Guidelines for selecting the dimensions of adhesively bonded end-loaded split joints: An approach based on numerical cohesive zone length

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Abstract

Recent attempts to measure the mode II fracture resistance of structural adhesive joints employing highly toughened adhesives have been hindered by the formation of extensive damage zones ahead of the crack tip. This can lead to strongly rising resistance curves being generated and to the steady-state not being attained. In this paper, a cohesive zone element formulation has been implemented in Abaqus v6.14 via a user element subroutine to determine the length of fracture process zone. It has been demonstrated that the existing analytical expressions cannot predict the length of fracture process zone accurately; hence a formulation for the length of the fracture process zone in an end-loaded split (ELS) joint specimen has been derived using a numerical parametric method. The results obtained from the proposed formulation correlate very well with those from numerical results. The results have allowed guidelines to be developed for selecting the dimensions of an ELS test specimen, comprising fibre-composite substrates bonded with a toughened adhesive, which ensure the attainment of the steady-state and hence a plateau in crack growth resistance curve is ensured. This removes a significant obstacle in the definition of a successful standard method test.

Keywords: Adhesively bonded joints; End-loaded split test specimen; Finite element analysis; Mode II fracture; Numerical cohesive zone length;

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## Nomenclature

**English Alphabet**

- \(a_p\)  \(\) Initial crack length after pre-cracking (\(m\))
- \(B\)  \(\) Width of the specimen (\(m\))
- CFRP  \(\) Carbon fibre reinforced polymer
- CZM  \(\) Cohesive zone modelling
- DCB  \(\) Double cantilever beam
- \(d\)  \(\) Damage variable
- ELS  \(\) End-loaded split
- \(E\)  \(\) Young’s modulus of adhesive (\(Pa\))
- \(E_i\)  \(\) Young’s modulus of substrate in the \(i\) direction (\(Pa\))
- \(E'_{ii}\)  \(\) Constant related to elastic properties of slender beams
- FPZ  \(\) Fracture process zone
- \(G\)  \(\) Shear modulus of adhesive (\(Pa\))
- \(G_{ij}\)  \(\) Shear modulus of substrate (\(Pa\))
- \(G_{ic}\)  \(\) Mode I critical energy release rate (\(J/m^2\))
- \(G_{Iic}\)  \(\) Mode II critical energy release rate (\(J/m^2\))
- GP  \(\) Gauss point
- \(h\)  \(\) Substrate thickness (\(m\))
- LEFM  \(\) Linear elastic fracture mechanics
- \(L_{cz}\)  \(\) Cohesive zone length (\(m\))
- \(L_{FPZ_{max}}\)  \(\) Fully developed fracture process zone (\(m\))
- \(L_{free}\)  \(\) Distance between the load point and clamp point (\(m\))
- \(L_{uncracked}\)  \(\) Distance between the crack tip and the clamp point (\(m\))
- \(l'_e\)  \(\) Total element length (\(m\))
- \(l_1\)  \(\) Height of the end-block (\(m\))
- \(l_2\)  \(\) Half-length of the end-block (\(m\))
- \(M\)  \(\) Cohesive zone model parameter
- \(P\)  \(\) Load at the centre of end block (\(N\))
- R-curve  \(\) Crack growth resistance curve
- TENF  \(\) Tapered end-notched flexure
- \(t_a\)  \(\) Thickness of the adhesive layer (\(m\))
- UEL  \(\) User element
- \(u_x\)  \(\) Displacement in x direction (\(m\))
- \(u_y\)  \(\) Displacement in y direction (\(m\))
- \(w_{ij}\)  \(\) Gauss point’s weighting factor

**Greek Alphabet**

- \(\alpha_k\)  \(\) Coefficients used in the parametric studies
1. Introduction

The use of adhesives to join component parts has become very popular over a wide range of industries including aerospace, automotive, electronics and construction. When compared to more traditional methods of joining, adhesive bonding can offer the advantages of reduced weight, a more even stress distribution, and consequently improved damage tolerance and fatigue resistance. As their use has increased, the need to characterise the fracture resistance of adhesively bonded joints has become more urgent. Under mode I, the tensile opening mode, an ASTM standard has existed since the early 1970s [1] and more recently an ISO standard was published [2], extending the application of corrected beam theory to the two popular mode I test specimens, the double cantilever beam and tapered double cantilever beam. The mode II, in-plane shear mode, is also an important mode of failure for adhesive joints because the direction of cracks in a bond-line of a joint will often be constrained by the nearby substrates to grow parallel to this constraint. Additionally, adhesive joints are usually designed in such a way that promotes mode II loading over mode I, to optimise the joint performance [3].

Several different mode II fracture test specimens have been proposed for composites laminates and adhesively bonded joints. These include the three-point loaded, end-notched flexure (3-ENF) test specimen, the four-point loaded, end-notched flexure (4-ENF) test specimen, the tapered end-notched flexure (T-ENF) and the end-loaded split (ELS) test specimen. However, whilst ISO and ASTM standards now exist for delamination in fibre-composites using the ELS and 3-ENF tests respectively [4, 5] to date, no international standard has been published for
adhesive joints. An important requirement for this type of test is that the deformation of the substrate beams remain elastic during the test. In the present work, the ELS test has been analysed following on from the successful ISO standard for laminates. The ELS test has a number of advantages including a more favourable stability condition, allowing stable crack propagation [6, 7, 8] when compared to the 3-ENF test.

