Effect of the polymer interlayer on the high-velocity soft impact response of laminated glass plates

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Abstract

The choice of the polymer interlayer is a key consideration for laminated aircraft windshields. Such windshields often employ chemically strengthened glasses and are required to withstand impact by birds, hail-stones and other foreign bodies. In the present study, windshields employing three different polymer interlayer materials were investigated under high-velocity impact by a soft projectile: Thermoplastic Polyurethane (TPU), Polyvinyl Butyral (PVB) and Ionoplast interlayer-SentryGlas\textsuperscript{®} Plus (SGP). Parameters such as the polymer interlayer type and thickness, multi-layering the interlayer and the sensitivity of the behaviour of the windshield to the environmental temperature were studied. The performance was assessed through a series of laboratory-scale impact experiments (using a bird-substitute material) and modelled via finite element simulations (using a smoothed particle hydrodynamics approach). The experimental and numerical results were found to be in good agreement for the three polymer interlayers investigated. The polymer interlayer type was found to have the most significant effect on both the deformation and the failure of the laminated glass windows at room temperature, i.e. 25 °C. However, the influence of the polymer interlayer type became
24 less pronounced at lower temperatures. The novel modelling that has been developed assists
25 in the choice of the best polymer interlayer, including the multi-layering of interlayers, for
26 complex windshield designs.
27
28 **Keywords:** Bird strike, Laminated glass, Soft impact, SPH, Digital Image Correlation
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30 **1-Introduction**
31
32 Windshields, like other forward-facing components of aircraft, are vulnerable to bird strike
during flight, especially at the time of take-off and landing [1]. A considerable fraction of all
33 fatal accidents caused by bird strikes involve aircraft windshields: for example, 10% of
34 accidents for executive jets and 52% of accidents for smaller aircraft [2]. Dennis and Lyle [3]
35 reported that, amongst the fifty-one fatal accidents identified between 1962 and 2009 caused
36 by bird strikes on the airframe, twenty-seven accidents occurred on the windshield with the
37 majority for smaller aircraft.
38
39 Depending on the type of aircraft and the location of the windshield, various materials
40 including thermally and chemically strengthened glass, polycarbonate, and cast and stretched
41 acrylic are used in monolithic and laminated forms. In the case of modern passenger aircraft,
42 the windshield is a complex structure consisting of several layers of glass and polymer
43 interlayers. In the aviation industry, various standards [4,5] have been developed over the
44 years for testing the resistance of windshields against bird strike. For example, according to
45 the ‘CS 25’ certification [5] for larger aircraft, the main windshield should withstand an
46 impact by a 1.8 kg bird when the velocity of the aircraft, relative to the velocity of the bird in
47 the direction of aircraft travel, is equal to the design cruising speed, \( V_c \), at sea level, or 0.85 \( V_c \)
48 at a height of 2438 m, whichever is the more critical. For smaller aircraft including utility,
aerobatics and commuter category aeroplanes, certification needs to be done based on an impact by a 0.91 kg bird at a velocity equal to the aircraft’s maximum approach flap speed, according to the ‘CS 23’ certification [4]. Furthermore, in all cases, the visibility through the damaged windshield should also be maintained to permit continued safe flight and landing.

As a result of the high cost of a full-scale experimental evaluation, there are limited experimental data available in the literature on the performance of the laminated glass windows against bird strike. Doubrava and Strnad [6] investigated the performance of laminated glass windows with a thickness of 14, 18 and 20 mm against impact by a 1.81 kg bird over the velocity range of 300-450 km h\(^{-1}\) (83-125 m.s\(^{-1}\)). The critical impact velocity, defined as the velocity at which the laminated glass window was fractured, increased linearly with the thickness of the windshield. Kangas and Pigman [7] performed impact tests on various windshields using different materials and types of construction. The tests were conducted using birds impacted at velocities of up to 725 km h\(^{-1}\) (208 m.s\(^{-1}\)). Their study suggested that the primary factor influencing the impact strength of laminated glass windows was the thickness of the polymer interlayer. Different methods of installation of the windshield to the cockpit, such as a flexible bolted edge and a clamped edge, were also investigated and were shown to have a strong effect on the impact strength of the windshield.

Numerical analyses, e.g. finite element analysis [8–13], have been used as an effective tool to reduce the cost of designing and testing new aircraft components against bird strike. Various computational methods, including Lagrangian [8,10,14–16], Eulerian [17,18], Arbitrary Lagrangian Eulerian (ALE) [19,20] and Smoothed Particle Hydrodynamics (SPH) [11,21–26], have been employed to model the soft and deformable ‘bird’ or ‘bird-substitute materials’. A detailed review of various numerical studies can be found in [27]. In the discussion below only the numerical studies concerned with bird strikes on windshields (or
bird-substitute materials on windshield components), as relevant to the present paper, will be reviewed.

Salehi et al. [8] investigated the response of various aircraft canopy windows using three different numerical methods, including the Lagrangian, ALE and SPH approaches. Single layer stretched acrylic, multi-walled stretched acrylic and laminated acrylic with Polyvinyl butyral (PVB) and Polyurethane (PU) interlayers windows were tested. Dar et al. [10] studied the response of windows made of monolithic Polymethyl methacrylate (PMMA). Parameters such as the mass, shape and velocity of the bird as well as the angle and location of the impact were investigated. Wang et al. [13] also investigated the response of PMMA windshields taking into account the influence of the environmental temperature, impact location and velocity.

Most of the numerical analyses (including [8–10]) in the literature concerning the impact response of windshields focus on windshields made of monolithic PMMA. Only a few numerical studies have been reported which consider the response of laminated glass windshields subjected to bird strike [11,12]. Grimaldi et al. [11] used the SPH method to parametrically investigate the response of a laminated glass window consisting of three layers of glass and two layers of PVB. They studied the effects of the target geometry, impact angle and plate curvature on the response of the windshield against bird strike. The impact angle was found to have the strongest influence on the impact performance. Changing the impact angle from 90° (i.e. normal impact) to 60° resulted in an approximately 40-50 % reduction in the amount of impact energy transferred to the windshield. Hedayati et al. [12] also used the SPH method for the selection of the best material for a helicopter windshield according to the ‘CS 29’ certification for large helicopters [28]. They suggested that a laminated glass windshield with a PVB interlayer showed the best performance. Mohagheghian et al. [18] studied laminated glass windows, made of chemically strengthened glass, subjected to impact
by a soft projectile with a velocity of up to 180 m.s\(^{-1}\). Finite element simulations were conducted using the Eulerian approach and the results were compared against experimental data obtained using high-speed 3D Digital Image Correlation (DIC). Good agreement was observed. Two impact damage threshold velocities were identified corresponding to the first failure of the front-facing glass plate and the subsequent failure of the inner glass plate. The degree of damage was found to be strongly influenced by the nose shape of the projectile. The effect of the type of front-facing glass and the order of the glass plates were also investigated. The order of the glass plates was found to have a significant influence on the impact failure of the structure. A laminated glass structure with a thinner front-facing glass performed the best. The type of front-facing glass plate was reported to have no significant effect on the impact velocity at which the inner glass plate broke.

Previous published research has not studied and modelled in detail the effect of different polymer interlayers, and combinations of polymer interlayers, on the performance of laminated glass. This is now possible with the present finite element simulation combined with a 3D DIC experimental approach, where the DIC experimental results can be used to validate the results from the simulation. Thus, in the present paper, the effect of the polymer interlayer on the impact damage caused by a soft projectile on laminated glass windows is measured and modelled using these techniques. The effects of polymer interlayer type, employing three commonly used polymer interlayers, the polymer interlayer thickness, multi-layering the polymer interlayer (i.e. using a combination of polymer interlayer materials), and the sensitivity of the impact response of the windows to the environmental temperature, are considered. Similar to [18], impact tests up to a velocity of 180 m.s\(^{-1}\) were performed using a bird-substitute (i.e. soft) material as the projectile. High-speed 3D DIC was employed to monitor experimentally the deformation and strain development, as a function of time, on the back face of the rear (i.e. non-impacted) glass plate of the windshield specimen during the
impact event. Finite element simulations, using the Smoothed Particle Hydrodynamics (SPH) method, were also employed to gain further insights into the development of the strains induced in the glass plates during the impact event.

2-Materials

The laminated glass window specimens used in the present study consisted of two layers of chemically strengthened glass and two or more layers of a similar, or different, polymer interlayers, which are sandwiched between two glass plates. The chemically strengthened glass plates were manufactured in the Beijing Institute of Aeronautical Materials (BIAM) by soaking float alumina silicate glass in a potassium salt solution, for ion exchange to occur, at 420°C for 5 hours. This is similar to the material used in [18,29]. For the polymer interlayer, three types were used: a Thermoplastic Polyurethane (TPU) (KRYSTALFEX®PE499 from Huntsman), a Polyvinyl Butyral (PVB) (Butacite® from DuPont) and an Ethylene/methacrylic acid copolymer containing small amounts of metal salts (SGP) (Ionoplast interlayer SentryGlas® Plus from DuPont).

