The behavior of rising bubbles covered by particles

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HIGHLIGHTS

• The velocity and aspect ratio of rising particle-laden bubbles was investigated.
• The buoyancy force and drag force exerted on the bubbles and the effect of particles were calculated.
• Particles attached strongly dampen the oscillations observed in bubble velocity and aspect ratio.
• A drag modification factor was identified to quantify the drag influence of particles on bubble velocity.
• A modified drag coefficient for uncoated and particle-laden bubbles was introduced for the first time.

GRAPHICAL ABSTRACT

A systematic investigation of the influence of particle coverage on the dynamics of rising bubbles was carried out using high-speed photography and image analysis techniques to study bubble behavior in terms of changes in velocity and aspect ratio. The buoyancy force and drag force exerted on the bubbles and the effect of particles were calculated to further understand their behavior. Results show that particles attached on the bubbles strongly dampen the oscillations observed in bubble aspect ratio and decrease its velocity and acceleration. The particles also render the bubbles more spherical and slow their velocity. It was found that the overall velocity of a bubble is directly correlated to its aspect ratio and inversely correlated to its particle coverage, while the acceleration and the aspect ratio and its change are inversely correlated. Interestingly, the trend observed in the oscillation and the oscillation period of particle-laden bubbles is similar for different levels of particle coating. A drag modification factor, which quantifies the drag influence of particles on bubble velocity, was identified from force analysis. A modified drag coefficient for uncoated and particle-laden bubbles was introduced, which allows, for the first time, to predict the behavior of rising bubbles in gas-liquid-particle systems.

1. Introduction

The rise of air bubbles is one of the most common gas-liquid flow phenomena [1–4]. Understanding the dynamic interaction between the gas and liquid phases is of great value for a wide range of industrial applications in chemical engineering, including bubble column reactors [5,6], gas-solid fluidized beds [7], froth flotation cells in mineral processing [8,9] and wastewater treatment tanks [10,11]. The behavior of rising bubbles, which includes their velocity and aspect ratio (i.e. shape), is determined by several acting forces. A bubble rises mainly because of the buoyancy force, which acts against the drag force. The density and viscosity of the gas in a bubble also
influence the bubble behavior, but these effects are not as significant as the drag force and buoyancy force exerted on the rising bubbles [2,3]. The drag coefficient, which is strongly influenced by factors such as the fluid properties and bubble diameter [12], is usually modified from classical models in order to better predict the velocity of the rising bubbles [13] or to improve the accuracy of CFD simulations [6].

For gas-liquid-solid systems, the presence of particles will also influence the behavior of a rising bubble, particularly if the particles are attached to the bubble’s surface. Due to the opaque and highly dynamic nature of the three-phase slurries, it is impossible to directly visualize the behavior of a particle-laden bubble. However, capillaries can be used to generate bubbles of controlled bubble size in a transparent experimental rig, which allows a clear visualization of their behavior. This technique has been used to generate bubbles that remain attached to the capillaries for the study of induction time [14], coalescence of bubbles [15,16], and attachment and detachment of particles to bubbles [17]. Capillaries have also been used to continuously generate bubbles in order to study bubble velocity and shape change of both rising bubbles [18,19] and bubbles as they bounce at the liquid-air interface [20].

Other influential factors for the behavior of rising bubbles include chemical adsorption [21], frother structure [22,23] and bubble size [24,25]. The behavior of particle-laden bubbles, however, has been only studied for fully coated bubbles [26] or bubbles partially coated with particles focusing on their bounce back from the liquid-air interface [27]. However, no previous study has addressed the behavior of rising bubbles with different levels of particle coverage, nor studied the modifications required for the drag coefficient in such cases.

The aim of this research is to use capillaries to generate bubbles of controlled size to explore the effect of attached particles on the behavior of rising bubbles. In addition, the forces exerted on these bubbles are calculated and the relationship between the velocity of a particle-laden bubble and its aspect ratio is analyzed. A modified drag coefficient model is obtained that fits the experiment data for particle-laden bubbles, which is essential to inform the modelling of industrially relevant processes where these bubble-particle aggregates play a key role.

