A Distributed Price-based Strategy for Flexible Demand Coordination in Multi-area Systems

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***Abstract*—This paper presents a novel distributed control strategy for large-scale deployment of price-responsive flexible demand. Differently from previous theoretical studies on the subject, the proposed analysis explicitly models multi-area systems, accounting for transmission lines of limited capacity and different locational marginal prices (LMP) throughout the network. A game-theoretic framework is adopted, designing a demand coordination scheme that converges to a stable market configuration (characterized as an aggregative equilibrium) through iterative price broadcasts. The performance of the proposed control strategy, that also ensures flattened generation profiles and reduced generation costs, is evaluated in simulation on a five-bus power system.**

***Index Terms*—Flexible demand, distributed control, multi-area systems, stable market configuration.**

I. Introduction

The increasing diffusion of new types of loads, such as electric vehicles and smart appliances, represents a crucial element in the ongoing transition of power systems towards the smart grid paradigm. The resulting increased flexibility in the power consumption of private customers could potentially be exploited for multiple purposes, such as energy cost reduction for domestic households or ancillary services provision for the system [1], [2]. Many distributed schemes have been proposed in the literature to achieve these benefits by properly coordinating flexible demand [3].

Within this context, a significant amount of research has focused on game-theoretic approaches for interruptible appliances, modelling the flexible devices as price-responsive rational agents that compete for power consumption at times with cheaper energy. For example, [4] proposes an iterative auction mechanism that coordinates the charging of electric vehicles so as to converge to an equilibrium at which both the benefit of each customer and the social welfare are maximized. A similar iterative pricing scheme is presented in [5], where the overnight demand valley is filled at the final stable market configuration. These approaches have been extended in multiple directions, evaluating alternative schemes for convergence to equilibrium [6], [7], considering more detailed models of the appliances [8] or proposing coordination schemes with incomplete information [9]. Note that all the cited papers [4]–[9] consider an abstraction of the electricity market where the electricity price is simply characterized as some monotone increasing function of total demand. Moreover, they do not consider the impact of flexible demand operation at a transmission level, which is a relevant factor in large-scale applications.

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This paper advances the state-of-the-art in flexible demand coordination by addressing the above shortcomings. A novel control strategy for multi-area systems is proposed, explicitly accounting for transmission lines of limited capacity and analytically quantifying the impact of flexible demand on the locational marginal prices (LMP). Using the theoretical framework in [7] as a starting point, it is proved that the proposed scheme always converges to a stable market configuration (characterized as an aggregative equilibrium) for all penetration levels and parameters of the flexible devices. The capability of the proposed strategy to flatten generation profiles and reduce system costs is tested in simulation, considering a 5-bus system with high penetration of electric vehicles.

The rest of the paper is structured as follows: Section II contains the main modelling elements of the flexible devices and the power system. Section III and Section IV present, respectively, the proposed control strategy and case studies. Conclusive remarks are provided in Section V.

II. Modelling Framework

The considered power system is composed of a finite set $\mathscr{K} = \{1,\dots,K\}$ of areas, connected by transmission lines of limited capacity. The population of flexible devices is denoted by $\mathscr{N} = \{1,\dots,N\}$, with $\mathscr{N}_k \subseteq \mathscr{N}$ denoting the set of devices operating in area k. Each device $j \in \mathscr{N}$ needs to complete an assigned task over the considered discrete time interval $\mathscr{T} = \{1,\dots,T\}$. The task of device $j \in \mathscr{N}$ is characterized by three parameters: its required energy E_j, its rated power P_j and its time availability window $\mathscr{A}_j \subseteq \mathscr{T}$. The set $\mathscr{U}_j$ of feasible power profiles $u_j : \mathscr{T} \to \mathbb{R}_+$ guaranteeing task completion can then be defined as:

$$\mathscr{U}_j := \left\{ u_j(\cdot) : \sum_{t=1}^{T} u_j(t)\Delta t = E_j, \right. \\ \left. 0 \le u_j(t) \le P_j \cdot \mathbb{1}_{\mathscr{A}_j}(t) \quad \forall t \in \mathscr{T} \right\} \tag{1}$$

where Δt denotes the time discretization step and $\mathbb{1}_{\mathscr{A}_j}(t)$ is the indicator function, with $\mathbb{1}_{\mathscr{A}_j}(t) = 1$ when $t \in \mathscr{A}_j$ and $\mathbb{1}_{\mathscr{A}_j}(t) = 0$ when $t \notin \mathscr{A}_j$. The equality in (1) indicates that the total energy consumed by device j is equal to the energy required to complete its task while the inequalities impose that its positive power consumption cannot exceed the rated power P_j for t within the availability interval $\mathscr{A}_j$ and must be zero for t outside it.

