Effect of pin arrangement on formed shape with sparse multi-point flexible tool for creep age forming

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Abstract:
The effect of forming pin arrangement on formed shape accuracy with sparse multi-point flexible (SMPF) tool has been experimentally and numerically investigated for creep age forming (CAF) process. An analytical method has been introduced to predict shape, stress and strain distributions of blanks loaded by SMPF tool with different pin configurations. Experiments and FE simulations of loading and CAF processes by SMPF tool with various pin number/interval conditions have been performed and the formed shapes after loading and CAF have been quantitatively analysed. The results show that increasing pin numbers in SMPF tool decreases shape errors and stress variations in the loaded blank, leading to lower deflections of the formed blank after CAF. With increasing pin numbers, the formed shape approaches the shape formed with corresponding surface tool. The shape error percentage in loaded blanks is significantly enlarged after CAF with SMPF tool, from 3% to more than 20% for singly-curved tool shapes with aluminium alloy 6082, and detailed value varies with tool shapes. Stresses in loaded blanks directly affect CAFed shapes and it has been found for the first time that there is a same stress discrepancy level between loaded blanks with SMPF tool and corresponding surface tool to achieve a particular shape accuracy after CAF with different tool shapes. It is proposed that stress discrepancy parameter in loaded blanks is used as a new and more efficient design criterion for pin arrangement in SMPF tool for CAF process. In addition, an asymmetric pin pattern, which reduces half of pins in SMPF tool and increases efficiency, has been proposed and its effectiveness for CAF process has been tested and discussed.

Keyword: Sparse multi-point forming tool; Flexible forming tool; Pin arrangement; Shape accuracy; Aluminium alloy; Creep age forming
### Nomenclatures:

<table>
<thead>
<tr>
<th>Variables</th>
<th>Unit</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>mm</td>
<td>Width of blank</td>
</tr>
<tr>
<td>( C_s )</td>
<td>-</td>
<td>Tool shape</td>
</tr>
<tr>
<td>( D_l, D_{CAF} )</td>
<td>mm</td>
<td>Maximum deflections in blank shapes after loading and CAF respectively</td>
</tr>
<tr>
<td>( e_l, e_{CAF} )</td>
<td>mm</td>
<td>Deflection errors between SMPF tool and surface tool in loaded and CAFed blanks respectively</td>
</tr>
<tr>
<td>( E, E_2 )</td>
<td>GPa</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>( F, F_i )</td>
<td>N</td>
<td>An array of forces on all forming pins and the force on the ( i )th forming pin respectively</td>
</tr>
<tr>
<td>( H, h )</td>
<td>mm</td>
<td>Thickness of blank and the distance to neutral plane through blank thickness respectively</td>
</tr>
<tr>
<td>( I )</td>
<td>mm(^4)</td>
<td>Second moment of area</td>
</tr>
<tr>
<td>( J_1 )</td>
<td>-</td>
<td>First deviatoric strain invariant for rubber material</td>
</tr>
<tr>
<td>( \bar{L} )</td>
<td>-</td>
<td>Normalised length of precipitates</td>
</tr>
<tr>
<td>( M )</td>
<td>N*m</td>
<td>Moment</td>
</tr>
<tr>
<td>( P_{e_l}, P_{e_{CAF}} )</td>
<td>%</td>
<td>Shape error percentages between SMPF tool and surface tool in loaded and CAFed blanks respectively</td>
</tr>
<tr>
<td>( SD_{\sigma} )</td>
<td></td>
<td>Stress discrepancy parameter, or standard deviation of stress differences between blanks loaded by SMPF tool and surface tool</td>
</tr>
<tr>
<td>( W )</td>
<td>J/m(^3)</td>
<td>Strain energy per unit volume for rubber material</td>
</tr>
<tr>
<td>( x, x_i )</td>
<td>mm</td>
<td>An array of ( x ) axis coordinates of forming pins and its ( i )th component respectively</td>
</tr>
<tr>
<td>( z, z_i )</td>
<td>mm</td>
<td>An array of ( z ) axis coordinates of forming pins and its ( i )th component respectively</td>
</tr>
<tr>
<td>( \varepsilon, \varepsilon_0 )</td>
<td>-</td>
<td>Strain and elastic strain at yield point of material respectively</td>
</tr>
<tr>
<td>( \varepsilon_{cr}, \varepsilon_p )</td>
<td>-</td>
<td>Creep strain and plastic strain respectively</td>
</tr>
<tr>
<td>( \bar{\rho} )</td>
<td>-</td>
<td>Normalised dislocation density</td>
</tr>
<tr>
<td>( \sigma, \sigma_i )</td>
<td>MPa</td>
<td>Surface stress in the loaded plate with SMPF tool and surface tool respectively</td>
</tr>
<tr>
<td>( \sigma_y )</td>
<td>MPa</td>
<td>Yield strength</td>
</tr>
<tr>
<td>( \sigma_A, \sigma_{dis}, \sigma_{ss} )</td>
<td>MPa</td>
<td>Strength components from precipitation, dislocation and solid solution hardening respectively</td>
</tr>
<tr>
<td><strong>Loaded blanks</strong></td>
<td></td>
<td>Blanks being fully loaded by SMPF tool</td>
</tr>
<tr>
<td><strong>CAFed blanks</strong></td>
<td></td>
<td>Blanks after CAF tests</td>
</tr>
</tbody>
</table>
1. Introduction

Creep age forming (CAF) is a sheet metal forming technology developed for manufacturing large/extra-large panels in the aerospace industry, such as wing skin panels of aircrafts [1, 2]. The technology combines creep and artificial aging processes to concurrently shape and strengthen aluminium alloys and can be applied to manufacture large panels in many other industries [2, 3]. As only part of elastic strain can be converted to creep strain to generate permanent deformation during the process [4, 5], large springback generally occurs after CAF, which leads to extremely high cost for large panels when conventional solid dies are used due to their large size, weight and high difficulty and cost in re-manufacturing for springback compensation. Hence, flexible forming tools which are lightweight and reconfigurable to generate different die shapes are preferred for CAF.

There are three main types of flexible forming tools for sheet metal forming processes, as summarised in Table 1, including discrete rib board tool, conventional multi-point flexible (MPF) tool and sparse multi-point flexible (SMPF) tool. Discrete rib board tool, which replaces solid die by discrete rib boards with specific contours, was firstly proposed by Levers [6] for forming wing skin panels due to its advantages of lower weight and less re-manufacturing cost than solid dies. It is a semi-flexible tool, as all rib boards need to be replaced for a different product. Conventional MPF tool was developed for sheet metal forming processes, with a matrix of continuously distributed forming pins whose height can be adjusted individually to generate any desired tool shapes [7, 8]. It has high flexibility, but the large number of continuously distributed forming pins leads to heavy weight and high complexity in arranging and controlling pins. SMPF tool was then proposed to reduce the weight and complexity of conventional MPF tool, in which the continuously distributed forming pins are replaced by sparse ones and a die sheet is added between the blank and sparse forming pins to maintain desired tool shapes, such as SMPF tool 1 in Table 1 [9]. Another type of SMPF tool was developed for large panels in CAF process, which replaces the die sheet by narrow splines for each array of pins [10]. Among these flexible tools, SMPF tool combines advantages of low weight and complexity from rib board tool and high flexibility from conventional MPF tool, making it a preferable choice for forming processes of large panels, such as CAF process.
Table 1. A brief summary of flexible forming tools for sheet metal forming.

