Experimental Quantum Fast Hitting on Hexagonal Graphs

Hao Tang[1,2,3], Carlo Di Franco[4,5], Zi-Yu Shi[1,3], Tian-Shen He[1], Zhen Feng[1,3], Jun Gao[1,3], Ke Sun[1], Zhan-Ming Li[1,3], Zhi-Qiang Jiao[1,3], Tian-Yu Wang[1], M. S. Kim[6,7] & Xian-Min Jin[1,2,3]

[1]*State Key Laboratory of Advanced Optical Communication Systems and Networks, School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China*

[2]*Institute of Natural Sciences, Shanghai Jiao Tong University, Shanghai 200240, China*

[3]*Synergetic Innovation Centre of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*

[4]*School of Physical and Mathematical Sciences, Nanyang Technological University, 637371, Singapore*

[5]*Complexity Institute, Nanyang Technological University, 637723, Singapore*

[6]*QOLS, Blackett Laboratory, Imperial College London, SW7 2AZ, UK*

[7]*Korea Institute of Advanced Study, Dongdaemoon-Gu, Seoul, 02455, South Korea*

Quantum walks are powerful kernels in quantum computing protocols that possess strong capabilities in speeding up various simulation and optimisation tasks. One striking example is given by quantum walkers evolving on glued trees[1] for their faster hitting performances than in the case of classical random walks. However, its experimental implementation is challenging as it involves highly complex arrangements of exponentially increasing number of nodes. Here we propose an alternative structure with a polynomially increasing number of nodes. We successfully map such graphs on quantum photonic chips using femtosecond laser

direct writing techniques in a geometrically scalable fashion. We experimentally demonstrate quantum fast hitting by implementing two-dimensional quantum walks on these graphs with up to 160 nodes and a depth of 8 layers, achieving a linear relationship between the optimal hitting time and the network depth. Our results open up a scalable way towards quantum speed-up in complex problems classically intractable.

Introduction

Adapting well-known classical mathematical models in a way to include quantum mechanical laws has shown the emergence of new interesting behaviours. In some cases, the modified protocols have revealed an advantage with respect to the original ones in solving specific problems. This has clearly triggered the interest of the scientific community in the quest for a better understanding and exploitation of these new tools[2]. A striking example is given by quantum walks, the adaptation of the classical random walk to the world of quantum mechanics[3]. Quantum walks have already found applications in several scenarios, including spatial search problems[4,5], the element distinctness problem[6], testing matrix identities[7], evaluating Boolean formulas[8], judging graph isomorphism[9,10], which all theoretically promise quantum speed-up and may inspire the breakthrough in real-life applications.

One feature of quantum walks on complex graphs that is key in quantum algorithms is their ability to propagate from a node to a distant one in an efficient way. This is often denoted as fast hitting. In particular, fast hitting on a structure known as glued tree is extremely charming due to its clear speed-up over the performance by classical random walks (*i.e.*, a particle following the laws of classical mechanics)[1,11]. A glued tree is obtained by connecting the 'final leaves' of two binary tree graphs[12] of the same depth, as shown in Fig. 1a. The process assumes a particle starting in the left-most vertex (called the Entry site), evolving through the graph, and finally hitting the right-most vertex (called the Exit site). It has been previously shown that, while for the normal glued tree, the classical random walk may have some fast algorithms, in the scenario where the end of each branch in one tree is randomly connected to two branches of the other tree, any algorithm exploiting a classical walker would require on average a time scaling exponentially with the graph depth to reach the Exit. On the other hand, a quantum walker will always require a time that scales only linearly[1,13,14]. Due to the close relation between binary trees and decision trees in computer science, this could generate enormous benefits if properly incorporated into real optimisation problems.

Unfortunately, an implementation of quantum walks on this class of graphs is not feasible with the current technology. The fact that the number of vertices grows exponentially with the size of the graph itself is one of the main hurdles for their realisation. However, even showing the speed-up by a quantum walker over a classical walker on a simpler graph (where, for instance, the number of vertices grows quadratically) is already of great interest: this would be a pioneering experimental demonstration of the quantum advantage in algorithms based on quantum walks on tree structures. So far, one-dimensional quantum walks have been successfully realised in various physical systems[15–23], and two-dimensional quantum walks have been demonstrated with time-polarisation degrees of freedom[24,25] or genuine spatial two-dimensional structures[26]. However, an experimental demonstration of the hitting time advantage given by quantum walks on complex structures has never been shown yet.

