A Numerical Investigation of Labyrinth Seal Flutter

Richard Phibel

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Statement of Originality

The work presented in the thesis is, to the best of the candidate’s knowledge and belief, original and the candidate’s own work, except as acknowledged in the text. The material has not been submitted, either in whole or in part, for a degree or comparable award of Imperial College or any other university or institution.

Richard Phibel
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Abstract

This thesis investigates numerically the phenomenon of flutter in labyrinth seals. Computational fluid dynamics (CFD) methods are used to predict the fluid forces produced in the labyrinth when one of the seal members is vibrating in its natural mode. The geometry of the seal, the vibrational characteristics and the flow characteristics are varied to determine their influence on the aeroelastic stability. The CFD results are used to develop a bulk-flow model for labyrinth seal flutter analysis. An aeroelastic design procedure for labyrinth seal is proposed.
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Dedication

This thesis is dedicated to my parents for their love and their belief in me.
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Nomenclature

Roman Symbols

\(a\)  Speed of sound
\(c\)  Seal clearance
\(E\)  Total energy
\(E_k\)  Kinetic energy
\(f\)  Frequency of vibration of structural modes
\(f_{ac}\)  Acoustic frequency
\(H\)  Total enthalpy
\(h\)  Cavity height
\(k\)  Wave number
\(K_e\)  Expansion loss coefficient
\(L\)  Cavity length
\(M\)  Mach number
\(m\)  Mass flow rate
\(n\)  Nodal diameter number
\(p\)  Seal pitch
<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$p_0$</td>
<td>Complex unsteady pressure</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Total pressure</td>
</tr>
<tr>
<td>$Pr_l$</td>
<td>Laminar Prandtl number</td>
</tr>
<tr>
<td>$Pr_t$</td>
<td>Turbulent Prandtl number</td>
</tr>
<tr>
<td>$R$</td>
<td>Cavity radius</td>
</tr>
<tr>
<td>$R_c$</td>
<td>Seal clearance radius</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$Sw$</td>
<td>Swirl ratio</td>
</tr>
<tr>
<td>$T$</td>
<td>Static temperature</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T_b$</td>
<td>Time taken by a fluid particle to go through an inter-fin cavity</td>
</tr>
<tr>
<td>$T_t$</td>
<td>Total temperature</td>
</tr>
<tr>
<td>$u_\theta$</td>
<td>Component of velocity in azimuthal direction</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Component of velocity in direction $x_i$</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity</td>
</tr>
<tr>
<td>$x_p$</td>
<td>Axial location of pivot point</td>
</tr>
<tr>
<td>$y^+$</td>
<td>Dimensionless wall distance</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational fluid dynamics</td>
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<td>Control volume</td>
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<td>HP</td>
<td>High-pressure</td>
</tr>
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<td>High-pressure side</td>
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<tr>
<td>LP</td>
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LPS  Low-pressure side

ND  Nodal diameter

RANS  Reynolds-averaged Navier-Stokes

Greek Symbols

\( \gamma \)  Ratio of specific heats

\( \delta \)  Logarithmic decrement

\( \theta \)  Azimuthal coordinate

\( \lambda_l \)  Laminar thermal conductivity

\( \lambda_t \)  Turbulent thermal conductivity

\( \mu \)  Total dynamic viscosity

\( \mu_l \)  Laminar dynamic viscosity

\( \mu_t \)  Turbulent dynamic viscosity

\( \nu \)  Kinematic viscosity

\( \nu_t \)  Turbulent kinematic viscosity

\( \rho \)  Density

\( \tau \)  Wall shear stress

\( \Omega \)  Rotational speed

\( f^* \)  Apparent frequency of vibration (frequency seen by the fluid)

Variables in two- and three-control-volume models

\( \delta_{SL} \)  Characteristic thickness for pressure gradient

\( \Phi \)  Viscous fluxes

\( \tau \)  Viscous stresses
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\begin{itemize}
  \item \[b\] Fin tip thickness
  \item \[F\] Inviscid fluxes
  \item \[h_1\] Height of control volume 1
  \item \[h_2\] Height of control volume 2
  \item \[h_3\] Height of control volume 3
  \item \[L\] Cavity length
  \item \[Q\] Vector of conservative variables
  \item \[w\] Velocity component in direction of z-axis
\end{itemize}

Subscripts in two- and three-control-volume models

\begin{itemize}
  \item \[1\] Control volume 1
  \item \[2\] Control volume 2
  \item \[3\] Control volume 3
  \item \[A\] Inlet of CV1
  \item \[B\] Outlet of CV1/Inlet of CV2
  \item \[C\] Outlet of CV2
  \item \[i\] Inlet
  \item \[o\] Outlet
  \item \[R\] Rotor surface
  \item \[S\] Stator surface
  \item \[SL\] Dividing streamline
\end{itemize}
Chapter 1

Introduction

Rotating machinery exhibits small radial clearances between rotating and stationary components. Leakage in radial clearances represents a loss to the working cycle and efficiency of the machine. To control this leakage flow, seals are introduced. Seals are found along the shaft, over rotor blade tips and between stages. In the primary gas path, they are used at the root of stator blades to prevent recirculation between stages and at compressor end to control the leakage flow extracted for purging and cooling purposes. In the secondary air system, the flow is largely controlled by labyrinth seals because of their simplicity, robustness and because they accommodate axial displacement.

A labyrinth seal geometry is presented in Fig. 1.1. The rotating part (rotor) or stationary part (stator) is equipped with a series of fins which reduce the flow area. The kinetic energy of the flow at the fin tips is dissipated in the cavities between the fins [18]. This process allows a significant reduction of the leakage flow in comparison with a simple annular slit (typically by 40%). The rotor is usually the inner part so that the seal clearance closes under centrifugal load and the teeth are placed on this component. The stator land is coated with an abradable material to tolerate rub from the teeth on the rotor.
Figure 1.1: Labyrinth seal geometry. The rotating part (rotor) is equipped with a series of fins which reduce the flow area.

There are a variety of seal configurations possible:

- straight-through labyrinth seals with teeth on rotor (Fig. 1.2a) or teeth on stator (Fig. 1.2b);
- step-up (Fig. 1.2c) or step-down (Fig. 1.2d) labyrinth seals;
- staggered labyrinth seals (Fig. 1.2e);
- interlocking labyrinth seals (Fig. 1.2f);
- double-sided labyrinth seals (Fig. 1.2g).

This list is not exhaustive. In addition, labyrinth seals can have straight or angled teeth as shown in Fig. 1.2h.

The major influence of seal performance on the overall performance of gas turbine engines was highlighted at a meeting sponsored by the Propulsion and
Energetics Panel of AGARD in 1978 [1]. It was noted at the time that research in seal technology was insufficient considering the potential gain from better seal design. Several issues are encountered in seals. Seal wear leads to an increase in clearance which causes performance deterioration. Seal clearance is also affected by thermal expansion.

While a large number of studies have been performed to investigate the leakage and rotordynamics characteristics of labyrinth seals, the studies on their aeroelastic stability remain scarce. The instability phenomena arising in these small clearance areas are ill-understood. Research in the 1960s and 1970s has shown that flutter could be a cause of fatigue failure in labyrinth seals. This research produced a frequency-based stability criterion which is still in use today but has limited validity. As a consequence, problems that were not predicted during design can occur in service, or machines are designed in an over-conservative way with excessively restrictive stability criteria. Such designs are heavier, less efficient, more expensive, so less competitive. Increasing the understanding of the flutter mechanism in labyrinth seal is the first goal of this research project. This will help in developing criteria to guide the design of this seal from an aeroelastic point of view.

The increase in computational power now allows the use of computational fluid dynamics (CFD) methods to investigate the aeroelastic stability of labyrinth seals. Such methods have been successfully used to predict seal flutter in a modern large-diameter aero engine ([16]). The computational time required for such an analysis is, however, almost prohibitive (several weeks). An attempt is made in the current research project to develop faster analysis methods.

The thesis is structured as follows: a review of the literature is presented in Chapter 2; a theoretical analysis of seal flutter is then carried out in Chapter 3; this is followed by a presentation of the numerical methods used in this thesis in Chapter 4; these methods are used to characterise the steady and unsteady flow in labyrinth seals in Chapter 5; the influence of different geometric and flow
parameters on seal flutter is investigated in Chapter 6; analytical methods developed for seal flutter predictions are presented in Chapter 7; Chapter 8 presents a practical application of the findings and developments made in this research project to an industrial seal design; finally, the conclusions and prospects are given in Chapter 9.

1.1 Terminology

Some of the terminology used in this thesis is explained in this section. Figure 1.3 presents a labyrinth seal geometry annotated with its main geometric characteristics which are:

- the seal clearance $c$;
- the seal pitch $p$;
- the cavity height $h$;
- the mean cavity radius $R$;
- the tip thickness $t$.

The term *mechanical* frequency is sometimes employed in this thesis to designate a natural frequency of vibration of the seal. This term is used to mark the distinction with the *acoustic* frequencies of the seal which are the frequencies of the acoustic circumferential modes of the inter-fin cavities.
1.1 Terminology

1.2a: Straight-through teeth-on-rotor.  1.2b: Straight-through teeth-on-stator.

1.2c: Step-up.  1.2d: Step-down.

1.2e: Staggered.  1.2f: Interlocking.

1.2g: Double-sided.  1.2h: Angled-teeth.

Figure 1.2: Labyrinth seal types. The black arrow indicates the direction of the flow.
1.1 Terminology

Figure 1.3: Seal geometry with main geometric characteristics: the seal clearance $c$, the seal pitch $p$, the cavity height $h$, the mean cavity radius $R$, the tip thickness $t$. 
Chapter 2

Review of the literature

2.1 Introduction

The literature dealing specifically with the phenomenon of labyrinth seal flutter is relatively scarce. A list of the most relevant literature is given in Table 2.1. In the present chapter, a review of the literature is presented to determine the state of the art of the theory and of the analysis techniques.
<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Approach</th>
<th>Seal configuration(s)</th>
<th>Parameters investigated</th>
<th>Stability criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alford [6]</td>
<td>Experimental/Analytical</td>
<td>Straight-through</td>
<td>Location of support</td>
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<td>Ehrich [20]</td>
<td>Bulk-flow model/Experimental</td>
<td>Unknown</td>
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<td>Location of support, mechanical/acoustic frequency ratio</td>
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<tr>
<td>Prokop'ev and Nazarenko [37]</td>
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<td>Volume of inter-fin cavities, pressure drop, clearance, seal stiffness, location of the support</td>
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<tr>
<td>Lewis et al. [31]</td>
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<td>Stepped</td>
<td>Clearance, pressure drop across first cavity</td>
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<td>Abbott [2]</td>
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<td>Stepped</td>
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<td>Srinivasan et al. [50]</td>
<td>Bulk-flow model</td>
<td>Unknown</td>
<td>Nodal diameter number, rotor/stator stiffness, swirl, mechanical/acoustic frequency ratio</td>
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<td>Schmidt [45]</td>
<td>Experimental</td>
<td>Straight-through</td>
<td>Location of support, mechanical/acoustic frequency ratio, inlet pressure, pressure drop, swirl</td>
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<td>Ziegler [54]</td>
<td>Bulk-flow model</td>
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<tr>
<td>Bonsell [13]</td>
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<td>Straight-through, stepped</td>
<td>Nodal diameter number</td>
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<tr>
<td>Roe [41]</td>
<td>Experimental</td>
<td>Straight-through</td>
<td>None</td>
<td>No</td>
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<tr>
<td>di Mare et al. [16]</td>
<td>CFD</td>
<td>Straight-through</td>
<td>Nodal diameter number, mechanical/acoustic frequency ratio</td>
<td>No</td>
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<tr>
<td>El-Aini [21]</td>
<td>Experimental/Analytical</td>
<td>Straight-through</td>
<td>Pressure drop, clearance, volume of cavities</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 2.1: Literature on seal flutter listed in chronological order.
2.2 Labyrinth seal failures and their causes

Fatigue failures in gas turbine labyrinth seals have been reported by many authors. Alford [6] discussed the failure of the stator component of a straight through labyrinth seal at compressor discharge. This failure was characterised by a long circumferential crack on the support of the stator and high vibration levels as demonstrated by the detachment of pieces of metal from the wall. Lewis et al. [31] discussed the failure of the rotor of a stepped labyrinth seal at the high-pressure compressor discharge of a turbofan engine. Cracks were found in the fin tips with some propagating in the rotor cylinder. More recently, high cycle fatigue cracks were found in two labyrinth seals at the turbine outlet section of the space shuttle main engine's high-pressure oxidizer turbopump as reported by Roe [41], Riha et al. [40] and Ehrgott et al. [19]. These failures can be caused by resonance or by an aeroelastic instability [6]. Two resonance phenomena are possible. The first involves a coincidence between the speed of rotation and the speed of the flexural travelling waves in the rotor or stator. This condition is expressed by the following relation:

\[ f = \frac{nN}{60} \]  

(2.2.1)

where \( n \) is the number of nodal diameters (ND) of the mode, \( f \) its frequency (Hz) and \( N \) the speed of rotation (RPM). The second resonance phenomenon corresponds to a coincidence between the speed of forward travelling waves in the stator and backward travelling waves in the rotor. This coincidence is given by the condition:

\[ f_s = \frac{nN}{60} - f_r \]  

(2.2.2)

where \( f_s \) and \( f_r \) are the stator and rotor frequency respectively.

In the case of aeroelastic instability, the exciting forces are generated by the seal motion itself and its interaction with the leakage flow. Due to the motion of the rotor or stator, there is a change in clearance in the labyrinth area. This
creates a pressure variation which can damp the seal motion or amplify it in a feedback loop [6]. Whether the pressure forces are stabilising or destabilising depends on several parameters, as discussed below.

2.3 Critical parameters for labyrinth seal flutter

The support side of the seal was the first critical parameter to be identified. Alford [6, 8] observed that all labyrinth seals that had failed due to aeroelastic instability were supported on the entrance or high-pressure (HP) side. He proposed an explanation by reasoning on how the change of inlet and outlet clearance during the seal motion was affecting the pressure in the labyrinth. The conclusions of this study were, however, incomplete because the circumferential flow was not considered in the analysis.

Taking into account the flow in the circumferential direction in his analytical model, Abbott [2] found that the aeroelastic stability depends on both the support side and the ratio between the natural frequencies and the frequencies of the inter-fin cavity acoustic modes. When the seal vibrates in a n nodal diameter mode, the acoustic mode to consider is the one with the same nodal pattern; its frequency is given by:

\[ f_{ac} = \frac{na}{2\pi R} \]  

(2.3.1)

where \( a \) is the speed of sound in the cavity and \( R \) the cavity radius. Abbott's analytical results showed that seals supported at the entrance side can only be unstable if the mechanical frequency is higher than the acoustic frequency, while seals supported at the exit side can only be unstable if the mechanical frequency is lower than the acoustic frequency. These results were confirmed by actual seal tests.

According to Ziegler [54] and Schmidt [45], the mode shape, rather than the
support side, should be considered when applying Abbott's stability criterion. They explained that the variation of the clearances at the fin tips could sometimes be different from what one would expect based on the support side due to the complex shape of the modes. They showed that the stability behaviour of actual seal designs is better explained if this is taken into account.

Using an analytical model and comparing a stable and an unstable seal of similar design, Ehrich [20] showed than the seal clearance has an influence on stability. This was further confirmed by Lewis et al. [31] who studied experimentally and analytically the stability of a turbofan engine labyrinth seal. Carrying a dimensional inspection of cracked and uncracked labyrinth seals, they determined that cracked seals tended to be on the tighter side of the clearance tolerances. Subsequent investigation with an aeroelastic analysis tool confirmed this observation and showed that a stable design could be obtained by increasing the clearance at the first fin. This solution was adopted and proved to be stable in engine test. The results of Ehrich's analytical model indicated that stability was improved for large seal clearances. Using an analytical model, Ziegler [54] showed that the fluid damping has a strongly nonlinear dependence on the clearance. He also observed that an unstable configuration could become stable at large clearances. Similar trends were indirectly observed in rig test. Industrial engine experience [13] also confirms this tendency but, to the author's best knowledge, this parameter has never been the subject of an explicit experimental investigation.

Srinivasan et al. [50] found that the inlet swirl was also an influential parameter for seal aeroelastic stability. Their analytical results indicated that an increase in inlet swirl had a stabilising effect. This finding was not confirmed by the experimental and analytical investigations of Schmidt [45] and Ziegler [54] whose results indicated that a general conclusion concerning the stabilising or destabilising effect of the swirl could not be drawn.

The pressure and pressure ratio (or pressure drop) across the seal were identified as influential parameters by several authors. Based on the results of his
analytical model, Ehrich [20] concluded that stability was enhanced for low inlet pressure level. Lewis et al. [31] determined analytically that the stability was particularly sensitive to the pressure drop across the first fin, an increase in this pressure drop increasing the instability. Their core engine tests confirmed this finding. The influence of the pressure drop was confirmed by Ziegler [54]. His analytical results showed that the complete fluid-structure system was stable for small pressure drops. For higher pressure drops, the system became unstable in frequency ranges in accordance with Abbott's frequency ratio criterion. These results were in agreement with the behaviour observed in rig tests.

Other parameters were determined as significant by Ehrich [20]. These parameters characterise the geometry of the labyrinth (length, seal teeth height). The influence of these parameters has not been investigated by other authors. One reason could be that constraints other than stability fix them. However, it would be interesting to investigate their influence.

2.4 Instability mechanism

Alford [6] was the first to propose an explanation for the instability mechanism. His analysis will be presented below.

We consider the case of a vibrating rotor supported at the entrance side, as illustrated in Fig. 2.1a. When the rotor is below its equilibrium position, the outlet clearance is greater than the inlet clearance. Alford argued that, in this situation, the outflow is greater than the inflow: the fluid mass in the labyrinth decreases and so does the pressure. When the rotor is above its equilibrium position, the outlet clearance is smaller than the inlet clearance, thus the outflow is smaller than the inflow and the pressure increases. If there is a delay in the response of the fluid to the seal motion (this delay is not explicitly introduced by Alford in his explanation but is needed to explain the stability), when the rotor returns to its equilibrium position after a downward motion, the
pressure is still lower than its equilibrium value. Thus, when moving upward from this equilibrium position, the rotor is subjected to an unsteady pressure force directed upward owing to the deficit of pressure. Similarly, when the rotor moves downward from its equilibrium position, the unsteady pressure force is in the direction of motion due to the excess pressure. This leads to a positive aerodynamic work over a cycle of vibration hence an aeroelastic instability.

When the seal is supported at the exit side as illustrated in Fig. 2.1b, the change in clearance during the seal motion is greater at the entrance than at the exit. For the same case of motion of the rotating part, when the rotor is below its equilibrium position, the inflow is greater than the outflow and the pressure increases in the labyrinth; when the rotor is above its equilibrium position, the inflow is lower than the outflow and the pressure decreases. In this case, the seal motion leads to a pressure variation in opposite phase compared to the case of entrance side support. Thus the aerodynamic work will be negative and the system aeroelastically stable.

This analysis led Alford to conclude that seals supported on the low-pressure side could not be unstable. But the analysis neglected the flow propagating in the circumferential direction. Considering this circumferential flow, Abbott [2] showed that Alford's conclusion is only true when the mechanical frequency is greater than the acoustic frequency. However, Abbott did not provide a physical explanation concerning this influence of the frequency ratio.

2.5 Analysis tools for labyrinth seal stability

Most of the analysis tools presented in the literature to investigate the aeroelastic stability of labyrinth seals are based on bulk-flow models. A comprehensive review of published work on bulk-flow models for rotordynamics applications is presented by Scharrer [44]. The successive contributions to the refinement of bulk-flow models are presented in this review. Childs [14] also gives a detailed
2.5 Analysis tools for labyrinth seal stability

Figure 2.1: Seal motion during vibration. For a seal supported at the entrance side, the change in clearance during the seal motion is greater at the exit than at the entrance. For a seal supported at the exit side, the change in clearance is greater at the entrance than at the exit.
presentation of bulk-flow models, including the governing equations and solution procedures. The review is not comprehensive but it includes some material published after Scharrer's review. The first bulk-flow models developed used a single control volume (CV) for the entire cavity (Alford [5], Kostyuk [30] Iwatsubo [27], Iwatsubo et al. [28], Childs and Scharrer [15]). Such models solve the continuity and circumferential momentum equations for the flow in a cavity. The shear stresses at the wall, appearing in the circumferential momentum equation, are evaluated using a Blasius-type friction factor model:

\[ c_f = n_f R e^{m_f} \]  \hspace{1cm} (2.5.1)

where \( n_f \) and \( m_f \) are empirical constants and \( Re \) is the Reynolds number. Instead of solving an axial momentum equation, leakage equations give the mass flow rate at a fin tip as a function of the pressures in the cavities upstream and downstream. Most of these models assume that the temperature is constant throughout the labyrinth. The pressure \( p_i \) and density \( \rho_i \) in a cavity are linked by the equation of state for a perfect gas:

\[ p_i = \rho_i R T \]  \hspace{1cm} (2.5.2)

where \( T \) is the gas temperature and \( R \) is the perfect gas constant. Iwatsubo [27], on the other hand, assumes that the flow changes are isentropic. Childs and Scharrer compared their model predictions to the measurements of Benckert and Wachter [10]. Their results showed a good agreement for the cross-coupled stiffness. Scharrer [44] has reported some discrepancies on the other rotordynamic coefficients using Childs and Scharrer's model. Recognising that the single-control-volume model did not take into account the fact that the flow in a labyrinth contains two flow regions, the through-flow and the vortex region, Wyssmann et al. [53] developed a two-control-volume model. This model is based on the work by Florjancic [23] for liquid annular seals. The model is of the "box-in-a-box" type with one control volume for the entire cavity and a second
control volume, contained in the first one, for the vortex region. Scharrer [44] used a more conventional approach with one control volume for the through-flow and one for the vortex region (Fig. 2.2). These two-control-volume models include a modelling of the shear stress at the interface between the through-flow and the cavity vortex. They combine the model of Abramovich [3] for the velocity profile in a jet flow and Prandtl's mixing length model to evaluate the shear stress. The axial velocity of the through-flow ($U_1$ in Fig. 2.2) and the recirculation velocity of the vortex flow in the case of Scharrer’s model ($U_2$ in Fig. 2.2) are used to evaluate the shear stresses. However, no axial momentum equation is solved: the axial velocity of the through-flow is obtained from a leakage equation and the recirculation velocity of the vortex flow assumed to be proportional to the former. As for single-control-volume models, the temperature is assumed constant throughout the labyrinth. Scharrer [44] reports improved predictions of the cross-coupled stiffness and direct damping with the two-control-volume model when compared to the single-control-volume model.
2.5 Analysis tools for labyrinth seal stability

model of Childs and Scharrer [15]. Extensive comparisons between experiments and predictions of the two-control-volume model of Scharrer are presented by Childs [14]. They are extracted from the work of Pelletti [36]. Good results are obtained at large clearances with reduced supply pressures and speeds, but discrepancies are observed at increased supply pressures and running speeds. Nordmann and Weiser [35] developed a three-control-volume model for rotordynamic predictions in labyrinth seals. The additional control volume is at the fin tip. In their model, they solve the equation of mass, axial momentum, circumferential momentum and energy for the fin tip control volume and added the radial momentum equation for the two control volumes in the cavity. The rotordynamic coefficients predicted by their model for two different straight labyrinth seals are in excellent agreement with experimental data; in particular, their model predicts the direct stiffness with good accuracy whereas the single-control-volume model of Childs [15] and the two-control-volume model of Wyssmann [53] gives results in poor correspondence with experiments.

Similar bulk-flow models have been developed to investigate the aeroelastic stability of labyrinth seals. The first attempt to use a bulk-flow model for seal flutter predictions was presented by Ehrich [20]. He used a single-cavity single-control-volume bulk-flow model to derive a stability parameter. In this model, the mass conservation equation is solved with leakage equations for the inlet and outlet flow. The outlet tooth is assumed choked which limits the validity of the model to high pressure ratios. The model does not take into account the circumferential flow and the temperature is assumed constant, so that the density is proportional to the pressure, as in many bulk-flow models discussed above. The motion of the seal rotor or stator is modelled as an elastic rotation about a virtual pivot point. When applying the stability parameter he had derived to actual seal designs, Ehrich found that he had to correct it to take into account the non-axisymmetry of the vibration mode. After correction, the stability parameter was able to make the distinction between a stable and an unstable design. However, he observed a more fundamental discrepancy in his model: the model
predicted an instability of the divergence (non-oscillatory) type whereas experimental evidence showed an oscillatory phenomenon. He suggested that the discrepancy could be due to the lack of modelling of the circumferential unsteady flow. Prokop’ev and Nazarenko [37] studied the self-excited vibration of a plane labyrinth seal model. They did not, however, suppose the outlet fin choked, thus extending the validity of the analysis to lower pressure ratios. Moreover their seal model contained multiple cavities. Their flow model uses a leakage equation at the fin tips and assumes isothermal variations in each cavity. The variation of the chamber area is neglected. The vibrating seal member is modelled as a cantilever plate and Galerkin method is used to discretise the equation of motion of the plate. After linearisation of the governing equations, the flutter determinant is formed. The Routh-Hurwitz criterion is used to investigate the sign of the real parts of the roots [38] and determine stability. Prokop’ev and Nazarenko investigated the influence of the volume of the chambers, the pressure drop across the seal, the clearance, the seal stiffness, the location of the support and type of support. From their analytical results, they concluded that stability could be increased by decreasing the pressure drop across the seal, introducing mechanical damping, decreasing the volume of the chambers and increasing the stiffness. Their results also showed no instability for low-pressure side support or support at both sides. The seal clearance was found to have no direct influence on the flutter boundaries. Of course, their model suffered from the same flaw as Ehrich’s model since it did not consider the tangential flow, making the validity of the results questionable for turbomachinery seals. The single-control-volume model used by Lewis et al. [31] included the circumferential flow in the analysis. The isothermal assumption was replaced by an isentropic relation between pressure and density. The leakage model of Egli [18] was used for the fin tip flow. However, this model did not include the contribution of the change in cross-sectional area during vibration; the only source of pressure perturbation came from the change in clearance at the fin tip. The mode shape was used to define the distribution of unsteady clearances and the
2.5 Analysis tools for labyrinth seal stability

cavity wall normal velocity, the latter being used to determine the aerodynamic work. Lewis reported good correlations between the predictions of his analytical model and the behaviour observed in engine test. Abbott [2] used a bulk-flow model similar to the model of Lewis. He successfully used this model to derive the frequency ratio stability criterion still in use today. Srinivasan et al. [50] extended the work of Lewis et al. [31] and Abbott [2] to take into account the flexibility of both seal members. He solved the complex eigenvalue problem of the coupled fluid-structure system composed of the seal stator, the seal rotor and the leakage flow to obtain the system modes and the logarithmic decrement. Using this model, he showed that, when there was a match between rotor and stator frequency, there was a sudden change in logarithmic decrement toward a less stable system. Unfortunately, no comparison of the results of his model with experimental data or with the results of other models were presented.

