Evaluating Grid-interactive Electric Bus Operation and Demand Response with Load Management Tariff

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Abstract

Electric Vehicles are expected to play a vital role in the transition of smart energy systems. Lots of recent research has explored numerous underlying mechanisms to achieve the synergetic interactions in the electricity balancing process. In this paper, the grid-interactive operation of electric buses is first time integrated within a dynamic market frame using the Distribution Locational Marginal Price algorithm for load congestion management. Since the defined problem correlates the opportunity charging flexibility with the bus mobility over a network, the tempo-spatial distribution of energy needs can be reflected in the dynamic of service planning. The interactions between bus operators and suppliers are quantitatively modelled by a bi-level optimisation process to represent the electric bus service planning and electricity market clearing separately. The effectiveness of the proposed load management has been demonstrated using data collected from an integrated real-world bus network. Experiments show that engagement of electric bus charging load in demand response is helpful to alleviate the network congestion and to reduce the power loss by 7.2\% in the distribution network. However, alleviated charging loads have exhibited counter-intuitive ability for load shifting. The restricted electric bus operational requirements leads to a 8.17\% loss of charging demand, while the reliance on large batteries has increased by 10.57\%. However, the sensitivity analysis also shows that as the battery cost declines, the such discourage implications on grid-interactive electric bus operation will decrease once the battery cost below 190/kWh. The optimal grid-ebus integration have to consider the trade-off between range add-up, reduced battery cost and additional benefits.

Keywords: Electric bus, Opportunity charging, Demand response, Distribution Locational Marginal Price, Congestion load management, Bi-level optimisation

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\delta$</td>
<td>Charging power rate</td>
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<tr>
<td>$\epsilon$</td>
<td>Fare per passenger boarding</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Maximum number of bus available in bus network</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Dual variables for network equality constraints of power balance</td>
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1. Introduction

With the growing popularity of Electric Vehicles (EVs) in recent years, the electrification of transportation drew increasing interests from researchers, industry players and consumers. EVs can help to alleviate a number of contemporary issues such as the reduced carbon emission, oil dependency and noise pollution [1]. However, uncoordinated EV charging will result in excess intervention (upgrading or even complete replacement) in the distribution power system [2]. If all buses in the UK were electrified, 850 MW of power, the same as one large nuclear reactor or two coal-fired turbine [3], has to be coordinately managed. If the storage capacity of the batteries in EVs can be engaged in the demand response programme, such as for personal use [4], for buildings [5] and for aggregated application through Vehicle-to-Grid [6], there could potentially be significant benefits for both grid operators and EV owners. As such, this necessitates the importance of introducing more intelligent vehicle-grid integration.

On the top of grid-interactive engagement of EVs is the estimation of flexibility, which is achieved using transportation features [7] and guided by Demand Response (DR) scheme. DR is usually introduced
in a manner to influence the consumption pattern to follow the generation pattern to better manage the overall grid balance. A review paper [8] categorised approaches examined in the existing works into the physical-based method and the market-based demand response. Compared to direct control methods for congestion management, market-based methods provide certain market places where prices are formed and products are traded, so that the overall social welfare can be quantified and maximised. For any scenarios, the challenge of predicting short-term EV demand profile is key to successful EV applications [9], as it depends on many factors, such as vehicle owner driving profiles, routes, charging behaviour and infrastructure features. In this sense, works such as [10] give the highest priority to the driving time windows and energy demand that have to be met in the re-scheduling process of demand response. However, commercial fleets with pre-specified schedules, especially public transport buses, could greatly simplify the problem of predictable coordination. Because the planning of bus activities (such as [11]), not only the service but the vehicles usages, should endogenously consider the driving profiles, route and State-of-Capacity (SoC) characteristics and charging availability in the optimisation.

In such a context of grid-interactive electric bus operation, the main challenges consist of the estimation of electric bus load profile, both temporally and spatially, and the mechanism to effectively enable the interactions between electric bus charging and electricity supplier.

The existing studies, which have explored the inter-connection between electric buses and power systems, can be categorised into two streams, as follows:

1. Involvement of variable tariffs as costs in the scheduling of electric buses

   This is the simplest way to take the implication of energy system into consideration. A paper [12] established a single depot vehicle scheduling model with specific constraints concerning the operation and charging features of electric buses. A paper [13] optimised electric bus recharging schedules, which can determine both the planning and operational decisions while minimising total annual costs. Later, a paper [11] further solved a vehicle and charging integrated scheduling problem and provide a solution of charging infrastructure planning. However, none of these works has realised the impacts of demand response on the economic operation of buses. Only two recent papers [14] and [15] explored the advantages to life cycle cost from the point of view of earning excess incomes by giving V2G service, in which they indicated scientifically a V2G-capable electric bus could turn into a net present advantage after five years of operation. This work [16] then advances the limited literature regarding the excessive benefits of electric buses. A more beneficial approach is to build a mini scale energy storage system based on Battery Swapping Station (BSS), such that the BSS can operate "either as an energy source when the batteries are fully charged or as an electrical load when the batteries need to be charged" [17].

2. Application of EV loads for smart energy system

   The application of EVs in the smart grid has been investigated for years [6], but the efforts of integrating electric bus loads are relatively limited. The study [18] examined directed-controlled electric bus charging strategies with energy procurements in joint market operation. The bus
operation are modelled as a virtual power plant to enable the provision of power system services. For the market-based energy system, recent works (e.g. [19]) proposed a step-wise structure to determine dynamic tariff, where the energy balance components in the LMP and congestion component are calculated separately based on lossless DCOPF. A paper [20] extended the locational marginal price (LMP) concept in transmission systems to the distribution-level electricity market to efficiently manage flexible loads. However, modelling the EV aggregator by a Linear Programming (LP) formulation resulted in a multiple solution issue [21]. The study [22] pointed out that different combination of charging loads spread throughout time intervals with flat Distribution Locational Marginal Prices (DLMPs) would result in the polar solution, but the total costs from the view of aggregator are identical. The work [23] then replaced the LP with a quadratic cost formulation with a proof to guarantee the global optima. It is yet realised that heavy-duty cycle vehicles propose significant diversity in electric mobility from individuals. For instance, electric buses that intend to be used as many as they can would present different convexity as it undertakes more restrictions.

