Contract design in agriculture supply chains with random yield

March 31, 2019

Edward Anderson*
University of Sydney Business School, University of Sydney, NSW 2006, Australia

Marta Monjardino
CSIRO Waite Campus, Urrbrae SA 5064, Australia

*Corresponding author: edward.anderson@sydney.edu.au (phone +61 2 9036 7546).

Abstract

In an agricultural setting it is natural to consider yield risk in the context of a three level supply chain: with a small number of suppliers, large numbers of growers, and a small number of buyers. In the cereal growing case that is our focus, there is a supplier of fertiliser, a potentially large number of growers of cereal crops and a buyer, who purchases grain from the growers. The yield depends both on the input level of fertiliser and also on random weather-related factors. We study the impact of a new type of contract structure in which the grower purchases inputs at a discount, but agrees to a reduced price for the crop. The buyer makes a payment to the supplier to compensate for the discount offered. We show how this can coordinate the supply chain and demonstrate the potential advantages of this contract form when producers are risk averse. We look in detail at the implications of the use of these contracts by Australian wheat growers using data generated by APSIM, a growth simulation tool, to understand the connection between yields, fertiliser use and the weather. By using APSIM we can estimate the distribution of yields implied by the grower’s decision on fertiliser application and hence estimate optimal fertiliser use for risk averse growers.

Key words: OR in agriculture, yield uncertainty, supply chain contracts, risk aversion

1 Introduction

In this paper we discuss contractual arrangements in supply chains with yield risk. Our motivation arises from work in agricultural supply chains and we begin by describing some aspects of the situation faced by grain producers. Our case study examples will be drawn from wheat growing regions in Australia. The major risks faced by cereal growers relate to the price of the crop, low yields due to poor weather, and disease and pest damage. We will concentrate on weather-related risks, where Australian producers are particularly vulnerable with crop yields being more volatile than in any comparable country (Kingwell 2011).

The problems caused by this “feast or famine” characteristic of farming activities are notorious. Smaller farms with fewer financial resources will find this particularly difficult to deal with. Insurance solutions have been popular in North America as a way of dealing with agricultural risk. Since yield risk is the hardest to hedge, this is most likely to be the focus of the insurance package – called Multi-Peril Crop Insurance (MPCI) in this context, since the insurance payouts are directly linked to low yields which may occur as the result of a large number of different types of risk. One difficulty
with this insurance is that administration costs are very high, since it is typically the case that an
on-farm evaluation is required in the case of a claim. It is generally thought that MPCI is unlikely to
be attractive to the grower without a significant level of government subsidy. See Coble et al. (2000)
for a general discussion of insurance in this context.

Besides more general difficulties caused by volatile income, there are management choices that
are impacted by risk aversion. It is common for capital investment to be reduced both directly as a
result of the uncertainty associated with paying for the investment, or indirectly through difficulties
in obtaining credit. But our focus is on a different type of investment related to the choice of input
(fertiliser) quantity.

The largest single input cost for a cereal grower is likely to be the cost of fertiliser, being primarily
nitrogen (N) in one form or another. Nitrogen costs are related to energy costs and have increased in
recent years. Application of N will usually increase yield in years in which there is substantial rainfall,
but will give negligible benefit in dry years. There is evidence that risk aversion by growers leads to
significant reductions in fertiliser application in comparison with a risk neutral case (Monjardino et
al. 2015). Figure 1 shows an example of the variation over a 15-year period of the yield achieved at
different levels of N application. This data is generated from a plant growth simulation using soil data
from Wongan Hills in Australia and actual weather data at that location (more details are given later
in the paper). The results show how yield varies dramatically from year to year especially at higher
levels of N application.

Figure 1 round here caption: Simulated yield as a function of N application at sowing for years
1950-1964

The hypothesis that growers under-invest because of risk concerns also gives other supply chain
partners an incentive to address the risk equation for the grower. The approach we will investigate
here is a contractual arrangement whereby the grain buyer pays the supplier for part of the grower’s
fertiliser purchase and at the same time offers a lower price for grain at harvest. This will give some
reduction in risk for the grower without the high costs associated with insurance at the farm level.
We will analyze the factors that determine the extent of the supply chain benefit. In fact we will
demonstrate potential benefits from this arrangement even in the risk neutral case. We show that
because of the concave shape of the average yield curve, these new contract arrangements can lead to
improved supply chain coordination. But our case study examples will show that the most significant
benefits occur when growers are risk averse. There are questions that arise over the nature of supply
chain coordination where one partner is risk averse (here the grower) and others are risk neutral. This
issue is addressed in a two tier supply chain by Gan et al. (2005).

In order to examine the implications of the proposed scheme we consider its application to wheat
growing in Australia. The requirement to model both average yield curves (as a function of N appli-
cation) and also the variation in yield that we expect to see as a result of annual variation in weather
conditions (heat, rainfall etc.) means that we cannot use empirical data from crop trials. These may
explore the results of decisions on different levels of N application, but they will not be run for a long
enough period to allow for the inherent variability of outcomes because of different weather conditions
during the growing season. For this reason we will need to use a crop simulation tool. The method of
choice here is the Agricultural Production Systems Simulator (APSIM) (Holzworth et al. 2014). This
is a simulation tool that has been used extensively in related studies on the impact of N application on
average yields (Hochman et al. 2012, Monjardino et al. 2013). This approach has also been applied
with risk aversion on the part of the grower, see Monjardino et al. (2013, 2015). We therefore have an
opportunity to repeat this work to ascertain whether a different contract structure can address this
contributor to overall yield gaps.

The contribution of this paper is three-fold. First we propose a novel contract arrangement in a
three-echelon supply chain with yield uncertainty, as typically occurs in agricultural settings. Second
we show that the new contract can achieve supply chain coordination, and we give conditions under
which the new contract will achieve higher supply chain benefits when the grower is risk averse. Thirdly
we demonstrate the effectiveness of the new contract arrangement with data for three different areas of wheat production in Australia.

2 Related literature

There has been previous work in the agribusiness area (Tang et al. 2016) looking at the use of partially guaranteed price contracts in an environment with price uncertainty, but no yield uncertainty. When considering crop yields it is often appropriate to model the dependence between price and yield. For example Kazaz (2004) considers an inverse relationship between olive oil yields and prices. In our model we will not consider this possibility, but assume that prices are fixed. This is partly justified by the possibility of growers entering into arrangements with their buyers under which the price risk is taken by the buyer. Moreover when considering grain production, prices are typically set in an international market and the correlation between an individual grower’s yield and the grain price is quite weak.

There has been a large amount of research related to yield uncertainty and how it impacts on production decisions. The paper by Yano and Lee (1995) gives a review of different versions of this problem focussing on the order size question. There are many different forms that uncertain yield may take. We will consider the interaction between an input (in our case fertiliser) and the characteristics of the resulting uncertainty. Hence we need to consider the stochastic behaviour of the production function (see Just and Pope 1978). It has been observed by several authors that, in an agricultural context, inputs like fertiliser are likely to lead to increased uncertainty for higher levels of output (e.g. Feder 1980). Research that considers Australian growers provides evidence that the yield gap is a key consequence of this with Australian growers achieving only about half of the yield potential of their crops (Hochman et al. 2016).

A key question arising with yield uncertainty concerns the mechanisms required to ensure that the overall supply chain can be coordinated. Previous work considers a supplier who makes a decision on how much to produce, when the amount of output that finally occurs is determined by a random yield (which may take either an additive or multiplicative form). Li et al. (2013) and Inderforth and Clemens (2014) are examples of papers that consider how contract designs that penalize shortfalls or overproduction can coordinate this supply chain. The different forms of contract lead to a redistribution of risk between the supply chain partners (Xia et al. 2011). Luo and Chen (2016) consider this type of arrangement in a two echelon supply chain with shortage and overstocking costs when there is uncertainty in both demand and supply. They show that a revenue sharing contract needs to be augmented with a surplus subsidy contract to achieve supply chain coordination in this case.