2. Experimental

2.1. Materials

Spherical soda lime glass beads (Sigmund Lindner GmbH) were used, with a characteristic size $d_{50} = 39 \, \mu m$ ($d_{90} = 64 \, \mu m$) and a mass density of 2.5 g cm$^{-3}$. Analytical grade myristyltrimethylammonium bromide, TTAB (Sigma-Aldrich Chemical Co), was used as a collector. All experiments were performed at a temperature of 15 °C under natural pH condition.

2.2. Setup

A schematic of the setup is shown in Fig. 1. A capillary placed in a transparent rectangular perspex chamber (50 mm $\times$ 50 mm $\times$ 125 mm) is connected to a programmable micro-syringe pump. This pump is used to achieve fine control of air injection into the capillary, which is essential for repeatable bubble generation. To minimize disturbances, the entire setup was placed on a vibration-free table. The behavior of rising bubbles was recorded by a high speed camera, an Olympus i-speed LT with photographic objective and magnification lenses, at a capture rate of 1000 frames per second. In addition, to analyze the coverage of particles on the bubbles, a second camera, a Canon EOS 600D, was set perpendicular to the Olympus camera; this guaranteed a close view and a clear image of the bubbles generated in the capillary. In both cases, LED back-lighting was used for illumination.

Fig. 1. Experimental configuration to investigate the behavior of a rising air bubble, showing the setup including the cameras and light arrangement.

2.3. Methodology

2.3.1. Experiment procedure

Two grams of glass beads particles were added into the transparent chamber, which was filled with 250 mL deionized water (resistivity larger than 15 MΩ cm). TTAB, at a concentration of $1 \times 10^{-6}$ M, was added and then mixed for 15 min with a magnetic stirrer at a speed of 700 rpm. After the suspended particles settled down, a single bubble was formed at the tip of a stainless steel capillary needle (inner diameter of 0.84 mm and outer diameter of 1.27 mm) using a programmable syringe pump to control the air injection with high precision.

For the uncoated bubble, the air injection rate was kept at 0.72 mL h$^{-1}$ until the bubble was released. For the coated bubble, after the bubble grew up to a diameter of approximately 2.4–2.5 mm (with the grids on the camera used as a guide), the magnetic stirrer was turned on at a speed of 350 rpm so the bubble could be covered by particles. A stainless wire mesh (750 μm) was inserted below the capillary in order to guarantee that particles were evenly dispersed during the stirring process and also to decrease the chance of bubbles detaching from the capillaries due to excessive turbulence. The stirrer was turned off once a bubble was coated to the required fraction of surface coverage for each experiment. Surface coverage was initially estimated visually, and photographs were taken for detailed analysis as described in Section 2.3.2. A settling time of 1 min was allowed for the liquid in the rig to become clear, after which additional air was injected to the capillary pinned bubble at a constant air flow rate of 0.72 mL h$^{-1}$ in order to release it from the tip of the capillary.

It is important to point out that only one bubble was generated for each experiment and its behavior was recorded and analyzed over a distance of 30 mm from the tip of the capillary. Footage of the rising bubble for each experiment was processed using ImageJ to obtain the position and dimensions of the bubbles. The tip of the needle was also photographed and used as a reference length to calculate the magnification of the optical system.

2.3.2. Image processing methods

The images of bubbles were analyzed for dimensional measurements first. The exact fractions of bubble surface covered by particles in each experiment were calculated in this work using the pendant drop method. First of all, the bubble was assumed symmetrical, and the images were rotated for the analysis. A series of the edge points on the right half of the rotated bubble were then detected. After that, the pendant drop equation model [28,29] was fitted with the location of the edge points. The analysis of the drop shape (i.e. the bubble shape in this case) was derived according to the Young-Laplace equation [30], which relates the radii of curvature $R_1$ and $R_2$, the surface tension $\sigma$, and
the pressure difference \( \Delta P \) across the curved liquid/gas interface:

\[
\Delta P = (P_{\text{ext}} - P_{\text{int}}) = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)
\]

where \( P_{\text{ext}} \) and \( P_{\text{int}} \) represent the internal and external pressure, respectively. This is shown schematically in Fig. 2.