<table>
<thead>
<tr>
<th>Forming tool</th>
<th>Illustration</th>
<th>Basic properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discrete rib board tool</td>
<td><img src="image" alt="Illustration" /></td>
<td>• Developed for forming of large panels [6]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Semi-flexible: replace rib boards for each component</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Light weight</td>
</tr>
<tr>
<td>Continuous multi-point flexible (MPF) tool</td>
<td><img src="image" alt="Illustration" /></td>
<td>• Developed for sheet metal forming [7]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• High flexibility</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Heavy weight</td>
</tr>
<tr>
<td>Sparse multi-point flexible (SMPF) tool 1</td>
<td><img src="image" alt="Illustration" /></td>
<td>• Developed for sheet metal forming [9]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• High flexibility</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Light weight</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Difficult for shape control</td>
</tr>
<tr>
<td>Sparse multi-point flexible (SMPF) tool 2</td>
<td><img src="image" alt="Illustration" /></td>
<td>• Developed for CAF of large panels [10]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• High flexibility</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Light weight</td>
</tr>
</tbody>
</table>

Many studies have been carried out to investigate shape properties of formed sheets by conventional MPF tools in sheet metal forming. Li et al. [11] summarised the general MPF tool for sheet metal forming processes and indicated that an intermediate pad between the multi-point forming die and blank can increase contact area and help to reduce defects, such as dimple and wrinkling, in test blank after forming. The same conclusion has been drawn through a set of numerical simulations of sheet metal forming by MPF tool with intermediate elastic cushions [12, 13]. The effect of thickness [14], hardness [15] and materials [16] of elastic cushions on formed quality of test blanks with MPF tool has been investigated and the results indicated that a higher forming shape accuracy can be achieved by elastic cushions with smaller thickness and greater hardness. However, the thickness of elastic cushions should be above a critical value to avoid apparent dimples in formed blanks [17]. Forming pin unit is another key factor.
to determine shape properties of formed blanks with conventional MPF tools [18]. More continuous forming pins and larger hemisphere radius of pins can avoid dimple and achieve high shape accuracy and uniform thickness in formed blank [19, 20]. While for SMPF tools, which replace continuous forming pins with sparse ones, forming pin arrangement, including numbers and intervals, becomes more important in determining shape accuracy of formed blanks. However, few related studies can be found.

CAF process is generally used to manufacture large panels with gentle curvatures and the test blank is loaded within the elastic region in many CAF cases [2]. Because of the gentle curvatures with comparatively low stresses in the test blank during CAF, defects (dimples and wrinkling) occurring in conventional sheet metal forming processes become less of a problem for CAF process with SMPF tools. The high springback characteristic of CAF requires a high shape accuracy in blanks after loading. Hence, the forming pin arrangement in SMPF tools, such as pin numbers and intervals, which directly determines the shape accuracy after loading, plays a crucial role for CAF process. However, their effect on shape accuracy after loading and CAF has rarely been investigated. Zhang et al. [21] successfully formed a spherical shaped sheet by a SMPF tool, in which forming pins distributed in four discrete circles with an interval of 45 mm. The effect of forming pin arrangement on formed shape was not mentioned. Lam et al. [10] developed a theoretical method to design forming pin arrangement according to the shape accuracy after loading by SMPF tool. However, no corresponding experiments was presented for verification. Furthermore, the shape properties of test blanks after CAF process by SMPF tools with different pin arrangements are still unknown and proper criteria for designing pin arrangement in SMPF tools for CAF process are still lacking currently.

In this study, the effect of forming pin arrangement on formed shapes with SMPF tools for CAF process has been experimentally and numerically investigated. An analytical method to predict the shape, stress and strain distributions of loaded blanks with SMPF tools for singly-curved tool shapes has been presented. Experiments and FE simulations of CAF process with various pin number and interval conditions in a SMPF tool have been performed. The effect of shape accuracy and stress distribution in blanks after loading with different pin arrangements on corresponding formed shapes after CAF has been quantitatively analysed, base on which, a proper criterion and strategy for pin arrangement design in SMPF tools for CAF process have been proposed and discussed. In addition, an asymmetric pin pattern, which can reduce half pins in a SMPF tool, has been proposed according to the force analysis of SMPF tools and its effectiveness in CAF applications has been discussed.
2. Theoretical analysis of sparse multi-point flexible tool

A theoretical analysis to predict the shape, force and stress and strain distributions in the blank loaded by SMPF tool for tool shapes with arbitrarily single curvatures is carried out in this section, in order to enable further quantitative analysis of effect of pin arrangement on formed shapes with SMPF tool for CAF process.

Fig. 1(a) illustrates the conceptual 2-D model of the SMPF tool for CAF. Sparse control points are arranged according to the designated tool shape. During CAF, the intermediate spline is elastically loaded to have full contact with all sparse control points to generate the tool shape ($Cs$). The test blank has the same shape as the spline during CAF. The shape accuracy (or deviation from the target shape) of loaded blanks by SMPF tool has been used as a criterion for designing pin arrangement by Lam et al. [10], however, the stress and strain distributions in the loaded blank will directly influence the CAFed shape and should be considered as well.

![Diagram of SMPF tool and theoretical model](image)

**Fig. 1.** (a) Concept of SMPF tool and (b) theoretical model of elastically loaded intermediate spline with sparse control points.