Our problem involves a decision on inputs (fertiliser) rather than on the planned production quantity, which would more closely correspond to the amount of land planted to a particular crop. However the underlying structure of the problem of input choice with uncertain yield remains the same. A more important difference arises from the nature of the demand process: these papers have demands that may not be known in advance but have a fixed value, and as a result there are additional costs, either when demand cannot be met, or when there is excess supply. In our model all output will be sold, so that these costs will not exist.

A number of papers such as Xu (2010) and Luo and Chen (2015) consider an environment where there is yield uncertainty and option contracts are allowed. Here the buyer has either a fixed or stochastic demand to meet and there is a spot market in which additional purchases can be made if required due to a low yield. A related problem of this type can occur with fresh produce (see Wang and Chen 2017) where losses can occur during transport (“circulation losses”).

There are not many papers that consider yield uncertainty in a supply chain with more than two echelons. An exception is the paper by He and Zhao (2012) who consider a three-tier supply chain with a raw-material supplier, a manufacturer, and a retailer. In their paper there is both supply uncertainty arising from the raw material supplier and demand uncertainty occurring at the retailer. Our model has a different structure: we examine a three-tier contract structure where the yield uncertainty arises in the middle tier of producers, rather than in the other tiers (suppliers and buyers). This type
of contract arrangement requires agreement from all the supply chain partners, and is described by Van Der Rhee et al. (2010) as a spanning revenue sharing contract. Formentini and Romano (2016) describe it as an extra dyadic contract (which goes beyond simply replicating dyadic contracts between different pairs of supply chain participants.)

The specific contract structure we consider has not, as far as we can tell, been examined by other researchers but is natural in an agricultural context. For example it closely mirrors the end point royalty scheme that is extensively used for seeds in Australia (see Arnold 2015). This acts as a form of risk sharing mechanism. Besides paying a price for the seed that they buy, growers are also charged an additional royalty on the tons of crop produced. In some cases this money is deducted by the grain buyer and a payment then made directly back to plant breeder who supplies the seeds. This will then be a close parallel to the type of contract arrangements we are proposing for fertiliser.

3 The model

In an agricultural supply chain, there are suppliers, growers and buyers. We simplify the setting by focusing on one supplier, one grower and one buyer. Assume that there is an input which is purchased or made by the supplier at a cost (to the supplier) of \( C \) per kg, and the unit price of crop sold by the buyer to purchasers in the commodity market is \( R \) per kg. These are prices external to the supply chain. The grower pays a higher price for the input and she receives a lower price for the crop. We will assume both \( C \) and \( R \) are fixed since we wish to focus on variability in the yield (which is often the dominant uncertainty for the grower).

The most critical component of our model is the yield behaviour. The yield \( Y(q, \varepsilon) > 0 \) is the amount of crop in kg and is a function depending both on the quantity of input \( q \) and a random variable \( \varepsilon \) that captures the uncertainty that is primarily related to the weather behaviour during the growing season.

Different specific models can be used, for example we may have a quadratic underlying yield with multiplicative uncertainty (which is more natural in this context than additive uncertainty). This gives

\[
Y(q, \varepsilon) = \varepsilon f(q) = \varepsilon (a + bq - dq^2),
\]

where \( \varepsilon \) is a positive random variable with mean \( \bar{\varepsilon} \) and variance \( \sigma^2 \). We can normalize the values of \( f(q) \) so that the noise term has mean \( \bar{\varepsilon} = 1 \). Our analysis, however, will carry through with quite general forms of yield uncertainty.

Our primary application area is to wheat crops with Nitrogen as an input. We can normalise to consider the yield per hectare, with a decision made on input of fertiliser as an amount per hectare. However the yield model we consider is common in agricultural settings. The input may be labour used to tend crops; insecticides and herbicides; or irrigation water. In all cases we can expect to see the yield as determined by a combination of a nonlinear function of the inputs together with random weather factors.

We write \( \overline{Y}(q) \) for the expected yield at input amount \( q \) kg per hectare, so \( \overline{Y}(q) = E_{\varepsilon}[Y(q, \varepsilon)] \). In the case of the multiplicative model and normalized values we simply have \( \overline{Y}(q) = f(q) \). Throughout this paper we will make the following assumption on \( \overline{Y} \):

**Assumption 1:** \( \overline{Y}(q) \) is increasing concave and bounded above, with \( \overline{Y}(0) > 0 \).

We begin by stating the baseline (or traditional) arrangement. The supplier charges \( \beta C \) to the grower for each kg of input where \( \beta > 1 \); so the supplier gets a fixed profit margin \( (\beta - 1) \). The buyer pays \( \delta R \) to the grower for one kg of crop, where \( 0 < \delta < 1 \). Thus the buyer has a profit margin \( (1 - \delta) \).

We wish to make a comparison between the baseline arrangement and a new supply chain contract which we call a double discount contract and that is determined by two constants \( \gamma \) and \( \eta \), and a function \( W \). In the new arrangement, for each kg of input purchased from the supplier, the grower pays \( (\beta - \eta)C \) where \( \eta < \beta \), so the supplier offers a discount of \( 100\eta/\beta \% \) on the cost of the input. For each kg of crop purchased by the buyer, the grower is paid \( (\delta - \gamma)R \), where \( \gamma < \delta \), so that the price
paid is reduced by a percentage $100\gamma/\delta \%$. In addition there is a further payment to the supplier from the buyer which is determined as a function $W(q, Y)$ of both the quantity purchased by the grower and the yield $Y$. This is illustrated in Figure 2.

By setting $W(q, Y) = \eta q C$ we have the buyer reimbursing the supplier for the discount given to the grower. This is a natural arrangement when the buyer is the supply chain leader and initiates the contract arrangements. In this case the supplier does not hold any of the risk from varying yield. Under this payment scheme there is no need for yield information to be transferred from the buyer to the supplier. An alternative arrangement has the supplier as supply chain leader. Then we may take $W(q, Y) = \gamma Y R$ so that the buyer makes payments to the supplier that match the amount that is not paid to the grower. In this case the supplier who holds the additional risk from the double discount contract associated with uncertain yield. Under these arrangements the buyer does not need to be given information on the amount of input purchased.

We will start with the risk neutral case. The expected profit for the grower in the baseline case is

$$\Pi_F(q) = \overline{Y}(q) \delta R - q \beta C$$

and the grower maximizes their expected profit by choosing $q = q_B$ defined from

$$\overline{Y}'(q_B) = \frac{\beta C}{\delta R}. \quad (2)$$

In the case of a quadratic function (1) we have $\overline{Y}'(q_B) = b - 2dq_B$, so (2) gives

$$q_B = \frac{1}{2d} \left( b - \frac{\beta C}{\delta R} \right).$$

We write $\alpha(q)$ for the ratio of outputs to inputs in dollar terms measured at the underlying cost of inputs and the final price of grain given a quantity $q$ of inputs. An important parameter is $\alpha(q_B)$, which is this ratio evaluated at the baseline input amount $q_B$, so

$$\alpha(q_B) = \frac{\overline{Y}(q_B) R}{q_B C}.$$  

This ratio may be estimated from the actual figures achieved by the grower operating in the baseline case. In the case of the quadratic function (1) we have

$$\alpha(q_B) = \left( \frac{a}{q_B} + b - dq_B \right) \frac{R}{C} = \frac{2dq_B \delta}{(bR - \beta C)} \frac{R^2}{2C} + \frac{bR}{2C} + \frac{\beta}{2d}.$$  

In this paper we will deal with three different alternatives to the baseline choice of input quantity $q_B$. First we will calculate the input quantity $q^*$ that occurs under the double discount contract (depending on the choices of parameters $\eta$ and $\gamma$). In the next section we will calculate the quantity $q^*_C$ that maximizes the supply chain profit (the sum of profits for supplier, farmer and buyer), and then in Section 5 we will define a risk averse choice $q_U$ that depends on the utility function for the buyer and occurs under the baseline contract arrangements.

