Comparative study of photo-produced ionosphere in the close environment of comets

A. Beth, M. Galand, and K. L. Heritier

Department of Physics, Imperial College London, Prince Consort Road, London SW7 2AZ, United Kingdom

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ABSTRACT

Context. Giotto and Rosetta gave us the unique opportunity to probe the close environment of cometary ionospheres, the ones of 1P and 67P. The conditions encountered at both comets were very different on a plasma perspective, mainly driven by different heliocentric distances – driving photo-ionisation rates – and outgassing activities – driving the neutral densities.

Aims. We assess the relative contribution of different plasma processes ongoing in the inner coma, namely photo-absorption, photo-ionisation, transport and electron-ion dissociative recombination. The main goal is to identify what processes are at play for a quantitative assessment of ionospheric density.

Methods. We provide a set of analytical formulas to describe the ionospheric number density profile for cometary environments taking into account some of these processes and discuss the validity of each model in the context of the Rosetta and Giotto missions.

Results. We show that transport is the dominant loss process at large cometocentric distances/low outgassing rates. Chemical plasma loss through $e^−$-ion dissociative recombination matters around 67P near perihelion and at 1P during the Giotto flyby: its effects increases as the heliocentric distance decreases, that is, at higher outgassing activity and higher photo-ionisation frequency. Photo-absorption is of importance for outgassing rates higher than $10^{16}$ s$^{-1}$ and only close to the cometary nucleus, well below the location of both spacecraft. Finally, whatever the processes we considered, the ion number density profile always follows a $1/r$ law at large cometocentric distances.

Key words. < Comets: general - Plasmas - Sun: UV radiation - Methods: analytical>

1. Introduction

European Space Agency’s Rosetta (Glassmeier et al. 2007) and Giotto (Reinhard 1986) missions gave us the best opportunities to scrutinise the inner coma and the ionosphere of two comets: 1P/Halley (hereinafter referred as 1P) and 67P/Churyumov-Gerasimenko (hereinafter referred as 67P). These two comets were probed under very different conditions and stages of their orbit. On the one hand, 1P was probed at 0.89 au (to compare with its perihelion at 0.58 au) by Giotto down to 596 km from the comet with a relative velocity of a few tens of kilometres per second over a few hours (Reinhard 1986). On the other hand, Rosetta examined 67P from 3.6 au (August 2014) to perihelion at 1.24 au (August 2015) to 3.8 au at the end of mission (30 September 2016) at a relative velocity of a few metres per second, and with the first in-situ plasma measurements down to the surface (Heritier et al. 2017b). During the escort phase, from a plasma perspective, a wide variety of conditions have been encountered: increase in the photo-ionisation by the solar Extreme Ultra-Violet (EUV) radiation, as the comet got closer to the Sun (Heritier et al. 2018), large dynamic range in the outgassing rate (Hansen et al. 2016), rich diversity and variability in neutral composition (Le Roy et al. 2015; Gasc et al. 2017).

In particular, at 1P, from observations of the High-Intensity Spectrometer (HIS) on-board Giotto, Balsiger et al. (1986) reported the total ion number density (more precisely from the mass 16-19 and 32 amu, the main contributors to the cometary ion number density) as a function of the cometocentric distance. They found that from the closest approach ($\sim$ 600 km) to the so-called contact surface ($\sim$ 4600 km), the ion number density profile followed a $1/r$ law. This dependence was “[...] predicted by photochemical equilibrium models in which the photo-ionisation of cometary neutrals is balanced by electron dissociative recombination [...]”. However, outside this region, further away, the dependence was different and followed $1/r^2$, preceded by a steep increase in the plasma density between both regions. At 67P, during pre-perihelion (February 2015), Edberg et al. (2015) reported a similar dependence in $1/r$ of the ion number density for cometocentric distances from a few kilometres up to 250 km. In contrast, the dependence was “[...] expected from ionisation of a neutral gas expanding radially from the comet nucleus and when there is no significant recombination or other loss source for the plasma [...]”. At both comets, similar behaviours were observed but the drivers seemed to be different.

Several analytical and numerical approaches have been proposed to explain the plasma behaviour as aforementioned or developed in preparation of these missions. Amongst those, the most comprehensive numerical models of cometary plasmas was developed by Marconi & Mendis (1983); Marconi & Mendis (1984) and Mendis et al. (1985). This 1D model included solved the continuity, momentum and energy equations for electron and each ion species considered. It includes a broad range of processes, such as the ion-neutral chemistry, photo-dissociation, photo-ionisation, radiative transfer, dissociative recombination, dust dynamics, elastic and inelastic collisions. The main feature on the plasma number density profile is a sharp increase at $2.5 \times 10^{5}$ km, linked to a steep rise of the electron temperature,
Fig. 1. Photo-electron production rate at the solar zenith angle $0^\circ$ as a function of the column density (left $y$-axis) or optical depth (right $y$-axis). Two cases are represented: 1) expanding coma: 1P during Giotto’s flyby (red solid line) and 67P at two different heliocentric distances (green and blue solid lines, see legend and Table 2), 2) atmospheres in hydrostatic equilibrium like Earth at 1 au (black, solid line). The number density scale height $H$ for the hydrostatic equilibrium case is set to 20 km and the same cross-section of photo-absorption is assumed for all three cases. The dotted line corresponds to the production rate below the cometary surface. The circles identify the nucleus surface and the dotted line corresponds to the hypothetical production rate below.

lessening the loss through dissociative recombination. Note that this model was designed in the context of Giotto and predated this mission. In comparison, 3D models in the context of 1P for 67P have been proposed as well, like Rubin et al. (2009, 2014), but at larger spatial resolution (larger than 100 km). Concerning analytical works, Cravens (1987) proposed that the cometary ionosphere, below the contact surface, is in photo-chemical equilibrium: photo-ionisation and dissociative recombination balance each other, while loss through transport is neglected. Based on these assumptions, he derived the theoretical size of the so-called diamagnetic cavity, characteristic magnetic field-free cometary region observed at 1P (Neubauer et al. 1986). More recently, Galand et al. (2016) proposed a model applied to comet 67P at large heliocentric distances during the Rosetta mission; photo-ionisation is balanced by transport. Finally, Gombosi (2015) proposed a model including both transport and $e^-$-ion dissociative recombination as loss processes. However, the nucleus was reduced to a point source. These three models are listed in Table 1 with their respective assumptions and applications. None of them includes the attenuation of the solar EUV radiation through the coma. Bhardwaj (2003) assessed the effect of this additional process on the photo-ionisation profile for large outgassing rates, and compared photo-ionisation rates with ionisation rate by photo-$e^-$ impact. He however did not evaluate the effect of photo-absorption on electron number density.

Fig. 1 compares the photo-ionisation rates in the case of cometary expanding coma (coloured lines) and in the case of an atmosphere under hydrostatic equilibrium like at Earth (black line). When close to the nucleus’ surface, the rates are always higher for comets (near 1 au) than for the hydrostatic equilibrium case. This is all the more true than $N_2$ ionisation rate is lower than that of water (used in all cases here). Furthermore, the photo-ionisation rates between both cases do not peak at the same column density or optical depth. One of the purpose of this paper is to shed lights on these differences.

In the present paper, we focus on the total plasma number density and the following processes occurring in inner comae and cometary ionospheres, that is, photo-ionisation, photo-absorption, transport and finally $e^-$-ion dissociative recombination. Ion-neutral chemistry is ignored as it barely affects the total plasma number density. Indeed, the latter is sensitive to ion composition only via the dissociative recombination of the terminal ions. However, the most likely terminal ions, including $H_3O^+$ and $NH_3^+$, have dissociative recombination kinetic coefficients of the same order of magnitude (Beth et al. 2016; Herriert et al. 2017a). In Section 2, we assess the solar deposition through the coma. In particular, in Section 2.1, we highlight the main difference between planetary and cometary ionospheres. In Section 2.2, we calculate the effect of the photo-absorption on the solar flux and photo-electron production profile. In Section 3, we provide an exhaustive list of analytical formulas for the electron density under different conditions. We introduce new models missing from the literature and capable of describing the electron density profile of the inner coma, for low and intermediate outgassing rates. At the same time, we link previous existing models with some of the analytical models we propose. Finally, in Section 4, we compare with the past literature and analytical modelling, to highlight which processes and models are relevant for 1P/Giotto and 67P/Rosetta and discuss additional processes which may be important as well.

2. Solar deposition

2.1. Gravitational effect on the photo-electron production rate

It is well established that the maximum, in solar energy deposition, at a given wavelength $\lambda$, and thus in photo-electron production, occurs at the optical depth $\tau_\lambda = 1$, in planetary atmospheres for which the plane parallel approximation and isothermal, hydrostatic equilibrium can be assumed (Chamberlain & Hunten 1987). If the total production of photo-electrons is integrated over the full solar EUV spectrum, we find that the maximum in solar energy deposition has its location at an optical depth of 1 for wavelengths around 30-35 nm at Earth. This is not surprising keeping in mind that the strongest solar EUV line is He $\Pi$ 30.4 nm.

However, comets are different from planets, for the following reasons:

- they are small bodies, with a radius of often a few kilometres. The plane parallel approximation is therefore not appropriate, especially near the surface and the inner coma, instead spherical symmetry is usually applied,
- the coma is mainly made of sublimated water, carbon dioxide and carbon monoxide (Hässig et al. 2015; Gasc et al. 2017), which are primarily escaping the body. There may be a contribution from the sublimation of dust grains (like at 103P, A’Hearn et al. 2011; 73P, Fougere et al. 2012; and 67P, Fougere et al. 2016) and photo-dissociation of, e.g., $H_2CO$ into CO, as inferred at 1P (Eberhardt 1999),
Table 1. List of analytical formulas for the ion number density profiles for the inner coma present in the past literature as well as provided in this paper. For each model, we list the different assumptions, the corresponding equations in this paper, as well as the extent of their respective applications. \( r \) refers to the cometocentric distance and N/A means “not applicable”.

<table>
<thead>
<tr>
<th>Analytical model</th>
<th>Comet size</th>
<th>Phot absorption</th>
<th>( e^- )-ion dissociative recombination</th>
<th>Transport</th>
<th>Analytical solution given in:</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cravens (1987)</td>
<td>N/A</td>
<td>( \times )</td>
<td>( \times )</td>
<td>( \times )</td>
<td>31</td>
<td>1P - Giotto</td>
</tr>
<tr>
<td>Gombosi (2015)</td>
<td>( \times )</td>
<td>( \times )</td>
<td>( \times )</td>
<td>( \times )</td>
<td>28</td>
<td>Large ( r )</td>
</tr>
<tr>
<td>Galand et al. (2016)</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>29</td>
<td>67P - Rosetta (&gt; 3 au)</td>
</tr>
</tbody>
</table>

This work:
- Section 3.1: \( \checkmark \)
- Section 3.2: \( \checkmark \)
- Section 3.3: \( \checkmark \)
- Section 3.4: \( \checkmark \)

Table 2. List of parameters for our modelling and of figures for which they have been used. Three cases have been considered: 1P during Giotto flyby, 67P at perihelion and at the beginning of the escort phase. The photo-ionisation frequency is that of water for averaged conditions during the Rosetta escort phase (7 \( \times \) 10\(^{-7}\) s\(^{-1}\) at 1 au, Heritier et al. (2018) and has been corrected for the heliocentric distance.

<table>
<thead>
<tr>
<th>Case study</th>
<th>1P/Halley (Giotto flyby)</th>
<th>67P/C-G (low activity)</th>
<th>67P/C-G (high activity, perihelion)</th>
<th>General case (this work)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cometary radius (km)</td>
<td>( \sim 5.5^{(4)} )</td>
<td>( \sim 2^{(4)} )</td>
<td>( \sim 2^{(4)} )</td>
<td>2</td>
</tr>
<tr>
<td>Heliocentric distance (au)</td>
<td>0.89</td>
<td>3.2 (2)</td>
<td>1.24</td>
<td>\ldots</td>
</tr>
<tr>
<td>Local photo-ionisation frequency (s(^{-1}))</td>
<td>8.6 ( \times ) 10(^{-7})</td>
<td>6.8 ( \times ) 10(^{-8})</td>
<td>4.5 ( \times ) 10(^{-7})</td>
<td>\ldots</td>
</tr>
<tr>
<td>Local outgassing rate (s(^{-1}))</td>
<td>6.9 ( \times ) 10(^{29}) (1)</td>
<td>6 ( \times ) 10(^{25}) (2)</td>
<td>6 ( \times ) 10(^{28}) (3)</td>
<td>10(^{25-10})</td>
</tr>
<tr>
<td>Neutral velocity (m.s(^{-1}))</td>
<td>900 (1)</td>
<td>600 (2.3)</td>
<td>900 (3)</td>
<td>600</td>
</tr>
<tr>
<td>Electron temperature (K)</td>
<td>[150-1000]</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Photo-absorption cross section (m(^2))</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>7.82 ( \times ) 10(^{-22})</td>
</tr>
</tbody>
</table>

Cases shown in the following figures:
- 1,11,12 |
- 1,11,12 |
- 1,11,12 |
- 2,3,4,5,8 |

References. (1) Krankowsky et al. (1986); (2) Galand et al. (2016); (3) Hansen et al. (2016); (4) Lamy et al. (2004)

- the neutral number density \( n(r) \) is a function of the cometocentric distance \( r \). It was observed to follow a \( 1/r^2 \)-dependence at the location of Rosetta, from a few kilometres from the surface (e.g., Hässig et al. 2015), which is consistent with the flux conservation at constant velocity and depart from the hydrostatic equilibrium. Possible contributions from extended sources on the neutral number density have been neglected thereafter in this paper,

- the neutral gas undergoes an adiabatic expansion (Huebner & Markiewicz 2000; Heritier et al. 2017b) – which is faster and thus critical close to the surface – and reaches supersonic speeds around 600-700 m.s\(^{-1}\) at a few hundreds of kilometres from the surface,

- the outgassing rate and the photo-ionisation rate evolve with respect to the heliocentric distance, which varies widely along the comet’s large eccentric orbit (Heritier et al. 2018).