From the review of the literature, it emerges that bulk-flow models have proven useful in the field of labyrinth seal flutter to determine the critical parameters. However, bulk-flow models heavily rely on empirical correlations [39] and there is a lack of validation of these tools for seal flutter predictions. On this subject, we can cite Lewis et al.’s work [31] who used their analytical model to redesign an unstable seal. The resulting design proved to be stable as expected.

More recently computational fluid dynamics (CFD) techniques have been used to study the unsteady flow in labyrinth seals. Nordmann and Weiser [34] used a two-dimensional finite-difference technique to compute the rotordynamic coefficients of labyrinth seals. The flow model was based on the Navier-Stokes equations coupled with the $k - \epsilon$ turbulence model of Launder and Spalding. They used a coordinate transformation and a perturbation analysis to reduce the problem to the solution of a system of zeroth order and first order differential equations depending only on the axial and (transformed) radial coordinate. The solution of the zeroth order equations gave the steady-state flow while the solution of the first order equations yielded the unsteady pressure field from which
the rotodynamic forces were computed. The $k - \epsilon$ equations were dropped from the first order solution under the assumption of small amplitude perturbations. The results presented showed good agreement with experimental data but were very limited in number. Rhode et al. [39] used a three-dimensional finite difference method to compute the rotodynamic force of an eccentric whirling rotor. They solved the three-dimensional Reynolds-averaged Navier-Stokes equations (3D RANS) coupled with a high Reynolds number $k - \epsilon$ turbulence model. Solving the equations in the frame of reference of the whirling rotor, they were able to transform the unsteady problem into a steady one. Rhodes et al. used such an approach to study the influence of the inlet swirl on the rotodynamic forces. They obtained detailed information on the distributions of the unsteady pressure, tangential velocity and axial velocity in the labyrinth and how they were affected by the swirl. Hirano et al. [24] used a three-dimensional finite volume CFD code to compute the rotodynamic forces in two labyrinth seals. Their results showed important discrepancies between CFD predictions and the predictions of a bulk-flow model. They suggested that these discrepancies could be caused by a reverse flow upstream of the labyrinth observed in CFD simulations which is not considered in the bulk-flow model. These results illustrated the influence of an exact description of the geometry and flow on the rotodynamics predictions. Similar observations were made by Moore [33]. Sayma et al. [43] used a time-accurate coupled fluid-structure approach to study the aeroelastic behaviour of an air riding seal. The fluid was modelled using an unsteady 3D RANS approach. The fluid mesh was moved at each time step to take into account the motion of the seal. Their study highlighted convergence issues when applying CFD techniques in labyrinth seals due to the simultaneous presence of low Mach number and transonic flows. All these studies considered a rigid motion of the seal. di Mare et al. [16] applied the method used by Sayma et al. to a flexible labyrinth seal. The motion of the seal was described by a sum of mode shapes computed using a finite element method (FEM). The three-dimensional flow over the whole seal annulus including the high-pressure
(HP) and low-pressure (LP) cavities was modelled. The aeroelastic stability of nodal diameter modes one to five was investigated. The results showed that the aeroelastic code was able to predict the stability behaviour observed in engine tests. The work also allowed quantifying the relative contribution of each cavity to the instability, this contribution being different for the different modes.

2.6 Design criteria for aeroelastic stability

There are two types of criteria presented in the literature to assess the aeroelastic stability of labyrinth seal designs. The first kind is based on a static aeroelastic analysis. Alford [6] established the value of the radial deflection of a flexible seal under a pressure load with \( n \) lobes around the circumference. This expression depends on the length of the seal, its radius, its mechanical frequency and the pressure drop:

\[
\frac{w}{R} = \frac{n^2 \Delta p L g}{n^2 + 14\pi f^2 W}
\]

(2.6.1)

where \( w \) is the maximum value of the radial deflection, \( R \) the mean radius of the seal, \( n \) the number of lobes around the circumference (nodal diameter number), \( \Delta p \) the pressure drop, \( L \) the length of the seal, \( g \) the acceleration of the gravity, \( f \) the mechanical frequency, and \( W \) the weight. The stability criterion is obtained by setting a limit value for the radial deflection based on engine experience.

The second type is based on the dynamic aeroelastic analysis as described by Ehrich [20] and Abbott [2]. Both authors analysed the unsteady flow in a labyrinth seal under a given seal motion. Ehrich supposed an axisymmetric seal motion and analysed the stability of the coupled fluid-structure system. He derived a theoretical stability parameter, \( S \), indicating if a seal motion was stable or unstable. As mentioned in the previous section, this parameter was only moderately successful due to assumption of axisymmetry made when deriving the model. Abbott extended the model of Ehrich to take into account the flow in the circumferential direction. He did not derive a theoretical expression for the
2.7 Conclusions from review of literature

Stability parameter but computed aerodynamic damping curves for labyrinth seals supported on their low-pressure side (LPS) and high-pressure side (HPS) as a function of the frequency of vibration. After an analysis of these curves, he established the frequency ratio stability criterion. This criterion was found to be in good correlation with test results. However, Abbott pointed out that the criterion could not be applied to some seals where the support type was neither a high-pressure nor a low-pressure side support (seals supported at both ends for example).

Based on engine experience, Bonsell [12] added a design criterion for the aeroelastic stability of labyrinth seals concerning the ratio of the mechanical frequency to the frequency of acoustic modes in the adjacent cavities. To avoid mechanical/acoustic resonance, this ratio should not be in the range 0.9 to 1.1. This criterion was added after seal failures that could be related to acoustic resonance of the downstream cavity were experienced [11]. Alford [4], [7] and Schuck et al. [47] also discussed the importance of mechanical-acoustic coupling in labyrinth seals, and resonance between seal natural vibration mode and adjacent cavity acoustic mode.

2.7 Conclusions from review of literature

The review of the literature shows that:

- the basic mechanism of labyrinth seal flutter is only partially understood; the motion of the seal generates unsteady pressures in the labyrinth due to variations of the clearances at the fin tips; but a physical explanation is still needed in terms of the ratio of the mechanical frequency to the acoustic frequency.

- The influence of parameters, such as the pressure drop or the swirl, on stability remains unclear.
• The reliability of prediction tools based on bulk flow models is not established.

• CFD techniques allow an accurate description of the seal geometry, the flow and the inclusion of actual mode shapes. They provide detailed information on the unsteady flow. They are a valuable tool to investigate the influence of parameters such as the swirl. They are able to predict correctly the aeroelastic stability of an aero engine seal design.

2.8 Objectives of present research project

The following needs have been identified as not being fully addressed from the review of the literature:

• a fundamental understanding of the flutter mechanism;

• an identification of the influence of flow and geometric parameters on stability;

• an assessment of the validity of low-order analysis tools for seal flutter predictions;

• a more reliable aeroelastic design criterion than Abbott's frequency ratio criterion which would include the influence of additional parameters.

Those shortcomings of the present state of the art have a direct impact on the ability of engine manufacturers to produce reliable and competitive engines. To address them, we propose to use the following approach. We will start by a theoretical analysis to gain a better fundamental understanding into the seal flutter mechanism. This is the subject of Chapter 3. This theoretical analysis will also allow to characterise qualitatively the influence of some parameters on stability. We will then use CFD techniques to obtain of more detailed view of the flutter mechanism. These techniques are presented in Chapter 4 and
applied to a typical seal configuration in Chapter 5. The influence of different geometric and flow parameters on stability will be quantified in Chapter 6 by using those same CFD techniques to make a parametric study. This study will help to validate or invalidate the conclusions of the theoretical analysis. Once a clearer picture of the characteristics of seal flutter has been drawn, we propose to investigate a low-order alternative to the use of CFD techniques for seal flutter predictions. The goal is to increase the speed of the analyses. This is the subject of Chapter 7. In Chapter 8, an aeroelastic design methods is proposed for labyrinth seals, based on the findings of the present research project, and applied to an actual seal design. The final Chapter summarises the conclusions from this research project and gives some prospects for future research.
Chapter 3

Theoretical analysis of circumferential waves in an annular cavity

3.1 Introduction

In this chapter, a theoretical analysis is carried out to investigate the characteristics of pressure waves in annular cavities. The walls of the cavities are deflected by mechanical waves travelling around the circumference. The critical influence of the mechanical-to-acoustic frequency ratio on the phase of the unsteady pressure is first demonstrated on a closed-cavity case. A simplified aeroelastic model of a single-cavity labyrinth seal is then derived and the influence of some structural and flow parameters on stability is briefly studied.
3.2 Closed cavity

As mentioned in the literature review, the mechanical-to-acoustic frequency ratio is one of the critical parameters for stability in labyrinth seals. It is instructive to study theoretically the case of a closed annular cavity to understand how the frequency ratio affects the acoustic response. We consider a cavity of square cross section representative of a labyrinth seal cavity. The control volume is presented in Fig. 3.1. We will assume here that viscous effects and heat conduction are negligible. The flow is thus isentropic. The conservation of mass and circumferential momentum in the control volume can be written:

\[
\frac{\partial (\rho Rh)}{\partial t} + \frac{\partial (\rho u_\theta h)}{\partial \theta} = 0 \tag{3.2.1}
\]

\[
\frac{\partial (\rho u_\theta Rh)}{\partial t} + \frac{\partial (\rho u_\theta^2 h)}{\partial \theta} + h \frac{\partial p}{\partial \theta} = 0 \tag{3.2.2}
\]

where \( p \) is the pressure, \( \rho \) the density, \( u_\theta \) the circumferential velocity, \( h \) the cavity height and \( R \) its mean radius. The isentropic law for a perfect gas states that the ratio \( \frac{p}{\rho^2} \) is constant or, in differential form, \( dp = \bar{a}^2 d\rho \), where \( \bar{a} \) is the mean speed of sound. The unsteady flow can be studied by carrying a perturbation analysis. The flow quantities may be decomposed into two parts,
one steady and the other unsteady:

\[ \rho = \bar{\rho} + \rho' \]
\[ u_{\theta} = u' \]
\[ p = \bar{p} + p' \]
\[ h = \bar{h} + h' \]
\[ R = \bar{R} + R' \]

where \( \bar{\cdot} \) denotes the steady component and \( ' \) the unsteady one. The first component is independent of \( \theta \) since the steady-state solution is a uniform pressure in the cavity with no flow. Assuming small perturbations, we may linearise the mass and momentum equations. We can then use the isentropic law in addition to these two equations to obtain a single partial differential equation for the pressure:

\[
\frac{\partial^2 p'}{\partial t^2} - \frac{\alpha^2}{R^2} \frac{\partial^2 p'}{\partial \theta^2} = -\bar{\rho} \alpha^2 \left( \frac{1}{h} \frac{\partial^2 h'}{\partial t^2} + \frac{1}{R} \frac{\partial^2 R'}{\partial t^2} \right)
\]  

(3.2.3)

In the above equation, the deformations of the upper and lower wall appear as source terms for the pressure waves via \( R' \) and \( h' \). If we consider the case of a lower wall deformed by a travelling wave and a rigid upper wall, which is representative of a vibrating rotor and fixed stator in a labyrinth seal, we have:

\[ R_2' = 0 \]  
(3.2.4a)
\[ R_1' = R_0 e^{i(2\pi ft + n\theta)} \]  
(3.2.4b)
\[ h' = R_2' - R_1' = -R_0 e^{i(2\pi ft + n\theta)} \]  
(3.2.4c)
\[ R' = \frac{R_1' + R_2'}{2} = \frac{R_0}{2} e^{i(2\pi ft + n\theta)} \]  
(3.2.4d)

where \( n \) and \( f \) are respectively the number of nodal diameters and the angular frequency of the considered mode of vibration. We look for a travelling wave solution for the pressure of the form \( p' = p_0 e^{i(2\pi ft + n\theta)} \), where \( p_0 \) is the amplitude.
3.3 Single-cavity labyrinth seal

function. After substitution in Eq. (3.2.3), we obtain:

\[ p_0 = -p_\infty H(r_f) \]  \hspace{1cm} (3.2.5a)

\[ p_\infty = \rho a^2 \left( 1 - \frac{k}{2R} \right) \frac{R_0}{h} \]  \hspace{1cm} (3.2.5b)

\[ H(r_f) = \frac{(r_f)^2}{1 - (r_f)^2} \]  \hspace{1cm} (3.2.5c)

\[ r_f = \frac{f}{f_{ac}} \]  \hspace{1cm} (3.2.5d)

\[ f_{ac} = \frac{n\dot{a}}{2\pi R} \]  \hspace{1cm} (3.2.5e)

\( p_\infty \) is the amplitude of the pressure wave at an infinite frequency ratio and \( r_f \) is the mechanical-to-acoustic frequency ratio for this particular case. The modulus and phase of \( p_0 \) are plotted in Fig. 3.2 against \( f/f_{ac} \). The unsteady pressure is in phase with the wall motion when the mechanical frequency is higher than the acoustic frequency, in phase opposition otherwise. Thus, when the frequency ratio is higher than one, the pressure in a section increases when the area of that section decreases, which is the variation we would have if the configuration was two-dimensional and no flow was possible in the tangential direction; this pressure variation tends to oppose the reduction of section and thus the wall motion. For a frequency ratio lower than one, the pressure decreases when the section decreases; this pressure variation is compliant to the wall motion. These results will be used when investigating the single-cavity labyrinth seal case in the following section. When both frequencies match, there is a resonance phenomenon.

3.3 Single-cavity labyrinth seal

3.3.1 Governing equations

We now proceed to analyse the case of a single-cavity labyrinth seal. The control volume is presented in Fig. 3.3. The lower wall rotates about a pivot point \( P \),
3.3 Single-cavity labyrinth seal

Figure 3.2: Variation of modulus and argument of unsteady pressure amplitude function \( p_0 \) with mechanical-to-acoustic frequency ratio \( f/f_{ac} \) in closed cavity with moving lower wall.

Figure 3.3: Control volume for single-cavity labyrinth seal (x-r plane view). The lower wall rotates about pivot point \( P \).
whose axial position $x_p$ is measured relative to the centre of the cavity $O$. The rotation causes a change in the clearances $c_1$ and $c_2$ and in area of the section. This structural model is representative of actual seal mode shapes. The mass and circumferential momentum conservation equations may be written:

$$\frac{\partial (\rho Rh)}{\partial t} + \frac{\partial (\rho u_\theta h)}{\partial \theta} = \frac{1}{L} (m_1 - m_2)$$ (3.3.1)

$$\frac{\partial (\rho u_\theta Rh)}{\partial t} + \frac{\partial (\rho u_\theta^2 h)}{\partial \theta} + h \frac{\partial p}{\partial \theta} = 0$$ (3.3.2)

where $m_1$ and $m_2$ are the inlet and outlet mass flow rate per unit angle which can be expressed as:

$$m_i = \rho_i v_i c_i R_{ci}$$ (3.3.3)

where $\rho_i$ is the average density on top of the fin, $v_i$ the bulk velocity, $R_{ci}$ the clearance radius and $c_i$ the clearance. The indices 1 and 2 will be used in the following to designate flow quantities on top of the inlet and outlet fin respectively. In the derivation of Eq. (3.3.2), the assumption has been made that there is no transport of circumferential momentum at the inlet and outlet. We assume that there is no inlet swirl; however, nothing prevents the circumferential momentum in the cavity from being transported by the mean flow at the outlet. In this section, we will neglect the latter contribution to simplify the analysis. The consequences of taking into account the transport of circumferential momentum at the outlet will be examined in Section 3.4. For small motions of the seal, the flow and geometric quantities may be decomposed into their steady component and a perturbation:

$$q = \bar{q} + q'$$

To simplify the notations the $\bar{}$ symbol will be dropped in the following and $u_\theta$ will simply be noted $u$. For small perturbations, the equations may be linearised. Combining Eq. (3.3.1) and (3.3.2) and assuming isentropic perturbations ($p' = \dots$)
3.3 Single-cavity labyrinth seal

\( a^2 \rho' \) yields to the following differential equation for the pressure perturbation:

\[
\frac{1}{\rho a^2} \left( \frac{\partial^2 p'}{\partial t^2} + \frac{2 u}{R} \frac{\partial^2 p'}{\partial t \partial \theta} + \frac{u^2 - a^2}{R^2} \frac{\partial^2 p'}{\partial \theta^2} \right) =
\]

\[
- \frac{1}{h} \left( \frac{\partial^2 h'}{\partial t^2} + \frac{2 u}{R} \frac{\partial^2 h'}{\partial t \partial \theta} + \frac{u^2}{R^2} \frac{\partial^2 h'}{\partial \theta^2} \right)
\]

\[
- \frac{1}{R} \left( \frac{\partial^2 R'}{\partial t^2} + \frac{u}{R} \frac{\partial^2 R'}{\partial \theta \partial t} \right)
\]

\[
+ \frac{1}{\rho LRh} \left( \frac{\partial(m_1' - m_2')}{\partial t} + \frac{u}{R} \frac{\partial(m_1' - m_2')}{\partial \theta} \right)
\]  (3.3.4)

For large-diameter aero engine \( R \gg h; \) as \( R' \sim h', \frac{R'}{R} \ll \frac{h'}{h} \) and the contribution of \( R' \) may be neglected. From Eq. (3.3.3), we have for the mass flow rate \( m \) at the inlet or outlet:

\[
\frac{dm}{m} = \frac{d\rho}{\rho} + \frac{dv}{v} + \frac{dc}{c} + \frac{dR_c}{R_c}
\]  (3.3.5)

The variation of the radial clearance \( dR_c \) is of the same order of magnitude as \( dc \) and, for a large-diameter seal, the term \( \frac{dR_c}{R_c} \) is negligible in comparison with \( \frac{dc}{c} \). \( d\rho \) is related to \( dp \) by the pressure-density isentropic relationship \( \frac{d\rho}{\rho} = \frac{1}{\gamma} \frac{dp}{p} \). \( dv \) can be determined by relating it to the total pressure variation. The total pressure \( p_t \) and the Mach number \( M \) are defined as follows:

\[
p_t = p \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}}
\]  (3.3.6)

\[
M = \frac{v}{\sqrt{\frac{\gamma p}{\rho}}}
\]  (3.3.7)

The differentiation of these relations gives:

\[
\frac{dp_t}{p_t} = \frac{dp}{p} + \frac{\gamma M dM}{1 + \frac{\gamma - 1}{2} M^2}
\]  (3.3.8)

\[
\frac{dM}{M} = \frac{dv}{v} - \frac{1}{2} \frac{dp}{p} + \frac{1}{2} \frac{dp}{\rho}
\]  (3.3.9)
Combining these two relations with the pressure-density isentropic relationship gives:

\[
\frac{dp_t}{\rho_t} = \frac{1}{1 + \frac{\gamma - 1}{2} M^2} \left( \frac{dp}{\rho} + \gamma M^2 \frac{dv}{v} \right) \quad (3.3.10)
\]

At the inlet fin, the total pressure is equal to the total pressure upstream of the cavity which is constant. So \( dp_{i1} = 0 \) and we have:

\[
\frac{dv_t}{v_t} = -\frac{1}{\gamma M_t^2} \frac{dp_t}{\rho_t} \quad (3.3.11)
\]

and:

\[
\frac{dm_{i1}}{m} = \frac{M_t^2 - 1}{\gamma M_t^2} \frac{dp_t}{\rho_t} + \frac{dc_{i1}}{c} \quad (3.3.12)
\]

The static pressure at the exit of the inlet fin is equal to the static pressure in the cavity. So:

\[
p_1 = p \quad (3.3.13)
\]

\[
p_1' = p' \quad (3.3.14)
\]

For an ideal seal, the kinetic energy at the inlet fin is dissipated in the cavity, and the total pressure upstream of the outlet fin is equal to the static pressure in the cavity. In actual seals, part of the kinetic energy is carried-over onto the next fin [18]. We can take this into account with a loss model for the total pressure:

\[
p_{01} - p_{02} = K_e (p_{01} - p) \quad (3.3.15)
\]

The coefficient \( K_e \) is an expansion loss coefficient [26], which is unity when there is no carry-over. This coefficient depends on the clearance and the expansion angle of the carry-over jet [52]. Though its value is not constant during vibration, we will neglect its variation here since we are only after qualitative results. The static pressure at the outlet fin is the discharge pressure of the seal which is supposed fixed; this is a reasonable assumption if the cavity downstream of the
3.3 Single-cavity labyrinth seal

Seal has a large volume and if the excitation frequency is not close to an acoustic frequency of this cavity. Hence $dp_2 = 0$. Using Eq. (3.3.10) and (3.3.15) at the outlet fin gives:

$$\frac{dm_2}{m} = \frac{1 + \frac{\gamma - 1}{2} M_2^2}{\gamma M_2^2} \frac{K_e dp}{K_e p + (1 - K_e)p_{01}} + \frac{dc_2}{c} \quad (3.3.16)$$

This relation is valid as long as the outlet fin is not choked. For a choked outlet fin, quantities must be evaluated at the sonic section since no information can travel upstream of this section. At the sonic section $M_2^* = 1$ and $dM_2^* = 0$. Eq. (3.3.8) and (3.3.9) give:

$$\frac{dp_2^*}{p_2^*} = \frac{dp_{02}}{p_{02}} \quad (3.3.17)$$

$$\frac{dv_2^*}{v_2^*} = \frac{1}{2} \frac{dp_2^*}{p_2^*} - \frac{1}{2} \frac{dp_2^*}{p_2^*} = \frac{\gamma - 1}{2\gamma} \frac{dp_2^*}{p_2^*} \quad (3.3.18)$$

where the pressure-density isentropic relationship has been used. And:

$$\frac{dm_2}{m_2} = \frac{dp_2^*}{p_2^*} + \frac{dv_2^*}{v_2^*} + \frac{dc_2}{c} = \frac{\gamma + 1}{2\gamma} \frac{dp_2^*}{p_2^*} + \frac{dc_2}{c} \quad (3.3.19)$$

The pressure-density isentropic relationship has again been used. Using Eq. (3.3.17) and (3.3.15), we finally obtain:

$$\frac{dm_2}{m} = \frac{\gamma + 1}{2\gamma} \frac{K_e dp}{K_e p + (1 - K_e)p_{01}} + \frac{dc_2}{c} \quad (3.3.20)$$

Taking $M_2 = 1$ in Eq. (3.3.16) gives the same result.

