Seismic Performance of Single Layer Steel Cylindrical Lattice Shells

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Abstract

This paper examines the elastic and inelastic seismic behaviour of single layer steel cylindrical lattice shells. The main dynamic characteristics for this form of structure are firstly examined through a parametric assessment, which also leads to proposed expressions for estimating the fundamental period and mode of vibration. The seismic response of five typical shell configurations, representing a wide range of rise to span ratios, is then assessed within the linear elastic range under selected earthquake excitations. Particular focus is given to the relative influence of the horizontal and vertical seismic components on the internal actions. In order to provide a means for evaluating the underlying inelastic behaviour, a simple pushover approach, which is suitable for this structural form, is suggested using the forces obtained from the fundamental mode shape. The peak angle change is proposed as a damage parameter within the nonlinear analysis for characterising the inelastic global and local demands in shells of different geometries. Incremental dynamic analysis is subsequently carried out in order to evaluate the detailed nonlinear time history response. The results provide detailed insights into the influence of the horizontal and vertical excitations on the nonlinear seismic response, and illustrate the suitability of the peak angle change as an inelastic deformation measure for shells of different geometric configurations. The main findings from the linear and nonlinear assessments are highlighted within the discussions, with a view to providing guidance for performance based assessment procedures as well as simplified design approaches.

Keywords: Seismic performance; dynamic characteristics; cylindrical lattice shells; pushover response; incremental dynamic analysis.

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1. Introduction

Shell structures represent an uncommon symbiosis between architecture and structural engineering, where the form or space created is defined by the forces themselves [1]. Single-layer cylindrical lattice shells are an example of these structures, and are commonly seen in train stations and sports facilities. They are formed by short bars, commonly circular hollow sections, arranged in a triangulated lattice held together by fixed nodes, so as to provide resistance against buckling.

Lattice shells are habitually designed with mainly gravity loads in mind, as they can span large distances efficiently without intermediate columns. Their general design against gravity loads and global buckling has been covered extensively in previous studies [2-10], for all sorts of geometries including elliptic paraboloids [11, 12], spherical domes [13, 14], and cylindrical forms [15-20]. Although these structures are mainly designed for gravity loads, horizontal forces produced by earthquakes can have a significant influence on various design and performance aspects.

A number of previous studies have examined the dynamic characteristics of cylindrical lattice shells. For example, Yamada [21] modelled the lattice shell as an orthotropic continuum, taking into account the angles between the members to determine the equivalent stiffness in each direction in order to determine the dynamic properties using classical shell theory. The influence of different geometrical configurations and boundary conditions on the dynamic characteristics of single layer cylindrical lattice shells was also explored in previous studies [22, 23]. However, these studies involved limited parameter ranges, and did not result in reliable prediction relationships or guidance for practical design application.

The seismic behaviour of lattice shells has also been the subject of some research interest in recent years. Nie et al [24] studied the performance of single layer domes, and proposed a damage model and an associated fragility analysis. Other aspects related to spherical domes under earthquake actions have also been examined. For example, Yang et al [25] proposed a theoretical strut model for use in the seismic analysis of latticed domes, for capturing the post-buckling behaviour in a single element, in order to improve computational efficiency. Liu and Ye [26] used a genetic simulated annealing algorithm to optimise shell collapse capacity to avoid dynamic instability, based on the failure modes identified by Zhi et al [27, 28]. The failure mechanisms were classified as either strength failure or dynamic instability, depending on how sudden or gradual the maximum displacement was reached as plasticity spread within the shell when subjected to two earthquake excitations of increasing intensity [29]. In a more recent study, Qi et al [30] examined the damage modes of a single layer lattice shells, including both spherical domes and cylindrical vaults. It was suggested that the plastic yielding of members is not the governing factor for failure of single layer cylindrical lattice shells, but rather the combined effect of the buckling of members leading to structural instability, and joint fractures leading to a change of structural topology.
In terms of providing tools for their design, Takeuchi et al. [31] proposed amplification factors to evaluate the seismic response of lattice shell roofs, and suggested a method to determine equivalent static loads for design. However, the shells considered were supported by substructures, in which most of the seismic demand was concentrated. To address the particularities of structures other than conventional framed buildings, Ohsaki et al. [32] performed a series of multimodal pushover analyses to predict the inelastic seismic response of long-span arches. In this study, the inadequacy of the conventional horizontal drift as a damage measure for spatial structures was noted, as it fails to capture the vertical displacements that occur. Instead, a measure referred to as the representative displacement was proposed, which takes into account the total displacement of the arches, noting that such parameter however does not appear to correspond to a direct physical interpretation.

Although several studies have been carried out on assessing the seismic response of spatial shell structures, there is still no consensus on an accepted approach for practical design [33]. Current European seismic design codes do not incorporate specific provisions for these types of structures [34]. The lack of codified guidance leaves the designer with no other choice but to perform complex seismic analysis to design these structures. There are no simplified expressions for period estimation that are readily available, or clear depictions of the mechanisms by which cylindrical shells are affected in seismic events, in terms of areas of maximum stresses, plastic zone development, or influence of geometry and loading.

This paper therefore aims to provide a detailed insight into the key parameters influencing the seismic response of single layer cylindrical lattice shells, with a view to identifying parameters and approaches suitable for consideration in practical design and assessment procedures. A detailed parametric assessment first highlights the main aspects that influence the dynamic characteristics, and culminates in simplified expressions for estimating the fundamental period of vibration and associated mode shape. The response of shells of different rise to span ratios in the linear elastic range under horizontal and vertical earthquake excitations are then examined, and the critical zones and members within the structure are identified in each case. The shells are subsequently assessed in the inelastic range by means of pushover analysis, and the peak angle change is proposed as a damage measure that is particularly suited for assessing the inelastic behaviour of this form of spatial structure. This parameter is shown to capture the global and local demand in a representative manner that provides a clear physical correspondence with the actual performance. Finally, incremental dynamic analysis is used to examine the full nonlinear seismic response up to collapse, with particular emphasis on the influence of the vertical and horizontal excitations as well as that of the geometric form on the behaviour.
2. Structural Configuration

This paper assesses the seismic behaviour of single layer steel cylindrical lattice shells of the form depicted in Figure 1. Based on numerous previous studies [2, 4, 5, 35], this three-way configuration of cylindrical lattice shell, with the longitudinal members parallel to the generatrix, or longitudinal axis of the cylinder, is deemed to be a highly efficient structural system under symmetrical and asymmetrical loads. This form also possesses a pleasing aesthetic appearance, and is consequently widely used in train stations, museums, or sports arenas. Due to these structural and architectural merits, single layer cylindrical lattice shells of this configuration have attracted significant research in the past [17, 18, 21-23, 29, 36-38], and this paper hence builds upon this work, with a focus on their seismic behaviour.

The key parameters that define the geometry of these shells are the span, B, the rise, H, and the length, L. The rise and the span define three points that form a circular arc on which the nodes of the edge arches are located at equal distances. The number of divisions in the arch plane are labelled $n_{\text{div}}$, and these nodes are then joined together by straight lines. These are then extruded along the length L, and divided longitudinally into $n_{\text{long}}$ divisions, to form a triangulated grid. The angle formed by the diagonal members and the longitudinal members is labelled $\alpha$. Finally, two important ratios derived from these key parameters are the rise-to-span ratio, $H/B$, and the length-to-span ratio, $L/B$. The X-direction herein corresponds to the transversal direction, parallel to the edge arches, the Y-direction is the longitudinal, parallel to the longitudinal members or the axis of the cylinder, and the Z-direction is the vertical.

The boundary conditions considered in this study correspond to those deemed to be found most frequently in practice. The longitudinal edges of the cylindrical shell are pinned, with all translational movements restrained, but free to rotate in any direction. The edge arches are unrestrained, free to move and rotate, simulating the condition where there is an opening in that plane, or a non-structural glass façade, as would often be the case in airplane hangars or train stations.

The loading is considered as a surface load, $q$, given in kN/m², as would be common in practice when accounting for superimposed dead loads, live loads, or snow loads. This surface load is then multiplied by the tributary area of each node, and the load is applied on the structure as vertical point loads in the Z-direction on the nodes, or intersections between elements. These loads contribute to the mass of the structure when performing dynamic analyses, where the masses are typically modelled as lumped at the nodes as well.

The open-source nonlinear structural analysis program OpenSees [39] is used for modelling the shells considered in this investigation. For the parametric assessments of the dynamic characteristics, elastic beam column elements were used. For the nonlinear analysis studies, force-based beam-column elements were adopted, including geometric nonlinearities by using the co-rotational geometric transformation. Material nonlinearities were included by using fibre sections to model the circular hollow sections, using the uniaxial material Steel02
[40]. This material has a more rounded transition into the yield plateau, which was deemed more representative of the behaviour of cold formed steel circular hollow sections, which are often employed for these types of shells, as they characteristically do not exhibit a defined yield point and transition smoothly into the strain hardening region. The material was modelled with a Young’s Modulus $E = 210,000$ N/mm$^2$, a yield strength $f_y = 275$ N/mm$^2$, and a strain hardening ratio of 0.5%, as a representative idealised value which approximates the actual constitutive response of the steel material adopted, as has been used in a number of previous studies [41, 42].