Under the double discount contract we have an expected profit for the grower of

$$\tilde{\Pi}_F(q) = \overline{Y}(q) (\delta - \gamma) R - q (\beta - \eta) C,$$

giving an optimal input choice $q^*$ satisfying

$$\overline{Y}'(q^*) = \frac{(\beta - \eta) C}{(\delta - \gamma) R}. \quad (3)$$
In the case of a quadratic function (1) we will have

\[ q^* = \frac{1}{2d} \left( b - \frac{(\beta - \eta) C}{(\delta - \gamma) R} \right). \]

We will need to make the following assumption on the behaviour of the slope of the expected yield function.

**Assumption 2:** \( Y'(0) > (\beta/\delta)(C/R). \)

This assumption ensures that under the baseline arrangements and starting with zero input level, there is an improvement in expected profit for the grower when small quantities of input are used.

**Proposition 1**

Under Assumptions 1 and 2, if \( \eta \) and \( \gamma \) are chosen so that

\[ \eta = \alpha(q_B)\gamma, \]

and either:

(a) \( W(q, Y) = \eta q C \) and

\[ \beta - \eta \geq \beta(\delta - \gamma); \]  

(b) \( W(q, Y) = \gamma Y R \) and

\[ \delta(\beta - \eta) \geq (\delta - \gamma); \]

then the supplier, grower and buyer will all have higher expected profit from the double discount contract than is obtained under the baseline contract.

Note that when \( \eta = \gamma = 0 \) both of the inequalities (5) and (6) will hold (since \( \beta > 1 > \delta \)). Hence by choosing \( \eta \) and \( \gamma \) small enough, and with the correct ratio, all the required conditions for this result will be met, and so all parties in the supply chain will see an improvement. This is true whichever form of \( W \) is used.

4 Supply chain optimality

Now we consider the supply chain as a whole. If we can achieve a supply chain optimal solution then there is the greatest possible expected profit to be distributed between the three supply chain partners. The total supply chain profits are given by

\[ \Pi_C(q) = Y(q)R - qC, \]

so that an optimal choice for the supply chain as a whole \( q_C^* \) is given by the solution to

\[ Y'(q_C^*) = \frac{C}{R}. \]

The grower will make the supply chain optimal choice for the amount of input to use whenever \( \beta - \eta = \delta - \gamma \), so that the ratio of the cost of input to the price for the output for the grower matches this ratio for the supply chain as a whole. Using values for \( \eta \) and \( \gamma \) that satisfy this will automatically lead to a good outcome for the supply chain as a whole.

However this choice will also mean that the inequalities (5) and (6) fail to hold. Hence we can have no guarantee of finding a solution satisfying the equation \( \eta = \alpha(q_B)\gamma \), that gives the best supply chain result and at the same time allows each of the three players to do better than they would under the baseline arrangement. This is true for both the \( W \) functions discussed, corresponding to either a buyer or supplier led contract structure. Often we find that, around the optimum level of \( N \) application, the increase in yield for a relatively large increase in \( N \) level is modest. In this case the supplier can...
see a substantial increase in profit from the additional N bought by the grower if the supply chain is coordinated, but there will only be limited benefits to the buyer. This can lead to difficulties in ensuring a better outcome for the buyer in the case that the buyer is the supply chain leader.

This leads us to consider using the flexibility arising from the function $W$ to formulate a double discount policy that provides a guarantee for each of the buyer and supplier of doing better than under the baseline policy in expectation. We do this using a parameter $\mu$ that determines how the risk is shared between the two players. Besides the restriction $\beta - \eta = \delta - \gamma$ that is necessary to give a supply chain optimal choice for the grower, we will retain the equation (4) for the ratio of $\eta$ and $\gamma$. Together these define the values for $\eta$ and $\gamma$. Then we can show that for a range of choices for $W(q, Y)$, determined by a parameter $\mu$, we will improve outcomes, in expectation, for all three supply chain partners.

**Proposition 2.**

Under Assumption 1, a double discount contract with

$$\gamma = (\beta - \delta) \frac{1}{\alpha(q_B) - 1},$$

$$\eta = (\beta - \delta) \frac{\alpha(q_B)}{\alpha(q_B) - 1},$$

$$W(q, Y) = \mu(\eta q + (1 - \beta)(q - q_B))C + (1 - \mu)(\gamma Y + (1 - \delta)(Y - \bar{Y}(q_B)))R,$$

for any $0 < \mu < 1$ has the property that a supply chain optimal solution is obtained and all parties make strictly greater profit in expectation than under the baseline arrangement.

The simplest version of this double discount policy has $\mu = 1/2$ which gives

$$W(q, Y) = ((1 - \beta + \eta)qC + (1 - \delta + \gamma)YR - (1 - \beta)q_B C - (1 - \delta)\bar{Y}(q_B)R)/2.$$

### 5 Risk aversion

We can also ask about the degree of risk faced by the grower under the new contract scheme. It is easy to see that the risk is reduced. Suppose that yield is variable in a range $Y(q) - \Delta, Y(q) + \Delta$. In the baseline case this will lead to profits that vary in the range $\Pi_F(q) \pm \Delta \delta R$. Under the double discount contract the variation in profit is reduced to a range $\pm \Delta \delta \gamma R$. So it is clear that with the new contract structure a grower who makes the same choice of input quantity has lower variability in profit and so a reduced risk.

However a full analysis of the situation needs to consider the fact that the choice of input quantity made by the grower will change if she is risk averse. We may suppose that a risk averse grower makes her decision on the basis of a utility function $U(\cdot)$. The grower will choose $q$ to maximize expected utility:

$$E[U(\Pi_F)] = E[E[U(Y(q, \varepsilon)\delta R - q\beta C)]] = E[U(\Pi)] = E[U(Y(q, \varepsilon)\delta R - q\beta C)].$$

Thus the grower will choose $q = q_U$ where

$$E[\varepsilon[U'(Y(q_U, \varepsilon)\delta R - q\beta C)] Y'(q_U, \varepsilon)\delta R - \beta C]] = 0.$$

We will find conditions under which $q_U < q_B$. Note that $U$ being concave is not enough on its own to ensure this result, since if the uncertainty decreased with higher $q$ then risk aversion would tend to mean a choice with $q_U > q_B$.

We say that the random yield is **divergent** if

$$Y(q, \varepsilon_1) < Y(q, \varepsilon_2) \implies \frac{\partial}{\partial q} Y(q, \varepsilon_1) < \frac{\partial}{\partial q} Y(q, \varepsilon_2).$$
Under this condition the difference between $Y(q, \varepsilon_1)$ and $Y(q, \varepsilon_2)$ increases with $q$. Note that this condition will hold if $Y(q, \varepsilon) = \varepsilon Y(q)$.

For the empirical results from APSIM runs, we cannot access the yield derivatives directly and this condition becomes harder to check. Note that with yield divergence the ordering between yields with different values of $\varepsilon$ remains unaltered as the input quantity $q$ varies. We can observe from plots such as Figure 1 that this is not always the case for APSIM yields, though it is common to see increasing differences between yields as $q$ increases.

