Assessing the performance of UK universities in the field of chemical engineering using data envelopment analysis

Andrés González-Garay\textsuperscript{a}, Carlos Pozo\textsuperscript{a}, Ángel Galán-Martín\textsuperscript{b}, Clemens Brechtelsbauer\textsuperscript{a}, Benoît Chachuat\textsuperscript{a}, Deesa Chadha\textsuperscript{a}, Colin Hale\textsuperscript{a}, Klaus Hellgardt\textsuperscript{a}, Andreas Kogelbauer\textsuperscript{a}, Omar K. Matar\textsuperscript{a}, Niall McDowell\textsuperscript{a}, Nilay Shah\textsuperscript{a}, Gonzalo Guillén-Gosálbez\textsuperscript{b}\textsuperscript{,}\textsuperscript{*}

\textsuperscript{a} Department of Chemical Engineering, Imperial College London, South Kensington, London SW7 2AZ, UK

\textsuperscript{b} Institute for Chemical and Bioengineering, Department of Chemistry and Applied Biosciences, ETH Zürich, Vladimir-Prelog-Weg 1, 8093, Zürich, Switzerland

\textsuperscript{*}Corresponding author. E-mail: gonzalo.guillen.gosalbez@chem.ethz.ch

Abstract

University rankings have become an important tool to compare academic institutions within and across countries. Yet, they rely on aggregated scores based on subjective weights which render them sensitive to experts’ preferences and not fully transparent to final users. To overcome this limitation, we apply Data Envelopment Analysis (DEA) to evaluate UK universities in the field of chemical engineering as a case study, using data retrieved from The Guardian University Guide 2018. DEA is a non-parametric approach developed for the multi-criteria assessment of entities that avoids the use of subjective weightings and aggregated scores; this is accomplished by calculating an efficiency index, on the basis of which universities can be classified as either ‘efficient’ or ‘inefficient’. Our analysis shows that the Higher Education Institutions (HEI) occupying the highest positions in the chemical engineering rankings might not be the most efficient ones, and vice versa, which highlights the need to complement the use of rankings with other analytical tools. Overall, DEA provides further insight into the assessment of HEIs, allowing institutions to better understand their weaknesses and strengths, and pinpoint sources of inefficiencies where improvement efforts must be directed.
1. Introduction

The assessment of academic institutions is thought to be key for improving the quality of education and making better use of the available resources. A common example of such assessments are university rankings, where the academic institutions are evaluated according to multiple criteria. Global university rankings compare Higher Education Institutions (HEIs) across countries on the basis of their research and teaching performance. Over recent years, rankings have attained prominence in higher education and policymaking, ultimately becoming an important tool for students and academics who wish to discriminate among academic institutions. Yet, despite their widespread use, rankings often fail to provide full transparency on how the assess and weight the categories evaluated. As a result, there is a clear interest in improving the way in which rankings are developed and evaluations are made so as to ensure a fair and transparent comparison between HEIs (Marginson and Van Der Wende 2006). This would allow HEIs to clearly understand how to ensure higher quality of education and undertake appropriate responses (Meredith 2004).

University rankings aim to evaluate HEIs according to various indicators or metrics, such as the number of publications and citations, teaching satisfaction, expenditure per student, and employability, among others. Academic quality is inherently multi-faceted, and therefore, difficult to assess using a single indicator. Because of the lack of a single, globally-accepted metric to evaluate HEIs objectively and systematically, several assessment methodologies have been put forward that differ both in scope and focus (Kivinen, Hedman, and Artukka 2017). For example, rankings such as the Academic Ranking of World Universities (ARWU – Shanghai ranking) (Liu and Cheng 2005) or the University Ranking by Academic Performance (URAP 2015) focus on research and academic performance. The Times Higher Education World University Rankings (2018) considers teaching, research, and reputation. The Performance Ranking of Scientific Papers for World Universities (NTU 2018) quantifies scientific performance in terms of publications, while the QS World University Rankings (Huang 2011) focuses on the success achieved in becoming or remaining a world-class institution.
Within national systems, highly-ranked universities are considered as symbols of national achievement and prestige. Within the United Kingdom (UK), the three most prominent examples of HEI rankings are The Complete University Guide (CUG) (2018), The Guardian University Guide (2017) and the Good University Guide jointly published by The Times, and The Sunday Times (2016). These rankings periodically analyse the quality of UK universities, making this information available to the public. These three rankings evaluate the performance of the HEIs in areas such as student satisfaction, entry standards, student to staff ratio, expenditure per student, and education added value. While The Guardian ranking focuses only on teaching experience, the Complete University Guide and The Times and The Sunday Times rankings also include research performance.

A common characteristic of these rankings is their reliance on aggregated scores to compute a single metric. This metric greatly simplifies the comparison among HEIs helping universities to benchmark their performance against each other in a straightforward manner. It is also useful for wider society and for policymakers to make comparisons between HEIs. This simplicity and ease of application, however, comes at the cost of introducing controversial and subjective weightings on which there is no general consensus (Marginson and Van Der Wende 2006; Huang 2011). Furthermore, any ranking is purpose-driven and very sensitive to the weightings, assumptions, methods, and data considered during the evaluation. Consequently, there is no perfect ranking and none of them provide a fully accurate assessment of the quality of a given HEI (Bougnol and Dula 2015).

In this article, we propose the assessment of HEIs including the application of Data Envelopment Analysis (DEA) (Charnes, Cooper, and Rhodes 1978), a mathematical technique originally developed in economics and research operations. DEA is based in the concept of efficiency, defined as the ratio between weighted outputs and weighted inputs of a given unit, which can also be seen as the conventional benefit/cost relationship. This methodology identifies the best practices and inefficiency sources among a group of peers based on the efficiency score calculated for each unit. The use of DEA as an alternative approach to university rankings offers several advantages. First,
DEA avoids the use of subjective weightings, as these are optimised during the application of the methodology rather than chosen beforehand. Second, compared with traditional rankings that provide little insight on how to enhance performance, DEA highlights potential sources of inefficiency that could provide directions for improvement of the lower-performing universities. Third, DEA accounts for the size of a university in a natural way, allowing fair comparisons between institutions and providing quantitative performance improvement targets. All these features make DEA a very appealing tool for comparing HEIs and an excellent complement to the standalone use of rankings.

