Does Size Matter? Bailouts with Large and Small Banks*

Eduardo Dávila[†] Ansgar Walther[‡]

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Abstract

We explore how large and small banks make funding decisions when system-wide bailouts are possible. We show that bank size, purely on strategic grounds, is a key determinant of banks' leverage choices, even when bailout policies treat large and small banks symmetrically. Large banks leverage more than small banks because they internalize that their decisions directly affect bailout policies. In equilibrium, this effect is amplified by strategic spillovers to small banks, since banks' leverage choices are strategic complements. Overall, the presence of large banks makes bailouts more likely. The optimal regulation features size-dependent policies that disproportionately restrict large banks' leverage.

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[†]Yale University and NBER. Email: eduardo.davila@yale.edu. Corresponding author. Address: Yale University, Department of Economics, 28 Hillhouse Avenue, New Haven, CT, 06520.

[‡]Imperial College London. Email: a.walther@imperial.ac.uk

1 Introduction

The differential treatment of large financial institutions has drawn substantial interest in recent financial regulatory discussions. In particular, several regulatory measures put in place after the 2008 financial crisis have singled out large banks as subjects of increased scrutiny. At the same time, the U.S. banking industry has become increasingly concentrated, as illustrated in Figure 1. The total number of U.S. banks has dropped from 25,000 in the 1920s, to 14,000 in the 1970s, to less than 6,000 as of today, while the top 10 bank holding companies now control more than 50% of total bank assets. In parallel, an active literature has highlighted that large firms have a disproportionate impact on aggregate outcomes if they are subject to granular firm-specific shocks (Gabaix, 2011). Taken together, these developments suggest that concerns about too-big-to-fail banks are now more important than ever before. However, since the consolidation wave coincided with a gradual overhaul of financial regulation, it is difficult to tell empirically whether bank size has had an independent effect on banks' behavior. And despite the prominence of the too-big-to-fail question in public debates – see, for instance, the forceful exposition of Stern and Feldman (2004) – the theoretical literature provides only limited guidance on the effects of bank size.

FIGURE 1 ABOUT HERE

In this paper, we formally study the effects of bank size on banks' funding decisions, and ultimately on system-wide risk. Rather than analyzing market power, we investigate the *strategic* effects of bank size in an environment with systemic bailouts. From a positive perspective, we seek to understand whether the current levels of bank concentration have consequences for aggregate banking stability. From a normative perspective, we seek to understand whether regulators directly need to address bank concentration *per se*, or whether size-independent regulations that apply to all banks are sufficient. We address both sets of questions theoretically and provide a quantitative illustration of the mechanisms at play.

A simple example illustrates the underlying mechanism behind our results. Is the government's decision problem different when it contemplates a bailout of 10 banks of size one versus a bailout of one bank of size 10? If we assume that the losses associated with bank failure are proportional to bank size, the naive answer to this question, from an ex-post perspective, is no. This can be called the too-many-to-fail critique to the too-big-to-fail problem, or the "clones" property of bailouts (e.g., Rogoff, 2010). The problem with this argument is that large banks are aware that their individual choices directly affect the likelihood and magnitude of a bailout, while small banks are individually unable to modify bailout policy responses. Therefore, anticipating the government's policy response and internalizing the effect of their size, large banks decide to be more aggressive at an ex-ante stage, increasing their leverage in equilibrium and, consequently, the likelihood of a bailout. Moreover, this effect is amplified by strategic spillovers to small banks. Aggressive leverage choices by large banks increase the implicit bailout subsidy for the banking sector as a whole. Small banks, encouraged by this shield, respond by increasing their leverage beyond what they would optimally choose in the absence of large banks. This mechanism is most salient when large banks experience granular shocks, that is, when idiosyncratic risk is not perfectly diversified within large banks. In this case, idiosyncratic shocks to a large bank become shocks to aggregate bank capital, strengthening strategic leverage incentives for large banks, spillovers among small banks and, consequently, increasing the likelihood of bailouts.

In our model, banks optimally choose their leverage trading off bankruptcy costs associated with default with costs of equity issuance, as in the canonical trade-off theory of capital structure. Ex-post, to avoid bankruptcy costs, the government may find it optimal to bail out banks, and we focus our attention on system-wide bailout policies. Examples of system-wide bailouts include asset purchase programs such as TARP, a significant share of which benefited smaller banks, or emergency lending programs with "broad-based eligibility" that are mandated by the Dodd-Frank Act. To highlight the strategic role of bank size, we make two conservative assumptions. First, banks have constant returns to scale, so that we can rule out size effects driven by technological differences. Second, the government decides its bailout policy considering all banks equally, that is, the government's objective is simply to minimize aggregate welfare losses, regardless of whether large or small institutions generate these losses. These assumptions guarantee that a too-many-to-fail scenario, in which multiple small banks fail, can provide as strong a motivation for a bailout as the failure of a single large institution of equal size.[1] If large banks enjoy cost advantages, or if distress in large banks is disproportionately costly, the strategic incentives that we highlight in this paper will be stronger. Our assumptions are therefore geared towards obtaining a lower bound on the relative importance of large banks.

Our first result shows that a large bank takes on more leverage than a small bank, purely on strategic grounds. Relative to a small bank, a large bank internalizes that its actions directly affect the magnitude of the government's optimal bailout, which generates an additional incentive to take on debt. Our second result establishes that small banks take on more leverage when large banks are

[1]This assumption is in line with recently developed empirical metrics of systemic risk, such as CoVaR (Adrian and Brunnermeier, 2016). CoVaR is defined so that many small banks generate the same measure of ex-post distress as one large bank, holding constant their asset exposures and leverage. Adrian and Brunnermeier (2016) further show that bank size predicts high contributions to CoVaR in the U.S. data. This is consistent with the equilibrium prediction of our model, since larger banks choose higher leverage and contribute more to systemic risk.

present. This behavior arises because banks' borrowing choices are strategic complements. Intuitively, when choosing their capital structure ex-ante, banks anticipate that, ex-post, the magnitude of any bailout increases with aggregate leverage. When other banks increase their leverage, each individual bank rationally anticipates larger bailouts, which provide an effective shield against bankruptcy costs and increase the marginal value of increasing its own leverage. Thus, leverage choices become strategic complements, introducing a coordination motive among banks. Because large banks have an additional incentive to take on more leverage, their presence makes small banks more willing to borrow. This strategic interaction generates amplification: Large banks take on yet more leverage in response to small banks' choices, small banks respond with a further round of increased borrowing, and so forth, until convergence. These effects arise even though bank managers do not directly exploit government bailouts. Instead, the incentive to take leverage is driven by the combination of managers' value maximizing goal with competitive capital markets, which value bank debt more generously when expected bailouts are larger.

When combined, our results about the behavior of large and small banks imply that system-wide leverage is higher in an economy in which large banks are present. Our results predict that all banks, large and small, take more leverage and default more frequently, also implying that the government relies on bailouts more frequently than when banks are small. We show that an increase in bank size is associated with a leverage multiplier that increases the quantitative significance of this mechanism.[2]

After establishing the positive predictions of bank size for the behavior of banks, we characterize the optimal ex-ante bank regulation. First, we show that when the government implements a constrained efficient outcome, all banks, large and small, choose the same level of leverage. This result implies that the optimal ex-ante regulation regarding quantities, which can be implemented through binding capital requirements, is identical for large and small banks. Subsequently, we show that the optimal regulation can be equivalently implemented with size-dependent Pigouvian taxes. Large banks are charged a supplement tax on borrowing, which counteracts their incentive to increase leverage so as to maximize government subsidies. Our normative results provide a formal rationale to regulate large banks differently from small banks, simply because of their size. Our results further imply that there is a natural interaction between financial regulation and policies that directly control industry structure (i.e., antitrust policy, merger regulation, etc.).

[2]The ideal experiment to test our results exploits a plausibly exogenous change in bank concentration and traces its impact on the leverage choices of all banks. It is not possible to assess our predictions using only time series correlations of concentration and leverage. For instance, while Figures 1a and 1b shows increases in concentration in the past 30 years, leverage regulation was tightened considerably over the same period as a result of the Basel Accords.

Even though in the baseline model banks' returns are only subject to aggregate risk, we show that our results extend easily to the more realistic scenario in which banks face idiosyncratic and aggregate risk. We can parametrize in our model whether idiosyncratic shocks cancel out when small banks merge into a large bank, by appealing to the Law of Large Numbers, or not, under a granularity hypothesis (Gabaix, 2011). Although our theoretical results remain valid in both scenarios, we show in our quantitative illustration of the results that the granular formulation – which is the empirically plausible one – substantially amplifies the strategic forces that we study in this paper.

Finally, we illustrate the predictions of our model when selecting parameters consistent with U.S. data over the period 1990Q1 to 2013Q4. We find an increasing and convex relation between the leverage choices of large and small banks and the share of assets held by the largest banks. We show that moderate increases in the share of assets held by the largest banks starting from the status quo, in which the top 5 largest banks hold around half of total bank assets, are associated with substantial increases in leverage by large and small banks. We also compute the associated "size tax" implied by the optimal policy, which increases when large banks become more prominent.

Related Literature This paper is most closely related to the growing literature that studies the implications of bank bailouts, and other system-wide government interventions in financial crises. The core idea underlying both earlier and most recent contributions, including those of Holmstrom and Tirole (1998), Freixas (1999), Schneider and Tornell (2004), Acharya and Yorulmazer (2007), Diamond and Rajan (2012), Bianchi (2016), Keister (2016), Nosal and Ordoñez (2016), Chari and Kehoe (2016), Bianchi and Mendoza (2017), and Gourinchas and Martin (2017) is that the lack of government commitment regarding ex-post optimal policies modifies the ex-ante behavior of banks, a phenomenon that is often described as moral hazard.

The strategic problem faced by banks in our model is most closely related to that in Farhi and Tirole (2012), who identify the strategic complementarity caused by systemic, non-targeted bailouts. They refer to this behavior as collective moral hazard. While our results build on theirs, we focus on size asymmetries, while they study an environment with symmetric (small) agents. Our results extend far beyond their informal discussion of size effects. Indeed, we show that bank size is relevant for bank decisions under the conservative assumption that the social consequences of bank failure (and the welfare weight on large banks) are independent of size. We therefore provide a distinct theory of how bank size influences bank decisions, which applies to a wider class of models.[3] Bianchi (2016) shows

[3]More broadly, our model and solution method contribute to the large literature on strategic complementarities (e.g., Cooper and John, 1988; Angeletos and Lian, 2016) and coordination games among heterogeneous agents (Corsetti et al., 2004; Sakovics and Steiner, 2012; Drozd and Serrano-Padial, 2016; Kacperczyk, Nosal and Sundaresan, 2017).

that non-targeted and systemic bailouts are preferred to targeted ones, since the latter exacerbate banks' ex-ante responses (moral hazard), and provides a full-fledged macroeconomic calibration. Our results highlight the importance of size asymmetries in a similar environment. Our results are also related to the growing literature on granular shocks, following Gabaix (2011).

Only a few papers refer to size asymmetries in the context of bank bailouts. Freixas (1999) rationalizes the presence of too-big-to-fail policies, through which a regulator responds more strongly to the actions of large banks, if the costs associated with bank failure are increasing in bank size. Acharya and Yorulmazer (2007) formalize the too-many-to-fail argument, showing that multiple failures by small banks can generate an identical bailout response as the failure of a single large bank. By design, our environment captures the too-many-to-fail effect. In their extension with asymmetric banks, the behavior of large banks does not affect the optimal bailout policy at the margin, which is crucial for our results. Nosal and Ordoñez (2016) show that uncertainty about the ex-post bailout policy can mitigate the ex-ante effects of lack of commitment, reducing the strength of strategic complementarities and consequently reducing the incentives of banks to take on excessive risk. They show that this effect is weakest for large banks. Stavrakeva (2018) analyzes fiscal subsidies in an environment with large but symmetric banks. The thorough reviews of Freixas and Rochet (2008) and Gorton and Winton (2003) do not discuss the effect of bank size on banks' funding decisions. Leaving aside the possibility of bank bailouts, there is scope to explore further how asymmetries in bank size affect bank's decisions. For instance, Hachem and Song (2017) show how large banks choose more liquid positions in an equilibrium model with endogenous interbank pricing and Egan, Hortaçsu and Matvos (2017) structurally estimate a strategic model of competition for bank deposits.

Within the sizable quantitative literature, the work of Cuciniello and Signoretti (2015) – see also Corbae (2015) – studies the effects of banks size in the context of the model of Corbae and D'Erasmo (2010), who provide a detailed quantitative analysis of banking dynamics in a strategic environment.

Finally, within the small but growing empirical literature that seeks to directly quantify the effects of bank size and government guarantees, the work of Kelly, Lustig and Van Nieuwerburgh (2016) is most relevant. By comparing the price of out-of-the-money put options on individual banks with the price of identical options on a bank index, they present evidence consistent with market participants who expect collective government guarantees, which is the starting point of our theory.

Outline Section 2 describes the environment and Section 3 characterizes the equilibrium of our model, introducing the main positive results. Section 4 conducts the normative analysis and describes the implication of our results for policy-making. Section 5 discusses idiosyncratic risk and granular shocks. Section 6 provides a quantitative illustration of our results, and Section 7 concludes. The Appendix contains all proofs and technical derivations.

2 Environment

Our model provides a parsimonious framework to study banks' borrowing decisions in the presence of government bailouts and heterogeneity in banks' size.

Agents and timing There are two dates $t = \{0, 1\}$ and a single consumption good (dollar), which serves as numeraire. There is a unit mass of risk-neutral financiers with a unit discount factor and a benevolent government. Financiers hold claims in a continuum of banks, which are indexed by $i \in [0, 1]$. We denote by $\mu(A)$ the share of capital stock managed by any (Borel) subset of banks $A \subset [0, 1]$. If bank i has positive point mass $\mu(i) > 0$, then we refer to it as a large bank. We refer to infinitesimal banks as small banks. Without loss of generality, we normalize total assets/capital held by banks to one unit. Figure 2 illustrates the timeline of events, which we proceed to describe.

FIGURE 2 ABOUT HERE

Banks' technology and capital structure At date 0, bank i sells claims to financiers by issuing debt with face value b^i and by selling equity. Banks choose a capital structure that maximizes market value $q^i b^i + e^i$, where q^i is the market price of debt and e^i is the market value of the bank's shares. Bond prices q^i and stock values e^i are endogenously determined in equilibrium. The funds raised by issuing debt and equity are used by the bank to make a fixed investment of one dollar per unit of capital at date 0.

At date 1, each bank's assets yield a random return $u \geq 0$ of date 1 dollars per unit of initial capital. The return u is common across banks. To clarify the exposition, we begin by analyzing the case where the aggregate shock u is the only source of uncertainty. This restriction does not affect our qualitative results. In Section 5, and in our quantitative assessment in Section 6, we additionally allow for idiosyncratic shocks to individual banks' returns.

At date 1, after u is realized and possibly after receiving a proportional government transfer $t^i \geq 0$, as described below, bank i's shareholders decide whether to default. If the bank defaults, shareholders receive nothing and creditors seize all of the bank's resources including government transfers and receive $\phi u + t^i$ per unit of capital, where the remainder $(1 - \phi) u$ measures the deadweight losses associated with default. If the bank does not default, creditors are paid b^i and shareholders receive the residual claim $(1 - \psi)\left(u + t^i - b^i\right)$ per unit of capital, where ψ captures the costs of equity issuance or tax advantages of debt. The positive costs of default, $1-\phi>0$, and of equity issuance, $\psi>0$, imply that the Modigliani and Miller (1958) theorem does not hold, and guarantee an interior capital structure decision (e.g., with $\psi=0$, the optimal choice would trivially be to issue only equity). Our setup therefore mirrors the classical "trade-off theory" of capital structure (Kraus and Litzenberger, 1973; Myers, 1984). Alternative assumptions that generate a well-defined capital structure (e.g., moral hazard among shareholders, a demand for "money-like" claims (Gorton and Pennacchi, 1990; Stein, 2012; DeAngelo and Stulz, 2015), or bank runs and market discipline (Diamond and Rajan, 2001)) would not modify our qualitative results.