Proposition 3

Suppose that $Y$ is divergent, $Y'(q, \varepsilon)$ is continuous and decreasing in $q$ for each $\varepsilon$, and $U$ is concave, differentiable and increasing. Then $q_U < q_B$ and the potential supply chain benefit available from the new contract is greater when the grower is risk averse than when the grower is risk neutral.

Obviously when $U$ is linear (so there is no risk aversion) $q_B$ will be chosen. The loss of overall profit in the supply chain occurs as a result of $q_B$ being below the supply chain optimal value $q^*_C$. Since the supply chain expected profit is concave in $q$, further reduction from $q_B$ to the value $q_U$ reduces the expected supply chain profit even further. The result then follows easily from establishing that we can choose $\eta$ large enough to get supply chain optimality in the risk averse case. This result holds even when individual $Y(q, \varepsilon)$ functions may start to decrease for large enough $q$.

However the exact behaviour of the grower’s choice of input quantity as a function of $\gamma$ and $\eta$ is hard to determine, and we cannot easily find $\eta$ and $\gamma$ parameters that lead to supply chain optimality. We will see in the case studies how the additional benefits that occur when the grower is risk averse are larger than the benefits that accrue from the concave expected yield curve, but even quite large discounts may still lead to an under-investment in inputs in comparison with the risk neutral case.

6 Case study

In this section we illustrate the discussion in the previous sections on three representative wheat growing areas of Australia. Our intention is to show what the parameters of the model will turn out to be in practice and to make some estimation of the potential of this approach.

We consider three different areas in Australia where wheat is grown. Wongan Hills (-30.88, 116.71) in the central wheatbelt of Western Australia, and Hart (-33.753, 138.416) in mid-north of South Australia are both located in the winter-dominant medium-rainfall region (350 to 500 mm). The third site is Temora (-34.533, 147.57) in New South Wales located in the high rainfall region (above 500 mm).

For each site we look at weather data for the period 1950 to 2010 and use soil data for that site to run an APSIM simulation for the yield that would be anticipated with different levels of N application. In practice it is common to use an additional N application part way through the growing season. For simplicity we have assumed that all N is applied at sowing.

We do not have access to information on the profit margins that occur in practice for the grain buyer or the supplier. The relevant figures are those that apply on the margin to increases in volume, so that higher fixed-costs for manufacturing (for the supplier) or distribution networks (for the buyer) will typically be associated with higher margins to make the business profitable overall. We use margins of 15% for the buyer and 25% for the supplier (corresponding to $\delta = 0.85$ and $\beta = 1.25$). Higher profit margins will lead to increased benefits from the contract structures we propose.

In the first set of experiments we consider the risk neutral case and compare the profits achieved under a baseline contract with those available under the double discount contract of Proposition 2 with the parameter $\mu = 1/2$. For these calculations we will need to make assumptions on the prices and costs involved in the farm operation.

We assume that the grower receives AU$283 per metric ton of wheat - based on the Australian Standard White price in 2010-2011 using data from ABARES (2014). The price of N is taken as AU$1224 per metric ton (using ABARES data on urea prices taken as 46% N). For the farming input
costs other than N we use the figures estimated by Monjardino et al. (2015) of 185, 190 and 203 in $AU per hectare for Wongan Hills, Hart and Temora respectively.

The APSIM settings used for the simulations are site specific and are described in more detail in Monjardino et al. (2015). The APSIM yield data is available at specific N application levels of 0, 7.5, 15, 30, 60, 90, 120 and 150 kg N per hectare. The range of values occurring for yield at the three sites at different levels of N are shown in Figure 3. For our purposes we need to estimate a smooth average yield curve, over the N application range of interest, since we need to access the derivative $Y'$. We do this by fitting a polynomial of degree 4 through the average yield (over all 61 simulated years) at the five N application levels from 30 to 150 kg.

Figure 3 round here caption: Variation in yield with Nitrogen at three sites

Table 1 below shows a comparison of the baseline profits and those available under the Proposition 2 values for each site.

<table>
<thead>
<tr>
<th></th>
<th>Wongan Hills</th>
<th>Hart</th>
<th>Temora</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grower’s optimal N (kg per ha)</td>
<td>114.9</td>
<td>128.6</td>
<td>102.9</td>
</tr>
<tr>
<td>Grower expected profit ($AU per ha)</td>
<td>684.6</td>
<td>775.5</td>
<td>1164.3</td>
</tr>
<tr>
<td>Buyer expected profit ($AU per ha)</td>
<td>178.3</td>
<td>198.1</td>
<td>263.5</td>
</tr>
<tr>
<td>Supplier expected profit ($AU per ha)</td>
<td>28.1</td>
<td>31.5</td>
<td>25.2</td>
</tr>
</tbody>
</table>

Double discount policy:

Parameters:

$\gamma = 0.042$  $\eta = 0.442$

Grower’s optimal N (kg per ha) | 128.1  | 137.7 | 110.6 |
Grower expected profit ($AU per ha) | 687.1 | 777.1 | 1165.8 |
Buyer expected profit ($AU per ha)  | 178.6 | 198.3 | 263.7 |
Supplier expected profit ($AU per ha) | 28.4  | 31.7  | 25.3  |

Observe that the average $\eta$ value across the three sites is 0.436. The grower sees a reduction in price from $B$ to $(\beta - \eta)C$, so that this $\eta$ value corresponds to a discount of 35% with the value $\beta = 1.25$ that we are using. There is a significant increase in N use under the new contract arrangements, which lead to supply chain optimal choices. The average increase over the three sites is 8.7%. However the increase in grower profit is small, an average of $1.86 per hectare. This is a result of the relatively flat behaviour of the yield function over the region of interest. The profit increases for the buyer and supplier are very small. Overall we can see that the improvements implied by the new contract structure in this risk neutral case are very limited.

Now we consider the case with risk aversion. A starting point is the utility function that the grower uses. We do not intend to give precise predictions of outcomes, but rather to give an indication of the size of the effects we are considering. So it makes sense to use a simple utility function and we choose a constant absolute risk aversion (CARA) model with $U(x) = 1 - \exp(-c_0x)$, with values of the risk aversion coefficient $c_0 = 0.01$ (moderate risk aversion) and $c_0 = 0.02$ (high risk aversion), see the discussion in Hardaker (2004a, 2004b). An implication of the CARA model is that choices will not depend on the wealth position of the grower. The CARA model with similar coefficient values has been used in Monjardino et al. (2015).

The grower is supposed to maximize expected utility. For the given values of N application for which we have APSIM simulation results we can calculate utility for each of the 61 years in the simulation. The average of these values then becomes the expected utility at that level of N application. To obtain the complete expected utility curve (on which we carry out an optimization) we take a polynomial
of degree 4 and fit it to the data points corresponding to N levels 30, 60, 90, 120 and 150 kg per ha. Table 2 gives the optimal N levels under this baseline case and we can see that they are dramatically reduced even with moderate risk aversion. The table also shows the expected profit figures. The grower is concerned not with profit but with expected risk, so changes in grower expected profit occur as a result of a risk calculation. However we can suppose the buyer and supplier to be relatively risk neutral since they are diversified across many growers. So the data in this table shows how badly the risk aversion of the grower impacts the buyer and supplier profitability, especially for Wongan Hills and Hart.