Under these conditions, we would expect a different response of the expanding neutral coma to the incoming solar EUV radiation. Assuming a coma with one dominant neutral species, here water, the production of electrons at a given wavelength \( \lambda \) is defined as:

\[
P_{e-\lambda}[r, \chi, Q] = v_0(\lambda) n(r) \exp\left[ -\sigma_{abs}(\lambda) N(r, \chi) \right]
\]

where \( v_0 \) stands for the unattenuated spectral photo-ionisation frequency, \( \sigma_{abs} \), for the total photo-absorption cross-section and \( N(r, \chi) \), for the upstream column density in the direction of the Sun of the dominant species, at the cometocentric distance \( r \) and at a solar zenith angle \( \chi \) (see Appendix A and Fig. A.1). The total production of electrons, integrated over all wavelengths, is given by:

\[
P_e[r, \chi, Q] = \int_{r_{\min}}^{r_{\max}} I_{\infty}(\lambda) \sigma_{ion}(\lambda) n(r) \exp\left[ -\sigma_{abs}(\lambda) N(r, \chi) \right] d\lambda
\]

(2)

where \( I_{\infty}(\lambda) \) stands for the unattenuated incoming solar spectral flux \([\text{photons.m}^{-2}.\text{s}^{-1}.\text{A}^{-1}]\) at the wavelength \( \lambda \), \( \sigma_{ion} \), for the photo-ionisation cross-section, \( \lambda_{\min} \), the ionisation threshold wavelength (98.2 nm for water) and \( \lambda_{\min} \), the lower bound for integration, usually 0.1 nm, as shorter wavelengths do not significantly contribute to the ionisation. Being given that the neutral number density is defined as:

\[
n[r, Q] = \frac{Q}{4\pi U_n r^2}
\]

where \( Q \) stands for the outgassing rate and \( U_n \), for the neutral gas outflow velocity. The neutral column density \( N \) is given by (Bhardwaj 2003; Beth et al. 2016):

\[
N[r, \chi, Q] = \frac{Q}{4\pi U_n r^2} \sin \chi
\]

and the optical depth \( \tau_{\chi}[r, \chi] \) at the wavelength \( \lambda \) at the cometocentric distance \( r \) and the solar zenith angle \( \chi \), is defined by:

\[
\tau_{\chi}[r, \chi, Q] = \sigma_{abs}(\lambda) N(r, \chi) = \frac{\sigma_{abs}(\lambda) Q}{4\pi U_n r^2} \sin \chi
\]

\[
= \frac{Q}{Q_{\chi}[r]} \frac{r_c}{r} = \tau_{\chi, r_c} \frac{r_c}{r}
\]

(3)
where \( r_c \) stands for the cometary radius, \( \tau_{c,\lambda}[\chi] = \tau_1[r_c, \chi] \), for the optical depth at the surface for a given solar zenith angle \( \chi \). We have introduced a new parameter \( Q_{c,\lambda} \), which has the dimension of an outgassing rate, as:

\[
Q_{c,\lambda}[\chi] = \frac{4\pi U_n r_c \sin \chi}{\sigma_{abs}(\lambda)} \chi = \frac{Q}{\tau_{c,\lambda}[\chi]} \text{[s}^{-1}] \tag{4}
\]

\( Q_{c,\lambda} \) corresponds to the outgassing rate for which the optical depth is 1 at the surface for a given \( \chi \) and a given \( \lambda \). Eq. 1:

\[
P_{e-,\lambda}[r, \chi, Q] = v_0(\lambda) n(r) \exp \left[ -\frac{\sigma_{abs}(\lambda) Q}{4\pi U_n r_c} \frac{\chi}{\sin \chi} \right] = v_0(\lambda) n(r) \exp \left[ -\frac{Q}{Q_{c,\lambda}[\chi]} \frac{r_c}{r} \right] \tag{5}
\]

At a given wavelength \( \lambda \), the maximum in electron production rate (Eq. 1) is located at \( \tau_1[r, \chi] = 2 \) in cometary comae, instead of \( \tau_1[r, \chi] = 1 \) in planetary atmospheres. A detailed derivation is given in Appendix A for a general case of a number density profile defined as: \( n(r) \propto r^{-\beta} \) where \( \beta = 2 \) for cometary expanding comae. We also show that for planetary isothermal atmospheres in hydrostatic equilibrium correspond to the case of very large \( \beta \) values.

As the number density at comets relies on the outgassing activity, the optical depth is a function of \( Q \) and \( \chi \); the spherical surface at which \( \tau_1[r, \chi] = 2 \) moves outwards, as \( Q \) increases and is above the surface for high enough \( Q \). Interestingly, for a fixed outgassing rate, the maximum of \( P_{e-,\lambda}[r, \chi] \), is given by:

\[
\left( \frac{\partial P_{e-,\lambda}[r, \chi, Q]}{\partial r} \right)_{Q,\chi,\lambda} = 0 \leftrightarrow P_{e-,\lambda}[r, \chi, Q]_{\text{max}} = \begin{cases} 
\frac{v_0(\lambda)Q_{c,\lambda}}{\pi U_n r_c^2} \exp[-2] & \text{if } \tau_{c,\lambda} \geq 2 \text{ (i.e., } Q \geq 2Q_{c,\lambda}) \\
\frac{v_0(\lambda)Q}{4\pi U_n r_c^2} \exp \left[ -\frac{Q}{Q_{c,\lambda}} \frac{r_c}{r} \right] & \text{if } \tau_{c,\lambda} < 2 \text{ (i.e., } Q < 2Q_{c,\lambda}) 
\end{cases} \tag{6}
\]

located at

\[
(r/r_c)_{\text{max}} = \begin{cases} 
\frac{Q}{2Q_{c,\lambda}} & \text{if } \tau_{c,\lambda} \geq 2 \\
1 & \text{if } \tau_{c,\lambda} < 2 
\end{cases} \tag{7}
\]

However, for a fixed cometocentric distance \( r' \), the maximum of electron production rate at \( r' \) occurs for \( Q = Q_{c,\lambda} r'/r_c \), such that

\[
\left( \frac{\partial P_{e-,\lambda}}{\partial Q} \right)_{r',\chi,\lambda} = 0 \leftrightarrow P_{e-,\lambda}[r', \chi, Q]_{\text{max}} = \frac{v_0(\lambda)Q_{c,\lambda}}{4\pi U_n r_c r'} \exp[-1] \tag{8}
\]

For a given cometary \( r' \), a given photo-ionisation frequency \( v_0(\lambda) \), a given neutral expansion velocity \( U_n \), and a given nucleus’ radius \( r_c \), Eq. 8 provides the maximum which the photo-electron production rate can reach when the full range of \( Q \) is considered. This bound is reached for \( Q = Q_{c,\lambda} r'/r_c \)

\[\text{Fig. 2. Photo-electron production rate in the coma at the solar terminator (} \chi = \pi/2) \text{[in dimensionless units], for different outgassing rates, as a function of the cometocentric distance [in cometary radii]. The production is monotonically decreasing with respect to } r \text{ for } Q < 2Q_c. \text{ For } Q > 2Q_c, \text{ the production increases up to the cometocentric distance } (r/r_c)_{\text{max}} = 0.5Q/Q_c \text{ and decreases beyond. The solid, black line corresponds to the upper bound of the photo-electron production rate for a given cometocentric distance. The grey part is the theoretical region which is not reachable by any } Q. \text{ The expanding velocity is set to } U_n = 600 \text{ m.s}^{-1}.\]

and results from photo-absorption of the solar radiation by the neutral coma.

Fig. 2 shows the electron production rate profiles as a function of the outgassing rate in dimensionless units. On the one hand, for optically thin comae or low outgassing activities (i.e., \( \tau_{c,\lambda} < 2 \) or \( Q < 2Q_{c,\lambda} \)), \( P_e^- \) is monotonically increasing when the cometocentric distance is decreasing. However, for optically thick comae or high outgassing activities (i.e., \( \tau_{c,\lambda} > 2 \) or \( Q > 2Q_{c,\lambda} \)), the electron production reaches a maximum at \( r = (0.5Q/Q_{c,\lambda}) r_c \) (i.e., where \( \tau_1[r, \chi] = 2 \)) and then decreases down to the surface. The maximum reached by \( P_e^- \) in that case decreases with respect to the outgassing rate \( Q \) as \( Q \) increases, the distance \( r \) at which \( \tau_1[r, \chi] = 2 \) increases, as \( \tau_1 \propto Q/r \). For a fixed cometocentric distance \( r' \), solar zenith angle \( \chi \), and photo-ionisation frequency \( v_0 \), the maximum in photo-electron production rate occurs at \( Q = Q_{c,\lambda} r'/r_c \) such that \( \tau_1[r', \chi] = 1 \) (see Eq. 8 and Fig. 2, solid, black line). The highest \( P_e^- \) is reached at the surface for \( Q = Q_{c,\lambda} \). This highlights how critical it is to include photo-absorption: for \( Q > \exp[-1]Q_{c,\lambda} \), it exists a range of cometocentric distances for which \( P_e^- \) is within the grey area if the photo-absorption is neglected, meaning that \( P_e^- \) takes forbidden values and is overestimated.

The difference between planetary atmospheres and comae raises from the neutral number and neutral column number densities. Under isothermal, hydrostatic equilibrium, the former have a column density proportional to the neutral number density
(i.e., $\tau_1[z, \chi] = \sigma_{\text{abs}}(\lambda)N[z, \chi] \propto n[z]H$, $H$ stands for the scale height constant regardless of the altitude for the isothermal atmosphere). Thus, $P_{e^-, \lambda}$ behaves as $\propto \tau_1 \exp[-\tau_1]$: 

$$P_{e^-, \lambda} \propto \sigma_{\text{abs}}(\lambda) N[z, \chi] \propto n[z] H.$$ 

$$= v_0(\lambda) n(z) \exp \left[ -\frac{\sigma_{\text{abs}}(\lambda) N(z) H}{\cos \chi} \right]$$

$$= v_0(\lambda) \cos \chi \frac{\sigma_{\text{abs}}(\lambda)}{H} \tau_1 \exp[-\tau_1]$$  \hspace{1cm} (9)

Unlike the planetary case, for comets $\tau_1[r, \chi, Q] = \sigma_{\text{abs}}(\lambda) N[r, \chi, Q] \propto 1/r \propto n(r)r$, such that $P_{e^-, \lambda} \propto \tau_1^2 \exp[-\tau_1]$:

$$P_{e^-, \lambda, \text{comets}}[r, \chi, Q] = v_0(\lambda) n(r) \exp \left[ -\frac{\sigma_{\text{abs}}(\lambda) n(r)r \chi}{\sin \chi} \right]$$

$$= \frac{v_0(\lambda) \sin \chi}{\sigma_{\text{abs}}(\lambda) r c} \tau_1 \exp[-\tau_1]$$

$$= \frac{4 \pi v_0(\lambda) U_0}{2} \frac{\sin \chi}{\chi^2} \tau_1^2 \exp[-\tau_1]$$

$$= v_0(\lambda) Q e^{-1}[\chi] \frac{\sin \chi}{\chi^2} \tau_1^2 \exp[-\tau_1]$$  \hspace{1cm} (10)

of which the maximum occurs at $\tau = 2$.

