The influence of the dynamic magnetoelastic effect on potential drop measurements

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Abstract

Alternating Current Potential Drop (ACPD) measurements are routinely used for monitoring crack length in laboratory-based fatigue tests, and so measurements will be taken on components which are exposed to cyclic dynamic stresses. It has been empirically observed that cyclic stresses cause a strong increase (above 10%) in measured resistance that is both AC inspection frequency and loading frequency dependent. The excess resistance will result in erroneous measurements; this paper investigates the cause and provides recommendations to limit the influence. Applied stresses influence ACPD measurements through the magnetoelastic effect; elastic strain induces magnetization in ferromagnetic materials, which in turn influences the magnetic permeability and therefore skin depth. Further, it has recently been realised that cyclic magnetization results in a frequency-dependent concentration of the magnetic flux at the surface of the component, and consequently a non-uniform spatial distribution of magnetic permeability. In this study it is found that the interaction between the non-uniform spatial distribution of both the current density and magnetic permeability results in significant non-linear modulation of the measurement signal. A combination of finite element simulations and experimental results are used to explore this phenomenon.

1. Introduction

Potential drop (PD) measurements are commonly used for monitoring crack growth in laboratory-based fatigue tests. A known current is injected through the test component using two electrodes while the resulting electrical potential is monitored through a separate pair of sensing electrodes, as illustrated in Fig. 1a) for a standard compact tension sample. The resulting impedance indicates the crack length (see ASTM standard E647 for more detail on PD measurements of crack length [1]).

This investigation is motivated by the empirical observation that in ACPD (alternating current potential drop) measurements unexpectedly large, PD inspection frequency $f_{\text{AC}}$ and loading frequency $f_{\text{L}}$ dependent, changes in impedance occur at relatively modest stress amplitudes, as shown in Fig. 1b); the test conditions are detailed in the figure caption. The excess resistance is not associated with crack growth and therefore will cause erroneous interpretation. This paper investigates the cause of the excess resistance with the aim of furthering our understanding of the underlying phenomenon and so that recommendations may be made to limit its influence.

Potential drop measurements are broadly categorised based on the use of alternating or direct current (ACPD and DCPD respectively). ACPD measurements are subject to the skin effect which electromagnetically concentrates current flux to the surface of the component.

The electromagnetic skin depth, $\delta$, is defined as the depth at which the current density is $1/e$ ($\sim 37\%$) of the surface density; in a homogeneous isotropic medium of resistivity, $\rho$, magnetic permeability, $\mu$, and AC frequency, $f_{\text{AC}}$, the skin depth may be calculated as:

$$\delta = \frac{\sqrt{\rho}}{\pi f_{\text{AC}} \mu}$$

Equation (1) indicates that ACPD measurements are sensitive to magnetic permeability through the skin effect. In ferromagnetic materials elastic strain induces magnetization through the reversible magnetoelastic effect (also known as inverse magnetostriction effect or Villary effect), which in turn alters the magnetic permeability; the stress induced permeability changes result in an undesired influence on the PD readings, causing erroneous measurements. The influence of static stresses on ACPD measurements has been examined previously [2-4], but the influence of dynamic cyclic stresses found in fatigue tests needs to be investigated.

The observed behaviour is unexpected as the impedance is measured using phase sensitive detection (a SR830 lock-in amplifier [5]) with an extremely narrow effective bandwidth (in this case < 0.1 Hz) and the PD inspection frequency, $f_{\text{AC}}$, and cyclic loading frequency, $f_{\text{L}}$, are well separated. The explanation of how the cyclic loading influences the measurement requires two steps. Firstly, in order for the cyclic stresses to influence the PD measurement the cyclic stresses must cause...
a quasi-static change in permeability, which we show may arise through non-linear parametric modulation. Secondly, it will be shown that in order for the quasi-static change in permeability to be dependent on the loading frequency, the loading must act through altering the spatial distribution of the permeability. It will be demonstrated that loading induces magnetization of the component, cycling the magnetization at increasing frequency concentrates the magnetic flux to the surface, and consequently modifies the magnetic permeability (a phenomenon recently mentioned in literature [6–11]). This study will investigate this ‘dynamic magnetoelastic effect’ and in particular show that the interaction of the effect with the interrogation current provides a compelling explanation of the observed empirical behaviour.

