-active polymers (EAPs). Compliant mechanisms, such as cellular and corrugated structures, and multi-stable mechanisms/structures were utilised to introduce the flexibility to the aerofoils/wings.

2) The theoretical thermomechanical behaviour of the SMA wires were investigated by analysing several SMA constitutive models, such as Tanaka, Liang and Rogers and Brinson SMA constitutive models. The Tanaka SMA constitutive model was decided to be implemented in the development of a new user defined material model (UMAT) of the SMA wires. Material properties of the SMA wires were obtained from series of DSC tests, tensile tests and DMTA tests on the SMA wires, and the supplier material datasheet.

3) A new user defined material model (UMAT) for shape memory alloy (SMA) wires was successfully developed in an explicit LS-DYNA, for actuation of various structures. The key approach was solving an actuation of a fundamental structure, consisted of a SMA wire and a linear spring connected in series. The UMAT was systematically developed, by validations of the SMA wire - linear spring structure and a SMA-actuated cantilever beam, and by an extensive virtual testing of the fundamental structure in a mesh sensitivity and a key parameter studies.

4) The UMAT of the SMA wires was used as a design tool to actuate various structures, such as several aluminium aerofoils, carbon fibre (CF) composite aerofoils, and pre-curved corrugated aluminium plates. The introduction of several corrugated sections with SMA
wires onto the aerofoils, and different arrangement of the SMA wires, produced various trailing edge deflections. Several aerofoil design were investigated, and a final aerofoil concept which consisted of a rigid D-nose spar, a centre corrugated section with SMA wires, and a rear trailing edge section, was chosen for fabrication, because of a high trailing edge deflection and a smooth variation of the aerofoil camber. The chosen morphing aerofoil concept should be covered by a very flexible structure, i.e. a thin compliant cellular honeycomb structure or an aerogel-based structure, on top and bottom parts of the corrugated section, which would form a complete aerofoil with the outer surfaces. However, the design of such structures was out of scope of this research.

5) A 3D-printing technology was used to manufacture the carbon fibre (CF) composite corrugated structure with a rear trailing edge (TE) section. Experimentally, actuation of the structure by a single NiTi SMA wire aligned parallel to the chord line, resulted in a maximum trailing edge deflection of 6.0 mm. 10 and 30 cyclic tests were carried out, and the trailing edge deflections were found to converge after a few cycles. Small permanent TE deflections were observed on cooling of the SMA wire, due to the residual stress/strain in the SMA, which was typically observed in various SMA-actuated structures from the literature. The experimental finding of the small permanent TE deflection on cooling of the SMA wire matched the predicted FE simulations.

6) A new approach of finite element modelling a 3D-printed composite structure was explored, by defining a user defined integration through the shell thickness. Using this method, composite layers made of different materials and thickness could be accurately modelled, which was very useful especially for the 3D-printed composite with a concentric fibre ring printing mode. Virtual 3-point bending tests on the carbon fibre (CF) composite beams were simulated, and the results of the flexural stiffness within the elastic region, matched the results obtained from the experimental tests. The FEA simulations of the CF composite beam, which was modelled as two beams and with a user defined integration through the shell thickness, validated the experimental finding on how the CF composite behaved after a delamination occurred.

7) A 1.25m-span CF composite morphing wing was analytically and numerically designed, and was then experimentally manufactured. The wing comprised of a CF composite D-nose spar, which was designed to be sufficiently strong and stiff to resist the main aerodynamic loading, and a rear section which consisted of a rigid rear section and a
7.1 Conclusions

flexible morphed rear section. The CF composite D-nose spar was built from 3D-printed CF composite spar flanges, spar web, front and rear ribs, and a moulded CF composite skin. The rear section was made of rigid and flexible foams, and 3D-printed CF composite rear ribs and SMA-actuated corrugated section.

7.2 Recommendations for Future Research

Based on the extensive research of the composite morphing wing, it is highly recommended for the future research to:

1) Design and develop a new and novel SMA spring, made of braided SMA wires, with an internal heater as a core element. Intuitively, the new braided SMA spring will have thermomechanical properties between a single SMA wire (high actuation force, low stroke) and a single SMA spring (low actuation force, high stroke). With an optimum design, the actuation force of the braided SMA spring can be altered to meet the force requirement, while the achievable stroke can be improved. The introduction of an internal heating element as a core will provide a more uniform and fast heating, compared to the direct resistive heating.

2) Utilise the UMAT of the SMA wire as a design tool for other types of morphing aircraft structures, such as winglet morphing, dihedral morphing, swept morphing, span morphing, engine inlet morphing and engine chevron morphing.

3) Exploit the UMAT of the SMA wire to design a morphing micro unmanned air vehicle (UAV), as the size or the cross-sectional area of the SMA wire can be reduced numerically to micrometres, depending on the size of the micro UAV.

4) Extend the UMAT to 2D and 3D FE simulations.

5) Investigate SMA wires with a higher SMA maximum recoverable strain. The experimental and numerical trailing edge (TE) deflections of the composite morphing aerofoils were achieved by using only a SMA wire with 1.6% maximum recoverable strain. The TE deflections are expected to amplify if SMA wires with a higher maximum recoverable strain, e.g. 4%, from other commercial suppliers are considered.

6) Experimentally optimise the trailing edge deflection of the SMA-actuated CF composite corrugated structure, by developing a SMA actuator consisted of the predicted optimum
54 SMA wires, which are arranged in parallel and embedded in a compliant elastomer. Alternatively, SMA plates/strips with an equal total cross-sectional area can be investigated.

7) Increase the amplitude/height of the corrugation, and hence increasing the distance of acting SMA wires away from the neutral axis or the chord line, which increase the applied moment and the resulted trailing edge deflection.

8) Design and manufacture a morphing skin made of a compliant structure, such as a thin flexible cellular honeycomb structure and an aerogel-based structure.

9) Machine two-pieces male and female solid aluminium moulds, by using a CNC machine, for the autoclave manufacturing of the CF composite skin. The lead time of the mould making is worth waiting, rather than using a single male/female solid wood mould which had a risk of burn and produced unsatisfactory end product that required multiple iterations. It would save a lot of expensive composite material.

10) Simulate Fluid Structure Interaction (FSI) simulations on the SMA-actuated composite morphing aerofoil/wing, to investigate the effect of morphing the aerofoil/wing on the surrounding air flow, and the effect of the changing air flow on the aerofoil/wing. This study will provide more understanding on the magnitude of the lift increment and the drag reduction due to morphing, compared to the conventional hinged aerofoil/wing.


Anaglyph Ltd 1995-2000 Laminate Analysis Program (LAP) 4.1 user manual.


Katwijk K V 1981 Buckling of flat rectangular plates (isotropic, orthotropic and laminated composite plates and sandwich panels), ESDU 81047.


Hexcel Corporation 2016 HexPly® 8552 epoxy matrix (180°C/356°F curing matrix), *Product Data Sheet*.


Katwijk K V 1981 Buckling of flat rectangular plates (isotropic, orthotropic and laminated composite plates and sandwich panels), *ESDU (Engineering Science Data Unit) 81047*.


Markforged Inc. 2016 Mechanical properties of continuous fibers and nylon, *Material Datasheet*.


SAES Getters 2008 SAES smart materials.

S.A.E.S Group (Societa Apparecchi Elettriche Scientifici – Electrical and scientific components company) 2009 SmartFlex wire and spring datasheet.


Schuerch H U 1968 Certain physical properties and applications of nitinol, NASA CR-1232.


TA Instruments 2014 RSA-G2 solids analyser.


Talling R J, Jackson M, Dashwood R, Iannucci L and Dye D 2007 Deformation of Ti-36Nb-2Ta-3Zr-0.3O (Gum metal) and applications to morphing wings, 2nd SEAS DTC (System Engineering and Autonomous Systems Defence Technology Centre) Technical Conference, Edinburgh.


www.fordgt.com

www.mercedesamgf1.com
APPENDIX A: EXPERIMENTAL TESTS ON SHAPE MEMORY ALLOY (SMA) WIRES

APPENDIX A1 - DIFFERENTIAL SCANNING CALORIMETRY (DSC) TESTS

Figure A1. Differential Scanning Calorimetry (DSC) tests on shape memory alloy wires with 0.1 mm diameter.
Figure A2. Differential Scanning Calorimetry (DSC) tests on shape memory alloy wires with 0.2 mm diameter.
Figure A3. Differential Scanning Calorimetry (DSC) tests on shape memory alloy wires with 0.5 mm diameter.
APPENDIX A2 – TENSILE TESTS

Figure A4. Captured images of NiTi SMA wires with a diameter of 0.1 mm and a gauge length of 200 mm, subjected to a tensile load at a loading rate of (a) 0.5 N/min and (b) 5 N/min, at the start of tests (left), at the maximum tensile load of 10N (middle), and at the end of tests (right).
Figure A5. Captured images of NiTi SMA wires with a diameter of 0.2 mm and a gauge length of 300 mm, subjected to a tensile load at a loading rate of (a) 1 N/min and (b) 5 N/min, at the start of tests (left), at the maximum tensile load of 30N (middle), and at the end of tests (right).
Figure A6. Captured images of NiTi SMA wires with a diameter of 0.5 mm and a gauge length of 200 mm, subjected to a tensile load at a loading rate of 5 N/min, at the start of tests (left), at the maximum tensile load of 200N (middle), and at the end of tests (right).
APPENDIX B: SMA TRANSFORMATION EQUATIONS

Martensitic transformation (austenite-to-martensite phase)

From transformation kinetics:

\[ \frac{\dot{\xi}}{1 - \xi} = b_M c_M \dot{T} - b_M \dot{\sigma} \]  

(B1)

With a condition \( b_M c_M \dot{T} - b_M \dot{\sigma} \geq 0 \).

With initial states of \( \xi = 0, T = M_s, \sigma = 0 \) in fully austenite condition, integration of the above Eq. B1 to current states of stress, temperature and martensite volume fraction, resulted in:

\[ \int_{0}^{\xi} \frac{\dot{\xi}}{1 - \xi} = \int_{M_s}^{T} b_M c_M \dot{T} - \int_{0}^{\sigma} b_M \dot{\sigma} \]

\[ -[\ln(1 - \xi) - \ln(1 - 0)] = b_M c_M (T - M_s) - b_M (\sigma - 0) \]

\[ \ln(1 - \xi) = b_M c_M (M_s - T) + b_M \sigma \]

\[ 1 - \xi = e^{b_M c_M (M_s - T) + b_M \sigma} \]

\[ \xi_{A \rightarrow M} = 1 - e^{b_M c_M (M_s - T) + b_M \sigma} \]  

(B2)

Reverse transformation (martensite-to-austenite phase)

From transformation kinetics:

\[ -\frac{\dot{\xi}}{\xi} = b_A c_A \dot{T} - b_A \dot{\sigma} \]  

(B3)

With a condition \( b_A c_A \dot{T} - b_A \dot{\sigma} \geq 0 \).