Fig. 3 shows the photo-electron production rate at comets from a pure water coma for different outgassing rates, scaled to the maximum production rate of the hydrostatic case. This case is derived assuming a constant scale height $H \approx 20$ km (typical for the terrestrial lower thermosphere) and the photo-ionisation frequency of water. For an atmosphere in hydrostatic equilibrium, such as Earth, the maximum photo-electron production occurs at $\tau_1 = 1$ and $\chi = 0$, which corresponds to:

$$P_{e^-, \text{Earth}}(\tau_1 = 1, \chi = 0) = \frac{v_0(\lambda) \sin \chi}{\sigma_{\text{abs}}(\lambda) r c} \exp[-1]$$  \hspace{1cm} (11)

independent of the neutral number density. At the surface of Earth, the optical depth in the EUV range is higher than $\tau_1 = 10^5$ (out of boundaries in Fig. 3 which covers for the hydrostatic case, $\tau_4 \in [10^{-4}; 10^2]$). For comets, the optical depth $c$ at the surface is given by $Q/c$. The photo-electron production rate at surface is given by:

$$P_{e^-, \lambda, \text{comets}}(\tau_4 = c, \chi = 0) = \left( \frac{H}{r c} \right)^2 \frac{v_0(\lambda)}{\sigma_{\text{abs}}(\lambda) r c} \exp \left[ -\frac{Q}{Q c} \right]$$

$$= \left( \frac{H}{r c} \right)^2 \frac{v_0(\lambda)}{\sigma_{\text{abs}}(\lambda) r c} \exp \left[ -\frac{Q}{Q c} \right]$$  \hspace{1cm} (12)

where $d_h$ stands for the heliocentric distance of the comet, in au. Eq.12 is represented by the solid black line in Fig. 3 for $d_h = 1$ au. This is the upper bound for which $P_{e^-, \lambda, \text{comets}}(\tau_4 < c, \chi = 0)$. Beyond it (dark grey region in Fig. 3), the photo-electron production is meaningless as it is below the surface.

Fig. 3 highlights that there are outgassing rates at which the photo-electron production rate is higher than at Earth. While Earth’s atmosphere is not dominated by water, that statement remains valid. Indeed, amongst the major terrestrial species, $N_2$, $O_2$ and $O_3$, only $O_3$ has a photo-ionisation frequency higher than that of $H_2O$ (Huebner & Mukherjee 2015). As the terrestrial $O_2$ mixing ratio decreases with altitude in the terrestrial upper atmosphere where the ionosphere is located, we can estimate an upper limit for the total photo-ionisation frequency corrected from a terrestrial composition with an $O_2$ mixing ratio of 0.2 (maximum reached). With a mixing of 80% $N_2$ and 20% $O_2$, the total photo-ionisation frequency is $4.09 \times 10^{-7}$ s$^{-1}$ for a quiet Sun and $1.07 \times 10^{-6}$ s$^{-1}$ for an active Sun, to be compared with the photo-ionisation rates of water in the same solar conditions, $4.05 \times 10^{-7}$ s$^{-1}$ and $1.04 \times 10^{-7}$ s$^{-1}$, respectively.

Let’s introduce the parameter:

$$\delta = \left( \frac{1 \text{ au}}{d_h} \right)^2 \frac{H}{r c} \frac{\tan \chi}{\chi}$$

For $\delta < 1$, whatever $Q$ is, the photo-electron production peak is always located lower down than at Earth in terms of optical depth or column density. For $\delta > 1$, there is a range of $Q$ for which the photo-electron production rate is higher than its maximum value at Earth (i.e., $P_{e^-, \lambda, \text{comets}}/P_{e^-, \text{Earth}}(\tau_4 = 1, \chi = 0) > 1$ in Fig. 3).
The extreme values are given by:

\[ Q_{\text{min}}[\delta] = -W_0 \left[ \frac{1}{\delta \exp(1)} \right] Q_c \chi \]

\[ Q_{\text{max}}[\delta] = \begin{cases} 
4 \exp(-1) \delta Q_c \chi & \text{if } \delta > \exp(1)/2 \\
-W_1 \left[ \frac{1}{\delta \exp(1)} \right] Q_c \chi & \text{if } \exp(1)/2 > \delta > 1 
\end{cases} \]

where \( W[x] \) stands for the Lambert-W function and \( Q_{\text{min}}[1] = Q_{\text{max}}[1] = Q_c \chi \). Conversely, as \( r_c \) and \( H \) are fixed, the constraint on \( \delta \) can be interpreted in terms of \( d_h \):

\[ \delta > 1 \Rightarrow \frac{H \tan \chi}{r_c} > \left( \frac{d_h}{1 \text{ au}} \right) \]

For \( \chi = 0 \), the necessary but not sufficient condition on the heliocentric distance \( d_h \) of the comet is \( d_h < \sqrt{H/r_c} \times 1 \text{ au} \approx 3.16 \text{ au} \) for having a photo-electron production rate from water higher than at Earth. The additional condition is on \( Q \in [Q_{\text{min}}, Q_{\text{max}}] \). For instance, at 1 au, \( Q_{\text{min}} \approx 7.4 \times 10^{26} \text{ s}^{-1} \) and \( Q_{\text{max}} \approx 2.8 \times 10^{29} \text{ s}^{-1} \) (using \( r_c, U_n \), and \( \sigma_{\text{abs}} \) in Table 2, last column, and \( H = 20 \text{ km} \)). Furthermore, Fig. 3 shows that for a given heliocentric distance, if close enough to the surface, the production rate at a comet is always higher than in the hydrostatic case for the same column density or optical depth (see also Fig. 1 but corrected from the heliocentric distance).

### 2.2. Attenuated solar spectral flux

In this subsection, we assess in more details one process under focus in this paper, namely the photo-absorption of solar EUV radiation by the neutral coma.

First of all, the coma is assumed to be pure water. Rosetta clearly observed variability in neutral composition throughout the escort phase with CO$_2$-dominated coma, especially over the southern hemisphere away from perihelion (Hässig et al. 2015; Hoang et al. 2017; Gasc et al. 2017). That may affect the plasma radiation by the neutral coma.

For strong outgassing rates (i.e., \( Q > 10^{30} \text{ s}^{-1} \)), soft X-rays start to be absorbed in the lower part of the coma, near the surface while radiation at longer wavelengths has been fully absorbed higher up. It should be highlighted that at short wavelengths, there are large uncertainties on the photo-ionisation:

- the spectral range from 0 to 5 nm covers a large range of energies, this requires a high spectral resolution in photo-absorption cross sections, including near 2.3 nm where the cross section exhibits a peak associated with the emission of two electrons, one of them is a so-called Auger electron. Indeed, photons of these energies can ionise electrons of the inner shells (e.g., K or L shells). Then, the hole in the inner shell is filled by an electron from an outer shell with a surplus energy. This energy can be released in two ways: either through X-ray fluorescence (the electron filling the hole emits a X-ray photon) or ionising an electron from the outer shell, letting the atom/molecule into a double-ionised state. Small uncertainties on the photo-absorption cross-section lead to large ones on the location of maximum energy deposition, and hence on the vertical structure of the photo-production rate.
- the solar spectral flux used and based on TIMED/SEE is provided over a 1-nm bin and cannot capture the structure of specific solar lines.

One of the main caveats when deriving the electron production profile in Section 2.1 is that we have neglected the electron-impact ionisation by photo-electrons (and their secondaries). We only include photo-ionisation. However, depending on their initial kinetic energy – which is equal to the photon energy minus the ionisation potential –, newborn photo-electrons may in their turn ionise neutrals through electron impact. Bhardwaj (2003) provided a quantitative analysis of the ionisation by these energetic electrons. Such electrons become the main ionisation source of 1 nm intervals. As the cross-sections and the solar EUV flux are not sampled over the same bins, we subdivide in smaller wavelengths (typically 0.01 nm wide or less) bins to take the structure of both into account. The two major sources of uncertainties are the cross-sections at short wavelengths (<5 nm) and from the binning of the solar EUV flux (1-nm bin).

The optical depth, \( \tau_{c,\lambda} = Q/Q_{c,\lambda} \) gives the efficiency of solar energy deposition at the surface (see Section 2.1):

- if \( \tau_{c,\lambda} < 2 \), the coma is optically thin at the wavelength \( \lambda \), thus the solar spectral flux around \( \lambda \) is barely attenuated through the coma.
- if \( \tau_{c,\lambda} > 2 \), the coma is optically thick at the wavelength \( \lambda \); the solar flux is efficiently absorbed, at least close to the surface. Moreover, in such a case, the maximum of solar deposition occurs at the cometocentric distance \( r = 0.5 \tau_{c,\lambda} r_c = (0.5Q/Q_{c,\lambda} \chi) r_c \) (see Eq. 7).

Fig. 4 shows the parameter \((r/r_c)\) at which the optical depth \( \tau_{c,\chi} = \pi/2 \), that is, from Eq. 7: \( (r/r_c) = 0.5 \tau_{c,\lambda} = (0.5Q/Q_{c,\lambda} \chi) \). We have also plotted the product between the photon solar spectral flux and the total photo-ionisation cross section for water as a function of \( \lambda \). For \( Q < 10^{28} \text{ s}^{-1} \), the maximum of solar energy deposition is below the surface (identified by the dashed, black line); in other words, the coma is optically thin \( \forall \lambda \). For \( Q = 10^{28} \text{ s}^{-1} \), the coma starts to be optically thick over the range [15, 98.2] nm, with the maximum of energy deposition between the surface and 10 \( r_c \).

For strong outgassing rates (i.e., \( Q > 10^{30} \text{ s}^{-1} \)), soft X-rays start to be absorbed in the lower part of the coma, near the surface while radiation at longer wavelengths has been fully absorbed higher up. It should be highlighted that at short wavelengths, there are large uncertainties on the photo-ionisation:

- the spectral range from 0 to 5 nm covers a large range of energies, this requires a high spectral resolution in photo-absorption cross sections, including near 2.3 nm where the cross section exhibits a peak associated with the emission of two electrons, one of them is a so-called Auger electron. Indeed, photons of these energies can ionise electrons of the inner shells (e.g., K or L shells). Then, the hole in the inner shell is filled by an electron from an outer shell with a surplus energy. This energy can be released in two ways: either through X-ray fluorescence (the electron filling the hole emits a X-ray photon) or ionising an electron from the outer shell, letting the atom/molecule into a double-ionised state. Small uncertainties on the photo-absorption cross-section lead to large ones on the location of maximum energy deposition, and hence on the vertical structure of the photo-production rate.
- the solar spectral flux used and based on TIMED/SEE is provided over a 1-nm bin and cannot capture the structure of specific solar lines.

One of the main caveats when deriving the electron production profile in Section 2.1 is that we have neglected the electron-impact ionisation by photo-electrons (and their secondaries). We only include photo-ionisation. However, depending on their initial kinetic energy – which is equal to the photon energy minus the ionisation potential –, newborn photo-electrons may in their turn ionise neutrals through electron impact. Bhardwaj (2003) provided a quantitative analysis of the ionisation by these energetic electrons. Such electrons become the main ionisation source of
the inner coma for outgassing rates above $10^{29} \text{ s}^{-1}$. So neglecting the effect of photo-electrons as a source of ionisation is valid for low $Q$, typically $<10^{29} \text{ s}^{-1}$. Finally, an additional source of plasma could be the production of cometary ions through charge exchange between cometary neutrals and the solar wind. However, this process is usually negligible, while the contribution of electron-impact as a source of ionisation was found to be significant at large heliocentric distances (Heritier et al. 2018).

We have investigated the validity of our assumption considering one single value for the photo-absorption cross-section and hence the presence of only one main ionisation peak. Fig. 5 shows the photo-electron production rate from the deposition of the soft X-ray to EUV solar spectral flux at 1 au. Depending on $\chi$ and $Q$, the photo-electron production rate exhibits one, two and even three distinct peaks above the cometary surface. Each peak is associated with the absorption of a different spectral band. The lower the photo-absorption cross-section is, the higher the column density of water needs to be to efficiently absorb the photons. The main, uppermost peak – identified by a blue triangle – corresponds to the absorption of photons mainly between 15 and 40 nm dominated by strong solar emission lines: He II (30.4 nm) and several Fe lines from different ionisation states. The mean cross section at this maximum is $(\sigma_{\text{abs}}) = 7.8 \times 10^{-22} \text{ m}^2$. The second one – identified by a yellow triangle – corresponds to the energy deposition of soft X-rays (1-10 nm) with a mean cross section $(\sigma_{\text{abs}}) = 3.8 \times 10^{-23} \text{ m}^2$. Finally, the third, least intense peak – identified by a red triangle – corresponds to the absorption of harder X-ray photons. Our secondary peak is more pronounced than the one derived by Bhardwaj (2003). It seems to result from the lower limit of 5 nm used by Bhardwaj (2003), while we extended our calculations for the solar spectral flux down to 0.1 nm. We also used updated cross-section values and at higher resolution, as described earlier in this Section.