A contentious issue is whether the costs of bank equity are private or social. We do not take a stand in this debate and allow for either case. We therefore assume that $\bar{\psi}\left(u+t^i-b^i\right)$ dollars are reimbursed to financiers as a lump sum if the bank does not default, where $0\leq\bar{\psi}\leq\psi$. The boundary case $\psi=\bar{\psi}$ corresponds to purely private costs, for example arising from tax considerations, while $\bar{\psi}<\psi$ implies that the private costs of bank equity issuance are also social costs.

To simplify our derivations, we assume that the costs of equity issuance materialize ex-post. We do not have a strong view on whether ex-ante or ex-post costs are more realistic. For example, if bank debt is "money-like", then the cost of issuing equity represents the cost of wasteful information acquisition after issuance. Similarly, in models with moral hazard (e.g., Innes, 1990), the cost of issuing outside equity (which deteriorates insiders' incentives) is in terms of lower average returns ex post. The tax-based private costs of equity finance are also paid after returns are realized. All of our results go through in the case where the costs of equity issuance are sunk ex ante, after a modification in the formulae describing optimal policy. In Section 4, we discuss this point in more detail.

Notice that asset values, as well as the costs of distress and equity issuance, exhibit constant returns and are independent of bank size. This allows us to define all bank-level (lower-case) variables *per unit of capital/assets*. For example, a large bank that manages a point mass $\mu\left(i\right)>0$ share of capital issues debt with face value $\mu\left(i\right)b^i$, earns a return $\mu\left(i\right)u$ and receives $\mu\left(i\right)t^i$ from the government. A subset S of small banks collectively issue debt with face value $\int_S b^i d\mu$, earn returns $u\mu\left(S\right)$ and receive transfers $\int_S t^i d\mu$. We can express aggregates of bank level variables a^i as the Lebesgue integral $\int_0^1 a^i d\mu$, which accounts for both large and small banks.

The assumption of constant returns reflects our goal to understand the role of banks' size separately, independently of technological differences. We also abstract from market power and model all banks as price-takers in funding and asset markets.[4]

Government policy The existence of deadweight losses associated with bank failure provides a rationale for a benevolent government to bail out banks at date 1. In this sense, the financial institutions in our model can represent both traditional banks and other levered institutions (e.g.: "shadow banks"), as long as widespread defaults cause social costs and induce the government to provide support. Formally, we allow the government to transfer funds to banks under the following three assumptions.

First, we assume that government transfers cannot be conditioned on bank characteristics. This assumption, which we discuss in more detail below, captures the fact that some bank support policies are provided to the banking system as a whole, consistently with empirical evidence in Kelly, Lustig and Van Nieuwerburgh (2016). Second, we assume that the government must decide on the level of the bailout transfer with imperfect information about the realization of the aggregate state. Formally, the government chooses a transfer $t(s)$ contingent on a signal s about the realization of aggregate returns u. Hence, each bank i receives a transfer $t^i = t(s)$ per unit of capital. This assumption captures the fact that bailout policies are often determined under uncertainty about fundamentals. From a formal standpoint, it guarantees that the problem solved by banks is smooth. Finally, we assume that government transfers are associated with a net deadweight loss of $\kappa(t)$ dollars, where $\kappa(t)$ is a weakly increasing and convex function that satisfies $\lim_{t\to\infty} \kappa(t) = \infty$. This assumption limits the magnitude of the optimal transfer chosen by the government. The deadweight loss of intervention can be literally interpreted as a fiscal distortion. A broader interpretation of $\kappa(t)$ not only captures the slack of the fiscal capacity of the government, but it can also capture the "type" of the government: hawkish vs. dovish (e.g., Diamond and Rajan, 2012).

Asset returns and government signals are drawn according to a joint distribution $F(u, s)$. We write $F_u(u|s)$ for the conditional distribution of asset values, with density $f_u(u|s)$, and $G(s)$ for the marginal distribution of signals. A mild regularity condition on the conditional c.d.f. $F_u(u|s)$ guarantees that the government chooses an interior transfer $t(s)$. Formally, we assume that the conditional density of asset returns satisfies the following condition for all marginal default states $u = b^i - t$:

$$\frac{d \log f_u(u|s)}{d \log u} = \frac{u f_u'(u|s)}{f_u(u|s)} > -1, \quad \forall s. \tag{1}$$

[4]Empirically, Kroszner and Strahan (2014) show that, despite the increase in nationwide banking concentration, local measures of market power have remained roughly constant over time. The empirical evidence on technological size effects is mixed, but overall suggests that large banks have weakly lower funding or operating costs than small ones (e.g. Gandhi and Lustig, 2015; Minton, Stulz and Taboada, 2017).

To understand this condition, suppose that the conditional density $f_u(u|s)$ is single-peaked in u. Then (1) holds strictly for all u below the conditional mode of the distribution (where $f_u' > 0$). Hence, this assumption holds as long as the conditional probability of default given public information is not too large. In the Appendix, we characterize the conditions under which the government's problem is well-behaved.

Equilibrium definition An equilibrium is defined as a set of bank capital structure decisions b^i and default decisions, prices for bank debt q^i and equity e^i, and a government bailout policy $t(s)$, such that (i) banks maximize their market value net of issuance costs, given the behavior of other banks and the government, (ii) financiers break even when purchasing debt or equity, and (iii) the government maximizes ex-post welfare at date 1 given their signal.

3 Equilibrium characterization

We characterize the equilibrium of the model in multiple steps. First, we characterize the ex-post optimal government policy. Next, we determine banks' default decisions at date 1 and their date 0 market value for a given government policy. Finally, we study banks' ex-ante funding decisions at date 0.

Ex-post optimal bailout policy

At date 1, the government observes the signal s of asset values and chooses transfers t to maximize expected social welfare. Given banks' borrowing choices $\boldsymbol{b} = (b^i)_{i=0}^1$, a choice of transfer t, and a signal s, we let $W_1(\boldsymbol{b}, t|s)$ denote expected social welfare from the perspective of date 1.

In our model, date 1 welfare corresponds to the sum of aggregate resources:

$$\begin{aligned} W_1(\boldsymbol{b}, t|s) = \mathbb{E}[u|s] - \kappa(t) \\ - (1-\phi)\int \Pr\left[\mathcal{D}^i|s\right]\mathbb{E}\left[u|\mathcal{D}^i, s\right] d\mu \\ - \left(\psi - \bar{\psi}\right)\int \Pr\left[\mathcal{N}^i|s\right]\mathbb{E}\left[t + u - b^i|\mathcal{N}^i, s\right] d\mu. \end{aligned} \tag{2}$$

where $\mathcal{D}^i$ denotes the event of default by bank i, and $\mathcal{N}^i$ is the complementary non-default region. The interpretation of Equation (2) is intuitive. The first line captures the present value of banks' assets and deadweight costs of taxation. The second and third lines respectively measure the deadweight costs of bank failure and equity issuance. Note that the latter accounts for the possibility that the costs of equity are partly or exclusively private costs when $\bar{\psi} > 0$.

Hence, the government's optimal bailout policy after observing the signal s is to maximize expected welfare. It corresponds to

$$t(\boldsymbol{b}|s) = \arg\max_{t\geq 0} W_1(\boldsymbol{b}, t|s). \tag{3}$$

The optimal bailout policy $t(\boldsymbol{b}|s)$ is characterized by the following first-order condition:

$$(1-\phi)\int f_u\left(b^i - t|s\right)\left(b^i - t\right)d\mu \leq \kappa'(t) + \left(\psi - \bar{\psi}\right)\int \Pr\left[\mathcal{N}^i|s\right]d\mu, \tag{4}$$

which is satisfied with equality when the optimal t is strictly positive. Intuitively, the left-hand side in Equation (4) measures the marginal benefit of transfers, which equals the marginal reduction in default costs. The right-hand side measures the marginal cost of the bailout which consists of two terms. First, the government incurs the direct marginal cost of taxation $\kappa'(t)$. Second, an indirect cost arises when a bailout is conducted but the bank remains solvent. In this case, the bailout constitutes a transfer to shareholders, who derive a lower marginal utility from this transfer due to the social costs $\left(\psi - \bar{\psi}\right)$ of equity. This second term only matters whenever bank i is solvent, which occurs with conditional probability $\Pr\left[\mathcal{N}^i|s\right]$. In the empirically relevant region where public news s is bad, this probability is low, and therefore bailouts are driven largely by the trade-off between the benefit of preventing bank failure and the cost of taxation.

The government's optimal response illustrates the key difference between small and large banks. A small bank's choice b^i has no impact on the integral in (4), and therefore no impact on ex-post optimal transfers. But if bank j is large – with point mass $\mu(j)$ – and the government chooses a strictly positive transfer, $t(\boldsymbol{b}|s) > 0$, after receiving a signal s, a marginal change in the large bank borrowing position b^j increases the optimal transfer, since

$$\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} \overset{\text{sign}}{=} f_u'\left(b^j - t|s\right)\left(b^j - t\right) + f_u\left(b^j - t|s\right) + \frac{\psi - \bar{\psi}}{1-\phi} f_u\left(b^j - t|s\right) > 0, \tag{5}$$

where the inequality is implied by our regularity condition. The formal derivation of (5) is in the Appendix, but the economics are clear. If a large bank takes more leverage, all else equal, the failure of this bank becomes more likely after adverse realizations of the public signal s. Therefore, to reduce the deadweight losses associated with bank failure, the government decides in favor of a larger bailout.

Banks' default decision and date 0 market value

At date 1, given a realization of banks' aggregate return u and after the government chooses a bailout policy $t(\boldsymbol{b}|s)$, a bank i defaults on its debt whenever $u + t(\boldsymbol{b}|s) < b^i$, and repays otherwise. As expected, holding u and $t(\boldsymbol{b}|s)$ constant, the likelihood of default increases with the level of bank borrowing b^i.

Formally, the payoff to debtholders corresponds to

$$\begin{cases} b^i, & \text{if } u + t(\boldsymbol{b}|s) \geq b^i \\ \phi u + t(\boldsymbol{b}|s), & \text{if } u + t(\boldsymbol{b}|s) < b^i, \end{cases} \tag{6}$$

while the payoff to shareholders corresponds to

$$\begin{cases} (1-\psi)\left(u + t(\boldsymbol{b}|s) - b^i\right), & \text{if } u + t(\boldsymbol{b}|s) \geq b^i \\ 0, & \text{if } u + t(\boldsymbol{b}|s) < b^i. \end{cases} \tag{7}$$

In equilibrium, because financiers are risk-neutral, the market value of bonds $q^i b^i$ is determined by the expected payoff to debtholders, while the market value of bank shares e^i is determined by the expected payoff to shareholders.

Therefore, taking expectations at date 0, the market value of a bank given a level of borrowing b^i and a set of possible transfers $t(\boldsymbol{b}|s)$ for each signal realization s, corresponds to

$$\begin{aligned} V\left(b^i, t(\boldsymbol{b}|s)\right) &\equiv q^i b^i + e^i \\ &= \mathbb{E}\left[u + t(\boldsymbol{b}|s)\right] - (1-\phi)\Pr\left[\mathcal{D}^i\right]\mathbb{E}\left[u|\mathcal{D}^i\right] - \psi \Pr\left[\mathcal{N}^i\right]\mathbb{E}\left[u + t(\boldsymbol{b}|s) - b^i|\mathcal{N}^i\right], \end{aligned} \tag{8}$$

where the optimal default and non-default regions are defined as

$$\begin{aligned} \mathcal{D}^i &= \left\{(u,s)|\, t(\boldsymbol{b}|s) + u < b^i\right\}, \\ \mathcal{N}^i &= \left\{(u,s)|\, t(\boldsymbol{b}|s) + u \geq b^i\right\}. \end{aligned} \tag{9}$$

Note that the expectations and probabilities in Equation (8) account for the realizations of the return u and the signal s received by the government. The value function $V\left(b^i, t(\boldsymbol{b}|s)\right)$ is therefore defined in terms of *all* possible realizations of these variables and of the corresponding government policy $t(\boldsymbol{b}|s)$. A detailed derivation of this function is in the Appendix. Equation (8) highlights that banks' leverage decisions are driven by competitive market forces. Bank managers do not directly benefit from government bailouts, and their objective function simply corresponds to their firm's market value at date 0. Nevertheless, markets generate implicit incentives to capture government bailouts, because the implicit subsidy is accounted for in security prices.

Equation (8), which effectively corresponds to the banks' objective function, further clarifies our previous remark on the difference between large and small banks. From a technological perspective, both large and small banks face an identical optimization problem. Large banks are different only in a strategic sense; unlike small banks, they expect their leverage choices to directly impact bailout policies $t(\boldsymbol{b}|s)$ in future states of the world. Equation (8) also highlights that banks' decisions are determined through classic trade-off theory: Only its second element, which corresponds to costs of distress, and its third element, which corresponds to the costs of equity issuance, are affected directly by the choice of b^i for a given value of $t(\boldsymbol{b}|s)$.

Banks' leverage choices

We now turn to banks' incentives when choosing their borrowing level b^i at date 0. Bank i's problem is

$$\max_{b^i \geq 0} V\left(b^i, t(\boldsymbol{b}|s)\right), \text{ subject to (3).} \tag{10}$$

The bank takes others' choices $(b^j)_{j \neq i}$ as given, and realizes that the government will respond to the collective choice $\boldsymbol{b}$ according to Equation (3). Then, it chooses its leverage b^i to maximize the joint value of debt and equity.

The first-order condition for a small bank i, which takes the government policy $t(\boldsymbol{b}|s)$ as given, is

$$\frac{\partial V\left(b^i, t(\boldsymbol{b}|s)\right)}{\partial b^i} = \psi \Pr\left[\mathcal{N}^i\right] - (1-\phi) \int\limits_{\partial \mathcal{D}^i} u dF = 0, \tag{11}$$

where $\partial D^i = \left\{(u,s) \,|t(\boldsymbol{b}|s) + u = b^i\right\}$ is the boundary of the default region.[5] This expression is familiar from the canonical trade-off theory of capital structure. The first term captures the deadweight costs of equity issuance, which encourage higher leverage and arise whenever the bank is in the solvent region $\mathcal{N}^i$. The second term measures the deadweight costs of default, which discourage higher leverage.

