Optimal supply chain resilience with consideration of failure propagation and repair logistics

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Abstract

The joint optimisation of investments in capacity and repair capability of production and logistics systems at risk of being damaged is an important aspect of supply chain resilience that is not sufficiently addressed by state-of-the-art modelling approaches. Furthermore, logistical issues of procuring repair resources impact speed of recovery but are not considered in most existing models. This paper presents a novel multi-stage stochastic programming model that optimizes pre-disruption investment decisions, as well as post-disruption dynamic adjustment of supply chain operations and allocation of repair resources. A case study demonstrates how the method can quantify the effects of pooling repair resources.

Keywords:
- supply chain networks
- resilience
- interdependency
- network flow modelling
- stochastic programming
- input-output modelling

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1. Introduction

Global sourcing, just-in-time logistics and lean production methods keep costs low in stable business environments but can increase risks and uncertainties when disruptions occur (Fiksel et al., 2015). Traditional supply chain risk management (SCRM) methods focus on identifying, avoiding and controlling risks (Heckmann et al., 2015; Ho et al., 2015). However, rising complexity, interconnectedness and uncertainty make the application of such methods increasingly difficult (Simichi-Levi et al., 2013). Furthermore, global efforts to reduce the environmental impact of supply chains have been found to also make them more sensitive to disruptions (Fahimnia et al., 2018).

In response, supply chain resilience planning seeks to minimize the overall disruption impacts by achieving a balance of robustness, redundancy and recovery capabilities (Ponomarov and Holcomb, 2009; Tukamuhabwa et al., 2015; Kamalahmadi and Mellat Parast, 2016). The increasing importance of strengthening the resilience of supply chains is also recognised by international organisations, such as the World Economic Forum (2013) and the International Transport Forum (2018). When operationalising resilience concepts, it is important to critically discuss “resilience of what, to what, for whom and over what time frame” (Helfgott, 2018).

Recent examples of major supply chain disruptions are presented in Table 1, which also illustrates how their impacts can be exacerbated by interdependencies between different systems. Such interdependencies can give rise to cascading failure, as observed, for example, in the chain of events that was triggered by a lightning strike in New Mexico, USA, in the year 2000. The resulting power fluctuations in the electricity grid lead to the failure of electrical equipment and a small fire at a factory for radio frequency chips.
Without power supply, the ventilation system was disrupted and smoke contaminated the facility’s clean rooms. The factory was a supplier to Ericsson (the mobile phone producer), for which the disrupted supply of radio frequency chips resulted in USD 200m costs of business interruption (Norrman and Jansson, 2004).

The 2011 Thailand flooding event illustrates that the geographic concentration of important supply chain capacities is a risk factor because spatial proximity means shared hazard exposure and correlated asset failure probabilities. The flooding affected more than 800 factories, among them some of the world’s largest producers of hard disk drives. Their production output decreased by 30 %, resulting in price increases of 80 to 190 % affecting the entire global IT value chain (Haraguchi and Lall, 2015).

Functional dependencies occur if one process requires an output from another, with the consequence of inadequate operation if the supply of the

<table>
<thead>
<tr>
<th>Trigger event (country, year)</th>
<th>Direct damage</th>
<th>Inter-dependency effects</th>
<th>Supply chain impacts</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lightning strike (USA, 2000)</td>
<td>power grid</td>
<td>electrical fire, clean room contamination, supply disruptions</td>
<td>Ericsson mobile phone production disrupted for months (USD 200m losses)</td>
<td>Norrmann and Jansson (2004)</td>
</tr>
<tr>
<td>Earthquake (Japan, 2011)</td>
<td>infrastructure, factories</td>
<td>supply disruptions</td>
<td>disruption of many industry sectors (e.g. 2-week closure of all Toyota plants)</td>
<td>Matsuo (2015)</td>
</tr>
<tr>
<td>Flooding (Thailand, 2011)</td>
<td>more than 800 factories</td>
<td>supply disruptions, co-location</td>
<td>USD 40bn damages, major loss of global HDD production capacity, increase of computer prices</td>
<td>Haraguchi and Lall (2015)</td>
</tr>
<tr>
<td>Hurricane Sandy (USA, 2012)</td>
<td>power grid, oil terminals, refineries, pipelines</td>
<td>recovery delay due to power and transport disruptions</td>
<td>fuel shortage at gas stations</td>
<td>The City of New York (2013)</td>
</tr>
</tbody>
</table>
required product or service is disrupted. For example, the 2011 Japan earthquake caused a two-week shutdown of all Toyota factories, even though most escaped damage. However, just-in-time production planning meant that they were immediately affected by disruptions in the supply of parts from other factories (Matsuo, 2015).

Functional dependencies can also delay recovery processes. This was observed, for example, after hurricane Sandy when the liquid fuel supply chain in New York City was disrupted due to flood damage to terminals, refineries and pipelines. Recovery required electric power supply to operate pumps and other equipment, and was, therefore, dependent on the restoration of the electric power network or provision of back-up power supply (The City of New York, 2013).

Empirical research by Ambulkar et al. (2015) and de Sá et al. (2019) suggests that firms can become more resilient against supply chain disruptions by investing in dedicated recovery resources and fostering the ability to reconfigure resource allocation. To plan such interventions, quantitative modelling, simulation and optimisation methods are needed.

Different modelling approaches have been developed in this field, using methods such as network theory (Geunes and Pardalos, 2003), system dynamics (Wilson, 2007), input-output modelling (Pant et al., 2011), agent-based simulation (Hou et al., 2018), queuing models (Gillen and Hasheminia, 2016), quality function deployment (Lam and Bai, 2016), cellular automata (Chen et al., 2015), game theory (Nagurney et al., 2016) and control theory (Ivanov and Sokolov, 2019). Such methods have been used to analyse pre-disruption mitigation decisions (e.g. supply chain network design, capacity investments) and post-disruption operational decisions (e.g. re-routing, adjustment of production). However, relatively few publications consider
optimal recovery strategies, which entail planning the repair of damaged assets and the logistics of providing repair resources.

This paper proposes a novel optimisation approach that considers three types of decisions: capacity planning, operational adjustments and recovery strategies. By considering these three decision domains in a multi-stage stochastic optimisation model, the proposed methodology is capable of analysing important-trade-offs (e.g. between investing in spare production capacity and repair capability) in greater detail compared to previous research. Another contribution of this paper is a generic formulation of two types of dependency relations (failure propagation and functional dependency), which enables the analysis of interdependency effects observed in the above-mentioned cases of global supply chain disruptions (correlated or cascading asset failure, unavailable production inputs or repair resources).

