Formulae for determining elastic local buckling half-wavelengths of structural steel cross-sections

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Abstract

Formulae for determining elastic local buckling half-wavelengths of structural steel I-sections and box sections under compression, bending and combined loading are presented. Knowledge of local buckling half-wavelengths is useful for the direct definition of geometric imperfections in analytical and numerical models, as well as in a recently developed strain-based advanced analysis and design approach [1, 2]. The underlying concept is that the cross-section local buckling response is bound by the theoretical behaviour of the isolated cross-section plates with simply-supported and fixed boundary conditions along their adjoined edges. At the isolated plate level, expressions for the half-wavelength buckling coefficient \( k_{Lb} \), which defines the local buckling half-wavelength of a plate as a multiple of its width \( b \), taking into account the effects of the boundary conditions and applied loading, have been developed based on the results of finite strip analysis. At the cross-sectional level, element interaction is accounted for through an interaction coefficient \( \zeta \) that ranges between 0 and 1, corresponding to the upper (simply-supported) and lower (fixed) bound half-wavelength envelopes of the isolated cross-section plates. The predicted half-wavelengths have been compared against numerical values obtained from finite strip analyses performed on a range of standard European and American hot-rolled I-sections and square/rectangular hollow sections (SHS/RHS), as well as additional welded profiles. The proposed approach is shown to predict the cross-section local buckling half-wavelengths consistently to within 10% of the numerical results.

1 Introduction

Formulae for determining elastic local buckling stresses of full structural cross-sections, allowing for the interaction between individual plate elements, were developed in a recent study [1]. Equivalent
formulae, drawing on those presented in [1], for the determination of elastic local buckling half-wavelengths are presented herein. Knowledge of elastic local buckling half-wavelengths is important for the direct definition of geometric imperfections in analytical and numerical models [3, 4], as well as in the application of a recently developed strain-based structural design approach [2]. In this design approach, strain limits are used to mimic the effects of local buckling in beam finite element models. Thus, in addition to the familiar benefits of advanced analysis [5–9], it is now also possible to control the spread of plasticity and level of moment redistribution within a structure. Furthermore, the beneficial effects of local moment gradients can be exploited by applying the strain limit to a strain that is averaged over the local buckling half-wavelength, rather than simply to the peak strain. Local buckling half-wavelengths can be determined numerically, for example using the finite strip method implemented in software such as CUFSM [10], but explicit formulae, such as those developed herein, are often more convenient for practical design situations. The underlying concept behind the formulae developed in the present paper is that the cross-section local buckling half-wavelength lies between the theoretical buckling half-wavelengths of the isolated cross-section plates with simply-supported and fixed boundary conditions along their adjoined edges. The effects of element interaction are accounted for through an interaction coefficient, which is used to determine where between the lower and upper bounds the cross-section response lies.

In this paper, formulae for determining the elastic local buckling stresses of flat plates and full cross-sections are first reviewed. Then, the concept adopted herein to predict the full cross-section elastic local buckling half-wavelength is outlined and the developed formulae are presented. Finally, the accuracy of the proposed expressions is assessed against the results of finite strip analysis and their application is demonstrated through a set of worked examples.

2 Elastic local buckling of plates and cross-sections

The elastic buckling of flat plates has been studied extensively [11–13]. The elastic critical buckling stress of an isolated plate $\sigma_{cr,p}$ of width $b$ and thickness $t$, made from an elastic material with Young’s modulus $E$ and Poisson’s ratio $\nu$, is given by Eq. (1). The effects of the applied stress distribution and boundary conditions are accounted for through the buckling coefficient $k$. Expressions for the plate buckling coefficient $k$ are readily available for common cases, such as flat rectangular plates with assorted boundary conditions (simple, fixed or free) along the longitudinal edges subjected to various in-plane loading conditions [1, 14]. The associated local buckling half-wavelengths, which are inherently related to the local buckling stress, are typically expressed as a function of the plate width [11–13].

$$\sigma_{cr,p} = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

At the cross-sectional level, the individual cross-section plates interact with each other. The resulting
boundary conditions along the adjoined edges typically lie between simply-supported and fixed. Element interaction between the individual plates of structural profiles arises in two scenarios [1]: (1) in cross-sections composed of individual plates of different slenderness (i.e. the buckling stress of the isolated and simply-supported flange $\sigma_{cr,f}^{SS}$ and web $\sigma_{cr,w}^{SS}$ are not equal) and (2) in cross-sections composed of equally slender flange and web plates that buckle at different half-wavelengths (i.e. $L_{b,f}^{SS} \neq L_{b,w}^{SS}$). The resulting elastic local buckling stress is typically greater than the buckling stress of the critical isolated and simply-supported plate [1], while the associated half-wavelength is usually less than the maximum cross-section dimension [15]. The effect of element interaction is shown schematically in Fig. 1 with reference to an I-section subjected to combined compression and major axis bending.

![Figure 1: Element interaction in structural cross-sections experiencing local buckling: (a) isolated flange and web plates buckling at individual critical stresses (i.e. $\sigma_{cr,f}^{SS} \neq \sigma_{cr,w}^{SS}$) with individual local buckling half-wavelengths (i.e. $L_{b,f}^{SS} \neq L_{b,w}^{SS}$) and (b) the full cross-section buckling locally at a single critical stress $\sigma_{cr,cs}$ with a single bucking half-wavelength $L_{b,cs}$.](image)

While the effects of element interaction on the elastic local buckling stress of different cross-section profiles have been examined in previous research [1, 16–22], the effects of element interaction on the local buckling half-wavelength have rarely been considered. Shen and Wadee [23] derived an expression for predicting the elastic local buckling half-wavelength of rectangular hollow sections subjected to pure compression. In the plastic regime, Lay [24] derived an expression to predict the inelastic local buckling half-wavelength of compact I-sections subjected to pure major-axis bending by accounting for the rotational support that the web provides to the flanges.

Explicit expressions for calculating the full cross-section local buckling stress of structural steel profiles subjected to compression, bending or combined loading, allowing for the effects of element interaction, have been developed in [1]. The underlying concept, from which the expressions were derived, is that the buckling stress of the full cross-section $\sigma_{cr,cs}$ is bound by the buckling stresses of the critical isolated plates with simply-supported $\sigma_{cr,p}^{SS}$ and fixed $\sigma_{cr,p}^{F}$ boundary conditions along the adjoined edges. This concept is expressed in general form by Eq. (2), where the subscripts p and cs refer to the isolated...
plates and full cross-section respectively, the superscript SS and F refer to simply-supported and fixed boundary conditions along the adjoined edges respectively, and \( \zeta \) is a coefficient to account for the effects of element interaction. The interaction coefficient \( \zeta \) ranges from 0 to 1, corresponding to the limiting buckling stresses of the isolated critical plates with simply-supported or fixed boundary conditions along the adjoined longitudinal edges respectively.

\[
\sigma_{cr,cs} = \sigma_{cr,p}^{SS} + \zeta \left( \sigma_{cr,p}^{F} - \sigma_{cr,p}^{SS} \right)
\]  

(2)

Gardner et al. [1] used the governing parameter \( \phi \) to categorise the local buckling of cross-sections into the cases of either flange-critical (\( \phi < 1 \)) or web-critical (\( \phi \geq 1 \)). The parameter \( \phi \) is defined as the ratio of the buckling stress of the isolated flange \( \sigma_{cr,f}^{SS} \) to that of the web \( \sigma_{cr,w}^{SS} \) with simply-supported boundary conditions along the adjoined edges, as given in Eq. (3), where the subscripts f and w refer to the isolated flange and web plates respectively. Note that, as described by Gardner et al. [1], any difference in maximum applied compressive stress \( \sigma_{max} \) between the constituent cross-section plates must also be accounted for through the correction factors \( \beta_f \) and \( \beta_w \), given by Eqs. (4) and (5) respectively, where \( \sigma_{max,f} \) is the maximum compressive stress in the flange, \( \sigma_{max,w} \) is the maximum compressive stress in the web and \( \sigma_{max,cs} \) is the maximum compressive stress in the cross-section i.e. \( \sigma_{max,cs} = \max(\sigma_{max,f}, \sigma_{max,w}) \).

