Seismic simultaneous inversion using a multi-damped subspace method

Jinyue Liu 1, and Yanghua Wang 2

1. Imperial College London, Department of Earth Science and Engineering, Centre for Reservoir Geophysics, London SW7 2BP, UK. Email: jinyue.liu13@imperial.ac.uk.
2. Imperial College London, Department of Earth Science and Engineering, and Resource Geophysics Academy, Centre for Reservoir Geophysics, London SW7 2BP, UK. Email: yanghua.wang@imperial.ac.uk (corresponding author).

ABSTRACT

Seismic inversion of amplitude variation versus offset (AVO) plays a key role in seismic interpretation and reservoir characterization. The AVO inversion shall be a simultaneous inversion which inverts for three elastic parameters simultaneously: the P-wave impedance, the S-wave impedance and the density. Using only seismic P-wave reflection data with a limited source receiver offset range, the AVO simultaneous inversion can obtain two elastic parameters reliably, but is difficult to invert for the third parameter, usually the density term. To tackle this difficulty in the AVO simultaneous inversion, we employed a subspace inversion method in which we partitioned the elastic parameters into different subspaces. We parameterized each single elastic parameter with a truncated Fourier series and inverted for the Fourier coefficients. Because the Fourier coefficients of different wavenumber components have different sensitivities, we grouped the Fourier coefficients of low, medium and high wavenumber components into different subspaces. We further assigned different damping factors to the Hessian matrix corresponding to different wavenumber components within each subspace. This inversion scheme is referred to as a multi-damped subspace
method. Both synthetic and field seismic data examples confirmed that the AVO simultaneous inversion with this multi-damped subspace method is capable to produce reliable estimation of the three elastic parameters simultaneously.

**Keywords**: AVO inversion, damping scheme, elastic parameters, Fourier coefficients, model parameterization, subspace method.

**INTRODUCTION**

Seismic inversion of amplitude variation versus offset (AVO) plays a key role in lithological interpretation and hydrocarbon detection (Smith and Gidlow, 1987; Rutherford and Williams, 1989; Fatti et al., 1994; Luo et al., 2017; Lu et al., 2018). The AVO inversion should be a simultaneous inversion that inverts for three elastic parameters: the P-wave impedance, the S-wave impedance and the density. The density is a vital elastic parameter and can be used as a direct indicator for lithology and fluid, if the density can be reliably estimated from the seismic data. Although the pre-stack elastic inversion can estimate the P-wave and S-wave impedances including the density, it is difficult to estimate the third elastic parameter, the density (de Nicolao et al., 1993; Ursin and Tjäland, 1996), especially when attempting to estimate together with P-wave and S-wave impedances. This is because the density parameter has a huge difference in sensitivity, when compared to other two parameters (Wang and Pratt, 1997). Therefore, the AVO simultaneous inversion is an ill-posed problem, in which seismic amplitudes have a very weak sensitivity to the density perturbations (Buland and Omre, 2003; Tang and Wang, 2017; Liang et al., 2017). For the ill-posed inverse problem, even a small data error may result in a huge error in the model estimate, and the inversion is unstable and the inversion results are not unique (Wang, 2016).

To tackle the issue, one solution is to include long-offset data in the AVO simultaneous inversion (Downton, 2005; Zhu and McMechan, 2012). A problem with this solution is that
the AVO simultaneous inversion becomes highly non-linear as the incident angle approaching the critical angle. Another solution is to use the PP-wave and PS-wave jointly, since the PS-converted wave brings extra angle information into the inversion (Margrave et al., 2001; Veire and Landrø, 2006; Tang and Wang, 2017; Zhi et al., 2018). A problem with the PP-PS joint inversion is the event registration of the PP- and PS- reflections. Nevertheless, it is necessary to find a technology to estimate three elastic parameters simultaneously using only the conventional P-wave seismic data with a limited range of source-receiver offsets.

In this paper, we adopt the subspace method as a robust solution to the AVO simultaneous inversion. The subspace method is capable of handling different functional dependencies on the various parameter types in a balanced way. Examples include seismic amplitude inversion inverting for the interface geometry and the velocity structure (Wang and Houseman, 1994, 1995; Wang and Pratt, 2000) and the joint inversion of traveltimes and amplitudes to estimate the interface geometry and the elastic parameters simultaneously (Wang, 1999b).

The subspace method reduces the size of inverse matrix from the large number of model parameters down to a small number of independent subspaces. The gradient vectors among different subspaces represent different model parameter types and are orthogonal (Kennett et al., 1988; Bodin and Sambridge, 2009). Therefore, the sensitivities of different elastic parameters can be independently measured according to their associated gradient vectors in the subspace domain.