One important phenomenon with mode II fracture tests is the presence of a large fracture process zone (FPZ) ahead of the crack tip compared to mode I or mixed mode fracture tests. The length of the FPZ is even larger when dealing with highly toughened adhesives i.e. those with high mode II critical energy release rate ($G_{IIc}$). Due to this, and the presence of microcracks ahead of the main crack, it is difficult to define the true crack tip and reach a plateau in the crack growth resistance curve (R-curve). The former, defining the true crack tip, has been addressed by Blackman et al. [9] via an effective crack length approach, where the method provides an insight into the likely errors encountered when attempting to measure mode II crack growth experimentally. However, with highly toughened adhesives, difficulties remain in reaching a steady-state and a plateau in the R-curve.

Alvarez [10] showed that using a relatively short ELS test specimen (of length about 250 mm) to test a joint bonded with a high toughness epoxy-film adhesive, stable crack growth and a plateau in the R-curve were not achieved. On the other hand, Sorensen et al. [11], manufactured much longer specimens, approximately 2 m in length, and found that using such long specimens, a steady-state together with a plateau in the R-curve could be achieved. The disadvantage of such a large specimen however, is the cost of making them and the availability of equipment to test them. What is required is a procedure to determine the minimum joint dimensions needed to attain a steady-state and provide a plateau in the R-curve of sufficient length for the confidence in the result to be high.

This has been investigated here and it has been demonstrated that, the length of the FPZ, and as a result the free length (the distance between the point load and the starting position of the clamp) of ELS test specimen, play an important role on attaining stable crack propagation and consequently reaching a plateau in R-curve. One way of calculating the length of the FPZ numerically is through cohesive zone modelling (CZM). Cohesive elements are specialised finite elements used to simulate crack initiation and propagation in numerical models. They have become increasingly used for modelling composites, particularly in relation to delamination [12, 13, 14, 15] and adhesive bond-line failure [16, 17, 18]. In this paper a
cohesive zone element formulation proposed by Alvarez et al. [19] has been adopted to capture the length of the FPZ. As a result, a formulation for the length of the FPZ has been derived through a detailed numerical parametric study. The formulation has been tested and the results are compared with analytical models [15, 20]. The expression has subsequently been used to develop guidelines for selecting the dimensions of an ELS test specimen made of composite substrates, which allow stable crack growth and attainment of a steady-state in R-curve.

2. End-loaded split test specimen

A schematic representation of the ELS geometry is shown in Figure 1.

![Figure 1](image)

Figure 1. Schematic representation of the CFRP-ELS test specimen (with constant width, B). A Polytetrafluoroethylene (PTFE) film is used to generate an initial defect.

While on one side of the specimen, load is applied vertically (upwards) through an end-block, the other side is constrained vertically but free to move horizontally. As the loading point, which is at the centre of the end-block, is free to rotate, with both arms sharing the same curvature, pure shear conditions are attained at the crack tip.

In Figure 1, \( h \) is the substrate thickness, \( L_{\text{free}} \) is the distance between the load point and clamp point, \( L_{\text{uncracked}} \) is the distance between the crack tip and the clamp point, \( B \) is the width of the specimen (which has been kept constant in this study), \( a_p \) is initial crack length after pre-cracking, \( t_a \) is the thickness of the adhesive layer, \( l_1 \) is the height of the end-block, \( l_2 \) the half-length of the end-block, \( P \) and \( \delta \) are the load and displacement respectively at the centre of end-block.

In all experimental results presented in the following sections, the ISO standard for determination of the mode II fracture resistance for composite delamination [4] has been taken as reference for the analysis. William [21, 22] proposed a stability criterion for the case where the resistance curve is constant and hence the steady-state plateau was reached. This criterion (under fixed displacement) when applied to ELS geometry leads to:
\[
\frac{a_p}{L_{\text{free}}} \geq 0.55 + 0.1\left(\chi h/L_{\text{free}}\right)
\]

where the correction parameter, \(\chi\) can be determined both experimentally and analytically, but in this study, the analytical value presented by Reeder et al. [23] is used, which can be found in [15]. Henceforth this criterion is applied to all experimental and numerical results in this paper.

3. Numerical Implementation

3.1. Cohesive zone modelling (CZM)

The cohesive zone model which was originally derived from the damage mechanics framework, relies on the premise that the damage mechanisms, within a region ahead of the crack tip that lead to fracture, may be denoted by the FPZ [24]. The material behaviour within the FPZ, when dealing with a mode II fracture, is characterized through a relationship of the form \(\sigma_{II} = \sigma_{II}(\delta_{II})\) where \(\sigma_{II}\) is mode II shear stress and \(\delta_{II}\) is mode II separation. The relationship is commonly referred to as the cohesive law or traction-separation law and can take a range of shapes.

Embedded into either finite or zero-thickness elements [25], connecting two surfaces as a contact function [26] or combined with finite volume methods [27], the CZM approach has become one of the favoured methods to simulate fracture. Specifically, cohesive or interface elements have been applied to a variety of damage initiation and/or propagation problems. Cohesive elements have also been used to investigate the response of an unidirectional carbon fibre reinforced polymer (CFRP) double cantilever beam (DCB) specimen (e.g. [15, 16]).

In addition, the CZM approach can be used when linear elastic fracture mechanics (LEFM) assumptions are violated [28]. The CZM approach is therefore suited to model problems where large scale deformation occurs within the FPZ [29], for example, those comprising of a substantial amount of plasticity.

3.2. Cohesive element formulation

In this study, the quadratic cohesive element formulation proposed by Alvarez et al. [19] has been implemented in Abaqus v6.14 via user element (UEL) subroutine. As a result, a two-dimensional revised interface element formulation, to model crack initiation and growth in adhesively bonded joints under quasi-static loading conditions, has been derived. The damage advancement and constitutive equations used by proposed quadratic cohesive elements [19] is
similar to that presented by Camanho et al. [30] and later advanced by Turón et al. [31]. However, it has been adapted for the finite thickness case to guarantee that the elements would capture the elastic behaviour, prior to damage initiation.

