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A METHOD FOR THE TREATMENT OF SECOND ORDER EFFECTS IN PLASTICALLY-DESIGNED STEEL FRAMES

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Abstract: *The susceptibility of steel frames to global second order effects, also referred to as sway effects, 'P–Δ' effects and global geometric nonlinearities, is traditionally assessed through the elastic buckling load amplifier* α_{cr}*. For elastic analysis, EN 1993-1-1 and other international steel design standards state that second order effects may be neglected provided* α_{cr} *is greater than or equal to 10. However, when plastic analysis is employed, yielding of the material degrades the stiffness of the structure, and hence a stricter requirement of* $\alpha_{cr} \geq 15$ *is prescribed in EN 1993-1-1 for second order effects to be neglected. Use of a single limit of 15 for any structural system is however considered to be overly simplistic. A more consistent and accurate approach is to determine the degree of stiffness degradation and hence the increased susceptibility to second order effects on a frame-by-frame basis. A parametric analysis to assess the stability of steel frames in the plastic regime is presented herein. A series of frames with varying geometries and load cases has been assessed. Based on the findings, a proposal for the calculation of a modified elastic buckling load factor* $\alpha_{cr,mod}$*, which considers the reduction in stiffness following plasticity on a frame-by-frame basis, is presented.*

Keywords: Frame stability; Global analysis; Numerical modelling; Plastic design; Second order effects

1 INTRODUCTION

Degradation of stiffness affects the characteristics of a structural system and the subsequent distribution of internal forces and moments. There are two key types of nonlinearity to consider in the global analysis of a structure: (i) geometric nonlinearity, and (ii) material nonlinearity. Fig. 1 shows typical load-lateral displacement paths determined from different types of structural analysis. The effects of geometric nonlinearities may be seen by comparing the first order (linear) elastic analysis (LA) and second order (geometrically nonlinear) elastic analysis (GNA) paths, while the effects of material nonlinearities may be observed by comparing the first order elastic (LA) and first order plastic (materially nonlinear) analysis (MNA) paths. Geometrical nonlinearities, also referred to as second order effects, may result either from global imperfections and deformations associated with frame instability, known as 'P–Δ' effects, or member imperfections and deformations associated with member instability, known as 'P–δ' effects. Material nonlinearity results from the development of inelastic strains i.e. the onset and spread of plasticity. There are two common forms of plastic analysis: plastic hinge analysis, in which material nonlinearity is considered through the development of zero-length plastic hinges within the structure, and plastic zone analysis, in which the development and spread of plasticity through the depth of the cross-section as well as along the length of the member is captured. Both are illustrated in Figure 1. It can be seen that both forms of nonlinearities can have a significant influence on the global response of the structure. Second order plastic analysis (i.e. geometrically and materially nonlinear) with imperfections (GMNIA) is assumed to provide the 'true' equilibrium path and failure load of the frame.

In common with other structural steel design standards, the stability of frames and the need to consider global second order effects is assessed in EN 1993-1-1 [1] on the basis of the value of the elastic buckling load factor α_{cr}. This represents the factor by which the applied loading would need to be increased to cause elastic instability of the frame in a global sway mode.

Second order effects are deemed sufficiently small that they may be ignored if the amplification of the internal forces and moments due to sway second order effects is no more than 10% of the original internal forces determined according to first order theory. This corresponds to a limit of $\alpha_{cr}\geq 10$ for elastic analysis. For plastic analysis, a stricter limit of $\alpha_{cr}\geq 15$ beyond which second order effects can be ignored is given in EN 1993-1-1; this increased limit for plastic analysis recognises the fact that frames have reduced stiffness following plasticity and therefore increased susceptibility to second order effects. Note that in prEN 1993-1-1 [2], new provisions for the assessment of second order effects in the plastic regime are included, where the limit of 10 is used for both elastic and plastic analysis, and the effect of the reduced stiffness is instead accounted for by reducing the critical load factor itself. This is discussed further in Section 3.1.2.

1.1 Global sway behaviour

Material yielding and the formation of plastic hinges results in a progressive deterioration of frame stability [3, 4], where the stiffness is progressively reduced with each hinge formed. This is illustrated in Fig. 2, which shows the behaviour of an idealised one-bay fixed-based frame considering the progressive formation of plastic hinges. It should be noted that the illustrated sequence of hinge formation relates to the loading conditions indicated in the figure; this sequence could of course differ under alternative loading conditions. Fig. 2a shows the second order elastic load-displacement paths for the frame with an increasing number of plastic hinges, from GNA_0, with zero hinges, to GNA_3, with 3 hinges. It can be seen that the corresponding elastic critical buckling load of the frame reduces as the number of hinges increases, reflecting the overall loss in stiffness of the frame. Fig. 2b shows the second order plastic behaviour of the frame. Initially the frame is fully elastic and the load displacement path remains essentially linear, since geometric nonlinearities are small at low load levels. As the frame starts to deflect, the loading induces second order forces and moments, and the frame response deviates from linearity following the path of the second order elastic analysis GNA_0. The internal forces and moments continue to increase until the most heavily loaded cross-section reaches its plastic moment capacity and a plastic hinge forms [5]. This is illustrated in Fig. 2b by the point marked 1. Once a hinge forms, the frame stiffness immediately reduces further and the frame now follows the path from a second order elastic analysis with a hinge at the location of the formed plastic hinge (GNA_1). As the loading increases further, another cross-section reaches its plastic moment capacity and a second hinge forms (point 2). The frame now follows the path of a second order elastic analysis with two hinges (GNA_2). This continues until a sufficient number of hinges have formed to cause a collapse mechanism [6, 7]. It should be noted that the frame remains elastic except at the plastic hinges; this is shown by the GMNA path matching exactly with the path made up of the second order elastic paths with an increasing number of hinges.

The assumption of zero-length plastic hinges is however an unconservative idealisation [8], which fails to capture the initiation and progressive spread of plasticity both through the cross-section depth and along the member length. The latter response is captured through plastic zone analysis. While plastic hinge analysis is more computationally efficient in comparison to plastic zone analysis and therefore currently more commonly used in practical applications, improvements in computational power and advances in software are allowing advanced analysis to become more viable for widespread use in design [9].

In this paper, the stability of steel frames and the treatment of second order effects in the plastic regime are examined. In Section 2, benchmark ultimate load factors are generated for a series of frames using geometrically and materially nonlinear analyses with imperfections (GMNIA). The results from first and second order elastic and plastic analyses are then compared, and the influence of material nonlinearity on the stability of frames considered. Finally, a proposal to account for the effect of plasticity on the generation of second order effects on a frame-by-frame basis is made in Section 3.

2 BENCHMARK FRAME MODELLING

As discussed in Section 1, there are two common types of plastic analysis – plastic hinge and plastic zone, and both are assessed in this study. For plastic zone analysis, beam finite element (FE) models were developed using the general purpose FE software ABAQUS [10]. Since ABAQUS does not allow for the combined influence of bending moments and axial force on the development of plastic hinges, the plastic hinge analyses were carried out using models developed in the MASTAN2 [11] structural analysis software.

Frames constructed using hot-rolled steel I-sections were simulated; the chosen cross-section geometry was that of a standard European HEB 340 section, which is Class 1 under all load cases for the considered material. This cross-section is therefore deemed capable of reaching and maintaining its full plastic moment resistance, and is thus suitable for plastic design. The 2-noded linear Timoshenko B31OS beam element, from the ABAQUS element library, was employed to create the models and was used in all numerical simulations in ABAQUS, while the 3D shear flexible quadratic beam elements were employed in MASTAN2. The frame geometries, boundary conditions and load cases were devised such that global instability featured prominently. Fig. 3 and Table 1 provide an overview of the frame configurations modelled in this study.

All members were connected via fixed multi-point constraint ties at their ends providing full continuity. The frames were restrained out-of-plane such that only in-plane major axis bending/buckling was considered. For the ABAQUS models, 100 elements were used to model each of the members, to accurately capture the spread of plasticity, and the modified Riks method [10] was used to trace the full load-deformation response of the frames. The load factors, displacements, section forces and section moments were extracted from all analyses at each load increment.

The stability of structural frames is assessed through the elastic buckling load factor α_{cr}. Throughout this study, α_{cr} is determined under the applied loading that causes collapse of the frame, as determined from GMNIA. In the following two subsections, the approach to determining elastic buckling load factors α_{cr} from a linear buckling analysis (LBA) and ultimate load factors α_u from a geometrically and materially nonlinear analysis with imperfections (GMNIA) of the considered frames are described.

2.1 Elastic buckling load factor α_{cr}

The elastic critical load factor α_{cr} of a frame may be determined by two common methods (i) an approximate method, originally proposed in [12, 13] and set out in EN 1993-1-1 whereby the elastic critical load factor is calculated on the basis of the lateral deflections of a frame under applied horizontal loading or (ii) using a linear buckling analysis (LBA) and taking α_{cr} as the eigenvalue corresponding to the lowest sway buckling mode. Both calculations may be performed under any level of applied loading and then factored by the collapse load factor α_u. Throughout this study, α_{cr} has been determined using linear buckling analysis of the frames under the loading corresponding to the ultimate load factor α_u, as predicted by GMNIA and outlined in Section 2.2.

2.2 Ultimate load factor α_u

In this subsection, the calculation of the ultimate load factors of the frames from geometrically and materially nonlinear analyses with imperfections (GMNIA) is outlined. These are taken as the 'true' failure load factors of the frames and are used in this study as benchmark load factors. A linear-elastic perfectly-plastic stress-strain relationship was employed with a yield stress f_y=235 N/mm^2 and a Young's modulus E=210000 N/mm^2. As recommended in EN 1993-1-1 [1], an initial member out-of-straightness in the form of a half-sine wave and with a magnitude of 1/1000 of the member length was assumed while, for the initial frame imperfection, an out-of-plumbness 1/200 of the frame height was assumed. The initial frame out-of-plumbness was applied as an equivalent horizontal force, to induce the first order deformations equivalent to the geometric imperfection [9], and included in all analyses. For each frame, the initial displacements were defined such that they provided the greatest destabilising effect considering the boundary conditions, frame geometry and load case. The ECCS residual stress distribution for hot-rolled I-sections with height-to-width ratio 1.2 [14] was incorporated into the finite element models, with each flange and web of the sections discretised into 33 section points to ensure that the residual stress distribution and subsequent spread of plasticity could be captured accurately [15]. The initial stress values were introduced at the section points through the SIGINI user subroutine [10]. For each frame and load case outlined in Table 1, the ultimate load factor α_u was directly taken as the peak load factor from a GMNIA.