Equations (3.3.12) and (3.3.16) give for the unsteady mass flow difference:

$$\frac{m'_1 - m'_2}{m} = -\psi \frac{p'}{p} + \frac{c'_1 - c'_2}{c} \quad (3.3.21)$$

with:

$$\psi = -\left( \frac{M_1^2 - 1}{\gamma M_1^2} - \frac{1 + \frac{\gamma - 1}{2} M_2^2}{\gamma M_2^2} \frac{K_e}{K_e + (1 - K_e)p_{01}} \right) \quad (3.3.22)$$
3.3 Single-cavity labyrinth seal

$h'$ can also be related to the change of inlet and outlet clearance:

$$h' = \frac{c'_1 + c'_2}{2} \quad (3.3.23)$$

Using these results yields the following differential equation for $p'$:

$$\frac{1}{\rho a^2} \left[ \frac{\partial^2 p'}{\partial t^2} + 2 \frac{u}{R} \frac{\partial p'}{\partial t} + \frac{u^2}{R^2} \frac{\partial^2 p'}{\partial \theta^2} + \chi \left( \frac{\partial p'}{\partial t} + \frac{u}{R} \frac{\partial p'}{\partial \theta} \right) \right] =$$

$$- \frac{1}{h} \left( \frac{\partial^2}{\partial t^2} + 2 \frac{u}{R} \frac{\partial}{\partial t} + \frac{u^2}{R^2} \frac{\partial^2}{\partial \theta^2} \right) \frac{c'_1 + c'_2}{2}$$

$$+ \frac{m}{\rho LRh} \frac{1}{c} \left( \frac{\partial}{\partial t} + \frac{u}{R} \frac{\partial}{\partial \theta} \right) (c'_1 - c'_2) \quad (3.3.24)$$

with:

$$\chi = \frac{\psi ma^2}{LRhp} \quad (3.3.25)$$

Using the fact that $m = \rho v c R_c$, and that, for large-diameter aero engine, $R_c \sim R$, gives for $\chi$:

$$\chi = \gamma c \frac{1}{h} \frac{1}{T_b} \psi \quad (3.3.26)$$

where $T_b = \frac{L}{v}$ is the time taken by a fluid particle to go through the cavity (transit time). We look for a travelling wave solution of the form:

$$c'_1 = c_{10} e^{i(2\pi ft + n\theta)} \quad (3.3.27a)$$

$$c'_2 = c_{20} e^{i(2\pi ft + n\theta)} \quad (3.3.27b)$$

$$p' = p_0 e^{i(2\pi ft + n\theta)} \quad (3.3.27c)$$

Inserting these relations in Eq. (3.3.24) yields the following expression for $p_0$:

$$p_0 = \rho a^2 \frac{c}{h} \left( \frac{f^*}{f_{ac}} \right)^2 \frac{c_{10} + c_{20}}{2c} + i \left( \frac{1}{2\pi f_{ac} T_b} \right) \left( \frac{f^*}{f_{ac}} \right) \frac{c_{10} - c_{20}}{c}$$

$$1 - \left( \frac{f^*}{f_{ac}} \right)^2 + i \gamma \frac{c}{h} \psi \left( \frac{1}{2\pi f_{ac} T_b} \right) \left( \frac{f^*}{f_{ac}} \right)$$

$$f_{ac} = \frac{an}{2\pi R}$$

$$f^* = f - \frac{un}{2\pi R}$$
For a seal supported at the high-pressure side, \( c_{20} > c_{10} \). For a seal supported at the low-pressure side, \( c_{10} < c_{20} \).

Let \( r' = r_0 \cos(2\pi t + n\theta + \phi_r) \) be the radial deflection of the seal rotor cylinder, positive downward. The aerodynamic work during a cycle of vibration is given by:

\[
W = \int_S \left( \int_0^T p' v' \, dt \right) \, dS
\]

\[
v' = \frac{\partial r'}{\partial t} = -r_0 \pi f \sin(2\pi ft + n\theta + \phi_r)
\]

\[
p' = \left| p_0 \right| \cos(2\pi ft + n\theta + \phi_p)
\]

\[
W = 2\pi RL \left( \pi r_0 \left| p_0 \right| \sin(\Delta \phi) \right)
\]

(3.3.29)

where \( \Delta \phi = \phi_p - \phi_r \) is the pressure-displacement phase difference. The average deflection of the rotor \( r_0 \) is \( \frac{c_{10} + c_{20}}{2} \).

### 3.3.2 Application

The present model has been applied to the flow in a single-cavity labyrinth seal for which CFD results were available. The geometry of the configuration is shown in Fig. 3.4. The steady-state computed with CFD is used to determine the steady flow values appearing in Eq. (3.3.28). The nominal conditions for this case are given in Table 3.1. All the results presented here are obtained at these conditions unless otherwise specified.

<table>
<thead>
<tr>
<th>Clearance ( c_{nom} ) (mm)</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity height ( h_{nom} ) (mm)</td>
<td>3.264</td>
</tr>
<tr>
<td>Cavity length ( L ) (mm)</td>
<td>4.2</td>
</tr>
<tr>
<td>Seal pitch ( p ) (mm)</td>
<td>4.5</td>
</tr>
<tr>
<td>Pressure ratio ( \Pi )</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 3.1: Nominal conditions for single-cavity labyrinth seal case.

In Fig. 3.5, the amplitude and phase of the unsteady pressure are plotted against
3.3 Single-cavity labyrinth seal

Figure 3.4: Single-cavity labyrinth seal geometry.

the mechanical-to-acoustic frequency ratio for different pivot point locations. A positive phase leads here to an aerodynamically unstable configuration as indicated in the figure. In the amplitude plot, the curves for \( x_p/p = \pm 1 \) and \( x_p/p = \pm 0.5 \) are on top of each other since, as shown below, the amplitude of the unsteady pressure depends only on the absolute value of \( x_p \). This amplitude reaches a maximum around a frequency ratio of 1 as could be expected due to the acoustic resonance. This amplitude decreases when the pivot point is moved away from the centre. For all pivot point locations, the amplitude of motion is chosen so that the clearance changes by 1%. It can be shown (see Appendix C) that the dependence of the magnitude of \( p_0 \) on the pivot point location is of the form:

\[
|p_0| \sim \sqrt{\frac{x_p}{p}^2 + \left(\frac{1}{2\pi f^* T_b}\right)^2 \left(\frac{x_p}{p} + \frac{1}{2}\right)}
\]  (3.3.30)

This function is plotted for different values of \( f^* T_b \) in Fig. 3.6. This function is strictly decreasing for \( |x_p|/p < 2/(2\pi f^* T_b)^2 \) and strictly increasing afterward.

In the present case, \( 2\pi f^* T_b \) varies between 0.036 and 0.14 when \( f^*/f_{oc} \) varies between 0.5 and 2; thus, for the range of pivot point locations investigated, we see only the decreasing part. These results illustrate the influence of the \( f^* T_b \) on the unsteady pressure. This parameter can be expressed differently.
3.3 Single-cavity labyrinth seal

Figure 3.5: Single-cavity labyrinth seal: unsteady pressure $p_0$. The top plot shows the variation of the amplitude of $p_0$ with the mechanical-to-acoustic frequency ratio $f/f_{ac}$ for different locations of the pivot point $x_p/p$. The bottom plot shows the variation of the phase. Nominal conditions, Table 3.1.
Figure 3.6: Single-cavity labyrinth seal: influence of value of parameter $f^*T_b$ on variation of unsteady pressure amplitude $|p_0|$ with pivot point location (from Eq. (3.3.30)). The amplitude is strictly decreasing for $|x_p|/p < 2/(2\pi f^*T_b)^2$. At the nominal conditions, $2\pi f^*T_b$ varies between 0.036 and 0.14 when $f^*/f_{ac}$ varies between 0.5 and 2.
Introducing the Mach number of the flow $M = v/a$, we have:

$$f^* T_b = \frac{1}{M} \frac{nL}{2 \pi R} f^* f_{ac}$$  \hspace{1cm} (3.3.31)

Even if the Mach number $M$ and the frequency ratio $f^*/f_{ac}$ are kept constant, $f^* T_b$ can still vary through the ratio $nL/R$. Thus $f^* T_b$ is independent of both $M$ and $f^*/f_{ac}$ and should be considered as an additional similarity parameter. This will be discussed further in Section 3.5.

The phase of the unsteady pressure $p_0$, relative to the phase of the seal motion $r_0$ (positive downward), is centred around 180 degrees for $x_p/p < 0$ (high-pressure support) and 0 degree for $x_p/p > 0$ (low-pressure support). The middle support configuration $x_p/p = 0$ is not plotted since, for this configuration, $r_0 = 0$ and we cannot define the phase of the displacement. The phases of the numerator and denominator in expression (3.3.28) are shown in Fig. 3.7 for $x_p/p = -1$ and $x_p/p = 1$. The phase of the numerator is around -90 degrees for $x_p/p = -1$ and +90 degrees for $x_p/p = 1$ over the whole frequency range. This indicates that the imaginary part, which is due to the unsteady mass flow at the inlet and outlet, is dominant. The change in sign comes from the change in sign of $c_{10} - c_{20}$. The denominator does not depend on the pivot point location. Its phase is around zero degree at very low frequency ratios and would be around 180 degrees at high frequency ratios. This is similar to the closed cavity case. At frequency ratios not too far from unity, the phase of the denominator is close to +90 degrees and its imaginary part, coming from the unsteady mass flow at the inlet and outlet, is dominant. The imaginary part of the numerator is the contribution of the clearance variation to this unsteady flow. That of the denominator is the contribution of the changes in density and velocity at the fin tips. The fact that they are both dominant for frequency ratios around unity indicates that the changes in density and velocity at the fin tips counterbalance the changes in clearance; thus the inlet/outlet mass flow difference remains small. This can also be shown by making an analysis of the order of magnitude
3.3 Single-cavity labyrinth seal

Figure 3.7: Single-cavity labyrinth seal: variation of phase of numerator and denominator of analytical expression for unsteady pressure \( p_0 \) (Eq. (3.3.28)) with mechanical-to-acoustic frequency ratio \( f/f_{ac} \). The pivot point is upstream of the cavity for \( x_p/p = -1 \) and downstream for \( x_p/p = 1 \). Nominal conditions, Table 3.1.
of the different terms in the mass conservation equations. To this end, we write the linearised mass conservation equation in dimensionless form:

$$\frac{i2\pi f \rho R_h L}{m} \left( \frac{\rho'}{\rho} + \frac{R'}{R} + \frac{h'}{h} \right) - in \frac{\rho a h L u'}{m} a = \frac{m'_1 - m'_2}{m}$$  \hspace{1cm} (3.3.32)

The linearised momentum equation reads:

$$i2\pi f \rho R u' - in p' = 0$$  \hspace{1cm} (3.3.33)

Dividing this equation by $in \rho a^2 = in \gamma p$ gives:

$$\frac{f u'}{f_{ac} a} = \frac{1}{\gamma} \frac{p'}{p}$$  \hspace{1cm} (3.3.34)

$R'/R$ and $h'/h$ are both negligible compared to $c'/c$. And $p'/p$ is of the same order of magnitude as $c'/c$ as can be seen from the expression of $p_0$. Thus $R'/R$ and $h'/h$ can be dismissed in Eq. (3.3.32). From the isentropic law, we have:

$$\frac{e'}{\rho} = \frac{1}{\gamma} \frac{e'}{p}.$$  \hspace{1cm} Using Eq. (3.3.3) for $m$, we have:

$$\frac{f \rho R_h L}{m} = \frac{h}{c} \frac{R}{R_c} f_{ac} T_b \frac{f}{f_{ac}}$$  \hspace{1cm} (3.3.35)

$$n \frac{\rho a h L}{m} = \frac{h}{c} \frac{R}{R_c} 2\pi f_{ac} T_b$$  \hspace{1cm} (3.3.36)

For the application presented in the preceding the group $\frac{h}{c} \frac{R}{R_c} 2\pi f_{ac} T_b$ is approximately 0.5. Eq. (3.3.32) can be rewritten:

$$i \frac{h}{c} \frac{R}{R_c} 2\pi f_{ac} T_b \left( \frac{f}{f_{ac}} - \frac{f_{ac}}{f} \right) \frac{1}{\gamma} \frac{p'}{p} = \frac{m'_1 - m'_2}{m}$$  \hspace{1cm} (3.3.37)

When the mechanical-to-acoustic frequency ratio is not too far from unity and the ratio $\frac{h}{c} \frac{R}{R_c} 2\pi f_{ac} T_b$ is of order unity or lower, the left-hand side of the mass conservation equation is negligible and we must have $m'_1 - m'_2 \approx 0$. To ensure this, the flow quantities at the fin tips will vary in such a way as to keep the inlet and outlet mass flow rates approximately equal.
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We can now explain the phase between the pressure and seal motion (for frequency ratios not too far from unity). For a high-pressure support mode, when the seal moves down (positive change of clearances), the outlet clearance increases more than the inlet clearance. This will induce an excess outflow. Eq. (3.3.12) shows that, to reduce this disequilibrium, the pressure must decrease to increase the inflow (since $M_i < 1$). For a low-pressure support mode, the mass flow disequilibrium will be reversed and an increase of the pressure will be necessary. Hence the phase relationship between $p_0$ and $r_0$.

We also notice that the phase changes sign around a frequency ratio equal to one for HP side and LP side support. We can use the analysis of a closed annular cavity carried out in the previous chapter to explain this phenomenon. To this end, we decompose the cavity between two fins into a through-flow area and a cavity vortex area separated by a dividing streamline as shown in Fig. 3.8. The vortex area is a closed annular cavity as the one previously studied. In this model, the change in inlet and outlet clearances during the seal motion will directly affect the pressure in the through-flow area. The change in pressure in the through-flow area will induce a motion of the dividing streamline that will affect the surface of the vortex area. The vortex area being a closed annular cavity, for a frequency ratio lower than one, the pressure variation in the vortex area will be compliant to the motion of the dividing streamline (which acts as the upper "wall" of the vortex area), while it will oppose that motion for a frequency ratio higher than one according to Section 3.2. Thus the interaction with the vortex area will induce a phase lead of the pressure over the seal motion for a frequency ratio lower than one and a phase lag for a frequency ratio higher than one, which is exactly what the change of sign of the phase of $p_0$ indicates. One could argue that the present analytical model does not decompose the cavity into two control volumes. Nor does it model the motion of a dividing streamline. This division was used here only to exploit the theoretical results available on a closed cavity directly. However, the proclivity of the cavity to retard or hasten the pressure variation depending on the frequency ratio can still be advocated.
3.3 Single-cavity labyrinth seal

Figure 3.8: Through-flow and vortex areas in a cavity of a labyrinth seal. The two flow areas are separated by the dividing streamline.
without using the dividing streamline as an intermediary. This explains why, even with a single control volume for the cavity, we are able to get the correct trends.

The two previous arguments concerning the influence of the support side and the mechanical-to-acoustic frequency ratio on the phase between pressure and displacement lead to the Argand diagram presented in Fig. 3.9.

Figure 3.9: Single-cavity labyrinth seal: Argand diagram showing the phase difference $\Delta \phi$ between unsteady pressure $p_0$ and seal downward motion $r_0$.

Knowing this phase relationship, one can explain qualitatively the observed influence of location of support and frequency ratio on labyrinth seal aeroelastic stability. For a high-pressure support mode for instance, a downward motion of the seal will induce a decrease in pressure. If the mechanical frequency is lower than the acoustic frequency, the pressure variation will lead the displacement. This means that, when the seal starts moving downward from its equilibrium position, the pressure has already decreased in the cavity. The negative unsteady pressure creates an upward unsteady pressure force which opposes the
wall motion. Thus the configuration is stable. This is illustrated in Fig. 3.10.

On the other hand, if the mechanical frequency is higher than the acoustic fre-

quency, the pressure lags behind the displacement and, when the seal starts its
downward motion, the unsteady pressure is still positive and creates a down-
ward unsteady pressure force helping the motion. This leads to an unstable
configuration.

Plots of the logarithmic decrement as a function of the frequency ratio, presented
in Fig. 3.11, agree well with our analysis: HP support is unstable when the
frequency of vibration is higher than the acoustic frequency, and LP support
when the frequency of vibration is lower than the acoustic frequency. These
results are also in agreement with those presented by Abbott [2]. We notice
that the logarithmic decrement does not change sign at a frequency ratio exactly
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Figure 3.11: Single-cavity labyrinth seal: logarithmic decrement $\delta$ vs mechanical-to-acoustic frequency ratio $f/f_{ac}$ for a HP support mode ($x_p/p = -1$) and a LP support mode ($x_p/p = 1$). Nominal conditions, Table 3.1.
equal to one. The limit of stability predicted by the analytical model is given by the following formula:

\[
\frac{f}{f_{ac}} = \sqrt{1 + \frac{x_p}{p} \frac{c}{\gamma H \psi}}
\]  

(3.3.39)

This frequency ratio is plotted against \(x_p/p\) in Fig. 3.12. The frequency ratio decreases monotonically when the pivot point is moved from upstream to downstream and tends towards zero when \(x_p/p\) tends toward infinity. Moreover, the ratio tends towards infinity when \(x_p/p\) tends towards \(-1/(\gamma H \psi)\) (\(\approx -3.3\) for the present application). When the pivot point is upstream of this axial location, there is no real solution for \(f/f_{ac}\) and the model predicts unconditional stability. When \(|x_p/p|\) increases, the inlet/outlet clearance difference \(c_{10} - c_{20}\), which is the source of the (potentially) destabilising aerodynamic force, decreases in magnitude. When \(|x_p/p| > 1/(\gamma H \psi)\), the destabilising force induced by the clearance difference is small enough to be compensated by the stabilising effect.

Figure 3.12: Single-cavity labyrinth seal: evolution of limit of stability with pivot point location \(x_p/p\) (from Eq. (3.3.39)). Nominal conditions, Table 3.1.
3.3 Single-cavity labyrinth seal

of the changes in density and velocity at the fin tips, which, as explained before, tend to oppose to disequilibrium caused by the clearance difference. Thus, there is no instability.

To evaluate the validity of the simplified analytical model derived in the previous section, we can compare its results to CFD results.

Figure 3.13 compares CFD and analytical results concerning the influence of the position of the pivot point at different frequency ratios. Results are in reasonable qualitative agreement except for $f/f_{ac} = 1$ where the analytical model fails to reproduce the stability characteristic obtained using CFD simulations. These discrepancies at $f/f_{ac} = 1$ are observed in all the results presented here. At this frequency ratio, where resonance would occur in a closed cavity, the modelling of the unsteady mass flow at the inlet and outlet of the cavity becomes critical. This can be seen clearly from Eq. (3.3.28). When the frequency ratio $f^*/f_{ac}$ is equal to one, the denominator reduces to its imaginary part. As explained earlier, the latter represents the contributions of the change in average density and velocity on top of the fins to the unsteady mass flow rate at the inlet and outlet of the cavity. The variation of the flow quantities at the fin tips has been linked to the variation of the pressure in the cavity via simplified relations like the loss model (3.3.15). Those simplifications appear to be inappropriate at "resonance", and a more accurate modelling would be needed. The situation is similar to the one encountered in structural vibrations where damping becomes prominent at resonance.

The analytical model can be used to investigate the influence of parameters whose influence is less well known on seal aeroelastic stability. Figure 3.14 shows results concerning the influence of the clearance. The values on the x axis are the clearances divided by the nominal clearance. Although not quantitative, the analytical results are again in qualitative agreement with CFD results (except at a frequency ratio of 1). Both analytical model and CFD predict an increase in magnitude of the logarithmic decrement when the clearance is reduced.
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Figure 3.13: Single-cavity labyrinth seal: influence of pivot point position \( x_p/p \) on logarithmic decrement \( \delta \). Nominal conditions, Table 3.1.
3.3 Single-cavity labyrinth seal

Figure 3.14: Single-cavity labyrinth seal: influence of clearance $c$ on logarithmic decrement $\delta$. Nominal conditions, Table 3.1, except for clearance. High-pressure support mode ($x_p/p = -1$).
3.3 Single-cavity labyrinth seal

The influence of the clearance on the curves of logarithmic decrement against frequency ratio is shown in Fig. 3.15. These curves show that the clearance affect primarily the magnitude of the logarithmic decrement. However, it can be seen that the clearance seems to have a slightly stabilising effect according to the theoretical model. This effect is more visible for high-pressure support for frequency ratios close to the limit of stability. The stabilising effect of an increase in clearance is in agreement with the observations of Lewis et al. [31] as mentioned in the review of the literature.

The influence of a significant increase in clearance for modes which are unstable at the nominal clearance is presented in Fig. 3.16. We see that in both cases, the analytical model predicts a stabilisation of the modes above a certain clearance. For a high-pressure support mode, a doubling of the clearance is enough to stabilise the mode, whereas for the low-pressure support mode the clearance needs to be increased tenfold. The stability at larger clearances is somewhat in agreement with findings by Lewis et al. [31]. By carrying out dimensional inspections on cracked seals, they found that their clearance was in the tightest range of the design clearance tolerance, which would imply that, in their case, instability occurred only for small clearances. Analysis of Eq. (3.3.28) shows that the stabilising effect of the clearance comes from the imaginary part of the denominator which becomes dominant for large clearances. Results obtained with CFD methods concerning the influence of the clearance will be presented in more detail in the parametric study chapter.

The predicted influence of the pressure ratio is presented in Fig. 3.17 and 3.18, for a high-pressure support mode and a low-pressure support mode at different frequency ratios. Looking at the results for \( f/f_{ac} = 1.5 \) for the high-pressure support mode and \( f/f_{ac} = 0.5 \) for the low-pressure support mode, we notice that both analytical model and CFD predict that below a certain pressure ratio the mode becomes stable. Fig. 3.19 shows how the curves of logarithmic decrement as a function of the frequency ratio evolve when the pressure ratio is modified.
3.3 Single-cavity labyrinth seal

Figure 3.15: Single-cavity labyrinth seal: logarithmic decrement $\delta$ vs mechanical-to-acoustic frequency ratio $f/f_{ac}$ for different values of the clearance $c$. Nominal conditions, Table 3.1, except for clearance. Top: high-pressure support mode ($x_p/p = -1$). Bottom: low-pressure support mode ($x_p/p = 1$).
Figure 3.16: Single-cavity labyrinth seal: influence of large increase in clearance on logarithmic decrement $\delta$ for two modes unstable at nominal clearance $c_{nom}$: high-pressure support mode ($x_p/p = -1$) at $f/f_{ac} = 1.5$ and low-pressure support mode ($x_p/p = 1$) at $f/f_{ac} = 0.5$. Nominal conditions, Table 3.1, except for clearance.
Figure 3.17: Single-cavity labyrinth seal: influence of pressure ratio $\Pi$ on logarithmic decrement $\delta$. Nominal conditions, Table 3.1, except for pressure ratio. High-pressure support mode ($x_p/p = -1$).
3.3 Single-cavity labyrinth seal

Figure 3.18: Single-cavity labyrinth seal: influence of pressure ratio $\Pi$ on logarithmic decrement $\delta$. Nominal conditions, Table 3.1, except for pressure ratio. Low-pressure support mode ($x_p/p = 1$).
At low pressure ratio, both high-pressure support and low-pressure support are stable over the whole frequency range and the magnitude of damping is low. As the pressure ratio is increased, the magnitude of the damping increases and the damping becomes negative at high frequency ratio for high-pressure support and low frequency ratio for low-pressure support. The range of frequency ratios where the damping is negative increases in size as the pressure ratio is further increased.

The origin of the stability at low pressure ratio/low Mach number can be investigated by analysing Eq. (3.3.28). The terms depending on the Mach number are $2\pi f_{ac} T_b$ and $\psi$. $1/(2\pi f_{ac} T_b)$ can be rewritten:

$$f_{ac} T_b = \frac{na L}{2\pi R v} = \frac{1}{M} \frac{L}{\lambda}$$

(3.3.40)

where $M$ is some average Mach in the through-flow area, and $\lambda = 2\pi R/n$ the wavelength of the mode. The inlet and outlet Mach number $M_1$ and $M_2$ appearing in $\psi$’s expression will be close to $M$ and can be replaced by $M$ for the purpose of the analysis and:

$$\psi = -\left( \frac{M^2 - 1}{\gamma M^2} - \frac{1 + \frac{2}{\gamma M^2} - \frac{K_e}{K_e + (1 - K_e) \frac{M_1}{M_2}}} \right)$$

Thus $\psi/(2\pi f_{ac} T_b)$ appearing in the denominator tends toward infinity as $1/M$ when $M$ tends towards zero and becomes the dominant term. At the numerator, $1/(2\pi f_{ac} T_b)$ tends towards zero and the imaginary part becomes negligible. Thus, in the limit of small Mach numbers, we have:

$$p_0 \approx \rho a^2 \frac{c_i}{c_0} \left( \left( \frac{f^*}{f_{ac}} \right)^2 \frac{c_{10} + c_{20}}{2c} \right)$$

$$\approx -i \frac{\rho a^2}{\gamma \psi} (2\pi f^* T_b) \frac{c_{10} + c_{20}}{2c}$$

$$\approx -i \frac{\rho a^2}{\gamma \psi} (2\pi f^* T_b) \frac{r_0}{c}$$
3.3 Single-cavity labyrinth seal

![Graphs](image)

Figure 3.19: Single-cavity labyrinth seal: logarithmic decrement $\delta$ vs frequency ratio $f/f_{ac}$ for different values of the pressure ratio $\Pi$. Nominal conditions, Table 3.1, except for pressure ratio. Top: high-pressure support mode ($x_p/p = -1$). Bottom: low-pressure support mode ($x_p/p = 1$).
where \( r_0 \) is the average radial deflection, positive downward. This expression shows that, in the limit of very low Mach number, there is no distinction between high-pressure support and low-pressure support since the term \( c_{10} - c_{20} \) has disappeared. This makes sense since in the absence of any mean flow, the configuration is symmetric. This can also be seen in Fig. 3.19 where the curves for high-pressure support and low-pressure support are identical for the lowest pressure ratio \( \Pi = 1.01 \). Using the notations of Eq. (3.3.29), we have \( \Delta \phi = -\pi/2 \) which implies that the aerodynamic work is negative and the seal is stable. In fact, this phase difference is the most favourable for stability.

From the previous analyses concerning the influence of the clearance and of the pressure ratio, we see that the \( \psi \) term at the denominator of \( p_0 \) provides aeroelastic stability when it becomes dominant. As mentioned previously, this term represents the contribution of the variation in density and velocity at the fin tips to the change in inlet and outlet mass flow rate. When the flow is effectively incompressible (and viscous effects are negligible), it is easy to show on physical grounds that this term opposes the variation of the pressure in the cavity. Indeed, when the pressure increases in the cavity, the pressure difference between the inlet and the cavity decreases and that between the cavity and outlet increases. Consequently (according to Bernoulli’s theorem for example), the inlet velocity decreases, the outlet velocity increases and the inlet/outlet mass flow rates vary in the same manner. Both changes in mass flow rate tend to induce a decrease in cavity pressure thus opposing the pressure increase that caused them. It follows from this analysis that the \( \psi \) term can be seen as a positive aerodynamic damping. Change in flow and geometric parameter contributing to its increase will lead to an increased stability (the mechanical-to-acoustic frequency ratio must be excluded because it influences significantly other potentially destabilising contributions). A decrease in cavity height \( h \) for example increases the magnitude of this term thus should have a stabilising influence. We verify in Fig. 3.20 that this is what the analytical model predicts. The instability mechanism being linked to acoustic waves in the cavity, it is
natural to think that reducing the depth of this cavity will improve the stability. However, this idea should be validated with the help of CFD simulations or experiments.