The material behaviour within the FPZ has been described through a bilinear traction separation law, which has been shown [10, 19] to be the best choice for the particular type of adhesives under consideration in this work. The chosen cohesive law also captures the global response of the structure together with a good prediction of the FPZ length and crack propagation in comparison to experimental test results [10].

To fully develop the traction separation law, three particular variables need to be selected. These being the initial elastic slope (prior to damage initiation), the critical energy release rate and the maximum or peak stress at the point of damage initiation. In this paper, the cohesive parameters are derived from the macroscopic mechanical properties of the adhesive material. The initial elastic stiffness is set equal to the shear modulus \( G \), corrected/divided by the thickness of the unstrained configuration, \( t_a \). It is difficult to establish an appropriate initiation criterion as the concept of damage onset could be linked to different phenomena (e.g. plastic yielding, cavitation in rubber-toughened systems, etc.) depending on the type of adhesive material. It has been demonstrated previously [19] that, by using the plastic yielding of the epoxy resin as the stress at damage initiation, a good correlation between the numerical and experimental results can be achieved under mode I loading. Therefore, in the present work the initiation of damage (for both epoxy-paste and epoxy-film adhesives) is assumed to be directly linked to plastic yielding of the epoxy resin \( \sigma_y \) and most importantly, the area under the traction separation law is set equal to the mode II critical energy release rate of the adhesive material \( G_{IIc} \).

3.3. Modelling of the ELS specimen

Using Abaqus/CAE v.6.14, a two-dimensional model was created for the ELS joint. The adhesive layer has been modelled using one layer of 0.5 \( mm \) long, finite thickness, six noded quadratic cohesive elements, with the number of integration points fixed to thirty in all cases. An in-depth investigation on the effects of the element size, order and number of integration points is presented in [10]. Also, further information on the modelling of the substrates and the end blocks, together with the kinematics and constitutive equations can be found in [10, 19].
Figure 2. Schematic representation of the boundary conditions used in modelling of the ELS test specimen where $u_x$ and $u_y$ are the horizontal and vertical displacements in xy-plane.

The ELS specimen model in Figure 2 illustrates the schematics of the loading and boundary conditions applied. To avert interpenetration of elements in the unbonded region, a frictionless, “hard” contact was specified between the bottom and upper crack faces in the ELS.

In view of large deflections being expected in ELS fracture testing, geometrical nonlinearities had to be addressed in the numerical analysis in order for errors to be minimised. For that reason, the "NLGEOM" feature was activated in Abaqus, which accounts for large deformations, in all the numerical analyses. Non-linear geometry was also taken into consideration in the cohesive element formulation through a suitable definition of the Jacobian matrix.

3.4. Cohesive zone length

Theoretical expressions, to estimate the cohesive zone length ($L_{CZ}$) in constant-width components with a crack covering the whole width, have been proposed by several authors [32, 33, 34]. Most of these expressions are derived from the analytical stress field attained for an infinite cracked body assuming linear elastic behaviour. They follow the form,

$$L_{CZ} = ME \frac{G_{IIc}}{(\tau_{II}^0)^2}$$ \hspace{1cm} (2)

where $E$ is the Young’s modulus of the material, $G_{IIc}$ is the critical strain energy release rate, $\tau_{II}^0$ is the maximum shear interfacial strength and $M$ is a parameter that depends on the cohesive zone model. The cohesive zone length is considered to be constant across the width. Similarly, suitable equations for slender bodies have been obtained using beam theory analysis [15, 35],

$$L_{CZ} = M \sqrt{\left(E_{II}^I \frac{G_{IIc}}{(\tau_{II}^0)^2}\right) h}$$ \hspace{1cm} (3)
where \( h \) is the laminate half thickness. Details on how to calculate \( E'_{II} \), which depends on the elastic properties, width and loading conditions are given in [36]. The assumptions made to obtain these expressions (equations (2) and (3)) make their suitability to adhesive joint problems arguable. Any substrate effects are ignored by the first approximation (equation (2)), whereas equation (3) completely overlooks the presence of the adhesive layer.

In addition, neither can explain the response of the constraint effects or the bondline thickness on the adhesive layer. Moreover, they both presume constant traction in the FPZ. For comparison reasons, equation (3) with \( M = 0.5 \) proposed by Harper and Hallett [15] and the expression proposed by M. Conroy et al. [20] where \( M = 0.65 \), has been used to compare the analytical solutions with the numerical cohesive length expression proposed in this paper. Finally, the results have also been compared with an empirical expression proposed by Soto et al. [37].

Harper and Hallett [15] proposed that the numerical cohesive zone length amounts to the distance ahead of the numerical crack tip, over which the interface elements lie on the softening part of the traction-separation law. The length associated with the integration points or Gauss points (GP) that experience irreversible damage (i.e. \( 0 < d < 1 \) where \( d \) is the damage variable) are thought to take part in the calculation of the numerical cohesive zone length.

Therefore, instead of considering whether an entire element is part of the cohesive zone, this method assesses the contribution of each integration point separately (refer to Figure 3 for an illustration of the numerical cohesive zone length schematics), therefore, the equivalent length associated with an integration point would be obtained from the total element length, \( l_e \) (length of the deformed mid-surface) and the corresponding Gauss weighting factor \( (w) \) used in the numerical integration,

\[
L_{cz} = \sum_{i=1}^{No. \ elements} \sum_{j=1}^{No. \ GP} \frac{W_j l_i}{2l_e} \tag{4}
\]

It is important to mention that large displacements and rotations are taken into account to calculate the element length, which is measured along the deformed mid-surface. In addition, throughout this paper the cohesive zone length \( (L_{cz}) \), also referred to as the FPZ length \( (L_{FPZ}) \), are used interchangeably.
Figure 3. Schematic representation of the numerical cohesive zone length, $L_{CZ}$ with square Gauss points experiencing irreversible damage and the circles points representing undamaged Gauss points.