3 RESULTS

In this section, the results of first and second order analyses are compared to study the response of the examined frames and to assess the rules for global stability design set out in EN 1993-1-1 [1]. In the elastic regime, Merchant [16] proposed an amplification factor k_{amp}, which is included in EN 1993-1-1 and given by Eq. (1), to estimate second order sway moments from a first order analysis. This amplification factor is applied to the horizontal loads and may be used to account for second order effects when $3 \leq \alpha_{cr} < 10$ [1]. For very slender frames when $\alpha_{cr} < 3$, a full second order analysis must be carried out [1].

$$k_{\mathrm{amp}} = \frac{1}{1 - \frac{1}{\alpha_{\mathrm{cr}}}} \quad (1)$$

where α_{cr} is the factor by which the applied loading must be increased to cause elastic instability of the frame in a global sway mode, as outlined in Section 2.1.

As discussed in Walport et al. [17], for elastic analysis, the amplification factor k_{amp} is equivalent to the ratio of internal forces from a second (GNA) to a first order (LA) analysis (M_{GNA}/M_{LA}) at any point in the frame. However, for plastic analysis, due to the redistribution of the internal forces and moments following material nonlinearity, k_{amp} must be calculated by determining the magnitude of the amplification of the horizontal loading in a first order analysis (MNA+ k_{amp}) required to align the sway deflections to those in a second order analysis (GMNA) at a given applied load factor, as shown in Fig. 4. At this level of amplification to the horizontal loading, the moments in the amplified first order analysis are equal, when sway effects are dominant, to the moments in the second order analysis i.e. $M_{GMNA}=M_{MNA+kamp}$. This required amplification factor has been determined herein at the benchmark ultimate load factor α_u for each frame, considering both elastic and plastic analyses.

For elastic analysis, the ratios of moments extracted from the FE frame models at α_u using a second order elastic analysis (GNA) and a first order elastic analysis (LA) are plotted in Fig. 5, along with Eq. (1). As expected, the results may be seen to follow the expression for the amplification factor k_{amp} accurately. At α_{cr}=10, the required amplification from first to second order analysis is around 10%, which is the threshold beyond which account must be taken of second order effects in EN 1993-1-1 [1]. As α_{cr} tends towards unity (i.e. as the applied loading F_{Ed} approaches the elastic buckling load F_{cr}), second order effects become increasingly significant.

For plastic analysis, the amplification factors calculated using the approach illustrated in Fig. 4, along with Eq. (1), are plotted against α_{cr} in Fig. 6. The results from both plastic hinge analysis and plastic zone analysis are shown. Unlike in Fig. 5, the results no longer match well with Eq. (1) and lie on the unsafe side of the curve (by 2% on average and by up to almost 20% for particular cases). This indicates that a greater amplification factor is required to take account of the second order effects in the plastic regime than is predicted with Eq. (1). While for the plastic analysis of frames where the sway mode is dominant, amplifying the horizontal force is accurate in accounting for second order effects, for frames where nonsway effects are significant, amplifying the horizontal loads in a first order plastic analysis does not result in the same forces and moments as those in a corresponding second order plastic analysis. This is in line with the guidance given in [18]. Similar observations were made by Demonceau et al. [19] who noted that amplifying a first order analysis will not always result in the same plastic mechanism as formed in a second order analysis due to the second order effects influencing differently the yielding of the structure. Therefore, it is concluded that it is only appropriate to use k_{amp} in conjunction with plastic analysis in cases where sway is dominant.

The amplification concept can alternatively be used to relate load factors for a given level of deflection. For elastic analysis, the relationship between load factors obtained from first order (α_{el1}) and second order (α_{el2}) analyses is given by Eq. (2) [16]. This expression applies at all load levels and provides a precise means of relating first and second order load factors, as illustrated in Fig. 7.

$$\left(\frac{\alpha_{\mathrm{el2}}}{\alpha_{\mathrm{el1}}}\right) = \frac{\alpha_{\mathrm{cr}}-1}{\alpha_{\mathrm{cr}}} \tag{2}$$

In the plastic regime, Lim et al. [20] proposed that the same concept could be applied to relate first order (α_{pl1}) and second order (α_{pl2}) plastic collapse loads through Eq. (3).

$$\left(\frac{\alpha_{\mathrm{pl2}}}{\alpha_{\mathrm{pl1}}}\right) = \frac{\alpha_{\mathrm{cr}}-1}{\alpha_{\mathrm{cr}}} \tag{3}$$

For plastic analysis, the ratios of the second order plastic collapse load factor, taken as the GMNIA ultimate load factor (i.e. $\alpha_{\mathrm{pl2}}=\alpha_{\mathrm{u}}$), to the first order plastic collapse load factor for all frames are plotted in Fig. 8 alongside Eq. (3), against the elastic critical load factor α_{cr}. It can be seen that, in similar conclusion to Fig. 6, the FE results do not match well with Eq. (3) and the majority of the points lie on the unsafe side relative to the Merchant-Rankine formula, with an average value of $(\alpha_{\mathrm{pl2}}/\alpha_{\mathrm{pl1}})/((\alpha_{\mathrm{cr}}-1)/\alpha_{\mathrm{cr}})$ of 0.97 and a minimum value of 0.85. This can be explained with reference to Fig. 9, which shows the load-lateral displacement paths from the different types of structural analysis for an idealised frame. In the elastic regime, it can be seen in Fig. 9a that applying the Merchant-Rankine formula provides a precise prediction of the reduction in load factor due to second order effects. However, if the (elastic) amplification concept is applied in the plastic regime (i.e. Eq. (3)) it can be seen in Fig. 9b that the reduction in load factor from first order analysis α_{pl1} is no longer sufficient for predicting the second order plastic collapse load α_{pl2}. For the amplification approach to accurately predict the second order plastic collapse load, it should no longer be based on the elastic critical load factor α_{cr} (i.e. Eq. (3)), but on a modified (reduced) critical load factor $\alpha_{cr,mod}$ as shown in Fig. 9c and described in Section 3.1. It is therefore concluded that the amplification concept, as given by Eq. (3), is not appropriate and that a modified form is required.

3.1 Proposal to account for the influence of plasticity on the development of second order effects

In the previous section it was shown that material yielding and the onset of plasticity results in a reduction in sway stiffness of the frame and hence an increased susceptibility to second order effects. The 'average' reduction in sway stiffness at a specified load level can be estimated through the ratio K_s/K of the load-lateral deflection curve from a first order plastic analysis (MNA), where K is the initial stiffness and K_s is the secant stiffness at a specified point (i.e. the applied load level), as shown in Fig. 10. The ratio K_s/K may be alternatively expressed as Δ_{el}/Δ_{pl}, facilitating its calculation – see Fig. 10. This ratio represents the reduced sway stiffness of the frame and can therefore be used to account for the increased susceptibility of the frame to global second order effects. However, at the same load level, the forces and moments in a frame obtained from a second order plastic analysis will be higher than those from a first order plastic analysis, so the actual degradation in stiffness will in fact be somewhat larger than that predicted using the method described above. This can be seen in Fig. 11, where the results of a GNA performed with the Young's modulus E reduced by factor K_s/K are shown to not fully capture the degradation in stiffness of the frame at α_u, as given by the GMNA results. A factor Y for the further loss of stiffness due to the additional plastification is proposed. Based on the analyses performed herein, for single storey portal frames, a factor of 0.9 for Y is proposed while for all other frames, a factor of 0.65 for Y is proposed. This greater reduction for more complex frames reflects the fact that more plastic hinges can form, at a given load level, between a first order and second order analysis, resulting in a greater degradation in stiffness. Fig. 12 highlights the frames where more hinges have formed in the second order analysis than in the first order analysis at α_u. The variability in the results is attributed to the wide range of frame geometries and load cases considered, resulting in different load-deformation histories, plastic hinge formations sequences and collapse mechanisms. The anomalous result at a value of α_{cr} around 15 is from the Case 3 10×10 frame with H=0.05V. This result can be explained by the nature of the collapse mechanism, which was a beam collapse mechanism rather than a sway mechanism, and therefore the influence of second order effects is small. Whether additional plastic hinges form at a given load level due to second order effects cannot be predicted from a first order analysis, and therefore the reduction based on the combination of K_s/K and the Y factor is proposed to provide a safe sided assessment of the loss of stiffness due to plasticity. For the typical portal frame shown in Fig. 11, the results of GNA with E reduced by 0.9K_s/K are shown to provide good agreement with the GMNA results at α_u, demonstrating that the influence of material degradation on the sway stiffness of the frame is accurately captured.

3.1.1 Proposed approach

As presented in Walport et al. [17] for stainless steel, the influence of plasticity on the sway stiffness of frames may be considered by defining a modified elastic buckling load factor $\alpha_{cr,mod}$, as given by Eq. (4). The ratio K_s/K accounts for the reduction in stiffness arising in a first order plastic analysis due to the material yielding, while the Y factor approximates the further loss of stiffness due to the additional plastification, and potential formation of additional plastic hinges, that occurs at the same load level when second order effects are considered. The modified critical load factor $\alpha_{cr,mod}$ may then be used to assess whether or not second order effects are significant; the assessment is made against a limiting value of 10, allowing for consistency with elastic analysis.

$$\alpha_{cr,mod} = \Upsilon \frac{K_s}{K} \alpha_{cr} \tag{4}$$

A modified reduction factor to account for the influence of plasticity can likewise be defined, utilising the modified elastic buckling load factor $\alpha_{cr,mod}$, as given by Eq. (5).

$$\left(\frac{\alpha_{pl2}}{\alpha_{pl1}}\right) = \frac{\alpha_{cr,mod}-1}{\alpha_{cr,mod}} \tag{5}$$

From the study on multi-storey frames (Frame cases 4, 9, 13-15, 17 in Table 1), it was found that the effects of material nonlinearity on the reduction in global sway stiffness should be assessed on a storey-by-storey basis, with the most critical storey (i.e. the lowest value of $\alpha_{cr,mod}$) used to represent the overall frame. This prevents the deleterious influence of plasticity on frame stability from being 'averaged out' and ensures safe sided estimates of $\alpha_{cr,mod}$. This approach of assessing frame stability based on the interstorey sway of the critical storey in the frame is similar to the approximate method originally developed by Horne [13] and used in the elastic regime in EN 1993-1-1 [1].