3. Dynamic Characteristics

A parametric assessment was firstly carried out to determine the influence of different geometric parameters on the vibration modes and natural periods of vibration of single layer cylindrical lattice shells. In order to build on and expand results from previous studies [22, 23], the reference span and length were set at $B = 15$ m and $L = 27$ m. These parameters were then varied to assess the influence of the rise-to-span ratio ($H/B$) and the length-to-span ratio ($L/B$) on the dynamic characteristics of the shells.

For the reference shell, the number of subdivisions in the arch plane is $n_{\text{div}} = 8$, and the number of divisions longitudinally is $n_{\text{long}} = 9$. The distance between supports in the longitudinal (Y) direction is 3 m. All of the members are circular hollow sections, and are joined together by rigid joints. Using similar initial sizes as those adopted in previous studies [23], the diagonal members have an external diameter of 127 mm, and a thickness of 4.5 mm, while longitudinal members have a diameter of 83 mm, and a thickness of 3.5 mm. Although in design the member sizes would change depending on applied load and geometry, for this initial study, the sizes are kept constant so as to reduce the number of changing variables, and provide a clearer picture of how each parameter influences the natural frequency.

3.1. Rise-to-Span Ratio

The dynamic characteristics of a single layer cylindrical lattice shell are most significantly affected by $H/B$. A wide range of $H/B$ ratios were considered herein, starting from a rise of 0.75 m, representing $H/B$ of 0.05, all the way up to 7.50 m, representing a full semi-circle of $H/B = 0.50$. The rise was varied in increments of 0.15 m, which corresponds to an incremental increase of 0.01 in $H/B$. The applied load was varied between no applied load and $q = 5.0$ kN/m$^2$, and the $L/B$ ratio was varied between 1.0 and 3.0 in order to assess their interdependence with $H/B$. The first eight modes and corresponding periods for selected shells with representative $H/B$ values are shown in Figure 2.

As shown in Figure 3, the fundamental natural period of single layer cylindrical lattice shells with free ends increases with $H/B$, at a relatively slow rate for $H/B$ between 0.05 (1/20) and 0.25 (1/4), and more rapidly afterwards. As expected, the larger the rise of the shell, the more
flexible it becomes laterally, as the inclination of the members close to the supports becomes more vertical. In other words, the stiffness with respect to horizontal loads, provided through axial action, is decreased. This trend is independent of L/B, which is expected given the shape of the first mode as indicated in Figure 2. The entire shell moves in the X direction in unison, with no differences in movement longitudinally. The H/B ratio does not change the shape of the first mode consisting of two half waves, with the largest displacements taking place at the quarter points of the shell.

The lateral response of the shells is typically dominated by the first mode, as illustrated in Figure 4 which depicts the modal participating mass ratio in the X-direction, as a function of H/B. The difference between the modal participating mass ratio of the first mode, and that of the sum of the first twenty-five modes, is relatively small. It is important to note that the modal participating mass ratio of the sum of the first twenty-five modes does not reach a value of over 0.90 until relatively high values of H/B are used. Shallow shells have a more complex dynamic response, and the influence of higher modes is more significant with lower H/B. For example, up to 90 modes need to be included for H/B = 0.05 in order to reach a total mass participation of 90% in the X direction. This contrasts with their vertical response, which reaches 90% within the first twenty-five modes, as evident in Figure 4, unlike deeper shells. This difference in behaviour is clearly important, as it influences the extent to which the vertical component of seismic action affects the response.

3.2. Length-to-Span Ratio

In a similar way to H/B, the L/B ratio was also varied across a wide practical length range to assess its influence on the dynamic characteristics. The length ranged from 15 m, representing L/B = 1.0, to 45 m, representing L/B = 3.0. In order to maintain the angle between the members, the length is only increased in 3 m increments, increasing n_{long} by one in each model, adding one grid of triangles, and increasing the L/B ratio by 0.2. In a similar fashion to the H/B study, the applied load was kept constant at q = 1.0 kN/m², and the H/B ratio was also varied to assess the interdependence.

The L/B ratio was found to have no influence on the fundamental natural period of the structure for these boundary conditions, as discussed before. The corresponding figures are therefore not presented herein for the sake of brevity. The natural period remains constant and is independent of the length in the case where the edge arches are unrestrained. As noted previously, this is due to the two dimensional arch-like mode shape of the shell with unrestrained edges, where the length of the shell has no effect on the mode shape.

3.3. Applied Mass

The mass was also changed to assess its influence on the dynamic properties, by varying the applied surface load on the structure. This was varied from a case of no super-imposed applied load, with the mass of the structure only based on the self-weight of the steel
members, to 5.0 kN/m² of applied load, increased in increments of 0.2 kN/m² in order to capture sufficient data points. In this study, the L/B ratio was maintained at 1.8 (i.e. L = 27m), while the H/B ratio was varied.

As expected, the fundamental natural period increases with the increase in applied load, as shown in Figure 5. The mass has a direct effect on the natural period of the shells, which increases proportionally with the square root of the mass, and having a larger influence on deeper shells of higher H/B ratios.

3.4. Diagonal-to-Longitudinal Angles

The angle between the longitudinal and diagonal members, $\alpha$, was varied in order to assess the influence of the topology and density of the geometric mesh on the dynamic response. As shown in Figure 6, this was examined in two ways. Firstly, the number of divisions in the arch plane, $n_{\text{div}}$, was varied from 4 to 30, increasing the density of members in the longitudinal direction. Secondly, the number of divisions into which the total length is divided longitudinally, $n_{\text{long}}$, was varied from 2 to 30, increasing the density of diagonal members in the transversal direction. The rise and length were kept constant at 3 m and 27 m respectively, i.e. H/B = 0.2 and L/B = 1.8, and the load was maintained as 1.0 kN/m².

As shown in Figure 7, the fundamental natural period decreases when the angle is increased by varying $n_{\text{div}}$. When the angle is varied by changing $n_{\text{long}}$, a similar trend is observed, although more pronounced. As the angle approaches 90º, the diagonals tend toward becoming a series of parallel arches. Since the fundamental mode implies movements in the transversal direction, the closer the members are to this direction the stiffer the structure becomes.

Increasing the angle by increasing $n_{\text{long}}$ is akin to increasing the density of the mesh in the transversal direction or, in other words, adding more diagonal members per length of shell, with a steeper angle. This adds stiffness to the shell in the transversal direction, hence reducing the natural period. Decreasing the angle by increasing $n_{\text{div}}$ means adding more longitudinal members to the shell, which provides an insignificant contribution to the lateral resistance. This produces an increase in mass with a minimal increase in stiffness in the transversal direction, resulting in an increase in natural period.

3.5. Proposed Prediction Relationship

Based on the results of the parametric assessments described above, it would be useful to provide simple expressions that can be used for estimating the fundamental period of vibration. Various previous studies have provided theoretical derivations for the natural frequencies of arches, shells, and plates [43]. The fundamental natural frequency of a cylindrical shell with free ends is similar in formulation to that of a plane arch, but with an added term to account for the bending stiffness of a shell. As discussed above, the length of a free ends shell does not affect its natural frequency, hence the similarity in formulation with a
plane arch. The relationship given by Hartog [44], and similarly by Blevins [45], for the fundamental natural frequency of a pinned circular arc is:

\[
f_1 = \frac{1}{2\pi(\theta R)^2} \left[ \frac{\theta^4 - 8\pi^2\theta^2 + 16\pi^4}{1 + \frac{3}{4}(\theta \frac{\pi}{R})^2} \right]^{1/2} \cdot \frac{EI}{\sqrt{m}} \tag{1}
\]

The variable \(\theta\) represents the central angle of the arc, and \(R\) its radius. These can be easily obtained from common values of rise, \(H\), and span, \(B\), through the following expressions:

\[
R = \frac{B^2 + 4H^2}{8H} \tag{2}
\]

\[
\theta = 2\sin^{-1} \left( \frac{4HB}{B^2 + 4H^2} \right) \tag{3}
\]

Substituting the bending stiffness of a beam, \(EI\), for the bending stiffness of a shell, \(D\), and the mass per unit length of the arc, \(m\), for \(\rho t\), representing the shell density and thickness, respectively, the fundamental natural frequency of a free ends cylindrical shell can be obtained as follows:

\[
f_1 = \frac{1}{2\pi(\theta R)^2} \left[ \frac{\theta^4 - 8\pi^2\theta^2 + 16\pi^4}{1 + \frac{3}{4}(\theta \frac{\pi}{R})^2} \right]^{1/2} \cdot \frac{D}{\sqrt{\rho t}} \tag{4}
\]