A number of research gaps about interconnection between electric buses and power system can be found in the state-of-art literature, which reduces the ability to address problems of proficient EV-grid integration. Firstly, an over-simplistic price-response assumption and the lack of distinction between different vehicle types may lead to a bias in the balancing strategy for the Distribution System Operator (DSO). Secondly, very few studies have considered the joint scheduling of electric bus operation and grid-interactive charging, which leaves a significant research gap for understanding the implications. Finally, how electric buses could interactively benefit from the demand response program is still a question.

This paper provides an in-depth insight into the synergetic interactions between electric buses and the demand response-enabled energy system by solving a Stackelberg game problem. The tempo-spatial estimation of bus charging flexibility is correlated within tactical bus operation planning problem and embedded in the upper-level problem of the bi-level framework, whereas the lower-level problem represents the distribution-level market clearing optimisation. The DSO in this case solves the social surplus maximisation problem and the bus operators solve the cost minimisation problem separately. The tempo-spatial load profile of electric bus fleets and DLMP can be iteratively updated and broadcast to electric bus charging stations at each feeder until each side cannot improve the performance by changing the decision influencing the other. In this way, both the operation decisions and demand response for the smart energy system can be quantitatively analysed.

The paper is organised as follows. Section 2.1 depicts the overall interaction between two participants. The formulation of bus operation planning in the tactical phase is given in Section 2.2 and the marketing clearing model in Section 2.3 with an iterative approximation of power losses are elaborated separately. Section 2.4 presents the derivation and decomposition of DLMP and Section 2.5 presents the overall implementation process of producing the multi-period Mathematical Program with Complementary Constraints (MPCC) conditions. A case study in Section 3 using real passenger data is employed to illustrate the effects of congestion load management through DLMP as well as the demand response.
along the day. Finally, the sensitivity is conducted with respect to the battery costs and unit vehicle running costs are carried out in Section 4.

2. Definition And Formulation Of Problems

2.1. General framework of interactions and assumptions

In the proposed bi-level framework, operators can balance the supply of electricity imported from the bulk grid and demand of electricity through solving an optimal power flow problem w.r.t network constraints. Parts of flexible loads consist of electric buses charging loads spreading through load points \( N \) in the city. Buses travelling throughout the network \( \{L, S\} \) are subject to provide transit service for passengers \( B_{i,j,l} \) in this area. After obtaining the optimal unit dispatch in the distribution network, the DLMP and its decomposition are calculated based on the first-order partial derivative of the Lagrangian function and broadcasted to the bus operator. The latter solves its own planning problem and submits the bids with demand profile into the day-head market. Fig 1 illustrates the structure of the proposed market and the interactions between two networks.

Figure 1: Proposed trading framework between bus operators and distribution network operators

In order to capture the dynamic in the temporal and spatial characteristic of aggregated energy demand, the tactic planning of bus operation is introduced in the upper-level. The estimation of energy demand has to consider route-specific and trip-dependent energy consumption and the charging characteristics. Output of the operational optimisation assigns numerical electric buses to each route and
constitutes the timetable that can be used to estimate the duration of opportunity charging. The overall cost is thus determined by the total service length and charging decision. In the lower-level, the optimisation task corresponds to the DSO’s market clearing process. The security constrained economic dispatch problem (SCED) for DSOs is modelled as the form of Direct-Current Optimal Power Flow (DCOPF) problem where the objective is meeting the load in the power system while maximising social surplus.

To sum up, the following assumptions are made to facilitate the decision-making of both electric bus operator and DSO:

1. As described above, bus operators are capable to retain sufficient capacity to participate in the day-ahead market and adjust the charging loads through the dwell time in response to the DLMP rate.
2. For simplicity, the road condition is assumed freeflow without any congestion, i.e. the arrival time can be estimated after departure.
3. Buses are assumed running bi-directional in each bus line, and only able to recharge when they are arriving at the stations at the end of the route.
4. It is assumed that the dwell time between departure and arrival time (referred as opportunity charging) are used for opportunity recharging.
5. The arrivals of passengers follow a Possion process and elastic to the service quality (wait time).
6. The voltage drop along the branch in distribution is close to zero since the distance between two adjacent nodes is normally very short.

2.2. Modelling electric bus operation

Most works in transportation studies set the optimal bus frequencies as an exercise of tactical operation planning that balance the passenger demand with the available supply of buses [24]. In this paper, the classic bus operational planning problem is extended by correlating the charging cost with the operational decision as an output of the frequency setting problem. The objective function is composed by four components: (1) operational cost relating to vehicle travel distance; (2) revenue collected from passengers that is sensitive to the service quality; (3) charging costs that relate to the lay-over time between arrival and departure time; and (4) the opportunity cost of not charging which is associated to the daily battery usage cost per kWh.

Consider a bus network \( \{L,S\} \), each with a number of stops \( s \) and two stations \( s' \) (Origin and Terminal). A set of variables \( f \) depict the bus dispatch scheduling in each line. Given that the arrivals of passengers follow a Possion process, the headway is the halved reverse of frequency: \( h_l = 1/2f_l \). For a bus route that has a round trip time of \( T \), the total required number of buses to that route as: \( T_lf_l \). In this way, we have to ensure that the number of buses allocated to that route is an integer number. However, this is a rare case and we need to round upwards the number of buses \( NB_l = [T_lf_l] \). Hence, the vehicle running cost can be written as Equation 1:

\[
Cost_{bus}(f) = \sum_{t \in L} \rho_t \theta[T_lf_l]T_{i,t}
\]
where \( \rho_o \) and \( \theta \) are the unit vehicle-running cost and average bus travelling speed respectively.