Typically, the maximum compressive stresses in the flange and web plates are equal e.g. box sections under any loading condition and I-sections under compression or bending about the major axis; in these common cases, both \( \beta_f \) and \( \beta_w \) are equal to unity and Eq. (3) simplifies to Eq. (6).

\[
\phi = \frac{\beta_f \sigma_{cr,f}^{SS}}{\beta_w \sigma_{cr,w}^{SS}} = \left( \frac{\sigma_{cr,f}^{SS}}{\sigma_{cr,w}^{SS}} \right) \left( \frac{\sigma_{max,w}}{\sigma_{max,f}} \right)
\]

(3)

\[
\beta_f = \frac{\sigma_{max,cs}}{\sigma_{max,f}}
\]

(4)

\[
\beta_w = \frac{\sigma_{max,cs}}{\sigma_{max,w}}
\]

(5)

\[
\phi = \frac{\sigma_{cr,f}^{SS}}{\sigma_{cr,w}^{SS}} \quad \text{if} \quad \sigma_{max,f} = \sigma_{max,w}
\]

(6)

The lower and upper bound stresses \( \sigma_{cr,p}^{SS} \) and \( \sigma_{cr,p}^{F} \) are defined as the minimum buckling stress of the isolated flange and web with simply-supported and fixed boundary conditions along the adjoined edges respectively, as given by Eqs. (7) and (8).

\[
\sigma_{cr,p}^{SS} = \min \left( \beta_f \sigma_{cr,f}^{SS}, \beta_w \sigma_{cr,w}^{SS} \right)
\]

(7)
\[ \sigma_{cr,p}^F = \min (\beta_f \sigma_{cr,f}^F, \beta_w \sigma_{cr,w}^F) \] (8)

3 Finite strip analysis

The elastic local buckling stresses and half-wavelengths of 1460 European and American square/rectangular hollow sections (SHS/RHS) and I-/H-sections (generally referred to hereinafter simply as I-sections) were obtained through finite strip analysis (FSA) using the finite strip software CUFSM v4.03 [10]. These data are used in Section 4 to underpin the development of predictive expressions for the elastic local buckling half-wavelength of full structural cross-sections and in Section 5 to evaluate the accuracy of the proposed formulae. In the analyses, each cross-section was simplified to its centreline geometry and individual cross-section plates were discretised into 10 strips. A summary of the considered profiles and the limiting normalised dimensions of the assessed cross-sections are given in Table 1. A Young’s modulus of \( E = 210000 \text{ MPa} \), a shear modulus \( G = 81000 \text{ MPa} \) and Poisson’s ratio \( \nu = 0.3 \) were used. Similar to Seif and Schafer [22], the unconstrained finite strip method was used to capture all the possible buckling modes that the cross-sections may experience; this led, in a small number of cases, to cross-section buckling stresses marginally below the theoretical plate buckling stress of the isolated critical plate with simply-supported boundary conditions [1]. Each cross-section was analysed under 17 combinations of axial compression and bending about either the major or minor axis, yielding a total of almost 45000 data points. The local buckling half-wavelengths were extracted from the so-called signature curve [25] at the minima corresponding to local buckling, which typically arose at lengths shorter than the maximum cross-section dimensions [15].

Table 1: Analysed cross-section profiles and range of normalised cross-section dimensions considered.

<table>
<thead>
<tr>
<th>Profile</th>
<th>Sections analysed</th>
<th>Number of sections</th>
<th>Normalised dimension range</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHS/RHS</td>
<td>European: SHS, RHS</td>
<td>376</td>
<td>1.00 ( \leq \frac{h}{t_f} \leq 6.25 )</td>
</tr>
<tr>
<td></td>
<td>American: HSS</td>
<td>367</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Welded sections ((t_f \neq t_w))</td>
<td>179</td>
<td></td>
</tr>
<tr>
<td>I/H</td>
<td>European: IPE, HEA, HEB, UKC, UKB</td>
<td>198</td>
<td>0.89 ( \leq \frac{h}{t_w} \leq 6.36 )</td>
</tr>
<tr>
<td></td>
<td>American: W, M, S, HP</td>
<td>340</td>
<td></td>
</tr>
</tbody>
</table>
4 Cross-section elastic local buckling half-wavelengths: Concept and formulae

4.1 Introduction and overview of concept

In this section, the adopted concept and expressions for the determination of the full cross-section local buckling half-wavelengths are presented. Analogous to the local buckling stress [1], it is assumed that the half-wavelength of the full cross-section $L_{b,cs}$ lies between the half-wavelength envelopes of the isolated critical plates with simply-supported $L_{b,p}^{SS}$ and fixed $L_{b,p}^F$ boundary conditions along the adjoined edges. The local buckling half-wavelength of the full cross-section $L_{b,cs}$ is expressed in general form by Eq. (9), where $\zeta$ is the interaction coefficient introduced in Section 2 that ranges from 0 to 1 to account for the level of element interaction.

$$L_{b,cs} = L_{b,p}^{SS} - \zeta (L_{b,p}^{SS} - L_{b,p}^F) \quad \text{where} \quad 0 \leq \zeta \leq 1$$  \hspace{1cm} (9)

Contrary to Eq. (2) for the calculation of the local buckling stress, the interaction term in Eq. (9) is subtracted from the simply-supported half-wavelength envelope $L_{b,p}^{SS}$. This is because the buckling half-wavelength $L_{b,p}$ of an isolated plate decreases as the edge restraint along the unloaded edges increases from simply-supported to fixed; hence, the simply-supported boundary conditions form the upper bound half-wavelength envelope. Expressions for the isolated plate buckling half-wavelengths and the interaction coefficient $\zeta$ are presented in the following subsections.

4.2 Lower and upper bound elastic buckling half-wavelengths of isolated plates

The adopted concept used to predict the cross-section local buckling half-wavelength requires knowledge of the buckling half-wavelengths of the isolated critical cross-section plates with simply-supported and fixed boundary conditions along their adjoined edges. For an isolated plate of width $b_p$, the local buckling half-wavelength can be defined in general form by Eq. (10), where $k_{lb}$ is a coefficient to account for the boundary conditions and applied stress distribution. The coefficient $k_{lb}$ is analogous to the buckling coefficient $k$ in Eq. (1) used to predict the local buckling stress $\sigma_{cr,p}$ of a plate.

$$L_{b,p} = k_{lb} b_p$$  \hspace{1cm} (10)

Simple expressions for the half-wavelength coefficient $k_{lb}$ have been derived based on the results of finite strip analyses performed on a series of rectangular plates with different boundary and loading conditions. The generated finite strip data and calibrated expressions for the half-wavelength coefficients
are shown in Fig. 2 for internal plates and Figs. 3 and 4 for outstand plates; note the internal and outstand elements are often referred to as stiffened and unstiffened elements in North American specifications. A summary of the expressions for $k_{Lb}$ is provided in Table 2, where $\psi = \sigma_2 / \sigma_1$ defines the stress distribution along the loaded edges of the plate, in which $\sigma_1$ is the maximum compressive stress (with compression taken as positive) and $\sigma_2$ is the minimum compressive or maximum tensile stress. Note that if the condition $\sigma_1 > 0$ is not satisfied then the element is subjected exclusively to tension and hence local buckling will not occur. The flange and web plates are labelled, irrespective of the applied loading direction, as shown in Table 3. For the common cases of plates under uniform compression or bending, the values of $k_{Lb}$ given in Table 2 accord with those previously presented [11,13].