Here we present a modified version of the glued tree structure that can be mapped into a photonic chip and keep excellent extendibility for larger complexity. We study the hitting efficiency against the evolution time when increasing the network layer depth, representing the network complexity, going from 2 to 8. We demonstrate that the time for optimal hitting increases linearly with the layer depth, giving a quadratic speed-up over the hitting performance by classical random walks.

Main

As is shown in Fig. 1a and Fig. 1b, the hexagonal structure proposed here resembles the glued binary tree as they are both obtained by gluing two tree-like structures. In our mapping into a three-dimensional waveguide scheme [shown in Fig. 1c], the cross section of the three-dimensional array corresponds to the desired graph, with each waveguide representing a node, while the longitudinal propagation direction corresponds to the evolution time.

When photons are injected into the Entry site, they propagate along this waveguide, and meanwhile evanescently couple to other waveguides[26,27]. As the coupling coefficient decays exponentially when the waveguide spacing increases[28], only the coupling between the most adjacent waveguides are considered here, representing a connected path in the graph [*e.g.*, Site A-B in Fig. 1a, Site D-E in Fig. 1b)]. Waveguides further apart can be considered disconnected due to the marginal coupling coefficient (e.g. Site A-C, Site D-F). In the hexagonal structure, the layer depth corresponds to the number of hexagons in the central column, as shown in Fig. 1b. When the layer depth increases, having an exponentially increasing number of waveguides disconnected in the photonic chips for the glued binary trees is clearly not feasible. On the other hand, we use the hexagonal structure that grows in a regular way and is possible to map on a photonic chip.

For a quantum walk that evolves along the waveguides, the propagation length z is proportional to the propagation time t according to $z = vt$, where v is the speed of light in the waveguide, and hence all the terms that are a function of t would use z instead in this paper for simplicity. The initial wavefunction $|\Psi(0)\rangle$ evolves according to

$$|\Psi(z)\rangle = e^{-iHz} |\Psi(0)\rangle , \tag{1}$$

where H is the Hamiltonian that contains the information on the couplings within the photonic network. The evolved wavefunction can be obtained by matrix exponential methods when H is known[26,29]. For classical random walks, we use the versatile quantum stochastic walk model[30] and set it to the purely classical domain without the quantum term. We obtain the continuous-time dynamics for classical random walks that can be compared to our continuous-time quantum walks. More theoretical details can be found in the Supplementary Note 1.

It is worth noting that quantum walks intrinsically yield non-stationary solutions[31], which means that there exists an optimal hitting scenario at a certain evolution length. We would take notes of the optimal hitting efficiency and its corresponding optimal evolution length z_o.

In light of theoretical predictions, we use femtosecond laser direct writing techniques[28,32–34] to fabricate 7 sets of hexagonal graphs with their layer depth varying from 2 to 8. For each layer depth i, we prepare 9 samples with their evolution length varying from z_{oi} - 4 mm to z_{oi} + 4 mm with intervals of 1 mm, where z_{oi} is the calculated optimal length for this structure. We characterise all samples and identify z_{oi} for each set of graphs by injecting a vertically polarised 780nm laser beam into the Entry site and capturing the evolution pattern with a CCD camera (see Supplementary Note 2-3, and Supplementary Fig. 1-8). With heralded single photons, we then directly observe the evolution pattern of the characterised z_{oi} (see Methods for the details of single-photon generation and measurement).

From the photographed patterns for 2-layered hexagonal graphs in Fig. 2a-e, the brightest spot at the Exit site occurs in Fig. 2c, corresponding to the optimal hitting efficiency among these figures. The hitting efficiency against z from the experiments agrees very well with the theoretical results, as shown in Fig. 2f, in terms of both the value of optimal hitting efficiency (of around 90%) and the evolution length at which the optimal hitting occurs (at around 25 mm). In order to show the advantage given by quantum walks, a comparison is made by adding in Fig. 2f the theoretical result of classical random walks on the same structure: the classical hitting efficiency of only 6.25% is outperformed by more than one order of magnitude. The low efficiency for the classical hitting is a result of the diffusive nature of the classical random walk, and the optimal hitting efficiency is in fact the inverse of the total number of sites. On the other hand, the impressive advantage in the quantum case for the hitting task in networks with many binary paths comes from the interference governing the quantum evolution of the walker. We further polish the chip to seek the even higher hitting efficiency suggested by the fitting result. The measured optimal evolution pattern at z of 25.2 mm with both laser beam and heralded single photons are shown in Fig. 2g-h. Our results confirmed that for one walker single photons and laser beam produce very consistent results. Furthermore, the implementation with genuine single photons represents a substantial step forward to a faithful realisation of quantum fast hitting.