In addition to these terms, large banks internalize the indirect effect of its leverage on the optimal bailout $t(\boldsymbol{b}|s)$, as characterized in Equation (5). Therefore, the first-order condition for a large bank j is to set the *total* derivative of its value function equal to zero:

$$\begin{aligned}\frac{dV\left(b^j, t(\boldsymbol{b}|s)\right)}{db^j} = & \frac{\partial V\left(b^j, t(\boldsymbol{b}|s)\right)}{\partial b^j} \\ & + \mathbb{E}\left[\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j}\left(1 - \psi \mathbf{1}\left(\mathcal{N}^j\right)\right)\right] + (1-\phi) \int\limits_{\partial \mathcal{D}^j} \frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} u dF = 0,\end{aligned} \tag{12}$$

where $\mathbf{1}\left(\mathcal{N}^j\right)$ denotes an indicator function for the non-default states of bank j. Comparing (11) and (12), it follows that *ceteris paribus,* a large bank has strictly greater incentives to take leverage than a small bank, measured by the two terms on the second line of (12). First, increasing leverage strictly increases the expected government transfer $\mathbb{E}\left[t(\boldsymbol{b}|s)\right]$, since $\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} > 0$. To the extent that

[5]Technically, the second term is defined as a "line integral," along a one-dimensional curve (i.e., along ∂D^i) in a two-dimensional space (i.e., in the space of signals s and asset returns u). In terms of Riemann integrals, this line integral is defined as $\int_{\partial \mathcal{D}^i} J(u,s)\, dF(u,s) = \int_{\mathcal{S}} J\left(b^i - t(\boldsymbol{b}|s), s\right) f\left(b^i - t(\boldsymbol{b}|s), s\right) ds$ for any function $J(u,s)$, where $\mathcal{S}$ denotes the support of signals s.

transfers are eventually distributed to shareholders, these benefits are adjusted for the deadweight costs of equity. Second, by increasing transfers, an increase in leverage b^j shifts the default boundary, reducing the bankruptcy region and consequently the deadweight losses associated with bankruptcy. We further show in the Appendix (Equation **??**) that the derivative of the transfer with respect to b^j scales proportionally with the measure $\mu(j)$ of bank j. This highlights that our results go beyond the distinction between infinitesimal and non-infinitesimal banks. Evidently, the measure of each large bank is important, because it is exactly proportional to the bank's strategic incentives to take leverage.

We next consider the impact of a large bank's leverage on small banks' payoffs and incentives. We show that, all else constant, small banks have greater market value, and a higher marginal incentive to borrow, when large banks borrow more.

Lemma 1. (Effect of large bank leverage on small banks' incentives) *Suppose that in some states a positive bailout is optimal, that is, $t(\boldsymbol{b}|s) > 0$ for some s. Then for a small bank i with borrowing level b^i, denoting by b^j the leverage choice of a large bank, we have*

$$\frac{dV\left(b^i, t(\boldsymbol{b}|s)\right)}{db^j} > 0 \tag{13}$$

$$\frac{d^2V\left(b^i, t(\boldsymbol{b}|s)\right)}{db^i db^j} > 0. \tag{14}$$

Lemma 1 shows that both the *value* of a small bank and its *marginal benefit* from leveraging up are increasing in the leverage choices of the large bank. The first result is intuitive. If a large bank takes more leverage, the government's optimal bailout policy is more generous. Financiers are now willing to pay more for claims on a small bank at date 0, because they expect to benefit from a higher bailout at date 1. Thus, other things equal, the market value of a small bank increases, which explains Equation (13).

The second result in Equation (14) shows that banks' payoffs are supermodular in each other's borrowing choices. The economic implication of supermodularity is that banks' leverage decisions become *strategic complements* in the sense of Bulow, Geanakoplos and Klemperer (1985) and Cooper and John (1988). Higher leverage for large banks strengthens the strategic incentives for small banks to increase leverage. Intuitively, small banks perceive that they are shielded by the large banks.

To understand this point, recall that a small bank's appetite for borrowing is driven by the trade-off between the costs of equity issuance and the costs of default. When a large bank levers up, bailouts are larger and, consequently, small banks remain solvent more frequently. This has two effects. First, the costs of equity issuance, which hit shareholders when the bank is solvent, become more salient, and the marginal benefit of issuing debt rises. Second, since default becomes less likely, the marginal costs of default fall, and borrowing becomes yet more attractive. Both effects increase small banks' appetite for borrowing. Moreover, the same logic applies if b^j in Equation (14) is interpreted as the borrowing among the whole sector of small banks, or at least among a subset of small banks with positive mass.

In summary, our model simultaneously allows for moral hazard in the classical sense, since large banks have incentives to take leverage in order to exploit government subsidies, and for collective moral hazard, since small banks have a bigger appetite for leverage when their peers are also levering up.

These observations lead to the two main positive results of this paper, which are stated in Propositions 1 and 2. Two technical challenges arise in characterizing equilibria. First, large banks' objective functions are not necessarily concave. Second, the strategic complementarity in leverage choices opens the door to multiple equilibria. Since we wish to keep the model general, we do not restrict the primitives to guarantee concavity or uniqueness. Without such restrictions, it is not possible to derive comparative statics by totally differentiating the relevant first-order conditions. Instead, our proofs rely on a modified version of the results on monotone comparative statics in Milgrom and Shannon (1994), which require only supermodularity of payoffs.

Proposition 1. (Large banks borrow more) *Large banks borrow strictly more than small banks in any equilibrium.*

This result follows from the observation that large banks are subject to moral hazard in the classical sense. Large banks internalize that their leverage decisions directly affect the magnitude of bailouts t and, therefore, that they can increase the market prices of their debt and equity at date 0 by borrowing more and boosting their implicit government subsidy. Small banks, by contrast, are not subject to classical moral hazard because bailouts depend exclusively on aggregate conditions, and small banks rationally consider their impact on aggregates to be infinitesimal. Therefore, large banks choose to borrow strictly more than small ones.

FIGURE 3 ABOUT HERE

Strategic complementarities also imply that large banks' appetite for leverage spills over to small banks. Indeed, the mere presence of large banks induces small banks to lever up more aggressively.

Proposition 2. (When large banks are present, small banks take more leverage) *Let b_0 be the smallest borrowing level which occurs in a symmetric equilibrium with only small banks. In any equilibrium with large banks, each small bank chooses strictly higher borrowing $b^i > b_0$.*

Proposition 2 is an instance of collective moral hazard, and follows naturally from Lemma 1. Consider a hypothetical market where all banks are small and choose to borrow b_0 in equilibrium, and suppose that a subset of these banks merge to form a large bank. As shown in Proposition 1, the large bank has an independent incentive to increase their leverage to $b > b_0$. Because banks leverage decisions are strategic complements, the marginal benefit of borrowing increases for the remaining small banks, and they follow suit by choosing $b^i > b_0$.

We can combine our positive results in the following Corollary, which follows directly from both Propositions.

Corollary. (Banking systems with larger banks are associated with higher system-wide leverage) *All else constant, when larger banks are present, aggregate leverage is higher and government bailouts are larger and more frequent.*

In summary, the presence of large banks leads to an unambiguous increase in system-wide leverage and financial sector risk, implying that the government is forced to rely more frequently on larger bank bailouts than when banks are small. These effects would be stronger in a model with multi-dimensional moral hazard, for example, if banks are able to increase the variance or correlation of their asset returns. In this sense, our results represent a lower bound on the impact of large banks on system-wide risk. A more nuanced feature of the model is that the effect of large banks on the likelihood of bank failures is ambiguous, since banks create more system-wide risk, but the government simultaneously responds with increased support. Our quantitative exercise below illustrates these competing forces.

In reality, banks differ not only on their size, but also on their degree of interconnectedness in the form of network centrality, or in richer dimensions regarding liquidity risk or maturity mismatch. While we have not modeled these factors explicitly, our analysis suggests that they are likely to lead to similar results. Indeed, Equation (12) shows that the driver of strategic leverage is the sensitivity $\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j}$ of bailouts to individual banks' choices. If this sensitivity depends on other bank characteristics such as interbank connections, then these factors will play a role that is analogous to size in our model, and ceteris paribus, the presence of large banks continues to lead to increased risk taking.

Figure 3 provides a simple graphical illustration of equilibrium when there are large and small banks. Consider an economy where there are N large banks with total point mass λ, so that each large bank controls a share $\frac{\lambda}{N}$ of aggregate capital, and a complementary mass $1-\lambda$ of identical small banks. In this case, each small bank takes as given the borrowing choices b^L of the large bank and the choices b^S of its small peers, and its best response is to set

$$b^i = \arg\max_b V\left(b, t\left(b^S, b^L|s\right)\right) \equiv BR^S\left(b^S, b^L\right). \tag{15}$$

Given a borrowing level b^L for large banks, the partial equilibrium choice of the sector of small banks is found by solving the fixed point problem $b^S = BR^S\left(b^S, b^L\right)$. Figure 3b shows this for two different levels b_0^L and b_1^L of large banks' borrowing choices. This induces the "collective best response" of the small bank sector as a whole, denoted $CBR^S\left(b^L\right)$ and shown as the red (solid) curve in Figure 3a.

Similarly, we can define a large bank's individual best response, if small banks are choosing b^S and other large banks are choosing b^L, as

$$b^j = \arg\max_b V\left(b, t\left(b^S, \left(b; b_{-1}^L\right)|s\right)\right) \equiv BR^L\left(b^S, b^L\right), \tag{16}$$

where b_{-1}^L denotes the $(N-1)$-vector of other large banks' symmetric choices. We can again define large banks' collective best response $CBR^L\left(b^S\right)$ as the solution to the fixed point problem $b^L = BR^L\left(b^S, b^L\right)$, which is shown as the blue (dotted) curve in Figure 3a.

The economy is in equilibrium if small banks and large banks are responding optimally to each others' choices, that is, where the best response curves intersect at point E. If there were no large banks, by contrast, the economy would be in equilibrium at point B, where small banks are responding optimally to each other.

Proposition 1 establishes that, regardless of parameters, any equilibrium will be below the 45-degree line where large banks choose more leverage $b^L > b^S$ than small ones. Proposition 2 shows that small banks will always choose more leverage in an equilibrium when large banks are present (e.g. point E) than in the benchmark case with only small banks (e.g. point B).

The arrows connecting points B and E illustrate an amplification mechanism that arises in our model. Starting from the benchmark B, when large banks enter the market, they increase their leverage for strategic reasons, and the economy moves to the right in the figure onto the collective best response CBR^L of large banks. As a result of strategic complementarities, small banks now increase their leverage, and we move upwards to the collective best response CBR^S of small banks. This move gives large banks an additional incentive to increase leverage, and so forth, until equilibrium is reached at point E. In the remainder of this section, we study the importance of this amplification mechanism when industry concentration increases.

Industry concentration and multiplier effects

We have established that the presence of large banks, or more generally increases in industry concentration, lead to higher system-wide leverage. By exploiting monotone methods, we have so far emphasized directional and qualitative results. We now show analytically that the quantitative implications of the mechanisms that we study in this paper are potentially significant because strategic complementarities amplify the initial impulse of an increase in concentration. In Section 6, we further explore the quantitative importance of our results by simulating the model for empirically plausible parameters.

For concreteness, suppose that there are N large banks with collective mass λ and that the economy has reached a stable equilibrium, such as point E in Figure 3a. Now consider an increase in industry concentration, that is, an increase in the size λ of the large bank sector. One interpretation of this change is a merger between each large bank and a subset of small banks.

First, hold constant the leverage choice b^L of the large bank, and consider the response of the remaining small banks. As before, this is determined by the fixed-point equation $b^S = BR^S\left(b^S, b^L; \lambda\right)$, where we have made explicit the dependence of best responses on industry structure. It is easy to see that, holding constant borrowing choices in the neighborhood of an equilibrium, the best response of a small bank is increasing in λ with $\partial BR^S/\partial\lambda > 0$. Intuitively, because $b^L > b^S$ in equilibrium, the increase in λ shifts weight from low-leverage to high-leverage banks, which increases aggregate leverage and leads to more generous government bailouts. By Lemma 1, this makes leverage more attractive for each small bank. However, once each small bank increases its leverage, aggregate leverage increases again, and we move into a second round of adjustments. Overall, we can characterize the response of the small bank sector as

$$\frac{\partial b^S}{\partial \lambda} = M^S \frac{\partial BR^S}{\partial \lambda} \tag{17}$$

where the small bank multiplier M^S is defined as

$$M^S \equiv \left(1 - \frac{\partial BR^S}{\partial b^S}\right)^{-1} > 1. \tag{18}$$

Even before large banks respond, small banks respond by increasing leverage, and this effect is amplified by strategic complementarities, as captured by the multiplier $M^S > 1$.

Second, large banks respond to the change by adjusting their own leverage, for two reasons. The first reason is that small banks have raised their borrowing, making borrowing more attractive. The second reason is that the large bank is now larger than before, and internalizes an even stronger effect of its choices on government bailouts. Computing total changes yields

$$\frac{db^L}{d\lambda} = \bar{M} \cdot \left(M^L M^S \frac{\partial BR^S}{\partial \lambda} \frac{\partial BR^L}{\partial b^S} + M^L \frac{\partial BR^L}{\partial \lambda}\right), \tag{19}$$

$$\frac{db^S}{d\lambda} = \bar{M} \cdot \left(M^L M^S \frac{\partial BR^L}{\partial \lambda} \frac{\partial BR^S}{\partial b^L} + M^S \frac{\partial BR^S}{\partial \lambda}\right), \tag{20}$$

where the large bank multiplier M^L is defined as

$$M^L \equiv \left(1 - \frac{\partial BR^L}{\partial b^L}\right)^{-1} > 1, \tag{21}$$

and the aggregate multiplier $\bar{M} > 1$, which itself is a function of the small and large bank multipliers, is defined as

$$\bar{M} \equiv \left(1 - M^L M^S \frac{\partial BR^L}{\partial b^S} \frac{\partial BR^S}{\partial b^L}\right)^{-1} > 1. \tag{22}$$

Equation (22) reveals that industry concentration has an effect on leverage that is amplified on two levels. First, within each subsector (of large or small banks), banks encourage each other to take more leverage, as reflected by the size-dependent multipliers M^S and M^L. Second, across bank sizes, strategic complementarities induce further amplification via the aggregate multiplier $\bar{M} > 1$ in Equation (22). The two effects reinforce each other since the aggregate multiplier $\bar{M}$ is itself increasing in M^S and M^L.

Size distribution of large banks

Our results so far stress the difference between small and large banks, but the size distribution among large banks is itself important. As a complementary exercise to the above, consider an increase in market concentration brought about by a decrease in N for a given λ. This change also represents increased market concentration (indeed, the Herfindahl index of banking concentration in this economy is $\sum_{i=1}^{N} \left(\frac{\lambda}{N}\right)^2 = \lambda^2/N$, since the squared market shares of all small banks are zero).

For given leverage choices b^S and b^L, a small bank's best response $BR^S\left(b^S, b^L\right)$ does not depend on N. Intuitively, the government bailout, and the shield it provides to small banks, only depends on the aggregate importance λ of large banks not on their number. By contrast, the best response $BR^L\left(b^S, b^L\right)$ of each large bank increases when N falls, because the strategic leverage effect in its first-order condition (12) is proportional to its size. In terms of Figure 3, the collective best response of large banks (the blue dotted line) therefore shifts to the right. Equilibrium leverage moves along small banks' best response curve (the red line), and equilibrium leverage increases for all banks. This comparative static highlights that increases in banking concentration generally cause a strategic, system-wide increase in leverage. The effect persists also when the size distribution among large banks consolidates.

Discussion of bailout policies

Bailouts in our model are systemic and not targeted. Equation (5) clarifies the relevance of this assumption in generating strategic complementarities among banks: Each large bank's leverage increases the aggregate bailout, and therefore provides a shield against default to other banks. Our insights therefore apply directly to government support policies that benefit all banks holding troubled assets. Some major bailout policies in the crisis of 2008 were accessible to all banks and had a system-wide component. For example, the Capital Purchase Program under TARP benefited 707 banks in total in 2008/09, with a total Treasury exposure of $205bn. Nine of the largest banks, who controlled 55% of aggregate assets, received about 61% of overall support (Calomiris and Khan, 2015). Hence, our assumption that benefits accrue to banks per unit of assets is a reasonable approximation within this program. The interpretation of system-wide assistance can also be broader. Strategic complementarities arise naturally if the government intervenes by lowering interest rates below their otherwise optimal level (a policy often referred to as the "Greenspan put").