A. Optimal Power Flow and Locational Marginal Prices

The electricity price p_k in each area $k \in \mathscr{K} = \{1, \dots K\}$ of the power system corresponds to the LMP associated to an optimal power flow problem. For simplicity, a DC optimal power flow (DCOPF) with no losses is considered. This choice is common in analytical works on multi-area systems, as it ensures satisfying approximations and efficient computability [10]. To present an analytical formulation, we denote the vector of demand values D_k in each area as $D = [D_1, \dots, D_K]$. Similar notation is used for the generation values $G = [G_1, \dots, G_K]$ and the voltage angles $\theta = [\theta_1, \dots, \theta_K]$. The DCOPF determines the minimum total generation costs $\varphi(D)$ for a certain demand vector D, subject to power balance constraints (3a), transmission line limits (3b) and generation limits (3c). This can be expressed as:

$$\varphi(D) = \min_{G,\theta} \sum_{k=1}^{K} f_k(G_k) \tag{2}$$

subject to:

$$D_k - G_k + \sum_{k'=1}^{K} F_{kk'} = 0 : \quad p_k \tag{3a}$$

$$F_{kk'} = (\theta_k - \theta_{k'})/X_{kk'} \leq F_{kk'}^{max} \tag{3b}$$

$$G_k^{min} \leq G_k \leq G_k^{max} \tag{3c}$$

for $k, k' = 1, 2, \dots, K$. Note that $f_k(G_k)$ represents the cost of generating G_k power units in area k and it is assumed to be a strictly convex function, p_k is the Lagrangian multiplier of the constraint (3a), whereas $X_{kk'}$ and $F_{kk'}^{max}$ denote the reactance and the maximum capacity, respectively, of the transmission line connecting areas k and k'. In order to formally characterize the electricity prices used in the subsequent analysis, some preliminary properties of the DCOPF are demonstrated.

Proposition 1: The minimized generation cost $\varphi(D)$ of problem (2)-(3) is a strictly convex function of the demand vector D on any compact set $[D_{min}, D_{max}]^K \subset \mathbb{R}_+^K$.

Proof: Given the maximum theorem under convexity [11], this proposition follows from the convexity of f_k and the linearity and convexity of the constraints in (3). ■

From Proposition 1, $\varphi(D)$ is Lipschitz continuous on any open interval $(D_{min}, D_{max})^K$ and differentiable almost everywhere, except for some zero-measure set $\mathscr{D} \subset (D_{min}, D_{max})^K$ [12]. The sub-differential of $\varphi(D)$ is now introduced:

$$\frac{\partial \varphi(D)}{\partial D_k} = \begin{cases} \varphi_k(D) & \text{if} \quad D \notin \mathscr{D} \\ [\varphi_k^-(D), \varphi_k^+(D)] & \text{if} \quad D \in \mathscr{D}. \end{cases} \tag{4}$$

where $\varphi_k^-(D)$ and $\varphi_k^+(D)$ are the left and right derivatives of $\varphi(D)$, respectively.

The Lagrangian multiplier p_k associated to (3a), i.e. the LMP in area k, is equal to $\varphi_k(D)$ if $\varphi(D)$ is differentiable at D. To account for the points $D \in \mathscr{D}$ of non-differentiability, we introduce the quantities $\underline{p}_k$ and $\bar{p}_k$, defined as follows:

$$\underline{p}_k = \Pi_k^-(D) = \begin{cases} \varphi_k(D) & \text{if} \quad D \notin \mathscr{D} \\ \varphi_k^-(D) & \text{if} \quad D \in \mathscr{D}. \end{cases} \tag{5a}$$

$$\bar{p}_k = \Pi_k^+(D) = \begin{cases} \varphi_k(D) & \text{if} \quad D \notin \mathscr{D} \\ \varphi_k^+(D) & \text{if} \quad D \in \mathscr{D}. \end{cases} \tag{5b}$$

The price $\bar{p}_k$ represents the marginal cost of providing an additional unit of power in area k, while $\underline{p}_k$ is the marginal saving of reducing by one unit the power supplied to area k. Given the convexity of $\varphi(D)$, $\underline{p}_k$ and $\bar{p}_k$ are strictly monotone increasing with respect to the demand D_k in area k.

Remark 1: The prices $\bar{p}_k$ and $\underline{p}_k$ are considered as distinct quantities in the proposed analysis to provide rigorous theoretical results. The outcome of this study is still valid if $\varphi(D)$ is assumed to be differentiable everywhere and, as a result, $\bar{p}_k$ and $\underline{p}_k$ identically coincide to p_k in (3a).

The price notation introduced in (5) can be extended to consider the price vectors $\underline{p}(t) = [\underline{p}_1(t), \dots, \underline{p}_K(t)]$ and $\bar{p}(t) = [\bar{p}_1(t), \dots, \bar{p}_K(t)]$ at time t as functions of the demand vector $D(t) = [D_1(t), \dots, D_K(t)]$ at the same time instant:

$$\underline{p}(t) = \Pi^-(D(t)) = [\Pi_1^-(D(t)), \dots, \Pi_K^-(D(t))] \tag{6a}$$

$$\bar{p}(t) = \Pi^+(D(t)) = [\Pi_1^+(D(t)), \dots, \Pi_K^+(D(t))]. \tag{6b}$$

Each component $D_k(t)$ of the demand vector $D(t)$ corresponds to the aggregate demand in area k at time t. It can be expressed as the sum of two distinct terms: the total power consumption $D_k^f(t)$ of flexible devices in area k and the total power demand $D_k^i(t)$ of inflexible loads in area k:

$$D_k(t) = D_k^f(t) + D_k^i(t) = \sum_{j \in \mathscr{N}_k} u_j(t) + D_k^i(t). \tag{7}$$

