Elasto-plastic behaviour and design of semi-compact circular hollow sections

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Highlights

- Existing cross-section and member buckling test data on CHS were collected;
- Numerical simulation was conducted to generate further data on CHS;
- New design rules were developed to capture the elasto-plastic behaviour of semi-compact CHS;
- Accuracy of the current Eurocode 3 and proposed methods were assessed and compared;
- Reliability of the proposed design rules was confirmed through statistical analyses.
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Abstract

Previous research has revealed shortcomings in the current Eurocode 3 (EC3) provisions for the design of semi-compact (Class 3) cross-sections. These shortcomings arise primarily from the lack of utilisation of partial plastification in bending, leading to a step in the design resistance function at the boundary between Class 2 and 3 cross-sections and an underestimation of the available capacity. This affects the accuracy of resistance predictions in bending and under combined loading, and applies at both cross-sectional and member level. To address this issue, the use of an elasto-plastic section modulus, which lies between the plastic and elastic section moduli, has been proposed and employed in the design of semi-compact I- and box sections. The aim of the present study is to develop new cross-section and member buckling design rules incorporating the elasto-plastic section modulus for semi-compact circular hollow sections (CHS), and to assess their accuracy against existing experimental and freshly generated numerical data. Firstly, an experimental database, consisting of previous cross-section and member buckling test results on steel CHS, was established. A comprehensive numerical simulation programme was subsequently carried out; in this programme, finite element (FE) models were developed, validated and used for parametric studies, where over 600 numerical structural performance data on semi-compact CHS were generated. New sets of cross-section and member buckling design expressions featuring elasto-plastic section properties were then proposed and assessed against the test and numerical data. The proposals were shown to offer improved accuracy and design efficiency over
the elastic EC3 methods. The reliability of the proposed elasto-plastic design rules was then confirmed through statistical analyses in accordance with EN 1990, demonstrating their suitability for inclusion into the next revision of EN 1993-1-1.

**Keywords:** circular hollow sections; cross-section design; member buckling design; numerical simulation; elasto-plastic section modulus; reliability analysis; semi-compact sections

1. **Introduction**

Structural steel circular hollow sections (CHS) have been since the early 19th century [1]. Owing to their unique aesthetic appearance and favourable structural properties, CHS have gained widespread use in the construction industry, serving as various structural components such as columns, bracing members and truss elements. The two most common production methods for structural steel CHS are hot-rolling and cold-forming. For larger diameter tubes, longitudinal or spiral welding [2] is typically used. Structural design rules for CHS are provided in EN 1993-1-1:2005 [3], while the design of shells is treated in EN 1993-1-6:2007 [4].

EN 1993-1-1:2005 [3] features the concept of cross-section classification to account for the influence of local instability. For Class 1 and 2 cross-sections, the design resistance is taken as the full plastic cross-section capacity; for Class 3 (semi-compact) cross-sections, only the elastic cross-section resistance is utilised due to premature local buckling. This leads to a discontinuity in the resistance function at the boundary between Class 2 and 3 cross-sections and an underestimation of the load-carrying capacity for Class 3 cross-sections, especially when the local slenderness approaches the Class 2 limit. This shortcoming is particularly pronounced for CHS due to their high shape factor (i.e. the plastic section modulus divided by the elastic section modulus). To address this issue, new design rules, featuring a linear transition between the elastic and plastic section capacities over the semi-compact range, as illustrated in Fig. 1(a) and 1(b), have been proposed in [5] for I-sections and box sections,
based on the findings of two recent European research projects – SEMI-COMP [6] and SEMI-COMP+ [7].

The aims of the present paper are to extend this new approach to the design of semi-compact CHS and to assess the accuracy and reliability of the proposals. A review of existing experimental work and the establishment of an experimental database for structural steel CHS are initially described in Section 2. Owing to the limited test data in the semi-compact range, a numerical simulation programme is conducted to expand the current test data pool, as presented in Section 3. Finite element (FE) models are established and validated against the test results, and parametric studies are subsequently carried out using the validated models for the generation of additional structural performance data. The design of semi-compact CHS is then addressed in Sections 4 and 5 at the cross-section and member buckling levels respectively. Following a brief description of the existing elastic EC3 design rules, new design expressions for semi-compact CHS, incorporating the elasto-plastic section capacity, are proposed. Assessment of the elastic EC3 approach and the proposed elasto-plastic approach is carried out through comparisons against the test and numerical data, from which the advantages of the elasto-plastic proposal over the elastic EC3 methods are revealed. Subsequent statistical analyses confirm the reliability of the proposed design methods, and thus demonstrate their suitability for incorporation into the next revision of EN 1993-1-1 [8]. The development and verification of equivalent rules for structural steel elliptical hollow sections (EHS) [9] have been presented in [10].

2. Existing experimental studies on steel CHS

2.1. General

A review of the existing laboratory tests on structural steel CHS is presented in this section. Previous tests on CHS with the local slenderness \( D / t \varepsilon^2 \leq 240 \) (where \( D \) is the outer diameter, \( t \) is the thickness and \( \varepsilon^2 = 235 / f_y \), with \( f_y \) being the material yield strength), which is the limit of applicability of the upcoming rules for CHS in prEN 1993-1-1:2018 [8], are considered, while those with the local
slenderness $D/te^2 > 240$ are beyond the scope of this review. The established experimental database includes a total of 626 test data made up of 438 cross-section tests and 188 member buckling tests, covering the load cases of axial compression, uniform bending and combined compression plus bending. The database is used for the calibration of numerical models and the assessment of the design methods in the following sections.

2.2. Cross-section tests

Early experimental investigations into the cross-sectional load-carrying capacity of CHS under axial compression were carried out by Greene [11] in 1924. In the 1970s, stub column tests were carried out by Bouwkamp [12] on seamless and welded CHS and by Marzullo and Ostapenko [13] and Chen and Ross [14] on fabricated cylindrical tubes. More recent years have witnessed extensive experimental work on hot-rolled and cold-formed CHS, with examples including Elchalakani, Zhao and Grzebieta [15], Ma, Chan and Young [16], Xiong, Xiong and Liew [17] and Meng and Gardner [18].

Korol and Huboda [19], Wilhoit Jr and Merwin [20] and Sherman [21] were among the first researchers to study the flexural behaviour of CHS by means of laboratory testing. A comprehensive testing programme investigating the rotation and load-carrying capacities of CHS beams was reported by Sedlacek et al. [22]. With the increasing application of high strength steels in the construction industry, tests on high strength steel CHS beams have been performed by Jiao and Zhao [23] and Ma, Chan and Young [24]. Note that relatively high levels of scatter are often observed in collections of bending test data due to the range of adopted test setups (e.g. three-point bending or four-point bending, where higher capacities are typically obtained in the former case because of the beneficial effect of the moment gradient [25]) and local stiffening methods at the points of load introduction and support, where inadequate stiffening can result in premature failure expedited by local deformation of the cross-section from concentrated transverse forces.