With initial states of \( \xi = 1, T = A_s, \sigma = 0 \) in fully martensite condition, integration of the above Eq. B3 to current states of stress, temperature and martensite volume fraction:

\[ \int_{1}^{\xi} -\frac{\dot{\xi}}{\xi} = \int_{A_s}^{T} b_A c_A \dot{T} - \int_{0}^{\sigma} b_A \dot{\sigma} \]

\[ -[\ln \xi - \ln 1] = b_A c_A (T - A_s) - b_A (\sigma - 0) \]

\[ \ln \xi = b_A c_A (A_s - T) + b_A \sigma \]

\[ \xi_{M \rightarrow A} = e^{b_A c_A (A_s - T) + b_A \sigma} \]  

(B4)
APPENDIX C: UMAT, FORTRAN & MATLAB CODES

APPENDIX C1 – UMAT OF SMA

In the section card in LS-DYNA, the default ‘Hughes-Liu element formulation with cross-section integration’ (ELFORM = 1) was assigned, while the integration rule ID (IRID) was linked to Integration_Beam, having circular standard cross section type (ICST = 8) and cross section dimension of 0.5 mm (D1 = 0.5). For output, option 4 in Database_Elout was set to 42, which was equal to the number of history variables used in the UMAT FORTRAN code, and Database_History_Beam was activated and assigned to the beam element of the SMA wire. The important keyword files in LS-DYNA that were used to simulate the UMAT of the SMA are listed in Table C1.
<table>
<thead>
<tr>
<th>Material Model</th>
<th>LS-DYNA Keywords</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Section Beam</strong></td>
<td>ELFORM (element formulation)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>CST (cross section type)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>TS1, TS2 (outer diameter, mm)</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>TT1, TT2 (inner diameter)</td>
<td>0</td>
</tr>
<tr>
<td><strong>Integration Beam</strong></td>
<td>ICST (standard cross section type)</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>D1 (cross section dimension, mm)</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Mat_User Defined Material Models</strong></td>
<td>RO (mass density, kg/mm$^3$)</td>
<td>7.89×10$^{-6}$</td>
</tr>
<tr>
<td></td>
<td>MT (user material type)</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>LMC (length of material constant)</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>NHV (number of history variables)</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>IBULK (address of bulk modulus)</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>IG (address of shear modulus)</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>IVECT (vectorization flag)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>ITERM (temperature flag)</td>
<td>1</td>
</tr>
<tr>
<td><strong>Shape Memory Alloy (SMA) Wires</strong></td>
<td>cm(1) Elastic modulus at martensite phase, $E_M$ (GPa)</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>cm(2) Poisson’s ratio, $\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>cm(3) Elastic modulus at austenite phase, $E_A$ (GPa)</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>cm(4) Martensite starting temperature (°C)</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>cm(5) Martensite finishing temperature (°C)</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>cm(6) Austenite starting temperature (°C)</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>cm(7) Austenite finishing temperature (°C)</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>cm(8) Martensite stress influenced coefficient (GPa/°C)</td>
<td>0.0082</td>
</tr>
<tr>
<td></td>
<td>cm(9) Austenite stress influenced coefficient (GPa/°C)</td>
<td>0.0082</td>
</tr>
<tr>
<td></td>
<td>cm(10) Maximum recoverable strain, $\varepsilon_r$ (mm/mm)</td>
<td>0.1/0.2/0.4/0.5</td>
</tr>
<tr>
<td></td>
<td>cm(11) SMA wire diameter (mm)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>cm(12) SMA wire length (mm)</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>cm(13) Spring stiffness (kN/mm)</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>cm(14) Maximum number of iteration (convergence)</td>
<td>1 or 2</td>
</tr>
<tr>
<td></td>
<td>cm(15)</td>
<td>0 or 1</td>
</tr>
<tr>
<td></td>
<td>cm(16) Bulk modulus, $B$ (GPa)</td>
<td>33.33</td>
</tr>
<tr>
<td></td>
<td>cm(17) Shear modulus, $G$ (GPa)</td>
<td>15.38</td>
</tr>
<tr>
<td></td>
<td>cm(18) Cases:</td>
<td>1 or 2</td>
</tr>
<tr>
<td></td>
<td>1 – Constant load case / loading before actuation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 – SMA actuation only</td>
<td></td>
</tr>
<tr>
<td></td>
<td>cm(19) Relaxation time before activation of SMA wires</td>
<td></td>
</tr>
<tr>
<td></td>
<td>cm(20) Monitoring, on (0) and off (1) print statements</td>
<td></td>
</tr>
<tr>
<td><strong>Load Thermal Load Curve</strong></td>
<td>LCID</td>
<td>1</td>
</tr>
<tr>
<td><strong>Database Elout</strong></td>
<td>Option4 (number of additional history variables for beam elements)</td>
<td>42</td>
</tr>
<tr>
<td><strong>Database History Beam</strong></td>
<td>Each ID for each beam element of SMA wires</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX C2 – FORTRAN CODE: UMAT

******************************************************************************

# USER DEFINED MATERIAL (UMAT) MODEL OF SHAPE MEMORY ALLOY (SMA) #
# PROJECT: COMPOSITE MORPHING STRUCTURES #
# RESEARCHER: WAN LUQMAN HAKIM BIN WAN A HAMID #
# SUPERVISORS: PROFESSOR LORENZO IANNUCCI & PROFESSOR PAUL ROBINSON #
# UNIVERSITY: IMPERIAL COLLEGE LONDON #
# RESEARCH PERIOD: 1ST MAY 2015 – 30TH APRIL 2019 #
******************************************************************************

subroutine umat41 (cm,eps,sig,eps, hsv,dt1,capa,etype,tt, 
  temper,failel,crv,cma,qmat,elsiz,idele)

  Livermore Software Technology Corporation (LSTC)
  Copyright 1987-2008 Livermore Software Tech. Corp
  All rights reserved

  isotropic elastic material (sample user subroutine)

  Variables

  cm(1)=first material constant, here young's modulus
  cm(2)=second material constant, here poisson's ratio
  cm(n)=nth material constant

  eps(1)=local x strain increment
  eps(2)=local y strain increment
  eps(3)=local z strain increment
  eps(4)=local xy strain increment
  eps(5)=local yz strain increment
  eps(6)=local zx strain increment

  sig(1)=local x stress
  sig(2)=local y stress
  sig(3)=local z stress
  sig(4)=local xy stress
  sig(5)=local yz stress
  sig(6)=local zx stress

  hsv(1)=1st history variable
  hsv(2)=2nd history variable
  hsv(n)=nth history variable

  dt1=current time step size
  capa=reduction factor for transverse shear
  etype:
  eq."solid" for solid elements
  eq."sph" for smoothed particle hydrodynamics
  eq."sld2d" for shell forms 13 (2D solids - plane strain)
  eq."sldax" for shell forms 14, and 15 (2D solids - axisymmetric)
  eq."sh1" for shell forms 25, 26, and 27 (shells with thickness stretch)
  eq."shell" for all other shell elements plus thick shell forms 1 and 2
  eq."tshel" for thick shell forms 3 and 5
  eq."bbeam" for beam element forms 1 and 11
  eq."tbeam" for beam element form 3 (truss)
  eq."dbeam" for beam element form 6 (discrete)
  eq."beam " for all other beam elements

  tt=current problem time.
  temper=current temperature

316
Appendix

c failel=flag for failure, set to .true. to fail an integration point,
c    if .true. on input the integration point has failed earlier

c crv= array representation of curves in keyword deck

c cm= additional memory for material data defined by LMCA at 6th field of 2nd card of
  *DATA_USER_DEFINED

c elsiz= characteristic element size

c idele= element id

c All transformations into the element local system are performed prior to entering this
    subroutine. Transformations back to the global system are performed after exiting this
    routine.

c All history variables are initialized to zero in the input phase. Initialization of
    history variables to nonzero values may be done during the first call to this
    subroutine for each element.

c Energy calculations for the dyna3d energy balance are done outside this subroutine.

c include 'nlqparm'
include 'bk06.inc'
include 'iounits.inc'
dimension cm(*),eps(*),sig(*),hsv(*),crv(lq1,2,*),cma(*),qmat(3,3)
logical failel
character*5 etype
#endif BIGID
integer*8 idele
#else
integer idele
#endif
if (ncycle.eq.1) then
    if (cm(16).ne.1234567) then
        call usermsg('mat41')
    endif
endif
*******************************************************************************************
## START: SHAPE MEMORY EFFECT OF SMA - BEAM WITH ONE INTEGRATION POINT ##
*******************************************************************************************
elseif (etype.eq.'beam') then
    SMA material constants (am bm aa ba)
        am=log(0.01)/(cm(4)-cm(5))
        bm=am/cm(8)    ! (Eq. 8)
        aa=log(0.01)/(cm(6)-cm(7))
        ba=aa/cm(9)   ! (Eq. 7)
        pi=3.141592654
        A=pi*(cm(11)*cm(11))/4    ! SMA cross-sectional area

c History Variables:
    hsv(1)= updated strain
    hsv(2)= updated stress
    hsv(3)= total strain
    hsv(4)= total stress
    hsv(5)= deltaT
    hsv(6)= To
    hsv(7)= deltaT/dt
    hsv(8)= mvf M-A (heating)
    hsv(9)= mvf A-M (cooling)
    hsv(10)= Young’s modulus E
    hsv(11)= shear modulus G
    hsv(12)= bulk modulus
    hsv(13)= Xiderivative(dx/dsig)
    hsv(14)= Ederivative(dE/dsig)
    hsv(15)= sig_ite
    hsv(16)= fsig
    hsv(17)= f derivative (df/dsig)
    hsv(18)= sig
    hsv(19)= mvf
    hsv(20)= E
    hsv(21)= strain
    hsv(22)= displacement
    hsv(23)= strain without load (spring)
    hsv(24)= delta sig
    hsv(25)= temperature
    hsv(26)= iteration no.
    hsv(27)= eps(1)-eps0
    hsv(28)= eps(1)
    hsv(29)= sig(1)-sig0
    hsv(30)= sig(1)
    hsv(31)= delteps/sig
    hsv(32)= eps0_AM
    hsv(33)= sig0_AM
    hsv(34)= T3
    hsv(35)= cycle no.
    hsv(36)= eps0_MA
    hsv(37)= sig0_MA
    hsv(38)= T1
    hsv(39)= T2
    hsv(40)= tempint

hsvo(1)= eps(1)
hsvo(2)= sig(1)
hsvo(4)= cm(1)* hsv(3)
hsv(5)=temper-hsv(6)
hsv(6)=temper
hsv(7)=hsv(5)/dt1

hsv(27)=eps(1)-hsv(28)
hsv(28)=eps(1)
hsv(29)=sig(1)-hsv(30)
hsv(30)=sig(1)
hsv(31)=hsv(27)/hsv(29)  ! delta_eps / delta_sig in derivative of stress functions (trials in the effort of generalizing SMA material model)

hsv(8)=exp(aa*(cm(6)-temper)+ba*sig(1)*0)
! IF T < As and T > Af
if(temper .le. cm(6))then
  hsv(8)=1
  hsv(23)=cm(10)*(hsv(8)-1)
elseif(temper .ge. cm(7))then
  hsv(8)=0
  hsv(23)=cm(10)*(hsv(8)-1)
end if

hsv(10)=cm(3)+hsv(8)*(cm(15)-cm(3))
hsv(11)=hsv(10)/(2*(1+cm(2)))
hsv(12)=hsv(10)/(3*(1-2*cm(2)))