Eq. 10 represents the photo-electron production rate as a function of $r$, $\chi$ and $Q$. It appears that for a constant outgassing rate $Q$: 

$$P_{e-,\lambda}[r, \chi, Q] = \left( \frac{\sin \chi}{\chi} \right)^2 P_{e-,\lambda} \left[ \frac{\sin \chi}{\chi} , 0 , Q \right]$$  \hspace{1cm} (13)

However, for a constant zenith angle $\chi$: 

$$P_{e-,\lambda}[r, \chi, Q] = \zeta P_{e-,\lambda}[\zeta r, \chi, \zeta Q]$$  \hspace{1cm} (14)

where $\zeta$ is a scaling factor ($\zeta = 10^{30}/Q$). Combining Eq. 13, Eq. 14 and the effect of the heliocentric distance $d_h$: 

$$P_{e-,\lambda}[r, \chi, Q] = \left[ R, 0, 10^{10} \right] \times \left( \frac{10^{30}}{Q} \right) \times \left( \sin \frac{\chi}{\chi} \right)^2 \times \left( \frac{1 \text{ au}}{d_h} \right)^2$$  \hspace{1cm} (15)

where 

$$R = \frac{10^{30} \sin \chi}{Q}$$  \hspace{1cm} (16)

From Fig. 5, it is possible to extend the photo-electron production rate for any $r$, $\chi$, $Q$ and $d_h$, applying the scaling relations given.
3. Electron density

In the present section, we investigate two mechanisms of interest in the production and loss of cometary ions in the inner coma:

- the photo-ionisation by the extreme ultraviolet solar radiation, which can be strongly attenuated through the coma,
- the $e^-$-ion dissociative recombination, which may be more significant than loss through transport at times.

This work is based on analytical solutions of the continuity equation, built upon six assumptions:

- there is no time dependency,
- the coma is assumed to have a spherical symmetry,
- the ions are moving radially at constant velocity $U_n$, which is assumed to be the same as the neutral velocity $U_n$,
- the solar flux is reduced to one single wavelength (see Section 2.2),
- no additional ionisation processes beside photo-ionisation are included; in particular, $e^-$-impact ionisation by photo-electrons and their secondaries or by the space environment has been neglected.
- the $e^-$-ion dissociative recombination rate is assumed constant throughout the coma, i.e., the electron temperature is taken constant.

The general continuity equation used here is:

$$\frac{1}{r^2} \frac{dn_i(r) U_n r^2}{dr} + \alpha n_i^2(r) = \frac{\nu_0 Q}{4\pi U_n r^2} \exp \left[ -\frac{\tau_c}{r} \right]$$

where $n_i$ stands for the ion/electron number density, $\nu_0$ for the photo-ionisation frequency, $Q$ for the total outgassing rate, $\tau_c = Q/Q_0$ for the optical depth at the cometary surface, and $\alpha$ for the $e^-$-ion dissociative recombination rate. We have decided to write the continuity equation in this unusual form, such that the loss terms (transport and $e^-$-ion dissociative recombination) on the left-hand side, and the production term, here the photo-ionisation — which can be attenuated through the coma — on the right-hand side. Considering a pure water coma and thus a cometary ionosphere solely made of water ions, the $e^-$-ion dissociative recombination rate corresponds to $4.3(\pm 0.6) \times 10^{-13}$ m$^3$ s$^{-1}$ (Rosen et al. 2000) for both ion species. The advantage is, even if there is proton transfer between H$_2$O$^+$ and water producing H$_3$O$^+$, the recombinant rate, and hence the total ion number density will not change.

3.1. Photo-chemical equilibrium including photo-absorption

In this section, we investigate the ion number density profile under photo-chemical equilibrium, i.e., by neglecting the loss through transport. As shown later in Section 4, this case applies to very active comets where the outgassing rate is high (see Section 4.2.1).

3.1.1. Theory

Neglecting transport, the continuity equation is reduced to the balance between the net chemical loss of charges and the ion production rate due to solar absorption:

$$\frac{1}{r^2} \frac{dn_i(r) U_n r^2}{dr} + \alpha n_i^2(r) = \frac{\nu_0 Q}{4\pi U_n r^2} \exp \left[ -\frac{\tau_c}{r} \right]$$

Hence, the ion number density profile is given by:

$$n_i(r) = \frac{U_n}{2 \alpha \pi r_c \nu_0} \sqrt{\frac{Q}{Q_0}} \exp \left[ -\frac{\tau_c}{r} \right]$$

where

$$Q_0 = \frac{\pi U_n^2}{\nu_0 \alpha} [s^{-1}]$$

$Q_0$ has been previously introduced by Gombosi (2015) and may be compared to the outgassing rate $Q$.

Several assumptions have been applied here:

- the finite size of the comet cannot be taken into account here. This leads to significant discrepancies near the surface compared with a full ionospheric model (e.g., see Section 3.4) because the plasma number density has to be 0 there,
- extra ionisation processes have been neglected, such as the $e^-$-impact by photo-electrons and their secondaries. For high outgassing rates, these electrons efficiently ionise the neutrals, particularly in the densest part, near the surface (Bhardwaj 2003).

3.1.2. Results

Fig. 6 shows the total ion number density resulting from photo-chemical equilibrium and including photo-absorption of the solar radiation (see Eq. 19). The dashed line corresponds to photo-chemical equilibrium in an optically thin coma (no photo-absorption). Close to the surface, the photo-absorption plays a critical role: as not all photons can penetrate deep into the coma, the plasma production rate exhibits a maximum. Hence, the plasma number density does not monotonically increase all the way down to the surface (as expected for a point source comet with an optically thin coma). Instead, it exhibits a maximum at $r = 0.5 \tau_c r_c$ (where the optical depth is 2, see Section 2.1) which is above the surface if $\tau_c \geq 2$. Thus, the ion number density peaks at the same location as the photo-electron production rate (see Section 2.2), as the transport has been neglected and $\alpha$ is constant here. At last but not least, the comet is treated as a point source here; as a result, for the optically thin case ($\tau_c = 0$, Cravens 1987), the ion plasma density tending towards $+\infty$ at $r = 0$, instead of dropping to zero at $r = 0$ for $\tau_c > 0$, which is more realistic. At larger cometocentric distances, the ion number density has the same profile regardless of the photo-absorption, as the coma is optically thin to the solar radiation.

3.2. Transport balancing photo-ionisation with photo-absorption

In this section, the ion number density profile results from the equilibrium between plasma transport and photo-ionisation, including photo-absorption; the chemical loss through $e^-$-ion dissociative recombination is neglected. This case applies when outgassing is low, that is, the transport time-scale is lower than time-scale associated with dissociative recombination (see Section 4).

3.2.1. Theory

By neglecting chemical loss, the continuity equation is reduced to:

$$\frac{1}{r^2} \frac{dn_i(r) U_n r^2}{dr} + \alpha n_i^2(r) = \frac{\nu_0 Q}{4\pi U_n r^2} \exp \left[ -\frac{\tau_c}{r} \right]$$

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The reader can notice that the solution requires the integration from the surface $r_c$ to the position $r$ of the right hand side, after having multiplied both sides by $r^2$. By the adequate substitution, one yields:

$$n_i(r) = \frac{v_0 Q}{4\pi U_0 r_c^3} \left( \frac{r}{r_c} \right)^2 \left( E_2 \left( \frac{Q}{Q_c} \frac{r_c}{r} \right) - E_2 \left( \frac{Q}{Q_c} \right) \right)$$

where $E_n$ is the generalised exponential integral which is defined by:

$$E_n[x] = x^{n-1} \int_{-\infty}^{\infty} \frac{\exp[-t]}{t^n} dt = \int_{-\infty}^{\infty} \frac{\exp[-xt]}{t^n} dt$$

$$nE_{n+1}[x] = \exp[-x] - xE_n[x]$$

Our model takes the finite size $r_c$ of the comet into account. For $r_c = 0$, Eq. 22 is reduced to the formula (44) given by Vigren & Eriksson (2017), with slight mathematical simplifications (using, for instance, the relation $E_2[x] = \exp[-x] - E_1[x]$).

The derivation of the number densities for two distinct ion species, $H_3O^+$ and $H_3O^+$, is given in Appendix B.

3.2.2. Results

Fig. 7 shows the location of the total ion number density maximum as a function of the optical depth $\tau_c$ at the comet surface. We remind the reader that the optical depth is a function of not only the outgassing rate, but also the solar zenith angle ($\tau_c[\chi] = \tau_c[0] \chi / \sin \chi$). Thus, for $\tau_c[0] \gg 1$, the maximum of the total ion number density depends on $\chi$ and the cometary ionosphere has no spherical symmetry but a cylindrical one instead: the maximum in $\chi$ of the plasma density peak occurs along the axis between the comet and the Sun. We have also assessed the location of the maximum total ion number density in the asymptotic regimes. We have found that:

- for $\tau_c, \lambda \ll 1$:
  - the maximum is located at $(r/r_c)_{\text{max}} = 2$ (see Fig. 7, blue line)
- for $\tau_c, \lambda \gg 1$:
  - the maximum is located at $(r/r_c)_{\text{max}} \sim \tau_c / T$ where $T$ is the solution of $2 \exp[T] E_2[T] = 1$, i.e., $T \approx 0.610055779\ldots$, the optical depth at the maximum (see Fig. 7, red line)

We would like to point out that $n_i(r) \ (\text{Eq.} \ 22)$ exhibits a least upper bound (see vertical, black line in Fig. 8) for increasing outgassing rates. Indeed, the plasma number density peaks at a constant optical depth for large outgassing rates. This least upper bound is given by:

$$n_i[r, \chi] < n_{i, \text{max}}[\chi] = \frac{\sin \chi v_0 T \exp[-T]}{2\tau_{\text{abs}}(\lambda) U_0}$$

In particular, for all $\chi$:

$$n_i[r, \chi] < 0.1657 \left( \frac{v_0}{\tau_{\text{abs}}(\lambda) U_0} \right) \times 10^{11} \frac{v_0 [10^{-6} \text{S}^{-1}]}{U_0 [10^3 \text{m.s}^{-1}]} \text{[m}^{-3}]$$

(24)

For a given photo-ionisation rate, even if $Q$ increases, the cometary plasma density has an upper limit depending on the solar zenith angle $\chi$. It has to be kept in mind that this limit only takes into account the photo-ionisation rate and not, for instance, extra ionisation processes, such as the electron-impact by energetic electrons from the space environment. Conversely, cometary observations of higher plasma number densities point...
out the need for including these additional processes (e.g., Galand et al. 2016; Heritier et al. 2017b). Finally, though the $e^-$-ion dissociative recombination is neglected here, this upper limit remains valid, as this additional process is a loss mechanism.

Fig. 8 shows the number densities for the total plasma (black), H$_2$O$^+$ (blue) and H$_3$O$^+$ (red). For small outgassing rates, the total ion number density is proportional to $Q$. As $Q$ increases, the total ion number density then reaches an upper limit $n_{\text{lim}}$, and its peak shifts outwards. Regarding the water ions, except at very low outgassing rates ($Q < 10^{25}$ s$^{-1}$), H$_2$O$^+$ is always the dominant ion species at low cometocentric distances while, further away, H$_3$O$^+$ becomes dominant. Moreover, we have determined the asymptotic profiles for both species: at large cometocentric distances, $n_{\text{H}_2\text{O}^+}(r) \sim 1/r$ whereas $n_{\text{H}_3\text{O}^+}(r) \sim \log(r)/r^2$; the latter is decreasing faster than the former.

Eq. 21 is not a first-order differential equation because the right hand side does not depend on the ion number density. This means that the total ion plasma number density is the integral over wavelength of the solutions, given by Eq. 22, i.e.:

$$n_i(r) = \frac{Q}{4\pi U_n^2} \frac{r^2}{r^2} \times \int_{\lambda_{\text{min}}}^{\lambda_{\text{th}}} I(\infty, \lambda) \sigma_{\text{ion}}(\lambda) \left( \frac{r}{r_c} \right) E_2 \left( \tau_{\text{d}}(r_c, \chi) \right) \left( \frac{r_c}{r} \right)^{\gamma - 1} (25)$$

Furthermore, Eq. 25 can be extended and applied to a multi-species coma, with a uniform coma composition in terms of cometocentric distance, considering the outgassing rate, the respective photo-absorption and photo-ionisation cross-sections for each neutral species $n$. Indeed, assuming the neutral composition independent of $r$, the different neutral species have the same density profile, following $\propto r^{-2}$, so that the respective cross-section can be replaced by:

$$\sigma_{\text{abs}}(\lambda) = \sum_n f_n \sigma_{\text{abs}}^n(\lambda)$$
$$\sigma_{\text{ion}}(\lambda) = \sum_n f_n \sigma_{\text{ion}}^n(\lambda)$$

where $f_n$ is the volume mixing ratio for the neutral species $n$.