Available test data on CHS under combined compression plus bending at the cross-sectional level are relatively scarce. Short beam-column tests were performed by Prion and Birkemoe [26], O'Shea and Bridge [27] and Pan [28] on fabricated cylindrical tubes, and by Nseir [29], Ma [30], Pournara et al.
[31] and Meng and Gardner [18] on hot-rolled and cold-formed CHS. A full list of the sources associated with the cross-section tests on CHS is shown in Table 1.

2.3. Member buckling tests

Early tests on slender CHS columns were carried out by Wilson [61] in 1937 to investigate the global buckling behaviour of large-diameter fabricated cylindrical tubes. Later, in the 1970s, further column buckling tests were reported by Bouwkamp [12] and Chen and Ross [14]; more recent experimental studies include those by Javidan et al. [62], Shi et al. [63], Ma [30] and Pournara et al. [31]. Laboratory testing has also been conducted by Young and Hartono [64] and Buchanan, Real and Gardner [65] on stainless steel CHS columns. The assembled column test data exhibit relatively high scatter; this is typical of collections of column buckling test results obtained from a wide range of sources and attributed to a number of causes, including variation in test boundary conditions and the sensitivity of column buckling resistance to global imperfections and initial loading eccentricities.

To understand the member buckling behaviour of CHS under compression plus bending, early beam-column tests were conducted by Wagner [66], featuring both constant bending moment and moment gradients. Prion and Birkemoe [26] went on to carry out further tests on fabricated CHS beam-columns, while rolled CHS specimens were tested by Linzell, Zureick and Leon [67]. In recent years, a number of beam-column tests on hot-rolled and cold-formed CHS were performed by Ma [30], Hayeck [40] and Pournara et al. [31]. A full list of the sources of the CHS member buckling tests is presented in Table 2.

3. Numerical simulations

3.1. General

In parallel to the collection of test data, a numerical simulation programme has been carried out to expand the test database. Finite element (FE) models of CHS were initially established and then
validated against the previous test data, with details described in Sections 3.2 and 3.3. Parametric studies were then performed using the validated FE models, from which structural performance data were generated over a wider spectrum of practical local slendernesses, material properties, load combinations and member lengths. The obtained numerical data, combined with the collated test results, are utilised for the assessment of the design methods in the following sections.

3.2. **Key modelling assumptions**

Geometrically and materially nonlinear analyses with imperfections (GMNIA) were performed using the static Riks solver in Abaqus 2016 [75] to simulate the structural response of the circular hollow section members. The key features of the models are described herein. The four-noded doubly curved shell element with reduced integration and finite membrane strains S4R [75] was employed, which is typically used for the numerical simulation of thin-walled structural elements [10,65,76]. The constitutive material properties were introduced into Abaqus in the form of true stress and logarithmic plastic strain. Symmetry about the mid-length plane and the plane perpendicular to the axis of bending was exploited by the use of quarter-models of the CHS, as illustrated in Fig. 2, with suitable symmetry boundary conditions applied to the mid-length section and two longitudinal edges of the models. All degrees of freedoms at the end sections were coupled to a reference point through kinematic coupling, and boundary conditions were applied to the reference point in accordance with the corresponding test conditions. A fine mesh with an element size equal to \(0.1\sqrt{D/t}\), where \(D\) is the outer diameter and \(t\) is the thickness, was found to be sufficiently refined to accurately capture the local buckling behaviour of the modelled CHS. This fine mesh was applied at the mid-span of the members where local buckling is expected, while for the remainder of the model, a coarser mesh with an element size of \(0.5\sqrt{D/t}\) was adopted. For the stub columns, the fine mesh was adopted over the full member length. The same meshing strategies were also used in [10] for EHS, which were shown to be able to accurately reproduce the structural behaviour while maintaining good computational efficiency.

Both local and global geometric imperfections were incorporated into the FE models. The elastic local buckling mode shapes from a linear bifurcation analysis (LBA) of the modelled sections, but with a
modified thickness \( t_{\text{mod}} = D/5 \) (referred to as LBA-\( t_{\text{mod}} \) mode shape), were used to represent the local imperfection patterns [10]; this avoids unrealistically short buckling wavelengths that were observed in the LBA results for CHS with relatively high \( D/t \) values. For comparison purposes, typical buckling mode shapes from LBA-\( t \) models (i.e. LBA with the real thickness \( t \)) and LBA-\( t_{\text{mod}} \) models (i.e. LBA with \( t_{\text{mod}} = D/5 \)) are displayed in Fig. 3. The local imperfection amplitudes \( \omega_l \) were generated from the predictive formula given by Eq. (1), which was proposed in [10] on the basis of experimental data. Further details on the adopted local imperfection modelling technique can be found in [10]. The global imperfection profile took the form of the lowest elastic global buckling mode. Explicit incorporation of residual stresses into the numerical models was deemed unnecessary for both hot-rolled and cold-formed CHS, for the same reasons as explained in [10].

\[
\omega_l = 0.01\sqrt{Dt}
\]  