! Applied stress before temperature cycles
if(cm(18) .eq. 1)then
  print *,’CASE 1 = SMA-spring’
  continue
elseif(cm(18) .eq. 2)then
  print *,’CASE 2 = constant load’
  if(hsv(7) == 0 .and. tt <= cm(19))then
    hsv(3)=hsv(3)+eps(1)
    hsv(8)=1
    hsv(10)=cm(1)
    hsv(15)=hsv(10)*hsv(3)
    hsv(36)=hsv(3)                                        ! eps0_MA
    hsv(37)=hsv(15)                                       ! sig0_MA
    hsv(38)=cm(6)+hsv(37)/cm(9)                           ! T1
    hsv(39)=cm(7)+hsv(37)/cm(9)                           ! T2
  endif
endif

*****
## Martensite-to-Austenite (Reverse Transformation) – Heating ##
*******************************************************************************************
! IF As < T < Af
if(hsv(7) .gt. 0 .and. temper >= (cm(6)+hsv(37)/cm(9)))then
  ! .and. ‘ to avoid unnecessary iteration at the beginning of if condition.
  ! Newton-Raphson iteration
  hsv(3)=hsv(3)+eps(1)

  * Statement indicating the start of Newton-Raphson iteration for heating stage *
  if(cm(20) .eq. 1)then
    print *,’=> Newton-Raphson iteration:’
  elseif(cm(20) .eq. 0)then
    continue
  endif
  * Statement ends *

  its = 0                 ! iteration counter
  maxits = cm(14)         ! maximum iterations
  converged = 0           ! convergence flag
  error = 1.0d-6          ! maximum error
hsv(15) = hsv(18)  ! initial stress

do while (converged == 0 .and. its < maxits)
  hsv(8)=exp(aa*(cm(6)-temper)+ba*hsv(15))  ! MVF Xi (Eq. 6)
  hsv(10)=cm(3)+hsv(8)*(cm(15)-cm(3))  ! E (Eq. 2)
  hsv(11)=hsv(10)/(2*(1+cm(2)))  ! G (Eq. 27)
  hsv(12)=hsv(10)/(3*(1-2*cm(2)))  ! B (Eq. 28)
  hsv(13)=ba*hsv(8)  ! dXi (Eq. 12)
  hsv(14)=(cm(15)-cm(3))*hsv(13)  ! dE (Eq. 13)

  ! GENERALIZE constitutive equation - NOT use A, cm(12) k, and cm(13) l.
  f3=hsv(10)*(hsv(3)-hsv(36))  ! f(s) (Eq. 10)
  f4=cm(10)*hsv(10)*(hsv(8)-1)  ! f(s) (Eq. 10)
  df2=(hsv(36)-hsv(3))*hsv(14)  ! df(s) (Eq. 11)
  df4=cm(10)*hsv(8)*hsv(14)  ! df(s) (Eq. 11)
  df5=cm(10)*hsv(14)  ! df(s) (Eq. 11)
  hsv(16)=hsv(15)-hsv(37)-f3+f4  ! f(s) (Eq. 10)
  hsv(17)=1+df2+df3+df4+df5  ! df(s) (Eq. 11)
  hsv(15)=hsv(15)-hsv(16)/hsv(17)  ! signew (Eq. 18)

  * Convergence status monitoring *
  if(cm(20) .eq. 1)then
    print *, 'iteration no = ', its
    print *, 'hsv(15):sig = ', hsv(15)
    print *, 'hsv(16):fsig = ', hsv(16)
  elseif(cm(20) .eq. 0)then
  endif
  * Statement ends *
  its = its + 1
  if (abs(hsv(16)) <= error) converged = 1
end do

* Convergence statement *
if(cm(20) .eq. 1)then
  if(converged==1)then
    print *, 'Newton_Raphson iteration converged'
  else
    print *, 'Newton_Raphson iteration did not converge'
  endif
elseif(cm(20) .eq. 0)then
  continue
endif
* Statement ends *

hsv(23)=cm(10)*(hsv(8)-1)  ! iteration no
hsv(26)=its

!!! HOW to make final strain as constant?
eps0_AM=hsv(3)
sig0_AM=cm(3)*hsv(3)+cm(10)*cm(3)
T3=cm(4)+sig0_AM/cm(8)
! ERRORS: All 3 above goes to zero in the next cycle

hsv(32)=hsv(3)  ! eps0_Am
hsv(33)=hsv(37)+cm(3)*(hsv(3)-hsv(36))+cm(10)*cm(3)  ! sig0_AM
hsv(34)=cm(4)+hsv(33)/cm(8)  ! T3 **signew

if HERE, final stress constant. see FEA 03. If inside next loop, final stress
increase. see FEA 02.
! ERROR: eps0_Am=-sig0_Am*A/(cm(13)*cm(12))
! ERROR: sig0_Am=cm(10)*cm(3)*cm(13)*cm(12)/(cm(13)*cm(12)+cm(3)*A)

*******************************************************************************************
## Austenite - to - Martensite (Forward Martensitic Transformation) - Cooling ##
elseif(hsv(7) .lt. 0)then
  ! .and. temper => cm(5)
  !!! Solution to increasing/decreasing strain on cooling !!!
hsv(3)=hsv(3)+eps(1)        !!!
if(cm(20) .eq. 1)then
  print *, 'eps0_AM = ', hsv(32)    ! eps0_AM
  print *, 'sig0_AM = ', hsv(33)    ! sig0_AM
  print *, 'T3 = ', hsv(34)         ! T3
elseif(cm(20) .eq. 0)then
  continue
endif
if(temper .ge. hsv(34))then
  hsv(9)=0
!elseif(temper .le. cm(5))then
!    hsv(9)=1
end if
hsv(10)=cm(3)+hsv(9)*(cm(15)
- cm(3))                   ! E
hsv(11)=hsv(10)/(2*(1+cm(2)))                         ! G
hsv(12)=hsv(10)/(3*(1
- 2*cm(2)))                       ! B
if(temper .le. hsv(34))then  ! temper .ge. cm(5) .and.
  !!!SOLUTION TO INCREASING
/DECREASING STRAIN ON COOLING!!!
! .and. to avoid unnecessary iteration at the beginning of if cond.
  ! Newton-Raphson iteration (cooling)

* Statement indicating the start of Newton-Raphson iteration for cooling stage *
if(cm(20) .eq. 1)then
  print *, '&=> Newton-Raphson iteration:'
elseif(cm(20) .eq. 0)then
  continue
endif
* Statement ends *
its = 0                 ! iteration counter
maxits = cm(14)         ! maximum iterations
converged = 0           ! convergence flag
error = 1.0d-6          ! maximum error
hsv(15) = hsv(18)       ! initial stress
do while (converged == 0 .and. its < maxits)
  ! GENERALIZE constitutive equation - NOT use A cm(12) k cm(13) l
  hsv(9)=1-exp(am*(cm(4)
- temper)+bm*hsv(15))      ! MVF Xi (Eq. 5)
hsv(10)=cm(3)+hsv(9)*cm(15)
- cm(3))                   ! E (Eq. 2)
hsv(13)=bm*(hsv(9)
- 1)                           ! dXi (Eq. 16)
hsv(14)=(cm(15)
- cm(3))*hsv(13)                  ! dE (Eq. 17)
f3=hsv(10)*hsv(32)-hsv(3)
+ hsv(15)-hsv(33)+f3+f4
  ! f(sig) (Eq. 14)
df2=hsv(32)*hsv(14)
df3=hsv(3)*hsv(14)
df4=cm(10)*hsv(10)*hsv(9)
hsv(16)=hsv(15)-hsv(33)+f3+f4
  ! f(sig) (Eq. 14)
df5=cm(10)*hsv(9)*hsv(14)
hsv(17)=1+df2-df3+df4+df5
hsv(15)=hsv(15)-hsv(16)/hsv(17)                 ! signew (Eq. 18)
* Convergence status monitoring *
if(cm(20) .eq. 1)then
  print *, '&iteration no = ', its
  print *, '&hsv(15):sig = ', hsv(15)
  print *, '&hsv(16):fsig = ', hsv(16)
elseif(cm(20) .eq. 0)then
  continue
endif
* Monitoring ends *
its = its + 1
if (abs(hsv(16)) <= error) converged = 1
end do

* Convergence statement *
if(cm(20) .eq. 1)then
if(converged==1)then
print *, 'Newton-Raphson iteration converged'
else
print *, 'Newton-Raphson iteration did not converge'
endif
elseif(cm(20) .eq. 0)then
continue
endif
* Statement ends *
end if

hsv(23)=cm(10)*(hsv(9)-1)
hsv(26)=its  ! iteration no
hsv(36)=hsv(3)  ! eps0_MA
hsv(37)=hsv(33)+cm(1)*hsv(3)-hsv(32))-cm(10)*cm(1)*hsv(3)+1  ! sig0_MA
hsv(38)=cm(6)+hsv(37)/cm(9)  ! T1
hsv(39)=cm(7)+hsv(37)/cm(9)  ! T2

!!! hsv(35)=hsv(35)+1                         ! cycle number

! Final output: MUST be outside if condition!
hsv(24)=hsv(15)-hsv(18)                     ! delta sig
hsv(18)=hsv(15)                          ! stress
hsv(19)=hsv(8)                          ! martensite volume fraction
hsv(20)=hsv(10)                          ! E
hsv(21)=eps(1)                           ! strain
hsv(22)=cm(12)+hsv(21)*cm(12)            ! displacement
hsv(25)=temper                            ! G shear modulus
hsv(11)=hsv(20)/(2*(1+cm(2)))          ! B bulk modulus
hsv(12)=hsv(20)/(3*(1-2*cm(2)))
cm(1)=hsv(20)
cm(16)=hsv(12)
cm(17)=hsv(11)

g2 =abs(hsv(20))/(1.+cm(2))  ! cm(1) to hsv(20)
g =.5*g2
q1 =hsv(20)*cm(2)/(1.0+cm(2))*1.0-2.0*cm(2))  ! cm(1) to hsv(20)
q3 =q1+2.0*g
gc =capa*g
deti =1.0/(q3*q3-q1*q1)
c22i = q3*deti
c23i = q1*deti
fac =c22i+c23i)*q1
eps(2)=eps(1)*fac-sig(2)*c22i-sig(3)*c23i  ! eps(1) to hsv(21)
eps(3)=eps(1)*fac-sig(2)*c23i-sig(3)*c22i  ! eps(1) to hsv(21)
davg =(-eps(1)-eps(2)-eps(3))/3.  ! eps(1) to hsv(21)
p =-davg*hsv(20)/1.2.*cm(2))  ! cm(1) to hsv(20)
sig(1)=hsv(18)
sig(2)=0.0
sig(3)=0.0
sig(4)=sig(4)+gc*eps(4)
sig(5)=0.0
sig(6)=sig(6)+gc*eps(6)