4. Significance of the length of the FPZ

In this section, the significance of the length of FPZ, to reach a steady-state or plateau in R-curve, has been highlighted through the consideration of two case studies using ELS test specimens.

In the first study, Blackman et al. [9] demonstrated that using epoxy-paste adhesive with relatively low critical energy release rate ($G_{IIc} = 4280 \ J/m^2$), a steady-state and hence plateau in the R-curve was achieved (Figure 4c). Figure 4a [9] illustrates the load-displacement trace for an ELS joint bonded with an epoxy-paste adhesive. Details on the specimen dimensions, material properties and ELS testing procedures followed, can be found in [9].

In order to test the robustness of the cohesive element formulation (set out in previous sections) for a low $G_{IIc}$, modelling analysis has been undertaken using the same ELS test specimen that was used by Blackman et al. [9]. For reason of comparison, the results of which are displayed in Figure 4b, with four critical points selected. The line, (1)-(2) in Figure 4b, relates to the initial slope prior to damage initiation. Point, (2), corresponds to the point at which damage initiates and Point (3) is the maximum load point. Finally, point (4) relates to the load at which the test was terminated.

The final crack length that was achieved through CZM, was about 108 $mm$ which is in good agreement with the result obtained from the test (final crack length of 100 $mm$ shown in Figure 4a). In addition, the results at the four critical points, when compared to those on the experimental result curve, also illustrate that the numerical analysis has provided a good correlation.
Figure 4. (a) Experimental load - displacement trace for an ELS joint bonded with the epoxy-paste adhesive [9] vs. (b) results obtained from modelling using CZM elements. (c) Demonstrates the plateau in R-curves obtained from the experimental results using simple beam theory (SBT), corrected beam theory (CBT), corrected beam theory with effective crack length (CBTE) and experimental compliance method (ECM) [9]

Figure 5. (a) Load - displacement trace for an ELS joint bonded with the epoxy-film adhesive. Experimental results [10] (dashed line) vs. results obtained from modelling using CZM elements (solid line). (b) Demonstrates the rise in R-curves (i.e. not achieving steady state) obtained from the experimental results using SBT, CBT and CBTE [10]
The second case study, is that undertaken by Alvarez [10], whereby a commercial epoxy-film adhesive with a relatively high critical energy release rate ($G_{Ic} = 17800 \text{ J/m}^2$) was used, a plateau in the R-curve was not attained (Figure 5b). The experimental load-displacement data is shown in Figure 5a and once again, the results of the numerical analysis are in good correlation with that of the experimental test.

The results also demonstrate that the CZM adopted can capture the experimental Load-displacement behaviour of the joints with high precision i.e. the compliance of the joint at each applied displacement and hence the amount of energy dissipated as a result of crack growth and the damage zone created ahead of the crack. Since the nonlinearity/reduction in stiffness in the Load-displacement plot is as the result of the crack propagation together with the damage zone in the adhesive layer, i.e. the compliance $C = C(a + L_{FPZ})$, it can be concluded that the model can predict the overall value $a + L_{FPZ}$ with good precision.

Figure 6. Results obtained from numerical analysis upon further investigation of the second case study. (a) Numerical Load-displacement trace of the ELS test specimen. (b) Presentation of crack extension and the FPZ development ahead of the initial crack tip as a function of applied displacement.

Figure 7. Results obtained from further numerical analysis where $L_{free}$ is increased to 300 $mm$. (a) Numerical load-displacement trace of the ELS test specimen. (b) Presentation of crack extension and FPZ development ahead of the initial crack tip as a function of applied displacement.
Upon further investigation of this case, through the use of CZM, in an attempt to attain a plateau in the R-curve, the displacement was further increased. However, from the results, an increase in stiffness is evident (Figure 6a). Upon further study, this increase, appears to coincide with the FPZ reaching the clamp point. Figure 6b illustrates the FPZ ahead of the crack tip reaches the clamp point, just prior to crack initiation. In this case, it appears that the FPZ reaching the clamp point has prevented a steady-state from being achieved. This provided the basis for further exploration.

In an attempt to prevent the FPZ reaching the clamp point prior to crack initiation, $L_{free}$ was increased numerically (from 130 mm to 300 mm) to allow the FPZ to fully develop (point of crack propagation), and also to permit the crack to propagate, before the clamp point was reached. As illustrated in Figure 7a, a notable difference is observed between the load displacement results from the new specimen when compared to the previous result. The maximum load was dropped from 1600 (N) to 1000 (N) but the displacement (at the point of maximum load) was increased from approximately 35 mm to about 105 mm.

The analysis was stopped when the displacement reached about 125 mm (when FPZ reaches close to the clamp point, Figure 7b) because after this point the load drop trend halts and it starts to increase again. If it was allowed to increase, the stiffness of the structure would also rise and steady-state would not be achieved. This, together with other numerical analysis, suggest that the clamping point should not be reached in order for the plateau in the R-curve to be attained. Therefore, the results of this study recommend that a window of approximately 10% of the initial $L_{uncraked}$ (Figure 1), between the end of FPZ and the clamp point should be considered.

The above outcome has highlighted the significance of prior knowledge of the length of the FPZ. Establishing the length is important in order to select appropriate dimensions that first and foremost allow the FPZ to fully develop, and then for the crack to propagate sufficiently for a steady-state to be reached. Therefore, a parametric study is presented in the section that follows.