<table>
<thead>
<tr>
<th></th>
<th>Wongan Hills</th>
<th>Hart</th>
<th>Temora</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0 = 0.01$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal N per ha</td>
<td>40.4</td>
<td>32.3</td>
<td>33.9</td>
</tr>
<tr>
<td>Grower profit ($/ha)</td>
<td>577.1</td>
<td>517.0</td>
<td>1012.4</td>
</tr>
<tr>
<td>Buyer profit ($/ha)</td>
<td>143.2</td>
<td>131.7</td>
<td>221.6</td>
</tr>
<tr>
<td>Supplier Profit ($/ha)</td>
<td>9.9</td>
<td>7.9</td>
<td>8.1</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0 = 0.02$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal N per ha</td>
<td>0</td>
<td>0</td>
<td>28.1</td>
</tr>
<tr>
<td>Grower profit ($/ha)</td>
<td>326.5</td>
<td>301.1</td>
<td>992.7</td>
</tr>
<tr>
<td>Buyer profit ($/ha)</td>
<td>90.3</td>
<td>86.7</td>
<td>217.7</td>
</tr>
<tr>
<td>Supplier Profit ($/ha)</td>
<td>0</td>
<td>0</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Table 2: The impact of risk aversion on the baseline case

We will consider the double discount contract structure with $\eta$ values of 0.4, 0.6 and 0.8 which corresponds to discounts on the price of N to the grower of 32%, 48% and 64% respectively. We set the $W$ values at $\eta q C$ corresponding to the buyer as the supply chain leader. In each case the $\gamma$ values are set from (4). Table 3 below shows the result of these contract structures on the expected utility maximizing choices of N and the consequences for expected profits. This table also includes the risk neutral case where the $\eta$ values of 0.6 and 0.8 would be larger than appropriate for optimizing supply chain profit.

We see from these results that, when there is risk aversion by the grower, significantly better overall results can be obtained by using double discount contracts. High levels of discount ($\eta = 0.8$) are preferable in most cases, and even where this is not the case (when the grower is risk neutral, or for Temora where the overall supply chain profit may not improve) there is no significant loss. The gains from the double discount contract can be really substantial. For the Wongan Hills case with moderate risk aversion ($c_0 = 0.01$) the reduction in risk afforded by taking $\eta = 0.8$ (64% discount) allows the grower to opt for a doubling of the N application level to 81.7 kg/ha with a corresponding increase in expected profit of 12% to $647. There are even larger increases for the buyer’s profit (increase of 17% to $168.2 per ha) and supplier’s profit (increase of 102% to $20 per ha).
Table 3: The impact of risk aversion with double discount contracts (all amounts per ha)

<table>
<thead>
<tr>
<th>$c_0, \eta$</th>
<th>Optimal N rate kg</th>
<th>Grower profit $</th>
<th>Buyer profit $</th>
<th>Supplier profit $</th>
<th>Supply Chain profit $</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wongan Hills:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0, 0.4</td>
<td>126.9</td>
<td>686.7</td>
<td>176.4</td>
<td>31.1</td>
<td>894.2</td>
</tr>
<tr>
<td>0, 0.6</td>
<td>132.7</td>
<td>689.3</td>
<td>172.0</td>
<td>32.5</td>
<td>893.7</td>
</tr>
<tr>
<td>0, 0.8</td>
<td>138.3</td>
<td>692.9</td>
<td>165.4</td>
<td>33.9</td>
<td>892.1</td>
</tr>
<tr>
<td>0.01, 0.4</td>
<td>61.3</td>
<td>620.5</td>
<td>158.2</td>
<td>15.0</td>
<td>793.7</td>
</tr>
<tr>
<td>0.01, 0.6</td>
<td>71.3</td>
<td>634.4</td>
<td>163.5</td>
<td>17.4</td>
<td>815.4</td>
</tr>
<tr>
<td>0.01, 0.8</td>
<td>81.7</td>
<td>647.0</td>
<td>168.2</td>
<td>20.0</td>
<td>835.1</td>
</tr>
<tr>
<td>0.02, 0.4</td>
<td>18.2</td>
<td>444.7</td>
<td>119.1</td>
<td>4.5</td>
<td>568.3</td>
</tr>
<tr>
<td>0.02, 0.6</td>
<td>35.5</td>
<td>525.8</td>
<td>138.7</td>
<td>8.7</td>
<td>673.2</td>
</tr>
<tr>
<td>0.02, 0.8</td>
<td>48.7</td>
<td>567.2</td>
<td>149.9</td>
<td>11.9</td>
<td>729.0</td>
</tr>
<tr>
<td><strong>Hart:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0, 0.4</td>
<td>136.8</td>
<td>776.8</td>
<td>196.8</td>
<td>33.5</td>
<td>1007.1</td>
</tr>
<tr>
<td>0, 0.6</td>
<td>141.6</td>
<td>778.6</td>
<td>193.5</td>
<td>34.7</td>
<td>1006.8</td>
</tr>
<tr>
<td>0, 0.8</td>
<td>146.9</td>
<td>781.4</td>
<td>187.9</td>
<td>36.0</td>
<td>1005.2</td>
</tr>
<tr>
<td>0.01, 0.4</td>
<td>45.0</td>
<td>564.1</td>
<td>146.2</td>
<td>11.0</td>
<td>721.4</td>
</tr>
<tr>
<td>0.01, 0.6</td>
<td>52.1</td>
<td>589.0</td>
<td>153.7</td>
<td>12.8</td>
<td>755.4</td>
</tr>
<tr>
<td>0.01, 0.8</td>
<td>60.1</td>
<td>615.2</td>
<td>161.2</td>
<td>14.7</td>
<td>791.2</td>
</tr>
<tr>
<td>0.02, 0.4</td>
<td>0.0</td>
<td>279.0</td>
<td>86.7</td>
<td>0.0</td>
<td>365.7</td>
</tr>
<tr>
<td>0.02, 0.6</td>
<td>12.8</td>
<td>361.8</td>
<td>105.9</td>
<td>3.1</td>
<td>470.9</td>
</tr>
<tr>
<td>0.02, 0.8</td>
<td>26.8</td>
<td>442.2</td>
<td>124.8</td>
<td>6.6</td>
<td>573.6</td>
</tr>
<tr>
<td><strong>Temora:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0, 0.4</td>
<td>110.1</td>
<td>1165.6</td>
<td>262.2</td>
<td>27.0</td>
<td>1454.8</td>
</tr>
<tr>
<td>0, 0.6</td>
<td>114.4</td>
<td>1167.3</td>
<td>259.1</td>
<td>28.0</td>
<td>1454.4</td>
</tr>
<tr>
<td>0, 0.8</td>
<td>119.7</td>
<td>1169.9</td>
<td>253.5</td>
<td>29.3</td>
<td>1452.7</td>
</tr>
<tr>
<td>0.01, 0.4</td>
<td>35.0</td>
<td>1000.2</td>
<td>223.5</td>
<td>8.6</td>
<td>1232.2</td>
</tr>
<tr>
<td>0.01, 0.6</td>
<td>36.5</td>
<td>996.6</td>
<td>224.9</td>
<td>8.9</td>
<td>1230.4</td>
</tr>
<tr>
<td>0.01, 0.8</td>
<td>38.5</td>
<td>995.4</td>
<td>226.7</td>
<td>9.4</td>
<td>1231.6</td>
</tr>
<tr>
<td>0.02, 0.4</td>
<td>30.9</td>
<td>982.8</td>
<td>219.7</td>
<td>7.6</td>
<td>1210.1</td>
</tr>
<tr>
<td>0.02, 0.6</td>
<td>31.9</td>
<td>976.4</td>
<td>220.6</td>
<td>7.8</td>
<td>1204.9</td>
</tr>
<tr>
<td>0.02, 0.8</td>
<td>33.3</td>
<td>972.1</td>
<td>221.9</td>
<td>8.1</td>
<td>1202.2</td>
</tr>
</tbody>
</table>

7 Discussion of results

The case study results confirm the theory we have developed. First we show that there are advantages in a double discount contract when growers are risk neutral, which follows from the concavity of the expected yield curve as estimated from APSIM using actual weather patterns at three different locations. It is important to note that in this case the improvement is relatively minor.

