Does Household Finance Matter? Small Financial Errors with Large Social Costs

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Households with familiarity biases tilt their portfolios toward a few risky assets. The resulting mean-variance loss from portfolio underdiversification is equivalent to only a modest reduction of about 1% per year in a household's portfolio return. However, once we consider also the effect of familiarity biases on the asset-allocation and intertemporal consumption-savings decisions, the welfare loss is multiplied by a factor of four. In general equilibrium, the suboptimal decisions of households distort also aggregate growth, amplifying further the overall social welfare loss. Our findings demonstrate that financial markets are not a mere sideshow to the real economy and that improving the financial decisions of households can lead to large benefits, not just for individual households, but also for society.

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One of the fundamental insights of standard portfolio theory (Markowitz, 1952, 1959) is to hold diversified portfolios. However, evidence from natural experiments (Huberman, 2001) and empirical work (Dimmock et al., 2014) shows households invest in underdiversified portfolios that are biased toward a few familiar assets.[1] Familiarity biases may be a result of geographical proximity, employment relationships, language, social networks, and culture (Grinblatt and Keloharju, 2001). Holding portfolios biased toward a few familiar assets forces households to bear more financial risk than is optimal. Calvet, Campbell and Sodini (2007) empirically study the importance of household portfolio return volatility for welfare.[2] They find that, within a static mean-variance framework, welfare costs for individual households arising from underdiversified portfolios are modest. We extend the static framework to a dynamic, general-equilibrium, production-economy setting to examine how underdiversification in household portfolios impacts the asset-allocation and intertemporal consumption decisions of individual households, and upon aggregation, real investment, aggregate growth, and social welfare.

In our paper, we address the following questions. How large are the welfare costs of underdiversification for individual households?[3] Do the consequences of household-level portfolio errors cancel out, or does aggregation amplify their effects, thereby distorting growth and imposing significant social costs? How large are the negative macroeconomic effects for the aggregate economy when households invest in underdiversified portfolios? Are pathologies such as familiarity biases in financial markets merely a sideshow, or do they impact the real economy?[4] In short, does household finance matter?

Our paper makes two contributions. First, we show that even if the welfare loss to a household from investing in an underdiversified portfolio is modest, once we incorporate the effect of familiarity biases on the household's decision to allocate wealth between risky and safe assets and on its consumption-savings decision, the welfare loss to the household is am-

[1] For surveys of the portfolio behavior of households, see Guiso, Haliassos and Jappelli (2002), Haliassos (2002), Vissing-Jorgensen (2003), Campbell (2006), and Guiso and Sodini (2013).

[2] The analogous question at the macroeconomic level has been studied by Lucas (1987, 2003).

[3] These welfare costs for individual households are an example of what is often referred to as an "internality" in public economics literature. Herrnstein et al. (1993, p. 150) use internality to refer to a "within-person externality," which occurs when a person ignores a consequence of her own behavior for herself.

[4] For a review of the literature on the interaction between financial markets and the real economy, see Bond, Edmans and Goldstein (2012).

plified by a factor of four. Thus, familiarity biases have three effects in partial equilibrium. One, there is underdiversification, which leads to excessive risk taking. Two, to mitigate this excessive risk taking, there is under-investment in the risky asset, which lowers the expected portfolio return, a channel that has not been emphasized in the literature. Three, the consumption-wealth ratio is distorted.

Our second contribution is to demonstrate that even if familiarity biases in portfolios cancel out across households, their implications for consumption and investment choices do not. Instead, household-level distortions to individual consumption are amplified further by aggregation and have a substantial effect on aggregate growth and social welfare in general equilibrium.[5] Overall, combining the impact of underdiversification on intertemporal consumption and aggregate growth amplifies social welfare losses by a factor of five. If households have heterogeneous preferences, then the desire for risk sharing is even greater, which further increases the welfare losses from underdiversification. Thus, financial markets are not a sideshow—excessive risk taking at the micro level arising from underdiversified household portfolios can create a macro-level general-equilibrium effect in the form of reduced economic growth.

To analyze the effects of underdiversification in household portfolios on the aggregate economy, we construct a model of a production economy that builds on the framework developed in Cox, Ingersoll and Ross (1985). As in Cox, Ingersoll and Ross, there are a finite number of firms whose physical capital is subject to exogenous shocks. But, in contrast with Cox, Ingersoll and Ross, we have heterogeneous households with Epstein and Zin (1989) and Weil (1990) preferences *with* familiarity biases. Each household is more familiar with a small subset of firms, with this subset varying across households. Familiarity biases create a desire to concentrate investments in a few familiar firms rather than holding a portfolio that is well diversified across all firms. Importantly, we specify the model so that familiarity biases cancel out across households. This assumption is similar to that made in Constantinides and Duffie (1996).

[5]This also distinguishes our paper from recent work by Hassan and Mertens (2011), who find that when households make small *correlated* errors in forming expectations, the errors are amplified with significant distortive effects on growth and social welfare. In contrast, we show that such effects can arise even when households' errors in portfolio choices cancel out.

We conceptualize the idea of greater familiarity with particular assets, introduced in Huberman (2001), via *ambiguity* in the sense of Knight (1921). The lower the level of ambiguity about an asset, the more "familiar" is the asset. To allow for *differences* in familiarity across assets, we start with the modeling approach in Uppal and Wang (2003) and extend it along three dimensions: 1) we distinguish between risk across states of nature and over time by giving households Epstein-Zin-Weil preferences, as opposed to time-separable preferences; 2) we consider a production economy instead of an endowment economy; 3) we consider a general-equilibrium rather than a partial-equilibrium framework.

We determine the optimal portfolio decision of each household in the presence of familiarity biases. Because of the familiarity-induced tilt, the portfolio return is excessively risky relative to the return of the optimally diversified portfolio without familiarity bias. The inefficient risk-return trade-off from the underdiversified portfolio reduces the mean-variance utility of the individual household; calibrating the model to the empirical findings in Calvet, Campbell and Sodini (2007) suggests that the resulting welfare loss to an individual household is modest and equivalent to a reduction of about 1% per year in a household's portfolio return. However, when we allow for asset-allocation and intermediate consumption, familiarity biases magnify the direct welfare loss from portfolio underdiversification by a factor of four. Upon aggregation, the biased consumption-savings decisions of individual households distort aggregate growth and lead to an even larger loss in social welfare compared to the direct loss to individual households. The overall effect on social welfare of allowing underdiversification to impact intertemporal consumption and aggregate growth is to multiply by more than five the individual mean-variance welfare losses.

Our results suggest that financial literacy, financial regulation, and financial innovation designed to mitigate familiarity biases could reduce investment mistakes by households and hence have a substantial impact on social welfare. Our work thereby provides an example of how improving the decisions made by households in financial markets can generate positive benefits for society; Thaler and Sunstein (2003) provide other examples of how public policy can be used to reduce the investment mistakes of households.

We now describe the related literature. There is a large literature documenting that households make mistakes in their financial decisions that reduce their welfare. For in-

stance, failure to contribute to 401(k) plans with matching employer contributions with the option to withdraw funds immediately for older employees (Choi, Laibson and Madrian, 2011), failure to locate taxable assets in nontaxable retirement savings accounts (Barber and Odean, 2003, Bergstresser and Poterba, 2004), and the failure to refinance fixed-rate mortgages when it is beneficial to do so (Campbell, 2006). Campbell (2016) provides a detailed discussion of these kind of mistakes, which occur because households lack basic financial literacy (Hastings, Madrian and Skimmyhorn, 2013, Lusardi and Mitchell, 2014).

In our study, we focus on the failure of households to hold diversified portfolios. There is a great deal of evidence showing that households hold poorly diversified portfolios. Guiso, Haliassos and Jappelli (2002), Haliassos (2002), Campbell (2006), and Guiso and Sodini (2013) highlight underdiversification in their surveys of household portfolios. Polkovnichenko (2005), using data from the Survey of Consumer Finances, finds that for households that invest in individual stocks directly, the median number of stocks held was two from 1983 until 2001, when it increased to three, and that poor diversification is often attributable to investments in employer stock, which is a significant part of equity portfolios. Barber and Odean (2000) and Goetzman and Kumar (2008) report similar findings of underdiversification based on data for individual households at a U.S. brokerage firm. In an influential paper, Calvet, Campbell and Sodini (2007) examine detailed government records covering the entire Swedish population. They find that of the households who participate in equity markets, many are poorly diversified and bear significant idiosyncratic risk. Campbell, Ramadorai and Ranish (2012) report that for their data on Indian households, "the average number of stocks held across all accounts and time periods is almost 7, but the median account holds only 3.4 stocks on average over its life." They also estimate that mutual fund holdings are between 8% and 16% of household direct equity holdings over the sample period.[6]

[6]Lack of diversification is a phenomenon that is present not just in a few countries, but across the world. Countries for which there is evidence of lack of diversification include: Australia (Worthington, 2009), France (Arrondel and Lefebvre, 2001), Germany (Börsch-Supan and Eymann, 2002) and Barasinska, Schäfer and Stephan (2008), India (Campbell, Ramadorai and Ranish, 2012), Italy (Guiso and Jappelli, 2002), the Netherlands (Alessie, Hochguertel and Van Soest, 2002), Norway (Fagereng, Gottleib and Guiso, 2017), and the United Kingdom (Banks and Smith, 2002).

Typically, the few risky assets that households hold are ones with which they are familiar. Huberman (2001) introduces the idea that households invest in familiar assets and provides evidence of this in a multitude of contexts; for example, households in the United States prefer to hold the stock of their local telephone company. Grinblatt and Keloharju (2001), based on data on Finnish households, find that households are more likely to hold stocks of Finnish firms that are located close to the household, communicate in the household's native language, and have a chief executive of the same cultural background. Massa and Simonov (2006) also find that households tilt their portfolios away from the market portfolio and toward stocks that are geographically and professionally close to the household. French and Poterba (1990) and Cooper and Kaplanis (1994) document that households bias their portfolios toward "home equity" rather than diversifying internationally. Dimmock et al. (2014) test the relation between familiarity bias and several household portfolio-choice puzzles. Based on a survey of U.S. households, they find that familiarity bias is related to stock-market participation, the fraction of financial assets in stocks, foreign-stock ownership, own-company-stock ownership, and underdiversification.[7] They also show that these results cannot be explained by risk aversion.

The most striking example of investing in familiar assets is the investment in "own-company stock," that is, stock of the company in which the person is employed. Haliassos (2002) reports extensive evidence of limited diversification based on the tendency of households to hold stock in the employer's firm. Mitchell and Utkus (2004) report that five million Americans have over 60% of their retirement savings invested in own-company stock and that about eleven million participants in 401(k) plans invest more than 20% of their retirement savings in their employer's stock. Benartzi et al. (2007) find that only 33% of

[7] The literature has proposed several other explanations for the tendency of households to hold only a small number of stocks in their portfolios. These include: the presence of fixed transaction costs (Brennan, 1975), awareness about only a subset of the available stocks (Merton, 1987), preference for skewness (Kraus and Litzenberger, 1976, Mitton and Vorkink, 2007, Barberis and Huang, 2008), loyalty toward the company where individual's work (Cohen, 2009), rank-dependent preferences (Polkovnichenko, 2005), and increasing returns to scale in information acquisition (Van Nieuwerburgh and Veldkamp, 2010). However, an information-based explanation will work only if households believes that the expected return on familiar assets will always be *higher* than that on other assets; but information about the familiar asset could also be negative, in which case the information-based explanation would suggest a *reduced* position in the familiar asset. Second, our model can generate a portfolio with investment in only a small *number* of assets, while in the information-based explanation there is always a component of the Markowitz portfolio with investments in all the assets. Third, our model can explain non-participation, which an information-based model cannot do.

the households who own company stock realize that it is riskier than a diversified fund with many different stocks. Remarkably, a survey of 401(k) participants by the Boston Research Group (2002) found that half of the respondents said that their company stock had the same or less risk than a money-market fund, even though there was a high level of awareness among the respondents about the experience of Enron's employees, who lost a substantial part of their retirement funds that were invested in Enron stock.[8]

Our paper is also related to the literature on risk sharing among heterogeneous agents with Epstein and Zin (1989) preferences; see, for example, Backus, Routledge and Zin (2007), Borovička (2015), Colacito and Croce (2010, 2011, 2012, 2013, 2014), Colacito et al. (2014), Colacito et al. (2015), Collin-Dufresne, Johannes and Lochstoer (2015, 2016*b*), Kollmann (2016), and Lustig (2000). We contribute to this literature by developing a model with multiple heterogeneous households and firms in which it is possible to obtain closed-form solutions for optimal consumption-portfolio decisions and asset prices.[9]

Typically, macroeconomic models have a single representative household or a large number of identical households with power utility or Epstein-Zin-Weil utility—see, for example, Lucas (1978) and Acemoglu (2009, Chapter 5). Our model differs in two respects. First, households in our model have Epstein-Zin utility *with familiarity biases*. Second, the households in our model are *heterogeneous* in their familiarity biases and preferences. The importance of studying models with heterogeneous agents rather than a representative agent has been recognized by both policy makers and academics. For instance, see the Ely lecture of Hansen (2007) and the presidential address to the American Economic Association by Sargent (2008). In addition to these two aspects of household preferences, in many macroeconomic models, households may have only limited access to financial markets, often just to the bond market; in our model, households can invest in both bonds and stocks. Finally, in

[8] At the end of 2000, 62% of Enron employees' 401(k) assets were invested in company stock; between January 2001 and January 2002, the value of Enron stock fell from over \$80 per share to less than \$0.70 per share.

[9] In contrast to our work, which focuses on risk sharing across agents and diversification across multiple stocks, there is also literature studying the decision of an individual household on how to allocate wealth between a single risky asset and a risk-free asset. For instance, Cocco (2005) examines how investment in housing affects the proportion of household wealth invested in the stock market, while Cocco, Gomes and Maenhout (2005) investigate the effect of labor income on the share of wealth invested in equities.

many macroeconomic models, households have labor income; while in our base-case model labor income is absent, we allow for labor income in an extension of the base-case model.

The rest of this paper is organized as follows. We describe the main features of our model in Section I. The choice problem of a household that exhibits a bias toward familiar assets is solved in Section II, and the general-equilibrium implications of aggregating these choices across all households are described in Section III. We evaluate the quantitative implications of our stylized model in Section IV. We discuss the sensitivity of our results to our modeling assumptions in Section V and conclude in Section VI. Proofs for all results are collected in Appendix A and the extension of the baseline model to allow for labor income is presented in Appendices B.

I. The Model

In this section, we develop a parsimonious model of a stochastic dynamic general equilibrium economy with a finite number of production sectors and household types. Growth occurs endogenously in this model via capital accumulation. When defining the decision utility of households, we show how to extend Epstein and Zin (1989) and Weil (1990) preferences to allow for familiarity biases, where the level of the bias differs across risky assets.

A. Firms

There are N firms indexed by $n \in \{1, \ldots, N\}$. The value of the capital stock in each firm at date t is denoted by $K_{n,t}$ and the output flow by

$$Y_{n,t} = \alpha K_{n,t}, \tag{1}$$

for some constant technology level $\alpha > 0$ that is common across all firms. The level of a firm's capital stock can be increased by investment at the rate $I_{n,t}$. We thus have the following capital accumulation equation for an individual firm:

$$dK_{n,t} = I_{n,t}\, dt + \sigma\, K_{n,t}\, dZ_{n,t},$$

where σ, the volatility of the exogenous shock to a firm's capital stock, is constant over time and across firms. The term $dZ_{n,t}$ is the increment in a standard Brownian motion and is firm-specific, which creates ex-post heterogeneity across firms. The correlation between $dZ_{n,t}$ and $dZ_{m,t}$ for $n \neq m$ is denoted by ρ, which is assumed to be constant over time and the same for all pairs $n \neq m$. For expositional ease, in the main text, we shall assume that $\rho = 0$, although in the proofs for all our results and also in our numerical work, we do not restrict ρ to be zero.

A firm's output flow is divided between its investment flow and dividend flow:

$$Y_{n,t} = I_{n,t} + D_{n,t}.$$

We can therefore rewrite the capital accumulation equation as

$$dK_{n,t} = \big(\alpha K_{n,t} - D_{n,t}\big)dt + \sigma\, K_{n,t}\, dZ_{n,t}. \tag{2}$$

B. The Investment Opportunities of Households

There are H households indexed by $h \in \{1, \ldots, H\}$. Households can invest their wealth in two classes of assets. The first is a risk-free asset, which has an interest rate i that we assume for now is constant over time—and we show below, in Section III.B, that this is indeed the case in equilibrium. Let $B_{h,t}$ denote the stock of wealth invested by household h in the risk-free asset at date t:

$$\frac{dB_{h,t}}{B_{h,t}} = i\, dt.$$

Additionally, households can invest in N risky firms, or equivalently, the equity of these N firms.[10] We denote by $K_{hn,t}$ the stock of household h's wealth invested in the n'th risky firm. Given that the household's wealth, $W_{h,t}$, is held in the risk-free asset and the N risky firms, we have:

$$W_{h,t} = B_{h,t} + \sum_{n=1}^{N} K_{hn,t}.$$

[10] Back (2010, p. 440) explains that "One can interpret a model with costless adjustment and constant returns to scale as one in which individuals invest in the equity of the firm, with the firm choosing the amount to invest, or as a model in which individuals invest directly in the production technology."

The *proportion* of a household's wealth invested in risky firm n is denoted by ω_{hn} and $\pi_{h,t} = \sum_{n=1}^{N} \omega_{hn,t}$ is the proportion of household h's wealth allocated to all the risky assets at date t. The wealth allocated to the risk-free asset is

$$B_{h,t} = \Big(1 - \sum_{n=1}^{N} \omega_{hn,t}\Big) W_{h,t} = (1 - \pi_{h,t}) W_{h,t}. \tag{3}$$

Denoting by $x_{hn,t} = \omega_{hn,t}/\pi_{h,t}$ the weight of risky asset n in the portfolio consisting of *only* risky assets and by $c_{h,t} = C_{h,t}/W_{h,t}$ the propensity of household h to consume, the dynamic budget constraint for household h is

$$\frac{dW_{h,t}}{W_{h,t}} = \Big(1 - \pi_{h,t}\Big)\, i\, dt + \pi_{h,t} \sum_{n=1}^{N} x_{hn,t} \Big(\alpha\, dt + \sigma\, dZ_{n,t}\Big) - c_{h,t} dt. \tag{4}$$

Because the shocks to capital are firm-specific, there are benefits from diversifying investments across firms. Given our assumption that the expected rate of return, α, is the same across the N firms, the diversification benefits manifest themselves solely through a reduction in risk—expected returns do not change with the level of diversification.

Thus, each household needs to choose three quantities, all of which appear in the dynamic budget constraint (4). One, the consumption-saving decision: what proportion of its wealth to consume; i.e., the choice of $c_{h,t} = C_{h,t}/W_{h,t}$. Two, the asset-allocation decision: how to allocate its savings between the risky assets and the safe asset; i.e., the choice of $\pi_{h,t}$. Three, the portfolio-diversification decision: how much to invest in each of the N risky assets; i.e., the choice of $\boldsymbol{x}_{h,t} = (x_{h1,t}, \ldots, x_{hN,t})^{\top}$.

C. Preferences and Familiarity Biases of Households

A household's utility is modeled by standard Epstein-Zin preferences. In the absence of familiarity biases, a household's date-t utility level, $U_{h,t}$, is defined as in Epstein and Zin (1989) by an intertemporal aggregation of date-t consumption flow, $C_{h,t}$, and the date-t certainty-equivalent of date $t + dt$ utility:

$$U_{h,t} = \mathcal{A}(C_{h,t}, \mu_{h,t}[U_{h,t+dt}]),$$

where $\mathcal{A}(\cdot,\cdot)$ is the time aggregator, defined by

$$\mathcal{A}(a,b) = \left[(1-e^{-\delta_h dt})a^{1-\frac{1}{\psi_h}} + e^{-\delta_h dt} b^{1-\frac{1}{\psi_h}}\right]^{\frac{1}{1-\frac{1}{\psi_h}}}, \tag{5}$$

in which $\delta_h > 0$ is the rate of time preference of household h, $\psi_h > 0$ is its elasticity of intertemporal substitution, and $\mu_{h,t}[U_{h,t+dt}]$ is the date-t certainty equivalent of $U_{h,t+dt}$.[11] We can exploit our continuous-time formulation to write the certainty equivalent of household utility an instant from now in the following intuitive fashion:

$$\mu_{h,t}[U_{h,t+dt}] = E_t[U_{h,t+dt}] - \frac{1}{2}\,\gamma_h\, U_{h,t}\, E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]. \tag{6}$$

The above expression[12] reveals that the certainty equivalent of utility an instant from now is just the expected value of utility an instant from now adjusted downward for risk. Naturally, the size of the risk adjustment depends on the relative risk aversion of the household, γ_h. The risk adjustment also depends on the volatility of the proportional change in household utility, given by $E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]$. Additionally, the risk adjustment is scaled by the current utility of the household, $U_{h,t}$.[13]

We now describe the motivation for the way we model the effect of familiarity biases on the decisions and utility of households. Typically, standard models of asset allocation and portfolio choice assume that households know the true expected return α on each capital stock. Such perfect knowledge would make each household fully familiar with every firm and the probability measure $\mathbb{P}$ would then be the true objective probability measure.[14] However, in practice households do not know the true expected returns, so they do not view $\mathbb{P}$ as the true objective probability measure—they treat it merely as a common reference measure.

[11] The only difference with Epstein and Zin (1989) is that we work in continuous time, whereas they work in discrete time. The continuous-time version of recursive preferences is known as Stochastic Differential Utility (SDU), and is derived formally in Duffie and Epstein (1992). Schroder and Skiadas (1999) provide a general proof of existence and uniqueness for the finite-horizon case. In our infinite-horizon setting, we establish existence constructively by deriving an explicit closed-form solution.

[12] The derivation of this result, and all the other results that follow, is provided in Appendix A.

[13] The scaling ensures that if the expected proportional change in household utility and its volatility are kept fixed, doubling current household utility also doubles the certainty equivalent. For a further discussion, see Skiadas (2009, p. 213).

[14] In continuous time, when the source of uncertainty is a Brownian motion, one can always determine the true volatility of the return on the capital stock by observing its value for a finite amount of time; therefore, a household can be uncertain only about the expected return.

The name "reference probability measure" reflects the idea that even though households do not observe true expected returns, they do observe the same data and use it to obtain identical point estimates for expected returns.

We assume that households are averse to their lack of knowledge about the true expected returns and respond by adjusting their point estimates. For example, household h will change the empirically estimated return for firm n from α to $\alpha+\nu_{hn,t}$, thereby *reducing* the magnitude of the firm's expected risk premium ($\nu_{hn,t} \leq 0$ if $\alpha > i$ and $\nu_{hn,t} \geq 0$ if $\alpha < i$). The size of the reduction depends on each household's familiarity with a particular firm—the reduction is smaller for firms with which the household is more familiar. Differences in familiarity across households lead them to use different estimates of expected returns in their decision making, despite having observed the same data. We can see this explicitly by observing that in the presence of familiarity, the contribution of risky portfolio investment to a household's expected return on wealth changes from $\pi_{h,t}\sum_{n=1}^{N} x_{hn,t}\,\alpha\,dt$ to $\pi_{h,t}\sum_{n=1}^{N} x_{hn,t}(\alpha+\nu_{hn,t})dt$. The *adjustment* to the expected return on a household's wealth stemming from familiarity biases is thus the difference between the above two expressions:

$$\pi_{h,t}\sum_{n=1}^{N} x_{hn,t}\nu_{hn,t}dt. = \pi_{h,t}\boldsymbol{\nu}_{h,t}^{\top}\boldsymbol{x}_{h,t}. \tag{7}$$

Without familiarity biases, the decision of a household on how much to invest in a particular firm depends solely on the certainty equivalent. Therefore, to allow for familiarity biases it is natural to generalize the concept of the certainty equivalent. We extend Uppal and Wang (2003) and define the *familiarity-biased certainty equivalent* by

$$\mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}] = \mu_{h,t}[U_{h,t+dt}] + U_{h,t}\times\left(\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\pi_{h,t}\boldsymbol{\nu}_{h,t}^{\top}\boldsymbol{x}_{h,t} + \frac{1}{2\gamma_h}\frac{\boldsymbol{\nu}_{h,t}^{\top}\Gamma_h^{-1}\boldsymbol{\nu}_{h,t}}{\sigma^2}\right)dt, \tag{8}$$

where $U_{W_{h,t}} = \frac{\partial U_{h,t}}{\partial W_{h,t}}$, $\boldsymbol{\nu}_{h,t} = (\nu_{h1,t},\ldots,\nu_{hN,t})^{\top}$, and $\Gamma_h = [\Gamma_{h,nm}]$ is the $N\times N$ diagonal matrix with $(1-f_{hn})/f_{hn}$ on the diagonal, where $f_{hn}\in[0,1]$ is a measure of how familiar the household is with firm, n. A larger value for f_{hn} indicates more familiarity, with $f_{hn}=1$ implying perfect familiarity, while $f_{hn}=0$ indicating no familiarity at all. We denote the mean (average) and variance of the familiarity biases across the N firms for household h

by, respectively:

$$\mu_{fh} = \frac{1}{N}\sum_{n=1}^{N} f_{hn} \quad \text{and} \quad \sigma_{fh}^2 = \frac{1}{N}\sum_{n=1}^{N} \left(f_{hn} - \mu_{fh}\right)^2 .$$

We now explain the expression in (8). The first component, $\mu_{h,t}[U_{h,t+dt}]$, is the pure certainty equivalent, which does not depend directly on the familiarity-bias adjustments. The first term of the second component is the scaling factor $U_{h,t}$, which is explained in footnote 13. The next term, $\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\pi_{h,t}\boldsymbol{\nu}_{h,t}^{\top}\boldsymbol{x}_{h,t}$, is the adjustment to the expected change in household utility. It is the product of the elasticity of household utility with respect to wealth, $\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}$, and the change in the expected return on household wealth arising from the adjustment made to returns, which is given in (7).

The tendency to make adjustments to expected returns is tempered by a penalty term, $\frac{1}{2\gamma}\frac{\boldsymbol{\nu}_{h,t}^{\top}\Gamma_h^{-1}\boldsymbol{\nu}_{h,t}}{\sigma^2}$, which captures two distinct features of household decision making. The first pertains to the idea that when a household has more accurate estimates of expected returns, it will be less willing to adjust them. The accuracy of the household's expected return estimates is measured by the standard errors of these estimates, which are proportional to σ.[15] With smaller standard errors, there is a stiffer penalty for adjusting returns away from their empirical estimates. The second feature pertains to familiarity, reflected by Γ_h^{-1}: when a household is more familiar with a particular firm, the penalty for adjusting its return away from its estimated value is again larger.

II. Portfolio Decision and Welfare of an Individual Household

We solve the model described above in two steps. First, we solve in partial equilibrium for decisions of an individual household that suffers from familiarity biases. To solve the individual household's intertemporal decision problem, we show that the portfolio-choice problem can be interpreted as the problem of a mean-variance household, where the familiarity biases in the household's utility are captured by adjusting expected returns. We then

[15] In our continuous-time framework, an infinite number of observations are possible in finite time, so standard errors equal the volatility of proportional changes in the capital stock, σ, divided by the square root of the length of the observation window.

show how the mean-variance portfolio decision impacts the asset-allocation and intertemporal consumption decisions of the household. Then, in the next section, we aggregate over all households to study the general-equilibrium effect of familiarity biases on social welfare.

A. The Intertemporal Decision Problem of an Individual Household

If a household did not suffer from familiarity biases, it would choose its consumption rate $C_{h,t}$, its asset-allocation policy $\pi_{h,t}$, and its portfolio-diversification policy $\boldsymbol{x}_{h,t}$ to maximize its utility

$$\sup_{C_{h,t}} \mathcal{A}\Big(C_{h,t}, \sup_{\pi_{h,t},\boldsymbol{x}_{h,t}} \mu_{h,t}[U_{h,t+dt}]\Big), \tag{9}$$

where $\mathcal{A}(\cdot,\cdot)$ is the time aggregator in (5).

In the presence of familiarity biases, the time aggregator in (5) is unchanged—and all that one needs to do is to replace the maximization of the standard certainty-equivalent, $\sup_{\pi_{h,t},\boldsymbol{x}_{h,t}} \mu_{h,t}[U_{h,t+dt}]$, with the combined maximization and minimization of the familiarity-based certainty equivalent, $\sup_{\pi_{h,t},\boldsymbol{x}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} \mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$, to obtain

$$\sup_{C_{h,t}} \mathcal{A}\Big(C_{h,t}, \sup_{\pi_{h,t},\boldsymbol{x}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} \mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]\Big). \tag{10}$$

A household, because of its familiarity bias, chooses $\boldsymbol{\nu}_{h,t}$ to *minimize* its familiarity-biased certainty equivalent; that is, the household adjusts expected returns more for firms with which it is less familiar, which acts to reduce the familiarity-biased certainty equivalent.[16] By comparing (9) and (10), we can see that once a household has chosen the vector $\boldsymbol{\nu}_{h,t}$ to adjust the expected returns of each firm for familiarity bias, the household makes consumption, asset-allocation, and portfolio decisions in the standard way.

Assuming a constant risk-free rate, our assumptions of homothetic preferences and constant-returns-to-scale production lead to an investment opportunity set that is constant over time, and hence, implies that for a given level of wealth the household's decisions

[16] In the language of decision theory, households are averse to ambiguity and so they minimize their familiarity-biased certainty equivalents.

are constant over time and maximized household utility is a constant multiple of household wealth. In this case, (10) reduces to a Hamilton-Jacobi-Bellman equation, which can be decomposed into two parts: (i) an *intertemporal* consumption-choice problem and (ii) a *mean-variance* asset-allocation and portfolio-diversification problem for a household with familiarity biases:

$$0 = \sup_{C_h} \left(\delta_h \, u_\psi \left(\frac{C_h}{U_h} \right) - \frac{C_h}{W_h} + \sup_{\pi_h, \boldsymbol{x}_h} \inf_{\boldsymbol{\nu}_h} MV_h(\pi_h, \boldsymbol{x}_h, \boldsymbol{\nu}_h) \right). \tag{11}$$

In the above expression, $MV_h(\pi_h, \boldsymbol{x}_h, \boldsymbol{\nu}_h)$ is the objective function of a mean-variance household with familiarity biases,

$$MV_h(\pi_h, \boldsymbol{x}_h, \boldsymbol{\nu}_h) = \big[i + \big(\alpha - i\big)\pi_h \big] - \frac{1}{2}\gamma_h \sigma^2 \pi_h^2 \boldsymbol{x}_h^\top \boldsymbol{x}_h + \pi_h \boldsymbol{\nu}_h^\top \boldsymbol{x}_h + \frac{1}{2\gamma_h} \frac{\boldsymbol{\nu}_h^\top \Gamma_h^{-1} \boldsymbol{\nu}_h}{\sigma^2}, \tag{12}$$

where the first component, $\big[i + \big(\alpha - i\big)\pi_h \big]$, is the expected portfolio return; the second component, $-\frac{1}{2}\gamma_h \sigma^2 \pi_h^2 \boldsymbol{x}_h^\top \boldsymbol{x}_h$, is the penalty for portfolio variance;[17] the third component, $\pi_h \boldsymbol{\nu}_h^\top \boldsymbol{x}_h$, is the adjustment to the portfolio's expected return arising from familiarity biases; and the last component, $\frac{1}{2\gamma_h} \frac{\boldsymbol{\nu}_h^\top \Gamma_h^{-1} \boldsymbol{\nu}_h}{\sigma^2}$, is the penalty for adjusting expected returns.[18]

B. Portfolio Diversification and Asset Allocation of a Household

In this section, we study the optimal portfolio-diversification and asset-allocation choices of an individual household that suffers from familiarity biases. These optimal choices are obtained by solving the first-order conditions for $\boldsymbol{\nu}_h$, $\boldsymbol{x}_h$, and π_h from (12).