To end this theoretical study, we shall investigate briefly how the addition of swirl affects the stability characteristics. This is illustrated in Fig. 3.21 where curves of logarithmic decrement against frequency ratio are plotted for a case with no air swirl \( u = 0 \) and two cases with air swirl \( u \neq 0 \); we consider a forward travelling wave in the first case and a backward travelling wave in the second case. The curves show that if a forward travelling mode is stabilised by the addition of air swirl, its backward travelling counterpart will be destabilised.
Figure 3.21: Single-cavity labyrinth seal: influence of air swirl Mach number $M_\theta$ on logarithmic decrement $\delta$ for forward travelling wave ($ND = +2$) and backward travelling wave ($ND = -2$). Nominal conditions, Table 3.1. Top: high-pressure support mode ($x_p/p = -1$). Bottom: low-pressure support mode ($x_p/p = 1$).
3.4 Single-cavity labyrinth seal with transport of circumferential momentum at the outlet

In this section, the assumption made in the previous analysis concerning the absence of transport of circumferential momentum at the outlet shall be removed. A new analytical solution will be obtained and the results compared to those of the previous section.

3.4.1 Governing equations

In the presence of circumferential momentum at the inlet and outlet, the equation of conservation of the circumferential momentum becomes:

$$\frac{\partial(pu_\theta R)_{h}}{\partial t} + \frac{\partial(pu_\theta^2 h)}{\partial \theta} + h \frac{\partial p}{\partial \theta} = \frac{1}{L}
(m_{1}u_{\theta 1} - m_{2}u_{\theta 2}) \quad (3.4.1)$$

where $u_{\theta 1}$ and $u_{\theta 2}$ are the average values of the circumferential velocity at the inlet and outlet respectively. We shall assume as in the previous section that there is no swirl at the inlet since this assumption was legitimate. The value of the average circumferential velocity at the outlet is written as a fraction $\alpha_2$ of the circumferential velocity in the cavity.

$$u_{\theta 2} = \alpha_2 u_{\theta} \quad 0 \leq \alpha_2 \leq 1 \quad (3.4.2)$$

Carrying the perturbation analysis and looking for a solution of the form $p' = p_0 e^{i(2\pi f + n\theta)}$ as in the previous section we obtain:

$$p_0 = \rho a^2 \frac{c}{h}
\left(\frac{\alpha_2}{h} \frac{1}{2\pi f_{ac} T_b} + i \frac{f^*}{f_{ac}}\right)
\left(\frac{1}{2\pi f_{ac} T_b} \frac{c_{10} - c_{20}}{c} - i \frac{f^*}{f_{ac}} \frac{c_{10} + c_{20}}{2c}\right)
\left(1 + \frac{\gamma_{\psi} c}{h} \frac{1}{2\pi f_{ac} T_b} + i \frac{f^*}{f_{ac}}\right)
\left(\frac{c}{h} \frac{1}{2\pi f_{ac} T_b} + i \frac{f^*}{f_{ac}}\right) \quad (3.4.3)$$
3.4.2 Application

Figure 3.22 shows the influence of $\alpha_2$ on the plots of logarithmic decrement versus frequency ratio for a high-pressure support mode and a low-pressure support mode. CFD simulations give a value of $\alpha_2$ around 0.15 for a single-cavity non rotating labyrinth seal. In both cases, increasing $\alpha_2$ reduces the frequency ratio at which the stability changes. The stability characteristics remain, however, similar when $\alpha_2$ is changed and the conclusions drawn in the previous section remain valid.

![Figure 3.22: Single-cavity labyrinth seal: logarithmic decrement $\delta$ vs mechanical-to-acoustic frequency ratio $f/f_{ac}$ for different values of parameter $\alpha_2$ from Eq. (3.4.2) for a high-pressure support mode ($x_p/p = -1$) and a low-pressure support mode ($x_p/p = 1$). Nominal conditions, Table 3.1.](image-url)
3.5 Similarity parameters for seal flutter

Several similarity parameters arise from the preceding theoretical analysis. Before giving them, the expression for \( p_0 \), will be rearranged to introduce the location of the pivot \( x_p/p \). When the the lower wall rotates about the pivot point \( P(x_p, r_p) \) by an angle \( \alpha_0 \), assumed to be small consistently with the hypotheses of the linear analysis, a point at coordinates \((x, r)\) is moved to coordinates \((x_1, r_1)\) such that:

\[
\begin{bmatrix}
  x_1 - x_p \\
  r_1 - r_p 
\end{bmatrix} =
\begin{bmatrix}
  \cos(\alpha_0) & -\sin(\alpha_0) \\
  \sin(\alpha_0) & \cos(\alpha_0) 
\end{bmatrix}
\begin{bmatrix}
  x - x_p \\
  r - r_p 
\end{bmatrix}
\]

\( \alpha_0 \) being small, the radial displacement \( \delta r = r_1 - r \) is given by:

\[
\delta r = (x - x_p) \alpha_0
\]

This gives for the change of clearance at the inlet and outlet of the cavity, which are located at \( x = -p/2 \) and \( x = p/2 \) respectively:

\[
c_{10} = \left( -\frac{p}{2} - x_p \right) \alpha_0 \quad (3.5.1)
\]

\[
c_{20} = \left( \frac{p}{2} - x_p \right) \alpha_0 \quad (3.5.2)
\]

Thus:

\[
\frac{c_{10} + c_{20}}{2} = -x_p \alpha_0 
\]

\[
c_{10} - c_{20} = -p \alpha_0 \quad (3.5.4)
\]

Using these relations, Eq. (3.3.28) becomes:

\[
p_0 = -p a^2 \frac{L \alpha_0}{h} \frac{\left( \frac{f^*}{f_{ac}} \right)^2 \frac{x_p}{p} + i \frac{1}{2\pi f_{ac} T_b} \left( \frac{f^*}{f_{ac}} \right)}{1 - \left( \frac{f^*}{f_{ac}} \right)^2 + i \gamma \frac{c}{h} \psi \frac{1}{2\pi f_{ac} T_b} \left( \frac{f^*}{f_{ac}} \right)}
\]

\[
(3.5.5)
\]
Using Eq. (3.5.5), we can list the following parameters:

- the ratios $\frac{L}{h}$, $\frac{c}{h}$ characterising the geometry;
- the Mach number $M$ (appearing in $\psi$) characterising the steady-state flow;
- the dimensionless pivot point location $\frac{x_p}{p}$ characterising the mode shape;
- the mechanical-to-acoustic frequency ratio $\frac{f^*}{f_{ac}}$;
- the parameter $f_{ac}T_b$ or equivalently the ratio of lengths $\frac{L}{\lambda}$ (see Eq. (3.3.40));

The pivot point location and the mechanical-to-acoustic frequency ratio are the classical seal flutter parameters. Eq. (3.5.5) suggests that the only influence of the swirl of the flow in the cavity is via the mechanical-to-acoustic frequency ratio. This should be checked in the parametric study. Other parameters like the Mach number $M$ (in the literature, the pressure ratio is considered instead of the Mach number) and the clearance $c$ are known to be influential (see review of literature). The remaining parameters have been investigated briefly above, but little is known about their influence on seal aeroelastic stability from the literature. More data, either from CFD or experiments, would be needed.

3.6 Summary

The critical influence of the mechanical-to-acoustic frequency ratio has been demonstrated analytically for a closed annular cavity. It was shown that, for a frequency ratio lower than one, the pressure variation was compliant to the wall motion, while it opposed the wall motion for a frequency ratio higher than one. To exploit this result for a seal cavity, the area in a cavity was decomposed into two areas: the through-flow area and the vortex area. The latter is assimilated to a closed annular cavity. These two areas interact across the dividing streamline. To explain the dependence of the pressure variation in a seal cavity on the
relative change of inlet and outlet clearance, the idea that, during vibration, the mass flow rates at the inlet and outlet of the cavity remains approximately equal was proposed. Using this idea to explain the change in pressure in the through-flow area and using the results obtained on a closed annular cavity to predict the response of the vortex area, it was possible to explain qualitatively the observed influence of the location of the support and the frequency ratio on the unsteady pressure phase. A number of predictions of the influence of several geometric and flow parameters on the stability of a single-cavity labyrinth seal have been made using a simplified analytical model. The clearance was predicted to influence primarily the magnitude of the logarithmic decrement, this magnitude decreasing with an increase in clearance. A slightly stabilising effect of an increase in clearance was also predicted. Concerning the influence of the pressure ratio, it was predicted that there was no instability below a certain pressure ratio. A physical explanation was proposed based on an analysis for an inviscid incompressible flow: for such a flow, according to Bernoulli’s theorem, the change in velocity at the fin tips must oppose the pressure variation in the cavity thus having a stabilising effect. Decreasing the cavity height was also predicted to be stabilising. Finally it was shown that the addition of swirl had opposite influence on a forward travelling mode and its backward counterpart, stabilising one while destabilising the other. A number of similarity parameters have been deduced from the theoretical analysis whose influence on stability needs further investigation. The conclusions of this chapter will be validated by using CFD techniques which are presented in the next chapter.
Chapter 4

Computational method

The CFD code AU3D developed at Imperial College has been used for CFD simulations. This chapter presents the physical models and numerical methods in the code.

4.1 Flow model

4.1.1 Governing equations

The three-dimensional unsteady Favre-averaged Navier-Stokes equations, cast in terms of the absolute velocity $u$ and written in an Arbitrary Lagrangian-Eulerian (ALE) conservative form for a control volume $\Omega$ with boundary $\Gamma = \partial \Omega$, take the form:

$$\frac{\partial}{\partial t} \int_{\Omega} U \, d\Omega + \int_{\Gamma} \left( F_i - \frac{1}{Re} G_i \right) \cdot n_i \, d\Gamma = \int_{\Omega} S \, d\Omega$$

(4.1.1)

where $n$ represents the outward unit normal of the element of surface $d\Gamma$. The viscous term $G_i$ on the left hand side of Eq. (4.1.1) has been scaled by a reference Reynolds number for non-dimensionalisation purposes. The solution vector $U$
4.1 Flow model

is given by:

\[
U = \begin{bmatrix}
\rho \\
p \rho u_1 \\
p \rho u_2 \\
p \rho u_3 \\
p \rho E
\end{bmatrix}
\] (4.1.2)

The inviscid flux term \( \vec{F} \) has the following components:

\[
F_i = \begin{bmatrix}
p(u_i - v_{gi}) \\
p u_1(u_i - v_{gi}) + p \delta_{1i} \\
p u_2(u_i - v_{gi}) + p \delta_{2i} \\
p u_3(u_i - v_{gi}) + p \delta_{3i} \\
p(u_i - v_{gi})H + pv_{gi}
\end{bmatrix}
\] (4.1.3)

where \( v_g \) is the velocity of the element of surface \( d\Gamma \). The pressure \( p \) and the total enthalpy \( H \) are related to the density \( \rho \), the absolute velocity \( u \) and the total energy \( E \) by the two perfect gas relations:

\[
p = (\gamma - 1) \rho \left( E - \frac{u^2}{2} \right)
\] (4.1.4)

\[
H = E + \frac{p}{\rho}
\] (4.1.5)

where \( \gamma \) is the constant specific heat ratio. The viscous flux \( \vec{G} \) has the following components:

\[
G_i = \begin{bmatrix}
0 \\
\sigma_{1i} \\
\sigma_{2i} \\
\sigma_{3i} \\
u_j \sigma_{ji} + \frac{\gamma}{\gamma - 1} \left( \frac{\mu_i}{\rho R} + \frac{\mu}{\rho R} \right) \frac{\partial T}{\partial x_i}
\end{bmatrix}
\] (4.1.6)

The viscous stress tensor \( \sigma \) is expressed using the eddy viscosity concept which assumes that, in analogy with the viscous stresses in laminar flows, the turbulent
4.1 Flow model

stresses are proportional to the mean velocity gradients:

\[ \sigma_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \lambda (\nabla \cdot \mathbf{u}) \delta_{ij} \]  \hspace{1cm} (4.1.7)

The total viscosity of the fluid is \( \mu = \mu_l + \mu_t \). The molecular viscosity \( \mu_l \) is given by Sutherland's formula, and the one-equation model of Spalart and Allmaras [49] is used to compute the turbulent viscosity \( \mu_t \).

The value of \( \lambda \) is given by the Stokes relation \( \lambda = -\frac{2}{3} \mu \), while the laminar and turbulent Prandtl numbers \( Pr_l \) and \( Pr_t \) are taken as 0.7 and 0.9 respectively.

The source term \( S \) in the right-hand side of Eq. (4.1.1) accounts for the rotation of the coordinate system at speed \( \omega \):

\[ S = \begin{bmatrix} 0 \\ 0 \\ -\rho \omega u_2 \\ \rho \omega u_3 \\ 0 \end{bmatrix} \]  \hspace{1cm} (4.1.8)

4.1.2 Spatial discretisation

The governing equations are solved using a finite volume approach. In this formulation, an approximation of the continuous solution is obtained by discretising the three-dimensional domain with a mesh. Each mesh cell \( I \) defines a control volume \( \Omega_I \) over which the flow variables are assumed constant. Writing equation (4.1.1) for control volume \( \Omega_I \) gives an ordinary differential equation:

\[ \frac{d}{dt} (U_1 \Omega_I) + \sum_{i=1}^{n_F} (F_i - G_i) \Gamma_i = S_I \Omega_I \]  \hspace{1cm} (4.1.9)

where \( n_F \) is the number of faces of cell \( I \), and \( F_i, G_i \) are the inviscid, respectively viscous fluxes at face \( i \). In the present code, general unstructured grids of mixed elements (tetrahedra, hexahedra, pyramids and prisms) are used to mesh the
4.1 Flow model

Figure 4.1: Median dual control volume. The control volume for node \( I \) is constructed by connecting the centres of the cells, cell faces and cell edges of all cells sharing node \( I \).

computational domain. The unknowns are defined at the vertices of the cells and the control volumes surrounding each vertex are constructed by connecting the centres of the cells, cell faces and cell edges (median dual mesh, Fig. 4.1). An edge-based data structure is used in the solver for computational efficiency. This representation allows an easy traversal through the mesh (Barth [9]). Using an edge-based formulation, the discrete equations can be written:

\[
\frac{d}{dt} (U_I \Omega_I) + \sum_{s=1}^{n_s} (F_{IJ_s} - G_{IJ_s}) \eta_{IJ_s} + B_i = S_I \Omega_I
\]  \hspace{1cm} (4.1.10)

where \( \eta_{IJ_s} \) are the edge weights and \( B_i \) is the boundary integral. More details can be found in [42]. The inviscid flux is computed using a central-difference scheme with addition of a matrix dissipation for stability considerations as proposed by Swanson and Turkel [51]:

\[
F_{IJ_s} = \frac{F_I + F_{J_s}}{2} - D_{IJ_s}
\]  \hspace{1cm} (4.1.11)
where $D_{IJ}$ denotes the artificial dissipation terms. The matrix dissipation is a blending of second-order and fourth-order terms:

$$D_{IJ} = \left[ \Phi U_{IJ} \right] \left( \Phi (U_{IJ} - U_I) + \epsilon_4 (1 - \Phi) (\nabla^2 U_J - \nabla^2 U_I) \right)$$

(4.1.12)

$\Phi$ is a flux limiter based on the pressure:

$$\Phi = \max(\Phi_I, \Phi_J)$$

(4.1.13)

$$\Phi_I = \frac{2 |p_J - p_I - S_{IJ} \nabla p_J|}{(1 - \omega) [ |p_J - p_I| + |p_I - p_J| + 2S_{IJ} \nabla p_J ]} + 2\omega [p_J + p_I + S_I \nabla p_I]$$

(4.1.14)

This limiter is of first order in space in smooth regions of the flow, for second order accuracy of the scheme, and of order unity near shocks to ensure the TVD property. This property prevents the occurrence of spurious oscillations. The second-order dissipation provided by the first term of Eq. (4.1.12) is insufficient in smooth regions of the flow. A fourth-order term needs to be added to damp high frequency oscillations and allow a convergence to a steady-state. This term must be switched off near shocks to prevent overshoots [29]. This is what the multiplication by $1 - \Phi$ accomplishes.

The viscous flux $G_{IJ}$ is expressed in a central-difference fashion; the gradients of the flow variables $\nabla U_{IJ}$ required to evaluate $G_{IJ}$ are defined as follows:

$$\nabla U_{IJ} = \frac{\nabla U_I + \nabla U_J}{2}$$

(4.1.15)

The diffusion coefficients are computed in a similar manner.

### 4.1.3 Integration scheme

A first-order accurate implicit scheme is used to advance the solution in time:

$$\frac{(\Omega U)^{n+1} - (\Omega U)^n}{\Delta t} = R(U^{n+1})$$

(4.1.16)
4.1 Flow model

where $R(U)$ designates the residuals and the index $n$ is used to indicate the solution at time $t = t^n$. Equation (4.1.16) is solved by using a dual-time stepping procedure. A pseudo-time step $\tau$ is introduced on the left-hand side as follows:

$$
\Omega^{n+1}\frac{U_t^{m+1} - U_t^m}{\Delta \tau} + \frac{(\Omega U)_t^{n+1} - (\Omega U)_t^n}{\Delta t} = R(U^{m+1})
$$

Linearising the right-hand side of Eq. (4.1.17) around $U^m$, we obtain the following scheme:

$$
\left( \left( \frac{\Omega^{n+1}}{\Delta \tau} + \frac{\Omega^n}{\Delta t} \right) \vec{I} - \vec{J}_m \right) \Delta U_t = R(U^m) - \frac{\Omega^{n+1}U^m - (\Omega U)^n}{\Delta t}
$$

where $\Delta U_t = U_t^{m+1} - U_t^m$ and $\vec{I}$ is the identity matrix. The physical time derivative $\Delta t$ is introduced on the left-hand side to ensure the stability of the scheme in regions of the flow where the ratio $\frac{\Delta \tau}{\Delta t}$ is large (Melson et al. [32]). A Jacobi iterative procedure is used to solve (4.1.18) and the scheme is advanced in pseudo-time step until the unsteady residuals $\Delta U_t$ are driven to small enough values. This dual-time stepping procedure allows the use of relatively large physical time steps, as dictated by the time scales of the physical phenomenon of interest rather than by stability considerations. Classical acceleration techniques such as local-time stepping and residual smoothing can be used to improve the convergence rate at the pseudo-time level.

4.1.4 Turbulence modelling

The turbulent viscosity $\mu_t$ is computed using the finite Reynolds number version of the Spalart-Allmaras model [49]. This model solves a transport equation for the quantity $\bar{v}$ equivalent to the kinematic turbulent viscosity $\nu_t$ far from the walls:

$$
\frac{D\bar{v}}{Dt} = c_{b1} \bar{S} \bar{\nu} \left( c_{w1} f_w - \frac{c_{b1}}{\kappa} \right) \left( \frac{\bar{\nu}}{d} \right) + \frac{1}{\sigma} \left[ \nabla \cdot \left( (\nu + \bar{\nu}) \nabla \bar{\nu} \right) + c_{b2} \nabla \bar{\nu} \cdot \nabla \bar{\nu} \right]
$$

(4.1.19)
where $d$ is the distance to the nearest wall. The turbulent viscosity is then computed as follows:

\[
\nu_t = f_{v1} \tilde{\nu} \tag{4.1.20}
\]

\[
f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}} \tag{4.1.21}
\]

\[
\chi = \frac{\tilde{\nu}}{\nu} \tag{4.1.22}
\]

The other terms appearing in Eq. (4.1.19) are given below:

\[
f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}} \tag{4.1.23}
\]

\[
\tilde{S} = \Omega + \frac{\tilde{\nu}}{\kappa^2 d^2} f_{v2} \tag{4.1.24}
\]

\[
f_w = g \left( \frac{1 + c_{w3}^6}{g^6 + c_{w3}^6} \right)^{1/6} \tag{4.1.25}
\]

\[
g = r + c_{w2} (r^6 - r) \tag{4.1.26}
\]

\[
r = \frac{\tilde{\nu}}{\tilde{S} \kappa^2 d^2} \tag{4.1.27}
\]

\[
\sigma = 2/3 \tag{4.1.28}
\]

\[
c_{v1} = 7.1 \tag{4.1.29}
\]

\[
c_{b1} = 0.1335 \tag{4.1.30}
\]

\[
c_{b2} = 0.622 \tag{4.1.31}
\]

\[
c_{w1} = \frac{c_{b1}}{\kappa^2} + \frac{1 + c_{b2}}{\sigma} \tag{4.1.32}
\]

\[
c_{w2} = 0.3 \tag{4.1.33}
\]

\[
c_{w3} = 2 \tag{4.1.34}
\]

where $\Omega$ is the magnitude of the vorticity and $\kappa$ the Kármán constant. The boundary condition at walls is $\tilde{\nu} = 0$. 

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4.1.5 Boundary conditions

When the HP and LP cavities are included in the model, the total pressure, total temperature and flow angle are imposed at the inlet while the static pressure is imposed at the outlet. When only the labyrinth area is modelled, Riemann invariants boundary conditions are imposed at the inlet and outlet.

At the wall boundaries, either no slip boundary conditions or standard wall functions with slip boundary conditions are applied. The latter are used to reduce mesh requirements. From a thermal point of view, the wall are supposed to be adiabatic. In reality, there is a significant heat exchange between the fluid and the structure in labyrinth seals. Taking this into account would require to solve the equation of conduction in the structure, in addition to the unsteady fluid dynamics problem. Such an approach presents several numerical difficulties and would severely limit the number of cases that could be studied. Since the objective of the thesis was primarily to investigate the flutter mechanism, the assumption of adiabaticity was deemed acceptable.

4.2 Structural and aeroelastic model

It is assumed that the behaviour of the structure can be modelled by a multi-degree of freedom system with proportional damping. The aeroelastic equations of motion of the structure can be written as:

\[ \ddot{x} + \zeta \dot{x} + \omega^2 x = F(t) \]  

(4.2.1)

where \( \ddot{M}, \ddot{C}, \ddot{K} \) are the mass, structural damping and stiffness matrices, \( x \) is the displacement vector, \( F(t) \) is the vector of fluid forces. At a specific node \( I \), the fluid force \( F_I(t) \) is given by:

\[ F_I(t) = -p_I(t) \Gamma_I n_I \]  

(4.2.2)
where $p_I(t)$ is the local pressure, $\Gamma_I$ the application area, $n_I$ the local normal unit vector on the surface of the structure. Having determined the natural frequencies $\omega_i$ and the mode shapes $\Phi_i$ of the structure, the structural equations of motion can be decoupled by a simple coordinate transformation from physical coordinates $x$ to principal coordinates $q$ with the help of the modal matrix $\Phi$:

$$x = \Phi q$$  \hspace{1cm} (4.2.3)

Inserting (4.2.3) in Eq. (4.2.1) and pre-multiplying by $\Phi^T$:

$$\Phi^T M \Phi \ddot{q} + \Phi^T C \Phi \dot{q} + \Phi^T K \Phi q = \Phi^T F(t) = n(t)$$  \hspace{1cm} (4.2.4)

where $\Pi(t)$ is the vector of modal forces. Using the orthogonality properties of the system matrices with respect to the mode shape matrix, and with mass-normalised mode shapes, one obtains the following system of decoupled equations:

$$\ddot{q}_i + 2\xi_i \omega_i \dot{q}_i + \omega_i^2 q_i = \Pi_i(t)$$  \hspace{1cm} (4.2.5)

where $q_i$, $\xi_i$, $\omega_i$ and $\Pi_i(t)$ are the modal displacement, modal damping, natural frequency and modal force for mode $i$. In this form, the time integration of the structural equations of motion is straightforward. For the present application, only one mode is considered and the structural damping is neglected. A purely harmonic motion is enforced by setting the modal forces to zero in Eq. (4.2.5). The fluid mesh is moved at each time step to follow the motion of the structure. The computed fluid unsteady pressure $p$ is used to evaluate the aerodynamic work $w$ and the logarithmic decrement $\delta$:

$$w = \int_0^T \int_{\Gamma} p v \Phi \cdot n d\Gamma dt$$  \hspace{1cm} (4.2.6)

$$\delta = \frac{-w}{2E_k}$$  \hspace{1cm} (4.2.7)

$$E_k = \frac{1}{2} v^2$$  \hspace{1cm} (4.2.8)
4.3 Single-passage approach

where $T$ is the period of the motion, $v = \dot{q}$ the modal velocity and $E_k$ the kinetic energy of the structure.

Usually a finite elements model (FEM) of the seal provides the mode shapes. In the work presented in this thesis, unless otherwise specified, the mode shapes are fictitious modes constructed using a Fortran program. These mode shapes are travelling waves in the circumferential direction (Fig. 4.2b). In an axial-radial plane, the applied deformation is a rotation of the seal rotor about a pivot point. This point can be located upstream or downstream of the labyrinth to simulate a seal supported respectively at the high-pressure side (HPS mode shape) or at the low-pressure side (LPS mode shape). This is illustrated in Fig. 4.2a. This approach was better suited than FEM for the parametric study that needed to be carried out in the present work.