2.1 Geometric shape of loaded blank with SMPF tool

An analytical model based on the Euler-Bernoulli beam theory has been developed to determine the loaded shape of spline and blank with sparse control points. For a designated shape of $Cs$ with $n$ sparse control points, the location of each point ($\mathbf{x} = [x_1, x_2, ..., x_n]$ and $\mathbf{z} = [z_1, z_2, ..., z_n]$) can be pre-determined. A set of forces $\mathbf{F} = [F_1, F_2, ..., F_n]$ from control points is defined to deform the intermediate spline to have full contact with all control points,
as shown in Fig. 1(b). The moment \( M(x) \) about the rightmost control point (\( n \)th pin in Fig. 1(b)) along the loaded spline can be expressed as:

\[
M(x) = F_1 (x - x_1) + F_2 (x - x_2) + \cdots + F_i (x - x_i) + \cdots + F_n (x - x_n)
\]  

(1)

where \( (x - x_i) \) is the Macaulay’s function, which is expressed as:

\[
(x - x_i) = \begin{cases} 
0 & x < x_i \\
(x - x_i) & x \geq x_i 
\end{cases}, \quad i = 1, 2, \ldots, n
\]  

(2)

According to the Euler-Bernoulli beam theory,

\[
EI \frac{d^2 z}{dx^2} = -M(x)
\]  

(3)

where \( E \) is Young’s modulus of spline material and \( l = bH^3/12 \) is the second moment of area of the spline, \( b \) and \( H \) are respectively the width and thickness of the spline. By double integrating Eq. (3) at each control point, the relationship between the deflection and force at each control point \( i \) can be expressed as:

\[
z_i = -\frac{1}{El} \int M(x_i) \, dx = -\frac{1}{El} \int (F_1 (x_i - x_1) + \cdots + F_n (x_i - x_n)) \, dx
\]  

(4)

The integration of Macaulay’s function is:

\[
\int_0^{x_i} (x - x_i)^n \, dx = \frac{(x - x_i)^{n+1}}{n+1}
\]  

(5)

Substituting Eq. (5) into Eq. (4):

\[
z_i = -\frac{1}{El} \left[ F_1 \frac{(x_i - x_1)^3}{6} + F_2 \frac{(x_i - x_2)^3}{6} + \cdots + F_i \frac{(x_i - x_i)^3}{6} + C_1 (x_i - x_1) + C_2 \right]
\]  

(6)

In addition, the model in Fig. 1(b) should obey the force and moment equilibriums:

\[
\sum_{i=0}^{n} F_i = 0
\]  

(7)

\[
\sum_{i=0}^{n} M_i = 0
\]  

(8)

Combining the \( n \) equations of Eq. (6) with Eqs. (7) and (8), the \( n + 2 \) unknown variables \( F_1 \) to \( F_n, C_1 \) and \( C_2 \) can then be calculated according to the matrix below:

\[
\begin{bmatrix}
z_1 \\
z_2 \\
\vdots \\
z_n \\
0 \\
0
\end{bmatrix} = -\frac{1}{El}
\begin{bmatrix}
\frac{(x_1 - x_1)^3}{6} & 0 & 0 & \cdots & 0 & \frac{(x_1 - x_1)^3}{6} \\
\frac{(x_2 - x_1)^3}{6} & \frac{(x_2 - x_2)^3}{6} & 0 & \cdots & \frac{(x_2 - x_1)^3}{6} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\frac{(x_n - x_1)^3}{6} & \frac{(x_n - x_2)^3}{6} & \frac{(x_n - x_3)^3}{6} & \cdots & 0 \\
1 & 1 & 1 & \cdots & 0 \\
x_n & x_{n-1} & x_{n-2} & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_n \\
C_1 \\
C_2
\end{bmatrix}
\]  

(9)
The deflections of loaded spline between each control point with $x_i \leq x \leq x_{i+1}$ is:

$$z(x) = -\frac{1}{EI} \left( F_1 \frac{(x-x_1)^3}{6} + F_2 \frac{(x-x_2)^3}{6} + \cdots + F_i \frac{(x-x_i)^3}{6} + C_1(x - x_1) + C_2 \right)$$  (10)

The test blank shares the same shape with spline and can be described by Eq. (10) as well.

2.2 Stress and strain distributions in test blank loaded with SMPF tool

The strain distributions through thickness of the loaded blank can be expressed as:

$$\varepsilon(x) = -\frac{d^2z}{dx^2} \ast h = -\frac{M(x)}{EI} \ast h; \quad -\frac{H}{2} \leq h \leq \frac{H}{2}$$  (11)

where $h$ represents the distance to the neutral plane through the thickness ($H$) of the blank. Meanwhile, stresses through thickness of the loaded blank can be calculated according to corresponding strain distributions and mechanical properties of the blank material, as:

$$\sigma(x) = \begin{cases} E_2 \ast \varepsilon(x) & \varepsilon(x) < \varepsilon_0 \\ \sigma_y + K[\varepsilon(x) - \varepsilon_0]^N & \varepsilon(x) \geq \varepsilon_0 \end{cases}$$  (12)

where $E_2$ and $\sigma_y$ are the Young’s modulus and yield strength of the blank material and $\varepsilon_0 = \sigma_y/E_2$ is the elastic strain at the yield point. $K$ and $N$ are material constants to characterise stress-strain relationship beyond yield of the blank material.

Based on the results from Eqs. (9) to (12), the shape, force and stress and strain distributions of the blank after loading by the SMPF tool with a particular tool shape and pin arrangement can be theoretically obtained. Their effect on the formed shape after CAF process will be quantitatively analysed through both CAF experiments and simulations in the following sections.

3. Experimental programme

3.1 Sparse multi-point flexible tool and test material

A prototype of SMPF tool was used for CAF tests in this study, as shown in Fig. 2. Studs with hemisphere nuts (Φ29.3 mm) were fitted on both base and upper plates, acting as forming pins to control the tool shape. A maximum number of 45 pins can be inserted into each plate, consisting an evenly distributed 9×5 matrix with an interval of 60 mm, as shown in Fig. 2(a). Four guide pillars in each corner of the tool were used to maintain the alignment of the base and upper plates and four locking nuts were used to lock the tool for creep-ageing after loading in CAF. 3 mm thick spring steel with a high yield strength of around 1100 MPa was selected as the spline material and was cut to a dimension of 540×30 mm. A silicone rubber with 60
Shore A hardness and with a dimension of 600×160×12 mm was used and placed between the bottom splines and test blank to reduce stress concentrations in the test blank from forming pins. All the components were assembled according to Figs. 2(b) and 2(c) for CAF tests.

A heat-treatable Al-Si-Mg alloy, AA6082, with a peak-aged initial temper of T6 was used for CAF tests in this study, whose main composition is Al-0.82Si-0.82Mg-0.64Mn-0.24Fe (wt%). The stress-relaxation and ageing behaviour of the material at 160 °C has been investigated and modelled by Rong et al. [22], and relevant results can be used. Hence, CAF tests of AA6082 were carried out at 160 °C in this study. The yield strengths at room temperature and 160 °C were respectively 290 and 225 MPa. The Young’s modulus was set as 68 and 66 GPa respectively at room temperature and 160 °C.

3.2 Experimental plan

Different pin arrangements in the SMPF tool were used to form two singly-curved products using CAF process. Four different pin arrangements with 5, 7 or 9 symmetric pairs of pins along x direction in the top and bottom dies, as shown in Figs. 3(a) to 3(d), were designed for CAF tests. For 5-pin and 9-pin symmetric configurations, pins were evenly arranged with intervals of 120 and 60 mm respectively. While with 7 symmetric pairs of pins, two types of pin configurations were used, one with additional pins at \( x = \pm 60 \) (7A-pin) and the other with additional pins at \( x = \pm 180 \) (7B-pin) based on the 5-pin configuration, as shown in Figs. 3(b) and 3(c) respectively.
Fig. 3. Illustration of different pin configurations for a singly-curved shape with (a) 5-pin, (b) 7A-pin, (c) 7B-pin and (d) 9-pin symmetric patterns.