The laminated glass window specimens with dimensions of 180 ×180 mm were manufactured by a hot-pressing method using an autoclave. The following steps were undertaken to manufacture the window specimens: (i) the glass plates and polymer interlayers were stacked in the correct order before being fastened with a heat-resistant tape to prevent them sliding apart; (ii) the lay-up was then placed in a vacuum bag and degassed for about 60 minutes at room temperature; (iii) the vacuum bag was then heated to 120°C before applying a pressure of 8 MPa; (iv) the temperature and pressure were held constant for 120 minutes; (v) the vacuum bag was removed from the autoclave and left cool; and (v) finally, after making sure that the temperature of the vacuum bag had fallen to room temperature, the
pressure was released to atmospheric pressure and the laminated glass window specimen was removed.

In total, five different window configurations were manufactured. The details of each configuration can be found in Table 1. In all cases, 2.2 and 4.0 mm chemically strengthened glass plates were used, with the thinner glass plate facing the projectile. Due to a limitation on the thickness of the polymer interlayer that is commercially available, more than one layer of polymer was used to achieve the required thickness, see Table 1. The thickness of the windows were measured after the lamination process at various locations. The average values for the thickness of the windows are reported in Table 1. The final average thicknesses of the polymer interlayers after lamination for the TPU, SGP and PVB layers were 3.03, 3.06 and 2.91 mm, respectively. Therefore, a maximum variation of 0.15 mm was observed for these values, which is only about 5% of the average polymer thickness of about 3.0 mm.

3-Experimental

The experimental method used is adopted from the techniques described in [18]. Laboratory-scale impact experiments were performed using a gas gun apparatus at a 90° incidence angle (i.e. the target was orientated normal to the barrel). RTV rubber, Mold Max ® 10T, was used for the projectile which had a density of 1.09 g cm⁻³ and a Shore Hardness A of 10. (RTV rubber is often used as a bird-substitute material as it has been demonstrated to create a pressure profile similar to that of a real bird [30]). The projectile was a flat-nosed cylinder with a diameter of 23.5 ± 0.05 mm and length of 50 ± 0.3 mm (i.e. an aspect ratio, defined as the projectile length divided by its diameter, of approximately two).

A schematic and photograph of the test set up are shown in Figures 1a and b. High-speed 3D Digital Image Correlation (DIC) was performed using two synchronised high speed cameras (both being Phantom Miro M/R/ LC310 cameras). The cameras were located at the back of
the target chamber with an approximate angle of 25° from each other (Figure 1a). They were both simultaneously triggered using a signal generated by the infra-red (IR) sensors located at the end of the barrel (Figure 1a). The cameras recorded images at a rate of 40,000 frame per second. For monitoring the deformation of the projectile, another high speed camera (a FASTCAM Mini UX50) was employed which recorded images at a rate of 20,000 frame per second. Halogen lamps were used to illuminate the target and were turned on just a few seconds before the test.

As described in [18], to prevent any shadow from the projectile affecting the DIC calculations, the front face of the front (i.e. impacted) glass plate was painted black. For DIC measurements, a random speckle pattern was applied onto the back face of the rear (i.e. the non-impacted) glass plate. To generate the maximum contrast, the speckles were produced using a permanent black marker on a surface which had been previously painted white. The average diameter of the speckles was 0.8 mm. A schematic of the test specimen and the clamping arrangement are shown in Figures 1c and 1d, respectively. Only an area of the most interest in the centre of the plate was speckled (as shown in Figure 1c). The test specimens were clamped to a thick metallic fixture using a clamping plate, with an opening of 150×150 mm, by using twelve ‘M8’ sized bolts. To avoid any direct contact between the glass plates and the metallic clamp, rubber gaskets were used (Figure 1d). The gaskets were compressed between the test specimen and the clamp by tightening the bolts. The amount of the compression in the rubber gaskets was controlled by a metallic spacer (Figure 1d). This was to ensure that the boundary condition remained the same for all experiments. For validating the DIC calculations, a strain gauge (a FLA-2-8 gauge from Techni Measure Ltd) was used to measure the deformation of the plate at a location 30 mm off-centre of the plate (Figure 1c). Data acquisition was conducted using a high-speed transducer amplifier (a FE-H379-TA, from FYLDE) in combination with an oscilloscope.
Dynamic Mechanical Analysis (DMA), as well as uniaxial tensile tests, were used to characterise the polymer interlayer materials. DMA tests were conducted using a TA Instruments Q800 DMA machine in the tension mode at a frequency of 1 Hz with a 0.01 N preload and an oscillation amplitude of 15 µm. The specimens had a rectangular geometry with a width of 6.17 mm. The thickness of the specimens was 1.27 mm for the TPU interlayer, 1.52 mm for the SGP interlayer and 0.76 mm for the PVB interlayer. The free length of each specimen was measured after fixing the specimen in the clamp. For temperature sweep tests, the specimens were left at -100 °C for 10 mins, to achieve thermal equilibrium through the specimen thickness, before heating up to 80 °C with a heating rate of 2 °C min⁻¹. Uniaxial tensile tests were performed at two strain rates: 2.38×10⁻³ s⁻¹ and 2.38×10² s⁻¹. The tests at the lower strain rate were conducted using a screw-driven test machine (an Instron 5800 series machine, UK) whilst the higher rate tests were performed using a high-speed servo-hydraulic test machine (an Instron VHS 8800 machine, UK). Dog-bone shaped tensile specimens were cut from the polymer sheets according to ASTM standard D412-15a [31]. The mechanical properties of these interlayers are shown in Figure 2. Figures 2a and 2b show the DMA results. As indicated by the locations of the peak in Tan δ (i.e. the tangent of the phase angle, defined as the ratio of storage, \(E'\), over the loss modulus) in Figure 2b, the polymer interlayers have very different glass transition temperatures, \(T_g\): -32 °C for the TPU, 27 °C for the PVB and 54 °C for the SGP interlayer. The differences in these values of \(T_g\) results in a significant difference in the stiffness of the polymer interlayers at room temperature (e.g. at 25 °C the storage modulus of SGP interlayer is nearly two orders of magnitude greater than that of the TPU, see Figure 2a.). The uniaxial tensile responses of the three polymer interlayers are also shown at the strain rate of 2.38×10⁻³ s⁻¹ in Figure 2c and at the strain rate of 2.38×10² s⁻¹ in Figure 2d. At both strain rates, the SGP polymer interlayer shows superior stiffness and strength compared to the other two
polymer interlayers. Whilst at the low strain rates the stiffness of PVB is similar to that of TPU (Figure 2c), at high strain rates it becomes more comparable to that of the SGP. The high strain rate sensitivity of PVB is attributed to its glass transition temperature, i.e. $T_g = 27^\circ$C, which is very close to room temperature, i.e. 25 $^\circ$C [29].

4-Numerical modelling

The finite element (FE) method was employed to simulate the response of the laminated glass windows subjected to an impact by a soft projectile at a relatively high velocity. The simulations were performed using Abaqus/explicit (Abaqus version 6.14). The window target, including the glass plates, polymer interlayer and rubber gaskets, were modelled in 3D and were discretised using brick elements with eight nodes and reduced integration, C3D8R (in the Abaqus notation). The FE model is schematically shown in Figure 3a. The mesh was refined near the central region of the frontal (i.e. impacted) glass plate (not shown in Figure 3a) with a typical element size of $1\times1\times0.55$ mm (with the smallest dimension in the $z$-direction, i.e. four elements through the thickness). The number of elements through the thicknesses varies for the different layers, e.g. six elements for the polymer interlayer with a thickness of approximately 3 mm and also six elements for the rear glass plate with a thickness of 4.0 mm. It was found that the simulation results became insensitive to the size of the mesh on using further mesh refinements.

The glass plates were modelled as elastic materials with $\rho = 2440$ kg m$^{-3}$, $E = 71.7$ GPa and $\nu = 0.21$ [32], where $\rho$, $E$ and $\nu$ are the density, elastic modulus and Poisson’s ratio, respectively. It should be noted that strain rate and temperature sensitivity were not included in the material model for the glass plates, since their elastic properties were not considered to change significantly over the test conditions being modelled in the present work. The modelling of crack initiation and propagation in the glass plates was also not considered in
the present study and therefore no failure model was employed for the glass plates in the present FE studies. However, as will be discussed later in Section 5, the simulations were performed for two separate situations: the first situation is when the velocity of projectile was insufficiently high to produce any cracking in any of the glass plates. In this case, the response of the window was considered to be simply elastic. The second situation considered is when the velocity of the projectile was sufficiently high to cause fracture only in the front glass plate. In order to simulate this situation, a preset fracture pattern was considered for the front glass plate. The validity of this assumption, and the effect of the fracture pattern on the final response of the laminated glass windows, will be discussed in Section 5-2.

The rubber gaskets were modelled using a hyperelastic material model (Mooney-Rivlin) [33] with a density of 1060 kg m$^{-3}$ and $C_{10}$ and $C_{01}$ (the Mooney-Rivlin material model constants) of 0.69 and 0.173 MPa respectively. The two free surfaces of the rubber gaskets were constrained in the $z$-direction, simulating the presence of the two clamps in Figure 1d. General frictionless explicit contact was used for modelling the contact between all surfaces, except for the interface between the glass plates and the polymer interlayer, and also between the various polymer layers in the case of a multi-interlayer, see Section 5-5, where a tie constraint was used. The validity of the tie constraint assumption will be discussed in Section 5-4.

