Evaluation of Parameters Affecting the Energy Accumulation in Longwall Mining

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ABSTRACT: The elastic strain energy accumulation in a retreating longwall mining panel is dependent on several parameters which have been identified using elastic theory for beams in an elastic supporting medium. The identified parameters were varied to identify the parameters that have a strong influence on the elastic strain energy accumulation. Upon sensitivity analysis, mining depth, length of cantilever roof in the goaf, coal seam thickness, roof thickness, Young’s modulus of coal seam and roof were identified as the main influencing parameters. The analytical equations were validated by comparing the results with previous studies and a case-specific numerical model. The results obtained from analytical equations were comparable with those obtained from a numerical model. The developed analytical equations will provide a handy tool to make daily, weekly and monthly prediction of areas of high elastic strain energy accumulations inside the solid coal pillar. Hence, the paper finds direct application in improving the safety of people working in coal mines, minimising machine downtime and production delays.

1. INTRODUCTION

The advancement in the production and support technologies like longwall mining with hydraulic powered supports has facilitated coal extraction from higher mining depths which was earlier unsafe and uneconomical. Deep mining depth conditions leads to the development of a high-stress state mostly due to the gravity loading of the overburden strata (Si et al., 2018). The coal seam and roof strata remain under a combined state of static and dynamic stresses at greater depths (Li et al., 2008, Zhu et al., 2010). Elastic strain energy develops in the coal and surrounding rock mass due to the amount of work done on the coal and rock mass system to compress the strata to the in-situ state (Gale, 2018).

Longwall mining is practised using hydraulic powered supports, with no permanent supports being used (Rezaei et al., 2015). The hydraulic powered supports are advanced as soon as the longwall shearer cuts the coal face. The advancement of hydraulic powered support results in the lowering of the unsupported roof under its own weight forming a cantilevering roof in the goaf. This leads to the development of abutment stresses at the active coal face.

Elastic strain develops in the hanging roof due to its lowering (deflection). The elastic strain in the cantilever roof and the increased abutment stress ahead of the active face contributes to the increased elastic strain energy accumulation in the roof and the coal seam (Wu, 1995). In case of a stiff, massive, strong and competent roof strata, the cantilever roof may hang for a long distance in the goaf resulting in an increased abutment stress and subsequent accumulation of a large amount of elastic strain energy in the active coal face which may result in rock bursts (Haramy and McDonnell, 1988; Zhao and Jiang, 2009; Calleja and Nemcik, 2016; Iannacchione and Zelanko, 1995; Maleki, 1995).

As the coal is extracted, the elastic strain energy stored in the in-situ stress state in the coal seam is released. This energy transfers to the surrounding rock mass (Brady and Brown, 2005). A fraction of the released elastic strain energy increases the elastic strain of the roof strata resulting in fracturing and caving of the surrounding rock mass. Enough fractures develop subsequently in the cantilever goaf roof, and the roof caves some distance behind the hydraulic powered support in the goaf area. The remaining fraction of the released elastic strain energy is transferred to the panel roof strata (Wu, 1995; Rezaei et al., 2015).
Elastic strain energy developed in the coal and surrounding rock mass has been the topic of research for many researchers (Cook, 1965; Kidbyinski, 1981; Salamon, 1984; Wu, 1995; Brady and Brown, 2005; Rezaei et al., 2015; Gale, 2018). Based on experimental results, Cook (1965) pioneered the theoretical analysis of fundamental energy accumulation and release during mining. Stephansson (1971) extended the theory of elastic beams on elastic supports to emphasise the effect of abutment compression on the deflection of the roof. He analysed seven different scenarios commonly observed in underground mining for horizontally bedded deposits and estimated their deflection. The scenario with the height of the roof less than one-fifth of the roof span is applicable in the longwall mining. Wu (1995) used this concept to develop a roof model to analyse the deflection and elastic strain energy stored in a cantilevering roof and the coal seam during periodic weighting phase in a retreating longwall mining panel. Wu (1995) considered the effect of abutment stresses in the supported section of the roof. However, he did not consider the effect of roof supports on the deflection of the roof and allowed the roof to deflect at the mine face (Karfakis and Wu, 1995).