3.3. Validation

In order to validate the developed FE models, the models were initially used to simulate selected tests and replicate the observed experimental responses including the ultimate loads, load-deformation histories and failure modes. Experimental data from 37 stub column tests, 32 four-point bending tests and fifteen short beam-column tests were used to validate the FE models for the simulation of the cross-sectional behaviour of CHS (referred to as cross-section models); the FE models for the simulation of CHS at the member buckling level (referred to as member models) were validated against 37 test data on CHS columns and 21 test data on CHS beam-columns. To model the material behaviour, the experimental stress-strain relationships were used when available; otherwise, the predictive models developed by Yun and Gardner [77] and Gardner and Yun [78] were employed to generate the stress-strain curves for hot-rolled and cold-formed steel sections respectively. The input parameters for the predictive models, including the Young’s modulus \( E \), yield strength \( f_y \), ultimate strength \( f_u \) and, if available, the corresponding ultimate strain \( \varepsilon_u \), were taken as the reported test values in the validation study.
The accuracy of the developed cross-section models is firstly assessed. Two types of local imperfection patterns – the LBA-\( t \) pattern (i.e. the local buckling mode shape associated with the modelled geometry with the actual thickness \( t \)), and the LBA-\( t_{\text{mod}} \) pattern (i.e. the local buckling mode shape associated with the modelled geometry but with a modified thickness \( t_{\text{mod}} = D/5 \)), as defined in [10], were compared; the local imperfection amplitudes \( \omega_l \) from the predictive formula of Eq. (3) were used to factor the local imperfection patterns. Geometrically and materially nonlinear analyses with no imperfections (GMNA), i.e. \( \omega_l = 0 \), were also carried out in parallel to investigate the sensitivity of the FE models to different forms of local imperfection pattern and different levels of imperfection amplitude. The average ratios of the predicted strength from the FE models to the test strength (i.e. \( u_{\text{FE}}/u_{\text{test}} \) and \( M_{u,\text{FE}}/M_{u,\text{test}} \)), together with the corresponding coefficient of variation (COV), are reported in Table 3 for various loading scenarios. For the case of axial compression (\( N \)), accurate resistance predictions were obtained for all types of local geometric imperfection, and the predictions were shown to be insensitive to variation of the local imperfection patterns and amplitudes in the considered local slenderness range. For the cases of bending (\( M \)) and combined compression plus bending (\( N + M \)), the predicted resistances associated with the LBA-\( t \) local imperfection pattern were shown to be, on average, overly conservative, with an unduly high degree of sensitivity to the local imperfection amplitude; however, the sensitivity was reduced when the LBA-\( t_{\text{mod}} \) pattern was employed, and excellent accuracy and consistency were achieved, as reflected by the mean and COV values. Good agreement in terms of the load-deformation curves between the test and FE results was also achieved with the LBA-\( t_{\text{mod}} \) pattern, as shown by the typical experimental and numerical load-deformation curves in Figs. 3(a) and 3(b). Overall, the FE models using the modified local imperfection pattern LBA-\( t_{\text{mod}} \) and the predictive formula for the imperfection amplitude given by Eq. (3) yielded the most accurate simulation of the cross-sectional responses of CHS, and these models are therefore deemed suitable for use in the parametric studies to generate further cross-section resistance data.

Having confirmed the accuracy of the cross-section models, the CHS member models are now assessed. In addition to the local imperfections, four groups of global imperfection amplitudes \( \omega_g \) were investigated: (1) combined measured \( \omega_g \) plus measured initial loading eccentricities \( e_0 \) (for CHS
columns only); (2) \( \omega_g = L / 2000 \); (3) \( \omega_g = L / 1000 \) (i.e. the value used for the formulation of the column buckling curves in Eurocode 3 [79,80]); and (4) \( \omega_g = L / 500 \) (i.e. a representative value of the out-of-straightness tolerance for hollow sections specified in EN 10210-2:2006 [81] and EN 10219-2:2006 [82]). The mean and COV values of the normalised member buckling strength predictions \( N_{u,FE} / N_{u,load} \) are summarised in Table 4. The statistical results indicate that the buckling strengths of the CHS columns were rather sensitive to the global imperfection amplitude, while the buckling strengths of the CHS beam-columns exhibited a reduced level of sensitivity. In general, excellent agreement between the test and predicted resistances in terms of accuracy and consistency was achieved by the FE models with \( \omega_g = L / 1000 \). Comparisons between typical load-deformation curves from the experiments and FE models are shown in Fig. 3(c), where excellent agreement is also observed.

The presented FE modelling approach has also been validated against a series of tests on both hot-rolled and cold-formed elliptical hollow sections, which feature similar material properties, geometric imperfections, residual stresses and general structural response to the CHS studied herein. Thus, the developed member models, along with the cross-section models, are considered to be suitable for utilisation in the parametric studies to generate further CHS cross-section and member buckling resistance data.

3.4. Parametric studies

Upon completion of the validation study, a parametric investigation was conducted using the validated FE models to produce numerical data on semi-compact CHS over a wider range of material properties, local and global slendernesses and load combinations. In the parametric studies, the outer diameter \( D \) of the CHS was set to be 100 mm for all the modelled members, with the thickness \( t \) varied to achieve a range of \( D / te^2 \) values. Two production routes – hot-rolled and cold-formed, and two material grades – S355 and S460, were considered; the corresponding stress-strain curves were generated from the predictive models of Yun and Gardner [77] and Gardner and Yun [78], with the input parameters \( E, f_y \) and \( f_u \) taken as the nominal values specified in EN 1993-1-1:2005 [3], as listed in Table 5. In the cross-
section models for axial compression and combined loading, the modelled member length was set to $3D$, which is deemed long enough for local buckling to freely develop but short enough to avoid any significant influence from global buckling [80]. To produce moment capacity data, the flexural length of the model was set to be sufficiently long to account for the ovalisation effect; this length was chosen such that $\Omega$ was equal to 7, where $\Omega$ is the dimensionless length of the CHS, as defined by Eq. (2), and $\Omega = 7$ corresponds to the long cylinder domain where the influence from ovalisation is stable and essentially at its most severe [76]. To expand the current CHS member buckling test data pool, in addition to the various material types and local slendernesses, a range of global slendernesses, with eight values of $\lambda$ from 0.25 to 2 for columns and four values of $\lambda$ from 0.5 to 2 for beam-columns, were considered, where $\lambda$ is the non-dimensional column buckling slenderness as defined in EN 1993-1-1:2005 [3]. In both the cross-section and member models under combined compression plus bending, different load combinations were achieved by varying the initial loading eccentricities, with nine eccentricities employed for each case to enable the shapes of the interaction curves to be accurately captured. In total, 660 structural performance data on semi-compact CHS were numerically generated from the parametric studies. A summary of the obtained test and numerical data is provided in Table 6.

$$\Omega = \frac{L}{D} \sqrt{\frac{8t}{D}}$$  \hspace{1cm} (2)

4. Cross-section design of semi-compact CHS

4.1. General

The cross-section design of semi-compact (Class 3) circular hollow sections (CHS) is addressed in this section. A summary of the new Class 3 slenderness limits for CHS to be included in the next revision of EN 1993-1-1 [8] is initially provided in Section 4.2, along with a description of the current elastic EC3 cross-section design rules for semi-compact CHS. Following this, a new design approach, featuring the previously introduced elasto-plastic design concept, is developed to exploit the additional cross-section resistance arising from the partial plastification and presented in Section 4.3. The elasto-plastic
design proposals are illustrated in Fig. 1(a) for pure bending and in Fig. 1(b) for combined loading, where \( M_{pl} \) and \( M_{el} \) are the elastic and plastic moment capacities respectively, \( M_{ep} \) is the proposed elasto-plastic moment capacity (defined in Section 4.3) and \( N_{pl} \) is the plastic load. The improvement of the new elasto-plastic cross-section design approach over the elastic EC3 design provisions is revealed in Section 4.4 through comparisons against the experimental and numerical results. Standard statistical analyses in accordance with EN 1990:2002 [83] are subsequently carried out on the design proposals to evaluate its reliability through the derivation of partial safety factors. Note that the design methods are assessed based on the measured (or modelled) material and geometric properties, unfactored resistance predictions and proportional loading.