This paper will be structured as follows. Background to non-linear parametric modulation and the dynamic magnetoelastic skin effect will be given to provide context for the following sections. A multiphysics finite element model of an ACPD measurement on a cyclically loaded cylindrical rod will then be presented. The model is based on simple ad hoc approximations of the underlying physics; the intention of this approach is to capture the key characteristics of the proposed dynamic magnetoelastic effect and subsequently how the effect influences ACPD measurements. Following from this a cyclic loading experiment on a cylindrical rod is presented; the experiment is similar to that already presented in Fig. 1, but the simple geometry allows straightforward comparison to the finite element results. Comparison between the finite element model and the experiment is given, followed by discussion on the implications of the effect and how it may be mitigated.

2. Magnetoelastic modulation

The behaviour observed in Fig. 1 is initially puzzling as the impedance is measured using a lock-in amplifier with an extremely narrow effective bandwidth at the potential drop inspection frequency. Fig. 2 illustrates the possible ways in which the loading frequency may influence the measurement signal, and consequently the measured resistance. The remainder of this section will discuss each effect in turn. It will be assumed throughout this study the effect of piezoresistivity is small and the magnetoelastic effect is dominant [2,4].

2.1. Linear superposition

Linear superposition is not modulation, but deserves mention as an obvious influence that should be immediately dismissed in the case under consideration. The influence of linearly superposing a signal is illustrated by Effect A in Fig. 2. Linear superposition may occur through interference from spurious signals; it is likely that strong linear superposition at the loading frequency will occur. Yet, linear superposition will only influence the measurement if it (or a higher harmonic) coincides with the inspection frequency; provided the loading and inspection frequencies are separated beyond the narrow bandwidth of the inspection frequency then linear superposition will not influence the measurement. In a simple check that linear superposition did not interfere with the measurement, the PD injection current was turned off and no remnant PD reading was present.

2.2. Linear magnetic modulation

As previously noted, elastic strain may alter the magnetic permeability through the magnetoelastic effect, which in turn will alter the skin-depth, and therefore modulate the potential drop signal. The influence of linear modulation is illustrated by Effect B in Fig. 2. For the purpose of illustration only, if we initially assume the simplest case that the magnetic permeability is modulated harmonically and linearly at the loading frequency, \( \omega L \), then,

\[
\Delta \mu(t) = \gamma_1 \cos(\omega_L t + \varphi_1)
\]

where \( \Delta \mu \) is the permeability change induced by the applied stress with respect to the permeability \( \mu_L \) in the initial state of the material and \( \gamma_1 \) and \( \varphi_1 \) are amplitude and phase coefficients, respectively. It should be
emphasised that the measured AC potential drop is a nonlinear function of the magnetic permeability. For example, at high frequencies this nonlinear function asymptotically approaches $V \propto \mu^{1/2}$, where $\mu = \mu_i + \Delta \mu(t)$. However, for weak modulation we can exploit the binomial approximation to obtain $V \propto \mu^{1/2} + \Delta \mu(t)/2\mu^{1/2}$ and conclude that a weak linear modulation of the magnetic permeability according to Eq. (2) indeed results in a weak linear modulation of the measured potential drop. The magnetic permeability modulation will only introduce frequency components of the measured signal at $f_{AC} \pm f_{o}$, as illustrated in Fig. 2. Since the measured resistance is proportional to the component of the voltage difference that is strictly inphase with the injected current, all harmonics of the permeability perturbation except any at $f_{AC}$ are suppressed and the change in the measured resistance is simply dependent on $\Delta \mu$, where the overbar indicates temporal average. The temporal average of magnetic permeability from Equation (2) is $\Delta \mu = 0$ and therefore linear parametric modulation has no effect at the inspection frequency, $f_{AC}$.

### 2.3. Non-linear magnetic modulation

The influence of non-linear modulation is illustrated by Effect C in Fig. 2. Now let us assume that the magnetic permeability of the material exhibits spatially uniform non-linear modulation at the loading frequency $f_{o}$ so that

$$\Delta \mu(t) \approx \sum_{m=0}^{\infty} \gamma_m \cos(m\omega_0 t + \varphi_m)$$

(3)

where $\gamma_m$ and $\varphi_m$ are amplitude and phase coefficients, respectively, in the Fourier series of $\Delta \mu(t)$. This is of course much more likely than the linear case due to any form of non-linearity between applied stress and magnetic permeability. The modulated signal will have possible frequency components at $f_{AC} \pm m f_{o}$, as illustrated in Fig. 2. A quasi-static term is introduced due to the non-linearity at $m = 0$. Assuming that $\varphi_0 = 0$, the temporal average of the permeability change is

$$\overline{\Delta \mu} = \gamma_0$$

(4)

Let us assume that the applied stress is the sum of a static component $\sigma_0$ and a dynamic component $\sigma(t)$, i.e., $\sigma(t) = \sigma_0 + \sigma(t)$. Fig. 3 illustrates schematically how harmonically applied stress may result in a quasi-static net change in permeability, $\overline{\Delta \mu}$.