if(cm(20) .eq. 1)then                         ! print statements after main equations, not at top
print *, 'dt1 = ',dt1
print *, 'tt = ',tt
print *, 'eps(1) = ',eps(1)
print *, 'current stress sig(1)= ', sig(1)
print *, 'current E = ', cm(1)
print *, 'temperature = ', hsv(25)
print *, '=> History variables:
print *, 'hsv(1): strain = ', hsv(1)
print *, 'hsv(2): stress = ', hsv(2)
print *, 'hsv(3): total strain = ', hsv(3)
print *, 'hsv(4): total stress = ', hsv(4)
print *, 'hsv(5): deltaT = ', hsv(5)
print *, 'hsv(6): To = ', hsv(6)
print *, 'hsv(7): deltaT/dt = ', hsv(7)
print *, 'hsv(8): martensite volume fraction M-A = ', hsv(8)
print *, 'hsv(9): martensite volume fraction A-M = ', hsv(9)
print *, 'hsv(10): young modulus = ', hsv(10)
print *, 'hsv(11): shear modulus = ', hsv(11)
print *, 'hsv(12): bulk modulus = ', hsv(12)
print *, '=> Final results:
print *, 'hsv(18): stress = ', hsv(18)
print *, 'hsv(19): mvf = ', hsv(19)
print *, 'hsv(20): E = ', hsv(20)
print *, 'hsv(21): strain = ', hsv(21)
write(*,100) 'sig(1)=', sig(1)
100format (A,F)
write(*,110) 'eps(1)=', eps(1)
110format (A,F)
write(*,120) 'E cm(1)=', cm(1)
120format (A,F)
write(*,130) 'B cm(16)=', cm(16)
130format (A,F)
write(*,140) 'G cm(17)=', cm(17)
140format (A,F)
print *, '----------------------------------------
elseif(cm(20) .eq. 0)then
continue
endif
c  ## END SHAPE MEMORY EFFECT OF SMA - BEAM WITH ONE INTEGRATION POINT ##
celse
c  write(iotty,10) etype
c  write(iohsp,10) etype
c  write(iomsg,10) etype
c  call adios(TC_ERROR)
cerdat(1)=etype
c  call lsmsg(3,MSG_SOL+1150,ioall,ierdat,rerdat,cerdat,0)
end

c10  format(/
c1  1 ' *** Error element type ',a,' can not be',
c2  2 ' run with the current material model. ')
return
end
APPENDIX C3– FORTRAN CODE: VECTORISED UMAT

******************************************************************************
# VECTORIZED USER DEFINED MATERIAL (UMAT) MODEL OF SHAPE MEMORY ALLOY (SMA) #
# PROJECT: COMPOSITE MORPHING STRUCTURES #
# RESEARCHER: WAN LUKMAN HAKIM BIN WAN A HAMID #
# SUPERVISORS: PROFESSOR LORENZO IANNUCCI & PROFESSOR PAUL ROBINSON #
# UNIVERSITY: IMPERIAL COLLEGE LONDON #
# RESEARCH PERIOD: 1ST MAY 2015 - 30TH APRIL 2019 #
******************************************************************************

subroutine umat41v(cm,d1,d2,d3,d4,d5,d6,sig1,sig2,sig3,sig4,sig5,sig6,epsps,hsvs,lft,llt,dt1siz,capa,etype,tt,temps,failels,nlqa,crv,cma,qmat,elsizv,idelev)

******************************************************************************
# Livermore Software Technology Corporation (LSTC)                          |
# Copyright 1987-2008 Livermore Software Tech. Corp                        |
# All rights reserved                                                      |
******************************************************************************

#include 'nlqparm'
#include 'nhisparm.inc'
dimension d1(*),d2(*),d3(*),d4(*),d5(*),d6(*)
dimension sig1(*),sig2(*),sig3(*),sig4(*),sig5(*),sig6(*)
dimension cm(*),epsps(*),hsvs(nlq,2,*),dt1siz(*)
dimension temps(*),crv(lq1,2,3),cma(*),qmat(nlq,3,3),elsizv(*)
#ifdef BIGID
integer*8 idelev(*)
#else
integer idelev(*)
#endif
logical failels(*)
character*5 etype
dimension sig(6),eps(6),hsv(NHISVAR)
do i=lft,llt

* Monitoring beam elements of SMA *
if(cm(20) .eq. 1)then
    print *, '***************************'
    print *, 'Element no. =',i
    print *, '***************************'
elseif(cm(20) .eq. 0)then
    continue
    endif
* Monitoring ends *

sig(1)=sig1(i)
sig(2)=sig2(i)
sig(3)=sig3(i)
sig(4)=sig4(i)
sig(5)=sig5(i)
sig(6)=sig6(i)
eps(1)=d1(i)
eps(2)=d2(i)
eps(3)=d3(i)
eps(4)=d4(i)
eps(5)=d5(i)
eps(6)=d6(i)

hsv(1)=hsvs(i,1)
hsv(2)=hsvs(i,2)
hsv(3)=hsvs(i,3)
hsv(4)=hsvs(i,4)
hsv(5)=hsvs(i,5)
hsv(6)=hsvs(i,6)
hsv(7)=hsvs(i,7)
hsv(8)=hsvs(i,8)
hsv(9)=hsvs(i,9)
hsv(10)=hsvs(i,10)
hsv(11)=hsvs(i,11)
hsv(12)=hsvs(i,12)
hsv(13)=hsvs(i,13)
hsv(14)=hsvs(i,14)
hsv(15)=hsvs(i,15)
hsv(16)=hsvs(i,16)
hsv(17)=hsvs(i,17)
hsv(18)=hsvs(i,18)
hsv(19)=hsvs(i,19)
hsv(20)=hsvs(i,20)
hsv(21)=hsvs(i,21)
hsv(22)=hsvs(i,22)
hsv(23)=hsvs(i,23)
hsv(24)=hsvs(i,24)
hsv(25)=hsvs(i,25)
hsv(26)=hsvs(i,26)
hsv(27)=hsvs(i,27)
hsv(28)=hsvs(i,28)
hsv(29)=hsvs(i,29)
hsv(30)=hsvs(i,30)
hsv(31)=hsvs(i,31)
hsv(32)=hsvs(i,32)
hsv(33)=hsvs(i,33)
hsv(34)=hsvs(i,34)
hsv(35)=hsvs(i,35)
hsv(36)=hsvs(i,36)
hsv(37)=hsvs(i,37)
hsv(38)=hsvs(i,38)
hsv(39)=hsvs(i,39)
hsv(40)=hsvs(i,40)

call umat41(cm,eps,sig,epsps(i),hsv,dt1siz(i),capa,etype,tt,
temps(i),faiiels(i),crv,cma,0,elsizv(i),idelev(i))
c
sig1(i)=sig(1)
sig2(i)=sig(2)
sig3(i)=sig(3)
sig4(i)=sig(4)
sig5(i)=sig(5)
sig6(i)=sig(6)
c
hsvs(i,1)=hsv(1)
hsvs(i,2)=hsv(2)
hsvs(i,3)=hsv(3)
hsvs(i,4)=hsv(4)
hsvs(i,5)=hsv(5)
hsvs(i,6)=hsv(6)
hsvs(i,7)=hsv(7)
hsvs(i,8)=hsv(8)
hsvs(i,9)=hsv(9)
hsvs(i,10)=hsv(10)
hsvs(i,11)=hsv(11)
hsvs(i,12)=hsv(12)
hsvs(i,13)=hsv(13)
hsvs(i,14)=hsv(14)
hsvs(i,15)=hsv(15)
hsvs(i,16)=hsv(16)
APPENDIX C4 – MATLAB CODE

%% REVERSE TRANSFORMATION (MARTENSITE-TO-AUSTENITE M-A: HEATING) %%
% This MATLAB code predicts a complete behaviour of a SMA wire connected to a %
% linear spring, during heating and cooling cycle, by applying Tanaka constitutive %
% model and its related expression of martensite volume fraction. %
%
% SMA wire dimensions:
d = 0.5; % SMA diameter (mm)
l = 100; % SMA length (mm)
A = pi * (d^2) / 4; % SMA cross-sectional area (mm^2)

% Material constants (SAES Datasheet):
Em = 40000; % SMA Young's Modulus in martensite (MPa)
Ea = 75000; % SMA Young's Modulus in austenite (MPa)
As = 60.0; % Austenite starting temperature (Degree Celsius)
Af = 65.0; % Austenite finishing temperature (Degree Celsius)
Ca = 8.2; % Austenite stress influence coefficient (MPa / Degree Celsius)
Ms = 47.0; % Martensite starting temperature (Degree Celsius)
Mf = 43.0; % Martensite finishing temperature (Degree Celsius)
Cm = 8.2; % Martensite stress influence coefficient (MPa / Degree Celsius)
strainL = 0.016; % Maximum recoverable strain - from experiment

k = 3.5; % Spring stiffness (N/mm) - k = 1.5737 N/mm for cantilever beam

% Final output: (1) stress, (2) martensite volume fraction, (3) strain, and (4) tip position/displacement of SMA wire as function of temperature
%
% Initial states of SMA wire:
sigma0 = 0; % Initial stress
strain0 = - sigma0 * A / ( k * l ); % Initial strain
Xi0 = 1; % Initial martensite volume fraction - fully martensite

% Preallocating to increase speed (save computational time)
T = zeros(1,70000);
Xi = zeros(1,1000000);
E = zeros(1,1000000);
Xi_final = zeros(1,70000);
sigma = zeros(1,70000);
sigma_ite = zeros(1,1000000);
f_sigma = zeros(1,1000000);
df_sigma = zeros(1,1000000);
strain = zeros(1,70000);
lf = zeros(1,70000);
E_plot = zeros(1,70000);
sigma_plot = zeros(1,70000);

sigma(1) = sigma0; % Initial stress
Xi_final(1) = Xi0; % Initial martensite volume fraction
strain(1) = strain0; % Initial strain
lf(1) = 1 + strain(1) * l; % Initial displacement
T(1) = As; % Initial temperature

for i = (1:70000)
    sigma_ite(i) = sigma(i);
end
% Stress value to start iteration; changes every cycle of complete iteration
for j = (1:1000000)
    % Martensite volume fraction (Tanaka exponential equation - heating)
    Xi(j) = exp ( aA * ( As - T(i) ) + bA * sigma_ite(j) );
    % SMA stiffness / Young's modulus
    E(j) = Ea - Xi(j) * ( Ea - Em );
    % Function 'f' obtained from SMA constitutive equation
    f_sigma(j) = sigma_ite(j) + E(j) * sigma_ite(j) * A / ( k * l ) + strainL * E(j) * Xi(j) - strainL * ( Em - Ea ) * bA * Xi(j); % New stress iterated from old stress
    sigma_ite(j+1) = sigma_ite(j) - f_sigma(j) / df_sigma(j);
    if ( (sigma_ite(j+1) - sigma_ite(j))^2 ) <= 0.000000000001^2
        break
    end
end