4.2. **Elastic EC3 cross-section design rules for semi-compact CHS**

The Class 3 slenderness limits for CHS will be updated in the next revision of EN 1993-1-1, following the recommendations of Chan and Gardner [84]. In the current version of EN 1993-1-1:2005 [3], the same Class 3 limit of \( D / t e^2 = 90 \) is used for all loading scenarios including axial compression, bending and combined compression plus bending. In the next revision of EN 1993-1-1 [8], the Class 3 slenderness limit in bending is due to be relaxed to 140, as proposed in [84] on the basis of the experimental and numerical data on CHS and EHS in bending. An intermediate slenderness limit between 90 (pure compression) and 140 (pure bending), based on the parameter \( \psi \), which is the ratio of the minimum stress to the maximum stress over the cross-section depth (with compression positive) assuming an elastic stress distribution, is to be introduced for the case of combined compression plus bending. The expression for \( \psi \) for CHS is given by Eq. (3), where \( N_{Ed} \) and \( M_{Ed} \) are the design applied compression and bending moment, \( A \) is the cross-sectional area and \( W_{el} \) is the elastic section modulus.

The existing and new Class 3 slenderness limits for CHS are summarised in Table 7. The new slenderness limits are used for the classification of the circular hollow sections throughout the present study.

\[
\psi = \frac{\left( \frac{N_{Ed}}{A} \right) - \left( \frac{M_{Ed}}{W_{el}} \right)}{\left( \frac{N_{Ed}}{A} \right) + \left( \frac{M_{Ed}}{W_{el}} \right)}
\] (3)
The elastic EC3 cross-section design rules for semi-compact CHS are now described. The resistance of a semi-compact section is characterised by first yield in the current EC3 cross-section design framework, with no account taken of the benefits from the partial spread of plasticity. More specifically, for the case of axial compression, the design resistance to axial load $N_{c,Rd}$ is equal to the plastic load $N_{pl,Rd}$, as given by Eq. (4), where the subscript $Rd$ denotes the design value of the resistance, $A$ is the cross-sectional area, $f_y$ is the yield strength and $\gamma_{Mo}$ is the partial factor for the resistance of cross-sections. For the case of pure bending, the cross-sectional bending moment resistance $M_{c,Rd}$ is limited to the elastic moment capacity $M_{el,Rd}$, as given by Eq. (5), where $W_{el}$ is the elastic section modulus. For semi-compact CHS under combined axial compression and bending, the linear interaction formula between $N_{pl,Rd}$ and $M_{el,Rd}$ given by Eq. (6) is employed, where the subscript $Ed$ denotes the design value of the effect of the actions.

$$N_{c,Rd} = N_{pl,Rd} = \frac{Af_y}{\gamma_{Mo}}$$  \hspace{1cm} (4)

$$M_{c,Rd} = M_{el,Rd} = \frac{W_{el}f_y}{\gamma_{Mo}}$$  \hspace{1cm} (5)

$$\frac{N_{Ed}}{N_{pl,Rd}} + \frac{M_{Ed}}{M_{el,Rd}} \leq 1$$  \hspace{1cm} (6)

### 4.3. Proposed elasto-plastic cross-section design rules for semi-compact CHS

As discussed previously, there exists a step in the current EC3 design resistance function at the boundary between Class 2 and 3 cross-sections due to the lack of account for partial plastification, which leads to overly conservative resistance predictions under pure bending and combined loading. To address this shortcoming, a new design approach featuring a linear transition between the elastic and plastic section properties has been developed [5] and has been included in Annex B of the upcoming revision to EN 1993-1-1 [8], covering I- and box sections. This elasto-plastic design approach is now extended to CHS herein and presented as follows. For axial compression, the EC3 design formula given by Eq. (4) remains applicable. For load cases involving bending, the elastic section modulus $W_{el}$ is replaced by the elasto-plastic section modulus $W_{ep}$ to exploit the additional resistance arising from partial plastification,
where \( W_{ep} \) is calculated based on a linear transition between \( W_{el} \) and the plastic section modulus \( W_{pl} \) with respect to the local slenderness \( D / \tau e^2 \), as given by Eqs (7) and (8).

\[
W_{ep} = W_{pl} - (W_{pl} - W_{el}) \beta_{ep} \tag{7}
\]

\[
\beta_{ep} = \max \left( \frac{D / \tau - 70e^2}{70e^2}; 0 \right) \text{ but } \beta_{ep} \leq 1 \tag{8}
\]

Having defined the elasto-plastic section modulus \( W_{ep} \), new design expressions employing \( W_{ep} \) in place of \( W_{el} \) are subsequently formulated. For the case of pure bending, the moment resistance is taken as the elasto-plastic design moment resistance \( M_{ep,Rd} \), as given by Eq. (9). For the case of combined compression plus bending, a similar linear interaction formula to Eq. (6), but with \( M_{ep,Rd} \) adopted in place of \( M_{el,Rd} \) as the end point, as given by Eq. (10), is proposed.

\[
M_{el,Rd} = M_{ep,Rd} = \frac{W_{ep} f_y}{\gamma_M} \tag{9}
\]

\[
\frac{N_{el}}{N_{pl,Rd}} + \frac{M_{el}}{M_{ep,Rd}} \leq 1 \tag{10}
\]

4.4. Assessment of design methods

The elastic EC3 cross-section design rules and the proposed elasto-plastic approach are assessed through comparisons between the test and FE resistances \( R_u \) and the design predictions \( R_{u,pred} \), as illustrated in Fig. 4. For a semi-compact CHS under axial compression, the design resistance is equal to the plastic resistance \( N_{pl} \) for both approaches. The ultimate loads derived from the tests and numerical simulations are normalised by \( N_{pl} \), arranged by production route and plotted against the corresponding local slenderness \( D / \tau e^2 \) in Fig. 5. The majority of the data points lie above unity (i.e. on the safe side), with the mean values of \( N_u / N_{u,pred} \) equal to 1.012 and 1.095 for hot-rolled and cold-formed CHS respectively and the corresponding COV values equal to 0.033 and 0.065, as reported in Table 8, where \( el \) and \( ep \) denote the elastic EC3 design approach and the proposed elasto-plastic approach respectively.