In this paper, the deflection of a cantilever roof and the supported roof is evaluated considering the effect of abutment stresses and hydraulic powered supports using fundamental elastic theory. The concept of elastic beams on elastic supports has been developed incorporating the effect of roof supports, to calculate the deflection of the cantilever and supported roof. The deflection so obtained has been used to evaluate the elastic strain energy stored in the cantilever roof, supported roof and the coal seam. Based on the elastic strain energy equations obtained, the main parameters contributing to the accumulation of elastic strain energy are parametrically analysed.

2. ANALYTICAL EQUATIONS

2.1 Deflection

When a coal seam is extracted, the lower section of the roof detaches itself from the rock layers above due to lowering under its own weight and a gravity loaded rock layer is formed (Stephansson, 1971). This layer is considered as a cantilever beam clamped at the edges by the overburden pressure. The deflection of such beams can be evaluated from the simple beam theory (Stephansson, 1971). To evaluate the effect of various parameters affecting the elastic strain energy build-up in the longwall mining, analytical equations based on the elastic beams on elastic supports are developed incorporating the effect of abutment load and hydraulic powered supports.

A vertical cross-section of a retreating longwall mining panel is shown in Fig. 1. The coal seam with a thickness of \( h_{\text{coal}} \), is assumed to be completely extracted. The immediate roof layer acts as a cantilever beam with a thickness of \( h_{\text{roof}} \), in the goaf area behind the hydraulic powered support as shown in Fig. 1. The roof is assumed to be clamped at point O by the overburden pressure and the installation of hydraulic powered supports. The hydraulic powered support prevents excessive roof deflection at the active face. The deflection of the supported roof compared to the height of the roof is assumed to be negligible for calculation purposes. Due to clamping at point O, a bending moment, \( M \), and a shear force, \( V \), develops at point O as shown in Fig. 1.

If coal is assumed to be an elastic medium, the maximum abutment load \( P_o \) occurs at the face and decreases exponentially ahead of the face in the solid coal pillar (Wu, 1995; Rezaei et al., 2015). In practice, the coal face comprises several cracks due to the high abutment stress and low confinement, hence, it cannot sustain high abutment load. The maximum abutment load occurs at a distance \(-0.015H\) inside the solid coal pillar (Wilson, 1972).

To determine the effect of in-situ and abutment load on the deflection of the overlying strata, several assumptions have been made to apply the Euler-Bernoulli theory of elastic beam (Galvin, 2016). It is assumed that the beam is isotropic, homogeneous, free of any discontinuities and defects, linearly elastic, perfectly straight along its axes, initially stress-free, loaded only normal to its faces, of uniform flexural rigidity and symmetric about an axis in the plane of bending (Galvin, 2016). It is also assumed that the length of each layer is more than two times the width of the layer. The maximum thickness of each layer is assumed to be less than half of the roof span \( l \) (Stephansson, 1971; Wu, 1995). The weight of each layer can thus be approximated by applying a uniformly
distributed load, \( p \), per unit width to its top surface (Galvin, 2016).

An elastic coal foundation is considered having a foundation modulus \( C \) (Eq. (1)) which provides a vertical reaction force \( q(x) \) to the abutment load in the elastic supporting medium (Stephansson, 1971). The symbols presented throughout the paper have the same notation as listed in Table 1.

\[
C = \frac{E_{\text{coal}}}{h_{\text{coal}}(1-\nu_{\text{coal}}^2)}
\]  

(1)

