Estimating stress wave velocity in granular materials: Apparent particle size dependency and appropriate excitation frequency range

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There is a lack of consensus in the literature on the influence of the median particle size on stress wave velocity in cohesionless soils. For assemblies of spherical particles with Hertzian contacts, the stress wave velocities should not depend on particle size. However, a link between particle size and stress wave velocity has been reported in laboratory experiments. In this study, to identify the reasons for the discrepancies, wave velocity measurements were performed using planar piezoelectric transducers on four different sizes of alkaline glass beads and natural silica sands. The experimental results indicate that shear and compression wave velocities are independent of the median particle size. In accordance with dispersion theory, both the experiments and discrete element simulations demonstrate that the maximum frequency that can propagate through a granular assembly (i.e., the lowpass frequency) reduces with increasing median particle size. The relationship between the lowpass frequency and the input signal frequency determines the quality of the received signal and hence the accuracy of the interpreted stress wave velocity data. To accurately estimate shear wave velocities, the selected input frequencies should match those frequencies which exhibit the largest gain factors and input frequencies should not exceed the half of lowpass frequency. To determine the compression wave velocity, it is suggested to adopt the start to start method and to choose an input frequency which is slightly lower than the lowpass frequency.

Keywords: Shear wave velocity, compression wave velocity, median particle size, planar piezoelectric transducers

Word count (main texts only): 2043

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Introduction

It is well known that the elastic wave velocity and thus the small-strain stiffness of granular materials depends on packing density and confining stress (Hardin & Richart, 1963; Lawrence, 1963 & 1965; Iwasaki & Tatsuoka, 1977; Kuwano & Jardine, 2002). The use of piezo-electric transducers to measure wave velocities is popular due to their simplicity of use (Shirley & Hampton, 1978; Dyvik & Madshus, 1985; Viggiani & Atkinson, 1995). From a theoretical perspective, the elastic wave velocities of assemblies of granular particles having Hertzian contacts should be independent of particle size ($D_{50}$). However, amongst experimental researchers there is no agreed consensus on the influence of $D_{50}$ on shear wave velocity ($V_s$) and small-strain shear modulus ($G_s$). Iwasaki & Tatsuoka (1977) reported a limited effect of $D_{50}$ on $G_s$. Sharifipour et al. (2004) stated that $V_s$ of glass beads increases with increasing $D_{50}$, whereas Bartake & Singh (2007) and Patel et al. (2008) found opposite results. Yang & Gu (2013) observed a slight reduction in $V_s$ of glass beads as $D_{50}$ increases; however, acknowledging experimental uncertainties, postulated that $V_s$ is independent of $D_{50}$. This contribution seeks to identify the source of the discrepancy between theory and experimental observations.

This contribution has used disk transducers (D Ts) to measure $V_s$ and compression wave velocities ($V_p$). The frequency domain responses were analyzed to provide recommendations on the selection of suitable input frequencies using DTs for both glass beads and silica sands having different $D_{50}$ values. Recognizing that it cannot be guaranteed in the laboratory that all other properties remain constant when the grain size changes, complementary numerical simulations using the discrete element method (DEM) were performed considering testing conditions equivalent to the laboratory tests to assess the S- wave frequency domain responses of glass beads. Compared to bender and extender elements, the DT assembly is suitable for coarse-grained particles, undisturbed and cemented specimens due to its non-invasive nature, and the DT can generate more planar waves.

Laboratory test setup

Samples of alkaline glass beads with four different $D_{50}$ values were tested ($D_{50} = 0.2, 0.5, 1, 1.8$ mm), having specific gravity ($G_s$), Young’s modulus, and Poisson’s ratio values of 2.5, 71.6 GPa, and 0.23, respectively. Measurement of the roughness (following the procedure outlined in Otsubo, 2016) of representative particles with $D_{50}$=0.2 and 1.8 mm gave root mean square roughness values $\approx$40 nm, confirming the surface roughness was size independent.