We parameterize each elastic parameter using a truncated Fourier series (Wang and Houseman, 1994, 1995), in which each Fourier coefficient represents a wavenumber component of the elastic parameter. We invert for the Fourier coefficients, instead of grids in a conventional discretization scheme. This Fourier series parameterization offers an
opportunity to evaluate sensitivities of an elastic parameter from low, to medium and to high
wavenumbers (Wang and Pratt, 1997, 2000; Wang et al., 2000). Then, in order to balance the
sensitivities of different wavenumber components within a subspace, we use a multi-damped
subspace inversion scheme.

**LINEARIZED AVO INVERSION**

Seismic AVO inversion estimates the elastic properties using the P-wave reflection-
transmission coefficients at the subsurface interfaces. The Zoeppritz equations describe the
exact reflection-transmission coefficients at an interface (Zoeppritz, 1919) and can be
expressed as (Wang, 1999a),

\[ R(\rho) = \frac{\alpha_4 + \alpha_5 p^2 + \alpha_6 p^4 - \alpha_4 p^6}{\alpha_1 + \alpha_2 p^2 + \alpha_3 p^4 + \alpha_4 p^6}, \]

where \( \alpha_1 - \alpha_7 \) are the coefficients depending on the P-wave velocity \( V_p \), S-wave velocity \( V_s \),
and density \( \rho \), and \( p \) is the ray parameter. According to Snell's law, the ray parameter \( p \)
holds a constant value at all interfaces, \( p = \sin \theta / V_p \), where \( \theta \) is the incident angle at the
interface.

Since the AVO inversion in this paper inverts for the P-wave impedance \( I_p \), S-wave
impedance \( I_s \) and density \( \rho \), we write the coefficients of \( \alpha_1 - \alpha_7 \) in terms of \( (I_p, I_s, \rho) \) as

\[ \alpha_1 = (\rho_2 q_{i,1} + \rho_1 q_{i,2})(\rho_2 q_{i,1} + \rho_1 q_{i,2}) \],
\[ \alpha_2 = -4(\rho_2 q_{i,1} q_{i,1} - \rho_1 q_{i,2} q_{i,2})\Delta\mu + (\Delta\rho)^2 + 4q_{i,1}q_{i,1}q_{i,1}(\Delta\mu)^2, \]
\[ \alpha_3 = 4(q_{i,1}q_{i,1} + q_{i,2}q_{i,2})(\Delta\mu)^2 - 4\Delta\mu\Delta\rho, \]
\[ \alpha_4 = 4(\Delta\mu)^2, \]
\[ \alpha_5 = (\rho_2 q_{i,1} - \rho_1 q_{i,2})(\rho_2 q_{i,1} + \rho_1 q_{i,2}) \],
\[ \alpha_6 = -4(\rho_2 q_{i,1} q_{i,1} + \rho_1 q_{i,2} q_{i,2})\Delta\mu - (\Delta\rho)^2 + 4q_{i,1}q_{i,1}q_{i,1}(\Delta\mu)^2. \]
\[ \alpha_7 = 4(q_{I_s,1}q_{I_s,1} + q_{I_s,2}q_{I_s,2})(\Delta \mu)^2 + 4\Delta \mu \Delta \rho, \]  

where \((I_{p1}, I_{s1}, \rho_1)\) are the three elastic parameters in the upper layer of the interface, \((I_{p2}, I_{s2}, \rho_2)\) are the three elastic parameters below the interface, \(q_{I_s,1} = \sqrt{\rho_1^2 / I_{p1}^2 - \rho^2}\), \(q_{I_s,2} = \sqrt{\rho_2^2 / I_{p2}^2 - \rho^2}\), \(q_{I_s,1} = \sqrt{\rho_1^2 / I_{s1}^2 - \rho^2}\), \(q_{I_s,2} = \sqrt{\rho_2^2 / I_{s2}^2 - \rho^2}\), \(\Delta \mu = I_{s2}^2 / \rho_2 - I_{s1}^2 / \rho_1\), and \(\Delta \rho = \rho_2 - \rho_1\).

For the AVO inversion the objective function is defined in a least-squares sense as

\[ J(m) = \|W r(m) - d\|^2, \]  

where \(d\) is a vector of the observed seismic data, \(W\) is a matrix of the seismic wavelet, and \(r(m)\) is a vector of the reflectivity series, and \(m = [I_p, I_s, \rho]^T\) is the model vector that needs to be estimated. The reflectivity series \(r(m)\) is linearized by the first order Taylor expansion,

\[ r(m) \approx r(m_0) + F \Delta m, \]  

where \(m_0\) is the current model vector, \(\Delta m = m - m_0\) is the model update, and \(F\) is the Fréchet matrix of the reflectivities with respect to the current model \(m_0\). The objective function is approximated to

\[ J(\Delta m) = \|WF \Delta m + [Wr(m_0) - d]\|^2. \]  