Since its introduction in the 1970s, DEA has found many applications in science and engineering, including the efficiency assessment of HEIs (Daraio, Bonaccorsi, and Simar 2015; Grosskopf, Hayes, and Taylor 2014; Taylor and Harris 2004). Taylor and Harris (2004) evaluated the efficiency of South African universities comparing models built using different inputs and outputs. Kuah and Wong (2011) presented a methodology for the assessment of universities’ teaching and research performance and applied it to hypothetical data. Other studies applied DEA to measure the efficiency of universities in Spain, Italy, Colombia, and Australia (Sellers-Rubio, Mas-Ruiz, and Casado-Díaz 2010; Visbal-Cadavid, Martínez-Gómez, and Guijarro 2017; Abbott and Doucouliagos 2003; Agasisti and Pérez-Esparrells 2009). The impact of size and specialisation in Austrian universities was studied by Leitner et al. (2007). DEA has also been applied to attain policy goals in higher education, as reported by Geva-May (2001) for the case of universities in Israel. In the context of British Universities, Flegg et al. (2004) presented an analysis of 45 HEIs in the period 1980/81 to 1992/93, studying the evolution of their efficiency over time. Building on these approaches, we have carried out a comparative efficiency assessment of HEIs in the UK by applying DEA to data taken from The Guardian University Guide 2019 and the Complete University Guide (CUG) 2019 for the subject of Chemical Engineering. We discuss how this DEA approach, complemented by a sensitivity and a multi period analysis, can add value to traditional rankings not only in terms of securing high positions on rankings but also in how to make an appropriate use of resources.
The rest of the paper is organised as follows. First, we introduce the data categories considered in the assessment along with the DEA methodology. Then, we present the DEA results along with a sensitivity analysis and the inclusion of a three-year period data in the assessment. Finally, we discuss the results and draw the conclusions of the study.

2. Methods

2.1 Data sources

To carry out the analysis, we used data retrieved from The Guardian University Guide 2019 (The-Guardian 2017) and The CUG 2019 (2018) for the subject area of Chemical Engineering. The Guardian evaluates 25 UK Universities in terms of eight indicators, defined as: staff-to-student ratio, expenditure per student, entry standards, course, teaching and feedback satisfaction, value-added scores, and graduate prospects. To generate the chemical engineering ranking, the CUG evaluates 30 UK Universities in terms of five indicators: entry standards, student satisfaction, research quality, research intensity, and graduate prospects. In order to include the largest number of Universities with the most significant indicators in the assessment, we considered the 25 institutions appearing in both rankings. The indicators included in the analysis are entry standards, research intensity, staff-to-student ratio, expenditure per student, student satisfaction, research quality, graduate prospects, and value added score. The only indicators reported in the rankings that were omitted from the analysis were course, teaching and feedback satisfaction (The Guardian), as they were aggregated in the indicator student satisfaction (CUG).

Following the general DEA paradigm, we model each university as an entity that converts multiple ‘inputs’ into multiple ‘outputs’. As explained in more detail in Section 2.2, indicators defined as ‘inputs’ are those for which lower values imply better performance, that is, those that should be ideally minimised, while indicators defined as ‘outputs’ should be ideally maximised (Belton and

---

1 We selected The Guardian University Guide and the Complete University Guide to consider indicators including quality of teaching and research. The subject of Chemical Engineering has been chosen as the expertise of the authors is related to this field, which we expect can provide better interpretation and discussion of the findings.
In this study, we perform an output-oriented model, which implies that the level of inputs remains at the same level while the outputs are maximised. Following these definitions, Fig. 1 shows the classification of these indicators in the analysis of each HEI. The ‘inputs’ are entry standards, research intensity, staff-to-student ratio, and expenditure per student. On the other hand, the ‘outputs’ include student satisfaction, research quality, graduate prospects, and value-added scores. A brief description of the indicators is provided next. For further details, we refer the reader to the ‘Methodology in the Guardian University Guide 2019’ (Hiely-Rayner 2016) and the ‘University League Tables Methodology’ (The Complete University Guide 2019).

**Input indicators.**

The first input are the entry scores of the students, expressed as the average UCAS tariffs of new undergraduate students. The second input is research intensity, which is a measure of the proportion of staff involved in research. This indicator is calculated from the number of staff submitted to the Research Excellence Framework (REF 2014) divided by the number who were eligible to give a proportion of those who were submitted. The third input refers to the staff-to-student ratio, which includes only those staff members who spend a significant portion of their time teaching. Hence, staff devoted primarily to research are excluded from this indicator. Furthermore, this indicator is treated as input on the basis that a given department has to allocate certain number of staff members to the corresponding number of students. Therefore, the input for the department is the quantity of staff per student. The fourth input refers to the expenditure per student, which quantifies the cost of delivering chemical engineering studies. As treated in the rankings, this indicator excludes the costs of the academic staff (accounted for in the staff-to-student ratio), but it does account for the costs associated with academic services such as library and computing facilities. The total amount is divided by the total number of students to enable a fair comparison between HEIs.

**Output indicators.**
Four different outputs are used to assess the performance of the chemical engineering departments in the different HEIs. The first refers to the students’ satisfaction after attending the degree course. This indicator was taken from the CUG, and aggregates the course, teaching, and feedback satisfaction as derived from the National Student Survey (NSS) and reported in The Guardian University Guide. The second output indicator refers to research quality, based on the results of the Research Excellence Framework (REF 2014). Here, each university department achieved a quality profile which gave the proportion of research in each of four categories from 4* to 1*. For the research assessment measure, the categories 4* to 1* were given a numerical value of 4 to 1 which allowed a grade point average to be calculated. An overall average was then calculated weighted according to the number of staff in each department. The third indicator pertains to graduate prospects, which are assessed by the employability of graduates who find graduate-level employment and/or study at a higher education or at the professional level six months after graduation. Note that this indicator does not take into account the type of job/further studies secured by the students (i.e., responsibility, wage, quality of the postgraduate studies, etc.), which may vary across graduates. However, it does include only employment in an area that normally recruits graduates was included. The fourth output indicator is based on value-added scores\textsuperscript{2} that ‘track students from enrolment to graduation. This indicator is taken as reported in The Guardian University Guide and compares qualifications upon entry against the award that students receive at the end of their studies.