Four different sizes of silica sands were also used with similar $D_{50}$ values and their shape factors such as sphericity and convexity are listed in Table 1. All the materials were uniformly graded having a coefficient of uniformity ($U_c$) of about 1.2. Cylindrical specimens, 75 mm in diameter and 150 mm in height, were prepared by means of a split mold using a consistent approach. The material was poured into the mold in five equal layers, and side tapping was
applied equally to each layer to achieve the desired void ratio \( e \). An isotropic confinement of 100 kPa was applied pneumatically to all samples; the samples were tested in dry conditions.

The overall test setup used for performing the wave measurements is illustrated in Fig. 1. The excitation wave signals were created using a digital function generator which was then amplified by a bipolar amplifier. The amplified input signal was sent to the transmitter element of the disk transducer. The input and output signals were recorded using an oscilloscope. Two conventional methods for obtaining the travel time \( T_{\text{travel}} \): the peak to peak (ptp) and start to start (sts) methods were adopted in this study (Fig. 2).

Referring to Fig. 1(b), DTs including both P- and S-type elements were developed based on Suwal & Kuwano (2013). Each element was 20 mm in diameter and 2 mm thick. A thin coating of epoxy resin was applied to the surface of the elements to avert damage due to water and abrasion caused by angular sand grains. The elements were placed inside a metal housing and supported using silicone and epoxy resin. The metal housing was then carefully inserted inside the top cap and bottom pedestal of the triaxial apparatus. The S-type element was placed nearer to the specimen.

**DEM simulation method**

For the DEM simulations, a modified version of LAMMPS (Plimpton, 1995) was used, considering the manufacturer-provided material properties for the glass beads, and a simplified Hertz-Mindlin contact model. The top and bottom boundaries were rigid walls with the glass bead material properties and the lateral boundaries were periodic. Spherical particles were randomly generated and compressed isotropically using a servo control with an inter-particle friction coefficient \( \mu \) of 0.1. Six samples were considered having \( D_{50} \) of 0.11, 0.2, 0.3, 0.5, 1.7, and 5.0 mm prepared at a narrow range of \( e = 0.609-0.627 \) with mean coordination numbers \( (CN) = 4.97-5.18 \), and subjected to an effective isotropic pressure \( (p') \) of 100 kPa. During the wave propagation simulations, the \( \mu \) value was increased to 0.5 to suppress slippage (Magnanimo et al., 2008). No additional damping was used during the wave propagation simulation as the wave velocities were found to be independent of damping; in agreement with the DEM study by Mouraille et al. (2006). Following Otsubo et al. (2017a) and Otsubo & O'Sullivan (2018), S-waves were generated by perturbing the transmitter wall horizontally in a sinusoidal manner with a double amplitude of 5 nm. The resultant changes in shear stress on both transmitter and receiver walls were recorded to determine \( V_S \).
S-wave measurement

Time domain responses

The S-wave signals for the glass bead samples obtained from experiments are shown in Fig. 3 considering input signal frequencies \( f_{in} \) of 5 and 20 kHz. The S-wave arrival times for samples with different \( D_{50} \) values are similar. However, for \( f_{in} = 20 \) kHz and larger \( D_{50} \) particles (1.8 and 1 mm), the period of the output waveform is greater than the input signal causing a distorted output response.

Referring to Fig. 4, for the silica sands, the sample with \( D_{50} = 1.8 \) mm which had a slightly larger \( e_r \) exhibits a lower \( V_s \) when compared to other smaller \( D_{50} \) samples \( (D_{50} = 0.3 \) and 0.5 mm), while \( T_{travel} \) for the finer sands are similar (Fig. 5). The greater sphericity for the 1.8 mm silica sand grains (Table 1) and slightly larger \( e_r \) may explain the lower \( V_s \) values observed; more angular particles exhibit higher \( V_s \) values under equivalent conditions (Shin & Santamarina, 2013; Liu & Yang, 2017).