### 3.3. Full ionospheric model in an optically thin coma

In this section, we rigorously assess the effect of the $e^-$-ion dissociative recombination on the plasma number density for an optically thin coma, i.e., $r_c = 0$. Such a situation has been encountered at comet 67P, at the location of Rosetta near perihelion (Heritier et al. 2018, see Section 4.2.2).

#### 3.3.1. Theory

By neglecting photo-absorption, the continuity equation is reduced to:

$$\frac{1}{r^2} \frac{dn_i(r) U_n r^2}{dr} + \alpha n_i^2(r) = \frac{\nu_0 Q}{4\pi U_n r^2} \exp \left[ -r_c \frac{r}{r_c} \right] (26)$$

Eq. 26 is a first-order ordinary differential equation, with the form of a Riccati equation. The derivation of the solution for Eq. 26 is cumbersome and provided in Appendix C. The final solution is given by:

$$n_i(r) = (\gamma - 1) \frac{U_n}{2\alpha r} \left( \frac{r}{r_c} \right)^{\gamma - 1} + \frac{1}{\gamma - 1} \left( 1 + \frac{\nu_0 Q r}{\pi U_n^3} \right) (27)$$

where $\gamma = \sqrt{1 + \frac{\nu_0 Q r}{\pi U_n^3}} = \sqrt{1 + \frac{Q}{Q_0}}$.

The location of the maximum is given in Appendix C.

For the case of a point source-like comet (i.e., $r_c = 0$) or of large cometocentric distances ($r \gg r_c$), Eq. 27 is reduced to the formula given by Gombosi (2015):

$$n_i(r) \approx \left( 1 + \frac{\nu_0 Q}{\pi U_n^3} - 1 \right) \frac{U_n}{2\alpha r} (28)$$

We have investigated both asymptotic regimes for Eq. 27 with respect to the outgassing rate $Q$:

- for $Q \ll Q_0$:

$$\gamma \sim 1 + \frac{1}{2} \frac{Q}{Q_0}$$

$$n_i(r) \approx \frac{1}{2} \frac{Q}{Q_0} \frac{U_n}{2\alpha r} \frac{r - r_c}{r_c} = \frac{\nu_0 Q}{4\pi U_n^2} \frac{r - r_c}{r^2} (29)$$

At large cometocentric distances or for a point source-like comet, Eq. 29 becomes:

$$n_i(r) \approx \frac{\nu_0 Q}{4\pi U_n^2} r (30)$$

- for $Q \gg Q_0$:

$$\gamma \sim \sqrt{\frac{Q}{Q_0}}$$

$$n_i(r) \approx \sqrt{\frac{Q}{Q_0}} \frac{U_n}{\sqrt{2\alpha r}} (31)$$

Both asymptotic profiles, Eq. 29 and 31, are decreasing as $1/r$ at large cometocentric distances. However, their respective amplitudes are different: the former is $\propto Q$ and the latter is $\propto \sqrt{Q}$.

#### 3.3.2. Results

Fig. 9 shows the comparison between existing models (Cravens 1987; Galand et al. 2016; Gombosi 2015, see also Table 1) and this work. We plot the dimensionless parameter $2\alpha n_i(r) r_c / U_n$ as a function of $r/r_c$ (see also Section 4). As the different profiles are asymptotically decreasing as $1/r$ (cf. Eq. 27, 28 and 31), the different profiles are parallel to each other for large cometocentric distances (i.e., $r \gg r_c$). Indeed, near surface, the least upper-bound is given by Galand et al. (2016) whereas further away, this is Gombosi (2015)’s profile.
Cravens (1987), Eq. 31: to put into context, originally, this model was developed for 1P. As 1P was close to the Sun during the Giotto flyby (0.89 au), the outgassing rate and the photo-ionisation rate were higher than at 67P so that \( Q/Q_0 \ll 0 \). The model proposed by Cravens (1987) is relevant when the transport is neglected against the loss by \( e^-\)-ion dissociative recombination, in particular for high outgassing rates. However, photo-absorption starts to be significant for \( Q > 10^{25} \) s\(^{-1}\), which is not included in this model. In addition, the model cannot take into account the finite size of the comet: \( n_1(r_c) \neq 0 \).

Galand et al. (2016), Eq. 29: this model was developed for 67P at large heliocentric distances (\( \geq 3 \) au) where the comet was not very active (\( Q \approx 10^{23} \sim 10^{24} \) s\(^{-1}\)). In this situation, the outward transport of plasma is the dominating net loss process. For that case, \( Q/Q_0 \ll 1 \). The finite size of the comet is taken into account in this model, which can then be used for estimating the plasma density close to the surface.

Gombosi (2015), Eq. 28: this model bridges between the two aforementioned models and their respective operational range (i.e. \( Q/Q_0 \ll 1 \) and \( Q/Q_0 \gg 1 \)) such that Eq. 28 is appropriate for intermediate outgassing rate (\( Q \sim Q_0 \)) as well, though at large cometocentric distances. Indeed, the comet is reduced to a point source and is not suitable for describing the plasma density close to the surface.

This work, Eq. 27: it is an improved version of Gombosi (2015) and Galand et al. (2016) as it is valid for any \( r \) and for any \( Q/Q_0 \) (as long as photo-absorption does not play a key role).

None of these models take into account photo-absorption. At the location of Giotto and Rosetta, even during perihelion, photo-absorption does not significantly affect the total ion number density (see Section 4). However, it may on the ion composition, which is beyond the scope of this paper.

All previous models agree or overestimate ours. Indeed, compared with Cravens (1987) and Galand et al. (2016), both models neglect one of the two loss mechanisms, either transport or the \( e^-\)-ion dissociative recombination. In comparison with Gombosi (2015), that is the size of the coma which matters. In our case, the plasma cannot be produced below the surface and be transported at larger cometocentric distances, where it contributes to the total plasma number density. However, as the plasma flows at constant velocity, the difference decreases as \( r \) increases.

We have performed a quantitative study on the relative error \( \varepsilon \) at large cometocentric distances for the different assumptions with respect to Eq. 28:

- under Cravens assumptions (Eq. 31), the relative error is less than \( \varepsilon \) if
  \[
  \frac{Q}{Q_0} > \frac{4(\varepsilon + 1)^2}{\varepsilon^2(\varepsilon + 2)^2}
  \]
  For instance, assuming a relative error less than 10%, Eq. 31 must be used down to \( Q/Q_0 \gg 48400/441 = 110 \). This has never happened during Rosetta escort phase, as the coma is dominated by transport. However, it might happen at 1P during the Giotto flyby (see Section 4).

- under Galand assumptions (Eq. 30), the relative error is less than \( \varepsilon \) if
  \[
  \frac{Q}{Q_0} < 4\varepsilon(\varepsilon + 1)
  \]
  For instance, assuming a relative error less than 10%, Eq. 30 must be used up to \( Q/Q_0 < 0.44 \), which happened at large
Fig. 9. Comparison between the different models (Cravens (1987) (red), Galand et al. (2016) (blue), Gombosi (2015) (circles) and our full model in a optically thin coma (stars) for different values of $Q/Q_0$: 0.01 (left), 4 (middle) and 100 (right). These plots are independent of $U_n$: $Q/Q_0$ is fixed for each plot and the abscissa is scaled.

Table 3 summarises the effect of the different parameters involved in this section and their respective effect on the plasma number density. For instance, the reader may notice that $\alpha$ and $U_n$ affect both $Q/Q_0$ and $U_n/2\alpha$, a scaling factor of $n_i(r)$ (Eq. 27). Regarding the dissociative recombination, it is a decreasing function of the electron temperature $T_e$. 

cometocentric distances during Rosetta escort phase but not during perihelion, as the dissociative recombination becomes a significant loss process (Heritier et al. 2018, see also Section 4).
Table 3. Effects of the different parameters on the ratio \(Q/Q_0\), \(2\alpha n_1(r)/U_n\) (see Fig. 9) and on the plasma number density \(n_1(r)\) (Eq. 27 and Eq. 28): \(\rightarrow\) means an increase and \(\downarrow\) a decrease.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(Q/Q_0)</th>
<th>(2\alpha n_1(r)/U_n)</th>
<th>(n_1(r))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_0)</td>
<td></td>
<td>(\rightarrow)</td>
<td>(\rightarrow)</td>
</tr>
<tr>
<td>(T_c)</td>
<td></td>
<td>(\rightarrow)</td>
<td>(\rightarrow)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td></td>
<td>(\rightarrow)</td>
<td>(\rightarrow)</td>
</tr>
<tr>
<td>(U_n)</td>
<td></td>
<td>(\rightarrow)</td>
<td>(\rightarrow)</td>
</tr>
</tbody>
</table>

3.4. Full ionospheric model in the general case

In this section, we take into account both plasma loss processes, that is, \(e^-\) ion dissociative recombination and transport. Furthermore, the photo-absorption of the solar EUV radiation is included. In particular, we assess the effect of the \(e^-\) ion dissociative recombination and of photo-absorption on the ion number density.

3.4.1. Theory

The continuity equation is given by Eq. 17. A few substitutions have been introduced in order to “simplify” it, which are:

\[
Y_i = \frac{4\pi U_r^2 n_i(r)}{v_0 Q} \left( \frac{r}{r_c} \right)^2
\]

\[
Z = \frac{r_c}{r} \rightarrow \frac{d}{dr} = -Z^2 \frac{d}{dz}
\]

Then, Eq. 17 becomes:

\[
\frac{dY_i}{dZ} = -\frac{1}{4} \frac{Q}{Q_0} Y_i^2 + \frac{1}{Z^2} \exp\left[-\frac{Q}{Q_c} Z\right]
\]

which has the form of the Riccati equation. Eq. 32 can be reduced to a second-order linear differential equation. Indeed, the function \(W(Z)\) defined as:

\[
W(Z) = \exp\left[ -\frac{1}{4} \frac{Q}{Q_0} \int Y_i(Z) dZ \right]
\]

is solution of:

\[
Z^2 W''(Z) - \frac{1}{4} \frac{Q}{Q_0} \exp\left[-\frac{Q}{Q_c} Z\right] W(Z) = 0
\]

To our knowledge, based on Polyanin & Zaitsev (2017), Eq. 33 does not have an analytical solution. However, we have performed numerical integration of Eq. 32 applying the explicit Runge-Kutta fourth-order method in order to derive \(n_i(r)\). Note that this form is appropriate neither for \(Q_0 = 0\) nor for \(Q_c = 0\). Moreover, the numerical solutions using Eq. 32 have been compared and agreed with the analytical solutions given by Eq. 22 when neglecting photo-absorption (i.e., for \(Q \ll Q_c\)), given by Eq. 27 when neglecting dissociative recombination (i.e., for \(Q \ll Q_0\)) and given by Eq. 30 when neglecting both processes.

3.4.2. Results

Fig. 10 shows the combined effect of the dissociative recombination and the photo-absorption on the ion number density profile. It clearly exhibits a competition between both mechanisms. On the one hand, the \(e^-\) ion dissociative recombination tends to lower the location of the maximum of the ion number density towards the comet. On the other hand, the photo-absorption tends to elevate it outwards as it reduces the electron production towards the comet. For outgassing rates below \(Q < 10^{28} s^{-1}\), there is not a clear difference between the different coloured profiles – associated with a given \(Q\) except in the first few kilometres above the surface. However, from \(Q = 10^{28} s^{-1}\) and above, we clearly see both effects. At \(Q = 10^{28} s^{-1}\), it is not possible to neglect the photo-absorption in the inner coma below \(10 r_c\), as this may lead to an overestimation by 2 or more orders of magnitude. However, including photo-absorption does not influence \(n_i\) at large cometocentric distances. Regarding the dissociative recombination, it does matter from \(Q = 10^{28} s^{-1}\), similarly to the photo-absorption. This makes sense as \(Q_c/\pi/2\) and \(Q_0\) are both around \(10^{24} s^{-1}\). Unlike the photo-absorption, the dissociative recombination still affects the ion number density at large cometocentric distances, preventing the possibility to neglect it at large outgassing rates.