**4-1 Modelling the soft projectile**

As discussed in the ‘Introduction’, various numerical approaches have been used in the literature for modelling the large deformation of the projectile (e.g. a bird or bird-substitute material) during impact. In the present paper, a Smoothed Particle Hydrodynamics (SPH) method was employed. The SPH approach is a meshless particle-based method in which a continuous field is represented by a set of discrete but interacting particles, as shown in
The technique, which was first introduced for solving astrophysics problems, has been used in various engineering applications. In the SPH method, each particle has a mass, velocity and a material law assigned to it. These properties are not localised in space for each particle but smoothed over a spatial distance (known as the ‘smoothing length’, see Figure 3c) using a smoothing kernel function [27]. The value of any quantity, $X$, at a point $r$ can be approximated by the following equation [34]:

$$X(r) = \sum_i m_i \frac{X_i}{\rho_i} W(|r - r_i|, h)$$ (1)

where $m$ and $\rho$ are the mass and density of each particle, $h$ is the smoothing length and $W$ is the kernel function. In Equation 1, the summation index $i$ denotes a particle label. The SPH method has several advantages over conventional approaches (e.g. the Lagrangian and Eulerian approaches). Examples of such advantages are: (i) a constant and a more stable time step for explicit solvers, compared to the Lagrangian approach, (ii) the need for fewer elements, (iii) avoiding the material interface problem, and (iv) a shorter simulation time as compared to the Eulerian approach [27].

The SPH needs constitutive equations for the material response and in the case of the high-velocity impact of a soft material, such as a rubber projectile, the Mie Gruneisen relationship [35] between the pressure and the density is sufficient without introducing elasticity. If a linear relationship between the velocity of the projectile, $V_0$, and the shock wave speed in the projectile material, $V_s$, is assumed, then:

$$V_s = c + sV_0$$ (2)

and the relationship between the pressure, $p$, and the nominal volumetric compressive strain, $\eta$, defined as $\eta = 1 - \rho_0 / \rho$, is [35]:

$$p = \frac{\rho_0 c^2 \eta}{(1 - s\eta)^2} \left(1 - \frac{\Gamma_0 \eta}{2}\right) + \Gamma_0 \rho_0 E_m$$ (3)
In the above equations the terms $\rho_0$, $\rho$, and $E_m$ are the initial density, the current density and internal energy per unit mass, respectively and $c$, $s$ and $I_0$ are material constants with values of $c = 1869 \text{ m.s}^{-1}$, $s = 0.5072$ and $I_0 = 0$ [36] in the present case.

### 4-2 Modelling the polymer interlayers

For modelling the polymer interlayers in the FE simulations, a linear viscoelastic material model (namely a generalised Maxwell model) was employed as follows [35]:

$$
E(t) = E_\infty + \sum_{i=1}^{n} E_i e^{-\left(\frac{t}{\tau_i}\right)}
$$  \hspace{1cm} (4)

where $E_\infty$ is the long-term modulus and $E_i$ is the elastic modulus associated with a relaxation time, $\tau_i$. Since, in the present paper, the deformation of the polymer interlayer is modelled only at impact velocities below the velocity needed to fracture both of the glass plates, only relatively small strains in the polymer layer are considered. The values for the material parameters for Equation 4 were extracted with a method similar to that used by Macaloney et al. [37]. In the DMA tests, the temperature was varied from $-45 \degree C$ to $70 \degree C$, in $5 \degree C$ steps. At each step, the specimen was kept at that temperature for two minutes before applying the load. This was done in order to achieve thermal equilibrium through the thickness of the sample. Frequency sweep tests were performed at each temperature using frequencies of 1, 3.2, 10, 31.6 and 100 Hz. An example of the test results for the TPU polymer interlayer is shown in Figure 4a.

In order to expand the response to frequencies beyond those achievable with DMA, the time-temperature superposition principle is employed, which suggests an equivalence between time (i.e. strain rate) and temperature on the response of the material. The results obtained at different temperatures, which were obtained over a limited range of frequencies (i.e. 1 to 100 Hz using DMA), can be shifted to generate a master curve at a single temperature but a wider
range of frequencies. In the present work, the William-Landel-Ferry (WLF) equation [38] is used:

\[
\log a_T = \frac{-C_1(T - T_0)}{C_2 + (T - T_0)}
\]  

(5)

where \( a_T \) is the shift factor, \( T \) and \( T_0 \) are the absolute current and reference temperatures, and the constants \( C_1 \) and \( C_2 \) are material parameters. For shifting the curves, the ‘Rheology Advantage Data Analysis’ Software, provided by TA Instruments, UK, was used. A non-linear least-squares method was employed in Figure 4b to fit Equation 5 into the measured shift factors for the TPU interlayer at a reference temperature of 25 °C (289.15 °K). A good fit is observed. The constructed master curve, by shifting individual curves (Figure 4a) by their corresponding shift factor (Figure 4b), is shown in Figure 4c for the TPU interlayer at the reference temperature of 25 °C. The resulting curve has now expanded to cover a wide range of frequencies (and consequently strain rates) spanning twenty-two orders of magnitude. Based on a series of preliminary impact simulations that were conducted, and the range of strain rates that the polymer interlayer had experienced, for the impact velocity range of interest, it was decided to limit the frequency range from \( 10^{-5} \) to \( 10^7 \) (rad/s). The master curves generated for the three polymer interlayers at \( T = 25 \) °C are shown in Figure 5a. The viscoelastic model may now be calibrated using the data in the frequency, \( \omega \), domain via the following equation and a non-liner least-square method:

\[
E'(\omega) = E_\infty + \sum_{i=1}^{m} \frac{\omega^2 \tau_i^2 E_i}{\omega^2 \tau_i^2 + 1}
\]  

(6)

It was found that twelve Maxwell elements were sufficient to accurately represent the response over the frequency range of interest. The fitting parameters used for the three polymer interlayers are listed in Table 2. The parameters are in the form of the shear modulus, \( G_i \), which is equal to approximately one-third of \( E_i \). The accuracy of the calibrated
model for the three polymer interlayers is demonstrated in Figure 5. Finally, densities of 1070, 950 and 1100 kg m$^{-3}$ were used in the FE model for the TPU, SGP and PVB polymer interlayers.

Depending on the speed of aircraft as well as the ambient air temperature, the aircraft windshield can experience various temperatures during take-off and landing. The mechanical properties of the polymer interlayer depends on its temperature (Figure 2a). Consequently, the mechanical behaviour of the laminated glass windows become temperature sensitive. Thus, the strain development, as well as damage, induced by a bird strike on the laminated glass window can be affected by a change in the windshield temperature. In order to investigate the effect of the environmental temperature on the impact performance of the laminated glass windows, master curves were generated for the three polymer interlayers again, but this time for a reference temperature of -10 °C. Thus, the same procedure, as was explained above using time-temperature superposition, was employed. The list of the fitted parameters for the three polymer interlayers at -10 °C can be found in Table 2.

5- Results

5-1 Introduction

An example of the experimental results is shown in Figure 6. The test was performed on a laminated glass window specimen with a TPU interlayer (Case 1, see Table 1) at an impact velocity of 157 m.s$^{-1}$. The out-of-plane displacement and principal major strain contours are displayed in Figure 6a. These parameters were measured over the ‘observation area’ on the back face of the rear glass plate, as defined in Figure 1c. The rear glass plate is displaced by a maximum value of 4.9 mm and a maximum principal strain of approximately 0.7% is observed to occur at the centre of the plate. The out-of-plane displacement and major
principal strain at the centre of the plate as a function of time are plotted in Figure 6b. The DIC measurements are compared against those obtained by the strain gauge in Figure 6c for a point located at 30 mm off-centre. The comparison is made for the strain along the y-direction, as indicated in Figure 1c. An excellent agreement exists between the two measurements which confirms the validity of the DIC results. Photographs taken of the faces of both glass plates (i.e. the frontal impacted and rear non-impacted plates) of the laminated glass window are shown in Figure 6d. At this impact velocity of 157 m.s\(^{-1}\) only the frontal glass plate is broken. The damage in this plate occurs at the very early stages of the deformation and is caused by the initial high-intensity pressure (known as the Hugoniot pressure, \(P_H\)) produced by a soft flat-nosed projectile \[18\]. The maximum level of Hugoniot pressure can be expressed as:

\[
P_H = \rho_0 V_0 V_s = \rho_0 \left( c V_0 + s V_0^2 \right)
\]  

(7)

For the impact conditions employed in the present tests, the velocity (and consequently the impulse transferred to the target) is insufficient to cause fracture in the rear glass plate. The impulse, \(I\), can be calculated by:

\[
I = \int_{t=0}^{t=H} P(t) \, dt = \int_{t=0}^{t=H} P_H(t) \, dt + \int_{t=H}^{t=C} P_S(t) \, dt
\]

(8)

The impulse, \(I\), depends on both the initial high intensity phase, \(P_H(t)\), which has a very short duration \(t_H\) (i.e. a few microseconds depending on the size and material of the projectile) and the steady-state phase, \(P_S(t)\), which has a lower intensity but longer duration (Figure 7). The steady-state pressure can be considered to have a constant value with a magnitude calculated from the Bernoulli equation:

\[
P_S = \frac{1}{2} \rho_0 V_0^2.
\]

(9)
By increasing the projectile velocity, the impulse transferred to the target also increases (i.e. both $P_H$ and $P_s$ are a function of $V_0$).