The data retrieved from the rankings are shown in Fig. 2. Following the same criteria as in the Guardian University Guide 2018, data gaps were covered by using the average value of the indicator across all the institutions and are enclosed in a green box.

\textsuperscript{2} According to the methodology reported, each full-time student is assigned with a probability to obtain a first or 2:1 degree, which is based on the entry qualifications of the students. If the student is in entry bands 20 and 50, the method considers the total percentage of good degrees expected for the student. An HEI scores high if it takes students with low-entry qualifications and the students exceed the expectations. It is important to recall that the quality of the course might not be considered, as the method to calculate the probability is not reported. Thus, a course with a higher level of difficulty, which may result in students with lower marks, might not score well in this indicator.
2.2 Data envelopment analysis

Data Envelopment Analysis (DEA) is a mathematical programming technique that identifies the best practices across similar organisations converting multiple inputs into multiple outputs. DEA links the outputs generated by each organisation to its corresponding inputs to determine efficiency scores and improvement targets. The first step of the methodology is to build a convex envelope of the entire set of assessed units. This envelope is also known as ‘efficient frontier’ or best ‘practice frontier’, and all the units lying on this frontier are classified as ‘efficient’, while the rest are identified as ‘inefficient’. The level of ‘efficiency’ is given by the calculated scores for each unit, which quantify the ratio of weighted outputs to weighted inputs. In a second step, DEA determines targets for the ‘inefficient’ units to become ‘efficient’ via its projection to the efficient frontier.

Computation of efficiencies and improvement targets.

We model the chemical engineering department of each HEI as a unit converting inputs into outputs following the scheme presented in Fig. 1. An HEI is deemed ‘efficient’ if for the same level of inputs (e.g. expenditure per student) there is no other institution that can provide higher outputs (e.g. quality of teaching). Conversely, an HEI is ‘inefficient’ if there is at least another institution, or combination of institutions, that consuming the same inputs can produce more outputs (i.e. the same expenditure per student while achieving a higher quality of teaching). DEA allows us to identify whether a given HEI is efficient or not, while providing insight on how to improve the inefficient ones. To this end, inefficient units are projected onto the efficient frontier formed by the efficient units. This projection can be made in various ways, with the input and output-oriented models being the most popular. The input-oriented model minimises the consumption of inputs maintaining the same level of outputs, while the output-oriented model maximises the outputs for a given level of inputs. In this work, we apply the output-oriented model, since the main goal of the analysis is to understand how to maximise the quality of teaching by making the best use of the resources (i.e. inputs) available. In other words, we aim to maximise the performance considering a given availability of resources.
The model of DEA is based on the concept of efficiency, which is defined as the ratio between weighted outputs and weighted inputs. The technical efficiency of a unit \( o \) being assessed among a group of \( n \) entities, consuming \( x_i \) inputs and \( y_r \) outputs, is given by the following BCC model, also known as Variable-Returns-to-Scale (VRS) (R.D. Banker 1984):

\[
\theta_o = \min \left( \frac{\sum_i v_i x_{io} - \mu_o}{\sum_r u_r y_{ro}} \right)
\]

\[
s.t. \quad \frac{\sum_i v_i x_{ij} - \mu_o}{\sum_r u_r y_{rj}} \geq 1; \quad j = 1, ..., n
\]

\[
v_i \geq 0, \quad u_r \geq 0; \quad \mu_o \text{ free in sign}
\]

where \( \theta_o \) is the technical efficiency score of unit \( o \) and indicates if the unit is efficient (\( \theta_o = 1 \)) or inefficient (\( \theta_o < 1 \)); \( u_r \) and \( v_i \) are variables optimised by the model, which represent the weights associated to inputs \( x_i \) and outputs \( y_r \) respectively; and \( \mu_o \) is a variable that imposes a convexity condition.

The original DEA model presented in Eqs. (1) to (3) is nonlinear, but can be reformulated into an equivalent linear program by applying fractional programming and duality theory (R.D. Banker 1984; Cook and Seiford 2009):

\[
\max \theta_o + \varepsilon \left( \sum_r s_r^+ + \sum_i s_i^- \right)
\]

\[
s.t. \quad \sum_j \lambda_j x_{ij} + s_i^- = x_{io}; \quad i = 1, ..., m
\]

\[
\sum_j \lambda_j y_{rj} - s_r^+ = \theta_o y_{ro}; \quad r = 1, ..., s
\]

\[
\sum_j \lambda_j = 1
\]

\[
\lambda_j, s_i^-, s_r^+ \geq 0; \quad \forall i, j, r; \quad \theta_o \text{ unconstrained}
\]
In addition to the efficiency score $\theta_o$, this model also provides improvement targets for the inefficient units, which are calculated from the optimal values of $\lambda_j$, $s^+_r$, and $s^-_i$. When inefficient units are determined, each one is compared against the subset of efficient units. The model then assigns positive weights $\lambda_j$ to the efficient units in such a way that their linear combination corresponds to the projection of the inefficient unit $o$ onto the efficient frontier. The slack variables $s^+_r$ and $s^-_i$ represent the surplus in which an input or output could be reduced or increased, respectively, to become a non-dominated solution (i.e. a strongly efficient unit). The non-Archimedean parameter $\varepsilon$ effectively allows the minimization over $\theta$. Thus, the optimisation can be computed in a two-stage process with maximal reduction of inputs being achieved first, via the optimal $\theta$; then, in the second stage, movement onto the efficient frontier is achieved via the slack variables. In linear programming, this value tends to be a very small value. If $\theta_o = 1$, and $s^+_r + s^-_i = 0$, unit $o$ is classified as strongly efficient. On the other hand, if $\theta_o = 1$, and $s^+_r + s^-_i > 0$, unit $o$ is classified as weakly efficient.