B. Game-theoretic formulation and aggregative equilibrium

Coordination of flexible demand is analyzed under a game-theoretic framework. Each device j in area k is modelled as a rational agent that aims to minimize its cost $C_{j,k}$. Denoting by $\bar{p}_k(t)$ the k-th element of the price vector (6b), we have:

$$C_{j,k} = \sum_{t=1}^{T} \bar{p}_k(t) \cdot u_j(t) \Delta t. \tag{8}$$

We wish to design an iterative distributed control scheme that ensures convergence to a stable configuration, characterized as the aggregative equilibrium notion defined below:

Definition 1: Consider the individual power profiles u_j^*, the demand vector D^* and the prices $\underline{p}^*$ and $\bar{p}^*$. These quantities correspond to an aggregative equilibrium if, for any area $k \in \mathscr{K}$, any agent $j \in \mathscr{N}_k$ and any feasible profile $u_j \in \mathscr{U}_j$, it holds:

$$\begin{aligned} \Delta J = & \sum_{t:\, u_j(t) \geq u_j^*(t)} \bar{p}_k^*(t) \left[u_j(t) - u_j^*(t)\right] \Delta t \\ & - \sum_{t:\, u_j(t) \leq u_j^*(t)} \underline{p}_k^*(t) \left[u_j^*(t) - u_j(t)\right] \Delta t \geq 0 \end{aligned} \tag{9}$$

$$D_k^*(t) = \sum_{j \in \mathscr{N}_k} u_j^*(t) + D_k^i(t) \quad \forall t \in \mathscr{T} \tag{10a}$$

$$\underline{p}^*(t) = \left[\underline{p}_1^*(t), \ldots, \underline{p}_K^*(t)\right] = \Pi^-\left(D^*(t)\right) \ \forall t \in \mathscr{T} \tag{10b}$$

$$\bar{p}^*(t) = \left[\bar{p}_1^*(t), \ldots, \bar{p}_K^*(t)\right] = \Pi^+\left(D^*(t)\right) \ \forall t \in \mathscr{T} \tag{10c}$$

The aggregative equilibrium is expressed as a fixed point. From (9), if any device $j \in \mathscr{N}_k$ in some area $k \in \mathscr{K}$ changes its power profile u_j^* (at equilibrium) to some other power profile u_j, the corresponding variation ΔJ of its cost will be non-negative. This cost variation is the difference between the costs of increased power consumption at certain time instants (priced at $\bar{p}_k^*$) and the savings from reduced power consumption at other times (priced at $\underline{p}_k^*$). From (10), the aggregate power consumption of flexible demand associated to the power profiles u_j^* of all devices $j \in \mathscr{N}_k$ will in turn lead to very same price profiles $\underline{p}_k^*$ and $\bar{p}_k^*$ considered in (9).

The aggregative equilibrium is a good approximation of the classical Nash equilibrium notion when single agents have negligible impact on the global quantities of the system and can therefore be considered as price-taking entities. This is reasonable in the present case since the power consumption u_j of a single device (in the order of KWs) is significantly smaller than total power demand (GWs). Note that the proposed formulation still takes into account the overall impact of the devices population through (10). Similar equilibrium notions are commonly adopted as design objective in analytical studies on flexible demand coordination [5], [7].

III. Distributed Control Strategy

The proposed scheme for coordination of flexible demand is presented in Algorithm 1. As discussed next, its implementation in practical contexts can be performed in a distributed manner through iterative price broadcasts and repeated power updates by the flexible devices. It is proven in Theorem 1 that such scheme always converges to a stable market configuration, i.e. to an aggregative equilibrium. The three main phases of Algorithm 1 are now described.

Algorithm 1 Iterative scheme for coordination of flexible demand in multi-area systems.