The effect of the projectile velocity on the deformation and damage development in the laminated glass with a TPU interlayer (Case 1) is shown in Figure 8. This figure shows (i) the maximum major principal strain at the centre, measured using DIC and (ii) the maximum $\varepsilon_y$, strain in the $y$-direction, according to the coordinates defined in Figure 1c, at a location 30 mm off-centre, measured using a strain gauge. Both values of strain are for the back face of the rear glass plate and are plotted against the initial velocity of the projectile. Three regions can be identified in Figure 8. For impact velocities below 131 m.s$^{-1}$, no damage was observed in any of the glass plates, since the intensity of the Hugoniot pressure (Equation 7) is not sufficiently high to cause failure in the front glass plate. The level of the Hugoniot pressure is, however, strongly dependent on the projectile initial velocity, $V_0$, and becomes sufficient to fracture and fragment the frontal glass plate at impact velocities of about 131 m.s$^{-1}$. However, since the exact impact damage threshold velocity at which fracture is initiated in the frontal glass plate is not precisely known, the region between 131 and 146 m.s$^{-1}$ is shown shaded (representing the region for the onset of fracture of the frontal glass layer) in Figure 8. Increasing the velocity further to about 179 m.s$^{-1}$ causes the magnitude of transferred impulse to the target (Equation 8) to become sufficient to now also fracture the rear glass plate. Similar to the first damage threshold, the area between the impact velocities of 168 and 179 m.s$^{-1}$ is shown shaded as a result of the uncertainty regarding the exact velocity corresponding to this second damage threshold.

5-2 Comparison between experimental and numerical results

In the present section, results from the FE simulations are compared to those obtained from the experiments for the laminated glass windows. The comparison is made at two regimes: (i)
at impact velocities for which no fracture occurs in either of the glass plates (i.e. impact velocities below about 131 m.s\(^{-1}\) in Figure 8), and (ii) at impact velocities for which only the frontal glass plate is broken (i.e. impact velocities between about 146 and 168 m.s\(^{-1}\) in Figure 8). In both cases, the deformation in the polymer interlayer is limited to small strains and the application of the linear viscoelastic material model for the polymer interlayer is therefore assumed to be valid.

Firstly, an example of the FE simulation results is shown in Figure 9 for a laminated glass window with a TPU interlayer (Case 1, see Table 1) impacted at the velocity of 118 m.s\(^{-1}\). The photographic images of the deformation of the projectile, as well as the out-of-plane displacement and major principal strain contours, measured using DIC on the back face of the rear glass plate, are compared with the FE simulation results. Only the area of most interest, as shown in Figure 1c, was monitored throughout the test using the DIC experimental method. The overall semi-quantitative results, for all above parameters, from the simulations and the experiments are in very good agreement.

The DIC experimental and FE numerical results are compared quantitatively in Figure 10 for the central out-of-plane displacement and the major principal strain (Figure 10a), the strain in the y-direction at the location of the strain gauge (i.e. 30 mm off-centre) (Figure 10b), and the out-of-plane displacement profile during the loading phase (Figure 10c). (All values correspond to the back face of the rear glass plate.) It is apparent from Figure 10 that the FE predictions generally agree quite well with the experimental results, especially for the loading phase (i.e. \(t < 0.5 \text{ ms}\)). Indeed, as is apparent in Figures 10a and c there is very good agreement for the out-of-plane displacement results. Further, good agreement between the strain values predicted by the FE model and those measured in the y-direction at the location of the strain gauge (i.e. 30 mm off-centre) is observed in Figure 10b. In these figures, the deviation of the FE modelling results from the experimental results becomes apparent only
towards the end of the loading phase, where the compression of the rubber gaskets around the window specimen becomes dominant [18].

Secondly, a comparison is made for velocities at which the frontal glass plate is certainly broken, i.e. at impact velocities between 146 and 168 m.s$^{-1}$. It should be emphasised that no model was included in our FE studies for the fracture of the glass plates. However, the failure in the frontal glass plate was modelled by replacing the glass plate with a series of glass fragments which were tied to the polymer interlayer surface (Figure 11a). Contact surfaces were defined between the sides of the glass fragments. This approach is based on considering the experimental observations that: (i) the glass frontal layer fractures almost instantaneously (i.e. in a few microseconds after the initial contact [18]) and (ii) most of the glass fragments remained attached to the polymer interlayer, at least in the loading phase [18]. Two patterns of glass fragments were used: the ‘fracture pattern 1’ where cracks were aligned in the horizontal and vertical directions (Figure 11b) and the ‘fracture pattern 2’ with cracks in the radial and circumferential directions (Figure 11c). It should be noted that the latter pattern matches more closely the experimentally observed fracture pattern (Figure 6d). For ‘fracture pattern 1’, horizontal and vertical cracks were considered to have occurred to create a uniform shape of square fragments with the size of $5 \times 5$ mm. The ‘fracture pattern 2’ was a representative pattern which was based on typical patterns which had been observed experimentally. However, depending on the impact velocity and the polymer interlayer material used, the density of the cracks was observed to vary significantly. Thus, capturing a detailed fracture pattern was not intended here and only a representative pattern was considered. Hence, in the ‘fracture pattern 2’, twelve radial cracks (separated by 30°) as well as four circumferential cracks were generated. A circular fragment with a diameter equal to that of the projectile was placed where the projectile impacted. The other circumferential cracks were separated by the distances of 12.5, 22.5 and 22.5 mm. In total forty-one glass
fragments were modelled. Finally, in order to investigate the effect of the density of the
fragments formed in the frontal glass plates on the FE modelling results, a separate study was
conducted based on ‘fracture pattern 2’ but now with twelve radial cracks and thirteen
circumferential cracks (separated by 5 mm) giving a total of one hundred and forty-nine glass
fragments. The results revealed that only a 3% difference in the maximum deflection of the
window was predicted between this fracture pattern being present compared with that of
‘fracture pattern 2’. This difference is considered insignificant and, for simplicity, ‘fracture
pattern 2’ was chosen, together with ‘fracture pattern 1’, for the modelling work reported
below for the fragmented frontal glass plate.

In Figure 12 the results from the FE simulations are shown from modelling the effects of the
fragmentation of the frontal glass plate for a laminated glass window using a TPU interlayer
(Case 1, see Table 1) (Figure 12a) and a laminated glass window using a SGP interlayer
(Case 3, see Table 1) (Figure 12b) impacted at a velocity of 165 m.s\(^{-1}\). Four conditions for the
frontal glass plate are considered: (i) an intact glass plate without fracture having occurred,
(ii) an intact glass plate without fracture having occurred but with an artificially reduced in-
plane stiffness (i.e. the in-plane stiffness is reduced to zero whilst retaining its through-
thickness stiffness), (iii) a broken glass plate with a ‘fracture pattern 1’ (Figure 11b) and (iv)
a broken glass plate with a ‘fracture pattern 2’ (Figure 11c). The DIC experimental results are
also given for a test where the frontal glass plate was impacted using the same velocity of 165
m.s\(^{-1}\). The FE and experimental results shown in Figures 12a and b clearly reveal that the
frontal glass plate still contributes significantly to the load carrying capacity of the structure
even after breaking into fragments. Reducing the in-plane stiffness of this glass plate to zero
in the FE model results in significantly increased values of the out-of-plane displacement,
especially when the SGP polymer interlayer was used. Considering the results of the two
fracture patterns (i.e. ‘fracture pattern 1’ and ‘fracture pattern 2’) in Figure 12, it is apparent
that, as a result of interactions between the glass fragments, the in-plane stiffness values for
the fragmented glass should lie between the two limits: (i) $E = 0$, giving zero in-plane
stiffness, and (ii) $E = 70$ GPa, considering no in-plane stiffness reduction. This observation is
indeed found to be correct. The contribution of the glass fragments from the fracture of the
frontal glass plate to the overall stiffness of the window can be explained by considering
Figure 11a. During bending the glass fragments come into contact with each other in the top
surface, as shown schematically in Figure 11a. Despite the fracture in the glass, this local
‘lock-up’ between the glass fragments increases the resistance against bending. It is also clear
from Figure 12, that the fracture pattern has a notable effect on the predicted stiffness of the
window, especially for windows which employed the relatively stiff polymer interlayer (i.e.
the SGP interlayer, see Figure 12b). Firstly, when ‘fracture pattern 2’ is employed in the FE
model for the frontal glass plate, the predicted values are in relatively good agreement with
the experimental values for both types of polymer interlayer. Secondly, the FE results, using
‘fracture pattern 2’, are compared against the experimental results for the central major
principal strain, as well as the strain in the y-direction at 30 mm off-centre, as a function of
time in Figures 13a and b, respectively, for the window using the TPU polymer interlayer.
Good agreement between the numerical and experimental results is observed. Therefore,
from the above observations, in all subsequent simulations where the frontal glass plate is
broken, only ‘fracture pattern 2’ will be used.