To further illustrate these concepts, let us consider a simple example involving five universities A, B, C, D, and E, that consume one input to produce one output. The values of the universities are shown in Fig. 3. The goal is to maximise the students’ satisfaction (output) for the given expenditure-per-student (input). The solution of Eqs. (4) to (8) identifies universities A, B, and C as efficient units ($\theta = 1$), given that they have the largest satisfaction for a given level of expenditure. These institutions form the strongly efficient frontier of the set (continuous green line in Fig. 3), representing the best practices. The weakly efficient frontier is an extension of the strongly efficient frontier (dashed green line in Fig. 3). On the other hand, universities D and E are inefficient, and are projected onto the efficient frontier, resulting in points ‘d’ and ‘e’, respectively. These points represent the maximum theoretical level of satisfaction that universities D and E could attain, based on how the existing HEIs operate. That is, universities D and E are inefficient, but they could become efficient by operating at the hypothetical points ‘d’ and ‘e’, respectively.
The efficiency score of an inefficient unit corresponds to the ratio between the projected and current levels of output. For instance, the efficiency score of university D is equal to $85.8/82.0=1.04$. As observed in Fig. 3, along the output-oriented projection trajectory of universities D and E, the same level of inputs is maintained while increasing the level of outputs until the efficient frontier is attained. The slack variables of the model are activated when the projection falls in the ‘weakly-efficient frontier’, an extension of the efficient frontier that occurs in the projection of E to ‘e’. This means, that an extra reduction of 1 unit in the expenditure per student (i.e. from 10 to 9) would be required for the projected university ‘e’ to become strongly efficient. Otherwise, it would remain dominated by university C, which shows the same level of outputs but lower input.

The improvement targets for the inefficient units are given by (Cooper, Lawrence, and Zhu 2011):

$$\hat{x}_{io} = \sum_j x_{ij} \lambda_j^*; \quad i = 1, \ldots, m$$  \hspace{1cm} (9)

$$\hat{y}_{ro} = \sum_j y_{rj} \lambda_j^*; \quad r = 1, \ldots, s$$  \hspace{1cm} (10)

where * refers to the optimal values of the linear weights; $\hat{x}_{io}$ and $\hat{y}_{ro}$ denote the coordinates of unit o after its projection onto the efficient frontier. In Fig. 3, the projection of university D onto the strongly-efficient frontier lies in the facet defined by universities B and C. As a result, the efficient student satisfaction (ss) value for D is a linear combination of the two efficient universities B and C, given by $\hat{ss}_d = ss_B \lambda_B^* + ss_C \lambda_C^*$. Therefore, university D could become efficient by achieving a student satisfaction of 85.8 % for the same expenditure of 8.5 units (point ‘d’ in Fig. 3). To put it differently, university D should take universities B and C as benchmarks since they show similar levels of expenditure but higher student satisfaction; the two efficient universities B and C are referred to as the ‘peer group’ of D.

*Analysing efficient units in DEA, the super-efficiency concept.*

The solution of Eqs. (4)-(8) identifies the efficient universities and assigns them an efficiency score of $\theta = 1$. Further assessment on how efficient universities perform might be necessary, but the
solution of the standard DEA model alone does not allow it. Instead, one can use the ‘super-efficiency’ score (Rajiv D Banker et al. 1980), which in an output-oriented model, corresponds to the extra productivity achieved by an efficient HEI. This is measured by the extent to which the efficient frontier changes upon discarding that HEI from the data set. The super-efficiency score is always less than or equal to one, so it can be used to compare the performance among efficient solutions. The lower the value of the super-efficiency score, the better the performance of the corresponding HEI.

The super-efficiency model is essentially the same as the one described in Eqs. (4)-(8), but in this formulation the summation of \( \lambda_j \) in Eqs. (5)-(6) excludes the efficient unit \( j' \) being assessed:

\[
\sum_{j,j' \neq j'} \lambda_j x_{ij} + s_i^- = x_{io}; \quad i = 1, \ldots, m
\]

\[
\sum_{j,j' \neq j'} \lambda_j y_{rj} - s_r^+ = \theta_o y_{ro}; \quad r = 1, \ldots, s
\]

*Returns to scale in DEA.*

The solution to the BCC VRS model generates a convex efficient frontier as depicted in Fig. 4, from which three subregions can be distinguished: increasing returns to scale (IRS), constant returns to scale (CRS), and decreasing returns to scale (DRS). A university in (or projected onto) the IRS subregion must increase its outputs proportionally more than its inputs in order to remain on the efficiency frontier; in the DRS subregion, increase its outputs proportionally less than its inputs; and in the CRS subregion, increase its inputs and outputs in the same proportion. In essence, efficient universities belonging to the CRS subregion (segment BC in Fig. 4) may thus obtain more output per unit of input invested than universities belonging to either the IRS or DRS subregion (segments AB and CD, respectively). This implies that universities operating at CRS have the most productive scale size. Universities in the IRS subregion should consider increasing their input level in order to improve their output-to-input ratio, while those in the DRS subregion should consider decreasing their input level.
Identifying the subregion of the efficient frontier for an HEI \( o \) entails the solution of the following auxiliary linear optimisation model (Cooper, Lawrence, and Zhu 2011):

\[
e_o = \max \sum_r u_r \hat{y}_{ro} - \mu_o^* \tag{13}
\]

\[
s. t. \sum_i v_i \hat{x}_{io} = 1 \tag{14}
\]

\[
\sum_r u_r \hat{y}_{rj} - \sum_i v_i \hat{x}_{ij} - \mu_o^* \leq 0, \quad \forall j \tag{15}
\]

\[
v_i, u_r \geq 0, \quad \forall r, i \tag{16}
\]

where \((\hat{x}_{io}, \hat{y}_{ro})\) denotes the solution of Eqs. (4)–(8), followed by the projection of inefficient HEIs onto the efficient frontier per Eqs. (9) and (10). In particular, the point \((\hat{x}_{io}, \hat{y}_{ro})\) belongs to the DRS subregion when \( \mu_o^* > 0 \) for all the optimal solutions; to the IRS subregion when \( \mu_o^* < 0 \) for all optimal solutions; and to the CRS subregion when \( \mu_o^* = 0 \) for at least one optimal solution.