The above relationships are conceived for solid isotropic shells. Various previous studies have used continuum shell analogies to represent the stiffness of lattice shells through an equivalent uniform shell thickness [8, 17]. The following expressions exist for lattice shells in which all members have equal cross sections, and the angle between the diagonal members and the longitudinal elements is \(\alpha = 60°\), forming equilateral triangles [8, 17]:

\[
D = \frac{E_{eq}t_{eq}^3}{12(1 - v^2)} \tag{5}
\]

\[
t_{eq} = 2\sqrt{3}i_g \tag{6}
\]

\[
E_{eq} = \frac{EA}{3l_0i_g} \tag{7}
\]

Hence, the bending stiffness of a cylindrical lattice shell would be:

\[
D = \frac{2\sqrt{3}EI}{3l_0(1 - v^2)} \tag{8}
\]

In addition, the density and thickness of the shell, \(\rho t\), are substituted, for convenience, by:

\[
\frac{2\sqrt{3}Ap}{l_0} \tag{9}
\]
Substituting these expressions into the formula given for the continuous shell, the fundamental natural frequency of a free ends cylindrical lattice shell is obtained as:

$$f_1 = \frac{1}{2\pi(\theta R)^2} \left[ \frac{\theta^4 - 8\pi^2 \theta^2 + 16\pi^4}{1 + \frac{3}{4} \left( \frac{\theta}{\pi} \right)^2} \right]^{1/2} \sqrt{\frac{EI}{3A(1 - \nu^2)\rho}}$$

(10)

Where $\theta$ = Central angle, $R$ = Radius, $I$ = Moment of inertia of section, $E$ = Young’s modulus ($E_{eq}$ = equivalent Young’s modulus), $A$ = Area of section, $\nu$ = Poisson’s ratio, $\rho$ = Density.

For convenience in seismic design, where the period is more frequently used, the above equation, expressed in terms of period, would be:

$$T_1 = 2\pi(\theta R)^2 \left[ \frac{1 + \frac{3}{4} \left( \frac{\theta}{\pi} \right)^2}{\theta^4 - 8\pi^2 \theta^2 + 16\pi^4} \right]^{1/2} \sqrt{\frac{3A(1 - \nu^2)\rho}{EI}}$$

(11)

The accuracy of this prediction equation was examined for a representative set of shells against the periods obtained through eigenvalue analysis using SAP2000 [46]. The proposed formula was found to slightly overestimate the periods, since the equivalent shell thickness and resulting bending stiffness from Equation 5 were conceived in a flat configuration for bending under classical shell theory. The curvature of the cylindrical shells adds a stiffness that results in lower periods, an effect which is more pronounced as the rise to span ratio increases, since this increases their curvature, hence deviating further from a flat surface. A correction factor, $C_\kappa$, is therefore proposed herein to account for this, as follows:

$$C_\kappa = 0.9 - 0.1 \cdot \left( \frac{H}{B} \right)$$

(12)

Multiplying the periods obtained from Equation 11 by this correction factor leads to a close prediction of the fundamental natural period. The results, which are compared in Figure 8, show very good agreement between the proposed prediction expression and the numerical values of the fundamental period, with the discrepancy consistently below 4%. This simplified approach offers a reliable initial estimate for the period, which is particularly suitable for preliminary design stages, as commonly adopted for other structural forms [34].

It is important to note that Equation 11 is derived for single layer cylindrical lattice shells with a boundary condition of free ends. It is derived for a shell where all the members are of the same section, and all the angles formed by the members are at 60°. As shown, it provides very close estimates if these conditions are met. Nevertheless, if, for instance, the longitudinal members are of a different size, making it heavier or lighter, or if the angles differ slightly, it will still provide a reasonable initial estimate for preliminary design, which is the intended use of the equation, as any detailed design would in any case use modal
analysis to determine the fundamental period. The influence of such changes is also examined in the paper, hence a designer would be aware of expected overestimation or underestimation.

To complement the prediction of the fundamental period, the first mode shape can also be estimated. The first mode shape of a single layer cylindrical lattice shell with free ends follows that of a pinned arch [45] which, with reference to Figure 9, can be determined as follows (noting that there is no variation in the Y-coordinates of the nodes):

\[
x = \sin \frac{2\pi n}{\theta} \quad (13)
\]

\[
z = \frac{\theta}{2\pi} \left( 1 - \cos \frac{2\pi n}{\theta} \right) \quad (14)
\]

The adequacy of this mode shape estimation is illustrated in Figure 10, comparing mode shapes from the estimation formulas, and those obtained numerically through eigenvalue analysis. It is shown that the estimation formula predicts the first mode shape most accurately for shells of low rise to span ratio. Nevertheless, even for the deepest shell of H/B = 0.5, the estimated mode shape nodal displacements differ within 8% when compared to the numerical results, which is considered a reasonable accuracy for preliminary design.

The period prediction relationships proposed in Equations 11 and 12, in conjunction with the first mode shape expressions provided in Equations 13 and 14, are particularly useful for preliminary design stages, and can be readily incorporated in codified design procedures. They enable the estimation of both the first mode shape and its corresponding fundamental period without the need for a detailed eigenvalue analysis. In addition, as discussed in subsequent sections of this paper, the fundamental mode shape can be used to create a load pattern that can capture the horizontal and vertical nature of the loading.

4. Linear Elastic Response

4.1. Design Considerations

To assess their seismic behaviour in detail, five shells were designed based on the general considerations in Eurocode 3 and Eurocode 8 [34, 47], along with available shell buckling recommendations [2, 8, 10, 15]. The five shells, with H/B ratios ranging from 0.1 to 0.5, were designed with the support of the program SAP2000 [40]. The span, B, was considered as 15 m and the length, L, as 27 m. The loads accounted for were the self-weight, a superimposed dead load of 1.0 kN/m² considering that these shells usually have lightweight finishes, and an additional live load of 1.0 kN/m². The shells were designed for an ultimate gravity combination of 1.35 times the dead load in addition to 1.50 times the live load, as per Eurocode specifications. Seismic loading was considered through an equivalent lateral force analysis, determined implicitly through the program by applying loads proportional to the height of the corresponding masses, using selected Eurocode 8 spectral parameters of $a_g = \ldots$
0.25 g, Spectrum Type 1, and Ground Type C. This was considered in a load combination of 1.00 Dead Load (DL) + 0.30 Live Load (LL) + 1.00 Earthquake Load (EL), again following Eurocode procedures. This approach is clearly intended for conventional building structures, but nonetheless serves to provide an initial consideration of seismic loading for the shells.

In addition to the simple gravity load case, a nonlinear analysis including P-Delta effects and large displacements was performed. The stiffness at the end of this nonlinear case was then used to evaluate a buckling factor by increasing the gravity load, in the following way: 1.35 DL + 1.50 LL + \( \lambda \cdot (1.35 \, \text{DL} + 1.50 \, \text{LL}) \), where a buckling load factor of at least \( \lambda = 1.0 \) was required to deem the shells adequate against buckling [15]. The members were sized either according to their utilisation ratio from the Eurocode 3 checks (strength driven), or according to the buckling analysis, to obtain a suitable buckling load factor (buckling driven). To replicate realistic design conditions, where members would likely be standardised, two sizes were adopted for each H/B ratio: one for the diagonals and another for the longitudinal elements. Circular hollow sections, 4 mm in thickness, and different diameters, were employed, as indicated in Table 1, together with the resulting dynamic characteristics, and governing factors in their design. The cumulative mass participation ratio for the first 100 modes in the X and Z directions for the five shells is also depicted in Figure 11. It can be inferred immediately that for the shell with H/B = 0.1, the vertical modes are expected to contribute to the response to a much greater extent compared to shells of higher aspect ratios.

4.2. Record Selection

Seven natural records were selected for the time history analyses from the PEER NGA West database [48], from events occurring in Europe and the Middle East. The records represent a medium seismic hazard scenario, using both seismological criteria, and a matching procedure to the Eurocode 8 target spectrum (\( a_g = 0.25 \, \text{g} \), Spectrum Type 1, Ground Type C), over a range of relevant periods. No frequency adjustments or modifications were carried out. To keep the scaling as low as possible, the following matching criterion that minimises the root mean squared (RMS) difference from the target spectrum was used [49]:

\[
D_{RMS} = \frac{1}{N} \sqrt{\frac{\sum_{i=1}^{N} \left( \frac{SA_0(T_i)}{PGA_0} - \frac{SA_S(T_i)}{PGA_S} \right)^2}{}}
\]

(15)

Where: \( N = \) number of periods, \( SA_0(T_i) = \) spectral acceleration of the record at period \( T_i \), \( SA_S(T_i) = \) spectral acceleration of the target spectrum at period \( T_i \), \( PGA_0 = \) zero period anchor point of the target spectrum, \( PGA_S = \) peak ground acceleration of the accelerogram.