Table 1. List of symbols.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abutment length</td>
<td>( l )</td>
<td>( m )</td>
</tr>
<tr>
<td>Bending moment</td>
<td>( M )</td>
<td>( N \cdot m )</td>
</tr>
<tr>
<td>Characteristic value</td>
<td>( \omega )</td>
<td>( m^{-1} )</td>
</tr>
<tr>
<td>Deflection</td>
<td>( y(x) )</td>
<td>( m )</td>
</tr>
<tr>
<td>Density of overburden</td>
<td>( \rho )</td>
<td>( kg/m^3 )</td>
</tr>
<tr>
<td>Elastic strain</td>
<td>( \varepsilon_x, \varepsilon_y, \varepsilon_z )</td>
<td>( m/m )</td>
</tr>
<tr>
<td>Elastic strain energy</td>
<td>( W_1, W_2, W_3 )</td>
<td>( J )</td>
</tr>
<tr>
<td>Foundation modulus</td>
<td>( C )</td>
<td>( kg/m^3 )</td>
</tr>
<tr>
<td>The height of the coal seam</td>
<td>( h_{\text{coal}} )</td>
<td>( m )</td>
</tr>
<tr>
<td>The height of the roof</td>
<td>( h_{\text{roof}} )</td>
<td>( m )</td>
</tr>
<tr>
<td>In-situ load</td>
<td>( P )</td>
<td>( N )</td>
</tr>
<tr>
<td>Length in the goaf</td>
<td>( L )</td>
<td>( m )</td>
</tr>
<tr>
<td>Load function factor</td>
<td>( \alpha )</td>
<td>( m^{-1} )</td>
</tr>
<tr>
<td>Mining depth</td>
<td>( H )</td>
<td>( m )</td>
</tr>
<tr>
<td>Peak abutment load</td>
<td>( P_o )</td>
<td>( N )</td>
</tr>
<tr>
<td>Poisson’s ratio of coal</td>
<td>( \nu_{\text{coal}} )</td>
<td>( m/m )</td>
</tr>
<tr>
<td>Shear force</td>
<td>( V )</td>
<td>( N )</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>( G_{\text{coal}} )</td>
<td>( Pa )</td>
</tr>
<tr>
<td>Shear stresses</td>
<td>( \tau_{xy}, \tau_{yz}, \tau_{xz} )</td>
<td>( Pa )</td>
</tr>
<tr>
<td>Shear strain</td>
<td>( \gamma_{xy}, \gamma_{yz}, \gamma_{xz} )</td>
<td>( m/m )</td>
</tr>
<tr>
<td>The second moment of inertia</td>
<td>( l )</td>
<td>( m^4 )</td>
</tr>
<tr>
<td>Uniformly distributed load</td>
<td>( p )</td>
<td>( N )</td>
</tr>
<tr>
<td>Vertical reaction force</td>
<td>( q(x) )</td>
<td>( N )</td>
</tr>
<tr>
<td>Young’s Modulus of coal</td>
<td>( E_{\text{coal}} )</td>
<td>( Pa )</td>
</tr>
<tr>
<td>Young’s Modulus of roof</td>
<td>( E )</td>
<td>( Pa )</td>
</tr>
</tbody>
</table>

As per the Euler-Bernoulli theory for elastic beams (Young et al., 2012; Galvin, 2016), the bending moment \( M \) for a deflection \( y(x) \) is given as:

\[
M = EI \left( \frac{d^2y}{dx^2} \right)
\]  

(2)

where \( I \) is the second moment of inertia given by \( I = h_{\text{roof}}^3/12 \) (Stephansson, 1971). The shear force developed across the fixed support is given as \( M'(x) = V(x) \), and the load per unit length \( P = M''(x) \). It is apparent that (Stephansson, 1971):

\[
P = M''(x) = EI \left( \frac{d^2y}{dx^2} \right)
\]  

(3)

As per Fig. 1, three different loading scenarios have been considered to analyse the deflection of the overlying roof strata spanning entire longwall mining panel, these are:

**Scenario 1: Cantilever beam section \((-L < x \leq 0)\)**

As the roof in the goaf detaches itself from other rock layers to form a gravity loaded cantilever beam behind the hydraulic powered supports in the goaf, a uniformly distributed load is considered mainly due to the weight of the cantilever roof thickness \( (p = \rho g h_{\text{roof}}) \). It is assumed that the beam is clamped at one end \( (x = 0) \) and is freely hanging at the other end \( (x = -L) \). A uniformly distributed load \( P(x) \) equal to \( p \) is assumed to be acting on the cantilever beam. The following boundary conditions are applied in the case of a cantilever beam (Kreyszig, 2006):

- Deflection \( y(x) \) at \( x = 0 \rightarrow y(0) = 0 \) (assumed).
- The slope of \( y(x) \) at \( x = 0 \rightarrow y'(0) = 0 \).
- The bending moment \( M \) at \( x = -L \rightarrow y''(-L) = 0 \) (free end).
- The shear force \( V \) at \( x = -L \rightarrow y''(-L) = 0 \) (free end).