Frequency domain responses and lowpass frequency

Figure 5 presents the variation in gain factor (ratio of the fast Fourier transforms of received signal to transmitted signal, \( FFT_{out}/FFT_{in} \)) with frequency for different \( D_{50} \) values. For both experiments and simulations, the gain factor observed at a given frequency reduces with increasing \( D_{50} \), corresponding to the higher amplitude of received signals observed in the smaller \( D_{50} \) samples (Figs. 3 and 4). A reduction in gain factor will lead to a lower signal-to-noise ratio.

Granular materials behave as a lowpass filter to applied seismic/stress/acoustic waves so that the high-frequency components of the signal attenuate with distance from the excitation source and eventually become negligible (Santamarina et al., 2001; Mouraille and Luding, 2008; Otsubo et al., 2017b). The lowpass frequency \( (f_{lp}) \) is the maximum signal frequency that can be transmitted through the granular packing. Low amplitude high-frequency signals (noise) inevitably appear during the wave measurement, and so \( f_{lp} \) is estimated by considering threshold gain factor values of 5×10⁻⁶ and 0.01 for the experiments and DEM simulations, respectively. The \( f_{lp} \) values decrease with increasing \( D_{50} \). Fig. 6 highlights that \( f_{lp}/f_{lp}(D=1mm) \) varies linearly with \( 1/D_{50} \); the \( f_{lp}(D=1mm) \) values for the DEM data were obtained by interpolation.

From the theoretical dispersion relation, the relationship between \( f \) and wave number (\( \kappa \)) for a regular array of monosized spheres is given as (Brillouin, 1946; Kittel, 2004; Mouraille, 2009; Otsubo et al., 2017b):

\[
f = \frac{1}{\pi} \sqrt{\frac{c}{m}} \left| \sin \left( \frac{\kappa}{2} \right) \right|
\]  

(1)
where, \( C = \) stiffness constant between neighboring layers, \( l = \) equivalent distance between particles which is related to \( D_{50} \), and \( m = \) particle mass. Considering long wave limit, i.e. \( \kappa \to 0 \), the wave velocity is derived as:

\[
V_{\text{longwave}} = \sqrt{\frac{C}{m} \frac{l}{s}}
\]  

(2)

Comparing Eq. 1 and 2 gives:

\[
V_{\text{longwave}} = \pi l f_{lp}
\]  

(3)

For a given \( V_{\text{longwave}} \), Eq. 3 indicates that \( f_{lp} \) is inversely proportional to \( l \); this provides a fundamental basis for the linear variation between \( f_{lp} \) and \( 1/D_{50} \) as illustrated in Fig. 6. Using DEM, Otsubo et al. (2017b) confirmed that for the disordered system, the linearity still holds.

Yamashita et al. (2009) noted that \( V_s \) measurement should be independent of the method of travel time estimation; thus the relative difference \( V_{s,\text{diff}} \) \(^{(\%)} \) \( = \frac{V_{s,\text{cts}} - V_{s,\text{ptp}}}{V_{s,\text{ptp}}} \) is taken here as a measure of signal quality. Referring to Fig. 7 and Tables 2 and 3, considering the experimental data, \( V_{s,\text{diff}} \) increases with \( f_{in} \) for all the test cases and for a given \( f_{in} \), \( V_{s,\text{diff}} \) increases with \( D_{50} \). When \( f_{in} \) is less than or equal to \( f_{lp}/2 \), \(|V_{s,\text{diff}}| \leq 3 \% \). However, when \( f_{in} \) exceeds \( f_{lp}/2 \), \( V_{s,\text{diff}} \) increases significantly, particularly in case of coarser particles (\( D_{50}=1.8 \) and 1 mm). Hence, to accurately estimate \( V_s \), the \( f_{in} \) values should correspond to those frequencies which have the maximum gain factors (in order to trigger the resonant frequencies) and \( f_{in} \) should not exceed \( f_{lp}/2 \).