5-3 Effect of thickness of the polymer interlayer

The effects of the thickness of the polymer interlayer on the impact performance of the
windows were investigated by increasing the polymer interlayer thickness from $3.18 \pm 0.01$
mm (Case 1) to $5.09 \pm 0.01$ mm (Case 2), see Table 1. For the Case 2 window configuration
the experiment results are shown in Figure 14. As shown in Figure 14a, the maximum strain
measured from the back face of the rear glass plate decreases upon increasing the polymer
interlayer thickness. The first damage threshold for the fracture and fragmentation of the
frontal glass plate, highlighted as a shaded area in Figure 14a, is identified between the
impact velocities of 136 and 145 m.s$^{-1}$; and for velocities up to 170 m.s$^{-1}$ fracture was only
observed in the frontal glass plate. Thus, the velocities for these events are very similar to
those for the windows using the thinner polymer interlayer (Case 1), as may be seen from
Figure 8. An example of the experimentally measured strains at 30 mm off-centre, via the
strain gauges, is shown in Figure 14b as a function of time for both Cases 1 and 2. (Note that
for the results shown in Figure 14b the frontal glass plate fractured and fragmented for both
window configurations.) For the thicker TPU polymer interlayer, the strain follows the
general trend as for the thinner interlayer, but with slightly lower values.

FE modelling was also used to simulate the response of these Case 1 and Case 2 window
configurations at the same impact velocity of 118 m.s$^{-1}$. (It should be noted that this velocity
is below the velocity for the first damage threshold and hence no pre-set fracture pattern for
the frontal glass plate was required in the FE modelling studies.) The central major principal
strain and the strain in the y-direction at the location of the strain gauge (i.e. 30 mm off-
centre) for the back surface of the rear glass plate were predicted numerically and are
compared in Figure 15. A reduction in the strain values for the window using a thicker
polymer interlayer (Case 2) is apparent in Figure 15.

5-4 Effect of the type of polymer interlayer

The performances of the window configurations using the other two types of polymer
interlayers, which are the SGP interlayer (Case 3, see Table) and the PVB interlayer (Case 4),
are compared against that of the TPU interlayer (Case 1) over a range of impact velocities in
Figures 16a and b. Both of the SGP and PVB window configurations have approximately the
same thickness of the polymer interlayer as that of the reference TPU interlayer. In general,
for both window configurations (i.e. Case 3 with SGP, see Figure 16a, and Case 4 with PVB, see Figure 16b) the levels of the maximum strains in the centre, and at the gauge at 30 mm off-centre, on the back face of the rear glass plate are somewhat lower than that for the TPU interlayer (Case 1) window configuration. No damage was observed in any of the frontal glass plates for the impact velocities of 124 and 138 m.s\(^{-1}\) for the Case 3 and 117 and 123 m.s\(^{-1}\) for the Case 4 configurations. For all velocities above 153 m.s\(^{-1}\) for the Case 3 and above 156 m.s\(^{-1}\) for the Case 4 configurations only the frontal glass plate broke. Similar to Figure 7a for the TPU interlayer (Case 1), the regions between 138 and 153 m.s\(^{-1}\) for the SGP interlayer (Case 3) and between 123 and 156 m.s\(^{-1}\) for the PVB interlayer (Case 4) are shown shaded, as the exact impact damage threshold velocities could not be identified. It is believed that the same failure mechanisms, as described in detail in [18], for a laminated glass window with a TPU interlayer are still operative. Namely, a combination of Rayleigh surface waves and localised bending stresses are believed to cause the damage in the frontal glass plate [18]. The failure occurs in the early stages of the hydrodynamic loading (i.e. a few microseconds after initial contact) and has similar characteristics to that observed for an impact by a liquid jet [39]. Figure 17a shows the cracks that developed in the first 25 µs after the impact for a SGP (Case 3) window configuration impacted at a velocity of 171 m.s\(^{-1}\). It should be noted that no DIC tests were conducted for this experiment as the main purpose here was to observe the fracture and fragmentation development in the frontal glass plates in the initial stages of the impact. As may be seen, a large number of circumferential cracks are formed in the frontal glass plate in a shape of a ring with an approximate initial diameter equal to the initial diameter of the projectile. These are very similar damage features as those described by Field [39] for a liquid jet impacting on a glass ceramic substrate (Figure 17b).

The damaged frontal glass plates of the laminated glass windows after impact are compared in Figure 18a for the TPU (Case 1) and the SGP (Case 3) window configurations subjected to
two different impact velocities of 165 and 179 m.s\(^{-1}\). For both configurations at the impact velocity of 165 m.s\(^{-1}\) only the frontal glass plate is fractured. Although the fracture initiation mechanisms of the frontal glass plates for both types of window configurations are similar, the higher stiffness of the SGP interlayer compared to the TPU interlayer reduces the local bending in the frontal glass plate. As a result, more circumferential and less radial cracks are seen in Figure 18a for the SGP (Case 3) window configuration. At the higher impact velocity of 179 m.s\(^{-1}\) only the frontal glass plate for the SGP (Case 3) window configuration is fractured and fragmented. However, both glass plates are broken for the TPU (Case 1) window configuration. The other notable feature shown in Figure 18a is the considerable number of glass fragments which are detached from the window with the SGP interlayer, especially at the velocity of 179 m.s\(^{-1}\). (See the white area in the photographs taken from the front). This could be due to relatively low adhesion between the glass and SGP interlayer (i.e. the Case 3 window configuration). The fractured and fragmented frontal glass plate for the PVB interlayer (Case 4) window configuration is shown in Figure 18b for an impact velocity of 160 m.s\(^{-1}\) and a large number of circumferential cracks are apparent.

Figure 19 shows the deformation and strain history of two laminated glass window configurations, with different types of polymer interlayers, i.e. TPU (Case 1) and SGP (Case 3). In both cases, this is for an impact velocity of 165 m.s\(^{-1}\). These data the DIC measured out-of-plane displacement (Figures 19a and b), the major principal strain history at the centre of the specimens (Figures 19c and d), and the off-centre strains measured via the strain gauge (Figure 19e). Similar to [18], the deformation is divided into four phases: Phase 1 is when the deformation is localised in the centre of the plate and both the strain and displacement are increasing; Phase 2 is the time period during which the flexural waves travel from the centre of the plate towards the plate boundary, and the displacement is still increasing but the strain is either increasing or decreasing; Phase 3 is when the strain is decreasing whilst the
displacement is still increasing; and finally Phase 4 is when both the displacement and strain are decreasing. Comparing the two types of glass windows configurations, the strain in the rear glass plate is significantly lower for the Case 3 window configuration with the SGP interlayer, especially in the first two phases of the deformation (compare Figures 19c and d).

In Phase 1, when the deformation is highly localised, the level of strain in the rear glass plate rises to a higher magnitude for the Case 1 configuration, with a TPU interlayer (Figure 19c), when compared with the Case 3, with an SGP interlayer (Figure 19d). The duration of Phase 1 is about 0.175 ms for the Case 1 window configuration compared to about 0.1 ms for the Case 3 window configuration. For the SGP (Case 3) window configuration, the displacement profile of the back face of the rear glass plate in Phase 1 (i.e. the solid lines in Figure 19b) has a much smoother shape as well as a lower value, which results in lower strain values in the centre of the plate being recorded. Whilst the strain in Phase 2 for the TPU (Case 1) configuration (Figure 19c) starts decreasing, the strain continues to rise for the SGP (Case 3) configuration (Figure 19d). The strain at the centre of the plate decreases in Phases 3 and 4 for both window configurations and returns to zero by the end of Phase 4. The strain measured from the strain gauge is compared for the two window configurations in Figure 19e. (For the SGP (Case 3) window configuration, the strain gauge terminals peeled off the glass surface at 400 µs.) Similar to Figures 19c and d, the main difference between the two window configurations can be seen in the first 0.2ms of the impact event.

It is clear from Figure 19 that the stiffness of the polymer interlayer has a significant effect on the duration of the contact, the maximum deflection and the development of strain in the window. The duration of the contact is shorter and the maximum displacement is smaller for the Case 3 configuration (i.e. a window with the stiffer interlayer of SGP) (Figure 19d). Although at velocity of 165 m.s\(^{-1}\), the frontal glass plate breaks for both the TPU (Case 1) and the SGP (Case 3) window configurations in the first few microseconds after initial contact,
the ‘locking-up’ of the glass fragments still contributes significantly to the in-plane stiffness of the broken frontal glass under bending for the remaining of the loading that occurs (as discussed in Section 5-2). It is also known that a stiffer interlayer (i.e. the SGP interlayer) transfers a higher amount of shear stresses between the two glass plates resulting in a stiffer response of the undamaged window [29]. Therefore, it is believed that the notable difference between the values of the stiffness of the two window configurations in Figure 19 can be explained by the difference in the amount of shear stress transfer, via the polymer interlayer, between the glass fragments in the frontal glass plate and the undamaged rear glass plate.

The validity of the FE model for the window configurations using the SGP (Case 3) and the PVB (Case 4) polymer interlayers are assessed in Figure 20 for impact velocities below the first damage threshold (i.e. where no damage occurs in either of the glass plates). The comparison is made for the strain in the y-direction at the location of the strain gauge (i.e. at 30 mm off-centre). For both types of interlayers there is very good agreement between the FE model and the experimentally measured strain values, especially in the loading phase. However, the duration of the impact event is somewhat over predicted by the FE modelling studies.