The remainder of this paper is structured as follows: Section 2 reviews existing network flow modelling methods for supply chains. Section 3 presents our modelling framework that considers supply chain assets as repairable systems and introduces the concept of repair resource networks. Section 4 presents a scenario tree generation method that uses input-output modelling to take into account the risk of failure propagation. Section 5 presents the mathematical formulation of the multi-stage stochastic programming model and Section 6 the application of the model to a real-world supply chain example.

2. Previous work on resilient supply chain network planning

A comprehensive review of quantitative supply chain resilience models is provided by Hosseini et al. (2019). In the following, we focus on three issues that are particularly relevant for the methodology proposed in our paper:
i) types of risk, ii) decision variables in optimisation models, and iii) dynamic modelling methods. Key characteristics of selected state-of-the-art models are summarised in Table 2.

2.1. Types of risk

The literature on supply chain planning addresses uncertainties of demand, capacity and costs (e.g. Santoso et al., 2005; Azaron et al., 2008; Mohammadi Bidhandi and Mohd Yusuff, 2011). These uncertainties are usually represented by a set of discrete scenarios. Two-stage models assume that the value of all uncertain parameters is revealed at the same time. Multi-stage models, on the other hand, consider multiple points in time at which the value of uncertain parameters becomes known and recourse decisions can be taken.

Interdependency entails that the probability distribution of parameters for one part of the supply chain may be dependent on the value of uncertain parameters for other parts. This can lead to instances of cascading failure or ripple effects (Dolgui et al., 2018). Masih-Tehrani et al. (2011) and Li et al. (2013) analyse such interdependencies arising from the correlation of capacity losses in different facilities. Rezapour et al. (2015) model the propagation of supply- and demand-side uncertainties, providing a method for calculating optimal service levels. A dynamic model with correlated disruptions is presented in Hasani and Khosrojerdi (2016).

However, the above-mentioned models do not address the issue of supply chain recovery. In Goldbeck et al. (2017), interconnected Markov chains were used to model failure as well as repair of supply chain assets, but this model treats recovery as a random process and does not consider the optimal scheduling of repair works and allocation of repair resources.
2.2. Decision variables

Pre-disruption decisions considered in existing models address different network design aspects, such as facility location, optimal capacities and asset protection. Post-disruption operational decisions include adjusting production rates, substituting suppliers, re-allocating customers, and re-routing vehicles.

Chen et al. (2017) and Zhalechian et al. (2018) also consider the problem of where to prioritize repair, given constraints such as an overall repair budget. This is important because the balance of asset protection and contingency planning is a key resilience aspect. However, these papers do not model the efficient management of repair resources, for example, staff, equipment, spare parts, which is a problem of great practical relevance for a quick recovery.

Pre-disruption decisions related to the management of repair resources can be seen as investments in repair capabilities, for example, hiring staff, agreeing repair service contracts and warehousing spare parts. Post-disruption decisions include the allocation of resources and scheduling of repair tasks. These should also take into account interdependencies, such as the optimal sequencing of repair task or the accessibility of damaged assets through transportation networks that may also be affected by the disruption. Such issues are not considered in the current modelling methods.

2.3. Dynamic flows

Most reviewed models use two-stage stochastic programming to model pre- and post-disruption decisions. However, such models cannot capture more intricate dynamic effects, for example, progressive escalation of cascading failure or scheduling of sequential and interdependent repair tasks.
Dynamic models with more than two time steps are presented, for example, by Hasani and Khosrojerdi (2016), Parajuli et al. (2017) and Ghavamifar et al. (2018). Such models are computationally difficult to solve and therefore of limited scalability, especially when they contain integer decision variables and non-linear equations. Furthermore, they do not consider repair. Nurre et al. (2012) present a dynamic network flow model including optimal repair scheduling but it is formulated for infrastructure and not supply chain networks.

Glockner and Nemhauser (2000) show that dynamic flows under uncertain network capacity can also be modelled using linear multi-stage stochastic programming models, which are easier to solve. We follow this approach in order to develop a model that can be used to analyse simulation periods with more than 100 time steps and 100 scenarios. This could not be achieved with integer programming approaches due to computational limitations.
<table>
<thead>
<tr>
<th>Publication</th>
<th>Description</th>
<th>Types of risk</th>
<th>Decisions</th>
<th># time steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Khaled et al. (2015)</td>
<td>Evaluation of freight railroad criticality using block-to-train assignment and train routing methods</td>
<td>✓ ✓</td>
<td>✓</td>
<td>1</td>
</tr>
<tr>
<td>Masih-Tehrani et al. (2011)</td>
<td>Single-period analysis of a two-echelon inventory system with dependent supply uncertainty</td>
<td>✓ ✓</td>
<td>✓</td>
<td>1</td>
</tr>
<tr>
<td>Salehi Sadghiani et al. (2015)</td>
<td>Optimizing facility location and coverage plans under disruption uncertainty</td>
<td>✓ ✓</td>
<td>✓</td>
<td>1</td>
</tr>
<tr>
<td>Li et al. (2013)</td>
<td>Reliable infrastructure location design under interdependent disruptions</td>
<td>✓ ✓</td>
<td>✓</td>
<td>1</td>
</tr>
<tr>
<td>Rezapour et al. (2015)</td>
<td>Optimal service levels under consideration of uncertainty propagation</td>
<td>✓ ✓ ✓</td>
<td>✓</td>
<td>1</td>
</tr>
<tr>
<td>Tsiakis et al. (2001)</td>
<td>Design of multi-echelon supply chain networks under demand uncertainty</td>
<td>✓ ✓</td>
<td>✓</td>
<td>2</td>
</tr>
<tr>
<td>Lu et al. (2011)</td>
<td>Product substitution and dual sourcing under random supply failures</td>
<td>✓ ✓</td>
<td>✓</td>
<td>2</td>
</tr>
<tr>
<td>Cui et al. (2016)</td>
<td>Optimal design of a two-echelon supply chain with different logistics services</td>
<td>✓ ✓</td>
<td>✓</td>
<td>2</td>
</tr>
<tr>
<td>Fahimnia and Jabbarzadeh (2016); Fahimnia et al. (2018)</td>
<td>Joint optimization of robustness and environmental impacts</td>
<td>✓ ✓ ✓</td>
<td>✓</td>
<td>2</td>
</tr>
<tr>
<td>Jabbarzadeh et al. (2016)</td>
<td>Robust optimization considering different types of risk</td>
<td>✓ ✓</td>
<td>✓</td>
<td>2</td>
</tr>
<tr>
<td>Zhalechian et al. (2018)</td>
<td>Bi-objective optimization of cost and resilience</td>
<td>✓ ✓</td>
<td>✓</td>
<td>2</td>
</tr>
<tr>
<td>Chen et al. (2017)</td>
<td>Resilience assessment for port-hinterland container transportation networks</td>
<td>✓ ✓</td>
<td>✓</td>
<td>2</td>
</tr>
<tr>
<td>Ghavamifar et al. (2018)</td>
<td>Design of a resilient competitive supply chain network</td>
<td>✓ ✓</td>
<td>✓</td>
<td>&gt;2</td>
</tr>
<tr>
<td>Parajuli et al. (2017)</td>
<td>Supply chain contingency planning using an attacker-defender model</td>
<td>✓ ✓</td>
<td>✓</td>
<td>&gt;2</td>
</tr>
<tr>
<td>Hasani and Khosrojerdi (2016)</td>
<td>Analysis of six resilience strategies to mitigate the risk of correlated disruptions</td>
<td>✓ ✓ ✓</td>
<td>✓</td>
<td>&gt;2</td>
</tr>
</tbody>
</table>
2.4. Summary of research gaps

State-of-the-art supply chain resilience models feature optimisation methods for capacity investments, operational planning and incident response. However, our review confirms the results of Hosseini et al. (2019) that the current literature does not sufficiently exploit the potential of mathematical programming approaches to optimise resilience-related decisions in dynamic and stochastic modelling frameworks.