Both the graphical and statistical results show that the current design expression is appropriate, and its reliability is verified by means of statistical analyses in the next subsection.
For the case of uniform bending, the test and FE moment resistances $M_u$ are normalised by $M_{el}$ for the elastic EC3 and by $M_{ep}$ for the proposed elasto-plastic approach and plotted against the corresponding $D/t_e^2$ values in Figs 6(a) and 6(b) for hot-rolled and cold-formed CHS respectively. The elastic EC3 design expression yields consistently conservative predictions of moment resistance over the semi-compact range, while the proposed expression improves the accuracy in the lower $D/t_e^2$ domain due to the utilisation of the additional resistance from partial plastification through incorporation of the elasto-plastic section modulus. This improvement is also revealed by comparisons of the statistical results in Table 8, where the mean values of $M_u/M_{u,pred}$ are reduced (i.e. improved) through application of the new elasto-plastic design approach from 1.259 to 1.108 and from 1.361 to 1.187 for hot-rolled and cold-formed CHS respectively. Note that the predictions from both approaches in the higher $D/t_e^2$ domain are still overly conservative, implying that the new Class 3 slenderness limit for CHS in bending, while offering a significant improvement over the existing provisions, could potentially be relaxed further. This is consistent with the findings of Meng and Gardner [10] for EHS, and further research is required on this issue.

For the case of combined compression plus bending, the $\theta$ parameter is defined to describe the ratio between axial compression and bending, as illustrated in Fig. 4 and defined by Eq. (11). The value of the $\theta$ parameter varies from $0^\circ$ to $90^\circ$, with $\theta = 0^\circ$ corresponding to pure uniaxial bending and $\theta = 90^\circ$ corresponding to pure axial compression. The cross-section resistances $R_u$ derived from the tests and numerical simulations are normalised by the resistance predictions $R_{u,pred}$ and plotted against the $\theta$ parameter in Figs 7(a) and 7(b) for the hot-rolled and cold-formed CHS respectively. It can be observed that both approaches are accurate in the higher $\theta$ domain (i.e. where compression is more dominant); however, the proposed elasto-plastic approach exhibits improved accuracy in the lower $\theta$ domain (i.e. where bending is more dominant) over the elastic EC3 approach due to the adoption of the more accurate end point (i.e. $M_{ep}$) to the interaction curve. This is further reflected in Table 8, where a considerable reduction in the mean and COV values of $R_u / R_{u,pred}$ is achieved through application of
the proposals made herein, confirming that the proposed elasto-plastic approach yields more accurate
and less scattered predictions than the elastic EC3 approach.

\[ \theta = \tan^{-1} \left( \frac{N_a / N_{e,pred}}{M_a / M_{e,pred}} \right) \]  

(11)

4.5. Reliability analysis

In the structural Eurocodes, partial safety factors \( \gamma_M \) are applied to the resistance functions in order to
ensure an appropriate level of reliability. In this section, the partial safety factors \( \gamma_{M0} \) are derived for the
proposed cross-section design expressions following the procedure outlined in EN 1990:2002 [83] to
assess their reliability. The values of the material over-strength \( f_{y,m} / f_{y,n} \), i.e. the ratio of the mean to
the nominal yield strengths, and the corresponding coefficient of variation \( V_{fy} \), were taken from Annex
E of the upcoming revision to EN 1993-1-1 [8], as summarised in Table 9. The COV of the cross-
sectional area \( V_A \) of the CHS was taken equal to 0.025; this value was calculated using the formulae set
out in [85] and the variability of dimensional parameters provided in Annex E of the upcoming revision
to EN 1993-1-1 [8]. To determine the mean value correction factor \( b \), the least squares method is
recommended in EN 1990:2002 [83]; however, this approach has been shown to bias the derived value
of \( b \) towards the data with higher failure loads and was therefore not employed in the present study.
Instead, as recommended by [86], the \( b \) parameter was calculated by means of averaging the ratios
between the experimental or numerical resistance \( r_{t,i} \) and the design resistance \( r_{e,i} \), as given by Eq. (12),
which ensures an equivalent weighting is given to each of the considered tests and simulations in the
calculation of \( b \). The design fractile factor \( k_{d,n} \), the COV of the test and FE resistances relative to the
predictions from the resistance model \( V_r \) and the combined COV incorporating the variability of the
resistance model and the basic variables \( V_r \) were determined in accordance with EN 1990:2002 [83].

Note that for the case of pure bending, since an increasing level of conservatism with the increase of
local slenderness is observed in Figs 6(a) and 6(b), the data were considered in two groups according
to their local slenderness – \( 70 < D / t e^2 \leq 105 \) and \( 105 < D / t e^2 \leq 140 \).

\[ b = \frac{1}{n} \sum_{j=1}^{n} \frac{r_{t,j}}{r_{e,j}} \]  

(12)
The statistical results are summarised in Tables 10(a) and 10(b) for the hot-rolled and cold-formed CHS respectively. Note that for the case of pure bending, the values of $b$, $V_\delta$ and $V_r$ based on each group were used for the derivation of $\gamma_{M0}$, while in Tables 10(a) and 10(b), only the mean values of these parameters are displayed for simplicity. Compared with the target partial safety factor of unity for cross-section design, the derived values of $\gamma_{M0}$ for axial compression are marginally higher (by less than 3%), while for other cases, the derived $\gamma_{M0}$ values are all below unity. Overall, the proposed elasto-plastic design expressions are considered to satisfy the reliability requirements of EN 1990:2002 [83] while offering improved accuracy over the elastic EC3 approach, and are therefore deemed suitable for inclusion in the next revision of EN 1993-1-1.

5. Member buckling design of semi-compact CHS

5.1. General

The stability design of semi-compact CHS members is addressed in this section. The existing EC3 design rules for semi-compact CHS columns and beam-columns are based on the elastic section properties, i.e. the full cross-sectional area $A$ and elastic section modulus $W_{el}$; a full description is given in Section 5.2. To exploit the benefits from partial plastification at the member buckling level, new design expressions, incorporating the elasto-plastic section modulus $W_{ep}$ in place of $W_{el}$, are proposed and presented in Section 5.3. The advantages of the new elasto-plastic member design proposals over the elastic EC3 approach are revealed in Section 5.4 by comparing the experimental and numerical results with the design predictions. The reliability of the new design expressions is then confirmed through statistical analyses in accordance with EN 1990:2002 [83], as presented in Section 5.5. Note that the design methods are assessed based on the measured (or modelled) material and geometric properties, unfactored resistance predictions and proportional loading.
5.2. Elastic EC3 member buckling design rules for semi-compact CHS