4.3 Single-passage approach

To reduce the computational cost of the simulations, one can simulate the flow on a sector instead of the whole annulus. This approach presents no difficulty for steady-state simulations since the flow is axisymmetric and periodic boundary conditions can be used on the circumferential boundaries of the domain. For flutter simulations, the periodic boundary conditions are replaced by phase-lagged boundary conditions for both flow variables and grid displacements. The methodology applied is the basic time-domain store of Erdos et al. [22] and is applied using auxiliary (shadow) points (Fig. 4.3). The shadow points represent geometrical projections of points near the periodic boundaries on the opposite side (pitchwise) of the passage. The points to be projected are chosen in such a way as to reconstruct the whole numerical stencil for points on the periodic boundary. Time histories of flow variables and grid velocities are collected on the master points and used to evaluate the desired quantities on the corresponding
4.3 Single-passage approach

4.2a: Schematic view in (x,r) plane.

4.2b: 3D view of a 2ND mode on the rotor surface.

Figure 4.2: Mode shape used for flutter simulations. In an axial-radial plane, the mode shape is a rotation about a pivot point.
4.3 Single-passage approach

Figure 4.3: Auxiliary shadow points for phase-lagged boundary conditions. Time histories of flow variables and grid velocities are collected on the master points and used to evaluate the desired quantities on the corresponding shadow points with a time-shift. The correspondence between master points and shadow points is indicated by the black arrows.
4.3 Single-passage approach

Figure 4.4: Information transfer between master and shadow points. When the time shift is positive, the information needs to be taken from the previous cycle at the master points.

Shadow points with a time-shift:

\[ U_{\text{shadow}}(t) = U_{\text{master}}(t - \Delta t) \]  \hspace{1cm} (4.3.1)

\[ \Delta t = \pm \frac{n}{fN_B} \]  \hspace{1cm} (4.3.2)

where \( f \) is the natural frequency of the mode, \( n \) is the associated nodal diameter pattern and \( N_B \) is the number of sectors in the assembly. The information transfer between master and shadow points is illustrated in Fig. 4.4 for a quantity \( q \). When the time shift \( \Delta t \) is positive, the information needs to be taken from the previous cycle at the master points. The correspondence between master points and shadow points is indicated by the black arrows in Fig. 4.3. In the current implementation, the quantities are stored for several consecutive cycles, and the reconstructed values are obtained as the average of the last \( N_C \) cycles, to remove spurious frequencies which arise in the flow at the beginning of the
4.4 Validation of single-passage approach

The single-passage approach has been tested on the seal configuration presented in Fig. 4.5. The pressure ratio across the seal is $\Pi = 2.86$. The same two-dimensional mesh has been extruded over sectors of 10, 30, 90 degrees and over a whole annulus. The convergence of the logarithmic decrement is plotted in Fig. 4.6 against the CPU time used for the four models. We check that all models converge towards the same value of the logarithmic decrement. However, there is no gain in CPU time with the sector models. It is believed that this is because the flow in the circumferential direction plays a major role on the aeroelastic mechanism as shown by Abbott [2]. When reducing the size of the sector, we have to wait more cycles for this circumferential unsteady flow to establish. Moreover, on the smallest sector, the convergence is affected by low-frequency oscillations. The origin of these oscillations has been investigated by...
4.4 Validation of single-passage approach

carrying a temporal Fourier analysis of the unsteady flow field computed with the ten-degree sector model. The pressure signal spectra for points located in the three inter-fin cavities and in the low-pressure cavity are shown in Fig. 4.7. It can be seen that there is a spurious frequency in the low-pressure cavity near the excitation frequency which is 400Hz. This leads to a beating phenomenon which is the source of the oscillations in the logarithmic decrement history.

In conclusion, it seems that the whole annulus approach should be preferred when it is possible to use it (size of the model no prohibitive). It has the additional advantage that all nodal diameters can be computed in one run whereas, with the sector model, one run per nodal diameter is needed. In the study presented in this thesis, the single-passage approach has been used due to its advantages for post-processing purposes.

![Figure 4.6](image)

Figure 4.6: Seal test case: convergence of logarithmic decrement vs CPU time consumed for different sector sizes. Nominal conditions, Table 5.1. 2ND mode.
4.4 Validation of single-passage approach

Figure 4.7: Seal test case: pressure spectrum in inter-fin cavities and LP cavity with 10-degree sector model. There is a spurious frequency in the LP cavity near the excitation frequency which is 400Hz. Nominal conditions, Table 5.1. 2ND mode.
4.5 Summary

The computational fluid dynamics methods used in this thesis have been presented. The fluid solver solves the Navier-Stokes equations coupled with the Spalart-Allmaras model for the turbulence closure. The spatial discretisation of the governing equations is achieved using a finite volume approach. A first-order accurate implicit scheme is used to advance the solution in time. A dual-time stepping technique is used for unsteady computation. A linear model is used for the structure. The mode of vibration is enforced and the motion is harmonic. The fluid mesh is moved at each time step to follow the motion of the structure. The computed fluid unsteady pressure is used to evaluate the aerodynamic work and the logarithmic decrement. The structural mode shape is a rotation about a pivot point in a meridional plane travelling around the circumference. A single-passage approach is used to reduce the computational cost. Numerical experiments have shown that this approach gives the same results as a whole annulus approach. However, there is no gain in CPU time compared to whole annulus calculations. It is believed that this is because the unsteady circumferential flow, which plays a major role on the aeroelastic mechanism, needs more cycles to establish itself on the sector model. The numerical methods presented in this chapter will be used to investigate the steady and unsteady flow in a typical labyrinth seal configuration in the next chapter.
Chapter 5

Analysis of steady and unsteady flow in labyrinth seal

5.1 Introduction

The purpose of this chapter is to present some characteristic features of the flow in labyrinth seals. The steady-state flow is first computed using CFD methods and examined. A vibration mode is then prescribed and the resulting unsteady flow is computed. The characteristics of the unsteady flow are compared with the predictions of the theoretical analysis. The distribution of the aerodynamic work is presented and analysed in detail.

5.2 Geometry and mesh

The seal considered here is straight-through and has four fins. The nominal conditions for this case are given in Table 5.1. This configuration was found
5.2 Geometry and mesh

<table>
<thead>
<tr>
<th>Nominal clearance (c)</th>
<th>0.5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch (p/c)</td>
<td>9</td>
</tr>
<tr>
<td>Height of cavity (h/c)</td>
<td>6.5</td>
</tr>
<tr>
<td>Pressure ratio (\Pi)</td>
<td>2.86</td>
</tr>
</tbody>
</table>

Table 5.1: Nominal conditions for 4-fin labyrinth seal case.

Unstable during rig tests. A study of this configuration was published by di Mare et al. [16]. The CFD model comprises the labyrinth seal, the high-pressure and the low-pressure cavity (Fig. 5.1). The geometry of the HP and LP cavities have been simplified.

Given the axisymmetry of the configuration, the meshing strategy consists in constructing a two-dimensional mesh in an axial-radial plane and extruding it in the circumferential direction. The extrusion is carried out over a ten-degree sector with a step size of one degree between each mesh layer. This corresponds
to 180 points per wavelength for a two-nodal diameter mode, as the one used in flutter simulations. The two-dimensional mesh is unstructured and is a mixture of triangular and quadrilateral elements near the walls to capture the boundary layers (Fig. 5.2). The mesh is refined near the fin tips with 15 quadrilateral elements at the fin tip surface. The height of the first quadrilateral layer at the walls is 3.2% of the clearance. There are five layers in total with a height ratio of 1.1 between consecutive layers giving a total height of 20% of the clearance. The $y^+$ value at the first nodes adjacent to the walls is below 10 almost everywhere with the highest values occurring at the fin tips. The final mesh has 135,234 points. The 3D mesh is presented in Fig. 5.3 with the boundary conditions. A finer mesh was generated to check the convergence of the solution on the first mesh (Fig. 5.4). This mesh contained 363,539 points. The first mesh will be referred to as “medium mesh” or “mesh 1” and the second as “fine mesh” or “mesh 2” in the following. The characteristics of the two meshes are summarised in Table 5.2.

<table>
<thead>
<tr>
<th></th>
<th>Mesh 1</th>
<th>Mesh 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of points</td>
<td>135,234</td>
<td>363,539</td>
</tr>
<tr>
<td>Number of elements on fin tip</td>
<td>15</td>
<td>23</td>
</tr>
<tr>
<td>Height of first wall quad layer (% of clearance)</td>
<td>3.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Height ratio between consecutive layers</td>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Number of quad layers</td>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>Total height of quad layers (% of clearance)</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 5.2: Characteristics of medium and fine mesh of 4-fin labyrinth seal configuration.

### 5.3 Steady-state flow characteristics

The steady-state simulations were carried out on 16 processors of a cluster of twin quad core Intel Xeon processors running at 2.8 GHz. The convergence of the simulations is assessed by looking at the evolution of the outlet total temperature as it was found to be the best indicator. This convergence is
5.3 Steady-state flow characteristics

5.2a: Entire computational domain.

5.2b: Zoom of labyrinth area.

5.2c: Zoom of fin tip area.

Figure 5.2: 4-fin labyrinth seal: mesh in x-r plane.
5.3 Steady-state flow characteristics

Figure 5.3: 4-fin labyrinth seal: 3D mesh and boundary conditions.

presented in Fig. 5.5 for the medium mesh at the nominal pressure ratio $\Pi = 2.86$. 100,000 iterations are required to obtain a fully converged solution from an initial state. This represented a restitution time of 11 hours. The slow convergence rate is due to the presence of large zones at low Mach number in the computational domain. It is well-known that the system of Euler/Navier-Stokes equations becomes stiff at low Mach number due to the disparity in the eigenvalues of the system, some being close to the speed of sound, others to the speed of the flow. A low Mach number preconditioning method could improve the speed of the convergence.

The computed seal characteristic is presented in Fig. 5.6. This characteristic is obtained by decreasing the outlet pressure while keeping the inlet total pressure and total temperature fixed. The flow function is defined as follows:

$$F = \frac{m\sqrt{T_t}}{AP_t} \quad (5.3.1)$$
5.3 Steady-state flow characteristics

Figure 5.4: 4-fin labyrinth seal: zoom of fine mesh in fin tip area.

Figure 5.5: 4-fin labyrinth seal: total temperature convergence. Nominal conditions, Table 5.1.
where $m$ is the mass flow rate, $T_t$ the inlet total temperature, $P_t$ the inlet total pressure and $A$ the clearance area. An experimentally-derived data point is also included at the nominal pressure ratio. There is a rapid increase in flow function for a pressure ratio between 1 and 1.6. Above this pressure ratio, the slope decreases rapidly until blockage is reached around the nominal pressure ratio. At this pressure ratio, the outlet fin is choked preventing further increase in mass flow as can be seen in the Mach number contours in Fig. 5.7. Simulation results are in good agreement with measured data at the nominal pressure ratio. The difference in flow function between experiment and simulations is around 4% for the medium mesh and 3% for the fine mesh, which is considered acceptable.

Some flow streamlines are shown in Fig. 5.8. The HP and LP cavities feature large recirculations driven by the leakage flow. The flow in the labyrinth area is characterised by a jet-like flow at high Mach number above the fin tips driving...
Figure 5.7: 4-fin labyrinth seal: Mach number distribution in labyrinth. Nominal conditions, Table 5.1. The outlet fin is choked.
5.3 Steady-state flow characteristics

Figure 5.8: 4-fin labyrinth seal: computed streamlines. Nominal conditions, Table 5.1.
a large vortex in the cavity below. The jet flow emanating from one fin tip impinges on the downstream fin, creating a local maximum of pressure at the stagnation point (Fig. 5.9). These characteristic features have been reported by several authors (Moore [33], Hirano et al. [24]). In our configuration, a small vortex is also visible on the rear face of the fins near the tip. In this region, the flow experiences an adverse pressure gradient due to the impingement of the vortex flow on the through-flow.

The pressure on the rotor surface of the labyrinth at the nominal pressure ratio is presented in Fig. 5.10. The results on the medium and fine mesh are in excellent agreement in the second and last cavity. The differences in the first cavity are probably due to a less accurate description of the strongly separated
flow on top of the first fin by the medium mesh. The highest pressure drops occur at the first and last fins. The high pressure drop at the first fin is due to the strong convergence of the streamlines and thus acceleration of the flow in this region. This convergence is further enhanced by the vena contracta as can be seen in Fig. 5.11a. We notice that there is no pressure drop at the second fin: this fin is ineffective because the flow almost passes above it without touching it (Fig. 5.11b); this is a consequence of the vena contracta at the first fin. Figure 5.12 shows the pressure distribution on the rotor at increasing pressure ratios. Above the nominal pressure ratio, a decrease in downstream pressure has no influence on the pressure in the labyrinth cavities: the outlet fin being choked, no downstream information can travel upstream of the outlet fin tip, and the pressure adaptation must take place at the outlet fin exit. It will be shown that this has some consequences on the flutter characteristics.

The swirl ratio in the labyrinth at the nominal pressure ratio is plotted in

Figure 5.10: 4-fin labyrinth seal: pressure distribution on rotor surface obtained with medium and fine mesh. Nominal conditions, Table 5.1.
5.3 Steady-state flow characteristics

5.11a: Vena contracta at first fin tip.

5.11b: Flow above second fin. The re-contraction is small.

Figure 5.11: 4-fin labyrinth seal: flow above fin tips. Nominal conditions, Table 5.1.
5.3 Steady-state flow characteristics

Figure 5.12: 4-fin labyrinth seal: pressure on rotor surface for increasing inlet/outlet pressure ratios. Above the nominal pressure ratio, the pressure distribution in the labyrinth remains the same since the outlet fin is choked. Nominal conditions, Table 5.1, except for pressure ratio.
Fig. 5.13. The swirl ratio is defined as follows:

\[ S_w = \frac{u_\theta}{R\Omega} \]  

(5.3.2)

where \( u_\theta \) is the tangential velocity, \( R \) the mean seal radius and \( \Omega \) the rotational speed. The swirl increases in the labyrinth from upstream to downstream due to the progressive entrainment by the rotating wall. This increase is retarded by the shear stresses exerted by the upper wall which is stationary. Profiles of the swirl ratio in the middle of the three cavities are plotted in Fig. 5.14. The upper part of the profile (through-flow area) is linear; in this region, the flow is similar to a Couette flow between an inner rotating cylinder and an outer stationary cylinder. The lower part of the profile (cavity area) exhibits a shape similar to a Taylor-Couette flow at large circumferential Reynolds number as reported by Smith et al. [48] and Dong [17]. The flow is characterised by a core region of constant circumferential velocity, the velocity gradient being restricted to the near wall region. The circumferential Reynolds number was defined by Dong as:

\[ Re = \frac{R_1 \Omega (R_2 - R_1)}{\nu} \]  

(5.3.3)

where \( R_1, R_2 \) are the inner and outer radii of the cavity respectively, and \( \nu \) is the kinematic viscosity of the fluid. In our case, \( Re \approx 8000 \) which is the same value as in the DNS from Dong. In the results presented by Smith et al. and Dong, the quantity that was found constant in the core region was the circumferential angular momentum \( Ru_\theta \), not the circumferential velocity. However, due to the large radius of the present seal compared to the cavity height, variation in circumferential momentum are equivalent to variation in circumferential velocity, the radius variation being small. This circumferential angular momentum is compared to the DNS data of Dong in Fig. 5.15 where the similarity in the profiles can be seen despite the obvious differences due to the presence of the through-flow in the present configuration. The axial velocity profiles at the same locations are plotted in Fig. 5.16. The jet like flow
in the through-flow area \( r_{adim} > 0.8 \) and the vortex flow in the cavity area \( r_{adim} < 0.8 \) are clearly visible. The profiles of axial and tangential velocities indicate fundamental differences in flow characteristics in the through-flow area and in the cavity area. For an accurate modelling of the flow in labyrinth seal with bulk flow methods, these differences should be taken into account. This will be discussed further in Chapter 7 which deals with the analytical seal flutter model.

Figure 5.13: 4-fin labyrinth seal: swirl ratio contours. Nominal conditions, Table 5.1.

5.4 Unsteady flow characteristics

Since steady-state results on the medium and fine meshes were in good agreement, flutter simulations were carried out only on the medium mesh at nominal pressure ratio \( \Pi = 2.86 \).
Figure 5.14: 4-fin labyrinth seal: swirl ratio profile in middle of cavities. Nominal conditions, Table 5.1.
Figure 5.15: 4-fin labyrinth seal: angular momentum profile in middle of cavities compared to DNS results of Dong [17] for Taylor-Couette flow between an inner rotating and an outer stationary cylinder. Nominal conditions, Table 5.1.
5.4 Unsteady flow characteristics

5.4.1 Mode shape

The mode shape considered here is a two-node-diameter travelling wave in the rotor (Fig. 5.17). For the results presented in this section, the pivot point is

Figure 5.16: 4-fin labyrinth seal: axial velocity profile in middle of second cavity. Nominal conditions, Table 5.1.
5.4 Unsteady flow characteristics

5.4.1 Mode shape

The mode shape considered here is a two-nodal-diameter travelling wave in the rotor (Fig. 5.17). For the results presented in this section, the pivot point is located on the LP side, at the base of the support below the third fin as shown in Fig. 5.18. The characteristics of the mode are given in Table 5.3. The deformation of the rotor at two different phases of the motion is presented in Fig. 5.19.

![Flow direction](image)

Table 5.3: Characteristics of LPS mode used for flutter simulations.

<table>
<thead>
<tr>
<th>Pivot point location $x_p/p$</th>
<th>1.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency ratio $f/f_{ac}$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Figure 5.17: 4-fin labyrinth seal flutter simulations: mode shape. LPS mode, Table 5.3.
Figure 5.18: 4-fin labyrinth seal flutter simulations: location of pivot point of mode shape. LPS mode, Table 5.3.
5.4 Unsteady flow characteristics

5.4.2 Results

The convergence of the flutter simulations is assessed by looking at the evolution of the logarithmic decrement (Fig. 5.20). Approximately 25 cycles are required to obtain a converged value of the logarithmic decrement. The results of a physical time step resolution study are presented in Table 5.4. 360 iterations per cycle represents a good compromise between accuracy and computational cost. The most critical aspect for accuracy of the simulations is to ensure that enough pseudo-time steps are performed to converge the unsteady residuals.

<table>
<thead>
<tr>
<th>No of physical time steps/cycle</th>
<th>Logarithmic decrement $\delta/\delta_0$</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>-1.10</td>
<td>-5%</td>
</tr>
<tr>
<td>360</td>
<td>-1.20</td>
<td>+3%</td>
</tr>
<tr>
<td>720</td>
<td>-1.16</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.4: 4-fin labyrinth seal flutter simulations: physical time step resolution study. Nominal conditions, Table 5.1, LPS mode, Table 5.3.

Figure 5.19: 4-fin labyrinth seal flutter simulations: rotor deformation at two different phases of the motion. LPS mode, Table 5.3.
5.4 Unsteady flow characteristics

The CFL number was fixed to a value as high as allowed by stability to increase the rate of convergence in pseudo-time steps. For the present case, this value was determined to be around 50 by numerical experiments. The influence of the number of pseudo-time steps on the computed logarithmic decrement is shown in Fig. 5.21. At least 40 pseudo-time steps are necessary for convergence of the unsteady residuals. This large number of pseudo-time steps is coherent with the slow convergence of the steady-state simulations. The results presented below were obtained with 360 iterations per cycle and 50 pseudo-time steps per iteration.

The magnitude of the unsteady pressure in the computational domain is presented in Fig. 5.22. This magnitude is significant only in the labyrinth area. Fig. 5.23 shows the unsteady pressure in the labyrinth area at different phases of the seal motion. The position of the rotor for $\Phi = 180$ and $\Phi = 360$ degrees is shown in Fig. 5.19. When $\Phi = 180$ degrees, the inlet clearance is larger than...
Figure 5.21: 4-fin labyrinth seal flutter simulations: influence on number of pseudo-time steps on computed logarithmic decrement. Nominal conditions, Table 5.1, LPS mode, Table 5.3.
Figure 5.22: 4-fin labyrinth seal flutter simulations: magnitude of unsteady pressure in computational domain. Nominal conditions, Table 5.1, LPS mode, Table 5.3.
Figure 5.23: 4-fin labyrinth seal flutter simulations: unsteady pressure in
labyrinth at different phases of the vibration (see Fig. 5.19 for relation between
value of phase of rotor displacement $\Phi$ and rotor position). Nominal conditions,
Table 5.1, LPS mode, Table 5.3.
the outlet clearance and the pressure in the labyrinth is increased while, for $\Phi = 360$ degrees, the outlet clearance is larger and the pressure is decreased. This is in agreement with the theoretical analysis presented in Section 3.3.2.

We also see from the slightly positive value of the unsteady pressure for $\Phi = 90$ degrees (or the slightly negative value for $\Phi = 270$ degrees) that the pressure leads the displacement causing it, which means that the mode is unstable. Here the frequency ratio is equal to 0.8. A phase lead was predicted by the theoretical analysis for a frequency ratio lower than one, which is confirmed by CFD results.

Fig. 5.24 shows the unsteady axial velocity in the labyrinth at different phases of the vibration. The red zones indicate an increase in axial velocity while the blue zones indicate a decrease. The most striking feature is that the axial velocities at the inlet and exit fins vary in opposite senses. When the inlet clearance is greater than the exit clearance ($\Phi = 180$ degrees), the axial velocity decreases at the inlet fin and increases at the outlet fin. This variation is in agreement with the idea of equalisation of the mass flow rates at the fin tips put forward in the theoretical analysis. The variations of the mass flow rates and clearances at the fin tips are shown in Fig. 5.25. If we focus our attention on the third and fourth fin tip for example, we see that the variation of the clearance at the fourth fin is in phase opposition with that at the third fin; on the other hand, the variation of the mass flow rate at the fourth fin is in phase with that at the third fin. As a consequence, the mass flow rate difference between the third and fourth fin is much lower than the one we would have if we had $m'/m = c'/c$. This is because the axial velocity at the fin tips vary in such a way as to reduce the mass flow rate difference between subsequent fins.
Figure 5.24: 4-fin labyrinth seal flutter simulations: unsteady axial velocity in labyrinth at different phases of the vibration (see Fig. 5.19 for relation between value of phase of rotor displacement $\Phi$ and rotor position). Nominal conditions, Table 5.1, LPS mode, Table 5.3.
5.4 Unsteady flow characteristics

Figure 5.25: 4-fin labyrinth seal flutter simulations: unsteady mass flow rates and clearances at the fin tips. Nominal conditions, Table 5.1, LPS mode, Table 5.3.
5.4 Unsteady flow characteristics

The magnitude of the unsteady tangential velocity is shown in Fig. 5.26.

![Unsteady flow characteristics graph](image)

Figure 5.26: 4-fin labyrinth seal flutter simulations: magnitude of the unsteady tangential velocity. Nominal conditions, Table 5.1, LPS mode, Table 5.3.

There is a clear distinction between the cavity vortex area and the through-flow area where the magnitude is small. In the through-flow area, the presence of a strong axial velocity prevents the unsteady tangential flow from establishing. This is an illustration of the distinct behaviour between the through-flow and the vortex flow during vibration.

5.4.3 Work distribution

The distribution of the aerodynamic work in the labyrinth is plotted in Fig. 5.27. The triangular symbols indicate the average value of the work in each cavity. There is a positive and negative work contribution in each cavity. The aerodynamic work (per unit area) is defined by the following expression:

\[ w = \left( \int_0^T -p'v'\, dt \right) \Psi \cdot n \]

\[ = \frac{p_0 v_0 T}{2} \sin(\phi) \Psi \cdot n \]

(5.4.1)

where \( T \) is the period of vibration, \( p' \) the local unsteady pressure, \( v' \) the modal velocity, \( \Psi \) the mode shape at the rotor wall, \( n \) the unit wall normal, \( p_0 \) the magnitude of the unsteady pressure, \( \phi \) its phase and \( v_0 \) the magnitude of the
5.4 Unsteady flow characteristics

Figure 5.27: 4-fin labyrinth seal flutter simulations: aerodynamic work (per unit area) distribution in the labyrinth. Nominal conditions, Table 5.1, LPS mode, Table 5.3.

modal velocity, the latter being a constant number. We can explain the work distribution by analysing the individual distribution of each quantity occurring in the definition of the work. The magnitude and phase of the first harmonic of the unsteady pressure along the rotor wall are presented in Fig. 5.28 and 5.29. The phase is relative to the phase of the displacement with the convention presented in Fig. 5.19. The magnitude of the unsteady pressure is practically uniform in a cavity. The magnitude in the first cavity is noticeably lower than in the other two cavities. So is the phase which indicates that the variation of pressure in the first cavity lags behind. As shown in the steady flow analysis, the main difference between the first cavity and subsequent cavities lies in the importance of the vena contracta at the first fin. From the theory of flow through an orifice, it is known that the minimum area at the vena contracta is a fraction $\alpha$ of the area of the opening. The coefficient $\alpha$ does not vary much for small changes in flow and geometry (Egli [18]). This means that, when the first fin
5.4 Unsteady flow characteristics

Figure 5.28: 4-fin labyrinth seal flutter simulations: unsteady pressure magnitude in labyrinth. Nominal conditions, Table 5.1, LPS mode, Table 5.3.