In the iterative inversion, the model update is given by

\[ \Delta m = -(H + \mu I)^{-1} g, \]

where \(g\) is the gradient vector, \(g = (WF)^T [Wr(m_0) - d]\), \(I\) is the identity matrix, \(H\) is the Hessian matrix, \(H = (WF)^T (WF)\), and \(\mu\) is a small positive-valued damping factor for stabilizing the inverse calculation of the Hessian matrix \(H\). We calculate the Hessian matrix \(H\) iteratively by the BFGS (Broyden-Fletcher-Goldfarb-Shanno) method (Nocedal, 1980; Fletcher, 1987; Rao and Wang, 2017), along with the iterative inversion procedure (Wang,
SENSITIVITY ANALYSES

Let us consider a linear relationship between the model perturbations and the data residuals,

$$\delta \mathbf{d} = \mathbf{S} \delta \mathbf{m},$$  \hspace{1cm} (7)

where $\delta \mathbf{m}$ is the vector of model perturbations, $\delta \mathbf{d}$ is the vector of seismic data residuals, and $\mathbf{S} \equiv \mathbf{WF}$ is the Fréchet derivatives of seismic responses with respect to the model parameters. In the following sensitivity analysis, we use the Fréchet matrix $\mathbf{S}$ to build the Hessian matrix $\mathbf{H} = \mathbf{S}^T \mathbf{S}$.

Resolution is the resolving power of the AVO inversion (Wang and Pratt, 1997). The singular value decomposition (SVD) of $\mathbf{H}$ is $\mathbf{H} = \mathbf{V} \Lambda \mathbf{V}^T$, where $\Lambda$ is the diagonal matrix containing the eigenvalues $\lambda_i$, and $\mathbf{V}$ is the matrix with the corresponding eigenvectors $\mathbf{v}_i$. The eigenvectors corresponding large eigenvalues represent well-determined parameter combinations, whereas the eigenvectors corresponding small eigenvalues represent poorly determined parameter combinations. However, for AVO inversion, the Hessian matrix $\mathbf{H}$ is nearly singular and does not have an exact inverse. A pseudo-inverse $\mathbf{H}^{\dagger}$ can be given by truncated $\mathbf{H}$ using SVD, as

$$\mathbf{H}^{\dagger} = \mathbf{V} \mathbf{S}^{\dagger} \mathbf{V}^T,$$  \hspace{1cm} (8)

where $\mathbf{S}^{\dagger}$ keeps first $k$ columns of $\mathbf{V}$ with remaining columns to be all zeros and $\mathbf{S}^{\dagger} = \text{diag}\{\lambda_1^{-1}, \lambda_2^{-1}, \lambda_3^{-1}, \lambda_4^{-1}, 0, L, 0\}$. The solution of the linearized inversion (Equation 7) can be written as

$$\delta \mathbf{m} = \mathbf{H}^{\dagger} (\mathbf{S}^T \delta \mathbf{d}) = \mathbf{H}^{\dagger} \mathbf{H} \delta \mathbf{m} = \mathbf{V} \mathbf{S}^{\dagger} \mathbf{V}^T \delta \mathbf{m},$$  \hspace{1cm} (9)

where $\mathbf{V} \mathbf{S}^{\dagger} \mathbf{V}^T$ is the model-resolution matrix. This model-resolution matrix maps the linear relationship between the true solution $\delta \mathbf{m}$ and estimate solution $\delta \mathbf{m}^{\text{est}}$ (Wang, 2016).
For the sensitivity analysis we build a 1D synthetic model (Figure 1). We adapt the P-wave and S-wave impedances based on Marmousi-II model and build the density to be uncorrelated with the P-wave and S-wave impedance. In the synthetic seismic traces for three ray-parameters 0.06, 0.12 and 0.18 ms/m, we show both noise-free synthetic traces (Figure 1d) and noisy synthetic traces (Figure 1e). For the noisy synthetic traces, we set the signal-to-noise ratio (S/N) to be 4. Here the S/N ratio is defined as the ratio of RMS (root mean square) value of the signal and RMS value of the noise.

Following Wang and Houseman (1994, 1995), we parameterize this 1D model using a truncated Fourier series:

\[ m(z) = a_0 + \sum_{j=1}^{M} [a_j \cos(j \Delta k z) + b_j \sin(j \Delta k z)], \]  

where \( \Delta k = \pi / Z \) is the basis wavenumber, and \( Z \) is the model coverage in the depth. We invert for the Fourier coefficients \( (\xi = a_0, a_j, b_j, \ j = 1, \ldots, M) \), corresponding to different wavenumber components. The final model of an elastic parameter is reconstructed via these Fourier coefficients. Hence for the sensitivity analysis, only cosine terms are examined, because the eigenvectors of cosine and sine terms show similar pattern and the only difference between cosine and sine terms is the phase (Wang and Pratt, 1997). Thus, there are 101 coefficients for each of the three elastic parameters. These coefficients represent various wavenumber components accordingly. There are 303 coefficients for three elastic parameters in total.

