Model Predictive Circulating Current Regulator for Single-Phase Modular Multilevel Converter

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Abstract—This paper describes a model predictive strategy to reduce the sub-module voltage oscillation in a single-phase modular multilevel converter. This task is accomplished by a predictive controller which solves an optimal control problem sequentially. By choosing the objective function weights appropriately, this controller can naturally trade-off sub-modules voltage ripple and recirculating current. It is shown that the sub-module voltage oscillation can be reduced without degrading the efficiency excessively, enhancing the performance of the overall converter. Additionally, it is guaranteed that the recirculating current can be regulated without exceeding the physical limitations of the device.

Index Terms—Model Predictive Control, Modular Multilevel Converter, Single Phase System

I. INTRODUCTION

One of the main challenges of the Modular Multilevel Converter (MMC) is its control design. Being several control strategies deeply discussed in the literature [1], the direct and indirect modulation strategies seem to be the most commonly implemented. However, recent advances in Field Programmable Gate Array (FPGA) devices and Digital Signal Processors (DSP) allow considering other control approaches to be implemented in power electronics devices. Amongst others, Model Predictive Control (MPC) is a promising alternative as it permits to foresee the impact of the control action over a defined horizon, handling multi-variable problems and system limitations easily.

The recent application of MPC into power electronics is discussed in [2] where its implementation is divided in two main groups: Continuous Control Set (CCS)-MPC and Finite Control Set (FCS)-MPC. This document introduces a CCS-MPC control structure that naturally reduces the voltage ripple of the sub-modules such that the size of the converter could be reconsidered, ultimately impacting in the cost of the device. Different papers studied this issue before by injecting harmonic into the recirculating current (e.g. [3]). However, the MPC approach brings an innovative methodology to perform this task optimizing the injection of recirculating current even during transients and guaranteeing that physical limits of the converter are never exceeded. Thus, the controller allows exploiting the full potential of the converter and permits to easily adjust the performance of the device by tuning the weight of the objective function appropriately.

II. SYSTEM DESCRIPTION AND SIMPLIFICATIONS

A detailed scheme of the system under study is shown in Fig. 1(a). The MMC considered is a single-phase converter composed of a unique phase connecting a DC link with an AC system. Conventionally, each phase of the MMC is named as leg, which at the same time is subdivided into two smaller groups named as arms (upper and lower arms are designated with sub-indices u and l respectively). Each arm is a stack of M Sub-Modules (SM), also called cells, and each SM is typically a half-bridge converter although other structures can be considered [4]. Additionally, each arm also contains an inductance $L_a$, added to limit the arm fault current, with its corresponding parasitic resistance $R_a$. Finally, note that the arms of the converter are designed to be symmetrical.

Because of the scalability of the MMC, each arm can be composed of hundreds of SM resulting in a very complex system. The computational burden of the resulting model and its control complexity are very high requiring the use of simplified versions that provides equivalent behavior and dynamic response. A common simplification is to consider a continuous model of the converter. Despite the fact that the number of SMs is limited, if the number of cells is large enough, the arm voltage $v_i$ and $v_l$ can be considered as a continuous variable and can be modeled as a controlled voltage source [5]. The voltage balancing of the cells inside the arm is achieved by a Balancing Control Algorithm (BCA) although this control block is generally treated independently of the energy control and it is out of the scope of this study.

The simplified version of the system used in this paper is shown in Fig. 1(b). The regulation of the DC side is not considered part of this study and, consequently, the DC side is modeled as two independent ideal voltage sources $V^{DC}_u$ and $V^{DC}_l$. Finally, an average model of the arm dynamics is adopted, being the DC side modeled as a current source in parallel with the equivalent arm capacitance and the AC side modeled as a controlled voltage source.