The performance of the FE model for velocities above the first threshold is shown for the SGP interlayer (Case 3), in Figures 21a and b, and for the PVB interlayer (Case 4), in Figures 21c and d. In general, good agreement exists between the experimental results and the FE predictions. However, it should be noted that there is some discrepancy between the experimental and numerical results shown in Figure 21 for the values of the central major principal strain for \( t > 0.3 \) ms. This is more notable for the window configuration using the more rigid SGP interlayer (Case 3), as shown in Figure 21a. This discrepancy can be explained by the earlier assumption in the numerical model discussed in Section 4 with respect to the ‘tie constraints’ that are employed for describing the interactions between the
glass plates and the polymer interlayer. Photographs of the damaged SGP interlayer (Case 3) window configuration are shown in Figure 18a and indicate that, for velocities above the first damage threshold, a considerable number of glass fragments became detached from the SGP polymer interlayer. High-speed photography during these experiments showed that such glass fragments remained attached during the loading phase, despite the complete fracture of the frontal glass plate, but that they became detached when the plate rebounded after the impact event. This can explain the underestimated predicted values for the strain in the FE model, compared to the experimental results for the Case 3 window configuration, for $t > 0.3$ ms. The FE model does not capture the degrading effect of glass fragments that become detached on the global stiffness of the window. As more glass fragments were detached from the SGP interlayer during the unloading phase, this discrepancy becomes more pronounced for windows using this interlayer in Figure 21a.

5-5 Effect of multi-interlayering the polymer layer

In this section, the effects of using a combination of polymers for the polymer interlayer for the windows is investigated, see Case 5 in Table 1. To study this a 2.28 mm thick SGP interlayer was sandwiched between two thin layers of TPU, each of 0.38 mm in thickness, and this multi-interlayer was laminated in the autoclave between the two glass plates to form the window specimen. (It should be noted that no adhesive was used between the polymer interlayers and they were bonded by hot pressing.) The nominal total thickness of this polymer interlayer (i.e. 3.04 mm) is therefore, close to the value for the monolithic TPU interlayer, which was 3.18 mm thick and the monolithic SGP interlayer which was 3.04 mm in thickness. The impact results of this multi-interlayer (Case 5) window configuration are plotted in Figure 22 and are compared with those for the windows with a monolithic TPU interlayer (Case 1) and a SGP interlayer (Case 3). For the multi-interlayer (Case 5) window configuration, below an impact velocity of about 150 m.s$^{-1}$ no damage was observed in either
of the glass plates and above about 150 m.s\(^{-1}\) damage was only observed in the frontal glass plate, see Figure 22a. In general, the level of maximum strain for the multi-layer (Case 5) window configuration lies between the values of the Case 1 and Case 3 configurations. An example of the results from the strain gauge for the Case 5 configuration is compared with that of the Case 1 configuration in Figure 22b. It can clearly be seen that the strain values are lower for the Case 5 window configuration where the multi-interlayer was employed. Therefore, exploiting the structural properties of the SGP interlayer can help to lower the strain in the rear glass plate. A front view of the fractured window is shown in Figure 22c and, as may be seen, the number of detached fragments is significantly reduced when using thin layers of TPU on either side of the SGP, due to the improved bonding between the polymer interlayer and the glass plates. This is in comparison to where a monolithic SGP interlayer was used, see Figure 18a.

The experimental results are compared against the FE model predictions in Figure 23 at a velocity of 150 m.s\(^{-1}\) for the multi-interlayer (Case 5) window configuration. It should be noted that no fracture was observed in either of the glass plates at this velocity. In the FE model, tie constraints were used between the polymer interlayers, thus no debonding was assumed to occur between them. Four elements through the thickness of the layer were used for the SGP interlayer, whilst only one element was used for each of the TPU interlayers either side of the SGP interlayer. Overall, very good agreement between the experimental results, from both the strain gauge and the DIC measurements, and the FE numerical simulations can be observed in Figure 23. However, as found in the results shown in Figure 21, the numerical FE predictions indicate an initial relatively high peak for the central major principal strain, which was not observed experimentally.

5-6 Effect of temperature
Following the good agreement that has been observed between the numerical FE simulations and the experimental results reported above for a test temperature of 25 °C, the impact performance of the laminated glass window is numerically simulated below, for the three polymer interlayers used, but now at a lower operational temperature of -10 °C. As discussed in Section 4, the small strain response of the polymer interlayer, obtained using DMA, can be extended to cover a wider range of frequencies. Thus, using the time-temperature superposition principal, the response of the interlayers can also be generated for different reference temperatures. For this study, \( T = -10 \) °C was selected as the reference temperature to investigate the effect of lowering the environmental temperature on the development of the strain in the laminated glass windows. The same approach, as discussed in Section 4-2, is used here to generate master curves for the three polymers at the temperature of \( T = -10 \) °C. The results are shown in Figure 5b for the same frequency range of \( 10^{-5} \) to \( 10^{7} \) rad/s. In contrast to room temperature, i.e. 25 °C, the response of the interlayers becomes more similar for the three polymers at relatively low temperatures, especially for frequencies above 1 rad/s. The material parameters used to fit the generalised Maxwell model at \( T = -10 \) °C are listed in Table 2.

The predicted effect of the temperature on the impact response of the three window configurations are shown in Figures 24a and b for temperatures of 25° and -10°C, respectively. The impact velocity that was modelled was 118 m.s\(^{-1}\). A comparison is made for the central major principal strain predicted at the back surface of the rear glass plate for three different window configurations. At \( T = 25 \) °C, the strain values are considerably lower for the window configuration with the SGP interlayer (Case 3). The strain values for the Case 4 window configuration with the PVB interlayer are also slightly lower than for the window configuration with the TPU interlayer (Case 4). At \( T = -10 \) °C the response is very similar to that at room temperature, i.e. 25 °C, for the window configuration with the SGP interlayer.
However, the strain values for the other two windows configuration are considerably reduced. From the results shown in Figure 24b it can be inferred that the response of the three window configurations to a soft impact becomes very similar at lower environmental temperature (i.e. \( T = -10 \, ^\circ C \)).

6- Conclusions

In the present paper, the effects of the type of polymer interlayer and the thickness of the interlayer on the impact response of laminated glass windows employing chemically strengthened glasses, as often used in the aviation industry for aircraft windshields, were investigated using experimental and numerical approaches. High-velocity impact tests, at velocities in the range of 100-180 m.s\(^{-1}\), were conducted. A soft projectile was employed which acted as a bird-substitute material. High-speed photography, as well as high-speed 3D digital image correlation, were employed to monitor the deformation of the back face of the rear glass plate during the impact event. Finite element simulations, using a smoothed particle hydrodynamics approach to model the soft and deformable bird-substitute material, were performed to gain a further understanding on the deformation of the laminated glass windows under high-velocity impact, as well to develop a predictive design tool.

The following main conclusions may be drawn:

- The type of polymer interlayer employed in the laminated windows has a significant effect on their impact performance. The level of strain in the back face of the rear glass plate is found to be significantly lower in the window configurations that employed the relatively stiff SGP interlayer. As first pointed out by Field [39], a combination of Rayleigh surface waves and localised bending stresses is believed to cause the damage in the frontal glass plate during the high-velocity impact. However, the appearance of the damage caused by such an impact event is slightly different
when using a relatively stiff polymer interlayer (i.e. using the SGP interlayer in comparison to the TPU interlayer). More circumferential and less radial cracks are observed to occur when employing the SGP (stiffer) interlayer. The number of glass fragments detaching from the interlayer is also significantly higher for the windows employing the SGP interlayer.

- The impact performance of the laminated glass window using a combination of the SGP and the TPU layers (i.e. the multi-interlayer window configuration) falls between the performance of windows using either the monolithic TPU or the monolithic SGP interlayers, as assessed by the strain values measured at the back face of the rear glass plate. However, by multi-layering the interlayer benefits from using the two different polymers (i.e. the two outer TPU layers and the inner SGP layer) can be exploited. For example, the window configuration using the multi-interlayer benefits from the better structural properties of the SGP inner layer, compared to the TPU interlayer. On the other hand, the two outer TPU layers exhibit relatively better adhesion to the glass plates, compared to the SGP interlayer, which leads to a lower number of glass fragments detaching from the window, using the multi-interlayer compared to the SGP interlayer, when the glass plates undergo fracture.

- Finite element simulations, using a smoothed particle hydrodynamics method to model the bird-substitute material, have been undertaken and validated against the values measured experimentally using the 3D DIC and strain gauge procedures. Overall very good agreement was observed between the theoretical FE numerical predictions and the experimental results. The numerical results revealed that, after fracture and fragmentation of the frontal glass plate at the higher impact velocities, there was a significant contribution of the broken glass fragments in the frontal glass plate to the subsequent load carrying capacity of the window. The ‘locking-up’ of the
glass fragments during bending of the window was found to be responsible for this finding.

- The sensitivity of the impact performance to the environmental temperature was highest for the windows made using either the TPU interlayer or the PVB interlayer. Whilst a clear distinction between the responses of the windows made using the three different interlayers was observed when they were tested at room temperature, a relatively similar response was predicted at an environmental temperature of -10°C. These observations essentially arise from the differences in the glass transition temperatures of these three polymer interlayers.

