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# Conformal transformation in bowtie nanoantennas and nanocavities: unveiling hidden symmetries

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***Abstract* – In this work, bowtie nanoantennas and nanocavities are analyzed using the conformal transformation technique. Their performance is studied in terms of the non-radiative Purcell enhancement and self-induced optical forces experienced by quantum emitters. It is demonstrated how these two geometrically different plasmonic nanoparticles can share the same non-radiative Purcell spectra. This hidden symmetric response is unveiled by properly applying the conformal transformation technique, demonstrating that both nanoparticles share the same transformed geometry.**

## I. INTRODUCTION

The unprecedented advance in nanofabrication techniques has enabled the experimental study of nanostructures with different geometries using a wide range of metals and dielectrics [1]–[4]. The comprehensive analysis of the coupling between quantum emitters and localized surface plasmons (LSP) resonances in plasmonic nanoparticles has become an important research field [5], [6]. To understand the performance of these nanoparticles, full-wave simulations have been widely used. However, the dispersion of metals at optical frequencies and sizes of the nanoparticles can increase both the computational burden and complexity of the numerical studies.

As it is known, analytical solutions for canonical cases such as a dipole next to a plasmonic nanosphere are accessible. However, to efficiently design plasmonic nanoparticles and gain physical insight on their performance, analytical solutions for more complex scenarios are needed. In this realm, the conformal transformation has become by its own merit as one of the preferred methods to study plasmonic nanoparticles [7], [8]. This technique was first proposed at microwave frequencies [9] and has demonstrated to be a powerful tool to gain physical understanding of plasmonic nanoparticles at optical frequencies, providing a high accuracy. It has been applied to different geometries such as cylinders, crescent-shaped nanoparticles [10], plasmonic gratings [11] and bowtie [12]–[14] and tripod nanoantennas [15].

Inspired by the importance of the conformal transformation technique and the necessity of analytical solutions for plasmonic nanoparticles, in this work, we study the coupling between quantum emitters (analytically treated as a point dipole) near bowtie nanoantennas and nanocavities. The Purcell enhancement and self-induced optical forces experienced by the nanoemitters are studied using bowtie nanoparticles with a length of $l'$ = 20 nm. As it will be shown, similar Purcell enhancement is achieved with both bowtie nanoantenna and nanocavities. This hidden symmetry is here unveiled and studied using the conformal transformation technique [16]. All the results are compared with numerical simulations using COMSOL Multiphysics®.

## II. CONFORMAL TRANSFORMATION TECHNIQUE

The schematic representations of the bowtie nanoantenna and nanocavity are shown in Fig. 1a,b, respectively, where it is shown how they are connected and disconnected at their center, respectively. Since both nanoparticles are geometrically different, non-radiative Purcell spectra can be expected (this performance will be studied in the following section and detailed during the conference).The nanoparticles (made of gold, Au) are immersed in vacuum and they are illuminated by a point dipole placed ($y'$ = 0, $x'$ = 1 nm) away from the center (0,0). Multiple conformal transformations can be cascaded depending on the nanoparticle to be studied [8]. Here, the single-step conformal transformation $z = ln(z'/a)$ is applied to both nanoparticles with $z = x + iy$ and $z' = x' + iy'$ as the coordinates in the transformed and original frames, respectively, and *a* as the distance between the nanoemitter and the coordinate center (0,0). By applying this conformal transformation to the bowtie nanoantenna in Fig. 1a, the resulting transformed space is the one shown in Fig. 1c, which can be treated analytically. As observed the metallic strips are connected by a metal wall at their left-hand side while there are disconnected in the other extreme (right-hand side). For the bowtie nanocavity shown in Fig. 1b, the transformed space is actually the same as the one shown in Fig. 1c. The difference is that for the nanocavity, the metal strips will be connected at the right hand side while disconnected in the other extreme (left-hand side). This performance reveals that the transformed multi-strip geometry in Fig. 1c is indeed the same for both bowtie nanoantennas and nanocavities, showing how two geometrically different nanoparticles can be transformed into the same multi-strip geometry. The performance of both bowtie nanoantennas and nanocavities is then studied in terms of the non-radiative Purcell enhancement. Since the material properties and potentials are preserved after applying the conformal transformation, the power dissipated by the bowtie nanoparticles is also the same as the one in the transformed geometries. Hence, the non-radiative Purcell enhancement is analytically calculated as the ratio between the non-radiative power emission in the transformed space $P_{nr} = -(1/2)\omega\{p_x^* E_{1x}^S(x,y) + p_y^* E_{1y}^S(x,y)\}$ (with $\omega$ as the angular frequency, $p_{x,y}$ and $E^S_{1x,y}(x,y)$ as the $x$ and $y$ components of the dipole moment and electric field, respectively) and the power radiated by the emitter $P_0$.