5. Parametric study

5.1. Variables under consideration

A parametric study was undertaken using the key variables that can affect the length of the fully developed FPZ. As a result, an expression of the form,
\[ L_{FPZ_{\text{max}}} = L_{FPZ_{\text{max}}} (E_i, G_{ij}, \nu_{ij}, h, L_{\text{uncracked}}, B, \sigma_y, G, G_{IIc}, t_a) \] (5)

has been considered, where, \( E_i \) is the Young’s modulus of substrate in the \( i \) direction (direction 1 or \( x \) is parallel to the fibre direction and 2 or \( y \) is perpendicular to fibres), \( \nu_{ij} \) is Poisson’s ratio of the substrate and \( G_{ij} \) is the shear modulus of the substrate and the remaining variables has been introduced previously.

As with the experimental test specimens (whereby unidirectional CFRP substrates were employed) the substrates are assumed to be transversely isotropic (\( yz \) is the plane of isotropic where the \( z \)-axis is out of plane). In addition, since two-dimensional analysis is undertaken (with constant specimen width, \( B \)) with a crack covering the whole width, the variation in the stress state across the width of the specimen is neglected. Therefore, the cohesive zone length is considered to be constant across the width.

Table 1. The range and increment for the parameters considered in this study.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range</th>
<th>Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adhesive Layer</td>
<td></td>
<td></td>
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<tr>
<td>( G ) (GPa)</td>
<td>0.4 – 4.9</td>
<td>0.5</td>
</tr>
<tr>
<td>( G_{IIc} ) (J/m²)</td>
<td>4050 – 18000</td>
<td>1550</td>
</tr>
<tr>
<td>( \sigma_y ) (MPa)</td>
<td>9.0 – 63.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Substrates</td>
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<td></td>
</tr>
<tr>
<td>( E_1 ) (GPa)</td>
<td>45 – 198</td>
<td>17</td>
</tr>
<tr>
<td>( E_2 = E_3 ) (GPa)</td>
<td>2 – 29</td>
<td>3</td>
</tr>
<tr>
<td>( G_{12} = G_{13} ) (GPa)</td>
<td>1 – 10</td>
<td>1</td>
</tr>
<tr>
<td>( \nu_{12} = \nu_{13} )</td>
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<td>0.05</td>
</tr>
<tr>
<td>( \nu_{23} )</td>
<td>0.2 – 0.5</td>
<td>0.05</td>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>( h ) (mm)</td>
<td>1.1 – 11.0</td>
<td>1.1</td>
</tr>
<tr>
<td>( L_{\text{uncracked}} ) (mm)</td>
<td>175 – 300</td>
<td>25</td>
</tr>
<tr>
<td>( t_a ) (mm)</td>
<td>0.1 – 1.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The range selected for each variable under consideration in this parametric study is presented in Table 1. These were chosen in order to keep the generality of the formulation as much as possible. These encompass, a wide range of material properties available for substrates, together with low and highly toughened adhesives. It also offers a good range for the geometric specimen dimensions. In order to execute a successful parametric study, a base model is required as a reference while the variables are changed within their allocated range (Table 1). The dimensions and material properties of the adhesive and substrates used as a reference for this, can be found in Table 2.
It is notable, from Table 2 that the shear modulus of $G = 0.414 \, (GPa)$, corresponds to the isotropic epoxy-film adhesive with Young’s modulus of $E = 1.1 \, (GPa)$ and $v = 0.34$. Based on the variables set out in this section, the section that follows presents the results of the parametric studies and the derived expression for the length of the fully developed FPZ.

Table 2. The dimensions and material properties of the base/reference model.

<table>
<thead>
<tr>
<th>$l_1$ (mm)</th>
<th>$l_2$ (mm)</th>
<th>$h$ (mm)</th>
<th>$B$ (mm)</th>
<th>$t_a$ (mm)</th>
<th>$L_{uncracked}$ (mm)</th>
<th>$\sigma_y$ (MPa)</th>
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<td>12.5</td>
<td>10.0</td>
<td>4.4</td>
<td>20</td>
<td>0.4</td>
<td>300</td>
<td>36</td>
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<table>
<thead>
<tr>
<th>$G$ (GPa)</th>
<th>$G_{11c}$ (J/m²)</th>
<th>$E_1$ (GPa)</th>
<th>$E_2 = E_3$ (GPa)</th>
<th>$G_{12} = G_{13}$ (GPa)</th>
<th>$\nu_{12} = \nu_{13}$</th>
<th>$\nu_{23}$</th>
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<td>0.414</td>
<td>18000</td>
<td>142</td>
<td>8.72</td>
<td>4.63</td>
<td>0.3</td>
<td>0.25</td>
</tr>
</tbody>
</table>

5.2. Results and derived expression for length of the FPZ

For each variable, the load, length of the FPZ and crack extension vs. applied displacement, have been analysed. From the plot of the length of the FPZ vs. applied displacement, the maximum length that the FPZ reaches is plotted against the variable under consideration.

For each parameter, approximately ten specimens have been considered and a power law trendline has been used to fit a curve through the results obtained.

As an example, Figure 8a-c demonstrates typical load, length of the FPZ and crack extension vs. applied displacement. Figure 8d then shows the maximum length of the FPZ vs. substrate thickness, $h$ and the power law trendline that is fitted to represent the results (the equation of the trendline and the correlation coefficient i.e. $R^2$ are also shown).

For reason of clarity, as Figure 8b demonstrates, the length of the FPZ ahead of the crack tip starts to contract once crack propagation initiates. This is due to the fact that in the case of the stiffer substrates the change in crack extension with respect to applied displacement is much larger than the growth of the FPZ as the displacement applied (i.e. $\frac{da}{d\delta} \gg \frac{dL_{FPZmax}}{d\delta}$ in Figure 8c). In this work, and the sections that follow, the length of the FPZ is assumed to remain constant and equal to the length of the fully developed FPZ.
Figure 8. Parametric study on the effect of substrate thickness on the joint’s (a) load-displacement plot (b) cohesive zone length vs. applied displacement plot (c) combination of the FPZ development and crack extension ahead of the initial crack tip against applied displacement plot (d) length of the fully developed FPZ as a function of substrate thickness.