Table 2: Proposed expressions for the plate buckling half-wavelength coefficient $k_{Lb}$ for internal and outstand plates with simply-supported and fixed edge conditions.

<table>
<thead>
<tr>
<th>Stress distribution</th>
<th>$\psi = \sigma_2 / \sigma_1$</th>
<th>Simply-supported edges</th>
<th>Fixed edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal plates</td>
<td>$\sigma_1$ $\sigma_2$</td>
<td>$1 \geq \psi \geq 0.25$</td>
<td>$k_{Lb} = 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.25 &gt; \psi \geq -1$</td>
<td>$k_{Lb} = 1 - 0.21(\psi - 0.25)^2$</td>
</tr>
<tr>
<td>Outstand plates</td>
<td>$\sigma_1$ $\sigma_2$</td>
<td>$1 \geq \psi \geq -1$</td>
<td>$k_{Lb} = \text{member length}^b$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 &gt; \psi \geq -1$</td>
<td>$k_{Lb} = \frac{0.818}{0.221 - \psi}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-1$</td>
<td>$k_{Lb} = 0.67$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-1 &gt; \psi \geq -3$</td>
<td>$k_{Lb} = 0.06\psi^2 + 0.39\psi + 1$</td>
</tr>
</tbody>
</table>

a Compression taken as positive (shaded grey).
b See Section 4.5.1 for practical limits to be used for isolated flanges of I-sections.

Table 3: Definitions of flange (f) and web (w) plates for each cross-section profile, irrespective of loading condition.

<table>
<thead>
<tr>
<th>Section</th>
<th>SHS/RHS</th>
<th>I-sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flange (f)</td>
<td>f</td>
<td>f</td>
</tr>
<tr>
<td>Web (w)</td>
<td>w</td>
<td>w</td>
</tr>
</tbody>
</table>
Figure 2: Local buckling half-wavelength coefficient $k_{LB}$ for internal plates with simply-supported and fixed boundary conditions along the adjoined edges.

Figure 3: Local buckling half-wavelength coefficient $k_{LB}$ for outstand plates with simply-supported and fixed boundary conditions along the adjoined edges and maximum compression at the supported edge.
Contrary to the elastic local buckling stress, the corresponding half-wavelength of an isolated plate restrained on both unloaded edges decreases as the plate is subjected to increasingly non-uniform compression (i.e. as $\psi$ reduces), as shown in Fig. 2. The half-wavelength also decreases as the level of edge restraint increases from simply-supported to fixed. Similar behaviour is observed in outstand plates where the maximum compression occurs at the supported edge, as shown in Fig. 3.

Simply-supported outstand plates with the maximum compression at the free edge present a special case, because the signature curve for these types of plates does not exhibit a local buckling minimum. Instead, as the plate (member) length increases, the half-wavelength follows and the local buckling stress asymptotically approaches a limiting value corresponding to a buckling coefficient of $k = 0.43$ [13]. The upper bound half-wavelength for simply-supported outstand plates is therefore equal to the plate (member) length, as shown for large $\psi$ values in Fig. 3. For the same reason, only the results for outstand plates with fixed boundary conditions along the supported edge are shown in Fig. 4, as the simply-supported plates have an undefined local buckling half-wavelength equal to the member length for all values of $\psi$. This would have the undesirable consequence of different half-wavelength predictions for members of the same cross-section but unequal lengths. However, I-sections, which contain outstand plates, typically exhibit distinct local buckling minima at practical half-wavelengths, implying that local buckling is not overly dependent on the member length – this is due to the restraining influence of the web. Practical upper bound limits for the simply-supported outstand flanges of I-sections can therefore be defined, as presented in Section 4.5.1. Note that in the case where the free edge is subjected to tension (i.e. $\psi < 0$ in Fig. 3), the (tensioned) free edge begins to behave like a supported edge and distinct local buckling half-wavelengths exist. This behaviour is shown in the local buckling modes depicted in Fig. 3 for the case of $\psi = -0.25$. 

Figure 4: Local buckling half-wavelength coefficient $k_{lb}$ for outstand plates with fixed boundary conditions along the adjoined edges and maximum compression at the free edge.
4.3 Half-wavelength envelope

Gardner et al. [1] showed that the elastic local buckling stress of a structural cross-section is bound by the envelopes defined by the isolated critical cross-section plates with simply-supported and fixed boundary conditions along the adjoined edges. The transition from web-critical to flange-critical local buckling occurs when the buckling stresses of the isolated flange and web plates with simply-supported boundary conditions are equal (i.e. when $\sigma_{cr,f}^{SS} = \sigma_{cr,w}^{SS}$ and hence $\phi = \sigma_{cr,f}^{SS} / \sigma_{cr,w}^{SS} = 1$). However, in this condition (i.e. $\phi = 1$), there is no requirement for the the associated half-wavelengths $L_{b,f}^{SS}$ and $L_{b,w}^{SS}$ to be equal. In fact, when $\phi = 1$, despite equal buckling stresses of the isolated plates, element interaction usually still exists due to the different half-wavelengths of the isolated plates [1]. The distinct half-wavelengths of the isolated flange and web result in a discontinuity in the half-wavelength envelope at $\phi = 1$, as shown schematically in Fig. 5(a). However, following the proposed concept, continuous lower and upper bound half-wavelength envelopes are required to predict the full cross-section response using the interaction coefficient $\zeta$.

![Figure 5](image)

(a) Discontinuity in half-wavelength envelope
(b) Proposed continuous half-wavelength envelope

Figure 5: Schematic representation of the local buckling half-wavelength envelope: (a) the discontinuity in the half-wavelengths of the isolated cross-section plates at $\phi = 1$ is overcome through (b) a weighted average envelope that considers the half-wavelength of both isolated plates.

To overcome the local buckling half-wavelength discontinuity at $\phi = 1$, a weighted average envelope that transitions smoothly from the isolated flange to the isolated web local buckling half-wavelength is used, as shown in Fig. 5(b). The continuous envelope is defined such that the flange half-wavelength dominates for $\phi \ll 1$ and the web half-wavelength is dominant for $\phi \gg 1$. The general form of the continuous half-wavelength envelopes for simply-supported $L_{b,p}^{SS}$ and fixed $L_{b,p}^{F}$ boundary conditions are defined by Eqs. (11) and (12) respectively, where $\eta$ is a cross-section profile dependent transition function that ranges from $0 \leq \eta \leq 1$. When $\eta = 0$, the upper and lower half-wavelength envelopes follow the half-wavelength of the isolated flange with simply-supported and fixed boundary conditions respectively.
Similarly, for $\eta = 1$, the envelopes are defined by the isolated web. At $\eta = 0.5$, the cross-section envelopes are equally dependent on the isolated flange and web. Note that the centreline dimensions of the constituent cross-section plates should be used to determine the upper and lower bounds.

\[
L_{b,p}^{SS} = L_{b,w}^{SS}\eta + L_{b,f}^{SS}(1 - \eta) \tag{11}
\]

\[
L_{b,p}^{F} = L_{b,w}^{F}\eta + L_{b,f}^{F}(1 - \eta) \tag{12}
\]

The expressions for the transition function $\eta$ for SHS/RHS and I-sections are given in Table 4 and shown graphically in Fig. 6. The flanges and webs of SHS/RHS are both internal plate elements and hence the transition function for these cross-sections is symmetrical (i.e. $\eta(1/\phi) = 1 - \eta(\phi)$) and centred at $\phi = 1$ (i.e. $\eta = 0.5$ when $\phi = 1$). To account for the different interaction between the outstand flanges and web of I-sections, the transition function for I-sections is centred at $\phi = 1.5$. An additional parameter $a_1$ (in which $\psi_f = \sigma_2/\sigma_1$ is the stress ratio across the flange width) is required to account for the shift in transition region for I-sections subjected to combined compression and minor axis bending (see Section 4.5.2 for further explanation).