The optimal adjustment to expected returns is

$$\nu_{hn} = -(\alpha - i)(1 - f_{hn}). \tag{13}$$

In this case it is easy to see that the size of a household's adjustment to a particular firm's return is smaller when the level of familiarity, f_{hn}, is larger; if $f_{hn} = 1$, then the adjustment vanishes altogether.

[17] In the absence of the simplifying assumption that the return correlations are zero, i.e. $\rho = 0$ and hence $\Omega = I$, the penalty for portfolio variance would be $-\frac{1}{2}\gamma_h \sigma^2 \pi_h^2 \boldsymbol{x}_h^\top \Omega \, \boldsymbol{x}_h$.

[18] The familiarity-bias adjustment is obtained from a minimization problem, so the associated penalty is positive, in contrast with the penalty for return variance.

The optimal weights in the portfolio consisting of only risky assets are given by

$$x_{hn} = \frac{f_{hn}}{\mathbf{1}^\top \boldsymbol{f}_h} = \frac{1}{N}\frac{f_{hn}}{\mu_{fh}}. \tag{14}$$

In the absence of a familiarity bias, i.e. if $f_{hn} = 1$ for all n, implying that $\mu_{fh} = 1$, the household's optimal investment in each risky asset would be $x_{hn} = \frac{1}{N}$. The intuition for the optimality of the equally weighted portfolio is that the returns of all risky assets have the same first and second moments. From (14), we see that familiarity biases, that is, $f_{hn} \neq \mu_{fh}$, tilt the portfolio weights away from $\frac{1}{N}$, creating an underdiversified portfolio with higher variance, which will lead to welfare losses for the individual household. Denoting by $\sigma^2_{1/N}$ the return variance of the fully diversified portfolio (absent familiarity biases) with each weight equal to $1/N$, the return variance $\sigma^2_{\boldsymbol{x}_h}$ of the familiarity-biased portfolio of household h can be expressed as

$$\sigma^2_{\boldsymbol{x}_h} = \sigma^2_{1/N}\left[1 + \left(\frac{\sigma_{fh}}{\mu_{fh}}\right)^2\right], \tag{15}$$

implying that in the presence of familiarity biases the return volatility of a household's portfolio will be higher than that of the unbiased portfolio: $\sigma_{\boldsymbol{x}_h} > \sigma_{1/N}$.

The familiarity biases in a household's portfolio decision impact also its asset-allocation decision; that is, the allocation of total wealth to all the risky assets, π_h, and to the risk-free asset, $1 - \pi_h$:

$$\pi_h = \left[\frac{1}{\gamma_h}\frac{\alpha - i}{\sigma^2_{1/N}}\right]\mu_{fh}. \tag{16}$$

In the absence of familiarity biases, i.e. $f_{hn} = 1$ implying that $\mu_{fh} = 1$, the unbiased allocation of wealth to risky assets, π^U_h, would be

$$\pi^U_h = \left[\frac{1}{\gamma_h}\frac{\alpha - i}{\sigma^2_{1/N}}\right]. \tag{17}$$

Comparing (16) and (17), we see that familiarity biases, i.e. $f_{hn} < 1$ leading to $\mu_{fh} < 1$, *reduce* the allocation to risky assets relative to the unbiased case: $\pi_h < \pi^U_h$.

C. Mean-Variance Welfare of an Individual Household

In a single-period mean-variance framework, the mean-variance utility achieved by an individual household is given by[19]

$$U_h^{MV}(\pi_h, \boldsymbol{x}_h) = \left[i + (\alpha - i)\pi_h\right] - \frac{1}{2}\gamma_h \pi_h^2 \sigma_{\boldsymbol{x}_h}^2 \tag{18}$$

$$= i + \frac{1}{2}\frac{1}{\gamma_h}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2 \left[1 - \sigma_{fh}^2 - (\mu_{fh} - 1)^2\right]. \tag{19}$$

Equation (18) shows that the welfare of a mean-variance household will be influenced through both the return volatility of the subportfolio of risky assets, $\sigma_{\boldsymbol{x}_h}$, and by the asset-allocation decision, π_h. The second term of (18), $-\frac{1}{2}\gamma_h \pi_h^2 \sigma_{\boldsymbol{x}_h}^2$, shows that the increase in $\sigma_{\boldsymbol{x}_h}$ because of familiarity biases reduces mean-variance utility. The reduction in π_h because of familiarity biases has two consequences for a household's welfare. One, as pointed out by Calvet, Campbell and Sodini (2007), the welfare loss from underdiversification in the second term of (18) is mitigated because less wealth is invested in the excessively risky subportfolio of risky assets. Two, there will be a new source of welfare loss from a reduction in the portfolio's expected return, $i + (\alpha - i)\pi_h$, caused by the reduction of wealth allocated to risky assets. Equation (19) shows that the magnitude of the mean-variance utility loss from under-diversification is driven by the variance of the familiarity biases, σ_{fh}^2, and the loss from under-allocation to risky assets is driven by the deviation of average familiarity from one, $\mu_{fh} - 1$.

D. Optimal Consumption of an Individual Household

When a household's asset-allocation decision is subject to familiarity biases, this will also impact her optimal propensity to consume, $c_h = C_{h,t}/W_{h,t}$. The first-order condition for consumption obtained from the Hamilton-Jacobi-Bellman equation in (11) allows one to show that the optimal consumption decision of a household with familiarity biases is the weighted average of the rate of time preference and the optimized mean-variance objective

[19] This is the actual utility a household would experience under the objective beliefs.

function given in (12) for a household that suffers from familiarity biases:

$$\frac{C_{h,t}}{W_{h,t}} = c_h = \psi_h \delta_h + (1-\psi_h) MV_h(\pi_h, \boldsymbol{x}_h, \boldsymbol{\nu}_h)$$

$$= \psi_h \delta_h + (1-\psi_h)\left[i + \frac{1}{2}\frac{1}{\gamma_h}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2 \mu_{fh} \right], \tag{20}$$

which shows that familiarity biases (i.e. $\mu_{fh} < 1$) distort the saving-consumption choice of a household. If elasticity of substitution $\psi_h > 1$, then a decrease in average familiarity μ_{fh} will increase consumption, thereby reducing savings, with the opposite effect if $\psi_h < 1$.

E. Lifetime Welfare of an Individual Household

In Section C, we considered welfare in a single-period mean-variance framework, which depended only on the portfolio-diversification and asset-allocation decisions of a household. In a multiperiod framework with intermediation consumption, the lifetime utility of a household per unit of current wealth, $\frac{U_{h,t}}{W_{h,t}}$, is given by

$$\frac{U_{h,t}}{W_{h,t}} = \kappa_h = \left[\frac{\psi_h \delta_h}{\psi_h \delta_h + \big(1-\psi_h\big)\big[U_h^{MV}(\pi_h, \boldsymbol{x}_h) - c_h\big]} \right]^{\frac{1}{1-1/\psi_h}} c_h, \tag{21}$$

in which the mean-variance welfare, $U_h^{MV}(\pi_h, \boldsymbol{x}_h)$, given in (18), is only one component of it. Lifetime welfare now depends also on the household's consumption choice, c_h, which is given in (20). We have already discussed above how $U_h^{MV}(\pi_h, \boldsymbol{x}_h)$ and c_h are affected by familiarity biases. To understand the overall expression in (21), observe that increasing current consumption is clearly beneficial but comes at the expense of reduced savings. The benefit is represented by the last term in (21), the c_h that appears outside the expression in square brackets. The negative impact on savings is captured by the term $[U_h^{MV}(\pi_h, \boldsymbol{x}_h) - c_h]$ in the denominator of the first component. The term $[U_h^{MV}(\pi_h, \boldsymbol{x}_h) - c_h]$ is the risk-adjusted expected return on a household's wealth, *net* of consumption. To obtain its impact on lifetime welfare, this expected return needs to be capitalized, as shown in the first term of (21). The capitalized value depends on the intertemporal aspects of the household's preferences, that is, the rate of time preference, δ_h, and the elasticity of intertemporal substitution, ψ_h.

In Section IV, we will evaluate quantitatively the magnitude of the gain in welfare of an individual household in partial equilibrium for reasonable parameter values if the household can eliminate the biases in its portfolio-diversification, asset-allocation, and consumption-saving decisions.

III. Social Welfare and Growth

In contrast with Section II, in which we examined how familiarity biases impact an *individual* household, in this section we investigate the general-equilibrium effects on aggregate growth and social welfare of the distortions in the decisions of individual households when we aggregate over all households and impose market clearing. To simplify the exposition, we assume that each f_{hn} is either 1 or 0; that is, each household is either fully familiar with a firm or not familiar at all.[20] We also assume that all households have the same preference parameters: subjective rate of time discount (δ), relative risk aversion (γ), and elasticity of intertemporal substitution (ψ); in Section III.E, we show that the insights from the homogeneous-economy setting extend in a straightforward manner to the setting where households are heterogeneous with respect to these preference parameters.[21]

A. Restricting Familiarity Biases

In this section, we explain the restrictions that we impose on the familiarity biases $\boldsymbol{f}_h$ to ensure that our results are not driven simply by portfolio biases at the aggregate level.

To guarantee that our results are not driven by aggregate biases in the portfolio-diversification decisions of households, $\boldsymbol{x}_h$, the first restriction we impose is that familiarity biases in $\boldsymbol{x}_h$ cancel out in the aggregate. We express this formally by writing household h's risky portfolio weight for firm n in (14) as

$$x_{hn} = \frac{1}{N}\frac{f_{hn}}{\mu_{fh}} = \frac{1}{N} + \epsilon_{hn},$$

[20] In case where each f_{hn} is restricted to equal either 1 or 0, the Γ_h matrix and its inverse are obtained by taking the limit as $f_{hn} \to 0$ or $f_{hn} \to 1$; in the appendices, the results are derived without restricting f_{hn}.

[21] For evidence on the heterogeneity in returns earned by households on their wealth, see Fagereng et al. (2016).

where $\frac{1}{N}$ is the unbiased portfolio weight and ϵ_{hn} is the bias of household h's portfolio when investing in firm n. We can now see that the familiarity biases cancel out in aggregate if and only if the following condition is true:

$$\frac{1}{H}\sum_{h=1}^{H}\epsilon_{hn}=0, \quad \forall n. \tag{22}$$

The above condition says that while it is possible for an individual household's portfolio to be biased, that is, to deviate from the unbiased $\frac{1}{N}$ portfolio, this bias must cancel out when forming the aggregate portfolio across all households.

To guarantee that our results are not driven by aggregate biases in the asset-allocation decisions of households, π_h, the second restriction we impose ensures that π_h is the same for each household. The second restriction, which we label the "symmetry condition," is that the mean (average) of the familiarity biases across firms be the same for each household; that is, for distinct households h and j:

$$\mu_{fh}=\mu_{fj}=\mu_f. \tag{23}$$

As a consequence of this restriction (and the assumption that all households have the same risk aversion), we see from (16) that π_h is the same for all households.[22] As shown below, market clearing for the risk-free bond then implies that each household has a zero position in the risk-free asset, implying that $\pi_h = 1$ for each household.

B. The Equilibrium Risk-Free Interest Rate

We now characterize the equilibrium in the economy by imposing market clearing for the risk-free bond. The risk-free bond is in zero net-supply, which implies that the demand for bonds aggregated across all households must be zero: $\sum_{h=1}^{H} B_{h,t} = 0$. The amount of wealth held in the bond by an individual household h is given by the expression in (3).

[22] The actual restriction that one needs is that the ratio of relative risk aversion to μ_{fh} is the same across all households; given our assumption that risk aversion is the same across all households, this restriction reduces to Equation (23). In the extensions to the basic model discussed below, we show how to solve for the quantities of interest when the preference parameters differ across households.

Summing this demand for bonds over all households gives

$$0 = \sum_{h=1}^{H} B_{h,t} = \sum_{h=1}^{H} (1 - \pi_h) W_{h,t} = \sum_{h=1}^{H} \left(1 - \frac{1}{\gamma} \frac{\alpha - i}{\sigma^2_{1/N}} \mu_{fh} \right) W_{h,t}.$$

As a consequence of the symmetry condition in (23), the market-clearing condition for the bond simplifies to

$$0 = \left(1 - \frac{1}{\gamma} \frac{\alpha - i}{\sigma^2_{1/N}} \mu_f \right) \sum_{h=1}^{H} W_{h,t}.$$

The equilibrium risk-free interest rate is thus given by the constant[23]

$$i = \alpha - \gamma \frac{\sigma^2_{1/N}}{\mu_f}, \tag{24}$$

where μ_f is the average familiarity bias across assets for each household. We can see immediately that the presence of familiarity biases (i.e. $\mu_f < 1$) amplifies the risk in the economy, leading to a greater precautionary demand for the risk-free asset, and hence, a decrease in the risk-free interest rate. Thus, in general equilibrium, the distortion in the asset-allocation decision is reflected in the interest rate. Even though households invest in different risky assets, the restriction in (23) implies that the demand for the risk-free asset is the same across all households and also constant over time.

C. Aggregate Growth

Substituting the equilibrium interest rate in (24) into the expression for the partial-equilibrium consumption-wealth ratio in (20) gives the consumption-wealth ratio in general equilibrium, which is common across households and is denoted by c:

$$\frac{C_{h,t}}{W_{h,t}} = c = \psi\delta + (1 - \psi) \left(\alpha - \frac{1}{2} \gamma \sigma^2_{\boldsymbol{x}} \right), \tag{25}$$

[23]If we did not impose the symmetry condition, the interest rate would not be constant over time. Then, the effect on portfolio choice of changes in investment opportunities arising from a stochastic interest rate would depend on the correlation between stock returns and the risk-free rate of interest. Over long-horizons, this correlation is close to zero. For instance, when we calculate the correlation between returns on S&P 500 stocks and the U.S. 3-month Treasury bill over the period 1928 to 2017, it is -0.0298 and over the period 1968 to 2017 it is 0.0319. Because this correlation is small, the effect of changing investment opportunities is small. Thus, we have constructed the model in a such a way that the interest rate is constant over time, which allows us to get closed-form expressions for all quantities of interest.

where the right-hand side of the above expression is the same across households and also constant over time.

Exploiting the result that the consumption-wealth ratio is constant across households and also over time allows us to obtain the ratio of *aggregate* consumption, $C_t^{\text{agg}} = \sum_{h=1}^{H} C_{h,t}$, to aggregate wealth, $W_t^{\text{agg}} = \sum_{h=1}^{H} W_{h,t}$:

$$\frac{C_t^{\text{agg}}}{W_t^{\text{agg}}} = c. \tag{26}$$

In equilibrium, the aggregate capital stock, $K_t^{\text{agg}} = \sum_{n=1}^{N} K_{n,t}$, equals the aggregate wealth of households, because the bond is in zero net supply: $K_t^{\text{agg}} = W_t^{\text{agg}}$. Therefore, from (26), the aggregate consumption-to-capital ratio is: $C_t^{\text{agg}}/K_t^{\text{agg}} = c$.

We now derive the aggregate investment-capital ratio. The aggregate investment flow, I_t^{agg}, is the sum of the investment flows into each firm, $I_t^{\text{agg}} = \sum_{n=1}^{N} I_{n,t}$. The aggregate investment flow must be equal to aggregate output flow less the aggregate consumption flow, i.e.,

$$I_t^{\text{agg}} = \alpha K_t^{\text{agg}} - C_t^{\text{agg}}.$$

Firms all have constant returns to scale and differ only because of shocks to their capital stocks. Therefore, the aggregate growth rate of the economy, g, defined by $g\,dt = E_t\left[\frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}}\right]$, is the aggregate investment-capital ratio, and so

$$g = \frac{I_t^{\text{agg}}}{K_t^{\text{agg}}} = \alpha - c = \psi(\alpha - \delta) - \frac{1}{2}(\psi - 1)\gamma\sigma_{\boldsymbol{x}}^2. \tag{27}$$

Underdiversification by individual households amplifies the risk in the economy, that is, increases $\sigma_{\boldsymbol{x}}$. If the substitution effect dominates ($\psi > 1$), the increase in risk will reduce the aggregate growth rate in (27) because, as we see from (25), households will choose to consume more of their wealth. On the other hand, if the income effect dominates ($\psi < 1$), the increase in portfolio risk will lead to an increase in the aggregate growth rate but, as we show below, even in this case welfare shall fall.

D. Social Welfare

We now study social welfare, that is, the aggregate welfare of all households. An individual household's utility level is given by $U_{h,t} = \kappa_h W_{h,t}$, where κ_h is defined in (21). Imposing market clearing and simplifying the resulting expression for κ_h, we obtain that when households are intertemporal consumers, social welfare is given by

$$U_t^{\text{social}} = \sum_{h=1}^{H} U_{h,t} = \kappa \sum_{h=1}^{H} W_{h,t} = \kappa\, K_t^{\text{agg}}, \tag{28}$$

where we obtain the last equality using the result that $W_t^{\text{agg}} = K_t^{\text{agg}}$ and where

$$\kappa = \begin{cases} \left[\frac{\psi\delta + (1-\psi)U^{MV}(\sigma_{\boldsymbol{x}})}{\delta^{\psi}}\right]^{\frac{1}{1-\psi}} & \psi \neq 0, \\ \\ U^{MV}(\sigma_{\boldsymbol{x}}) & \psi = 0, \end{cases} \tag{29}$$

in which

$$U^{MV}(\sigma_{\boldsymbol{x}}) = \delta + \frac{1}{\psi}\left(g - \frac{1}{2}\gamma\sigma_{\boldsymbol{x}}^2\right) = \alpha - \frac{\gamma}{2}\sigma_{\boldsymbol{x}}^2, \tag{30}$$

with the endogenous aggregate growth rate g given in (27) and where $U^{MV}(\sigma_{\boldsymbol{x}})$ denotes the welfare of a mean-variance household after imposing market clearing, which is obtained by substituting into (18) the equilibrium interest rate from (24) and the condition that $\pi_h = 1$ for each household.[24]

We know that households benefit from their own individual financial education if it allows them to overcome their familiarity biases.[25] But how significant would be the gains to society of widespread financial education, financial innovation, and financial regulation that lead households to invest in better diversified portfolios? To answer this question, we need to understand that the welfare gains take place via two different channels. One is a micro-level effect, whereby a household's welfare is increased purely from choosing a better diversified set of investments—the return on a household's financial wealth then becomes

[24] Calvet, Campbell and Sodini (2007) study the mean-variance case in partial equilibrium.

[25] Education in finance theory is not widespread. Guiso and Viviano (2015) report that households lack basic knowledge of the financial concepts that are required for making financial decisions; additional evidence is presented in Lusardi and Mitchell (2011).

less risky, which also reduces its consumption-growth volatility. The second is a macro-level general-equilibrium effect, which raises the welfare of all households. From where does this macroeconomic effect arise? Its source lies in the decline of risk in every household's portfolio. If the substitution effect dominates the income effect ($\psi > 1$), households prefer to consume less today and invest more in risky firms; therefore, aggregate investment increases, raising the trend growth rate of the economy, and increasing social welfare. If the income effect dominates ($\psi < 1$), households prefer to consume more today and invest less in risky production, thereby reducing trend growth, but still increasing welfare.

We now show analytically how to disentangle the micro-level channel from the macro-level general-equilibrium channel. In equilibrium, the level of social welfare is given by (28), where κ is the social welfare per unit of aggregate capital. The micro-level positive effect stems from a reduction in household portfolio risk, brought about by financial education, innovation, and regulation. The reduction in risk stems from improved diversification:

$$\Delta\sigma_{\boldsymbol{x}}^2 = -\underbrace{(\sigma_{\boldsymbol{x}}^2 - \sigma_{1/N}^2)}_{\text{diversification}} < 0.$$

The macro-level general-equilibrium effect manifests itself via a change in expected aggregate consumption growth, g, which we can write as follows:

$$\Delta g = -\frac{1}{2}(\psi - 1)\gamma\Delta\sigma_{\boldsymbol{x}}^2 = \frac{1}{2}(\psi - 1)\gamma\underbrace{(\sigma_{\boldsymbol{x}}^2 - \sigma_{1/N}^2)}_{\text{diversification}}.$$

The micro-level positive effect and the macro-level general-equilibrium effect combine to give the total impact on the welfare of a mean-variance household and hence social welfare as follows

$$\begin{aligned}\frac{d\ln\left(\frac{U_t^{\text{social}}}{K_t^{\text{agg}}}\right)}{d(\sigma_{\boldsymbol{x}}^2)} &= \frac{1}{\psi\delta + (1-\psi)U^{MV}(\sigma_{\boldsymbol{x}})}\frac{dU^{MV}(\sigma_{\boldsymbol{x}})}{d(\sigma_{\boldsymbol{x}}^2)} \\ &= \frac{1}{c}\frac{dU^{MV}(\sigma_{\boldsymbol{x}})}{d(\sigma_{\boldsymbol{x}}^2)},\end{aligned}$$

where

$$\frac{dU^{MV}(\sigma_{\boldsymbol{x}})}{d(\sigma_{\boldsymbol{x}}^2)} = \frac{\partial U^{MV}(\sigma_{\boldsymbol{x}})}{\partial(\sigma_{\boldsymbol{x}}^2)} + \frac{\partial U^{MV}(\sigma_{\boldsymbol{x}})}{dg}\frac{\partial g}{\partial(\sigma_{\boldsymbol{x}}^2)}. \tag{31}$$

The first term on the right-hand side of (31) captures the micro-level effect and the second term gives the macro-level general-equilibrium effect. Computing the relevant derivatives gives

$$\frac{d\ln\left(\frac{U_t^{\text{social}}}{K_t^{\text{agg}}}\right)}{d(\sigma_{\boldsymbol{x}}^2)} = -\frac{1}{2}\,\frac{\gamma}{c}\Big(\underbrace{\frac{1}{\psi}}_{\substack{\text{micro-level}\\ \text{effect}}} + \underbrace{1-\frac{1}{\psi}}_{\substack{\text{macro-level}\\ \text{effect}}}\Big). \tag{32}$$

We can see that a decline in the risk of household portfolios always increases social welfare. The relative importance of the micro- and macro-level channels is determined by the elasticity of intertemporal substitution, ψ. If $\psi > 1$, a reduction in risk at the microeconomic level has a greater follow-on impact at the macroeconomic level, because households are more willing to adjust their consumption intertemporally. For the special case of $\psi = 1$, the income and substitution effects offset each other exactly, and so the reduction in risk at the microeconomic level does not have further macroeconomic effects.

E. Social Welfare with Preference Heterogeneity

In our analysis above, we have assumed that all households have the same preference parameters. In this section, we extend the model to allow for households who are heterogeneous not only with respect to their familiarity biases but also with respect to their rate of time preference, elasticity of intertemporal substitution, and relative risk aversion.[26]

When households are heterogeneous, then social welfare per unit of aggregate wealth at date t is given by the wealth-weighted average of κ_h across all households, in contrast to the case where households had identical preferences and social welfare per unit of aggregate wealth was given by (the common) κ:

$$\frac{U_t^{\text{social}}}{W_t^{agg}} = \sum_{h=1}^{H} \kappa_h \left(\frac{W_{h,t}}{\sum_{j=1}^{H} W_{j,t}}\right), \tag{33}$$

[26]This is achieved by generalizing the condition in equation (23) so that even in the presence of heterogeneity each household has the same demand for the risk-free bond. In general equilibrium, where aggregate demand for the risk-free asset needs to be zero, this then implies that each household has zero invested in the risk-free bond, which then allows us to solve the model in closed form even when household preferences are heterogeneous.

where

$$\kappa_h = \begin{cases} \left[\frac{\psi_h \delta_h + (1-\psi_h) U^{MV}}{\delta_h^{\psi_h}}\right]^{\frac{1}{1-\psi_h}} & \psi_h \neq 0, \\ U^{MV} & \psi_h = 0, \end{cases},$$

and

$$U^{MV} = \alpha - \frac{\mathcal{R}}{2}\sigma^2_{1/N}, \quad \text{with} \quad \frac{1}{\mathcal{R}} = \frac{\mu_{fh}}{\gamma_h}, \forall h \in \{1, \ldots, H\}.$$

If we assume that all households have equal date-t wealth, equation (33) simplifies to

$$\frac{U_t^{\text{social}}}{W_t^{\text{agg}}} = \frac{1}{H}\sum_{h=1}^{H} \kappa_h, \tag{34}$$

where date-t aggregate wealth is given by $W_t^{\text{agg}} = \sum_{h=1}^{H} W_{h,t} = H\, W_{h,t}$.

In the next section, we will evaluate quantitatively the magnitude of the social welfare gain if households can eliminate the effects of familiarity biases in their portfolio-diversification, asset-allocation, and consumption-saving decisions. In this exercise, we will study the magnitude of the microeconomic and macroeconomic effects on welfare and also the consequences of preference heterogeneity across households.

IV. Implications of Financial Policy for Social Welfare

Our main goal in this section is to make statements about welfare, both in partial and general equilibrium, for a plausibly parameterized version of our stylized model. Below, we first explain our choice of parameter values, and then undertake a simple quantitative exercise to calculate the implications for welfare and explain the economic intuition for our findings. Recall our key modeling assumption in general equilibrium that the average familiarity level, μ_{fh}, is the same across all households; this assumption then implies that households have the same demand for the bond, which, with market clearing, leads to zero bond holdings by each household in equilibrium.

A. Parameter values

In this section, we explain how we estimate the parameters needed to compute the gain in welfare if a particular household were to switch to holding a more diversified portfolio.

To estimate the improvement in household welfare from holding a better diversified portfolio, we need an estimate of the portfolio volatility for a household that is underdiversified ($\sigma_{\boldsymbol{x}_h}$) and an estimate of portfolio volatility if the household were holding a well-diversified portfolio ($\sigma_{1/N}$). For both parameters, we use the estimates in Calvet, Campbell and Sodini (2007, Table 3), in which the portfolio volatility of the median underdiversified household is $\sigma_{\boldsymbol{x}_h} = 19.5\%$ per year and the volatility of a household's portfolio if it had invested in a diversified portfolio would be $\sigma_{1/N} = 14.7\%$ per year. Note that the estimate of 19.5% per year accounts for the investment by the median household of about half its wealth in well-diversified mutual funds. Then, using the result that $\mu_{fh} = (\sigma_{1/N}/\sigma_{\boldsymbol{x}_h})^2$ that is derived in (A27), one can identify $\mu_{fh} = \mu_f = 0.5682$.

The next parameter we need is the expected rate of return on stocks, α. Calvet, Campbell and Sodini (2007) estimate that the equity risk premium, $\alpha - i$, is 6.7% per year and they use an interest rate of $i = 3.7\%$ per year, which implies that $\alpha = 10.4\%$ per year.[27]

Finally, we need an estimate of relative risk aversion. For the partial-equilibrium setting, we identify this using the finding in Calvet, Campbell and Sodini (2007, Table 1) that for the typical household the proportion of wealth invested in risky assets is $\pi_h = 53\%$,[28] which from the expression for π_h in (16) implies that

$$\pi_h = \left[\frac{1}{\gamma_h}\frac{\alpha - i}{\sigma^2_{1/N}}\right]\mu_{fh} \tag{35}$$

$$0.53 = \left[\frac{1}{\gamma_h}\frac{0.067}{0.147^2}\right]0.5682,$$

which gives us an estimate of $\gamma_h = 3.3245$ for the typical household. For the general-equilibrium setting, we again use (35) but now, because of market clearing and the symmetry

[27] The interest rate of 3.7% is not reported in their paper but was obtained via private communication.

[28] The 53% is obtained from the last column of Table 1 in Calvet, Campbell and Sodini (2007), by considering the ratio of investments in domestic stocks, international stocks, and mutual funds to the total investment in financial assets, excluding capital insurance and bonds and derivatives: $(0.2090 + 0.018 + 0.223)/(0.351 + 0.055 + 0.209 + .018 + 0.223) = 53\%$.

condition, we specify $\pi_h = 1$ for all households, which leads to an estimate of $\gamma = 1.762$ for the economy-wide level of relative risk aversion.[29]

These parameters allow us to compute the mean-variance utility of a household. However, to compute lifetime utility, we need to specify also the preference parameters for the subjective rate of time preference, δ, and the elasticity of intertemporal substitution, ψ. In the macroeconomics literature, the value used for δ typically ranges from 2% to 4% per year (see, for instance, Bansal and Yaron (2004) and Collin-Dufresne, Johannes and Lochstoer (2016*a*)) so for our base case we use the value of 3% per year and report results for δ ranging around the base-case value.

For the parameter ψ that governs the elasticity of intertemporal substitution (EIS), there is less agreement in the literature. For instance, several papers estimate ψ and find that it is well in excess of 1.5; see, for instance, Attanasio and Weber (1989), Beaudry and van Wincoop (1996), Vissing-Jorgensen (2002), Attanasio and Vissing-Jorgensen (2003), and Gruber (2013). However, Hall (1988) and Campbell (1999) estimate ψ to be below 1, although their results are based on a model without fluctuating economic uncertainty and Bansal and Yaron (2004) show that ignoring this biases the estimate downwards. In terms of microeconomics, the assumption that $\psi > 1$ is more plausible. That is, if households are to have an asset holding (or saving) function that is increasing in the perceived return of the asset, then this requires $\psi > 1$. Similarly, from an asset-pricing perspective, a value of $\psi > 1$ is more reasonable because if $\psi < 1$ (and risk aversion $\gamma > 1$), then an increase in the volatility of output growth would lead to an *increase* in the value of assets—a counterintuitive and counterfactual outcome (see Bansal and Yaron (2004) for a further discussion of this point). Consequently, many papers in the recent literature, such as Barro et al. (2013), Bansal, Kiku and Yaron (2013), Collin-Dufresne, Johannes and Lochstoer (2013), Bansal et al. (2014), and Iachan, Nenov and Simsek (2016), use a value of EIS that is greater than 1. Therefore, we adopt a base-case value of $\psi = 1.20$ but report results for a range of values around the base case, including values where $\psi < 1$.

[29] This means that the partial-equilibrium and general-equilibrium results should not be compared because the level of risk aversion used in the two settings is different; in the partial-equilibrium setting, the level of risk aversion is chosen to match the allocation to the risky assets reported in Calvet, Campbell and Sodini (2007) while in the general-equilibrium setting the level of risk aversion is chosen to match the interest rate used in Calvet, Campbell and Sodini (2007).

B. Partial Equilibrium

Using the above parameter values, we compute the increase in welfare for an individual household in partial equilibrium from reducing the effect of familiarity biases on: portfolio diversification ($\boldsymbol{x}_h$), asset allocation (π_h), and consumption (c_h).

We consider two cases when the household has mean-variance utility. In the first case, we measure the gain from removing the bias in portfolio-diversification ($\boldsymbol{x}_h$) but with asset-allocation (π_h) still biased. In the second case, we measure the gain from removing both the bias in $\boldsymbol{x}_h$ and π_h.

For the first case, in which a household shifts from an underdiversified portfolio $\boldsymbol{x}_h$ with a volatility of 19.5% per year to a diversified portfolio $\boldsymbol{x}_{1/N}$ with a volatility of only 14.7% per year while keeping π_h fixed at 0.53, we see from (18) that the mean-variance welfare of that household per unit of the household's wealth would increase by:

$$\begin{aligned} U_h^{MV}(\pi_h, \boldsymbol{x}_{1/N}) - U_h^{MV}(\pi_h, \boldsymbol{x}_h) &= -\frac{\gamma_h}{2}\pi_h^2(\sigma_{1/N}^2 - \sigma_{\boldsymbol{x}_h}^2) \\ &= \frac{1}{2}\frac{1}{\gamma}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2 \sigma_{fh}^2 \\ &= 0.0077, \end{aligned} \tag{36}$$

which can be interpreted as an increase of 0.77% in the annualized expected return on the aggregate wealth of household h, which is only a modest increase.