3.1.2 Wood approach

As discussed in Section 1.1, frame stiffness decreases with the formation of plastic hinges and the structural stability is governed by the remaining elastic parts of the frame. prEN 1993-1-1 [2] includes a revised approach to the assessment of second order effects for plastic global analysis that, similarly to the proposal made in this study, retains the limiting value of 10 from elastic analysis and requires the calculation of $\alpha_{cr,mod}$. The approach is based on the research carried out by Wood [3] which investigated frame stability and plastic hinge formation, and concluded that a modified critical load factor, or deteriorated critical load factor, may be calculated by carrying out a linear buckling analysis of a frame with hinges at the location of the plastic hinges. Conservatively, this approach can be applied to the frame at the point at which the penultimate plastic hinge, prior to the creation of a collapse mechanism, forms, since in practice the number of hinges formed at the design load level may be initially unknown. More accurately, the number of hinges formed at the load level of interest may be substituted into the linear buckling analysis. This method may only be carried out with plastic hinge analysis.

3.1.3 Implementation using plastic hinge analysis

The structural analysis software MASTAN2 [11] was employed to carry out the plastic hinge analyses as discussed in Section 2, with the geometry and loading of all frames developed in accordance with the ABAQUS [10] models. Fig. 13 shows a comparison of the modified critical load factors for a 15×10 m portal frame calculated using the different methods described above. As expected, as the number of hinges in the linear buckling analysis increases, the modified critical load factor decreases. The method of including hinges in the linear buckling analysis allows the influence of plasticity on sway stability to be assessed on a frame-by-frame basis, which is rational, but yields very conservative results when compared to Eq. (3) – this can also be clearly seen in Figs. 14a and 14b for all considered frames in the study. This means that the calculated value of $\alpha_{cr,mod}$ will fall more frequently (than necessary) below the limit of 10, requiring a second order analysis to be performed. In addition to the proposed modified critical load factor in Section 3.1.1, a critical load factor based solely on the secant stiffness reduction has been plotted (see Fig. 14c for all frames). It can be seen that while the secant stiffness reduction factor K_s/K captures the majority of the influence of the plastification, the Y factor, in this example equal to 0.9, is also required to reflect the full degree of stiffness degradation (see Fig. 14d for all frames).

Using the Wood approach, the elastic buckling load factor of the frame is reduced substantially more than necessary for both considered cases for the number of hinges formed. This is because the Wood approach assumes that the stiffness reduction due to the formation of the plastic hinges begins at the onset of loading. Considering Fig. 2b, it can be seen that in reality, the frame initially has its full elastic stiffness and it is not until the onset of plasticity that the gradual degradation of stiffness begins. This is reflected in the proposed approach, which explains the considerably more accurate results; for the example frame shown in Fig. 13, $\alpha_{cr,mod} \geq 10$ according to the proposed approach and therefore a first order analysis is in fact sufficient.

Fig. 14 shows the results from the plastic hinge analyses plotted against the modified elastic buckling load factor for all frames considered in this study. The modified critical load factors obtained from the proposed approach (Eq. (4)), as well as from the alternative approaches, as included in Fig. 13, are shown. The conclusions from Fig. 13 may be seen to be true for all frames in Fig. 14. As well as the conclusions previously made, it can be seen that applying the Wood proposal with the number of hinges formed at the GMNIA collapse load (see Fig. 14b) can result in some unsafe predictions. This is because the number of hinges is based on those formed in a first order plastic analysis and does not account for the increased number that may form when second order effects are considered. Plotting the frame results against the proposed modified critical load factor (see Fig. 14d) yields close agreement with the Merchant-Rankine formula. The average value of $(\alpha_{pl2}/\alpha_{pl1})/((\alpha_{cr,mod}-1)/\alpha_{cr,mod})$ is 1.01. The requirement that for $\alpha_{cr,mod} \geq 10$, a first order analysis is sufficient, and for $\alpha_{cr,mod} < 10$, second order effects are significant and must be considered in the analysis, is therefore considered to be suitable. Note that for frames failing in the elastic range, the first order secant stiffness reduction (K_s/K) would be equal to unity, but the Y factor would still apply.

3.1.4 Implementation using plastic zone analysis

Fig. 15 shows the frame results from the plastic zone analyses plotted against the modified elastic buckling load factor. The modified critical load factors based solely on the secant stiffness reduction are shown in Fig. 15a, while the modified critical load factors obtained from the proposed approach (Eq. (4)) are shown in Fig. 15b. It can be seen that while the secant stiffness reduction factor K_s/K captures the majority of the influence of the plastification, the Y factor is also required, as described earlier. As for the plastic hinge analysis results, the plastic zone analysis results, when plotted against the proposed modified critical load factor, show good agreement with the modified Rankine-Merchant formula. The unsymmetrical pinned-based two-storey moment resisting (U-P36H) Ziemian [21] frame and the six-storey Vogel [22] frame have also been assessed and included in Fig. 15, with both resulting in safe sided, although very conservative, predictions. These two frames, along with the Case 3 10×10 frame with H=0.05V discussed in Section 3.1, have values of $\alpha_{cr,mod}$ in the range of 3 to 6, and can be seen to not fit the trend of results and the modified Merchant-Rankine. These frames failed with beam collapse mechanisms and therefore the influence of second order effects at collapse is small. In accounting for the reduction in stiffness due to the influence of plasticity by redefining the critical load factor on a frame-by-frame basis, the limit of 10 may now be used for plastic analysis as well as elastic analysis. As presented for plastic hinge analysis in Section 3.1.3, for plastic zone analysis, when $\alpha_{cr,mod} \geq 10$ a first order analysis is deemed sufficient, while for $\alpha_{cr,mod} < 10$, second order effects are considered to be significant (i.e. greater than 10% of the first order effects) and must be considered in the analysis. A reduction to the first order plastic collapse load factor based on Eq (5) has been shown to provide a good approximation to the second order plastic collapse load factor, but it is nonetheless recommended that a full second order plastic analysis is carried out for $\alpha_{cr,mod} < 10$.

4 SUMMARY OF PROPOSALS

A summary of the existing and proposed methods for accounting for second order effects is given in Table 2. The key steps involved in the proposed method for assessing the stability and treatment of second order effects in plastically-designed steel frames is as follows:

1. Calculate the load factor to cause elastic instability in a global sway mode α_{cr} under the applied factored loading.
2. Perform a first order plastic analysis (MNA) and extract the horizontal frame displacement Δ_{pl} at this load level.
3. Determine the secant stiffness reduction factor $K_s/K=\Delta_{el}/\Delta_{pl}$ based on the minimum value of K_s considering both sway displacement (i.e. left or right side of the frame) and loading history. For multi-storey frames, K_s/K should be based on the critical storey.
4. Calculate the modified critical load factor $\alpha_{cr,mod}=\Upsilon(K_s/K)\alpha_{cr}$, where Υ is equal to 0.9 for portal frames and 0.65 for all other frames.
5. If $\alpha_{cr,mod}\geq 10$ a first order analysis (MNA) may be employed. If $\alpha_{cr,mod}<10$, a second order analysis (GMNA) is required.

5 CONCLUSIONS

The onset of plasticity results in degradation of stiffness and hence enhanced second order effects in steel frames. If plasticity is considered in the global analysis of a frame, greater deflections ensue due to the reduction in material stiffness. At present, EN 1993-1-1 [1] accounts for this increased degree of flexibility by applying a stricter requirement of $\alpha_{cr}\geq 15$ before second order effects can be neglected. The use of a single limit of 15 for all structural systems no matter the degree of plastic deformation is however considered by the authors to be overly simplistic. Instead, a new method to account for the increased susceptibility to second order effects due to plasticity, whereby the elastic buckling load factor of the frame α_{cr} is reduced to $\alpha_{cr,mod}$ on the basis of the secant stiffness from a first order plastic analysis has been proposed and shown to yield accurate results. Based on the proposed approach, for plastically-designed frames, when $\alpha_{cr,mod}\geq 10$, second order effects may be neglected and a first order plastic analysis (MNA) can be carried out. A reduction to the first order plastic collapse load factor based on $\alpha_{cr,mod}$ has been shown to provide a good approximation to the second order plastic collapse load factor, but it is nonetheless recommended that for $\alpha_{cr,mod}<10$ a full second order plastic analysis (GMNA) should be carried out. Assessing frame stability on a frame-by-frame method is deemed to be a more consistent and accurate approach for any structural system than the current use of a single limit for all frames. Furthermore, the proposed approach provides a consistent treatment of second order effects between elastic and plastic global analysis i.e. a limit of 10 on both α_{cr} and $\alpha_{cr,mod}$, deeming second order effects sufficiently small to be ignored if the amplification of the internal forces and moments due to sway second order effects is no more than 10% of the original internal forces determined according to first order theory. This method leads to more accurate and less conservative results than carrying out a linear buckling analysis with hinges, as suggested in [3, 4], and included in prEN 1993-1-1 [2], because this assumes that the stiffness reduction due to the formation of the plastic hinges begins at the onset of loading. While the method is quick and convenient to carry out, it often results in overly pessimistic predictions. The proposed method may be used for both plastic hinge and plastic zone analysis and is straightforward to apply since it does not rely on knowledge of the plastic hinge locations or the order of their formation.