The effect of the transition region is considered to be significant in the range of $0.37 \geq \phi \geq 2.70$ for SHS/RHS, $0.87 \geq \phi \geq 3.17$ for I-sections subjected to combined compression plus major axis bending and $0.0 \geq \phi \geq 3.17$ for I-sections subjected to combined compression plus minor axis bending. Outside these ranges, the influence of the non-critical plate on the half-wavelength envelope is less than 5% and for simplicity the half-wavelength envelopes of the critical isolated plate may be used for hand-calculations.

Table 4: Proposed expressions for the transition function $\eta$ for SHS/RHS and I-sections subjected to combined compression plus major axis $(N + M_y)$ or minor axis $(N + M_z)$ bending.

<table>
<thead>
<tr>
<th>Profile</th>
<th>Load case</th>
<th>$N + M_y$</th>
<th>$N + M_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHS/RHS</td>
<td>$\eta = 1 - \frac{1}{\phi^3 + 1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I-sections</td>
<td>$\eta = 1 - \frac{1}{(\phi - 0.5)^3 + 1}$ $\geq 0$</td>
<td>$\eta = 1 - \frac{1}{(\phi - 0.5a_1)^3 + 1}$ $\geq 0$</td>
<td>where $a_1 = 2\psi_f - 1 \geq -0.6$</td>
</tr>
</tbody>
</table>
4.4 Interaction coefficient $\zeta$

The level of element interaction for each cross-section is defined through the interaction coefficient $\zeta$. Explicit functions for $\zeta$ to define the stress based level of element interaction (i.e. to determine $\sigma_{cr,cs}$) have been fitted to back-calculated results obtained from finite strip analyses performed on structural cross-sections [1]. It is initially assumed that the level of element interaction is the same in the buckling half-wavelength domain as it is in the buckling stress domain. Hence, the same interaction coefficient $\zeta$ can be used to determine the full cross-section elastic local buckling stress [1] and half-wavelength. The validity of this assumption is assessed based on the behaviour of an isolated plate with different degrees of edge restraint between simply-supported and fixed subjected to pure compression. For each level of edge restraint, imposed through the application of rotational springs of different stiffness $k_{spring}$, the interaction coefficient $\zeta$ can be back-calculated from the buckling stress by rearranging Eq. (2) or from the half-wavelength by rearranging Eq. (9). The results are presented in Fig. 7, where it can be seen that the level of interaction in both the buckling stress and half-wavelength domains are very similar, and hence the above assumption is considered acceptable. The expressions, developed in [1], for calculating $\zeta$ are provided in Tables 5 and 6 for SHS/RHS and I-sections respectively.
Figure 7: Back-calculated interaction coefficients $\zeta$ based on the local buckling stress and half-wavelength for an isolated internal plate subjected to pure compression and varying levels of elastic edge restraint.
Table 5: Interaction coefficient $\zeta$ for square and rectangular hollow sections.

<table>
<thead>
<tr>
<th>Section</th>
<th>Geometry$^a$</th>
<th>Load case</th>
<th>Flange critical ($\phi &lt; 1$)</th>
<th>Web critical ($\phi \geq 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHS/RHS</td>
<td>$\sigma_{1,f}$ $\sigma_{2,f}$ $\sigma_{1,w}$ $\sigma_{2,w}$</td>
<td>Compression and major axis bending</td>
<td>$\zeta = \frac{t_w}{t_f} \left(0.24 - a_r \phi\right)^{0.6}$</td>
<td>$\zeta = \frac{t_f}{t_w} \left(0.53 - a_w \phi\right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_r = 0.24 - 0.1 \left(\frac{t_f}{t_w}\right)^2 \left(\frac{h}{b} - 1\right)$</td>
<td>$a_w = 0.63 - 0.1 \frac{h}{b}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>but $a_r \leq 0.24$</td>
<td>but $a_w \leq 0.53$</td>
</tr>
</tbody>
</table>

Compression is positive and shaded grey.

For hot-rolled SHS/RHS with $t_f = t_w$, and for welded SHS and RHS with $t_f \geq t_w$, the flange is never critical under compression and minor axis bending loading cases.

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$^a$ Compression is positive and shaded grey.

$^b$ For hot-rolled SHS/RHS with $t_f = t_w$, and for welded SHS and RHS with $t_f \geq t_w$, the flange is never critical under compression and minor axis bending loading cases.
**Table 6: Interaction coefficient ζ for I- and H-sections.**

<table>
<thead>
<tr>
<th>Section</th>
<th>Geometry(^a) (only for (t_f \geq t_w))</th>
<th>Load case</th>
<th>Flange critical ((\phi &lt; 1))</th>
<th>Web critical ((\phi \geq 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>I/H</td>
<td><img src="image" alt="Diagram of I-section" /></td>
<td>Compression and major axis bending</td>
<td>(\zeta = 0.15 \frac{t_f}{t_w} \phi \geq \frac{t_w}{t_f} (0.4 - 0.25\phi))</td>
<td>(\zeta = \frac{t_f}{t_w} \left(0.45 - \frac{0.3}{\phi^2}\right))</td>
</tr>
</tbody>
</table>

\(a_f = 0.9 - 0.75\psi_f \leq 0.35\)
\(a_{f1} = 1 - 0.6\psi_f \leq 0.7\)

\(\zeta = a_{f} \frac{t_f}{t_w} \phi \geq \frac{t_w}{t_f} (a_{f1} - 0.25\phi)\)

\(\zeta = a_{w} \frac{t_f}{t_w} \left(0.45 - \frac{a_w}{\phi^2}\right)\)

\(a_w = 0.8\psi_f - 0.5 \geq 0.1\)

\(\sigma_{2,f} = \text{stress in web}\)

\(^a\) Compression is positive and shaded grey.
4.5 Further considerations for I-sections

For I-sections, the adopted concept of defining an envelope around the cross-section local buckling response is somewhat challenging due to the undefined upper bound for simply-supported outstand flanges, as mentioned in Section 4.2. Additionally, it was found that the variation in local buckling half-wavelength of I-sections is significant depending on the axis of bending. These two issues are addressed in the following two subsections.