The non-linear modulation function, previously expressed as a Fourier series in Equation (3), is now written as a Taylor expansion as follows,

$$\mu(t) = \sum_{n=0}^{\infty} \frac{\mu^{(n)}(\sigma_0)}{n!} \sigma^n = \sum_{n=0}^{\infty} A_n \sigma^n \cos^n(\omega_0 t)$$

(5)

where $\mu^{(n)} = \delta^n \mu/\delta \sigma^n$ denotes the nth derivative of $\mu(\sigma)$, $A_n = \mu^{(n)}(\sigma_0)/n!$, and the dynamic part of the stress is denoted by $\sigma(t) = \sigma_0 \cos(\omega_0 t)$. The zero order term is only dependent on the mean stress and is therefore independent of the loading frequency. The higher order terms will provide frequency components of the modulated signal at $f_{AC} \pm m f_{o}$ ($m = 1, 2, \ldots$). Critically, the odd terms do not affect the resistance measured at $f_{AC}$ since their time average is zero, while even terms introduce frequency content at $m = 0$ and thereby a quasi-static change in permeability

$$R = A_0 + A_2 \sigma_0^2 + \frac{3A_4}{8} \sigma_0^4 + \ldots$$

(6)

where $A_0 = \mu(\sigma_0)$. The above equation informs us that in order for the stress modulation to influence the potential drop measurement the stress modulation must be non-linear. It also indicates that only the even terms in the stress modulation need to be considered. It is therefore possible that non-linear modulation may influence the measurement, yet this term is independent of the loading frequency unless this quasi-static net change in magnetic permeability, $\gamma_0$, somehow becomes a function of $f_0$.

### 2.4. Spatially inhomogeneous non-linear magnetic modulation

It is suggested that the loading frequency dependence originates from the spatial distribution of the magnetic permeability that modulates the potential drop signal. A uniform elastic strain will result in effectively uniform magnetization. Under quasi-static loading conditions a uniform magnetic flux density will be induced, but this uniformity cannot be maintained as the loading frequency increases; eddy currents form in response to the alternating magnetic flux, due to Lenz’s law the magnetic field from the eddy current will oppose the magnetoising flux, this effect is strongest in the center of the conductor forcing the magnetic flux to the surface. The loading frequency dependent spatial distribution of the magnetic flux density will influence the spatial distribution of magnetic permeability, which modulates the PD signal.

This idea has precedent in previous literature relating to energy harvesting and shape memory alloys [6–11]. The most directly relevant analysis of a dynamic magnetoelastic ‘skin effect’ is provided by Scheidler & Dapino [8]. Fig. 4 is adapted from Ref. [8] and shows how a stress induced axial magnetic flux density distribution changes with the normalized radius at different frequencies. In this illustration a cylindrical rod of $R = 12.7$ mm radius made of a ferromagnetic material of $\mu_r = 200$ relative magnetic permeability and $\rho = 2 \times 10^{-7}$ $\Omega$m electric resistivity was modelled. The magnetic flux density was normalized to the uniform remanence level under static loading.

Non-linear parametric modulation may therefore provide a quasi-static change in magnetic permeability, this change may become loading frequency dependent by altering the spatial distribution of permeability.

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**Fig. 3.** Illustration of a non-linear relationship between imposed stress and permeability. In order for the cyclic stresses to exert an influence at the inspection frequency, it will need to cause a change in the quasi-static time-averaged permeability. This schematic chart illustrates how the permeability normal to the stress typically increases with stress (while the permeability parallel to the stress decreases rather than increases with increasing stress).