Figure 5.29: 4-fin labyrinth seal flutter simulations: unsteady pressure phase in labyrinth. Nominal conditions, Table 5.1, LPS mode, Table 5.3.
moves downward, the dividing streamline will move upward relative to the fin at the vena contracta to keep the area ratio $\alpha$ approximately constant. Conversely when the first fin moves upward, the dividing streamline moves downward. This is illustrated in Fig. 5.30 where the motion of the dividing streamline at the vena contracta relative to the fin motion is plotted together with the motion of the fin. Since the motion of the dividing streamline opposes the change in effective clearance (the effective clearance being the clearance at the vena contracta), this simultaneously reduces and retards the unsteadiness. This explains the observed magnitude and phase of the unsteady pressure in the first cavity. The uniformity of the magnitude of the unsteady pressure in a cavity and the absence of change of sign of the phase are at odds with the distribution of the fluid work in a cavity. We must turn to other contributions to explain the observed shape of the distribution, in particular the change of sign. The axial and radial components of the mode shape are plotted in Fig. 5.31. The mode shape has a strong axial

Figure 5.30: 4-fin labyrinth seal flutter simulations: motion of inlet fin and relative motion of dividing streamline at vena contracta. Nominal conditions, Table 5.1, LPS mode, Table 5.3.
5.4 Unsteady flow characteristics

Figure 5.31: 4-fin labyrinth seal flutter simulations: axial component $\Phi_x$ and radial component $\Phi_r$ of mode shape. LPS mode, Table 5.3.

component. This component is positive and is a linear function of the radial coordinate. The radial component is a linearly decreasing function of the axial coordinate which changes sign around the axial location of the pivot point. Due to the location of the pivot point in the mode studied here, the downstream part of the last cavity moves radially in phase opposition with the upstream part (and with the other cavities). The normal vectors at the cavity wall are represented in Fig. 5.32a, their axial and radial components in Fig. 5.32b. The axial component displays a strong similarity with the work distribution. Indeed it changes sign around the middle of the cavity and is the cause of the change of sign of the aerodynamic work in a cavity. Since the fluid work due to axial motion will have both a positive and negative contribution in each cavity, the stability will be determined (mostly) by the work due to radial motion of the seal. This work is proportional to $\Psi_r$, where $\Psi_r$ is the radial components of the mode shape. This explains why the average value of the work is small in the last cavity since, for this particular cavity, $\Psi_r$ changes sign inside the cavity, thus the
work due to radial motion will have both a positive and negative contribution.

5.5 Energy transfer

5.5.1 Introduction

In this section, we will investigate the energy transfer process between the fluid and the structure during vibration in a labyrinth seal. To simplify the analysis, since we are after a basic understanding of the energy transfer mechanism, we will consider a single-cavity non-rotating seal. This geometry will be used extensively in the parametric study presented in the next chapter. The results presented here have been obtained using the model with HP and LP cavities shown in Fig. 6.1b. The geometric and flow conditions can be found in Table 3.1 of Section 3.3.2. The energy transfer in the labyrinth during the steady-state will first be analysed before presenting results concerning the energy transfer during vibration.

5.5.2 Steady-state energy balance

We begin our discussion by considering the steady-state energy balance. For the entire flow area in the labyrinth, this balance over a (small) interval of time $\delta t$ can be written:

$$\dot{m} \delta t (H_2 - H_1) = \delta W_{wall} + \delta Q_{wall}$$

(5.5.1)

where $H_1$, $H_2$, $\delta W_{wall}$ and $\delta Q_{wall}$ are respectively the inlet total enthalpy, the outlet total enthalpy, the work and the heat exchanged with the walls. For a non-rotating seal with adiabatic walls, both $\delta W_{wall}$ and $\delta Q_{wall}$ are zero and the total enthalpy is conserved. Further discretising the flow area into two control volumes, the through-flow and the cavity flow, we can investigate how energy is
Figure 5.32: 4-fin labyrinth seal: surface normal vector for rotor wall.
exchanged between these two volumes. For the through-flow, we have:

$$\dot{m}\delta t (H_2 - H_1) = \delta W_{SL} + \delta Q_{SL} \quad (5.5.2)$$

where $\delta W_{SL}$ and $\delta Q_{SL}$ are the work and heat exchanged between the two control volumes along the dividing streamline. The work $\delta W_{SL}$ is due to the viscous entrainment of the cavity flow by the through-flow. Since $H_2 = H_1$, this work must be balanced by the heat given by the cavity flow to the through-flow. The work done and heat addition along the dividing streamline are given by:

$$\delta W_{SL} = \delta t \int_{S} \tau \cdot v dS \quad (5.5.3)$$
$$\tau = (\mu_t + \mu_e) \left( \nabla v + \nabla v^T - \frac{2}{3} \nabla \cdot v I \right) n \quad (5.5.4)$$
$$\delta Q_{SL} = \delta t \int_{S} - (\lambda_l + \lambda_t) \nabla T . n dS \quad (5.5.5)$$

The steady-state work and heat (per unit time) along the dividing streamline computed from CFD simulation results are presented in Fig. 5.33. The values of the work and heat are non-dimensionalised by a work evaluated using the Karman-Nikuradse correlation for the friction factor. We see that there is a local equilibrium between work and heat along the dividing streamline.

### 5.5.3 Energy balance during vibration

When one seal member is vibrating, there is a transfer of mechanical energy between the fluid and the structure, and the energy balance for the entire seal flow area over a period of vibration is given by:

$$\Delta mH = W_{wall} \quad (5.5.6)$$

where $\Delta mH = \int_{0}^{T} (\rho_1 u_1 H_1 - \rho_2 u_2 H_2) dt$ is the inlet/outlet enthalpy increase, and $W_{wall}$ is the work exchanged with the moving seal member. The contribution of the viscous stresses to this work is usually negligible; Rhode et al. [39]
Figure 5.33: Single-cavity labyrinth seal: steady-state work and heat along the dividing streamline. Nominal conditions, Table 3.1.

determined that, in the case of rotor whirl, the shear stress contribution is less than 1% of the total force on the rotor. Thus $W_{wall}$ can be approximated by:

$$W_{wall} = \int_0^T \int_S -p\mathbf{v} \cdot n dS$$

(5.5.7)

Writing this energy balance for the through-flow and cavity flow gives:

$$\Delta H = Q_{SL} + W_{SL}$$

(5.5.8)

$$0 = W_{wall} - W_{SL} - Q_{SL}$$

(5.5.9)

$W_{SL}$ includes here an inviscid contribution due to the motion of the dividing streamline:

$$W_{SL} = \int_0^T \int_S (\tau \cdot \mathbf{v} - p\mathbf{v} \cdot \mathbf{n}) dS$$

(5.5.10)

Using CFD flutter simulations, the values of the inviscid work at the cavity wall and along the dividing streamline have been assessed. $W_{wall}$ is the work done...
by the moving cavity wall on the cavity flow and $W_{\text{stream}}$ the work done by the cavity flow on the through-flow along the dividing streamline. The results are presented in Table 5.5.3 for two different modes of vibration of a single-cavity labyrinth seal. The values are presented as a percentage of the enthalpy flux.

<table>
<thead>
<tr>
<th></th>
<th>HPS mode</th>
<th>LPS mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{\text{wall}}/mc_pT_0$ (%)</td>
<td>-0.00157</td>
<td>0.00211</td>
</tr>
<tr>
<td>$W_{\text{stream}}/mc_pT_0$ (%)</td>
<td>-0.00144</td>
<td>0.00194</td>
</tr>
</tbody>
</table>

Table 5.5: Single-cavity labyrinth seal flutter simulations: work done at the wall and along the dividing streamline during one cycle of vibration. Nominal conditions, Table 3.1.

Through the seal. We first notice that the values are small due to the small area of application of the forces and the small amplitude of motion (the amplitude of motion is a fraction of the clearance which is typically of the order of 1 mm). The computed value of the work along the dividing streamline is close to that of the work at the wall indicating that the motion of the dividing streamline is the primary mode of energy transfer between the cavity flow and the through-flow. In both cases, the work along the dividing streamline is lower than the work at the wall. The difference, which is around 8% of the work at the wall, could be due to the contribution of the viscous work or of the heat transfer along the dividing streamline.

### 5.6 Summary

In this chapter, the steady-state and unsteady flow in a labyrinth seal have been investigated. A slow convergence of the steady-state simulations is observed due to the presence of large regions at low Mach number in the flow. The flow in the labyrinth area is characterised by a jet-like flow above the fin tips driving a large vortex in the cavity below. For the cases studied, an important vena contracta is also observed at the first fin tip. Above the pressure ratio at which the outlet fin is choked, there is no more change in pressure distribution in the
labyrinth. An analysis of the axial and tangential velocity profiles shows that there are significant differences in flow characteristics between the through-flow area and the cavity area.

A large number of pseudo-time steps are required to converge the inner iterations of the dual-time stepping procedure during flutter simulations. This is coherent with the slow convergence of the steady-state simulations. The flutter simulation results indicate a variation of the unsteady pressure in the labyrinth in agreement with the theoretical results of Chapter 3. The variation of the axial velocities at the fin tips seems to validate the idea of equalisation of the mass flow rates at the fin tips advanced in the same chapter. A clear difference in magnitude of the unsteady tangential velocity is observed between the cavity vortex area and the through-flow area, the magnitude in the latter being small. This is due to the strong axial velocity in the through-flow area which prevents the unsteady tangential flow from settling down. There is positive and negative work contribution in each cavity. This is due to the change of sign of the axial component of the wall normal in the middle of a labyrinth cavity. An investigation of the energy transfer in a labyrinth cavity shows that the motion of the dividing streamline is the primary mode of energy transfer between the cavity flow and the through-flow. Having presented the main characteristics of the flow in a labyrinth seal, both during the steady-state and during flutter, we will investigate in the next chapter the influence of different flow and geometric parameters on aeroelastic stability.
Chapter 6

Parametric study of labyrinth seal flutter

6.1 Introduction

The purpose of this chapter is to study the influence of different geometric and flow parameters on labyrinth seal aeroelastic stability. The parameters investigated are the location of the support, the mechanical to acoustic frequency ratio, the clearance, the pressure ratio across the seal, the inlet swirl, the inlet total pressure and temperature, the pitch and the cavity height. Three configurations are considered: the 4-fin labyrinth seal investigated in the previous chapter, a single-cavity labyrinth seal and a 5-fin labyrinth seal. The single-cavity case is obtained by isolating a cavity from the 4-fin configuration. The 5-fin labyrinth is a simplified geometry created specifically to study the influence of the pitch and cavity height. The small size of the single-cavity model allows an extensive study of the influence of the different parameters to be carried out. The results on the 4-fin labyrinth will be less comprehensive. However, it will be seen that the results on both configurations are remarkably similar and most of the conclusions drawn on the single-cavity case apply to the 4-fin labyrinth as well.
For the single-cavity and the 4-fin cases, two models have been constructed: a model containing only the labyrinth area and a model including the HP and LP cavities. The model without HP and LP cavities allows the fundamental flutter mechanism related to the local flow field of the seal labyrinth to be studied. The validity of these results can be checked on the model with cavities which is more realistic and the eventual influence of the HP and LP cavities can be investigated. The omission of the HP and LP cavities is also beneficial from a computational point of view: it reduces significantly the size of the computational domain and improves the convergence of the simulations since large regions of low Mach number flow are detrimental to the integration scheme. The two meshes for the 4-fin labyrinth case are identical in the labyrinth area. For the single-cavity case, the two meshes are of different: the 2D mesh, which is extruded in the circumferential direction to construct the 3D mesh, is finer in the fin region for the model with HP and LP cavities and contains quadrilateral elements near the walls (boundary layer mesh) while the other mesh contains only triangular elements. The latter mesh was generated for the purpose of carrying out the extensive parametric study, which would have been impracticable on the former mesh. It will be shown that the mesh with only triangular elements gives results whose accuracy is acceptable compared to the results obtained with the boundary layer mesh. The two meshes are shown in Fig. 6.1. These two-dimensional meshes are extruded over a ten-degree sector to generate the final three-dimensional meshes. The meshes have eleven layers in the circumferential direction. This results in 180 points per wavelength for the two-nodal-diameter mode used in flutter simulations. The influence of the location of the support is studied by moving the location of the pivot point of the mode from upstream to downstream. For each location of the pivot, simulations are performed for a range of frequency ratios in order to obtain a full matrix. The same approach is used to study the influence of the other geometric and flow parameters on stability. The range of some parameters, like the location of the pivot point for example, has been extended outside of the matrix a posteriori to check the
6.1 Introduction

6.1a: Model without HP and LP cavities

6.1b: Model with HP and LP cavities

6.1c: Model with HP and LP cavities in labyrinth area

Figure 6.1: Single-cavity labyrinth seal mesh.
6.2 Influence of pivot point location

Influence of larger values. The range of the parameters investigated for the three seal configurations is summarized in Table 6.1. The nominal conditions for the single-cavity labyrinth seal case can be found in Table 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Single-cavity</th>
<th>4-fin</th>
<th>5-fin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_p/p$</td>
<td>min -4</td>
<td>-10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>max 4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$f/f_{ac}$</td>
<td>min 0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>max 2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$c/h$</td>
<td>min 0.1</td>
<td></td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>max 0.3</td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>min 1.01</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>max 2.5</td>
<td>3.5</td>
<td>3.0</td>
</tr>
<tr>
<td>$M$</td>
<td>min 0.4</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>max 1.1</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>$M_{\theta}$</td>
<td>min 0.0</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>max 0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p/c$</td>
<td>min</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td></td>
<td>16</td>
</tr>
</tbody>
</table>

Table 6.1: Range of parameters investigated (empty columns indicate that the parameter was not investigated for the corresponding configuration).

6.2 Influence of pivot point location

Simulations have been performed for five pivot point locations and for thirteen frequency ratios equally spaced between 0.5 and 2.0 on the single-cavity case. The five pivot point locations are shown in Fig. 6.2. A map of the logarithmic decrement against the frequency ratio and pivot point location is presented in Fig. 6.3 for the single-cavity labyrinth seal at the nominal pressure ratio $\Pi = 1.7$. The flutter boundary (isoline $\delta/\delta_0 = 0$) is displayed with a thick black line. Two unstable zones are visible. These zones are at high frequency for high-pressure support (negative $x_p/p$) and low frequency for low-pressure support (positive $x_p/p$). This is in agreement with Abbott's criterion. For low-pressure support, there is a maximum of positive damping around a frequency ratio equal to one, which corresponds to the resonant condition. The magnitude
6.2 Influence of pivot point location

Figure 6.2: Single-cavity labyrinth seal - Locations of pivot point.

of the first harmonic of the unsteady pressure is shown in Fig. 6.4 as a function of the frequency ratio for different locations of the pivot point. We see that the magnitude reaches a maximum in the neighbourhood of a frequency ratio equal to one for the different pivot point locations. This explains the maximum of positive damping for low-pressure support. The absence of an extremum of damping for high-pressure support at resonance is due to the fact that the phase difference between pressure and displacement is close to 180 degrees for $f/f_{ac}$ close to 1 as can be seen in Fig. 6.5. This means that the phase between the pressure and the velocity is close to 90 degrees, thus the aerodynamic work and consequently the damping are small. For high-pressure support, the extremum of damping occurs around a frequency ratio of 1.5, the phase of the unsteady pressure being more favourable. The evolution of the phase with the frequency ratio for high-pressure and low-pressure support is qualitatively in agreement with the results of the theoretical analysis presented in Fig. 3.5.

We notice that the stable range of frequency ratios increases in size when the
6.2 Influence of pivot point location

Figure 6.3: Single-cavity labyrinth seal - Influence of pivot point location $x_p/p$ and frequency ratio $f/f_{ac}$ on logarithmic decrement $\delta$ - Nominal conditions, Table 3.1.
6.2 Influence of pivot point location

Figure 6.4: Single-cavity labyrinth seal - Magnitude of first harmonic of unsteady pressure in middle of cavity - Nominal conditions, Table 3.1.

Figure 6.5: Single-cavity labyrinth seal - Phase of first harmonic of unsteady pressure in middle of cavity - Nominal conditions, Table 3.1.
6.2 Influence of pivot point location

pivot point is moved away from the centre for both high-pressure and low-pressure support. Additional simulations have been performed to determine the evolution of this limit on an extended range of pivot point location. The results are presented in Fig. 6.6. The limit of stability seems to tend to zero when \( x_p/p \) tends to \( +\infty \), and to \( +\infty \) when \( x_p/p \) tends to \( -\infty \). The theoretical analysis predicted a similar evolution although it was predicted that the limit of stability tended to \( +\infty \) at a finite negative value of \( x_p/p \), around \(-3\). This is not observed in the range of \( x_p/p \) investigated in simulations. It is difficult to say if this phenomenon does not occur at all or if it occurs at a \( x_p/p \) much larger in magnitude than predicted by the theoretical analysis.

The evolution of the limit of stability makes sense since, when the pivot point is at infinity either on the HP or LP side, we obtain the same mode shape which is purely radial and uniform axially. Thus we should obtain the same stability
6.2 Influence of pivot point location

at both limits. The stability map seems to indicate that this pure radial mode will be stable at all frequency ratios. This is confirmed by simulation results for a pure radial mode presented in Fig. 6.7 where we see that the logarithmic decrement is always positive.

Figure 6.7: Single-cavity labyrinth seal - Logarithmic decrement for pure radial mode \((x_p/p = \pm \infty)\) - Nominal conditions, Table 3.1.

A comparison between results obtained on the models with and without HP and LP cavities at a unit frequency ratio is presented in Fig. 6.8. The results are in good qualitative agreement with each other. The observed differences are probably due to the absence of boundary layer mesh in the model without HP and LP cavities which leads to differences in the steady-state pressure distribution in the labyrinth as shown in Fig. 6.9.

A stability map for the 4-fin labyrinth seal case is presented in Fig. 6.10. The map is very similar to that of the single-cavity case and the analysis made for the single-cavity seal is equally valid for this case. Compared to the single-cavity case, the flutter boundary is shifted towards higher frequency ratios as shown
6.2 Influence of pivot point location

Figure 6.8: Single-cavity labyrinth seal - Influence of pivot point location $x_p/p$ on logarithmic decrement $\delta$ - Comparison between models with and without HP and LP cavities - Nominal conditions, Table 3.1, $f/f_{ac} = 1.0$.

Figure 6.9: Single-cavity labyrinth seal - Steady-state pressure distribution on rotor surface - Nominal conditions, Table 3.1.
6.2 Influence of pivot point location

Figure 6.10: 4-fin labyrinth seal - Influence of pivot point location $x_p/p$ and frequency ratio $f/f_{ac}$ on logarithmic decrement $\delta$ - Nominal conditions, Table 5.1.
6.2 Influence of pivot point location

in Fig. 6.11. It will be noticed that when the pivot point is in the middle of the

labyrinth \( x_p/p = 0 \), the logarithmic decrement is small. When the pivot point

is in the middle, one half of the rotor will be moving radially out of phase with

the other half. Assuming that the phase of the unsteady pressure in adjacent

cavities remains close, which is usually the case, this means that if one half of

the rotor is subjected to a positive aerodynamic work, the other half will be

subjected to a negative one as shown in Fig. 6.12. This explains the smallness

of the logarithmic decrement. A comparison between results obtained on the

models with and without HP and LP cavities at a frequency ratio of 0.8 is

presented in Fig. 6.13. There are some small differences between the results

of the two models which can be attributed to the contributions of the HP and

LP cavities to the aerodynamic work. The results remain, however, in good

qualitative agreement.

Fig. 6.14 to 6.17 show the distribution of the phase of the unsteady pressure in
6.2 Influence of pivot point location

Figure 6.12: 4-fin labyrinth seal - Aerodynamic work distribution for pivot point in the middle of the labyrinth ($x_p/p = 0$) - Nominal conditions, Table 5.1, $f/f_{ac} = 0.75$. 
Figure 6.13: 4-fin labyrinth seal - Influence of pivot point location $x_p/p$ on logarithmic decrement $\delta$ - Comparison between models with and without HP and LP cavities - Nominal conditions, Table 5.1, $f/f_{ac} = 0.8$. 
6.2 Influence of pivot point location

the labyrinth for different pivot point locations and frequency ratios. There

is a sharp change of the phase at the fin tips when passing from one cavity

to another. The phase difference between adjacent cavities increases with the

frequency ratio as shown in Fig. 6.18 to 6.21. The dependence on the frequency

ratio is linear for the phase difference between the second and third cavity and

almost linear for the phase difference between the first and second cavity. This

is consistent with a phase difference of the form:

\[ \Delta \phi = 2\pi f \Delta t \] (6.2.1)

where \( f \) is the frequency of vibration and \( \Delta t \) is a constant interval of time.

The variation of the phase difference between two adjacent cavities with the

pitch is shown in Fig. 6.22. The results have been obtained on the 5-fin labyrinth

seal geometry (cf. Section 6.7). The phase difference is obtained from the sim-

ulation results by computing the average value of the phase along the rotor

Figure 6.14: 4-fin labyrinth seal - Unsteady pressure phase distribution - Nom-

inal conditions, Table 5.1, \( x_p/p = -2 \).
6.2 Influence of pivot point location

Figure 6.15: 4-fin labyrinth seal - Unsteady pressure phase distribution - Nominal conditions, Table 5.1, $x_p/p = -1$.

Figure 6.16: 4-fin labyrinth seal - Unsteady pressure phase distribution - Nominal conditions, Table 5.1, $x_p/p = 1$. 

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6.2 Influence of pivot point location

![Figure 6.17: 4-fin labyrinth seal - Unsteady pressure phase distribution - Nominal conditions, Table 5.1, $x_p/p = 2$.](image)

The phase difference between adjacent cavities increases with the pitch. The unsteady pressure in a labyrinth cavity is influenced by perturbations coming from upstream and downstream. The distance between the inlet of the labyrinth and the exit of cavity $n$ is $np$ where $p$ is the pitch. Pressure perturbations travel downstream at a speed $a + u$ where $a$ is the (average) speed of sound and $u$ is speed of the flow. Thus, the time taken by a pressure perturbation coming from the inlet of the labyrinth to affect cavity $n$ is:

$$t_{I,n} = \frac{np}{a + u}$$

(6.2.2)

Similarly, the distance between the exit of the labyrinth and the inlet of cavity $n$ is $(N - n + 1)p$ where $N$ is the total number of cavities in the labyrinth. Pressure perturbations travel upstream at a speed $a - u$. Thus, the time taken by a pressure perturbation coming from the exit of the labyrinth to affect cavity
Figure 6.18: 4-fin labyrinth seal - Influence of frequency ratio on phase difference between adjacent cavities. Solid red line: phase difference between first and second cavity. Dashed green line: phase difference between second and third cavity - Nominal conditions, Table 5.1, $x_p/p = -2$. 
Figure 6.19: 4-fin labyrinth seal - Influence of frequency ratio on phase difference between adjacent cavities. Solid red line: phase difference between first and second cavity. Dashed green line: phase difference between second and third cavity - Nominal conditions, Table 5.1, $x_P/p = -1$. 
Figure 6.20: 4-fin labyrinth seal - Influence of frequency ratio on phase difference between adjacent cavities. Solid red line: phase difference between first and second cavity. Dashed green line: phase difference between second and third cavity - Nominal conditions, Table 5.1, $x_p/p = 1$. 
Figure 6.21: 4-fin labyrinth seal - Influence of frequency ratio on phase difference between adjacent cavities. Solid red line: phase difference between first and second cavity. Dashed green line: phase difference between second and third cavity - Nominal conditions, Table 5.1, $x_p/p = 2$. 
Figure 6.22: 5-fin labyrinth seal - Influence of pitch on phase difference between adjacent cavities - Nominal conditions, Table 6.2.

\[ n =: t_{E,n} = \frac{(N - n + 1)p}{a - u} \]  

(6.2.3)

Cavity \( n \) will have been affected by pressure perturbations coming from both the inlet and the exit after a time:

\[ t_n = \max \left( t_{I,n}, t_{E,n} \right) \]  

(6.2.4)

For two adjacent cavities \( n \) and \( n + 1 \), a theoretical phase difference based on these considerations will be:

\[ \Delta \phi_{n,n+1} = 2\pi f \left( t_n - t_{n+1} \right) \]  

(6.2.5)

The evolution of this theoretical phase difference with the pitch is presented in Fig. 6.22. We see that the slope of the theoretical phase difference is in good agreement with simulation results. However, the intercept obtained by
extrapolating the simulation results is not zero for zero pitch. This indicates that there is another contribution to the phase difference between adjacent cavities apart from the axial propagation of the acoustic waves. The flow in a labyrinth cavity is a largely separated flow. It is reasonable to think that, once perturbed, this flow requires a certain time to settle down. This phenomenon could account for the non-propagative part of the phase difference between cavities.