Some publications specifically address the issue of interdependency risks (Masih-Tehrani et al., 2011; Li et al., 2013; Rezapour et al., 2015; Hasani and Khosrojerdi, 2016). However, the current literature falls short of providing a comprehensive model for optimal supply chain recovery under consideration of interdependency. Those models that consider optimal asset repair (Chen et al., 2017; Zhaelechian et al., 2018) do not capture the logistics of repair resources, which are likely to be affected by interdependency effects. Moreover, current models assume fixed repair budgets and cannot optimise pre-disruption investments in the supply chain’s recovery capability, a key resilience aspect.

In this paper, we address these limitations by proposing a multi-stage optimisation method for initial investments in network capacity and subsequent dynamic network flows. We show that the logistics of repair resources can be modelled in a similar way as the core supply chain logistics. This approach enables the proposed method to provide more comprehensive decision support for improving the resilience of interdependent supply chains.
3. Model overview

We consider a directed graph $G = (V, E)$, with disjoint sub-graphs representing the flow of different commodities. Nodes represent production, storage and demand locations, while links represent processes over time and space, such as assembly or transportation steps. The combined set $V \cup E$ is referred to as the set of supply chain assets.

The uncertainty considered in this paper stems from interdependent failure events that can reduce the capacity of individual supply chain assets. Interdependent failure events could be triggered internally (e.g. by technical failures) or externally (e.g. by environmental hazard events or malicious attacks). The initial impact of such disruptions is modelled with probability distributions for first-order asset damage. Higher-order or cascading failure is also considered, by specifying probability distributions for the propagation of asset damage (e.g. between co-located assets).

Commodities in the supply chain can be raw materials, intermediate products or final products in the supply chain. Furthermore, they can include resources that are required to repair damaged supply chain assets. Sub-graphs of $G$ that model the flow of these repair resources are referred to as repair resource networks. Supply chain commodities and repair resources can be tangible (e.g. raw materials) or intangible (e.g. services, labour, energy, information).

3.1. Problem statement

The problem addressed in this paper is to minimise the expected costs of the supply chain (comprising cost of capacity provision, operational costs and penalties for unmet demand and service levels) for a period of $T$ time steps and a set of asset failure scenarios. The following decisions are con-
sidered in the optimisation problem:

- investments in production, storage and transportation capacities in the supply chain
- investments in repair capabilities
- planning of supply chain operations, including production rates, inventory levels and commodity transportation
- supply chain recovery strategies, including the procurement of repair resources and their allocation and transportation to damaged supply chain assets

This optimisation problem can also be described as finding the optimal initial capacity configuration for $G$ and optimal dynamic flows for all asset failure scenarios.

3.2. Scenario generation and stochastic programming model

Figure 1 illustrates the two basic steps of our modelling approach. The first step is the generation of a scenario tree that captures the uncertainty of asset failure over time steps $t = 1 \ldots T$. For each time step, the nodes of the scenario tree are indexed with $s = 1 \ldots S_t$, with $S_1 = 1$ and $S_{t+1} \geq S_t$. The total number of asset failure scenarios is $S_T$, which is the number of leaves in the scenario tree.

A scenario is a path through the scenario tree from the root node to a leave node. Scenarios are defined by inoperability variables $x_{i}^{t,s}$ and $x_{j,k}^{t,s}$. They denote the fraction of capacity that is lost at nodes and links in time steps $t = 1 \ldots T$ due to asset failures. For example, $x_{i}^{s,t} = 0.5$ means that in all scenarios that include scenario tree node $s$ in time step $t$, an asset failure occurs at node $i$ and reduces its production and storage capacity by 50%.
The second step involves the formulation of a stochastic programming model with \( T+1 \) stages. In stage \( t=0 \), decisions are taken regarding the initial capacities in the combined supply chain and repair resource network \( G \). In stages \( t=1\ldots T \), the model seeks to optimize link flows, production rates, storage and capacity repair, under the constraints of global capacity limits and scenario-specific capacity losses.

### 3.3. Variables

An overview of all model variables is presented in Table 3. Scenario-specific variables are indexed with super-scripts \( t, s \), denoting a scenario tree node. Scenarios are defined by the set of stochastic inoperability variables \( x_{i}^{t,s} \) and \( x_{j,k}^{t,s} \), as described in the previous section.

Decision variables \( u_{i}^{t,s} \) indicate the commodity amount that is supplied to end-users at node \( i \) in time step \( t \) and scenario \( s \). The amount by which this supply falls below end-user demands is captured with slack variables \( r_{i}^{t,s} \).
Table 3: Model variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global decision variables</td>
<td></td>
</tr>
<tr>
<td>$\bar{f}_{j,k}$</td>
<td>Nominal link flow capacities</td>
</tr>
<tr>
<td>$\bar{g}_{i}$</td>
<td>Nominal production capacities</td>
</tr>
<tr>
<td>$\bar{h}_{i}$</td>
<td>Nominal storage capacities</td>
</tr>
<tr>
<td>Stochastic variables</td>
<td></td>
</tr>
<tr>
<td>$x_{t,s,j,k}$</td>
<td>Damage to links</td>
</tr>
<tr>
<td>$x_{t,s,i}$</td>
<td>Damage to nodes</td>
</tr>
<tr>
<td>Time- and scenario-dependent decision variables</td>
<td></td>
</tr>
<tr>
<td>$\tilde{f}_{t,s,j,k}$</td>
<td>Available link flow capacities</td>
</tr>
<tr>
<td>$\tilde{f}_{t,s,j,k}$</td>
<td>Repair of link flow capacities</td>
</tr>
<tr>
<td>$f_{t,s,j,k}$</td>
<td>Link flows</td>
</tr>
<tr>
<td>$\tilde{g}_{t,s,i}$</td>
<td>Available production capacities</td>
</tr>
<tr>
<td>$\tilde{g}_{t,s,i}$</td>
<td>Repair of production capacities</td>
</tr>
<tr>
<td>$g_{t,s,i}$</td>
<td>Production rates</td>
</tr>
<tr>
<td>$\tilde{h}_{t,s,i}$</td>
<td>Available storage capacities</td>
</tr>
<tr>
<td>$\tilde{h}_{t,s,i}$</td>
<td>Repair of storage capacities</td>
</tr>
<tr>
<td>$h_{t,s,i}$</td>
<td>Inventory levels</td>
</tr>
<tr>
<td>$u_{t,s,i}$</td>
<td>Supply to end-users</td>
</tr>
<tr>
<td>$v_{t,s,i}$</td>
<td>Supply to dependent assets</td>
</tr>
<tr>
<td>$q_{t,s,i}$</td>
<td>Supply shortfalls below service level targets</td>
</tr>
<tr>
<td>$r_{t,s,i}$</td>
<td>Supply shortfalls below demand</td>
</tr>
</tbody>
</table>