The above equation was converted to a set of three first-order differential equations, formed by the spatial positions \( x \) and \( z \) and turning angle \( \Phi \) of the bubble interface as a function of the arc length \( s \) of the bubble shape. With three boundary values \( x(s = 0) = 0, z(s = 0) = 0, \Phi(s = 0) = 0 \), the equations were solved numerically, assuming \( R = R_1 = R_2 \):

\[
\frac{d\Phi}{ds} = -\frac{\sin\Phi}{x} + 2R \frac{\Delta \rho g c}{\sigma} \quad (2)
\]

\[
\frac{dx}{ds} = \cos\Phi \quad (3)
\]

\[
\frac{dz}{ds} = \sin\Phi \quad (4)
\]

The numerical fit of the bubble shape can yield the surface tension and the radii of the bubbles. By integrating this model, the surface coverage of particles was calculated.

Fig. 3 shows the bubble edge detection for a bubble before and after coated with particles. Fig. 4 compares the edge points of a bubble with the fitted pendant drop models. Firstly, the pendant drop model was fitted to the edge points of a bubble prior (Model-uncoated) and after (Model-coated) it was covered with particles. However, the model fitted well with the edge points of the uncoated bubble but not with the coated one. Therefore the diameter of the coated particles could not be neglected. In order to solve the above problem, the point from which particle coating occurred was detected, as can be seen from Fig. 4(a) (blue circle). Afterwards, the edge points of the bubble coated with particles were moved inside the bubble by a length equal to the diameter of the particles, and then a new fitting curve of the coated bubble was generated, as was shown in Fig. 4(b). The surface area of the bubbles was calculated by rotating the fitting model around the vertical axis. The covered area was determined by the average of the integrals of different heights of the particles covering the bubble. It is important to note that once a bubble is released from the capillary, its surface area becomes larger (since additional air is added to release it) and therefore the actual percentage of particle coverage has to be reassessed. Dividing the particle coverage obtained originally using the pendant drop method by the surface area of the released bubble yields the actual percentage of the surface area of the released bubble that is covered by particles. During this process, the detachment of particles is negligible.

### 2.3.3. Bubble velocity and aspect ratio calculation

Fig. 5 shows a schematic of the bubble image analysis procedure and how the diameters \( a \) and \( b \) of the bubble are determined. Assuming the bubble was an oblate spheroid [31] (symmetrical about the rising direction), the bubble equivalent diameter, \( d \), was calculated from the diameters \( a \) and \( b \), the bubble diameters parallel and perpendicular to its moving direction, respectively:

\[
d = \sqrt{ab^2} \quad (5)
\]

The aspect ratio (AR), a parameter that reflects the dynamic behavior of rising bubbles, was calculated as the ratio between the diameters \( b \) and \( a \) [31]:

\[
AR = \frac{b}{a} \quad (6)
\]

It has been reported that with an increase in the concentration of frother, bubbles tend to be more spherical and bubble rise velocity decreases. Oscillations in aspect ratio and velocity for uncoated rising bubbles have been found to be inversely related [32].

Here, only the vertical direction of the bubble motion was considered. The rise velocity \( u \) was calculated from the vertical central position of the bubble over two consecutive frames \( (i + 1, i) \) as:

\[
u = \frac{y_{i+1} - y_i}{\Delta t} \quad (7)
\]

where \( \Delta t \) is the time interval between two frames (1 ms for the recorded data).

The bubble releasing point was 30 mm below the air and liquid interface. The zero time point was taken to be the point one frame prior to the release of a bubble.