**Analysing efficiency over changes in data, a sensitivity analysis.**

To account for uncertainty in the DEA calculations, we performed a sensitivity analysis on the output indicators while fixing the inputs. The motivation for this is that outputs might be affected by some degree of uncertainty, as some correspond to subjective assessments, while inputs are obtained from direct measures provided by the HEIs that are immune to uncertainties.

To perform the analysis, we generated 100 scenarios by randomly varying each output indicator in the range of ±20% according to a uniform distribution. Then, the calculations were repeated for each sample using the super-efficiency model presented in eqs. (4),(7)-(8), (11)-(12).

**Analysing efficiency over time, the Malmquist approach.**

We analyse the productivity change over time from 2017 to 2019 using the Malmquist productivity index (Fare, Grosskopf, and Lovell 1993). These years were considered due to data availability. The Malmquist index measures the changes in the technology frontier and technical efficiency between periods \( t \) and \( t + 1 \). Its calculation require two single period and two mixed period
efficiencies. The single period measure $D_o^t(x_o^t, y_o^t)$ for time $t$ can be solved using the following output-oriented VRS DEA model (Färe et al. 1994):

\[
D_o^t(x_o^t, y_o^t) = \max \theta_o \quad (17)
\]

\[
s.t. \quad \sum_j \lambda_j x_j^t \leq x_o^t \quad (18)
\]

\[
\sum_j \lambda_j y_j^t \geq \theta_o y_o^t \quad (19)
\]

\[
\sum_j \lambda_j = 1 \quad (20)
\]

\[
\lambda_j \geq 0; \quad \forall j; \quad \theta_o \text{ unconstrained} \quad (21)
\]

The single period measure $D_o^{t+1}(x_o^{t+1}, y_o^{t+1})$ for time $t + 1$ can be calculated replacing $t$ by $t + 1$ in the previous model.

The first mixed efficiency $D_o^t(x_o^{t+1}, y_o^{t+1})$ is obtained from the solution of the following model:

\[
D_o^t(x_o^{t+1}, y_o^{t+1}) = \max \theta_o \quad (22)
\]

\[
s.t. \quad \sum_j \lambda_j x_j^t \leq x_o^{t+1} \quad (23)
\]

\[
\sum_j \lambda_j y_j^t \geq \theta_o y_o^{t+1} \quad (24)
\]

\[
\sum_j \lambda_j = 1 \quad (25)
\]

\[
\lambda_j \geq 0; \quad \forall j; \quad \theta_o \text{ unconstrained} \quad (26)
\]

This model compares the efficient frontier at time $t$ with the unit $o$ at time $t + 1$. In a similar way, we calculate the second mixed efficiency $D_o^{t+1}(x_o^t, y_o^t)$, which compares the efficient frontier at time $t + 1$ with the unit $o$ at time $t$.

The Malmquist productivity index, can be finally expressed by:
\[
M_o = TEC_o \times FS_o
\]  
(27)

where

\[
TEC_o = \frac{D_o^{t+1}(x_o^{t+1}, y_o^{t+1})}{D_o^t(x_o^t, y_o^t)}
\]  
(28)

\[
FS_o = \left(\frac{D_o^t(x_o^t, y_o^t) D_o^{t+1}(x_o^{t+1}, y_o^{t+1})}{D_o^{t+1}(x_o^{t+1}, y_o^{t+1}) D_o^t(x_o^t, y_o^t)}\right)^{1/2}
\]  
(29)

The term \(M_o\) measures the productivity change between periods \(t\) and \(t + 1\). \(M_o > 1\) indicates productivity decline, \(M_o = 0\) indicates no change, and \(M_o < 1\) indicates productivity improvement.

An additional advantage of the Malmquist index, is the disaggregation of the efficiency change into technical efficiency change \((TEC_o)\) and frontier shift \((FS_o)\). A value of \(TEC_o < 1\) reflects an improvement in technical efficiency, whereas \(TEC_o = 0\) reflects no change, and \(TEC_o > 1\) reflects a decline. If \(FS_o > 1\), there is a regress in the frontier technology, \(FS_o = 0\) reflects no change, and \(FS_o < 1\) reflects a progress in the frontier technology.

Limitations of the methodology.

DEA also present some limitations, which are particularly relevant in the context of higher education. The first relates to the measure of efficiency and concerns the fact that it compares institutions between themselves rather than using an absolute scale. As a result, the efficiency of the HEIs will be dictated by those with the best performance, while no clear guidelines others than the peer group members whose performance should be mimicked is provided to improve the efficiency.

A second limitation of DEA as applied to the HEIs data is to assume that each unit in the indicators is identical across universities, omitting quality differences, such as the type of job secured by the students when assessing the employability output. It is also important to mention that the indicators used in the methodology affect drastically the DEA outcome. This means that if the indicators considered do not fully reflect the overall mission of the universities, the insight generated might not be that meaningful.
3. Results

We first provide a preliminary assessment of the data before applying the DEA methodology.

3.1 Preliminary assessment

Fig. 5 shows a graphical representation of the data taken from The Guardian University Guide and The CUG for the year 2019, where data were normalised according to the number of standard deviations away from the mean (z-score). The figure corresponds to a radar diagram where every polyline represents one indicator and the axes represent the different HEIs. As shown in the figure, it is quite hard to get any valuable insight from a simple visual inspection of this multi-dimensional dataset.

As previously stated, an HEI is a complex entity where the multiple inputs and outputs interact with each other. Before applying DEA, we first carried out a statistical analysis that quantifies the correlation between indicators. This analysis helps to identify significant interactions among them, and sets the ground to understand how an action over one indicator may affect the performance of the HEIs in other correlated criteria. Fig. 6 shows the Spearman correlation values. Those marked with a star were found to be statistically correlated considering a 5% significance level.