Another set of slack variables, $q_{t,s,i}$, captures the shortfall of supply below service level targets. In addition to supplying end-users, nodes can also supply commodities to other nodes and links, the amount of which is indicated with variables $v_{t,s,i}$.

Link flows are denoted by $f_{t,s,j,k}$, production rates by $g_{t,s,i}$, and inventory levels by $h_{t,s,i}$. For these supply chain operations, there exist capacity limits, which are also included in the decision variable of the stochastic programming model, because they are impacted by the repair strategies.

We distinguish between nominal and available capacity. Nominal ca-
capacities $\bar{f}_{j,k}$, $\bar{g}_i$, and $\bar{h}_i$ are fixed for the entire simulation period and all scenarios, whereas available capacities $\tilde{f}_{j,k}^t$, $\tilde{g}_i^t$, and $\tilde{h}_i^t$ can change over time due to scenario-dependent damage and repair. Repair processes are modelled with decision variables $\hat{f}_{j,k}^t$, $\hat{g}_i^t$, and $\hat{h}_i^t$, measuring the amount of capacity that is restored through repair during time step $t$ in scenario $s$.

The change of available capacity over time through damage and repair is captured in the following equation for updating available link flow capacities:

$$\tilde{f}_{j,k}^t = (1 - x_{j,k}^t) \left( \tilde{f}_{j,k}^{\Delta_t,1} + \hat{f}_{j,k}^{\Delta_t,1} \right)$$

The predecessor function $\Delta_t : t \mapsto (t_{\text{pred}}, s_{\text{pred}})$ outputs the scenario tree node $t_{\text{pred}}, s_{\text{pred}}$ that is a predecessor to $t, s$ with distance $\delta_t$ time steps. Thus, $\tilde{f}_{j,k}^{\Delta_t,1} + \hat{f}_{j,k}^{\Delta_t,1}$ is the available capacity plus capacity repair in $t - 1$, which is multiplied with the damage factor $(1 - x_{j,k}^t)$ to obtain the available capacity in $t$. Production and storage capacities are updated in the same way. Figure 2 depicts two example scenarios in which a node partially loses its production capacity and is repaired over multiple time steps.

The proposed model of nominal and available capacity is appropriate for the purpose of this paper because it provides a generic method for representing the variable capacity of production, storage and transportation facilities. Moreover, it enables the model to capture different types of capacity fluctuations, for example, variable throughput rates, increased processing times or intermittent asset availability.

3.4. Dependency relations

We consider two generic types of dependency relations in this paper. Failure propagation dependencies mean that the probability distributions for the stochastic asset inoperability variables $x_{i}^{t,s}$ and $x_{j,k}^{t,s}$ is dependent on the
failure of other assets. Thus, failure propagation impacts the scenario tree generation. It is modelled using an input-output approach (ref. Section 4).

Functional dependencies, on the other hand, mean that supply chain operations (production, storage and transportation of commodities) and asset repairs require commodities provided by other systems as inputs. These dependency relations are modelled as constraints in the optimisation problem that is used to calculate dynamic network flows (ref. Section 5). They lead to a coupling of flows in different sub-graphs of $G$.

**3.5. Repair resource networks**

Repair resource networks model the allocation and transportation of equipment, spare parts, specialized staff and similar repair resources to damaged supply chain assets. Note that the distinction between repair resource networks and the core supply chain is not always unequivocal, as some commodities may be required both as repair resources and regular production inputs.

The level of disaggregation in modelling repair resources depends on the specific model application and availability of data. In the most aggregated
case, one (abstract) commodity serves as a proxy for all repair resources. The granularity of the model can be increased by modelling different repair resources and infrastructure through which they are provided, for example, suppliers, depots, and transportation networks.

Modelling repair resource networks as sub-graphs in $G$ means that the optimization of network capacities as well as flows, production rates, storage and repair rates also applies to them. Hence, the proposed model considers the same logistical aspects for repair resources as it does for supply chain commodities. This enables the model to capture important resilience aspects, including the pre-disruption optimization of repair capabilities. Post-disruption, the model captures optimal deployment of repair resources, recovery delays caused by shortages of staff, equipment or parts, and potential damage to the repair infrastructure itself.

### 3.6. Resilience measures

While there is no standard measure for supply chain resilience, there are many approaches for quantifying general system resilience (Hosseini et al., 2016). The area of the resilience loss triangle (RLT) is one of the most widely used measures because it captures various resilience aspects, including robustness, redundancy, recovery time and resourcefulness (Bruneau et al., 2003). It is calculated as

$$
\text{RLT} = \int_{t=0}^{T} (1 - Q(t)) \, dt
$$

where $Q(t)$ is a system performance function with values between 0 and 1.

In this paper, we use the RLT metric as main resilience measure and quantify the performance of a supply chain based on its ability to meet end-user demands, given by parameters $\bar{v}_i^t$ for nodes $i \in V$. As a means of
weighting demands for different commodities at different locations, we introduce parameters $c^i_r$ for the economic value of satisfying one unit of demand at node $i$. These parameters should be at least the company’s revenue and could also include, for example, contractual penalties or monetized estimates of reputation losses for non-fulfilment.

The supply to end-users is denoted with variables $u^{t,s}_i$. Using discretisation for scenario tree nodes with probabilities $p^{t,s}$, we can calculate the expected RLT as follows:

$$\text{RLT} = \sum_{t=1}^{T} \sum_{s=1}^{S_t} p^{t,s} \left( 1 - \frac{\sum_{i \in V} c^i_r u^{t,s}_i}{\sum_{i \in V} c^i_r \tilde{u}^t_i} \right)$$

The proposed model considers the effect that supply chain disruptions often lead to increased unit costs in so far as it will leave a demand unsatisfied if the unit costs exceed the economic value of fulfilling it. Cost increases that merely reduce profit do not affect the RLT measure, but such effects could be analysed separately.