The eigenvalues and the corresponding eigenvectors of the Hessian matrix \( \mathbf{H} \) (Figure 2a) reveal that the model parameters with highest wavenumber components show the highest sensitivity in the inversion. The first eigenvector (first column), associated to the largest eigenvalue, indicates the most sensitive combination of the model parameters, which are the highest wavenumber components of the P-wave impedance, the highest wavenumber
components of the S-wave impedance, and the highest wavenumber components of the density. When the eigenvalues decreasing the eigenvectors show a pattern that gradually goes to lower wavenumbers. This pattern indicates that, lower wavenumber components of the model parameters have weaker sensitivity.

The eigenvalues decrease quite quickly. The first 194 eigenvalues are larger than $-60$ dB. The remaining small eigenvalues to the right of the dashed line effectively become useless null space, which makes little contributions during the inversion. In the following sensitivity analyses, those eigenvalues that are smaller than $-60$ dB are neglected.

Neglecting the null space, we calculate the resolution matrix to evaluate how well each model parameter can be resolved during the inversion. In the ideal case, the resolution matrix should become an identity matrix with a value of 1 in the main diagonal. Only the main diagonal in the P- and S-wave impedance parts is focused and close to 1, indicating the reliability for P- and S-wave impedance recovery. The main diagonal in the density part (blue arrow) is smeared with small values. This observation indicates the low reliability and accuracy for density recovery. Sub-diagonals also exist in the P- and S-wave impedance area for the density recovery. These sub-diagonals represent the dependences between density and P- and S-wave impedances. Such dependences make it difficult for inversion to resolve density.

**A MULTI-DAMPING SCHEME**

The previous section showed that the different wavenumber components of any single model parameter have different sensitivities. Thus, we apply a multi-damping scheme to deal with the different sensitivities of the model parameters (Wang and Pratt, 2000).

In the inversion, each of model coefficients is divided into ten equal groups and thus the first group consists of the lowest wavenumber components and the last group consists of the
highest wavenumber components. In order to balance the sensitivities of different wavenumber components, different damping factors can be applied to different groups in the main diagonal of the Hessian matrix $H$ (Wang and Pratt, 2000). A damped Hessian matrix $H_{\text{damp}}$ is given by,

$$H_{\text{damp}} = H + Q,$$

where $Q = \text{diag}\{\mu^{(i)}\} , \mu^{(i)}$ is the damping factor corresponding to $i$ th group. These damping factors associated to each group are designed as follows,

$$\mu^{(1)} = 0.0001, \quad \mu^{(2)} = 0.0001,$$

$$\mu^{(3)} = 0.0001, \quad \mu^{(4)} = 0.0005,$$

$$\mu^{(5)} = 0.001, \quad \mu^{(6)} = 0.002,$$

$$\mu^{(7)} = 0.005, \quad \mu^{(8)} = 0.008,$$

$$\mu^{(9)} = 0.01, \quad \mu^{(10)} = 0.015,$$

where $\mu^{(i)}$ corresponding to the associated model group $m^{(i)}$. The same set of damping factors are applied to all three elastic parameters. According to Wang and Pratt (1997), these damping factors are wavenumber related. The groups containing lower wavenumber components have smaller damping factors and those groups containing higher wavenumber components have larger damping factors.

The damping factors can neither be too small nor too large. Large damping factors can prevent the inversion procedure from obtaining the desired results. However, small damping factors, especially for high wavenumber groups, might not be able to stabilize the inversion.

The sensitivity analysis of the multi-damped Hessian matrix $H_{\text{damp}}$ is carried out (Figure 2b). The eigenvalues for large index have been greatly enhanced, which results in a smaller null space. The resolution matrix shows that the main diagonal in the density area (blue arrow) is more focused, suggesting that the inversion can resolve the density better. The sub-diagonals in the P- (red arrow) and S-wave (orange arrow) impedance are much weaker than that when there is no damping (Figure 2a), and indicate weaker dependence between P-wave,
S-wave impedance and the density.

Figure 3 compares the diagonal values of resolution matrix from undamped Hessian matrix $H$ (blue) and multi-damped Hessian matrix $H_{\text{damp}}$ (red). The evident improvement of the diagonal values, especially for the density, clearly demonstrates that all three model parameters are likely to be reliably recovered by applying the multi-damping scheme.

**DAMPED SUBSPACE INVERSION METHOD**

The subspace method is used to invert for three model parameters simultaneously. In the subspace inversion, the different wavenumber groups are projected into the different independent subspaces, in which the different sensitivities of the different wavenumber groups can be handled without interference.