- The combined experimental and modelling approach developed (i.e. 3D DIC experimental measurements coupled with the FE-SPH predictive modelling) has provided a detailed understanding of the performance of the different configurations of the laminated glass windows when subjected to a high-velocity impact by a soft projectile. This approach has made it possible to understand the influence of parameters such as the type of polymer interlayer and its thickness, multi-layering the polymer interlayer and predicting the sensitivity of the impact performance of the windows to the environmental temperature.

- There are distinct advantages from multi-layering the polymer interlayer to optimise the performance of laminated glass windows and this concept is likely to attract much industrial interest. Indeed, the novel model developed in the present work can provide the basis of a viable design tool for aircraft windshields in the future.
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Figure and Table Captions:

Figure 1: The gas gun experimental test set-up: (a) schematic and (b) photograph of the high velocity 3D DIC test set-up, (c) schematic of the test specimen, and (d) the clamping arrangement.

Figure 2: Polymer interlayer characterisation results: (a) the storage modulus and (b) the value of tan δ plotted against temperature as measured using DMA. Uniaxial tensile results: (c) at the lower strain rate of $2.38 \times 10^{-3}$ s$^{-1}$ and (d) at the higher strain rate of $2.38 \times 10^{2}$ s$^{-1}$ for the three polymer interlayers studied.

Figure 3: Finite element model: (a) a section of laminated glass window assembly, (b) the discretised projectile using particles, and (c) schematic of the influence of each particle on its neighbouring region.

Figure 4: Generating a master curve for the TPU interlayer: (a) the storage modulus, $E'$, plotted against frequency at different test temperatures, (b) manually measured shift factors at a reference temperature of 25 °C plotted against temperature, and (c) a master curve for the storage modulus at a reference temperature of 25 °C spanning a wide range of frequencies.

Figure 5: Storage modulus master curves for the three polymer interlayers: TPU, SGP and PVB at (a) $T = 25$ °C and (b) $T = -10$ °C with the results from the fitted curve using a generalised Maxwell model.

Figure 6: An example of the experimental impact results on a laminated glass window: (a) out-of-plane displacement and major principal strain contours, measured using 3D DIC over the observation area on the back face of the rear glass plate, (b) the central out-of-plane displacement and major principal strain history, (c) comparison between the strain results obtained using the strain gauge and those measured using DIC at the location of the gauge,
and (d) photographs taken from the impacted and non-impacted sides of the damaged window. (Tests was performed using a laminated glass window with a TPU interlayer (Case 1, see Table 1) and impacted at a velocity of 157 m.s⁻¹.)

**Figure 7:** Schematic of a typical pressure versus time trace for a typical high-velocity soft impact loading.

**Figure 8:** The impact performance of laminated glass window with a TPU interlayer (Case 1, see Table 1) at various impact velocities. Solid symbols show the maximum strain obtained from the back face of the rear glass plate at the centre of the specimen calculated from the DIC measurements. (Open symbols show the maximum strain obtained from a strain gauge mounted on the back face the rear glass plate at a 30 mm off-centre.)

**Figure 9:** Comparisons between experimental and FE numerical results for the impact on a laminated glass window with a TPU interlayer (Case 1, see Table 1) at a velocity of 118 m.s⁻¹. The comparison is made for deformation of the projectile and the out-of-plane displacement and the major principal strain, ε, contours over the observation area, as highlighted by the white dashed line, at the back surface of rear glass plate. (V₃ is the velocity in the direction of travel of the projectile.)

**Figure 10:** Comparisons between the experimental and FE numerical results for (a) the central out-of-plane displacement and major principal strain history, (b) the strain in the y-direction at the location of the strain gauge (i.e. 30 mm off-centre), and (c) the deformation profile during the loading phase (the time increment between each profile is 0.0025 ms). All values are obtained from the back face of the rear glass plate. The comparison is made for a laminated glass window with a TPU interlayer (Case 1, see Table 1) impacted at a velocity of 118 m.s⁻¹. (Note: no damage occurred in any of the glass plates at this impact velocity.)
Figure 11: Schematic of (a) glass fragments tied to the polymer interlayer surface, and (b) ‘fracture pattern 1’ and (c) ‘fracture pattern type 2’ as used in the finite element modelling studies.

Figure 12: Comparisons between the results from the finite element simulations using various conditions to model the fragmented frontal glass plate for (a) a laminated glass window using a TPU interlayer (Case 1, see Table 1), and (b) a laminated glass window using a SGP interlayer (Case 3, see Table 1) impacted at a velocity of 165 m.s\(^{-1}\). Different conditions assumed for the frontal glass layer are: (i) an intact glass plate (i.e. with no fragmentation having occurred), (ii) reducing the in-plane stiffness to zero whilst retaining the through-thickness stiffness, (iii) a fracture pattern for the fragmented frontal glass plate with horizontal and vertical cracks (i.e. ‘fracture pattern 1’), and (iv) a fracture pattern for the fragmented frontal glass plate with radial and circumferential cracks (i.e. ‘fracture pattern 2’).

Figure 13: Comparisons between the experimental and numerical results at (a) the centre, and (b) 30 mm off-centre of the back face of the rear glass plate for a laminated glass window (with a TPU interlayer (Case 1), see Table 1) impacted at a velocity of 165 m.s\(^{-1}\). The ‘fracture pattern 2’ was used for the frontal glass plate for the finite element simulations.

Figure 14: Effect of the polymer interlayer thickness on the impact performance of a laminated glass window with a TPU interlayer: (a) maximum strain at the back face of the rear glass plate-plotted against the projectile’s initial velocity for two window configurations with a polymer interlayer thickness of 3.18 (Case 1, see Table 1), and 5.09 mm (Case 2, see Table 1). (The solid symbols are the maximum major strain at the centre of the specimen calculated by DIC and open symbols show the maximum strain, obtained from a strain gauge mounted on the back face of the rear glass plate at 30 mm off-centre. And (b) a comparison
between the experimental strain gauge measurements of the two window configurations at an impact velocity of 169±1 m.s\(^{-1}\) as a function of the time of loading.

**Figure 15:** The results of the strain versus time of loading from the finite element simulations for two laminated glass windows configurations using a TPU polymer interlayer of thickness of 3.18 mm (Case 1, see Table 1), and 5.09 mm (Case 2, see Table 1) impacted at a velocity of 118 m.s\(^{-1}\). The results are obtained numerically for two locations on the back face of the rear glass plate: at the centre of the plate and at 30 mm off-centre.

**Figure 16:** The experimental maximum strain at the back face of rear glass plate plotted against the projectile’s initial velocity for two laminated glass windows with (a) a SGP polymer interlayer (Case 3, see Table 1), and (b) a PVB polymer interlayer (Case 4, see Table 1). (Solid symbols are the maximum major strain at the centre of the specimen calculated from the DIC measurements and the open symbols show the maximum strain obtained from a strain gauge mounted on the back face of the rear glass plate at 30 mm off-centre. The results for a laminated glass window with a TPU interlayer (Case 1, see Table 1) are also included for comparison.)

**Figure 17:** Comparison between (a) the impact damage 25 µs after the initial contact, in a laminated glass window with a SGP interlayer (Case 3, see Table 1) impacted at a velocity of 171 m.s\(^{-1}\), and (b) fracture and erosion by a liquid jet in a glass ceramic [39].

**Figure 18:** Comparison between the impact damage in laminated glass windows with (a) a TPU interlayer and a SGP interlayer (Cases 1 and 3, respectively, see Table 1) impacted at velocities of 165 and 179 m.s\(^{-1}\), respectively, and (b) a PVB interlayer (Case 4, see Table 1) impacted at a velocity of 160 m.s\(^{-1}\).
Figure 19: Comparison between laminated glass windows with a TPU interlayer and a SGP interlayer (Cases 1 and 3, respectively, see Table 1) impacted at a velocity of 165 m.s\(^{-1}\). For (a-b) the out-of-plane displacement profile during the loading phase, (c-d) the time history of the central out-of-plane displacement and major principal strain and (e) strain gauge values.

Figure 20: Comparison between experimental results (obtained using the strain gauge) and finite element simulation results for (a) a laminated glass window with a SGP interlayer (Case 3, see Table 1) impacted at a velocity of 124 m.s\(^{-1}\), and (b) a laminated glass window with a PVB interlayer (Case 4, see Table 1) impacted at a velocity of 117 m.s\(^{-1}\). (Note: no damage occurred in any of the glass plates at these velocities.)

Figure 21: Comparison between the experimental and finite element simulation results for (a-b) a laminated glass window with a SGP interlayer (Case 3, see Table 1) impacted at a velocity of 165 m.s\(^{-1}\), and (c-d) a laminated glass window with a PVB interlayer (Case 4, see Table 1) impacted at a velocity of 167 m.s\(^{-1}\). (Note: ‘fracture pattern 2’ was used in the FE simulations.)

Figure 22: Effect of multi-layering the interlayer: (a) the maximum strain at the back face of rear glass plate plotted against the projectile’s initial velocity for a laminated glass window using a multi-interlayer configuration (TPU/SGP/TPU, Case 5, see Table 1). The values of the laminated glass window with a monolithic polymer interlayer of a TPU interlayer and a SGP interlayer (Cases 1 and 3, respectively, see Table 1) are plotted for comparison. (Solid symbols are the maximum major strain at the centre of the specimen calculated from DIC measurements and the open symbols show the maximum strain, obtained from a strain gauge mounted on the back face of the rear glass plate, at 30 mm off-centre.). (b) shows the comparison between the strain gauge measurements as a function of time of window configurations with a monolithic TPU interlayer (Case 1, see Table 1) and a multi-interlayer
(Case 5, see Table 1) and (c) shows the damage in the frontal glass plate of a laminated glass window using the multi-interlayer impacted at a velocity of 175 m.s\(^{-1}\).