### 3. Results and discussion

#### 3.1. Comparing uncoated bubbles with particle-laden bubbles

Fig. 6(a) and (b) show the aspect ratio of an uncoated bubble and a coated bubble (with 59.9% surface coverage), respectively. It is clearly seen that after being released from the capillary, the oscillation pattern of both bubbles were similar over the first 30 ms, and afterwards particles exerted a significant effect on the shape stability of the coated bubble. Apart from some minor decrease due to oscillations, the aspect ratio of the uncoated bubble rose steadily until it reached the maximum value of 2.82 at 66 ms, after which the bubble aspect ratio began to decrease due to the rotation of the bubble. For the coated bubble, the aspect ratio firstly increased to 1.29 over the first 11 ms, and then decreased to 1.13 at 18 ms prior to reaching the peak value of 1.39 at 30 ms. After that, the aspect ratio experienced a minor decrease and then changed slowly afterwards. In terms of velocity, as is shown in Fig. 7, there are more data points frames for the coated bubble than for the uncoated bubble within the same viewing area, due to the rising velocity of the former being lower. For the uncoated bubble, its velocity rose to 0.212 mm/ms at 17 ms and then oscillated within a range of 0.166 mm/ms to 0.251 mm/ms until 86 ms, after which the velocity decreased. The coated bubble, on the other hand, firstly rose to a velocity of 0.162 mm/ms at 20 ms, then experienced a minor decrease prior to rise again to 0.183 mm/ms at 34 ms. After that, although with
small variation, the velocity showed an overall stable tendency until 95 ms, after which it decreased due to the rotation of the bubble. According to the literature, bubble velocity and aspect ratio periodically reach top stable values and then slow down due to the bubble rotation (i.e. a > b). The amplitude of the velocity is mainly determined by the overall hydrodynamic forces. The comparison of the influence of particles on the bubble velocity and shape change was carried out for the initial time period when the angle (θ) between the bubble’s minor axis and its moving direction was lower than 10°, normally during the first 80 ms. Overall, the coated bubble showed a lower aspect ratio and velocity compared to the uncoated bubble. Without particles, bubble shape and velocity pulsated more strongly.

To explore the relationship between the parameters discussed above, the rising velocity (both for the uncoated bubble and the coated bubble) is plotted against its corresponding aspect ratio, as shown in Fig. 8. Overall, the rising bubble velocity was directly correlated to its aspect ratio, i.e. when the bubble shape becomes more spherical, the bubble slows down and vice versa. This has been shown before for uncoated bubbles, but had not been explored for particle-laden bubbles. The results confirm that the overall behavior is similar although the distribution of aspect ratio values for the uncoated bubble was wider than that of the coated bubble.

Having high velocities for the high aspect ratios and vice versa is counterintuitive as high aspect ratios are associated to high drag configurations. The reason for this apparent contradiction is because this is a dynamic situation in which bubbles are oscillating in terms of aspect ratio. At the maximum aspect ratio, the bubble will be experiencing a deceleration and will return to a round shape and lower drag, with this oscillation then repeating itself. These will be explained in more detail later.

![Fig. 3. Images rotated for edge detection of a bubble (a) before and (b) after being coated with particles. The red line shows the detected edge points. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)](image)

![Fig. 4. Curve fitting to the uncoated and coated bubble edge points: (a) detection of the point where particles began to coat the bubble, marked here with a blue circle; (b) fitting after the particle diameter was reduced. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)](image)

![Fig. 5. Image analysis procedure: (a) bubble image, (b) image after threshold applied, (c) edge detected and diameters a and b used to determine the aspect ratio.](image)
The data for acceleration of the uncoated bubble and the coated bubble are shown in Fig. 9(a). The noise in the raw data makes it difficult to analyze, the acceleration difference between the uncoated bubble and the coated bubble can be seen more clearly after applying a moving-average filter. This is done in Fig. 9(b), in which the averages of each six continuous acceleration data points (window size 6) are shown. As can be seen, the acceleration amplitude of the uncoated bubble was higher than that of the coated bubble, which indicated that the latter was dampened due to the attached particles.

Further to the previous discussion. The acceleration and aspect ratio of the uncoated bubble and coated bubble are plotted and shown in Fig. 10(a) and (b), respectively, after applying a moving-average filter for the acceleration (window size 6). Overall, the acceleration and the aspect ratio were inversely correlated: local peak value of acceleration corresponded to a local minima of aspect ratio. It can be concluded that high aspect ratio causes high drag which will decelerate the bubble motion and vice versa.

For more in-depth analysis, the acceleration and aspect ratio change of the uncoated bubble are also compared and shown in Fig. 11(a), after applying a moving-average filter for both data (window size 6). As can be seen, the acceleration and the aspect ratio change were also inversely correlated: each peak value of acceleration corresponded to a local minima of aspect ratio change. The same trend can be seen for the coated bubble, as shown in Fig. 11(b). Both the acceleration and the aspect ratio change of the coated bubble were smaller compared to those of the uncoated bubble.

3.2. Influence of surface coverage of particles on bubble behavior

To explore the effect of particle surface coverage on bubble behavior, bubbles with seven different levels of particle coating were produced. Fig. 12 shows those bubbles attached to the capillary, which once released resulted in the following percentages of surface coverage: 0.0%, 17.9%, 26.5%, 31.9%, 48.0%, 57.9% and 59.9%. Values for bubble aspect ratio and rising bubble velocity are compared in Figs. 13 and 14, respectively. The diameter of the released uncoated bubbles was 2.8–2.9 mm, and the diameter of the released coated bubbles was 2.9–3.0 mm. It should be noted that the diameter is calculated from the rising particle-bubble aggregates, which is relatively bigger if more particles were attached to the bubble surface.