% Final output:
% Martensite volume fraction corresponding to the final iteration stress
% Pass final value of stress 'sigma_ite(j+1)' into variable 'sigma' for next iteration
strain(i+1) = - sigma_ite(j+1) * A / ( k * l ); % Strain, calculated from stress
lf(i+1) = l + strain(i+1) * l; % Deflection, calculated from strain
T(i+1) = T(i) + 0.001;

% Condition to end iteration, get out from for loop
break
end

% -----------------------------------
% Newton-Raphson iteration ends
% -----------------------------------

% Final output:
% Martensite volume fraction corresponding to the final iteration stress
% Pass final value of stress 'sigma_ite(j+1)' into variable 'sigma' for next iteration

% Austenite to Martensite A-M (Cooling)

% Material constants
aM = log (0.01) / ( Ms - Mf );
bM = aM / Cm;

% Preallocating to increase speed (save computational time)
T_AM = zeros(1,70000);
Xi_AM = zeros(1,100000);
E_AM = zeros(1,100000);

sigma0_AM = ( strainL * Ea * k * l ) / ( k * l + Ea * A ); % Initial stress
strain0_AM = - sigma0_AM * A / ( k * l ); % Initial strain
Xi_final_AM = zeros(1,70000);
sigma_AM = zeros(1,70000);
sigma_ite_AM = zeros(1,100000);
f_sigma_AM = zeros(1,100000);
df_sigma_AM = zeros(1,100000);
Xii_plot_AM = zeros(1,70000);
strain_AM = zeros(1,70000);
lf_AM = zeros(1,70000);
E_AM_plot = zeros(1,70000);
sigma_AM_plot = zeros(1,70000);

sigma_AM(1) = sigma0_AM; % Initial stress
Xi_final_AM(1) = Xi0_AM; % Initial martensite volume fraction
strain_AM(1) = strain0_AM; % Initial strain
lf_AM(1) = 1 + strain_AM(1) * l; % Initial displacement

T3 = Ms + sigma0_AM / Cm;
T_AM (1) = T3; % Initial temperature

for i = (1:70000)

sigma_ite_AM(1) = sigma_AM(i);
% Stress value to start iteration; changes every cycle of complete iteration

% ------------------ Newton-Raphson iteration starts ------------------
% for j = (1:100000)
% Martensite volume fraction (Tanaka exponential equation - cooling)
Xi_AM(j) = 1 - exp ( aM * ( Ms - T_AM(i) ) + bM * sigma_ite_AM(j) );
% SMA stiffness / Young’s modulus
E_AM(j) = Ea - Xi_AM(j) * ( Ea - Em );
% Function 'f' obtained from SMA constitutive equation
f_sigma_AM(j) = sigma_ite_AM(j) - sigma0_AM - E_AM(j) * ( - sigma_ite_AM(j) * A / ( k * l ) - strain0_AM ) + strainL * E_AM(j) * Xi_AM(j);
% Differentiation of Function 'f'
df_sigma_AM(j) = 1 - ( Em - Ea ) * bM * ( Xi_AM(j) - 1 ) * ( - sigma_ite_AM(j) * A / ( k * l ) - strain0_AM - strainL * Xi_AM(j) ) + E_AM(j) * A / ( k * l ) + strainL * E_AM(j) * bM * ( Xi_AM(j) - 1 );
% New stress iterated from old stress
sigma_ite_AM(j+1) = sigma_ite_AM(j) - f_sigma_AM(j) / df_sigma_AM(j);
if ( (sigma_ite_AM(j+1) - sigma_ite_AM(j))^2 ) <= 0.000000000001^2
% Condition to end iteration, get out from for loop break
end
%
% Newton-Raphson iteration ends -------------------
%
% Final output:

Xi_final_AM(i+1) = Xi_AM(j);
% Martensite volume fraction corresponding to the final iteration stress
sigma_AM(i+1) = sigma_ite_AM(j+1);
% Pass final value of stress 'sigma_ite_AM(j+1)' into variable 'sigma_AM' for next iteration
strain_AM(i+1) = - sigma_ite_AM(j+1) * A / ( k * l );
% Strain, calculated from stress
lf_AM(i+1) = 1 + strain_AM(i+1) * l; % Deflection, calculated from strain

T_AM(i+1) = T_AM(i) - 0.001;
% Increase temperature by 0.001 Degree Celsius, for next cycle/iteration
sigma_ite_AM = zeros(1,100000); % Zeros, otherwise iteration will be disturbed
end

% ---------------------------- END COOLING ----------------------------- %
APPENDIX D: EXPERIMENTAL TESTS ON 3D-PRINTED CF COMPOSITE

APPENDIX D1 - VOLUME FRACTION

From a broader view of the carbon fibre (CF) composite specimen, the estimated CF volume fraction within the unidirectional composite specimen was calculated by dividing the cross-sectional area of the CF layers with the total cross-sectional area of the unidirectional composite (CF and onyx):

Cross-sectional area of CF = 1.19 mm × 0.865 mm × 15 = 15.44 mm².

Cross-sectional area of composite = 15.32 mm × 2.09 mm = 32.02 mm².

Estimated fibre volume fraction:

\[ V_f = \frac{Volume\ CF}{Volume\ Composite} = \frac{A_{CF} \times L}{A_{UD\ composite} \times L} = \frac{15.44}{32.02} = 48.2\% \quad (D1) \]

For the unidirectional (UD) CF composite specimens used in the 3PBTs, the CF volume fraction was roughly estimated by dividing the cross-sectional area of the CF layers with the total cross-sectional area of the unidirectional composite (CF and onyx):

Cross-sectional area of CF = 1.075 mm × 2 × 12.93 mm = 27.80 mm².

Cross-sectional area of composite = 13.13 mm × 3.02 mm = 39.65 mm².

Estimated fibre volume fraction:

\[ V_f = \frac{Volume\ CF}{Volume\ Composite} = \frac{A_{CF} \times L}{A_{UD\ composite} \times L} = \frac{27.80}{39.65} = 70.1\% \quad (D2) \]
APPENDIX D2 - FLEXURAL STRESS AND STRAIN IN 3-POINT BENDING TESTS

**Flexural stress**

\[ \sigma = \frac{My}{I} \]


\[ M = \left( \frac{P}{2} \right) \left( \frac{L}{2} \right) = \frac{PL}{4} \]

\[ y = \frac{h}{2} \]

\[ I = \frac{bh^3}{12} \]

\[ \sigma = \frac{\left( \frac{PL}{4} \right) \left( \frac{h}{2} \right)}{\left( \frac{bh^3}{12} \right)} \]

\[ \sigma = \frac{3PL}{2bh^2} \]

**Flexural strain**

From Hooke’s Law,

\[ \sigma = E \varepsilon \]

We know the centre deflection of the simply supported beam as (Hibbeler, 2011)

\[ \delta = \frac{PL^3}{48EI} \]

\[ I = \frac{bh^3}{12} \]

\[ E = \frac{PL^3}{48\delta \left( \frac{bh^3}{12} \right)} = \frac{PL^3}{4\delta bh^3} \]

\[ \varepsilon = \frac{\sigma}{E} = \frac{\left( \frac{3PL}{2bh^2} \right)}{\left( \frac{PL^3}{4\delta bh^3} \right)} \]

\[ \varepsilon = \frac{6\delta h}{L^2} \]
Figure D1. The developed temperature measurement system of the SMA wire, using the NI 9211 thermocouple module and LabVIEW from National Instruments: (a) block diagram and (b) panel view.
## APPENDIX E: FEA SIMULATIONS OF VIRTUAL 3-POINT BENDING TESTS ON CF COMPOSITE BEAM – TWO BEAMS FEA MODEL

**Table E1.** Details of the integration points used in the FEA simulation of the CF composite beam, which was modelled as two beams, with 6 integration points (IPs) across the thickness (3 IPs per beam), subjected to 3-point bending load.

<table>
<thead>
<tr>
<th>CF Composite Beams</th>
<th>Integration Points</th>
<th>Materials</th>
<th>Layer Thickness (mm)</th>
<th>Coordinate of IP, S (-1 to 1)</th>
<th>Weighting Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Beam</td>
<td>1</td>
<td>Onyx</td>
<td>0.250</td>
<td>-0.833333</td>
<td>0.166667</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>CF</td>
<td>1.075</td>
<td>0.05</td>
<td>0.716667</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Onyx</td>
<td>0.175</td>
<td>0.883333</td>
<td>0.116667</td>
</tr>
<tr>
<td>Top Beam</td>
<td>4</td>
<td>Onyx</td>
<td>0.175</td>
<td>-0.883333</td>
<td>0.116667</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>CF</td>
<td>1.075</td>
<td>-0.05</td>
<td>0.716667</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>Onyx</td>
<td>0.250</td>
<td>0.833333</td>
<td>0.166667</td>
</tr>
</tbody>
</table>

**Table E2.** Details of the integration points used in the FEA simulation of the CF composite beam, which was modelled as two beams, with 8 integration points (IPs) across the thickness (4 IPs per beam), subjected to 3-point bending load.

<table>
<thead>
<tr>
<th>CF Composite Beams</th>
<th>Integration Points</th>
<th>Materials</th>
<th>Layer Thickness (mm)</th>
<th>Coordinate of IP, S (-1 to 1)</th>
<th>Weighting Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Beam</td>
<td>1</td>
<td>Onyx</td>
<td>0.2500</td>
<td>-0.833333</td>
<td>0.166667</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>CF</td>
<td>0.5375</td>
<td>-0.308333</td>
<td>0.358333</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>CF</td>
<td>0.5375</td>
<td>0.408333</td>
<td>0.358333</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Onyx</td>
<td>0.1750</td>
<td>0.883333</td>
<td>0.116667</td>
</tr>
<tr>
<td>Top Beam</td>
<td>5</td>
<td>Onyx</td>
<td>0.1750</td>
<td>-0.883333</td>
<td>0.116667</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>CF</td>
<td>0.5375</td>
<td>-0.408333</td>
<td>0.358333</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>CF</td>
<td>0.5375</td>
<td>0.308333</td>
<td>0.358333</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>Onyx</td>
<td>0.2500</td>
<td>0.833333</td>
<td>0.166667</td>
</tr>
</tbody>
</table>

**Table E3.** Details of the integration points used in the FEA simulation of the CF composite beam, which was modelled as two beams, with 10 integration points (IPs) across the thickness (5 IPs per beam), subjected to 3-point bending load.