**Scenario 2: Abutment load section \((0 < x \leq l)\)**

The mining-induced stresses redistribute upon coal extraction, and abutment stresses are observed at the active coal face. An exponentially decreasing load function, \( P(x) = P_o e^{-ax} \), is considered in the abutment load section with its peak, \( P_o = 2P \), at the coal face (Brady and Brown, 2005). Previous research suggests that the abutment load decreases to the in-situ load \( P \) at a distance \( l = 0.12H \) inside the solid coal pillar (Sheorey, 1993). The exponentially decreasing load function has a maximum value \( P_o \) at \( x = 0 \) and a minimum value \( P \) at \( x = l \), one gets,

\[
P = P_o e^{-at} \rightarrow a = -\frac{\ln p}{l}
\]  

(4)

A vertical reaction force \( q(x) \) proportional to the deflection \( y(x) \) acts against the abutment load \( P(x) \) in the elastic supporting medium (Stephansson, 1971). The following boundary conditions are applied in the abutment load section:

- Vertical reaction force: \( q(x) = C y(x) \)
- The deflection \( y(x) \) at \( x = 0 \rightarrow y(0) = 0 \)
- The slope of \( y(x) \) at \( x = 0 \rightarrow y'(0) = 0 \)

**Scenario 3: Normal load section \((x > l)\)**

In the normal load section, the effect of abutment load is not appreciable, the coal seam is loaded only under the in-situ load. The vertical reaction force \( q(x) = C y(x) \) acts against the in-situ load, \( P \), in the elastic supporting medium. However, for practical purposes, this section would be far inside the solid coal pillar and hence, the deflection and elastic strain energy accumulated in this
region will not affect the coal face substantially. This observation is consistent with the observations made by other researchers (Cao et al., 2018). Cao et al. (2018) observed that the effect of elastic strain energy is mostly limited within ~100 m ahead of the coal face as confirmed by field microseismic monitoring. The load in different sections of the roof can be summarised as,

$$P(x) = \begin{cases} 
   p, & -L < x \leq 0 \\
   P_0 e^{-ax}, & 0 < x \leq l \\
   P, & x > l 
\end{cases}$$  \tag{5}

Using Eq. (5), solving the differential equation, Eq. (3), and substituting the boundary conditions for different sections of a retreating longwall mining panel, the deflection $y(x)$ of the immediate roof can be determined. The deflection of the roof in different scenarios is listed in Table 2.

Table 2. Deflection $y(x)$ for different loading scenarios in a retreating longwall mining panel.

<table>
<thead>
<tr>
<th>Range</th>
<th>Load</th>
<th>Deflection $y(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-L &lt; x \leq 0$</td>
<td>$p$</td>
<td>( \frac{1}{24EI}px^2(x^2 + 4Lx + 6l^2) )</td>
</tr>
<tr>
<td>$0 &lt; x \leq l$</td>
<td>$P_0 e^{-ax}$</td>
<td>( \frac{P_0}{(EIa^4 + C)} \left[ e^{-ax} + e^{-ax} \left( \frac{A}{a} \sin(\omega x) - \cos(\omega x) \right) - \sin(\omega x) \right] )</td>
</tr>
</tbody>
</table>

\( \omega \) is the characteristic value obtained in the general solution of the ordinary differential equation by solving the Euler’s identity, which is given by $\omega = \sqrt{C/4EI}$ (Stephansson, 1971).

The deflection equations derived for different loading scenarios in a retreating longwall mining panel as listed in Table 2 are different from those developed by Wu (1995). It is also apparent from equations in Table 2, that the length of goaf $L$ is a direct manifestation of the rate of face advance, which has not been studied before. Hence, these relationships provide updated equations for measuring the deflection of the longwall roof in the practical mining scenario as observed in a retreating longwall mining panel. These equations have been used to calculate the elastic strain energy accumulated in different sections of the roof and the coal seam as discussed in the next section.

2.2 Elastic Strain Energy

The mechanical energy stored in an elastically stressed system is termed as elastic strain energy. When static loading is applied to an elastic system, the external work done by the static load as they increase from zero to the maximum value is equal to the elastic strain energy developed in the system (Young et al., 2012). As mining progresses, the in-situ stress field is disturbed leading to mining-induced stresses around a retreating longwall mining panel. Strain energy is proportional to the square of the stresses accumulated in the roof and the coal seam (Young et al., 2012). Analytically, the elastic strain energy accumulation occurring due to cantilevering beams over the working faces can be evaluated using elastic beam theory (Wu, 1995).