The seven records selected were all from events with a moment magnitude \( M_w \) between 5.0 and 6.5. The distance from the fault ranged from 10 km to 100 km, with a shear wave velocity, \( V_{S30} \), from 180 m/s to 800 m/s. The matching procedure was set to minimise \( D_{RMS} \) over a period range of 0.2 s to 2.0 s. The selected records and their characteristics are listed in
Table 2. The corresponding acceleration response spectra for (a) the horizontal and (b) vertical components of the selected records, along with their mean, and the Eurocode 8 spectrum they were scaled to, are shown in Figure 12.

4.3. Response to Horizontal Excitations

The linear elastic response of the five shells to the horizontal earthquake excitations was analysed using OpenSees [39]. The co-existing gravity loads were based on the unfactored dead load and 30% of the imposed load, and the mass was also consistently assumed according to these values. Rayleigh damping was used, with a damping ratio $\zeta = 0.02$. A series of seven time history analyses were performed on each of the five shells using the selected records. The results discussed below are based on the average response obtained.

Figure 13 shows the axial force utilisation for a quarter of the shell in plan (shown on the right for all five shells), and the edge arch in elevation (shown on the left for all five shells), as indicated in the key diagram shown in the top left corner of the figure. Utilisation is defined as the ratio of axial force in the member to its axial force capacity, i.e. $N/N_{pl}$, where $N_{pl} = A \cdot f_y$. As shown in Figure 13, under the horizontal excitations, the diagonal elements are the main load resisting members of the shell, with negligible forces induced in the longitudinal elements. The axial forces are highest towards the bottom members, as they accumulate load to carry it down to the supports, and they increase towards the corners as well, with the highest force found in the diagonal that meets the edge arch.

The distribution of bending moments across the shell is similar in all cases, but their relative values differ significantly, as shown in Figure 14 which depicts the bending moment utilisation, defined as the ratio of bending moment in the member to its plastic bending moment capacity, i.e. $M/M_{pl}$, where $M_{pl} = W_{pl} \cdot f_y$. The maximum bending moments consistently occur in the edge arches, and more specifically at the intersection of the first and second members. Longitudinally across the shell, the highest bending moments occur at the intersection between the second and third row of diagonals, at the quarter points of the shells.

The H/B ratio influences the behaviour significantly. As expected, the axial forces are highest in the shallow shells with low H/B, whereas the bending moments are highest in the shells with high H/B ratios. As shown in Figure 15, shallower circular arcs resemble closely the geometry of an inverted catenary. Hence, the load is carried more efficiently through axial action. As H/B increases, the circular arc deviates further from the perfect thrust line for gravity loads [50]. The eccentricity between the thrust line and the actual geometry produces higher bending moments in the deeper shells.

The values of section utilisation produced by bending moments are comparatively higher than those produced by axial forces. In the shells of H/B = 0.4 and above, values of around 0.9 of the plastic moment capacity are reached. Conversely, the highest axial utilisation is found in
the shell of $H/B = 0.1$, with a value of just 0.17 of the axial capacity of the section. The high values of bending moments indicate clearly their governing role in terms of design.

4.4. **Response to Vertical Excitations**

To provide further insight into the seismic response, the natural recorded vertical components of the selected seven records were firstly used in isolation. For this purpose, the records were scaled such that the vertical peak ground acceleration $a_{vg}$ is 0.90 of the horizontal $a_g$, in accordance with Eurocode 8 provisions [34].

As shown in Figure 16, the axial forces in the shells subjected to the vertical excitations resemble that of a gravity case. Shells of relatively low $H/B$ largely carry the load axially through membrane action within their entire surface, whereas shells of higher $H/B$ resemble a framed structure, with a larger increase in forces towards the lower, more vertical, members.

The bending moments produced by the vertical excitations, shown in Figure 17, are again highest in the edge arches. They present more localised concentrations than the horizontal component, with two peaks, one at the intersection between the first and second member, and another at the apex of the arch. The $H/B$ ratio again plays an important role in the response under vertical excitations. Shallower shells experience larger values of axial force utilisation, whereas deeper shells of higher $H/B$ ratio exhibit higher values of bending moment utilisation. Again, the bending effects are comparatively larger than the axial effects.

4.5. **Comparative Assessments**

It is useful at this stage to compare the actions produced by the two earthquake components and those from the ultimate gravity case. Firstly, the axial forces along the key diagonals (marked in green in Figure 1) are compared, as these are the members where the highest axial forces occur, and would hence govern the member sizes. The bending moments along the edge arch (marked in red in Figure 1) where the maximum values occur, are also compared. The results for an additional case in which the horizontal and vertical seismic excitations are applied simultaneously, is also considered for comparison purposes.

As shown in Figure 18 (for shells with three representative values of $H/B$), in all shells the axial forces produced by the ultimate gravity load case are higher than those experienced under both earthquake component cases as well as their combination. The reduction in the applied gravity load considered in the seismic cases decreases the axial demands significantly, and the considered level of seismic hazard does not produce axial forces greater than those from the gravity load case. For all cases, the vertical seismic case produces higher axial forces than the horizontal, especially for lower $H/B$. The axial forces are greatest for shells of lower $H/B$, ranging between 0.15 and 0.30 of section axial utilisation, compared to values between 0.03 and 0.13 for shells of $H/B = 0.5$.

Figure 19 shows the maximum utilisation ratios for bending moments. The horizontal earthquake case typically produces higher bending moments than the other two cases. The
vertical earthquake case produces higher bending moments than the gravity load case for H/B = 0.1 as expected, due to its sensitivity to vertical effects. Nevertheless, the overall bending moments in shells of low H/B are smaller (reaching up to only 0.30 of $M_{pl}$) than those found in shells of higher H/B (which reach up to 0.90 of $M_{pl}$).

The distribution of bending moments also differs between the horizontal earthquake case and the gravity and vertical earthquake cases. Although in all cases the maximum bending moments occur at the intersection between the first and second members of the edge arch, there are some differences. For the horizontal earthquake case, the entire member and the adjacent joint experience bending moments that are similar, and then decrease towards the centre. These correspond to the members with the largest displacement in the first mode shape, hence subjected to the highest bending moments. On the other hand, for the gravity and vertical earthquake load cases, a second more concentrated peak occurs in the centre.

The internal forces produced by the simultaneous action of the horizontal and vertical components are also worth discussing. The axial forces produced by the ‘Horizontal+Vertical’ case are not very different to the Vertical case alone. Conversely, the bending moments from the horizontal component of the earthquake are not dissimilar from those in the ‘Horizontal+Vertical’, indicating that the horizontal component is largely producing these moments. This also shows that the vertical and horizontal components are largely uncoupled in terms of dominant frequency ranges.

To illustrate the relative influence of the seismic-to-gravity load cases, the maximum axial forces and bending moments produced by the seismic load cases are divided by those from the gravity load case, and the average of these ratios are presented in Figure 20. A value greater than 1.0 implies that the forces produced by the seismic load cases are larger than those produced by the gravity case. The bending moments from the horizontal seismic case are consistently larger than those from the gravity load case, and up to five times as much in shells of low H/B. The vertical seismic case is notable in shells of low H/B, for which the bending moments are larger than those from gravity.

The maximum overall internal forces are also plotted against H/B in Figure 21. As noted before, the axial forces (Figure 21 a) from the seismic cases are smaller than those from the ultimate gravity load case, with the axial forces decreasing as H/B increases. The vertical seismic case clearly produces higher axial forces than the horizontal seismic case. In contrast, the bending moments (Figure 21 b) increase as H/B increases. The bending moments are smaller in shallower shells (low H/B), but the increase they experience from the horizontal seismic case is larger. On the other hand, in shells of higher H/B, the increase in bending moment produced by seismic cases is not significant, but the magnitude is higher.

The above comparisons illustrate clearly the zones and members exhibiting the highest axial forces and bending moments under different loading cases. This is important for design and assessment purposes, as well as possible optimisation procedures, as it identifies the key
elements that govern the member sizes for different geometries and loading combinations. In addition to the important role played by the aspect ratio of the shell, represented by H/B, the above assessments show that the horizontal and vertical responses are largely uncoupled, due to the differences in dominant response frequencies as well as the affected critical members.

5. Nonlinear Static Behaviour

5.1. Pushover Methodology

Previous studies on the seismic behaviour of shell structures have not made much use of pushover analysis as it is, in its typical form, clearly more suited for conventional building structures. When it has been employed, multiple modes have often been resorted to [32, 51], which adds significant complexity to the process, particularly for use in practical design and assessment approaches. To this end, a simple yet reliable methodology for pushover analysis of shell structures is presented herein in order to represent the key inelastic mechanisms and assess the local demands within critical elements.