A. System Modelling

The analysis of the upper and lower meshes of the continuous model represented in Fig. 1(b) leads to the following expressions.
The main objective of the MMC is to transfer power from one side to the other ensuring that the system is kept stable. The leg of the MMC works as a buffer interface between both sides, being possible to store some energy in the cells' capacitor, potentially increasing their voltage. Then, for stable operation, the voltage of SMs must be controlled. Note that the interaction of the current through the arms and the modulating signal originates voltage oscillations to the SMs capacitors at fundamental and higher frequencies. Different control structures deal with this phenomena either by suppressing the recirculating current [6] or by using it to reduce the ripple of the cells' voltage [3], [7]. This issue is tackled by the MPC structure proposed here, which determines the modulation of each arm in order to deliver the desired grid current \( i_g \) while controlling the internal recirculating current to reduce the oscillation of the SMs capacitors. The injection of recirculating current does not only depends on pre-calculated values but it is adjusted by the MPC controller according to the value of the systems states at each sampling instant. For instance, in case of eventual disturbances, the controller would rectify the injection of recirculating current while guaranteeing the physical limitations of the system, modeled as constraints in the MPC algorithm, are not exceeded.

The control structure proposed resembles an indirect modulation in closed-loop. The total voltage of the arms is measured and fed back in order to prevent unbalances or to start accumulating excessive energy in the arms. The modulation references of each arm are obtained using the MPC regulator which determines the optimal control trajectory according to the weight given to each state. One of the main difficulties of implementing this strategy is the time required to solve the optimization problem online. A workaround towards an experimental implementation could be the approach discussed in [8], where an MPC controller is cascaded with a current controller. In this approach, the arm current trajectories obtained from the MPC block would be used as a reference signals to a lower current controller. Doing so, the sampling of the MPC could be decreased as the inner current controller would act quickly in case of grid disturbances until a new reference from MPC block is available. The simulations results presented hereinafter are based on a switching model of the converter and emulates the performance of this control structure.

### A. Definition of the MPC algorithm

In a general way, the discrete-time optimal control problem solved at each sampling time is defined as following:

\[
J \triangleq \sum_{k=0}^{N} \frac{R}{\psi_1} (i_{u[k]} - i_{c[k]})^2 + \sum_{k=0}^{N} \frac{R}{\psi_1} (i_{l[k]} - i_{d[k]})^2 + \sum_{k=0}^{N} \frac{S}{\psi_2} \left[ v_{uc[k]} - v_{de[k]} \right]^2 + \sum_{k=0}^{N} \frac{L}{\psi_1} (v_{u[k]})^2 + \sum_{k=0}^{N} \frac{L}{\psi_1} (v_{l[k]})^2
\]
subject to:
\[ i_{u[k+1]} = i_{u[k]} + \frac{\Delta t}{L_a} (V_{DC[k]}^u - v_{cu[k]} - v_{g[k]}) \]
\[ -R_a i_{u[k]} - R i_{g[k]} - \frac{L}{\Delta t} (i_{g[k+1]} - i_{g[k]}) \]
\[ i_{l[k+1]} = i_{l[k]} + \frac{\Delta t}{L_a} (v_{cu[k]} + v_{g[k]}) \]
\[ -R_a i_{l[k]} + R i_{g[k]} + \frac{L}{\Delta t} (i_{g[k+1]} - i_{g[k]}) \]
\[ v_{c(u)}[k+1] = v_{c(u)}[k] + \frac{\Delta t}{C_a} v_{c(u)}[k] \]
\[ i_{c(u)}[k+1] = i_{c(u)}[k] + v_{c(u)}[k] \]
\[ i_{u}[k] = i_{u}[k] - i_{l}[k] \]
\[ \hat{v}_c \leq v_{c(u)}[k] \leq \hat{v}_c \]
\[ \hat{v}_c \leq v_{c(u)}[k] \leq \hat{v}_c \]
\[ \hat{v}_c \leq v_{c(u)}[k] \leq \hat{v}_c \]

where the sub-index \([k]\) describes the discrete time instant and \(N\) is the number of points of the control horizon. Then, (5) accounts for the terms of the objective function. Note that \(\psi_1, \psi_2, \psi_3\) are scaling factors; and \(R, S\) and \(U\) are Objective Function (OF) weights. By applying a forward Euler discrete approximation, (6) and (7) are obtained from analyzing the upper and lower arm meshes independently (see Fig. 1b). Also, (8) defines the capacitor dynamics of each arm. Then (10) ensures that the grid current is as expected and (9) models power balance within the device, which cannot be violated. Finally, (11), (12) and (13) bound the values that the states can take. Note that it is expected that the modulated voltage should not exceed the voltage available in the arm’s capacitor. Thus, constraint (11) bounds the voltage of the arm’s capacitor to be higher than the modulated signal plus a threshold value \(\mathcal{V}\).