1) **Initialization**
   $u_j^{(0)} \in \mathscr{U}_j \quad D_k^{(0)}(t) = \sum_{j=1}^{N_k} u_j^{(0)}(t) + D_k^i(t) \quad \forall k \in \mathscr{K} \quad \forall j \in \mathscr{N}_k$
   $\underline{p}^{(0)} = \Pi^-(D^{(0)}) \qquad \bar{p}^{(0)} = \Pi^+(D^{(0)}) \qquad l = 0 \qquad flag = 1$
2) **Power scheduling update**
   WHILE $(flag = 1)$
   a) $flag = 0$
   b) **FOR** $k = 1$ to K
   c) **FOR** each $j \in \mathscr{N}_k$
      i) $l = l + 1$
      ii) $u_j^{(l)}(\cdot) = u_j^{(l-1)}(\cdot) \quad D_k^{(l)}(\cdot) = D_k^{(l-1)}(\cdot) \quad \forall k \in \mathscr{K} \quad \forall j \in \mathscr{N}_k$
      $\underline{p}^{(l)}(\cdot) = \underline{p}^{(l-1)}(\cdot) \quad \bar{p}^{(l)}(\cdot) = \bar{p}^{(l-1)}(\cdot)$
      iii) **FIND** $t_1, t_2 \in \mathscr{A}_j$ such that:
      $u_j^{(l-1)}(t_1) < P_j \qquad u_j^{(l-1)}(t_2) > 0 \qquad \bar{p}_k^{(l-1)}(t_1) < \underline{p}_k^{(l-1)}(t_2)$
      iv) **IF** t_1, t_2 exist:
      $$\Delta = \min\left(\left\{P_j - u_j^{(l-1)}(t_1), u_j^{(l-1)}(t_2)\right\}\right)$$
      $u_j^{(l)}(t_1) = u_j^{(l-1)}(t_1) + \Delta \qquad D_k^{(l)}(t_1) = D_k^{(l-1)}(t_1) + \Delta$
      $u_j^{(l)}(t_2) = u_j^{(l-1)}(t_2) - \Delta \qquad D_k^{(l)}(t_2) = D_k^{(l-1)}(t_2) - \Delta$
      $\underline{p}^{(l)}(t_1) = \Pi^-\left(D^{(l)}(t_1)\right) \qquad \bar{p}^{(l)}(t_1) = \Pi^+\left(D^{(l)}(t_1)\right)$
      $\underline{p}^{(l)}(t_2) = \Pi^-\left(D^{(l)}(t_2)\right) \qquad \bar{p}^{(l)}(t_2) = \Pi^+\left(D^{(l)}(t_2)\right)$
      $$flag = 1$$
      v) **IF** $\bar{p}_k^{(l)}(t_1) > \underline{p}_k^{(l)}(t_2)$
      $$\bar{\Delta} = \sup\left\{\delta \in [0, \Delta] : \Pi_k^+\left(D^{(l-1)}(t_1) + \delta \cdot \hat{e}_k\right) \leq \Pi_k^-\left(D^{(l-1)}(t_2) - \delta \cdot \hat{e}_k\right)\right\}$$
      $u_j^{(l)}(t_1) = u_j^{(l-1)}(t_1) + \bar{\Delta} \qquad D_k^{(l)}(t_1) = D_k^{(l-1)}(t_1) + \bar{\Delta}$
      $u_j^{(l)}(t_2) = u_j^{(l-1)}(t_2) - \bar{\Delta} \qquad D_k^{(l)}(t_2) = D_k^{(l-1)}(t_2) - \bar{\Delta}$
      $\underline{p}^{(l)}(t_1) = \Pi^-\left(D^{(l)}(t_1)\right) \qquad \bar{p}^{(l)}(t_1) = \Pi^+\left(D^{(l)}(t_1)\right)$
      $\underline{p}^{(l)}(t_2) = \Pi^-\left(D^{(l)}(t_2)\right) \qquad \bar{p}^{(l)}(t_2) = \Pi^+\left(D^{(l)}(t_2)\right)$
   END (Corresponding to **FOR** each $j \in N_k$)
   END (Corresponding to **FOR** $k = 1$ to K)
   END (Corresponding to **WHILE**)
3) **Final results**
   $u_j^*(\cdot) = u_j^{(l)}(\cdot) \quad D_k^*(\cdot) = D_k^{(l)}(\cdot) \quad \underline{p}_k^*(\cdot) = \underline{p}_k^{(l)}(\cdot)$
   $\bar{p}_k^*(\cdot) = \bar{p}_k^{(l)}(\cdot) \quad \forall k \in \mathscr{K} \quad \forall j \in \mathscr{N}_k$

1) **Initialization:** The central entity collects the feasible power consumption profiles $u_j^{(0)}$ that have been initially scheduled by each device. On the basis of this information, it calculates the resulting aggregate demand profile $D_k^{(0)}$ and price profiles $\underline{p}_k^{(0)}$ and $\bar{p}_k^{(0)}$ in each area $k \in \mathscr{K}$. The iteration counter l and the flag variable $flag$ are initialized. The latter is used to detect whether a change in power scheduling has occurred at the latest iteration in phase 2.

2) **Power scheduling update:** The power scheduling is modified in succession from Area 1 to Area K (Step 2.b). The devices within a certain area k sequentially update their power profiles (Step 2.c). When a single device j updates its power profile $u_j^{(l)}$, all other devices retain their strategies from the previous iteration (Step 2.c.ii). The device j verifies whether it can switch a feasible amount Δ of its power consumption from a time t_2 with a higher electricity price to a time t_1 with a lower price (Step 2.c.iii), thus reducing its energy cost. If this is the case, the power swap is performed (the power profile $u_j^{(l)}$ is updated), and the power amount Δ switched from t_2 to t_1 will be communicated to the central entity, which will in turn update the demand profile $D_k^{(l)}$ in area k and the price profiles $\underline{p}^{(l)}$ and $\bar{p}^{(l)}$ (Step 2.c.iv). In addition, since a power swap has occurred, the flag variable $flag$ is set to 1. Step 2.c.v makes sure that the price order at times t_1 and t_2 is not reversed as a result of the power swap performed at the previous step, which increases $\bar{p}_k^{(l)}(t_1)$ and decreases $\underline{p}_k^{(l)}(t_2)$. In this case, the swapped power is capped to a smaller quantity $\bar{\Delta}$. Given the monotonicity of $\Pi^-(D)$ and $\Pi^+(D)$ in (5), which follows from the convexity of $\varphi(D)$ established in Proposition 1, $\bar{\Delta}$ can be easily calculated through a bisection method.