**Fig. 4.** Normalized magnetic flux density distribution in the axial, $z$, direction at $z = 0$ calculated from the analytical solution of Scheidler & Dapino [8].
It is hypothesised that the combination of non-linear parametric modulation and the cyclic loading frequency dependent spatial distribution of magnetic permeability may explain the observed empirical behaviour. A multi-physics finite element approach will be used to investigate how these phenomena influence potential drop measurements, validating the hypothesis.

3. A multi-physics finite element model of the dynamic magnetoelastic effect

In order to investigate the influence of the magnetoelastic skin effect on ACPD measurements, ad hoc approximations of the underlying physical phenomena will be incorporated into finite element simulations. The intention is to provide a model that captures the key characteristics and provides a compelling explanation for the cause of the empirical observations.

The approach requires two successive stages: 1) Modelling the dynamic magnetoelastic effect and 2) Modelling the influence of the dynamic magnetoelastic effect on ACPD measurements. For simplicity, a basic geometry of a cylindrical bar, as illustrated in Fig. 5, will be used both in the modelling and subsequent experimental section.

3.1. Modelling the dynamic magnetoelastic effect

In a ferromagnetic material the magnetic flux density \( \mathbf{B} \) is proportional to the sum of the magnetic field \( \mathbf{H} \) and the magnetization \( \mathbf{M} \) of the material, i.e., \( \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \), where \( \mu_0 \) denotes the magnetic permeability of free space. The basis of the parametric modulation discussed in the previous section is the magnetoelastic effect by which elastic strain causes alignment of the otherwise randomly oriented magnetic domains that leads to macroscopic magnetization \( \mathbf{M} \). Dynamic stress \( \sigma \) leads to dynamic magnetization \( \mathbf{M} \) of similar spatial distribution independent of the loading frequency \( f_r \). However, according to Faradays Law the resulting dynamic magnetic flux density \( \mu_0 \mathbf{M} \) produces an induced electric field \( \mathbf{E} \) that, in a conducting material, leads to an eddy current distribution \( \mathbf{J} = \mathbf{E}/\rho \). Because the dynamic magnetic field \( \mathbf{H} \) of the eddy current opposes the effect of stress-induced magnetization, the spatial distribution of the dynamic magnetic flux density \( \mathbf{B}(\mathbf{r}) \) is very different from that of the stress-induced magnetization \( \mathbf{M}(\mathbf{r}) \). Effectively, the magnetic flux density is squeezed out from the interior of the specimen towards its surface in a way that is inherently dependent on the loading frequency \( f_r \).

As we are concerned only with quasi-static changes induced by dynamic stresses it is adequate to only model the magnetoelastic effect as an even function, as discussed in Section 2.4. The magnetoelastic effect is known to be complex, involving sensitivity to many different parameters. Due to the complexity, for practical purposes it is appropriate to adopt a simplified model depending on the objectives at hand. For the present situation, where we are concerned with only even terms of parametric modulation, it is sufficient to assume the most generic even function, namely a piecewise linear absolute value function without the higher order quadratic term proposed by Brown [12],

\[
\beta| = \lambda| \tag{7}
\]

where \( \lambda \) is an arbitrary coefficient. This represents the simplest possible model of the magnetoelastic effect, yet we will find that it is sufficient in order to capture the main characteristics of the empirically observed behaviour, without making the modelling too onerous. For more in depth studies into magnetoelastic behaviour, the reader is referred to the experimental work of Craik & Wood [13] or the analytical model of Jiles [14].

In order to model the influence of stress induced magnetization on magnetic permeability, \( \mu \), we start by considering the relative permeability in the axial z direction, with the arbitrary expression,

\[
\mu_{zz}(B_z) = \mu_{\infty}(1 + \frac{\partial \mu_{\infty}}{\partial B_z} \Delta B_z) \tag{8}
\]

where the subscript \( r \) denotes relative permeability, \( z \) denotes the axial direction, and \( 0 \) denotes an initial unstressed condition. At a given mean \( B_z \) and for small \( \Delta B_z \), we may assume that \( \partial \mu_{\infty}/\partial B_z \) is a constant. We can then combine terms in Equation (8) to give,

\[
\mu_{zz}(B_z) = \mu_{\infty}(1 - 2\alpha \Delta B_z) \tag{9}
\]

The choice of the arbitrary coefficient as \( -2\alpha \) (with physical units m²/Vs) is chosen for convenience, as will become evident later. Given that from Equation (7) we expect \( \Delta B_z \) to always be positive then it is useful to consider the sign of the lumped coefficient. The incremental relative permeability is defined as \( \mu_r = \mu^{-1} \partial B_z/\partial H \). Except for initial magnetization from a demagnetized state, as the magnetic flux density increases and starts to saturate the incremental permeability typically decreases, consequently \( \partial \mu_{\infty}/\partial B_z \) is negative. As \( \mu_{\infty} \) must be positive then we expect the coefficient term to be negative; \( \alpha \) is assumed therefore to be positive.