4.5.1 Practical upper bound half-wavelength limits for outstand flanges

An upper bound half-wavelength coefficient \( k_{Lb,f} \) is sought for isolated simply-supported outstand flanges to enable the conservative prediction of the buckling half-wavelength of I-sections, irrespective of the member length. It was observed that the upper bound half-wavelength of the flange plate is influenced by the geometry of the cross-section, as well as the relative buckling stresses of the isolated flange and web plates (i.e. the ratio \( \phi \)). The proposed expression for \( k_{Lb,f}^{SS} \) is given by Eq. (13), with coefficient \( a_2 \) defined by Eq. (14). The cross-section geometry dependent limit in Eq. (13) (i.e. \( k_{Lb,f}^{SS} = 2.8 + 0.3a_2 \left( \frac{2h}{b} \right) \left( \frac{t_f}{t_w} \right) \)) was calibrated to the back-calculated upper bound values of \( k_{Lb,f}^{SS} \) obtained from the finite strip results for flange critical (i.e. \( \phi < 1 \)) I-sections subjected to combined compression and major axis bending. As shown in Fig. 8, the half-wavelength buckling coefficient \( k_{Lb,f}^{SS} \) increases with the normalised cross-section dimension \( (2h/b)(t_f/t_w) \), where \( b \) refers to the full flange width of an I-section as shown in Table 6. To capture the reduced level of element interaction, and the resulting increase in half-wavelength, in the transition region near \( \phi = 1 \), a second \( \phi \) dependent limit for the upper bound half-wavelength coefficient (i.e. \( k_{Lb,f}^{SS} = 2 + 3a_2\phi \)) is specified. The parameter \( a_2 \) accounts for the different behaviour observed for combined compression plus minor axis bending, as described in Section 4.5.2.

\[
\begin{align*}
  k_{Lb,f}^{SS} &= \max \left\{ \begin{array}{c} 
    2.8 + 0.3a_2 \left( \frac{2h}{b} \right) \left( \frac{t_f}{t_w} \right), \\
    2 + 3a_2\phi 
  \end{array} \right. \\
  a_2 &= \begin{cases} 
    1 & \text{for } N + M_y \\
    2\psi_f \leq 2.6 - 1.6\psi_f & \text{for } N + M_z 
  \end{cases}
\end{align*}
\]
Figure 8: Back-calculated upper bound half-wavelength coefficient $k_{SS}^{Lb,f}$ for the outstand plates for flange critical (i.e. $\phi < 1$) I-sections subjected to combined compression and major axis bending.

The effects of the cross-section geometry and $\phi$ dependent limits in Eq. (13) are illustrated in Fig. 9 for the example case of an HEA 200 section subjected to combined compression and major axis bending. Fig. 9(a) shows that the geometry dependent upper bound limit for $k_{SS}^{Lb,f}$ is able to accurately capture the buckling response of the HEA 200 for bending dominated load cases ($\psi_w < 0.4$). However, a constant limit does not account for the increasing half-wavelength as the level of edge restraint provided by the web decreases in compression dominated cases ($\psi_w > 0.4$). On the contrary, the $\phi$ dependent limit for $k_{SS}^{Lb,f}$ is able to capture the increasing half-wavelength in compression dominated cases, but yields erroneous results in bending dominated cases, as shown in Fig. 9(b). The full behaviour of the HEA 200 cross-section is accurately captured across the full loading range from axial compression ($\psi_w = 1$) to major axis bending ($\psi_w = -1$) when the greater of the cross-section geometry and $\phi$ dependent upper bound limit for $k_{SS}^{Lb,f}$ is considered, as shown in Fig. 9(c).
Figure 9: Effects of the upper bound half-wavelength coefficient $k_{l.b,f}^{SS}$ on the prediction of the half-wavelength for an HEA 200 subjected to combined compression and major axis bending; the greater of the (a) geometry dependent and (b) $\phi$ dependent upper bound half-wavelength limit is taken to (c) accurately capture the elastic local buckling half-wavelength of the full cross-section.

### 4.5.2 Combined compression and minor axis bending

From continuity, the predictive formulae for $L_{b,cs}$ for the cases of combined compression plus major axis bending and combined compression plus minor axis bending should yield the same result for the special case of pure compression. This is readily achieved for the doubly symmetric SHS and RHS, where the same symmetrical transition function (i.e. $\eta = 1 - (\phi^3 + 1)^{-1}$) is adopted for both principal loading directions. However, this is not the case for I-sections and the previously introduced expression requires modification to account for the different interaction between the flange and web plates subjected to loading about each principal axis. More specifically, two coefficients, $a_1$ and $a_2$, are introduced to account for the shift in transition region, as well as for the reduced upper bound half-wavelength of the outstand flange plate for I-sections subjected to combined compression and minor axis bending.

Fig. 10(a) shows the local buckling half-wavelength response for an UB 406×140×46 subjected to
combined compression and minor axis bending, as well as the predicted local buckling half-wavelengths using the transition function for I-sections loaded about the major axis (i.e. $a_1 = 1$ in the expression listed in Table 4). Fig. 10(a) shows that the predictions do not accurately capture the transition from web to flange dominated buckling half-wavelengths. The factor $a_1$ shifts the transition from web to flange dominated half-wavelength envelopes towards the bending dominated cases, thereby reflecting the cross-section response more accurately, as shown in Fig. 10(b). Note that the half-wavelength predictions in Fig. 10 include the modified upper bound half-wavelength for the isolated flange plate based on the coefficient $a_2$, the effects of which are described below.

![Figure 10: Effect of the coefficient $a_1$ on the local buckling half-wavelength prediction of I-sections subjected to combined compression and minor axis bending demonstrated on a UB 406×140×46: (a) the observed shift in transition region is accounted for by (b) including the coefficient $a_1$ in the transition function $\eta$, as defined in Table 4.](image)

Under pure minor axis bending, the web of an I-section is not subjected to any meaningful level of compression and is therefore not susceptible to local buckling. As a result, the elastic restraint provided to the outstand flanges increases, reducing the corresponding outstand flange half-wavelength. The coefficient $a_2$ is introduced to account for this observed change in behaviour, by modifying the practical upper bound half-wavelength buckling coefficient $k_{SS}^{Lb,f}$ for simply-supported outstand flange plates. Fig. 11(a) shows the local buckling half-wavelength response of an HEA 260 subjected to combined compression and minor axis bending. It can be seen that, when the upper bound half-wavelength is not modified by the coefficient $a_2$, the overall behaviour is not captured accurately, except for the case of pure compression. Fig. 11(b) shows that through the introduction of coefficient $a_2$, defined by Eq. (14), the general behaviour of the cross-section is reflected more accurately; both the increased half-wavelength under compression dominated loading, as well as the significantly reduced half-wavelength for minor axis bending dominated cases, are captured. Note that the coefficient $a_1$ is included in the transition function in Fig. 11.
Figure 11: Effect of the coefficient $a_2$ on the local buckling half-wavelength prediction of I-sections subjected to combined compression and minor axis bending demonstrated on an HEA 260: (a) the observed change in the upper bound half-wavelength of the outstand flange is accounted for by (b) including the coefficient $a_2$ in the buckling half-wavelength coefficient $k_{SS,Lb,f}^{SS}$, as defined in Eqs. (13) and (14).

For the case of combined compression and biaxial bending, a similar approach to that described in [1], whereby the non-critical plate is conservatively assumed to be in uniform compression, and thus providing less restraint to the critical plate than the actual case of biaxial bending case, may be adopted to estimate the local buckling half-wavelength. Further verification of this approach in the wavelength domain is recommended though.

5 Evaluation of elastic local buckling half-wavelength $L_{b,cs}$ predictions

In this section, the predictions for the full cross-section local buckling half-wavelength are assessed against the results of finite strip analyses. Comparisons are made for predictions based on the the critical buckling stress $\sigma_{cr,cs}$ obtained from CUFSM, as well as the predicted buckling stresses based on the expressions presented by Gardner et al. [1]. Probability distributions of the predicted half-wavelengths versus the numerical half-wavelengths obtained from CUFSM are shown in Table 7 for the considered cross-sections and load cases. A numerical summary of the comparisons is provided in Table 8. In both Tables 7 and 8, values less than 1.0 indicate conservative results (i.e. under-predictions of half-wavelengths), while lower coefficients of variation (CoV) indicate higher accuracy in the prediction of the finite strip results.