A second way to measure welfare is in terms of the percentage increase in initial aggregate wealth, λ_h, which is required to raise welfare with a familiarity-biased portfolio to that under the fully diversified portfolio:

$$W_h \times U_h^{MV}(\pi_h, \boldsymbol{x}_{1/N}) = W_h(1 + \lambda_h) \times U_h^{MV}(\pi_h, \boldsymbol{x}_h)$$

$$\lambda_h = \frac{U_h^{MV}(\pi_h, \boldsymbol{x}_{1/N})}{U_h^{MV}(\pi_h, \boldsymbol{x}_h)} - 1,$$

where W_h is the level of initial wealth of household h. According to this measure, reported in the second column of the first row of Table 1, a shift from a portfolio with a volatility of 19.5% to a portfolio with a volatility of only 14.7% is equivalent to a $\lambda_h = 14\%$ increase in the initial wealth of household h.

TABLE 1—INDIVIDUAL HOUSEHOLD WELFARE GAIN

	in levels	in %	in % × (1/T)
Mean-variance welfare gain			
$\boldsymbol{x}_h$ unbiased, π_h biased	0.0077	14.00	0.77
$\boldsymbol{x}_h$ and π_h unbiased	0.0135	24.63	1.35
Lifetime utility welfare gain			
$\boldsymbol{x}_h$ unbiased, π_h and c_h biased	0.0273	36.90	2.02
$\boldsymbol{x}_h$ and π_h unbiased, c_h biased	0.0560	75.80	4.15
$\boldsymbol{x}_h$, π_h, and c_h, unbiased	0.0568	76.78	4.20

Notes: In this table, we report the increase in welfare of an individual household in partial equilibrium from reducing the effect of familiarity biases on: portfolio-diversification ($\boldsymbol{x}_h$), asset-allocation (π_h), and the consumption rate (c_h). The first panel measures welfare using mean-variance utility while the second panel measures welfare using intertemporal lifetime utility. The parameter values used are: $\sigma_{1/N} = 14.7\%$ p.a.; $\sigma_{\boldsymbol{x}_h} = 19.5\%$ p.a.; $\alpha = 10.4\%$ p.a.; $i = 3.7\%$ p.a.; $\gamma_h = 3.3245$; $\delta_h = 0.03$ p.a.; $\psi_h = 1.20$.

A third way to measure welfare is to relate this increase in the initial capital stock to an annualized return via the following simple expression: $\frac{14\%}{T} = 0.77\%$, implying that an increase in the initial wealth of 14% is equivalent to an annualized return of 0.77% per year over $T = 18.2632$ years.[30] This is reported in the last column of the first row of numbers in Table 1.

For the second case, we consider the welfare gain for a mean-variance household that overcomes the effect of familiarity biases on its portfolio diversification ($\boldsymbol{x}_h$) *and* also its asset allocation (π_h). That is, we compute

$$U_h^{MV}(\pi_h^U, \boldsymbol{x}_{1/N}) - U_h^{MV}(\pi_h, \boldsymbol{x}_h) = \frac{1}{2}\frac{1}{\gamma}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2 \left[\sigma_{fh}^2 + (\mu_{fh} - 1)^2\right], \tag{37}$$

where π_h^U is the optimal level of wealth allocated to risky assets in the absence of familiarity biases. Comparing (37) to (36), we see that now there will be an additional improvement in household welfare because of the improvement in asset allocation, which is reflected by the last term in (37), $(\mu_{fh} - 1)^2$. As a result of this, the welfare gain is 1.35% instead of 0.77% for the first case where only the bias in portfolio diversification was eliminated. Measured as the percentage increase in the initial wealth, the welfare gain is now 24.63% instead of

[30] For the details of this calculation, observe that $1 + \lambda_h = e^{rT}$, where r is the annualized return over T years for household h, and so $r = \frac{1}{T}\ln(1 + \lambda_h) \approx \frac{1}{T}\lambda_h$.

14% for the first case. Thus, including the impact on asset allocation increases the welfare gains from removing familiarity biases by about three quarters.

Next, we look at welfare when a household is not just a mean-variance optimizer but also wishes to smooth consumption intertemporally. These results are reported in Panel B of Table 1. For the case where we have "$\boldsymbol{x}_h$ unbiased, π_h and c_h biased," the welfare gain measured in levels is 0.0273, instead of 0.0077 for the mean-variance case when accounting for only the effect on portfolio diversification. The welfare gain measured in percentage is 36.90% (instead of 14% when accounting only for portfolio diversification) and the welfare gain scaled over T periods is 2.02% per year (instead of 0.77% per year from just portfolio diversification). In contrast, for the case reported in the last row where $\boldsymbol{x}_h$, π_h, and c_h are all unbiased, the welfare gain is even larger: in levels it is 0.0568, in percentage terms it is 76.78%, and scaled over T periods it is 4.20% per year.

In summary, the above results for the partial-equilibrium setting indicate that the modest increase in the welfare of an individual household from holding a better diversified portfolio is substantially larger once we allow for the possibility that households can adjust also their asset allocation and intertemporal consumption. The lifetime welfare gain of 4.20% per year reported in the last column of Panel B of Table 1 from eliminating the effect of familiarity biases in portfolio diversification, asset allocation, and intertemporal consumption is more than five times the gain of 0.77% per year reported in the last column of Panel A of Table 1 in mean-variance welfare from eliminating the effect of familiarity biases on just portfolio diversification.

C. General Equilibrium

In this section, we consider the effects on *aggregate* social welfare from removing familiarity biases of households. That is, in contrast to the partial-equilibrium analysis considered in the previous section, we impose the market-clearing condition that aggregate borrowing is zero and we consider the effects on macroeconomic growth arising from aggregating the consumption-savings decisions of individual households.

Using the parameter values described in Section A, we compute social welfare per unit of aggregate capital stock for three cases in general equilibrium, as summarized in the three rows of numbers in Table 2. In the first case, we consider mean-variance households; in our general-equilibrium model we see from (29) that this corresponds to the special case in which all households have zero elasticity of intertemporal substitution: $\psi = 0$. If households were to shift from underdiversified portfolios with a volatility of 19.5% to diversified portfolios with a volatility of only 14.7%, we see from (30) that social welfare per unit capital stock would increase by:

$$U^{MV}(\sigma_{1/N}) - U^{MV}(\sigma_{\boldsymbol{x}}) = -\frac{\gamma}{2}(\sigma_{1/N}^2 - \sigma_{\boldsymbol{x}}^2) = 0.0145,$$

which can be interpreted as an increase of 1.45% in the annualized expected return on the aggregate capital stock (i.e. aggregate wealth).

Just like for the partial-equilibrium setting, one could also compute the percentage increase in the initial aggregate capital stock, λ, which is required to raise social welfare with poorly diversified portfolios to that under perfectly diversified portfolios:

$$K^{\text{agg}} \times U^{MV}(\sigma_{1/N}) = K^{\text{agg}}(1+\lambda) \times U^{MV}(\sigma_{\boldsymbol{x}})$$

$$\lambda = \frac{U^{MV}(\sigma_{1/N})}{U^{MV}(\sigma_{\boldsymbol{x}})} - 1,$$

where K^{agg} is the initial level of the aggregate capital stock. According to this measure, reported in the second column of the first row of numbers in Table 2, a shift from portfolios with a volatility of 19.5% to portfolios with a volatility of only 14.7% is equivalent to a $\lambda = 20.51\%$ increase in the initial capital stock. This increase in the initial capital stock can be related to an annualized return via the following simple expression: $\frac{20.51\%}{T} = 1.45\%$, implying that an increase in the aggregate capital stock of 20.51% is equivalent to an annualized return over $T = 14.18$ years of 1.45%, which is reported in the last column of the first row of numbers in Table 2.[31]

Next, we look at social welfare when households desire to smooth consumption intertemporally ($\psi > 0$). These results are reported in the row titled "Intertemporal household: $\boldsymbol{x}, c$ unbiased but growth exogenous" of Table 2. We find that if $\psi = 1.20$, shifting from a portfolio with a volatility 19.5% to a portfolio with a volatility of 14.7%, keeping the aggregate

[31] For the details of this calculation, see footnote 30.

TABLE 2—SOCIAL WELFARE GAINS FROM HOUSEHOLD PORTFOLIO DIVERSIFICATION

	in levels	in %	in % $\times (1/T)$
PANEL A: FULL REMOVAL OF FAMILIARITY BIASES			
Mean-variance welfare gain			
$\boldsymbol{x}$ unbiased	0.0145	20.51	1.45
Lifetime utility welfare gain			
$\boldsymbol{x}, c$ unbiased but growth exogenous	0.1145	79.15	5.58
$\boldsymbol{x}, c$ unbiased and growth endogenous	0.1491	103.05	7.26
PANEL B: PARTIAL REMOVAL OF FAMILIARITY BIASES			
Mean-variance welfare gain			
$\boldsymbol{x}$ less biased	0.0070	9.92	0.70
Lifetime utility welfare gain			
$\boldsymbol{x}, c$ less biased but growth exogenous	0.0455	31.46	2.22
$\boldsymbol{x}, c$ less biased and growth endogenous	0.0566	39.11	2.76

Notes: In this table, we report the potential gains to social welfare if households diversify their portfolios. There are two panels: in Panel A, we consider the welfare gains from the full removal of all familiarity biases so that the volatility of each household's portfolio decreases from $\sigma_{\boldsymbol{x}} = 19.5\%$ per year to $\sigma_{1/N} = 14.7\%$ per year, while in Panel B, we consider the welfare gains from the partial removal of familiarity biases so that the volatility of each household's portfolio decreases partially from $\sigma_{\boldsymbol{x}} = 19.5\%$ to 17.3% per year. In each panel, we report results for three cases. In the first case, households have mean-variance utility. In the second case, households are intertemporal consumers, but growth is exogenous. In the third case, households are intertemporal consumers and growth is endogenous. The parameter values are: $\sigma_{1/N} = 14.7\%$ p.a.; $\sigma_{\boldsymbol{x}} = 19.5\%$ p.a.; $\alpha = 10.4\%$ p.a.; $\gamma = 1.762$; $\delta = 0.03$ p.a.; $\psi = 1.20$.

growth rate fixed exogenously (at the level at which the portfolio volatility of each household is equal to 19.5%), raises social welfare per unit capital stock by 0.1145 rather than the 0.0145 for the mean-variance case that ignores the effect on intertemporal consumption. The social welfare gain is more than seven times larger now because the households benefit not just from reducing the volatility of their portfolios, but also from the utility gain from smoothing intertemporal consumption. This larger increase in social welfare is similar in magnitude if one were to measure the gain in terms of the required increase in the initial aggregate capital stock; that is, $\lambda = 79.15\%$ instead of the 20.51% for the mean-variance case. Similarly, the change in initial capital stock is equivalent to an annualized expected return of 5.58% over $T = 14.18$ years, compared to the 1.45% for the mean-variance case.

Finally, we look at the case in which aggregate growth is endogenous, and hence, it changes when all households shift to a portfolio with less risk. These results are reported in Table 2, in the row titled "Intertemporal household: $\boldsymbol{x}, c$ unbiased and growth endogenous" We find that if $\psi = 1.20$, shifting from portfolios with a volatility 19.5% to portfolios with a volatility of 14.7% raises social welfare per unit of capital stock by 0.1491 instead of the 0.1145 for the case with exogenous growth, and 0.0145 for the mean-variance case. The social welfare gain is much larger now because households benefit not just from reducing the volatility of their portfolio and the resulting smoothing of intertemporal consumption, but also from the increase in aggregate growth.

The social welfare gain of 0.1491 compared to 0.1145 is of course similar in magnitude to what one would obtain by measuring the gain in terms of the required increase in initial capital stock: $\lambda = 103.05\%$ compared to 79.15%, which is reported in the second column of Table 2. Similarly, using the same scaling of $T = 14.18$ years as for the mean-variance case, we find that the change in initial capital stock is equivalent to an annualized expected return of 7.26% with endogenous growth, in comparison to 5.58% with exogenous growth, and 1.45% for the mean-variance case.

Panel B of Table 2 reports the welfare gains for the case where, instead of removing familiarity biases entirely, these biases are removed only partially. Specifically, instead of the volatility of the households portfolio dropping all the way from 0.195 to 0.147, it drops only halfway to about 0.173. The insights in this case are exactly the same as for Panel A, where we removed familiarity biases completely: the welfare gain after incorporating the effects of intertemporal consumption and endogenous growth is much larger than the direct gain from just the reduction in portfolio volatility.

Thus, the above results suggest that the modest increase in social welfare for mean-variance households from holding a better diversified portfolio is substantially larger once we allow for the possibility that households can smooth consumption intertemporally and that the aggregate effects of these changes could lead to an increase in growth.

We now examine the sensitivity of the above results to the impatience parameter, δ, and the elasticity of intertemporal substitution parameter, ψ. As in all dynamic models,

the effect of the intertemporal allocation of capital becomes less pronounced as households become more impatient. The effect of an increase in δ is illustrated in the first panel of Figure 1 below, in which we plot the social welfare gain for the three settings examined in the three rows of the table above as households' impatience changes. The figure reports the social welfare gain per unit capital stock in percentage terms based on an annualized return over $T = 14.18$ years. The solid (black) line in the figure shows the welfare gains from diversification for a mean-variance household: this line is flat because the mean-variance utility does not depend on δ. The dotted (blue) line shows the gains from portfolio diversification for a household with intertemporal consumption when growth is exogenous: this line shows that for patient households (low δ) with a strong willingness to postpone consumption to future dates, the welfare gains can be amplified by an order of magnitude relative to the mean-variance case. The dashed (red) line shows the welfare gain to society from portfolio diversification when growth is endogenous: this line shows that the social welfare gains exceed the private gains to individual households, with the gap between the two increasing as δ decreases. These lines show that a large part of the amplification stems not from the direct welfare gain arising from a reduction in micro-level volatility via portfolio diversification, but instead from the smoothing of intertemporal consumption and the macro-level effect on aggregate growth.

In the second panel of Figure 1, we plot the social welfare gain per unit of capital stock for the three settings examined in the three rows of the table above as the household's elasticity of intertemporal substitution (ψ) changes. The solid (black) line shows the welfare gains from diversification for a mean-variance household: this line is flat because mean-variance utility does not depend on ψ. The dotted (blue) line shows the gains from portfolio diversification for a household with intertemporal consumption when growth is exogenous: this line shows that the welfare gain from portfolio diversification for an individual household exceeds that of the mean-variance household. The dashed (red) line shows that the welfare gain to society from portfolio diversification when growth is endogenous. This line intersects the dotted (blue) line at the special case of $\psi = 1$ because at that point the income and substitution effects offset each other exactly, so society chooses not to adjust aggregate investment at all. For the region where $\psi > 1$, society is willing to consume less today and

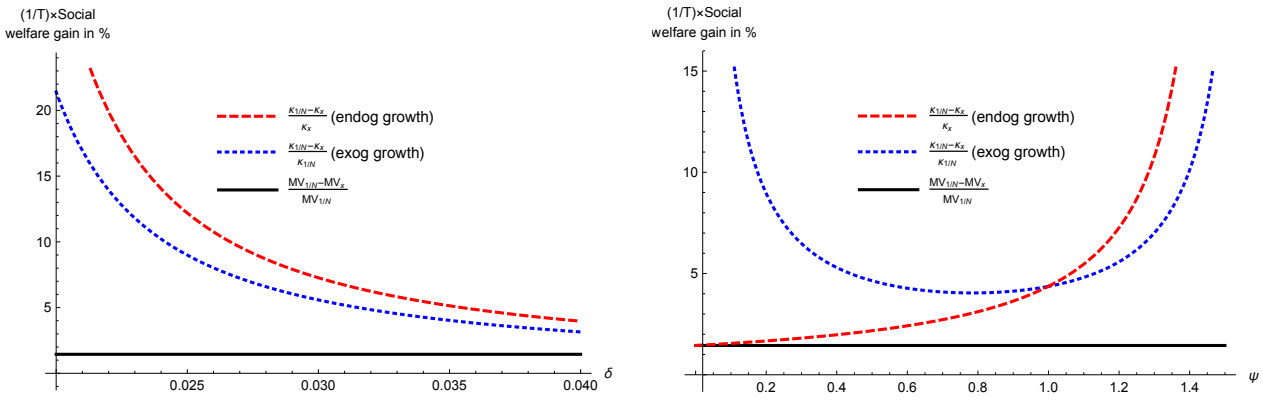

FIGURE 1. SOCIAL WELFARE WITH CHANGES IN δ AND ψ

Notes: In the panel on the left, we plot welfare as the impatience parameter, δ, varies and in the panel on the right, we plot welfare as the elasticity of intertemporal substitution, ψ, varies. In both panels, we report the social welfare gain per unit capital stock in percentage terms based on an annualized return over $T = 14.18$ years. Each panel has three curves: the solid (black) line shows the welfare gains from diversification for a mean-variance household; the dotted (blue) line shows the gains from portfolio diversification for a household with intertemporal consumption but with growth being exogenous; and the dashed (red) line shows the welfare gain from portfolio diversification when growth is endogenous.

invest more, leading to positive growth effects; thus, in this region the social gains, given by the dashed (red) line, exceed the private gains, given by the dotted (blue) line. The reverse is true for $\psi < 1$, with the social welfare gains coinciding with the gains for a mean-variance household when $\psi = 0$.

We conclude this section by examining the effects of preference heterogeneity on the results reported above. The results for the case where households differ in their preference parameters are based on the expression in (34) and are given in Table 3, which has 4 panels. Panel A gives the results for the base case, where there is no heterogeneity and familiarity is reduced partially (exactly as in Panel B of Table 2). Panels B, C, and D of Table 3 are for the cases where there is heterogeneity in subjective discount rates, elasticities of intertemporal substitution, and relative risk aversion, respectively. Heterogeneity is modeled as a uniform distribution centered on the parameter value for the base case of that parameter.

We see from this table that the social welfare gains increase with heterogeneity. For example, in Panel A where all households have the same preference parameters, the social

TABLE 3—WELFARE GAINS WHEN δ_h, ψ_h, OR γ_h ARE HETEROGENEOUS

	in levels	in %	in % $\times$ $(1/T)$
PANEL A: BASE CASE WITHOUT HETEROGENEITY			
Mean-variance welfare gain			
$\boldsymbol{x}$ less biased	0.0070	9.92	0.70
Lifetime utility welfare gain			
$\boldsymbol{x}, c$ less biased but growth exogenous	0.0455	31.46	2.22
$\boldsymbol{x}, c$ less biased and growth endogenous	0.0566	39.11	2.76
PANEL B: HETEROGENEITY IN SUBJECTIVE DISCOUNT RATES			
Mean-variance welfare gain			
$\boldsymbol{x}$ unbiased	0.0070	9.92	0.70
Lifetime utility welfare gain			
$\boldsymbol{x}, c$ unbiased but growth exogenous	0.0965	37.29	2.63
$\boldsymbol{x}, c$ unbiased and growth endogenous	0.1231	46.98	3.31
PANEL C: HETEROGENEITY IN ELASTICITY OF INTERTEMPORAL SUBSTITUTION			
Mean-variance welfare gain			
$\boldsymbol{x}$ unbiased	0.0070	9.92	0.70
Lifetime utility welfare gain			
$\boldsymbol{x}, c$ unbiased but growth exogenous	0.0628	36.13	2.55
$\boldsymbol{x}, c$ unbiased and growth endogenous	0.0919	49.72	3.51
PANEL D: HETEROGENEITY IN RELATIVE RISK AVERSION			
Mean-variance welfare gain			
$\boldsymbol{x}$ unbiased	0.0090	12.80	0.90
Lifetime utility welfare gain			
$\boldsymbol{x}, c$ unbiased but growth exogenous	0.0618	42.71	3.01
$\boldsymbol{x}, c$ unbiased and growth endogenous	0.0777	53.71	3.79

Notes: In this table, we report the potential gains to social welfare from reducing households' familiarity biases partially. Our results are reported in four panels. Panel A gives the results for the case where there is no heterogeneity. Panels B, C, and D are for the cases where there is heterogeneity in subjective discount rates, elasticities of intertemporal substitution, and relative risk aversion levels, respectively. Heterogeneity of 1001 households is modeled as a uniform distribution centered on the parameter value for the base case of that parameter in Panel A, in which $\delta = 0.03$ p.a., $\psi = 1.20$, and $\gamma = 1.762$. In Panel B, δ_h ranges from 0.02 to 0.04. In Panel C, ψ_h ranges from 0.90 to 1.50. In Panel D, γ_h ranges from 1.262 to 2.262. The parameter values we have assumed are: $\sigma_{1/N} = 14.7\%$ p.a.; $\sigma_{\boldsymbol{x}} = 19.5\%$ p.a.; and $\alpha = 10.4\%$ p.a..

welfare gain is 39.11%, but it is 46.98% in Panel B where all households have different subjective time discount rates, it is 49.72% in Panel C where all households have different EIS parameters, and it is 53.71% in Panel D where all households have different levels of

risk aversion. The intuition for why the impact of household heterogeneity increases the welfare gains is that when households have heterogeneous preference parameters, the desire for risk sharing and consumption smoothing is even greater; thus, underdiversification is even more costly in this setting. This intuition is evident also from Figure 1 where we plot the level of welfare gain for different values of the preference parameters. These figures show that the welfare gains are convex functions of the household preference parameters; thus, when households have heterogeneous preferences, because of Jensen's inequality, the welfare gains for society (that is, the welfare gain averaged across households) are at least as great as the welfare gains for the average household (that is, the welfare gain for the average value of a preference parameter).

V. Discussion of Modeling Choices

In this section, we discuss how our results depend on the various modeling choices that we have made. We start by making a general observation. Note that our main insight is about the *change in welfare* from removing familiarity biases being larger than just the gain from improving portfolio diversification if one accounted also for the asset-allocation implications and intertemporal effects at the individual level and the growth effects at the aggregate level. To make this point, we are *comparing the change* in the welfare levels when accounting for these additional effects. That is, we are looking at the *difference in difference* of welfare levels. Because we are looking at the difference in difference, variations in the baseline model will largely wash out.

A. Labor Income

In our analysis of the effect of familiarity biases on the lifetime utility of a household, we have ignored the presence of uninsurable background risks from sources such as labor income, employer stock holdings, and restrictions on pension investments. In this section, we discuss how the presence of such risks would affect our results, with a special focus on the effect of labor income. A more comprehensive survey of both the theoretical and empirical work on the effect of background risk on stock-market participation is presented in Curcuru

et al. (2010), who find that "including background income risk can make it more difficult to explain non-participation in the stock market, or low levels of stock holdings."

We start by reporting the textbook solution for the optimal portfolio weight in the presence of labor income:[32] Campbell (2018, Equation (10.15), p. 311) shows that for the case of CRRA utility and the choice between a single risky asset and the risk-free asset in partial equilibrium, the share of wealth invested in the risky asset, using our notation and for the setting in continuous time, can be written as:

$$\omega_h = \underbrace{\frac{1}{\hat{\gamma}_h}\frac{\alpha - i}{\sigma^2}}_{\text{myopic component}} - \underbrace{\left(\frac{\gamma_h}{\hat{\gamma}_h} - 1\right)\beta}_{\text{intertemporal component}}, \tag{38}$$

where $\hat{\gamma}_h = \gamma_h \times \rho^{\text{adj}}$ can be interpreted as the risk-aversion of the household adjusted for labor income, $\rho^{\text{adj}} < 1$ is the elasticity of consumption with respect to financial wealth, and β is the covariance of the household's labor income with the return on the risky asset divided by the variance of the risky asset return.

In our setting, where households have recursive utility with familiarity biases, and there are multiple risky assets, one can derive an expression for the optimal portfolio policy that is similar to (38). To simplify the derivation, we assume that each household works for a single firm and that the labor income of this household is more highly correlated with the stock returns of this particular firm, compared to the correlation with the stock returns for all the other firms. Denoting by ϵ the proportional increase in the correlation between the labor income of a particular household and the stock return of the firm where this household is employed, we show that the optimal portfolio vector is:

$$\boldsymbol{\omega}_h = \Psi_h^{-1}\left[\underbrace{\frac{1}{\widehat{\gamma}_h}\frac{\alpha - i}{\sigma^2}\mathbf{1}}_{\text{myopic component}} - \underbrace{\left(\frac{\gamma_h}{\widehat{\gamma}_h} - 1\right)\beta(\mathbf{1} + \epsilon\,\boldsymbol{e}_h)}_{\text{intertemporal component}}\right], \tag{39}$$

where Ψ_h is the correlation matrix for the returns of the N risky assets adjusted for familiarity biases, $\mathbf{1}$ is the $N \times 1$ vector of ones, and $\boldsymbol{e}_h \in \mathbb{R}^H$ is the column vector with zeros in all entries except entry h. The expression in (39) is derived in Appendix B.

[32] For an excellent summary of the key theoretical results about the effect of labor income on portfolio choice, see Campbell (2018, Ch. 10) and Guiso and Sodini (2013). While most of the empirical results are for U.S. data, the results are similar for other countries: Guiso, Jappelli and Terlizzese (1996) find that background risk has only a small affect on portfolio choice for Italian households and Fagereng, Guiso and Pistaferri (2018) find that the size of uninsurable wage risk is small in administrative data for Norway.

To understand the effect of labor income on the effects of familiarity biases, note that there are four possibilities for the stochastic nature of labor income. The first possibility is that labor income is *deterministic*. Equation (39) shows that in this case, because β is zero, the second component of the portfolio weight is zero. Therefore, the only effect of labor income is to *increase* the share of wealth the household would like to invest in the risky asset (because $\rho^{\text{adj}} < 1$), which would only exacerbate the effect of familiarity biases in household portfolios.[33]

The second possibility is that labor income is *risky but uncorrelated* with stock returns, which is what many studies find empirically.[34] In this case again, β is zero, and therefore, labor income has no impact on the portfolio composition of the household and its only effect is to reduce the demand for the risk-free asset. For instance, Campbell (2018, p. 311) concludes that: "An investor with risky but uncorrelated labor income always invests a larger share of his financial wealth in the risky asset than an investor with no labor income." The intuition is that because of the presence of labor income, a one percent negative shock in financial wealth does not fully translate into a one percent decrease in consumption growth; thus, households should hold riskier portfolios.

The third possibility is that labor income is *risky but positively correlated* with stock returns. Thus, the returns on these stocks would be low when labor income was low, which implies that it would be optimal for the household to hedge the risky labor income by *reducing* the holding in the familiar firm.[35] Thus, in this case a bias in portfolios toward familiar assets has *even more negative consequences* for welfare.

The fourth possibility is that labor income is *negatively correlated* with stock returns. In this case, it would be optimal for each household to hold a portfolio that is biased *toward* the familiar firm; thus, only in this case will the welfare cost of the familiarity bias in portfolios be smaller than for the case without labor income. However, the case of labor income being

[33]Bodie, Merton and Samuelson (1992) show that when future labor income is certain, it is optimal for households to hold proportionally *more* stocks in their portfolios—because deterministic labor income is similar to receiving interest from holding a risk-free bond.

[34]See, for example, Botazzi, Pesenti and van Wincoop (1996), Cocco, Gomes and Maenhout (2005, page 529), and Davis and Willen (2000*a*).

[35]See Viceira (2001, Prop. 3) for a more detailed discussion.

negatively related to stock returns is unlikely because this would imply that a household's labor income *increases* when the firm where the household is employed performs poorly.[36]

Overall, the size of the hedging-demand component is likely to be small. The reason for this is that the β of the household's labor income with respect to the return on a risky asset is small because individual stock-return volatility is about 0.30 per year and much higher than labor income volatility, which is only about 0.03 or 0.04 per year, and therefore, even if the correlation between labor income and stock returns is assumed to be 0.10 or 0.20, it will be reduced substantially when it is multiplied by the ratio of the volatility of labor income to the volatility of individual stock returns. Therefore, when we evaluate numerically the expression in (39) for the portfolio weights, we find that in the absence of familiarity biases, the intertemporal-hedging component of the portfolio is small—only about 5% of the mean-variance component; moreover, in the presence of familiarity biases, the size of the intertemporal-hedging component decreases further by about half.

B. Other Sources of Income

In our analysis, we have assumed that household portfolios consist of investments only in financial assets. However, many households invest a major share of their wealth in real estate (in fact, the investment in real estate is typically levered) and some households invest also in entrepreneurial ventures. These investments would imply that household portfolios are even less well diversified than we have assumed above. Consequently, the social welfare gains from improved diversification would be even larger than the ones reported above.

C. Other Approaches to Modeling Production

In our analysis, we have used the Cox, Ingersoll and Ross (1985) production model that has constant returns to scale. Instead, if one has a Cobb-Douglas production function, growth is not endogenous and diversification impacts the steady-state level of aggregate capital. One can show that even in this case diversification across production technologies

[36] Davis and Willen (2000*b*, their Table 3) report R^2 values for regressions of earnings innovations on the returns on the S&P 500 and find that the R^2 values are small for all groups, and always less than 10 percent.

(that is, firms) is equivalent to a permanent increase in the level of technology, which increases the steady-state level of aggregate capital.

On the other hand, in a model where technological progress is endogenous and depends on innovation by entrepreneurs, familiarity biases will have the benefit of *increasing* the rate of innovation, while still having all the costs that we have documented in our paper. But, as we explained above, because we are looking at the difference in difference of welfare levels, the insights we get in our model would be similar to the ones in a model with endogenous technological progress.

Finally, consider an overlapping-generations (OLG) model. In such a model, because each generation of households is short-lived, the long term growth of the economy would matter less for their welfare than in our current set up. One way to see this in the context of our model is to increase the subjective discount rate, δ: the first panel of Figure 1 shows this. However, in the OLG model, if instead of measuring the welfare of a particular generation one were to consider a social planner who cares about the welfare of many generations, then it would move the results of the OLG model closer to the ones that we report.

D. Speed of Capital Adjustment

We have assumed that firms can adjust their investment policies instantly and at no cost; if the adjustment of physical capital takes time, then the magnitude of the macroeconomic-level growth effect we have identified will be smaller. To study the impact of assuming that investment levels can be adjusted instantaneously, one can use the approach in Obstfeld (1994, p. 1325), in which it is assumed that the annual welfare gain converges toward the long-run annual gain at an instantaneous rate of y percent. Barro, Mankiw and Sala-i-Martin (1992) estimate that the convergence rate when adjusting physical capital *across* countries is about 2.2% per year. Assuming that the rate of convergence when adjusting physical capital across sectors *within* a country is double of 2.2%, then the actual capitalized social welfare gain from the change in growth, $\lambda_g^{\text{actual}}$, is related to the reported social welfare

gain from the change in growth, which we label λ_g, as follows:

$$\lambda_g^{\text{actual}} = \int_0^\infty i\,\lambda_g(1 - e^{-y\,t})e^{-i\,t}dt = \lambda_g \frac{y}{i+y}.$$

Using the per annum real interest rate of 0.56% for the U.S., $\frac{y}{i+y} = 89\%$, suggesting that the actual social welfare gain from a change in growth would be close to the one we computed. Alternatively, using the Swedish interest rate of 3.7%, $\frac{y}{i+y} = 54\%$, which suggests that the actual social welfare gain from the macroeconomic-level change in growth would be about half of the one we computed.