Two singly-curved tool shapes with radius of $R = 700$ mm (R700) and 500 mm (R500) were selected for CAF tests. In order to characterise the shape, stress and strain properties in loaded blanks and quantify their effect on subsequent CAF process, two separate sets of tests, loading tests and CAF tests, were carried out. Table 2 shows the experimental plan.

<table>
<thead>
<tr>
<th>Pin arrangement</th>
<th>Pin number</th>
<th>Test conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric pattern</td>
<td>5</td>
<td>Tool shape: $R = 700$ mm or 500 mm</td>
</tr>
<tr>
<td></td>
<td>7A</td>
<td>Test type: Loading and CAF</td>
</tr>
<tr>
<td></td>
<td>7B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Asymmetric pattern</td>
<td>9</td>
<td>Tool shape: $R = 700$ mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Test type: CAF</td>
</tr>
</tbody>
</table>

The required external force at each control point (forming pin position) to load the blank to a particular shape with SMPF tool can be obtained according to Eq. (9). For the R700 tool shape with 9-pin pattern in SMPF tool investigated in this study, the external force at each control point ($F = [F_1, F_2, ..., F_9]$) has been calculated and listed in Table 3. The obtained external force at each control point has a specific direction, either upward or downward, as indicated in Fig. 4(a). For the symmetric pattern, upper and bottom forming pins are placed at each control point in the actual SMPF tool (Fig. 4(b)). However, it can be inferred that only one forming pin at each pin position is necessary to provide the required external force with particular direction, e.g. the bottom pin can provide external force with upward direction, while the upper pin only applies external force with downward direction during and after loading.
Therefore, a new configuration of asymmetric pattern is proposed, with which only the pins that provide required external forces are used, as shown in Fig. 4(b) for the 9-pin asymmetric pattern in SMPF tool. This new asymmetric pin configuration theoretically shares the same loaded shape, force and stress and strain conditions with the conventional symmetric one, but reduces half of the forming pins for SMPF tool. Hence, an additional CAF test with this asymmetric pin configuration was performed to analyse its effectiveness. The test condition is listed in Table 2.

Table 3. Loading forces from pins in SMPF tool with 9 control points for singly-curved R700 tool shape. (Positive represents upward direction and negative represents downward direction.)

<table>
<thead>
<tr>
<th>Tool shape</th>
<th>Loading force in each control point (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_1$</td>
</tr>
<tr>
<td>R700</td>
<td>6.85</td>
</tr>
</tbody>
</table>

Fig. 4. (a) Schematic of external force directions at each control point for a blank loaded by 9-pin configuration and (b) concept of pin arrangement transition from symmetric to asymmetric 9-pin pattern in SMPF tool according to force directions.
3.3 Experimental procedures

Test blanks were prepared to a dimension of 550×120×4 mm, which were cut from an as-received 4 mm plate with length direction perpendicular to the rolling direction of the plate. Two adjacent rows of pins along x axis were used for one test plate. In order to maintain the equilibrium of the tool during loading, two plates were tested at the same time using the four rows of pins indicated in Fig. 2(a). The thin blanks were prepared by cutting, which may introduce some initial deformation into the prepared blanks. The initial deformation of all blanks were measured using a coordinate measurement machine and the maximum initial deflections of the blanks from the measurements were all below 0.12 mm, which is very small and has little effect on CAF [23]. Hence, the effect of initial deflection in the test blanks was ignored in this study.

CAF tests of AA6082 blanks were carried out with the following steps:

i) Tool preparation: arrange the designated pin configuration and adjust their heights to generate the desired tool shape. Assemble all components as shown in Fig. 2.

ii) Loading: load the upper plate with a specified displacement and lock the tool. The displacement are respectively 44.43 and 61.03 mm for R700 and R500 tool shapes.

iii) Heating and creep-ageing: heat the whole tool to 160 °C and hold for 2 h for CAF. Four thermocouples were attached on four corners of the blank to record temperatures, which are controlled within 160 ± 3 °C during tests.

iv) Cooling and unloading: open the furnace, cool down the tool to room temperature and unload the CAFed blank.

v) Measuring: measure z axis deflections of the CAFed blank using a coordinate measuring machine with an accuracy of 0.01 mm.

For loading tests, the load was directly removed after steps i and ii, and deflections of loaded blanks were measured according to step v.

4. FE numerical programme

4.1 FE model with SMPF tool

Fig. 5 shows the FE model of half SMPF tool with 9×2 pins from PAM-STAMP for CAF process simulation. The basic geometry, mesh and mechanical properties of each part in the FE model are listed in Table 4.
Fig. 5. Schematic showing FE model for loading and CAF tests: (a) SMPF tool model with half of 9x2 pins and (b) boundary conditions of blank set; and (c) surface tool model.

4.1.1 Forming pins, splines and rubber

The geometric and physical models of the forming pins, splines and rubber in SMPF tool for CAF process simulation have been introduced and validated in a previous study [24] and the settings are used in this study.

Forming pins were modelled as hemisphere rigid surfaces with rigid shell elements. Bottom pins were fully constrained as the fixed bottom die and only $z$ translational freedom ($T_z$) was left in upper pins to apply load during simulations. There is no $x$- or $y$-movement for all the pins during the entire process.

Splines were modelled by shell elements with constraints in $x$ translational ($T_x$) and $y$ and $z$ rotational freedoms ($R_y$ and $R_z$) of all nodes with $x = 0$, and $T_y$, $R_x$ and $R_z$ constraints in all nodes with $y = \pm 30$, as shown in Fig. 5(b). As splines remained elastic during and after loading in CAF, an elastic material was defined with Young’s modulus of 210 GPa.
Rubber sheet was modelled as deformable solid elements with constraints shown in Fig. 5(b).

Rubber sheet was defined as an incompressible material by the Neo-Hookean model as:

$$W = C_{10} (J_1 - 3)$$  \hspace{1cm} (13)

where $W$ is the strain energy per unit volume and $J_1$ is the first deviatoric strain invariant. $C_{10}$ is a material constant and equals to 1.45 MPa for the rubber material used in this study [25].