Secondly we confirm that in the presence of risk averse growers the changes in input by the growers are far larger and the corresponding benefits to supply chain profits also larger. This happens even though the requirement of the theory for the random yield to be divergent is not strictly true for the yield results generated by APSIM.

It is interesting that even with $\eta = 0.8$ the N application level still falls well below the supply chain optimal amount. In fact with this data and with risk aversion parameter $c_0 = 0.01$ discount levels of more than 95% would be required to push the grower into making a supply chain optimal decision on inputs.

The detailed results from the case study show that the best choice of parameters for total supply chain profit varies according to both the site (with different soil properties and weather variation) and
also the risk aversion of the grower. When the grower is risk neutral then we should use relatively low values of discount percentage, but with higher risk aversion then higher levels of discount become worthwhile. In practice, however, we may need to apply a blanket choice of discount percentages (for input cost and grain price) across different grain growing regions and with growers who have different degrees of risk aversion. The analysis we provide suggests that it may be possible to find a common set of parameters that give reasonably good outcomes in many different circumstances.

8 Conclusions and future developments

We have shown how a form of double discount contract involving all three tiers of the supply chain can achieve coordination when there is non-linear yield curve combined with yield uncertainty, as will occur in an agricultural context when considering fertiliser as an input. This is a new form of contract arrangement with discounts offered by the supplier, reduced prices paid by the buyer, and a balancing payment made from the buyer to the supplier. There are two distinct ways in which an improvement occurs. First, when supply chain participants are risk neutral benefits occur from the concavity of the expected yield curve. Secondly benefits arise when there is risk aversion, in the case that increasing levels of inputs correspond to increased variability in yield. We have used data from crop simulations to explore appropriate settings within the new double discount contract and assess the changes in average profit that are likely. We find that in practice growers’ risk aversion is far more important than concave yield curves in leading to an overall improvement.

Overall, given the significant wheat yield gap in Australia, which can be more than 1.7 t/ha or 50% of the water-limited yield (Hochman et al. 2012), and which is attributable to sub profit-maximising input levels due to risk and risk aversion in many major grain growing regions, there is a real opportunity to increase supply chain profit and reduce grower risk with higher N applications resulting from double discount contracts. This beneficial outcome could have a positive social impact on other risky grain-growing regions of the world with even larger yield gaps.

For economic and environmental reasons there is increasing emphasis on the efficient use of N fertilizers. Certainly the combined effects of climate change and rising input costs have led to renewed interest in this area (Chen et al. 2008; Lobell 2007), for several reasons:

(A) A substantial amount of N applied is removed at harvest as a component of the harvested grain (up to 300 kg/ha of N every year) with grain protein a highly desirable trait; (B) the remainder of N applied (approximately 60% ) is either slowly available to crop plants or lost in highly polluting forms, that may contribute to the degradation of water, air and soil quality; and (C) the increased frequency of cereal cropping in modern broadacre farming prevents legume crops and pastures from supplying sufficient N to the system, requiring more attention to be paid to soil fertility as an asset to be restored and maintained.

There are many cases where growers form a consortium or some sort of cooperative for the purchase of required inputs (like fertiliser or pesticides). In other cases there may be a cooperative owned by growers that acts as buyer. The range and role of agricultural cooperatives/consortia around the world has been discussed widely (e.g. Bijman et al. 2014, McBride 1986, Ortmann & King 2007). In many supply chain frameworks the common ownership of different components of the supply chain will resolve coordination problems. However, in cases where growers jointly own some other parts of the supply chain, there remains a risk-based decision taken by the grower on how much input to purchase. This is impacted by the margins for the buyer and supplier, that will in this case be a mechanism for recovering the fixed costs of operation, with profits handed back to the growers in some form of dividend. Nevertheless, the decisions taken by the grower will depend on the marginal prices for both input and output. Simply having some shareholding or ownership rights in relation to the buyer or supplier does not in this case remove the benefits of a double discount policy in moving towards a supply chain optimal solution, at a reduced level of risk.

There are a number of practical questions where more work is required before the contract design proposed is implemented, and we deal with some of these here.

In the proposed contract arrangements the commercial offer to the grower is that they receive a
substantial discount on the costs of inputs provided that they sign up to sell their crop to a specific grain buyer (and accept a reduced price for their grain). In practice the introduction of this new type of supply chain contracts requires a single supply chain coordinator. This is most likely to be the grain buyer who already in many cases offers risk management services through setting forward prices for crops. Moreover it may be easier for a buyer to fulfil this role since payments are made from the buyer to the supplier rather than vice versa. In one version of the scheme we propose the payment made from the buyer to the supplier is independent of the yield achieved in a given year, and depends only on the discount offered by the supplier to the grower. The form of the balancing payment will determine which of the supply chain partners will bear the risk of a poor crop. Under the arrangements we propose if there is widespread crop failure then the grain buyer is at risk with payments due to the suppliers and insufficient opportunity to recoup this from reduced payments to the growers. In fact, quite independent of this supply chain contract proposal, grain buyers face a risk to their business if there is a poor harvest. The increased risk would need to be factored into the calculation of whether there is sufficient geographical diversification, or whether the grain buyer should insure against low overall crop volumes.

There is a benefit to the grower associated with the timing difference: she receives a discount at the beginning of the growing season and in a sense does not pay for this until later, with the reduced amount received at harvest. The implied cost to the supplier needs to be considered when determining the level of payments $W$ from buyer to supplier, but will not have a significant effect on the overall benefits of the new contract structure.

An important issue in practice for almost all agricultural supply chains is related to product quality. Yield is rarely straightforwardly a matter of quantity produced. Weather conditions that reduce yield may often lead to lower quality even in the crops that are left. In the case of wheat the protein content of the grain may vary and is one factor in determining prices. The existence of different prices for different quality makes the analysis more complex.

It would be possible to design contracts which discount the price paid to the grower only when yield became high enough. This would be a more effective way of responding to the risk concerns of the grower, however it would probably require the grower to commit in advance to a certain number of hectares of planting to a given crop. Our proposal is simpler and can work independently of the size of the farm or decisions on crop rotation.

Our case study data using APSIM simulation does not take account of all sources of variability that a grower faces. Essentially it gives an assessment of the variability that arises from purely weather related factors. In addition the grower will face some uncertainty that arises from the unknown impact of other factors such as pest and disease. In as much as these other factors also operate in a multiplicative, rather than additive, way they will tend to further depress the level of N applied by a risk averse grower, correspondingly increasing the benefits available from the new contract structure.

The practical benefits of the double discount contract structure are dependent on the decisions made by growers in response to risk, and this is therefore an important area for further study. How would this new form of contract be perceived by growers in practice?

Another area of practical concern is the link between forecast weather and N application. A grower can consider long term weather forecasts in making a decision on N application, and can also delay some of the fertiliser application until part way through the growing season. This makes the decision problem faced by the grower more complex than is considered in our analysis.

Finally we note that in many cases, where the market for a crop is more local, there will be a negative correlation between prices and yield. So that a year with low yield will mean a higher price, on average, than a year with higher yield. This requires a more sophisticated model to capture the dependence between yield and price.