3.2 DEA results

Computation of efficiencies.

The correlation analysis helps us to understand relationships between indicators. To further extend this analysis to the comparison across universities, we applied the DEA models described in Section 2 using the data presented in Fig. 2. The results are presented in Fig. 7, where we show the efficiency score of the 25 HEIs by means of bubbles, where the size of the bubble indicates the efficiency performance (the smaller the more efficient) and the position of the bubble denotes the location of the corresponding HEI. At the top right of the figure, we present the histogram of the efficiency scores. A total of 21 institutions were found to be efficient, while the inefficient universities showed efficiency scores close to one (from 1.01 to 1.03). This shows that, generally, universities
perform quite well in the way that they convert resources and human capital into satisfied, well-educated, and employable graduates across the UK while maintaining a focus on research. The 21 efficient universities are Aston, Bath, Birmingham, Bradford, Cambridge, Heriot-Watt, Hull, Imperial College, Lancaster, Leeds, London South Bank, Loughborough, Nottingham, Portsmouth, Queen’s Belfast, Sheffield, Strathclyde, Swansea, Teesside, and West of Scotland. The five HEIs identified as inefficient are Edinburgh, Surrey, Newcastle and University College London. As for the geographical location of efficient and inefficient HEIs, no trend was identified from our analysis.

Fig. 8 shows the ranking positions given by the Guardian University Guide on the x-axis, by the CUG on the y-axis, and by the super-efficiency score as the coloured bubble representing each university.

Returns to scale in HEIs.

As described in section 2.2, the most productive scale corresponds to the constant returns to scale subregion (CRS). This was the case for Bradford, Hull, London South Bank, and Teesside universities. The remaining HEIs operate in the DRS subregion, which suggests that the increment in student satisfaction, research quality, graduate prospects, and added-value for a given increment in the level of inputs diminishes after a certain point.

Improvement targets and members of the peer group.

An inherent advantage of DEA over traditional rankings is its capability to define improvement targets for those units identified as inefficient. As mentioned in the Methods section, inefficient units are projected onto the efficient frontier, and by measuring the distance between the actual performance of the inefficient unit and its projection, it is possible to determine those improvement targets. These targets help to identify the main sources of inefficiency and take suitable corrective actions. Focusing on the inefficient institutions, we show in Fig. 9 the improvement targets obtained from the solution of Eq. (10) after identifying efficient and inefficient units from Eqs. (4) - (8).
The projection of an inefficient unit to the efficient frontier lies on a facet determined by other efficient units, which are defined as the ‘peers’ (see Fig. 3). In Fig. 10, we report the ‘peer groups’ of the inefficient HEIs to reach their improvement targets. Recall that the targets indicate the necessary increments needed by an inefficient unit in order to become efficient. Since our DEA model is output-oriented, we assume fixed inputs and seek to increase the outputs to the same levels as those on the efficient frontier. In practice, an inefficient HEI should observe the practices of its peers, which present similar levels of inputs, and apply suitable policies so as to attain the output targets.

*Analysis of efficiency over change of data and time.*

In Fig. 11, we present the results of the sensitivity analysis using box plots, where the bottom and top of the box represent the 25th and 75th percentiles, respectively, while the red line represents the median. The whiskers extend to ±2.7σ, and the outliers are plotted individually using the '+' red symbol. The deterministic values are depicted using blue diamonds. Fig. 12 shows the results for the change of efficiency over time. The first two columns refer to the change in technical efficiency $TEC_o$. That is, if the institution being analysed change from efficient to inefficient or vice versa. The third and fourth columns refer to shifts in the frontier from one year to the other $FS_o$, and the last two columns refer to the total change of productivity $M_o$ of each unit.

4. Discussion

4.1 Correlation of indicators.

Analysing Fig. 6, we observe a correlation between the entry standards and research intensity and research quality. This correlation might indicate that the universities having the most stringent entry criteria have also a major focus on research development, which is also reflected in a major expenditure per student. The additional correlation observed is between the graduate prospects and the expenditure per student and research intensity. This correlation could imply that universities with a focus on research motivate graduates enough to be enrolled in a postgraduate course, and the research intensity correlation is just the result of the research-focus of the department. A different
view, however, is that universities with larger resources and research outputs, possess a higher prestige among employers, proving their graduates with an advantage against colleagues from other institutions. In any case, the graduate prospects behaviour is difficult to predict. As pointed out by the Wakeham Review of STEM Degree Provision and Graduate Employability (Wakeham 2016), students in chemical engineering possess a great variety of options and reasonable good salaries, allowing graduates to consider their employment options upon completion of their degree.

The staff-to-student ratio considers the staff primarily devoted to teaching. Hence, we could expect that this metric would correlate with the students’ satisfaction. However, this trend was not observed, implying that the group size might have little impact on how students perceive the quality of the course. This conclusion is in agreement with ‘The 2016 Student Academic Experience Survey’ (Buckley, Soilemetzidis, and Hillman 2015), where it was identified that smaller class sizes are not one of the main drivers of a positive overall experience. They also observed that this statement holds despite students saying that they benefit from being part of smaller cohorts. In addition, the added-value shows no correlation with any other indicator. From this, we could conclude that this indicator is either the result of the interaction between all the different metrics or an independent indicator that is not related at all with the other metrics.