If there is no feasible power swap, the variable $flag$ remains equal to 0 (step 2.a) throughout the two nested **FOR** cycles. The **WHILE** cycle consequently terminates.

3) **Final results:** At the end of the **WHILE** cycle, the values of $u_j^{(l)}$, $D_k^{(l)}$, $\underline{p}_k^{(l)}$ and $\bar{p}_k^{(l)}$ at the last iteration are returned as the final results u_j^*, D_k^*, $\underline{p}_k^*$ and $\bar{p}_k^*$, for all $k \in \mathscr{K}$ and all $j \in \mathscr{N}_k$.

Note that, if the initial demand $D_k^{(0)}$ in each area is known or estimated with sufficient precision, the proposed coordination scheme preserves the privacy of the agents. In fact, they do not need to divulge their parameters $(E_j, P_j, \mathscr{A}_j)$ but only communicate, at each update, the time instants t_1 and t_2 and the amount Δ of their power swap.

Remark 2: Faster implementations, with a one-shot broadcast of a price signal and negligible degradation of the equilibrium performance, can alternatively be considered. These are not presented for length reasons but are discussed in [7] for the case of single-area systems.

It is now demonstrated that Algorithm 1 always converges to a stable market configuration, where each device has no interest in unilaterally changing its scheduled power profile.

Theorem 1: For a power system composed of K areas, each endowed with a population of flexible devices, the proposed Algorithm 1 asymptotically converges to the aggregative equilibrium introduced in Definition 1.

Proof: See Appendix I. ■

The algorithm not only guarantees convergence to an aggregative equilibrium but it also ensures a consistent reduction of the total generation costs. As demonstrated in Appendix I and shown in the simulations of Section IV, the total generation costs of the system are reduced at each iteration of the algorithm and converge asymptotically to some minimum value. In an extended journal version of this work, we demonstrate that this minimum is a global optimum.

IV. Simulation Results

The proposed coordination strategy is applied to the PJM 5-bus system [13], whose topology and parameters are shown in Fig. 1. The generation cost f_m of each generator m is a

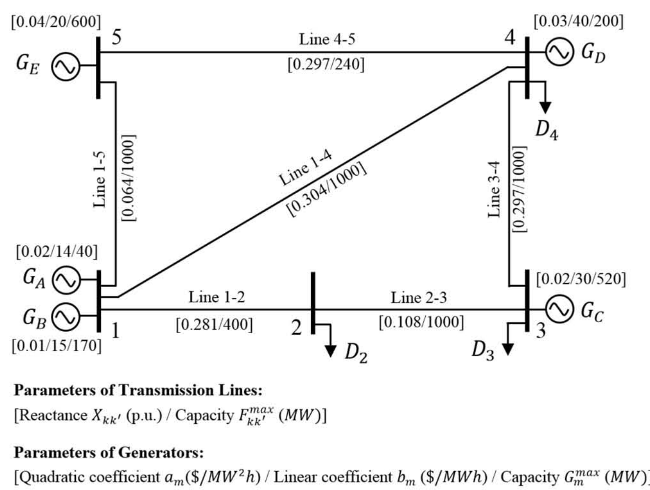

Fig. 1. The PJM 5-bus system.

quadratic function of power generation G_m, with $f_m(G_m) = a_m G_m^2 + b_m G_m$. It is assumed that the populations $\mathscr{N}_k$ of electric vehicles (EVs) in Area 2, 3 and 4 aim to charge their batteries during night time. The energy E_j required by the EVs $j \in \mathscr{N}_k$ in each area k follows a Gaussian distribution with mean value μ_k^E and standard deviation σ_k^E. In addition, it is assumed that all the EVs in area k have equal rated power P_k. The chosen parameter values are listed below:

$$|\mathscr{N}_2| = 15000 \quad \mu_2^E = 30kWh \quad \sigma_2^E = 1.0kWh \quad P_2 = 11kW$$
$$|\mathscr{N}_3| = 20000 \quad \mu_3^E = 29kWh \quad \sigma_3^E = 1.5kWh \quad P_3 = 10kW$$
$$|\mathscr{N}_4| = 20000 \quad \mu_4^E = 30kWh \quad \sigma_4^E = 1.5kWh \quad P_4 = 11kW$$