For values of surface coverage below 50%, the oscillation trend, shown in Fig. 13, appeared to be similar in frequency but different in amplitude over the first 85 ms. The uncoated bubble showed larger values of aspect ratio (from 0.80 to 2.82) than particle-laden bubbles.
The presence of particles clearly stabilized the bubble shape, particularly dampening the shape oscillation. This damping effect was more obvious when the particle surface coverage was higher (i.e. surface coverage higher than 50%).

Fig. 14 quantifies how the bubble velocity changed as the bubble rose at different levels of particle coverage. Overall, the bubble velocity decreased with the increase in surface coverage. The velocities of bubbles with the three lowest levels of surface coverage followed the same oscillation trend over the first 85 ms. The velocity oscillation amplitude was more noticeably damped for surface coverages of 48.0%, 57.9% and 59.9%. Nevertheless, comparing Figs. 13 and 14, the overall correlation between shape and velocity oscillation was maintained: with a higher proportion of particles coating the surface of a bubble, the bubbles tended to have lower aspect ratio and velocity. These results thus extend previous findings for uncoated bubbles [31,32] to different levels of particle coverage. Bubbles rose with oscillations of both velocity and aspect ratio. The longer oscillations were determined by the hydrodynamic force and on top of these longer overall oscillations were small vibrations caused by the capillary oscillation [12].

3.3. Oscillation period

As described in the previous section, the rise of a bubble is accompanied by a fluctuation of both velocity and aspect ratio. In order to better understand the influence of the particles on the bubble shape change, the oscillation periods [33], $T$, were calculated and compared with the experimental data using Eq. (8):

$$T = 2\pi \frac{\Delta \rho R^4}{6 \gamma}$$

where the bubble radius ($R$) is 1.45 mm, the density difference ($\Delta \rho$) of gas phase and liquid phase is 1000 kg m$^{-3}$, and the surface tension ($\gamma$) tested is 74 mN. Table 1 shows the times at which aspect ratio peaks occurred for different levels of particle coverage. It can be concluded that the oscillation periods were within a range of 13–21 ms, which is in good agreement with the theoretical calculated value of 17 ms [34,35].

The capillary force causes the small oscillation in aspect ratio while the overall aspect ratio change is determined by the hydrodynamic force. The oscillation of bubble aspect ratio follows a similar trend under the action of capillary forces; since capillary forces are related to surface tension, it can be concluded that the surface tension does not change significantly when bubbles are covered by particles. The hydrodynamic forces thus play a major role in the phenomena observed and need to be analyzed in further detail.

3.4. Force analysis

For an ideal bubble, to simplify the calculation, it is assumed that the bubble is rigid and only influenced by the buoyancy and drag forces after it is released from the capillary. The drag force includes both viscous and pressure drag on the body in the direction of the slip...
For the bubble-water interface, the drag force can be written as:

\[ F_D = \frac{C_D \rho u^2 A}{2 \cos \theta} \]  

where \( C_D \) is the drag coefficient, \( \rho \) is the density of the liquid, \( A \) is the cross-sectional area of the bubble perpendicular to the moving direction and \( \theta \) is the angle between the bubble’s minor axis and its moving direction. For spherical objects, \( A = \frac{4}{3} \pi d^3 \). The widely used Schiller-Nauman drag coefficient is selected \([37]\) to calculate the drag force, which is given as:

\[ C_D = \begin{cases} 
24 & \text{if } Re < 1000 \\
1 + 0.15 Re^{0.87} & \text{otherwise} 
\end{cases} \]

where \( Re = \frac{\rho u d}{\mu} \) is the Reynolds number, dependent on \( d \), the equivalent diameter of the bubble, and \( \mu \), the dynamic viscosity of the fluid. The added mass force of the bubble can therefore be expressed as:

\[ F = m^* \frac{du}{dt} = F_B - F_D \]  

\[ \text{Fig. 11. Acceleration and aspect ratio change of the (a) uncoated bubble and (b) coated bubble. Original data processed with a moving-average filter (window size 6).} \]

\[ \text{Fig. 12. Different fractions of bubbles covered by particles.} \]

\[ \text{Fig. 13. Aspect ratio as a function of different fractions of bubbles covered by particles.} \]
Fig. 14. Velocity as a function of different fractions of bubbles covered by particles.