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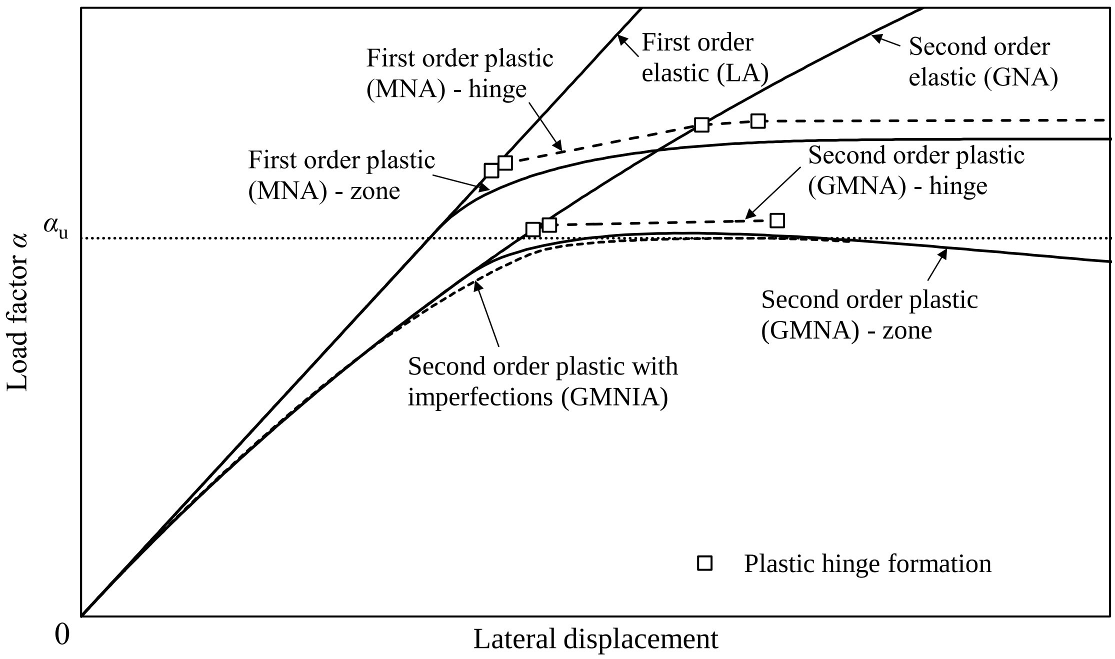
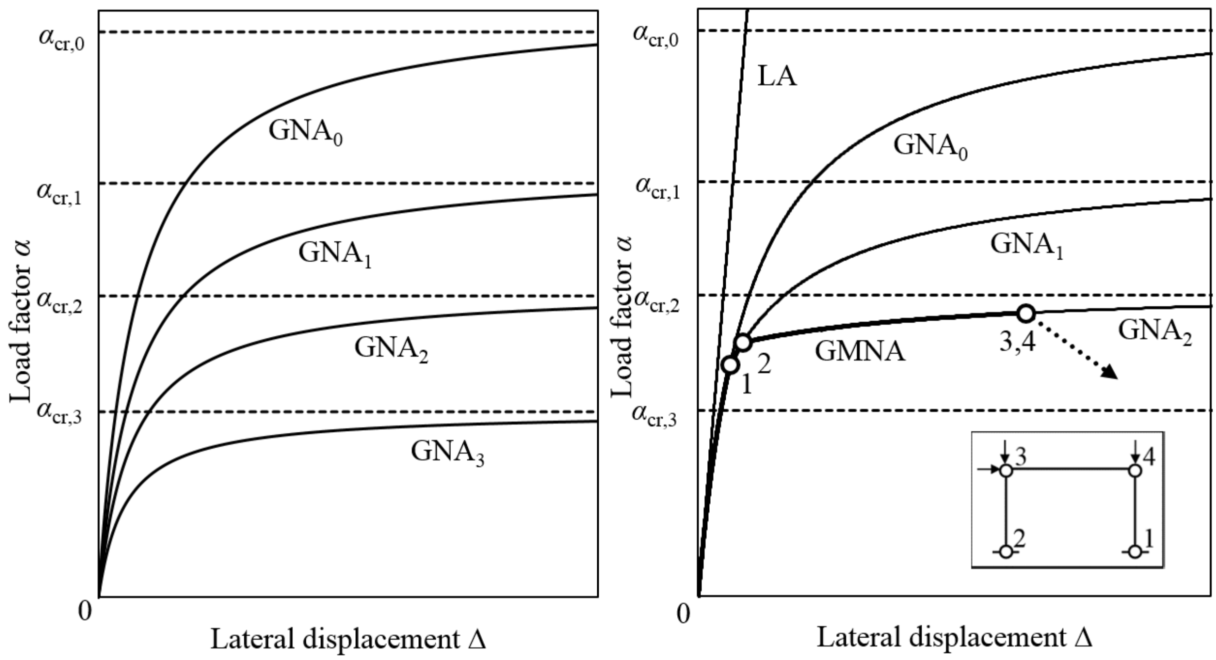


Figure 1: Methods of structural analysis (LA = Linear Analysis; GNA = Geometrically Nonlinear Analysis; MNA = Materially Nonlinear Analysis; GMNA= Geometrically and Materially Nonlinear Analysis; GMNIA = Geometrically and Materially Nonlinear Analysis with Imperfections).



a) Second order elastic frame response with an increasing number of plastic hinges

b) Progressive formation of plastic hinges

Figure 2: Representative frame sway behaviour with the onset of plastic hinge formation.

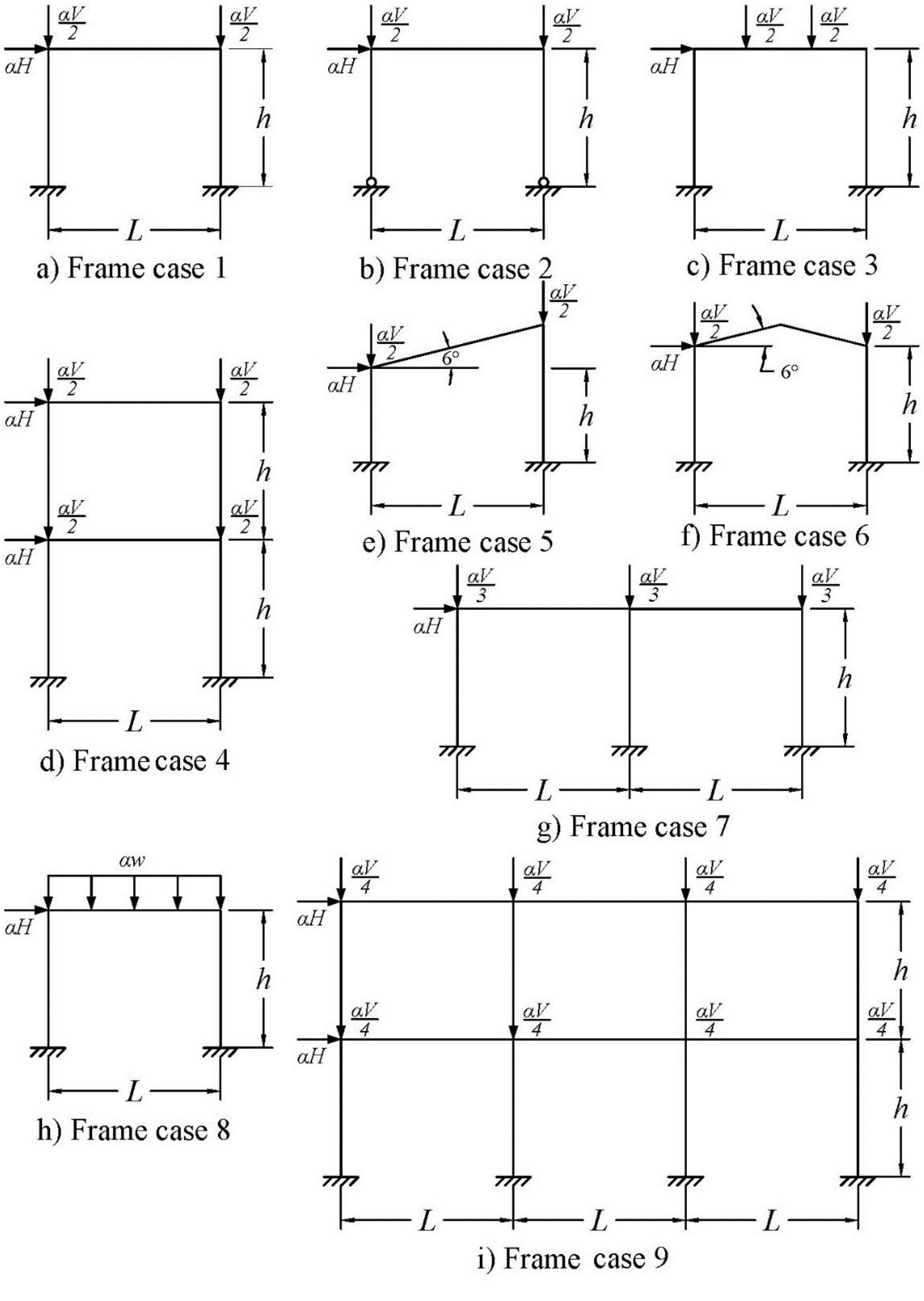


Figure 3: Details of portal frames modelled, where α is the load factor, as discussed in Sections 2.1 and 2.2.