In general, the patronage change as a result of key service elements change (e.g. headway, fares). An increase in waiting time that associated with the headway leads to a decrease in patronage and vice versa. If assuming the average fare is constant, the revenue is then inversely related to the headway. Elasticity is here defined as the expression of such a relationship. In the simplest form the value is the ratio of the proportional change in patronage to the proportional change in waiting time. In order to retain the convexity in the demand function while restoring the sensitivity of passengers to the bus service quality, a linear elastic demand function is introduced as many other studies did [25].

However, one should note that elasticity are dynamic, varying over modes, time over which it is being examined and specific circumstances. The work [26] and the practical guide for bus planning [27] suggested an average value of elasticity \( e \) should be -0.64 for long-term operations, and Furth’s work [24] suggested a differentiated value for the off-peak and peak period since demand elasticity with respect to waiting time is higher during off-peak periods. By assuming the value during the peak period to be -0.2 and off-peak to be -0.64, i.e. we have a 0.2/0.64% decrease in passengers’ demand if 1% waiting time increases. The following equations 2 and 3 show that the expected passengers’ demand and revenues by routes will increase because they are sensitive to the service improvements:

\[
\Delta B_l(h_l) = e \frac{0.5 \ast h_l - w_b}{w_b} \tag{2}
\]

\[
B_l(f_l) = B^h_l \ast \left( \frac{e}{2w_b \ast f_l} + 1 - e \right) \tag{3}
\]

As afore-mentioned, the revenue from passengers is equal to the fare times the total patronage, the negative sign can unify the income as the same type of cost in Equation 4:

\[
Cost_{pass} = -\epsilon \ast B^h_l \ast \left( \frac{e}{2w_b \ast f_l} + 1 - e \right) \tag{4}
\]

where \( w_b \) and \( \epsilon \) represent the benchmark of wait time that passengers are willing to accept and fare per passenger respectively. \( B^h_l \) is the historical counts of passenger boarding in the whole route.

For a bus route with a round trip time \( T \) and bus dispatching frequency of \( f \), buses will be scheduled to leave every \( 1/f \) minutes, i.e. from the start of planning period \( t = t_0 \), the timetable for each bus \( n \in NB \) to leave is given in Equation 5:

\[
t_0 + \frac{n}{f}, \forall n = 0, 1, 2, 3...NB \tag{5}
\]

The bus that has completed its journey will arrive at the origin station by Equation 6

\[
t_{arr} = t_0 + T + \frac{n}{f} + \xi, \forall n = 0, 1, 2, 3...NB \tag{6}
\]

where \( \xi \) represents any delays caused by passenger boardings/alightings and other traffic conditions. One should note that the arrival time often subjects to some factors, such as the road condition and the total boarding/alighting time affected by the passengers. Electric bus operators have to provide extra buses in order to maintain the original timetabling [28]. Since there are many factors affecting \( \xi \), it was relaxed
by adding one more bus in the estimation of layover between arrive time and the next departure time.

This paper only considers the planning in the tactical phase with the assumption of an uncongested road network, and the total boarding and alighting times are neglected in the modelling. The total request number of buses in the approximation of charging duration becomes \( N = \lceil T_l f_l + 1 \rceil \) and thereby yielding the timetables for the next round of dispatch is given by Equation 7:

\[
t_{\text{next}} = t_o + \frac{N + n}{f}, \quad \forall n = 0, 1, 2, 3...N
\]  

(7)

A second rule is introduced that if the layover time \( I_l(f) \) is less than 3 minutes, operators will not use this short period to recharge buses. Hence, the layover which represents as the time interval between the first departure and the next departure is:

\[
I_l(f) = \lceil T_l f_l + 1 \rceil f_l - T_l
\]  

(8a)

\[
ct_l(f) = \begin{cases} 
0 & \text{if } I_l(f) < 0.05 \text{ hour} \\
I_l(f) & \text{if } I_l(f) \geq 0.05 \text{ hour}
\end{cases}
\]  

(8b)

The above function is piece-wise non-linear and hence undifferentiated at the point; another drawback of the above equation is that the output will fluctuate along with the value in several intervals of inputs and will drop to 1 when \( T_l f_l \) reaches an integer. In order to take into consideration of the case that more precise data of arriving time to be integrated into the model, we introduce a least square method to approximate the mapping of bus dispatching frequency to the maximum charging time for each bus route, while preserving the convexity of both the function of charging time and charging loads. The new charging time function is expressed in Equation 9:

\[
ct'_l(f_l) = p_1 f_l + p_2
\]  

(9)

Figure 2 illustrates the characteristics of the new charging time function of a bus routewith the outbound and inbound length of 6.68 miles (10.75 km) and 6.32 miles (10.17 km) respectively. In this figure, the approximated response function with respect to \( p_1 \) and \( p_2 \) is shown as the yellow curve, which is convex.

By introducing the second set of decision variable for controlling the charging process, \( x_{s'} \) represents the percentage of charging in the lay-over time which can be achieved by either shortening the duration connected to the electric grid or adjusting the charging rates. The load submitted to the DSO and the charging cost for bus operators can be expressed as:

\[
L_{t,s'}(f, x) = \delta * x_{t,s'} * ct'_l(f_l)
\]

\[
\text{cost charging} = \sigma_{t,n} * \delta * x_{t,s'} * ct'_l(f_l)
\]

(10)

where the \( \delta \) is the charging power rate and \( \sigma_{t,n} \) represent the dynamic tariff rate at time \( t \) at station \( n \).

The opportunity charging, although requiring extremely high power output, would reduce the interruption to the least level as well as achieve valuable cost-savings on the battery packs. Theoretically
speaking, if the charging power is extremely high enough that could complement the energy consumed in one service trip in a short time, the required battery capacity should equal to the energy consumption along this trip. Contrasting to the case that bus operator schedules all the charging events to the overnight and requires a large battery, the savings on battery investment - which indeed constitutes over 40% of the total capital cost of the electric bus - is achieved via the trade-off between intermittent recharging and an opportunity cost of requiring incremental battery capacity. In addition, as unveiled in the work [29], the inter-temporal discontinuity of SoC which has the non-linear feature in the equality and inequality constraints would make the problem non-convex.