The deflection $y(x)$ of the cantilevering roof under a uniformly distributed load $p$, can be used to determine the elastic strain energy stored in the cantilever roof. The deflection $y(x)$ at the point of loading in the direction of load is equal to twice the strain energy divided by the load (Young et al., 2012). The deflection $y(x)$ of the roof listed in Table 2 is used to calculate the elastic strain energy developed in the cantilevering and supported roof and the coal seam by evaluating the work done by the load.

In longwall mining, the total elastic strain energy accumulated in the system comprises of primarily three components,

(i) Elastic strain energy stored in the cantilever roof, $W_1$
(ii) Elastic strain energy stored in the supported roof, $W_2$, and
(iii) Elastic strain energy stored in the coal seam, $W_3$

For a bending beam, the flexural stress and strain can be evaluated using the following equations (Wu, 1995; Young et al., 2012):

$$\sigma_x = \frac{My}{I}, \quad \varepsilon_x = \frac{\sigma_x}{E}, \quad l = \int_{area} y^2 \, dA$$

$$W_l = \frac{1}{2} \int_{volume} \sigma_x^2 \, dV \rightarrow W_l = \frac{1}{2} \int_0^L \frac{M^2}{EI} \, dx \tag{6}$$

2.2.1 Elastic strain energy stored in a cantilever beam

Taking the deflection $y(x)$, as listed in Table 2, for the range $-L < x \leq 0$, the bending moment $M$ (Eq. (2)) can be calculated as:

$$M = \frac{p}{2} \left( x^2 + 2Lx + L^2 \right) \tag{7}$$

Substituting Eq. (7) in Eq. (6) and integrating over the range $-L < x \leq 0$, one gets:

$$W_1 = \frac{p^2L^5}{40EI} \tag{8}$$

Eq. (8) is the same as reported in previous research work to calculate the energy accumulated in a cantilever beam (Wu, 1995; Young et al., 2012; Galvin, 2016). This confirms that the boundary conditions considered in Section 2.1 to calculate the deflection of the roof is correct.

2.2.2 Elastic strain energy stored in the supported roof

The deflection $y(x)$, as listed in Table 2, in the supported roof range $0 < x \leq l$, having an abutment load $P_0 e^{-ax}$, the bending moment $M$ (Eq. (2)), of the supported beam, can be calculated as:
\[ M = \frac{P_0 E_I}{(a + x)^2} [\alpha^2 e^{-ax} - 2\omega e^{-\omega x}((\alpha - \omega)\cos(\omega x) + \omega \sin(\omega x))] \tag{9} \]

Substituting Eq. (9) in Eq. (6) over the range \(0 < x \leq l\), one gets:

\[ W_2 = \frac{1}{2} \int_0^l \frac{M^2}{E_I} \, dx \tag{10} \]

### 2.2.3 Elastic strain energy stored in the coal seam

The elastic strain energy stored in the coal seam in the abutment load range \(0 < x \leq l\), having abutment load \(P_0 e^{-ax}\) and deflection \(y(x)\), can be calculated using (Wu, 1995; Wang and Park, 2001; Young et al., 2012; Gale, 2018):

\[ W_3 = \frac{1}{2} (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy} \gamma_{xy} + \tau_{yz} \gamma_{yz} + \tau_{zx} \gamma_{zx}) \tag{11} \]

Following Hooke’s law, the stress-strain relationship for elastic beams can be described as (Wu, 1995; Young et al., 2012):

\[ \varepsilon_x = \frac{1}{E} [\sigma_x - u_{coal} (\sigma_y + \sigma_z)] \]
\[ \varepsilon_y = \frac{1}{E} [\sigma_y - u_{coal} (\sigma_x + \sigma_z)] \]
\[ \varepsilon_z = \frac{1}{E} [\sigma_z - u_{coal} (\sigma_x + \sigma_y)] \]
\[ \gamma_{xy} = \frac{\tau_{xy}}{G_{coal}}, \gamma_{yz} = \frac{\tau_{yz}}{G_{coal}}, \gamma_{zx} = \frac{\tau_{zx}}{G_{coal}} \tag{12} \]