6.3 Influence of clearance

A map of the logarithmic decrement against clearance and frequency ratio is plotted in Fig. 6.23 for the single-cavity case. The clearance is non-dimensionalised by the nominal clearance $c_{nom}$. The clearance affects primarily the magnitude of the logarithmic decrement: reducing the clearance increases the magnitude of the logarithmic decrement. The evolution of the logarithmic decrement with the clearance at fixed frequency ratios is presented in Fig. 6.24 for a HPS mode ($x_p/p = -1$) and a LPS mode ($x_p/p = 1$). The tendency of the magnitude of the logarithmic decrement to decrease with an increase in clearance is visible at the lowest and at the two highest frequency ratios. At frequency ratios of 0.75 and 1.0, the evolution is not monotonic. For the present case, an increase in clearance decreases the range of stable frequencies for a HPS mode and increases it for a LPS mode. Consequently, from this point of view, an increase in clearance can neither be said to be stabilising or destabilising. In the theoretical study, a stabilising effect of an increase in clearance was predicted; this theoretical study did not take into account the effect of the clearance on the steady-state, thus would only give the local slope of the curve $\delta = f(c)$ around a fixed steady-state. The Mach number contours in the labyrinth at the nominal clearance and twice the nominal clearance are shown in Fig. 6.25. The maximum value of the Mach number changes from 0.96 at the nominal clearance to 1.17 at twice the nominal clearance. The variation of the flow function with the clearance at the nominal pressure ratio is shown in Fig. 6.26. The flow function varies by 14% when the
6.3 Influence of clearance

Figure 6.23: Single-cavity labyrinth seal - Influence of seal clearance $c$ and frequency ratio $f/f_{ac}$ on logarithmic decrement $\delta$ - Nominal conditions, Table 3.1, except for clearance.

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6.3 Influence of clearance

Figure 6.24: Single-cavity labyrinth seal - Logarithmic decrement $\delta$ vs clearance $c$ for a HPS mode ($x_p/p = -1$) and a LPS mode ($x_p/p = 1$) - Nominal conditions, Table 3.1, except for clearance.
Figure 6.25: Single-cavity labyrinth seal - Influence of seal clearance on steady-state flow - Nominal conditions, Table 3.1, except for clearance which is $2c_{nom}$ for Fig. 6.25b.
6.4 Influence of pressure ratio/Mach number

A change in pressure ratio across the seal will induce a change in the Mach number of the through-flow. The Mach number is a more characteristic parameter of the state of the flow than the pressure ratio. Indeed, when the Mach number reaches unity at the outlet fin, the flow will be blocked, and for Mach number lower than 0.3, the flow can be considered incompressible. For these reasons, most of the results will be presented here in term of Mach number instead of pressure ratio. The Mach number is evaluated at the outlet fin tip. A stability map showing the combined influence of the Mach number and the frequency ratio is presented in Fig. 6.27 for the single-cavity case. From the map, we see
6.4 Influence of pressure ratio/Mach number

Figure 6.27: Single-cavity labyrinth seal - Influence of Mach number and frequency ratio $f/f_{ac}$ on logarithmic decrement $\delta$ - Nominal conditions, Table 3.1, except for pressure ratio which varies between 1.2 and 2.5.
6.4 Influence of pressure ratio/Mach number

that the Mach number has a strong influence on stability. Simulation results clearly indicate that there is no instability at low Mach number. This is in agreement with the theoretical predictions in Section 3.3.2. The unstable zone is at high Mach number/high frequency ratio for high-pressure support, and high Mach number/low frequency ratio for low-pressure support. The trend at high pressure ratio/high Mach number ($M > 0.5$) is that the magnitude of the logarithmic decrement increases with the Mach number. This can be seen more clearly in Fig. 6.28, where the logarithmic decrement is plotted as a function of the pressure ratio for a HPS mode ($x_p/p = 1$) and LP support ($x_p/p = -1$). We also see that the logarithmic decrement reaches a plateau at high pressure ratio. This is due to the fact that, when the outlet fin is choked, the state of the flow in the labyrinth does not change anymore with a decrease in outlet pressure as explained in Section 5.3.

Fig. 6.29 shows the map of logarithmic decrement against pivot point location.
6.4 Influence of pressure ratio/Mach number

and frequency ratio at low Mach number \((M = 0.4)\). The stability map is

significant altered compared to the one at the nominal pressure ratio presented in Fig. 6.3. The unstable zones are reduced in size and concentrated around two zones which correspond here to the locations of the entrance and exit fin.

Results for the 4-fin labyrinth concerning the influence of the Mach number are presented in Fig. 6.30. The results have been obtained on the configuration including the HP and LP cavities. The mode shape is the same as the one studied in Section 5.4 and the frequency ratio is equal to 0.8. At this frequency ratio, the mode is unstable at the nominal pressure ratio (cf. 5.4). The results are similar to the ones on the single-cavity labyrinth. The logarithmic decrement which is negative at high Mach number becomes positive at low Mach number indicating a stabilisation of the mode. Moreover, it reaches a plateau at a Mach
6.5 Influence of inlet swirl

When the air in the cavity possesses a tangential velocity \( u \), an apparent mechanical frequency \( f \) must be evaluated in the frame of reference of the swirling air:

\[
    f = f_n - \frac{nu}{2\pi R}
\]

where \( f_n \), \( n \) and \( R \) are the natural frequency, the nodal diameter number and the mean cavity radius respectively. Figure 6.31 shows a map of the logarithmic decrement of the 4-fin labyrinth seal as a function of Mach number, nominal conditions, Table 5.1, except for pressure ratio which varies between 1.1 and 3.5.

The stability of labyrinth seals at low Mach number, predicted by both theory and simulations appears to be a distinctive property of seal flutter; thus, the Mach number should be added to the list of critical parameters.

Figure 6.30: 4-fin labyrinth seal - Influence of Mach number on logarithmic decrement - Nominal conditions, Table 5.1, except for pressure ratio which varies between 1.1 and 3.5.
6.6 Influence of inlet total temperature and pressure

decrement as a function of the frequency ratio $f/f_{ac}$ and of the inlet swirl Mach number. The limit of stability is not modified by the value of the swirl; however, the swirl influences the value of the logarithmic decrement. If we plot the aerodynamic work instead of the logarithmic decrement (Fig. 6.32), the influence of the inlet swirl disappears: the value of the aerodynamic work is independent of the value of the swirl and depends only on the frequency ratio.

The change in logarithmic decrement at a fixed frequency ratio for different values of the swirl is due to the difference between the natural frequency and the frequency seen by the fluid: the fluid produces a work which is dependent of the apparent mechanical frequency; on the other hand the kinetic energy of the structure depends on the natural frequency. Hence the change in logarithmic decrement with the swirl. This implies that the same natural mode of the structure will be damped/amplified differently by the fluid for different values of the air swirl. The swirl also introduces an asymmetry between forward and backward travelling waves since, according to Eq. (6.5.1), they will have different frequency ratios: one of them will be increased while the other will be reduced by the swirl. This can lead to one being stabilised while the other is destabilised by the presence of air swirl.

6.6 Influence of inlet total temperature and pressure

Simulations have been carried out at modified inlet total temperature and pressure on the single-cavity case to check their influence on the stability characteristics. In the following, we will denote $T_{t,ref}$ and $P_{t,ref}$, the inlet total temperature and pressure of the reference case. The first case is at the same inlet total pressure as the reference case but a modified inlet total temperature $T_{t,mod}$. The ratio between this inlet total temperature and that of the reference case is $T_{t,mod}/T_{t,ref} = 0.48$. The second case is at the same inlet total tem-
6.6 Influence of inlet total temperature and pressure

6.31a: HPS mode \( \frac{x_p}{p} = -1 \)

6.31b: LPS mode \( \frac{x_p}{p} = 1 \)

Figure 6.31: Single-cavity labyrinth seal - Influence of seal inlet swirl Mach number \( M_\theta \) and frequency ratio \( \frac{f}{f_{ac}} \) on logarithmic decrement \( \delta \) - Nominal conditions, Table 3.1.
Figure 6.32: Single-cavity labyrinth seal - Influence of seal inlet swirl Mach number $M_\theta$ and frequency ratio $f/f_{ac}$ on aerodynamic work $w$ - Nominal conditions, Table 3.1.
6.6 Influence of inlet total temperature and pressure

perature as the reference case but an inlet total pressure $P_{t,\text{mod}}$ which is half that of the reference case. For the second case, the outlet static pressure is also modified to keep the pressure ratio across the seal constant. The mode of vibration used in flutter simulations is a low-pressure support mode. The frequency is varied to obtain stability characteristics. The stability characteristics at the reference and modified inlet total temperature are compared in Fig. 6.33. On the left figure, the logarithmic decrement is presented and on the right figure the aerodynamic work. The abscissa is the mechanical to acoustic frequency ratio. Since the total temperature is changed, the acoustic frequency differs between the two cases and at the same frequency of vibration, we obtain different frequency ratios. The change in inlet total temperature affects the magnitude of the logarithmic decrement; however, at a given frequency ratio, we obtain the same aerodynamic work in both cases. We can use the same argument as in the previous section: at a given value of the frequency ratio, the value of the me-

![Figure 6.33: Single-cavity labyrinth seal - Influence of inlet total temperature $T_t$ on logarithmic decrement and aerodynamic work - Nominal conditions, Table 3.1. Left: logarithmic decrement. Right: aerodynamic work.](image-url)
6.6 Influence of inlet total temperature and pressure

The influence of the inlet total temperature and pressure on the stability characteristics is presented in Fig. 6.34. The stability characteristics are similar at the reference and modified inlet total pressure. In the right figure, the logarithmic decrement is divided by the value of the inlet total pressure. This artifice renders the two stability curves quasi-identical. This shows that the logarithmic decrement is proportional to the value of the inlet total pressure. This was to be expected since the aerodynamic force and thus the work is proportional to the pressure.
6.7 Influence of pitch

The investigation on the influence of the pitch is carried out on a 5-fin labyrinth seal geometry. Four values of the pitch-to-clearance ratio $p/c$ are considered: 4, 8, 12 and 16. All the other geometric parameters and the boundary conditions are identical for the four configurations. The nominal conditions are given in Table 6.2. The geometry and mesh for the four configurations are shown in Fig. 6.35. The steady-state pressure contours in the labyrinth at a pressure ratio across the seal of 1.7 are compared in Fig. 6.36. The pressure is non-dimensionalised by the inlet total pressure. The pitch influences significantly the value of the pressure in the first cavity. This pressure decreases as the pitch is decreased. The pressure distribution on the lower wall of the labyrinth is shown is Fig. 6.37. This figure confirms the observations made on the contour plots: at the two lowest values of the pitch, the pressure in the first cavity is lower than in the second cavity. In the four cavities, the value of the pressure decreases as the pitch is decreased, but the first cavity is the most affected.

The Mach number contours presented in Fig. 6.38 show that the Mach number of the through-flow increases as the pitch is decreased. This is due to the fact that when the cavity length is reduced, the expansion losses are diminished. The flow pattern in the cavities is shown in Fig. 6.39. The pitch has a strong influence on the flow pattern inside the cavity. The flow pattern appears to be controlled by the aspect ratio of the cavity. At the smallest pitch, there are three large recirculations stacked one on top of the other in the cavity. For $p/c = 8$, where the cavity has a square aspect ratio, there is only one large recirculation.

<table>
<thead>
<tr>
<th>Nominal clearance $c$</th>
<th>0.5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch $p/c$</td>
<td>from 4 to 16</td>
</tr>
<tr>
<td>Height of cavity $h/c$</td>
<td>6</td>
</tr>
<tr>
<td>Pressure ratio $H$</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 6.2: Nominal conditions for 5-fin labyrinth seal case for study of influence of pitch.
6.7 Influence of pitch

Figure 6.35: 5-fin labyrinth seal - Influence of pitch - Geometries and meshes.
6.7 Influence of pitch

Figure 6.36: 5-fin labyrinth seal - Steady-state pressure contours for different values of the pitch - Nominal conditions, Table 6.2.
in the cavity with small secondary recirculations at the cavity corners. At the two highest values of the pitch, there are two large recirculations inside the cavity: the main recirculation driven by the through-flow and an additional recirculation occupying the upstream side of the cavity.

The flutter simulations have been carried out for a HPS mode \((x_p/p = -2.5)\) and a LPS mode \((x_p/p = 2.5)\). The pivot point was at the labyrinth entrance and exit respectively. The frequency ratio was 1.5 for the HPS mode and 0.5 for the LPS mode. At these frequency ratios, we expect the two modes to be unstable according to Abbott's stability criterion. The evolution of the logarithmic decrement with the pitch at a pressure ratio of 1.7 is presented in Fig. 6.40. The two curves have the same trend: the logarithmic decrement reaches a minimum at \(p/c = 8\) then increases. For the HPS mode, the logarithmic decrement becomes positive at \(p/c = 16\). The increased stability at larger values of the pitch can be explained by the decrease of the Mach number. Indeed, it has been
6.7 Influence of pitch

Figure 6.38: 5-fin labyrinth seal - Steady-state Mach number contours for different values of the pitch - Nominal conditions, Table 6.2.
6.7 Influence of pitch

Figure 6.39: 5-fin labyrinth seal - Steady-state streamlines for different values of the pitch - Nominal conditions, Table 6.2.
Figure 6.40: 5-fin labyrinth seal - Influence of pitch on logarithmic decrement for a HPS mode \((x_p/p = -2.5)\) at a frequency ratio \(f/f_{ac} = 1.5\) and a LPS mode \((x_p/p = 2.5)\) at a frequency ratio \(f/f_{ac} = 0.5\) - Nominal conditions, Table 6.2.
shown in Section 6.4 that decreasing the Mach number is beneficial for stability. To confirm this hypothesis, the logarithmic decrement is plotted against the flow function at the four pitch-to-clearance ratios in Fig. 6.41 for the HPS mode. The flow function is a measure of the Mach number of the flow. This plot shows that, at a pressure ratio $\Pi = 1.7$, the flow function is reduced by 30% when the pitch-to-clearance is changed from 4 to 16. All curves show that the stability increases when the flow function decreases; this is consistent with the increased stability at low Mach number already mentioned. Thus, part of the change in stability observed on the curves in Fig. 6.40, presenting the logarithmic decrement as a function of the pitch-to-clearance ratio at a fixed pressure ratio, is due to the change in the steady-state Mach number. However, since the points at the different values of the pitch-to-clearance ratio do not fall on the same curve in Fig. 6.41, the pitch-to-clearance ratio also has a direct effect on stability. As shown in Section 6.2, the pitch influences the phase difference between adjacent cavities. Moreover, as discussed in Section 5.3, for small values of the pitch-to-clearance ratio the second fin can be ineffective because the flow passes above it without touching it. This is what happens for $p/c = 4$ and $p/c = 8$. Consequently, even if the flow function has the same value, we will have a different flow pattern at small and high value of the pitch-to-clearance ratio: the first cavity is not closed by the dividing streamline for small pitch-to-clearance ratio. This could explain the influence of the pitch on stability.

6.8 Influence of cavity height

The influence of the cavity height on the stability characteristics has been investigated on the configuration used in the previous section to study the influence of the pitch. The configuration of pitch-to-clearance ratio $p/c = 8$ was chosen. Three height-to-clearance ratios have been considered: $h/c = 2$, $h/c = 4$ and $h/c = 6$. The nominal conditions are given in Table 6.3. The geometries and
Figure 6.41: 5-fin labyrinth seal - Logarithmic decrement vs flow function for different values of the pitch - Nominal conditions, Table 6.2, except for pressure ratio which varies between 1.1 and 3.0 - HPS mode \((x_p/p = -2.5), f/f_{ac} = 1.5\).
6.8 Influence of cavity height

<table>
<thead>
<tr>
<th>Nominal clearance $c$</th>
<th>0.5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch $p/c$</td>
<td>8</td>
</tr>
<tr>
<td>Height of cavity $h/c$</td>
<td>from 2 to 6</td>
</tr>
<tr>
<td>Pressure ratio $H$</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 6.3: Nominal conditions for 5-fin labyrinth seal case for study of influence of cavity height.

Meshes of the three configurations are presented in Fig. 6.42. The steady-state pressure and Mach number contours in the labyrinth are compared in Fig. 6.43 and 6.44. The pressure distribution on the lower wall is shown is Fig. 6.45. The value of the static pressure in the labyrinth decreases when the cavity height is increased. The static pressure values are close at the two highest cavity heights. However, there is a noticeable difference at the smallest height. The smaller value of the Mach number of the through-flow at the smallest height indicates that the losses in the cavity are increased at this height ratio. This is confirmed by the contours of turbulent viscosities presented in Fig. 6.46 which show that the levels of turbulence increase as the height of the cavity is decreased. The flow pattern in the cavities is shown in Fig. 6.47. At the smallest cavity height, there is a large recirculation occupying most of the cavity and a smaller recirculation in the left corner of the cavity. At the intermediate cavity height, the flow is similar to the one observed in the previous section at the two highest values of the pitch, with two large recirculations in the cavity. At the highest cavity height, where the ratio of the cavity length to the cavity height is close to one, there is only one large recirculation filling the cavity.

Flutter simulations have been carried out for a high-pressure support mode at a frequency ratio of 1.5 and a low-pressure support mode at a frequency ratio of 0.5. The influence of the cavity height on the logarithmic decrement is shown in Fig. 6.48. Decreasing the cavity height has a stabilising effect: the modes which are unstable at the highest cavity height become stable at the intermediate height and have an even greater damping at the smallest height. Such a stabilising effect of the decrease of the height-to-clearance ratio $h/c$ was
Figure 6.42: 5-fin labyrinth seal - Influence of cavity height - Geometries and meshes.
Figure 6.43: 5-fin labyrinth seal - Steady-state pressure contours for different values of the cavity height - Nominal conditions, Table 6.3.
Figure 6.44: 5-fin labyrinth seal - Steady-state Mach number contours for different values of the cavity height - Nominal conditions, Table 6.3.
Figure 6.45: 5-fin labyrinth seal - Steady-state pressure distribution on lower wall for different values of the cavity height - Nominal conditions, Table 6.3.

predicted in the theoretical study presented in Section 3.3.2 and is confirmed by simulation results.

When the cavity height is changed, there is a change in both geometry and flow parameters. Both changes can affect the stability. To separate these two influences, the logarithmic decrement is plotted against the flow function at the three cavity heights in Fig. 6.49 for the HPS mode. It is clear from this plot, that the stability at the lowest cavity height cannot be attributed to a change in flow function. This confirms that reducing the height-to-clearance ratio $h/c$ improves stability.

6.9 Summary

The influence of different geometric and flow parameters on the aeroelastic stability of labyrinth seals has been investigated in this chapter. The influence
Figure 6.46: 5-fin labyrinth seal - Steady-state turbulent viscosity for different values of the cavity height - Nominal conditions, Table 6.3.
6.47a: $h/c = 2$

6.47b: $h/c = 4$

6.47c: $h/c = 6$

Figure 6.47: 5-fin labyrinth seal - Steady-state streamlines for different values of the cavity height - Nominal conditions, Table 6.3.
Figure 6.48: 5-fin labyrinth seal - Influence of cavity height on logarithmic decrement for a HPS mode \((x_p/p = -2.5)\) at a frequency ratio \(f/f_{ac} = 1.5\) and a LPS mode \((x_p/p = 2.5)\) at a frequency ratio \(f/f_{ac} = 0.5\) - Nominal conditions, Table 6.3.
Figure 6.49: 5-fin labyrinth seal - Logarithmic decrement vs flow function for different values of the cavity height - Nominal conditions, Table 6.3, except for pressure ratio which varies between 1.1 and 3.0 - HPS mode ($x_p/p = -2.5$), $f/f_{ac} = 1.5$. 

The logarithmic decrement is approximately proportional to the value of the flow function, which reflects the dependence of the flow function on the pressure ratio. The cavity height has a stabilizing effect as evidenced in Chapter 5. The CFD techniques used for the present parameter study are not suited for research at present.
of the location of the pivot point and of the mechanical to acoustic frequency ratio are found in agreement with Abbott's criterion. The stable range of frequency ratio increases when the pivot point is moved away from the centre of the labyrinth for both high-pressure and low-pressure support. As a consequence, a purely uniform radial mode (pivot point at infinity) is stable over the whole frequency range. The clearance affects the magnitude of the logarithmic decrement, reducing the clearance increasing the magnitude of the logarithmic decrement. The dependence of the logarithmic decrement on the clearance is nonlinear: the sensitivity is large at small clearances and decreases with an increase in clearance. A stabilising effect of an increase in clearance is observed in some cases. This is in agreement with the theoretical results of Chapter 3. However, general conclusions for large changes in clearance are difficult to make since these changes are accompanied by changes in flow parameters (like the Mach number) which also affect the stability. Simulation results clearly indicate that there is no instability at low pressure ratio, or equivalently at low Mach number. The logarithmic decrement reaches a plateau at high pressure ratio. This is a consequence of the choking of the outlet fin. The aerodynamic work is independent of the value of the swirl and depends only on the frequency ratio (when the swirl value is taken into account to evaluate this frequency ratio). The change in logarithmic decrement at a fixed frequency ratio for different values of the swirl is due to the difference between the natural frequency, which fixes the kinetic energy of the structure, and the frequency seen by the fluid, which depends on the value of the swirl. Similarly, the aerodynamic work is independent of the value of the inlet total temperature at a given frequency ratio. The logarithmic decrement is (approximately) proportional to the value of the inlet total pressure, which reflects the dependence of the aerodynamic force on the pressure. The pitch affects stability via its influence on the steady-state flow and via its influence on the phase difference between adjacent cavities. Decreasing the cavity height has a stabilising effect as predicted in Chapter 3. While the CFD techniques used for the present parametric study are useful for research
purposes, they are still too costly for routine use in industry. The purpose of the next chapter is to present a cheaper alternative based on bulk-flow models.
Chapter 7

Analytical model

7.1 Introduction

This section presents analytical models which have been developed for labyrinth seal flutter predictions. The models are of the bulk-flow type. Two models are presented. In the first one, the labyrinth area is discretised using two control volumes, one for the through-flow and one for the vortex area. This model is an improvement over the one presented in Section 3.3 where only one control volume was used to describe the flow in a labyrinth cavity. Steady-state and unsteady flow results presented in Chapter 5 clearly indicate that the vortex area and the through-flow area behave in a distinct manner. The second model adds an additional control volume for the fin tip. This model was devised because of qualitative discrepancies observed between CFD results and the analytical results of the two-control-volume model which seemed to originate at the fin tips. The distinctive feature of these models is that they use the dividing streamline as a limit between the through-flow and the vortex control volumes.

The presentation will follow the approach used in the development of these models. The governing equations of the two-control-volume model will first be
introduced. Some results obtained by applying this model to the single-cavity and 4-fin labyrinth seal cases will then be presented. The discrepancies with CFD results will be highlighted. The three-control-volume model will then be described. The derivation of a model for the motion of the dividing streamline will be explained. A correction of the modelling of the vortex control volume to take into account the possibility of a flow out of this control volume during vibration will be discussed next. The treatment of the fluxes between the control volumes and of the boundary conditions will be described before presenting some results obtained on the single-cavity case and on the 4-fin labyrinth case. The equations of the models are derived for a seal with a moving rotor (lower) wall. The case of a moving stator wall does not present any additional difficulty.

7.2 Two-control-volume model

7.2.1 Governing equations

The equations will be written for a two-dimensional flow in the \((x, z)\) plane. The treatment of the axisymmetric case is similar. Here we will adopt the larger definition of a two-dimensional flow as a flow which is invariant by translation in the transverse direction \((y)\). This means that the flow can have a velocity component in the \(y\) direction.

This model divides the inter-fin cavity into two control volumes separated by the streamline originating from the leading edge of the entrance fin: the through-flow area and the cavity vortex area. The application of the principles of mass, circumferential momentum and energy conservation to these two control volumes (Fig. 7.1) yields the following equations:
7.2 Two-control-volume model

Figure 7.1: Control volumes for two-control-volume model.

**CV1:**

\[
\begin{align*}
\frac{\partial (\rho_1E_1h_1)}{\partial t} + \frac{\partial (\rho_1v_1h_1)}{\partial y} &= \frac{1}{L}(m_i - m_o) \\
\frac{\partial (\rho_1v_1h_1)}{\partial t} + \frac{\partial (\rho_1v_1^2h_1)}{\partial y} + h_1\frac{\partial p_1}{\partial y} &= \frac{1}{L}(m_i v_i - m_o v_o) \\
\frac{\partial (\rho_1E_1h_1)}{\partial t} + \frac{\partial (\rho_1v_1H_1h_1)}{\partial y} &= \frac{1}{L}(m_iH_i - m_oH_o) + p_{SL}w_{SL}
\end{align*}
\]

**CV2:**

\[
\begin{align*}
\frac{\partial (\rho_2h_2)}{\partial t} + \frac{\partial (\rho_2v_2h_2)}{\partial y} &= 0 \\
\frac{\partial (\rho_2v_2h_2)}{\partial t} + \frac{\partial (\rho_2v_2^2h_2)}{\partial y} + h_2\frac{\partial p_2}{\partial y} &= 0 \\
\frac{\partial (\rho_2E_2h_2)}{\partial t} + \frac{\partial (\rho_2v_2H_2h_2)}{\partial y} &= p_2w_R - p_{SL}w_{SL}
\end{align*}
\]

The motion of the dividing streamline will be driven by the normal pressure.
gradient along the dividing streamline. This can be modelled by the following equation:

\[ \rho_{SL} \frac{\partial w_{SL}}{\partial t} = \frac{p_2 - p_1}{\delta_{SL}} \]  

(7.2.7)

where \( \delta_{SL} \) is a characteristic thickness, such as for example the mixing layer thickness between the through-flow and the vortex flow.