<table>
<thead>
<tr>
<th>CF Composite Beams</th>
<th>Integration Points</th>
<th>Materials</th>
<th>Layer Thickness (mm)</th>
<th>Coordinate of IP, S (-1 to 1)</th>
<th>Weighting Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Beam</td>
<td>1</td>
<td>Onyx</td>
<td>0.2500</td>
<td>-0.833333</td>
<td>0.166667</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>CF</td>
<td>0.3583</td>
<td>-0.427778</td>
<td>0.238889</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>CF</td>
<td>0.3583</td>
<td>0.05</td>
<td>0.238889</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>CF</td>
<td>0.3583</td>
<td>0.527778</td>
<td>0.238889</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Onyx</td>
<td>0.1750</td>
<td>0.883333</td>
<td>0.116667</td>
</tr>
<tr>
<td>Top Beam</td>
<td>6</td>
<td>Onyx</td>
<td>0.1750</td>
<td>-0.883333</td>
<td>0.116667</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>CF</td>
<td>0.3583</td>
<td>-0.527778</td>
<td>0.238889</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>CF</td>
<td>0.3583</td>
<td>-0.05</td>
<td>0.238889</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>CF</td>
<td>0.3583</td>
<td>0.427778</td>
<td>0.238889</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>Onyx</td>
<td>0.2500</td>
<td>0.833333</td>
<td>0.166667</td>
</tr>
</tbody>
</table>
APPENDIX F: COMPOSITE MORPHING WING

APPENDIX F1 – LIFT DISTRIBUTION ALONG A FINITE WING

The vortices along the wing span were first obtained.

\[ \Gamma(\theta) = 2bV_\infty \sum_{n=1}^{\infty} A_n \sin n\theta \]  

(F1)

where \( \Gamma \) was the vortices and \( \theta \) was the wing span, ranges from 0 to \( \pi \) (0 < \( \theta < \pi \)). The constants \( A_n \) were determined by solving the following equation in the Fourier series form, for several points along the wing span.

\[ C_i(y_0) = m_0 (\alpha_{eff}(y_0) - \alpha_{L0}(y_0)) = m_0 [\alpha(y_0) - \alpha_i(y_0) - \alpha_{L0}(y_0)] \]

\[ \alpha(y_0) = \frac{C_i(y_0)}{m_0} + \alpha_{L0}(y_0) + \alpha_i(y_0) \]

\[ \alpha(y_0) = \frac{2\Gamma(y_0)}{m_0 V_\infty c(y_0)} + \alpha_{L0}(y_0) - \frac{1}{4\pi V_\infty} \int_{-b/2}^{b/2} \left( \frac{d\Gamma}{dy} \right)_{y_0} dy \]  

(F2)

\[ \alpha(\theta) = \frac{4b}{m_0 c(\theta)} \sum_{n=1}^{\infty} A_n \sin n\theta + \alpha_{L0}(\theta) + \sum_{n=1}^{\infty} nA_n \frac{\sin n\theta}{\sin \theta} \]  

(F3)

where \( \alpha_{eff}, \alpha_{L0} \) and \( \alpha_i \) were the effective angle of attack (AOA), zero-lift AOA, and induced AOA, respectively. \( m_0 \) is the slope of section lift, while \( y_0 \) was the distance from the wing root to the location of vortices in interest.

The higher the number of \( n \) and \( A_n \), the higher the accuracy of the vortices and lift distributions along the wing span. Because the wing was symmetrical, all of the \( A_n \)'s values for even \( n \)'s were zero. By using Eq. F3, the determination of the \( A_n \)'s for \( n = 3 \) and \( n = 7 \) for a wing at 3° angle of attack is shown as follows.
Appendix

**n = 3 (three terms with even term equal to zero):**

- At \( y = 0 \) or \( \theta = \pi/2 \), \( c = 0.3 \) and \( \alpha = 3^\circ \)

\[
3 \left( \frac{\pi}{180} \right) = \frac{4(5)}{(2\pi)(0.3)} \left[ A_1 \sin \frac{\pi}{2} + A_3 \sin \frac{3\pi}{2} \right] + 0 + \left[ (1)A_1 \frac{\sin \frac{\pi}{2}}{\sin \frac{\pi}{2}} + 3A_3 \frac{\sin \frac{3\pi}{2}}{\sin \frac{3\pi}{2}} \right]
\]

\[
\frac{\pi}{60} = \left( \frac{100}{3\pi} + 1 \right)A_1 + \left( -\frac{100}{3\pi} - 3 \right)A_3 \quad (i)
\]

- At \( y = b/4 \) or \( \theta = \pi/3 \), \( c = 0.3 \) and \( \alpha = 3^\circ \)

\[
3 \left( \frac{\pi}{180} \right) = \frac{4(5)}{(2\pi)(0.3)} \left[ A_1 \sin \frac{\pi}{3} + A_3 \sin \frac{3\pi}{3} \right] + 0 + \left[ (1)A_1 \frac{\sin \frac{\pi}{3}}{\sin \frac{\pi}{3}} + 3A_3 \frac{\sin \frac{3\pi}{3}}{\sin \frac{3\pi}{3}} \right]
\]

\[
\frac{\pi}{60} = \left( \frac{100}{3\pi} \sin \frac{\pi}{3} + 1 \right)A_1 \quad (ii)
\]

Solving equations (i) and (ii) simultaneously,

\( A_1 = 5.1390 \times 10^{-3} \)

\( A_3 = 5.3673 \times 10^{-4} \)

Hence, the circulation (Eq. F1) and the lift distribution along the wing span for the solution of \( n = 3 \), are given by

\[
\Gamma(\theta) = 2bV_\infty \sum_{n=1}^{\infty} A_n \sin n\theta = 10V_\infty (5.1390 \times 10^{-3} \sin \theta + 5.3673 \times 10^{-4} \sin 3\theta)
\]

\[
l = \rho_\infty V_\infty \Gamma = 10\rho_\infty V_\infty^2 (5.1390 \times 10^{-3} \sin \theta + 5.3673 \times 10^{-4} \sin 3\theta) \quad (F4)
\]
\[ n = 7 \text{ (seven terms with even terms equal to zero):} \]

- At \( y = 0 \) or \( \theta = \pi/2 \), \( c = 0.3 \) and \( \alpha = 3^\circ \)

\[
3 \left( \frac{\pi}{180} \right) = \frac{4(5)}{(2\pi)(0.3)} \left[ A_1 \sin \frac{\pi}{2} + A_3 \sin \frac{3\pi}{2} + A_5 \sin \frac{5\pi}{2} + A_7 \sin \frac{7\pi}{2} \right] + 0 \\
+ \left[ (1)A_1 \frac{\sin \frac{\pi}{2}}{\sin \frac{\pi}{2}} + 3A_3 \frac{\sin \frac{3\pi}{2}}{\sin \frac{\pi}{2}} + 5A_5 \frac{\sin \frac{5\pi}{2}}{\sin \frac{\pi}{2}} + 7A_7 \frac{\sin \frac{7\pi}{2}}{\sin \frac{\pi}{2}} \right]
\]

\[
\frac{\pi}{60} = (\frac{100}{3\pi} + 1) A_1 + (- \frac{100}{3\pi} - 3) A_3 + (\frac{100}{3\pi} + 5) A_5 + (- \frac{100}{3\pi} - 7) A_7 \quad (i)
\]

- At \( y = b/8 \) or \( \theta = 0.4196\pi \), \( c = 0.3 \) and \( \alpha = 3^\circ \)

\[
3 \left( \frac{\pi}{180} \right) = \frac{4(5)}{(2\pi)(0.3)} \left[ A_1 \sin 0.4196\pi + A_3 \sin(3 \times 0.4196\pi) + A_5 \sin(5 \times 0.4196\pi) \right. \\
+ A_7 \sin(7 \times 0.4196\pi) \bigg] + 0 \\
+ \left[ A_1 + 3A_3 \frac{\sin(3 \times 0.4196\pi)}{\sin 0.4196\pi} + 5A_5 \frac{\sin(5 \times 0.4196\pi)}{\sin 0.4196\pi} \\
+ 7A_7 \frac{\sin(7 \times 0.4196\pi)}{\sin 0.4196\pi} \bigg]
\]

\[
\frac{\pi}{60} = 11.2733A_1 - 9.9577A_3 + 4.7801A_5 + 3.4968A_7 \quad (ii)
\]

- At \( y = b/4 \) or \( \theta = \pi/3 \), \( c = 0.3 \) and \( \alpha = 3^\circ \)

\[
3 \left( \frac{\pi}{180} \right) = \frac{4(5)}{(2\pi)(0.3)} \left[ A_1 \sin \frac{\pi}{3} + A_3 \sin \frac{3\pi}{3} + A_5 \sin \frac{5\pi}{3} + A_7 \sin \frac{7\pi}{3} \right] + 0 \\
+ \left[ A_1 + 3A_3 \frac{\sin \frac{3\pi}{3}}{\sin \frac{\pi}{3}} + 5A_5 \frac{\sin \frac{5\pi}{3}}{\sin \frac{\pi}{3}} + 7A_7 \frac{\sin \frac{7\pi}{3}}{\sin \frac{\pi}{3}} \right]
\]

\[
\frac{\pi}{60} = 10.1888A_1 - 14.1888A_5 + 16.1888A_7 \quad (iii)
\]
• At \( y = 3b/8 \) or \( \theta = 0.2300\pi \), \( c = 0.3 \) and \( \alpha = 3^\circ \)

\[
3 \left( \frac{\pi}{180} \right) = \frac{4(5)}{(2\pi)(0.3)} \left[ A_1 \sin(0.2300\pi) + A_3 \sin(3 \times 0.2300\pi) + A_5 \sin(5 \times 0.2300\pi) \\
+ A_7 \sin(7 \times 0.2300\pi) \right] + 0
\]

\[
+ \left[ A_1 + 3A_3 \frac{\sin(3 \times 0.2300\pi)}{\sin 0.2300\pi} + 5A_5 \frac{\sin(5 \times 0.2300\pi)}{\sin 0.2300\pi} \\
+ 7A_7 \frac{\sin(7 \times 0.2300\pi)}{\sin 0.2300\pi} \right]
\]

\[
\frac{\pi}{60} = 1.1338A_1 + 9.3994A_3 + 25.6527A_5 + 49.8731A_7
\]

Solving equations (i) – (iv) simultaneously,

\[
A_1 = 5.9571 \times 10^{-3}
\]

\[
A_3 = 1.9110 \times 10^{-3}
\]

\[
A_5 = 7.6885 \times 10^{-4}
\]

\[
A_7 = 1.5880 \times 10^{-4}
\]

Hence, the circulation and the lift distribution along the wing span were obtained.

\[
\Gamma(\theta) = 2bV_\infty \sum_{n=1}^{\infty} A_n \sin n\theta
\]

\[
\Gamma(\theta) = 10V_\infty (5.9571 \times 10^{-3} \sin \theta + 1.9110 \times 10^{-3} \sin 3\theta + 7.6885 \times 10^{-4} \sin 5\theta \\
+ 1.5880 \times 10^{-4} \sin 7\theta)
\]

\[
l = \rho_\infty V_\infty \Gamma
\]

\[
l = 10\rho_\infty V_\infty^2 (5.9571 \times 10^{-3} \sin \theta + 1.9110 \times 10^{-3} \sin 3\theta + 7.6885 \times 10^{-4} \sin 5\theta \\
+ 1.5880 \times 10^{-4} \sin 7\theta)
\]

\[(F5)\]
APPENDIX F2 – CIRCULATION ALONG A FINITE WING

\[ \int_{-b/2}^{b/2} \Gamma(y) \, dy = \int_{\pi}^{0} \left[ 2b V_\infty \sum_{n=1}^{\infty} A_n \sin n \theta \right] \left[ \frac{b}{2} (- \sin \theta) d\theta \right] \]

\[ \int_{-b/2}^{b/2} \Gamma(y) \, dy = -b^2 V_\infty \int_{\pi}^{0} \sum_{n=1}^{\infty} A_n \sin n \theta \sin \theta \, d\theta \]

The integral of \( \sin n \theta \sin \theta \) from \( \pi \) to zero for \( n \neq 0 \) are zero, and therefore:

\[ \int_{-b/2}^{b/2} \Gamma(y) \, dy = -b^2 V_\infty \int_{\pi}^{0} A_1 \sin \theta \sin \theta \, d\theta = -b^2 V_\infty \int_{\pi}^{0} A_1 \sin^2 \theta \, d\theta \]

\[ \int_{-b/2}^{b/2} \Gamma(y) \, dy = -b^2 V_\infty \int_{\pi}^{0} A_1 \left( \frac{1 - \cos 2\theta}{2} \right) \, d\theta \]

\[ \int_{-b/2}^{b/2} \Gamma(y) \, dy = -\frac{b^2 V_\infty A_1}{2} \left[ \theta - \frac{\sin 2\theta}{2} \right]_{\pi}^{0} = -\frac{b^2 V_\infty A_1}{2} (-\pi) \]

\[ \int_{-b/2}^{b/2} \Gamma(y) \, dy = \frac{\pi}{2} b^2 V_\infty A_1 \]
APPENDIX F3 – DESIGN OF COMPOSITE D-NOSE SPAR

An example of the sizing calculation of the spar caps/flanges for a section of the composite D-nose spar at the wing root, which was section S1, is described below.