In order to gain a fundamental insight into the nonlinear inelastic behaviour of the shells, a series of nonlinear static (pushover) analyses were carried out, using the parameters outlined above in Section 2. In the pushover analysis, the gravity part of the loading (i.e. 1.30 kN/m²) is firstly applied. Subsequently, the lateral loads are applied, in a pattern proportional to the first mode shape, as illustrated in Figure 22 for three selected shells of different H/B. As indicated in the figure, unlike conventional building structures, the first mode shape includes significant vertical actions, especially for lower H/B ratios. This enables appropriate representation of the influence of the vertical seismic component, which are particularly important in relatively shallow shells. As noted in Section 3.1, lattice shells possess a complex modal response involving numerous modes. The aim of using the first mode is to examine whether a simplified pushover using just the first mode shape as the load pattern is adequate enough for preliminary design purposes. This will be assessed in the following sections by comparing the results with those obtained using nonlinear dynamic analysis, which inherently takes into account the full response and influence of all modes.

A displacement control analysis is adopted, by controlling the horizontal displacement of the top node of the edge arch. The horizontal drift is presented in terms of the horizontal displacement of the top node of the edge arch divided by the rise of the shell. In addition, the peak angle change is also proposed herein as a damage measure, or inelastic deformation parameter, that is specifically more suited to shell structures. The angles between the members of the edge arches in the undeformed structure are equal, as shown in Figure 23. As the loads are applied and the structure deforms, the changes in the angles can be determined, with the peak angle change defined as the maximum value recorded out of the seven angle changes.
Figure 24 shows the pushover curves for the five shells considered, in terms of base shear versus the top node drift (Figure 24a) and the peak angle change (Figure 24b), for comparison purposes. Table 3 also gives the numerical results of the same pushover analyses of the five shells. The key limit points are marked on the curves in Figure 24. The first point (in green) corresponds to any one member in the edge arch reaching its elastic moment capacity, $M_{el}$. The second point (in red) takes into account the plastic interaction of forces (referred to as P.I.) through the following relationship [16]:

$$\frac{M_y}{M_{pl}} + \frac{M_z}{M_{pl}} + \left(\frac{N}{N_{pl}}\right)^2 \leq 1$$  \hspace{1cm} (16)

The third point (in cyan) corresponds to when the moment in a member of the edge arch reaches the full plastic moment capacity, $M_{pl}$. Finally, the last point (in dark blue) marks the point at which the predicted critical strain, $\varepsilon_{cr}$, corresponding to local buckling, is first reached in a member. This strain depends on the diameter, $D$, and thickness, $t$, of the circular hollow section, and is estimated from the elastic critical buckling stress for a circular hollow section, using the following relationship [52]:

$$\varepsilon_{cr} = \frac{\varepsilon_y}{f_y} \cdot \frac{E}{\sqrt{3(1-\nu^2)}} \cdot \frac{2t}{D}$$  \hspace{1cm} (17)

5.2. Characteristics of Pushover Response

From Figure 24, it is clear that reaching the elastic moment does not correspond to significant deviation from linearity, which only occurs after the plastic interaction yield criteria from Equation 16 is satisfied. In shells of low H/B, this point is close to that corresponding to the full plastic moment capacity. In deeper shells, these points are further apart, indicating that the minor axis bending effects are more significant, due to their spatial geometric configuration. There is also a smaller dispersion in peak angle change at this level of yielding. It follows that the point corresponding to reaching the plastic interaction of forces in Equation 16 is suitable to determine the first yield, $V_1$, which is used as a reference in subsequent dynamic analysis.

Comparing the response in Figure 24 (a) and (b), it is shown that the peak angle change is a particularly suitable measure for representing the nonlinear inelastic response of the shells, with less dispersion between the curves compared to drift. In terms of the dispersion in drift at the point of first yield, the results range from 2.31% to 4.17%, compared to between 0.051 and 0.060 radians when considering the peak angle change. This results in a coefficient of variation of 19.4% for first yield using drift versus just 6.2% for the peak angle change, which can be particularly useful when determining performance limits for seismic design. In addition, the peak angle change captures the nonlinear behaviour in more detail for shells of low H/B. The shallow shells experience very small incremental horizontal displacements in their top node once a certain level of loading is reached, yet they still continue to deform.
This is more realistically represented through the peak angle change, which takes into account both horizontal and vertical displacements.

In order to assess the suitability of the peak angle change as a predictor of local demands, much like inter-storey drifts in conventional buildings, the local rotations in the members were determined. The rotation demand, $R$, representing local damage, was thus evaluated in each member of the edge arches using the following relationship [53-55]:

$$ R = \frac{\kappa}{\kappa_p} - 1 $$  \hspace{1cm} (18)

Where $\kappa$ is the curvature, and $\kappa_p$ is the curvature corresponding to the plastic moment capacity, defined by:

$$ \kappa_p = \frac{M_{pl}}{EI} $$  \hspace{1cm} (19)

The peak $R$, or the maximum rotation from all members in the edge arch, is plotted against base shear in Figure 25. The points shown in red mark successive plastic hinge formation in members of the edge arch of the shell, considering the plastic interaction of forces from Equation 16. This signifies when different performance damage states are reached. It should be noted that $R$ is influenced by the slenderness of the cross-section (i.e. diameter to the thickness ratio) [54]. A value of $R = 6$, marked with a vertical dashed line in Figure 25, can serve as a Collapse Prevention limit state, as suggested in FEMA P-58 [56], since it corresponds to significant inelastic deformation levels, yet without reaching the rotation capacity limits of the members according to their cross-section slenderness [54].

Figure 26 shows a comparison of the relationship between the rotation $R$ when plotted against the drift or the peak angle change. It is evident again that, compared to the peak angle change, the results for drift are significantly more dispersed between shells of different $H/B$. When considering values at $R = 6$, as shown in Table 4, the coefficient of variation is considerably lower when using the peak angle change. As a result, a simple linear regression relationship (shown in red in Figure 26b) could for example be used to estimate the maximum value of local rotation $R$ expected in a member, based on the peak angle change, $\Delta \theta_{max}$ as follows:

$$ R = 82.6 \cdot \Delta \theta_{max} - 2.9 $$  \hspace{1cm} (20)

5.3. **Plastic Mechanisms**

The nonlinear behaviour and failure mechanisms of the shells is highly dependent on the $H/B$ ratio. Due to their geometry, shallow shells are prone to buckling and snap-through once yielding is initiated, whereas deeper shells behave more like a framed structure and thus exhibit a more ductile behaviour when loaded with forces proportional to their first mode shape. In addition, the vertical components of the forces applied in shallow shells are larger than those applied in deeper shells, due to the shape of the first mode. These downward forces have a detrimental effect on shells of low $H/B$, and lead to early collapse.
Figure 27 shows the interaction of forces in the edge arches of the five shells at the point of first yield. The first plastic hinge is formed in the same location for all shells, at the quarter point, consistent with the areas of highest bending moment indicated in the linear elastic studies in Section 4. Figure 28 also depicts the curvature in the shells at the point when the critical strain, $\varepsilon_{cr}$, is reached and corresponding to significant inelasticity levels. The shell with H/B = 0.1 has plasticity concentrated in the four plastic hinges formed around its quarter points. For higher H/B, plasticity spreads more throughout the shells, forming more plastic hinges in other members, and hence enhancing their ductility.

To further illustrate the failure mechanisms, Figure 29 depicts the progression of plastic hinges, based on the plastic interaction from Equation 16, in the shells with the five different H/B ratios, with the hinge locations and sequence indicated. As mentioned above, the first hinge forms in the same location for all shells; however, the spread of plasticity varies and is dependent on H/B. For the shallow shell with H/B = 0.1, only four hinges are formed, all of them at the quarter point, concentrating all the plasticity in this region. The medium shells of H/B = 0.2 and 0.3 form the first four hinges at these points as well, but then continue forming hinges elsewhere, spreading to nearby joints. Finally, the deeper shells of H/B = 0.4 and 0.5 form a hinge at the joint between the first and second member after the first hinge is formed. It is clear from this figure that the central portion of the shells is not initially expected to undergo significant levels of plasticity.

The above discussions clearly illustrate the significant influence of H/B on the levels of inelastic deformations exhibited by the shell structures as well as their typical plastic mechanisms. It is shown that the interaction of major and minor axis bending moments together with the axial forces provides a reliable representation for the point of first yield. Additionally, the proposed peak angle change parameter serves as a more appropriate damage measure for the seismic analysis of shell structures, compared to the drift typically used in conventional building structures. This parameter is able to capture the effects of the global horizontal and vertical displacements as well as the local rotational demands within the structure in a representative manner, whilst providing a clear physical correspondence with the actual performance. Overall, characterisation of the failure mechanisms in typical shell configurations also enables identification of critical elements, which is useful for design optimisation and intervention purposes, as well as for considering computationally efficient nonlinear modelling and analysis procedures.