It is known that elastic moduli (or wave velocities) depend on CN (Makse et al., 1999; Agnolin and Roux, 2007; Magnanimo et al., 2008); however, it is difficult to quantify CN in experiments. In the current study, under a consistent specimen preparation method, a void ratio correction function \( f(e) \) as proposed by Hardin and Richart (1963) is adopted to isolate the influence of \( e \) on \( V_s \).

\[
f(e) = B - e
\]  

(4)

where, \( B = 1.28 \) for glass beads assembled in a specific way only (Otsubo & O’Sullivan, 2018) and \( B = 1.74 \) for silica sands based on \( V_s-e \) data developed in the current study.

Figure 8 portrays the variation of \( V_s/f(e) \) normalized by their average \( ([V_s/f(e)]_{\text{avg}}) \) for each material type with \( l/D_{50} \). The \( V_s \) values which have the smallest \( V_{s,\text{diff}} \) were selected. The \( V_s/f(e) \) values are consistently within 5\%, 9\% and 2\% of the average \( V_s/f(e) \) values for the glass beads, silica sands, and DEM data, respectively. There is no evidence of a consistent dependency of \( V_s \) on \( D_{50} \). The slightly higher discrepancy observed in case of silica sands may be ascribed to the influence of varying shapes or roughness, i.e. a departure of the inter-particle response from a Hertzian contact.
**P-wave measurement**

Figures 9 gives the P-wave time domain responses of glass beads from experiments. The arrival times for the P-wave signals for samples with different $D_{50}$ values are comparable. In the experiments, for the smallest particle size considered ($D_{50} = 0.2$ mm), the received signals contain high-frequency components that are not evident in the signals for the larger $D_{50}$. This is due to the triggering of a resonant vibration mode at a higher frequency (around 95 kHz) for 0.2 mm glass beads which was identified from the gain factor vs. frequency response of P-waves.

Referring to Fig. 10 and Tables 4 and 5 for P-wave propagation, $V_{p,pp} > V_{p,ts}$ at smaller $f_{in}$, while for higher $f_{in}$ ($< f_{lp}$), both methods are in good agreement. The $V_{p,ts}$ data are less sensitive to $f_{in}$ than the $V_{p,pp}$ data. Therefore, using the sts method and selecting a $f_{in}$ value slightly smaller than $f_{lp}$ for P-wave propagation is recommended; this agrees with Brignoli et al. (1996).

**Conclusions**

In this contribution, planar piez-electric transducers were used to obtain $V_s$ and $V_p$ for samples of glass beads and silica sands having different $D_{50}$. Complementary DEM simulations were also performed to assess the frequency domain responses of glass beads. For the uniformly graded materials ($U_e = 1.2$) considered here at a stress level of 100 kPa, the following conclusions can be made:

- $V_s$ and $V_p$ are insensitive to $D_{50}$; variations reported in the literature may be attributed to the dependency of the quality of the received signals obtained on the $f_{in}$ value used in the tests.
- The lowpass frequency ($f_{lp}$) reduces with increasing $D_{50}$ so that $f_{lp} \propto 1/D_{50}$.
- For $V_s$ measurements, $f_{in}$ should correspond to those frequencies which have the maximum gain factors and $f_{in}$ should not exceed $f_{lp}/2$.
- For $V_p$ measurements, adopting the start to start method and selecting a $f_{in}$ that is slightly lower than $f_{lp}$ is recommended.

**References**


Fig. 1 (a) Schematic of assembly for wave measurement (b) design of disk transducer.

Fig. 2 Assessment of travel time from time domain signals.
Fig. 3 S-wave signals for glass beads from experiments at $p' = 100$ kPa.