VI. Conclusion

Our results indicate that the impact on household and social welfare of financial policy, through education, innovation, and regulation, can be substantial—the potential gains are equivalent to an increase in the return on aggregate wealth of about 5%. Most of this gain arises from a multiplier effect applied to the gains for a household with mean-variance utility. This multiplier effect is driven by the impact of improved portfolio diversification on intertemporal consumption smoothing at the microeconomic level and on aggregate growth at the microeconomic level. The analysis in our paper suggests that the answer to the question posed in the title is a resounding "yes." Household finance matters a great deal because small improvements in the financial decisions of individual households have the potential to generate large economic gains for society: a small step for households can be a giant leap for society.

Thaler and Sunstein (2003) recommend "nudges" that gently guide people in a direction that increases welfare. Similarly, one could consider a variety of policies that could ameliorate the familiarity biases of households. One such policy measure is to introduce default portfolios that are well diversified. There is substantial evidence that the choice of a default option can be important (Samuelson and Zeckhauser, 1988), because when a particular choice is designated as the default, it attracts a disproportionate market share. For example, households could be offered a small number of portfolios to choose from, with

the portfolios having different levels of risk, but all of them being well diversified.[37] Carlin and Robinson (2012) and Carlin and Davies (2015) analyze how the right menu of portfolio options should be chosen based on the financial sophistication of households and their behavioral biases. In the context of our model, the default fund would be one that was diversified across the N risky assets.

A second policy measure is financial education. For example, households could be educated about the benefits of diversification. Empirical evidence suggests that financial literacy can play an important role in improving decisions made by households. For instance, Bayer, Bernheim and Scholz (2008) find that both participation in and contributions to voluntary savings plans are significantly higher when employers offer frequent seminars about the benefits of planning for retirement. Dimmock et al. (2014) also find that, while general education has only a small effect in reducing familiarity bias, an increase in financial competence does reduce this bias. Carlin and Robinson (2012) also find that financial education can have a strong positive effect on investment decisions. Financial education could also inform households about the benefits of investing in broadly diversified funds, such as mutual funds and ETFs, which do not require familiarity with particular assets.

A third alternative is to introduce financial regulation to limit the tendency of households to bias portfolios toward a few familiar assets. For example, financial regulation could be introduced to prohibit companies from providing employees own-company stock when matching the pension contributions of employees. Financial regulation could also prohibit the use of own-company stock in 401(k) plans. Concurrently, one could require mutual funds to simplify investment procedures in order to lower the barrier to entry and increase investments in these diversified assets.

[37] Similar to the policy advocated by Benartzi and Thaler (2004), in which people commit in advance to allocating a proportion of their future salary increases toward retirement savings, one could design sensible default options that encourage households to invest in portfolios that are diversified across equities and asset classes. Madrian and Shea (2001) study the impact of automatic enrollment on 401(k) savings behavior. They find that participation is significantly higher under automatic enrollment and that a substantial fraction of the participants retain the default contribution rate and fund allocation. Cronqvist and Thaler (2004) find similar results in Sweden, where the government introduced a private plan for social security savings, with a "default" fund that was diversified internationally.

A. Appendix

In this appendix, we provide full derivations for all the results in the main text. The title of each subsection below indicates the particular equation(s) derived in that subsection. To make it easier to read this appendix without having to go back and forth to the main text, we reproduce the key equations to be derived as propositions and also rewrite any equations from the main text that are needed; these equations are assigned the same numbers as in the main text.

A.1. The certainty equivalent in (6)

Definition A.1.1 *A certainty equivalent amount of a risky quantity is the equivalent risk-free amount in static utility terms, i.e.*

$$u_{\gamma_h}\left(\mu_{h,t}\left[U_{h,t+dt}\right]\right) = E_t[u_{\gamma_h}(U_{h,t+dt})], \tag{A1}$$

where $u_{\gamma_h}(\cdot)$ is the static utility index defined by the power utility function[38]

$$u_{\gamma_h}(x) = \begin{cases} \frac{x^{1-\gamma_h}}{1-\gamma_h}, & \gamma_h > 0,\ \gamma_h \neq 1 \\ \ln x, & \gamma_h = 1, \end{cases} \tag{A2}$$

and the conditional expectation $E_t[\cdot]$ is defined relative to a reference probability measure $\mathbb{P}$.

Proposition A.1.1 *The date-t certainty equivalent of household h's date-$t+dt$ utility is given by*

$$\mu_{h,t}[U_{h,t+dt}] = E_t[U_{h,t+dt}] - \frac{1}{2}\,\gamma_h\,U_{h,t}\,E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]. \tag{6}$$

Proof: The definition of the certainty equivalent in (A1) implies that

$$\mu_{h,t}[U_{h,t+dt}] = E_t\left[U_{h,t+dt}^{1-\gamma_h}\right]^{\frac{1}{1-\gamma_h}} = E_t\left[U_{h,t}^{1-\gamma_h} + d(U_{h,t}^{1-\gamma_h})\right]^{\frac{1}{1-\gamma_h}}.$$

Applying Ito's Lemma, we obtain

$$d(U_{h,t}^{1-\gamma_h}) = (1-\gamma_h)U_{h,t}^{-\gamma_h}dU_{h,t} - \frac{1}{2}(1-\gamma_h)\gamma U_{h,t}^{-\gamma_h-1}(dU_{h,t})^2$$

[38] In continuous time the more usual representation for utility is given by $J_{h,t}$, where $J_{h,t} = u_{\gamma_h}(U_{h,t})$, with the function u_{γ_h} defined in (A2).

$$= (1-\gamma_h) U_{h,t}^{1-\gamma_h} \left[\frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2}\gamma_h \left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right].$$

Therefore,

$$\mu_{h,t}[U_{h,t+dt}] = E_t \left[U_{h,t+dt}^{1-\gamma_h} \right]^{\frac{1}{1-\gamma_h}} = U_{h,t} \left(E_t \left[1 + (1-\gamma_h) \left[\frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2}\gamma_h \left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right] \right)^{\frac{1}{1-\gamma_h}}$$
$$= U_{h,t} \left(1 + (1-\gamma_h) \left[E_t \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2}\gamma_h E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right] \right)^{\frac{1}{1-\gamma_h}}.$$

Hence, expanding the above expression, and using the notation $g = o(dt)$ to denote that $g/dt \to 0$ as $dt \to 0$, one obtains:

$$\mu_{h,t}[U_{h,t+dt}] = U_{h,t} \left(1 + E_t \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2}\gamma_h E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right) + o(dt),$$

which, in the continuous-time limit, leads to the expression in (6). ■

A.2. The familiarity-biased certainty equivalent in (8)

While (8), giving the familiarity-biased certainty equivalent, is given as a definition within the main text of the paper, we can derive it from more primitive assumptions. To do so, we need additional definitions and lemmas.

We start by defining the measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$.

Definition A.2.1 *The probability measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ is defined by*

$$\mathbb{Q}^{\boldsymbol{\nu}_h}(A) = E[1_A \xi_{h,T}],$$

where E is the expectation under $\mathbb{P}$, A is an event realized at date T, and $\xi_{h,t}$ is the exponential martingale (under the reference probability measure $\mathbb{P}$) given by

$$\frac{d\xi_{h,t}}{\xi_{h,t}} = \frac{1}{\sigma} \boldsymbol{\nu}_{h,t}^\top \Omega^{-1} d\boldsymbol{Z}_t,$$

where $\Omega = [\Omega_{nm}]$ is the $N \times N$ correlation matrix of returns on firms' capital stocks

$$\Omega_{nm} = \begin{cases} 1, & n = m, \\ \rho, & n \neq m, \end{cases}$$

and

$$\boldsymbol{Z}_t = (Z_{1,t}, \ldots, Z_{N,t})^\top.$$

Recall that when a household is less familiar with a particular firm, it adjusts its expected return, which is equivalent to changing the reference measure to a new measure, denoted by $\mathbb{Q}^{\boldsymbol{\nu}_h}$. Applying Girsanov's Theorem, we see that under the new measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$, the evolution of firm n's capital stock is given by

$$dK_{n,t} = [(\alpha + \nu_{hn,t})K_{n,t} - D_{n,t}]dt + \sigma K_{n,t} dZ^{\boldsymbol{\nu}_h}_{n,t},$$

where $Z^{\boldsymbol{\nu}_h}_{n,t}$ is a standard Brownian motion under $\mathbb{Q}^{\boldsymbol{\nu}_h}$, such that

$$dZ^{\boldsymbol{\nu}_h}_{n,t} dZ^{\boldsymbol{\nu}_h}_{m,t} = \begin{cases} dt, & n = m. \\ \rho dt, & n \neq m. \end{cases}$$

Before motivating the definition of the penalty function, we make the following additional definition, so we can measure information losses stemming from biases with respect to a specific firm.

Definition A.2.2 *The probability measure $\mathbb{Q}^{\nu_{h,n}}$ is defined by*

$$\mathbb{Q}^{\nu_{h,n}}(A) = E[1_A \xi_{h,n,T}],$$

where E is the expectation under $\mathbb{P}$, A is an event realized at date T, and $\xi_{h,n,t}$ is the exponential martingale (under the reference probability measure $\mathbb{P}$) given by

$$\frac{d\xi_{h,n,t}}{\xi_{h,n,t}} = \frac{1}{\sigma} \nu_{h,n,t} dZ_{n,t}.$$

The probability measure $\mathbb{Q}^{\nu_{h,n}}$ is just the probability measure associated with familiarity bias with respect to firm n. Familiarity bias along this factor is equivalent to using $\mathbb{Q}^{\nu_{h,n}}$ instead of $\mathbb{P}$, which leads to a loss in information. The rate of information loss stemming from familiarity bias with respect to firm n can be quantified via the Kullback-Leibler divergence (per unit time) between $\mathbb{P}$ and $\mathbb{Q}^{\nu_{h,n}}$, given by

$$D^{KL}[\mathbb{P}|\mathbb{Q}^{\nu_{h,n}}] = \frac{1}{2} \frac{\nu^2_{h,n}}{\sigma^2}.$$

We can now think about how to measure the total rate of information loss from familiarity biases with respect to all N firms. We can form a simple weighted sum of the date-t

conditional Kullback-Leibler divergences for familiarity bias with respect to each individual firm, i.e.,

$$\hat{L}_{h,t} = \sum_{n=1}^{N} \mathcal{W}_{h,n} D^{KL}[\mathbb{P}|\mathbb{Q}^{\nu_{h,n}}],$$

in which $\mathcal{W}_{h,n}$ is a household-specific weighting matrix. We can think of the matrix $\mathcal{W}_{h,n}$ as a set of weights for information losses, analogous to the weights used in the generalized method of moments.

The choice of weighting matrix depends on how a household weights information losses, which we assume depends on the household's level of familiarity bias. For illustration, consider the simple case where $\mathcal{W}_{h,n} = \frac{f_{h,n}}{1-f_{h,n}}$, $\rho = 0$ so shocks to firm-level returns are mutually orthogonal, and the household h is completely unfamiliar with all firms save firm 1. In this case,

$$\mathcal{W}_{h,n} = \begin{cases} \frac{f_1}{1-f_1}, & n = 1 \\ 0, & n \neq 1. \end{cases}$$

Our expression for total rate of information loss from familiarity biases with respect to all N firms then reduces to

$$\hat{L}_{h,t} = \frac{f_1}{1-f_1} D^{KL}[\mathbb{P}|\mathbb{Q}^{\nu_{h,1}}].$$

So, we can see that if a household is completely unfamiliar with a particular firm, the information loss associated with deviating from the reference measure $\mathbb{P}$ is assigned a weight of zero. The more familiar a household is with a firm, the greater the weight on the information loss for that firm caused by deviating from the reference measure.

Motivated by the above discussion, we now define a penalty function for using the measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ instead of $\mathbb{P}$.

Definition A.2.3 *The penalty function for household h associated with its familiarity biases is given by*

$$\hat{L}_{h,t} = \frac{1}{2\sigma^2} \boldsymbol{\nu}_{h,t}^{\top} \Gamma_h^{-1} \boldsymbol{\nu}_{h,t}.$$

We can see that information losses linked to the firms with which the household is totally unfamiliar are not penalized in the penalty function. The household is penalized only for

deviating from $\mathbb{P}$ with respect to a particular firm if it has some level of familiarity with that firm. If it has full familiarity with a firm, the associated penalty becomes infinitely large, so when making decisions involving this firm, the household will not deviate at all from the reference probability measure $\mathbb{P}$.

Theorem A.2.1 *The date-t familiarity-biased certainty equivalent of date-$t+dt$ household utility is given by*

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] + U_{h,t}L_{h,t}dt, \tag{A3}$$

where $\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ is defined by

$$u_\gamma\left(\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]\right) = E_t^{\mathbb{Q}^{\boldsymbol{\nu}_h}}\left[u_\gamma\left(U_{h,t+dt}\right)\right], \tag{A4}$$

and

$$L_{h,t} = \frac{1}{\gamma}\frac{\boldsymbol{\nu}_{h,t}^\top \Gamma_h^{-1}\boldsymbol{\nu}_{h,t}}{\sigma^2} = \frac{1}{\gamma}\hat{L}_{h,t}, \tag{A5}$$

where $\boldsymbol{\nu}_{h,t} = (\nu_{h1,t},\ldots,\nu_{hN,t})^\top$ is the column vector of adjustments to expected returns, and $\Gamma_h = [\Gamma_{h,nm}]$ is the $N\times N$ diagonal matrix defined by

$$\Gamma_{h,nm} = \begin{cases} \frac{1-f_{hn}}{f_{hn}}, & n = m, \\ 0, & n \neq m, \end{cases}$$

and $f_{hn}\in[0,1]$ is a measure of how familiar the household is with firm, n, with $f_{hn}=1$ implying perfect familiarity, and $f_{hn}=0$ indicating no familiarity at all.

Proof: Using the penalty function given in Definition A.2.3, the construction of the familiarity-biased certainty equivalent of date-$t+dt$ utility is straightforward—it is merely the certainty-equivalent of date-$t+dt$ utility computed using the probability measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ plus a penalty. The household will choose its adjustment to expected returns by minimizing the familiarity-biased certainty equivalent of its date-$t+dt$ utility—the penalty stops the household from making the adjustment arbitarily large by penalizing it for larger adjustments. The size of the penalty is a measure of the information the household loses by deviating from the common reference measure, adjusted by its familiarity biases, and so

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] + U_{h,t}L_{h,t}dt,$$

where $\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ is defined by (A4) and $L_{h,t}$ is given in (A5). ■

Equation (8) follows from Theorem A.2.1, so we restate the equation formally as the following corollary before giving a proof.

Corollary A.2.1 *The date-t familiarity-biased certainty equivalent of date-t+dt household utility is given by*

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \mu_{h,t}[U_{h,t+dt}] + U_{h,t} \times \left(\frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}^{\top}_{h,t} \pi_{h,t} \boldsymbol{x}_{h,t} + \frac{1}{2\gamma_h} \frac{\boldsymbol{\nu}^{\top}_{h,t} \Gamma_h^{-1} \boldsymbol{\nu}_{h,t}}{\sigma^2} \right) dt, \tag{8}$$

where $U_{W_{h,t}} = \frac{\partial U_{h,t}}{\partial W_{h,t}}$ *is the partial derivative of the utility of household h with respect to its wealth.*

Proof: The date-t familiarity-biased certainty equivalent of date-$t+dt$ household utility is given by (A3), (A4), and (A5). We can see that $\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ is like a certainty equivalent, but with the expectation taken under $\mathbb{Q}^{\boldsymbol{\nu}_h}$ in order to adjust for familiarity bias. From Lemma A.1.1, we know that

$$\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = U_{h,t} \left(1 + E_t^{\mathbb{Q}^{\boldsymbol{\nu}_h}} \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2} \gamma_h E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right) + o(dt).$$

We therefore obtain from (A3)

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = U_{h,t} \left(1 + E_t^{\mathbb{Q}^{\boldsymbol{\nu}_h}} \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2} \gamma_h E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] + L_{h,t} dt \right) + o(dt). \tag{A6}$$

Applying Ito's Lemma, we see that under $\mathbb{Q}^{\boldsymbol{\nu}_h}$,

$$dU_{h,t} = W_{h,t} \frac{\partial U_{h,t}}{\partial W_{h,t}} \frac{dW_{h,t}}{W_{h,t}} + \frac{1}{2} W^2_{h,t} \frac{\partial^2 U_{h,t}}{\partial W^2_{h,t}} \left(\frac{dW_{h,t}}{W_{h,t}} \right)^2,$$

where

$$\frac{dW_{h,t}}{W_{h,t}} = \left(1 - \sum_{n=1}^{N} \omega_{hn,t} \right) i dt + \sum_{n=1}^{N} \omega_{hn,t} \left((\alpha + \nu_{h,t}) dt + \sigma dZ^{\mathbb{Q}^{\nu_h}}_{n,t} \right) - c_h dt.$$

Hence, from Girsanov's Theorem, we have

$$E_t^{\mathbb{Q}^{\nu_h}} \left[\frac{dU_{h,t}}{U_{h,t}} \right] = E_t \left[\frac{dU_{h,t}}{U_{h,t}} \right] + \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \pi_{h,t} \boldsymbol{x}^{\top}_{h,t} \boldsymbol{\nu}_{h,t} dt.$$

We can therefore rewrite (A6) as

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = U_{h,t} \left(1 + E_t \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2} \gamma_h E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] + L_{h,t} dt + \frac{W_{h,t}}{U_{h,t}} \frac{\partial U_{h,t}}{\partial W_{h,t}} \pi_{h,t} \boldsymbol{x}^{\top}_{h,t} \boldsymbol{\nu}_{h,t} dt \right) + o(dt).$$

Using (6) we obtain

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \mu_{h,t}[U_{h,t+dt}] + U_{h,t}\left(\frac{W_{h,t}}{U_{h,t}}\frac{\partial U_{h,t}}{\partial W_{h,t}}\pi_{h,t}\boldsymbol{x}^{\top}_{h,t}\boldsymbol{\nu}_{h,t} + \frac{1}{2\gamma}\frac{\boldsymbol{\nu}^{\top}_{h,t}\Gamma^{-1}_{h}\boldsymbol{\nu}_{h,t}}{\sigma^2}\right)dt + o(dt),$$

and hence (8). ■

A.3. The Bellman Equation and Mean-Variance Choice in (11) and (12)

We state the Hamilton-Jacobi-Bellman equation as the following proposition.

Proposition A.3.1 *The utility function of a household with familiarity biases is given by the following Hamilton-Jacobi-Bellman equation:*

$$0 = \sup_{C_{h,t}}\left(\delta_h\, u_{\psi_h}\left(\frac{C_{h,t}}{U_{h,t}}\right) + \sup_{\pi_{h,t},\,\boldsymbol{x}_{h,t}}\inf_{\nu_{h,t}}\frac{1}{U_{h,t}}\mu^{\boldsymbol{\nu}}_{h,t}\left[\frac{dU_{h,t}}{dt}\right]\right), \tag{A7}$$

where the function

$$u_{\psi_h}(x) = \frac{x^{1-\frac{1}{\psi_h}} - 1}{1-\frac{1}{\psi_h}}, \psi_h > 0,$$

and

$$\mu^{\boldsymbol{\nu}}_{h,t}\left[dU_{h,t}\right] = \mu^{\boldsymbol{\nu}}_{h,t}\left[U_{h,t+dt} - U_{h,t}\right] = \mu^{\boldsymbol{\nu}}_{h,t}\left[U_{h,t+dt}\right] - U_{h,t},$$

with $\mu^{\boldsymbol{\nu}}_{h,t}\left[U_{h,t+dt}\right]$ *given in* (8).

Proof: Writing out (10) explicitly gives

$$U^{1-\frac{1}{\psi_h}}_{h,t} = (1-e^{-\delta_h dt})C^{1-\frac{1}{\psi_h}}_{h,t} + e^{-\delta_h dt}\left(\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]\right)^{1-\frac{1}{\psi_h}},$$

where for ease of notation sup and inf have been suppressed. Now,

$$\begin{aligned}\left(\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]\right)^{1-\frac{1}{\psi_h}} &= \left(U_{h,t} + \mu^{\boldsymbol{\nu}}_{h,t}[dU_{h,t}]\right)^{1-\frac{1}{\psi_h}} \\ &= U^{1-\frac{1}{\psi_h}}_{h,t}\left(1 + \mu^{\boldsymbol{\nu}}_{h,t}\left[\frac{dU_{h,t}}{U_{h,t}}\right]\right)^{1-\frac{1}{\psi_h}} \\ &= U^{1-\frac{1}{\psi_h}}_{h,t}\left(1 + \left(1-\frac{1}{\psi_h}\right)\mu^{\boldsymbol{\nu}}_{h,t}\left[\frac{dU_{h,t}}{U_{h,t}}\right]\right) + o(dt).\end{aligned}$$

Hence,

$$U_{h,t}^{1-\frac{1}{\psi_h}} = \delta_h C_{h,t}^{1-\frac{1}{\psi_h}} dt + U_{h,t}^{1-\frac{1}{\psi_h}} \left(1 + \left(1 - \frac{1}{\psi_h}\right) \mu_{h,t}^{\boldsymbol{\nu}} \left[\frac{dU_{h,t}}{U_{h,t}}\right]\right) - \delta_h U_{h,t}^{1-\frac{1}{\psi_h}} dt + o(dt),$$

from which we obtain (A7). ■

Equations (11) and (12) are obtained from the following proposition by setting $\rho = 0$.

Proposition A.3.2 *The household's optimization problem consists of two parts, a mean-variance optimization*

$$\sup_{\pi_{h,t}, \boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} MV_h(\pi_{h,t}, \boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}),$$

and an intertemporal consumption choice problem

$$0 = \sup_{C_{h,t}} \left(\delta_h \, u_{\psi_h}\left(\frac{C_{h,t}}{U_{h,t}}\right) - \frac{C_{h,t}}{W_{h,t}} + \sup_{\pi_{h,t}, \boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} MV_h(\pi_{h,t}, \boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t})\right), \tag{11}$$

where

(12)

$$MV(\pi_{h,t}, \boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) = i + \big(\alpha - i\big)\pi_{h,t} - \frac{1}{2}\gamma_h \sigma^2 \pi_{h,t}^2 \boldsymbol{x}_{h,t}^\top \Omega \boldsymbol{x}_{h,t} + \boldsymbol{\nu}_{h,t}^\top \pi_{h,t} \boldsymbol{x}_{h,t} + \frac{1}{2\gamma_h} \frac{\boldsymbol{\nu}_{h,t}^\top \Gamma_h^{-1} \boldsymbol{\nu}_{h,t}}{\sigma^2}.$$

Proof: Assuming a constant risk-free rate, homotheticity of preferences combined with constant returns to scale for production implies that we have $U_{h,t} = \kappa_h W_{h,t}$, for some constant κ_h. Equations (11) and (12) are then direct consequences of (8) and (A7). ■

A.4. Adjustment to expected returns and portfolio choice in (13)–(16)

Proposition A.4.1 *For a given portfolio, $\boldsymbol{\omega}_{h,t} = \pi_{h,t}\boldsymbol{x}_{h,t}$, adjustments to firm n's expected return are given by*

$$\nu_{hn,t} = -\frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \Big(\frac{1}{f_{hn}} - 1\Big)\sigma^2 \, \gamma_h \, \pi_{h,t} x_{hn,t}, \; n \in \{1, \ldots, N\}. \tag{A8}$$

Proof: From (8), we can see that

$$\inf_{\boldsymbol{\nu}_{h,t}} \mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}]$$

is equivalent to

$$\inf_{\boldsymbol{\nu}_{h,t}} \frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^\top \pi_{h,t} \boldsymbol{x}_{h,t} + \frac{1}{2\gamma_h \sigma^2} \boldsymbol{\nu}_{h,t}^\top \Gamma_h^{-1} \boldsymbol{\nu}_{h,t}.$$

The minimum exists and is given by the first-order condition,

$$\frac{\partial}{\partial \boldsymbol{\nu}_{h,t}}\left[\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\boldsymbol{\nu}_{h,t}^{\top}\pi_{h,t}\boldsymbol{x}_{h,t}+\frac{1}{2\gamma_h\sigma^2}\boldsymbol{\nu}_{h,t}^{\top}\Gamma_h^{-1}\boldsymbol{\nu}_{h,t}\right]=0.$$

Carrying out the differentiation and exploiting the fact that Γ_h^{-1} is symmetric, we obtain

$$0=\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\pi_{h,t}\boldsymbol{x}_{h,t}+\frac{1}{\gamma_h\sigma^2}\Gamma_h^{-1}\boldsymbol{\nu}_{h,t}.$$

Hence,

$$\boldsymbol{\nu}_{h,t}=-\gamma_h\sigma^2\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\Gamma_h\,\pi_{h,t}\boldsymbol{x}_{h,t}.$$

Therefore, we obtain (A8). ■

Proposition A.4.2 *For a given portfolio decision, the optimal adjustment to firm-level expected returns is given by*

(A9) $$\boldsymbol{\nu}_{h,t}=-\gamma_h\sigma^2\,\Gamma_h\,\pi_{h,t}\boldsymbol{x}_{h,t}.$$

Each household then faces the following mean-variance portfolio problem:

(A10)

$$\sup_{\pi_{h,t},\,\boldsymbol{x}_{h,t}}\inf_{\boldsymbol{\nu}_{h,t}}MV(\pi_{h,t},\boldsymbol{x}_{h,t},\boldsymbol{\nu}_{h,t})=\left(i+\big(\alpha+\frac{1}{2}\boldsymbol{\nu}_{h,t}^{\top}\boldsymbol{x}_{h,t}-i\big)\pi_{h,t}\right)-\frac{1}{2}\gamma_h\sigma^2\pi_{h,t}^2\boldsymbol{x}_{h,t}^{\top}\Omega\boldsymbol{x}_{h,t}.$$

Proof: Because household utility is a constant multiple of wealth, the expression for the optimal adjustment to expected returns in (A8) simplifies to (A9). Substituting (A9) into (12), we see that each household faces the mean-variance portfolio problem in (A10).

For the special case in which a household is fully familiar with all firms, Γ_h is the zero matrix, and from (A9) we can see the adjustment to expected returns is zero and the portfolio weights are exactly the standard mean-variance portfolio weights. For the special case in which the household is completely unfamiliar with all firms, each $\Gamma_{h,nn}$ becomes infinitely large and $\pi_h=0$: complete unfamiliarity leads the household to avoid any investment in risky firms, in which case we get non-participation in the stock market in this partial-equilibrium setting. ■

Proposition A.4.3 *The optimal adjustment to expected returns made by a household with familiarity biases is*

(A11) $$\boldsymbol{\nu}_h=-(\alpha-i)\,[I+\Omega\Gamma_h^{-1}]^{-1}\mathbf{1},$$

where $\mathbf{1}$ *is the* N *by 1 vector of ones. The optimal vector of optimal portfolio weights is* $\boldsymbol{\omega}_h = \pi_h \boldsymbol{x}_h$*, where*

$$\pi_h = \frac{\mu_{qh}}{\gamma_h} \frac{\alpha - i}{\sigma^2_{1/N}}, \tag{A12}$$

$$\boldsymbol{x}_h = \frac{1}{\mu_{qh}} \frac{1}{N} \boldsymbol{q}_h, \tag{A13}$$

$\sigma^2_{1/N}$ *is the variance of the fully diversified portfolio i.e.*

$$\sigma^2_{1/N} = \sigma^2 (\boldsymbol{x}_h^U)^\top \Omega \, \boldsymbol{x}_h^U = \frac{\sigma^2}{N}[1 + (N-1)\rho],$$

and $\boldsymbol{q}_h$ *is the following* N *by 1 vector,*

$$\boldsymbol{q}_h = (1 + (N-1)\rho)(\Omega + \Gamma_h)^{-1}\mathbf{1}, \tag{A14}$$

the entries of which have the following arithmetic mean

$$\mu_{qh} = \frac{1}{N} \mathbf{1}^\top \boldsymbol{q}_h.$$

For the special case of $\rho = 0$ *used in the main text, we obtain equations* (13)*,* (14)*, and* (16) *in the main text:*

$$\boldsymbol{\nu}_h = -(\alpha - i)\,(\mathbf{1} - \boldsymbol{f}_h), \tag{13}$$

$$\boldsymbol{x}_h = \frac{\boldsymbol{f}_h}{\mu_{fh}} \frac{1}{N} \mathbf{1}, \tag{14}$$

$$\pi_h = \frac{\mu_{fh}}{\gamma_h} \frac{\alpha - i}{\sigma^2_{1/N}}, \tag{16}$$

where $\mu_{fh} = \frac{1}{N} \mathbf{1}^\top \boldsymbol{f}_h$*.*

Proof: Minimizing (12) with respect to $\nu_{h,t}$ gives (A9). Substituting (A9) into (12) and simplifying gives

$$MV_h = i + (\alpha - i)\pi_h - \frac{1}{2}\gamma_h \sigma^2 \pi_h^2 \boldsymbol{x}_h^\top (\Omega + \Gamma_h) \boldsymbol{x}_h. \tag{A15}$$

We find $\boldsymbol{x}_h$ by minimizing $\sigma^2 \boldsymbol{x}_h^\top (\Omega + \Gamma_h) \boldsymbol{x}_h$, so we can see that $\boldsymbol{x}_h$ is household h's minimum-variance portfolio adjusted for familiarity bias. The minimization we wish to perform is

$$\min_{\boldsymbol{x}_h} \frac{1}{2} \boldsymbol{x}_h^\top (\Omega + \Gamma_h)\, \boldsymbol{x}_h,$$

subject to the constraint

$$\mathbf{1}^\top \boldsymbol{x}_h = 1.$$

The Lagrangian for this problem is

$$\mathcal{L}_h = \frac{1}{2}\boldsymbol{x}_h^\top (\Omega + \Gamma_h)\, \boldsymbol{x}_h + \lambda_h(1 - \mathbf{1}^\top \boldsymbol{x}_h),$$

where λ_h is the Lagrange multiplier. The first-order condition with respect to $\boldsymbol{x}_h$ is

$$(\Omega + \Gamma_h)\, \boldsymbol{x}_h = \lambda_h \mathbf{1}.$$

Hence

$$\boldsymbol{x}_h = \lambda_h (\Omega + \Gamma_h)^{-1} \mathbf{1}.$$

The first order condition with respect to λ_h gives us the constraint

$$\mathbf{1}^\top \boldsymbol{x}_h = 1,$$

which implies that

$$\lambda_h = \left[\mathbf{1}^\top (\Omega + \Gamma_h)^{-1} \mathbf{1}\right]^{-1}.$$

Therefore, we have

$$\boldsymbol{x}_h = \frac{(\Omega + \Gamma_h)^{-1}\mathbf{1}}{\mathbf{1}^\top (\Omega + \Gamma_h)^{-1}\mathbf{1}} = \frac{\boldsymbol{q}_h}{\mathbf{1}^\top \boldsymbol{q}_h},$$

where $\boldsymbol{q}_h$ is defined in (A14). Hence

$$\lambda_h = \frac{1 + (N-1)\rho}{\mathbf{1}^\top \boldsymbol{q}_h}.$$

Substituting the optimal choice of $\boldsymbol{x}_h$ back into $\boldsymbol{x}_h^\top (\Omega + \Gamma_h)\boldsymbol{x}_h$ implies that

$$\boldsymbol{x}_h^\top (\Omega + \Gamma_h)\boldsymbol{x}_h = \lambda_h.$$

Therefore, to find the optimal π_h, we need to minimize

$$MV_h = i + (\alpha - i)\pi_h - \frac{1}{2}\gamma_h \sigma^2 \pi_h^2 \lambda_h.$$

Hence,

$$\pi_h = \frac{1}{\lambda_h}\frac{1}{\gamma_h}\frac{\alpha - i}{\sigma^2} = \frac{\mathbf{1}^\top \boldsymbol{q}_h}{\gamma_h}\frac{\alpha - i}{\sigma^2[1 + (N-1)\rho]} = \frac{\frac{\mathbf{1}^\top \boldsymbol{q}_h}{N}}{\gamma_h}\frac{\alpha - i}{\frac{\sigma^2}{N}[1 + (N-1)\rho]},$$

which gives us the result in (A12). Substituting (A12) and (A13) into (A9) and simplifying gives (A11). Settinng $\rho = 0$ in these expressions gives us the results in the main text.