4. Scenario tree generation using input-output modelling

A scenario tree generation algorithm that captures asset damage uncertainty has to consider the risk of cascading failure, represented by failure propagation dependencies. If the network of such dependencies does not include cycles, failure probabilities can be calculated recursively. However, some of the most prevalent failure propagation dependencies are bi-directional, for example, those related to spatial proximity. Therefore, we use a method based on the inoperability input-output model (Haimes and Jiang, 2001), which is capable of modelling bi-directional and cycling dependencies.
The inoperability input-output model represents the risk of inoperability for individual systems with a vector $x$, measuring “the joint effect of the probability (likelihood) and degree (percentage) of the inoperability” (Haimes and Jiang, 2001). The central formula is

$$x = Ax + c$$

(4)

where $A$ is the interdependency matrix describing the degree of dependency between systems, and $c$ is the perturbation vector describing inherent or independent inoperability. Assuming $(I - A)$ is non-singular, the equation can be solved for $x$:

$$x = (I - A)^{-1}c$$

(5)

Input-output modelling has been used in several publications to investigate supply chain resilience. For example, Santos and Haimes (2004) analyse the propagation of demand perturbations among US industry sectors. Barker and Santos (2010) use a dynamic version of the model to evaluate how inventory policies affect economic resilience to productivity degradation after disasters. Pant et al. (2014) analyse the resilience of interdependent infrastructure and industry sectors.

4.1. Adaptations of the input-output method

The above-cited applications of the input-output method consider interdependencies between entire industries, whereas we consider interdependency between individual supply chain assets. For ease of notation, we concatenate variables $x_{i,s}^{t,s}$ for damage to nodes and $x_{j,k}^{t,s}$ for damage to links into combined inoperability vectors $x^{t,s} \in [0,1]^{|V|+|E|}$.

Typically, input-output models abstract from different types of interdependency and assume that the matrix $A$ captures their overall effect. In
our hybrid modelling framework, however, functional dependencies are captured more realistically by the network flow model because it takes into account operational aspects, such as capacity and cost. Therefore, we use the input-output model only for dependency relations that cannot be modelled as network flows, namely stochastic failure propagation or correlation effects.

Another adaptation is that our model distinguishes between the probability and degree of inoperability so that it can be used for generating random samples rather than calculating expected outcomes. This is achieved by considering the perturbation vector $c$ and interdependency matrix $A$ to be composed of random variables. The probability that a random variable in $c$ or $A$ is greater than zero corresponds to the probability of initial damage or failure propagation, respectively. The observed value in a simulation experiment is the severity of the corresponding effect.

Random variables in $c$ and $A$ can be discrete, continuous or mixed-type. The choice of probability distributions depends on the method for quantifying the risks of initial damage and failure propagation, for example, by statistical analysis of historical data or by elicitation of subject matter expertise.

4.2. Sampling method

To generate a scenario tree, we begin with sampling $n$ time series of perturbation vectors and interdependency matrices, denoted with $c^{t,s}$ and $A^{t,s}$ for $t = 1 \ldots T$ and $s = 1 \ldots n$. Using Equation (5), we calculate the corresponding $n$ inoperability vector time series:

$$x^{t,s} = (I - A^{t,s})^{-1} c^{t,s}$$ (6)
Each scenario has a probability of $1/n$. From the set of $n$ scenarios, we create a scenario tree by iterating through time steps $t_c = 1 \ldots T$ and grouping scenarios that have identical inoperability vectors for all $t = 1 \ldots t_c$. These scenario groups are the nodes of the scenario tree, indexed $s = 1 \ldots S_t$, with their probabilities $p^{t,s}$ calculated as the number of scenarios in the group divided by $n$.

An important question when applying the proposed model is how to set the value for the parameter $n$ such that the resulting scenario tree sufficiently captures the uncertainty. An effective yet computationally expensive approach is to increment $n$ until the solution of the stochastic programming model converges. Variance reductions techniques, such as conditional sampling (Shapiro, 2003), may be considered to enhance convergence, but this is beyond the scope of our paper. An alternative, more practical approach is to generate samples until the most extreme asset damage scenarios correspond to edge cases that subject matter experts have identified based on the supply chain’s risk appetite as reasonable basis for a resilience strategy.

5. Stochastic resilience planning model

In this section, we formulate the multi-stage stochastic programming model that optimizes the preparedness and recovery of supply chain networks, given a scenario tree that captures asset damage uncertainty.

5.1. Sequential decisions

The stochastic programming model has $1 + T$ stages. In the first stage, decisions are taken regarding the investment in nominal network capacities, including capacities to provide repair resources. With these decisions fixed, the aim in the subsequent $T$ stages is to optimize network flows for each scenario defined in the scenario tree.
Similar to the dynamic network flow model with uncertain arc capacities in Glockner and Nemhauser (2000), our approach models that network flow decisions are made under certainty regarding past damage and uncertainty regarding future damage. Each node in the scenario tree is associated with a separate set of decision variables for the respective time step. This ensures a realistic level of anticipation without the need to formulate non-anticipativity constraints, based on the assumption that the branching probabilities in the scenario tree can be estimated by the decision makers.

5.2. Functional dependencies

Functional dependencies model that supply chain operations (production, storage and transportation of commodities) and asset repair may require certain resource inputs provided by different sub-graphs in $G$. Following Holden et al. (2013), the method used in this paper assumes linear input functions. This means that required inputs are proportional to the production rates, inventory levels and repair rates of dependent nodes, as well as flow and repair rates of dependent links. Thus, the parameters for an input dependency from node $i$ (provides commodity) to link $j,k$ (consumes commodity) are as follows:

- coefficient $\alpha_{i,j,k}^{f}$ for flow rates $f_{t,s}^{j,k}$
- coefficient $\alpha_{i,j,k}^{\tilde{f}}$ for flow capacity repair rates $\tilde{f}_{t,s}^{j,k}$

An input dependency relation from node $i$ to node $k$ has the following parameters:

- coefficient $\alpha_{i,k}^{g}$ for commodity production rates $g_{k}^{t,s}$
- coefficient $\alpha_{i,k}^{h}$ for inventory levels $h_{k}^{t,s}$
- coefficient $\alpha_{i,k}^{\tilde{g}}$ for production capacity repair rates $\tilde{g}_{k}^{t,s}$

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• coefficient $\alpha_{i,k}^h$ for storage capacity repair rates $\tilde{h}_{k}$

Let $D$ denote the set of all functional dependencies. The total amount of a commodity supplied by node $i \in V$ to all dependent nodes and links is denoted with variables $v_{i}^{t,s}$ and calculated as follows:

$$
v_{i}^{t,s} = \sum_{(j,k) \in E: (i,j,k) \in D} \left( \alpha_{i,j,k}^{f} f_{j,k}^{t,s} + \alpha_{i,j,k}^{\tilde{f}} \tilde{f}_{j,k}^{t,s} \right) + \\
\sum_{k \in V: (i,k) \in D} \left( \alpha_{i,k}^{g} g_{k}^{t,s} + \alpha_{i,k}^{\tilde{g}} \tilde{g}_{k}^{t,s} + \alpha_{i,k}^{h} h_{k}^{t,s} + \alpha_{i,k}^{\tilde{h}} \tilde{h}_{k}^{t,s} \right)
$$

(7)

The linearity assumption is justified because our method is primarily targeted at modelling assembly processes along the supply chain with fixed ratios of input commodities. If required by different model applications, Equation (7) could be generalized to consider non-linear input functions.