Instead, a new definition is introduced in this section. The difference between the add-up energy per flash charge and the energy consumed on roads would require the same amount of battery capacity to support so that this difference can be charged in the next plan period or overnight. This difference accumulated along time is defined as the battery capacity deficit:

$$\text{Deficit}_{\text{battery}} = \theta \ast \varrho \ast T_i \ast f_l - L_i(f_l, x)$$ (11)

Hence, the opportunity cost of not using flash charging is defined as the levelised ownership cost of battery per unit in days times the battery capacity deficit. In this paper, the battery is conservatively assumed with a five year lifetime and a zero second-hand value after degradation, i.e.:

$$\text{Cost}_{\text{battery}}(f_l, x) = \rho_b \ast (\theta \ast \varrho \ast T_i \ast f_l - L_i(f_l, x))$$ (12)

where $\varrho$ is the average energy consumption rate per km.

Given the above notations and assumptions, the final expression used by bus operators in the plan period can be expressed by combining Equation 1, 4, 10 and 12:

$$\varphi_{\text{bus}}(f) = \sum_{l \in L} \rho_o \beta T_i [T_i f_l] - \epsilon B_i^h \left( \frac{e}{2 w_b \ast f_l} + 1 - \epsilon \right) + \sigma_{t, \lambda} \delta x_{l,s} c t_i' (f_l) + \rho_b (\theta \ast \varrho \ast T_i \ast f_l - L_i(f_l, x))$$ (13a)
where $\Gamma$ is the maximum number of available buses in total and $f_{min}$ is the regulated bus service level. For simplicity, bus speed $\theta$, fare $\epsilon$ and maximum charging rate $\delta$ are assumed constant for each ride.

Bus operators commonly have to conform to a set of bus operating constraints, which are given the highest priority to meet in the optimisation. First, the number of buses allocated to the bus routes should not exceed the total available number of vehicles. Equation 13b, and constraint Equation 13c guarantee the sufficient transit capacity for the passenger demand. In addition, there can be minimum and maximum requirements for the service frequency due to the regulation of service.

2.3. Electricity Market Clearing

Withstanding the findings in the first section, DCOPF is still the most commonly used methodology for LMP due to its high computation efficiency and good accuracy. ACOPF is much slower in computation and requires special care in input data to make it converge [30]. Considering the large number of nodes in the distribution network and the loss caused by low voltage, we employ DCOPF with an iterative process of evaluating power losses and Fictitious Nodal Demand (FND) (see [30, 31]) to improve the accuracy while keeping the convexity of the whole problem traceable.

First, for each electricity market settlement $t \in pp$ (for simplicity, $t$ is avoided in the following formulation but one should note that this is a multi-period optimisation problem), the optimal generation and demand dispatch are achieved by optimising social cost function including the cost of purchasing active power from the wholesale energy market. It is assumed that total active power imported must satisfy the inflexible demand plus the electric load submitted by the bus operator. The voltage magnitudes are assumed to be unity and reactive power is ignored. For a distribution network $\{N, C\}$, the distribution-level day ahead lossless DCOPF problem that is constrained by a nodal balance 14b and power flow limits in branch 14c is formulated as follows.

$$\min \sum_{n \in N} \rho_n^f \times P_n$$

s.t.

$$\sum_{n \in N} (P_n - D_n) - \sum_{s \in S} L_{t,s}(f, x) = 0$$

$$\sum_{n \in N} SF_{c,n}(P_n - D_n - M_{s-n} \times L_{t,s}(f, x)) \leq PF_{c}^{max}, \forall c \in C$$

$$\sum_{n \in N} SF_{c,n}(P_n - D_n - M_{s-n} \times L_{t,s}(f, x)) \leq -PF_{c}^{max}, \forall c \in C$$
average system loss, which results in a mismatch that absorbed in the slack busbar only. Therefore, fictitious nodal demand is introduced to distribute the total system loss to each line. By applying the FND approach, the power loss is equally halved and attached to both connector of power lines, which can be written as:

$$E_{\text{FND}}^n = \sum_{c=1}^{M_n} \frac{1}{2} R_k (PF_c^*)^2$$

where $E_{\text{FND}}^n$ is applied at each end of the individual line to avoid the situation that most mismatch of power loss are absorbed in the slack busbar. With * indicates constant value of power flow obtained from previous iteration of sub-optimisation of Equation 14. Now the line flow can be viewed as the aggregation of the contribution from all power sources minus the power consumed by the FND, yielding:

$$PF_c = \sum_{n=1}^{N} SF_{c-n} (P_n - D_n - E_{\text{FND}}^n)$$

The Loss Factor is then defined as the sensitivity of system loss to the change of net injection at feeder; and the deliver factor can be derived by the FND algorithm:

$$P_{\text{loss}} = \sum_{c \in C} PF_c^2 R_C$$

$$LF_n = \frac{\partial P_{\text{loss}}}{\partial L_n} = \sum_{c=1}^{C} 2R_c \cdot PF_c \frac{\partial PF_c}{\partial P_n} = \sum_{c=1}^{C} 2R_c \cdot SF_{c-n} (P_n - D_n - E_{\text{FND}}^n) \left( \sum_{n=1}^{N} SF_{c-n} \right)$$

$$DF_n = 1 - LF_n$$

Experiments in reference [30] show that the power loss in FND algorithm quickly converges in 45 iterations for the PJM 5-bus case within much high accuracy. The new iterative DCOPF formulation, which replaces the Equation 14 can be expressed as:

$$\min \sum_{n \in N} \rho_n^g \cdot P_n$$

subject to:

$$\sum_{n \in N} DF_n P_n - \sum_{n \in N} DF_n (D_n + M_{s-n} \times L_{l,s}(f,x)) + P_{\text{loss}} = 0, \lambda$$

$$\sum_{n \in N} SF_{c-n} (P_n - D_n - E_{\text{FND}}^n - M_{s-n} \times L_{l,s}(f,x)) \leq PF_{\text{max}}^c, \mu_c^+, \forall c \in C$$

$$\sum_{n \in N} SF_{c-n} (P_n - D_n - E_{\text{FND}}^n - M_{s-n} \times L_{l,s}(f,x)) \leq -PF_{\text{max}}^c, \mu_c^-, \forall c \in C$$

where $\lambda, \mu^+$ and $\mu^-$ are dual variables for network constraints. Note the value of $P_{\text{loss}}$ is positive and used to offset the doubled system loss caused in the calculation of loss factor, such proof can be found in [30] as well.