Given a projection matrix $A = [a^{(1)}, a^{(2)}, \ldots, a^{(l)}]$, which is spanned by $l$ basis vectors $\{a^{(i)}, i = 1, 2, L, l\}$, we construct the model update $\Delta m$ with these basis vectors, by

$$\Delta m = -A[A^T(H + Q)A]^{-1}A^T g.$$  \hspace{1cm} (13)

Following Wang and Houseman (1994, 1995), we partition the gradient vector $g$ into $l$ sub-vectors $\{g^{(1)}, g^{(2)}, \ldots, g^{(l)}\}$, and build the projection matrix $A$ as

$$A = \begin{bmatrix} g^{(1)} & 0 & L & 0 \\ 0 & g^{(2)} & M & M \\ M & M & O & 0 \\ 0 & L & 0 & g^{(l)} \end{bmatrix},$$ \hspace{1cm} (14)

where $g^{(i)}$ is the normalized sub-vector of $g^{(i)}$.

To demonstrate the effectiveness of the subspace method, we compare its result with other methods using both noise-free (Figure 4) and noisy (Figure 5) synthetic datasets. It can be observed that the noise level clearly affects the inversion results and the input data with higher S/N can produce more reliable inversion results.
In the conventional simultaneous inversion (Figure 4a and Figure 5a), the density is unstable, although P- and S-wave impedances are well recovered. Especially under the presence of noise, the conventional simultaneous inversion produces even a worse density result (Figure 5a). Since it is difficult to obtain three different model parameters at the same time, we perform a sequential inversion in which we invert for the P-wave impedance (the most sensitive model parameter) in the first step, the S-wave impedance second, and the density (the least sensitive model parameter) in the third step. In the first step inverting for the P-wave impedance, we use the low ray parameter (near-offset) traces, which mainly contain the information of the P-wave impedance, and less information about the S-wave impedance and the density. In the second step inverting for the S-wave impedance, we apply a large damping value ($\mu = 1$) to the P-wave impedance, and a relatively small damping value ($\mu = [0.01, 0.1]$) to the S-wave impedance. The third step inverting for the density, we attempt to keep the previous P- and S-wave impedance results unchanged by applying a large damping value to both the P- and the S-wave impedances. We apply a small damping value to the density and enable to recover the density. According to Figures 4b and 5b, the sequential inversion is able to produce P- and the S-wave impedances and the density step by step reliably. Thus, we use this inversion result as the reference for various inversions.

Figures 4c and 5c present the subspace inversion without damping. For the noise-free synthetic example (Figure 4c), all the three elastic parameters are successfully recovered and the result is comparable to the sequential inversion. However, for the noisy synthetic example (Figure 5c), the S-wave impedance and density still show some degree of fluctuation. In order to mitigate this fluctuation, we apply the multi-damping scheme, as illustrated in the previous section, to different wavenumber groups in the subspace domain.

In the subspace domain, each of the elastic parameter has 201 harmonic terms when $M$ is set to be 100. Each of model coefficients ($\xi_p, \xi_s, \xi_d$) has been divided into ten groups
evenly. There are 30 groups in total in the subspace domain. Therefore, the size of the projection matrix $A$ is $(201 \times 3)$ by $(10 \times 3)$ in the subspace inversion. We apply the damping scheme to these groups, using the damping values of equation 12. Using this multi-damped subspace method (Figures 4d and 5d), all the three elastic parameters are recovered from the noisy data. In particular, Figure 5d shows that the inversion result is less affected by the data noise, and the method is more robust. Furthermore, the multi-damped subspace method is even more reliable than the sequential inversion method, since this method deals better with the sensitivities of different wavenumber components.

FIELD DATA APPLICATION

When applying the multi-damped subspace inversion method to the field seismic data, we implement the AVO inversion in the ray parameter domain, following the ray impedance concept (Wang, 2003; Zhang et al., 2012).

This field dataset was acquired from the Ordos Basin, China (Figure 6). The study area is made up of six structural units: the Yimeng uplift, western edge overthrust belt, Tianhuan depression, Yishan ramp, Jinxi flexural belt, and Weibei uplift (Du et al., 2016). In the study area, there are a series of coal-bearing sedimentary rocks deposited in Formation Taiyuan (T) of the Upper Carboniferous and Formation Shanxi (S) of the Lower Permian. The thickness of Formation T varies in 30-60 m, while the thickness of Formation S varies between 100-140 m. Both formations consist of medium to coarse-grained quartz sandstone, thick coal beds, and mudstones, which have been proved as the two most important gas production units in this area.

We transform the migrated common-reflection-point (CRP) gathers from the offset domain (Figure 7a) to the ray-parameter domain (Figure 7b). The transform is a ray-tracing based method, using the same velocity model in migration for generating the CRP gathers. In
order to preserve the lateral continuity and signal-to-noise ratio of the data, it is necessary to locally stack around neighbouring traces for a single ray parameter. For instance, the constant ray parameter at $p = 0.05$ ms/m stacks the neighbouring ray-parameter traces from 0.045 to 0.055 ms/m.