A time interval $T = 24h$ and a time discretization step $\Delta t = 0.25h$ are considered. In addition, the plug-in time and plug-off time of each vehicle j are represented by t_j and $t_j + d_j$, respectively. It is assumed that t_j and d_j of a vehicle $j \in \mathscr{N}_k$ in area k also follow Gaussian distributions with mean value μ_k^t and μ_k^d and standard deviation σ_k^t and σ_k^d, respectively. The chosen values are the following:

$$\mu_2^t = 20{:}30h \quad \sigma_2^t = 1.5h \quad \mu_2^d = 10h \quad \sigma_2^d = 1.0h$$
$$\mu_3^t = 21{:}30h \quad \sigma_3^t = 1.5h \quad \mu_3^d = 11h \quad \sigma_3^d = 2.0h$$
$$\mu_4^t = 21{:}00h \quad \sigma_4^t = 1.0h \quad \mu_4^d = 11h \quad \sigma_4^d = 1.5h$$

The availability window $\mathscr{A}_j$ of the single device j can now be expressed in discrete time as:

$$\mathscr{A}_j = \left\{ t \in \mathscr{T} : t_j \leq t \cdot \Delta t \leq t_j + d_j \right\}. \tag{11}$$

The inflexible demand profile D_k^i in area k is chosen according to historical data [14]. In the initialization phase of the algorithm, it is assumed that the initial power profile $u_j^{(0)}$ of each EV j corresponds to charging at constant power throughout its whole availability window $\mathscr{A}_j$. On the basis of these power profiles, the resulting demand profiles $D_k^{(0)}$ and price profiles $\underline{p}_k^{(0)}$ and $\bar{p}_k^{(0)}$ of all areas are obtained. The final results of the proposed algorithm (**WE**) are compared with the ones obtained with a price-greedy strategy (**PG**) and the case of no EVs (**NoE**). In the **PG** scenario, EVs will minimize their energy costs in response to the electricity prices obtained by only considering the inflexible demand profiles D_k^i.

The demand and price profiles in each area, under the three scenarios, are shown in Fig. 2. In the **PG** case, when the EVs simply schedule power consumption in response to the prices of inflexible demand, peaks appear in demand profiles D_k^{PG} and consequently in the prices $\bar{p}_k^{PG}$. Conversely, the demand profiles D_k^{WE} obtained in the **WE** case, with the application of the proposed algorithm, are much flatter and the price profiles $\bar{p}_k^{WE}$ are completely flattened between 22:00 and 7:00. Fig. 3 shows the generation profiles in the three scenarios. A similar trend emerges: the generation profiles in the **PG** case have peaks and oscillations whereas the ones in the **WE** scenario are completely flat. Fig. 4 shows the power flows on Line 1-2 and Line 4-5, with maximum capacity of $400MW$ and $240MW$, respectively. Line 4-5 is always congested, leading to price differentials between each area. The comparison of the power flows on Line 1-2 shows that the power exchanged between areas has reduced oscillations in the **WE** case.

Finally, Fig. 5 shows the performance of the proposed strategy (**WE** case) from a system perspective. Fig. 5a compares the total generation profile G_T of the system in the

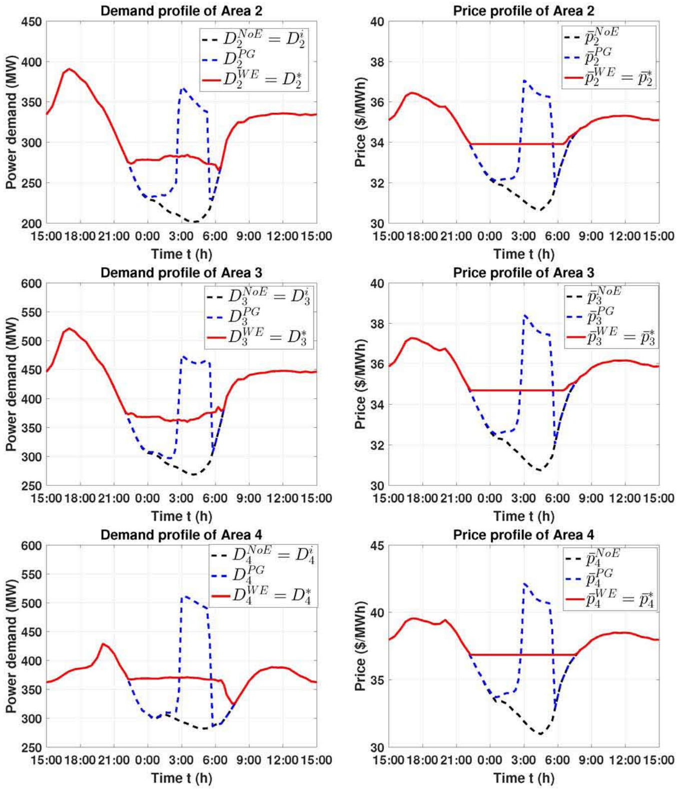

Fig. 2. Demand profiles (left) and price profiles (right).