where $m^*$ is the added mass \[38,39\], and $F_B$, the buoyancy force is defined as:

$$F_B = \frac{1}{6} \pi \rho d^3 g$$

(12)

where $g$ is the gravity acceleration constant. It should be noted that the aspect ratio of the bubble is closest to 1 immediately after the bubble is released (in the first 5 ms). It is its diameter at this time that was used for the calculation of Eq. (12). The added mass is:

$$m^* = C_m \times \frac{1}{6} \pi \rho d^3$$

(13)

The added mass coefficient ($C_m$) is a function of the bubble-deformation ratio, i.e., the aspect ratio (AR):

$$C_m = 0.62 \times AR - 0.12$$

(14)

Using Eqs. (5) and (6), we can express $a$ and $b$ in terms of $d$ and $AR$ as follows:

$$a = d \times AR^{-\frac{1}{2}}$$

(15)

$$b = d \times AR^{-\frac{3}{2}}$$

(16)

Therefore the cross section area can be defined as:

$$A = \pi \left( \frac{b}{2} \right)^2 = \frac{\pi d^2}{4} AR$$

(17)

and the acceleration of the bubble can be expressed as:

$$\frac{du}{dt} = \frac{1}{m^*} (F_B - F_D) = \frac{1}{C_m} \left( g - \frac{3C_D \rho d^2 AR^2}{4 \cos \theta} \right)$$

(18)

The Schiller–Naumann drag coefficient was developed for rigid solid spherical particles and also it was extended to various shape of particles, drops and bubbles \[13,39–41\]. Since the bubble shape oscillates during the rising period, and also the drag coefficient of a real bubble is lower than the one of a spherical bubble \[1,12,42\]. In order to fit the calculated velocity to the experimental one, the value of $C_D$ has to be adjusted.

A drag modification parameter, $\eta_d$, is added to adjust the drag coefficient for the uncoated bubble, so Eq. (18) can be changed to

$$\frac{du}{dt} = \frac{1}{m^*} (F_B - F_D) = \frac{1}{C_m} \left( g - \frac{3C_D \rho d^2 AR^2}{4 \cos \theta} \right)$$

(19)

where $C_D$ is the modified drag coefficient of the uncoated bubble according to the experimental results:

$$C_D = \eta_d C_D$$

(20)

The existence of adsorbed reagents on the bubble surface can exert a significant influence on the surface forces and the drag coefficient, which finally affect the bubble floating velocity \[40\].

For the calculation of the coated bubble, the overall force that the particles exert on the bubble, $F_i$, must be taken into account. Since particles move together with the bubble, and to simplify the process, the drag force $F_{Dp}$ from the particles can be regarded as a force that makes the bubble less mobile, which can be expressed as:

$$F_{Dp} = \eta_d F_D$$

(21)

$$m_p \frac{du}{dt} = F_i + F - m_p g - F_{Dp}$$

(22)

where $F_{Dp}$ represents the buoyancy force on the particles, $m_p$ is the mass of the particles calculated from the surface coverage and particles being regarded as one layer evenly distributed onto the bubble surface without any aggregation. $\eta_d$ is a drag modification factor corresponding to different levels of particle surface coverage. Eq. (11) can therefore be changed to

$$\frac{du}{dt} = \frac{1}{m^*} (F_B - F_D - F_i).$$

(23)

Substituting $F_i$ from Eq. (22) leads to

$$\frac{du}{dt} = \frac{1}{m_p + m^*} (F_B + F_{Dp} - F_D - F_{Dp} - m_p g).$$

(24)

Further, substituting $F_D$ and $F_{Dp}$ from Eqs. (19) and (21) into Eq. (24) gives:

$$\frac{du}{dt} = \frac{1}{m_p + m^*} \left( F_B + F_{Dp} - m_p g - \frac{C_D \rho d^2 \eta_d g d^2 AR^2}{8 \cos \theta} (1 + \eta_d) \right)$$

(25)

$$C_D = \eta_d C_D (1 + \eta_d)$$

(26)

where $C_D = \eta_d C_D$ represents the modified drag coefficient of the particle-laden bubbles. In order to fit the force calculation to the experimental data, least squares regression analysis was used and the influence of particles on the bubble behavior was determined. As the bubble detaches from the capillary, its velocity fluctuates for the first 10 ms, after which the velocity increases before further fluctuation. For the purposes of these calculations, the first 10 ms were not taken into account. Similarly to the previous section, data is only considered for the initial period during which the angles between the bubble’s minor axis and its moving direction do not exceed 10°.