As the input of the AVO simultaneous inversion, totally ten of the constant ray-parameter profiles from $p = 0.01$ to 0.10 ms/m with the interval of 0.01 ms/m are used. The time window of the inversion is set to be 1.3 to 1.6 s, as this time length covers both formations. Figure 7 also presents the trimmed CRP gathers from 1.3s to 1.6 s. The top of T and S horizons are highlighted by blue and black curves respectively.

Figure 8 displays three of the constant ray-parameter seismic profiles with the ray-parameter $p = 0.02, 0.06, 0.10$ ms/m, respectively. Similarly, the top of Formation S and the top of Formation T are picked as blue and black curves respectively. Both the tops of the S and T formations appear as medium to strong peaks. The thickness of Formation S is about 50 to 60 ms while the thickness of Formation T is about 15 to 25 ms.

In this field dataset from the two-way travel time 1.3s to 1.6 s, the average P-wave velocity is around 5000 m/s and thus the maximum ray-parameter of 0.10 ms/m is about incident angle of 30 degrees. These three profiles clearly show that the dominant frequency is generally decreasing when the $p$ value increases, because of $k_x = \omega p$ in which, for a given horizontal wavenumber $k_x$, the ray parameter $p$ and the frequency $\omega$ are reversely proportional (Wang and Houseman, 1997).

For the inversion, we parameterize the 2D model $m(x, z)$ by the 2D Fourier series (Wang and Houseman, 1995; Wang, 1999b):
\( m(x, z) = a_0 + \sum_{i=1}^{N} [a_{i0} \cos(i\Delta k_x x) + c_{i0} \sin(i\Delta k_x x)] \\
+ \sum_{j=1}^{M} [a_{0j} \cos(j\Delta k_z z) + c_{0j} \sin(j\Delta k_z z)] \\
+ \sum_{i=1}^{N} \sum_{j=1}^{M} [a_{ij} \cos(i\Delta k_x x)\cos(j\Delta k_z z) \\
+ b_{ij} \cos(i\Delta k_x x)\sin(j\Delta k_z z) \\
+ c_{ij} \sin(i\Delta k_x x)\cos(j\Delta k_z z) \\
+ d_{ij} \sin(i\Delta k_x x)\sin(j\Delta k_z z)], \) (15)

where \( \Delta k_x = \pi / X \) and \( \Delta k_z = \pi / Z \) are the horizontal and vertical wavenumber interval, respectively, and \( X \) and \( Z \) are the model coverage in \( x \) and \( z \) directions. Hence, in the 2D case, a total number of \( (4MN + 2M + 2N + 1) \) Fourier coefficients to be determined for each elastic parameter. To invert for P-wave impedance, S-wave impedances and the density, we have \( 3 \times (4MN + 2M + 2N + 1) \) Fourier coefficients to be estimated simultaneously. The benefit of using 2D Fourier series is that lateral variances of neighboring traces are taken into consideration, which may well improve the lateral continuity of the inversion results.

Figure 9 shows the initial models for the three elastic parameters. These initial models are first built near the well location based on a low-pass filtered well logging data and then extrapolated along the top of Formation S. Since the bandwidth of the seismic profiles is between 6 to 60 Hz, the low cut frequency of the low-pass filter is set to be 6 Hz. In the inversion, we extract the wavelets statistically corresponding to each constant ray-parameter profile using a mixed-phase wavelet estimation method (Lü and Wang, 2007), which is computed iteratively by minimizing the difference between the fourth-order moment of the estimated wavelet and the windowed fourth-order cumulant of the seismic trace from the constant-phase wavelet.

The coefficients \( \{a, b, c, d\} \) of 2D Fourier series are divided into 16 groups based on the horizontal wavenumber \( k_x \) and vertical wavenumber \( k_z \). We have applied generally large
damping parameters to the high wavenumber components in order to produce robust inversion results. The damping parameters for the 2D field data application are designed as,

\[
\mu = \begin{bmatrix}
0.001 & 0.004 & 0.010 & 0.050 \\
0.004 & 0.020 & 0.050 & 0.085 \\
0.010 & 0.050 & 0.080 & 0.100 \\
0.050 & 0.085 & 0.100 & 0.150
\end{bmatrix}.
\] (16)

Figures 10 and 11 compare the inversion results between conventional AVO simultaneous inversion method and the multi-damped subspace method. Figure 10 shows only the P-wave impedance have been adequately obtained using the conventional AVO simultaneous inversion method. The inversion results of both S-wave impedance and the density show some unstable fluctuations, especially for the latter. This is because that the conventional AVO inversion is difficult to handle the different sensitivities of the three parameters during the inversion. However, the inversion results produced by the multi-damped subspace method (Figure 11) are more reliably estimated with high resolution and good lateral continuity, which may prove that the multi-damped subspace method can effectively balance the different sensitivities of the three parameters. Furthermore, as the multi-damped subspace method estimates the elastic parameters from the 2D Fourier coefficients indirectly, the inversion results take the variances of neighboring traces into consideration and the lateral continuity of the inversion profiles can been greatly enhanced.