B. Linear Time-Varying (LTV) small-signal approximation

The non-linear dynamics of the system, as a consequence of the power balance within the device, restrict the choice of the optimization algorithms applicable and increases the computational burden. Therefore, this section discusses an approach to tackle this issue with the aim of simplifying the Optimal Control Problem (OCP).

Similarly to the small-signal analysis in Linear Time-Invariant (LTI) systems where a perturbation is applied around an equilibrium operating point, the LTV approach is based on describing the non-linear behavior of the system around bias trajectory varying in time [9]. Considering that the steady-state trajectories of the system’s states are periodic, the system variables can be generically described as:

\[ x = \bar{x} + \delta x \]
\[ u = \bar{u} + \delta u \]

where \(x\) designates the system states and \(u\) the system control variables. In addition, the accent (~) describes steady periodic trajectories that vary over time and the symbol (\(\delta\)) designates variables that model disturbances around the steady bias point. If the system states described in (1)-(3) are generically described as:

\[ \frac{d}{dt} x = f(x, u) \]

the state trajectories can be linearized around the bias trajectories as:

\[ \frac{d}{dt} x = \frac{d}{dt} (\bar{x} + \delta x) = \frac{d}{dt} \bar{x} + \frac{d}{dt} \delta x = f(x, u) \]

\[ \approx f(\bar{x}, \bar{u}) + \frac{\partial f}{\partial x} (x - \bar{x}) + \frac{\partial f}{\partial u} (u - \bar{u}) \]

\[ = f(\bar{x}, \bar{u}) + \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial u} \delta u \]

where the term \(A_0\) describes the steady-state trajectory given the desired operating point and the other terms are partial derivatives of the system evaluated at the expected operating point.
C. Equivalent LTV-MPC Controller

The equivalent optimal control problem to be solved by the MPC algorithm once applied the LTV approximation is as following:

\[
\begin{align*}
\min_{x, u} \sum_{k=0}^{N} \frac{S}{\psi_k} (\delta_{ecu[k]} + v_{ecu[k]})^2 + \sum_{k=0}^{N} \frac{S}{\psi_2} (\delta_{vcl[k]} + v_{vcl[k]})^2 \\
+ \sum_{k=0}^{N} \frac{R}{\psi_1} (\delta_{iu[k]} + i_{iu[k]})^2 + \sum_{k=0}^{N} \frac{R}{\psi_1} (i_{il[k]} + i_{il[k]})^2 \\
+ \sum_{k=0}^{N} \frac{L}{\psi_1} (\delta_{vcl[k]} + v_{vcl[k]})^2 + \sum_{k=0}^{N} \frac{L}{\psi_1} (v_{l[k]} + v_{cl[k]})^2 \quad (18a)
\end{align*}
\]

s.t.

\[
\begin{align*}
\delta_{iu[k+1]} &= (1 - \Delta t \frac{R_i}{L_i}) \delta_{iu[k]} - \Delta t \frac{L_i}{C_i} \delta_{vcl[k]}, \quad k = 0, \ldots, N - 1 \\
\delta_{il[k+1]} &= (1 - \Delta t \frac{R_i}{L_i}) \delta_{il[k]} - \Delta t \frac{L_i}{C_i} \delta_{vcl[k]}, \quad k = 0, \ldots, N - 1 \\
\delta_{vcl[k+1]} &= (1 - \Delta t \frac{L_i}{C_i}) \delta_{vcl[k]} - \Delta t \frac{C_i}{L_i} \delta_{vcl[k]}, \quad k = 0, \ldots, N - 1 \\
\delta_{vcl[k+1]} &= (1 - \Delta t \frac{L_i}{C_i}) \delta_{vcl[k]} - \Delta t \frac{C_i}{L_i} \delta_{vcl[k]}, \quad k = 0, \ldots, N - 1
\end{align*}
\]