### 2.4. DLMP calculation and decomposition

The LMP is defined as the sensitivity of total marginal cost (Lagrangian function) with respect to the change of demand at each busbar/load point. Therefore, after obtaining the optimal solution of DCOPF, the LMP at any point can be calculated with the Lagrangian function, shown as:

$$\min \sum_{n \in N} \rho_n^g \cdot P_n$$

subject to:

$$\sum_{n \in N} DF_n P_n - \sum_{n \in N} DF_n (D_n + M_{s-n} \times L_{l,s}(f,x)) + P_{\text{loss}} = 0, \lambda$$

$$\sum_{n \in N} SF_{c-n} (P_n - D_n - E_{\text{FND}}^n - M_{s-n} \times L_{l,s}(f,x)) \leq PF_{\text{max}}^c, \mu_c^+, \forall c \in C$$

$$\sum_{n \in N} SF_{c-n} (P_n - D_n - E_{\text{FND}}^n - M_{s-n} \times L_{l,s}(f,x)) \leq -PF_{\text{max}}^c, \mu_c^-, \forall c \in C$$

$$\sum_{n \in N} SF_{c-n} (P_n - D_n - E_{\text{FND}}^n - M_{s-n} \times L_{l,s}(f,x)) \leq -PF_{\text{max}}^c, \mu_c^-, \forall c \in C$$
\[ \Psi = \sum_{n \in N} \rho_n^g \cdot P_n - \lambda \left( \sum_{n \in N} DF_n P_n - \sum_{n \in N} DF_n (D_n + M_{s-n} \times L_{l,s}(f,x)) + P_{\text{loss}} \right) \]

\[ - \sum_{c \in C} \mu_c^+ \left( \sum_{n \in N} S_F c-n (P_n - D_n - E^{FND}_n - M_{s-n} \times L_{l,s}(f,x)) - PF_{c}^{\max} \right) \]

\[ - \sum_{c \in C} \mu_c^- \left( \sum_{n \in N} S_F c-n (P_n - D_n - E^{FND}_n - M_{s-n} \times L_{l,s}(f,x)) + PF_{c}^{\max} \right) \]

Equation 20

The DLMP is the first-order partial derivative of the Lagrangian w.r.t. the active load demand, yield the price at the load point \( n \) as:

\[ \sigma_n = \frac{\partial \Psi}{\partial L_n} = \lambda \cdot DF_n + \sum_{c \in C} (\mu_c^- - \mu_c^+) \times S_F c-n \]

Equation 21

In Equation 21, the DLMP can be decomposed into energy price, loss price and congestion price. Incentivised by the price signals of network congestion, the electric bus operator can adjust their own operation plans and charging activities, which will support system operation. In the short-term, this proposed DLMP is effective in congestion alleviation.

2.5. MPCC conversion and implementation process

The work [32] proved that the bi-level problem is NP-Hard. The original bi-level problem is solved after converting to a Mathematical Program with Complementary Constraints (MPCC) when the Lagrange multipliers are introduced, and followed by simplification that replace the lower level problem with its equivalent Karush-Kuhn-Tucker (KKT) optimality conditions. Since for any \( \bar{y} \in \mathbb{R} \), KKT condition is the necessary step to test if it is one of the optimal solutions. Therefore, the bi-level problem can be transformed to Equation 22:
\[
\varphi_{\text{bus}}(f) = \sum_{l \in L} \rho_{\delta} T_l \left[ f_l \right] - \epsilon B_l^b \left( \frac{e}{2w_b * f_l} + 1 - e \right) + \sigma_{l,n} \delta x_{l,s'} c'_l(f_l) \\
+ \rho_0 (\theta * g * T_l * f_l - L_l(f_l, x))
\]

subject to:

\[
\sum_{l \in L} \left[ T_l f_l \right] \leq \Gamma \\
B_l(f_l) \leq \sum_{l \in L} \tau f_l, \forall s \in S, l \in L \\
B_l(f_l) = B_l^b \left( \frac{e}{2w_b * f_l} + 1 - e \right) \\
\sigma_{l,n} = \lambda + \alpha \cdot (DF_{t,n} - 1) + \sum_{c \in C} \mu^+_{t,c} \times SF_{c,n} \\
\rho_{t,n}^g + \lambda * DF_{t,n} + \sum_{c \in C} \mu^+_{t,c} \times SF_{c,n} = 0, \forall n \in N
\]