Extending the size of the hexagonal structure to a layer depth up to 8, we are able to show the quantum advantage in fast hitting for graphs of higher complexity. The evolution patterns with an optimal hitting efficiency out of the nine samples for each structure are measured using the heralded single photons, see Fig. 3a-f. We can see that the Exit site attracts more light than the others, and at least 50% of the sites have barely any probability, a situation very different from the even distribution in the classical hitting.

We then analyse the performance of quantum and classical hitting on the hexagonal structures as a function of the layer depth. The optimal quantum hitting efficiency from both experiments and theory always remains more than one order of magnitude larger than the classical case, as shown in Fig. 4a. In this hexagonal structure, the total number of sites is $2n^2 + 4n$, where n is the layer depth. Hence, the classical optimal hitting efficiency scales as n^{-2}. Fig. 4b presents the evolution length at which the optimal hitting efficiency occurs. The panel shows a clear linear trend for the quantum scenario, while classical hitting requires a quadratically larger evolution length.

Discussion

In conclusion, we have demonstrated the quantum fast hitting on hexagonal structures in photonic chips and experimentally observed that the time for optimal quantum hitting increases linearly with the layer depth. In comparison, the classical scenario is characterised by a quadratic relation; our investigation is therefore a demonstration of the speed-up given by the interference that governs the evolution of the quantum walker, a key point in many tasks based on quantum walks. Overall there is a very good agreement between the experimental data and the theoretical predictions, for both the optimal efficiency and its corresponding evolution length. This work was made possible through the precise and versatile techniques of fabricating three-dimensional integrated photonic chips using femtosecond laser direct writing. Such capability paves the way for a broader and useful exploitation of quantum walks on complex graphs.

In the future, it would be interesting to investigate the role of defects, asymmetry and photon bunching effect[27] for quantum fast hitting. Experimental demonstration can also be extended to other interesting structures, *e.g.* the Sierpinski gaskets[35], to check other possible quantum advantages in fast hitting. Finally, as binary trees are closely related to decision trees in computer science, we may utilize the quantum speed-up to improve the performance for tasks such as optimisation, management and information searching.

Methods

Fabrication of the hexagonal graph chip. The three-dimensional layout of hexagonal structure is designed according to the characterised coupling coefficients and programed by inputting the (x,y) coordinates for each site and the evolution length z identically for all sites in each sample. We feed a 513-nm femtosecond laser (up converted from a pump laser of 10 W, 1026 nm, 290 fs pulse duration, 1 MHz repetition rate) into a spatial light modulator (SLM) to shape the laser pulse in the temporal and spatial domain. We then focus the pulse onto a borosilicate substrate with a 50X objective lens (numerical aperture of 0.55) at a constant velocity of 10 mm/s. The waveguide arrays are prepared in a hexagonal structure by mapping it into the cross-section of the chip and the designed evolution length determines the array length. Power and SLM compensation are used to improve the uniformity[34].

Heralded single-photon preparation and single-photon-level imaging. We use a frequency doubled 390-nm fs laser pump a 2-mm-thick BBO crystal to generate degenerate 780-nm photon pairs via type-II spontaneous parametric downconversion process in the beam-like scheme[36]. The photons are then filtered by a 3-nm band pass filter and guided to the photonic chip. We inject the vertically polarised photon into the Entry waveguide in the photonic chip, while the horizontally polarised photon is connected to a single photon detector that sets a trigger for heralding the horizontally polarised photons on an ICCD camera[37] with a time slot of 10 ns. Without the external trigger, the measured patterns would come from the thermal-state light rather than single-photons. The ICCD camera captures each evolution pattern with a certain evolution length, after accumulating in the 'external' mode for 1-1.5 hours.

Hitting efficiency acquisition. When collecting the data from experiments, we obtain the corresponding ASCII file, which is essentially a matrix of pixels. We create a 'mask' that contains the pixel coordinate of the circle centre and the radius in pixels for each waveguide, and sum up the light intensity for all the pixels within each circle using Matlab. The normalised proportion of light intensity for each circle represents the probability at the corresponding waveguide. The hitting efficiency is the proportion of light intensity at the Exit site.

1. Childs, A. M., Farhi, E. & Gutmann, S. An example of the difference between quantum and classical random walks. *Quantum Inf. Process.* **1**, 35–43 (2002).