If the potential drop measurement is taken ‘parallel’ to the loading direction (the axial z direction, as in Fig. 6) then the current flux is predominantly axial and the magnetic field resulting from the current flux will be azimuthal; consequently, the azimuthal permeability is of the greatest interest. Magnetization, and therefore the change in magnetic permeability, in the axial direction must come at the expense of the orthogonal directions, a concept previously utilised for stress measurement [3,4,15]. Since macroscopic magnetization implies the reorientation of already spontaneously magnetized microscopic domains, we assume that the decrease in permeability is approximately balanced by an equal increase in the orthogonal directions, though the exact factor will become inconsequential,

\[
\mu = \mu_{\infty} \begin{bmatrix} 1 + 2\alpha \Delta B_z & 0 & 0 \\ 0 & 1 + 2\alpha \Delta B_z & 0 \\ 0 & 0 & 1 - 2\alpha \Delta B_z \end{bmatrix} \tag{10}
\]

Following from Equation (7) we only concern ourselves with the even behaviour of \( \Delta B_z \). Combining Equations (7) and (10) gives,

\[
\mu = \mu_{\infty} \begin{bmatrix} 1 + \alpha \Delta B_z & 0 & 0 \\ 0 & 1 + \alpha \Delta B_z & 0 \\ 0 & 0 & 1 - 2\alpha \Delta B_z \end{bmatrix} \tag{11}
\]

The dynamic magnetoelastic effect can then be modelled using a multi-physics finite element package, in this case Comsol [16]. A cylindrical equivalent to that shown in Fig. 5 was modelled with properties \( \mu_{\infty} = 200, \rho = 2 \times 10^{-7} \Omega \text{m} \) \( \alpha = 5 \text{ m}^2/\text{Vs} \). For the limited purposes of this simplistic demonstration, the result of the magnetoelastic effect was modelled directly without including strain in the model. It was...
arbitrarily assumed that a cyclic uniaxial stress acting in the axial direction of the cylinder produces a uniform magnetization oscillation with \( M \approx 796 \) A/m amplitude that is equivalent to \( B_r = 1 \) mT remanence amplitude. Only the azimuthal component of Equation (11) is of consequence and therefore only that term is incorporated into the model. Incorporating Equation (11) leads to the non-uniform magnetic flux density and permeability.

The results of the simulation are illustrated in Fig. 6. The magnetic permeability has a significantly frequency dependent spatial distribution; at higher loading frequency the magnetic permeability increase is concentrated to the surface.

Fig. 7 shows the axial magnetic flux density along the radius at the center of the rod \((z = 0)\) for a range of loading frequencies. Despite the simplicity of the models used, the results of Fig. 7 closely reflect the analytical solution of Scheidler & Dapino, shown in Fig. 4.

The resulting radial distribution of the azimuthal relative permeability is shown in Fig. 8. At quasi-static loading frequencies there is a uniform increase of \( \Delta \mu_r \approx 8 \) in relative magnetic permeability from \( \mu_{r0} = 200 \); this is the effect that is investigated in Ref. [2]. At higher frequency the permeability at the surface increases at the expense of the permeability at the center.

3.2. Modelling the influence of the dynamic magnetoelastic effect on ACPD measurements

The magnetic permeability from the previous stage of the study can be used as the basis for the subsequent investigation of the influence of dynamic loading on the potential drop measurement. The potential drop measurement configuration illustrated in Fig. 5 was added to the
finite element model. A range of AC frequencies were modelled with a range of loading frequencies.

Before moving onto the present issue of dynamic loading, the relatively well understood case of quasi-static loading where the permeability is assumed to be uniform at $\mu_r = 200$ is considered for background. Fig. 9 shows the current distribution in a 25 mm diameter rod of $\rho = 2 \times 10^{-7}$ Ωm electric resistivity along the radius at the center of the rod ($z = 0$) for a range of PD inspection frequencies. At the higher frequencies the skin depth is smaller than the rod radius and the current is concentrated to the surface according to Equation (1). At low frequencies the skin depth is larger than the rod radius so the current is essentially uniformly distributed.