For a column with a semi-compact (Class 3) section, the cross-section is assumed to be fully effective, and the same column design procedure as that used for Class 1 and 2 sections is followed. For a semi-compact CHS beam-column, owing to its symmetric geometry, biaxial bending is treated in the same manner as uniaxial bending, leading to the simplified interaction formula of Eq. (13), where $\lambda$ and $\chi$ are the non-dimensional column slenderness and column buckling reduction factor respectively, $k$ is the interaction factor as calculated from Eqs (14) and (15), and $C_m$ is the equivalent uniform moment factor and is taken as unity since only uniform bending is considered in the present study.

$$\frac{N_{Ed}}{\chi N_{pl,Rd}} + k \frac{M_{Ed}}{M_{el,Rd}} \leq 1 \quad (13)$$

$$k = C_m \left( 1 + 0.6 \frac{N_{Ed}}{\chi N_{pl,Rd}} \right) \text{ for } \lambda \leq 1\quad (14)$$

$$k = C_m \left( 1 + 0.6 \frac{N_{Ed}}{\chi N_{pl,Rd}} \right) \text{ for } \lambda > 1 \quad (15)$$

5.3. Proposed elasto-plastic member buckling design rules for semi-compact CHS

Since the moment resistance is not used in the column buckling design procedure, the existing design rules remain applicable. For a semi-compact CHS beam-column, an improved design approach is proposed to exploit the additional resistance arising from consideration of the partial spread of plasticity. The elasto-plastic moment resistance $M_{ep,Rd}$ is employed in place of $M_{el,Rd}$ to achieve a more accurate bending end point, leading to the new interaction formula given by Eq. (16); the shape of the interaction curve is also modified by the use of different $k$ values, as given by Eqs (17) and (18), which are used for the design of beam-columns with Class 1 and 2 sections in the current EC3 approach.

$$\frac{N_{Ed}}{\chi N_{pl,Rd}} + k \frac{M_{Ed}}{M_{ep,Rd}} \leq 1 \quad (16)$$

$$k = C_m \left( 1 + 0.2 \frac{N_{Ed}}{\chi N_{pl,Rd}} \right) \text{ for } \lambda \leq 1\quad (17)$$

$$k = C_m \left( 1 + 0.8 \frac{N_{Ed}}{\chi N_{pl,Rd}} \right) \text{ for } \lambda > 1 \quad (18)$$
5.4. Assessment of design methods

The accuracy of the elastic EC3 and proposed elasto-plastic design approaches is evaluated in this subsection using the previously established experimental and numerical results. The column design rules are firstly evaluated. Comparisons of the design predictions with the test and FE results are illustrated by plotting the normalised test and FE buckling strength $N_u / N_{u,\text{pred}}$ against the non-dimensional column slenderness $\bar{A}$ in Fig. 8, and then quantified in terms of the mean and COV values of $N_u / N_{u,\text{pred}}$, as reported in Table 11. The data points generally lie above unity, with the mean values of $N_u / N_{u,\text{pred}}$ equal to 1.060 and 1.080, and COV values equal to 0.033 and 0.050, for the hot-rolled and cold-formed CHS respectively, showing that the current column design approach yields good accuracy and consistency for semi-compact CHS.

The elastic EC3 and proposed elasto-plastic design approaches for semi-compact CHS beam-columns are now assessed. The experimental and numerical buckling loads are normalised by the design predictions and plotted against the $\theta$ parameter in Figs 9(a) and 9(b) for hot-rolled and cold-formed CHS respectively, where $\theta$ describes the ratio between compression and bending, as defined by Eq. (11) in Section 4.4. The elastic EC3 design approach yields accurate predictions in the higher $\theta$ domain (where compression is more dominant); however, with decreasing values of $\theta$ (i.e. as bending becomes more dominant), an increasing level of conservatism is observed, which is primarily attributed to the neglect of the contribution from partial plastification. Through the incorporation of the elasto-plastic section modulus, the proposed approach improves the accuracy in the lower $\theta$ domain, and subsequently reduces the scatter in the design predictions. This is also shown quantitatively by the reduced mean and COV values of $N_u / N_{u,\text{host}}$ from the proposed elasto-plastic approach compared with those from the elastic EC3 approach, as reported in Table 11.

5.5. Reliability analysis

The reliability of the proposed elasto-plastic member buckling design expressions is assessed in this subsection through the derivation of the partial safety factor $\gamma_M$ in line with the EN 1990:2002 [83]
approach, as set out in Section 4.5. The test and FE data were grouped by non-dimensional column slenderness \( \bar{\lambda} < 0.5, \ 0.5 \leq \bar{\lambda} < 1, \ 1 \leq \bar{\lambda} < 1.5 \) and, for the beam-columns, also by local slenderness \( 70 < D / t e^2 \leq 105 \) and \( 105 < D / t e^2 \leq 140 \); the values of \( b \) and \( V_\delta \) were calculated from each group and then used to determine the overall partial safety factor \( \gamma_{M1} \).

Tables 12(a) and 12(b) report the key results from the reliability analyses for the hot-rolled and cold-formed CHS respectively, where the mean values of \( b, V_\delta \) and \( V_r \) are displayed for simplicity. All the derived values of \( \gamma_{M1} \) are marginally above the target value of unity for member buckling design in EN 1993-1-1, but by less than 5%, which is deemed acceptable. The proposed member buckling design expressions are shown to improve the accuracy of the elastic EC3 approach while maintaining an appropriate level of reliability, and are therefore considered suitable for inclusion into the next revision of EN 1993-1-1.

6. Conclusions

Shortcomings in the elastic Eurocode 3 (EC3) design methods for semi-compact (Class 3) CHS members have been highlighted, particularly in relation to the neglect of the additional resistance arising from partial plastification. The consequent underestimation of cross-section and member buckling capacities is especially significant for circular hollow sections (CHS) owing to their high shape factors.

The present research is aimed at overcoming this issue in the elastic cross-section and member buckling design of semi-compact CHS. A review of existing experimental studies was firstly carried out, from which a total of 626 experimental data points on CHS, consisting of 438 cross-section test data points and 188 member buckling test data points, were collected. Following this, a comprehensive numerical simulation programme was conducted. Finite element (FE) models were initially established, and a validation study was then performed to evaluate the accuracy of the established models and seek the most appropriate forms and amplitudes of local and global geometric imperfections. The validated FE models were used in subsequent parametric studies to expand the experimental database, where a total
of 660 numerical data were produced. Upon completion of the numerical simulation programme, a new
design approach for semi-compact CHS was sought; in particular, expressions for the elasto-plastic
section modulus $W_{ep}$ of CHS were proposed to account for the partial plastification. New elasto-plastic
cross-section and member buckling design rules based on the elastic EC3 design formulae, but
incorporating $W_{ep}$ in place of $W_{el}$, were developed. The accuracy of the elastic EC3 and the proposed
elasto-plastic design approach was evaluated through comparisons against the test and numerical results.
The elastic EC3 methods were shown to yield overly conservative predictions when bending was
present, while the proposed approach improved the overall accuracy owing to the exploitation of the
partial spread of plasticity. Standard reliability analyses of the proposed design expressions were
conducted following the steps set out in EN 1990:2002; the derived partial safety factors were either
below or marginally above the target value, confirming that an appropriate level of safety is achieved
by the proposed design methods. The present research demonstrates the reliability and the improved
accuracy of the proposed elasto-plastic cross-section and member buckling design methods, and
therefore confirms their suitability for inclusion into the next revision of EN 1993-1-1.