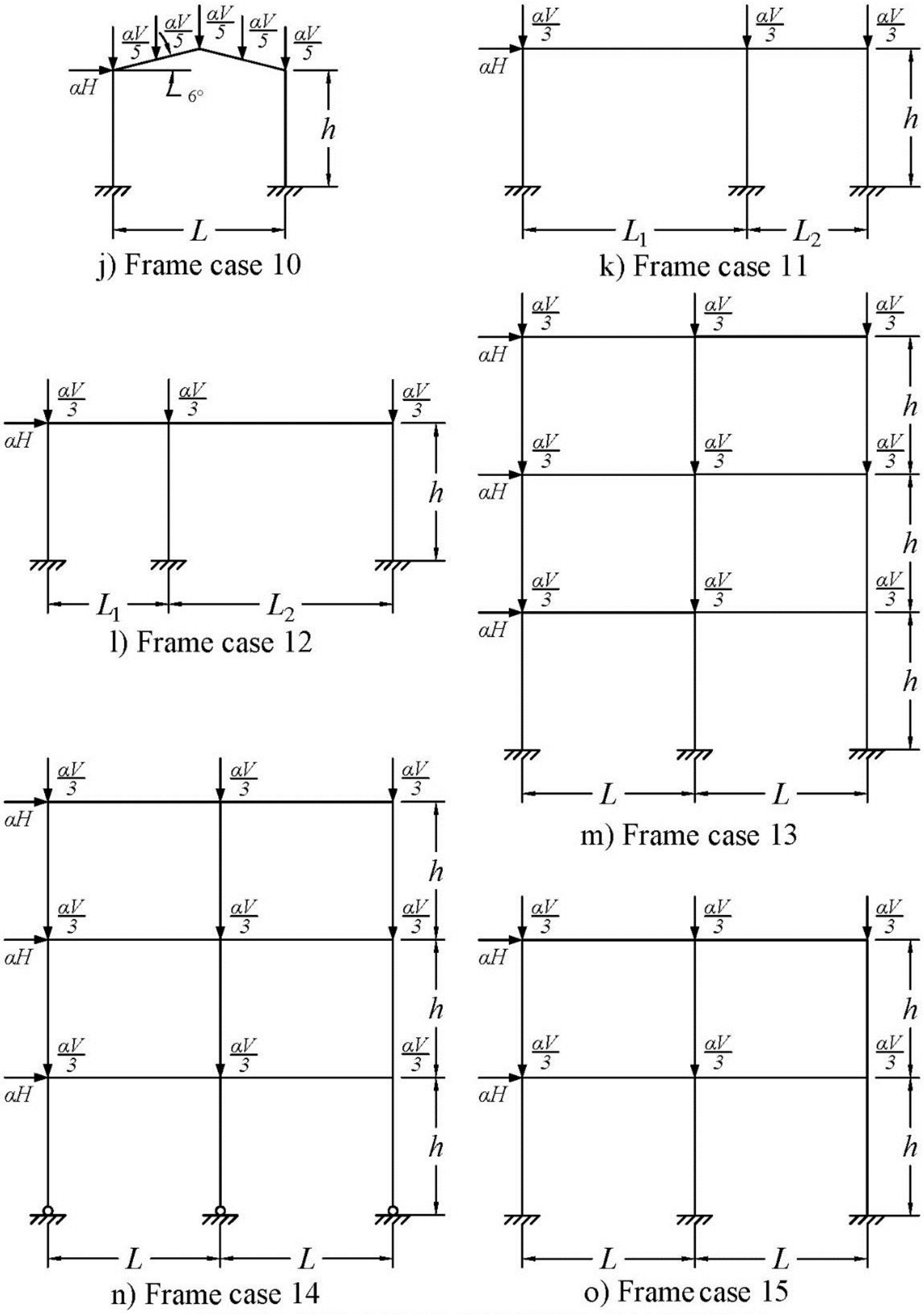
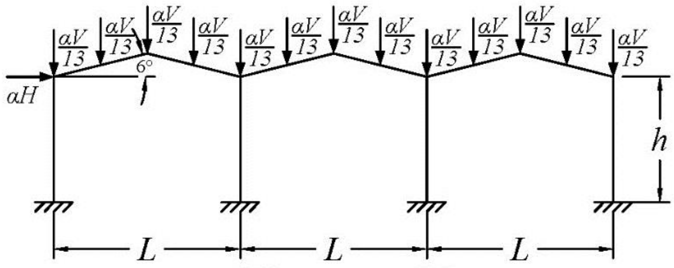
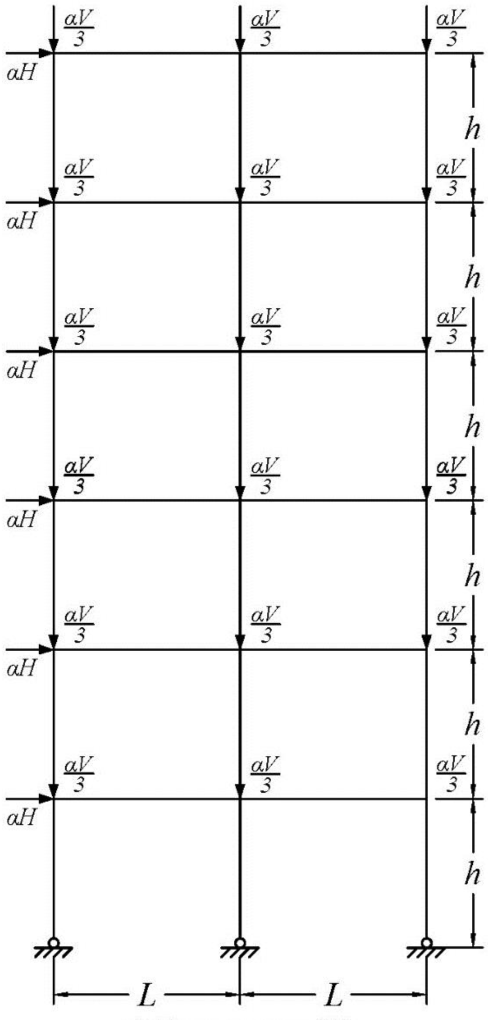


Figure 3 cont.: Details of portal frames modelled, where α is the load factor, as discussed in Sections 2.1 and 2.2.



p) Frame case 16



q) Frame case 17

Figure 3 cont.: Details of portal frames modelled, where α is the load factor, as discussed in Sections 2.1 and 2.2.

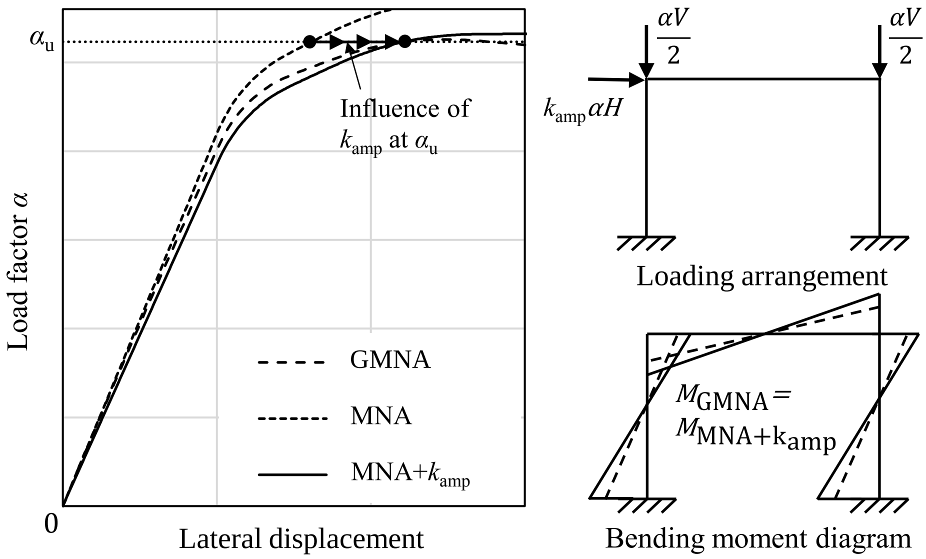


Figure 4: Determination of k_{amp} from frame FE models.

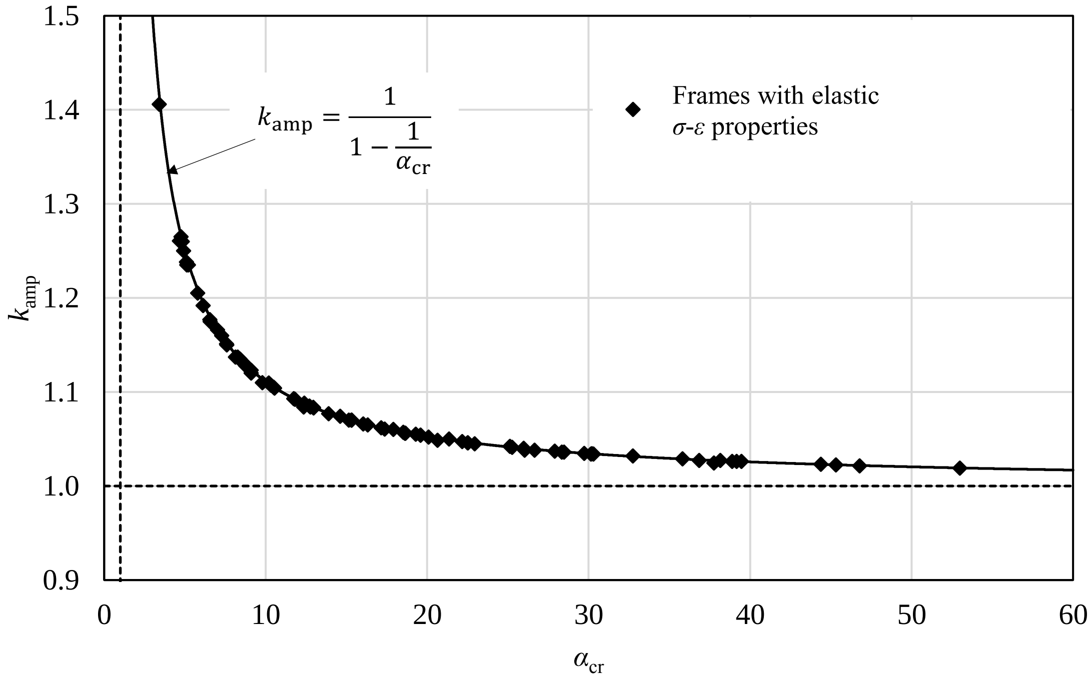


Figure 5: Amplification factor from first to second order elastic analyses at α_u versus α_{cr}.

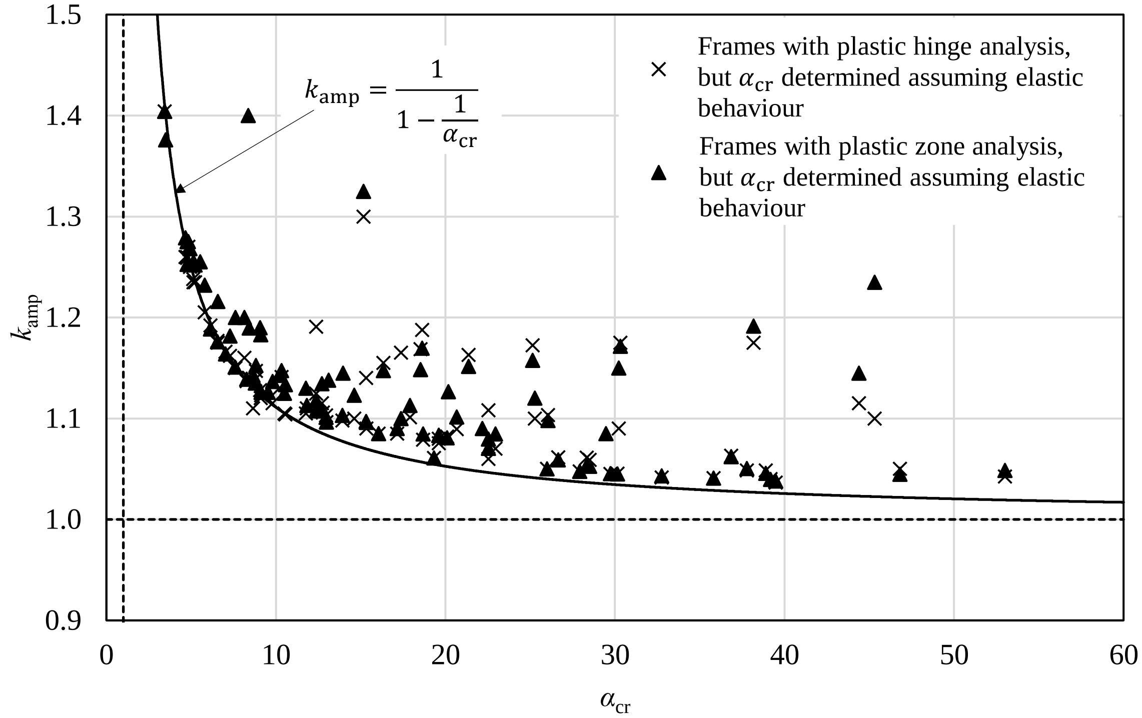


Figure 6: Amplification factor from first to second order plastic analyses at α_{u} versus α_{cr}.