From the lift force, the shear forces, the bending moments and the boom areas of the spar caps/flanges were calculated based on the compressive strength of the Markforged carbon fibre (CF) composite, which was 320 MPa (Markforged Inc., 2016). The compressive strength was used as a design limit, instead of the CF composite tensile strength of 700 MPa, as the former was lower than the later. Then from the boom area, the thickness of the CF composite spar cap was calculated based on a constant flange width of 36 mm. Then, the number of layers of the CF composite plies were calculated based on one-ply thickness of 0.125 mm.

The bending moment acted on the section S1 of the D-nose spar was calculated from the total shear/lift force acted on sections S1 to S8, multiplied with the distance between the inner side of the section S1 and the loading point which was approximated from the lift vs. wing span curve \((n = 7)\) in Fig. 6.1 in Chapter 6.

\[
M = F \cdot d = (448.5937 \, N)(1227.7078 \, mm) = 550742.03 \, Nmm
\]

The area of the booms shown in Fig. F1 was calculated based on the resulted bending moment, the vertical distance from the neutral axis, and the compressive strength of the carbon fibre (CF) composite.

\[
\sigma = \frac{My}{I_{xx}} = \frac{My}{2Ad^2} \quad (F6)
\]

\[
A = \frac{My}{2d^2 \sigma} = \frac{(550742.03 \, Nmm)(18 \, mm)}{2(18 \, mm)^2(320 \, N/mm^2)} = 47.81 \, mm^2
\]
Next, the thickness of the CF composite spar caps/flanges was calculated based on the previously obtained booms area and a constant caps/flanges width of 36 mm.

\[ t = \frac{A}{w} = \frac{47.81 \text{ mm}^2}{36 \text{ mm}} = 1.33 \text{ mm} \]

As the thickness of one layer of the 3D-printed CF composite was approximately 0.125 mm, the total thickness of the CF composite spar caps/flanges had to be a multiplication of 0.125 mm, and had to be higher than the above thickness, to avoid a compressive failure. For 11 CF layers,

\[ t = 11 \text{ layers} \times 0.125 \text{ mm/layer} = 1.375 \text{ mm} \]

The above-described calculations for all 8 sections of the composite D-nose spar are summarised in Table F1. Sections S1 to S8 were the sections from the wing root to the wing tip, respectively. For structural safety of the composite morphing wing, the size of the CF composite spar caps/flanges was also calculated for factor of safety (FOS) of 5.625 (1.25 \( n_l \)) and 6.75 (1.5 \( n_l \)). Here, \( n_l \) was 4.5, which was typical for a semi-aerobatic aircraft (Megson, 2017).

In addition to the sizing calculation of the CF composite spar flanges (area, width, thickness and number of CF layers) to ensure the strength was adequate, the tip deflection of the CF composite D-nose spar (hence the CF composite wing) was also calculated to ensure the spar was sufficiently stiff and no excessive deformation occurred when the CF composite D-nose spar was subjected to the external load. The calculations of the wing tip deflection, which was contributed by the section S1, are shown below.

The external moment on the section S1 was

\[ M = 550742.03 \text{ Nmm} \]

The virtual moment on the section S1 was

\[ m = F.d = (1)(1227.7078 \text{ mm}) = 1227.71 \text{ mm} \]

The updated area of the booms/flanges of the section S1, after the calculated final thickness, was

\[ A = w \times t = (36 \text{ mm}) \times (1.375 \text{ mm}) = 49.5 \text{ mm}^2 \]

The second moment of area was

\[ I_{xx} = 2Ad^2 = 2(49.5 \text{ mm}^2)(18 \text{ mm})^2 = 32076 \text{ mm}^4 \]
For the wing semi-span of 2.5 m, the length of each of sections S1-S8 was 312.5 mm. The tensile/compressive modulus of the CF composite was taken from the datasheet, as 54 GPa (Markforged Inc., 2016). Hence, the tip deflection, which was contributed by the section S1 of the CF composite spar flange was

$$\delta = \int_0^{312.5} \frac{M \cdot m}{E I_{xx}} = \frac{(550742.03 \text{ Nmm})(1227.71 \text{ mm})(312.5 \text{ mm})}{(54000 \text{ N/mm}^2)(32076 \text{ mm}^4)} = 121.99 \text{ mm} \quad (F7)$$

The tip deflections contributed by the CF composite spar caps/flanges of all of the other sections S2-S8, were calculated and listed in the Table F1. They were also calculated for factor of safety (FOS) of 5.625 and 6.75, and were included in the table. The total tip deflection contributed by all of the sections S1-S8 of the CF composite spar caps/flanges, was summation of the tip deflection for all eight sections (Eq. F8). For FOS of 1, 5.625 and 6.75, the total tip deflections were 484.20 mm, 538.31 mm, and 544.59 mm, respectively.

$$\delta_{\text{tip}} = \delta_{S1} + \delta_{S2} + \delta_{S3} + \delta_{S4} + \delta_{S5} + \delta_{S6} + \delta_{S7} + \delta_{S8} \quad (F8)$$
Table F1. Design and sizing of the spar caps/flanges of the CF composite D-nose spar.

<table>
<thead>
<tr>
<th>Sections</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF Composite Spar Caps/Flanges</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear Force (N)</td>
<td>448.59</td>
<td>392.66</td>
<td>336.88</td>
<td>281.25</td>
<td>225.63</td>
<td>169.38</td>
<td>110.88</td>
<td>49.00</td>
</tr>
<tr>
<td>Bending Moment (Nmm)</td>
<td>550,742.03</td>
<td>420,551.60</td>
<td>307,963.71</td>
<td>212,905.12</td>
<td>135,229.34</td>
<td>74,741.06</td>
<td>31,479.34</td>
<td>6,498.87</td>
</tr>
<tr>
<td>Boom Area (mm²)</td>
<td>47.81</td>
<td>36.51</td>
<td>26.73</td>
<td>18.48</td>
<td>11.74</td>
<td>6.49</td>
<td>2.73</td>
<td>0.56</td>
</tr>
<tr>
<td>CF layers</td>
<td>11</td>
<td>9</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>1.375</td>
<td>1.125</td>
<td>0.75</td>
<td>0.625</td>
<td>0.375</td>
<td>0.25</td>
<td>0.125</td>
<td>0.125</td>
</tr>
<tr>
<td>Tip Deflection (mm)</td>
<td>121.99</td>
<td>99.32</td>
<td>93.12</td>
<td>63.97</td>
<td>53.62</td>
<td>32.73</td>
<td>17.74</td>
<td>1.71</td>
</tr>
<tr>
<td>CF Composite Spar Caps/Flanges (FOS = 1.25n₁ = 5.625)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sections</td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
<td>S4</td>
<td>S5</td>
<td>S6</td>
<td>S7</td>
<td>S8</td>
</tr>
<tr>
<td>CF layers</td>
<td>60</td>
<td>46</td>
<td>34</td>
<td>24</td>
<td>15</td>
<td>9</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>7.5</td>
<td>5.75</td>
<td>4.25</td>
<td>3</td>
<td>1.875</td>
<td>1.125</td>
<td>0.5</td>
<td>0.125</td>
</tr>
<tr>
<td>Tip Deflection (mm)</td>
<td>125.80</td>
<td>109.31</td>
<td>92.44</td>
<td>74.97</td>
<td>60.32</td>
<td>40.91</td>
<td>24.94</td>
<td>9.62</td>
</tr>
<tr>
<td>CF Composite Spar Caps/Flanges (FOS = 1.5n₁ = 6.75)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sections</td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
<td>S4</td>
<td>S5</td>
<td>S6</td>
<td>S7</td>
<td>S8</td>
</tr>
<tr>
<td>CF layers</td>
<td>72</td>
<td>55</td>
<td>41</td>
<td>28</td>
<td>18</td>
<td>10</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>9</td>
<td>6.875</td>
<td>5.125</td>
<td>3.5</td>
<td>2.25</td>
<td>1.25</td>
<td>0.625</td>
<td>0.125</td>
</tr>
<tr>
<td>Tip Deflection (mm)</td>
<td>125.80</td>
<td>109.71</td>
<td>91.99</td>
<td>77.11</td>
<td>60.32</td>
<td>44.18</td>
<td>23.95</td>
<td>11.55</td>
</tr>
</tbody>
</table>
Appendix

An example of sizing calculations of the thickness of the cell walls for a section of the D-nose spar, at the wing root (section S1), is shown below.

\[ q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r + q_{s,0} \]  \hspace{2cm} (F9)

\[ I_{xx} = 2Ad^2 = 2(47.81)(18)^2 = 30,980.88 \text{ mm}^4 \]

where \( q_s \) and \( q_{s,0} \) were the total shear flow and the close section shear flow, respectively. \( S_y, I_{xx}, B_r \) and \( y_r \) were the external applied shear force or the lift force, the second moment of area, the boom area and the distance of the boom from the neutral axis of the D-nose section. The boom area, \( A \) was obtained from the previous subsection of the CF composite spar caps/flanges calculation (Table F1). The wall 21 was cut as shown in the above Fig. F2, and the basic shear flow (the first term of Eq. 6.14) for both walls 12 and 21 were calculated as

\[ q_{b,21} = 0 \]

\[ q_{b,12} = -\frac{448.59 N}{30980.88 mm^4}(47.81 mm^2)(18 mm) = -12.46 \text{ N/mm} \]

To obtain the close section shear flow, \( q_{s,0} \), by taking moment about point O,

\[ M = q_{b,12}(l_{12})(d) + 2A'q_{b,21} + 2Aq_{s,0} \]

\[ 0 = (-12.46)(36)(16) + 2A'(0) + 2(2614.15)q_{s,0} \]

\[ q_{s,0} = 1.2869 \text{ N/mm} \]

Finally, the total shear flow for the cell wall 12 and cell wall 21 were found as

\[ q_s = q_b + q_{s,0} \]

\[ q_{12} = q_{b,12} + q_{s,0} = -12.46 + 1.2869 = -11.1731 \text{ N/mm} \]
\[ q_{21} = q_{b,21} + q_{s,0} = 0 + 1.2869 = 1.2869 \, N/mm \]

The negative sign indicated that the shear flow was in the opposite direction to the one shown in Fig. F2. From the obtained shear flow and the CF composite shear strength of 30 MPa, the wall thickness were calculated as

\[ t_{12} = \frac{q_{12}}{\tau} = \frac{11.1731 \, N/mm}{30 \, N/mm^2} = 0.37 \, mm \]

\[ t_{21} = \frac{q_{21}}{\tau} = \frac{1.2869 \, N/mm}{30 \, N/mm^2} = 0.04 \, mm \]

Calculations for the shear flow and thickness of the cell walls of all 8 sections of the CF composite D-nose spar are summarised in Table F2. Again, for structural safety, the walls thickness were also calculated for factor of safety (FOS) of 5.625 (1.25 \( n_1 \)) and 6.75 (1.5 \( n_1 \)), with a typical \( n_1 \) of 4.5, for a semi-aerobatic aircraft.