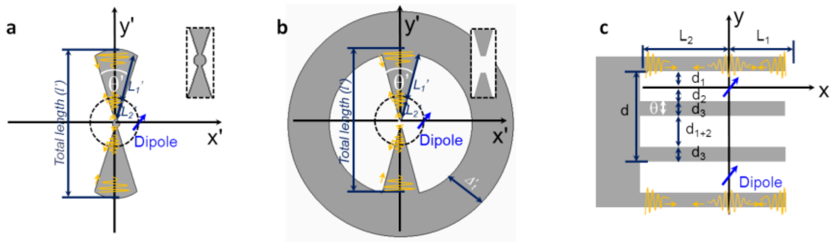


Fig. 1. Schematic representation of the bowtie nanoantenna (a) and nanocavity (b) along with the transformed multi-strip geometry after applying the conformal transformation to the bowtie nanoantenna (c). The transformed geometry for the bowtie nanocavity is the same as in panel c but with air and metal at the left and right hand sides of the metallic strips, respectively.

## III. Results and Discussion

The numerical and analytical results of the non-radiative Purcell enhancement spectra for the bowtie nanoantennas and nanocavities with $l'$ = 20 nm and varying $\theta'$ are shown in Fig. 2b,c and Fig. 2f,g, respectively. Here, a horizontal polarization of the dipole emitter is considered (the response using vertical polarization will be presented during the conference). An agreement between the numerical and analytical results is clearly observed. Moreover, it is interesting to note how the LSP modes for both nanoparticles appear at the same spectral location. This performance is more evident in Fig. 2d,h where the non-radiative Purcell enhancement for three values of $\theta'$ are presented for completeness. These results demonstrate how two geometrically different nanoparticles can have the same spectral response. This hidden symmetry can be explained based on the conformal transformation shown in Fig. 1c: they share the same spectral response due to the fact that both nanoparticles are transformed into the same multi-strip geometry. Details about polarization, magnitude of the non-radiative Purcell enhancement, self-induced optical forces and influence on the position of the dipole emitter will be presented during the conference. High dielectric index bowtie nanoantennas are now under study and the results will be presented in the near future.

## VI. Conclusion

In this communication, the conformal transformation technique has been applied to bowtie nanoantennas and nanocavities in order to gain physical understanding of their performance when they are illuminated with a dipole nanoemitter. The nanoparticles were evaluated in terms of the non-radiative Purcell enhancement and forces demonstrating a good agreement with numerical calculations. Apparently non-existent symmetries have been unveiled for the first time between both bowtie nanoantennas and nanocavities, demonstrating how the same spectral response can be obtained with these two geometrically different nanoparticles. This performance has been explained using the conformal transformation technique where it has been shown how both nanoparticles share the same transformed geometry. The results here presented can open new avenues in the design of plasmonic applications using bowtie nanoantennas and nanocavities where their performance can be efficiently studied before experimental characterizations.

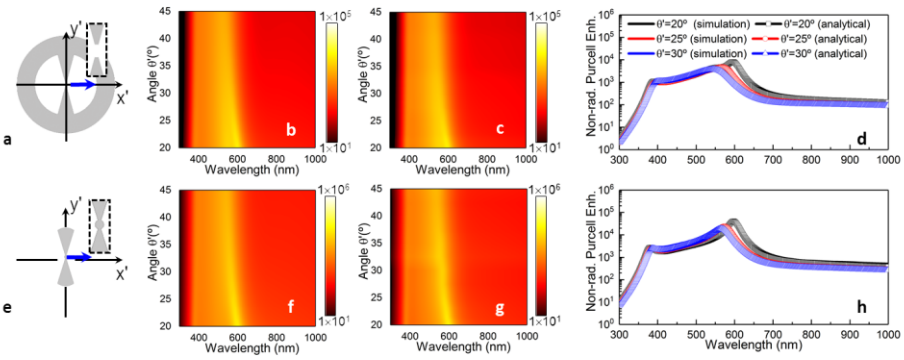


Fig. 2. Non-radiative Purcell enhancement spectra for bowtie nanoantennas (a-d) and nanocavities (e-h) as a function of the aperture angle $\theta'$: numerical (b,f) and analytical results (c,g).

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