(22)

\[
\lambda_t \cdot \left( \sum_{n \in N} DF_{t,n} P_{t,n} - \sum_{n \in N} DF_{t,n} (D_{t,n} + \sum_{l \in L} L_{l,n}(f_l, x)) + P_{\text{loss,t}} \right) = 0 \\
\mu^+_{t,c} \cdot (PF_{c}^{\text{max}} - \sum_{n \in N} SF_{c,n}(P_{t,n} - D_{t,n} - E_{t,n}^{\text{FND}} - \sum_{l \in L} L_{l,n}(f_l, x))) = 0, \forall c \in C \\
\mu^-_{t,c} \cdot (PF_{c}^{\text{max}} - \sum_{n \in N} SF_{c,n}(P_{t,n} - D_{t,n} - E_{t,n}^{\text{FND}} - \sum_{l \in L} L_{l,n}(f_l, x))) = 0, \forall c \in C \\
\sum_{n \in N} DF_{t,n} P_{t,n} - \sum_{n \in N} DF_{t,n} (D_{t,n} + \sum_{l \in L} L_{l,n}(f_l, x)) + P_{\text{loss,t}} = 0 \\
\sum_{n \in N} SF_{c,n}(P_{t,n} - D_{t,n} - E_{t,n}^{\text{FND}} - \sum_{l \in L} L_{l,n}(f_l, x)) \leq PF_{c}^{\text{max}} \\
\forall f \in Z : f_{\text{min}}, \forall x \in (0, 1), \lambda > 0, \mu^-, \mu^+ \geq 0, \forall t \in pp
\]

In the multi-period optimisation framework, since the resolution of lossy DCOPF of one hour is not equal to the resolution of bus planning, a nested conversion of the DCOPF problem to the MPCCs has to be embedded in broader planning horizons of electric bus operation. This imperatively increases the computational burdens to satisfy all the network constraints. Searching for the optimal solution set that includes the \((f, x)\) in the upper level problem and the \(P\) in the lower level problem as well as an additional set of variables of the dual variables \((\lambda, \mu)\) associated with the power system constraints would result in an effort to gradually minimise the electric bus operational cost and hopefully satisfying the relaxed power system constraints.

If the power system constraints are satisfied, the dual variable along with the latest bus dispatch and generation responses constitutes a market equilibrium according to the relevant definition, since each party maximises/minimises their economic surplus/cost and system constraints are satisfied. The fact that this solution minimises the social cost according to the properties of the duality theory is justified by the economic theory. The implementation process is summarised and shown in Figure 3.
3. Case Studies

To test the applicability of proposed bi-level grid-interactive model, and gain insights into real-world implementation, the optimization model was applied to a virtual joint transit and energy network with partly real data, as shown in Figure 4. This is because of the lack of real-world bilateral trade market.

3.1. Network description and data preparation

The virtual network is built based on a combination of selected 8 bus routes network in Shenzhen, China, and the Busbar 4 distribution network of the Roy Billinton Test System (RBTS) [33, 34]. The charging demand in the 16 bus stations were aggregated by their Euclidean distance and constituted three ebus loads. It is assumed that those aggregated bus charging demands are respectively supplied by the three supply points in the RBTS. The selected 8 bus routes comprise 304 bus stops and 14 bus...
stations. Among those bus routes, 3 west-to-east and 1 north-to-east long-distance bus routes take over 5 hours to complete a round-trip journey; 4 medium-distance routes take less than 3 hours to complete the service. To facilitate the understanding of bus movement, the direction of buses from origin to terminal is defined as “Outgoing”, and bus returning from the terminal is defined as the direction of ”Return”. The passenger patterns and bus routes extracted from smart cards and General Transit Feed Specification (GTFS) data are presented in Figure 5, where it can be seen that the peak service time occurs between 6:00 am to 10:00 am in the morning and 5:00 pm to 9:00 pm in the afternoon.

![Figure 5: Examples of passenger demand variations along a typical day in two directions (a) direction of outgoing; (b) direction of return](image)

In the distribution network, 38 groups of consumers are connected to the load point via transformers, and received power from 7 feeders (11 kV) that connected to 3 supply points at medium voltage level (33 kV). The feeders are operated as radial feeders but connected as a mesh through normal open sectionalising. The impedance per km length is assumed uniform for all connectors on a 100 MVA p.u. base, thus yielding the impedance by its length, voltage rate and type of use. The single line diagram of the distribution system is shown in Figure 4 (a) and the parameters of connecting power lines including the peak/average residential and commercial loads can be found in [33, 34]. The coupled inflexible demand varying along the day would results in different network conditions and the data can be found in the IEEE Reliability Test System [35].

The scope of the case studies in this paper is limited to the electric mobility, loads and locational price in the distribution network only. As presented in Figure 4 (b), major parts of original service cover a compact geographical area that enables an evident clustering of bus stations. From the views of DSOs, the flexible demands of electric bus fleets examined in the presented case studies spread over the 3 feeders with differentiated amount of stations (charging facilities). The allocation of aggregated chargers in each station was carried out through the mapping presented in Table 1, in which $O$ represents the originating station of the ongoing direction and $T$ represents the originating station of the opposite direction.

The rest parameters are summarised in Table 2. Miles et.al. [36] suggested an average rate of energy consumption should be within the range from 1.6 to 2.14 kWh/km for practice in the UK’s case. On a similar consideration of the weather condition in this case study, this rate is set to 1.25 kWh/km. The work [28] also provided a valuable reference for the assumption of flash chargers, in which two inductive
charging systems are installed with a grid-to-vehicle efficiency 80%. Based on this demonstration project, the charger power is set to 140 kWh and the grid-to-vehicle charging rate is 112 kWh. In terms of battery cost, McKinsey [37] forecasts that the current projections will put EV battery pack prices below $190/kWh by the end of the decade. With an assumption of a 5 year lifetime of battery, this paper uses the present-day battery cost of 300 $/kWh, i.e. 0.16 £/kWh and a value of 0.08 £/kWh by the end of the decade is used in the sensitivity analysis. Lastly, since the fuel accounts for 9% of the total running cost and 27% of all non-labour costs respectively [38], the $\rho_b$ (variable cost per km associated with the total vehicle travel kms for service) is estimated as £2.3 per km based on the fact that each bus travels approximately 64000 kms with fuel consumption of 5 mpg.