### Table 4. Basic geometric and mechanical properties of parts in FE model with SMPF tool and surface tool.

<table>
<thead>
<tr>
<th>Part name</th>
<th>Element type</th>
<th>Global mesh size</th>
<th>Dimensions (mm)</th>
<th>Young’s modulus (GPa)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMPF tool</td>
<td>Pin</td>
<td>Rigid</td>
<td>1 mm</td>
<td>Φ29.3</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Spline</td>
<td>Shell</td>
<td>5 mm</td>
<td>540×30×3</td>
<td>210</td>
</tr>
<tr>
<td></td>
<td>Rubber</td>
<td>Solid</td>
<td>5 mm</td>
<td>600×160×12</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Blank</td>
<td>Shell</td>
<td>5 mm</td>
<td>550×120×4</td>
<td>68/66</td>
</tr>
<tr>
<td>Surface tool</td>
<td>Die</td>
<td>Rigid</td>
<td>5 mm</td>
<td>600×200</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Blank</td>
<td>Shell</td>
<td>5 mm</td>
<td>550×120×4</td>
<td>68/66</td>
</tr>
</tbody>
</table>

#### 4.1.2 Test blank

Blank was modelled by shell elements with the same constraints for the rubber model, as shown in Fig. 5(b). The stress-strain relationship of AA6082 at room temperature was used for loading simulations, described by a power-law equation:

$$\sigma = \sigma_y + K \varepsilon_p^N$$  \hspace{1cm} (14)

where $\sigma$ is the applied stress, $\varepsilon_p$ is the plastic strain and $K$ and $N$ are material constants. For AA6082-T6 used in the study, $\sigma_y = 290$ MPa, $K = 151.1$ MPa and $N = 0.684$ at room temperature, which were obtained by fitting Eq. (14) to room temperature tensile test data of AA6082, as shown in Fig. 6.
CAF process is generally treated as an isothermal forming process for FE simulations as creep stains are mainly generated in the isothermal creep-ageing stage during CAF [2, 5]. Hence, material properties of the blank at creep-ageing temperature (160 °C) were used for CAF simulations in this study. For the loading stage of CAF tests, material properties of blank were defined by fitting the stress-strain curve of AA6082 at 160 °C with Eq. (14), as shown in Fig. 6, from which \( \sigma_y = 225 \text{ MPa}, K = 20.1 \text{ MPa} \) and \( N = 0.540 \).

For the creep-ageing stage of CAF tests, a unified constitutive model which can predict both creep strain and yield strength evolutions during CAF [5, 26] was used and implemented into PAM-STAMP [27] to simulate the stress-relaxation behaviour of AA6082 during CAF. The constitutive model for AA6082 used in this study has been calibrated and validated for CAF process simulation in previous studies [22, 24]. The constitutive equations are listed below:

\[
\dot{\varepsilon}_c = A_1 \sinh \left[ B_1 \left| \sigma \right| (1 - \bar{\rho}) - k \sigma_y \right] \text{sign}(\sigma) \tag{15}
\]

\[
\dot{\rho} = A_3 (1 - \bar{\rho}) |\dot{\varepsilon}_c| - C_p \bar{\rho}^{m_4} \tag{16}
\]

\[
\dot{\sigma}_A = C_A (1 - \bar{L}) \dot{\bar{L}}^{m_1} \tag{17}
\]

\[
\dot{\bar{L}} = C_r (Q - \bar{L})^{m_2} (1 + \gamma_0 \bar{\rho}^{m_3}) \tag{18}
\]

\[
\dot{\sigma}_{dis} = A_2 n \bar{\rho}^{n-1} \dot{\bar{\rho}} \tag{19}
\]

\[
\dot{\sigma}_{ss} = 0 \tag{20}
\]

\[
\sigma_y = \sigma_{ss} + \sqrt{\sigma_A^2 + \sigma_{dis}^2} \tag{21}
\]

\[
\dot{\bar{\rho}} = -E \dot{\varepsilon}_c \tag{22}
\]
The model relates microstructural variables (normalised dislocation density $\bar{\rho}$ and normalised precipitate length $\bar{L}$) to macro-properties of the material (creep strain $\varepsilon_{cr}$ and yield strength $\sigma_y$) during CAF. Creep strain rate $\dot{\varepsilon}_{cr}$ is determined by the applied stress $\sigma$, normalised dislocation density $\bar{\rho}$ and yield strength $\sigma_y$ of the material, while yield strength is comprised of three components: precipitation hardening $\sigma_A$, solid solution hardening $\sigma_{ss}$ and dislocation hardening $\sigma_{dis}$. Initial values of process variables and material constants used in this study are from [22], as listed in Table 5.

In addition, for the unloading stage of CAF simulations, the blank material was defined as an elastic material with basic properties listed in Table 4.

**Table 5.** Initial values of process variables and material constants for CAF of AA6082-T6 at 160 °C [22].

<table>
<thead>
<tr>
<th>$A_1$ ($h^{-1}$)</th>
<th>$B_1$ (MPa)</th>
<th>$k$ (-)</th>
<th>$C_A$ (MPa)</th>
<th>$m_1$ (-)</th>
<th>$C_r$ (-)</th>
<th>$Q$ (-)</th>
<th>$m_2$ (-)</th>
<th>$\gamma_0$ (-)</th>
<th>$m_3$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2e-5</td>
<td>0.0525</td>
<td>0.375</td>
<td>0.4</td>
<td>0.5</td>
<td>0.03</td>
<td>10</td>
<td>1.4</td>
<td>1.5</td>
<td>1.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$A_2$ (MPa)</th>
<th>$n$ (-)</th>
<th>$A_3$ (-)</th>
<th>$C_p$ (-)</th>
<th>$m_d$ (-)</th>
<th>$\sigma_{A0}$ (MPa)</th>
<th>$\sigma_{a0}$ (MPa)</th>
<th>$\sigma_{dis0}$ (MPa)</th>
<th>$l_0$ (-)</th>
<th>$\tilde{\rho}_0$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>0.54</td>
<td>550</td>
<td>0.035</td>
<td>1.1</td>
<td>150</td>
<td>140</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

4.1.3 Contact definition

Surface-to-surface contact was selected to define contacts between each part. The friction coefficients for the contacting pairs of metal-rubber and aluminium-steel were defined as 0.2 and 0.25 respectively [28].

4.2 FE model with surface tool

For comparison, FE model with singly curved surface tools of R700 and R500 was also established. Fig. 5(c) shows the main components of the model. The basic geometry, mesh and mechanical properties of dies and blank are listed in Table 4. The same material models defined in section 4.1.2 were used for the blank.

4.3 FE simulations

FE simulations were performed for all experiments in Table 2, including both loading tests and CAF tests. Furthermore, in order to quantify the effect of forming pin arrangement on CAFed shapes, an additional set of FE simulations for CAF tests was performed, in which SMPF tool with evenly distributed forming pins ranging from 3 to 11 were used for singly curved R700 and R500 tool shapes, as listed in Table 6.
Table 6. Evenly distributed pins for CAF FE simulations with SMPF tool.

<table>
<thead>
<tr>
<th>Tool shape</th>
<th>R500 and R700</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pin number</td>
<td>R500</td>
</tr>
<tr>
<td></td>
<td>R700</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>240</td>
<td>180</td>
</tr>
</tbody>
</table>

FE simulation of loading tests includes three steps: i) initialising position of the model; ii) loading and iii) unloading. While for CAF tests, four steps were performed: i) initialising position of the model; ii) loading, iii) creep-ageing, during which creep-ageing constitutive model was active, and iv) unloading [23].