**Figure 23:** Comparison between the experimental and finite element simulation results for a laminated glass window using a multi-interlayer (Case 5, see Table 1) impacted at a velocity of 150 m.s\(^{-1}\). The comparison is made for (a) the central out-of-plane displacement and the major principal strain, and (b) the strain in the y-direction at the location of the strain gauge (i.e. 30 mm off-centre). (Note: no damage occurred in either of the glass plates at this velocity.)

**Figure 24:** Comparison between the finite element simulation results of laminated glass windows using the TPU (Case 1), SGP (Case 3) and PVB (Case 4) interlayers impacted at a velocity 118 m.s\(^{-1}\) for two environmental temperatures: (a) \(T = 25^\circ\text{C}\) and (b) \(T = -10^\circ\text{C}\). The comparison is made for the central major principal strain at the back face of the rear glass plate.

**Table 1:** Different configurations of laminated glass windows used in the present study.

**Table 2:** Prony series material-constants extracted for the three polymer interlayers used (i.e. the TPU, SGP and PVB interlayers) at a test temperature of 25 °C and -10 °C.
References:


EASA. Certification Specifications for Large Rotorcraft CS-29 (Amendment 3) 2012:132.


Figure 1:

(a) Target chamber
   Lamp
   High speed camera
   Distance between cameras (410 mm)
   Working distance (925 mm)
   Triggering high speed camera

(b) Image of experimental setup

(c) Observation area for DIC
    Strain gauge
    M8 bolts
    Laminated glass plate

(d) Clamp
    Rubber gasket
    Metallic spacer
    M8 bolt
    Laminated glass
Figure 2:

(a)\ E (MPa)\n\begin{align*}
\text{Temperature (°C)} & \quad 10^{-14} \\
& \quad 10^{-13} \\
& \quad 10^{-12} \\
& \quad 10^{-11} \\
& \quad 10^{-10} \\
& \quad 10^{-9} \\
& \quad 10^{-8} \\
& \quad 10^{-7} \\
& \quad 10^{-6} \\
& \quad 10^{-5} \\
& \quad 10^{-4} \\
& \quad 10^{-3} \\
& \quad 10^{-2} \\
& \quad 10^{-1} \\
& \quad 10^0 \\
& \quad 10^1 \\
& \quad 10^2 \\
& \quad 10^3 \\
& \quad 10^4 \\
& \quad 10^5 \\
& \quad 10^6 \\
& \quad 10^7 \\
& \quad 10^8 \\
& \quad 10^9 \\
& \quad 10^{10} \\
& \quad 10^{11} \\
& \quad 10^{12} \\
& \quad 10^{13} \\
& \quad 10^{14} \\
\end{align*}

(b)\ \tan \delta\n\begin{align*}
\text{Temperature (°C)} & \quad 0 \\
& \quad 0.2 \\
& \quad 0.4 \\
& \quad 0.6 \\
& \quad 0.8 \\
& \quad 1.0 \\
& \quad 1.2 \\
\end{align*}

(c)\ \text{Engineering stress (MPa)}
\begin{align*}
0 & \quad 20 \\
& \quad 40 \\
& \quad 60 \\
& \quad 80 \\
& \quad 100 \\
\end{align*}

(d)\ \text{Engineering strain}
\begin{align*}
0 & \quad 20 \\
& \quad 40 \\
& \quad 60 \\
& \quad 80 \\
& \quad 100 \\
\end{align*}

---

TPU \quad PVB \quad SGP
Figure 3:

(a) Rubber gaskets
(b) Glass
(c) Polymer interlayer

Projectile

Smoothing function
Smoothing length
Neighbouring particle
Figure 4

(a) Storage modulus (MPa) vs. Frequency (Hz)

(b) Log(a) vs. Temperature (K)

Equation 5: \( C_1 = 59.65, C_2 = 400.7 \) K

(c) Storage modulus (MPa) vs. Frequency (rad/s)

\( T = 25 \) °C (298.15 K)
Figure 5

(a) Storage modulus (MPa) vs. frequency (rad/s)

(b) Storage modulus (MPa) vs. frequency (rad/s)
Figure 6

(a) Out-of-plane displacement  Major principal strain

(b) Central out-of-plane displacement (mm)

(c) Strain (%)  Time (ms)

(d) Impacted face  Back face
Figure 8:

![Graph showing maximum strain vs. projectile initial velocity]

- Case 1 (3.2 mm TPU)
  - No breakage
  - Frontal layer broken
  - Both layers broken

- Symbols:
  - Max $\varepsilon_y$ in 30 mm off-centre strain gauge
  - Max major strain from DIC at the centre

- Key areas:
  - First damage threshold
  - Second damage threshold

- Axes:
  - Maximum strain (%)
  - Projectile initial velocity (m s$^{-1}$)
Figure 9:
Figure 10:
Figure 11:
Figure 12:

(a) Central out-of-plane displacement (mm) vs. Time (ms)

(b) Central out-of-plane displacement (mm) vs. Time (ms)

Legend:
- Exp (DIC)
- FE (No fracture)
- FE (Fracture pattern 1)
- FE (No in-plane stiffness)
- FE (Fracture pattern 2)
Figure 13:

(a) Case 1 (TPU interlayer)

(b) Exp (strain gauge)
Exp (DIC)
FE

Strain (%)

Time (ms)

Central major principal strain

Central out-of-plane displacement (mm)
Figure 14:

(a) No breakage | Frontal layer broken

First damage threshold

(b) Strain in the gauge (%)
Figure 15:

![Graph showing strain over time for two cases, labeled Case 1 and Case 2. The graph indicates major strain at the centre and ε_y at 30 mm off-centre.](image_url)
Figure 16:

(a) 

(b) 

Case 3 (3 mm SGP)

No breakage | Frontal layer broken

First damage threshold

Case 1 (TPU interlayer)
Case 3 (SGP interlayer)

Case 4 (3 mm PVB)

No breakage | Frontal layer broken

First damage threshold

Case 1 (TPU interlayer)
Case 4 (PVB interlayer)
Figure 17:

(a)  

Front view  

Back view  

(b)  

Ref [39]
Figure 18:

(a) Velocity of 165 m s⁻¹
Front
Front after removing the paint

Case 1 (TPU interlayer)

Case 3 (SGP interlayer)

Clamping area

(b) Velocity of 160 m s⁻¹

Case 4 (PVB interlayer)
Figure 19:
Figure 20:

(a) Strain gauge trace
(b) Strain gauge trace
Figure 21:
Figure 22:

(a) Maximum strain (%) vs. Projectile initial velocity (m s\(^{-1}\)).

(b) Strain in the gauge (%) vs. Time (ms).

(c) Image showing the result of Case 5 (multi-interlayer TPU/SGP/TPU) with no breakage at the frontal layer.
Figure 23:

(a) Case 5 Multilayer (TPU/SGP/TPU)

Central out-of-plane displacement (mm)

Time (ms)

(b) Exp (strain gauge)  Exp (DIC)  FE

Strain (%)
Figure 24:

(a) Central major principal strain (%)

(b) Central major principal strain (%)

Time (ms)
Table 1:

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Glass plates and polymer interlayers (nominal thicknesses given)</th>
<th>Average window thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>2.2 mm CS(^{(i)})/1.27+1.91 mm TPU(^{(ii)})/4.0 mm CS</td>
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<td>2.2 mm CS/1.27+1.91+1.91 mm TPU/4.0 mm CS</td>
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<td>Case 3</td>
<td>2.2 mm CS/1.52+1.52mm SGP(^{(iii)})/4.0 mm CS</td>
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<td>Case 4</td>
<td>2.2 mm CS/1.52+1.52mm PVB(^{(iv)})/4.0 mm CS</td>
<td>9.11</td>
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<td>Case 5</td>
<td>2.2 mm CS/0.38 mm TPU/2.28 mm SGP/0.38 mm TPU/4.0 mm CS</td>
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</tr>
</tbody>
</table>

\(^{(i)}\) Chemically strengthened glass plate

\(^{(ii)}\) Thermoplastic polyurethane interlayer (KRYSTALFEX®PE499)

\(^{(iii)}\) Ionoplast interlayer (SentryGlas® Plus)

\(^{(iv)}\) Polyvinyl Butyral interlayer (Butacite®)
Table 2:

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<th>$i$</th>
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<th>$T= 25 \degree C$</th>
<th>$T= -10 \degree C$</th>
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<td></td>
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* $G_o$ is the instantaneous shear modulus and it value is equal to: $G_o + \sum_{i=1}^{n} G_i$. $G_o$ is 94.6 MPa for TPU, 274.1 MPa for SGP, and 213.6 MPa for PVB.

** $G_o$ is the instantaneous shear modulus and it value is equal to: $G_o + \sum_{i=1}^{n} G_i$. $G_o$ is 1.137 GPa for TPU, 1.170 GPa for SGP, and 1.565 GPa for PVB.