Substituting Eq. (12) in Eq. (11) and rearranging, one gets:

\[ W_3 = \frac{1}{2E_{coal}} \left( \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \right) - \frac{u_{coal}}{E_{coal}} (\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_x \sigma_z) + \frac{1}{2G_{coal}} \left( \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 \right) \tag{13} \]

The vertical reaction force \(q(x) = Cy(x)\) acts against the abutment load in the elastic supporting medium. Hence, the stress in the coal seam are assumed to be biaxial:

\[ \sigma_z = Cy, \quad \sigma_x = \sigma_y = \frac{u_{coal}}{(1 - u_{coal})} \sigma_z = \frac{u_{coal}}{(1 - u_{coal})} Cy \tag{14} \]

Substituting Eq. (14) in Eq. (13), neglecting shear strain \(\tau_{xy} = \tau_{yz} = \tau_{zx} = 0\), integrating over the abutment load range, and rearranging, one gets:

\[ W_3 = \frac{E_{coal}}{2} \left[ \frac{(1 - u_{coal})}{(1 - 2u_{coal})} \right] \int_0^l y^2 \, dx \tag{15} \]

The total elastic strain energy stored in a retreating longwall mining panel near to the working face is calculated by adding Eqs. (8), (10) and (15):

\[ W_{\text{total}} = W_1 + W_2 + W_3 \tag{16} \]

Analysing Eqs. (8), (10) and (15), it can be found that the main parameters affecting the accumulation of elastic strain energy in a retreating longwall mining are mining depth \((H)\), the height of the coal seam \((h_{coal})\), the height of the roof \((h_{roof})\), the length of the cantilever roof \((L)\), Young’s modulus of coal \((E_{coal})\), Young’s modulus of the roof \((E)\), and Poisson’s ratio of coal \((\nu_{coal})\). These parameters were evaluated for their severity as discussed in the next section.

### 3. SENSITIVITY ANALYSIS

The deflection of the roof and elastic strain energy accumulated in the cantilevering roof, supported roof and the coal seam in a retreating longwall mining panel was analysed by varying the parameters listed in Section 2.2. The parametric variation undertaken in the sensitivity analysis is listed in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L), m</td>
<td>10, 20, 30</td>
</tr>
<tr>
<td>(H), m</td>
<td>600, 800, 1000, 1200</td>
</tr>
<tr>
<td>(h_{coal}), m</td>
<td>2, 4</td>
</tr>
<tr>
<td>(h_{roof}), m</td>
<td>4, 8, 12, 16, 20</td>
</tr>
<tr>
<td>(E_{coal}), GPa</td>
<td>1.00, 1.50, 2.00, 2.50, 3.00, 3.50</td>
</tr>
<tr>
<td>(E), GPa</td>
<td>4.00, 8.00, 12.00, 16.00</td>
</tr>
<tr>
<td>(\nu_{coal})</td>
<td>0.20, 0.25, 0.30, 0.35, 0.40</td>
</tr>
</tbody>
</table>

Previous research suggests that the abutment stresseses rapidly change up to \(\sim 10\) m inside the solid coal pillar (Yang et al., 2011) and the maximum liability of rock bursts can occur at a distance up to four times the height of coal extraction, beyond which the elastic strain energy build-up does not increase rock burst liability at the coal face (Kidybinski, 1981; Haramy and McDonnell, 1988). Hence, energy accumulation in the parametric analysis was analysed at a distance within \(10\) m ahead of the coal face.

As per the assumption of the application of elastic theory for beams, the maximum height of the roof layer should be less than half of the length of the roof span, i.e., \(H < l/2\). Three different scenarios for the length of the cantilevering roof have been considered, i.e., for a \(10\) m length of the cantilever roof, the roof thickness of \(4\) m was considered. Similarly, for a \(20\) m and \(30\) m length of the cantilever roof, the roof thickness of \(8\) m and \(12\) m were considered respectively.

It can be observed from Fig. 2 that for a \(2\) m thick coal seam, the deflection of the cantilever roof is around \(90\) mm for a cantilever roof of \(30\) m length in the goaf. It can also be observed from Fig. 2, that due to the bending moment, \(M\), at the clamped coal face, the roof in the supported roof section tends to move upward, which increases the abutment load and elastic strain energy accumulation in the supported roof. Fig. 3 shows the variation in the total elastic strain energy developed inside the solid coal pillar for varying length of cantilever roof.
hanging in the goaf. It can be observed from Fig. 3, that the total elastic strain energy increases for an increase in the cantilevering roof length. The sensitivity analysis presented in this paper analyses the total elastic strain energy accumulation in a retreating longwall mining panel of a cantilever roof of 10 m length hanging in the goaf.