We can express $\boldsymbol{\omega}_h = \pi_h \boldsymbol{x}_h$ in terms of the familiarity-biased adjustment made to expected returns:

$$\boldsymbol{\omega}_h = \frac{1}{\gamma_h}\Omega^{-1}\frac{\alpha\mathbf{1} + \boldsymbol{\nu}_h - i\mathbf{1}}{\sigma^2}.$$

Substituting the expressions for the portfolio choices and the Lagrange multiplier λ_h into the mean-variance objective function with familiarity biases gives:

$$MV_h = i + \frac{1}{2}\frac{1}{\gamma_h}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2 \frac{\mathbf{1}^\top \boldsymbol{q}_h}{N}.$$

Hence

$$MV_h = i + \frac{1}{2}\frac{1}{\gamma_h}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2 \mu_{qh}. \tag{A16}$$

■

A.5. Mean-Variance Welfare in (19)

The following proposition summarizes results on how familiarity biases impact a household's mean-variance welfare.

Proposition A.5.1 *Mean-variance welfare evaluated using the portfolio policy which is optimal in the presence of familiarity biases is given by*

$$i + \frac{1}{2\gamma_h}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2 \left(1 - (\mu_{qh} - 1)^2 - \sigma_{qh}^2\right). \tag{A17}$$

The increase in mean-variance welfare from removing familiarity biases is given by

$$\frac{1}{2\gamma_h}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2 \left((\mu_{qh} - 1)^2 + \sigma_{qh}^2\right), \tag{A18}$$

where $\frac{1}{2\gamma_h}\left(\frac{\alpha-i}{\sigma_{1/N}}\right)^2\sigma_{qh}^2$ *is the increase in mean-variance welfare obtained by first removing familiarity biases in the choice of composition of the subportfolio of risky assets, and* $\frac{1}{2\gamma_h}\left(\frac{\alpha-i}{\sigma_{1/N}}\right)^2(\mu_{qh}-1)^2$ *is the subsequent increase in mean-variance welfare obtained by removing also familiarity biases in the capital allocation decision, i.e. the choice of which proportion of wealth to invest in risky assets.*

Proof: We start by giving both the mean-variance objective function in the presence of familiarity biases and the mean-variance welfare function in terms of general, not necessarily optimal, portfolio choices.

Mean-variance welfare is given as a function of the proportion of wealth invested in risky assets, π_h, and the subportfolio of risky assets $\boldsymbol{x}_h$ by (A15). Substituting in the household's decisions, given in (A12) and (A13) into the above expression and simplifying gives

$$U_h^{MV}(\pi_h,\boldsymbol{x}_h)=i+\frac{1}{2\gamma}\left(\frac{\alpha-i}{\sigma_{1/N}}\right)^2\left[2\mu_{qh}-\frac{\frac{1}{N}\boldsymbol{q}_h^\top\Omega\boldsymbol{q}_h}{\frac{1}{N}\mathbf{1}^\top\Omega\mathbf{1}}\right],$$

where

$$\frac{1}{N}\mathbf{1}^\top\Omega\mathbf{1}=1+(N-1)\rho.$$

Defining

$$\sigma_{qh}^2=\frac{\frac{1}{N}\boldsymbol{q}_h^\top\Omega\boldsymbol{q}_h}{\frac{1}{N}\mathbf{1}^\top\Omega\mathbf{1}}-\mu_{qh}^2,$$

we obtain

$$U_h^{MV}(\pi_h,\boldsymbol{x}_h)=i+\frac{1}{2\gamma}\left(\frac{\alpha-i}{\sigma_{1/N}}\right)^2\left[1-(1-\mu_{qh})^2-\sigma_{qh}^2\right].$$

Setting $\rho=0$ gives expression (19) in the main text.

Without familiarity biases, mean-variance welfare is given by

$$U_h^{MV}(\pi_h^U,\boldsymbol{x}_h^U)=i+\frac{1}{2\gamma_h}\left(\frac{\alpha-i}{\sigma_{1/N}}\right)^2.$$

Hence, the increase in mean-variance welfare obtained from removing familiarity biases is given by (A18).

We now study how mean-variance welfare changes when the biases in the subportfolio of risky assets are eliminated, followed by eliminating the biases in the proportion of wealth

invested in risky assets. Denote the biased portfolio choices by $\pi_h, \boldsymbol{x}_h$ and the unbiased portfolio choices by $\pi_h^U = \pi_h + \Delta\pi_h$, $\boldsymbol{x}_h^U = \boldsymbol{x}_h + \Delta\boldsymbol{x}_h$, i.e.

$$\pi_h = \frac{1}{\gamma_h}\frac{\alpha - i}{\sigma_{1/N}^2}\mu_{qh}, \tag{A19}$$

$$\boldsymbol{x}_h = \frac{1}{\mu_{qh}}\frac{1}{N}\boldsymbol{q}_h, \tag{A20}$$

$$\pi_h^U = \pi_h + \Delta\pi_h = \frac{1}{\gamma_h}\frac{\alpha - i}{\sigma_{1/N}^2}, \tag{A21}$$

$$\boldsymbol{x}_h^U = \boldsymbol{x}_h + \Delta\boldsymbol{x}_h = \frac{1}{N}\mathbf{1}. \tag{A22}$$

Observe that

$$\begin{aligned}
&MV^e(\pi_h + \Delta\pi_h, \boldsymbol{x}_h + \Delta\boldsymbol{x}_h) - MV^e(\pi_h, \boldsymbol{x}_h) \\
&= -\frac{1}{2}\gamma_h\sigma^2\pi_h^2\left[(\boldsymbol{x}_h + \Delta\boldsymbol{x}_h)^\top\Omega(\boldsymbol{x}_h + \Delta\boldsymbol{x}_h) - \boldsymbol{x}_h^\top\Omega\boldsymbol{x}_h\right] \\
&\quad + (\alpha - i)\Delta\pi_h - \frac{1}{2}\gamma_h\sigma^2\left[[(\pi_h + \Delta\pi_h)^2 - \pi^2](\boldsymbol{x}_h + \Delta\boldsymbol{x}_h)^\top\Omega(\boldsymbol{x}_h + \Delta\boldsymbol{x}_h)\right],
\end{aligned}$$

where $\frac{1}{2}\gamma_h\sigma^2\pi_h^2\left[(\boldsymbol{x}_h + \Delta\boldsymbol{x}_h)^\top\Omega(\boldsymbol{x}_h + \Delta\boldsymbol{x}_h) - \boldsymbol{x}_h^\top\Omega\boldsymbol{x}_h\right]$ is the change in mean-variance welfare when the biases in the subportfolio of risky assets are eliminated, and $(\alpha - i)\Delta\pi_h - \frac{1}{2}\gamma_h\sigma^2\left[[(\pi_h + \Delta\pi_h)^2 - \pi_h^2](\boldsymbol{x}_h + \Delta\boldsymbol{x}_h)^\top\Omega(\boldsymbol{x}_h + \Delta\boldsymbol{x}_h)\right]$ is the change in mean-variance welfare by then eliminating the biases in the proportion of wealth invested in risky assets. Using the expressions in (A19), (A20), (A21), and (A22), it follows that

$$-\frac{1}{2}\gamma_h\sigma^2\left[\pi_h^2[(\boldsymbol{x}_h + \Delta\boldsymbol{x}_h)^\top\Omega(\boldsymbol{x}_h + \Delta\boldsymbol{x}_h) - \boldsymbol{x}_h^\top\boldsymbol{x}_h]\right] = \frac{1}{2\gamma_h}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2\sigma_{qh}^2,$$

and

$$(\alpha - i)\Delta\pi_h - \frac{1}{2}\gamma_h\sigma^2\left[[(\pi_h + \Delta\pi_h)^2 - \pi_h^2](\boldsymbol{x}_h + \Delta\boldsymbol{x}_h)^\top\Omega(\boldsymbol{x}_h + \Delta\boldsymbol{x}_h)\right] = \frac{1}{2\gamma_h}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2(\mu_{qh} - 1)^2.$$

■

A.6. Optimal consumption in (20)

The following proposition summarizes results on optimal consumption choice.

Proposition A.6.1 *A household's optimal consumption-to-wealth ratio is given by*

$$\frac{C_{h,t}}{W_{h,t}} = c_h = \psi_h \delta_h + (1-\psi_h)\left(i + \frac{1}{2}\frac{1}{\gamma_h}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2 \mu_{qh}\right). \tag{20}$$

Proof: Mean-variance utility subject to familiarity biases and with the household's decisions is given by (A16). Hence, we can rewrite (11) as

$$0 = \sup_{C_{h,t}} \left(\delta_h u_{\psi_h}\left(\frac{C_{h,t}}{U_{h,t}}\right) - c_h + MV_h(\pi_h, \boldsymbol{x}_h, \boldsymbol{\nu}_h)\right). \tag{A23}$$

The first-order condition with respect to consumption is

$$\delta_h \left(\frac{C_{h,t}}{U_{h,t}}\right)^{-\frac{1}{\psi_h}} \frac{1}{U_{h,t}} - \frac{1}{W_{h,t}} = 0.$$

Hence, we obtain

$$c_h = \delta_h^{\psi_h}\left(\frac{U_{h,t}}{W_{h,t}}\right)^{1-\psi_h},$$

which implies that

$$\sup_{C_{h,t}} \left(\delta_h\, u_{\psi_h}\left(\frac{C_{h,t}}{U_{h,t}}\right) - c_h + MV_h(\pi_h, \boldsymbol{x}_h, \boldsymbol{\nu}_h)\right) = \frac{c_h - \psi_h \delta_h}{\psi_h - 1} + MV_h,$$

where $C_{h,t}/W_{h,t}$ is the consumption-wealth ratio chosen by household h and MV_h is her resulting mean-variance utility subject to familiarity biases. It follows from (A23) that

$$c_h = \psi_h \delta_h + (1-\psi_h) MV_h = \psi_h \delta_h + (1-\psi_h)\left(i + \frac{1}{2}\frac{1}{\gamma_h}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2 \mu_{qh}\right)$$

from which one can get the expression in the text by setting $\rho = 0$. ■

A.7. Welfare in (21)

Proposition A.7.1 *Welfare is given by a function of the proportion of wealth invested in risky assets, π_h, the subportfolio of risky assets $\boldsymbol{x}_h$, and the consumption-wealth ratio, $c_h = C_{h,t}/W_{h,t}$ by*

$$U_{h,t} = \kappa_h\left(c_h, \pi_h, \boldsymbol{x}_h\right) W_{h,t},$$

where

$$\kappa_h\left(c_h,\pi_h,\boldsymbol{x}_h\right)=\kappa_h=\left[\frac{\delta_h\psi_h}{\psi_h\delta_h+\left(1-\psi_h\right)\left(U^{MV}(\pi_h,\boldsymbol{x}_h)-c_h\right)}\right]^{\frac{1}{1-\frac{1}{\psi_h}}}c_h, \tag{21}$$

in which $c_h=C_{h,t}/W_{h,t}$. *The impact of a one percent change in* $U_h^{MV}(\pi_h,\boldsymbol{x}_h)$ *on the percentage change in welfare is given by the following elasticity*

$$\frac{\partial\ln\kappa_h}{\partial\ln U_h^{MV}(\pi_h,\boldsymbol{x}_h)}=U_h^{MV}(\pi_h,\boldsymbol{x}_h)\frac{1}{\delta_h-\left(1-\frac{1}{\psi_h}\right)\left(U_h^{MV}(\pi,\boldsymbol{x})-c_h\right)}>0.$$

The size of the above elasticity beyond one captures the size of the additional intertemporal effect of a change in mean-variance utility on lifetime welfare.

The impact of a one percent change in the consumption-wealth ratio on the percentage change in welfare is given by the following elasticity

$$\frac{\partial\ln\kappa_h}{\partial\ln(c_h)}=\frac{\psi_h\delta_h+(1-\psi_h)U_h^{MV}(\pi_h,\boldsymbol{x}_h)-c_h}{\psi_h\delta_h+(1-\psi_h)(U_h^{MV}(\pi_h,\boldsymbol{x}_h)-c_h)}.$$

When removing familiarity biases the resulting percentage change in welfare is always positive, i.e. $\frac{\partial\ln\kappa_h}{\partial\ln(c_h)}\frac{\Delta(c_h)}{(c_h)}>0$, *where* $\frac{\Delta(c_h)}{(c_h)}$ *is the percentage change in the consumption-wealth ratio.*

Proof: We start from the recursive equation for welfare

$$U_{h,t}=\mathcal{A}(C_{h,t},\mu_{h,t}[U_{h,t+dt}]).$$

Hence

$$\begin{aligned}
\left(U_{h,t}\right)^{1-\frac{1}{\psi_h}} &= (1-e^{-\delta_h dt})C_{h,t}^{1-\frac{1}{\psi_h}}+e^{-\delta_h dt}\left(\mu_{h,t}\left[U_{h,t+dt}\right]\right)^{1-\frac{1}{\psi_h}}\\
&= \delta_h dt(C_{h,t})^{1-\frac{1}{\psi_h}}+(1-\delta_h dt)(U_{h,t}+\mu_{h,t}[dU_{h,t}])^{1-\frac{1}{\psi_h}}\\
&= \delta_h dt(C_{h,t})^{1-\frac{1}{\psi_h}}+(1-\delta_h dt)(U_{h,t})^{1-\frac{1}{\psi_h}}\left(1+\mu_{h,t}\left[\frac{dU_{h,t}}{U_{h,t}}\right]\right)^{1-\frac{1}{\psi_h}}\\
&= \delta_h dt(C_{h,t})^{1-\frac{1}{\psi_h}}+(1-\delta_h dt)(U_{h,t})^{1-\frac{1}{\psi_h}}\left(1+\left(1-\frac{1}{\psi_h}\right)\mu_{h,t}\left[\frac{dU_{h,t}}{U_{h,t}}\right]+o(dt)\right)\\
&= \delta_h C_{h,t}^{1-\frac{1}{\psi_h}}dt+(U_{h,t})^{1-\frac{1}{\psi_h}}\left(1+\left(1-\frac{1}{\psi_h}\right)\mu_{h,t}\left[\frac{dU_{h,t}}{U_{h,t}}\right]-\delta_h dt\right)-(U_{h,t})^{1-\frac{1}{\psi_h}}+o(dt)\\
0 &= \delta_h C_{h,t}^{1-\frac{1}{\psi_h}}dt+(U_{h,t})^{1-\frac{1}{\psi_h}}\left(1+\left(1-\frac{1}{\psi_h}\right)\mu_{h,t}\left[\frac{dU_{h,t}}{U_{h,t}}\right]-\delta_h dt\right)+o(dt).
\end{aligned}$$

Hence, in the continuous time limit, we obtain

$$0 = \delta_h C_{h,t}^{1-\frac{1}{\psi_h}} + (U_{h,t})^{1-\frac{1}{\psi_h}} \left[\left(1 - \frac{1}{\psi_h}\right) \frac{1}{U_{h,t}} \mu_{h,t}\left[\frac{dU_{h,t}}{dt}\right] - \delta_h \right].$$

Treating $U_{h,t}$ as a function of $W_{h,t}$, we have via Ito's Lemma

$$\frac{dU_{h,t}}{U_{h,t}} = \frac{W_{h,t}\frac{\partial U_{h,t}}{\partial W_{h,t}}}{U_t}\frac{dW_{h,t}}{W_{h,t}} + \frac{1}{2}\frac{W_{h,t}^2\frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2}}{U_{h,t}}\left(\frac{dW_{h,t}}{W_{h,t}}\right)^2.$$

Assuming that

$$U_{h,t} = \kappa_h W_{h,t},$$

where κ_h is a constant, we obtain

$$\frac{dU_{h,t}}{U_{h,t}} = \frac{dW_{h,t}}{W_{h,t}}.$$

Hence

$$\begin{aligned}\mu_{h,t}\left[\frac{dU_{h,t}}{U_{h,t}}\right] &= \mu_{h,t}\left[\frac{dW_{h,t}}{W_{h,t}}\right] \\ &= \left[i + (\alpha - i)\pi_h - \frac{1}{2}\gamma_h\sigma^2\pi_h^2 \boldsymbol{x}_h^\top \Omega \boldsymbol{x}_h - c_h\right] dt,\end{aligned}$$

where we assume that i, π_h, $\boldsymbol{x}_h$, and c_h are constants. Therefore

$$\begin{aligned}0 &= \delta_h C_{h,t}^{1-\frac{1}{\psi_h}} + (U_{h,t})^{1-\frac{1}{\psi_h}} \left[\left(1 - \frac{1}{\psi_h}\right)\left(i + (\alpha - i)\pi_h - \frac{1}{2}\gamma_h\sigma^2\pi_h^2\boldsymbol{x}_h^\top\Omega\boldsymbol{x}_h - c_h\right) - \delta_h\right] \\ 0 &= \delta_h (c_h)^{1-\frac{1}{\psi_h}} + (\kappa_h)^{1-\frac{1}{\psi_h}} \left[\left(1 - \frac{1}{\psi_h}\right)\left(U_h^{MV}(\pi_h, \boldsymbol{x}_h) - c_h\right) - \delta_h\right] \\ 0 &= \psi_h\delta_h (c_h)^{1-\frac{1}{\psi_h}} - (\kappa_h)^{1-\frac{1}{\psi_h}} \left[\psi_h\delta_h + (1 - \psi_h)\left(U_h^{MV}(\pi_h, \boldsymbol{x}_h) - c_h\right)\right]\end{aligned}$$

$$\begin{aligned}(\kappa_h)^{1-\frac{1}{\psi_h}} &= \frac{\psi_h\delta_h (c_h)^{1-\frac{1}{\psi_h}}}{\psi_h\delta_h + (1 - \psi_h)\left(U_h^{MV}(\pi_h, \boldsymbol{x}_h) - c_h\right)} \\ \kappa_h &= \left[\frac{\psi_h\delta_h}{\psi_h\delta_h + (1 - \psi_h)\left(U^{MV}(\pi_h, \boldsymbol{x}_h) - c_h\right)}\right]^{\frac{1}{1-\frac{1}{\psi_h}}} c_h.\end{aligned}$$

Therefore, we obtain (21).

For a given consumption-wealth ratio, we now consider the impact of changes in $\boldsymbol{x}_h$ and π_h on percentage changes in the utility-wealth ratio and hence on κ_h, that is we compute $\frac{\partial \ln \kappa_h}{\partial \ln U^{MV}(\pi_h, \boldsymbol{x}_h)}$:

$$\frac{\partial \ln \kappa_h}{\partial \ln U_h^{MV}(\pi_h, \boldsymbol{x}_h)} = U_h^{MV}(\pi_h, \boldsymbol{x}_h) \frac{1}{\delta_h - \left(1 - \frac{1}{\psi_h}\right)\left(U_h^{MV}(\pi_h, \boldsymbol{x}_h) - c_h\right)}.$$

Observe that a necessary condition for κ_h to be well-defined is that

$$\delta_h - \left(1 - \frac{1}{\psi_h}\right)\left(U_h^{MV}(\pi_h, \boldsymbol{x}_h) - c_h\right) > 0.$$

Hence, we can see that a percentage decrease in $U_h^{MV}(\pi_h, \boldsymbol{x}_h)$ is multiplied by the factor $U_h^{MV}(\pi_h, \boldsymbol{x}_h) \frac{1}{\delta_h - \left(1 - \frac{1}{\psi_h}\right)\left(U_h^{MV}(\pi_h, \boldsymbol{x}_h) - c_h\right)} > 0$. The size of this elasticity beyond one captures the size of the additional intertemporal effect of a change in mean-variance utility on lifetime welfare.

Now note that

$$\begin{aligned}
\frac{\Delta \kappa_h^e}{\kappa_h^e} &\approx \frac{\partial \ln \kappa_h}{\partial \ln(c_h)} \frac{\Delta(c_h)}{c_h} + \frac{c_h^2}{\kappa_h^e} \frac{1}{2} \frac{\partial^2 \kappa_h^e}{\partial c_h^2} \left(\frac{\Delta c_h}{c_h}\right)^2 \\
&= \frac{\psi_h \delta_h + (1 - \psi_h) U_h^{MV}(\pi, \boldsymbol{x}) - c_h}{\psi_h \delta_h + (1 - \psi_h)(U^{MV}(\pi_h, \boldsymbol{x}_h) - c_h)} \frac{\Delta c_h}{c_h} \\
&\quad + \psi_h \left(\psi_h - \frac{1}{2}\right) \frac{c_h^2}{\left[\psi_h \delta_h + (1 - \psi_h)(U_h^{MV}(\pi_h, \boldsymbol{x}_h) - c_h)\right]^2} \left(\frac{\Delta c_h}{c_h}\right)^2.
\end{aligned}$$

Hence, we can see that to first order, increasing c_h increases utility if $c_h < \psi_h \delta_h + (1 - \psi_h) U_h^{MV}(\pi_h, \boldsymbol{x}_h)$. ■

A.8. Condition for no-aggregate-biases across households in (22)

We start by formally stating the "no aggregate bias" condition.

Definition A.8.1 *Suppose household h's risky portfolio weight for firm n is given by*

$$x_{hn} = \frac{1}{N} + \epsilon_{hn},$$

where $\frac{1}{N}$ is the unbiased portfolio weight and ϵ_{hn} is the bias of household h's portfolio when investing in firm n. The biases ϵ_{hn} "cancel out in aggregate" if

$$\forall n, \quad \frac{1}{H} \sum_{h=1}^{H} \epsilon_{hn} = 0.$$

Proposition A.8.1 *The following condition holds*

$$\forall n,\ \frac{1}{H}\sum_{h=1}^{H}\frac{1}{\mu_{qh}}q_{hn}=1,$$

if and only if portfolio biases cancel out in aggregate.

Observe that the above condition reduces to (22) *for the special case of* $\rho=0$.

Proof: The no aggregate bias condition is equivalent to

$$\sum_{h=1}^{H}\boldsymbol{x}_h=\sum_{h=1}^{H}\frac{1}{N}\mathbf{1},$$

which is equivalent to

$$\frac{1}{H}\sum_{h=1}^{H}\boldsymbol{x}_h=\frac{1}{N}\mathbf{1}.$$

Because the optimal risky portfolio with familiarity biases is given by (A13), the above condition can be rewritten as

$$\frac{1}{H}\sum_{h=1}^{H}\frac{1}{\mu_{qh}}\boldsymbol{q}_h=\mathbf{1},$$

i.e.

$$\forall\, n\in\{1,\ldots,N\},\ \frac{1}{H}\sum_{h=1}^{H}\frac{1}{\mu_{qh}}q_{hn}=1.$$

Now suppose that

$$\frac{1}{H}\sum_{h=1}^{H}\frac{1}{\mu_{qh}}\boldsymbol{q}_h=\mathbf{1}.$$

It follows that

$$\frac{1}{H}\sum_{h=1}^{H}\boldsymbol{x}_h=\frac{1}{N}\mathbf{1},$$

which is equivalent to the no aggregate bias condition.

Therefore,

$$\forall\, n\in\{1,\ldots,N\},\ \frac{1}{H}\sum_{h=1}^{H}\frac{1}{\mu_{qh}}q_{hn}=1.$$

holds if and only if the no aggregate bias condition holds. ■

A.9. The symmetry condition in (23)

In order to derive a closed-form expression for the equilibrium interest rate, we impose the following "symmetry condition".

Definition A.9.1 *The "symmetry condition" states that for distinct households, h and j, we have*

$$\mu_{qh} = \mu_{qj} = \mu_q.$$

Observe that for the special case of $\rho = 0$ used in the text, the symmetry condition reduces to (23).

Proposition A.9.1 *The following condition is equivalent to the combination of the symmetry condition and the no aggregate bias condition:*

$$\frac{1}{H}\sum_{h=1}^{H} q_{hn} = \frac{1}{N}\sum_{n=1}^{N} q_{hn}. \tag{A24}$$

Proof: Because the LHS of (A24) is independent of h, it follows that $\mu_{qh} = \frac{1}{N}\sum_{n=1}^{N} q_{hn}$ is independent of h, which is the symmetry condition. Hence,

$$\frac{1}{H}\sum_{h=1}^{H} q_{hn} = \mu_q,$$

which implies that the no aggregate bias condition holds.

Now suppose that both the symmetry condition and the no aggregate bias condition hold. No aggregate bias implies that

$$\forall\, n \in \{1, \ldots, N\},\ \frac{1}{H}\sum_{h=1}^{H} \frac{1}{\mu_{qh}} q_{hn} = 1.$$

Using the symmetry condition, the above expression becomes

$$\forall\, n \in \{1, \ldots, N\},\ \frac{1}{H}\sum_{h=1}^{H} \frac{1}{\mu_q} q_{hn} = 1,$$

which reduces to

$$\forall\, n \in \{1, \ldots, N\},\ \frac{1}{H}\sum_{h=1}^{H} q_{hn} = \mu_q,$$

which is equivalant to (A24). ■

A.10. Equilibrium interest rate in (24)

The following proposition summarizes the equilibrium interest rate.

Proposition A.10.1 *The equilibrium risk-free interest rate is given by the constant*

$$i = \alpha - \gamma \frac{\sigma_{1/N}^2}{\mu_q}. \tag{24}$$

Proof: Market clearing in the bond market implies that

$$\sum_{h=1}^{H} B_{h,t} = 0, \tag{A25}$$

where the amount of wealth held in the bond by household h is given by

$$B_{h,t} = (1 - \pi_{h,t}) W_{h,t}.$$

Using the expression for $\pi_{h,t}$ given in (A12), we can rewrite the market clearing condition (A25) as

$$\sum_{h=1}^{H} \left(1 - \frac{\mu_{qh}}{\gamma} \frac{\alpha - i}{\sigma_{1/N}^2} \right) W_{h,t} = 0.$$

Hence,

$$\sum_{h=1}^{H} W_{h,t} = \frac{1}{\gamma} \frac{\alpha - i}{\sigma_{1/N}^2} \sum_{h=1}^{H} \mu_{qh} W_{h,t}$$

$$i = \alpha - \frac{\sum_{h=1}^{H} W_{h,t}}{\sum_{h=1}^{H} \mu_{qh} W_{h,t}} \gamma \sigma_{1/N}^2,$$

which reduces to

$$i = \alpha - \gamma \frac{\sigma_{1/N}^2}{\mu_q},$$

if the symmetry condition holds, and upon setting $\rho = 0$ it gives the expression for the interest rate in (24) in the main text. ■

A.11. Equilibrium macroeconomic quantities in (26) and (27)

Proposition A.11.1 *The general equilibrium economy-wide consumption-wealth ratio is given by*

$$\frac{C_t^{agg}}{W_t^{agg}} = c = \alpha - g = \psi\delta + (1-\psi)\Big(\alpha - \frac{1}{2}\gamma\sigma_{\boldsymbol{x}}^2\Big), \tag{26}$$

where g, the aggregate growth rate of the economy, is equal to the aggregate investment-capital ratio, which is given by

$$g = \frac{I_t^{agg}}{K_t^{agg}} = \alpha - c = \psi(\alpha - \delta) - \frac{1}{2}(\psi - 1)\gamma\sigma_{\boldsymbol{x}}^2.$$

Proof: Substituting the equilibrium interest rate in (24) into the expression in (20) for the consumption-wealth ratio for each individual gives the general-equilibrium consumption-wealth ratio:

$$c_h = c = \psi\delta + (1-\psi)\Big(\alpha - \frac{1}{2}\gamma\frac{\sigma_{1/N}^2}{\mu_q}\Big), \tag{A26}$$

where μ_q is constant across households because of the symmetry condition. Observe that

$$\begin{aligned}
\sigma_{\boldsymbol{x}_h}^2 &= \sigma^2 \boldsymbol{x}_h^\top \Omega \boldsymbol{x}_h \\
&= \sigma^2 \frac{\boldsymbol{q}_h^\top \Omega \boldsymbol{q}_h}{(\mathbf{1}^\top \boldsymbol{q}_h)^2} \\
&= \sigma^2 \frac{1}{N\mu_q^2} \frac{\mathbf{1}^\top \Omega \mathbf{1}}{N} \frac{\frac{\boldsymbol{q}_h^\top \Omega \boldsymbol{q}_h}{N}}{\frac{\mathbf{1}^\top \Omega \mathbf{1}}{N}} \\
&= \sigma^2 \frac{1}{N\mu_q^2} \frac{\mathbf{1}^\top \Omega \mathbf{1}}{N} (\sigma_{qh}^2 + \mu_q^2) \\
&= \sigma_{1/N}^2 \left(1 + \left(\frac{\sigma_{qh}}{\mu_q}\right)^2\right).
\end{aligned}$$

For the case $\rho = 0$ and under the condition that each familiarity coefficient f_{hn} can be either 1 or 0, we have that $\mu_{fh} = \frac{1}{N}\sum_{n=1}^N f_{hn} = \frac{1}{N}\sum_{n=1}^N f_{hn}^2$, implying that $\sigma_{fh}^2 = \frac{1}{N}\sum_{n=1}^N f_{hn}^2 - \mu_{fh}^2 = \mu_{fh} - \mu_{fh}^2$. Therefore, using (15), we get:

$$\sigma_{\boldsymbol{x}_h}^2 = \frac{\sigma_{1/N}^2}{\mu_{fh}}, \tag{A27}$$

which under the symmetry condition that $\mu_{fh} = \mu_f$ implies that $\sigma^2_{\boldsymbol{x}_h} = \sigma^2_{\boldsymbol{x}}$ is identical across all households, leading to

$$c_h = c = \psi\delta + (1-\psi)\Big(\alpha - \frac{1}{2}\gamma\sigma^2_{\boldsymbol{x}}\Big).$$

Observe that in the expression above, all the terms on the right-hand side of the second equality are constants, implying that the consumption-wealth ratio is the same across households. Exploiting the fact that the consumption-wealth ratio is constant across households allows us to obtain the ratio of aggregate consumption-to-wealth ratio, where aggregate consumption is $C_t^{\text{agg}} = \sum_{h=1}^{H} C_{h,t}$ and aggregate wealth is $W_t^{\text{agg}} = \sum_{h=1}^{H} W_{h,t}$:

$$\frac{C_t^{\text{agg}}}{W_t^{\text{agg}}} = c,$$

which is the second equality in (26).