\[
\begin{align*}
\delta_{iu[k]} - \delta_{il[k]} - \delta_{iu[k]} - \delta_{il[k]} - \delta_{iu[k]} - \delta_{il[k]} \quad k = 1, \ldots, N \\
\gamma_{u} \leq \gamma_{u} + \delta_{iu[k]} \leq \gamma_{u} \quad k = 0, \ldots, N - 1 \\
\gamma_{l} \leq \gamma_{l} + \delta_{il[k]} \leq \gamma_{l} \quad k = 0, \ldots, N - 1 \\
\gamma_{v} + \delta_{vcl[k]} + \delta_{vcl[k]} + \delta_{vcl[k]} + \delta_{vcl[k]} + \delta_{vcl[k]} \leq \gamma_{v} \quad k = 0, \ldots, N - 1 \\
\gamma_{v} \leq \gamma_{v} + \delta_{vcl[k]} + \delta_{vcl[k]} \leq \gamma_{v} \quad k = N \\
\gamma_{v} \leq \gamma_{v} + \delta_{vcl[k]} \leq \gamma_{v} \quad k = N
\end{align*}
\]

where \(i_{ecu}, i_{eld}, v_{ecu}, v_{eld}, v_{ecu} \) and \(v_{eld} \) are variables that represent the difference between the bias trajectory and the reference to be tracked (if any). For instance, \(v_{ecu} = v_{cu} - v_{eu} \). Note that the bias trajectories are obtained solving the steady-state of the system and inferring it through the prediction horizon.

D. MPC-LTV Controller Tuning

The MPC-LTV algorithm presented in this document is comparable to the controller introduced in [8], where an MPC strategy is described to control a single-phase inverter. Likewise, [8] discusses the choice of the prediction horizon length \(T_s \) and the sampling frequency \(f_s \) considering the impact on the algorithm performance and the computational burden.\(^1\)

1The efficiency of the device has been approximated considering that \(R_a \) accounts for all the resistive losses of the converter and that the switching losses are modelled as introduced in [10] (it has been assumed that each arm has a single half-bridge sub-module and that its output voltage is synthesized using a Phase-Width Modulation (PWM) technique). The parameters of the switching device in [11] has been considered as a reference for this analysis.

This analysis, performed in a system of similar characteristics, studied how the controller could offer additional capabilities in order to enhance the system’s final performance. To do so, the MPC controller is tuned considering a prediction horizon at least equal to one grid period and a sampling frequency high enough to capture harmonic components higher than the grid fundamental. A similar \textit{ad-hoc} strategy has been conducted here to determine these parameters.

In addition, the performance of the LTV-MPC controller highly depends on the weighting factors of the OF terms. In [12], guidelines to choose the weights of MPC strategies applied to power electronics converters are discussed. It evaluates the difficulty of having terms of different nature in the OF and considers the use of scaling factors to equalize them. Therefore, parameters \(\psi_1, \psi_2 \) and \(\psi_3 \) scale the different terms of the OF (see Tab. 1). Once the different terms are of comparable magnitude, their weights are chosen considering their effect on the system.

1) Weight \(\mathcal{R} \): This weight regulates the injection of current into the arms. Considering that the MPC algorithm is defined such that the grid current is tracked explicitly, a higher or lower weight might result on the injection of a different amount of recirculating current inside the device but not into the grid.

2) Weight \(\mathcal{S} \): This weight regulates the voltage of the arms capacitor. Therefore, it is expected that a higher value should decrease the deviation of the arms voltage with respect to the tracking reference. Nevertheless, a minimum value of \(\mathcal{S} \) is set to guarantee a correct regulation of the arms’ capacitor voltage.

3) Weight \(\mathcal{U} \): This weight regulates the modulated voltage \(v_u \) and \(v_l \) but it does not seem to have a large impact on the final performance of the algorithm as long as the control action is feasible. Therefore, this weight is set to be small.