2. Mohseni, M. *et al.* Commercialize quantum technologies in five years. *Nature* **543**, 171–174 (2017).

3. Childs, A. M. & van Dam, W. Quantum algorithms for algebraic problems. *Rev. Mod. Phys.* **82**, 1–52 (2008).

4. Szegedy, M. Quantum speed-up of markov chain based algorithms. *Proceedings of the 45th Annual IEEE Symposium on Foundations of Computer Science* 32–41 (IEEE, 2004).

5. Childs, A. M. & Goldstone, J. Spatial search by quantum walk. *Phys. Rev. A* **70**, 022314 (2004).

6. Aaronson, S. & Shi, Y. Quantum lower bounds for the collision and the element distinctness problems. *J. ACM* **51**, 595–605 (2004).

7. Buhrman, H. & Špalek, R. Quantum verification of matrix products. *Proceedings of the seventeenth annual ACM-SIAM symposium on Discrete algorithm* 880–889 (SIAM, 2006).

8. Farhi, E., Goldstone, J. & Gutmann, S. A quantum algorithm for the hamiltonian nand tree. *Theory Comput.* **4**, 169–190 (2008).

9. Douglas, B. L. & Wang, J. B. A classical approach to the graph isomorphism problem using quantum walks. *J. Phys. A: Math. Theor.* **41**, 075303 (2008).

10. Bruderer, M. & Plenio, M. B. Decoherence enhances performance of quantum walks applied to graph isomorphism testing. *Phys. Rev. A* **94**, 062317 (2016).

11. Childs, A. M. *et al.* Exponential algorithmic speedup by a quantum walk. *Proceedings of the thirty-fifth annual ACM symposium on Theory of computing* 59–68 (ACM, 2003).

12. Farhi, E. & Gutmann, S. Quantum computation and decision trees. *Phys. Rev. A* **58**, 915–928 (1998).

13. Douglas, B. L. & Wang, J. B. Efficient quantum circuit implementation of quantum walks. *Phys. Rev. A* **79**, 052335 (2009).

14. Carneiro, I. *et al.* Entanglement in coined quantum walks on regular graphs. *New J. Phys.* **7**, 156 (2005).

15. Schmitz, H. *et al.* Quantum walk of a trapped ion in phase space. *Phys. Rev. Lett.* **103**, 090504 (2009).

16. Karski, M. *et al.* Quantum walk in position space with single optically trapped atoms. *Science* **325**, 174–177 (2009).

17. Broome, M. *et al.* Discrete single-photon quantum walks with tunable decoherence. *Phys. Rev. Lett.* **104**, 153602 (2010).

18. Sansoni, L. *et al.* Two-particle bosonic-fermionic quantum walk via integrated photonics. *Phys. Rev. Lett.* **108**, 010502 (2012).

19. Cardano, F. *et al.* Quantum walks and wavepacket dynamics on a lattice with twisted photons. *Sci. Adv.* **1**, e1500087 (2015).

20. Du, J. *et al.* Experimental implementation of the quantum random-walk algorithm. *Phys. Rev. A* **67**, 042316 (2003).

21. Perets, H. B. *et al.* Realization of quantum walks with negligible decoherence in waveguide lattices. *Phys. Rev. Lett.* **100**, 170506 (2008).

22. Peruzzo, A. *et al.* Quantum walks of correlated photons. *Science* **329**, 1500–1503 (2010).

23. Preiss, P. M. *et al.* Strongly correlated quantum walks in optical lattices. *Science* **347**, 1229–1233 (2015).

24. Schreiber, A. *et al.* A 2d quantum walk simulation of two-particle dynamics. *Science* **336**, 55–58 (2012).

25. Jeong, Y. C., Di Franco, C., Lim, H. T., Kim, M. S. & Kim, Y. H. Experimental realization of a delayed-choice quantum walk. *Nat. Commun.* **4**, 2471 (2013).

26. Tang, H. *et al.* Experimental two-dimensional quantum walk on a photonic chip. *Sci. Adv.* **4**, eaat3174 (2018).

27. Gao, J. *et al.* Non-classical photon correlation in a two-dimensional photonic lattice. *Opt. Express* **24**, 12607–12616 (2016).

28. Szameit, A., Dreisow, F., Pertsch, T., Nolte, S. & Tünnermann, A. Control of directional evanescent coupling in fs laser written waveguides. *Opt. Express* **15**, 1579–1587 (2007).

29. Izaac, J. A. & Wang, J. B. pyctqw: A continuous-time quantum walk simulator on distributed memory computers. *Comput. Phys. Commun.* **186**, 81–92 (2015).