In practice, system-wide support is often provided at the request of the participating bank. While we model bailouts parsimoniously as direct cash injections, the resulting incentives are likely to be very similar. Suppose, for instance, that the government offers to buy assets from banks at a given price (e.g., via TARP), or makes cheap loans with assets as collateral (e.g., via a discount window). The bank's optimal use of the scheme generates a per-unit-of-assets subsidy that is formally equivalent to t in our model.

Of course, not all bailout policies in reality are system-wide. To further illuminate the mechanism behind our results, we discuss two features of policies in practice that can generate different effects. First, bailout policies often impose an overall limit on government funds. The limit of $700bn imposed in the US in 2008 was never binding, but in other countries and time periods, fiscal capacity has been much more limited. A binding fiscal constraint leads to different incentives from those in our model, especially if large banks are the first to receive support. In this case, strategic spillovers from large to small banks become weaker and can even turn into strategic substitutes. As an extreme example, consider the limiting case where the government always runs out of money before small banks are served. Then, there are no bailouts for small banks, and, by a parallel argument to Proposition 2, small banks take *less* leverage than they would if there were no large banks in the economy.

Second, consider a policy such as deposit insurance, which is a function of each bank's individual capital shortfall. Formally, suppose the government in our model can choose a separate bailout t^i for each bank. Then the government's optimality condition for choosing t^i at date 1, which replaces (4), is

$$\frac{\partial W_1(\boldsymbol{b}, t|s)}{\partial t^i} = (1-\phi)\, f_u\left(b^i - t|s\right)\left(b^i - t\right) - \left(\psi - \bar{\psi}\right)\left[1 - F_u\left(b^i - t|s\right)\right] - \kappa'(T) = 0 \tag{23}$$

where $T = \int t^i d\mu$ denotes the total bailout. If the fiscal cost $\kappa(T)$ is convex, then a larger aggregate bailout increases the marginal cost of supporting bank i and therefore lowers the government's optimal choice of t^i, all else equal. This implies that size effects can reverse. Indeed, a large bank internalizes the effect of its choices on T, which dampens the strategic incentive to take leverage. Moreover, leverage choices may become strategic substitutes: If j is a large bank (or a set of small ones) we have $\frac{\partial t^i}{\partial t^j} < 0$ because of the marginal cost term, implying $\frac{\partial t^i}{\partial b^j} < 0$.

However, at the same time as weakening strategic complementarities, targeted bailouts can exacerbate moral hazard, because they give all individual banks the incentive to take strategic leverage. For example, consider the case where costs are close to linear with $\kappa' \to \bar{\kappa}$. Now there are no interactions and each bank acts like a monopolist in the baseline model. In this case, strategic leverage is strictly higher than in our baseline model. This illustrates a broader point: Imperfect targeting can be an optimal arrangement when governments lack commitment. Indeed, this is in line with existing work which demonstrates that perfectly targeted bailouts exacerbate moral hazard ex-ante (Bianchi, 2016), and force the government to provide distortive information rents to banks if it does not perfectly observe bank-level performance (Farhi and Tirole, 2012). Imperfect targeting can also be beneficial because it generates constructive ex-ante ambiguity (Nosal and Ordoñez, 2016).

In summary, the strategic effects we highlight apply to a wide range of government support policies that are used in practice and priced by financial markets. There exist alternative policies that can dampen or reverse strategic complementarities between large and small banks, but system-wide support is likely to remain important in reality. In particular, the results in Kelly, Lustig and Van Nieuwerburgh (2016) provide robust empirical evidence consistent with the notion that market participants expect overall system-wide government guarantees.

4 Optimal ex-ante policies

In our model, there is a clear case for prudential regulation. In particular, banks do not internalize the impact of their leverage choices on the social cost of ex-post bailouts. On the contrary, they endeavor to attract bailouts because funding markets at date 0 reward them for large implicit subsidies.

Constrained efficient choices

We study a constrained efficient benchmark for policy in which a benevolent social planner can dictate banks' borrowing choices $\boldsymbol{b}$ at date 0, but cannot overcome financing frictions, that is, the costs of default and the social cost of equity issuance.

We first compute a general expression for expected social welfare at date 0. Given banks' choices $\boldsymbol{b}$ and a state-contingent bailout policy $t(s)$ at date 1 (not necessarily equal to the ex-post optimal policy $t(\boldsymbol{b}|s)$), expected welfare at date 0 is

$$
\begin{aligned}
W_0(\boldsymbol{b}, t(s)) = {} & \mathbb{E}[u] - \mathbb{E}[\kappa(t(s))] \\
& - (1-\phi) \int \Pr\left[\mathcal{D}^i\right] \mathbb{E}\left[u|\mathcal{D}^i\right] d\mu \\
& - \left(\psi - \bar{\psi}\right) \int \Pr\left[\mathcal{N}^i\right] \mathbb{E}\left[t(s) + u - b^i|\mathcal{N}^i\right] d\mu.
\end{aligned} \tag{24}
$$

This is intuitive: The constrained social planner measures welfare as the expected value of asset returns, less the deadweight social costs of bailout transfers, bank default, and equity issuance. Comparing (24) with banks' private objective function in (8) highlights the rationale for regulation. In particular, the government transfer $t(s)$ enters negatively in social welfare due to deadweight costs $\kappa(t(s))$. This cost is not accounted for in banks' market values. Moreover, $t(s)$ enters positively in banks' market values because it represents a subsidy from taxpayers to the owners of bank debt and equity.

Two maximization problems are of potential interest. First, we can assume that the planner can commit to any state-contingent bailout policy $t(s)$ at date 0. Then he solves

$$
W^c = \max_{\{\boldsymbol{b}, t(s)\}} W_0(\boldsymbol{b}, t). \tag{25}
$$

Second, we can assume that the planner cannot commit to transfers, and chooses them optimally ex-post as in Section 3. In this case he solves

$$
W^{nc} = \max_{\{\boldsymbol{b}, t(s)\}} \left\{ W_0(\boldsymbol{b}, t(s)) \text{ subject to } t(s) = t(\boldsymbol{b}|s) \right\}, \tag{26}
$$

where $t(\boldsymbol{b}|s)$ is the government's ex-post best response, defined in (3). Clearly, the loss of commitment in (26) cannot increase the maximized value of welfare, so that $W^c \geq W^{nc}$. Moreover, we can show that a lack of commitment does not reduce welfare in this instance:

Lemma 2. (Commitment is irrelevant when the planner controls banks) *When the planner perfectly controls banks' borrowing decisions* $\boldsymbol{b}$*, the solutions to problems* (25) *and* (26) *coincide and satisfy* $W^c = W^{nc}$.

Intuitively, commitment provides no additional value when the planner controls the banks, because in any case, the solution with commitment involves choosing bailouts that are ex-post optimal. When the planner does not control the banks, by contrast, commitment may be beneficial because it allows the planner to distort bailouts away from ex-post optimality in order to curb moral hazard.

Lemma 2 allows us to characterize the planner's optimal borrowing choices:

Proposition 3. (Efficient leverage is independent of bank size) *The constrained efficient choice is to set* $b^i = b^\star$ *for all* i *except possibly for a measure zero set, regardless of the distribution* μ *of bank sizes, and regardless of whether the planner can commit to a transfer policy.*

Proposition 3 shows that the planner chooses symmetric policies for all banks, and in particular, that the socially optimal borrowing level b^i is independent of bank size. This is a natural consequence of assuming identical technologies for large and small banks and constant returns to scale: A large bank has no technological reason to take more or less leverage than a small bank. This policy is in contrast to the laissez-faire equilibrium of Section 3, where large banks unambiguously chose larger borrowing levels. These equilibrium choices were motivated not by technological or contractual differences, but by the desire to maximize an implicit government subsidy. Since the planner internalizes that such a subsidy is socially wasteful, he does not respond to the same incentives. More generally, there may be other unmodeled reasons that may lead to efficient leverage choices that are heterogeneous. Therefore, Proposition 3 should be understood as showing that leverage disparities caused by size differences are inefficient.

Before considering ways to implement the optimal allocation, it is useful to explicitly characterize the best borrowing level $b^\star$. When all banks choose the same borrowing level b, we can write ex-ante welfare as

$$\bar{W}_0\left(b, t\left(s\right)\right) = V\left(b, t\left(s\right)\right) + \bar{\psi}\Pr\left[\mathcal{N}\right]\mathbb{E}\left[t\left(s\right) + u - b|\mathcal{N}\right] - \mathbb{E}\left[t\left(s\right) + \kappa\left(t\left(s\right)\right)\right], \tag{27}$$

where $V\left(b, t\left(s\right)\right)$ is the private value of an individual bank in Equation (8), and $\mathcal{N} = \left\{\left(u, s\right)|t\left(s\right) + u \geq b\right\}$ is the event of non-default. The first term is aligned with the bank's objective. The second term captures the fact that the costs of equity issuance are not fully social costs if $\bar{\psi} > 0$. The last term arises because, unlike the bank, the social planner internalizes the full cost $t\left(s\right) + \kappa\left(t\left(s\right)\right)$ of the bailout to taxpayers.

Since the welfare maximization problems with and without commitment are equivalent, we can assume that the planner is already committed to the optimal ex-post transfer $t(b^\star|s)$. Then the choice of $b^\star$ must maximize welfare taking this commitment as given, thus solving the fixed point equation

$$b^\star = \arg\max_b \bar{W}_0\left(b, t\left(b^\star|s\right)\right). \tag{28}$$

The first-order condition in this problem gives a characterization of socially optimal borrowing:

$$\frac{\partial V\left(b^\star, t\left(b^\star|s\right)\right)}{\partial b} - \bar{\psi}\Pr\left[\mathcal{N}^\star\right] = 0, \tag{29}$$

where $\mathcal{N}^\star = \left\{\left(u, s\right)|t\left(b^\star|s\right) + u > b^\star\right\}$ is the non-default event under the optimal policy. Intuitively, the planner considers the impact of debt choices on the value of the bank, adjusted for the part of private equity issuance costs that is explained by transfers. One might expect the planner to also consider the impact of debt choices on the ex-post transfer. Formally, Equation (29) includes the term

$\mathbb{E}\left[\frac{\partial W_0}{\partial t(b^\star|s)}\frac{\partial t(b^\star|s)}{\partial b}\right]$. However, since transfers are chosen optimally, marginal changes in transfers have only a second-order welfare effect, and this term vanishes after an application of the envelope theorem.

This argument clarifies how our analysis would change if the costs of equity issuance were incurred ex-ante. In this case, the social planner has a time-consistency problem: She takes the costs of equity into account when choosing taxes at date 0, but not when choosing bailouts at date 1. In this setting, the envelope argument behind Equation (29) cannot be applied, and the optimal tax will contain the additional term $\mathbb{E}\left[\frac{\partial W_0}{\partial t(b^\star|s)}\frac{\partial t(b^\star|s)}{\partial b}\right]$, driven by the planner's desire to discipline her future self. The optimal policy would impose this additional tax on both large and small banks.

Optimal capital requirements

Dictating banks' choices b^i is tantamount to quantity regulation. Indeed, the constrained efficient choice $b^\star$ can be achieved by imposing binding capital requirements, as included in the Basel III Accord, which limit borrowing as a fraction of risky assets (recall that in our model, borrowing choices b^i are already expressed per unit of risky capital). The following result is a direct corollary of Proposition 3.

Corollary. (Optimal capital requirements are independent of bank size) *The constrained efficient allocation can be implemented by imposing binding capital requirements that are independent of bank size.*

It follows that strategic differences between large and small banks are insufficient to justify size-dependent capital regulation. Even though the incentives to take leverage remain stronger for large banks when capital requirements are in place (the Lagrange multiplier on a large bank's capital constraint will be greater), the optimal level of capital that a regulator wishes to enforce is independent of bank size, as long as technologies exhibit constant returns to scale. As noted when introducing Proposition 3, if efficient leverage choices were to be heterogeneous, this corollary implies that binding capital requirements should not be a direct function of bank size per se. Naturally, this conclusion changes when policy is conducted via Pigouvian taxes, which we examine next.

Optimal Pigouvian taxes

We now assume that the social planner cannot directly control banks' borrowing quantities $\boldsymbol{b}$. Moreover, we focus on the case where the planner has no commitment power; in particular, he cannot credibly announce a date 1 bailout policy at date 0. Market participants instead expect that the date 1 bailout policy will be chosen to maximize welfare ex-post.

The socially optimal choices $b^\star$ can be implemented by taxing banks in proportion to their borrowing choices at date 0. We let τ^i denote the Pigouvian tax levied on bank i, who consequently pays $b^i\tau^i$ to the government; tax revenues are rebated to financiers as a lump sum. Therefore, the value of τ^i should be interpreted as a tax on the bank's face value of debt. Under Pigouvian taxation, the bank maximizes its value net of tax, $V\left(b^i, t\left(\boldsymbol{b}|s\right)\right) - b^i\tau^i$. As before, small banks take the bailout policy $t\left(\boldsymbol{b}|s\right)$ as fixed. Large banks, on the other hand, realize that the government lacks commitment and take into account their impact on the ex-post optimal bailout. A standard argument then leads to optimal taxes:

Proposition 4. (Optimal Pigouvian taxes) *The following Pigouvian taxes implement the social planner's choice $b^i = b^\star$ in equilibrium:*

– *If i is a small bank, then set*

$$\tau^i = \bar{\psi} Pr\left[\mathcal{N}^\star\right]. \tag{30}$$

– *If j is a large bank, then set*

$$\tau^j = \tau^i + \mathbb{E}\left[\left.\frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^j}\right|_{\boldsymbol{b}=b^\star}\left(1 - \psi\mathbf{1}\left(\mathcal{N}^\star\right)\right)\right] + (1-\phi)\int_{\partial\mathcal{D}^\star}\left.\frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^j}\right|_{\boldsymbol{b}=b^\star}\left(b^\star - t\left(b^\star|s\right)\right)dF, \tag{31}$$

where $\partial D^\star = \left\{t\left(b^\star|s\right) + u = b^\star\right\}$ is the boundary of the optimal default region.

To implement the efficient outcome, each small bank is charged in proportion to the expected wedge between private and social costs of equity issuance. Second, large banks are charged a size top-up tax in order to account for their risk taking incentives. The "size tax" is designed to counteract the part of large banks' incentives that stems from their desire to maximize implicit subsidies. In particular, the second term in (31) offsets a large bank's incentive to attract larger subsidies for its financiers, while the third term offsets the incentives generated by the fact that larger subsidies reduce the expected costs of default. Note that these two terms are equivalent to the final two terms in the large bank's first-order condition (12), evaluated at the optimal choice $b^\star$.

Policy implications

The prescription of the optimal policy is as follows. If large banks are present, regulators need to be wary of significant and amplified increases in risk taking incentives. If binding capital requirements can be imposed to correct all externalities, then constant returns to scale still guarantee that there is no need to make capital requirements size-dependent. However, if regulators choose to use price instruments to combat systemic risk, then a top-up tax in line with Equation (31) should be charged to large banks.

Further implications arise if financial regulation is imperfect: First, if the optimal Pigouvian tax or capital rule cannot be imposed, then a stricter antitrust policy can be welfare-enhancing at the margin because it prevents excessive leverage when large banks are formed. Second, although the first-best quantity regulation is size-independent, stricter capital requirements for large banks would form part of the second-best policy if banks are able to circumvent capital requirements (e.g., by gaming regulatory risk weights, which large banks have a stronger incentive to do). Exploring these effects would require a more explicit treatment of banks' market power and incentives.