Overall, the proposed expressions lead to accurate predictions of the local buckling half-wavelengths of SHS/RHS and I-sections. When using the critical buckling stress obtained from CUFSM to determine the level of element interaction, the mean ratio of predicted-to-numerical local buckling half-wavelength for all analysed cross-sections is 1.000 with a CoV of 0.054. Using the local buckling stress obtained
from Gardner et al. [1] yields a marginally higher mean prediction of 1.018 and CoV of 0.063. This is explained by the fact that, on average, the buckling stress predictions from Gardner et al. [1] are slightly conservative and hence, the corresponding half-wavelength predictions are slightly larger. In general, the predictions for SHS/RHS are more accurate and show less scatter, with over 98% of the half-wavelength predictions being within ±10% of the numerical value. The scatter in the half-wavelength predictions for I-sections is slightly higher, with around 85% of all predictions being within ±10% of the CUFSM half-wavelengths. The increased scatter is mainly attributed to the difficulty in defining the upper bound half-wavelength for isolated outstand flanges in I-sections and the resulting approximations made.

Table 7: Probability distributions for predicted full cross-section local buckling half-wavelengths versus numerical (CUFSM) results.

<table>
<thead>
<tr>
<th>Section</th>
<th>Compression + major axis bending</th>
<th>Compression + minor axis bending</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHS/RHS</td>
<td><img src="image1.png" alt="Graphs" /></td>
<td><img src="image2.png" alt="Graphs" /></td>
</tr>
<tr>
<td>I-sections</td>
<td><img src="image3.png" alt="Graphs" /></td>
<td><img src="image4.png" alt="Graphs" /></td>
</tr>
</tbody>
</table>

Legend: Predictions based on $\sigma_{cr}$ from Gardner et al. [1]. Predictions based on $\sigma_{cr}$ from CUFSM.

Table 8: Assessment of accuracy of elastic buckling half-wavelength predictions using a back-calculated interaction coefficient $\zeta$ from CUFSM and the proposed expressions for $\zeta$ from Gardner et al. [1] versus the numerical (CUFSM) results.

<table>
<thead>
<tr>
<th>Section</th>
<th>Load case</th>
<th>$\sigma_{cr}$ from CUFSM</th>
<th>$\sigma_{cr}$ from Gardner et al. [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prediction / CUFSM</td>
<td></td>
<td>Prediction / CUFSM</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>CoV</td>
<td>within ±10%</td>
</tr>
<tr>
<td>SHS/RHS</td>
<td>$N + M_y$</td>
<td>1.010</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>$N + M_z$</td>
<td>1.039</td>
<td>0.022</td>
</tr>
<tr>
<td>I-sections</td>
<td>$N + M_y$</td>
<td>0.973</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>$N + M_z$</td>
<td>0.961</td>
<td>0.072</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>1.000</td>
<td>0.054</td>
</tr>
</tbody>
</table>

The local buckling half-wavelength responses for representative SHS and RHS subjected to combined
loading are shown in Fig. 12. The discontinuity in the local buckling half-wavelength of the isolated plates at \( \phi = 1 \) is shown in Fig. 12(a) and (b) for an RHS 500×300×10 and RHS 200×150×7.1 subjected to combined compression and major axis bending. The transition function \( \eta \), defined in Table 4, forms a continuous envelope across the discontinuity and provides clear upper and lower bounds to the numerically obtained half-wavelengths. Using these bounds, the interaction coefficient \( \zeta \) is able to accurately predict the full cross-section response, even in the transition region near \( \phi = 1 \). Fig. 12(a) and (b) also show that at \( \phi = 1 \), the level of element interaction is at a minimum since both the numerical and predicted full cross-section half-wavelengths are closest to the simply-supported upper bound envelope. No element interaction occurs in square hollow sections (SHS) subjected to pure compression and consequently the local buckling half-wavelength is equal to the (centreline) plate width, as shown in Fig. 12(c) for an SHS 70×70×4. For rectangular hollow sections (RHS) subjected to minor axis bending, the web (as defined in Table 3) is always critical and as a result the local buckling half-wavelength remains relatively constant, as shown in Fig. 12(d) for an RHS 180×100×8. Overall, the proposed approach is able to accurately predict the different types of local buckling behaviour of SHS and RHS subjected to combined loading about the major or minor axis.
Figure 12: Prediction of local buckling half-wavelength for (a) an RHS 300×300×10, (b) an RHS 200×150×7.1 and (c) an SHS 70×70×4 subjected to combined compression and major axis bending; as well as for (d) an RHS 180×100×8 subjected to combined compression and minor axis bending. Note that, where applicable, the lower and upper bound half-wavelengths of the isolated flange ($\phi < 1$) and web ($\phi \geq 1$) are shown for reference.

Fig. 13 shows the variation in local buckling half-wavelength for representative I- and H-sections under different loading conditions. Similar to SHS/RHS, the proposed isolated plate envelopes form clear upper and lower bounds to the cross-section response obtained from CUFSM. For I-sections subjected to combined compression and major axis bending, the transition functions accurately capture the change from web-dominated to flange-dominated buckling modes, as shown for an IPE 180 and an HEB 550 in Fig. 13(a) and (b) respectively. Similarly, the proposed adjustments described in Section 4.5.2 capture the different behaviour of I- and H-shaped sections subjected to combined compression and minor axis bending, as shown in Fig. 13(c) and (d) for an IPE 240 and a W8×31 respectively. Overall, the proposed approach is able to accurately predict the wide range of responses observed in I- and H-shaped cross-sections subjected to combined loading.
The probability distributions shown in Table 7 highlight less accurate half-wavelength predictions for some I-sections relative to the numerically obtained results from CUFSM. The source of these outlier results can be explained with reference to Fig. 14. The local buckling half-wavelengths of the majority of I-sections change gradually in response to changes in the applied stress distribution; typically, this change is from a web-dominated buckling mode under pure compression to a flange-dominated buckling mode under pure major axis bending. An example of this type of behaviour is shown in Fig. 14(a) on an IPE 140 cross-section through a series of signature curves obtained from CUFSM for different loading conditions. By linking the local buckling minima, the continuous change in local buckling half-wavelength can be visualised. The same type of behaviour is also shown in Fig. 13(a) on an IPE 180. However, some combinations of flange-to-web proportions result in a ‘jump’ in local buckling half-wavelength, despite a smooth and continuous change in local buckling stress, as shown in Fig. 14(b) for an HEB 450. This ‘jump’ is due to the existence of two local buckling modes with the same buckling stresses but distinct
half-wavelengths. The resulting discontinuity in half-wavelength is difficult to capture using the proposed concept and results in the over- and under-prediction of the local buckling half-wavelength in a small range of cases. It should be emphasised that only around 5% of the considered cross-sections exhibit this type of behaviour, and that for these cases, only a small range of load combinations are affected. Furthermore, the resulting discrepancies in half-wavelength predictions remain tolerable, with peak errors generally less than ±20% for CUFSM based $\sigma_{cr,cs}$ predictions and less than ±30% for predictions using the interaction coefficient $\zeta$ from [1].

![Signature curves and local buckling minima illustrating the change in local buckling half-wavelength for different load cases: (a) smooth change in half-wavelength for an IPE 140 and (b) a ‘jump’ in the response for an HEB 450.](image)

Figure 14: Signature curves and local buckling minima illustrating the change in local buckling half-wavelength for different load cases: (a) smooth change in half-wavelength for an IPE 140 and (b) a ‘jump’ in the response for an HEB 450.