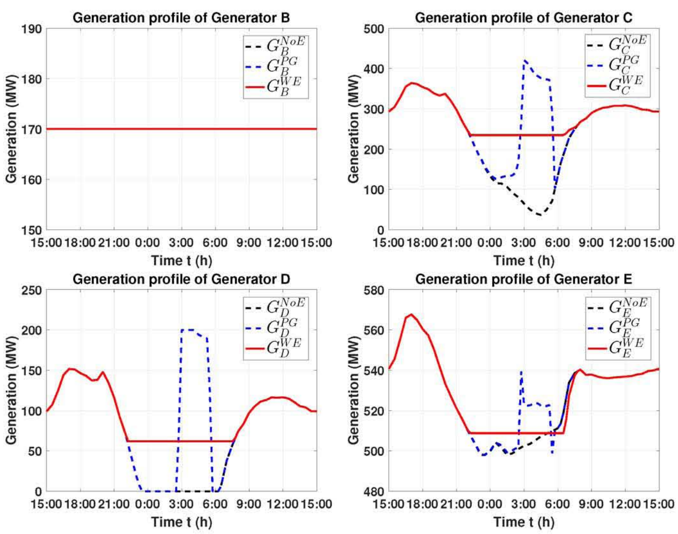

Fig. 3. Generation profiles.

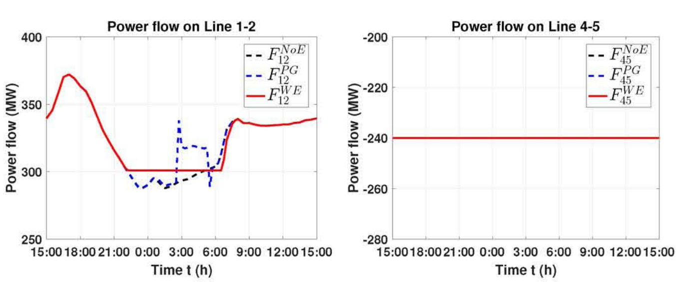

Fig. 4. Power flows.

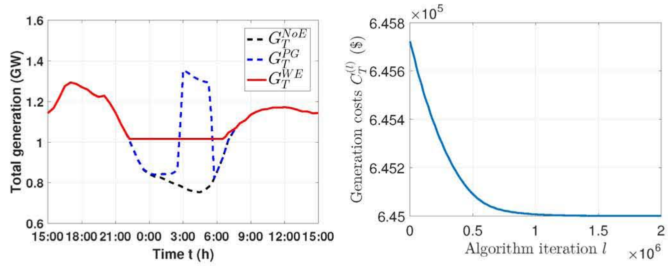

Fig. 5. System performance of the proposed algorithm: (a) Profiles of total generation; (b) Total generation costs as function of algorithm iteration l.

different cases. Note that the total generation (equivalently, total demand) profile G_T^{WE} in the **WE** case is flat, despite the oscillations of the demand profiles D_k^{WE} in the single areas. Given the convexity of the generation cost functions f_k, this result strongly hints to a minimization of total generation costs with the proposed coordination scheme. This is corroborated by Fig. 5b, showing system generation costs C_T as a function of the algorithm iteration l, as expressed below:

$$C_T^{(l)} = \sum_{t=1}^{T} \varphi\left(D^{(l)}(t)\right)\Delta t \tag{12}$$

where $D^{(l)}(t)$ is the demand profile vector at l-th algorithm iteration and φ is the result of the OPF in (2). As expected from the proof of Theorem 1, $C_T^{(l)}$ decreases at each iteration and reaches a minimum value at the aggregative equilibrium, where the algorithm terminates.

V. Conclusion

This paper presents a distributed strategy for the coordination of flexible demand in multi-area systems, taking network topology and capacity constraints of transmission lines into account. The strategy converges to a stable market configuration (characterized as an aggregative equilibrium) through iterative price broadcasts and power updates. Simulations are carried out on a five-area system with high penetration of electric vehicles, showing that the proposed strategy can flatten generation profiles and reduce total generation costs.

Appendix A
Proof of Theorem 1

Proof of convergence: Consider the total generation cost $C_T^{(l)}$ at the l-th algorithm iteration, defined in (12). Firstly, it is verified that the cost C_T is reduced at each iteration l of the algorithm in which a power swap is performed:

$$C_T^{(l)} < C_T^{(l-1)}. \tag{13}$$

This reduction implies the asymptotic convergence of C_T to some limit value, since C_T denotes the total generation cost of the system and is therefore lower bounded. At such value, the algorithm does not perform any additional power swap, since this would correspond to a further cost reduction. As a result, the variable $flag$ remains equal to zero throughout the nested **FOR** cycles and the algorithm terminates. To prove (13) consider that, if at a certain iteration l there exist t_1 and t_2 as specified in step 2.c.iii, a power swap between these two time instants is performed by some device j in a certain area k. As a result, it holds:

$$D_k^{(l)}(t_1) = D_k^{(l-1)}(t_1) + \Delta^* \quad D_k^{(l)}(t_2) = D_k^{(l-1)}(t_2) - \Delta^* \quad (14)$$

where Δ^* is either equal to Δ or $\bar{\Delta}$ and $D_k^{(l)}(t) = D_k^{(l-1)}(t)$ at all the other time instants. Since $\varphi(D)$ is Lipschitz continuous, following the mean value theorem for non-smooth functions [15], there exist $\Delta' \in (0, \Delta^*)$ and $\bar{p}'_k(t_1)$ such that:

$$\bar{p}'_k(t_1) \in \left[\Pi_k^- \left(D^{(l-1)}(t_1) + \Delta' \cdot \hat{e}_k\right), \Pi_k^+ \left(D^{(l-1)}(t_1) + \Delta' \cdot \hat{e}_k\right)\right] \quad (15)$$

and for which it holds:

$$\begin{aligned} \varphi\left(D^{(l)}(t_1)\right) &= \varphi\left(D^{(l-1)}(t_1) + \Delta^* \cdot \hat{e}_k\right) \\ &= \varphi\left(D^{(l-1)}(t_1)\right) + \bar{p}'_k(t_1) \cdot \Delta^*. \end{aligned} \quad (16)$$

Given the strict convexity of φ established in Proposition 1 and recalling (4) and (5), it holds:

$$\begin{aligned} &\bar{p}_k^{(l-1)}(t_1) = \Pi_k^+ \left(D^{(l-1)}(t_1)\right) < \Pi_k^- \left(D^{(l-1)}(t_1) + \Delta' \cdot \hat{e}_k\right) \leq \bar{p}'_k(t_1) \\ &\leq \Pi_k^+ \left(D^{(l-1)}(t_1) + \Delta' \cdot \hat{e}_k\right) < \Pi_k^+ \left(D^{(l-1)}(t_1) + \Delta^* \cdot \hat{e}_k\right) = \bar{p}_k^{(l)}(t_1) \end{aligned} \quad (17)$$

With similar arguments, there exists $\underline{p}'_k(t_2)$ such that:

$$\varphi\left(D^{(l)}(t_2)\right) = \varphi\left(D^{(l-1)}(t_2)\right) - \underline{p}'_k(t_2) \cdot \Delta^* \quad (18)$$

$$\underline{p}_k^{(l)}(t_2) < \underline{p}'_k(t_2) < \underline{p}_k^{(l-1)}(t_2). \quad (19)$$

As $\bar{p}_k^{(l)}(t_1) \leq \underline{p}_k^{(l)}(t_2)$ is always satisfied by construction of the algorithm, combining (17) and (19) yields:

$$\bar{p}'_k(t_1) \leq \underline{p}'_k(t_2). \quad (20)$$

Moreover, since $D^{(l)}$ and $D^{(l-1)}$ only differ at t_1 and t_2, by applying (16) and (18), we obtain:

$$C_T^{(l)} - C_T^{(l-1)} = \left(\bar{p}'_k(t_1) - \underline{p}'_k(t_2)\right) \cdot \Delta^* \cdot \Delta t < 0 \quad (21)$$

where the inequality straightly follows from (20). This result proves (13), thus ensuring convergence.

Proof of equilibrium: It is now shown that the final result u^* returned by the algorithm fulfills (9) in Definition 1 (note that (10) holds by construction) and therefore corresponds to an aggregative equilibrium. To prove (9), it is useful to consider that any feasible u_j can be expressed as the sum of u_j^* and a finite number Q of feasible power swaps δ_q:

$$u_j(\cdot) = u_j^*(\cdot) + \sum_{q=1}^{Q} \delta_q(\cdot). \quad (22)$$

Each δ_q corresponds to shifting a certain amount Δ_q of power consumption from time t_2^q to time t_1^q:

$$\delta_q(t) = \begin{cases} \Delta_q & \text{if } t = t_1^q \\ -\Delta_q & \text{if } t = t_2^q \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

where $\Delta_q > 0$ and $t_1^q, t_2^q \in \mathscr{A}_j$. Moreover t_1^q and t_2^q can always be chosen so as to ensure:

$$u_j^*(t_1^q) < P_j \qquad u_j^*(t_2^q) > 0. \quad (24)$$

Albeit not formally proved for length reasons, it can be shown that the representation in (22) and (23) is always well defined and (24) holds. By replacing (22) and (23) in (9), an equivalent equilibrium condition can be provided:

$$\Delta J = \sum_{q=1}^{Q} \left(\bar{p}_k^*(t_1^q) - \underline{p}_k^*(t_2^q)\right) \cdot \Delta_q \Delta t \geq 0. \quad (25)$$

The equilibrium is now proved by contradiction. If (25) does not hold, since $\Delta_q > 0$ in (23), there must exist $k \in \mathscr{K}$, $j \in \mathscr{N}_k$, $q \in \{1, \ldots, Q\}$ and $(t_1^q, t_2^q) \in \mathscr{A}_j \times \mathscr{A}_j$ such that:

$$\bar{p}_k^*(t_1^q) < \underline{p}_k^*(t_2^q). \quad (26)$$

Note that (26) and (24) are equivalent to the **IF** condition at step 2.c.iii of the algorithm. Since the algorithm has terminated and has returned the power scheduling u^*, we can conclude that its internal variable $flag$ remains equal to zero in the two nested **FOR** cycles, implying that (26) and (24) never hold, thus proving (25) and (9) and ensuring that the final result u^* is an aggregative equilibrium.

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