5 Idiosyncratic risk

In our baseline environment, to simplify the exposition, we assume that bank returns are fully determined by an aggregate shock. In this section, we assume that bank returns have both an aggregate and an idiosyncratic component. This version of the model highlights the need to make assumptions on how the distribution of banks' profitability varies when there are changes in industry concentration.

Modified environment Formally, we preserve the rest of the assumptions, but we now assume that bank i's assets yield a random return u^i at date 1 of the form

$$u^i = v + w^i, \tag{32}$$

where the first component v is an aggregate shock, while the second component w^i is an idiosyncratic shock to bank i, which is independent across banks and independent of v, with mean $\mathbb{E}\left[w^i\right] = 0$, variance $\mathbb{V}ar\left[w^i\right] < \infty$, and density $h^i\left(w^i\right)$.

As in the baseline model, the government receives a signal s of the common shock v, and we write $f_v\left(v|s\right)$ for the conditional density of v. Therefore, the value of the aggregate capital stock at date 1 is given by

$$\bar{u} = \int u^i d\mu\left(i\right) = v + \sum_{j \in \mathcal{L}} \mu\left(j\right) w^j, \tag{33}$$

where $\mathcal{L} = \{j : \mu\left(j\right) > 0\}$ denotes the set of large/non-infinitesimal banks. This setup allows for a flexible relationship between idiosyncratic risk and bank size. Intuitively, the idiosyncratic shocks w^i of small banks integrate to zero by the Law of Large Numbers. By contrast, idiosyncratic shocks w^j to large banks, who have a strictly positive point mass $\mu\left(j\right) > 0$, directly affect aggregate output. In particular, aggregate output is more volatile when large banks are present as long as the variance of large bank's idiosyncratic return component is non-zero, since

$$\mathbb{V}ar\left[\bar{u}\right] = \mathbb{V}ar\left[v\right] + \sum_{j \in \mathcal{L}} \mu\left(j\right)^2 \mathbb{V}ar\left[w^j\right]. \tag{34}$$

Therefore, depending on the assumptions on how the variance of large banks, $\mathbb{V}ar\left[w^j\right]$, is determined, our formulation allows us to capture different views of how idiosyncratic risk aggregates within entities. On the one hand, one can assume, appealing to the Law of Large Numbers, that when a continuum of small banks merge to form a large bank, their idiosyncratic risk cancels out. In that case, the variance of the idiosyncratic return component for large banks becomes zero, that is, $\mathbb{V}ar\left[w^j\right] = 0$ if $j \in \mathcal{L}$. On the other hand, one can assume that the idiosyncratic return component is realized at the bank level (e.g., each bank has a CEO, and the idiosyncratic shock is driven by the CEO's individual decisions). In that case, which we refer to as the granular scenario, as in Gabaix (2011), the variance of the idiosyncratic return component for otherwise comparable large and small banks should be identical.

Formally, consistently with our assumption that all banks are otherwise identical but for their size, we modulate the extent to which the idiosyncratic shocks to large bank returns are granular by defining a parameter $\zeta \in [0, 1]$, such that,

$$\zeta = \frac{\mathbb{V}ar\left[w^{\text{large}}\right]}{\mathbb{V}ar\left[w^{\text{small}}\right]} \qquad \text{(Granularity parameter)}. \tag{35}$$

Under this formulation, when $\zeta = 0$, idiosyncratic shocks cancel out at the bank level and the Law of Large of Numbers applies, which corresponds to the first scenario we just described. When $\zeta = 1$ instead, idiosyncratic shocks occur at the bank level, consistently with the granular scenario. Intermediate levels of ζ simply represent a combination between both extremes.

Equilibrium with idiosyncratic risk In the presence of idiosyncratic risk, the conditional c.d.f. of total asset values in bank i is given by

$$\Pr\left[u^i \leq u|s\right] = \int_0^\infty \left[\int_{-\infty}^{u-v} h^i(w)\, dw\right] f_v(v|s)\, dv \equiv F_u^i(u|s)\,. \tag{36}$$

In Appendix C, we show formally that our main results remain valid under mild regularity conditions after this change of variable. Intuitively, the government has less information about individual banks' performance when there is idiosyncratic risk. In general, the impact of idiosyncratic risk on the level of bailouts is therefore ambiguous: After good aggregate news, the government is less confident that individual banks are safe, and may increase its bailout, while after bad aggregate news, the government is less confident that a bailout is necessary, since a positive measure of individual banks are certain to recover. However, for any given level of bailout, it remains true that (i) only large banks internalize the marginal impact of their leverage choices on government policy, and that (ii) leverage choices are strategic complements across banks. Therefore, large banks continue to choose greater leverage in any equilibrium, as in Proposition 1, and small banks choose greater leverage in response, as in Proposition 2. We provide further insights into how changes in the granularity parameter ζ affect our results in Section 6.3.

6 Quantitative illustration

Finally, we illustrate how bank size affects aggregate leverage and the magnitude of government bailouts in equilibrium by solving our model using parameters consistent with U.S. data over the period 1990Q1 to 2013Q4. We describe in detail the choice of parameters in the Appendix. We work with the richer formulation with idiosyncratic risk developed in Section 5, since it is more realistic and easier to discipline by observables. To capture long-term funding decisions, we map a period in our model to a two-year time horizon.

In order to explicitly solve the model, we make the following functional form assumptions. First, we assume that there exists a finite number N of large banks that hold a share λ of total bank assets, and a set of small banks that hold a share $1-\lambda$ of total assets, as illustrated in Figure 4. This is the same formulation used to draw Figure 3.

FIGURE 4 ABOUT HERE

Second, we assume that the social cost of government bailouts $\kappa(t)$ takes the following exponential-affine form $\kappa(t) = \frac{\kappa_1}{\kappa_2}\left(e^{\kappa_2 t} - 1\right)$,where $\kappa_1, \kappa_2 \geq 0$. The parameter κ_1 represents the marginal cost of public funds for a small intervention, since $\kappa'(0) = \kappa_1$, while the parameter κ_2 is a measure of curvature, since $\frac{\kappa''(t)}{\kappa'(t)} = \kappa_2$, $\forall t$. This formulation implies that $\kappa(0) = 0$.

Finally, we assume that the aggregate component v and the idiosyncratic component w^i of banks' returns are normally distributed as follows: $v \sim N(\mu_v, \sigma_v)$ and $w^i \sim N(0, \sigma_w)$, where $\mathbb{C}ov\left[v, w^i\right] = 0$, $\forall i$, and $\mathbb{C}ov\left[w^j, w^i\right] = 0$, $\forall i \neq j$. We also assume that the government receives a signal s with the following structure: $s = v + \varepsilon_s$, where $\varepsilon_s \sim N(0, \sigma_s)$.

6.1 Impact of industry concentration

Figure 5a summarizes our results for the baseline calibration ($\lambda = 0.5$) and illustrates how changes in industry composition modify the leverage decisions of large and small banks in the unregulated equilibrium studied in Section 3; we turn to optimal policy below.

FIGURE 5 ABOUT HERE

Our baseline parametrization generates values for average debt to assets of 0.906, with large banks choosing 0.910 and small banks 0.901. Compared to the average debt-to-asset ratios for top 5 banks and the remaining banks during the period considered, which respectively correspond to 0.935 and 0.914 in the Call Reports data, our model accounts for around half of the difference in debt-to-asset choices between large and small banks, although it somewhat understates average leverage. The difference in leverage choices was not directly targeted by our parameter choices. Since large and small banks are otherwise identical, any difference in the behavior of large banks relative to small banks is due to the strategic effect that we identify in this paper.

Figure 5a shows that increases in the level of industry concentration in the form of an increase in the share of assets held by the top 5 banks are associated with a significant effect on the leverage choices of both large and small banks. Our results show that increases in industry concentration have a significant impact on system-wide borrowing, in particular for values of λ higher than 0.5. For instance, our model implies that an increase in concentration in which the largest 5 banks hold 70% of assets (λ moves from 0.5 to 0.7) would be associated with an increase in system-wide borrowing of 3.5 percentage points, from 90.1% to 93.6%, with the difference between large and small banks borrowing increasing slightly to around 2%.

Figure 5b shows that the magnitude of bailouts in equilibrium increases monotonically with the level of industry concentration. Intuitively, given that both large and small banks borrow more, all else equal, the probability of failure is larger, which makes it more appealing for the government to bail out banks in bad aggregate states. In the baseline scenario, the magnitude of the government's transfer, conditional on a significant intervention, which we define as a transfer greater than 2% of bank value, corresponds to just under 3% of banks' assets, as targeted by our calibration. The unconditional value of government guarantees is roughly 0.6% of banks' total asset value. A shift from $\lambda = 0.5$ to $\lambda = 0.7$ would be associated with a 25% increase in the magnitude of significant intervention, from 2.75% to 3.5%, and with a four-fold increase in the average expected transfer, from 0.6% to roughly 2.5% of banks' value.

Consistently with our theoretical results, Figures 5a and b jointly illustrate that, through the mechanism that we study in this paper, increases in banking concentration have the potential to substantially affect banks' choices, especially when large banks are sufficiently large.

6.2 Optimal policy

Building on the results described in Section 4, Figure 6 quantitatively assesses the magnitude of the optimal Pigouvian policy, for different values of concentration. Since we have assumed that the wedge $\bar{\psi}$ between the private and social costs of equity issuance is non-zero, the planner finds it optimal to set a non-zero corrective tax for both large and small banks.

FIGURE 6 ABOUT HERE

The optimal Pigouvian tax for the baseline parametrization is of the order of 7.5% of the value of the debt issued by banks, and is directly related to the value of $\bar{\psi}$, as implied by the optimal tax characterization in Equation (30). The optimal debt-to-asset choice for the planner in the baseline scenario for both large and small banks is 0.874. In particular, large and small banks respectively face taxes of 7.69% and 7.29%. However, the main object of interest for us is the differential tax charged to large banks relative to small banks. In our baseline scenario, large banks face an additional 0.4% tax relative to small banks under the optimal ex-ante policy: we refer to this difference in taxes as a "size tax". The optimal size tax grows approximately linearly with λ. For instance, our model implies that an increase in concentration in which the largest 5 banks hold 70% of assets (λ moves from 0.5 to 0.7) would be associated a 50% increase in the optimal size tax, from 0.4% to about 0.6%.

We further evaluate the constrained efficient choice for our baseline parametrization. The optimal debt-to-asset ratio is 87.4% and, consistently with Proposition 3, this choice is independent of bank size. Under perfect enforcement, the planner's choice can then be decentralized by imposing the capital requirement that $\frac{\text{Equity}}{\text{Assets}} = \frac{\text{Assets} - \text{Debt}}{\text{Assets}} \leq 12.6\%$.

6.3 Sensitivity analysis: granularity parameter

Finally, we explore the sensitivity of our results to the granularity parameter ζ, which plays a crucial role when considering changes in bank concentration. Figure 7 assesses the sensitivity of our quantitative results to ζ. Recall that $\zeta = 1$ corresponds to the case where idiosyncratic shocks occur at the bank level, consistent with the granular hypothesis, while $\zeta \simeq 0$ corresponds to the case where large banks are perfectly diversified. The high levels of leverage associated with our baseline calibration ($\zeta = 1$), and the strategic effects of bank size, arise only for ζ sufficiently close to 1, and are small for $\zeta < 0.5$.

FIGURE 7 ABOUT HERE

FIGURE 8 ABOUT HERE

Figure 8 illustrates the deeper reasons for this result by examining large banks' objective function and expected government transfers. As ζ falls, large banks face lower idiosyncratic risk and are less likely to default for given leverage choices. Hence, bailout transfers become less relevant relative to trade-off theory considerations, dampening the strategic differences between large and small banks, and inducing lower aggregate leverage in equilibrium. This occurs because the expected government transfer becomes less sensitive to large banks' borrowing choices, which decreases large banks' marginal benefit from borrowing, consistently with banks' optimality conditions (11) and (12).

The granular case $\zeta \simeq 1$ is empirically plausible: Minton, Stulz and Taboada (2017), for example, report a marginally *higher* standard deviation of return on assets for the largest banks in the Call Reports data – our own unreported calculations replicate their conclusions. This is consistent with estimates for large companies across industries in Gabaix (2011). Therefore, the sensitivity of our results to ζ arises in an empirically unlikely region of the parameter space. However, our analysis highlights an interesting feature of the model. The impact of granular shocks on system-wide risk is amplified by the strategic responses of large banks. Therefore, and in addition to the failure of the Law of Large Numbers highlighted by Gabaix (2011), granular shocks further increase aggregate volatility due to the behavioral response of large firms in equilibrium.

7 Conclusion

We have shown that the size distribution of financial institutions does matter for the ex-ante determination of leverage when bailouts are possible. Large banks, by internalizing that their actions affect the government's bailout response, find it optimal to increase their leverage in equilibrium. Their increased leverage increases the magnitude of bailouts, thereby encouraging small banks to take on more leverage. Both effects are mutually reinforcing, and generate further increases system-wide leverage in equilibrium. Hence, aggregate leverage and the magnitude of government bailout interventions are larger when large banks are present.

Our results rely on the fact that system-wide policy responses induce strategic complementarities but do not hinge on the exact nature of how banks determine their capital structure. Our findings support the notion that regulators and policymakers must pay special attention to large financial institutions, since they have a direct motive to take on more risk and also because their behavior disproportionally influences the decisions of small players in equilibrium. In this model, a regulator that closely monitors and restricts funding decisions of large financial institutions arises as a natural optimal policy.

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Appendix

A Section 3: Proofs and derivations

Government's problem

For a given signal realization s, the government optimally chooses a transfer $t(\boldsymbol{b}|s)$ at date 1 to maximize:

$$\max_{t\geq 0} W_1(\boldsymbol{b},t|s), \tag{37}$$

where $W_1(\boldsymbol{b},t|s)$ is given by

$$W_1(\boldsymbol{b},t|s) = \int_0^\infty u dF_u(u|s) - (1-\phi)\int\left[\int_0^{b^i-t} u dF_u(u|s)\right]d\mu - \left(\psi-\bar{\psi}\right)\int\left[\int_{b^i-t}^\infty \left(t+u-b^i\right)dF_u(u|s)\right]d\mu - \kappa(t), \tag{38}$$

which corresponds to Equation (2) in the text. Note that the outer integrals in the second and third terms are cross-sectional integrals over banks i, while the inner ones are integrals over the possible realizations of u given the signal s.

The first-order and second-order conditions to this problem correspond to

$$\frac{\partial W_1(\boldsymbol{b},t|s)}{\partial t} = (1-\phi)\int f_u\left(b^i-t|s\right)\left(b^i-t\right)d\mu - \left(\psi-\bar{\psi}\right)\int\left[1-F_u\left(b^i-t|s\right)\right]d\mu - \kappa'(t) \tag{39}$$

$$\frac{\partial^2 W_1(\boldsymbol{b},t|s)}{\partial t^2} = -(1-\phi)\int\left[f_u'\left(b^i-t|s\right)\left(b^i-t\right)+f_u\left(b^i-t|s\right)\right]d\mu - \left(\psi-\bar{\psi}\right)\int f_u\left(b^i-t|s\right)d\mu - \kappa''(t) \leq 0. \tag{40}$$

Note that the second-order condition is always satisfied with strict inequality under our regularity condition (1). It immediately follows that $\lim_{t\to\infty}\frac{\partial W_1(\boldsymbol{b},t|s)}{\partial t} < 0$, which guarantees that the optimal t is bounded above. Note further that

$$\left.\frac{\partial W_1(\boldsymbol{b},t|s)}{\partial t}\right|_{t=0} = (1-\phi)\int f_u\left(b^i|s\right)b^i d\mu - \left(\psi-\bar{\psi}\right)\int\left[1-F_u\left(b^i|s\right)\right]d\mu - \kappa'(0), \tag{41}$$

which implies that the optimal transfer $t(\boldsymbol{b}|s)$ is zero when $(1-\phi)\int f_u(b^i|s)\,b^i d\mu \leq \kappa'(0) + \left(\psi-\bar{\psi}\right)\int\left[1-F_u(b^i|s)\right]d\mu$, and determined uniquely by the first-order condition otherwise.