Similarly as in [8], weights \(\mathcal{R} \) and \(\mathcal{S} \) are adjusted to trade off the injection of current harmonics and the voltage error of the arm capacitor. These weights are found to be opposing each other and, because of the scaling of the OF terms, it is set that \(\mathcal{S} + \mathcal{R} = 1 \). In order to quantify the impact of the weighting, Fig. 3 shows the expected arms’ voltage ripple reduction against the system losses. Note that the system losses are chosen as a parameter to quantify the impact of having higher recirculating current within the device, considering that the physical limitations of the device cannot be exceeded.

E. LTV-MPC Controller Analysis

First, the performance of the MPC controller described in (18a)-(18l) is analyzed considering an ideal system where the current and voltage trajectories of the MPC regulator matches the state trajectories of the system. This analysis is used to assess the performance of the controller. Next section provides detailed simulations to evaluate the performance of the controller under a more realistic scenario.

The results shown in Fig. 4 are obtained using the Operator Splitting Quadratic Program (OSQP) solver [13], which successfully solved the MPC problem online requiring a computational time between 0.5 and 1 ms. These results are obtained running the same scenario (1 p.u. and unity power

\[
\begin{align*}
\text{where } i_{ecu}, i_{eld}, v_{ecu}, v_{eld}, v_{ecu} \text{ and } v_{eld} \text{ are variables that represent the difference between the bias trajectory and the reference to be tracked (if any). For instance, } v_{ecu} = v_{cu} - v_{eu}.\text{ Note that the bias trajectories are obtained solving the steady-state of the system and inferring it through the prediction horizon.}
\end{align*}
\]
factor) under different weights. Because of symmetry, only the trajectories of the upper arm are shown. Besides, note that the range of $S$ is chosen according to the information provided in Fig. 3, discarding the areas that are not relevant for the operation of the converter. These plots show how the MPC regulator adjust the arms current to trade-off voltage ripple error and the injection of recirculating current into the arm. Interestingly, the injection of additional harmonics into the arms is constrained by the physical limitations of the device as shown in Fig. 4(b) (see Tab. I).

Additionally, Fig. 5 shows the harmonic decomposition of the arm current ($i_u$) and the grid current ($i_g$) for different values of $S$. Note that the grid current only contains the fundamental component, which matches with the reference given to the MPC controller. Nevertheless, the arm current contains DC component, used to exchange power with the DC grid, AC component at 50 Hz, used to exchange power with the AC grid, and AC components at even frequencies, which do not exchange net power neither with the AC or DC grid nor the upper and lower arms but shape the waveform of the arms’ capacitor voltage. However, this causes in a higher recirculating current which decreases the efficiency of the device. The mapping between the reduction of the arms capacitor ripple and the detriment of the system’s efficiency is depicted in Fig. 3.

**IV. Simulation Results**

This section evaluates the performance of the control algorithm in a simulation model that approximates the real system in higher detail. To do so, a simulation model considering the time-delay of the MPC block and other non-ideal characteristics of the system elements, such as the switching behavior of the IGBTs or the current controller dynamics, are included. First, the steady-state analysis is presented in Fig. 6. After, the dynamic performance of the LTV-MPC algorithm is compared with a conventional controller tuned following the guidelines described in [14].

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SIMULATION PARAMETERS.</th>
</tr>
</thead>
<tbody>
<tr>
<td>System Data</td>
<td>Value</td>
</tr>
<tr>
<td>$L_a$</td>
<td>7.3 mH</td>
</tr>
<tr>
<td>$R_a$</td>
<td>0.1 Ω</td>
</tr>
<tr>
<td>$L_s$</td>
<td>1.8 mH</td>
</tr>
<tr>
<td>$C_u$</td>
<td>0.19 mF</td>
</tr>
<tr>
<td>$P_N$</td>
<td>4.6 kW</td>
</tr>
<tr>
<td>$V_{SRMS}$</td>
<td>230 V</td>
</tr>
</tbody>
</table>