From the data in Fig. 14, $\eta_d$ can be calculated using the velocity of the uncoated bubble, which results in a value of 0.81. For bubbles coated at different particle coverage, given $\eta_d$ of 0.81, the corresponding values of $\eta_d$ can be calculated from Eqs. (25) and (26). These are shown in Fig. 15, from which it can be observed that the drag modification factor $\eta_d$ increases from 0.13 to 1.05 as the surface coverage of
particles increases from 0.0% to 59.9%. There are two reasons for this increase, one is the drag due to roughness and the other is the drag due to reduced mobility of interface.

The drag coefficient calculated from the experimental data ($C_{D_k}$) as well as the modified drag coefficient ($C_{Dm}$) from the least-squares fit to the experimental data are shown in Fig. 16(a). Here we should note that the $C_{Dk}$ is also calculated from Eq. (10) and Eq. (20), but the calculation is based on the velocity from the experimental data, while the calculation of $C_{Dm}$ is based on the velocity from the fitting calculation of Eq. (19). For the uncoated bubble, the ideal drag coefficient from the Schiller-Naumann model as well as the modified drag coefficient from the least-squares fit to the experimental data as a function of Reynolds number are displayed in Fig. 16(b). It was found that both the drag coefficient predicted by the Schiller-Naumann model and that from the fitting to the data decrease with Reynolds number. It can also be observed that the Schiller-Naumann model over-predicts the real drag coefficients for the rising uncoated bubble. The modified drag coefficient of bubbles covered with different levels of particles as a function of Reynolds number are summarized in Fig. 16(c). It is clear that with the increase of particle surface coverage, the modified drag coefficient of particle-laden bubbles increases.

Fig. 15. Particle influence on the drag coefficient according to different fractions of bubbles covered by particles.

Fig. 16. (a) Modified drag coefficient compared with the drag coefficient calculated from the experimental data for an uncoated bubble as a function of the time. (b) Modified drag coefficient compared with the drag coefficient predicted from Schiller-Naumann for an uncoated bubble as a function of the Reynolds number. (c) Modified drag coefficients of the uncoated bubble and bubbles loaded at different levels of particle coverage as a function of the Reynolds number.
4. Conclusions

The behavior of bubbles after being released from a capillary tip was investigated as a function of their particle surface coverage. The pendant drop model was used and fitted to the edge points of the bubbles to calculate their particle surface coverage. Results show that the velocity and aspect ratio of bubbles decrease as the surface coverage of particles increases. The particles exert a strong damping effect on the amplitude of the oscillation observed in bubble velocity, acceleration and aspect ratio. In general, the rising bubble velocity is directly correlated to its aspect ratio, regardless of whether the bubble is coated or not. The acceleration and the aspect ratio and its change are found to be inversely correlated: each peak value of acceleration corresponds to a local minimum value of the aspect ratio (change), and vice versa.

The oscillation of bubbles coated with different levels of particles followed a similar trend, indicating the surface tension was not significantly affected by a bubble being coated or not. A drag modification factor $\eta_{D}$, which quantifies the drag influence of particles on the bubble velocity, was identified from force analysis. Results show that the drag modification factor follows an increasing trend rising from 0.13 to 1.05 as the surface coverage of particles increases from 0.0% to 59.9%. This drag modification has allowed to obtain a modified drag coefficient for particle-laden bubbles for the first time.

This work contributes to a further understanding of the behavior of particle-laden bubbles, in particular with regards to the increase in the modified drag coefficient experience by the bubble-particle aggregate with an increase in surface coverage of particles. Understanding and being able to predict the bubble hydrodynamics in these complex three-phase systems is essential to improve the performance of industrial processes that rely on particle-laden bubbles.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.cej.2019.02.005.