The resulting impedance can be divided into quasi-DC and high-frequency regimes, separated by a transition zone, as illustrated in Fig. 10. As expected, there is essentially perfect agreement between the results of the FE simulation and the exact analytical solution of Giacoletto [17], which is included for comparison. The experimental data, which will be discussed in the following section, exhibits the same general behaviour, but the transition zone happens to be shifted towards slightly higher frequencies most likely because the uncertain magnetic permeability of the S275 steel specimen is lower than the $\mu_r = 200$ assumed in the analytical and numerical calculations.

We now move on to the case of cyclic stress induced non-uniform magnetic permeability. The potential drop measurements will be dependent on the interaction between the non-uniform current density and non-uniform magnetic permeability; this interaction will depend on both the frequency of loading and that of the potential drop inspection current.

Fig. 11 shows how the real (resistive) part of the impedance varies with AC and loading frequencies; parts a) and b) show the same data but plotted with different independent variable. Both parts of Fig. 11 show non-monotonic changes in the frequency dependences of resistance. An interesting characteristic of Fig. 11 a) is that the peak resistance changes occur when the loading frequency is around three-six times higher than the PD inspection frequency; as the loading frequency increases, the permeability change becomes increasingly concentrated to a thin surface layer representing a smaller fraction of the interrogated material, and consequently the excess resistance reduces.

4. Experimental study of ACPD measurements on a cyclically loaded rod

An experiment was carried out using an S275 steel (with nominal yield stress of at least 275 MPa) cylindrical bar of diameter 25 mm and length 200 mm, a simple geometry similar to the finite element simulations. The rod was sinusoidally loaded with a constant range of $P_{max} = 10kN$ (20 MPa) and $P_{min} = 1 kN$ (2 MPa) at a range of frequencies between 0.4 and 20 Hz using a tensile testing machine. During each loading frequency stage ACPD measurements were also taken at a range of inspection frequencies between 0.6 and 100 Hz.

Like the computational model, current was injected near the top and bottom of the cylindrical bar, while the potential difference was measured at the centre of the bar, as illustrated in Fig. 5. Loading and ACPD frequencies were selected such that the fundamental frequency or harmonics do not coincide and cause spurious interference; as with the original experiment the measurement settings were such that the equivalent noise bandwidth was < 0.1 Hz [5]. The loading frequency range is restricted in comparison to the finite element simulation due to the limitation of the tensile testing machine. Additionally, there is a practical low frequency limit for both loading and AC due to the long time periods involved.

Fig. 12 shows the PD frequency response under static mean load of...
A range of inspection frequencies are shown in the legend (in Hz). All values are normalized to the static mean load, 0.6 Hz inspection frequency resistance.

5.5 kN. It shows the same behaviour as anticipated from the quasi-static FE and analytical results. A slight shift in the frequency response arises from the estimated material properties used in the FE and analytical models, yet the quasi-DC – AC transition frequency is sufficiently similar for the present purposes.

Fig. 12 shows how the experimentally obtained resistance varies with loading and inspection frequencies; as with Fig. 11, parts a) and b) show the same data but plotted with different independent variables. Fig. 12 a) shows the dependence of the normalized resistance on the loading frequency at a number of inspection frequencies; the experimental results capture the same features as the finite element simulation; the normalized resistance increases until the loading frequency approximately matches the AC frequency, beyond which the resistance gradually begins to decline.

Fig. 12 b) shows the dependence of the normalized resistance on the inspection frequency at a number of loading frequencies. These results are broadly consistent with the finite element simulation up to approximately 10 Hz, but also show an additional decrease in resistance at higher inspection frequencies above approximately $f_{AC} = 20$ Hz. This non-monotonic behaviour is believed to be a result of magnetic saturation that occurs in an increasingly shallow surface layer. This behaviour is not predicted by the simplistic magnetization relationship of Equation (7) introduced for our demonstration purposes; it may be observed in Fig. 8 that the increase in permeability is unbound.

An interesting feature of the dynamic magnetoelastic skin effect is the level of linearity and repeatability. Fig. 13 shows the influence of cyclic loading amplitude on resistance (for a given combination of loading and AC frequency and constant mean stress). Measurements are taken as the loading amplitude is increased and decreased; the influence is reasonably linear and very repeatable with negligible hysteresis. The linearity over this range of stresses indicates that, despite the simplicity, Equation (7) is a reasonable approximation.