6 Summary of proposals and worked examples

A summary of the developed method and calculation steps required to determine full cross-section elastic local buckling half-wavelengths $L_{b,cs}$ is provided in Table 9. The developed functions are a continuation of the work presented by the authors on the elastic local buckling stress of structural cross-sections [1] and also lend themselves to programming and tabulation. It should be emphasised that the functions have been calibrated based on a finite set of cross-sections (see Table 1) and that further investigation is recommended when applying the same concept to other types of cross-sections (e.g. monosymmetric I-sections). Two worked examples are presented to illustrate the application of the developed formulae for the determination of the local buckling half-wavelength of a cross-section. Worked example 1 considers an RHS 200×100×5 subjected to major axis bending and worked example 2 considers an HEB 140 subjected to minor axis bending. The examples have been chosen to highlight the different elements of the developed expressions, including the transition function $\eta$ to bridge the discontinuity from flange to web-critical local buckling half-wavelengths, as well as the modification factors required for I-sections under minor axis bending. Note that the centreline geometry of cross-sections is used in the worked
examples and the effects of fillets and corner radii are ignored.

Table 9: Overview of proposed method to determine the full cross-section elastic local buckling half-wavelength $L_{b,cs}$.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Establish the stress distributions $\psi_w$ and $\psi_f$ along the centreline dimensions of the web and flange plates.</td>
</tr>
<tr>
<td>2*</td>
<td>Calculate the elastic buckling stresses of the isolated web ($\sigma^{SS}<em>{cr,w}$ and $\sigma^{F}</em>{cr,w}$) and flange ($\sigma^{SS}<em>{cr,f}$ and $\sigma^{F}</em>{cr,f}$) with simply-supported and fixed boundary conditions from Eq. (1).</td>
</tr>
<tr>
<td>3*</td>
<td>If required, determine load correction factors $\beta_f$ and $\beta_w$ from Eqs. (4) and (5).</td>
</tr>
<tr>
<td>4*</td>
<td>Calculate $\phi = \frac{\beta_f \sigma^{SS}<em>{cr,f}}{\beta_w \sigma^{SS}</em>{cr,w}}$ and determine which plate is critical: Flange is critical when $\phi &lt; 1$; web is critical when $\phi \geq 1$.</td>
</tr>
<tr>
<td>5</td>
<td>Determine the half-wavelength buckling coefficient $k_{L_b}$ for the isolated web and flange plates with simply-supported and fixed boundary conditions from Table 2.</td>
</tr>
<tr>
<td>6</td>
<td>Calculate the elastic local buckling half-wavelengths of the isolated web ($L^{SS}<em>{b,w}$ and $L^{F}</em>{b,w}$) and flange ($L^{SS}<em>{b,f}$ and $L^{F}</em>{b,f}$) with simply-supported and fixed boundary conditions from Eq. (10).</td>
</tr>
<tr>
<td>7</td>
<td>Determine the value of the transition function $\eta$ for the cross-section from Table 4.</td>
</tr>
<tr>
<td>8</td>
<td>Determine the upper and lower bound local buckling half-wavelength envelopes $L^{SS}<em>{b,p}$ and $L^{F}</em>{b,p}$ from the isolated plate half-wavelengths $L_{b,p}$ using Eqs. (11) and (12).</td>
</tr>
<tr>
<td>9</td>
<td>Calculate the interaction coefficient $\zeta$, given in Tables 5 and 6 for SHS/RHS and I-sections respectively.</td>
</tr>
<tr>
<td>10</td>
<td>Calculate the full cross-section local buckling half-wavelength from $L_{b,cs} = L^{SS}<em>{b,p} - \zeta(L^{SS}</em>{b,p} - L^{F}_{b,p})$.</td>
</tr>
</tbody>
</table>

*See [1] for further details.

6.1 Worked example 1: RHS under major axis bending

An RHS 200×100×5 in S355 steel with $E = 210000$ MPa and $\nu = 0.3$ is subjected to a pure major axis bending moment $M_{y,Ed} = 30$ kNm. Determine the full cross-section local buckling half-wavelength $L_{b,cs}$.

**RHS 200×100×5**

$A = 2900$ mm$^2$

$I_y = 1.52 \times 10^7$ mm$^4$

$f_y = 355$ MPa

**Loading**

$M_{y,Ed} = 30.0$ kNm

![Figure 15: Worked example 1: RHS 200×100×5 subjected to pure major axis bending.](image)

Firstly, determine the stress distribution $\psi_w$ along the web of the cross-section. Since the cross-section is symmetric about the axis of bending and subjected to pure bending, $\psi_w = -1$. Then, the local buckling response of the cross-section must be classified as flange- or web-critical, based on the parameter $\phi$. 

26
\[ \phi = 0.167 \left( \frac{h-t}{b-t} \right)^2 = 0.167 \left( \frac{200-5}{100-5} \right)^2 = 0.706 \] (15)

\[ \therefore \text{Flange is critical since } \phi < 1 \]

Next, determine the interaction coefficient \( \zeta \) based on the full cross-section local buckling stress \( \sigma_{ct,cs} \), accounting for the effects of element interaction. In this example, the expressions developed by the authors [1] for \( \zeta \) for the case of RHS subjected to major axis bending are used:

\[ \zeta = (0.24 - a_f \phi)^{0.6} = (0.24 - 0.218 \times 0.706)^{0.6} = 0.229 \]

where \( a_f = 0.24 - \left[ 0.1 \left( \frac{200}{100} - 1 \right) \right]^{1/0.6} = 0.218 \) (\(< 0.24\)) (16)

The calculations up until now follow the steps outlined in [1]. To determine the full cross-section local buckling half-wavelength, the lower and upper bound half-wavelength envelopes are required. The local buckling half-wavelengths of the isolated flange and web plates with simply-supported and fixed boundary conditions are calculated using the half-wavelength buckling coefficient \( k_{lb} \) given in Table 2.

For the flange (\( \psi_f = 1.0 \)), \( k_{lb,f}^{SS} = 1.0 \) and \( k_{lb,f}^{F} = 0.66 \). The centreline dimension of the isolated flange is equal to \( b_f = 100 - 5 = 95 \) mm. Hence:

\[ L_{b,f}^{SS} = k_{lb,f}^{SS} \times b_f = 1.0 \times 95 = 95 \text{ mm} \] (17)

\[ L_{b,f}^{F} = k_{lb,f}^{F} \times b_f = 0.66 \times 95 = 62.7 \text{ mm} \] (18)

For the web (\( \psi_w = -1.0 \)), \( k_{lb,w}^{SS} = 0.672 \) and \( k_{lb,w}^{F} = 0.473 \). The centreline dimension of the isolated web is equal to \( b_w = 200 - 5 = 195 \) mm. Hence:

\[ L_{b,w}^{SS} = k_{lb,w}^{SS} \times b_w = 0.672 \times 195 = 131.0 \text{ mm} \] (19)

\[ L_{b,w}^{F} = k_{lb,w}^{F} \times b_w = 0.473 \times 195 = 92.1 \text{ mm} \] (20)

A value of \( \phi = 0.706 \) lies within the limits for which the both the flange and web influence the local buckling half-wavelength envelopes of the full cross-section. Hence, the transition function \( \eta \) must be determined.

\[ \eta = 1 - \frac{1}{\phi^3 + 1} = 1 - \frac{1}{0.706^3 + 1} = 0.260 \] (21)

It follows that the half-wavelength envelopes for simply-supported and fixed boundary conditions are:

\[ L_{b,p}^{SS} = L_{b,w}^{SS} \eta + L_{b,f}^{SS} (1 - \eta) = 131.0 \times 0.260 + 95(1 - 0.260) = 104.4 \text{ mm} \] (22)
\[ L_{b,p}^F = L_{b,w}^F \eta + L_{b,f}^F (1 - \eta) = 92.1 \times 0.260 + 62.7(1 - 0.260) = 70.3 \text{ mm} \]  

(23)

Finally, the full cross-section local buckling half-wavelength \(L_{b,cs}\) is calculated.

\[ L_{b,cs} = 104.4 - 0.229(104.4 - 70.3) = 96.6 \text{ mm} \]  

(24)

The finite strip analysis result from CUFSM is 97.7 mm, only 1.13% higher than the predicted value using the presented formulae.