4. Discussion

For each of the three cases introduced in Table 2, we review the key processes driving the solar deposition and ionospheric densities, in the context.
Fig. 11. Dimensionless parameter $2\alpha n_i(r)/U_n$ (Eq. 27) as a function of $Q/Q_0$ ($Q_0$ defined by Eq. 20). The ion number density $n_i$ is derived from: Cravens (1987) (red, solid line, Eq. 31); Galand et al. (2016) (Eq. 29) assuming $r = 1.1 r_c$ (dotted, blue line), $r = 2 r_c$ (dashed, blue line) and $r \gg r_c$ (solid, blue line, Eq. 30); Gombosi (2015) or this work assuming $r \gg r_c$ (black circles, Eq. 28); this work assuming $r = 1.1 r_c$ (blue upward-pointing triangles, ▲), $r = 2 r_c$ (orange downward-pointing triangles, ▼). Conditions at 1P during the Giotto encounter and at 67P during the Rosetta escort phase are over-plotted as grey areas. We provide a broad range of values for 1P as uncertainties remained on the electron temperature and, therefore, on the $e^-$-ion dissociative recombination rate depends on the electron temperature, which was assumed to be between 150 K (upper bound) and 1000 K (lower bound) (see Table 2). However, for 67P, the dynamic range of $Q/Q_0$ is primarily driven by the outgassing rate $Q_0$ from $10^{25}$ (end of mission) to $6 \times 10^{28}$ s$^{-1}$ (perihelion) and the heliocentric distance, affecting the photo-ionisation frequency.

4.1. Comparison with analytical models in literature

Through three decades now, several analytical models have been proposed to describe the ion number density profile in the inner part of the cometary environment. In particular, this section is dedicated to the comparison of the pros and cons between some of them, namely models proposed by Cravens (1987), Gombosi (2015), Galand et al. (2016) and in this work.

Fig. 11 shows the comparison between previous analytical models (Cravens 1987; Galand et al. 2016; Gombosi 2015) and this work (Eq. 27). We have plotted the dimensionless parameter $\eta(r) = 2\alpha n_i(r)/U_n$ (see Section 3.3) as a function of $Q/Q_0$ – where $Q_0$ is defined by relation 20 (see Section 3.1). As the different profiles are asymptotically or exactly decreasing as $1/r$ (cf. Eq. 27, 28, 29, 31 and Fig. 9), the parameter $\eta(r)$ either is or tends, at large cometocentric distances, towards a constant, which is a function of solely $Q/Q_0$ (cf. Table 4).

Fig. 11 allows to visualise the validity of each model in different regimes, i.e., for different values of the parameter $Q/Q_0$. We use Eq. 27 as a fiducial, reference model because it takes into account the finite size of the comet, plasma transport and dissociative recombination, while the other models disregard one of these characteristics or processes. Note that under high outgassing rates, close to the nucleus, photo-absorption affects plasma density. This process is not included in any of the models compared in this section but its effect at 1P during Giotto and 67P during the Rosetta escort phase is assessed in Section 3, respectively.

We review here the strengths and weaknesses of the three published models compared with the reference model and assess their domain of validity in terms of cometocentric distances and values of outgassing activity parametrised by $Q/Q_0$.


Cravens (1987), who developed a plasma density model for application to high outgassing conditions, assumed a balance between ionisation of the coma and chemical plasma loss through $e^-$-ion dissociative recombination. He neglected transport. As a result, it is expected to be suitable for high outgassing conditions, that is, when chemical plasma loss through $e^-$-ion dissociative...
recombination dominates over transport. We indeed find that the model (Fig. 11, solid red line) agrees with our reference model, for large values of \( Q/Q_0 \) (\( Q/Q_0 \approx 1 \)) and large cometary distances: \( Q/Q_0 \geq 30 \) for \( r \gg r_c \) (circles) and \( r = 2r_c \) and for \( Q/Q_0 > 1000 \) for \( r = 1.1r_c \). The photo-chemical equilibrium approach applied to Cravens (1987) does not allow to take the size of the nucleus into account. This explains the departure of this model from ours at small cometary distances. However, for small outgassing rates (\( Q/Q_0 < 5 \)), the model from Cravens (1987) considerably overestimates the ion number density by a factor \( \approx 2 \sqrt{Q_0/Q}(r-r_c)/r \). When \( Q \gg Q_0 \) during high outgassing, the plasma density depends on \( \sqrt{Q} \) (Eq. 31). The reason is that the loss term associated with dissociative recombination is quadratic to the ion number density (\( \propto n_i^2 \)) and the production term associated with photo-ionisation is \( \propto Q \). In absence of transport (\( Q_0 \rightarrow 0 \)), there is only balance between these two terms leading, to \( n_i \propto \sqrt{Q} \).

4.1.2. Galand et al. (2016)

Galand et al. (2016) proposed a plasma density model for application to low outgassing rate, close to the nucleus. This model (Fig. 11, solid, dashed and dotted blue lines) agrees with the reference model for small values of \( Q/Q_0 \) (\( Q/Q_0 \ll 1 \)), regardless of the cometary distance. Indeed, the triangles follow well the blue lines for the different cometary distances in this regime. Both models take into account the finite size of the comet. However, for large \( Q/Q_0 \), the model proposed by Galand et al. (2016) considerably overestimates the ion number density by a factor \( \approx 0.5 \sqrt{Q/Q_0}(r-r_c)/r \). Interestingly, this factor, when it is of the order of unity, allows to identify the upper cometary distance at which the model still remains valid:

\[
\frac{1}{2} \sqrt{\frac{Q}{Q_0}} \frac{r_0 - r_c}{r_0} \approx 1 \rightarrow r_0 \approx \frac{1}{1 - 2 \sqrt{Q/Q_0}} \quad (34)
\]

This means that the model by Galand et al. (2016) is valid:

- for \( Q/Q_0 \leq 4 \ \forall r \),
- or for \( Q/Q_0 \gg 1 \) with the condition on \( r \in [r_c; r_0] \) (Eq. 34).

As Giotto approximatively flew by 1P at 596 km (\( \approx 108 r_c \)), the plasma density measured in-situ was not significantly sensitive to photo-absorption (see Fig. 12). Moreover, the attenuation by photo-absorption is further damped when dissociative recombination is included. Fig. 12 shows a comparison of the plasma density calculated with (no circles) and without (circles) photo-absorption. At a given cometary distance, this process affects more significantly the ion number density in absence of the dissociative recombination.

1P was at the limits of our model validity concerning photo-absorption, as we have assumed one plasma density peak driven by solar EUV radiation. For \( Q = 6.9 \times 10^{28} \text{ s}^{-1} \), a secondary peak in ionisation is expected to be present near the nucleus surface (cf. Fig. 5; at 1P, the surface is located at \( R = 3 \times 8 \times 10^3 \text{ km depending on the solar zenith angle near the yellow triangle} \). Using the mean photo-absorption cross section associated with the secondary peak (i.e., \( \sigma_{\text{abs}} = 3.8 \times 10^{-22} \text{ m}^2 \), see Section 2.2), it yields \( Q_c[0] \approx 1.6 \times 10^{10} \text{ s}^{-1} \), close to the outgassing rate at 1P/Halley. Note that Giotto flew by a large range of solar zenith angles from 0 to \( \chi_{\text{max}} \approx 110^\circ \), with the associated value \( Q_c[\chi_{\text{max}}] \approx 0.5Q_c[0] \).


The model proposed by Gombosi (2015) (circles) agrees well with our reference model at any values of \( Q/Q_0 \), but only for \( r \gg r_c \) or for \( r = r_c \). Indeed, Table 4 shows that the parameter \( \eta(r) \) is the same for the model from Gombosi (2015) as for ours under these assumptions. However, at finite and low cometary distance (e.g., \( r = 1.1r_c \) and \( 2r_c \)), the model from Gombosi (2015) departs from our reference model for small \( Q/Q_0 \), typically less than 10. This discrepancy is due to the point-source approximation applied by Gombosi (2015).

Dissociative recombination In Fig. 11, we have over-plotted the condition encountered at 1P in terms of the parameter \( Q/Q_0 \) (grey areas). It shows that \( Q/Q_0 \) was between \( \approx 62 \) (\( T_e = 1000 \text{ K} \)) and 161 (\( T_e = 150 \text{ K} \)) during the Giotto’s flyby. Therefore, transport can be neglected against dissociative recombination. Physically, the ions do not have enough time to be transported further away from the comet, they rather quasi-instantly recombine with locally produced electrons. This also provides another underlying assumption here: the ion velocity is close to 0 or is very small with respect to the neutral velocity, leading to a significant ion-neutral friction, at the origin, a priori, of the diamagnetic cavity (Cravens 1987; Ip & Axford 1987).

When both photo-absorption and dissociative recombination are included, our results are in good agreement with those of Marconi & Mendis (1984). For similar input parameters (\( Q = 4.7 \times 10^{29} \text{ s}^{-1} \), \( v_{\text{1 au}} \approx 4 \times 10^{-7} \text{ s}^{-1} \), \( U_n \approx 600 \text{ m.s}^{-1} \), \( r_c \approx 3 \text{ km} \), and \( T_e \approx 150 \text{ K} \) not shown here), we get a maximum plasma density of \( 4.6 \times 10^{10} \text{ m}^{-3} \) at \( r \approx 46 \text{ km} \) and \( \chi = \pi/2 \), to be compared with \( 1.9 \times 10^{10} \text{ m}^{-3} \) at \( r \approx 47 \text{ km} \) from Marconi & Mendis (1984). Possible reasons of discrepancy may be related to photo-absorption effects (e.g., different \( \chi \) and dust),
photo-absorption cross-sections combined with different neutral compositions (pure water here compared with 85% H2O, 10% CO2 and 5% N2 from Marconi & Mendis 1984), ion velocity and electron temperature profiles (constant through the coma here unlike in Marconi & Mendis 1984).

Summary and comparison with observations Scrutinising the nucleus’ size should be considered. Therefore, the derivation to be included for estimating such as at the location of Giotto. Photo-absorption need however show how ever approach (black crosses, Altwegg et al. 1993), as shown in Fig. 10 of plasma density measured during the Giotto flyby near closest approach of 1P (solid, red line) with observations the electron number density closer to the nucleus. refined model (see Fig. 12, green solid line), in order to assess the differential equation proposed in Section 3.4, for a more density in the inner coma probed in-situ by Giotto, – to solve photo-absorption) if one is only interested in the electron number density in the inner coma at 1P, we recommend: – to use Eq. 27 as an analytical model (with a finite size for the nucleus but without including coma at 1P, we recommend: – to use Eq. 27 as an analytical model (see Fig. 12, green solid line), in order to assess the electron number density closer to the nucleus.

4.2.2. 67P

Photo-absorption The case study of 67P is quite different from 1P. Through its two-year escort phase, Rosetta probed comet 67P over a large range of local outgassing rates, between ~1026 and ~6 × 1028 s⁻¹ (Hansen et al. 2016), which has to be compared with the parameter Q_e[π/2] (Eq. 4). As Rosetta spent most of the time in the terminator plane (i.e., γ = π/2), Q_e[π/2] = 2Q_e[0]π/2 ≈ 1.2 × 1028 s⁻¹. Based on the outgassing rate at 67P during the mission, Q was greater than Q_e[π/2] for a short period of time, during July and August 2015, approximatively 2 months near perihelion, between 1.24 and 1.3 au. This represents less than 10% of the escort phase. Outside this period, the photo-absorption does not need to be taken into account, including the inner coma, between the location of Rosetta and the surface. Furthermore, at the location of Rosetta, even during perihelion, photo-absorption does not affect significantly the local plasma density as Rosetta was located at cometocentric distances beyond 150 km which corresponds to r/r_c ≈ 75 (see Fig. 4 in Heritier et al. 2017a, and Fig. 12).

Dissociative recombination Excluding the uncertainties on the dissociative recombination rate (i.e., on T_e), the conditions met at 67P covered a broad range of values for the less dimensionless parameter Q_e/Q0. As the outgassing rate and the photo-ionisation go hand in hand with the heliocentric distance, the ratio Q_e/Q0 (∝ ν_0Q) ranges from ~1.7 × 10⁻⁴ to ~7.2, assuming an electron temperature between 150 and 1000 K. The former provides a safe upper limit: Q_e/Q0 is a decreasing function of the electron temperature and 150 K is of the order of the neutral temperature (Davidsson & Gutiérrez 2005; Tenishev et al. 2008; Heritier et al. 2017a). The lower bound could be even smaller as plasma instruments have reported electron energies around ~10 eV (Eriksson et al. 2017). According to the photo-ionisation frequency along the escort phase (Fig. 3 in Heritier et al. 2018) and the outgassing rate, the parameter Q_e/Q0 was of the order of the unity and above for the period 2015 August-November, assuming T_e ~ 200 K (Fig 6 in Heritier et al. 2018). Therefore, the dissociative recombination is not significant apart from this period. A detailed and quantitative analysis is made in Heritier et al. (2018) throughout the escort phase and showed that at the location of Rosetta, dissociative recombination affects by more than 20% the plasma density from February 2015 to February 2016.