**Acknowledgements**

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Fig. 1. Existing EC3 design curves and proposed design curves for CHS under (a) pure bending and (b) combined compression and bending.
Fig. 2. Quarter-model of CHS with non-uniform mesh

Fig. 3. Typical elastic buckling mode shapes of CHS 100×5 (in mm) under various loading scenarios:
(a) LBA-\(t\) mode shapes and (b) LBA-\(t_{mod}\) mode shapes
Fig. 4. Typical experimental and numerical load-deformation curves for (a) stub column tests, (b) four-point bending tests and (c) beam-column tests
Fig. 4. Definition of $R_u$, $R_{u,pred}$ and $\theta$

Fig. 5. Comparisons of cross-section test and FE resistances under axial compression with predicted design resistances from EC3
Fig. 6. Comparisons of cross-section test and FE resistances under pure bending with predicted design resistances for (a) hot-rolled CHS and (b) cold-formed CHS.
Fig. 7. Comparisons of cross-section test and FE resistances under compression plus bending with predicted design resistances for (a) hot-rolled CHS and (b) cold-formed CHS
Fig. 8. Comparisons of member buckling test and FE resistances under axial compression with predicted design resistances from EC3
Fig. 9. Comparisons of member buckling test and FE resistances under compression plus bending with predicted design resistances for (a) hot-rolled CHS and (b) cold-formed CHS
**Table 1** Summary of previous cross-section tests on structural steel CHS

<table>
<thead>
<tr>
<th>Test type</th>
<th>No. of data points</th>
<th>Hot-rolled</th>
<th>Cold-formed</th>
<th>Fabricated (or unspecified)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stub column tests</td>
<td>261</td>
<td>[12,17,29,31–40]</td>
<td>[12,15,16,18,29,32,35,41–46]</td>
<td>[11,13,14,26–28,32,47–52]</td>
</tr>
<tr>
<td>Four-point bending tests</td>
<td>113</td>
<td>[21,22,53–56]</td>
<td>[19-24,54,55,57–59]</td>
<td>[56,60]</td>
</tr>
<tr>
<td>Short beam-column tests</td>
<td>64</td>
<td>[29,31]</td>
<td>[18,29,30]</td>
<td>[26–28]</td>
</tr>
</tbody>
</table>

**Table 2** Summary of previous member buckling tests on structural steel CHS

<table>
<thead>
<tr>
<th>Test type</th>
<th>No. of data points</th>
<th>Hot-rolled</th>
<th>Cold-formed</th>
<th>Fabricated (or unspecified)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column tests</td>
<td>143</td>
<td>[12,31,35,67,69,70]</td>
<td>[12,32,35,62,68,71,72]</td>
<td>[14,61,63,73,74]</td>
</tr>
<tr>
<td>Beam-column tests</td>
<td>45</td>
<td>[31,40,67,69]</td>
<td>[30,66]</td>
<td>[26]</td>
</tr>
</tbody>
</table>

**Table 3** Comparisons of cross-section test results with FE results for varying local imperfection patterns and amplitudes

<table>
<thead>
<tr>
<th>Loading scenario</th>
<th>No. of test data</th>
<th>Evaluation parameter</th>
<th>$\omega_l = 0$</th>
<th>LBA-$t$, $\omega_l = 0.01(Dt)^{1/2}$</th>
<th>LBA-$t_{mod}$, $\omega_l = 0.01(Dt)^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$N_{u,FE} / N_{u,\text{test}}$</td>
<td>$M_{u,FE} / M_{u,\text{test}}$</td>
<td>$N_{u,FE} / N_{u,\text{test}}$</td>
<td>$M_{u,FE} / M_{u,\text{test}}$</td>
</tr>
<tr>
<td>$N$</td>
<td>37</td>
<td>Mean</td>
<td>1.006</td>
<td>1.005</td>
<td>1.003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>COV</td>
<td>0.063</td>
<td>0.063</td>
<td>0.063</td>
</tr>
<tr>
<td>$M$</td>
<td>32</td>
<td>Mean</td>
<td>0.992</td>
<td>1.000</td>
<td>0.987</td>
</tr>
<tr>
<td></td>
<td></td>
<td>COV</td>
<td>0.021</td>
<td>0.024</td>
<td>0.020</td>
</tr>
<tr>
<td>$N + M$</td>
<td>15</td>
<td>Mean</td>
<td>0.972</td>
<td>0.935</td>
<td>0.987</td>
</tr>
<tr>
<td></td>
<td></td>
<td>COV</td>
<td>0.021</td>
<td>0.024</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4 Comparison of member buckling test results with FE results for varying global imperfection amplitudes

<table>
<thead>
<tr>
<th>Loading scenario</th>
<th>No. of test data</th>
<th>Evaluation parameter</th>
<th>$\omega_g + \varepsilon_0$</th>
<th>$\omega_g = L / 2000$</th>
<th>$\omega_g = L / 1000$</th>
<th>$\omega_g = L / 500$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>37</td>
<td>Mean</td>
<td>1.001</td>
<td>1.035</td>
<td>1.008</td>
<td>0.967</td>
</tr>
<tr>
<td></td>
<td></td>
<td>COV</td>
<td>0.050</td>
<td>0.058</td>
<td>0.051</td>
<td>0.050</td>
</tr>
<tr>
<td>$N + M$</td>
<td>21</td>
<td>Mean</td>
<td>-</td>
<td>1.010</td>
<td>0.996</td>
<td>0.971</td>
</tr>
<tr>
<td></td>
<td></td>
<td>COV</td>
<td>-</td>
<td>0.032</td>
<td>0.035</td>
<td>0.045</td>
</tr>
</tbody>
</table>

Table 5 Input parameters for predictive models of stress-strain curves used in parametric study