Totally differentiating the first-order condition with respect to b^j, we have

$$\begin{aligned}0 &= \frac{\partial^2 W_1(\boldsymbol{b},t|s)}{\partial t^2}\times\frac{\partial t}{\partial b^j}+\frac{\partial^2 W_1(\boldsymbol{b},t|s)}{\partial t\partial b^j} \\ &= \frac{\partial^2 W_1(\boldsymbol{b},t|s)}{\partial t^2}\times\frac{\partial t}{\partial b^j}+\mu(j)\times\left[(1-\phi)\left(f_u'\left(b^j-t|s\right)\left(b^j-t\right)+f_u\left(b^j-t|s\right)\right)+\left(\psi-\bar{\psi}\right)f_u\left(b^j-t|s\right)\right],\end{aligned} \tag{42}$$

and it follows that

$$\frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^{j}}=\left(-\frac{\partial^{2}W_{1}\left(\boldsymbol{b},t|s\right)}{\partial t^{2}}\right)^{-1}\mu\left(j\right)\left[\left(1-\phi\right)\left(f_{u}'\left(b^{j}-t|s\right)\left(b^{j}-t\right)+f_{u}\left(b^{j}-t|s\right)\right)+\left(\psi-\bar{\psi}\right)f_{u}\left(b^{j}-t|s\right)\right].\tag{43}$$

Note that first factor is strictly positive, given the second-order condition, which implies Equation (5).

Banks' market value

The ex-post payoffs to debtholders and shareholders are defined in Equations (6) and (7) in the text. Taking expectations under the joint distribution of returns and signals $F\left(u,s\right)$ yields the date 0 value of bonds and shares for bank i, respectively given by

$$q^{i}b^{i}=\int\limits_{\mathcal{N}^{i}}b^{i}dF+\int\limits_{\mathcal{D}^{i}}\left(\phi u+t\left(\boldsymbol{b}|s\right)\right)dF\tag{44}$$

$$e^{i}=\left(1-\psi\right)\int\limits_{\mathcal{N}^{i}}\left(u+t\left(\boldsymbol{b}|s\right)-b^{i}\right)dF,\tag{45}$$

where the default and repayment regions are defined as $\mathcal{D}^{i}=\left\{\left(u,s\right)|t\left(\boldsymbol{b}|s\right)+u<b^{i}\right\}$ and $\mathcal{N}^{i}=\left\{\left(u,s\right)|t\left(\boldsymbol{b}|s\right)+u\geq b^{i}\right\}$. Adding up both, we can express the market value of the bank as follows

$$\begin{aligned}q^{i}b^{i}+e^{i}&=\int\limits_{\mathcal{N}^{i}}\left[u+t\left(\boldsymbol{b}|s\right)-\psi\left(u+t\left(\boldsymbol{b}|s\right)-b^{i}\right)\right]dF+\int\limits_{\mathcal{D}^{i}}\left[u+t\left(\boldsymbol{b}|s\right)-\left(1-\phi\right)u\right]dF\\&=\int\left(u+t\left(\boldsymbol{b}|s\right)\right)dF-\left(1-\phi\right)\int\limits_{\mathcal{D}^{i}}udF-\psi\int\limits_{\mathcal{N}^{i}}\left(u+t\left(\boldsymbol{b}|s\right)-b^{i}\right)dF\\&=\mathbb{E}\left[u+t\left(\boldsymbol{b}|s\right)\right]-\left(1-\phi\right)\Pr\left[\mathcal{D}^{i}\right]\mathbb{E}\left[u|\mathcal{D}^{i}\right]-\psi\Pr\left[\mathcal{N}^{i}\right]\mathbb{E}\left[u+t\left(\boldsymbol{b}|s\right)-b^{i}|\mathcal{N}^{i}\right],\end{aligned}\tag{46}$$

which corresponds to the definition of $V\left(b^{i},t\left(\boldsymbol{b}|s\right)\right)$ in Equation (8).

Banks' leverage choices

The bank maximizes its market value. Let $\mathcal{S}$ denote the support of the signal s, and recall that $G(s)$, $s\in\mathcal{S}$, is the marginal distribution of signals. We can write the bank's value, from (46), more explicitly as

$$V\left(b^{i},t\left(\boldsymbol{b}|s\right)\right)=\mathbb{E}\left[u\right]+\int\limits_{\mathcal{S}}\left[t\left(\boldsymbol{b}|s\right)-\left(1-\phi\right)\int\limits_{0}^{b^{i}-t(\boldsymbol{b}|s)}udF_{u}\left(u|s\right)-\psi\int\limits_{b^{i}-t(\boldsymbol{b}|s)}^{\infty}\left(u+t\left(\boldsymbol{b}|s\right)-b^{i}\right)dF_{u}\left(u|s\right)\right]dG(s).\tag{47}$$

Partially differentiating with respect to b^i gives

$$\frac{\partial V}{\partial b^i} = \int\limits_{\mathcal{S}} \left[-(1-\phi)\left(b^i - t\left(\boldsymbol{b}|s\right)\right) f_u\left(b^i - t\left(\boldsymbol{b}|s\right)|s\right) + \psi \int\limits_{b^i - t(\boldsymbol{b}|s)}^{\infty} dF_u\left(u|s\right)\right] dG(s)$$
$$= \psi \Pr\left[\mathcal{N}^i\right] - (1-\phi) \int\limits_{\partial \mathcal{D}^i} u dF, \tag{48}$$

which corresponds to Equation (11) in the text. The total effect of increasing b^i further takes into account the effect of b^i on optimal transfers, and is given by

$$\frac{dV}{db^i} = \frac{\partial V}{\partial b^i} + \int\limits_{\mathcal{S}} \frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^i} \left[1 + (1-\phi)\left(b^i - t\left(\boldsymbol{b}|s\right)\right) f_u\left(b^i - t\left(\boldsymbol{b}|s\right)|s\right) - \psi \int\limits_{b^i - t(\boldsymbol{b}|s)}^{\infty} dF_u\left(u|s\right)\right] dG(s)$$
$$= \frac{\partial V}{\partial b^i} + \mathbb{E}\left[\frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^i}\left(1 - \psi \mathbf{1}\left(\mathcal{N}^i\right)\right)\right] + (1-\phi)\int\limits_{\partial \mathcal{D}^i} \frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^i} u dF, \tag{49}$$

which corresponds to Equation (12) in the text. The first-order conditions (11) and (12) follow by noting that $\frac{\partial t(\boldsymbol{b}|s)}{\partial b^i} = 0$ for small banks.

Proof of Lemma 1 (Effect of large bank leverage on small banks' incentives.)

Proof. Differentiating the bank's value function in (8) with respect to b^j gives

$$\frac{dV\left(b^i, t\left(\boldsymbol{b}|s\right)\right)}{db^j} = \mathbb{E}\left[\frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^j}\left(1 - \psi \mathbf{1}\left(\mathcal{N}^i\right)\right)\right] + (1-\phi)\int\limits_{\partial \mathcal{D}^i} \frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^j} u dF > 0, \tag{50}$$

where the inequality follows from the properties of optimal bailouts. Indeed, (5) implies that $\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} \geq 0$, with strict inequality for the (positive measure) set of public signals s such that $t\left(\boldsymbol{b}|s\right) > 0$.

Differentiating again with respect to b^i gives

$$\frac{d^2 V\left(b^i, t\left(\boldsymbol{b}|s\right)\right)}{db^i db^j} = \psi \int\limits_{\partial \mathcal{D}^i} \frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^j} dF + (1-\phi)\int\limits_{\partial \mathcal{D}^i} \frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^j}\left\{1 + \frac{u f_u'\left(u|s\right)}{f_u\left(u|s\right)}\right\} dF > 0, \tag{51}$$

where the inequality follows from our regularity condition (1). □

Proof of Proposition 1 (Large banks borrow more.)

Proof. Take any equilibrium with leverage choices $\hat{\boldsymbol{b}}$ and ex-post optimal transfer $\hat{t}(s) = t\left(\hat{\boldsymbol{b}}|s\right)$. Suppose that j is a large bank, and let $t^j\left(b^j|s\right) = t\left(\boldsymbol{b}|s\right)\big|_{\boldsymbol{b}^{-j}=\hat{\boldsymbol{b}}^{-j}}$ be the transfer which becomes ex-post optimal if a large bank j chooses borrowing level b^j, while all other banks play their equilibrium strategy (note that $t^j(\hat{b}^j|s) \equiv \hat{t}(s)$).

First, suppose that $\hat{b}^i > \hat{b}^j$ for a small bank i and a large bank j. Optimality implies that neither bank has a profitable deviation by copying the other's strategy. Bank i's market value in equilibrium is $V\left(\hat{b}^i, \hat{t}(s)\right)$. If bank i deviates from equilibrium by copying bank j's choice $\hat{b}^j$, the bailout policy is unchanged because bank i is small, and i's market value becomes $V\left(\hat{b}^j, \hat{t}(s)\right)$. Therefore, optimality for bank i implies

$$V\left(\hat{b}^i, \hat{t}(s)\right) \geq V\left(\hat{b}^j, \hat{t}(s)\right). \tag{52}$$

Bank j's market value in equilibrium is $V\left(\hat{b}^j, \hat{t}(s)\right)$. If bank j deviates to $\hat{b}^i$, then the bailout policy changes to $t^j\left(\hat{b}^i|s\right)$, and j's market value becomes $V\left(\hat{b}^i, t^j\left(\hat{b}^i|s\right)\right)$. Therefore, optimality for bank j requires that

$$V\left(\hat{b}^j, \hat{t}(s)\right) \geq V\left(\hat{b}^i, t^j\left(\hat{b}^i|s\right)\right). \tag{53}$$

Combining the two optimality conditions, we have $V\left(\hat{b}^i, \hat{t}(s)\right) \geq V\left(\hat{b}^i, t^j\left(\hat{b}^i|s\right)\right)$, or equivalently,

$$0 \geq V\left(\hat{b}^i, t^j\left(\hat{b}^i|s\right)\right) - V\left(\hat{b}^i, t^j\left(\hat{b}^j|s\right)\right) = \int_{\hat{b}^j}^{\hat{b}^i} \left.\frac{dV\left(\hat{b}^i, t(\boldsymbol{b}|s)\right)}{db^j}\right|_{\boldsymbol{b}^{-j}=\hat{\boldsymbol{b}}^{-j}} db^j, \tag{54}$$

which contradicts Lemma 1. Therefore, $\hat{b}^j \geq \hat{b}^i$.

Second, suppose that $\hat{b}^j = \hat{b}^i$. We begin by showing that the small bank makes an interior choice in equilibrium, with $0 < \hat{b}^i < \infty$. Taking limits of the derivative in (11), we get

$$\lim_{b^i \to 0} \frac{\partial V\left(b^i, \hat{t}(s)\right)}{\partial b^i} = \psi \Pr\left[\mathcal{N}^i\right] - (1-\phi) \int_{\partial \mathcal{D}^i} \left(-\hat{t}(s)\right) dF > 0. \tag{55}$$

Thus the small bank must choose $\hat{b}^i > 0$. Moreover, we have

$$\lim_{b^i \to \infty} \frac{\partial V\left(b^i, \hat{t}(s)\right)}{\partial b^i} = -(1-\phi) \lim_{b^i \to \infty} \int_{\partial \mathcal{D}^i} \left(b^i - \hat{t}(s)\right) dF. \tag{56}$$

Note that the integral is strictly positive for a given b^i if and only if $b^i > \mathbb{E}\left[\hat{t}(s)|\mathcal{D}^i\right]$. The convexity of the cost $\kappa(t)$ of bailouts, combined with our assumption that $\lim_{t\to\infty} \kappa(t) = \infty$, implies that $\hat{t}(s)$ is bounded above. Since the small bank takes $\hat{t}(s)$ as given, it follows that the integral is positive for large enough b^i, so that $\lim_{b^i\to\infty} \frac{\partial V(b^i, \hat{t}(s))}{\partial b^i} < 0$, which establishes that the optimal choice $\hat{b}^i < \infty$. Therefore small banks are at an interior solution with $\hat{b}^i \in (0, \infty)$, and the first-order condition

$$\frac{\partial V\left(\hat{b}^i, \hat{t}(s)\right)}{\partial b^i} = 0. \tag{57}$$

holds with equality. The large bank's first-order condition, using the conjecture $\hat{b}^j = \hat{b}^i$, can in turn be written as

$$\frac{\partial V\left(\hat{b}^i, \hat{t}(s)\right)}{\partial b^i} + \frac{dV\left(\hat{b}^i, t(\boldsymbol{b}|s)\right)}{db^j} = 0. \tag{58}$$

Lemma 1 implies that the second term is strictly positive, and we obtain $\frac{\partial V\left(\hat{b}^i, \hat{t}(s)\right)}{\partial b^i} < 0$, a contradiction. $\square$

Proof of Proposition 2 (When large banks are present, small banks take more leverage.)

Proof. Let $t_0(b|s)$ denote the ex-post optimal transfer when there are only small banks who play the symmetric strategy $b^i = b$, and let $BR_0(b) = \arg\max_{b^i} V(b^i, t_0(b|s))$ be a small bank's best response to this transfer. b is a symmetric equilibrium if $b \in BR_0(b)$. Since $\left.\frac{\partial V(b^i,t)}{\partial b^i}\right|_{b^i=0} > 0$ regardless of transfers, we know that $BR_0(0) > 0$. By Tarski's fixed point theorem, the smallest equilibrium b_0 exists, and moreover, we have $\inf BR_0(b) > b$ for all $b < b_0$.

Now take any economy with large banks, and any equilibrium (not necessarily symmetric) with leverage choices $\hat{\boldsymbol{b}}$ and ex-post optimal transfer $\hat{t}(s) = t\left(\hat{\boldsymbol{b}}|s\right)$. Let the lowest borrowing level arising in equilibrium be $b_1 \equiv \inf_i \hat{b}^i$. Proposition 1 implies that a positive measure of banks (at least all large banks) chooses to borrow strictly more than b_1. Moreover, a parallel argument to Lemma 1 implies that incentives to borrow for small banks are strictly stronger under the equilibrium transfer $\hat{t}(s)$, than under the transfer $t_0(b_1|s)$ which would obtain if everybody chose b_1:

$$\frac{\partial V\left(b^i, \hat{t}(s)\right)}{\partial b^i} > \frac{\partial V\left(b^i, t_0(b_1|s)\right)}{\partial b^i}. \tag{59}$$

We need to show that $b_1 > b_0$.

First, suppose that $b_1 < b_0$. Let $\tilde{b}_1 = \inf BR_0(b_1)$ be the lowest borrowing level which small banks pick if everybody else chooses b_1. We know that $\tilde{b}_1 > b_1$, because our assumption $b_1 < b_0$ implies that $BR_0(b_1) > b_1$. Since $\tilde{b}_1$ maximizes bank value given the transfer $t_0(b_1|s)$, we have

$$V\left(\tilde{b}_1, t_0(b_1|s)\right) \geq V\left(b_1, t_0(b_1|s)\right) \tag{60}$$

Moreover, since b_1 maximizes some (small) bank's value given the equilibrium transfer $\hat{t}(s)$, we have

$$V\left(b_1, \hat{t}(s)\right) \geq V\left(\tilde{b}_1, \hat{t}(s)\right). \tag{61}$$

Combining,

$$V\left(\tilde{b}_1, t_0(b_1|s)\right) - V\left(b_1, t_0(b_1|s)\right) \geq 0 \geq V\left(\tilde{b}_1, \hat{t}(s)\right) - V\left(b_1, \hat{t}(s)\right), \tag{62}$$

which implies

$$\int_{b_1}^{\tilde{b}_1} \left(\frac{\partial V\left(b^i, t_0(b_1|s)\right)}{\partial b^i} - \frac{\partial V\left(b^i, \hat{t}(s)\right)}{\partial b^i}\right) db^i \geq 0, \tag{63}$$

contradicting (59).