The control loop has the sequence described in Fig. 2. Thus, the desired current expected to be injected into the AC grid is calculated based on the measurements obtained from the system. This reference is generated considering the limitations of the physical system. After, this data, together with other available measurements, is passed to the MPC regulator block which generates current references according to the tuning parameters of the OCP. Note that the control trajectory of the MPC controller is not fed directly into the PWM block.
as the update of the control trajectory is too slow in case of grid disturbances. The low output sampling of the MPC regulator is noticed in Fig. 6(a), where the output of this block is plotted as discrete dots labelled as “mpc”. Thus, the current trajectory provided to the current controller is obtained by interpolating the next expected current point with the following point of the prediction horizon. Doing so, the current trajectory is smoothed as it is observed in Fig. 6(a) (red line named as “intrp.”). The current controller is a $H_\infty$ controller based on the guidelines described in [15]. It is tuned to have a high gain at the harmonic components that the MPC algorithm is likely to inject but to reject higher frequencies to avoid the controller to be affected by the switching ripple of the system. Finally, the resulting arm current $i_u$ is also shown and a detail of the resulting ripple is provided.

In addition, Fig. 6(b) and Fig. 6(c) compare the trajectories of the arm capacitor voltage for different weights of $S$. This comparison is done running the same scenario used Section III-E in the same ideal model, in an average model of the converter and in a switching model of the converter. Note that the switching model is based on a single sub-module per arm whose output voltage is obtained using a PWM strategy. It is noticed that despite the added complexity of the switching and the average models, the trajectories obtained closely follow the results obtained by the ideal scenario, validating the results described in Section III-E.

Next, Fig. 7 compares the performance of the system under disturbances. The simulation starts from steady-state (load 0.5 p.u and at unity power factor). At a time 0.05 s., the power demanded undergoes a step change from 0.5 p.u. to 1 p.u. (power factor equal to 1). The performance of the system when it is controlled using a conventional controller [14] or the MPC controller is shown in Fig. 7(a). It is observed that the MPC

Fig. 6. Comparison of the steady-state trajectories for different simulating models. Operating point: nominal load and unity power factor.

Fig. 7. Transitory scenario; comparison of the MPC-LTV algorithm vs. a conventional controller.
controller reduces the transient as the algorithm foresees the dynamics of the system ones this happened.

Besides, the results in steady state denote how the MPC algorithm achieves a lower ripple of arm capacitor voltage. The trajectory of the grid current before and after the power step change is shown in Fig. 7(b).

Later, at time 0.4 s., the system undergoes a grid fault. The voltage $v_g$ drops to 30% of its nominal value and the phase shifts by 15°. Likewise, Fig. 7(a) depicts the transient of both controllers. It is seen how the current controller in the MPC structure adjust the current as soon as the fault occurs, impeding large excursions of the arm capacitor voltage even if the MPC sampling is relatively low. Besides, it is also noticed how the arm capacitor ripple is kept small before the inception of the fault and that later on is controlled to kept the converter stable. The transitory of the grid current, as well as grid voltage, are plotted in Fig 7(b) and Fig. 7(c).

V. Conclusions

This document has introduced a novel control structure based on an MPC algorithm to control the arms voltage of a single-phase MMC device such that the grid current is perfectly tracked whereas the internal recirculating current is regulated to reduce the voltage oscillation of the SMs. First, the algorithm has been stated and an approach to simplify the algorithm has been proposed. The performance of the LTV-MPC controller has been analyzed under steady-state conditions and it has been shown that under nominal operating conditions, the SMs voltage ripple can be reduced by around 35% by only decreasing the system efficiency by 1%. This fact allows reconsidering the capacitive energy storage requirements of the converter.

The algorithm has been shown to autonomously adjust the recirculating current of the controller in order to obtain the optimal current inject according to the system characteristics. Thus, the controller has been shown to be operated close to the constraints and to apply some of the control trick found in the literature in a natural way. Finally, a transient simulation has been conducted to observe the performance of the algorithm when a power step-change and voltage fault occurs. The results show a good converter dynamics performance, denoting some improvement with respect to a conventional controller.

REFERENCES