5. Results

5.1 Shapes after loading tests with different pin arrangements

For the R700 tool shape, both experimental and numerical results show that all blanks remain flat after loading tests with SMPF tool (5, 7A, 7B and 9-pin configurations) and surface tool, indicating that blanks are loaded in the elastic region for R700 case, whereas plastic deformation occurs in blanks after loading tests for R500 tool shape. Fig. 7 shows the deflections of half of the blank after loading tests with different tool configurations from both experiments and FE simulations. A good agreement has been achieved between experiments and corresponding FE results, which indicates that the developed FE model is applicable for loading simulation. A similar shape is formed after loading tests by SMPF tool with 5-pin and 7A-pin configurations with the maximum deflection of 4.31 and 3.90 mm respectively from experiments. Meanwhile, SMPF tool with 7B-pin and 9-pin configurations also lead to a similar formed shape after loading tests, with the maximum deflection of 2.99 and 2.69 mm respectively, which are smaller than the shapes formed by 5-pin and 7A-pin configurations. It can be seen that although 7A-pin and 7B-pin configurations have the same pin number, apparent differences exist in the formed deflections after loading because of their different pin locations. In addition, the formed shape after loading with surface tool from FE modelling is also plotted in Fig. 7, which shows the smallest deflection after loading test. These results are related to the stress and strain distributions in the blank after loading and will be discussed later.
Fig. 7. Comparison of experimental (symbols) and numerical (lines) results for blank shape
deflections of AA6082 after loading tests with SMPF tool (5, 7A, 7B and 9-pin symmetric
configurations) and with surface tool for tool shape of $R = 500$ mm.

5.2 Shapes after CAF tests with different pin arrangements

Fig. 8 compares deflections of CAFed blanks with different SMPF tool and surface tool from
experiments and FE models. Good agreement of CAFed deflections between experiments and
corresponding FE models has been achieved for all tests conditions, indicating the effectiveness
of the CAF FE model in springback prediction. Similar to the shape results after loading tests
in Fig. 7, the results with 5-pin and 7A-pin configurations are very close to each other in both
R700 and R500 cases. The same phenomenon is also observed in CAFed shapes with 7B-pin
and 9-pin configurations. In addition, larger deflections in CAFed shapes are observed with 5-
pin and 7A-pin configurations than that with 7B-pin and 9-pin configurations. FE model with
the surface tool predicts the lowest deflections of CAFed blank. It can be seen in Fig. 8 that
with increasing pin numbers in SMPF tool, lower deflections are occurring in CAFed shapes,
which become closer to the CAFed shape with corresponding surface tool.
5.3 Shapes after CAF tests with asymmetric pin pattern

Experimental and numerical results of CAFed shapes with asymmetric and symmetric 9-pin configurations in SMPF tool are compared in Fig. 9. Similar deflections are observed for both cases and the largest difference in formed shape deflections from experiments is 0.20 mm, about 4.8% of the maximum CAFed deflection. The results indicate that the asymmetric pin pattern in SMPF tool can achieve a similar shape accuracy with corresponding symmetric one for CAF, but reduces 50% numbers of pins in the SMPF tool, largely decreasing the tool weight, tool preparation and controlling complexity.
6. Discussion

6.1 Effect of forming pin number on blank characteristics after loading tests

The shape of loaded blank and stress/strain distributions directly affect the final formed shape after CAF tests with SMPF tool. The shape error between the blanks loaded by SMPF tool and corresponding surface tool is defined as $e_l$, as illustrated in Fig. 10. The percentage of the maximum shape error after loading $\max(e_l)$ to the maximum deflection of corresponding surface tool shape ($D_l$) is defined as the loaded shape error percentage $P_{e_l}$ in this study to characterise the shape accuracy of the loaded blank with SMPF tool:

$$P_{e_l} = \frac{\max(e_l)}{D_l} \times 100\% \tag{23}$$

Fig. 10. Schematic of ideal shape and loaded shape with 5-pin configuration in SMPF tool.

Fig. 11 compares the shape error percentage $P_{e_l}$ of the loaded blanks from analytical results (Eq. (10)) and FE simulations for R700 and R500 tool shapes. The FE and analytical results are close to each other. All results with different pin configurations used in this study show a
comparatively small shape error percentage \( P_{el} < 4\% \) in loaded blanks. The results show that loaded blanks with 5-pin and 7A-pin configurations have a similar shape, while 7B-pin and 9-pin configurations generate another similar loaded shape with smaller \( P_{el} \) values. Although having the same pin number, 7A-pin and 7B-pin configurations show apparently different \( P_{el} \) values for the same tool shape and hence, leading to the different shape deflections shown in Figs. 7 and 8.

![Fig. 11. Comparison of shape error percentage \( P_{el} \) between loaded blanks with indicated pin configuration in SMPF tool and with corresponding surface tool, from analytical (Ana.) and FE results.](image)

Fig. 11 illustrates the analytical and numerical results of effective stress and plastic strain distributions in the loaded blank with different pin configurations and tool shapes. Analytical results share the same trend with numerical results for all test conditions. However, analytical results predict higher levels of stress and strain concentrations at forming pin areas due to the ignorance of rubber sheet in analytical calculations, which is designed to release severe stress concentrations in the loaded blank [9].

![Fig. 12. Illustrates the analytical and numerical results of effective stress and plastic strain distributions in the loaded blank with different pin configurations and tool shapes.](image)
Fig. 12. Comparison of effective stress and plastic strain distributions along $y = 0$ in loaded blanks from analytical (Ana., solid lines) and FE (symbols) results with indicated tool shapes and pin configurations.

For R700 tool shape, no plastic strain occurs in the blank after loading and hence no deformation is observed in corresponding plates after loading tests, as mentioned in section 5.1. The effective stress distributions along $y = 0$ for different pin configurations and corresponding surface tools are compared in Figs. 12(a) and 12(b) for the R700 case. Both analytical and numerical results show similar stress distributions for 5-pin and 7A-pin configurations, and for 7B-pin and 9-pin configurations as well. The similar shapes and stress distributions in the loaded blanks then lead to similar formed shapes after both loading and CAF tests, as shown in Figs. 7 and 8. Figs. 12(c) and 12(d) show effective stress and plastic strain distributions in loaded blanks with 5-pin and 9-pin configurations in SMPF tool and with corresponding surface tool for the R500 case. Stress in some areas near forming pins exceeds the yield strength after loading, leading to plastic strains shown in Fig. 12(d). Analytical results predict a higher maximum plastic strain with 9-pin configuration. However, when considering the stress relief effect from rubber in SMPF tool, higher plastic strains are observed with 5-pin configuration from FE simulation (Fig. 12(d)). Furthermore, a slightly larger average plastic strain is obtained with 5-pin configuration than that with 9-pin one from both analytical and FE results, leading to the larger deflections with 5-pin configuration after loading tests shown in Fig. 7. The lowest plastic strains are obtained with surface tool condition, which generate the smallest deflections in the blank after the loading test (Fig. 7).