Second, suppose that $b_1 = b_0$. Since b_0 is optimal if everybody else chooses b_0, we have the first-order condition for a small bank

$$\frac{\partial V\left(b_0, t_0(b_0|s)\right)}{\partial b^i} = \frac{\partial V\left(b_1, t_0(b_1|s)\right)}{\partial b^i} = 0. \tag{64}$$

Moreover, since some small bank optimally chooses b_1 in equilibrium, we have

$$\frac{\partial V\left(b_1, \hat{t}(s)\right)}{\partial b^i} = 0 = \frac{\partial V\left(b_1, t_0\left(b_1|s\right)\right)}{\partial b^i}, \tag{65}$$

again contradicting (59). $\square$

B Section 4: Proofs and derivations

Proof of Lemma 2 (Commitment is irrelevant when the planner controls banks.)

Proof. Taking expectations over signals in (2) and comparing to (24) we obtain, for any (not necessarily optimal) bailout policy $t(s)$,

$$W_0\left(\boldsymbol{b}, t(s)\right) = \mathbb{E}\left[W_1\left(\boldsymbol{b}, t(s)|s\right)\right]. \tag{66}$$

Suppose that $W^c > W^{nc}$. Then the constraint in W^{nc} must bind in some states, that is, the policy achieving W^c is not ex-post optimal for a set of signals s with positive probability measure. Call this policy $\{\boldsymbol{b}^c, t^c(s)\}$ and consider replacing the transfer policy with the ex-post best response to $\boldsymbol{b}^c$, setting $t'(s) = \arg\max_t W_1(\boldsymbol{b}^c, t|s)$ for almost all s. By ex-post optimality, $W_1\left(\boldsymbol{b}^c, t'|s\right) \geq W_1\left(\boldsymbol{b}^c, t^c|s\right)$, with strict inequality for a positive measure of signals. The policy $\{\boldsymbol{b}^c, t'(s)\}$ yields ex-ante welfare $W_0(\boldsymbol{b}^c, t'(s)) = \mathbb{E}\left[W_1\left(\boldsymbol{b}^c, t'(s)|s\right)\right] > \mathbb{E}\left[W_1\left(\boldsymbol{b}^c, t^c(s)|s\right)\right]$, contradicting optimality of $\{\boldsymbol{b}^c, t^c(s)\}$. $\square$

Proof of Proposition 3 (Efficient leverage is independent of bank size.)

Proof. Lemma 2 shows that the maximization problems with and without commitment are equivalent. We consider the problem (25) with commitment. Suppose policy $\{\boldsymbol{b}^c, t^c(s)\}$ solves this problem. Note that we may write welfare in terms of bank value, the wedge between private and social equity issuance costs, and transfers:

$$W_0\left(\boldsymbol{b}, t(s)\right) = \int\left[V\left(b^i, t(s)\right) + \bar{\psi}\Pr\left[\mathcal{N}^i\right]\mathbb{E}\left[t(s) + u - b^i|\mathcal{N}^i\right]\right]d\mu - \mathbb{E}\left[t(s) + \kappa\left(t(s)\right)\right]. \tag{67}$$

Since $t(s)$ may be chosen independently of $\boldsymbol{b}$, the first-order condition for the optimal $\boldsymbol{b}$ is obtained by differentiating pointwise under the integral, and we obtain

$$\frac{\partial V\left(b^{c,i}, t^c(s)\right)}{\partial b^i} = \bar{\psi}\Pr\left[\mathcal{N}^i\right] \tag{68}$$

for all i. But under our regularity condition, the pointwise maximization problem is strictly concave in b^i so that we have $b^{c,i} = b^{c,j} = \bar{b}$ for almost all i, j. $\square$

C Section 5: Proofs and derivations

Equation (36) defines the distribution of total asset returns $u^i = v + w^i$ of bank i. We denote its density conditional on the public signal s by $f_u^i(u|s)$ as before. We impose the equivalent of our previous regularity condition (1) on this density: In marginal default states where $u^i = b^i - t$, we require that

$$\frac{d\log f_u^i(u|s)}{d\log u} > -1, \quad \forall s, i. \tag{69}$$

Note that this is generally a weaker condition than our requirement in (1) for the baseline model. Indeed, (1) holds when public signals about the aggregate state are sufficiently noisy. Since idiosyncratic risk effectively introduces noise into the government's signal of bank health, (69) is generally an even milder condition than (1).

In the model with idiosyncratic risk, welfare at date 1 is

$$W_1(\boldsymbol{b}, t|s) = \mathbb{E}[v|s] - (1-\phi)\int\left[\int_0^{b^i - t} u dF_u^i(u|s)\right] d\mu - \left(\psi - \bar{\psi}\right)\int\left[\int_{b^i - t}^{\infty}\left(t + u - b^i\right) dF_u^i(u|s)\right] d\mu - \kappa(t), \tag{70}$$

The ex-post optimal bailout policy $t(\boldsymbol{b}|s)$ can be characterized exactly as in the text. In particular, for a large bank j, we have the analogue to Equation (5):

$$\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} \overset{\text{sign}}{=} \left[f_u^{j,\prime}\left(b^j - t|s\right)\left(b^j - t\right) + f_u^j\left(b^j - t|s\right) + \frac{\psi - \bar{\psi}}{1-\phi} f_u^j\left(b^j - t|s\right)\right]\Bigg|_{u = b^j - t(\boldsymbol{b}|s)} \geq 0, \tag{71}$$

with strict inequality whenever $t(\boldsymbol{b}|s) > 0$. Now writing down the market value of bank i, we have

$$V\left(b^i, t(\boldsymbol{b}|s)\right) = \int_{\mathcal{S}}\left[u + t(\boldsymbol{b}|s) - (1-\phi)\int_0^{b^i - t(\boldsymbol{b}|s)} u dF_u^i(u|s) - \psi\int_{b^i - t(\boldsymbol{b}|s)}^{\infty}\left(u + t(\boldsymbol{b}|s) - b^i\right) dF_u^i(u|s)\right] dG(s). \tag{72}$$

Given this characterization, we can establish that the key qualitative properties of the bank's objective function V are as in the baseline model without idiosyncratic risk. In particular, if j is a large bank and i is a small bank, then

1. Bank j has strictly higher incentives to take leverage, other things equal:

$$\frac{dV\left(b^j, t(\boldsymbol{b}|s)\right)}{db^j} > \frac{dV\left(b^i, t(\boldsymbol{b}|s)\right)}{db^i} \tag{73}$$

2. Banks' leverage choices are strategic complements:

$$\frac{d^2 V\left(b^i, t(\boldsymbol{b}|s)\right)}{db^i db^j} > 0. \tag{74}$$

The derivations of these properties are identical to those in Appendix A and in the Proof of Lemma 1. We can now apply the arguments of Propositions 1 and 2 without further modification.

D Section 6: Choice of parameters

Table 1 summarizes the parameter choices in our baseline parametrization. Unless explicitly stated, any reference to measures of actual banks' performance is drawn from U.S. Call Reports data, as distributed by Drechsler, Savov and Schnabl (2016, 2017). We need to assign values to twelve parameters, which can be classified into three broad categories: banks' profitability, industry composition, and government related parameters. Our strategy is to select parameters to target average cross-sectional values, letting the model endogenously generate differences in behavior between large and small banks. We take a conservative stance and discipline all parameters using directly observable or already available information, except for σ_v and κ_2, which we jointly determine by matching two key statistics of our model. We adopt this approach because obtaining direct measures of the volatility σ_v of aggregate shocks to the banking sector is difficult, since such shocks (i.e. financial crises) are rare events. Bianchi (2016) and Mendicino, Nikolov and Suarez (2017) follow a similar approach by calibrating aggregate volatility parameters to match the empirical frequency of crises.

The first set of parameters determines banks' profitability and their capital structure. We set the value of deadweight losses associated with default to be $1 - \phi = 20\%$, of banks' value. This choice is consistent with the existing literature on capital structure, as shown by Davydenko, Strebulaev and Zhao (2012) and Strebulaev and Whited (2012). We set the value of $\psi = 0.15$, which is on the high end of estimates used in Hennessy and Whited (2005), but which is necessary to generate the large levels of leverage observed in the banking sector. Consistent with our agnostic view on the social vs. private costs of equity we set $\bar{\psi}$ equal to $\frac{\psi}{2}$. We set $\mu_v = 1.02$ and $\sigma_w = 0.06$ to match the average standard deviation across all banks during the period of interest in the Call Reports data. In line with the findings of Gabaix (2011), and consistently with our own calculations using the Call Reports data, we set the granularity parameter to $\zeta = 1$, implying that the idiosyncratic return risk is identical across banks of all sizes. Finally, we choose $\sigma_v = 0.02$ (jointly with κ_2) to target a probability of a significant intervention, defined as the probability of receiving a transfer higher than 2%, of 10%, which corresponds to a crisis episode in 20 years, as in Mendicino, Nikolov and Suarez (2017) – see also Reinhart and Rogoff (2009) – and to target an average intervention conditional on a bailout occurring of 3% of bank value, which is slightly above the median for developed countries in Laeven and Valencia (2013), but substantially below the median for emerging markets, and in line with bailouts granted to major global banks during the crisis of 2008, as reported by Hüttl and Schoenmaker (2016).

TABLE 1 ABOUT HERE

The second set of parameters pins down the industry composition. Consistently with our description of recent U.S. data in Figure 1, we set values of $\lambda = 0.5$ and $N = 5$. Our choice of N seeks to capture the persistence in the size rankings of a handful of banks. In particular, Bank of America, Citibank, JP Morgan Chase, and Wells Fargo have persistently ranked among the top 5 banks by asset size for the whole period considered. Our benchmark choice of λ is consistent with current levels of concentration, although our central counterfactual exercise explores in detail how alternative values of λ affect the outcomes of the model.

The third set of parameters determines the magnitude of the bailout policy.[6] Given that σ_s and σ_w cannot be separately identified when $\zeta = 1$, and to remain parsimonious, we set $\sigma_s = 0$, implying that the government perfectly observes the aggregate state. We set the value of κ_1 to match a (net) marginal cost of public funds for small interventions of 13%, which is a standard estimate in the literature (Dahlby, 2008). Finally, as described above, we choose $\kappa_2 = 10$ (jointly with σ_v) to target a probability of a significant intervention every 20 years, and an average intervention conditional on a bailout occurring of 3% of bank value.

E Section 6: Bank size distribution

Lastly, we evaluate whether our results are sensitive to our approximation of the bank size distribution. Our baseline results use the size distribution shown in Figure 4, with 5 large banks controlling 50% of aggregate assets. For a more detailed calibration, consistent with the Call Reports data (see Figure 1a), we introduce 15 additional "type 2" large banks that jointly control 20% of aggregate assets. The remaining 30% is managed by a continuum of small banks.

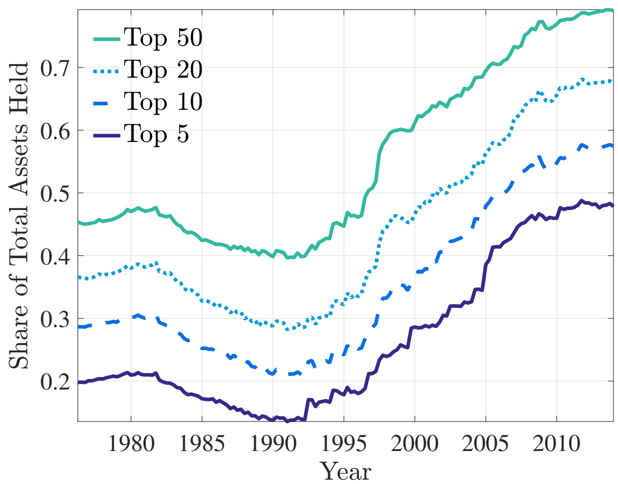
Figure 9 illustrates leverage choices in equilibrium. We vary the share λ of assets controlled by the 5 largest banks while holding constant the relative shares of remaining bank types. In the baseline case $\lambda = 0.5$, the largest banks choose debt-to-assets equal to 0.910, while "type 2" banks and small banks select 0.902 and 0.901 respectively. Accordingly, the figure shows that the relationship between λ and aggregate leverage is also similar to our baseline calibration.

FIGURE 9 ABOUT HERE

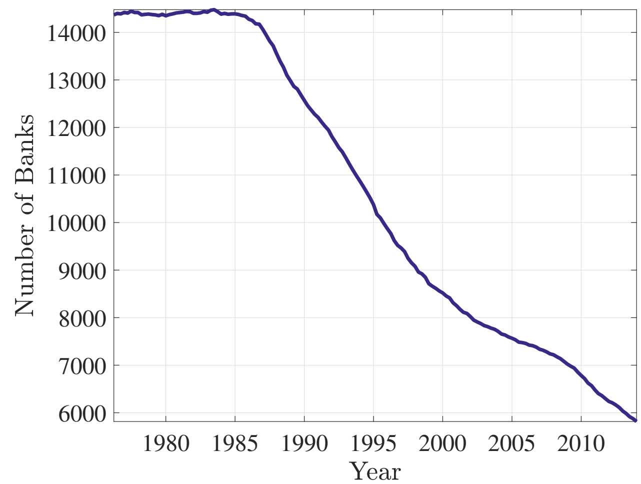
[6] Note that, after observing the signal s, the government perceives the return v to be distributed as $v|s \sim N\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}s + \frac{\sigma_s^2}{\sigma_v^2+\sigma_s^2}\mu_v, \frac{\sigma_v^2\sigma_s^2}{\sigma_v^2+\sigma_s^2}\right)$. Although assuming a Gaussian structure for returns and signals simplifies the government inference problem, its drawback is the possibility of experiencing negative payoffs. Our parametrization is such that the probability of u taking negative values is negligible. Similar results emerge assuming that returns are log-normally distributed. Since we allow for idiosyncratic risk, we are not restricted in the choice of σ_s. In particular, the government's problem remains smooth even when the aggregate state is perfectly observed with $\sigma_s = 0$.

Intuitively, the similarity in behavior of medium-sized and small banks suggests that strategic leverage incentives quickly diminish when considering large banks outside the top 5. Since our model is designed to isolate strategic effects, there appears to be little loss in considering the stylized size distribution in Figure 4.

Tables and Figures



(a) Share of total assets held by largest banks



(b) Total number of U.S. banks

Figure 1: Measures of bank concentration

Note: Figure 1a shows the share of total assets held by the 5, 10, 20, and 50 largest U.S. banks in terms of assets from 1976Q1 until 2013Q4. Figure 1b shows the total number of U.S. banks each year over the same period. Note that the secular increase in concentration reaches back further. The U.S. economy had more than 25,000 banks in the 1920's; see, e.g., Davison and Ramirez (2014). Both figures are based on U.S. Call Reports data, as distributed by Drechsler, Savov and Schnabl (2016, 2017). See also Janicki and Prescott (2006) and Fernholz and Koch (2016), who study the dynamics of the distribution of bank assets and document the sustained increase in concentration in the U.S. banking sector over the last decades. Bank concentration in other countries is even higher than in the U.S., as documented by Laeven, Ratnovski and Tong (2014).