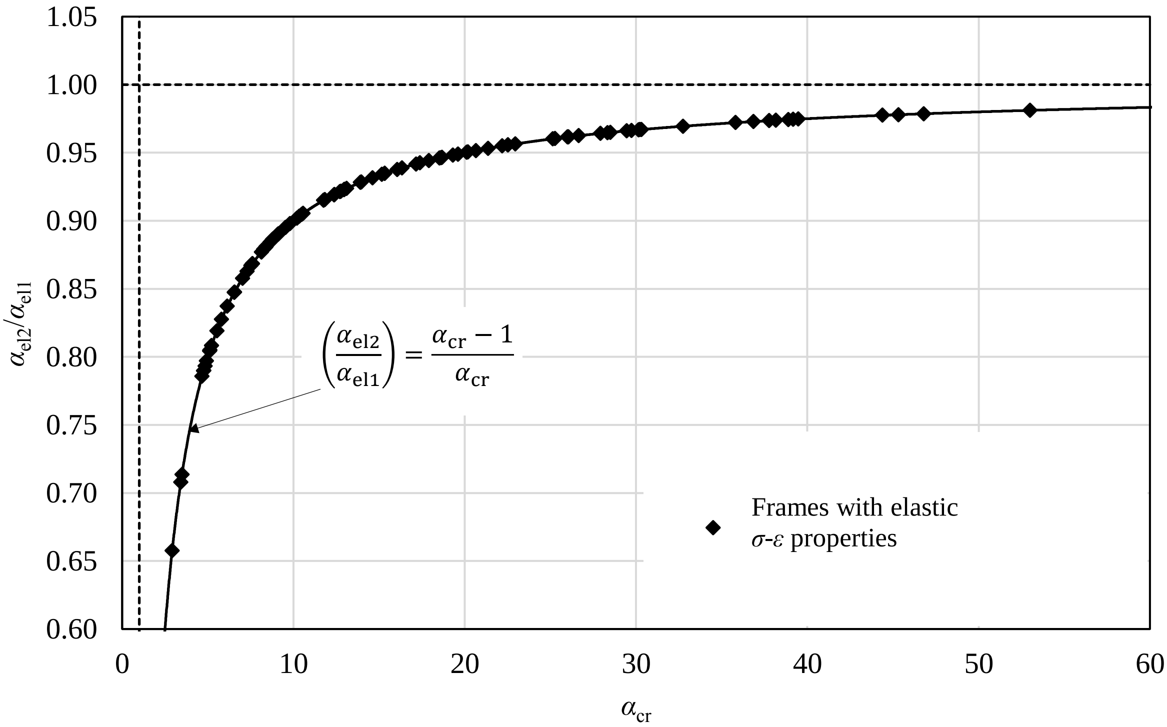


Figure 7: Reduction of load factor from first to second order elastic analyses versus α_{cr}.

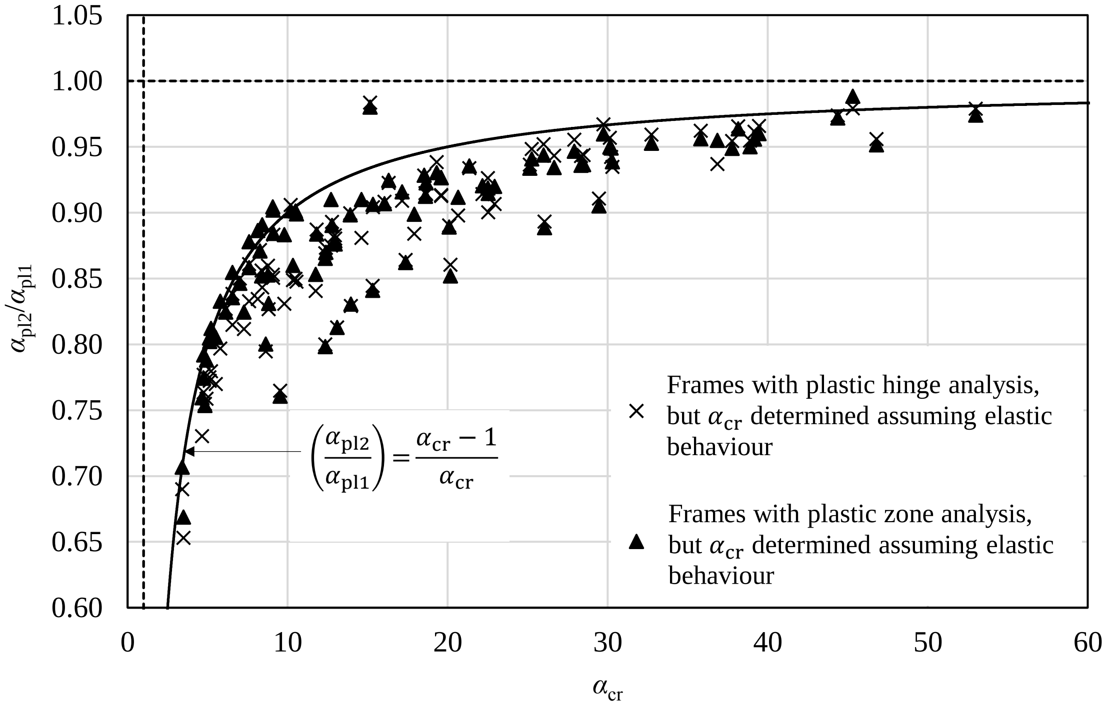
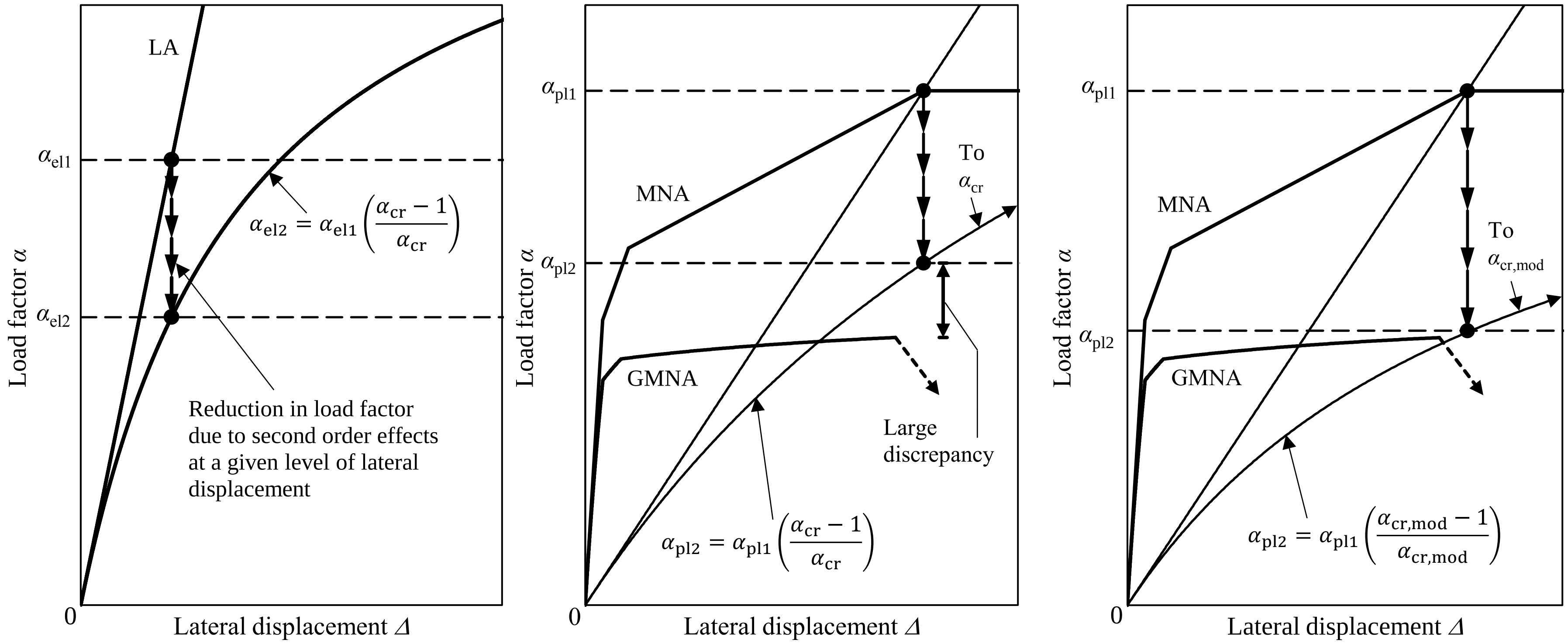


Figure 8: Reduction of load factor from first to second order plastic analyses at α_u versus α_{cr}.



(a) Reduction in load factor from first order to second order elastic analysis, showing precise match.

(b) Reduction from first order to second order plastic collapse load factor based on α_{cr}, showing a discrepancy in the prediction.

(c) Reduction from first order to second order plastic collapse load factor based on $\alpha_{cr,mod}$, showing an accurate prediction.

Figure 9: Assessment of amplification concept for an idealised frame.