5. Discussion

The aim of this study was to investigate and provide an explanation for the empirically observed sensitivity of ACPD measurements to dynamic cyclic stresses. The dynamic magnetoelastic skin effect has been modelled and results capture the main features of the observed behaviour. The models developed in this study are based on ad hoc approximations and are not intended to be a comprehensive description of the full complexity of the underlying physics, though the general approach could be further developed if more accurate predictions are required.

There are of course plausible alternative explanations, for example the temperature may be modulated by the so-called thermoelastic and viscous dissipation effects [18]. Undoubtedly, the heat generated by viscous dissipation could also cause a quasi-static change in both permeability and even conductivity. However, the effect would be cumulative with the number of cycles until thermal equilibrium is reached with the surroundings and the temperature change would be stronger in the interior than close to the surface. This is not consistent with the observed empirical response, where resistance changes are near-instantaneous.

The sensitivity of potential drop measurements to dynamic cyclic loading is undesired, as changes attributed to the effect will compete with the desired sensitivity to geometry (usually crack length) undermining resistance-geometry inversions. The improved insight into the influence of cyclic loading on ACPD measurements informs strategies on how to mitigate the effect. While Fig. 13 indicates the effect is stable at a given set of conditions, throughout a destructive material test the effective stress may increase and therefore should not simply be assumed to be constant. With reference to Fig. 10 the most reliable strategy is to reduce the AC frequency such that the electromagnetic skin depth is much larger than the component thickness and so sensitivity to magnetic permeability is suppressed. Fig. 14 re-plots the 1 Hz AC inspection frequency data from Fig. 12, illustrating that for this example the

![Fig. 12. Experimental results showing resistive (real) part of impedance as a function of a) loading frequency $f_l$ (a range of inspection frequencies are shown in the legend) and b) PD inspection frequency $f_{AC}$ (a range of loading frequencies are shown in the legend in Hz). All values are normalized to the static mean load, 0.6 Hz inspection frequency resistance.](image1)

![Fig. 13. Experimental results showing the influence of dynamic stress amplitude on the normalized potential drop resistance for a range of inspection frequencies.](image2)

![Fig. 14. Experimental results showing the influence of the loading frequency on the excess potential drop resistance for the 1 Hz AC inspection frequency.](image3)
influence of dynamic loading can be reduced to $< 1\%$. It is also worth recalling that the influence of the effect appears to peak when the loading frequency approximately matches the AC frequency; the loading frequency should therefore be increased so that it is higher than the AC frequency. Unfortunately, utilising lower AC frequencies presents a compromise; as explored in Refs. [19,20] reducing the measurement frequency increases the flicker noise considerably. It may therefore be advisable to conduct a preliminary investigation prior to a destructive test, quantifying the influence of cyclic loading at a range of PD inspection frequencies and then selecting a PD inspection frequency that is as high as possible (and therefore results in lower noise) whilst achieving an adequate suppression of the loading effects.

A pragmatic alternative approach is to build in pauses in loading that allow measurements under static conditions, thereby eliminating dynamic effects. Due to the quasi-static nature of the cyclically induced permeability change, simple measurement synchronisation with the loading will not be effective. It should be emphasised that the dynamic magnetoelastic effect will only be prominent in ferromagnetic materials and the authors have not observed a comparable effect in non-ferromagnetic steels. Finally, it is expected that the effect will be strongly materials dependent.

6. Conclusions

Cyclic loading influences ACPD measurements conducted on ferromagnetic materials, clearly this is particularly problematic for fatigue tests. The current distribution of ACPD measurements is sensitive to the magnetic permeability through the electromagnetic skin effect. A cyclic magnetoelastic effect results in a non-uniform magnetic flux density concentrated to the surface of the component, which in turn leads to a non-uniform magnetic permeability. The ACPD inspection frequency dependent electromagnetic skin effect and loading frequency dependent dynamic magnetoelastic effect interact through the spatial distribution of the magnetic permeability and electrical current, resulting in a spurious excess resistance.

To mitigate the influence of dynamic loading on ACPD measurements it is advised that sufficiently low AC inspection frequency is utilised with a high loading frequency to mitigate spurious sensitivity to the loading level and loading frequency.

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Appendix A. Supplementary data

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References