30. Whitfield, J. D., Rodríguez-Rosario, C. A. & Aspuru-Guzik, A. Quantum stochastic walks: A generalization of classical random walks and quantum walks. *Phys. Rev. A* **81**, 022323 (2010).

31. Sánchez-Burillo, E., Duch, J., Gómez-Gardenes, J. & Zueco, D. Quantum navigation and ranking in complex networks. *Sci. Rep.* **2**, 605 (2012).

32. Crespi, A. *et al.* Integrated multimode interferometers with arbitrary designs for photonic boson sampling. *Nat. Photon.* **7**, 545–549 (2013).

33. Chaboyer, Z., Meany, T., Helt, L. G., Withford, M. J. & Steel, M. J. Tunable quantum interference in a 3d integrated circuit. *Sci. Rep.* **5**, 9601 (2015).

34. Feng, Z. *et al.* Invisibility cloak printed on a photonic chip. *Sci. Rep.* **6**, 28527 (2016).

35. Darázs, Z., Anishchenko, A., Kiss, T., Blumen, A. & Mülken, O. Transport properties of continuous-time quantum walks on sierpinski fractals. *Phys. Rev. E* **90**, 032113 (2014).

36. Kim, Y. H. Quantum interference with beamlike type-ii spontaneous parametric down-conversion. *Phys. Rev. A* **68**, 013804 (2003).

37. Sun, K. *et al.* Mapping and measuring large-scale photonic correlation with single-photon imaging. *arXiv Preprint* arXiv:1806.09569 (2018).

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Author contributions X. -M. J. and M. S. K. conceived and supervised the project. H. T. and X. -M. J. designed the experiment. C.Di F., M. S. K. and T.-S. H. conducted the theoretical work. H. T., Z.-Y. S., J. G., K. S., Z. -Q. J. and X. -M. J. performed the single-photon experiment. H. T., Z.-M. L. and T.-Y. W. analysed the experimental data. Z. F. and Z.-Y. S. conducted chip fabrication. H. T., C. Di F., M. S. K. and X.-M.J. wrote the paper with input from all the other authors.

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Correspondence Correspondence and requests for materials should be addressed to Xian-Min Jin (email: xianmin.jin@sjtu.edu.cn).

Figure 1 Theoretical graphs and their implementation on photonic chips. **a**, Schematic diagram of a glued binary tree. **b**, Schematic diagram of the proposed hexagonal graph. **c**, Schematic diagram of quantum fast-hitting experiment on hexagonal graphs based on femtosecond laser written waveguide arrays.

Figure 2 Fast hitting on a 2-layered hexagonal graph. **a-e**, Photographed evolution patterns for a 2-layered hexagonal graph at different evolution lengths: **a**, 20.7mm; **b**, 22.7mm; **c**, 24.7mm; **d**, 26.7mm; **e**, 28.7mm. **f**, The hitting efficiency against the evolution length for quantum hitting and classical hitting. The evolution patterns for the same sample with an evolution length equal to 25.2mm, by injecting (**g**) laser beam and (**h**) heralded single photons, respectively.

Figure 3 Increasing the complexity of hexagonal graphs. **a-f**, Photographed evolution patterns for hexagonal graphs of different layer depths from 3 to 8. Each panel shows the optimal hitting scenario among the nine samples of the same layer depth. The single-photon-level imaged evolution patterns for: **a**, 3-layered graph at an evolution length of 30.4mm; **b**, 4-layered graph at 43.7mm; **c**, 5-layered graph at 48.4mm; **d**, 6-layered graph at 61.8mm; **e**, 7-layered graph at 70.8mm; **f**, 8-layered graph at 85.8mm. The experimental dynamics of hitting efficiency against the evolution time agrees well with our simulation.

Figure 4 Comparison between quantum hitting and classical hitting. **a**, Optimal hitting efficiency and **b**, the evolution length at which the optimal hitting occurs for hexagonal graphs of different layer depths. The error bars for the experimental optimal distance, which may not be very visibly clear in the double-logarithmic axes, are 1 mm above and below the measured value because the real optimal length may occur in between two samples that have discrete length values with an interval of 1 mm. As for classical hitting, it slowly converges to a stationary value P_a that equals to the inverse of the number of nodes in the glued tree, so we regard that classical hitting becomes optimal when the probability of each waveguide has a maximum deviance of no more than $10^{-4}P_a$, from P_a . The error bars for classical hitting scenarios correspond to a range of criteria for judging the convergence: the upper bound corresponds to a deviance threshold of $10^{-5}P_a$ and the lower bound of $10^{-3}P_a$.