Equation (1) implies

$$\sum_{n=1}^{N} Y_{n,t} = \alpha \sum_{n=1}^{N} K_{n,t},$$

and Equation (2) implies

$$d\left(E_t\left[\sum_{n=1}^{N} K_{n,t}\right]\right) = E_t\left[d\sum_{n=1}^{N} K_{n,t}\right] = \alpha\sum_{n=1}^{N} K_{n,t} - \sum_{n=1}^{N} D_{n,t}dt.$$

In equilibrium $\sum_{n=1}^{N} K_{n,t} = W_t^{\text{agg}}$ and $\sum_{n=1}^{N} D_{n,t} = C_t^{\text{agg}}$. Therefore,

$$\frac{dW_t^{\text{agg}}}{W_t^{\text{agg}}} = \left(\alpha - \frac{C_t^{\text{agg}}}{W_t^{\text{agg}}}\right)dt.$$

We also know that

$$\frac{dW_t^{\text{agg}}}{W_t^{\text{agg}}} = \frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}},$$

so

$$g\,dt = E_t\left[\frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}}\right] = \left(\alpha - \frac{C_t^{\text{agg}}}{W_t^{\text{agg}}}\right)dt = (\alpha - c)dt.$$

Rearranging terms leads to the first equality in (26):

$$c = \alpha - g. \tag{26}$$

We now derive the aggregate investment-capital ratio. The aggregate investment flow must be equal to aggregate output flow less the aggregate consumption flow:

$$I_t^{\text{agg}} = \alpha K_t^{\text{agg}} - C_t^{\text{agg}}.$$

It follows that the aggregate investment-capital ratio is given by

$$\frac{I_t^{\text{agg}}}{K_t^{\text{agg}}} = \alpha - \frac{C_t^{\text{agg}}}{K_t^{\text{agg}}} = \alpha - c = \psi\delta - (\psi - 1)\frac{1}{2}\gamma\sigma_{\boldsymbol{x}}^2.$$

Finally, we relate trend output growth to aggregate investment. Firms all have constant returns to scale and differ only because of shocks to their capital stocks. Therefore, the aggregate growth rate of the economy is the aggregate investment-capital ratio:

$$g = \frac{I_t^{\text{agg}}}{K_t^{\text{agg}}},$$

which gives us the expression for g in (27). ■

A.12. Social welfare per unit of aggregate capital in (29) and (30)

Proposition A.12.1 *Welfare is given in terms of the endogenous growth rate of the economy, g, by*

$$\text{(A28)}\quad \kappa_h = \left[\frac{\delta}{\delta - \left(1 - \frac{1}{\psi}\right)\left(g - \frac{1}{2}\gamma\,\sigma_{1/N}^2\left(1 + \frac{\sigma_{qh}^2}{\mu_q^2}\right)\right)}\right]^{\frac{1}{1-\frac{1}{\psi}}} \left[\delta - \left(1 - \frac{1}{\psi}\right)\left(g - \frac{1}{2}\gamma\frac{\sigma_{1/N}^2}{\mu_q}\right)\right],$$

where

$$\text{(A29)}\qquad g = \psi(\alpha - \delta) - \frac{1}{2}(\psi - 1)\gamma\,\frac{\sigma_{1/N}^2}{\mu_q}.$$

Proof: We impose the symmetry condition. Because

$$g = \psi(\alpha - \delta) - \frac{1}{2}(\psi - 1)\gamma\,\frac{\sigma_{1/N}^2}{\mu_q},$$

and

$$\alpha - i = \gamma\frac{\sigma_{1/N}^2}{\mu_q},$$

it follows that

$$c_h = \psi\delta + (1-\psi)\left[i + \frac{1}{2\gamma}\left(\frac{\alpha - i}{\sigma_{1/N}}\right)^2 \mu_{qh}\right] = \delta - \left(1 - \frac{1}{\psi}\right)\left(g - \frac{1}{2}\gamma\frac{\sigma_{1/N}^2}{\mu_q}\right).$$

Hence,

$$\begin{aligned} & c_h - \Big(1 - \frac{1}{\psi}\Big)\frac{1}{2\gamma}\Big(\frac{\alpha - i}{\sigma_{1/N}}\Big)^2 \big(\mu_q - \sigma_{qh}^2 - \mu_q^2\big) \\ &= \delta - \Big(1 - \frac{1}{\psi}\Big)\Big(g - \frac{1}{2}\frac{\sigma_{1/N}^2}{\mu_q}\Big) - \Big(1 - \frac{1}{\psi}\Big)\frac{1}{2\gamma}\Big(\frac{\sigma_{1/N}}{\mu_q}\Big)^2 \big(\mu_q - \sigma_{qh}^2 - \mu_q^2\big) \\ &= \delta - \Big(1 - \frac{1}{\psi}\Big)\Big(g - \frac{1}{2}\gamma\sigma_{1/N}^2\big(1 + \frac{\sigma_{qh}^2}{\mu_q^2}\big)\Big) \end{aligned}$$

Hence, we obtain (A28). ■

We now look at the special case where the familiarity coefficients f_{hn} are restricted to be either 0 or 1.

Proposition A.12.2 *If $\rho = 0$, σ_{fh} is independent of h and the familiarity coefficients f_{hn} are restricted to be either 0 or 1, then κ_h is independent of h and is given by*

$$\kappa = \begin{cases} \left[\frac{\psi\delta + (1-\psi)U^{MV}(\sigma_{\boldsymbol{x}})}{\delta^{\psi}}\right]^{\frac{1}{1-\psi}} & \psi \neq 0, \\ \\ U^{MV}(\sigma_{\boldsymbol{x}}) & \psi = 0, \end{cases} \tag{29}$$

and

$$U^{MV}(\sigma_{\boldsymbol{x}}) = \delta + \frac{1}{\psi}\left(g - \frac{1}{2}\gamma\sigma_{\boldsymbol{x}}^2\right) = \alpha - \frac{\gamma}{2}\sigma_{\boldsymbol{x}}^2,$$

with the endogenous aggregate growth rate g given in (27) *and where we use $U^{MV}(\sigma_{\boldsymbol{x}})$ to denote the utility of a mean-variance household after imposing market clearing, which is obtained by substituting into* (18) *the equilibrium interest rate from* (24) *and the condition that $\pi_h = 1$ for each household.*

Proof: We assume that $\rho = 0$, σ_{fh} is independent of h and the familiarity coefficients f_{hn} are restricted to be either 0 or 1. Consequently, (A26) reduces to

$$c_h = c = \psi\delta + (1-\psi)\Big(\alpha - \frac{1}{2}\gamma\sigma_{\boldsymbol{x}}^2\Big).$$

Furthermore, substituting the equilibrium interest rate from (24) into (A17) and simplifying gives

$$U^{MV}(\sigma_{\boldsymbol{x}}) = \alpha - \frac{1}{2}\gamma\sigma_{\boldsymbol{x}}^2.$$

Therefore (21) reduces to (29). ■

A.13. Disentangling the micro- and macro-level effects in (32)

Proposition A.13.1 *Suppose that σ_{qh} is independent of h. A reduction in familiarity biases changes social welfare per unit capital stock as follows:*

$$d\ln\left(U_t^{social}/K_t^{agg}\right) = d\ln\kappa = d\ln\kappa|_{micro\text{-}level} + d\ln\kappa|_{macro\text{-}level},$$

where $d\ln\kappa|_{micro\text{-}level}$ captures the effect of a reduction in familiarity biases at the micro-level, that is, a reduction in σ_q^2 and an increase in μ_q for individual households, whereas $d\ln\kappa|_{macro\text{-}level}$ gives the macro-level effect of a change in the equilibrium growth rate driven by an increase in μ_q for individual households.

The micro-level effect of a reduction in familiarity biases on social welfare is given by

$$d\ln\kappa|_{micro\text{-}level} = \frac{1}{2}\gamma\,\sigma_{1/N}^2\left[v_q^2 k_i(-d\ln\sigma_{qh}^2 + 2d\ln\mu_q) - \left(1-\frac{1}{\psi}\right)\frac{k_c}{\mu_q}\,d\ln\mu_q\right],$$

where k_i captures the intertemporal effects and k_c captures the effects arising from current consumption:

$$k_i = \frac{1}{\delta - \left(1-\frac{1}{\psi}\right)\left(g - \frac{1}{2}\gamma\,\sigma_{1/N}^2(1+v_q^2)\right)}$$
$$k_c = \frac{1}{\delta - \left(1-\frac{1}{\psi}\right)\left(g - \frac{1}{2}\gamma\,\frac{\sigma_{1/N}^2}{\mu_q}\right)},$$

where

$$v_q^2 = \left(\frac{\sigma_q}{\mu_q}\right)^2.$$

The macro-level effect of a reduction in familiarity biases on social welfare is given by

$$d\ln\kappa|_{macro\text{-}level} = \left[k_i - \left(1-\frac{1}{\psi}\right)k_c\right]dg,$$

where

$$dg = \frac{1}{2}(\psi - 1)\gamma \frac{\sigma_{1/N}^2}{\mu_q} d\ln \mu_q. \tag{A30}$$

If $\rho = 0$ and $f_{hn} \in \{0, 1\}$, then the percentage change in social welfare per unit of aggregate capital stock stemming from a change in familiarity biases is given by

$$\frac{d\ln \kappa}{d\sigma_{\boldsymbol{x}}^2} = \frac{1}{c}\frac{dU^{MV}(\sigma_{\boldsymbol{x}})}{d\sigma_{\boldsymbol{x}}^2},$$

where

$$\begin{aligned}\frac{dU^{MV}(\sigma_{\boldsymbol{x}})}{d\sigma_{\boldsymbol{x}}^2} &= \frac{\partial U^{MV}(\sigma_{\boldsymbol{x}})}{\partial g}\frac{\partial g}{\partial \sigma_{\boldsymbol{x}}^2} + \frac{\partial U^{MV}(\sigma_{\boldsymbol{x}})}{\partial \sigma_{\boldsymbol{x}}^2} \\ &= -\frac{1}{2}\gamma\Bigg[\underbrace{\left(1 - \frac{1}{\psi}\right)}_{\textit{macro-level effect}} + \underbrace{\frac{1}{\psi}}_{\textit{micro-level effect}}\Bigg].\end{aligned}$$

Proof: Define the square of the coefficient of variation

$$v_{qh}^2 = \left(\frac{\sigma_{qh}}{\mu_q}\right)^2.$$

From (A28), we can see that

$$\kappa_h = (\delta k_i)^{\frac{1}{1-\frac{1}{\psi}}}\frac{1}{k_c}.$$

Therefore

$$d\ln \kappa_h = \frac{1}{1 - \frac{1}{\psi}} d\ln k_i - d\ln k_c,$$

and

$$\begin{aligned}\frac{\partial \ln k_i}{\partial \ln v_{qh}^2} &= -\frac{1}{2}\gamma v_{qh}^2 \sigma_{1/N}^2 \left(1 - \frac{1}{\psi}\right) k_i, \\ \frac{\partial \ln k_c}{\partial \ln \mu_q} &= \frac{1}{2}\frac{\gamma}{\mu_q}\sigma_{1/N}^2 \left(1 - \frac{1}{\psi}\right) k_c, \\ \frac{\partial \ln k_i}{\partial \ln g} &= \left(1 - \frac{1}{\psi}\right) g k_i, \\ \frac{\partial \ln k_c}{\partial \ln g} &= \left(1 - \frac{1}{\psi}\right) g k_c.\end{aligned}$$

Hence

$$\frac{\partial \ln \kappa_h}{\partial \ln v_{qh}^2} = \frac{1}{1-\frac{1}{\psi}} \frac{\partial \ln k_i}{\partial \ln v_{qh}^2} = -\frac{1}{2}\gamma v_{qh}^2 \sigma_{1/N}^2 k_i$$

$$\frac{\partial \ln \kappa_h}{\partial \ln \mu_q} = -\frac{1}{2}\frac{\gamma}{\mu_q}\sigma_{1/N}^2 \left(1-\frac{1}{\psi}\right) k_c.$$

Therefore

$$\begin{aligned}
d\ln \kappa_h|_{\text{micro-level}} &= \frac{\partial \ln \kappa_h}{\partial \ln v_{qh}^2} d\ln v_{qh}^2 + \frac{\partial \ln \kappa_h}{\partial \ln \mu_q} d\ln \mu_q \\
&= -\frac{1}{2}\gamma v_{qh}^2 \sigma_{1/N}^2 k_i d\ln v_{qh}^2 - \frac{1}{2}\frac{\gamma}{\mu_q}\sigma_{1/N}^2 \left(1-\frac{1}{\psi}\right) k_c d\ln \mu_q \\
&= -\frac{1}{2}\gamma \sigma_{1/N}^2 \left[v_{qh}^2 k_i \, d\ln v_q^2 + \frac{k_c}{\mu_q}\left(1-\frac{1}{\psi}\right) d\ln \mu_q \right] \\
&= -\frac{1}{2}\gamma \sigma_{1/N}^2 \left[v_{qh}^2 k_i \left(d\ln \sigma_q^2 - 2d\ln \mu_q\right) + \frac{k_c}{\mu_q}\left(1-\frac{1}{\psi}\right) d\ln \mu_q \right] \\
&= \frac{1}{2}\gamma \sigma_{1/N}^2 \left[v_{qh}^2 k_i \left(-d\ln \sigma_q^2 + 2d\ln \mu_q\right) - \frac{k_c}{\mu_q}\left(1-\frac{1}{\psi}\right) d\ln \mu_q \right].
\end{aligned}$$

Also

$$\begin{aligned}
d\ln \kappa_h|_{\text{macro-level}} &= \frac{\partial \ln \kappa_h}{\partial \ln g} d\ln g \\
&= \frac{1}{1-\frac{1}{\psi}} \frac{\partial \ln k_i}{\partial \ln g} d\ln g - \frac{\partial \ln k_c}{\partial \ln g} d\ln g \\
&= g\left[k_i - \left(1-\frac{1}{\psi_h}\right) k_c \right] d\ln g \\
&= \left[k_i - \left(1-\frac{1}{\psi_h}\right) k_c \right] dg.
\end{aligned}$$

Equation (A30) then follows from (A29).

If σ_{qh} is independent of h, then κ_h is independent of h and social welfare per unit wealth is given by $\kappa = \kappa_h = \sum_{h=1}^{H} \frac{U_{h,t}}{W_{h,t}} = \frac{U_t^{\text{social}}}{W_t^{\text{agg}}}$. Hence, the increase in social welfare per unit wealth from infinitesimally small changes in familiarity biases is given by $d\kappa = d\kappa_h$.

We now impose the assumptions that $\rho = 0$ and $f_{hn} \in \{0, 1\}$, which implies that

$$1 + v_{fh}^2 = \frac{1}{\mu_f}.$$

The above expression tells us that the independence of μ_f from h implies that v_{fh} and hence σ_{fh} is independent of h. We can thus see from that (A28) that κ_h becomes independent of h and

$$\kappa = \begin{cases} \left[\frac{\psi\delta+(1-\psi)U^{MV}(\sigma_{\boldsymbol{x}})}{\delta^{\psi}}\right]^{\frac{1}{1-\psi}} & \psi \neq 0, \\ U^{MV}(\sigma_{\boldsymbol{x}}) & \psi = 0, \end{cases}$$

in which

$$U^{MV}(\sigma_{\boldsymbol{x}}) = \delta + \frac{1}{\psi}\left(g - \frac{1}{2}\gamma\sigma_{\boldsymbol{x}}^2\right) = \alpha - \frac{\gamma}{2}\sigma_{\boldsymbol{x}}^2,$$

where

$$\sigma_{\boldsymbol{x}}^2 = \frac{\sigma_{1/N}^2}{\mu_f} = \sigma_{1/N}^2(1+v_f^2).$$

Therefore

$$\begin{aligned} \frac{d\ln\kappa}{d\sigma_{\boldsymbol{x}}^2} &= \frac{1}{\psi\delta + (1-\psi)U^{MV}(\sigma_{\boldsymbol{x}})}\frac{dU^{MV}(\sigma_{\boldsymbol{x}})}{d\sigma_{\boldsymbol{x}}^2} \\ &= \frac{1}{c}\frac{dU^{MV}(\sigma_{\boldsymbol{x}})}{d\sigma_{\boldsymbol{x}}^2}, \end{aligned}$$

where

$$\begin{aligned} \frac{dU^{MV}(\sigma_{\boldsymbol{x}})}{d\sigma_{\boldsymbol{x}}^2} &= \frac{\partial U^{MV}(\sigma_{\boldsymbol{x}})}{\partial g}\frac{\partial g}{\partial \sigma_{\boldsymbol{x}}^2} + \frac{\partial U^{MV}(\sigma_{\boldsymbol{x}})}{\partial \sigma_{\boldsymbol{x}}^2} \\ &= -\frac{1}{2}\gamma\Bigg[\underbrace{\left(1-\frac{1}{\psi}\right)}_{\text{macro-level effect}} + \underbrace{\frac{1}{\psi}}_{\text{micro-level effect}}\Bigg]. \end{aligned}$$

■

A.14. Social welfare with preference heterogeneity in (33)

The proposition below shows that when households are heterogeneous, then social welfare per unit of aggregate wealth is given by κ_h *averaged* across all households, in contrast to the case where households had identical preferences and social welfare per unit of aggregate wealth was given by (the common) κ.

Proposition A.14.1 *We assume that the following symmetry condition holds*

$$\frac{1}{\mathcal{R}} = \frac{\mu_{qh}}{\gamma_h}, \forall h \in \{1, \ldots, H\}.$$

Social welfare per unit of aggregate wealth at date t is given by the wealth-weighted average of κ_h:

$$\frac{U_t^{social}}{W_t^{agg}} = \frac{\sum_{h=1}^{H} U_{h,t}}{\sum_{h=1}^{H} W_{h,t}} = \sum_{h=1}^{H} \kappa_h \frac{W_{h,t}}{\sum_{j=1}^{H} W_{j,t}},$$

where

$$\kappa_h = \left[\frac{\psi_h \delta_h}{\psi_h \delta_h + (1-\psi_h)\left[g_h - \frac{1}{2}\gamma_h \sigma_{1/N}^2 \left(1 + \frac{\sigma_{qh}^2}{\mu_{qh}^2} \right) \right]} \right]^{\frac{1}{1-\frac{1}{\psi_h}}} \left[\delta_h - \left(1 - \frac{1}{\psi_h} \right) \left(g_h - \frac{1}{2}\mathcal{R}\sigma_{1/N}^2 \right) \right],$$

and

$$g_h = \psi_h(\alpha - \delta_h) - \frac{1}{2}(\psi_h - 1)\mathcal{R}\sigma_{1/N}^2.$$

For the special case of $\rho = 0$ and with the assumption that $f_{hn} \in \{0, 1\}$, κ_h is given by

$$\kappa_h = \begin{cases} \left[\frac{\psi_h \delta_h + (1-\psi_h) U^{MV}}{\delta_h^{\psi_h}} \right]^{\frac{1}{1-\psi_h}} & \psi_h \neq 0, \\ U^{MV} & \psi_h = 0, \end{cases},$$

where

$$U^{MV} = \alpha - \frac{\mathcal{R}}{2}\sigma_{1/N}^2.$$

If we assume that all households have equal date-t wealth, we obtain

$$\frac{U_t^{social}}{W_t^{agg}} = \left(\frac{1}{H} \sum_{h=1}^{H} \kappa_h \right),$$

where date-t aggregate wealth is given by

$$W_t^{agg} = \sum_{h=1}^{H} W_{h,t} = H\, W_{h,t}.$$

Proof: If the investment opportunity set is constant, that is, the interest rate is constant (below we will specify the condition that ensures this indeed is the case), then the vector of optimal portfolio weights of household h is given by

$$\boldsymbol{\omega}_h = \frac{\alpha - i}{\gamma_h \sigma_{1/N}^2} \frac{\boldsymbol{q_h}}{N}.$$

Furthermore, the date-t optimal consumption rate of household h is given by

$$\frac{C_{h,t}}{W_{h,t}} = \psi_h \delta_h + (1 - \psi_h) \left(i + \frac{1}{2\gamma_h} \left(\frac{\alpha - i}{\sigma_{1/N}} \right)^2 \mu_{qh} \right). \tag{A31}$$

In equilibrium the bond market clears and so

$$\sum_{h=1}^{H} (1 - \pi_h) W_{h,t} = 0.$$

Therefore,

$$\sum_{h=1}^{H} \pi_h W_{h,t} = \sum_{h=1}^{H} W_{h,t}.$$

Hence,

$$\frac{\alpha - i}{\sigma_{1/N}^2} \sum_{h=1}^{H} \frac{\mu_{qh}}{\gamma_h} W_{h,t} = \sum_{h=1}^{H} W_{h,t},$$

and so

$$i = \alpha - \mathcal{R}\, \sigma_{1/N}^2, \tag{A32}$$

where

$$\mathcal{R} = \left(\frac{\sum_{h=1}^{H} \frac{\mu_{qh}}{\gamma_h} W_{h,t}}{\sum_{h=1}^{H} W_{h,t}} \right)^{-1}.$$

We now impose the symmetry condition that for distinct households h and j:

$$\frac{\mu_{qh}}{\gamma_h} = \frac{\mu_{qj}}{\gamma_j}.$$

We hence obtain

$$\mathcal{R} = \frac{\gamma_h}{\mu_{qh}}. \tag{A33}$$

Substituting (A32) into (A31) and using (A33) gives

$$c_h = \frac{C_{h,t}}{W_{h,t}} = \psi_h \delta_h + (1 - \psi_h) \left(\alpha - \frac{1}{2} \mathcal{R} \, \sigma_{1/N}^2 \right).$$

Observe that the symmetry condition implies that for every household$h$:

$$\pi_h = 1.$$

Household h's experienced utility level is given by (A15). It follows that in equilibrium with the symmetry condition, we have

$$U_h^{MV}(\sigma_{\boldsymbol{x}_h}) = \alpha - \frac{\gamma_h}{2} \sigma_{\boldsymbol{x}_h}^2,$$

where

$$\sigma_{\boldsymbol{x}_h}^2 = \sigma_{1/N}^2 \left(1 + \frac{\sigma_{qh}^2}{\mu_{qh}^2} \right).$$

Therefore $\kappa_h = \frac{U_{h,t}}{W_{h,t}}$ is given by

$$\kappa_h = \left[\frac{\psi_h \delta_h}{\psi_h \delta_h + (1 - \psi_h) \left(U^{MV}(\sigma_{\boldsymbol{x}_h}) - c_h \right)} \right]^{\frac{1}{1 - \frac{1}{\psi_h}}} c_h.$$

Now observe that

$$U^{MV}(\sigma_{\boldsymbol{x}_h}) - c_h = g_h - \frac{1}{2} \gamma_h \sigma_{1/N}^2 \left(1 + \frac{\sigma_{qh}^2}{\mu_{qh}^2} \right),$$

where

$$g_h = \frac{1}{W_{h,t}} E_t \left[\frac{dW_{h,t}}{dt} \right] = \psi_h (\alpha - \delta_h) - \frac{1}{2} (\psi_h - 1) \mathcal{R} \sigma_{1/N}^2.$$

We also have

$$g_h - \frac{1}{2} \mathcal{R} \sigma_{1/N}^2 = \psi_h \left(\alpha - \delta_h - \frac{1}{2} \mathcal{R} \sigma_{1/N}^2 \right),$$

and so

$$c_h = \delta_h - \left(1 - \frac{1}{\psi_h} \right) \left(g_h - \frac{1}{2} \mathcal{R} \sigma_{1/N}^2 \right).$$

Therefore

$$\kappa_h = \left[\frac{\psi_h \delta_h}{\psi_h \delta_h + (1-\psi_h)\left[g_h - \frac{1}{2}\gamma_h \sigma^2_{1/N}\left(1 + \frac{\sigma^2_{qh}}{\mu^2_{qh}}\right)\right]} \right]^{\frac{1}{1-\frac{1}{\psi_h}}} \left[\delta_h - \left(1 - \frac{1}{\psi_h}\right)\left(g_h - \frac{1}{2}\mathcal{R}\sigma^2_{1/N}\right)\right]$$

For the special case of $\rho = 0$ and with the assumption that $f_{hn} \in \{0, 1\}$, we have $\sigma^2_{f,h} = \mu_{f,h} - \mu^2_{f,h}$, and so

$$U^{MV} = U^{MV}_h(\sigma_{\boldsymbol{x}_h}) = \alpha - \frac{\mathcal{R}}{2}\sigma^2_{1/N},$$

while $U^{MV}(\sigma_{\boldsymbol{x}_h}) - c_h$ simplifies to give

$$U^{MV} - c_h = \psi_h \left(\alpha - \delta_h - \frac{1}{2}\mathcal{R}\sigma^2_{1/N}\right).$$

Therefore

$$\frac{\psi_h \delta_h}{\psi_h \delta_h + (1-\psi_h)\left(U^{MV}(\sigma_{\boldsymbol{x}_h}) - c_h\right)} = \frac{\delta_h}{c_h},$$

where

$$c_h = \psi_h \delta_h + (1-\psi_h)\left(\alpha - \frac{1}{2}\mathcal{R}\,\sigma^2_{1/N}\right).$$

Hence, we obtain

$$\kappa_h = \left[\frac{\psi\delta + (1-\psi)U^{MV}}{\delta^\psi} \right]^{\frac{1}{1-\psi}},$$

where

$$U^{MV} = \alpha - \frac{\mathcal{R}}{2}\sigma^2_{1/N}.$$

■

B. Labor Income

In this section, we first provide the details of the model with labor income and then provide the proofs for all the propositions.

B.1. Labor Income: Details

A household's dynamic budget constraint in the presence of labor income is given by

$$\frac{dW_{h,t}}{W_{h,t}} = \Big(1 - \pi_{h,t}\Big)\, i\, dt + \pi_{h,t} \sum_{n=1}^{N} x_{hn,t}\Big(\alpha\, dt + \sigma\, dZ_{n,t}\Big) - c_{h,t}dt + \frac{Y_{h,t}}{W_{h,t}}dt,$$

where $c_{h,t} = C_{h,t}/W_{h,t}$, $Y_{h,t}$ is the date-t labor income flow of household h, and

$$\frac{dY_{h,t}}{Y_{h,t}} = \theta_Y \left(m_Y - \ln \frac{W_{h,t}}{Y_{h,t}}\right) dt + \sigma_Y dZ_{Y,h,t},$$

where $Z_{Y,h}$ is a standard Brownian motion under the reference measure $\mathbb{P}$ such that

$$\begin{aligned} dZ_{Y,h,t} dZ_{h'\,t} &= \rho_{YK}(1 + \epsilon\delta_{hh'})dt, \\ dZ_{Y,h,t} dZ_{Y,h',t} &= \delta_{hh'}dt. \end{aligned}$$

We can define a new vector of Brownian motions, consisting of the Brownian motions driving labor income shocks, i.e.

$$\boldsymbol{Z}_{Y,t} = (Z_{Y,1,t}, \ldots, Z_{Y,H,t})^\top.$$

The correlation matrix for the combined vector of Brownian shocks $(d\boldsymbol{Z}_t^\top,\, (d\boldsymbol{Z}_{Y,t})^\top)^\top$ is denoted by Ω_A, that is

$$(d\boldsymbol{Z}_t^\top,\, (d\boldsymbol{Z}_{Y,t})^\top)^\top (d\boldsymbol{Z}_t^\top,\, (d\boldsymbol{Z}_{Y,t})^\top) = \Omega_A dt,$$

where

$$\Omega_A = \begin{pmatrix} \Omega & \rho_{YK}(J_N + \epsilon I_N) \\ \rho_{YK}(J_N + \epsilon I_N) & I_N \end{pmatrix},$$

and J_N is the $N \times N$ matrix in which every element is a one.

B.2. Labor Income: Propositions and Proofs

We start by extending the definition of the probability measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ to make clear that the expected labor income flow to a household is unaffected by familiarity biases.

Definition B.2.1 *The probability measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ is defined by*

$$\mathbb{Q}^{\boldsymbol{\nu}_h}(A) = E[1_A \xi_{h,T}],$$

where E is the expectation under $\mathbb{P}$, A is an event, and $\xi_{h,t}$ is an exponential martingale (under the reference probability measure $\mathbb{P}$)

$$\frac{d\xi_{h,t}}{\xi_{h,t}} = \frac{1}{\sigma}(\boldsymbol{\nu}_{h,t}^\top, \mathbf{0}_H^\top)\Omega_A^{-1}(d\boldsymbol{Z}_t^\top, (d\boldsymbol{Z}_{Y,t})^\top)^\top,$$

where $\mathbf{0}_H$ is the $H \times 1$ vector of zeros.