<table>
<thead>
<tr>
<th>Production route</th>
<th>Grade</th>
<th>$E$ N/mm$^2$</th>
<th>$f_y$ N/mm$^2$</th>
<th>$f_u$ N/mm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot-rolled</td>
<td>S355NH/NLH</td>
<td>210000</td>
<td>355</td>
<td>490</td>
</tr>
<tr>
<td></td>
<td>S460NH/NLH</td>
<td>210000</td>
<td>460</td>
<td>560</td>
</tr>
<tr>
<td>Cold-formed</td>
<td>S355NH/NLH</td>
<td>210000</td>
<td>355</td>
<td>470</td>
</tr>
<tr>
<td></td>
<td>S460NH/NLH</td>
<td>210000</td>
<td>460</td>
<td>550</td>
</tr>
</tbody>
</table>

Table 6 Summary of test and FE data obtained within semi-compact range

<table>
<thead>
<tr>
<th>Level</th>
<th>Loading scenario</th>
<th>No. of test data</th>
<th>No. of FE data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N$</td>
<td>29</td>
<td>40</td>
</tr>
<tr>
<td>Cross-sectional</td>
<td>$M$</td>
<td>36</td>
<td>56</td>
</tr>
<tr>
<td>level</td>
<td>$N + M$</td>
<td>16</td>
<td>132</td>
</tr>
<tr>
<td>Member buckling</td>
<td>$N$</td>
<td>12</td>
<td>128</td>
</tr>
<tr>
<td>level</td>
<td>$N + M$</td>
<td>15</td>
<td>304</td>
</tr>
</tbody>
</table>
Table 7 Current and upcoming Class 3 slenderness limits for CHS in EC3

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>90</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>90</td>
<td>140</td>
<td></td>
</tr>
<tr>
<td>$N + M$</td>
<td>90</td>
<td>2520</td>
<td>$5\psi + 23$</td>
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</table>

Table 8 Comparisons of cross-section test and FE resistances with predicted design resistances for different production routes and loading scenarios

<table>
<thead>
<tr>
<th>Loading scenario</th>
<th>No. of data</th>
<th>Evaluation parameter</th>
<th>Hot-rolled CHS</th>
<th>Cold-formed CHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>69</td>
<td>Mean COV</td>
<td>1.012</td>
<td>1.095</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.033</td>
<td>0.065</td>
</tr>
<tr>
<td>$M$</td>
<td>92</td>
<td>Mean COV</td>
<td>1.259</td>
<td>1.361</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.108</td>
<td>1.187</td>
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<td></td>
<td></td>
<td></td>
<td>0.043</td>
<td>0.073</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.059</td>
<td>0.096</td>
</tr>
<tr>
<td>$N + M$</td>
<td>148</td>
<td>Mean COV</td>
<td>1.249</td>
<td>1.461</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.136</td>
<td>1.326</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.079</td>
<td>0.081</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.049</td>
<td>0.064</td>
</tr>
</tbody>
</table>

Table 9 Summary of $f_{y,m}/f_{y,n}$ and $V_{fy}$ values for different steel grades from prEN 1993-1-1:2018 [8]

<table>
<thead>
<tr>
<th>Steel grade</th>
<th>$f_{y,m}/f_{y,n}$</th>
<th>$V_{fy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S235 &amp; S275</td>
<td>1.25</td>
<td>0.055</td>
</tr>
<tr>
<td>S355 &amp; S420</td>
<td>1.20</td>
<td>0.050</td>
</tr>
<tr>
<td>S460</td>
<td>1.15</td>
<td>0.045</td>
</tr>
<tr>
<td>Above S460</td>
<td>1.10</td>
<td>0.035</td>
</tr>
</tbody>
</table>

Table 10 Summary of reliability analysis results for proposed cross-section design approach for (a) hot-rolled CHS and (b) cold-formed CHS

(a) Hot-rolled CHS

<table>
<thead>
<tr>
<th>Loading scenario</th>
<th>$n$</th>
<th>$k_{d,n}$</th>
<th>$b$</th>
<th>$V_\delta$</th>
<th>$V_r$</th>
<th>$\gamma_{H0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>28</td>
<td>3.482</td>
<td>1.012</td>
<td>0.033</td>
<td>0.061</td>
<td>1.021</td>
</tr>
<tr>
<td>$M$</td>
<td>38</td>
<td>3.369</td>
<td>1.108</td>
<td>0.038</td>
<td>0.067</td>
<td>0.946</td>
</tr>
<tr>
<td>$N + M$</td>
<td>66</td>
<td>3.245</td>
<td>1.136</td>
<td>0.049</td>
<td>0.070</td>
<td>0.963</td>
</tr>
</tbody>
</table>
Table 11 Comparisons of member buckling test and FE results with design resistance predictions for different production routes and loading scenarios

<table>
<thead>
<tr>
<th>Loading scenario</th>
<th>No. of data</th>
<th>Evaluation parameter</th>
<th>Hot-rolled CHS</th>
<th>Cold-formed CHS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$R_u / R_{u,el}$</td>
<td>$R_u / R_{u,ep}$</td>
</tr>
<tr>
<td>$N$</td>
<td>140</td>
<td>Mean COV</td>
<td>1.060</td>
<td>1.080</td>
</tr>
<tr>
<td>$N + M$</td>
<td>319</td>
<td>Mean COV</td>
<td>1.146</td>
<td>1.079</td>
</tr>
</tbody>
</table>

Table 12 Summary of reliability analysis results for proposed member buckling design approach for (a) hot-rolled CHS and (b) cold-formed CHS

(a) Hot-rolled CHS

<table>
<thead>
<tr>
<th>Loading scenario</th>
<th>$n$</th>
<th>$k_{d,n}$</th>
<th>$b$</th>
<th>$V_δ$</th>
<th>$V_r$</th>
<th>$γ_{M1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>66</td>
<td>3.245</td>
<td>1.060</td>
<td>0.016</td>
<td>0.051</td>
<td>1.022</td>
</tr>
<tr>
<td>$N + M$</td>
<td>167</td>
<td>3.149</td>
<td>1.079</td>
<td>0.040</td>
<td>0.066</td>
<td>1.041</td>
</tr>
</tbody>
</table>

(b) Cold-formed CHS

<table>
<thead>
<tr>
<th>Loading scenario</th>
<th>$n$</th>
<th>$k_{d,n}$</th>
<th>$b$</th>
<th>$V_δ$</th>
<th>$V_r$</th>
<th>$γ_{M1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>74</td>
<td>3.224</td>
<td>1.080</td>
<td>0.038</td>
<td>0.062</td>
<td>1.045</td>
</tr>
<tr>
<td>$N + M$</td>
<td>152</td>
<td>3.155</td>
<td>1.075</td>
<td>0.028</td>
<td>0.058</td>
<td>1.014</td>
</tr>
</tbody>
</table>