Summary At the location of Rosetta, photo-absorption did not affect significantly the plasma number density. Furthermore, except a period near perihelion, covering the period from February 2015 to February 2016, the dissociative recombination did not significantly influence the ion number density profile of the inner coma at Rosetta. Hence:

Table 4. Parameter η(r) – defined by η(r) = 2an_0(r)r/U_n – and its asymptotic value function of Q/Q_0 – where Q_0 is defined by Eq. 20, for the different analytical models compared.

<table>
<thead>
<tr>
<th>Analytical models</th>
<th>Cravens (1987)</th>
<th>Galand et al. (2016)</th>
<th>Gombosi (2015)</th>
<th>This work (Eq. 27)</th>
</tr>
</thead>
<tbody>
<tr>
<td>η(r)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>\lim_{r \to 0} \eta(r) or r_c = 0</td>
<td>\sqrt{Q \over Q_0}</td>
<td>\sqrt{1 + Q \over Q_0 - 1}</td>
<td>\sqrt{1 + Q \over Q_0 - 1} + r_c^{-1}</td>
<td></td>
</tr>
</tbody>
</table>

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away from perihelion, the most appropriate and simplest analytical model remains Eq. 29 (Galand et al. 2016),

around perihelion, as 67P was getting closer to the Sun, both photo-ionisation frequency and the outgassing activity increased: dissociative recombination became more efficient and \( Q/Q_0 \) increased to be of the order of unity, considered as an intermediate value. Neither Eq. 29 nor Eq 31 are appropriate, as the discrepancies from the reference profile given by Eq. 27 is maximum for \( Q/Q_0 = 4 \) (see Fig. 11). Furthermore, Eq. 27 \( vr \) or Eq. 28 for large cometocentric distances should be a priori suitable.

There are however caveats when comparing the analytical models with observations as discussed in the next paragraph.

Comparison with in-situ electron density measurements by Rosetta Fig. 12 shows a comparison between plasma measurements at 67P by Rosetta Plasma Consortium (RPC)/Mutual Impedance Probe (MIP) (Trotignon et al. 2007) at 3.2 au on October 2014 (black pulses, Galand et al. 2016) and modelled plasma density profiles (blue) for the corresponding conditions (see Table 2). We have considered a period when ionisation was primarily driven by EUV solar radiation, i.e. when Rosetta was above the northern, summer hemisphere pre-perihelion. Rosetta was close to the nucleus, between 10 \((r/r_c \approx 5)\) and 20 km \((r/r_c \approx 10)\), and the outgassing rate was low. Any model including transport agrees well with the observations, within the errors of input parameters. Photo-absorption and dissociative recombination play a negligible role. The plasma density results from the balance between transport and photo-ionisation at that time.

At large heliocentric distances, during pre-perihelion over the winter hemisphere and post-perihelion over both hemispheres, multi-instrument analysis of RPC and ROSINA sensors highlighted that \( e^-\)-impact is an additional source of ionisation (Galand et al. 2016; Heritier et al. 2017b, 2018) and has to be considered during periods of low activity. The origin of these ionising electrons is not well understood yet but seems to be of external origin (Deca et al. 2017). In addition, plasma measurements down to the surface for the end of mission at 3.8 au showed that the acceleration of the neutral gas, in an adiabatic expansion, must be taken into account, at least for the first kilometres above the surface (Heritier et al. 2017b). At larger cometocentric distances, its effect on the plasma density depletes and is negligible as the neutral gas quickly reaches a steady velocity.

Near perihelion, the main source of ionisation is EUV solar radiation, and dissociative recombination can be significant unlike photo-absorption (Heritier et al. 2018). However, even considering these processes, there are still discrepancies between measured plasma densities and the predicted ones. Indeed, Henri et al. (2017) reported in-situ plasma observations and ion-to-neutral ratio for April 2015-February 2016 period, from 1.24 au to 2.4 au. Over this period, they found that:

\[
\frac{n_i}{n_n} \approx 1.2 \times 10^{-10} r
\]  

(35)

As near perihelion, dissociative recombination is significant but photo-absorption can be neglected, Eq. 28 can be be used to derive \( n_i/n_n \). The asymptotic ion-to-neutral ratio is:

\[
\frac{n_i}{n_n} \approx \frac{2Q_0}{Q} \left( \sqrt{\frac{1 + Q}{Q_0} - 1} \right) \frac{v_0}{U_n} r \leq \frac{v_0}{U_n} r
\]  

(36)

where \( (v_0/U_n)r \) represents the ratio at large cometocentric distances when only photo-ionisation and transport are included. Even if the dissociative recombination is included, using values provided in Table 2 for 67P at perihelion, we get \( n_i/n_n \approx 2.6 \times 10^{-10} r \). It is difficult to perform a thorough, quantitative comparison. Indeed, the relation 35 covered time period during which both the heliocentric distance and the outgassing rate varied. The overestimation by our theoretical approach with respect to the observations may be linked to over/underestimations of several parameters, such as:

- an underestimation of \( Q \) (unlikely as we use the largest outgassing rates),
- an overestimation of \( v_0 \),
- an underestimation of \( \alpha \), meaning an overestimation of \( T_e \) (unlikely, as the assumed \( T_e \) is already very low ),
- an overestimation of \( U_i \) (unlikely as it is in the range observed by MIRO, Heritier et al. 2018),
- an underestimation of \( U_n \), here supposed to be equal to \( U_n \).

Some are addressed in the following paragraphs.

Overestimation of \( v_0 \) near perihelion: Johansson et al. (2017) reported the photo-emission current from EUV solar radiation by the dual Langmuir Probe (LAP) (Eriksson et al. 2007) on-board Rosetta. It appeared that while they agree away from perihelion, the measured and expected photo-emission current at Rosetta differed by a factor up to 2 near perihelion: it is correlated with the outgassing activity and the heliocentric distance. Nanograins of \(~20 \text{ nm} \) size could be responsible for attenuating the solar radiation. Consequently, we would expect two effects on the plasma:

- the photo-absorption cross-section should be increased, depending on the size distribution of the nanograins,
- by lessening the photo-electron production, the ion number density may be lower than anticipated, yielding a lower effect of dissociative recombination on the plasma density.

Underestimation of \( U_i \) near perihelion: From the analyses of Rosetta Plasma Consortium (RPC, Carr et al. 2007) and ROSINA dataset, Vigren et al. (2017) reported ion velocities in the range \(~2-8 \text{ km.s}^{-1} \), significantly larger than the neutral velocity or the one assumed in this work. Assuming such a velocity leads to several corrections in the different analytical formulas we have provided so far in this paper. Indeed, for \( U_i \neq U_n \) and constant,

- in Section 3.1, 3.3 and 3.4: the parameter \( Q_0 \) changes and should be replaced by

\[
Q_0 = \frac{\pi U_n U_i^2}{v_0 \alpha}
\]  

(37)

where \( U_i \) is the ion velocity, taken constant. As \( U_i \) is higher than \( U_n \), \( Q/Q_0 \) is lower than plotted in Fig. 11, reducing the effect of the dissociative recombination against transport. In addition, the factor \( U_n/2\alpha \) has to be replaced by \( U_i/2\alpha \),

- in Section 3.2, the denominator \( U_n^2 \) has to be replaced by \( U_i^2 \),
- in Eq. 36, \( v_0/U_n \) should be replaced by \( v_0/U_i \).

By increasing the ion velocity, the loss by transport is higher than assumed in this paper such that we give here an overestimation of the ion number density near perihelion.
Fig. 12. Comparison of the plasma number density profiles with respect to the scaled cometocentric distances between 1P (red, 0.89 au) and 67P (blue, 3.2 au, and green, 1.24 au). The profiles have been generated under different assumptions: both dissociative recombination (DR) and photo-absorption (PA) included (thick, solid lines), PA excluded (circles), DR excluded (dotted lines). The dashed line corresponds to the plasma density profile from Cravens (1987) with $T_e = 1000$ K. The black crosses with the error bars are the in-situ plasma density measurement by the ion mass spectrometer onboard Giotto at 1P/Halley during the Giotto flyby (Altwegg et al. 1993). The black pluses are in-situ plasma density observations by RPC-MIP onboard Rosetta at 3.2 au (Galand et al. 2016).

5. Conclusion

In this paper, we assess key processes driving the structure of cometary ionospheres for low and intermediate activities. In Section 2, for the first time, we show that in cometary ionospheres, the production of photo-electrons peaks where the optical depth is 2, unlike at planetary atmospheres under hydrostatic equilibrium where it peaks at an optical depth of 1. Moreover, in the limit of outgassing rates lower than $10^{29}$ s$^{-1}$, the mean photo-absorption cross-section is constant. In Section 3, we provide analytical formulas for the ion number density profile in the inner ionosphere of comets, completing and comparing with the past literature. In particular, we have assessed the effects, combined or not, of the photo-absorption and the dissociative recombination in the coma, and assess their respective contribution to the plasma density in the context of the Giotto and Rosetta missions (Section 4). At 1P during the Giotto flyby, dissociative recombination was dominant over transport and photo-absorption was significant close to the nucleus. Nevertheless, the latter affected the lower part of the ionosphere, which was not sampled by the Giotto in-situ measurements. At 67P, photo-absorption and, away from perihelion, dissociative recombination, do not affect significantly the plasma density at the location of Rosetta. Photochemical equilibrium models as developed for 1P/Halley (e.g., Cravens 1987) are not suitable for application to 67P/Churyumov-Gerasimenko. Plasma transport is always significant in the coma of comet 67P probed by Rosetta.

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Fig. A.1. Schematic of the configuration between the Sun (yellow), the comet (black) and a water molecule, to determine the column density at a given \((r, \chi)\) in the coma. \(s\) is the abscissa along the line between the molecule and the Sun, \(s_0\) is the lower bound of integration \((s_0 < 0\) and \(\chi > \pi/2\) in the figure as \(s\) is increasing sunward), \(r_s\) is the cometocentric distance for a given \(s\) which is \(r_s = \sqrt{s^2 + r^2 \sin^2 \chi}\).

\[ s = +\infty \quad r \sin \chi \]

\[ s = 0 \]

\[ s_0 = r \cos \chi \]

\[ \text{Appendix A: Optical depth on comets} \]

In this appendix, we demonstrate analytically that the maximum photo-ionisation rate occurs at \(\tau_A = 2\), as the neutral number density decreases as \(\sim r^{-2}\) (Hässig et al. 2015). However, we first derive a general result assuming a more general profile for neutrals given by:

\[ n(r) = n_0 (r/r_0)^{-\beta} \]  \hspace{1cm} (A.1)

where \(r\) is the cometocentric distance, \(n_0\) is the number density at \(r_0\), and \(\beta > 1\). The coma is assumed here to be reduced to one neutral species and to be spherically symmetric.

In order to evaluate the optical depth, the column number density \(N(r, \chi)\) has to be assessed. This quantity depends on the cometocentric \(r\) and on the solar zenith angle \(\chi\). The column number density at the point \((r, \chi)\) in polar coordinates \((\chi = 0\) is along the Sun-comet line on the dayside) – of which the case \(\beta = 2\) is available in Beth et al. (2016) – is given by (see Fig. A.1):

\[ N[r, \chi] = n_0 \beta \int_{s_0=0}^{s=\infty} \frac{1}{r_0^2 \sin^2 \chi + s^2} s^{-\beta/2} ds \]

\[ = n_0 \beta (r \sin \chi)^{-\beta} \int_{r \cos \chi}^{\infty} \left(1 + \frac{s^2}{r^2 \sin^2 \chi}\right)^{-\beta/2} ds \]  \hspace{1cm} (A.2)

valid even if \(\chi > \pi/2\) (cf. Fig. A.1). By applying the following substitution:

\[ X = \frac{s}{r \sin \chi}, \]

\[ dX = \frac{ds}{r \sin \chi} \]

This yields:

\[ N[r, \chi] = n_0 \beta (r \sin \chi)^{-\beta} \int_{1/\tan \chi}^{\infty} (1 + X^2)^{-\beta/2} dX \]  \hspace{1cm} (A.3)

The right hand side is then separable, i.e., it is the product between a function of solely \(r\), \(F(r)\), and of solely \(\chi\), \(G(\chi)\):


where

\[ F[r] = n_0 r^\beta \]

\[ G[\chi] = \frac{\sin \beta + 1}{\sin \beta} \int_{1/\tan \chi}^{\infty} (1 + X^2)^{-\beta/2} dX \]

and

\[ G[\chi] = \sin^{-\beta+1} \chi \int_{1/\tan \chi}^{\infty} (1 + X^2)^{-\beta/2} dX \]

\[ = \sin^{-\beta+1} \chi \int_0^{\chi} \sin \beta - 2 \Theta d\Theta \]

\[ = \sin^{-\beta+1} \chi \left( C_\beta - \cos \chi \frac{3 - \beta}{2} \cdot \frac{\sin^2 \chi}{2} \right) \]

where (see Formula 3.249.8, Gradsteyn & Ryzhik 2015)

\[ C_\beta = \int_0^{\infty} (1 + X^2)^{-\beta/2} dX = \frac{2\pi \Gamma[\beta - 1]}{2^\beta \Gamma[\beta/2]} = \frac{\sqrt{\pi} \Gamma[\beta/2]}{\Gamma[\beta]} = G[\pi/2] \]