Figure 9 displays the results obtained from an extensive parametric study on all variables affecting the length of the fully developed FPZ. From the results, it can be observed that parameters such as the Young’s modulus in x-direction or fibre direction \(E_1\), the mode II critical energy release rate \(G_{IIc}\), adhesive thickness \(t_a\), the yield stress of the adhesive \(\sigma_y\) and the substrate thickness \(h\) influence the length of the FPZ the most, when compared with the other parameters.

Additionally, as previously described, for each study, a trendline using a power law has been used as a curve to fit the data and the equations of which can therefore be seen on each plot. The logic for using a power law as the fitting equation is that in doing so, the equations can be more simply used to derive an expression for the maximum length of the FPZ, through multiplication of all terms. Thus, providing the expression,

\[
L_{FPZ_{max}} = L_{FPZ_{max}}(E_i, G_{ij}, \nu_{ij}, h, L_{uncracked}, B, \sigma_y, G, G_{IIc}, t_a) = \prod_{k=1}^{10} \alpha_k \tag{6}
\]

where the terms associated with coefficients, \(\alpha_k\) can be found in Table 3.
Figure 9. The results from parametric studies on the variables effecting the maximum FPZ length.
It can be observed that the coefficients, $\alpha_k$, in Table 3 are not quite the same as those shown in Figure 9. This is a deliberate step to ensure the equality of the dimensions on both sides of this equation, and therefore by performing dimensional analysis on equation (6), they have been approximated.

It is important to mention that since a power law of the form $y_i = A_i x_i^{B_i}$ (where $A_i$ and $B_i$ are constants and $x_i$ is the variable under consideration) has been used to fit through the results obtained. Therefore, using equation (6),

$$L_{FPZ_{\text{max}}} = \prod A_i x_i^{B_i} = A_1 x_1^{B_1} \cdot A_2 x_2^{B_2} \cdots \cdot A_{10} x_{10}^{B_{10}} \quad (7)$$

Performing parametric studies, requires all variables except for the one under consideration to be fixed, as a result, a base model is always required. For example, if study on $x_{10}$ is performed,

$$L_{FPZ_{\text{max}}} = A_1 x_1^{B_1} \cdot A_2 x_2^{B_2} \cdots \cdot A_{10} x_{10}^{B_{10}} = C_{10} x_{10}^{B_{10}} \quad (8)$$

Where $C_{10}$ is a constant and changes value if a different base model is used but the trend i.e. $B_{10}$ will remain the same. Hence using a different baseline model should not affect the expression for $L_{FPZ_{\text{max}}}$ (equation (6)) once all parameters are studied, one by one.

To demonstrate this, a different baseline model is used. Hence, some variables that have the greatest effect on the overall $L_{FPZ_{\text{max}}}$ have been altered, i.e. $G_{l/c} = 8000 \left( \frac{1}{m^2} \right)$, $G = 4.0 \ (GPa)$ and $E_1 = 100 \ (GPa)$ and the expression proposed in this paper (equation (6)) has been used to compare the results (for simplicity two variables that have largest effect on $L_{FPZ_{\text{max}}}$, have been used at to demonstrate this i.e. $\sigma_y$ and $E_1$).

Figure 10 demonstrates that the same relationship is obtained with the new baseline model, showing that the expression proposed in equation (6) is independent of the baseline model used.

Finally, in order to validate equation (6), which accounts for severe geometrical nonlinearity, a comparison with the empirical expression proposed by Soto et al. [37] and the analytical expression equation (3), whereby different authors suggested different values for the constant parameter $M$ depending on the problem under consideration, has been undertaken.
The analytical expression was originally derived by Yang et al. [35, 36] (with $M = 1$) where a crack in slender beams, of orthotropic material, was considered. Harper and Hallett [15] argued that due to the uncertainties surrounding the analytical equation (3) a different value for the constant parameter, $M$, should be chosen.

Figure 10. The results obtained using a different base model. It demonstrates that the expression proposed for $L_{FPZ_{max}}$ is independent from the base model used.
The original expression significantly over predicts the numerical cohesive zone length and therefore, by selecting a different value for the constant parameter, there is greater potential to identify a better fit to numerical results. It was shown that using $M = 0.5$ would provide a reasonable and generally conservative correlation with numerical results.

However, Conroy et al. [20] questioned the generality of this value, especially when dealing with asymmetric geometries. In order to identify a suitable value for $M$, they proposed that in the case of the linear softening cohesive zone, the measured cohesive zone lengths from the numerical test cases should be plotted against predicted cohesive zone lengths. Then, for each case, $M$ was varied until an optimum least squares fit (between the measured against predicted data and the theoretical linear prediction ($x = y$)) is obtained (for full details refer to [20]).

It was found that $M = 0.65$ provides an optimum prediction for the linear softening cohesive zone. For comparative purposes the analytical solutions have been included to provide a process of validation for the expression derived in this paper. However, the analytical solutions do require problem tuning to fit the prediction from the analytical equations with the numerical data, which is not required in the case of the proposed expression in this paper. Figure 11 displays the comparison between the three analytical expressions and the expression derived in this paper against the numerical results.