### 3.2. Effectiveness of demand response through DLMP

First of all, two scenarios were built: one provides the baseline result and the other tests the performance of the proposed market mechanism in a circumstances of load congestion in the 4 feeder. The detailed description of the two scenarios are given as below:

a. **Bus operation is optimised irrespect of the energy network congestion.** The optimal objective was £85686.8, which was made up of the charging cost of £3066.7. The total charging demand over 18 hours is 46687.6 kWh and 51.52 MW load spreading in three feeders. As depicted in Figure 7a, a great proportion of loads were distributed in periods between 13:00 to 18:00 and 21:00 to 8:00 and caused a significant overloading in the connected feeder, yielding a high power loss rate of 12.4%. As a result, a
price spike was generated to penalise the load. At peak hours of 21:00 - 22:00 in feeder 4, the charging load from electric bus resulted in the 40% additional loads on the basis of inflexible demand, which took a proportion of 28.67% of the total loading.

b. With the bi-level model, the bus operation is optimised to minimise net operational cost while facilitate the relief network congestion by responding to DLMPs. The optimal objective was £85462.24, which consisted of the charging cost of £2273.04. The total charging demand over 18 hours is 42873.2 kWh and 43.8 MW load spreading in three feeders. The electric loading and balanced price are shown in Figure 7c and 7d, from which one can observe the alleviation of energy network congestion. The variety of electricity price along time and locations can be accounted for the unevenly distributed power loss in the whole distribution network. The power loss is approximately 5.2% of the total loading, which is much less than the other case.

Table 3: Difference of Net Cost, Charging Cost, Demand and Load between scenario with/without DR mechanism

<table>
<thead>
<tr>
<th></th>
<th>Net Cost (£)</th>
<th>Charging Cost (£)</th>
<th>Demand (kWh)</th>
<th>Net Load (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario A: Without DR</td>
<td>85686.80</td>
<td>3066.7</td>
<td>46687.6</td>
<td>51.52</td>
</tr>
<tr>
<td>Scenario A: With DR</td>
<td>85462.24</td>
<td>2273.0</td>
<td>42873.2</td>
<td>43.8</td>
</tr>
<tr>
<td>Change</td>
<td>0.26%</td>
<td>25.88%</td>
<td>8.17%</td>
<td>14.98%</td>
</tr>
</tbody>
</table>

The effectiveness of DLMP for congestion management is demonstrated by comparing the line loading and DLMPs in two scenarios, as shown in Figure 7. From a combined viewing of the tempo-spatial distribution of DLMP and the loading shown in Figure 7c and 7d, highly priced DLMP during network congestion discourages the bus charging in the congested feeder until the total loading declined below the limit. By enabling the interaction between consumers and supplier, the total energy system operates below the network constraints. Meanwhile, the new spike of DLMP that occurs at 10:00 am is approximately 30% lower than the case of uncoordinated charging, which is £136.18/MWh between 13:00 pm to 16:00 pm.

Nevertheless, a counter-intuitive observation is the difficulty of load shifting for electric buses, which distinguished from the acknowledged purpose to encourage EVs’ participation in demand response. In the case of electric bus, the proposed DLMP results in a drop of 8.17% charging demand that cannot be shifted to other time horizons. The interrupted charging undertake a restricted flexibility to restore, as electric buses have the priority to conform the generated schedules. Therefore, the interrupted energy demand can only be replenished during the overnight charge.

3.3. Impacts on electric bus operations

The concept of battery capacity deficit defined in Section 2.2 could provide an indicator revealing the battery capacity required to sustain the full day operation. The peak value of battery capacity deficit in the whole day can be understood as the least level of battery capacity that bus operators have to consider. Figure 6 deficits the variation of battery capacity deficit in Scenario a. The maximum level of
average battery capacity deficit is 67.1 kWh and if an awareness level of 20% is considered, a minimum of 83.88 kWh battery should be considered for bus operating with high-power opportunity charging. In the scenario b, the add-up range per dwell time is influenced by the network congestion, thus raising the battery capacity deficit 92.75 kWh. As a result, 10.57% larger batteries would be necessary to prevent from the shortage of energy.

From the perspective of economic operation of electric bus, the optimisation results suggested a reduced service, which is 13.45% less in terms of vehicle-kms and 13.7 % less int terms of dispatched buses when electric buses are more encouraged to participant in the demand response. In other words, bus operators have to suffer higher operational costs to maintain original service quality. As the bus charging during congested hours reduces, the maximal level of battery capacity deficit increase by 12.7% in the peak hours and 16.3% in the off-peak hours respectively. Both the increased reliance on battery and reduced service quality indicate an increased burden to support the grid.

Electric bus can still received benefits through the DLMP scheme. The charging cost is cut in the grid-interactive manner. By help alleviating the energy network congestion, the charging cost reduced by 25.8%. Such a reduction is partly contributed by the interrupted daytime charge (8.17%) and the reduced average DLMP price.

Table 4: Impacts of energy system congestion on bus operational activities in peak hours and off-peak hours

<table>
<thead>
<tr>
<th></th>
<th>Peak (6:00 to 9:00 pm)</th>
<th>Off-peak (12:00 to 3:00 pm)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without DR</td>
<td>With DR</td>
<td>Change</td>
</tr>
<tr>
<td>Vehicle-kms (km)</td>
<td>6548.07</td>
<td>5667.07</td>
<td>13.4%</td>
</tr>
<tr>
<td>No. of bus</td>
<td>219</td>
<td>189</td>
<td>13.7%</td>
</tr>
<tr>
<td>Average battery</td>
<td>72.72</td>
<td>81.95</td>
<td>12.7%</td>
</tr>
<tr>
<td>capacity deficit (kWh)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 7: Line loading and System DLMP in Scenario a and b
To sum up, the practicability of opportunity charging as a range remedy approach in electric bus operation depends on the power network that bus stations affiliated to. The high power rate of opportunity charger would cause significant balancing burdens at the bottleneck of power line capacity when congested. The proactive engagement of electric bus in the demand response could promise effectiveness in removing the exceeding loads. However, due to the restricted flexibility of heavy-duty bus operation, the unmet charging demand will not rebound to periods with lower price. The loss of cost-oriented demand response engagement has exceed the benefits can be received. For electric bus fleets operators, the electricity, as well as the electric loading condition of the power network where the bus station is located, should be taken into consideration in the design process of electric bus batteries. The optimal service adjustment have to be achieved as a trade-off between reduced range add-up, increased battery cost and improved charging cost.