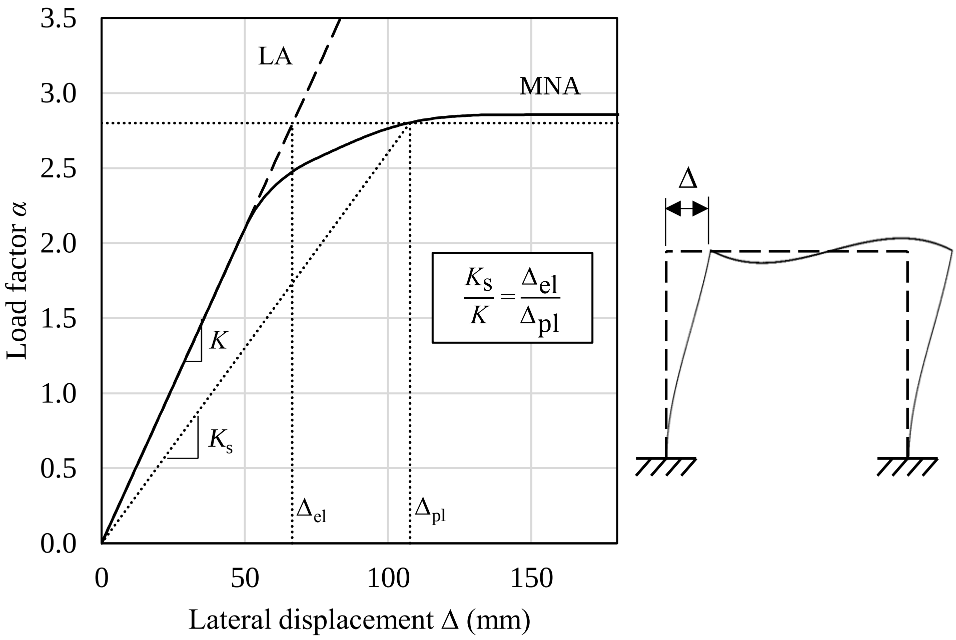


Figure 10: Typical load factor versus lateral displacement response of a frame showing reduction to sway stiffness due to material nonlinearity from first order analysis.

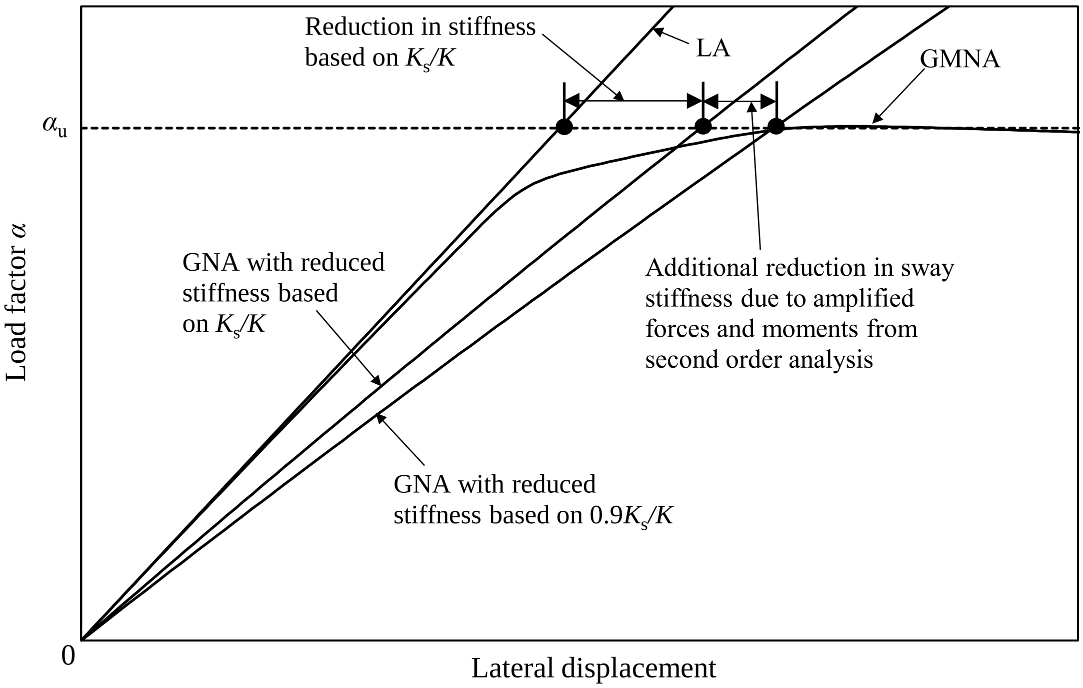


Figure 11: Load-displacement paths showing the full influence of material degradation on the sway stiffness of a typical structural frame.

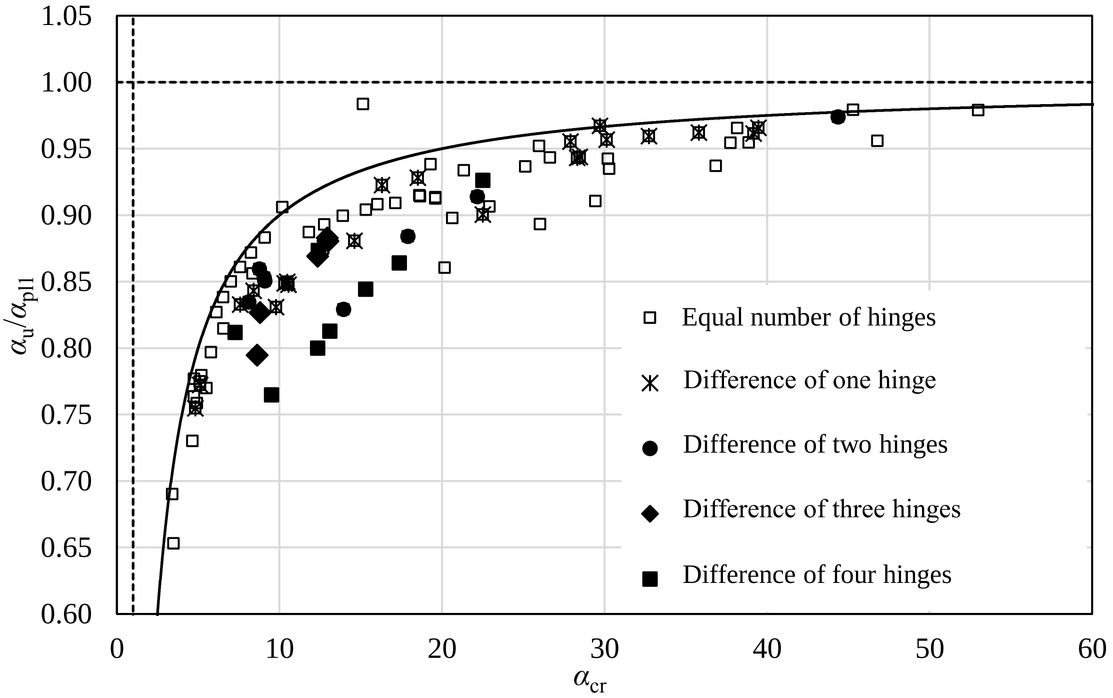


Figure 12: Increased number of hinges formed in second order plastic analysis (GMNA) to those formed in first order plastic analysis (MNA).

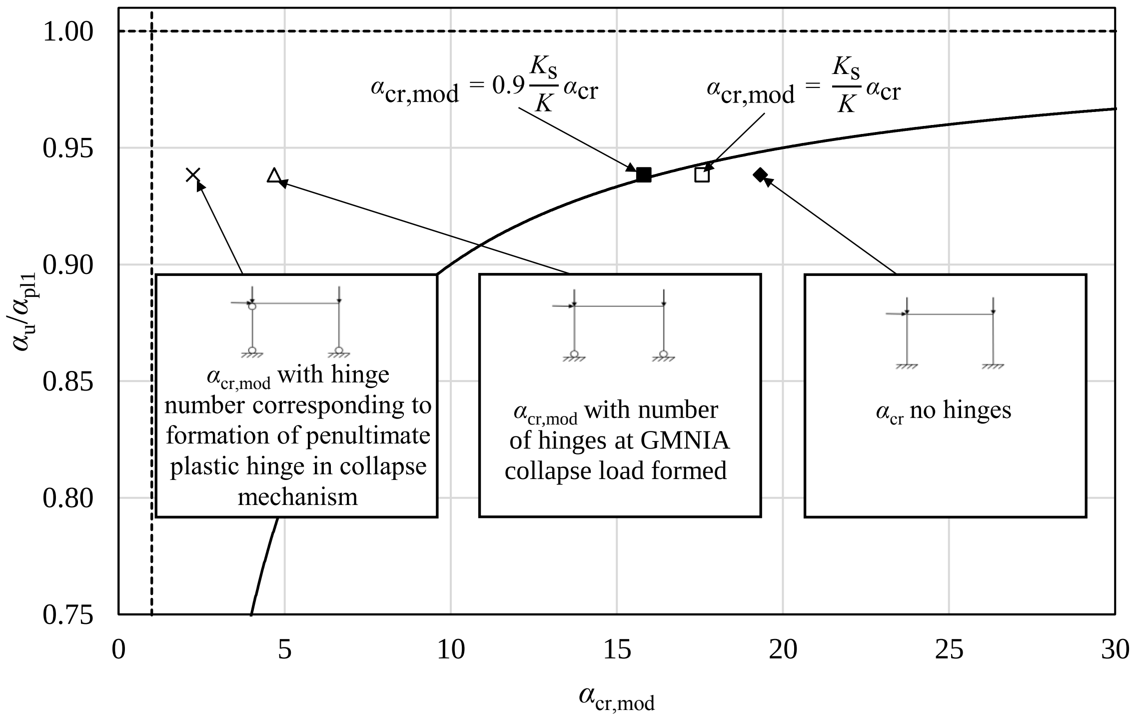
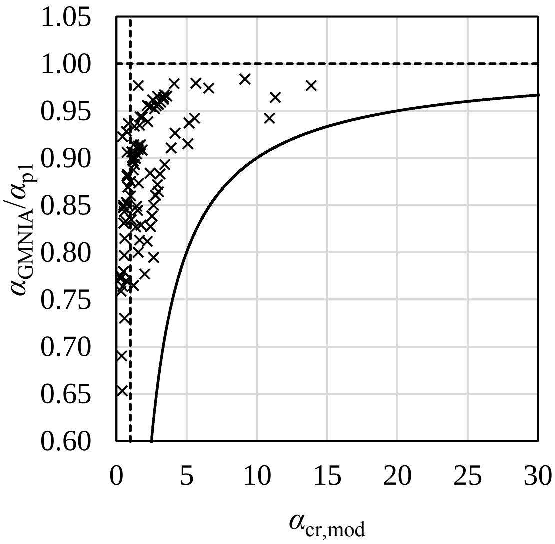
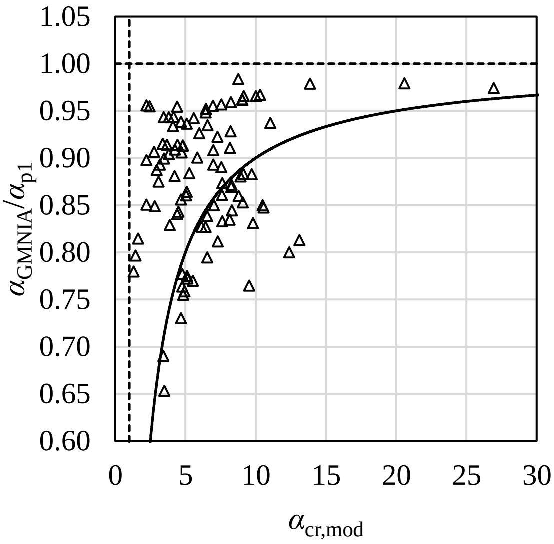