From the results shown in Figure 11, it can be seen that the expression derived in this paper correlates very well with the numerical results when compared with the analytical solutions. Among the expressions, that proposed by Soto et al. [37] produces the strongest result with an average percentage difference of about 11 % when compared to the numerical results. The maximum difference was about 31 % which corresponds to the analysis on the effect of the mode II critical energy release rate ($G_{IIc}$) on the length of fully developed FPZ. The expression with the highest average percentage difference, and the least correlation with numerical results, is that proposed by Harper and Hallett [15], with an average of about 37 %.

The maximum difference with this expression, is associated to the prediction of the length of FPZ against shear modulus of the adhesive ($G$). The maximum difference, for the expression derived by Yang et al. [35, 36], corresponds to the prediction of length of FPZ against mode II critical energy release rate ($G_{IIc}$), with percentage difference of approximately 80 %.
Figure 11. Comparison between the fully developed FPZ results obtained from analytical expressions proposed by Yang et al. [35, 36], Harper and Hallett [15], Conroy et al. [20] and Soto et al. [37] with the expression proposed in this paper and the numerical results using CZM.
Table 4. Comparison between the analytical methods and the expression proposed in this paper with the numerical results.

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>Minimum % difference</td>
<td>6.3 %</td>
<td>9.9 %</td>
<td>1.4 %</td>
<td>0.9 %</td>
<td>0.002 %</td>
</tr>
<tr>
<td>Maximum % difference</td>
<td>80.1 %</td>
<td>46.9 %</td>
<td>30.9 %</td>
<td>30.7 %</td>
<td>4.6 %</td>
</tr>
<tr>
<td>Average</td>
<td>26.4 %</td>
<td>36.8 %</td>
<td>18.4 %</td>
<td>11.3 %</td>
<td>1.2 %</td>
</tr>
</tbody>
</table>

A comparison of the percentage differences provided by all methods has shown that the expression proposed in this paper correlated exceptionally well with numerical results. The maximum percentage difference was about 5 % and on average only about 1 %. These results have been shown in Table 4.

6. Geometrical dimension selection guidelines

6.1. Development of guidelines

In this section, the expression derived for the fully developed FPZ is used to propose guidelines for the selection of the dimensions (in particularly the substrates thickness, $h$ and free length, $L_{free}$) of an ELS test specimen, which ensures the attainment of a steady-state and provides adequate crack propagation.

It is important to mention that $G_{IIc}$ is one of the variables required to determine $L_{FPZ_{max}}$ using the expression proposed in this paper (equation (6)). Since initially this value ($G_{IIc}$) is unknown, it is recommended that a value five times the mode I critical energy release rate ($G_{Ic}$), is selected (i.e. $G_{IIc, initial} = 5 \times G_{Ic}$) to manufacture the initial ELS test specimen, when following the geometrical dimension selection guidelines below. Subsequently a more precise and updated value for $G_{IIc}$ can be extracted using the experimental data and by adopting relevant analytical methods and following the same procedures (below) for the manufacturing of the final specimen.

Initially equation (6) has been reformatted and as a result,

$$ L_{FPZ_{max}} = \lambda \alpha_1 = \lambda h^{0.41} \quad \rightarrow \quad \lambda = \prod_{j=2}^{10} \alpha_j $$

where parameters $\alpha_j$ can be found from Table 3. Different values of $\lambda$ will result in different curves when the $L_{FPZ_{max}}$ is plotted against $h$. These curves are now referred to as characteristic curves, which corresponds to different material properties chosen for the substrates and adhesive, together with the thickness of the adhesive layer, $t_a$ and $L_{uncracked}$. It is considered
that most materials that may be used in an ELS test specimen (substrates and adhesives) will fall between $0.5 \leq \lambda \leq 1.0$. The results of this are displayed as characteristic curves in Figure 12.

For each characteristic curve shown in Figure 12, a model has been constructed with the lowest possible value for the substrate thickness, $h$ and run and analysed. It was concluded that for very compliant ELS test specimens with low $h$ values crack propagation was not achieved or excessive/impractical displacement had to be applied. After completing the analysis for each characteristic curve, it was established that a linear line of the form, $L_{FPZ_{max}} = 50h - 0.69$ fits through all the data corresponding to limits on what can be used as $h_{min}$ in each case (Figure 12).

Furthermore, with regards to choosing an appropriate substrate thickness, it is recommended that very small or very large substrates thickness should be avoided. This is due to the fact that the former requires very large applied displacement and in the case of the latter, a very high rate of crack propagation ($da/d\delta$) would be experienced.

![Figure 12. Characteristic curves (black), plotted using equation (9), corresponding to the range of materials available in an ELS test specimen (the green and brown lines correspond to the boundaries to these curve).](image)

In addition, $L_{uncracked}$, should be selected to be long enough to allow at the least, (i) the FPZ to fully develop, (ii) desirable crack growth i.e. $\Delta a$ to be achieved and (iii) a length of approximately 10% of $L_{uncracked}$ to the clamp point to be included. Therefore, the inequality,

$$L_{uncracked} \geq \frac{L_{FPZ_{max}} + \Delta a}{0.9}$$  \hspace{1cm} (10)

should be satisfied. The $L_{FPZ_{max}}$ is a function of $L_{uncracked}$ (equation (6)), so either equation $L_{uncracked} = L_{FPZ_{max}} + \Delta a/0.9$ should be solved or a suitable value for $L_{uncracked}$ that satisfies inequality (10) should be selected.
From Figure 1, $L_{\text{uncracked}} = L_{\text{free}} - a_p$ and by substituting equation (1) into this, the free length of the specimen can be calculated using,

$$L_{\text{free}} = \frac{L_{\text{uncracked}} + 0.1(\chi h)}{0.45} \quad (11)$$

Below is the step by step procedure that can be followed for selecting appropriate dimensions for the ELS test specimen,