4.2 Efficiency scores and ranking performance.

One of the highlights when comparing the scores of the rankings against those of the super efficiency (Fig. 8), is the lack of patterns among them. This, however, reinforces the previous statements about the fact that rankings are purpose driven and their results are highly dependent on the indicators included in the assessment. While the Guardian University Guide is said to be a ranking focused on the quality of teaching, the CUG includes research criteria in their assessment. As a result, despite we observe some departments in similar positions in both rankings, we also note striking differences for some others. This is the case of West of Scotland or Surrey, which are ranked fifth by the Guardian and below the 20th position by the CUG. The contrary occurs with Manchester or Strathclyde, which received high positions in the CUG but positions 23rd and 24th by The Guardian.
When compared to the super-efficiency ranking, we can observe significant differences. Let us take the case of Bradford, which is the most efficient department, holding the third position by The Guardian, and the 20th by the CUG. From Fig. 2, we can observe that it has the lowest expenditure per student (2), and very low inputs for entry standards (124) and research intensity (0.3). In terms of outputs generated, it has the largest score in students’ satisfaction (4.44) and value-added (10). From these values, it is evident its high output/input ratio generated. If we consider the objectives of the rankings, the positions given to Bradford seems reasonable, as it performs extremely well in students’ satisfaction and value-added (indicators related to only-teaching evaluation), but poorly in research related indicators (indicators only considered by the CUG). A different case is London South Bank, which is the second most efficient department and is ranked 22nd by The Guardian and 19th by the CUG. In this case, London South Bank has the lowest inputs in entry standards (108), staff-to-student ratio (0.037), and expenditure per student (2), while producing reasonably good outputs for students’ satisfaction (4.11), graduate prospects (84), and added-value (8). Again, the ratio output/input is high, mainly due to the low consumption of inputs. This low value in the inputs is also the reason to perform poorly in both rankings, as they operate under the concept of ‘the higher, the better’. We present the final case with Strathclyde, which is the third most efficient department. The high efficiency of Strathclyde is observed from its low values in expenditure per student (3) and staff-to-student ratio (0.040) and high values in the outputs research quality (3.03) and graduate prospects (84). While its high entry standards (225), research intensity (0.87), research quality and graduate prospects allow it to obtain the sixth position in the CUG, its low students’ satisfaction (3.55), added-value (2), expenditure per student and staff-to-student ratio result in a very low position in The Guardian (24th).

The previous analysis aims to clearly present one of the main differences between rankings and DEA. While DEA is based on the efficiency to transform inputs into outputs, traditional rankings operate under the concept of ‘the higher the value for all the indicators, the higher the position achieved’. Hence, rankings can assign low (or high) positions to HEIs regardless of whether they make an efficient (or inefficient) use of their resources. Additionally, we can also observe how
through the use of appropriate indicators, DEA defines as efficient universities those succeeding in the transformation of inputs to outputs, despite the fact that they might have different missions, such as teaching-focused, research-focused, or both.

The DEA results also show that chemical engineering departments in the UK perform mostly in an efficient manner, with 21 universities being efficient, and the rest having efficiencies very close to one (<1.03). The efficient universities operating at Constant Returns to Scale (CRS) are Bradford, London South Bank, Hull, and Teesside. Overall, from the analysis of the CRS subregion, we observe that universities with lower resources and educating students with lower entry UCAS tariff seem to succeed at satisfying students’ expectations and guaranteeing their employability. The remaining efficient universities operating at DRS are Strathclyde, Swansea, Aston, Bath, Cambridge, Imperial College, Manchester, West of Scotland, Birmingham, Nottingham, Portsmouth, Sheffield, Lancaster, Loughborough, Heriot-Watt, Queen’s Belfast, and Leeds. Further improvements can be attained by these HEIs if they increase their output levels or decrease their level of inputs to operate at CRS.

The universities identified as inefficient are Edinburgh (1.01), Surrey (1.02), Newcastle (1.02), and UCL (1.03). These values, however, are very close to one, implying that their performance is not significantly worse. To become efficient, these HEIs should improve their output values according to the values reported in Fig. 9. As an example, Surrey should increase the student’s satisfaction, research intensity, and value-added by 2%, while graduate prospects should increase by 17%. From Fig. 10, the main peers to achieve these targets would be Bath, Bradford, Imperial College, Loughborough, and Swansea. Taking the example of Newcastle, minor improvements in all of the output indicators would be required (<7%), and could be ideally accomplished taking Bath, Bradford, Imperial College, Nottingham, and Swansea as peers. For Edinburgh, efforts should be placed so as to increase the graduate prospects performance. In the case of UCL, an increase in students’ satisfaction would be required.
When an inefficient HEI looks at the practices of the peers in order to increase its level of outputs, it must be careful in the way it implements any new policy resulting from benchmarking its operation against that of its peers. As already highlighted in the methods section, DEA provides no specific guidelines on how to attain the improvement targets sought. Hence, a subsequent step an additional analysis should be carried out to delineate the necessary improvement actions. In any case, the study of best practices in similar institutions (‘peers’) can certainly provide insights and hints into how to improve.

4.3 Efficiency performance over data variation and time

From Fig. 11, we observe that the super efficiency values do not change so drastically (differences between the 75th and 25th percentiles corresponding to the sizes of the boxes around 14% and 8% of the nominal values, respectively), with similar trends as in the deterministic case are found. The results in Fig. 12, we can observe that the changes in the index are relatively low, indicating certain stability in the performance of the entire system. Given that this analysis can be particularly helpful for the HEIs identified as inefficient, let us first analyse the case of Edinburgh. For this institution, we can observe a slight decrease of efficiency in 2017-2018 but a constant performance from 2018-2019. In the period 2017-2018, there was an improvement in technical efficiency. However, the frontier presented a decline in productivity, meaning that similar HEIs worsened their performance. As a result, the productivity index decreased. For 2018 to 2019, both the technical efficiency and the frontier remained constant, so although Edinburgh emerged as inefficient in 2019, its performance did not worsen compared to 2018. In the case of Newcastle, its performance decreased since 2017, mainly caused by the decline in productivity of the frontier. In the case of Surrey, a decline in productivity took place from 2017-2018, but it increase in 2018-2019. Finally, UCL shows slight improvements over the two periods. In any of the previous cases, the Malmquist approach provides insight as to whether improvements/reductions in efficiency are the result of new policies and/or specific changes, e.g. recruitment of new staff, new facilities, innovation in teaching, etc., or are due to general trends across HEIs reflected in moves of the entire Pareto front.
5. Conclusions

This work analysed the performance of UK HEIs offering chemical engineering studies by applying DEA to data published in The Guardian University Guide and The Complete University Guide 2019. An output-oriented analysis was carried out, and an efficiency measure of converting ‘inputs’ into ‘outputs’ was introduced; departments with efficiency values of unity or lower are classified as ‘efficient’. We have found that, on average, HEIs are efficient in the way they educate students, with 21 out of 25 HEIs being efficient and the inefficient ones showing scores close to unity. From the solution of the returns-to-scale model, we have also found that most of the HEIs lie on the Decreasing returns to Scale (DRS) subregion, implying that marginal improvements in performance might require a significant increase in inputs. This confirms that excelling in education is increasingly demanding as we move toward higher standards.