5.3. Mathematical formulation

The objective function (8) seeks to minimize capital expenditure in the first stage and operational costs in the subsequent $T$ stages. Capital expenditure is calculated as the sum of first stage decision variables, weighted by cost coefficients $c_{i}^{f}$, $c_{i}^{g}$ and $c_{i}^{h}$, which are per unit capital costs adjusted to a planning horizon of $T$ time steps.

The operational expenditure is calculated as the sum over all time steps and scenarios, weighted by the scenario probabilities $p^{t,s}$. Operational costs include those for link flows, commodity production and inventory holding. Furthermore, the objective function includes the penalty $c_{i}^{q} q_{i}^{t,s}$ for unmet service level targets and the lost revenue $c_{i}^{r} r_{i}^{t,s}$. 

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minimize \[ \sum_{(j,k) \in E} c^f_{j,k} f^t_{j,k} + \sum_{i \in V} \left( c^g_i \bar{g}_i + c^h_i \bar{h}_i \right) \]  
\[ + T \sum_{t=1}^{T} \sum_{s=1}^{S} f^t_{j,k} \left( \sum_{(j,k) \in E} c^f_{j,k} f^t_{j,k} + \sum_{i \in V} \left( c^g_i \bar{g}_i + c^h_i \bar{h}_i \right) \right) \]  
subject to  
(9)  
(10)  
(11)  
(12)  
(13)  
(14)  
(15)  
(16)  
(17)  
(18)  
(19)  
(20)  
(21)  
(22)  

Constraints (9) to (11) model that capacity utilisation has to be less than or equal to the available capacity, which in turn has to be less than or equal to the nominal capacity, which in turn has to be less than or equal to the maximum capacity parameter. Constraints (12) to (14) model the loss and recovery of capacity based on Equation (1).
Equation (15) enforces the balance of flows and ensures for each node that the net flow over connected links, net inventory change, production, supply to end-users and dependent assets would sum up to zero.

We use parameters $\beta_i \in [0,1]$ to indicate service level targets for nodes $i \in V$, measured as fraction of the demand that has to be met. Slack variables $q^{t,s}_i$, calculated in Equation (16), measure by how much the supply to end-users $u^{t,s}_i$ falls below the service level target $\beta_i \hat{u}^t_i$. The second set of slack variables $r^{t,s}_i$, calculated in Equation (17), measure by how much the supply $u^{t,s}_i$ falls below the demand $\hat{u}^t_i$.

The supply to dependent nodes is considered in Equation (18) based on the definition of functional dependencies in Equation (7). Equations (19) to (22) set all decision variables to be non-negative.

6. Numerical example

In this section, we demonstrate the modelling method proposed above with an example from the real-world multi-echelon supply chains published by Willems (2008). This dataset has widely been used to test and benchmark supply chain optimisation models, especially models that consider operational aspects, such as inventory management (e.g. Humair et al., 2013; Kök and Shang, 2014). To the authors’ best knowledge, there is no comparable dataset in the public domain that is more recent. Despite many technological advances over the past decade, the fundamental mechanisms of supply chain disruptions have not changed and, therefore, the dataset by Willems (2008) is still useful for demonstrating models like the one proposed in this paper.
6.1. Overview of data and modelling assumptions

Supply chain example number 3 from the dataset was chosen for the case study in this paper because it allows the demonstration of all important modelling aspects while its relatively small size and clear structure facilitate the analysis and interpretation of simulation results.

The supply chain (Figure 3) consists of four demand nodes, four factories and five suppliers. Each supplier and factory is represented by an ensemble of two nodes and one link in order to model the duration of the production process and the ability to store the product. Transport links T1 to T4 are for parts and intermediate products, whereas transport links D1 to D4 model the distribution of final products to end-users.

The dataset published in Willems (2008) contains the expected demand as well as expected durations and unit costs for manufacturing and transportation steps. The value of intermediate and final products can be calculated by summing up the costs of all previous manufacturing and transportation steps. Some additional input parameters for our model have been

Figure 3: Structure of the supply chain network used in the case study
derived based on the following assumptions:

**Value of lost revenue** The market value of final products is calculated as the total cost of production multiplied by an assumed profit margin of 25%.

**Penalty for unmet service level targets** Service level targets are given in Willems (2008) as $\beta_i = 95\%$. The penalty parameter is set to the market value of the commodity because we assume that substitute products are bought from the market if the supply chain is unable to meet its service level targets.

**Inventory cost** Data on inventory costs are not included in the dataset, and we assume the cost to be 1% of the commodity value per day.

**Opex and capex** Willems (2008) does not distinguish between fixed and variable costs. In order to demonstrate the capacity optimization in our model, we assume a fixed-cost ratio of 50%.

An overview of all model parameters is provided in Appendix A.1 and Appendix A.2. As Willems (2008) primarily intended to test models for inventory optimization under demand uncertainty, some additional assumptions were made in this study regarding asset hazard exposure, failure propagation risks and repair resources.

6.1.1. **Hazard and failure propagation dependencies**

The structure of the supply chain example suggests that there are three geographic regions. All of the suppliers and factories F1 and F2 are assumed to be located in the same region because they are not separated by
transportation links. In this case study, we seek to analyse and optimize the resilience of the supply chain against a disruptive event in this region.

We consider a simulation period of length $T = 260$ business days. For demonstration purposes and without loss of generality, we assume that the hazardous event occurs at time $t_h = 100$ and damage occurs only during this time step, i.e. the scenario tree has one single branch for $t = 1 \ldots t_h - 1$ and $S_{th}$ branches for $t = t_h \ldots T$.

The direct damage vector $c$ and failure propagation matrix $A$ for $t_h = 100$ were populated with mixed-type random variables that have a certain probability of being non-zero, and if they are non-zero their values are normally distributed. This resembles parametrising the model based on risk information provided by subject matter experts who would be asked to estimate i) the likelihood that a certain event occurs, and ii) its expected impact. In the absence of subject matter expertise, this case study assumes that all assets in the affected region are exposed to the same level of risk and the parameter values for the random variables in $c$ are presented in Table 4.