According to Figure 11, the top of Formation S is characterized by an increase in the P-wave impedance and density along with a small increase in the S-wave impedance. The top of Formation T is well displayed on the inversion profiles by a strong increase in the P-wave impedance, S-wave impedance and the density. As the gas-bearing layer is usually characterized as the low P-wave impedance, low S-wave impedance, and low density (Marvko et al., 2009), the area with lower P-wave impedance, S-wave impedance, and density may be likely to represent the gas-bearing reservoirs. A decrease in the P-wave
impedance, S-wave impedance and the density can be observed in both formations, which may well indicate that both formations have the potential targets for gas exploration.

Figure 12 compares the conventional results (green curves) and multi-damped subspace inversion results (red curves), the initial models (blue curves), well logs (grey curves) and 60 Hz-high-cut filtered well logs (brown curves) near a well (at the distance of approximately 11 km) for three elastic parameters, P-wave impedance, S-wave impedance and the density, respectively. Although P-wave and S-wave impedances show good agreements with the well logs when using both conventional and multi-damped subspace methods, there are unstable fluctuations appearing 1.46s to 1.49s, which deviates from both the initial models and the well logs. However, the multi-damped subspace method can produce more robust inversion results under the constraint of the initial models than those produced by the conventional method. We have also used the RMS error to quantify the inversion results. The RMS errors between the initial models and the well logs for P-wave impedance, S-wave impedance and the density are 1.314 km/s·g/cm$^3$, 0.736 km/s·g/cm$^3$ and 0.118 g/cm$^3$, while the RMS errors between the inverted models using multi-damped subspace method and the well logs decreased to 0.974 km/s·g/cm$^3$, 0.495 km/s·g/cm$^3$ and 0.095 g/cm$^3$. Such decreases in term of the RMS error may well indicate the effectiveness of the subspace inversion method. In addition, we have compared 60 Hz-high-cut filtered well logs, which is the same as the highest input seismic frequency, to compare with the inversion results. And we can observe that the results from the multi-damped subspace method show a better match to the high-cut filtered well logs than those using the conventional method.

Figure 13 presents the correlation coefficient between inverted results and well logs by using the multi-damped subspace method and the conventional method. The correlation coefficients from the conventional method are 0.916, 0.859 and 0.808 for P-wave impedance, S-wave impedance and the density, respectively. However, by using multi-damped subspace
method, these correlation coefficients have increased to 0.961, 0.947 and 0.872 for P-wave impedance, S-wave impedance and the density, respectively. The improvement in the correlation coefficients may also validates the robustness of the multi-damped method.

CONCLUSION

In seismic AVO inversion, the multi-damped subspace method is capable to invert for three elastic parameters simultaneously. In the subspace implementation, we have parameterized the elastic parameters using the truncated Fourier series and partitioned the coefficients of different ranges from low- to high-wavenumber components into different subspaces. We have also applied the multi-damping scheme to balance the different sensitivities during the inversion procedure in the subspace domain. For the inversion of field seismic data, we have performed the AVO simultaneous inversion in the ray-parameter domain. The results have shown a high resolution and a good lateral continuity of the elastic parameters, which is very useful for reservoir characterization.

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**FIGURE CAPTION**

**Figure 1.** A 1D model: The P-wave impedance, S-wave impedance, density, and the synthetic seismic traces.

**Figure 2.** (a) Sensitivity analysis of the Hessian matrix $H$. (b) Sensitivity analysis of the multi-damped Hessian matrix $H_{damp}$. In each subfigures, the top are the eigenvalues and the corresponding eigenvectors, and the bottom is the resolution matrix.

**Figure 3.** Comparison of diagonal values of the resolution matrices obtained from undamped Hessian matrix $H$ (blue) and multi-damped Hessian matrix $H_{damp}$ (red), respectively. The multi-damped Hessian matrix shows higher resolution in general.

**Figure 4.** Comparison of inversion results using noise-free synthetic datasets. (a) The conventional inversion result. (b) The sequential inversion result. (c) The subspace inversion without damping. (d) The subspace inversion with multi-damping. The blue curve represents the true model, the red curve represents the inversion results and the black dashed curve is the initial model. Each model consists of P-wave impedance, S-wave impedance and density.

**Figure 5.** Comparison of inversion results using noisy synthetic datasets. (a) The conventional inversion result. (b) The sequential inversion result. (c) The subspace inversion without damping. (d) The subspace inversion with multi-damping. The blue curve represents the true model, the red curve represents the inversion results and the black dashed curve is the initial model. Each model consists of P-wave impedance, S-wave impedance and density.