6.2 Effect of forming pin number on blank characteristics during CAF tests

SMPF tool aims to form blanks with the same shape as that when corresponding surface tool is used. Hence, in order to investigate shape quality after CAF process with different SMPF tools, the shapes of CAFed blanks with corresponding surface tools from FE simulations are
used as a reference in this study. The CAFed shape error ($e_{CAF}$) between CAFed blanks with SMPF tool and with corresponding surface tool was obtained in the same way for load shape error ($e_l$) in Fig. 10. To characterise the shape accuracy of CAFed blanks, a CAFed shape error percentage ($P_{e_{CAF}}$) is defined by the percentage of the maximum CAFed shape error $\max(e_{CAF})$ to the maximum deflection of CAFed blank with surface tool ($D_{CAF}$):

$$P_{e_{CAF}} = \frac{\max(e_{CAF})}{D_{CAF}} \times 100\%$$ (24)

Table 7 lists the shape error percentages in loaded blanks and CAFed blanks for all test cases with SMPF tool. It can be seen that for both R700 and R500 tool shapes, although only 3 to 4% of $P_{e_l}$ exists in the loaded blanks, $P_{e_{CAF}}$ is above 20% in CAFed blanks. When $P_{e_l}$ value decreases to about 1%, $P_{e_{CAF}}$ value decreases to below 8%. These results indicate that shape error percentage in loaded shape with SMPF tool will be significantly enlarged after subsequent creep-ageing in CAF process, which is related to the high springback of the blank after CAF. Hence, in order to obtain a good shape accuracy in CAFed blanks, the forming pins in SMPF tool need to be arranged to achieve a much higher relative shape accuracy in loaded blanks. The details of this phenomenon will be quantitatively investigated in the following section.

**Table 7.** Shape error percentage in loaded ($P_{e_l}$) and CAFed ($P_{e_{CAF}}$) blanks with indicated symmetric pin configurations in SMPF tool for R500 and R700 tool shapes.

<table>
<thead>
<tr>
<th>Tool shape</th>
<th>R700</th>
<th>R500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pin configuration</td>
<td>5</td>
<td>7A</td>
</tr>
<tr>
<td>$P_{e_l}$ (%)</td>
<td>3.02</td>
<td>2.97</td>
</tr>
<tr>
<td>$P_{e_{CAF}}$ (%)</td>
<td>21.48</td>
<td>20.11</td>
</tr>
</tbody>
</table>

Stress or strain distributions in the loaded blank are the key factors that directly affect subsequent creep-ageing behaviour of the material and the formed blank shape after CAF process. Fig. 13 compares distributions of effective stress and plastic strain in a quarter of the test blank after loading and creep-ageing stages during CAF process with R700 and R500 tool shapes. It can be observed that with a small change in the loaded shapes ($\max(e_l)$) changes from 1.17 mm to 0.30 mm for R700 case and from 1.95 mm to 0.52 mm for R500 case with 5-pin and 9-pin configurations respectively), the stress distributions after loading stage in Fig. 13(a) show apparent differences. Much higher stress concentrations in the forming pin areas are observed with 5-pin configuration than that with 9-pin one for both tool shapes, leading to more stress-relaxation and plastic strains after CAF. The total plastic strains generated after CAF
determine the CAFed shapes. Apparent larger total plastic strain distributions can be observed with 5-pin configuration in SMPF tool, leading to larger formed deflections after CAF process, as shown in Fig. 8. In addition, with more forming pins in the SMPF tool, the stress and strain distributions in the loaded blank tend to be closer with those loaded by corresponding surface tools, making the shape error percentages decrease with increasing forming pins, as indicated in Table 7.

Fig. 13. Comparison of (a) effective stress and (b) plastic strain evolutions in a quarter of the test blanks after loading and creep-ageing stages with SMPF tool (5-pin and 9-pin symmetric configurations) and with surface tool.
6.3 Effect of forming pin arrangement on CAFed shape

The FE and analytical models presented earlier are used to further quantitatively analyse the detailed effect of forming pin arrangement on CAFed shape accuracy in this section. In addition to the shape error percentage of the loaded blanks \( P_{ei} \) defined in Eq. (23), which is the conventional design criterion for forming pin arrangement [10], the stress distributions in loaded blanks, which directly affect the formed shapes after CAF, are also considered in this study. The standard deviation of stress difference \( SD_{\sigma} \) between loaded blanks with SMPF tool \( \sigma \) and corresponding surface tool \( \sigma_i \) along x axis is introduced in this study to represent the stress discrepancy between blanks loaded by SMPF tools and corresponding surface tool, as:

\[
SD_{\sigma} = \sqrt{\frac{\sum_{i=1}^{m} (\sigma - \sigma_i)^2}{m-1}}
\]  

(25)

where \( (\sigma - \sigma_i) \) represents the average value of the stress difference \( \sigma - \sigma_i \) throughout the loaded blank. Fig. 14 shows the variations of shape error percentage \( P_{ei} \) and stress discrepancy parameter \( SD_{\sigma} \) in loaded blanks with evenly distributed pin numbers ranging from 3 to 11 in SMPF tool from analytical calculations. With more pins, a lower value of \( P_{ei} \) and \( SD_{\sigma} \) can be observed, making the shapes and stress distributions be more similar to those with corresponding surface tool. Similar \( P_{ei} \) values are obtained with the same pin number for different tool shapes for R500 and R700, while \( SD_{\sigma} \) values show apparent differences and for the same pin number, a higher \( SD_{\sigma} \) value is observed for R500 tool shape, which has a smaller radius and larger deformation than R700 case.
**Fig. 14.** Variations of (a) shape error percentage ($P_{el}$) and (b) stress discrepancy parameter ($SD_{\sigma}$) in loaded blanks with evenly distributed pin numbers in SMPF tool for R500 and R700 tool shapes.

Fig. 15 demonstrates variations of formed shape error percentage after CAF tests ($P_{eCAF}$) and $SD_{\sigma}$ values in loaded blanks versus evenly distributed pin numbers from FE simulations. Some experimental results are also plotted in Fig. 15 for verification. With increasing pin numbers, $P_{eCAF}$ decreases linearly at a higher rate first and then turns to a lower rate when $P_{eCAF}$ reaches about 10% for both R500 and R700 tool shapes. However, with different tool shapes, the number of pins needed to achieve the same CAFed shape accuracy is different. In order to achieve a $P_{eCAF}$ value below 10%, at least 6 and 8 evenly distributed pins are respectively needed for R700 and R500 cases, and the maximum shape error percentages $P_{el}$ required in loaded shapes are respectively 1.82% and 1.09% according to Fig. 14(a).