We have observed significant differences between the positions granted to one institution by the different rankings. Analysing them in terms of efficiencies, we found that universities securing high positions in the rankings might not operate in an efficient manner, and vice versa, and that the purpose of the ranking highly affects the positions awarded. While in rankings weights are subjectively defined, in the DEA methodology they are optimised for each HEI so that its efficiency score is maximised (output-oriented model), which leads to different results. This mismatch highlights the potential pitfalls of using weights when assessing the performance of HEIs while reinforcing the need for alternative analytical approaches to complement conventional rankings.

The use of DEA using historical data provided insight into how the efficiency of the HEIs changed over time, allowing for a deeper analysis and the observance of the impact that the application of new policies might bring. In a society where the education system is part of an open market, as in the UK, adequate use of resources is needed to be competitive. In this context, analysing the efficiency of HEIs is key for identifying sources of inefficiency and opportunities for improvements. The use of DEA, therefore, could become an alternative to traditional rankings and lead to better practices in education aiming at a continuous quality improvement.
6. References


Fig. 1 Classification of performance indicators into inputs and outputs.
<table>
<thead>
<tr>
<th>University</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aston</td>
<td>0.61</td>
<td>4.01</td>
</tr>
<tr>
<td>Bath</td>
<td>0.88</td>
<td>4.09</td>
</tr>
<tr>
<td>Birmingham</td>
<td>1.00</td>
<td>4.01</td>
</tr>
<tr>
<td>Bradford</td>
<td>0.28</td>
<td>4.44</td>
</tr>
<tr>
<td>Cambridge</td>
<td>0.089</td>
<td>4.35</td>
</tr>
<tr>
<td>Edinburgh</td>
<td>0.91</td>
<td>4.01</td>
</tr>
<tr>
<td>Heriot Watt</td>
<td>0.86</td>
<td>3.94</td>
</tr>
<tr>
<td>Hull</td>
<td>0.53</td>
<td>3.79</td>
</tr>
<tr>
<td>Imperial College</td>
<td>0.99</td>
<td>4.23</td>
</tr>
<tr>
<td>Lancaster</td>
<td>1.00</td>
<td>3.94</td>
</tr>
<tr>
<td>Leeds</td>
<td>0.83</td>
<td>3.81</td>
</tr>
<tr>
<td>London South Bank</td>
<td>0.91</td>
<td>4.11</td>
</tr>
<tr>
<td>Loughborough</td>
<td>0.93</td>
<td>4.32</td>
</tr>
<tr>
<td>Manchester</td>
<td>0.97</td>
<td>3.96</td>
</tr>
<tr>
<td>Newcastle</td>
<td>0.74</td>
<td>3.82</td>
</tr>
<tr>
<td>Nottingham</td>
<td>0.86</td>
<td>4.14</td>
</tr>
<tr>
<td>Portsmouth</td>
<td>0.58</td>
<td>3.89</td>
</tr>
<tr>
<td>Queens Belfast</td>
<td>1.00</td>
<td>3.77</td>
</tr>
<tr>
<td>Sheffield</td>
<td>0.88</td>
<td>4.01</td>
</tr>
<tr>
<td>Strathclyde</td>
<td>0.87</td>
<td>3.55</td>
</tr>
<tr>
<td>Surrey</td>
<td>0.80</td>
<td>4.19</td>
</tr>
<tr>
<td>Swansea</td>
<td>0.83</td>
<td>4.04</td>
</tr>
<tr>
<td>Teesside</td>
<td>0.26</td>
<td>4.11</td>
</tr>
<tr>
<td>UCL</td>
<td>0.98</td>
<td>3.63</td>
</tr>
<tr>
<td>West of Scotland</td>
<td>0.45</td>
<td>4.28</td>
</tr>
</tbody>
</table>

**Fig. 2** Inputs and outputs employed in the assessment of the UK Chemical engineering departments.

(The-Guardian 2017; Complete University Guide 2018)
Fig. 3 A DEA illustrative example. Universities in blue are ‘efficient’, whereas those in red are ‘inefficient’. Inefficient universities can theoretically become efficient via projection onto the efficient frontier.
Fig. 4 Illustration of the concept of ‘returns to scale’ showing increasing returns to scale (IRS), constant returns to scale (CRS), and decreasing returns to scale (DRS) along A-B, B-C, and C-D, respectively.
Fig. 5 Radar diagram showing the inputs (blue) and outputs (red) for the 25 HEIs reported by The Guardian University Guide and the Complete University Guide in the subject of Chemical Engineering. Values next to HEIs represent the z-score scale of the corresponding axis.
Fig. 6 Spearman correlation coefficients for the inputs and outputs reported in The Guardian University Guide 2018. Coefficients marked with an asterisk indicate a significant correlation.
Fig. 7 Histogram of efficiency scores and geographical location of the 25 HEIs analysed. [The size of the bubble indicates the efficiency performance - the smaller the more efficient]
**Fig. 8** DEA and rankings scores for the chemical engineering departments.
Fig. 9 Improvement targets (%) for the inefficient HEIs
**Fig. 10** Peers used by the inefficient departments to attain their improved targets
Fig. 11 Sensitivity analysis of the super-efficiency score for the change of output indicators in ±20%.
Fig. 12 Results of the Malmquist productivity index for the UK Chemical Engineering Departments in the period from 2017 to 2019.