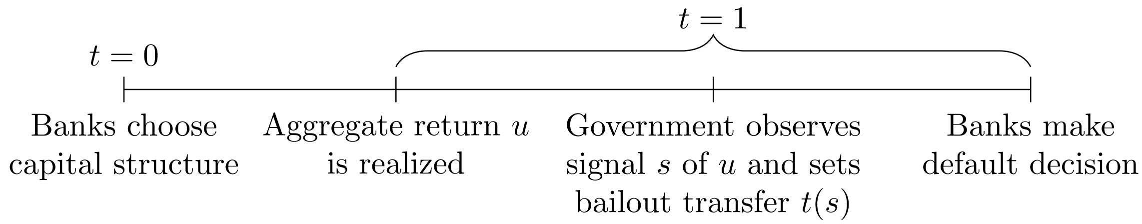


Figure 2: Timeline of events

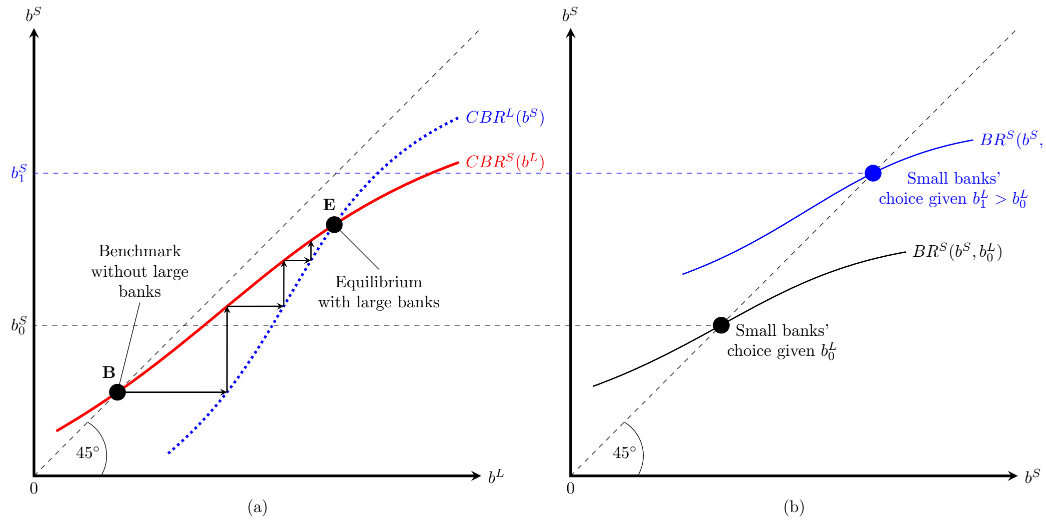


Figure 3: Equilibrium determination

Note: Figure 3a shows the best response of large banks (blue dashed line), given the borrowing choices of small banks, and the aggregate best response of small banks (red solid line), given the best responses of large banks. Figure 3b shows the best responses of a small bank given the borrowing choices of other small banks for two given values of large bank borrowing (b^L_0 and b^L_1). By varying the level of b^L_0 in Figure 3b, we can trace the aggregate best response of small banks (solid red line) in Figure 3a.

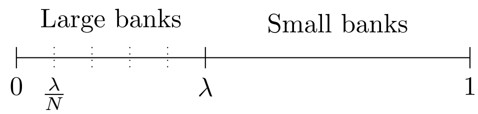
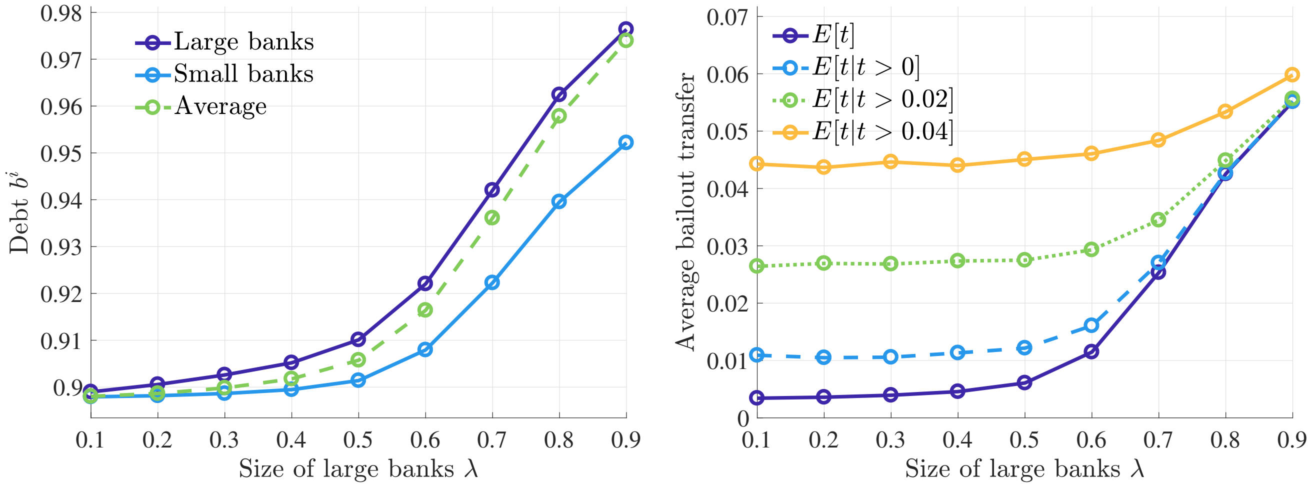


Figure 4: Distribution of banks' assets

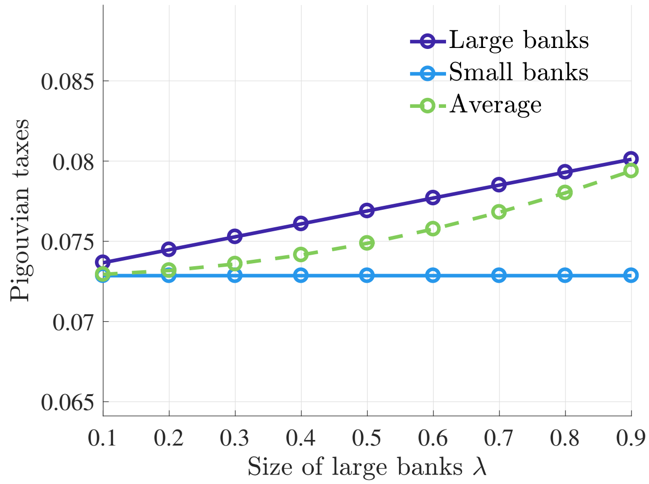


(a) Banks' borrowing choices

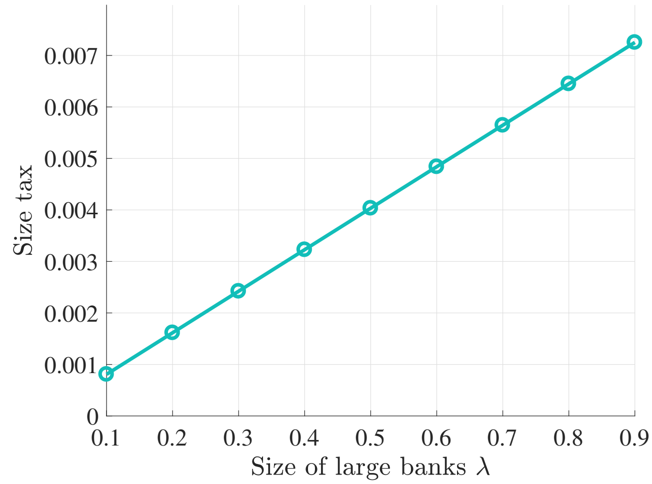
(b) Average bailout transfer

Figure 5: Equilibrium borrowing choices and government response

Note: Figure 5a shows the optimal equilibrium debt-to-asset ratio chosen by large banks (dark blue solid line) and by small banks (light blue solid line) for different levels of the share of assets held by large banks $\lambda \in [0.1, 0.9]$. It also shows the average debt-to-asset ratio in the economy (green dashed line). Note that, mechanically, the average debt-to-asset ratio tends to the debt-to-asset ratio of small banks when $\lambda \to 0$ and to the debt-to-asset ratio of large banks when $\lambda \to 1$. Figure 5b shows the equilibrium level of average bailout transfers, conditional on the size of the intervention.



(a) Optimal Pigouvian taxes



(b) Optimal size tax

Figure 6: Optimal regulation

Note: Figure 6a shows the optimal Pigouvian tax levied on large banks (dark blue solid line), small banks (light blue solid line), as well as the average tax (green solid line) in our baseline calibration. The optimal tax for small banks is uniform and independent of bank size, consistently with our results in Proposition 4. Figure 6b shows the optimal size tax, which corresponds to the difference between the taxes levied on large and small banks in Figure 6a.

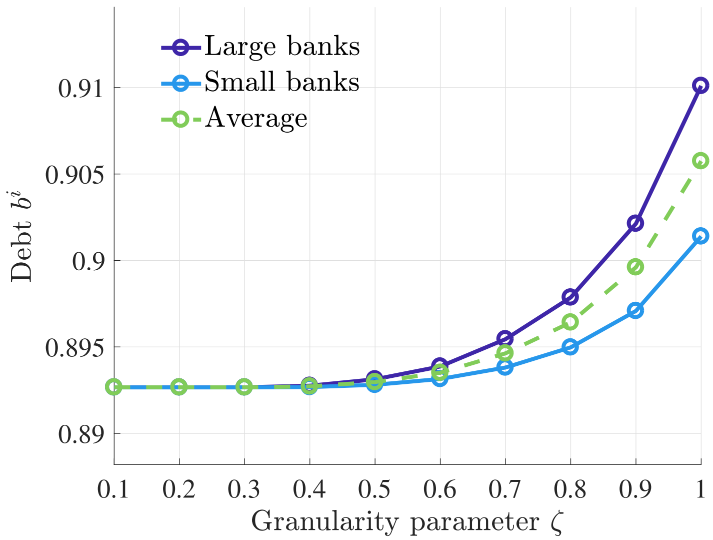


Figure 7: Sensitivity analysis

Note: Figure 7 shows the effect of varying the granularity parameter ζ on banks' borrowing choices in our baseline calibration.

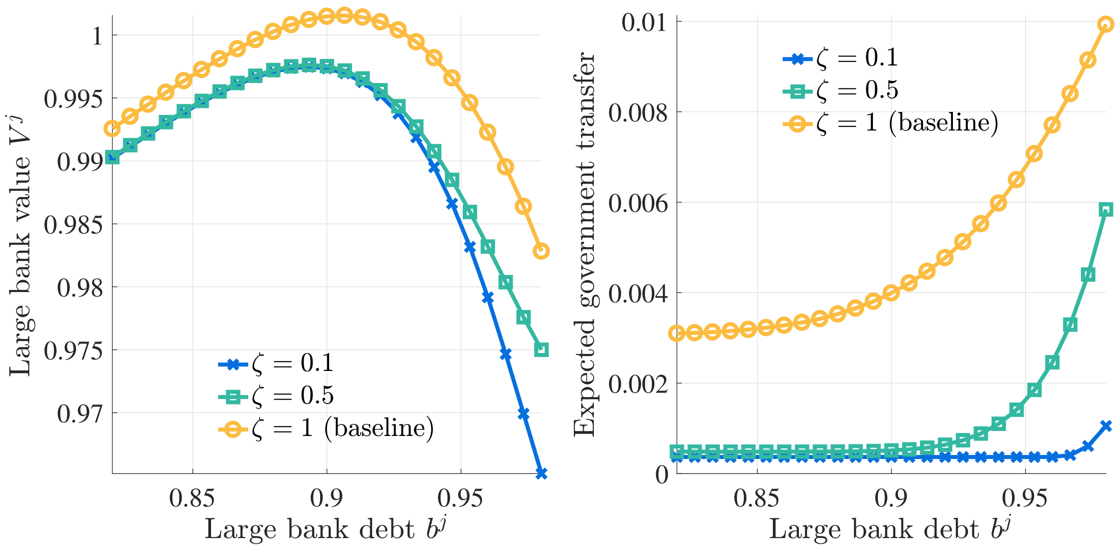


Figure 8: Large banks' incentives and the granularity parameter ζ

Note: Figure 8 illustrates the effect of varying the granularity parameter ζ on large banks' objective function. The left panel plots the value $V\left(b^j, t\left(\boldsymbol{b}|s\right)\right)$ of a large bank as a function of its borrowing b^j, for different levels of ζ, assuming that all other banks select $\boldsymbol{b}^{-j} = 0.9$. The right panel plots the corresponding expected government transfer $\mathbb{E}\left[t\left(\boldsymbol{b}|s\right)\right]$ as a function of b^j.

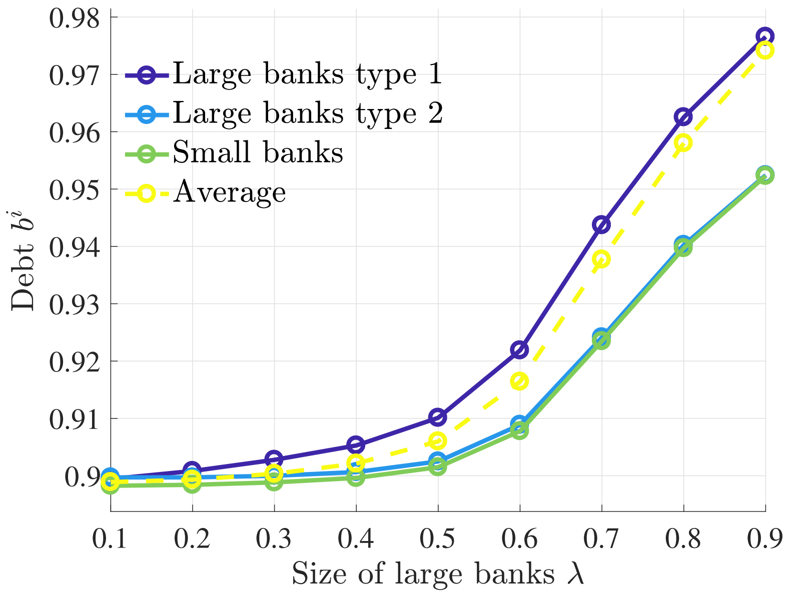


Figure 9: Sensitivity analysis: Bank size distribution

Note: Figure 9 shows the optimal equilibrium debt-to-asset ratio chosen by the 5 largest banks (type 1, dark blue solid line), the 15 additional large banks (type 2, light blue solid line) and by small banks (green solid line) for different levels of the share of assets held by type 1 large banks $\lambda \in [0.1, 0.9]$. The relative asset shares of remaining banks are held constant at the baseline level, so that type 2 large and small banks control shares $0.4(1-\lambda)$ and $0.6(1-\lambda)$ respectively. The figure also shows the average debt-to-asset ratio in the economy (yellow dashed line).

Table 1: Parameter values

Parameter	Definition	Value
Banks' profitability and capital structure determinants		
$1-\phi$	Default Deadweight Loss	0.20
ψ, $\bar{\psi}$	Cost of Equity Issuance	0.15, 0.075
μ_v	Average ROA	1.02
σ_v	Standard Deviation ROA (aggregate)	0.02
σ_w	Standard Deviation ROA (idiosyncratic)	0.06
ζ	Granularity Parameter	1
Industry composition		
λ	Large Banks' Share	0.5
N	Number of Large Banks	5
Government		
κ_1, κ_2	Bailout Costs	0.13, 10
σ_s	Government's Signal Precision	0