6. Incremental Dynamic Analysis

6.1. Analysis Procedures

To assess the nonlinear behaviour under realistic earthquake loading, the response of the same five shells described above was examined through incremental dynamic analysis (IDA) [57, 58]. Each shell was subjected to the seven records described in Section 4.2, increasing
the intensity of the applied accelerations incrementally until collapse was reached. The collapse in the IDA is considered when an infinitesimal increase of the ground motion intensity results in an unbounded deformation (i.e. a flat line in the IDA curve for an individual strong-motion record) [59]. Damping was modelled using Rayleigh damping, with a damping ratio of $\zeta = 0.02$, based on the tangent stiffness matrix and updated throughout the analysis, where the proportionality constants are updated to maintain the specified critical damping percentage for the inelastic vibration modes, as recommended in PEER/ATC 72-1 [60]. This practical approach was adopted in order to control excessive or spurious damping forces at large inelastic deformations, whose limit state is a negative global stiffness slope [61, 62]. In addition, in analyses where abrupt changes in stiffness were expected, as the structure was approaching collapse, equilibrium was ensured by using a greatly reduced time step [63].

In order to assess the influence of the vertical earthquake component on the results, two sets of analyses were carried out. In the first, only the horizontal component was applied while, in the second, both the horizontal and vertical components were used. The actual vertical component from each respective record was used and scaled by the same factor as the horizontal component, to maintain the proportion from the real event.

Comparative results in terms of peak angle change in the form of IDA curves are presented in Figure 30 (a-e) for the five shells of different H/B ratios. The peak angle change was chosen as the most representative damage measure, as discussed before. The peak ground acceleration was chosen as the intensity measure, as initial analysis indicated that it provided less dispersion for different records when compared to the first mode spectral acceleration. The thin solid lines represent the individual results for each of the seven records, where the grey lines are for the horizontal component only whilst the red lines include both the horizontal and vertical components. The median curves, as well as the 16% and 84% percentile curves, are also included. In addition, Figure 31 shows a summary plot with the median curves for the five shells, including their results with and without the vertical component, in order to facilitate a direct comparison.

6.2. Inelastic Demands

The values of peak angle change vary depending on H/B. Higher values are reached in deeper shells, as also observed in the pushover analysis. For several records, as captured by the medians in Figure 30, hardening is observed; having reached a certain peak angle change at a given PGA, some shells exhibit a lower value of peak angle change at a higher PGA. This is due to the nature of IDA where, as the records are scaled upwards an early cycle from a record that initially did not cause yielding triggers inelasticity earlier on hence changing the response for subsequent cycles of that same record. In some cases, this response results in a smaller damage measure, as the structure is somewhat protected from subsequent cycles of higher amplitude by the early yielding.
When comparing the response under the horizontal component only against that under the simultaneous horizontal and vertical components, the geometry of the shells once again plays an important role. As noted before, shells of low H/B exhibit the highest difference, as indicated in Figure 30 (a, b). The vertical component leads to a full collapse of the structure at a relatively lower value of PGA for shells of H/B = 0.1 (a) and 0.2 (b) when compared to the case with the horizontal component only. This difference exists for H/B = 0.3 (c) and 0.4 (d), but is not as pronounced. For H/B = 0.5 (e), as noted before, a significant deviation from the catenary shape makes the vertical component accentuate the bending moments produced by this geometrical eccentricity from the perfect thrust line.

When plotted together (Figure 31), the differences in response become more evident, particularly in terms of the influence of the vertical component as a function of H/B. The collapse of shells of higher H/B ratios is brought about at a much lower absolute value of PGA. This is due to the overstrength in the shallow shells, where buckling is a more decisive factor in governing the member sizes at the design stage.

To account for these differences, and in order to compare shells of different H/B more directly, the results are normalised using a scaling factor (SF) [41] obtained for a target intensity factor, \( q' \), as follows:

\[
SF = q' \cdot \frac{V_1}{S_a(T_1) \cdot m \cdot \gamma_1}
\]  

(21)

where \( V_1 \) is the base shear at first yield, as defined before in Section 5.2, \( S_a(T_1) \) is the first mode spectral acceleration for a given record, \( m \) is the seismic mass, and \( \gamma_1 \) is the first mode effective mass participation ratio. Rearranging these values, and considering the scaling factors used in the IDA, the corresponding intensity factor, \( q' \), can be obtained. This is a measure of the actual inelasticity in the structure, as it represents multiples of the base shear required to produce a first plastic hinge. Similarly, the peak angle change can be normalised by the value at the formation of that first plastic hinge. These values are obtained from the pushover analysis, as given in Table 3.

The results of the normalised intensity and damage measures for all five shells are depicted in Figure 32, with the normalised peak angle change given in Figure 32 (a) and the normalised drift in Figure 32 (b). The dashed lines represent the results for the horizontal component only, while the solid lines include both the horizontal and the vertical components. The grey dotted line represents the equal displacement assumption typically used in Eurocode 8 [34].

The differences between the analyses including the vertical component and those without can be observed at inelasticity levels above \( q' = 4 \). The largest differences between the two occur for H/B = 0.1 and H/B = 0.5, due to their respective geometries. The relatively shallow shells are affected as their response is more dominated by vertical components of vibration modes, whilst in the deep shells the eccentricity between the circular shape and the ideal catenary is accentuated by the vertical component. As mentioned previously, the equal displacement rule is shown to be conservative for all values of \( q' \) below 4. For higher intensity levels, some
shells experience increased peak angle changes, particularly when including the vertical component in the analysis. For comparison, the same procedure was carried out, but using the horizontal drift of the top node as a damage measure, as depicted in Figure 32 (b). This shows how the horizontal drift can be a deceptive damage measure for shell structures. All the drifts remain below those expected from the equal displacement rule. Thus, the levels of inelastic demand may be under-predicted if the horizontal drift is used as a damage measure.

Finally, to evaluate the validity of the peak angle change as a useful damage indicator, as well as the adopted pushover analysis procedure for predicting the inelastic behaviour, the different limit points are compared from the pushover and IDA. The peak angle change at which first yield in the edge arch occurs was obtained from each individual analysis from the IDA including both horizontal and vertical components. The same was carried out for the first time a rotation of $R = 6$ was registered in the edge arch, and the first time the critical strain $\varepsilon_{cr}$ was reached. The median value of the seven records was considered, as listed in Table 5, compared to the corresponding values of peak angle change obtained from the pushover analyses. Good agreement was generally obtained between the values of peak angle change from the pushover analysis at different limit points when compared to those from the IDA. This is illustrated in Figure 33, showing the individual points for the IDAs as well as the median values as vertical dashed lines, compared to the predicted values from the pushover analysis indicated as vertical solid lines. It is evident that, whilst IDA may be adopted for verification in important structures, the proposed pushover analysis methodology can serve as a reasonably good predictor of key limits, and that the peak angle change is a suitable damage measure for this form of shell structure.

The above discussions emphasise the important role played by the vertical seismic component in the response, particularly for shells of relatively low or comparatively high rise to span ratios. They also illustrate that the peak angle change is a suitable measure for representing the inelastic seismic response of shell structures. Importantly, the pushover methodology discussed in Section 5 was shown to offer a good representation of the nonlinear dynamic response obtained from the IDA assessments, and can therefore be used in defining limit states for the purpose of performance based assessment and design procedures.

7. Concluding Remarks

This paper has examined the detailed seismic behaviour of single layer cylindrical lattice shells. A parametric analysis was firstly carried out to assess the influence of different geometric and loading parameters on the dynamic characteristics of the shells. It was found that the rise-to-span ratio has a most significant influence on the fundamental natural period. Simplified expressions for estimating the fundamental period of vibration and corresponding mode shape were proposed. These are particularly useful for preliminary design stages, and can be readily incorporated in codified design procedures. The mode shape can also be used to create a load pattern that can capture the horizontal and vertical nature of the loading.
The seismic behaviour of the shells was assessed in the linear elastic range in order to obtain an initial insight before considering their nonlinear inelastic performance. Key elements that govern member sizes for different geometries and loading combinations were identified, which is important for design and assessment purposes as well as possible optimisation procedures. It was shown that the diagonal members are the main load carrying elements in the shells, and that the highest internal forces occur in the edge arches. The rise-to-span ratio was shown to have a most significant effect on the response, with the shallower shells exhibiting higher axial forces, and the deeper shells displaying higher bending moments. The bending moments as a whole were more significant than axial forces, with peaks occurring towards the quarter points of the shells. Additionally, the important influence of the vertical component of the earthquake was illustrated, which is particularly significant in shells of low rise-to-span ratio. The effects of the horizontal and vertical components were nonetheless shown to be largely uncoupled, due to the differences in dominant response frequencies as well as the affected critical members. The response was shown to be dominated by one of the components, depending primarily on the rise-to-span ratio.