For failure propagation, we use two sets of parameter values: one for failure propagation within the same site (e.g. from the production to the storage node of the same supplier) and another for failure propagation between sites (e.g. from one supplier to another). The corresponding parameter values for random variables in $A$ are also given in Table 4.

6.1.2. Repair network

In this case study, we model one abstract repair resource that can be seen as a proxy for staff, equipment and parts required to repair damaged supply chain assets. This repair resource is measured in monetary units.

As depicted in Figure 4, the example network contains a repair resource node for each of the five suppliers and two factories that are at risk of dam-
Table 4: Random variable parametrisation for asset damage and failure propagation risk

<table>
<thead>
<tr>
<th>Variable</th>
<th>Assets</th>
<th>Probability of damage or propagation</th>
<th>Severity of damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>S1-5, F1-2, T1-4</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>A</td>
<td>Within site (e.g. S1 to Out S1)</td>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Within region (e.g. S1 to F2)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

age. A joint repair node is considered for the four transport links leaving the affected region because they are assumed to make use of the same physical infrastructure asset, for example, a sea- or airport.

In order to demonstrate how the proposed method can be used to assess alternate contingency plans for providing repair resources, we analyse two alternative configurations of the repair network:

**Individual repair resources** Each repair node provides repair resources exclusively to the asset it is directly associated to.

**Shared repair resources** A repair hub is introduced and connected to each repair node so that repair resources can be shipped from one repair node to another.

With these two alternative configurations, we can investigate how the sharing of repair resources would affect the resilience of the supply chain and optimal capacity investments. Pooling of repair resources could also be virtual, similar to the concept of virtual stockpile pooling (Liu et al., 2016).

The repair nodes are connected to the associated node or link in the supply chain network via functional dependencies. In the absence of further information on the types of assets and repair resources, it is sensible to assume that the amount of repair resources required is related to the capital value of the asset. Therefore, we set the input dependency coefficients $\alpha_{i,j,k}^\tilde{f}$. 
\(\alpha_{i,k}^g\) and \(\alpha_{i,k}^h\) to the capital cost of flow, production and inventory capacity, multiplied with a markup of 10% to take into account that the cost for emergency capacity restoration is likely to be higher compared to planned procurement.

### 6.2. Model implementation

The linear program formulated in Equations (8) to (21) has \((10n+2)|V| + (3n+1)|E|\) decision variables, where \(n = \sum_{t=1}^{T} |S_t|\) is the number of scenario tree nodes. For the case study presented here, \(T = 260\) and \(S_T = 100\). With the branching at \(t_b = 100\), the resulting scenario tree has \(n = 16,199\) nodes.

The combined supply chain and repair network has \(|V| = 34\) nodes and \(|E| = 33\) links, resulting in a linear program instance with approximately 7.1 million decision variables. However, the problem size can be reduced to 3.4 million decision variables by excluding decision variables with an upper and lower bound of zero. The linear programming solver Gurobi 8.1 is able
to solve the model with a barrier method in 4 hours 22 minutes on a desktop computer with an Intel i7 quad-core CPU (3.40 GHz) and 32 GB RAM.

6.3. Results

Results are generated for three cases: i) baseline case with no hazard occurrence, ii) individual repair resources, and iii) shared repair resources. The results include optimal network capacities and the expected supply chain performance in each of the 100 scenarios.

For the case of individual repair resources, Figure 5 shows the supply chain performance, measured as the total value of products supplied divided by the total value of products demanded, as described in Section 3.6. Using Equation (2), the RLT metric was found to be 2.36, which corresponds to the shaded area between the expected and full performance curves in Figure 5. In other words, the model estimates that the expected impact of the assumed hazard is equivalent to a complete disruption of the supply chain for 2.36 days.
Figure 6: Cumulative probability function for the resilience loss triangle area

Figure 6 shows the distribution of RLT values for the cases of individual and shared repair resources. The worst-case RLT value with shared repair resources is 12.94, which is higher than in the case of individual repair resources (11.30). However, the expected RLT decreases from 2.36 to 1.66 and the 95% percentile decreases from 7.66 to 5.94. Thus, the results suggest that sharing recovery resources leads to a better performance of the supply chain in almost all disruption scenarios, but comes at the cost of slightly worse performance in the most severe disruption scenario.

Figure 7 shows the optimal capacities calculated for all three cases. Optimal capacities for production nodes and transport links to the demand nodes are almost the same in all three cases. However, there are substantial differences in optimal storage capacities. As expected, the optimal supply chain configuration in the no-hazard case is a just-in-time operation, which eliminates the need for storage. In the two other cases, storage is needed at the import and supplier nodes.

Optimal storage capacity is unevenly distributed among the five suppli-
ers, with a particularly high value for node Out S3 (284 and 64 commodity units with individual and shared repair resources respectively). This can be explained by differences in the average volume of parts needed from these suppliers and by the difference in their value. To satisfy the entire demand, 299 parts from S2 and S3 are needed, compared to only 126 from S1. Furthermore, parts supplied by S3 have a lower value than all other parts, which translates into lower costs for storage capacity and inventory holding. Considering that the damage risk is the same for the facilities of all suppliers, it is plausible to invest in storage for those parts that are needed in greater quantities and can be stored at relatively low costs.

The total storage capacity throughout the supply chain is 54% lower in the case of shared repair resources compared to the case of individual repair resources. This indicates the efficiency gains that can be achieved by pooling repair capabilities for several facilities at risk.

Optimal capacities to provide repair resources are depicted in Figure 8. The results suggest that sharing repair resources leads to a more equal distribution of repair capacity among the five suppliers and two factories. Moreover, the total capacity for providing repair resources throughout the supply chain decreases by 63%. Thus, it has been shown, so far, that the supply chain analysed in this case study is more resilient with overall lower repair capacity if this capacity is shared through a common hub for exchanging repair resources.
Figure 7: Optimal capacities in the supply chain network
Figure 8: Optimal capacities in the repair network
Finally, we analyse the costs predicted by the simulation model. Figure 9 shows cost differences compared to the no-disruption baseline scenario. The expected total costs are USD 18.3 million higher in the case of individual repair resources, and USD 15.1 million higher in the case of shared repair resources, compared to the no-disruption case.

The main driver for the cost increase is in both cases operational expenditure for asset repair. The main sources of the USD 3.2 million cost savings in the case of shared repair resources are lower investments in repair capacity as well as lower revenue losses and penalties.

In summary, we find that sharing repair resources has two effects in our example supply chain: First, the supply chain’s ability to mitigate impacts of asset failures improves, which leads to higher reliability of supply for end customers. Secondly, repair resources are used more efficiently, which means that the overall investment in capacity for providing repair resources can be reduced.