4. Sensitivity Analysis

The main responsibility of bus operation is to provide sufficient transit capacity for passengers. The last section highlighted the effectiveness of demand response in alleviating the energy system congestion. However, it also found that engagement of demand response during congestion would damage the ability of opportunity charging to replenish consumed energy as well as the economic feasibility to provide low-cost service. As the cost of battery declining, a sensitivity analysis w.r.t. the battery cost will facilitate both the bus operators and DSOs to understand how would this implication varies.

First, for the peak period only, the unit value of daily battery usage cost is set to 10 p/kWh and 8 p/kWh for the next decades. For each of the three generated scenarios, the bus operation planning problem is solved within the bi-level framework with a congested power line. The total operational cost/surplus, total travelling vehicle-kms, charging loads and battery capacity deficit are compared and summarised in Figure 8.

The main observations from Figure 8 for the peak period planning are:

1. The reduction of 37.5 % cost on battery from 16 p/kWh to 10 p/kWh contributes to an extension of vehicle running distance by 13.5% as well as an improvement of operating surplus by 10.87 %.
2. The further declining battery cost from 10 p/kWh to 8 p/kWh is found to have minor impacts on the economic vehicle-running distance. The optimal frequencies in the scenario of 10 p/kWh and 8 p/kWh are identical.
3. The charging loads and battery capacity deficit to the reduction of battery cost is more sensitive when battery cost declined below the 10 p/kWh. The reduction of charging load is 34.17 % in the range of 10 p/kWh to 8 p/kWh and 1.6 % in the range of 16 p/kWh to 10 p/kWh respectively, while the increased needs for battery capacity is 108.94 % in the range of 10 p/kWh to 8 p/kWh and 51.14 % in the range of 16 p/kWh to 10 p/kWh.

Three similar scenarios are carried out for the off-peak period, of which the same indicators are summarised in Figure 9. The main observations from Figure 9 for the off-peak period planning are:
Figure 8: Sensitivity to changes of $\rho_b$ associated with battery daily usage cost in peak period

1. Contrasting to the bus operation in the peak hours, the vehicle running distance and operating surplus in the off-peak hours is more sensitive to the reduction of cost on battery, since the same reduction of 37.5% contributed to an extension of vehicle running distance by 20% as well as an improvement of operational surplus by 81.7%. The ratio in the off-peak scenario is 18.2% and 70.83% higher than the peak scenario.

2. Similar to the trend of peak time, the further declining battery cost from 10 p/kWh to 8 p/kWh also has minor impacts on the economic vehicle running distance and operating surplus in the off-peak time.

3. There is a greater possibility for electric bus to recharge the previously unmet demand during the off-peak times, because the battery capacity deficits remained negative. However, when the battery cost declined below the 10 p/kWh, the reduction of charging loads and increased needs for battery capacity are 74.3% and 434.32% higher respectively.

The above findings can facilitate the decision-making process of the bus operators by providing information regarding the economic operation distance and reliance on batteries when the power network congestion is considered. It can be concluded that, as the cost on battery declining, the impacts of demand response on bus service will be vanished, and a larger battery that required to ensure the uninterruptible
bus operation would facilitate the grid-interactive operation.
5. Conclusions

This paper has developed a optimisation-based framework that jointly considers the trade-off within the bus operation among providing services to passengers, optimising cost structures and alleviating power congestion to local networks at the bottleneck. The dynamic of electric bus charging activities is innovatively integrated within the electricity market clearing model by solving a bus planning problem. The interaction between electric bus fleets and Distribution System Operators is realised by the Distribution Locational Marginal Pricing algorithm that encourages electric buses to respond in a beneficial way to the grid. Following the problem formulation, an Mathematical Program with Complementary Constraints based approach is developed to solve the problem using data extracted from Shenzhen and RBTS in the case studies. It has been demonstrated that the proposed framework can promise effectiveness in removing the excessive charging loads from electric buses while providing a good example for the optimal electric bus operation.

By building a joint transit network and energy network based on real-world data, the proposed work addresses the specific challenges and opportunities arising with the engagement of electric bus fleets into demand response. It provided in-depth insight into the profound implication of the bilateral interaction. Experiments show that engagement of electric bus charging load in demand response is helpful to alleviate the network congestion and to reduce the power loss by 7.2% in the distribution network. However, alleviated charging loads have exhibited counter-intuitive capability for load shifting. The restricted electric bus operational requirements leads to a 8.17% loss of charging demand, while the reliance on large batteries has increased by 10.57%. However, the sensitivity analysis also shows that as the battery cost declines, the such discourage implications on grid-interactive electric bus operation will decrease once the battery cost below 190/kWh. The optimal grid-ebus integration have to consider the trade-off between range add-up, reduced battery cost and additional benefits.

However, the proposed framework also exhibits some limitations. First, due to the limitation of the opportunity charging model that is based on a tactical level of frequency setting, only the buses employed on the road for providing service are included in the modelling of electric mobility. Vehicle optimisation is a very complicated problem in which buses that completed their trips from the peak periods will not just park in the depots and stayed off-duty. Furthermore, the other efficient way of using vehicles, interlining also known as cross-overs, cannot be included in the model. Interlining is a technique involved in bus scheduling where a single bus is scheduled to perform multiple bus routes. A great challenge embodies in the modelling of the tempo-spatial characteristics of electric mobility, both in operation and charging loads, along with uncertainty in multiple routes. For the power networks, the linearised DC power flow model employed in this work has great limitations on the accuracy in the low-voltage distribution system. Further works include the development of a more rigorous pricing structure that involves the voltage constraints and reactive power flow constraints as well as the full restoration of the dynamics in bus scheduling.
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References