The seismic behaviour of single layer cylindrical lattice shells was examined in the inelastic range, firstly through nonlinear static analysis. A pushover methodology based on the fundamental mode shape was proposed, in conjunction with a parameter reflecting the peak angle change. The suggested approach was shown to capture the deformations of the shells in a representative manner compared to conventional procedures. The pushover analysis clearly illustrated the significant influence of the rise to span ratio on the intrinsic plastic mechanisms and levels of global and local inelastic deformations. The characterisation of the failure mechanisms in typical shell configurations also enables identification of critical elements, which is useful for design optimisation and intervention purposes, as well as for considering computationally efficient nonlinear modelling and analysis procedures.

Finally, a series of time history analyses were carried out using incremental dynamic analysis. The peak angle change was again shown to be a suitable measure for representing the inelastic seismic response of shell structures. The vertical component of the earthquake was again shown to have a significant influence on the response, particularly for shells of relatively low or comparatively high rise to span ratios. Importantly, the suggested pushover methodology was shown to offer a good representation of the nonlinear dynamic response obtained from the incremental dynamic assessments, and can therefore be used in defining limit states for the purpose of performance based assessment and design procedures.

The main aim of this paper was to provide detailed insights into the seismic behaviour of single layer cylindrical lattice shells. In terms of practical applications, the fundamental period and mode shape estimation are of direct use for preliminary design stages, while the guidance on pushover and IDA provide approaches for more detailed evaluations and verifications, including the influence of vertical excitations. Key recommendations, such as the use of the peak angle change as a damage measure as well as the suggested limit states, are of direct relevance for detailed performance based assessments.
REFERENCES


List of notations

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<tr>
<th>Symbol</th>
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<tr>
<td>A</td>
<td>Area</td>
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<tr>
<td>a_g</td>
<td>Peak ground acceleration</td>
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<tr>
<td>a_v</td>
<td>Peak vertical ground acceleration</td>
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<tr>
<td>B</td>
<td>Span of the shell</td>
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<tr>
<td>C_x</td>
<td>Correction factor due to curvature</td>
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<td>D</td>
<td>Diameter of cross-section</td>
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<td>D_s</td>
<td>Shell bending stiffness</td>
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<td>DL</td>
<td>Dead Load</td>
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<td>Root mean square difference</td>
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<td>Young’s modulus</td>
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<td>Eurocode 8</td>
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<td>Earthquake Load</td>
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<td>f_1</td>
<td>Fundamental natural frequency</td>
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<td>f_y</td>
<td>Yield strength of steel</td>
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<td>h</td>
<td>Interstorey height</td>
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<td>H</td>
<td>Rise of the shell</td>
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<td>H/B</td>
<td>Rise to span ratio</td>
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<td>I</td>
<td>Moment of inertia</td>
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<td>Incremental dynamic analysis</td>
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<td>Peak ground velocity</td>
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<td>Plastic interaction</td>
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<td>Total gravity load</td>
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<td>Applied surface load</td>
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<td>q'</td>
<td>Intensity factor</td>
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<td>Radius of the shell</td>
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<td>R</td>
<td>Rotation demand</td>
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<td>Spectral acceleration of the record at the period T_i</td>
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<td>Spectral acceleration of the target spectrum at the period T_i</td>
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<tr>
<td>UX</td>
<td>Translational mode in X direction</td>
</tr>
<tr>
<td>UZ</td>
<td>Translational mode in Z direction</td>
</tr>
<tr>
<td>V_1</td>
<td>Base shear at first yield</td>
</tr>
<tr>
<td>V_max</td>
<td>Maximum base shear</td>
</tr>
<tr>
<td>V_s30</td>
<td>Shear wave velocity</td>
</tr>
<tr>
<td>V_tot</td>
<td>Total seismic storey shear</td>
</tr>
<tr>
<td>W_pl</td>
<td>Plastic section modulus</td>
</tr>
<tr>
<td>a</td>
<td>Longitudinal-diagonal angle</td>
</tr>
<tr>
<td>γ_1</td>
<td>First mode mass participation ratio</td>
</tr>
<tr>
<td>Δ_0,i</td>
<td>Individual changes in angle</td>
</tr>
<tr>
<td>Δ_0,max</td>
<td>Peak angle change</td>
</tr>
<tr>
<td>ε_cr</td>
<td>Critical strain</td>
</tr>
<tr>
<td>ε_y</td>
<td>Yield strain</td>
</tr>
<tr>
<td>ζ</td>
<td>Damping ratio</td>
</tr>
<tr>
<td>θ</td>
<td>Central angle of the shell</td>
</tr>
<tr>
<td>κ</td>
<td>Curvature</td>
</tr>
<tr>
<td>κ_pl</td>
<td>Curvature at plastic moment</td>
</tr>
<tr>
<td>λ</td>
<td>Buckling load factor</td>
</tr>
<tr>
<td>ν</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>ρ</td>
<td>Density</td>
</tr>
</tbody>
</table>
### Tables

**Table 1:** Shell characteristics and member sizes (dimensions in mm)

<table>
<thead>
<tr>
<th>Rise/Span Ratio (H/B)</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal members (D×t)</td>
<td>60×4</td>
<td>60×4</td>
<td>60×4</td>
<td>60×4</td>
<td>60×4</td>
</tr>
<tr>
<td>Diagonal members (D×t)</td>
<td>165×4</td>
<td>141×4</td>
<td>140×4</td>
<td>151×4</td>
<td>174×4</td>
</tr>
<tr>
<td>1st Mode Period, T₁ (s)</td>
<td>0.558</td>
<td>0.877</td>
<td>1.212</td>
<td>1.547</td>
<td>1.812</td>
</tr>
<tr>
<td>1st Mode Mass Participation, UX</td>
<td>0.122</td>
<td>0.389</td>
<td>0.640</td>
<td>0.809</td>
<td>0.901</td>
</tr>
<tr>
<td>Buckling Factor, λ</td>
<td>1.03</td>
<td>1.03</td>
<td>1.03</td>
<td>1.03</td>
<td>1.14</td>
</tr>
<tr>
<td>Maximum Utilisation Ratio</td>
<td>0.422</td>
<td>0.666</td>
<td>0.872</td>
<td>1.00</td>
<td>0.993</td>
</tr>
<tr>
<td>Governing design aspect</td>
<td>Buckling</td>
<td>Buckling</td>
<td>Buckling</td>
<td>Strength</td>
<td>Strength</td>
</tr>
</tbody>
</table>

**Table 2:** Record selection used in the time history analysis

<table>
<thead>
<tr>
<th>Earthquake Name</th>
<th>Date</th>
<th>Magnitude</th>
<th>Distance to Fault (km)</th>
<th>V$_{S30}$ (m/s)</th>
<th>PGA (g)</th>
<th>PGA$_v$ (g)</th>
<th>PGV (cm/s)</th>
<th>Horiz. Scale Factor</th>
<th>Vert. Scale Factor</th>
<th>D$_{RMS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kalamata, Greece</td>
<td>13/09/86</td>
<td>6.2</td>
<td>11.2</td>
<td>339</td>
<td>0.26</td>
<td>0.20</td>
<td>27.18</td>
<td>1.23</td>
<td>1.10</td>
<td>0.188</td>
</tr>
<tr>
<td>Friuli, Italy</td>
<td>11/09/76</td>
<td>5.5</td>
<td>15.1</td>
<td>339</td>
<td>0.05</td>
<td>0.03</td>
<td>4.86</td>
<td>7.55</td>
<td>7.76</td>
<td>0.197</td>
</tr>
<tr>
<td>Irpinia, Italy</td>
<td>23/11/80</td>
<td>6.2</td>
<td>41.7</td>
<td>500</td>
<td>0.04</td>
<td>0.02</td>
<td>3.45</td>
<td>9.83</td>
<td>9.34</td>
<td>0.226</td>
</tr>
<tr>
<td>Irpinia, Italy</td>
<td>23/11/80</td>
<td>6.2</td>
<td>22.7</td>
<td>530</td>
<td>0.11</td>
<td>0.07</td>
<td>11.56</td>
<td>3.49</td>
<td>3.35</td>
<td>0.23</td>
</tr>
<tr>
<td>Dinar, Turkey</td>
<td>01/10/95</td>
<td>6.4</td>
<td>35.6</td>
<td>339</td>
<td>0.04</td>
<td>0.03</td>
<td>4.65</td>
<td>7.16</td>
<td>7.17</td>
<td>0.245</td>
</tr>
<tr>
<td>Friuli, Italy</td>
<td>15/09/76</td>
<td>5.9</td>
<td>14.3</td>
<td>339</td>
<td>0.11</td>
<td>0.07</td>
<td>11.37</td>
<td>3.03</td>
<td>3.04</td>
<td>0.246</td>
</tr>
<tr>
<td>Lazio-Abruzzo, Italy</td>
<td>07/05/84</td>
<td>5.8</td>
<td>45.5</td>
<td>339</td>
<td>0.04</td>
<td>0.03</td>
<td>3.55</td>
<td>9.25</td>
<td>7.01</td>
<td>0.254</td>
</tr>
</tbody>
</table>
### Table 3: Results of pushover analysis