(i) Select appropriate materials for the substrates and the adhesive
(ii) Select appropriate adhesive thickness, $t_a$ and uncracked length, $L_{\text{uncracked}}$
(iii) Calculate $\lambda$ from equation (9)
(iv) From Figure 12, use the characteristic curve that corresponds to $\lambda$, then choose an appropriate substrate thickness, $h$ and its corresponding fully developed FPZ length, $L_{FPZ_{\text{max}}}$.
(v) Select a desirable crack growth, i.e. $\Delta a$. Does $L_{\text{uncracked}}$, $\Delta a$ and $L_{FPZ_{\text{max}}}$ satisfy inequality (10)? If yes, move to the step (vii) otherwise move to step (vi)
(vi) Repeat stage (iv) by decreasing $h$ until stage (v) is satisfied. If stage (v) is not satisfied at any values of $h$ within its limit on the characteristic curve then move to step (ii) (increase either $t_a$ or $L_{\text{uncracked}}$ or both) and if, ultimately, this does not satisfy the condition in step (v) then move to step (i) (choose appropriate material for substrates that result in lower values of $\lambda$).
(vii) Using equation (11), calculate the free length of the specimen, $L_{\text{free}}$

6.2. Application of guidelines

To test the robustness of the above proposed guidelines, the dimensions of the specimen in section 4 (second case study) were re-evaluated.

Following the proposed guidelines above,

- From equation (9), $\lambda$ was calculated to be approximately 0.7.
- From Figure 12, using the characteristic curve that corresponds to $\lambda = 0.7$, a substrate of thickness, $h = 0.01 \, m$ was chosen and as a result its corresponding fully developed FPZ length, was obtained ($L_{FPZ_{\text{max}}} = 0.104 \, m$).
- Allowing minimum crack extension of $\Delta a = 0.02 \, m$, the inequality (10) is satisfied.
• Using equation (11) the free length of the new specimen is calculated to be, \( L_{\text{free}} = 0.340 \, \text{m} \)

• Also the stability criterion shown in equation (1) is considered and as a result, the initial crack length after precracking was calculated to be \( a_p = 0.182 \, \text{m} \).

The new specimen, with dimensions and material properties specified in Table 5, was modelled using Abaqus, following the same procedures in section 3. The test was stopped when the FPZ reached a distance of about 10% of \( L_{\text{uncracked}} \) to the clamp point.

The numerical load-displacement plot (Figure 13a) is now observed to show a maximum value. Note that previous (Figure 5a and Figure 6a) no maximum value was reached. This corresponds to attainment of a steady-state plateau in the R-curve when using the new specimen.

Table 5. New dimensions and material properties for the specimen in section 4.

<table>
<thead>
<tr>
<th>( l_1 ) (mm)</th>
<th>( l_2 ) (mm)</th>
<th>( h ) (mm)</th>
<th>( B ) (mm)</th>
<th>( t_a ) (mm)</th>
<th>( L_{\text{uncracked}} ) (mm)</th>
<th>( \sigma_y ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
<td>10.0</td>
<td>10.0</td>
<td>20</td>
<td>0.4</td>
<td>140</td>
<td>36</td>
</tr>
<tr>
<td>( G ) (GPa)</td>
<td>( G_{\mu} ) (J/m²)</td>
<td>( E_1 ) (GPa)</td>
<td>( E_2 = E_3 ) (GPa)</td>
<td>( G_{12} = G_{13} ) (GPa)</td>
<td>( v_{12} = v_{13} )</td>
<td>( v_{23} )</td>
</tr>
<tr>
<td>0.414</td>
<td>17800</td>
<td>142</td>
<td>8.72</td>
<td>4.63</td>
<td>0.3</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Figure 13. Results obtained for the new specimen, following the guidelines proposed. (a) load-displacement plot (b) combination of the FPZ development and crack extension ahead of the initial crack tip against applied displacement plot.

Furthermore, as Figure 13b demonstrates, the maximum length of the FPZ attained (corresponding to applying approximately 43 mm of displacement) was about 109 mm. This is in good correlation to the predicted \( L_{FPZ_{\text{max}}} \) value of about 104 mm (a percentage difference
of about $4.6\%$). In addition, from the same figure, a crack extension of $22\ mm$ was attained when running the model which meets the minimum desirable crack extension of $20\ mm$.

7. Conclusion

A cohesive zone model was adopted, in conjunction with a mesh independent quadratic cohesive element formulation to model the fracture of a mode II, ELS adhesive joint specimen. Initially, it was demonstrated that the chosen cohesive law captures the global response of the structure together with a good prediction of fracture process zone (FPZ) length and crack propagation in comparison to experimental test results. The importance of the length of maximum FPZ and its position with reference to the clamp point in reaching steady-state was highlighted. It was shown the plateau in the R-curve could not be achieved in previous studies due the fact that the FPZ reached the clamp point. Therefore, it was proposed that a window of approximately $10\%$ of the initial $L_{uncraked}$, between the end of FPZ and the clamp point, should be accommodated.

An expression was proposed for the prediction of the fully developed FPZ which has been derived through parametric studies. For each parameter, approximately ten specimens were considered and a power law trendline was used to fit a curve through the results obtained. The proposed expression was validated through comparison with three analytical expressions and the results obtained from numerical simulations. It was demonstrated that the results obtained from the expression correlate very well with the numerical results when compared with those gained from analytical expressions. The maximum percentage difference was found to be about $5\%$ and on average only about $1\%$ when compared with the numerical simulations.

Finally, the derivation of the expression formed the basis of, and allowed, guidelines for the selection of the dimensions (in particularly the substrates thickness, $h$ and free length, $L_{free}$) of an ELS test specimen, to be proposed. This represents a significant step towards the successful development of a standard test method for determining the mode II fracture resistance of adhesive joints bonded with highly toughened adhesives.

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References