\(G[\chi]\), the Gamma function, and \(2F_1\) is the Gauss hypergeometric function. The optical depth \(\tau_A\) at the wavelength \(\lambda\) is given by:

\[ \tau_A[r, \chi] = \sigma_{\text{abs}}[\lambda] N[r, \chi] \propto r^{-\beta+1} G[\chi] \]

with \(\sigma_{\text{abs}}[\lambda]\) stands for the photo-absorption cross-section at the wavelength \(\lambda\). Considering the Beer-Lambert-Bouguer law, the electron production rate, \(P_{e^{-}\lambda}\) at a given wavelength \(\lambda\), is:

\[ P_{e^{-}\lambda}[r, \chi] = v_0 n(r) \exp[-\tau_A(r, \chi)] \]  \hspace{1cm} (A.4)

with \(v_0\) the photo-ionisation rate prior to any absorption. Hence, the maximum of \(P_{e^{-}\lambda}\) reached at a cometocentric distance \(r_{\text{max}}\) for a given \(\chi\) within the coma fulfills:

\[ \frac{\partial P_{e^{-}\lambda}}{\partial r} = 0 \rightarrow \frac{dn}{dr} - n(r) \frac{\partial \tau_A}{\partial r} = 0 \]  \hspace{1cm} (A.5)

However, from Eq. A.1:

\[ \frac{dn}{dr} = -\frac{\beta}{n} r (n) \]

and from Eq. A.3:

\[ \frac{\partial \tau_A}{\partial r} = \frac{1 - \beta}{r} \tau_A \]

Hence, Eq. A.5 implies:

\[ \frac{\beta}{n} r (n) + \frac{1 - \beta}{r} \tau_A[r, \chi] n(r) = 0 \]

This leads to the following expression for the optical depth, \(\tau_{\text{max}, \lambda}\) at \(r_{\text{max}}\):

\[ \tau_{\text{max}, \lambda} = \frac{\beta}{\beta - 1} \]  \hspace{1cm} (A.6)

which is independent of \(\chi\). In fact, if we express \(P_{e^{-}\lambda}\) as a function of \(\tau_A\), instead of \(r\), for a given \(\chi\) we get:

\[ P_{e^{-}\lambda} \propto \tau_A^{\frac{\beta}{\beta - 1}} \exp[-\tau_A] \]

The maximum occurs at \(\beta/(\beta - 1)\), consistent with Eq. A.6.
For $\beta = 1$, the number density is $\propto 1/r$ and the column number density is infinite; this leads to an undetermined value of $\tau_{\text{max},1}$. For comets, $\beta = 2$ which yields: $\tau_{\text{max},1} = 2$.

For large $\beta$, $\tau_\text{f}$ converges towards 1. This corresponds to what is obtained for an isothermal atmosphere in hydrostatic equilibrium. Indeed, if we consider that the surface is at $r_0$ ($r_0 > 0$, not a point source) and the altitude $z$ is such that: $z = r - r_0$, Eq. A.1 becomes:

$$n(z) = n_0 \left(1 + \frac{z}{r_0}\right)^{-\beta} \quad (A.7)$$

However, the density scale height $H$ is defined as:

$$H(r) = \frac{dn}{dr} = \frac{r}{\beta}$$

from Eq. A.4, this yields: $H(r) = r/\beta$. Hence, in the neighbourhood of $r = r_0$ and thus of $z = 0$, Eq. A.7 becomes:

$$n(z) \approx n_0 \left(1 + \frac{z}{\beta H(r_0)}\right)^{-\beta} \quad (A.8)$$

such that

$$\lim_{\beta \to \infty} n_0 \left(1 + \frac{z}{\beta H(r_0)}\right)^{-\beta} = n_0 \exp\left[-\frac{z}{H(r_0)}\right] \quad (A.9)$$

which corresponds to the number density for an isothermal atmosphere in hydrostatic equilibrium.

**Appendix B: Analytical ion number density profiles of $H_2O^+$ and $H_3O^+$**

To go a bit further, it is possible to distinguish two water ion-species, $H_2O^+$ and $H_3O^+$, for a pure water coma. Keeping in mind that the net chemical charge loss (e-ion dissociative recombination) is ignored, the two main chemistry channels are:

- $H_2O + h\nu \rightarrow n_0 \quad H_2O^+ + e^-$
- $H_2O^+ + H_2O \quad \Rightarrow \quad H_3O^+ + HO$

and their respective continuity equations:

$$\frac{dn_{H_2O^+}(r)^2}{dr} = \frac{\nu_0 Q}{4\pi U_n} \exp\left[-r c / \beta r\right] - \frac{k_1 Q}{4\pi U_n^2} n_{H_2O^+}(r) \quad (B.1)$$

$$\frac{dn_{H_3O^+}(r)^2}{dr} = \frac{k_1 Q}{4\pi U_n^2} n_{H_2O^+}(r) \quad (B.2)$$

Eq. B.1 is a simple first-order linear differential equation and can be solved analytically:

$$n_{H_2O^+}(r) = \frac{\nu_0 Q}{4\pi U_n^2 r c} \exp\left[\frac{r c}{\beta r}\right] \times$$

$$\left\{ \begin{array}{l} \frac{r}{r c} \left[ \frac{Q}{Q_1} + \frac{Q_0}{Q_1} \right] E_2 \left[ \frac{Q}{Q_1} + \frac{Q_0}{Q_1} \right] \end{array} \right\} \quad (B.3)$$

where the parameter $Q_1 = 4\pi U_n^2 r c / k_1 \approx (2.5 - 5.6) T \times 10^{23} \text{[s}^{-1}]$ (for $U_n = 600 - 900 \text{ m.s}^{-1}$, $k_1$ from Huntress & Pinizzotto 1973, $T$ is the temperature, in kelvins, of the neutrals supposed to be 150 K), with the dimension of an outgassing rate. As the total ion number density $n_i(r)$ is given by Eq. 22, the number density of $H_2O^+(r)$ is obtained from $n_{H_2O^+}(r) = n_i(r) - n_{H_3O^+}(r)$, yielding:

$$n_{H_3O^+}(r) = \frac{\nu_0 Q}{4\pi U_n^2 r c} \times$$

$$\left\{ \begin{array}{l} \frac{r}{r c} \left[ \frac{Q}{Q_1} + \frac{Q_0}{Q_1} \right] - \exp\left[\frac{Q}{Q_1}\right] E_2 \left[ \frac{Q}{Q_1} + \frac{Q_0}{Q_1} \right] \end{array} \right\} \quad (B.4)$$

**Appendix C: Solution for Section 3.3**

In absence of photo-absorption by the coma, Eq. 17 is reduced to:

$$\frac{dn_i(r)^2}{dr} = \frac{\nu_0 Q}{4\pi U_n^2} - \frac{\alpha n_i^2(r)^2}{U_n} \quad (C.1)$$

which is a Riccati equation. There is no straightforward solution for such an equation and a few manipulations have to be performed to get one. The first substitution is:

$$y \equiv n_i(r) \quad (C.2)$$

Eq. C.1 becomes:

$$r \frac{dy}{dr} = \frac{\nu_0 Q}{4\pi U_n^2} - \frac{\alpha y^2}{U_n} - y \quad (C.3)$$

Eq. C.3 is completely separable and can be rewritten as follows:

$$-\frac{U_n}{\alpha} \frac{dy}{y^2} + \frac{U_n}{\alpha} \frac{dy}{y} = \frac{\nu_0 Q}{4\pi U_n^2 \alpha} \quad (C.4)$$

The left-hand side only depends on the variable $y$, whereas the right-hand side, only on the variable $r$. The right-hand side is easy to integrate unlike the left-hand side. First, we have to perform a partial fraction decomposition and find the roots of the following quadratic polynomial:

$$y^2 + \frac{U_n}{\alpha} y - \frac{\nu_0 Q}{4\pi U_n^2 \alpha} = 0 \quad (C.5)$$

which are

$$y_s = \frac{U_n}{2\alpha} \pm \frac{U_n}{2\alpha} \sqrt{1 + \frac{\nu_0 Q \alpha}{\pi U_n^2 \alpha}} = \frac{U_n}{2\alpha} (1 \pm \gamma) \quad (C.6)$$

where

$$\gamma = \sqrt{1 + \frac{\nu_0 Q \alpha}{\pi U_n^2 \alpha}} \approx 1 + \frac{Q}{Q_0} \quad (C.7)$$

where

$$Q_0 = \frac{\pi U_n^2}{\nu_0 \alpha} \quad \text{(C.8)}$$

Thus, Eq. C.4 becomes:

$$-\frac{1}{\gamma} \left( \frac{dy}{y - Y_s} - \frac{dy}{y - Y_s} \right) = \frac{dr}{r} \quad (C.9)$$

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The integration of both sides leads to:

$$\frac{-1}{\gamma} \log \left[ \frac{y - Y_c}{y_0 - Y_c} \right] - \log \left[ \frac{y - Y_c}{y_0 - Y_c} \right] = \log \left[ \frac{r}{r_0} \right] \tag{C.10}$$

where $y_0$ is the initial value of $y$ at $r = r_0$. By assuming that the plasma number density is 0 at the surface ($y_0 = y(r_0) = y(r_c) = 0$), it turns that:

$$\frac{1}{\gamma} \log \left[ \frac{Y_c(y - Y_c)}{Y_c(y - Y_c)} \right] = \log \left[ \frac{r}{r_c} \right] \tag{C.11}$$

which is simply:

$$Y_c(y - Y_c) = Y_c(y - Y_c)r_c^\gamma \tag{C.12}$$

Some rearrangements are made to get the final expression for $y$:

$$y(Y, r - Y_c r_c^\gamma) = Y_c(Y, r - Y_c r_c^\gamma) \tag{C.13}$$

That is, from Eq. C.6:

$$\frac{U_n}{2\alpha} y (r^\gamma - r_c^\gamma + r\gamma r + r_c^\gamma) = -\frac{v_0 Q}{4\pi U_n} (r^\gamma - r_c^\gamma) \tag{C.14}$$

$$y (r^\gamma - r_c^\gamma + r\gamma r + r_c^\gamma) = \frac{2\nu_0 Q}{4\pi U_n} (r^\gamma - r_c^\gamma) \tag{C.15}$$

which yields:

$$y = \frac{\nu_0 Q}{2\pi U_n^2 r (1 + \gamma) r^\gamma - (1 - \gamma) r_c^\gamma} \tag{C.16}$$

According to the relation between $y$ and $n_i$ given by Eq. C.2, the total ion density is given by:

$$n_i(r) = \frac{\nu_0 Q}{2\pi U_n^2 r (1 + \gamma) r^\gamma - (1 - \gamma) r_c^\gamma} \tag{C.17}$$

Hence, using the previously defined quantity $Q_0$:

$$n_i(r) = \frac{Q}{Q_\infty} \frac{U_n}{2\alpha r} \left( \frac{r}{r_c} \right)^\gamma - 1 \tag{C.18}$$

From Eq. C.7 and C.18:

$$\frac{1}{\gamma + 1} \frac{Q}{Q_0} = \frac{\gamma - 1}{\gamma - 1} \frac{Q}{Q_0} = \frac{\gamma - 1}{\gamma - 1} \frac{Q}{Q_0} = \gamma - 1 \tag{C.19}$$

Therefore:

$$n_i(r) = \frac{U_n}{2\alpha r} (\gamma - 1) \left( \frac{r}{r_c} \right)^\gamma - 1 \tag{C.20}$$

The location of the maximum is:

$$\left( \frac{r}{r_c} \right)_{\text{max}} = \sqrt[2\gamma + 1]{\frac{2\gamma + 1}{\gamma + 1}} + \frac{\gamma}{\gamma + 1} \sqrt[2\gamma + 3]{\gamma^2 + 3} \tag{C.21}$$

of which:

$$\lim_{\gamma \to -1} \sqrt{\frac{\gamma^2 + 1}{\gamma + 1}} + \frac{\gamma}{\gamma + 1} \sqrt{\gamma^2 + 3} = 2 \tag{C.22}$$

$$\lim_{\gamma \to +\infty} \sqrt{\frac{\gamma^2 + 1}{\gamma + 1}} + \frac{\gamma}{\gamma + 1} \sqrt{\gamma^2 + 3} = 1 \tag{C.23}$$

such that $1 < (r/r_c)_{\text{max}} < 7$ whatever $\gamma$ is.

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