**Figure 6.** Location map of the study area (red box), highlighting the shape of the Ordos basin.
with its six main structural units: the Yimeng uplift, western edge overthrust belt, Tianhuan depression, Yishan ramp, Jinxi flexural belt, and Weibei uplift.

**Figure 7.** Common-reflection-point (CRP) gathers are transformed from the offset domain. (a) A CRP gather in the offset domain and a trimmed portion from 1.3 to 1.6 s. (b) The CRP gather in the ray-parameter domain and the trimmed portion from 1.3 to 1.6 s. The top of Formation T and the top of Formation S are marked as black and blue curves, respectively.

**Figure 8.** Constant ray-parameter profile with (a) $p = 0.02$ ms/m, (b) $p = 0.06$ ms/m, (c) $p = 0.10$ ms/m. Note that only these three constant ray parameter profiles are selected for display. The top of Formation T and the top of Formation S are marked as black and blue curves, respectively.

**Figure 9.** The initial models used in the subspace inversion. (a) The P-wave impedance. (b) The S-wave impedance. (c) The density.

**Figure 10.** The conventional AVO inversion results. (a) The P-wave impedance. (b) The S-wave impedance. (c) The density. The top of the Taiyuan (T) and Shanxi (S) formation are delineated as black and blue curves, respectively.

**Figure 11.** The inversion results of the multi-damped subspace method at seismic line 823. (a) The P-wave impedance. (b) The S-wave impedance. (c) The density. The top of the Taiyuan (T) and Shanxi (S) formation are delineated as black and blue curves, respectively.
Figure 12. Comparison of conventional results (green curves) and multi-damped subspace inversion results (red curves), the initial models (blue curves), well logs (grey curves) and 60 Hz high-cut filtered well logs (brown curves) near a well (at the distance of approximately 11 km). (a) P-wave impedance. (b) S-wave impedance. (c) The density.

Figure 13. Comparison of correlation coefficients between inverted results and well logs by using the multi-damped subspace method and the conventional method for three elastic parameters, P-wave impedance, S-wave impedance and the density, respectively.
Figure 1. A 1D model: The P-wave impedance, S-wave impedance, density, and the synthetic seismic traces.

158x61mm (300 x 300 DPI)
Figure 2. (a) Sensitivity analysis of the Hessian matrix $H$. (b) Sensitivity analysis of the multi-damped Hessian matrix $H_{damp}$. In each subfigure, the top are the eigenvalues and the corresponding eigenvectors, and the bottom is the resolution matrix.

150x170mm (300 x 300 DPI)
Figure 3. Comparison of diagonal values of the resolution matrices obtained from undamped Hessian matrix $H$ (blue) and multi-damped Hessian matrix $H_{damp}$ (red), respectively. The multi-damped Hessian matrix shows higher resolution in general.
Figure 4. Comparison of inversion results using noise-free synthetic datasets. (a) The conventional inversion result. (b) The sequential inversion result. (c) The subspace inversion without damping. (d) The subspace inversion with multi-damping. The blue curve represents the true model, the red curve represents the inversion results and the black dashed curve is the initial model. Each model consists of P-wave impedance, S-wave impedance and density.
Figure 5. Comparison of inversion results using noisy synthetic datasets. (a) The conventional inversion result. (b) The sequential inversion result. (c) The subspace inversion without damping. (d) The subspace inversion with multi-damping. The blue curve represents the true model, the red curve represents the inversion results and the black dashed curve is the initial model. Each model consists of P-wave impedance, S-wave impedance and density.
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119x158mm (300 x 300 DPI)
Figure 9. The initial models used in the subspace inversion. (a) The P-wave impedance. (b) The S-wave impedance. (c) The density.

106x126mm (300 x 300 DPI)
Figure 10. The conventional AVO inversion results. (a) The P-wave impedance. (b) The S-wave impedance. (c) The density. The top of the Taiyuan (T) and Shanxi (S) formation are delineated as black and blue curves, respectively.

110x127mm (300 x 300 DPI)
Figure 11. The inversion results of the multi-damped subspace method at seismic line 823. (a) The P-wave impedance. (b) The S-wave impedance. (c) The density. The top of the Taiyuan (T) and Shanxi (S) formation are delineated as black and blue curves, respectively.
Figure 12. Comparison of conventional results (green curves) and multi-damped subspace inversion results (red curves), the initial models (blue curves), well logs (grey curves) and 60 Hz high-cut filtered well logs (brown curves) near a well (at the distance of approximately 11 km). (a) P-wave impedance. (b) S-wave impedance. (c) The density.

106x87mm (300 x 300 DPI)
Figure 13. Comparison of correlation coefficients between inverted results and well logs by using the multi-damped subspace method and the conventional method for three elastic parameters, P-wave impedance, S-wave impedance and the density, respectively.

73x42mm (300 x 300 DPI)