Proposition B.2.1 *The stochastic optimal control problem for a household with familiarity biases and exogenous labor income can be solved via the following Hamilton-Jacobi-Bellman equation*

$$\sup_{\widehat{C}_{h,t},\, \pi_{h,t},\, \boldsymbol{x}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} \delta_h u_{\psi_h}\left(\frac{\widehat{C}_{h,t}}{\widehat{U}_{h,t}}\right) + \frac{\delta_h - k_{1h,t}}{1 - \frac{1}{\psi_h}}$$

(B1)

$$+ \frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\left[k_{2h,t} + \pi_{h,t}(\alpha + \boldsymbol{x}_h^\top \boldsymbol{\nu}_h - \gamma_h \rho_{YK}\sigma\sigma_Y(1+\epsilon x_{hh,t}) - i) - \frac{\widehat{C}_{h,t}}{\widehat{W}_{h,t}} + \frac{1}{\widehat{W}_{h,t}}\right]$$

$$+ \frac{1}{2}\left(\frac{\widehat{W}_{h,t}^2\widehat{U}_{ht,\widehat{W}_h\widehat{W}_h}}{\widehat{U}_{h,t}} - \gamma_h\left(\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\right)^2\right)(\sigma^2\pi_{h,t}^2 \boldsymbol{x}_{h,t}^\top \Omega \boldsymbol{x}_{h,t} - 2\rho_{YK}\sigma_Y\sigma\pi_{h,t}(1+\epsilon x_{hh,t}) + \sigma_Y^2)$$

$$+ \frac{1}{2\gamma_h}\frac{\boldsymbol{\nu}_{h,t}^\top \Gamma_h^{-1}\boldsymbol{\nu}_{h,t}}{\sigma^2},$$

where $\widehat{U}_{h,t} = U_{h,t}/Y_{h,t}$, $\widehat{W}_{h,t} = W_{h,t}/Y_{h,t}$, and

$$k_{1h,t} = \delta_h + \frac{1}{\psi_h}\mu_{Y,h,t} - \frac{1}{2}\left(1 + \frac{1}{\psi_h}\right)\gamma_h\sigma_Y^2 + \gamma_h\sigma_Y^2 - \mu_{Y,h,t},$$
$$k_{2h,t} = i + \gamma_h\sigma_Y^2 - \mu_{Y,h,t},$$
$$\mu_{Y,h,t} = \theta_Y\left(m_Y - \ln\frac{W_{h,t}}{Y_{h,t}}\right).$$

Proof: We now define

$$\widehat{W}_{h,t} = \frac{W_{h,t}}{Y_{h,t}}.$$

Hence, using Ito's Lemma

$$
\begin{aligned}
\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}} &= \frac{dW_{h,t}}{W_{h,t}} + \frac{d(Y_{h,t}^{-1})}{Y_{h,t}^{-1}} + \frac{dW_{h,t}}{W_{h,t}}\frac{d(Y_{h,t}^{-1})}{Y_{h,t}^{-1}} \\
&= \frac{dW_{h,t}}{W_{h,t}} + \frac{-Y_{h,t}^{-2}dY_{h,t} + \frac{1}{2}2Y_{h,t}^{-3}(dY_{h,t})^2}{Y_{h,t}^{-1}} + \frac{dW_{h,t}}{W_{h,t}}\frac{(-Y_{h,t}^{-2})dY_{h,t}}{Y_{h,t}^{-1}} \\
&= \frac{dW_{h,t}}{W_{h,t}} - \frac{dY_{h,t}}{Y_{h,t}} + \left(\frac{dY_{h,t}}{Y_{h,t}}\right)^2 - \frac{dW_{h,t}}{W_{h,t}}\frac{dY_{h,t}}{Y_{h,t}} \\
&= idt + \pi_{h,t}(\alpha - i)dt - c_{h,t}dt + \frac{Y_{h,t}}{W_{h,t}}dt + \sigma\pi_{h,t}\sum_{n=1}^{N} x_{hn,t}\, dZ_{n,t} \\
&\quad - \mu_{Y,h,t}dt - \sigma_Y dZ_{Y,h,t} + \sigma_Y^2 dt \\
&\quad - \sigma\sigma_Y\pi_{h,t}dZ_{Y,h,t}\sum_{n=1}^{N} x_{hn,t}dZ_{n,t} \\
&= \pi_{h,t}(\alpha - i)dt - c_{h,t}dt + \frac{Y_{h,t}}{W_{h,t}}dt + \sigma\pi_{h,t}\sum_{n=1}^{N} x_{hn,t}\, dZ_{n,t} \\
&\quad - \mu_{Y,h,t}dt - \sigma_Y dZ_{Y,h,t} + \sigma_Y^2 dt - \sigma\sigma_Y\pi_{h,t}\left(\rho_{YK}\sum_{n=1}^{N} x_{hn,t} + \rho_{YK}\epsilon x_{hh,t}\right)dt \\
\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}} &= \left[i + \pi_{h,t}[\alpha - \rho_{YK}\sigma_Y\sigma(1+\epsilon x_{hh,t}) - i] - \frac{\widehat{C}_{h,t}}{\widehat{W}_{h,t}} + \frac{1}{\widehat{W}_{h,t}} - \left(\mu_{Y,h,t} - \sigma_Y^2\right)\right]dt \\
&\quad + \sigma\pi_{h,t}\sum_{n=1}^{N} x_{hn,t}\, dZ_{n,t} - \sigma_Y dZ_{Y,h,t} \\
&= \left[i + \pi_{h,t}[\alpha - \rho_{YK}\sigma_Y\sigma(1+\epsilon \boldsymbol{e}_h^\top \boldsymbol{x}_{h,t}) - i] - \frac{\widehat{C}_{h,t}}{\widehat{W}_{h,t}} + \frac{1}{\widehat{W}_{h,t}} - \left(\mu_{Y,h,t} - \sigma_Y^2\right)\right]dt \\
&\quad + (\sigma\pi_{h,t}\boldsymbol{x}_{h,t}^\top, -\sigma_Y\boldsymbol{e}_h^\top)(d\boldsymbol{Z}_t^\top, d\boldsymbol{Z}_{Y,t}^\top)^\top \\
&= \left[i + \pi_{h,t}(\widehat{\alpha} - \epsilon\rho_{YK}\sigma_Y\sigma\boldsymbol{e}_h^\top\boldsymbol{x}_{h,t} - i) - \frac{\widehat{C}_{h,t}}{\widehat{W}_{h,t}} + \frac{1}{\widehat{W}_{h,t}} - \left(\mu_{Y,h,t} - \sigma_Y^2\right)\right]dt \\
&\quad + (\sigma\pi_{h,t}\boldsymbol{x}_{h,t}^\top, -\sigma_Y\boldsymbol{e}_h^\top)(d\boldsymbol{Z}_t^\top, d\boldsymbol{Z}_{Y,t}^\top)^\top
\end{aligned}
$$

where

$$
\begin{aligned}
\mu_{Y,h,t} &= \theta_Y\left(m_Y - \ln\frac{W_{h,t}}{Y_{h,t}}\right) = \theta_Y\left(m_Y - \ln\widehat{W}_{h,t}\right) \\
\widehat{\alpha} &= \alpha - \rho_{YK}\sigma_Y\sigma.
\end{aligned}
$$

For algebraic simplicity, we define

$$\tau_0 = \theta_Y m_Y$$
$$\tau_1 = -\theta_Y.$$

We now derive the dynamics of $\widehat{W}_{h,t}$ under the probability measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ by using Girsanov's Theorem. Hence, we obtain

$$\begin{aligned}\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}} = &\left[i + \pi_{h,t}[\widehat{\alpha} + \boldsymbol{x}_{h,t}^\top(\boldsymbol{\nu}_{h,t} - \epsilon\rho_{YK}\sigma_Y\sigma\boldsymbol{e}_h) - i] - \frac{\widehat{C}_{h,t}}{\widehat{W}_{h,t}} + \frac{1}{\widehat{W}_{h,t}} - \left(\mu_{Y,h,t} - \sigma_Y^2\right)\right]dt \\ &+ (\sigma\pi_{h,t}\boldsymbol{x}_{h,t}^\top, -\sigma_Y\boldsymbol{e}_h^\top)((d\boldsymbol{Z}_t^{\boldsymbol{\nu}})^\top, d\boldsymbol{Z}_{Y,t}^\top)^\top,\end{aligned}$$

where

$$d\boldsymbol{Z}_t^{\boldsymbol{\nu}} = d\boldsymbol{Z}_t - \pi_{h,t}\boldsymbol{x}_{h,t}^\top\boldsymbol{\nu}_{h,t}.$$

The dynamics of $Y_{h,t}$ remain the same under $\mathbb{Q}^{\boldsymbol{\nu}_h}$, because shocks to labor income are orthogonal to shocks to the exponential martingale $\xi_{h,t}$.

We start from the recursive definition of the utility function

$$U_{h,t} = \mathcal{A}(C_{h,t}, \mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}]).$$

Defining

$$\widehat{U}_{h,t} = \frac{U_{h,t}}{Y_{h,t}},$$

we obtain

$$\widehat{U}_{h,t} = \mathcal{A}\left(\widehat{C}_{h,t}, \mu_{h,t}^{\boldsymbol{\nu}}\left[\frac{Y_{h,t+dt}}{Y_{h,t}}\widehat{U}_{h,t+dt}\right]\right).$$

Observe that under both the reference probability measure $\mathbb{P}$ and $\mathbb{Q}^{\boldsymbol{\nu}_h}$

$$\frac{dY_{h,t}}{Y_{h,t}} = \mu_{Y,h,t}dt + \sigma_Y dZ_{Y,h,t}.$$

and

$$Y_{h,u} = Y_{h,t}e^{\int_t^u \mu_{Y,h,s}ds}e^{-\frac{1}{2}\sigma_Y^2(u-t) + \sigma_Y(Z_{Y,h,u} - Z_{Y,h,t})},$$

and so

$$\left(\frac{Y_{h,u}}{Y_{h,t}}\right)^{1-\gamma} = e^{(1-\gamma)\left(\int_t^u \mu_{Y,s}ds - \frac{1}{2}\gamma_h\sigma_Y^2(u-t)\right)}e^{-\frac{1}{2}(1-\gamma_h)^2\sigma_Y^2(u-t) + (1-\gamma_h)\left[\sigma_Y(Z_{Y,h,u} - Z_{Y,h,t})\right]}$$

$$= e^{(1-\gamma_h)\left(\int_t^u \mu_{Y,s}ds - \frac{1}{2}\gamma_h\sigma_Y^2(u-t)\right)}\frac{M_{Y,h,u}}{M_{Y,h,t}},$$

where

$$M_{Y,h,t} = e^{-\frac{1}{2}(1-\gamma_h)^2\sigma_Y^2 t + (1-\gamma_h)\sigma_Y Z_{Y,h,t}},$$

is an exponential martingale with respect to both the reference probability measure $\mathbb{P}$ and $\mathbb{Q}^{\boldsymbol{\nu}_h}$.

Therefore

$$\begin{aligned}
\mu^{\boldsymbol{\nu}}_{h,t}\left[\frac{Y_{h,t+dt}}{Y_{h,t}}\widehat{U}_{h,t+dt}\right] &= \left(E_t^{\mathbb{Q}^{\boldsymbol{\nu}_h}}\left[\left(\frac{Y_{h,t+dt}}{Y_{h,t}}\right)^{1-\gamma_h}\widehat{U}^{1-\gamma_h}_{h,t+dt}\right]\right)^{\frac{1}{1-\gamma_h}} + \widehat{U}_{h,t}L_{h,t}dt \\
&= \left(E_t^{\mathbb{Q}^{\boldsymbol{\nu}_h}}\left[e^{(1-\gamma_h)\left(\mu_{Y,h,t}dt - \frac{1}{2}\gamma_h\sigma_Y^2 dt\right)}\frac{M_{Y,h,t+dt}}{M_{Y,h,t}}\widehat{U}^{1-\gamma_h}_{h,t+dt}\right]\right)^{\frac{1}{1-\gamma_h}} + \widehat{U}_{h,t}L_{h,t}dt \\
&= e^{\left(\mu_{Y,h,t} - \frac{1}{2}\gamma_h\sigma_Y^2\right)dt}\left(E_t^{\mathbb{Q}^{\boldsymbol{\nu}_h}}\left[\frac{M_{Y,h,t+dt}}{M_{Y,h,t}}\widehat{U}^{1-\gamma_h}_{h,t+dt}\right]\right)^{\frac{1}{1-\gamma_h}} + \widehat{U}_{h,t}L_{h,t}dt \\
&= e^{\left(\mu_{Y,h,t} - \frac{1}{2}\gamma_h\sigma_Y^2\right)dt}\left(E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\widehat{U}^{1-\gamma_h}_{h,t+dt}\right]\right)^{\frac{1}{1-\gamma_h}} + \widehat{U}_{h,t}L_{h,t}dt,
\end{aligned}$$

where the probability measure $\mathbb{Q}_Y^{\boldsymbol{\nu}_h}$ is defined by the martingale $M_{Y,h}$. Now

$$\begin{aligned}
\left(E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\widehat{U}^{1-\gamma_h}_{h,t+dt}\right]\right)^{\frac{1}{1-\gamma_h}} &= \left(E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[(\widehat{U}_{h,t} + d\widehat{U}_{h,t})^{1-\gamma_h}\right]\right)^{\frac{1}{1-\gamma_h}} \\
&= \widehat{U}_{h,t}\left(E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\left(1 + \frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right)^{1-\gamma_h}\right]\right)^{\frac{1}{1-\gamma_h}} \\
&= \widehat{U}_{h,t}\left(E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\left(1 + (1-\gamma_h)\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}} - \frac{1}{2}(1-\gamma_h)\gamma_h\left(\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right)^2\right)\right] + o(dt)\right)^{\frac{1}{1-\gamma_h}} \\
&= \widehat{U}_{h,t}\left(1 + E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right] - \frac{1}{2}\gamma_h E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\left(\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right)^2\right] + o(dt)\right).
\end{aligned}$$

Therefore

$$\widehat{U}_{h,t}^{1-\frac{1}{\psi_h}} = (1 - e^{-\delta_h dt})\widehat{C}_{h,t}^{1-\frac{1}{\psi_h}}$$

$$+ e^{-\delta_h dt}\widehat{U}_{h,t}^{1-\frac{1}{\psi_h}} e^{\left(1-\frac{1}{\psi_h}\right)\left(\mu_{Y,h,t}-\frac{1}{2}\gamma_h\sigma_Y^2\right)dt}\left(1+E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right]-\frac{1}{2}\gamma_h E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\left(\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right)^2\right]+o(dt)\right)^{1-\frac{1}{\psi_h}}$$

$$0=\delta_h\left(\frac{\widehat{C}_{h,t}}{\widehat{U}_{h,t}}\right)^{1-\frac{1}{\psi_h}}dt$$

$$-k_{1h,t}dt+\left(1-\frac{1}{\psi_h}\right)\left(E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right]-\frac{1}{2}\gamma_h E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\left(\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right)^2\right]+L_{h,t}dt\right)$$

$$+o(dt),$$

where

$$k_{1h,t}=\delta_h+\frac{1}{\psi_h}\mu_{Y,h,t}-\frac{1}{2}\left(1+\frac{1}{\psi_h}\right)\gamma_h\sigma_Y^2+\gamma_h\sigma_Y^2-\mu_{Y,h,t}.$$

Hence, in the continuous time limit, we obtain

$$0=\delta_h\left(\frac{\widehat{C}_{h,t}}{\widehat{U}_{h,t}}\right)^{1-\frac{1}{\psi_h}}$$

$$-k_{1h,t}+\left(1-\frac{1}{\psi_h}\right)\frac{1}{dt}\left(E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right]-\frac{1}{2}\gamma_h E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\left(\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right)^2\right]+L_{h,t}dt\right),$$

which can be rewritten as

$$\delta_h u_h\left(\frac{\widehat{C}_{h,t}}{\widehat{U}_{h,t}}\right)+\frac{\delta_h-k_{1h,t}}{1-\frac{1}{\psi}_h}+\frac{1}{dt}\left(E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right]-\frac{1}{2}\gamma_h E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\left(\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right)^2\right]+L_{h,t}dt\right).$$

It follows from Girsanov's Theorem that under probability measure $\mathbb{Q}_Y^{\boldsymbol{\nu}_h}$, we have

$$\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}}=\left[k_{2h,t}+\pi_{h,t}(\alpha+\boldsymbol{x}_h^\top\boldsymbol{\nu}_h-\gamma_h\rho_{YK}\sigma\sigma_Y(1+\epsilon x_{hh,t})-i)-\frac{\widehat{C}_{h,t}}{\widehat{W}_{h,t}}+\frac{1}{\widehat{W}_{h,t}}\right]dt$$
$$+(\sigma\pi_{h,t}\boldsymbol{x}_{h,t}^\top,-\sigma_Y\boldsymbol{e}_h^\top)((d\boldsymbol{Z}_t^{\boldsymbol{\nu}})^\top,d\boldsymbol{Z}_{Y,t}^\top)^\top,$$

where

$$k_{2h,t}=i+\gamma_h\sigma_Y^2-\mu_{Y,h,t}.$$

Hence

$$\frac{1}{dt}E_t^{\mathbb{Q}_Y^{\nu_h}}\left[\left(\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}}\right)^2\right] = \sigma^2\pi_{h,t}^2\boldsymbol{x}_{h,t}^\top\Omega\boldsymbol{x}_{h,t} - 2\rho_{YK}\sigma_Y\sigma\pi_{h,t}\boldsymbol{x}_{h,t}^\top(J_N+\epsilon I_N)\boldsymbol{e}_h + \sigma_Y^2\boldsymbol{e}_h^\top I_N\boldsymbol{e}_h$$

$$= \sigma^2\pi_{h,t}^2\boldsymbol{x}_{h,t}^\top\Omega\boldsymbol{x}_{h,t} - 2\rho_{YK}\sigma_Y\sigma\pi_{h,t}(1+\epsilon x_{hh,t}) + \sigma_Y^2.$$

Also,

$$E_t^{\mathbb{Q}_Y^{\nu_h}}\left[\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right] = \frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}E_t^{\mathbb{Q}_Y^{\nu_h}}\left[\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}}\right] + \frac{1}{2}\frac{\widehat{W}_{h,t}^2\widehat{U}_{ht,\widehat{W}_h\widehat{W}_h}}{\widehat{U}_{h,t}}E_t^{\mathbb{Q}_Y^{\nu_h}}\left[\left(\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}}\right)^2\right]$$

$$E_t^{\mathbb{Q}_Y^{\nu_h}}\left[\left(\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right)^2\right] = \left(\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\right)^2 E_t^{\mathbb{Q}_Y^{\nu_h}}\left[\left(\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}}\right)^2\right],$$

and so

$$E_t^{\mathbb{Q}_Y^{\nu_h}}\left[\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right] - \frac{1}{2}\gamma_h E_t^{\mathbb{Q}_Y^{\nu_h}}\left[\left(\frac{d\widehat{U}_{h,t}}{\widehat{U}_{h,t}}\right)^2\right]$$

$$= \frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}E_t^{\mathbb{Q}_Y^{\nu_h}}\left[\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}}\right] + \frac{1}{2}\left(\frac{\widehat{W}_{h,t}^2\widehat{U}_{ht,\widehat{W}_h\widehat{W}_h}}{\widehat{U}_{h,t}} - \gamma_h\left(\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\right)^2\right)E_t^{\mathbb{Q}_Y^{\nu_h}}\left[\left(\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}}\right)^2\right]$$

$$= \frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\left[k_{2h,t} + \pi_{h,t}(\alpha + \boldsymbol{x}_h^\top\boldsymbol{\nu}_h - \gamma_h\rho_{YK}\sigma\sigma_Y(1+\epsilon x_{hh,t}) - i) - \frac{\widehat{C}_{h,t}}{\widehat{W}_{h,t}} + \frac{1}{\widehat{W}_{h,t}}\right]dt$$

$$+ \frac{1}{2}\left(\frac{\widehat{W}_{h,t}^2\widehat{U}_{ht,\widehat{W}_h\widehat{W}_h}}{\widehat{U}_{h,t}} - \gamma_h\left(\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\right)^2\right)(\sigma^2\pi_{h,t}^2\boldsymbol{x}_{h,t}^\top\Omega\boldsymbol{x}_{h,t} - 2\rho_{YK}\sigma_Y\sigma\pi_{h,t}(1+\epsilon x_{hh,t}) + \sigma_Y^2)dt.$$

Therefore, we obtain (B1). ■

Proposition B.2.2 *The FOC's of the Hamilton-Jacobi-Bellman equation* (B1) *give the following expressions for the optimal controls in terms of the normalized value function* $\widehat{U}_{h,t}$*:*

1. *The optimal consumption-wealth ratio* $\widehat{c}_{h,t}$ *is given by*

$$\widehat{c}_{h,t} = \frac{\widehat{C}_{h,t}}{\widehat{W}_{h,t}} = \delta_h^{\psi_h}\left(\frac{\widehat{U}_{h,t}}{\widehat{W}_{h,t}}\right)^{1-\psi_h}\left(\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\right)^{-\psi_h}.$$

2. The optimal portfolio policy is given by $\boldsymbol{\omega}_{h,t} = \pi_{h,t}\boldsymbol{x}_{h,t}$*, where*

$$\boldsymbol{\omega}_{h,t} = \Psi_{h,t}^{-1}\left[\frac{1}{\widehat{\gamma}_{h,t}}\frac{\alpha - i}{\sigma^2}\mathbf{1} - \beta\left(\frac{\gamma_h}{\widehat{\gamma}_{h,t}} - 1\right)(\mathbf{1} + \epsilon\boldsymbol{e}_h)\right],$$

and

$$\beta = \frac{\rho_{YK}\sigma_Y\sigma}{\sigma^2}$$

$$\widehat{\gamma}_{h,t} = \gamma_h\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}} + \frac{-\frac{\widehat{W}_{h,t}^2\widehat{U}_{ht,\widehat{W}_h\widehat{W}_h}}{\widehat{U}_{h,t}}}{\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}},$$

$$\Psi_{h,t} = \Omega + \frac{\gamma_h}{\widehat{\gamma}_{h,t}}\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\Gamma_h.$$

3. The optimal adjustment to the vector of expected returns is given by $\boldsymbol{\nu}_{h,t}$*, where*

$$\boldsymbol{\nu}_{h,t} = -\left(I + \widehat{\gamma}_{h,t}\left(\gamma_h\frac{\widehat{W}_h\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\right)^{-1}\Omega\Gamma_h^{-1}\right)^{-1}\left[(\alpha - i)\mathbf{1} - \rho_{YK}\sigma\sigma_K\left(\gamma_h - \widehat{\gamma}_{h,t}\right)(\mathbf{1} + \epsilon\boldsymbol{e}_h)\right].$$

Proof: The FOC for consumption can be solved to give

$$\widehat{c}_{h,t} = \frac{\widehat{C}_{h,t}}{\widehat{W}_{h,t}} = \delta_h^{\psi_h}\left(\frac{\widehat{U}_{h,t}}{\widehat{W}_{h,t}}\right)^{1-\psi_h}\left(\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\right)^{-\psi_h},$$

and so using the optimal consumption choice we obtain

$$\delta_h u_{\psi_h}\left(\frac{\widehat{C}_{h,t}}{\widehat{U}_{h,t}}\right) - \frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\frac{\widehat{C}_t}{\widehat{W}_t} = \frac{\frac{1}{\psi_h}\delta^{\psi_h}(\widehat{U}_{ht,\widehat{W}_h})^{1-\psi_h} - \delta_h}{1 - \frac{1}{\psi_h}}.$$

Hence

$$\frac{\delta^{\psi_h}(\widehat{U}_{ht,\widehat{W}_h})^{1-\psi_h} - \psi_h\delta_h}{\psi_h - 1} + \frac{\psi_h\delta_h - \psi_h k_{1h,t}}{\psi_h - 1}$$

$$+ \frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\left[k_{2h,t} + \pi_{h,t}(\alpha + \boldsymbol{x}_h^\top\boldsymbol{\nu}_h - \gamma_h\rho_{YK}\sigma\sigma_Y(1 + \epsilon x_{hh,t}) - i)) + \frac{1}{\widehat{W}_{h,t}}\right]$$

$$+\frac{1}{2}\left(\frac{\widehat{W}_{h,t}^2\widehat{U}_{ht,\widehat{W}_h\widehat{W}_h}}{\widehat{U}_{h,t}}-\gamma_h\left(\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\right)^2\right)(\sigma^2\pi_{h,t}^2\boldsymbol{x}_{h,t}^\top\Omega\boldsymbol{x}_{h,t}-2\rho_{YK}\sigma_Y\sigma\pi_{h,t}(1+\epsilon x_{hh,t})+\sigma_Y^2)$$

$$+\frac{1}{2\gamma_h}\frac{\boldsymbol{\nu}_{h,t}^\top\Gamma_h^{-1}\boldsymbol{\nu}_{h,t}}{\sigma^2}.$$

The FOC for $\boldsymbol{\nu}_{h,t}$ can be solved to give

$$\boldsymbol{\nu}_{h,t}=-\gamma_h\sigma^2\frac{\widehat{W}_h\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\pi_{h,t}\Gamma_h\boldsymbol{x}_{h,t},$$

and so

$$\begin{aligned}
&\frac{\delta^{\psi_h}(\widehat{U}_{ht,\widehat{W}_h})^{1-\psi_h}-\psi_h\delta_h}{\psi_h-1}+\frac{\psi_h\delta_h-\psi_h k_{1h,t}}{\psi_h-1}\\
&+\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\left[k_{2h,t}+\pi_{h,t}(\alpha-\gamma_h\rho_{YK}\sigma_Y\sigma\pi_{h,t}(1+\epsilon x_{hh,t})-i)+\frac{1}{\widehat{W}_{h,t}}\right]\\
&+\frac{1}{2}\frac{\widehat{W}_{h,t}^2\widehat{U}_{ht,\widehat{W}_h\widehat{W}_h}}{\widehat{U}_{h,t}}(\sigma^2\pi_{h,t}^2\boldsymbol{x}_{h,t}^\top\Omega\boldsymbol{x}_{h,t}-2\rho_{YK}\sigma_Y\sigma\pi_{h,t}(1+\epsilon x_{hh,t})+\sigma_Y^2)\\
&-\frac{1}{2}\gamma_h\left(\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\right)^2(\sigma^2\pi_{h,t}^2\boldsymbol{x}_{h,t}^\top(\Omega+\Gamma_h)\boldsymbol{x}_{h,t}-2\rho_{YK}\sigma_Y\sigma\pi_{h,t}(1+\epsilon x_{hh,t})+\sigma_Y^2).
\end{aligned}$$

We now rewrite the above expression as

$$\begin{aligned}
&\frac{\delta^{\psi_h}(\widehat{U}_{ht,\widehat{W}_h})^{1-\psi_h}-\psi_h k_{1h,t}}{\psi_h-1}\\
&+\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\left[k_{2h,t}+\frac{1}{\widehat{W}_{h,t}}+\pi_{h,t}(\alpha-i)-(\gamma_h-\widehat{\gamma}_{h,t})\rho_{YK}\sigma_Y\sigma\pi_{h,t}(1+\epsilon x_{hh,t})\right.\\
&\left.-\frac{1}{2}\widehat{\gamma}_{h,t}(\sigma^2\pi_{h,t}^2\boldsymbol{x}_{h,t}^\top\Psi_{h,t}\boldsymbol{x}_{h,t}+\sigma_Y^2)\right],
\end{aligned}$$

where

$$\widehat{\gamma}_{h,t}=\gamma_h\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}+\frac{-\frac{\widehat{W}_{h,t}^2\widehat{U}_{ht,\widehat{W}_h\widehat{W}_h}}{\widehat{U}_{h,t}}}{\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}},$$

$$\Psi_{h,t} = \Omega + \frac{\gamma_h}{\widehat{\gamma}_{h,t}} \frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\Gamma_h.$$

If $\widehat{U}_{h,t}$ is a concave function of $\widehat{W}_{h,t}$, then the optimal portfolio is given by the following optimization problem:

$$\sup_{\boldsymbol{\omega}_{h,t}}(\alpha - i)\mathbf{1}^\top\boldsymbol{\omega}_{h,t} - (\gamma_h - \widehat{\gamma}_{h,t})\rho_{YK}\sigma_Y\sigma(\mathbf{1}^\top\boldsymbol{\omega}_{h,t} + \epsilon\omega_{hh,t}) - \frac{1}{2}\widehat{\gamma}_{h,t}\sigma^2\boldsymbol{\omega}_{h,t}^\top\Psi_{h,t}\boldsymbol{\omega}_{h,t}.$$

The FOC for the optimal portfolio policy $\boldsymbol{\omega}_{h,t}$ is therefore

$$\sigma^2\widehat{\gamma}_{h,t}\Psi_{h,t}\boldsymbol{\omega}_{h,t} = (\alpha - i)\mathbf{1} - (\gamma_h - \widehat{\gamma}_{h,t})\rho_{YK}\sigma_Y\sigma(\mathbf{1} + \epsilon\boldsymbol{e}_h),$$

and so

$$\boldsymbol{\omega}_{h,t} = \Psi_{h,t}^{-1}\left[\frac{1}{\widehat{\gamma}_{h,t}}\frac{\alpha - i}{\sigma^2}\mathbf{1} - \beta\left(\frac{\gamma_h}{\widehat{\gamma}_{h,t}} - 1\right)(\mathbf{1} + \epsilon\boldsymbol{e}_h)\right],$$

where

$$\beta = \frac{\rho_{YK}\sigma_Y\sigma}{\sigma^2}.$$

It follows that

$$\begin{aligned}&\sup_{\boldsymbol{\omega}_{h,t}}(\alpha - i)\mathbf{1}^\top\boldsymbol{\omega}_{h,t} - (\gamma_h - \widehat{\gamma}_{h,t})\rho_{YK}\sigma_Y\sigma(\mathbf{1}^\top\boldsymbol{\omega}_{h,t} + \epsilon\omega_{hh,t}) - \frac{1}{2}\widehat{\gamma}_{h,t}\sigma^2\boldsymbol{\omega}_{h,t}^\top\Psi_{h,t}\boldsymbol{\omega}_{h,t}\\ &= \frac{1}{2}\widehat{\gamma}_{h,t}\sigma^2\boldsymbol{\omega}_{h,t}^\top\Psi_{h,t}\boldsymbol{\omega}_{h,t}.\end{aligned}$$

Therefore at the optimum

$$0 = \frac{\delta^{\psi_h}(\widehat{U}_{ht,\widehat{W}_h})^{1-\psi_h} - \psi_h k_{1h,t}}{\psi_h - 1} + \frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\left[k_{2h,t} + \frac{1}{\widehat{W}_{h,t}} + \frac{1}{2}\widehat{\gamma}_{h,t}(\sigma^2\pi_{h,t}^2\boldsymbol{x}_{h,t}^\top\Psi_{h,t}\boldsymbol{x}_{h,t} - \sigma_Y^2)\right].$$

Consequently,

$$\begin{aligned}\boldsymbol{\nu}_{h,t} &= -\gamma_h\sigma^2\frac{\widehat{W}_h\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\Gamma_h\Psi_{h,t}^{-1}\left[\frac{1}{\widehat{\gamma}_{h,t}}\frac{\alpha - i}{\sigma^2}\mathbf{1} - \beta\left(\frac{\gamma_h}{\widehat{\gamma}_{h,t}} - 1\right)(\mathbf{1} + \epsilon\boldsymbol{e}_h)\right]\\ &= -\gamma_h\sigma^2\frac{\widehat{W}_h\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}\left(\frac{\gamma_h}{\widehat{\gamma}_{h,t}}\frac{\widehat{W}_{h,t}\widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}}I + \Omega\Gamma_h^{-1}\right)^{-1}\left[\frac{1}{\widehat{\gamma}_{h,t}}\frac{\alpha - i}{\sigma^2}\mathbf{1} - \beta\left(\frac{\gamma_h}{\widehat{\gamma}_{h,t}} - 1\right)(\mathbf{1} + \epsilon\boldsymbol{e}_h)\right]\end{aligned}$$

$$= -\gamma_h \sigma^2 \left(\frac{\gamma_h}{\widehat{\gamma}_{h,t}} I + \left(\frac{\widehat{W}_h \widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}} \right)^{-1} \Omega \Gamma_h^{-1} \right)^{-1} \left[\frac{1}{\widehat{\gamma}_{h,t}} \frac{\alpha - i}{\sigma^2} \mathbf{1} - \beta \left(\frac{\gamma_h}{\widehat{\gamma}_{h,t}} - 1 \right) (\mathbf{1} + \epsilon \boldsymbol{e}_h) \right]$$

$$= - \left(I + \widehat{\gamma}_{h,t} \left(\gamma_h \frac{\widehat{W}_h \widehat{U}_{ht,\widehat{W}_h}}{\widehat{U}_{h,t}} \right)^{-1} \Omega \Gamma_h^{-1} \right)^{-1} \left[(\alpha - i)\mathbf{1} - \rho_{YK} \sigma \sigma_K \left(\gamma_h - \widehat{\gamma}_{h,t} \right) (\mathbf{1} + \epsilon \boldsymbol{e}_h) \right].$$

■

Proposition B.2.3 *If we look for an approximate loglinear solution of the form*

$$\widehat{U}_{h,t} = \kappa_h \widehat{W}_{h,t}^{a_h},$$

then the optimal consumption-wealth ratio is given by

$$\widehat{c}_{h,t} = \frac{\widehat{C}_{h,t}}{\widehat{W}_{h,t}} = \delta_h^{\psi_h} \kappa_h^{1-\psi_h} a_h^{-\psi_h} \widehat{W}_{h,t}^{(1-a_h)(\psi_h - 1)},$$

and the optimal portfolio policy by

$$\begin{aligned} \boldsymbol{\omega}_{h,t} &= \boldsymbol{\omega}_h \\ &= \Psi_h^{-1} \left[\frac{1}{\widehat{\gamma}_h} \frac{\alpha - i}{\sigma^2} \mathbf{1} - \beta \left(\frac{\gamma_h}{\widehat{\gamma}_h} - 1 \right) (\mathbf{1} + \epsilon \boldsymbol{e}_h) \right], \end{aligned} \tag{B2}$$

where

$$\widehat{\gamma}_h = a_h \gamma_h + 1 - a_h,$$
$$\Psi_h = \Omega + \frac{\gamma_h a_h}{\widehat{\gamma}_h} \Gamma_h.$$

Also the vector of adjustments to expected returns is given by

$$\begin{aligned} \boldsymbol{\nu}_{h,t} &= \boldsymbol{\nu}_h \\ &= - \left(I + \frac{\widehat{\gamma}_h}{\gamma_h a_h} \Omega \Gamma_h^{-1} \right)^{-1} \left[(\alpha - i)\mathbf{1} - \rho_{YK} \sigma_Y \sigma \left(\gamma_h - \widehat{\gamma}_h \right) (\mathbf{1} + \epsilon \boldsymbol{e}_h) \right]. \end{aligned}$$

Furthermore,

$$\kappa_h = \left[\left(\frac{\delta_h}{a_h} \right)^{\psi_h} \frac{1}{\widehat{c}_h^*} \right]^{\frac{1}{\psi_h - 1}} (\widehat{W}_h^*)^{1 - a_h},$$

where

$$\widehat{c}_h^* = \frac{-\tau_1}{a_h} + \frac{1}{1 - a_h} (\widehat{W}_h^*)^{-1}.$$

and a_h *and* $\widehat{W}_h^*$ *can be determined in terms of exogenous variables by solving*

$$\begin{aligned}\frac{-\tau_1}{a_h}+\frac{1}{1-a_h}(W_h^*)^{-1} &= \frac{1}{\widehat{W}_h^*}+k_{2h}^* \\ &\quad +\widehat{\gamma}_h\left[\sigma^2\pi_h^2\boldsymbol{x}_h^\top\Omega\boldsymbol{x}_h-\rho_{YK}\sigma_Y\sigma\pi_h(1+\epsilon x_{hh})\right], \\ -\tau_1+\frac{a_h}{1-a_h}(\widehat{W}_h^*)^{-1} &= \psi_h k_{1h}^* \\ &\quad +(1-\psi_h)a_h\left[k_{2h}^*+(\widehat{W}_h^*)^{-1}+\frac{1}{2}\widehat{\gamma}_h(\pi_h^2\sigma^2\boldsymbol{x}_h^\top\Psi_h\boldsymbol{x}_h-\sigma_Y^2)\right],\end{aligned}$$

where $k_{ih}^*=k_{ih,t}|_{\widehat{W}_{h,t}=\widehat{W}_h^*}$, $i\in\{1,2\}$.