6.2 Worked example 2: HEB 140 subjected to minor axis bending

An HEB 140 in S355 steel with \(E = 210000 \text{ MPa}\) and \(\nu = 0.3\) is subjected to a pure minor axis bending moment \(M_{y,Ed} = 15 \text{ kNm}\). Determine the full cross-section local buckling half-wavelength \(L_{b,cs}\).

\[ \sigma_{1,w} = \sigma_{2,w} = \psi_w = 1.0 \]

\[ \beta_f = 1.0 \]  

(25)

\[ \beta_w = \frac{\sigma_{\max,cs}}{0} = \infty \text{ (undefined)} \]  

(26)

The buckling stresses of the isolated web and flange plates do not need to be calculated to determine that the cross-section response is flange-critical and that the parameter \(\phi\) is equal to:
\[ \phi = \frac{\sigma_{cr,f}^{SS}}{\sigma_{cr,w}^{SS}} \leq 0 \]  \hspace{1cm} (27)

∴ Flange is critical since \( \phi < 1 \)

Next, the interaction coefficient \( \zeta \) is determined from the appropriate column for minor axis bending in Table 6.

\[ \zeta = a_f \frac{t_f}{t_w} \phi \geq \frac{t_w}{l_f} (a_f - 0.25\phi) \]
\[ = 0 \geq \frac{7}{12} (0.7 - 0.25 \times 0) \]
\[ = 0.408 \]  \hspace{1cm} (28)

where \( a_f = 0.9 - 0.75\psi_f = 0.9 - 0.75 \times 0 = 0.9 \) but \( \leq 0.35 \) \( \therefore a_f = 0.35 \)
and \( a_f = 1 - 0.6\psi_f = 1 - 0.6 \times 0 = 1 \) but \( \leq 0.7 \) \( \therefore a_f = 0.7 \)

To determine the full cross-section local buckling half-wavelength, the lower and upper bound half-wavelength envelopes are required. The local buckling half-wavelengths of the isolated flange and web plates with simply-supported and fixed boundary conditions are calculated using the half-wavelength buckling coefficient \( k_{Lb} \) given in Table 2. Since the half-wavelength buckling coefficient for outstand flanges with simply-supported boundary conditions and compression at the free plate edge is not defined, the practical limit for I-section flanges presented in Section 4.5.1 is used. Hence,

\[ k_{Lb,f}^{SS} = \max \begin{cases} 2.8 + 0.3 \times 0 \times \left( \frac{2 \times 140}{140} \right) \left( \frac{12}{7} \right) = 2.8 \\ 2 + 3 \times 0 \times 0 = 2 \end{cases} \]
\[ = 2.8 \]  \hspace{1cm} (29)

where \( a_2 = 2\psi_f = 0 \) \( (\leq 2.6 - 1.6\psi_f \leq 2.6) \)

Hence, for the flange \( (\psi_f = 0) \), \( k_{Lb,f}^{SS} = 2.8 \) and \( k_{Lb,f}^{F} = 1.65 \). The centreline dimension of the isolated flange is equal to \( b_f = 140/2 = 70 \) mm. Thus:

\[ L_{SS}^{b,f} = k_{Lb,f}^{SS} \times b_f = 2.8 \times 70 = 196 \text{ mm} \]  \hspace{1cm} (30)
\[ L_{F}^{b,f} = k_{Lb,f}^{F} \times b_f = 1.65 \times 70 = 115.5 \text{ mm} \]  \hspace{1cm} (31)

For the web, the stress distribution is taken as \( \psi_w = 1 \), though recall that for the case of pure minor axis bending no external stresses are actually applied to the web. Hence, \( k_{Lb,w}^{SS} = 1 \) and \( k_{Lb,w}^{F} = 0.66 \). The centreline dimension of the isolated web is equal to \( b_w = 140 - 12 = 128 \) mm. Hence:

\[ L_{SS}^{b,w} = k_{Lb,w}^{SS} \times b_w = 1.0 \times 128 = 128 \text{ mm} \]  \hspace{1cm} (32)
\[
L_{b,w}^F = k_{Lb,w}^F \times b_w = 0.66 \times 128 = 84.5 \text{ mm} \tag{33}
\]

The transition function \( \eta \) for I-sections subjected to minor axis bending is calculated as follows for a value of \( \phi = 0 \) and \( \psi_f = 0 \).

\[
\eta = 1 - \frac{1}{(\phi - 0.5a_1)^3 + 1} = 1 - \frac{1}{(0 - 0.5 \times (-0.6))^3 + 1} = 0.026
\tag{34}
\]

where \( a_1 = 2\psi_f - 1 = 2 \times 0 - 1 = -1 \) but \( \geq -0.6 \quad \therefore a_1 = -0.6 \)

Hence, the upper and lower bound half-wavelength envelopes are:

\[
L_{b,p}^{SS} = L_{b,w}^{SS} \eta + L_{b,f}^{SS} (1 - \eta) = 128 \times 0.0 + 196(1 - 0.026) = 194.2 \text{ mm} \tag{35}
\]

\[
L_{b,p}^{F} = L_{b,w}^{F} \eta + L_{b,f}^{F} (1 - \eta) = 84.5 \times 0.0 + 115.5(1 - 0.026) = 114.7 \text{ mm} \tag{36}
\]

Finally, the full cross-section local buckling half-wavelength \( L_{b,cs} \) is calculated as:

\[
L_{b,cs} = 194.2 - 0.408 (194.2 - 114.7) = 161.7 \text{ mm} \tag{37}
\]

The finite strip analysis result from CUFSM is 158.4 mm. Hence, the predicted half-wavelength is only 2.1% longer than the numerical value.

## 7 Conclusions

Expressions for calculating the elastic local buckling half-wavelength of structural I-sections and SHS/RHS have been presented in this paper. The underlying concept assumes that the cross-section local buckling half-wavelength lies between the local buckling half-wavelengths of the critical isolated cross-section plates with simply-supported and fixed boundary conditions along the adjoined edges, analogous to the concept presented by the authors for the local buckling stress of cross-sections [1]. The proposed expressions account for the effects of element interaction through an interaction coefficient \( \zeta \) that ranges between 0 (no element interaction) and 1 (full element interaction). To determine the lower and upper bound half-wavelength envelopes, expressions for the buckling half-wavelength of isolated internal and outstand plates subjected to a range of loading conditions have been calibrated to the results of finite strip analysis. Transition functions have been devised for I-sections and SHS/RHS to form continuous half-wavelength envelopes for the full cross-section response. Almost 1500 European and American hot-rolled and welded steel profiles have been analysed under combined loading conditions using the finite strip method. The predicted half-wavelengths have been assessed against the finite strip results and typically predict the numerical value to within 10%. The developed expressions can be used for the prediction of elastic local buckling half-wavelengths of I- and box sections, suitable for the definition of geometric imperfections.
in analytical and numerical models and for use in a recently developed advanced strain-based design method [2].

References