Fig. 15(b) shows the effect of stress discrepancy parameter on CAFed shape accuracy, and unlike the effect from pin numbers in Fig. 15(a), it is observed that a same $SD_{\sigma}$ value in blanks loaded with SMPF tool leads to a similar formed shape error of $P_{eCAF}$ after CAF process, no matter how many pins and what tool shapes are used. The CAFed shape error percentage $P_{eCAF}$ has a bi-linear relationship with $SD_{\sigma}$ and experiences a higher decreasing rate with decreasing $SD_{\sigma}$ values until reaching a critical level, below which a much lower decreasing rate is observed, as shown in Fig. 15(b). The critical level of $SD_{\sigma}$ is about 68 MPa with $P_{eCAF}$ of about 10% for both R700 and R500 cases.

These results indicate that to obtain a particular formed shape accuracy after CAF tests with SMPF tool, the required loaded shape error percentage ($P_{el}$) is dependent on the target tool shapes and a lower loaded shape error percentage is needed for a tool shape with larger deformations, e.g. 1.09% of $P_{el}$ for R500 case and 1.82% of $P_{el}$ for R700 case is required to achieve 10% of $P_{eCAF}$. The required stress discrepancy parameter ($SD_{\sigma}$), however, can be a constant for different tool shapes. Hence, for pin arrangement design in SMPF tool for CAF manufacture, the stress discrepancy parameter, such as $SD_{\sigma}$ proposed in this study, could be a more proper criterion than the generally used loaded shape error percentage ($P_{el}$). As $SD_{\sigma}$ value can be a constant for tool shapes with different deformation levels, the results, validated by CAF tests with particular tool shapes in this study, could be used to guide the pin arrangement design for other tool shapes with arbitrarily curvatures, whose stress distributions can be conveniently determined through theoretical calculations and FE models developed in this study. For example, to manufacture AA6082 singly-curved plates with varying curvatures by
CAF process with CAFed shape error percentage ($P_{eCAF}$) within 10%, the pin arrangement in SMPF tool should be designed to have a $SD_\sigma$ value lower than 68 MPa according to the results from this study. In addition, a nearly linear relationship between $P_{eCAF}$ and $SD_\sigma$ exists when $SD_\sigma < 68$ MPa, which can act as a criterion to achieve higher shape accuracy by CAF process with SMPF tool. All these criteria can be determined through the method and models developed in this study, which can be extended to CAF applications of other aluminium alloys with SMPF tool.

Fig. 15. Variations of shape error percentages of CAFed blanks ($P_{eCAF}$) with different (a) pin numbers and (b) stress discrepancy parameter ($SD_\sigma$) in loaded blanks with SMPF tools.

6.4 Asymmetric forming pin arrangement

An asymmetric forming pin arrangement pattern of SMPF tool has been proposed in this study and corresponding experimental results in Fig. 9 have shown that a similar CAFed shape has been formed by SMPF tools with both symmetric and asymmetric 9-pin configurations. Fig. 16 compares the effective stress and plastic strain distributions in the blanks with symmetric and asymmetric 9-pin configurations for R700 tool shape after loading and creep-ageing stages. Stress and strain show similar distributions, resulting in the nearly same shape after CAF process. Both theoretical calculations and simulation results demonstrate that the symmetric and corresponding asymmetric pin patterns in SMPF tool share similar shape, stress and strain distributions in the loaded blanks. The similar formed shapes after CAF tests have demonstrated the effectiveness of the new asymmetric pin configuration of SMPF tool for CAF process. Hence, it is reasonable to state that the asymmetric configuration could be a better pin arrangement method for SMPF tool for CAF manufacture, as it can retain CAFed shape accuracy and greatly decrease pin numbers, reducing weight and complexity of the tool
compared with the corresponding symmetric configuration. There might be a requirement of minimum pin numbers for the asymmetric pattern, as with only half pins in both top and bottom sides of SMPF tool compared with the symmetric one, the stress/strain evolutions in the blank during the loading stage may be affected [29], which may change the final shape of formed blanks after CAF. More related studies are needed to fully understand the implication of asymmetric configuration.

![Comparison of effective stress and plastic strain distributions after loading and creep-ageing stages with symmetric and asymmetric 9-pin patterns in SMPF tools.](image)

**Fig. 16.** Comparison of effective stress and plastic strain distributions after loading and creep-ageing stages with symmetric and asymmetric 9-pin patterns in SMPF tools.

### 7. Conclusions

The effect of forming pin arrangements on formed shape accuracy with sparse multi-point flexible (SMPF) tool has been investigated experimentally and numerically for CAF process. An analytical method has been proposed for the prediction of the shape, force and stress and strain distributions in the loaded blank with SMPF tool. CAF experiments and corresponding FE simulations have been performed with different pin configurations in SMPF tool. The shape and stress/strain properties in the blank after loading and CAF from analytical, experimental and numerical results show a good agreement among each other. The following conclusions can be drawn:

1. Increasing pin numbers in SMPF tool will reduce the shape errors and stress variations in the loaded blank, leading to lower deflections in the formed blank after CAF process and a shape closer to the one formed by corresponding surface tool.

2. The shape accuracy of formed blanks after CAF is highly sensitive to the loaded shape with SMPF tool due to the high springback nature of the process. For singly-curved tool shapes with radii of 500 and 700 mm, a 3% shape error in loaded blanks can lead to more than 20% shape error in formed blanks after CAF.
3. Stress distribution in the loaded blanks is found to be the key factor affecting the shape accuracy of formed plates after CAF with SMPF tool, as it directly determines the initial plastic deformation after loading and influence the stress-relaxation during subsequent creep-ageing. Increasing pin number from 5 to 9 for R500 and R700 tool shapes leads to only minor changes of shape error percentage (about 2%) in the loaded blanks, however, largely different stress distributions occur, resulting in the significant difference (more than 15%) in shape errors of the formed plates after CAF.

4. A stress discrepancy parameter ($SD_\sigma$) is proposed and validated as a proper parameter for pin arrangement design in CAF process, as it is the key factor to determine the formed shape error percentage ($P_{e,CAF}$) and a same $SD_\sigma$ value exists for different tool shapes to achieve a particular CAFed shape accuracy. In addition, a bi-linear relationship exists between $SD_\sigma$ and $P_{e,CAF}$. For CAF of AA6082 singly-curved shapes with SMPF tool, pin arrangement can be design with $SD_\sigma$ value lower than 68 MPa to control CAFed shape error within 10%.

5. An asymmetric pin arrangement is proposed for SMPF tool based on particular external force directions at each forming pin position from analytical analysis presented in this study. The asymmetric pin arrangement shares the similar shape accuracy, stress and strain distributions in loaded and CAFed blanks to those from the corresponding symmetric arrangement and reduces 50% of forming pins, largely decreasing the weight, complexity and setup time of the tool. It is suggested as a better choice for SMPF tool applications.

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