References


**Appendix: Proofs of the Propositions**

**Proof of Proposition 1**

From the concavity of $Y$ we know $Y'$ is decreasing, so the assumption $Y'(0) > (\beta/\delta)(C/R)$ will imply that $q_B > 0$ from (2). Moreover the concavity of $Y$ implies that

$$Y(0) < Y(q_B) - q_B Y'(q_B),$$

and hence, using $Y(0) > 0$ and (2),

$$\frac{\eta}{\gamma} = \frac{Y(q_B) R}{q_B C} > \frac{Y'(q_B) + Y(0) R}{q_B C} > \frac{\beta}{\delta}.$$

Thus

$$\frac{(\beta - \eta)}{(\delta - \gamma)} < \frac{\beta}{\delta}.$$

So from (3), $Y'(q^*) < Y'(q_B)$. Hence $q^* > q_B$ and $Y(q^*) > Y(q_B)$ since $Y$ is strictly increasing and concave.
Also

\[ 
\Pi_F(q_B) = \Upsilon(q_B)\delta R - q_B\beta C \\
> \Upsilon(0)\delta R + q_B\Upsilon'(q_B)\delta R - q_B\beta C \\
> 0. 
\]

Now notice that the profit made by the grower at \( q_B \) is unchanged under the new contract, since from (4),

\[ 
\bar{\Pi}_F(q_B) = \bar{\Upsilon}(q_B)(\delta - \gamma)R - q_B(\beta - \eta)C \\
= \bar{\Upsilon}(q_B)\delta R - \eta q_B C - q_B(\beta - \eta)C \\
= \bar{\Upsilon}(q_B)\delta R - q_B\beta C = \Pi_F(q_B). 
\]

But \( q^* \) optimizes \( \bar{\Pi}_F(q_B) \) and so \( \bar{\Pi}_F(q^*) \geq \bar{\Pi}_F(q_B) = \Pi_F(q_B) \) with strict inequality following from strict convexity of \( \bar{\Upsilon} \). Thus the grower does better under the new contract.

Now consider case (a) when \( W(q, Y) = \eta q C \), then since the payment from the buyer to the supplier leaves the supplier profit as

\[ 
\Pi_S = q C(\beta - 1) 
\]

under both arrangements, the supplier is better off under the new contract (as \( q^* > q_B \)).

It remains to consider the buyer’s profit. From concavity

\[ 
\bar{\Upsilon}(q^*) - \bar{\Upsilon}(q_B) > (q^* - q_B)\bar{\Upsilon}'(q^*) = (q^* - q_B)\frac{(\beta - \eta)C}{(\delta - \gamma)R}. 
\] (7)

The buyer in the baseline contract achieves an expected profit of

\[ 
\Pi_B = \bar{\Upsilon}(q_B)(1 - \delta)R
\]

and in the new arrangement the buyer has expected profit:

\[ 
\bar{\Pi}_B = \bar{\Upsilon}(q^*)(1 - \delta + \gamma)R - q^*\eta C.
\]

The change in profit is

\[ 
\bar{\Pi}_B - \Pi_B = \bar{\Upsilon}(q^*)(1 - \delta + \gamma)R - q^*\eta C - \bar{\Upsilon}(q_B)(1 - \delta)R \\
= (\bar{\Upsilon}(q^*) - \bar{\Upsilon}(q_B))(1 - \delta + \gamma)R - q^*\eta C + \bar{\Upsilon}(q_B)\gamma R \\
= (\bar{\Upsilon}(q^*) - \bar{\Upsilon}(q_B))(1 - \delta + \gamma)R - (q^* - q_B)\eta C.
\]

Hence from (7) and since \( \delta < 1 \),

\[ 
\bar{\Pi}_B - \Pi_B > (q^* - q_B)C\left(\frac{(\beta - \eta)(1 - \delta + \gamma) - \eta}{(\delta - \gamma)}\right) \\
= (q^* - q_B)C\left(\frac{\beta - \eta - \beta(\delta - \gamma)}{(\delta - \gamma)}\right) > 0
\]

using the assumption (5).

Next we consider case (b) when \( W(q, Y) = \gamma Y R \). Since the payment from the buyer to the supplier leaves the buyer profit as

\[ 
\Pi_S = Y R(1 - \delta)
\]

under both arrangements, the buyer is better off in expectation under the new contract as \( \bar{\Upsilon}(q^*) > \bar{\Upsilon}(q_B) \).

The supplier in the baseline contract has expected profit

\[ 
\Pi_S = q_B(\beta - 1)C
\]
and in the new arrangement this becomes
\[ \bar{\Pi}_S = q^*(\beta - \eta - 1)C + \gamma \bar{Y}(q^*)R. \]

Thus the change in profit is
\[
\bar{\Pi}_S - \Pi_S = q^*(\beta - \eta - 1)C + \gamma \bar{Y}(q^*)R - q_B(\beta - 1)C
\]
\[
= (q^* - q_B)(\beta - 1)C - q^*\eta C + \gamma \bar{Y}(q^*)R.
\]

Using (7) we know
\[
\bar{Y}(q^*) > \bar{Y}(q_B) + (q^* - q_B)\frac{(\beta - \eta)C}{(\delta - \gamma)R} = q_B\frac{\eta C}{\gamma R} + (q^* - q_B)\frac{(\beta - \eta)C}{(\delta - \gamma)R}.
\]

Thus
\[
\bar{\Pi}_S - \Pi_S > (q^* - q_B)(\beta - 1)C - q^*\eta C + \gamma R \left( q_B\frac{\eta C}{\gamma R} + (q^* - q_B)\frac{(\beta - \eta)C}{(\delta - \gamma)R} \right)
\]
\[
= (q^* - q_B)(\beta - 1)C - q^*\eta C + \left( \eta q_B + (q^* - q_B)\frac{\gamma(\beta - \eta)}{(\delta - \gamma)} \right)C
\]
\[
= (q^* - q_B)C \left( \frac{\delta(\beta - \eta) - (\delta - \gamma)}{(\delta - \gamma)} \right) > 0
\]
using the assumption (6). \hfill \Box

**Proof of Proposition 2**

The values of $\gamma$, $\eta$ imply that
\[
\beta - \eta = \frac{\beta(\alpha(q_B) - 1) - (\beta - \delta)\alpha(q_B)}{\alpha(q_B) - 1} = \frac{\delta(\alpha(q_B) - 1) - (\beta - \delta)}{\alpha(q_B) - 1} = \delta - \gamma.
\]

So from (3) we have $q^* = q^*_C$, and the supply chain optimal solution is achieved by the grower’s choice under the double discount contract.

Note that
\[
\eta = \alpha(q_B)\gamma = \frac{\bar{Y}(q_B)R}{q_BC}\gamma.
\]

First consider the growers expected profit at $q_B$ under the new contract:
\[
\bar{\Pi}_F(q_B) = \bar{Y}(q_B)(\delta - \gamma)R - q_B(\beta - \eta)C
\]
\[
= \bar{Y}(q_B)\delta R - q_B\beta C = \Pi_F(q_B).
\]

Thus the optimal choice for the grower, $q^*$, has $\bar{\Pi}_F(q^*) > \bar{\Pi}_F(q_B) = \Pi_F(q_B)$.