<table>
<thead>
<tr>
<th>Rise to Span Ratio, H/B</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height, h (m)</td>
<td>1.50</td>
<td>3.00</td>
<td>4.50</td>
<td>6.00</td>
<td>7.50</td>
</tr>
<tr>
<td>Base Z Reaction, P_{int} (kN)</td>
<td>542</td>
<td>571</td>
<td>630</td>
<td>711</td>
<td>815</td>
</tr>
<tr>
<td>Max Base Shear, V_{max} (kN)</td>
<td>263</td>
<td>290</td>
<td>279</td>
<td>262</td>
<td>265</td>
</tr>
<tr>
<td>Base shear, V_{1} (kN)</td>
<td>210</td>
<td>200</td>
<td>211</td>
<td>204</td>
<td>220</td>
</tr>
<tr>
<td>Top Node X Displacement (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.035</td>
<td>0.089</td>
<td>0.155</td>
<td>0.228</td>
<td>0.313</td>
<td></td>
</tr>
<tr>
<td>Top Node X Drift</td>
<td>2.31%</td>
<td>2.98%</td>
<td>3.44%</td>
<td>3.80%</td>
<td>4.17%</td>
</tr>
<tr>
<td>At 1st Yield</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Angle Change (rad)</td>
<td>0.051</td>
<td>0.056</td>
<td>0.060</td>
<td>0.060</td>
<td>0.060</td>
</tr>
<tr>
<td>Peak R</td>
<td>1.661</td>
<td>0.761</td>
<td>0.629</td>
<td>0.593</td>
<td>0.619</td>
</tr>
<tr>
<td>V_{max}/V_{1}</td>
<td>1.25</td>
<td>1.32</td>
<td>1.32</td>
<td>1.28</td>
<td>1.20</td>
</tr>
</tbody>
</table>

### Table 4: Values of peak angle change and drift measured at a level of R = 6 for the five shells

<table>
<thead>
<tr>
<th>Rise/Span Ratio, H/B</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Angle Change at R = 6</td>
<td>0.098</td>
<td>0.116</td>
<td>0.112</td>
<td>0.107</td>
<td>0.114</td>
<td>0.109</td>
<td>0.007</td>
<td>6%</td>
</tr>
<tr>
<td>Top Node Drift at R = 6</td>
<td>3.5%</td>
<td>5.2%</td>
<td>5.7%</td>
<td>6.4%</td>
<td>6.7%</td>
<td>5.5%</td>
<td>1.1%</td>
<td>20%</td>
</tr>
</tbody>
</table>

### Table 5: Comparison of peak angle change values at different key limit points, obtained from the pushover analyses and the IDAs.

<table>
<thead>
<tr>
<th>H/B</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>1st Yield Pushover</th>
<th>IDA</th>
<th>R = 6 Pushover</th>
<th>IDA</th>
<th>$\varepsilon_{cr}$ Pushover</th>
<th>IDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.051</td>
<td>0.056</td>
<td>0.060</td>
<td>0.060</td>
<td>0.060</td>
<td>0.045</td>
<td>0.068</td>
<td>0.083</td>
<td>0.069</td>
<td>0.073</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.098</td>
<td>0.116</td>
<td>0.112</td>
<td>0.107</td>
<td>0.114</td>
<td>0.072</td>
<td>0.110</td>
<td>0.126</td>
<td>0.108</td>
<td>0.091</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>0.287</td>
<td>0.265</td>
<td>0.235</td>
<td>0.207</td>
<td>0.359</td>
<td>0.242</td>
<td>0.309</td>
<td>0.222</td>
<td>0.202</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figures

Basic parameters that define the shell
B  Span
H  Rise
L  Length
n_{div}  Number of divisions in the arch plane
n_{long}  Number of divisions longitudinally

Values derived from the basic parameters
\( \alpha \)  Angle formed by diagonals and horizontal bars
H/B  Rise/Span ratio
L/B  Length/Span ratio

Figure 1: Parameters used for the geometrical definition of the shell
Figure 2: First eight mode shapes and associated periods for five selected values of H/B

Figure 3: Fundamental natural period versus H/B ratio, for values of q from 0.0 to 5.0 kN/m²
Figure 4: Modal participating mass ratio versus rise to span ratio

Figure 5: Fundamental natural period against applied load, for H/B ratios from 0.05 to 0.50 (L/B = 1.8)

Figure 6: Variation of angle $\alpha$ by varying $n_{\text{div}}$ (left), and by varying $n_{\text{long}}$ (right)
Figure 7: Fundamental natural period versus angle between longitudinal and diagonal members

Figure 8: (a) Fundamental natural period versus H/B obtained with the estimation formula and SAP2000. (b) Comparison between the numerical and predicted results
Figure 9: Parameters defining the first mode shape

Figure 10: Comparison of mode shapes obtained numerically, and using the prediction expression
Figure 11: Cumulative mass participation in horizontal and vertical directions for the first 100 modes.

Figure 12: (a) Horizontal acceleration response spectra for the seven records selected, with the mean and the Eurocode 8 target spectrum. (b) Vertical acceleration response spectra.
Figure 13: Axial force utilisation ratios for the horizontal earthquake load case
Figure 14: Bending moment utilisation ratios for the horizontal earthquake load case
Figure 15: Difference in geometry between a circular arc (in black) and an inverted catenary (in red) (showing no difference for shells of low H/B ratio, but significant differences for deeper shells of higher H/B ratio)
Figure 16: Axial force utilisation ratios for the vertical earthquake load case
Figure 17: Bending moment utilisation ratios for the vertical earthquake load case
Figure 18: Comparison of axial force utilisation in specified members for four load cases for shells of (a) $H/B = 0.1$, (b) $H/B = 0.3$, and (c) $H/B = 0.5$
Figure 19: Comparison of bending moment utilisation in edge arches for four load cases for shells of (a) H/B = 0.1, (b) H/B = 0.3, and (c) H/B = 0.5
Figure 20: Ratios of average internal forces from earthquake to gravity (a) axial forces, (b) bending moments

Figure 21: Maximum internal force utilisation: (a) axial forces, (b) bending moments
Figure 22: Force profiles proportional to the first mode shape used to load the shells in the pushover analyses, shown for three representative values of H/B.

Figure 23: Peak angle change considered as the maximum change in angle (b) from the undeformed shape (a). Peak Angle Change, $\Delta_{\theta_{\text{max}}} = \max(\Delta_{\theta_1}, \Delta_{\theta_2}, \Delta_{\theta_3}, \Delta_{\theta_4}, \Delta_{\theta_5}, \Delta_{\theta_6}, \Delta_{\theta_7})$

\[\begin{align*}
\Delta_{\theta_1} &= \theta'_1 - \theta_1 \\
\Delta_{\theta_2} &= \theta'_2 - \theta_2 \\
\Delta_{\theta_3} &= \theta'_3 - \theta_3 \\
\Delta_{\theta_4} &= \theta'_4 - \theta_4 \\
\Delta_{\theta_5} &= \theta'_5 - \theta_5 \\
\Delta_{\theta_6} &= \theta'_6 - \theta_6 \\
\Delta_{\theta_7} &= \theta'_7 - \theta_7
\end{align*}\]
Figure 24: Pushover curves for five shells, presented in terms of (a) base shear against drift, and (b) base shear against peak angle change.
Figure 25: Base shear versus peak rotation capacity

Figure 26: Comparison of peak rotation against (a) drift and against (b) peak angle change
Figure 27: Interaction of forces in the edge arch for the five shells at the point the plastic interaction is first reached in one of the members.

Figure 28: Curvature in the edge arch at the point of failure, when $\varepsilon_{cr}$ is reached in one of the members.

Figure 29: Spread of plasticity and development of plastic hinges in the five shells.
Figure 30: IDA curves showing median and 16/84 percentiles, in terms of PGA against peak angle change, for shells of five rise to span ratios, with and without the vertical component.
Figure 31: Comparison of the five median IDA curves, showing PGA against peak angle change, with and without the vertical component.

Figure 32: Inelasticity level against (a) normalised peak angle change, and (b) normalised top node X drift.
Figure 33: Comparison of predictions from pushover analysis with median results from individual IDA curves