Fig. 4 S-wave signals for silica sand at $p' = 100$ kPa
Fig. 5 Relationship between gain factor and $f$ (a) Experiments on glass beads (b) DEM simulations (c) Experiments on silica sand.
Fig. 6 Variation of $f_{lp}/f_{lp(D=1mm)}$ with $I/D_{50}$ ($f_{lp(D=1mm)}$ are $f_{lp}$ obtained for $D_{50} = 1$ mm).

Fig. 7 Variation of relative difference between $V_{s,sts}$ and $V_{s,pp}$ with $f_{in}$ for (a) glass beads (from experiments) and (b) silica sand.
Fig. 8 Variation of $\left[\frac{V_s}{f(e)}\right]/\left[\frac{V_s}{f(e)}\right]_{avg}$ with $I/D_{50}$ for each material type ($\left[\frac{V_s}{f(e)}\right]_{avg}$ is average obtained from various $D_{50}$ when $V_s$ obtained from ptp and sts are comparable i.e. $V_{s,dig}$ is the least).
Fig. 9 P-wave signals for glass beads from experiments at $p' = 100$ kPa (a) $f_{in} = 5$ kHz (b) $f_{in} = 20$ kHz.

Fig. 10 Sensitivity of $V_p$ with $f_{in}$ for glass beads (from experiments) and silica sand at $p' = 100$ kPa.
Table 1: Properties of Silica Sand Samples (Sphericity and convexity data were obtained from QicPic analyses (Altuhaifi et al. (2013)).

<table>
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<th>$D_{50}$</th>
<th>$G_s$</th>
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<th>Convexity</th>
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<tbody>
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<td>mm</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.3</td>
<td>2.63</td>
<td>0.864</td>
<td>0.924</td>
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<tr>
<td>0.5</td>
<td>2.64</td>
<td>0.857</td>
<td>0.936</td>
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<tr>
<td>1.0</td>
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</tr>
<tr>
<td>1.8</td>
<td>2.64</td>
<td>0.885</td>
<td>0.967</td>
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Table 2: Summary of $V_s$ for different sizes of glass beads (from experiments) and input frequencies ($p’=100$ kPa, $e_o=0.616-0.617$)

<table>
<thead>
<tr>
<th>$D_{50}=$ 1.8 mm</th>
<th>$D_{50}=$ 1 mm</th>
<th>$D_{50}=$ 0.5 mm</th>
<th>$D_{50}=$ 0.2 mm</th>
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<td>$V_{s,ptp}$</td>
<td>$V_{s,sts}$</td>
<td>$V_{s,diff}$*</td>
</tr>
<tr>
<td>kHz</td>
<td>m/sec</td>
<td>%</td>
<td>m/sec</td>
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<tr>
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<td>287.4</td>
<td>10.3</td>
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<tr>
<td>30</td>
<td>262.9</td>
<td>295.3</td>
<td>12.3</td>
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* $V_{s,diff} = \frac{V_{s,sts} - V_{s,ptp}}{V_{s,ptp}}$

Table 3: Summary of $V_s$ for different sizes of silica sand and input frequencies ($p’=100$ kPa, $e_o=0.902-0.921$)

<table>
<thead>
<tr>
<th>$D_{50}=$ 1.8 mm</th>
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<th>$D_{50}=$ 0.5 mm</th>
<th>$D_{50}=$ 0.3 mm</th>
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<td>$V_{s,sts}$</td>
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<tr>
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Table 4: Summary of $V_p$ for different sizes of glass beads (from experiments) and input frequencies ($p’=100$ kPa, $e_o=0.616-0.617$)

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<tr>
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\[ V_{p,diff} = \frac{V_{p,sts} - V_{p,ptp}}{V_{p,ptp}} \]

Table 5: Summary of \( V_p \) for different sizes of silica sand and input frequencies (\( p' = 100 \text{ kPa}, e_o=0.902-0.921 \))

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<th>( D_{50} )</th>
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<th>( V_{p,sts} )</th>
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\( f_{in} \) is in (kHz), \( V_p \) is in (m/sec), and \( V_{p,diff} \) is in (%).