Now consider the buyer’s expected profit under the new contract. Since the supply chain profit is maximized at $q^*_C$,
\[
\bar{Y}(q^*_C)R - q^*_C C > \bar{Y}(q_B)R - q_BC.
\]

Hence, as $(1 - \delta + \gamma) = (1 - \beta + \eta) > 0$ this implies
\[
(1 - \beta + \eta)(q^*_C - q_B)C < (1 - \delta + \gamma)(\bar{Y}(q^*_C) - \bar{Y}(q_B)).
\]

Thus, using $\eta q_B C = \gamma \bar{Y}(q_B)R$, we have
\[
(\eta q^*_C + (1 - \beta)(q^*_C - q_B))C = \eta q_B C + (1 - \beta + \eta)(q^*_C - q_B)C
\]
\[
\leq \gamma \bar{Y}(q_B)R + (1 - \delta + \gamma)(\bar{Y}(q^*_C) - \bar{Y}(q_B))R
\]
\[
= (\gamma \bar{Y}(q^*_C) + (1 - \delta)(\bar{Y}(q^*_C) - \bar{Y}(q_B)))R. \tag{8}
\]
Thus
\[ \tilde{\Pi}_B(q_C^*) = \mathbb{V}(q_C^*)(1 - \delta + \gamma)R - \mathbb{E}[W(q_C^*, Y(q_C^*, \varepsilon))] \]
\[ = \mathbb{V}(q_C^*)(1 - \delta + \gamma)R - \mu(q_C^* + (1 - \beta)(q_C - q_B))C - (1 - \mu)(\gamma \mathbb{V}(q_C^*) + (1 - \delta)(\mathbb{V}(q_C^*) - \mathbb{V}(q_B)))R \]
\[ \geq \mathbb{V}(q_C^*)(1 - \delta + \gamma)R - (\gamma \mathbb{V}(q_C^*) + (1 - \delta)(\mathbb{V}(q_C^*) - \mathbb{V}(q_B)))R \]
\[ = \mathbb{V}(q_B)(1 - \delta)R. \]

Also, again using (8),
\[ \tilde{\Pi}_S(q_C^*) = q_C^*(\beta - \eta - 1)C + \mathbb{E}[W(q_C^*, Y(q_C^*, \varepsilon))] \]
\[ = q_C^*(\beta - \eta - 1)C + \mu(q_C^* + (1 - \beta)(q_C - q_B))C + (1 - \mu)(\gamma \mathbb{V}(q_C^*) + (1 - \delta)(\mathbb{V}(q_C^*) - \mathbb{V}(q_B)))R \]
\[ \geq q_C^*(\beta - \eta - 1)C + (\eta q_C^* + (1 - \beta)(q_C - q_B))C \]
\[ = q_B(\beta - 1)C. \]

Hence we have established that all the participants in the supply chain do at least as well under this double discount policy. \( \square \)

**Proof of Proposition 3**

We prove the result for a discrete distribution of weather variables \( \varepsilon \in \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_M\} \) where \( \varepsilon_i \) has probability \( p_i \). The argument is easily generalized to an arbitrary random variable. Observe from the definition of \( q_B \) that
\[ E_q[Y'(q_B, \varepsilon)\delta R - \beta C] = \sum_{i=1}^{M} p_i \left( \delta R \frac{\partial}{\partial q} Y(q_B, \varepsilon_i) - \beta C \right) = 0. \]

We let \( Z = \{ i : \delta R \frac{\partial}{\partial q} Y(q_B, \varepsilon_i) > \beta C \} \), then
\[ \sum_{i \in Z} p_i \left( \delta R \frac{\partial}{\partial q} Y(q_B, \varepsilon_i) - \beta C \right) = \sum_{i \notin Z} p_i \left( \beta C - \delta R \frac{\partial}{\partial q} Y(q_B, \varepsilon_i) \right). \]
\[ Y(q, \varepsilon_1) < Y(q, \varepsilon_2) \implies \frac{\partial}{\partial q} Y(q, \varepsilon_1) < \frac{\partial}{\partial q} Y(q, \varepsilon_2). \]

Now since \( Y \) is divergent, and \( Z \) is the set of indices with \( \frac{\partial}{\partial q} Y(q_B, \varepsilon_i) > \beta C / (\delta R) \) we deduce that for any \( i \in Z \) and \( j \notin Z \), \( Y(q_B, \varepsilon_i) > Y(q_B, \varepsilon_j) \), so
\[ \delta R Y(q_B, \varepsilon_i) - q_B \beta C > \delta R Y(q_B, \varepsilon_j) - q_B \beta C. \]

Hence we may find a constant \( k_0 \) with
\[ 0 < U'(\delta R Y(q_B, \varepsilon_i) - q_B \beta C) < k_0 \] for \( i \in Z \);
\[ k_0 < U'(\delta R Y(q_B, \varepsilon_j) - q_B \beta C) \] for \( j \notin Z \).

Noting that all the terms in (9) are positive we have
\[ \sum_{i \in Z} p_i U'(\delta R Y(q_B, \varepsilon_i) - q_B \beta C) \left( \delta R \frac{\partial}{\partial q} Y(q_B, \varepsilon_i) - \beta C \right) \]
\[ < k_0 \sum_{i \in Z} p_i \left( \delta R \frac{\partial}{\partial q} Y(q_B, \varepsilon_i) - \beta C \right) \]
\[ = k_0 \sum_{i \notin Z} p_i \left( \beta C - \delta R \frac{\partial}{\partial q} Y(q_B, \varepsilon_i) \right) \]
\[ < \sum_{i \notin Z} p_i U' \left( \delta R Y(q_B, \varepsilon_i) - q_B \beta C \right) \left( \beta C - \delta R \frac{\partial}{\partial q} Y(q_B, \varepsilon_i) \right). \]
So $E[\frac{\partial}{\partial q} U(\Delta RY(q_B, \varepsilon_i) - q_B \beta C)] < 0$ and in maximizing the utility $U(\Delta RY(q, \varepsilon_i) - q \beta C)$ we will choose a $q$ value less than $q_B$.

To establish that the supply chain benefit available from the new contract is greater when the grower is risk averse, we show that a supply chain optimal solution can be obtained in the risk averse case, and that the solution under the baseline case is worse than when the grower is risk neutral. The latter statement follows from observing that the supply chain profits $Y(q)R - qC$ are a concave function of $q$ maximized at $q_C^\ast$. Since we have shown $q_U < q_B < q_C^\ast$ the risk averse baseline case corresponding to $q_U$ achieves lower supply chain profits than $q_B$.

Now consider the case that $\eta = \beta$, so the grower obtains the input for free. Now $U'$ is decreasing in $Y(q_C^\ast, \varepsilon)$ and $U'(q_C^\ast, \varepsilon)$ is decreasing in $Y(q_C^\ast, \varepsilon)$. Suppose that for $Y(q_C^\ast, \varepsilon) > y_0$ we have $Y'(q_C^\ast, \varepsilon) < 0$ so $U'(Y(q_C^\ast, \varepsilon)(\delta - \gamma)R)Y'(q_C^\ast, \varepsilon) > U'(y_0(\delta - \gamma)R)Y'(q_C^\ast, \varepsilon)$, for $Y(q_C^\ast, \varepsilon) > y_0$. But when $Y(q_C^\ast, \varepsilon) \leq y_0$ then $Y'(q_C^\ast, \varepsilon) \geq 0$ and $U'(Y(q_C^\ast, \varepsilon)(\delta - \gamma)R)Y'(q_C^\ast, \varepsilon) > U'(y_0(\delta - \gamma)R)Y'(q_C^\ast, \varepsilon)$. Thus

$$E_\varepsilon[U'(Y(q_C^\ast, \varepsilon)(\delta - \gamma)R)(Y'(q_C^\ast, \varepsilon)(\delta - \gamma)R)] \geq U'(y_0(\delta - \gamma)R)E[|Y'(q_C^\ast, \varepsilon)|(\delta - \gamma)R]$$

$$= U'(y_0(\delta - \gamma)R)(\delta - \gamma)C > 0.$$

Hence the expected utility is increasing at $q_C^\ast$ when $\eta = \beta$, and so the (risk averse) grower selects a value greater than $q_C^\ast$.

Now consider slowly varying $\eta$ and $\gamma$ from zero (corresponding to the baseline case) allowing $\eta$ to increase to $\beta$. We assume that $Y'(q, \varepsilon)$ is continuous for each $\varepsilon$, and $U$ is concave, differentiable and hence has a continuous derivative, and this is enough to ensure that the optimal choice of $q$ as a function of $\eta$ and $\gamma$ is continuous. Thus by the intermediate value theorem there are values of $\eta$ and $\gamma$ that will result in the grower choosing the supply chain optimal quantity in the risk averse case. □