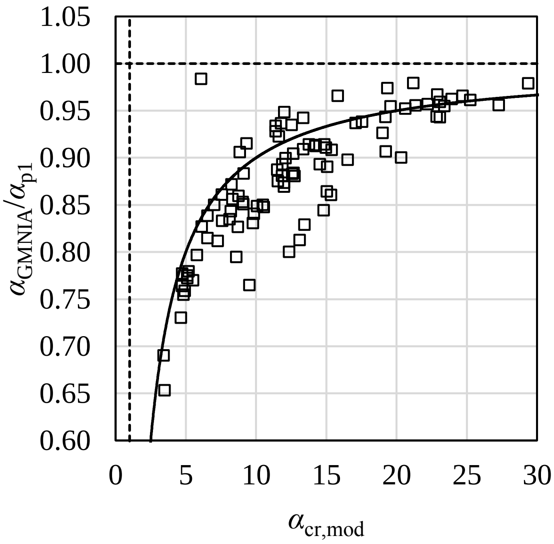
Figure 13: Comparison of the different methods of calculation of the modified critical load factor $\alpha_{cr,mod}$ for an example frame (15×10 m; H=0.5V).



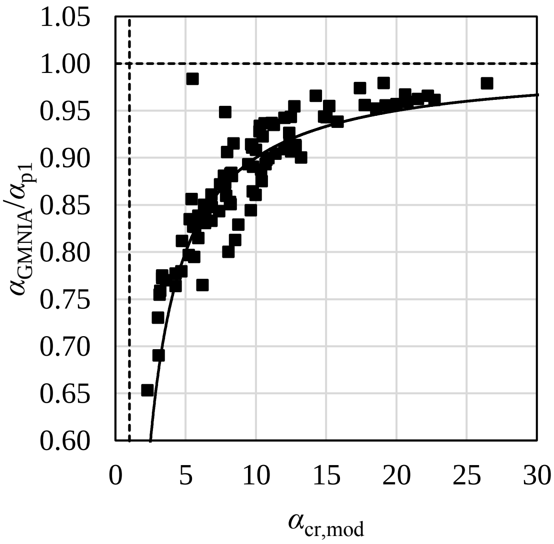
(a) Frames with plastic hinge analysis, but $\alpha_{cr,mod}$ determined from LBA with number of hinges corresesponding to penultimate plastic hinge in collapse mechanism



(b) Frames with plastic hinge analysis, but $\alpha_{cr,mod}$ determined from LBA of frames with number of hinges at GMNIA collapse load

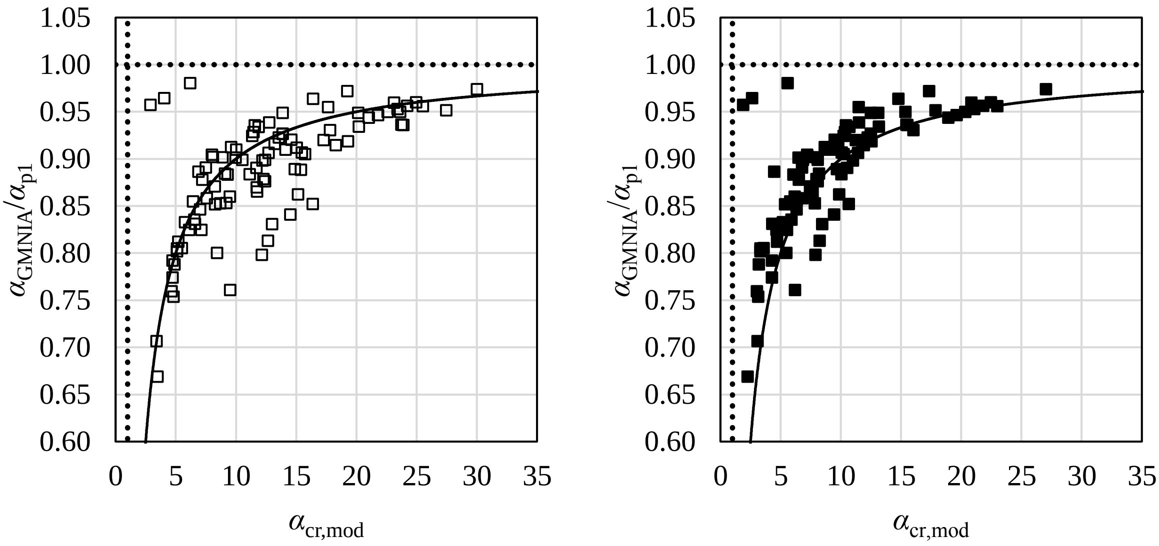


(c) Frames with plastic hinge analysis, but $\alpha_{cr,mod}$ calculated as $(K_s/K)\alpha_{cr}$



(d) Frames with plastic hinge analysis, but $\alpha_{cr,mod}$ calculated as $\Upsilon(K_s/K)\alpha_{cr}$ (Eq. (4))

Figure 14: Ratios of plastic collapse loads for the different methods of calculation of the modified critical load factor $\alpha_{cr,mod}$, accounting for influence of plasticity, from FE plastic hinge analyses.



(a) Frames with plastic zone analysis, but $\alpha_{\mathrm{cr,mod}}$ calculated as $(K_{\mathrm{s}}/K)\alpha_{\mathrm{cr}}$

(b) Frames with plastic zone analysis, but $\alpha_{\mathrm{cr,mod}}$ calculated as $\Upsilon(K_{\mathrm{s}}/K)\alpha_{\mathrm{cr}}$ (Eq. (4))

Figure 15: Ratios of plastic collapse loads for the different methods of calculation of the modified critical load factor $\alpha_{\mathrm{cr,mod}}$, accounting for influence of plasticity, from FE plastic zone analyses.

Table 1: Frame cases considered.

<table>
<tr><th>Frame case no.</th><th>No. of frames</th><th>Boundary conditions</th><th>Horizontal loading H</th><th>Storey height(s) h (m)</th><th colspan="2">Bay width(s) L (m)</th></tr>
<tr><td>1</td><td>21</td><td>Fixed</td><td>0.05V, 0.2V, 0.5V</td><td rowspan="2">5, 6, 7, 8, 9, 10, 15</td><td colspan="2" rowspan="10">10</td></tr>
<tr><td>2</td><td>7</td><td>Pinned</td><td>0.2V</td></tr>
<tr><td>3</td><td>6</td><td rowspan="12">Fixed</td><td rowspan="3">0.05V, 0.2V, 0.5V</td><td>5, 10</td></tr>
<tr><td>4</td><td>12</td><td>5, 8, 10, 15</td></tr>
<tr><td>5</td><td>3</td><td rowspan="2">5</td></tr>
<tr><td>6</td><td>3</td><td>0.05V, 0.1V, 0.2V</td></tr>
<tr><td>7</td><td>6</td><td>0.05V, 0.2V, 0.5V</td><td>5, 10</td></tr>
<tr><td>8</td><td>7</td><td>0.2V</td><td>5, 6, 7, 8, 9, 10, 15</td></tr>
<tr><td>9</td><td>3</td><td>0.1V, 0.2V, 0.41V</td><td>10</td></tr>
<tr><td>10</td><td>1</td><td>0.2V</td><td>5</td></tr>
<tr><td>11</td><td>6</td><td rowspan="2">0.05V, 0.2V, 0.5V</td><td rowspan="2">5, 10</td><td>10</td><td>5</td></tr>
<tr><td>12</td><td>6</td><td>5</td><td>10</td></tr>
<tr><td>13 (a)</td><td>1</td><td>0.13V</td><td>10</td><td colspan="2" rowspan="7">10</td></tr>
<tr><td>13 (b)</td><td>3</td><td>0.27V</td><td>5, 8, 10</td></tr>
<tr><td>14 (a)</td><td>1</td><td rowspan="2">Pinned</td><td>0.13V</td><td>10</td></tr>
<tr><td>14 (b)</td><td>2</td><td>0.27V</td><td>5, 10</td></tr>
<tr><td>15</td><td>3</td><td rowspan="3">Fixed</td><td>0.13V</td><td>5, 8, 10</td></tr>
<tr><td>16</td><td>2</td><td>0.13V, 0.27V</td><td>5</td></tr>
<tr><td>17</td><td>1</td><td>0.2V</td><td>5</td></tr>
</table>

Table 2: Comparison of design methods

Type of analysis	Approach	Analysis type: 1st order	Analysis type: Amplified 1st order or 2nd order	Analysis type: 2nd order	Method of accounting for plasticity
Elastic analysis	All	$\alpha_{\mathrm{cr}} \geq 10$	$3 \leq \alpha_{\mathrm{cr}} < 10$	$\alpha_{\mathrm{cr}} < 10$	-
Plastic analysis	EN 1993-1-1 [1]	$\alpha_{\mathrm{cr}} \geq 15$	-	$\alpha_{\mathrm{cr}} < 15$	Increased limit on α_{cr}
	prEN 1993-1-1 [15] (Wood approach)	$\alpha_{\mathrm{cr}} \geq 10$	-	$\alpha_{\mathrm{cr}} < 10$	Frame-by-frame basis – reduced critical load factor based on substituting hinges into LBA
	Proposal	$\alpha_{\mathrm{cr,mod}} \geq 10$	-	$\alpha_{\mathrm{cr,mod}} < 10$	Frame-by-frame basis – reduced critical load factor $\alpha_{\mathrm{cr,mod}}$ based on secant stiffness at design load level