![Figure 9: Cost compared to the no-disruption scenario](image-url)
6.4. Discussion

We have presented a case study that demonstrates the application of the proposed method to a real-world supply chain from a benchmarking dataset that provides topology, demand, cost and process duration parameters. The cost breakdown into capital and operational expenditure, as well as penalty costs for unmet demand, are parameter assumptions that could easily be replaced by an analyst with access to business sensitive data.

Parameter assumptions regarding the risk of initial damage and failure propagation could not be verified with the information provided by the generic benchmarking dataset. Considering that the method is geared towards analysing extreme event scenarios with little or no historical precedence, it is expected that the required parameters will be set based on subject matter expertise rather than statistical analysis of previous incidents. Extensive sensitivity analysis should be carried out before making decisions based on the simulation results.

In comparison with state-of-the-art supply chain models, we find that the method proposed in this paper provides additional insights that are useful for improving supply chain resilience. For instance, Chen et al. (2017) and Zhalechian et al. (2018) also provide a method for calculating optimal priorities for recovery tasks but these models include repair budgets as exogenous parameters. In contrast, the method proposed in this paper calculates the optimal repair capability based on savings from avoided disruptions of supply to end customers.

Another difference to the previous research by Chen et al. (2017) and Zhalechian et al. (2018) is that the methodology proposed in our paper considers supply chain recovery as a multi-stage optimisation problem. Similar dynamic models for analysing supply chain planning have been pub-
lished previously (e.g. Hasani and Khosrojerdi, 2016; Parajuli et al., 2017; Ghavamifar et al., 2018), but did not address the problem of optimising repair strategies. In the case study presented above, sequential decisions over 260 time steps were analysed. The dynamics of system performance degradation and recovery are very different across the 100 scenarios analysed, highlighting the importance of a higher temporal resolution.

In the related research area of infrastructure resilience, Nurre et al. (2012) provide a dynamic network flow model that also includes network design variables and optimises repair task scheduling. The methodology presented in our paper enhances this modelling approach by integrating the procurement, allocation and transportation of repair resources into the dynamic network flow model. This leads to a model that can be used to analyse logistical aspects of repair strategies, which are not considered in previous research.

The scenario generation method used in this paper considers the risk of failure propagation, similar to previous research on correlated failure probabilities in supply chains (e.g. Masih-Tehrani et al., 2011; Li et al., 2013; Rezapour et al., 2015; Hasani and Khosrojerdi, 2016). However, to the authors’ best knowledge, this paper is the first to provide a dynamic model for optimal network design and recovery strategies under consideration of interdependent asset failures.

It is worth noting that the multi-stage stochastic programming model used in this paper is based on the assumption that decisions are taken based on knowledge of the full scenario tree, i.e. that all possible asset failure scenarios and their probabilities are known to the decision makers. Therefore, the realism of the model depends on two factors: First, supply chain managers are assumed to have the same information that is used to gen-
erate the scenario tree. This would usually be the case when applying the model because the scenario tree generation method relies on subject matter expertise to estimate failure and failure propagation probabilities.

Second, the scenario tree should fully capture the uncertainty of asset failure. This can be difficult to achieve if the number of assets at risk is large. In this regard, it is an important advantage that the proposed model avoids integer variables and can, therefore, be solved for larger scenario trees compared to computationally more costly mixed-integer models.

A limitation of the model presented in this paper is that distributed decision making of different stakeholders is not considered. For this reason, the proposed methodology is most suitable for the analysis of supply chains that are sufficiently integrated, such that all stakeholders have a mutual interest and ability to share information and collaborate in order to achieve system-optimal supply chain recovery.

Finally, we note that the proposed model is capable of considering non-linear cost functions, such as additional penalties if the supply falls below a certain service level threshold. Moreover, the model captures economies of scale, as it distinguishes between capital and operational expenditure, and seeks to optimize capacity utilisation. However, modelling economies of scale resulting from increasing marginal returns from resource inputs for production or repair processes would lead to a non-convex optimization problem, which is substantially more difficult to solve.

7. Conclusions

Supply chain disruptions pose major threats to business continuity, especially when they involve cascading failure and asset damage requiring extensive repair work. This paper contributes in three important ways to ad-
vancing the mathematical modelling methodology for analysing supply chain resilience: i) The proposed model jointly optimises investments in network capacities and repair capabilities, as well as supply chain operations and repair works. ii) It considers logistical issues of providing repair resources. iii) It models the planning of network capacities, supply chain operations and repair works as sequential decisions using a multi-stage stochastic programming approach.

Supply chain managers can use the model presented in this paper for decision support in the following key areas of resilience planning:

1. Capacity planning for new and existing supply chains: The proposed model indicates optimal capacities of supply chain facilities for a given hazard, cost structure and service level target. This can be valuable guidance both for designing new supply chains and for adapting existing ones, for example, to changing environmental hazards.

2. Planning the provision of recovery resources: The model also indicates optimal capacities for providing repair resources. Importantly, it also considers the trade-off between investments in increased recovery capability and redundant capacity provision.

3. Operational and incident response planning: The results of the optimal network flow model can inform decisions on safety stock management, reconfiguration of production and inventory plans after disruptions, and scheduling of repair tasks.

4. What-if analysis: By analysing different supply chain topologies, the model can be used to evaluate how changes in component provision, manufacturing processes, transportation infrastructure
and repair logistics would affect the supply chain’s resilience.

The proposed methodology uses generic models for production facilities, storage locations, transportation infrastructure, asset failure and repair processes. These models can easily be parametrised according to the needs of various industries. Users of the proposed modelling method should carefully evaluate data availability (e.g. from historic records or provided by subject matter experts) before deciding on the appropriate granularity of their analysis, for example, whether to model one aggregated or multiple individual repair resources.

In future research, the methodological framework presented in this paper can be extended to consider different failure modes, use arbitrarily sized simulation time steps, and distinguish between consumable and non-consumable repair resources. Another promising direction would be to continue the investigation of interdependencies between supply chains and infrastructure systems. In particular, the dependency of supply chain and repair resource logistics on transport and energy systems, as well as the dependence of critical infrastructure on supply chains providing repair resources, are yet to be analysed in detail using dynamic and stochastic modelling methods. Such studies are likely to uncover non-linear relationships and may have to solve computationally challenging optimization problems or resort to heuristic or approximation methods.

8. Acknowledgements

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Appendix A. Model parameters

Table Appendix A.1: Node parameters

<table>
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<tr>
<th>Node</th>
<th>$c_i$</th>
<th>¯$g_i$</th>
<th>$c_i$</th>
<th>¯$h_i$</th>
<th>$h_i$</th>
<th>$u_i$</th>
<th>$c_i = c_i'$</th>
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Table Appendix A.2: Link parameters

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References