It can be observed from Fig. 3, that the elastic strain energy developed inside the solid coal pillar increases immediately behind the coal face. This can be attributed to the effect of bending moment at the clamped face and the abutment load. The energy decreases subsequently with an increase in the distance inside the solid coal pillar validating the observations of previous researchers (Yang et al., 2011).

Fig. 4 shows the variation of total elastic strain energy accumulation due to an increase in the mining depth. It can be observed from Fig. 4, that the elastic strain energy increases with an increase in mining depth following the same trend due to increasing overburden load. It can be observed from Fig. 5 that with an increase in the thickness of the coal seam, the elastic strain energy accumulation increases slightly.

Fig. 5 shows the variation of total elastic strain energy due to variation in Young’s modulus of the coal seam. It can be observed from Fig. 5, that for a lower value of Young’s modulus of the coal seam, the total elastic strain energy accumulated inside the solid coal pillar is more. It can be observed from Fig. 6, that for a thicker roof (5 m), more elastic strain energy was accumulated as compared to that for a thinner roof. This observation is obvious since massive roof strata are difficult to cave, it would develop more abutment load and accumulate more elastic strain energy in the solid coal pillar.

Fig. 7 shows the variation of total elastic strain energy due to variation in Young’s modulus of the roof. It can be observed from Fig. 7, that for a lower value of Young’s modulus of the roof, the total elastic strain energy accumulated inside the solid coal pillar is more. It can be observed from Fig. 8, that the total elastic strain energy accumulated inside the solid coal pillar is more for a higher value of Young’s modulus of the roof. This suggests that a stiff, massive and competent roof will accumulate more elastic strain energy which is commonly observed in coal mines. The variation in Poisson’s ratio
of the coal seam does not have any significant impact on the total elastic strain energy accumulation inside the solid coal pillar as shown in Fig. 9.

Fig. 6. Variation in the elastic strain energy accumulated in the coal seam and the supported roof due to variation in roof thickness.

Fig. 7. The effect of the variation in Young’s modulus of coal to the total elastic strain energy accumulated.

Based on the observations made from the sensitivity analysis, the total elastic strain energy increases with an increase in mining depth. However, for the same mining depth, a thick coal seam will have a slightly more elastic strain energy accumulation as compared to a thin coal seam.

The elastic strain energy accumulation increases with an increase in the roof thickness. Softer coal (lower value of Young’s modulus) tends to accumulate more elastic strain energy. However, a stiff, massive and competent roof (higher value of Young’s modulus) tend to accumulate more elastic strain energy.

To validate the observations made from the analytical equations, a 3-dimensional numerical model has been set up and implemented as discussed in the next section.

4. NUMERICAL MODELLING

A 3-dimensional model has been developed in FLAC\textsuperscript{3D} with a model dimension of $300 \times 50 \times 120$ m having a grid size of $1 \times 1 \times 1$ m along X-, Y-, and Z- directions respectively. A coal seam of 2 m was simulated at a depth of 800 m with a modeled roof of 70 m and a floor of 48 m in the numerical model. A total of 1,800,000 zones have been generated in the basic model.

The vertical displacement (Z-direction) of the model was restricted at the bottom plane by a pinned boundary. The horizontal displacement orthogonal to the plane of the model (X- and Y- directions) was restricted along each side-plane by roller boundaries. The top surface of the model was free to deform. A vertical load was applied at the top surface to simulate the overburden load of 800 m. The boundary conditions applied to the model is shown in Fig. 10. The model is loaded under gravity with an average density of 2,360 kg/m\textsuperscript{3}. 

Fig. 8. The effect of the variation in Young’s modulus of the roof to the total elastic strain energy accumulated.

Fig. 9. The effect of the variation in Poisson’s ratio of coal to the elastic strain energy accumulation.
The roof, coal and floor were assigned elastic constitutive material properties as it has been reported (Yang et al., 2011; Rezaei et al., 2015) that abutment stresses changes rapidly ~10 m ahead of the coal face, where the coal is likely to remain elastic (intact) (Cao et al., 2018). The rock properties used in numerical modelling are listed in Table 4.