The ratio of utility to wealth when $\widehat{W}_{h,t}=\widehat{W}_h^*$ *is given by*

$$\frac{U_h^*}{W_h^*}=\left[\left(\frac{\delta_h}{a_h}\right)^{\psi_h}\frac{1}{\widehat{c}_h^*}\right]^{\frac{1}{\psi_h-1}}.$$

Note that $\widehat{W}_h^*$ *is the level of the wealth-income ratio such that*

$$E_t^{\mathbb{Q}_Y^{\boldsymbol{\nu}_h}}\left[\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}}\right]=0,$$

where for an event A *realized at date* T

$$\mathbb{Q}_Y^{\boldsymbol{\nu}_h}(A|\mathcal{F}_t)=E_t^{\mathbb{Q}^{\boldsymbol{\nu}_h}}\left[\frac{M_{Y,h,T}}{M_{Y,h,t}}1_A\right],$$

and

$$M_{Y,h,t}=e^{-\frac{1}{2}(1-\gamma_h)^2\sigma_Y^2t+(1-\gamma_h)\sigma_YZ_{Y,h,t}}.$$

Proof: If we look for an approximate loglinear solution of the form

$$\widehat{U}_{h,t}=\kappa_h\widehat{W}_{h,t}^{a_h},$$

we see that the optimal consumption policy is given by

$$\widehat{c}_{h,t}=\frac{\widehat{C}_{h,t}}{\widehat{W}_{h,t}}=\delta_h^{\psi_h}\kappa_h^{1-\psi_h}a_h^{-\psi_h}\widehat{W}_{h,t}^{(1-a_h)(\psi_h-1)},$$

and the optimal portfolio policy by

$$\boldsymbol{\omega}_{h,t}=\boldsymbol{\omega}_h$$

$$= \Psi_h^{-1}\left[\frac{1}{\widehat{\gamma}_h}\frac{\alpha-i}{\sigma^2}\mathbf{1} - \beta\left(\frac{\gamma_h}{\widehat{\gamma}_h}-1\right)(\mathbf{1}+\epsilon\boldsymbol{e}_h)\right].$$

where

$$\widehat{\gamma}_h = a_h\gamma_h + 1 - a_h,$$
$$\Psi_h = \Omega + \frac{\gamma_h a_h}{\widehat{\gamma}_h}\Gamma_h.$$

If we define

$$\boldsymbol{b}_h = \Psi_h^{-1}\mathbf{1}$$
$$\overline{\boldsymbol{b}}_h = \Psi_h^{-1}\boldsymbol{e}_h,$$

then

$$\boldsymbol{\omega}_h = \frac{1}{\widehat{\gamma}_h}\frac{\alpha-i}{\sigma^2}\boldsymbol{b}_h - \beta\left(\frac{\gamma_h}{\widehat{\gamma}_h}-1\right)(\boldsymbol{b}_h + \epsilon\overline{\boldsymbol{b}}_h).$$

Also

$$\begin{aligned}
\boldsymbol{\nu}_{h,t} &= \boldsymbol{\nu}_h \\
&= -a_h\gamma_h\sigma^2\Gamma_h\boldsymbol{\omega}_h \\
&= -a_h\gamma_h\sigma^2\Gamma_h\Psi_h^{-1}\left[\frac{1}{\widehat{\gamma}_h}\frac{\alpha-i}{\sigma^2}\mathbf{1} - \beta\left(\frac{\gamma_h}{\widehat{\gamma}_h}-1\right)(\mathbf{1}+\epsilon\boldsymbol{e}_h)\right] \\
&= -\gamma_h a_h\sigma^2\left(\frac{\gamma_h}{\widehat{\gamma}_h}a_h I + \Omega\Gamma_h^{-1}\right)^{-1}\left[\frac{1}{\widehat{\gamma}_h}\frac{\alpha-i}{\sigma^2}\mathbf{1} - \beta\left(\frac{\gamma_h}{\widehat{\gamma}_h}-1\right)(\mathbf{1}+\epsilon\boldsymbol{e}_h)\right] \\
&= -\sigma^2\left(\frac{1}{\widehat{\gamma}_h}I + \frac{1}{\gamma_h a_h}\Omega\Gamma_h^{-1}\right)^{-1}\left[\frac{1}{\widehat{\gamma}_h}\frac{\alpha-i}{\sigma^2}\mathbf{1} - \beta\left(\frac{\gamma_h}{\widehat{\gamma}_h}-1\right)(\mathbf{1}+\epsilon\boldsymbol{e}_h)\right] \\
&= -\widehat{\gamma}_h\sigma^2\left(I + \frac{\widehat{\gamma}_h}{\gamma_h a_h}\Omega\Gamma_h^{-1}\right)^{-1}\left[\frac{1}{\widehat{\gamma}_h}\frac{\alpha-i}{\sigma^2}\mathbf{1} - \beta\left(\frac{\gamma_h}{\widehat{\gamma}_h}-1\right)(\mathbf{1}+\epsilon\boldsymbol{e}_h)\right] \\
&= -\left(I + \frac{\widehat{\gamma}_h}{\gamma_h a_h}\Omega\Gamma_h^{-1}\right)^{-1}\left[(\alpha-i)\mathbf{1} - \rho_{YK}\sigma_Y\sigma\left(\gamma_h - \widehat{\gamma}_h\right)(\mathbf{1}+\epsilon\boldsymbol{e}_h)\right].
\end{aligned}$$

Hence,

$$a_h\widehat{c}_h = \psi_h k_{1h,t} + (1-\psi_h)a_h\left\{k_{2h,t} + \frac{1}{\widehat{W}_{h,t}} + \frac{1}{2}\widehat{\gamma}_h(\sigma^2\pi_{h,t}^2\boldsymbol{x}_{h,t}^\top\Psi_h\boldsymbol{x}_{h,t} - \sigma_Y^2)\right\}.$$

The steady-state value of the wealth-labor income ratio, $\widehat{W}_h^*$, is defined by

$$E_t^{\mathbb{Q}_Y^{\nu_h}}\left[\frac{d\widehat{W}_{h,t}}{\widehat{W}_{h,t}}\right]\Bigg|_{\widehat{W}_{h,t}=\widehat{W}_h^*} = 0,$$

so we see that at the steady-state, where variables are denoted by an $*$, we have

$$\hat{c}_h^*(\widehat{W}_h^*, a_h) = k_{2h}^* + \pi_h(a_h)(\alpha + \boldsymbol{x}_h(a_h)^\top \boldsymbol{\nu}_h(a_h) - \gamma_h \rho_{YK}\sigma\sigma_Y(1 + x_{hh}(a_h)) - i) + \frac{1}{\widehat{W}_h^*},$$

which using the expression for the optimal portfolio vector in (B2), can be rewritten as

(B3)
$$\hat{c}_h^*(\widehat{W}_h^*, a_h) = k_{2h}^* + \frac{1}{\widehat{W}_h^*} + \widehat{\gamma}_h(a_h)\left[\sigma^2\pi_h(a)^2\boldsymbol{x}_h(a_h)^\top\Omega\boldsymbol{x}_h(a_h) - \rho_{YK}\sigma_Y\sigma\pi_h(a_h)(1 + \epsilon x_{hh}(a_h))\right]$$

Note also that

(B4)
$$\hat{c}_h^*(\widehat{W}_h^*, a_h) = \frac{\widehat{C}_h^*}{\widehat{W}_h^*} = \delta_h^{\psi_h}\kappa_h^{1-\psi_h}a_h^{-\psi_h}(W_h^*(a_h))^{(1-a_h)(\psi_h-1)}$$
$$k_{2h}^*(\widehat{W}_h^*) = i + \gamma_h\sigma_Y^2 - (\tau_0 + \tau_1 \ln \widehat{W}_h^*)$$

Defining

$$\widehat{z}_{h,t} = \ln\left(\frac{\widehat{W}_{h,t}}{\widehat{W}_h^*}\right),$$

we see that at the steady state $\widehat{z}_{h,t} = 0$. At the optimum, we have

(B5)
$$a_h\hat{c}_{h,t}e^{(\psi_h-1)(1-a_h)\hat{z}_{h,t}} = \psi_h k_{1h,t} + (1-\psi_h)a_h\left[k_{2h,t} + \frac{1}{\widehat{W}_{h,t}} + \frac{1}{2}\widehat{\gamma}_{h,t}(\sigma^2\pi_{h,t}^2\boldsymbol{x}_{h,t}^\top\Psi_{h,t}\boldsymbol{x}_{h,t} - \sigma_Y^2)\right].$$

If we expand (B5) around $\widehat{z}_{h,t} = 0$, we obtain

$$a_h\hat{c}_h^*(\widehat{W}_h^*, a_h)[1 + (1-a_h)(\psi_h - 1)\widehat{z}_{h,t}]$$

$$= \psi_h k_{1h,t} + (1-\psi_h)a_h\left[k_{2h,t} + (W_h^*)^{-1}(1 - \widehat{z}_{h,t}) + \frac{1}{2}\widehat{\gamma}_h(\pi_h^2\sigma^2\boldsymbol{x}_h(a_h)^\top\Psi_h\boldsymbol{x}_h(a_h) - \sigma_Y^2)\right] + O(\widehat{z}_h^2).$$

By comparing coefficients of $\widehat{z}_h$, we obtain

$$a_h\hat{c}_h^*(\widehat{W}_h^*, a_h) = \psi_h k_{1h}^*(W_h^*)$$
(B6)
$$+ (1-\psi_h)a_h\left[k_{2h}^*(W_h^*) + (W_h^*)^{-1} + \frac{1}{2}\widehat{\gamma}_h(\pi_h^2\sigma^2\boldsymbol{x}_h(a_h)^\top\Psi_h\boldsymbol{x}_h(a_h) - \sigma_Y^2)\right],$$

(B7)
$$-(1-a_h)a_h\hat{c}_h^*(\widehat{W}_h^*, a_h) = \tau_1(1-a_h) - a_h(W_h^*)^{-1}$$

Rearranging (B7), we have

$$\hat{c}_h^*(\widehat{W}_h^*, a_h) = \frac{-\tau_1}{a_h} + \frac{1}{1-a_h}(\widehat{W}_h^*)^{-1}. \tag{B8}$$

Using (B8), we rewrite (B3) and (B6) as

$$\begin{aligned}\frac{-\tau_1}{a_h} + \frac{1}{1-a_h}(W_h^*)^{-1} &= \frac{1}{\widehat{W}^*} + k_2^*(\widehat{W}_h^*) \\ &+ \widehat{\gamma}_h(a_h)\left[\sigma^2\pi_h(a)^2\boldsymbol{x}_h(a_h)^\top\Omega\boldsymbol{x}_h(a_h) - \rho_{YK}\sigma_Y\sigma\pi_h(a_h)(1+\epsilon x_{hh}(a_h))\right]\end{aligned} \tag{B9}$$

$$\begin{aligned}-\tau_1 + \frac{a_h}{1-a_h}(\widehat{W}_h^*)^{-1} &= \psi_h k_{1h}^*(\widehat{W}_h^*) \\ &+ (1-\psi_h)a_h\left[k_{2h}^*(\widehat{W}_h^*) + (\widehat{W}_h^*)^{-1} + \frac{1}{2}\widehat{\gamma}_h(a)(\pi_h^2\sigma^2\boldsymbol{x}_h(a_h)^\top\Psi_h\boldsymbol{x}_h(a_h) - \sigma_Y^2)\right]\end{aligned} \tag{B10}$$

To summarize, we can find an approximate loglinear solution by solving (B9) and (B10) numerically to obtain a and W^*. We can then use (B8) to obtain $\widehat{c}^*$, the steady-state consumption-wealth ratio. We can rearrange (B4) to obtain

$$\kappa_h = \left[\left(\frac{\delta_h}{a_h}\right)^{\psi_h}\frac{1}{\widehat{c}_h^*}\right]^{\frac{1}{\psi_h - 1}}(\widehat{W}_h^*)^{1-a_h}.$$

It follows that

$$\begin{aligned}\frac{U_h^*}{W_h^*} &= \frac{\widehat{U}_h^*}{\widehat{W}_h^*} \\ &= \kappa_h(\widehat{W}_h^*)^{1-a_h} \\ &= \left[\left(\frac{\delta_h}{a_h}\right)^{\psi_h}\frac{1}{\widehat{c}_h^*}\right]^{\frac{1}{\psi_h-1}}.\end{aligned}$$

■

Observe that $\widehat{W}_h^*$ is a stochastic steady state level of household wealth, which accounts for the long term pricing of risk as in Hansen and Scheinkman (2009), because the expected rate of change of wealth is zero at $\widehat{W}_{h,t} = \widehat{W}_h^*$ under the probability measure $\mathbb{Q}_Y^{\boldsymbol{\nu}_h}$, as opposed to under the reference probability measure $\mathbb{P}$. That is, it is the probability measure $\mathbb{Q}_Y^{\boldsymbol{\nu}_h}$ that adjusts for the long term pricing of risk.

Proposition B.2.4 *The utility of a household making biased consumption-portfolio choices is given by*

$$u^*_{e,h} = \frac{\widehat{U}^*_{e,h}}{\widehat{W}^*_{e,h}} = \left(\frac{\delta_h \psi_h}{\psi_h k^*_{e,1h} + (1-\psi_h) a_{e,h} \left(LQ^*_{e,h} - \widehat{c}^*_{e,h} \right)} \right)^{\frac{1}{1-\frac{1}{\psi_h}}} \widehat{c}^*_{e,h},$$

where

$$\begin{aligned} LQ^*_{e,h} &= k^*_{e,2h} \\ &+ \pi_{d,h}(\alpha - \gamma_h \rho_{YK} \sigma \sigma_Y (1 + \epsilon x_{d,hh}) - i) \\ &+ (\widehat{W}^*_{e,h})^{-1} - \frac{1}{2} \left[a_{e,h} \gamma_h + (1 - a_{e,h}) \right] (\sigma^2 \pi^2_{d,h} \boldsymbol{x}^\top_{d,h} \Omega \boldsymbol{x}_{d,h} - 2\rho_{YK} \sigma_Y \sigma \pi_{d,h} (1 + \epsilon x_{d,hh}) + \sigma^2_Y). \end{aligned}$$

The constant $a_{e,h}$ is the solution of

$$\begin{aligned} &\delta_h \psi_h b_{e,h} \\ &= \psi_h k^*_{e,1h} \\ &+ (1-\psi_h) a_{e,h} \Big\{ k^*_{e,2h} + \pi_{d,h}(\alpha - \gamma_h \rho_{YK} \sigma \sigma_Y (1 + \epsilon x_{d,hh}) - i) - \widehat{c}^*_{e,h} + (\widehat{W}^*_{e,h})^{-1} \\ &- \frac{1}{2} \left[a_{e,h} \gamma_h + (1 - a_{e,h}) \right] (\sigma^2 \pi^2_{d,h} \boldsymbol{x}^\top_{d,h} \Omega \boldsymbol{x}_{d,h} - 2\rho_{YK} \sigma_Y \sigma \pi_{d,h} (1 + \epsilon x_{d,hh}) + \sigma^2_Y) \Big\}, \end{aligned}$$

where $b_{e,h}$. The constant $a_{d,h}$ together with $\pi_{d,h}$, $\boldsymbol{x}_{d,h}$ define the biased decisions of the household.

The asterisk $*$ indicates that all values are computed at the steady state $\widehat{W}_{h,t} = \widehat{W}_{h,t}$ defined by

$$E^{\mathbb{P}_Y}_t \left[\frac{d\widehat{W}_{e,h,t}}{\widehat{W}_{e,h,t}} \right] \Bigg|_{\widehat{W}_{e,h,t} = \widehat{W}^*_{e,h}} = 0,$$

where for an event A realized at date T

$$\mathbb{P}_Y(A|\mathcal{F}_t) = E^{\mathbb{P}}_t \left[\frac{M_{Y,h,T}}{M_{Y,h,t}} 1_A \right],$$

and

$$M_{Y,h,t} = e^{-\frac{1}{2}(1-\gamma_h)^2 \sigma^2_Y t + (1-\gamma_h)\sigma_Y Z_{Y,h,t}}.$$

Proof: We start by computing welfare for a household without familiarity biases, using the consumption and portfolio policy for the household with familiarity biases. The consumption policy is of the form

$$\widehat{c}_{e,h,t} = \frac{\widehat{C}_{e,h,t}}{\widehat{W}_{e,h,t}} = \delta_h^{\psi_h} \kappa_{d,h}^{1-\psi_h} a_{d,h}^{-\psi_h} \widehat{W}_{e,h,t}^{(1-a_{d,h})(\psi_h-1)},$$

where $\kappa_{d,h}$ and $a_{d,h}$ have subscripts d to make it clear that they pin down the approximate optimal controls for a household with familiarity biases, but not for a household without such biases. Furthermore, $\widehat{c}_{e,h,t}$, $\widehat{C}_{e,h,t}$, and $\widehat{W}_{e,h,t}$ contain the subscript e to make it clear that they apply to a household without familiarity biases. Hence,

$$\widehat{c}_{e,h,t} = \frac{\widehat{C}_{e,h,t}}{\widehat{W}_{e,h,t}} = \delta_h^{\psi_h} \kappa_{d,h}^{1-\psi_h} a_{d,h}^{-\psi_h} (\widehat{W}_{e,h}^*)^{(1-a_{d,h})(\psi_h-1)} e^{(1-a_{d,h})(\psi_h-1)\widehat{z}_{e,h,t}},$$

where $\widehat{W}_{e,h}^*$ is the steady-state value of the wealth-labor income ratio for a household without familiarity biases, but whose controls are taken from the optimal problem for a household with familiarity biases, and also,

$$\widehat{z}_{e,h,t} = \ln \frac{\widehat{W}_{e,h,t}}{\widehat{W}_{e,h}^*}.$$

Consequently, when $\widehat{W}_{e,h,t} = \widehat{W}_{e,h}^*$, we have $\widehat{z}_{e,h,t} = 0$. Expanding around $\widehat{z}_{e,h,t} = 0$, we see that

$$\widehat{c}_{e,h,t} = \frac{\widehat{C}_{e,h,t}}{\widehat{W}_{e,h,t}} = \widehat{c}_{e,h}^* e^{(1-a_{d,h})(\psi_h-1)\widehat{z}_{e,h,t}},$$

where

$$\widehat{c}_{e,h}^* = \delta_h^{\psi_h} \kappa_{d,h}^{1-\psi_h} a_{d,h}^{-\psi_h} (\widehat{W}_{e,h}^*)^{(1-a_{d,h})(\psi_h-1)}. \tag{B11}$$

We assume the utility-labor income ratio is given by

$$\widehat{U}_{e,h,t} = \kappa_{e,h} \widehat{W}_{e,h,t}^{a_{e,h}},$$

where $\kappa_{e,h}$ and $a_{e,h}$ are endogenous constants we need to determine. Therefore

$$\begin{aligned}
\widehat{u}_{e,h,t} &= \frac{\widehat{U}_{e,h,t}}{\widehat{W}_{e,h,t}} \\
&= \kappa_{e,h} \widehat{W}_{e,h,t}^{a_{e,h}-1} \\
&= \kappa_{e,h} (\widehat{W}_{e,h}^*)^{a_{e,h}-1} e^{(a_{e,h}-1)\widehat{z}_{e,h,t}}
\end{aligned}$$

$$= \widehat{u}^*_{e,h} e^{(a_{e,h}-1)\widehat{z}_{e,h,t}},$$

where

$$\widehat{u}^*_{e,h} = \kappa_{e,h} (\widehat{W}^*_{e,h})^{a_{e,h}-1}.$$

Hence

$$\frac{\widehat{C}_{e,h,t}}{\widehat{U}_{e,h,t}} = \frac{\widehat{c}^*_{e,h}}{\widehat{u}^*_{e,h}} e^{[(1-a_{d,h})(\psi_h-1))-(a_{e,h}-1)]\widehat{z}_{e,h,t}}.$$

The evolution of the wealth-labor income ratio is determined by the consumption-portfolio policies, which are biased. Consequently, the steady-state shall be as before, that is, given by

$$\hat{c}^*_{d,h} = k^*_{2h} + \pi_{d,h}(\alpha + \boldsymbol{x}^\top_{d,h}\boldsymbol{\nu}_h - \gamma_h \rho_{YK}\sigma\sigma_Y(1 + x_{d,hh}) - i) + \frac{1}{\widehat{W}^*_h},$$

The steady-state value of the wealth-labor income ratio, $\widehat{W}^*_{e,h}$, is defined by

$$E^{\mathbb{P}_Y}_t \left[\frac{d\widehat{W}_{e,h,t}}{\widehat{W}_{e,h,t}} \right] \Bigg|_{\widehat{W}_{e,h,t}=\widehat{W}^*_{e,h}} = 0,$$

where for an event A realized at date T

$$\mathbb{P}_Y(A|\mathcal{F}_t) = E^{\mathbb{P}}_t \left[\frac{M_{Y,h,T}}{M_{Y,h,t}} 1_A \right],$$

and

$$M_{Y,h,t} = e^{-\frac{1}{2}(1-\gamma_h)^2\sigma^2_Y t + (1-\gamma_h)\sigma_Y Z_{Y,h,t}}.$$

So we see that at the steady-state, we have

$$\hat{c}^*_{e,h}(\widehat{W}^*_{e,h}) = k^*_{e,2h} + \pi_{d,h}(\alpha - \gamma_h \rho_{YK}\sigma\sigma_Y(1 + \epsilon x_{d,hh}) - i) + \frac{1}{\widehat{W}^*_{e,h}}.$$

Therefore, using (B11), we obtain

$$\delta^{\psi_h}_h \kappa^{1-\psi_h}_{d,h} a^{-\psi_h}_{d,h} (\widehat{W}^*_{e,h})^{(1-a_{d,h})(\psi_h-1)} = k^*_{e,2h} + \pi_{d,h}(\alpha - \gamma_h \rho_{YK}\sigma\sigma_Y(1 + \epsilon x_{d,hh}) - i) + \frac{1}{\widehat{W}^*_{e,h}}.$$

We can solve the above equation numerically to obtain $\widehat{W}^*_{e,h}$ in terms of exogenous constants. We can then use (B11) to obtain $\hat{c}^*_{e,h}$ in terms of exogenous constants. From (B1), we can see that

$$\frac{\delta_h \left(\frac{\widehat{c}^*_{e,h}}{\widehat{u}^*_{e,h}} \right)^{1-\frac{1}{\psi_h}} e^{\left(1-\frac{1}{\psi_h}\right)[(1-a_{d,h})(\psi_h-1))-(a_{e,h}-1)]\widehat{z}_{e,h,t}} - k_{1h,t}}{1-\frac{1}{\psi_h}}$$

$$+ a_{e,h}\left[k_{2h,t} + \pi_{d,h,t}(\alpha - \gamma_h \rho_{YK}\sigma\sigma_Y(1+\epsilon x_{d,hh,t}) - i) - \widehat{c}^*_{e,h} e^{(1-a_{d,h})(\psi_h - 1)\widehat{z}_{e,h,t}} + (\widehat{W}^*_{e,h})^{-1} e^{-\widehat{z}_{e,h,t}}\right]$$

$$-\frac{1}{2} a_{e,h}\left[a_{e,h}\gamma_h + (1-a_{e,h})\right](\sigma^2 \pi^2_{d,h,t} \boldsymbol{x}^\top_{d,h,t} \Omega \boldsymbol{x}_{d,h,t} - 2\rho_{YK}\sigma_Y \sigma \pi_{d,h,t}(1+\epsilon x_{d,hh,t}) + \sigma^2_Y),$$

which we can rewrite as

$$\delta_h \psi_h \left(\frac{\widehat{c}^*_{e,h}}{\widehat{u}^*_{e,h}}\right)^{1-\frac{1}{\psi_h}} e^{\left(1-\frac{1}{\psi_h}\right)[(1-a_{d,h})(\psi_h - 1)) - (a_{e,h}-1)]\widehat{z}_{e,h,t}}$$

$$= \psi_h k_{1h,t}$$

$$+ (1-\psi_h) a_{e,h}\Big\{k_{2h,t} + \pi_{d,h,t}(\alpha - \gamma_h \rho_{YK}\sigma\sigma_Y(1+\epsilon x_{d,hh,t}) - i) - \widehat{c}^*_{e,h} e^{(1-a_{d,h})(\psi_h - 1)\widehat{z}_{e,h,t}} + (\widehat{W}^*_{e,h})^{-1} e^{-\widehat{z}_{e,h,t}}$$

$$-\frac{1}{2}\left[a_{e,h}\gamma_h + (1-a_{e,h})\right](\sigma^2 \pi^2_{d,h,t} \boldsymbol{x}^\top_{d,h,t} \Omega \boldsymbol{x}_{d,h,t} - 2\rho_{YK}\sigma_Y \sigma \pi_{d,h,t}(1+\epsilon x_{d,hh,t}) + \sigma^2_Y)\Big\}$$

For ease of notation, define

$$b_{e,h} = \left(\frac{\widehat{c}^*_{e,h}}{\widehat{u}^*_{e,h}}\right)^{1-\frac{1}{\psi_h}},$$

and so

$$\delta_h \psi_h b_{e,h} e^{\left(1-\frac{1}{\psi_h}\right)[(1-a_{d,h})(\psi_h - 1)) - (a_{e,h}-1)]\widehat{z}_{e,h,t}}$$

$$= \psi_h k_{1h,t}$$

$$+ (1-\psi_h) a_{e,h}\Big\{k_{2h,t} + \pi_{d,h,t}(\alpha - \gamma_h \rho_{YK}\sigma\sigma_Y(1+\epsilon x_{d,hh,t}) - i) - \widehat{c}^*_{e,h} e^{(1-a_{d,h})(\psi_h - 1)\widehat{z}_{e,h,t}} + (\widehat{W}^*_{e,h})^{-1} e^{-\widehat{z}_{e,h,t}}$$

$$-\frac{1}{2}\left[a_{e,h}\gamma_h + (1-a_{e,h})\right](\sigma^2 \pi^2_{d,h,t} \boldsymbol{x}^\top_{d,h,t} \Omega \boldsymbol{x}_{d,h,t} - 2\rho_{YK}\sigma_Y \sigma \pi_{d,h,t}(1+\epsilon x_{d,hh,t}) + \sigma^2_Y)\Big\}.$$

Expanding the above equation around $\widehat{z}_{e,h,t} = 0$ up to first order and comparing coefficients gives

(B12) $\delta_h \psi_h b_{e,h}$

$$
\begin{aligned}
&= \psi_h k^*_{e,1h} \\
&+ (1-\psi_h) a_{e,h} \left\{ k^*_{e,2h} + \pi_{d,h}(\alpha - \gamma_h \rho_{YK} \sigma \sigma_Y (1+\epsilon x_{d,hh}) - i) - \widehat{c}^*_{e,h} + (\widehat{W}^*_{e,h})^{-1} \right. \\
&\left. -\frac{1}{2}\left[a_{e,h}\gamma_h + (1-a_{e,h})\right](\sigma^2 \pi^2_{d,h} \boldsymbol{x}^\top_{d,h} \Omega \boldsymbol{x}_{d,h} - 2\rho_{YK}\sigma_Y \sigma \pi_{d,h}(1+\epsilon x_{d,hh}) + \sigma^2_Y) \right\},
\end{aligned}
$$

and

$$
\begin{aligned}
&\delta_h \psi_h b_{e,h} \left(1 - \frac{1}{\psi_h}\right)\left[(1-a_{d,h})(\psi_h - 1)\right) - (a_{e,h}-1)] \\
&= \psi_h \left(\frac{1}{\psi_h} - 1\right)\tau_1 \\
&+ (1-\psi_h) a_{e,h} \left\{ -\tau_1 - \widehat{c}^*_{e,h}(1-a_{d,h})(\psi_h - 1) - (\widehat{W}^*_{e,h})^{-1} \right\},
\end{aligned} \tag{B13}
$$

where

$$
\begin{aligned}
k^*_{e,1,h} &= \delta_h + \left(\frac{1}{\psi_h} - 1\right)(\tau_0 + \tau_1 \ln \widehat{W}^*_{e,h}) - \frac{1}{2}\left(1 + \frac{1}{\psi_h}\right)\gamma_h \sigma^2_Y + \gamma_h \sigma^2_Y \\
k^*_{e,2,h} &= i + \gamma_h \sigma^2_Y - (\tau_0 + \tau_1 \ln \widehat{W}^*_{e,h}).
\end{aligned}
$$

We can make $b_{e,h}$ the subject of (B13) as follows:

$$
\begin{aligned}
&\delta_h b_{e,h} (\psi_h - 1)\left[(1-a_{d,h})(\psi_h - 1)\right) - (a_{e,h}-1)] \\
&= (1-\psi_h)\tau_1 \\
&+ (1-\psi_h) a_{e,h} \left\{ -\tau_1 - \widehat{c}^*_{e,h}(1-a_{d,h})(\psi_h - 1) - (\widehat{W}^*_{e,h})^{-1} \right\} \\
&\delta_h b_{e,h}[(1-a_{d,h})(\psi_h - 1)) - (a_{e,h}-1)] \\
&= -\tau_1 - a_{e,h} \left\{ -\tau_1 - \widehat{c}^*_{e,h}(1-a_{d,h})(\psi_h - 1) - (\widehat{W}^*_{e,h})^{-1} \right\}
\end{aligned}
$$

$$
b_{e,h} = \frac{-\tau_1 - a_{e,h}\left\{ -\tau_1 - \widehat{c}^*_{e,h}(1-a_{d,h})(\psi_h - 1) - (\widehat{W}^*_{e,h})^{-1} \right\}}{\delta_h[(1-a_{d,h})(\psi_h - 1) - (a_{e,h}-1)]}. \tag{B14}
$$

Substituting (B14) into (B12) gives a nonlinear algebraic equation for a_e, which can be solved numerically.

It follows from (B12) that

$$
u^*_{e,h} = \frac{\widehat{U}^*_{e,h}}{\widehat{W}^*_{e,h}} = \left(\frac{\delta_h \psi_h}{\psi_h k^*_{e,1h} + (1-\psi_h) a_{e,h} \left(LQ^*_{e,h} - \widehat{c}^*_{e,h} \right)} \right)^{\frac{1}{1-\frac{1}{\psi_h}}} \widehat{c}^*_{e,h},
$$

where

$LQ^*_{e,h} = k^*_{e,2h}$

$$+ \pi_{d,h}(\alpha - \gamma_h \rho_{YK} \sigma \sigma_Y (1 + \epsilon x_{d,hh}) - i)$$
$$+ (\widehat{W}^*_{e,h})^{-1} - \frac{1}{2} \left[a_{e,h} \gamma_h + (1 - a_{e,h}) \right] (\sigma^2 \pi^2_{d,h} \boldsymbol{x}^\top_{d,h} \Omega \boldsymbol{x}_{d,h} - 2\rho_{YK} \sigma_Y \sigma \pi_{d,h} (1 + \epsilon x_{d,hh}) + \sigma^2_Y).$$

■

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