Besides the sizing calculation of the CF composite spar webs and D-nose skin (the wall thicknesses) to ensure the strength was adequate to resist the shear stress, the tip deflection of the CF composite D-nose spar (hence the CF composite wing) which was contributed by the CF composite spar webs and D-nose skin, was also calculated to ensure the D-nose spar was sufficiently stiff and no excessive deformation occurred when the D-nose spar was subjected to the external load. The calculations are shown below.

The total shear flow and the thickness of wall 12, which were obtained from the previous calculation, were

\[ q_{12} = -11.1731 \, N/mm, t_{12} = 0.3725 \, mm \]

The total shear flow and the thickness of wall 21, which were obtained from the previous calculation, were

\[ q_{21} = 1.2869 \, N/mm, t_{21} = 0.0429 \, mm \]

The shear force acted on section S1, due to the external aerodynamic lift, was

\[ S_y = 448.59 \, N \]

The length of walls 12 and 21 were

\[ l_{12} = 36 \, mm, l_{21} = 189.95 \, mm \]
Therefore, the tip deflection, which was contributed by the CF composite spar web (wall 12) and CF composite D-nose skin (wall 21) of section S1, was

\[
\delta = \int_0^L \frac{(q_{12})^2 l_{12}}{S_y G t_{12}} \, ds + \int_0^L \frac{(q_{21})^2 l_{21}}{S_y G t_{21}} \, ds
\]

where

\[
\begin{align*}
\delta &= \frac{(11.1731)^2 (36) (312.5)}{(448.59)(7000)(0.3725)} + \frac{(1.2869)^2 (189.95) (312.5)}{(448.59)(7000)(0.0429)} \\
&= 1.9308 \text{ mm}
\end{align*}
\]

The tip deflections which were contributed by the CF composite spar web (wall 12) and CF composite D-nose skin (wall 21) of all of the other sections S2-S8, were calculated and listed in the Table F2. They were also calculated for factor of safety (FOS) of 5.625 and 6.75, and were included in the same table. Surprisingly, the tip deflection for all sections were equal, even though the total shear flow and the wall thickness for each section were different. More surprisingly, the tip deflections which were contributed by all sections S1-S8, were constant even though the FOS was increased to 5.625 and 6.75. The total tip deflection which was contributed by all of the sections S1-S8 of the CF composite spar web (wall 12) and CF composite D-nose skin (wall 21), for FOS of 1, 5.625 and 6.75, were constant at 15.45 mm.

The possible reason was that when the FOS increased, which caused an increase in the shear force and the internal shear flow, the thickness of the CF composite spar webs and the CF composite D-nose skin also increased, and hence the summation of the two terms resulted in a constant value. In fact, the first term of the Equation F10 for all FOS was equal, and the second term of the equation for all FOS was also equal.

To evaluate the total tip deflection of the CF composite D-nose spar, summation of the tip deflection which was contributed by the CF composite spar flanges and the tip deflection which was contributed by the CF composite spar webs and skins was calculated.

\[
\delta = \delta_{\text{flanges}} + \delta_{\text{sparweb & skin}}
\]

For a normal flight condition, the estimated total tip deflection was

\[
\delta = 484.20 \text{ mm} + 15.45 \text{ mm} = 499.65 \text{ mm}
\]

which was about 9.99% of the wing span.

For FOS of 5.625, the estimated total tip deflection was
\[ \delta = 538.31 \, mm + 15.45 \, mm = 553.76 \, mm \]

which was about 11.07\% of the wing span.

For FOS of 6.75, the estimated total tip deflection was

\[ \delta = 544.59 \, mm + 15.45 \, mm = \textbf{560.04 mm} \]

which was about 11.20\% of the wing span.

The results of the sizes the CF composite spar flanges, spar web and D-nose skin, listed in Table F1 and Table F2 were only a preliminary guideline for the initial design of the CF composite D-nose spar. They were then refined by FEA simulations in LS-DYNA to check the stability of the CF composite D-nose spar, in case the composite parts buckled under the applied aerodynamic load.
### Table F2. Design and sizing of the spar web and D-nose skin of the CF composite D-nose spar.

<table>
<thead>
<tr>
<th>Sections</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Flow $q_{12}$ (N/mm)</td>
<td>-11.1739</td>
<td>-9.7806</td>
<td>-8.3911</td>
<td>-7.0056</td>
<td>-5.6200</td>
<td>-4.2189</td>
<td>-2.7618</td>
<td>-1.2205</td>
</tr>
<tr>
<td>Shear Flow $q_{21}$ (N/mm)</td>
<td>1.2870</td>
<td>1.1265</td>
<td>0.9665</td>
<td>0.8069</td>
<td>0.6473</td>
<td>0.4859</td>
<td>0.3181</td>
<td>0.1406</td>
</tr>
<tr>
<td>Wall Thickness $t_{12}$ (mm)</td>
<td>0.3725</td>
<td>0.3260</td>
<td>0.2797</td>
<td>0.2335</td>
<td>0.1873</td>
<td>0.1406</td>
<td>0.0921</td>
<td>0.0407</td>
</tr>
<tr>
<td>Wall Thickness $t_{21}$ (mm)</td>
<td>0.0429</td>
<td>0.0376</td>
<td>0.0322</td>
<td>0.0269</td>
<td>0.0216</td>
<td>0.0162</td>
<td>0.0106</td>
<td>0.0047</td>
</tr>
<tr>
<td>Tip Deflection (mm)</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sections</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Flow $q_{21}$ (N/mm)</td>
<td>7.2395</td>
<td>6.3367</td>
<td>5.4365</td>
<td>4.5388</td>
<td>3.6412</td>
<td>2.7334</td>
<td>1.7893</td>
<td>0.7908</td>
</tr>
<tr>
<td>Wall thickness $t_{12}$ (mm)</td>
<td>2.0951</td>
<td>1.8339</td>
<td>1.5733</td>
<td>1.3135</td>
<td>1.0538</td>
<td>0.7910</td>
<td>0.5178</td>
<td>0.2288</td>
</tr>
<tr>
<td>Wall thickness $t_{21}$ (mm)</td>
<td>0.2413</td>
<td>0.2112</td>
<td>0.1812</td>
<td>0.1513</td>
<td>0.1214</td>
<td>0.0911</td>
<td>0.0596</td>
<td>0.0264</td>
</tr>
<tr>
<td>Tip Deflection (mm)</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sections</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Flow $q_{12}$ (N/mm)</td>
<td>-75.4240</td>
<td>-66.0190</td>
<td>-56.6402</td>
<td>-47.2878</td>
<td>-37.9353</td>
<td>-28.4777</td>
<td>-18.6419</td>
<td>-8.2386</td>
</tr>
<tr>
<td>Shear Flow $q_{21}$ (N/mm)</td>
<td>8.6874</td>
<td>7.6041</td>
<td>6.5238</td>
<td>5.4466</td>
<td>4.3694</td>
<td>3.2801</td>
<td>2.1472</td>
<td>0.9489</td>
</tr>
<tr>
<td>Wall thickness $t_{12}$ (mm)</td>
<td>2.5141</td>
<td>2.2006</td>
<td>1.8880</td>
<td>1.5763</td>
<td>1.2645</td>
<td>0.9493</td>
<td>0.6214</td>
<td>0.2746</td>
</tr>
<tr>
<td>Wall thickness $t_{21}$ (mm)</td>
<td>0.2896</td>
<td>0.2535</td>
<td>0.2175</td>
<td>0.1816</td>
<td>0.1456</td>
<td>0.1093</td>
<td>0.0716</td>
<td>0.0316</td>
</tr>
<tr>
<td>Tip Deflection (mm)</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
<td>1.9308</td>
</tr>
</tbody>
</table>

*Note: FOS = 1.25\(n_1\) = 5.625, FOS = 1.5\(n_1\) = 6.75.*
APPENDIX F4 – LS-DYNA KEYWORDS

CONTROL_ENERGY
CONTROL_HOURGLASS
CONTROL_SHELL
CONTROL_TERMINATION
BOUNDARY_PRESCRIBED_MOTION_SET
BOUNDARY_SPC_SET
DAMPING_GLOBAL
DATABASE_ASCII_NODFOR
DATABASE_BNDOUT
DATABASE_ELOUT
DATABASE_GLSTAT
DATABASE_MATSUM
DATABASE_NODFOR
DATABASE_SPCFORC
DATABASE_BINARY_D3PLOT
DATABASE_EXTEND_BINARY
DATABASE_NODAL_FORCE_GROUP
DATABASE_HISTORY_BEAM
DEFINE_CURVE
ELEMENT_BEAM
ELEMENT_SHELL
INTEGRATION_BEAM
INTEGRATION_SHELL
LOAD_NODE_SET
LOAD_NODE_POINT
LOAD_THERMAL_LOAD_CURVE
MAT_001_ELASTIC
MAT_002_MAT_ORTHOTROPIC_ELASTIC
MAT_020_RIGID
MAT_COMPOSITE_DAMAGE
MAT_USER_DEFINED_MATERIAL_MODELS
SECTION_BEAM
SECTION_SHELL
SET_NODE_LIST
APPENDIX F5 - MANUFACTURING AND ASSEMBLY OF CF COMPOSITE D-NOSE SPAR

Figure F3. The burned CF composite prepregs and wood female mould on the first trial of manufacture of the CF composite D-nose skin.

Figure F4. (a) The Nabertherm furnace and (b) the wood with the applied adhesive spray and with the applied filler before (top) and after (bottom) heated in the furnace for 3 hours.
Figure F5. Assembly of 3D-printed CF composite rear ribs.