Table 4. Rock properties used in modelling.

<table>
<thead>
<tr>
<th>Rock layer</th>
<th>Roof</th>
<th>Coal</th>
<th>Floor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk modulus, GPa</td>
<td>6.67</td>
<td>2.45</td>
<td>6.67</td>
</tr>
<tr>
<td>Shear modulus, GPa</td>
<td>3.07</td>
<td>0.94</td>
<td>3.07</td>
</tr>
</tbody>
</table>

A flow chart of the numerical modelling procedure is presented in Fig. 11. After the basic model was developed, the model was loaded to initialise the stress conditions as shown in Fig. 12. Headgate and tailgate were developed as per the dimensions as shown in Fig. 13 to form a longwall panel and rib pillars.

The coal was extracted sequentially from the longwall panel, 180 × 1 × 2 m along X-, Y- and Z- directions respectively was removed in each excavation step by using the null constitutive model in FLAC3D. The hydraulic powered supports were installed along the longwall face and were advanced sequentially after each excavation step as shown in Fig. 14.

The deflection of the roof and the total elastic strain energy accumulated inside the solid coal pillar was calculated from the equilibrated model to analyse the effect of different influencing parameters. A comparative numerical model was simulated to compare the deflection, stress and total elastic strain energy developed in the numerical model with the analytical solutions for a mining depth of 800 m. Other parameters were set as follows: $h_{coal} = 2$ m, $h_{roof} = 4$ m, $E_{coal} = 2.5$ GPa, $E = 8$ GPa, and $v_{coal} = 0.3$. Hydraulic powered supports applied a vertical reaction force of 10 MPa in the roof and the floor at the working face.
The roof deflection found in the numerical model is shown in Fig. 15. It can be observed from Fig. 15 that the deflection for a 10 m cantilever beam is around 40 mm while that calculated in analytical solution is around 10 mm. In an analytical solution, the weight of the cantilever beam is the only load acting on it. However, in the practical scenario, the overburden load will also have some influence on the loading of the cantilever roof resulting in larger deflection. The stress redistribution ahead of the coal face is shown in Fig. 16. It can be seen from Fig. 16 that the maximum abutment occurs within ~10 m of the coal face and subsequently decreases with an increasing distance inside the solid coal pillar.

Fig. 17 compares the total elastic strain energy developed in the numerical model with the analytical solution for a distance up to 10 m inside the solid coal pillar. It can be observed from Fig. 17, that total elastic strain energy accumulated in the numerical model is slightly higher than that calculated from the analytical solution. This observation is obvious, as the deflection of the cantilever beam is more in the numerical model, hence, more energy will be accumulated ahead of the face. The total elastic strain energy decreases with an increase in the distance inside the solid coal pillar. This observation is similar to previous research which suggests that the abutment stresses rapidly change up to ~10 m inside the solid coal pillar (Yang et al., 2011).

5. CONCLUSIONS

Based on the sensitivity analysis of the main parameters influencing the deflection and accumulation of elastic strain energy, it can be concluded that the mining depth, length of cantilever roof in the goaf, coal seam thickness, roof thickness, Young’s modulus of coal and roof have a strong influence on the total elastic strain energy accumulation. However, the Poisson’s ratio of coal does not affect elastic strain energy accumulation significantly.

Comparison of the results between analytical and numerical model suggests that the trend observed is similar. However, the total elastic strain energy calculated from the numerical model is slightly higher as compared to the analytical model. Hence, the developed analytical equations can be used on a day-to-day basis to determine the total elastic strain energy accumulated inside the solid coal pillar.

The parameters used in analytical equations are readily available in every mine. The equations presented in this
paper are very handy and can be solved on any computer without the expert knowledge of simulation software. These equations will provide a useful tool to evaluate daily, weekly and monthly situations in terms of elastic strain energy build-up and will thus help in identifying areas that may become hazard-prone in near future. Site-specific threshold limits can be determined depending on mining conditions, beyond which the risk of rock burst increases. The paper thus finds direct application in improving the safety of mine workers in deep coal mines, minimising machine downtime and production delays.

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REFERENCES