For small motion of the rotor, a perturbation analysis can be carried out. At first order, we obtain:

CV1:

\[ \rho_1 \frac{\partial h'_1}{\partial t} + h_1 \frac{\partial p'_1}{\partial t} + \rho_1 v_1 \frac{\partial h'_1}{\partial y} + \rho_1 h_1 \frac{\partial v'_1}{\partial y} + v_1 h_1 \frac{\partial p'_1}{\partial y} = \]

\[ \frac{1}{L} \left( m'_i - m'_o \right) \]  

(7.2.8)

\[ \rho_1 v_1 \frac{\partial h'_i}{\partial t} + \rho_1 h_1 \frac{\partial v'_i}{\partial t} + v_1 h_1 \frac{\partial p'_i}{\partial t} + \rho_1 v_1 \frac{\partial h'_1}{\partial y} + 2 \rho_1 v_1 h_1 \frac{\partial v'_i}{\partial y} + v_1^2 h_1 \frac{\partial p'_1}{\partial y} + h_1 \frac{\partial p'_i}{\partial y} = \]

\[ \frac{1}{L} \left[ m (v'_i - v'_o) + v_i m'_i - v_o m'_o \right] \]  

(7.2.9)

\[ \rho_1 E_1 \frac{\partial h'_1}{\partial t} + h_1 \frac{\partial (\rho_1 E_1 \gamma)}{\partial t} + \rho_1 v_1 H_1 \frac{\partial h'_1}{\partial y} + v_1 h_1 \frac{\partial (\rho_1 H_1 \gamma)}{\partial y} + \rho_1 H_1 h_1 \frac{\partial v'_1}{\partial y} = \]

\[ \frac{1}{L} \left[ m (H'_i - H'_o) + H_i m'_i - H_o m'_o \right] + p_{SL} w'_{SL} \]  

(7.2.10)
7.2 Two-control-volume model

CV2:

\[
\rho_2 \frac{\partial h'_2}{\partial t} + h_2 \frac{\partial p'_2}{\partial t} + \rho_2 v_2 \frac{\partial h'_2}{\partial y} + \rho_2 h_2 \frac{\partial v'_2}{\partial y} + v_2 h_2 \frac{\partial p'_2}{\partial y} = 0
\]  \hspace{1cm} (7.2.11)

\[
\rho_2 v_2 \frac{\partial h'_2}{\partial t} + \rho_2 h_2 \frac{\partial v'_2}{\partial t} + v_2 h_2 \frac{\partial p'_2}{\partial y} + 2 \rho_2 v_2 h_2 \frac{\partial v'_2}{\partial y} + v_2^2 h_2 \frac{\partial p'_2}{\partial y} + h_2 \frac{\partial p'_2}{\partial y} = 0
\]  \hspace{1cm} (7.2.12)

\[
\rho_2 E_2 \frac{\partial h'_2}{\partial t} + h_2 \frac{\partial (\rho_2 E_2)'}{\partial t} + \rho_2 v_2 H_2 \frac{\partial h'_2}{\partial y} + v_2 h_2 \frac{\partial (\rho_2 H_2)'}{\partial y} + \rho_2 H_2 h_2 \frac{\partial v'_2}{\partial y} = p_2 w'_R - p_{SL} w'_{SL}
\]  \hspace{1cm} (7.2.13)

Dividing streamline:

\[
\frac{\partial w'_{SL}}{\partial t} = \frac{p'_2 - p'_1}{\rho_{SL} \delta_{SL}}
\]  \hspace{1cm} (7.2.14)

The unsteady mass flow rates at the entrance and exit are related to the unsteady static and total pressure at the fin tips and to the variation of the clearance by using the same approach as in Section 3.3:

\[
m' = \frac{1}{\gamma M^2} \left[ \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \frac{p'_0}{p_0} + (M^2 - 1) \frac{p'}{p} \right] + \frac{c'}{c}
\]  \hspace{1cm} (7.2.15)

The total pressure loss model gives:

\[
p'_{0i} - p'_{0o} = K_e (p'_{0i} - p'_1)
\]  \hspace{1cm} (7.2.16)

Moreover \( h'_1 \) and \( h'_2 \) are related to the clearance variation by the following relation

\[
h'_1 + h'_2 = h' = \frac{c'_o + c'}{2}
\]  \hspace{1cm} (7.2.17)

The combination of all the relations allows the complete solution of the problem.

An analytical solution can be obtained if we look for a particular type of solution:
a travelling wave in the tangential direction. This solution is of the form:

\[ q = q_0 e^{i(2\pi ft - ky)} \]  

(7.2.18)

where \( f \) is the frequency and \( k \) the wave number. Writing all the perturbed quantities in this form, we obtain an algebraic system of equations that can be inverted directly.

7.2.2 Application of the model

Single cavity labyrinth seal

The model has been applied to the single-cavity labyrinth seal studied in the previous chapter. The nominal conditions for this case can be found in Table 3.1. Fig. 7.2 to 7.4 present results concerning the influence of the pivot point position, the clearance and the pressure ratio. The results of the single-control-volume model are also included. Discrepancies similar to those observed with the single-control-volume model are observed with the two-control-volume model at a frequency ratio \( f/f_{ac} = 1 \).

Four-fin labyrinth seal

Fig. 7.5 shows results obtained on the 4-fin labyrinth seal concerning the influence of the pivot point location at a frequency ratio equal to 1. The evolution of the logarithmic decrement with the location of the pivot point is recovered qualitatively with the analytical model. However, the model predicts an unstable range of pivot point locations around \( x_p/p = -0.5 \) that is not observed in CFD results. If we compare the aerodynamic work in each cavity predicted by the model and obtained with CFD for different pivot point locations (Fig. 7.6), we see that there are some discrepancies for the first and last cavity. In particular, for \( x_p/p = -0.5 \), the model predicts a positive (destabilising) work in the
7.2 Two-control-volume model

Figure 7.2: Single-cavity labyrinth seal - Influence of pivot point location $x_p/p$ on logarithmic decrement $\delta$ - Nominal conditions, Table 3.1.

Figure 7.3: Single-cavity labyrinth seal - Influence of clearance $c$ on logarithmic decrement $\delta$ for a HPS mode ($x_p/p = -1$) - Nominal conditions, Table 3.1, except for clearance.
Figure 7.4: Single-cavity labyrinth seal - Influence of pressure ratio $\Pi$ on logarithmic decrement $\delta$ for a HPS mode ($x_p/p = -1$) - Nominal conditions, Table 3.1, except for pressure ratio.
last cavity whereas CFD results show that the work is negative in this cavity. The discrepancies observed on the aerodynamic work can also be seen on the unsteady pressure phase (Fig. 7.7). We notice in particular that the phase difference between adjacent cavities is much smaller in the analytical model than in CFD simulations. The problem lies probably in the modelling of the unsteady flow at the fin tips. This is confirmed by comparing the magnitude of the unsteady density, velocity, mass flow rate at the fin tips for \( x_p/p = -0.5 \) (Fig. 7.8). Important discrepancies can been at the first and last fin tip for the axial velocity. Relation 7.2.15 used to model the flow at the tips appears to induce a strong coupling of the unsteady pressure in adjacent cavities. In simulations, the change of phase occurs at the fin tips. A better modelling of the fin tip flow could help in reproducing the sharp phase change between adjacent cavities. This is the main reason behind the development of the three-control-volume model presented in the next section.

Figure 7.5: 4-fin labyrinth seal - Influence of pivot point location \( x_p/p \) on logarithmic decrement \( \delta \) - \( f/f_{ac} = 1 \) - Nominal conditions, Table 5.1.
7.2 Two-control-volume model

Figure 7.6: 4-fin labyrinth seal - Influence of pivot point location $x_p/p$ on aerodynamic work in each cavity $w - f/f_{ac} = 1$ - Nominal conditions, Table 5.1.

Figure 7.7: 4-fin labyrinth seal - Influence of pivot point location $x_p/p$ on unsteady pressure phase distribution in labyrinth - $f/f_{ac} = 1$ - Nominal conditions, Table 5.1.
7.2 Two-control-volume model

7.2.3 Unsteady flow at fin tips

The flow variations at the fin tips predicted by the analytical model are obtained in CFD simulations are compared in Figs. 7.8 and 7.10 for the unsteady case. The first figure shows results for a HPS model (rpm = 41), the second
for LPS model (rpm = 11). The frequencies both are equal, but for both models the agreement at the outlet is not very good, considering the most fast fluid and the velocity. The model requires that the density and pressure are normalised through
the steady-state pressure. The total pressure 11 = p p at the outlet is equal to the mean pressure. If we use as input in the steady-state model the pressure at the outlet, the normalised pressure is the
same for all models, and the agreement is good. If we use as input in the steady-state model the pressure at the outlet, the normalised pressure is the
same for all models, and the agreement is good.

Figure 7.8: 4-fin labyrinth seal - Magnitude of unsteady clearance c', density p', axial velocity u' and mass flow rate m' at fin tips normalised by steady-state value - x_p/p = -0.5, f/f_ac = 1 - Nominal conditions, Table 5.1.
7.2.3 Unsteady flow at fin tips

The flow variations at the fin tips predicted by the analytical model and obtained in CFD simulations are compared in Fig. 7.9 and 7.10 for the single-cavity seal case. The first figure shows results for a HPS mode \( x_p/p = -1 \), the second for LPS mode \( x_p/p = 1 \). The frequency ratio is equal to 1. For both modes, the agreement at the outlet fin is reasonable concerning the mass flow rate and the velocity. The model assumes that the density and pressure are constant at the outlet fin. In simulations, we observe variations of the density at the outlet fin but the amplitude remains small. At the inlet fin, the model overestimates significantly the amplitude of the velocity and density variations. Looking at the distribution of unsteady pressure amplitude predicted by CFD (Fig. 7.11 and 7.12), we see that the amplitude on top of the inlet fin is not equal to the amplitude in the cavity as assumed in the analytical model. In the steady-state flow, the pressure on top of the first fin is also different from the pressure in the cavity as shown in Fig. 7.13. This is because of the vena contracta. If we look at the distribution of the unsteady pressure amplitude for the 4-fin labyrinth (Fig. 7.14), we see that, for the subsequent fin tips, the assumption made in the analytical model is valid.

7.2.4 Flow modelling at first fin

The validity of the modelling at the first fin tip has been checked with the help of CFD results. The total pressure variation at the first fin tip is presented in Fig. 7.15. The amplitude of variation is around 0.07% of the inlet total pressure; the amplitude of variation of the static pressure is 0.5%. Thus, the variation of total pressure at the first fin is not completely negligible. However, it remains a small fraction of the pressure variation. To check the validity of the isentropic assumption, the evolution of the quantities \( p'/\bar{p} \) and \( \gamma p'/\bar{p} \) is compared, where \( p', \bar{p}, \rho, \bar{\rho} \) are the unsteady pressure, steady-state pressure,
7.2 Two-control-volume model

unsteady density and steady-state density respectively. The results, presented in Fig. 7.16, show that the isentropic assumption is justified at the first fin. Using Eq. 3.3.10, we see that the variation of the velocity at a fin tip can be resolved in two contributions, one coming from the variation of static pressure, the other from the variation of total pressure. The unsteady velocity obtained by considering only the contribution of the static pressure and both contributions are compared to the actual unsteady velocity at the first fin in Fig. 7.17. The unsteady velocity at the first fin is better predicted if the contribution of the total pressure variation at the fin tip is taken into account, as is the unsteady mass flow rate presented in Fig. 7.18. We see from these results that the predictions of the present model could probably be improved by removing the assumption of constant total pressure at the first fin. The variation of the total pressure at the first fin comes from the work done by the entrance section of the seal, upstream of the first fin tip. Indeed, this section is moving as well when the

Figure 7.9: Single-cavity labyrinth seal - Unsteady flow time history at fin tips and in cavity for HPS mode \( (x_p/p = -1, \ f/f_{ac} = 1) \) - Nominal conditions, Table 3.1.
7.3 Three-control-volume model

As discussed in Section 7.2.2, a sharp change of phase of the unsteady pressure is observed at the fin tips in simulation. This feature could be reproduced by the two-control-volume model. The present model was developed to remove this discrepancy.

Figure 7.10: Single-cavity labyrinth seal - Unsteady flow time history at fin tips and in cavity for LPS mode ($x_p/p = 1$, $f/f_{ac} = 1$) - Nominal conditions, Table 3.1.

Seal vibrates, thus some work is done along its wall causing the change in total pressure. We could consider adding a control volume for this entrance section.
Figure 7.11: Single-cavity labyrinth seal - Amplitude of first harmonic of unsteady pressure for HPS mode ($x_p/p = -1$, $f/f_{ac} = 1$) - Nominal conditions, Table 3.1.
Figure 7.12: Single-cavity labyrinth seal - Amplitude of first harmonic of unsteady pressure for LPS mode \((x_p/p = 1, f/f_{ac} = 1)\) - Nominal conditions, Table 3.1.

Figure 7.13: Single-cavity labyrinth seal - Steady-state pressure contours - Nominal conditions, Table 3.1.
7.3 Three-control-volume model

Figure 7.14: 4-fin labyrinth seal - Amplitude of first harmonic of unsteady pressure for LPS mode \(x_p/p = 2, f/f_{ac} = 1\) - Nominal conditions, Table 5.1.
7.3 Three-control-volume model

Figure 7.15: 4-fin labyrinth seal - Unsteady total pressure at first fin tip for a LPS mode ($x_p/p = 2$, $f/f_{ac} = 1$) - Nominal conditions, Table 5.1.

Figure 7.16: 4-fin labyrinth seal - Unsteady pressure and density at first fin tip for a LPS mode ($x_p/p = 2$, $f/f_{ac} = 1$) - Nominal conditions, Table 5.1.
7.3 Three-control-volume model

Figure 7.17: 4-fin labyrinth seal - Unsteady velocity at first fin tip for a LPS mode \( (x_p/p = 2, f/f_{ac} = 1) \) - Nominal conditions, Table 5.1.

Figure 7.18: 4-fin labyrinth seal - Unsteady mass flow rate at first fin tip for a LPS mode \( (x_p/p = 2, f/f_{ac} = 1) \) - Nominal conditions, Table 5.1.
7.3 Three-control-volume model

7.3.1 Control volumes

In this model, the flow in a labyrinth seal is described using three control volumes (Fig. 7.19):

1. the fin tip area (CV1);
2. the through-flow area (CV2);
3. the cavity vortex area (CV3).

The last two control volumes are separated by the dividing streamline originating from the leading edge of the entrance fin (Fig. 7.20). The advantage of this approach is that there is no mass flow between CV2 and CV3, which simplifies the governing equations. Moreover this division represents more closely the physical mechanism which is linked to an exchange of energy between the

Figure 7.19: Control volumes for three-control-volume model.
7.3 Three-control-volume model

Figure 7.20: Dividing streamline used as a boundary between CV2 and CV3.

cavity area and the through-flow area. The difficulty lies in the modelling of the dividing streamline motion. This modelling is addressed later in Section 7.3.4.

7.3.2 Governing equations

As for the two-control-volume model, the equations will be written for a two-dimensional flow in the \((x, z)\) plane. For the first and second control volumes, we write the conservation of mass, axial momentum, tangential momentum and total energy. In the third control volume, there is no mean axial flow, and only the conservation of mass, tangential momentum and total energy are written.
7.3 Three-control-volume model

Control volume 1

The conservation of mass, momentum and energy in CV1 can be written in vector form:

\[
\frac{\partial (Qc)}{\partial t} + \frac{\partial (Qvc)}{\partial y} + c \frac{\partial F_y}{\partial y} + F_{Ey} \frac{\partial c}{\partial y} = \frac{1}{b} [c_A F_A - c_B F_B + (c_B - c_A) F_{Rz}] + F_{Rz} + \Phi_R + \Phi_S
\]  

(7.3.1)

with:

\[
Q = \begin{bmatrix}
    \rho \\
    \rho u \\
    \rho v \\
    \rho E
\end{bmatrix},
F_y = \begin{bmatrix}
    0 \\
    0 \\
    p \\
    p u
\end{bmatrix},
F_{Ey} = \begin{bmatrix}
    0 \\
    0 \\
    p \\
    p u
\end{bmatrix},
F_A = \begin{bmatrix}
    \rho_A u_A \\
    \rho_A u_A^2 + p_A \\
    \rho_A u_A u_A \\
    \rho_A u_A H_A
\end{bmatrix},
F_B = \begin{bmatrix}
    \rho_B u_B \\
    \rho_B u_B^2 + p_B \\
    \rho_B u_B v_B \\
    \rho_B u_B H_B
\end{bmatrix},
F_{Rx} = \begin{bmatrix}
    p_R \\
    0 \\
    0 \\
    p_R w_R
\end{bmatrix},
F_{Rz} = \begin{bmatrix}
    0 \\
    0 \\
    \tau_{Rx} \\
    \tau_{Ry}
\end{bmatrix},
\Phi_R = \begin{bmatrix}
    0 \\
    \tau_{Rx} \\
    \tau_{Ry} \\
    \tau_{Rx} u + \tau_{Ry} v
\end{bmatrix},
\Phi_S = \begin{bmatrix}
    0 \\
    \tau_{Sz} \\
    \tau_{Sy} \\
    \tau_{Sz} u + \tau_{Sy} v
\end{bmatrix}
\]  

(7.3.2)

Control volume 2

The equations for CV2 are similar to those of CV1, the clearance \( c \) being replace by the height \( h_2 \) of CV2, and the tip thickness \( b \) by the cavity length \( L \):

\[
\frac{\partial (Qh_2)}{\partial t} + \frac{\partial (Qvh_2)}{\partial y} + h_2 \frac{\partial F_y}{\partial y} + F_{Ey} \frac{\partial h_2}{\partial y} = \frac{1}{L} [c_B F_B - c_C F_C + (c_C - c_B) F_{SLz}] + F_{SLz} + \Phi_{SL} + \Phi_S
\]  

(7.3.3)
7.3 Three-control-volume model

with:

\[ F_C = \begin{bmatrix} \rho c_{uc} & 0 & 0 \\ \rho c u^2_c + p_c & p_{sl} & 0 \\ \rho c u_c v_c & 0 & p_{slw_{sl}} \end{bmatrix}, \quad F_{SLx} = \begin{bmatrix} 0 \\ p_{sl} \\ 0 \end{bmatrix}, \quad F_{SLz} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \]

\[ \Phi_{SL} = \begin{bmatrix} 0 \\ \tau_{SLx} \\ \tau_{SLy} \\ \tau_{SLz} u + \tau_{SLy} v \end{bmatrix} \] (7.3.4)

Control volume 3

In CV3, there is no (mean) axial flow and only the conservation of mass, tangential momentum and energy are written:

\[ \frac{\partial (Q h_3)}{\partial t} + \frac{\partial (Q v h_3)}{\partial y} + \frac{\partial h_3}{\partial y} + F_{Ey} \frac{\partial h_3}{\partial y} = F_R - F_{SL} + \Phi_R - \Phi_{SL} \] (7.3.5)

with:

\[ Q = \begin{bmatrix} \rho \\ \rho v \\ \rho E \end{bmatrix}, \quad F_y = \begin{bmatrix} 0 \\ p \\ pv \end{bmatrix}, \quad F_{Ey} = \begin{bmatrix} 0 \\ pv \\ p_{Rw_R} \end{bmatrix}, \quad F_R = \begin{bmatrix} 0 \\ 0 \\ p_{slw_{sl}} \end{bmatrix}, \quad F_{SL} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Phi_R = \left(1 + 2 \frac{h_2}{L} \right) \begin{bmatrix} 0 \\ \tau_{Ry} \\ \tau_{Ry} v \end{bmatrix}, \quad \Phi_{SL} = \begin{bmatrix} 0 \\ \tau_{SLy} \\ \tau_{SLy} v \end{bmatrix} \] (7.3.6)
7.3 Three-control-volume model

7.3.3 Perturbation analysis

Assuming small motion of the rotor, the flow in the labyrinth can be written as the sum of the steady-state flow and a perturbation:

\[ Q = \bar{Q} + Q' \quad (7.3.7) \]

with

\[
\bar{Q} = \begin{bmatrix} \bar{\rho} \\ \bar{\rho}u \\ \bar{\rho}v \\ \bar{\rho}E \end{bmatrix}, \quad Q' = \begin{bmatrix} \rho' \\ \bar{\rho}u' + \rho' \bar{u} \\ \bar{\rho}v' + \rho' \bar{v} \\ \bar{\rho}E' + \rho' \bar{E} \end{bmatrix}, \quad (7.3.8)
\]

At first order, we obtain the following equations for the perturbed flow:

CV1:

\[
\bar{Q} \frac{\partial c'}{\partial t} + \bar{c} \frac{\partial Q'}{\partial t} + \bar{Q} \bar{v} \frac{\partial c'}{\partial y} + \bar{Q} \bar{c} \frac{\partial v'}{\partial y} + \bar{v} \frac{\partial Q'}{\partial y} + \bar{c} \frac{\partial F'}{\partial y} + F' \frac{\partial Q}{\partial y} = \\
\frac{1}{b} \left[ c_A F_A' + c_A' \bar{F}_A - c_B F_B' - c_B' \bar{F}_B + (c_B' - c_A') F_R \right] + F'R + \phi'R + \phi'S \quad (7.3.9)
\]

CV2:

\[
\bar{Q} \frac{\partial h_2'}{\partial t} + \bar{h}_2 \frac{\partial Q'}{\partial t} + \bar{Q} \bar{v} \frac{\partial h_2'}{\partial y} + \bar{Q} \bar{c} \frac{\partial v'}{\partial y} + \bar{v} \frac{\partial Q'}{\partial y} + \bar{c} \frac{\partial F'}{\partial y} + \bar{F}_E \frac{\partial h_2'}{\partial y} = \\
\frac{1}{L} \left[ c_B F_B' + c_B' \bar{F}_B - c_C F_C' - c_C' \bar{F}_C + (c_C' - c_B') F_{SL} \right] + F'_{SL} + \Phi'_{SL} + \Phi'S \quad (7.3.10)
\]

CV3:

\[
\bar{Q} \frac{\partial h_3'}{\partial t} + \bar{h}_3 \frac{\partial Q'}{\partial t} + \bar{Q} \bar{v} \frac{\partial h_3'}{\partial y} + \bar{Q} \bar{c} \frac{\partial v'}{\partial y} + \bar{v} \frac{\partial Q'}{\partial y} + \bar{c} \frac{\partial F'}{\partial y} + \bar{F}_E \frac{\partial h_3'}{\partial y} = \\
F_R' - F'_{SL} + \Phi_R' - \Phi'S_{SL} \quad (7.3.11)
\]
The viscous contributions are neglected in the present model. The variation of the clearance at the fin tips $c'$ is known from the mode shape. $h'_2$ and $h'_3$ are part of the unknowns. They are linked by the following relation:

$$h'_2 + h'_3 = h'$$  \hspace{1cm} (7.3.12)

where $h'$ is the variation of the cavity height which is known from the mode shape. An additional relation is required to complete the solution. It is provided by the equation governing the motion of the dividing streamline.

### 7.3.4 Equation governing the motion of the dividing streamline

**Equation of motion of a fluid in streamline coordinates**

If the viscous stresses can be neglected, the equation of motion for a fluid reads in vector form:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{(v \cdot \nabla)} \mathbf{v} = -\frac{1}{\rho} \nabla p$$  \hspace{1cm} (7.3.13)

At any point in the fluid, we can define a streamline coordinate system as follows (Fig. 7.21):

- $s$ is the coordinate along the streamline;
- $n$ is the coordinate normal to the streamline;
- $l$ is the bi-normal coordinate.

The associated unit vectors are $s$ in the direction of the flow, $n$ pointing away from the local centre of curvature and $l$. In this coordinate system, the velocity vector has only one component:

$$\mathbf{v} = vs$$
7.3 Three-control-volume model

Figure 7.21: Streamline coordinates.

\[
\nabla = \frac{\partial}{\partial s} s + \frac{\partial}{\partial n} n + \frac{\partial}{\partial t} t
\]

\[v \cdot \nabla = v \frac{\partial}{\partial s}
\]

In streamline coordinates the convective acceleration is:

\[
(v \cdot \nabla)v = v \frac{\partial v}{\partial s} s + v^2 \frac{\partial s}{\partial s}
\]

\[= \frac{\partial}{\partial s} \left( \frac{v^2}{2} \right) s - \frac{v^2}{R_c} n
\]

since \(\frac{\partial s}{\partial \alpha} = -\frac{n}{R_c}\), where \(R_c\) is the local radius of curvature. Concerning the local acceleration, we have:

\[
\frac{\partial v}{\partial t} = \frac{\partial v}{\partial t} s + v \frac{\partial s}{\partial t}
\]

\[= \frac{\partial v}{\partial t} s + v \frac{\partial \alpha}{\partial t} n
\]

\[= \frac{\partial v}{\partial t} s + \frac{\partial v_n}{\partial t} n
\]

where \(\frac{\partial \alpha}{\partial t}\) is the local rate of change of the flow angle. Thus we have the following equations in the s- and n-direction:

\[
\frac{\partial v}{\partial t} + \frac{\partial}{\partial s} \left( \frac{v^2}{2} \right) = -\frac{1}{\rho} \frac{\partial p}{\partial s} \tag{7.3.14}
\]

\[
\frac{\partial v_n}{\partial t} - \frac{v^2}{R_c} = -\frac{1}{\rho} \frac{\partial p}{\partial n} \tag{7.3.15}
\]
Perturbation analysis for dividing streamline

For small perturbations about the steady-state, we can obtain a first order accurate formulation of these equations:

\[
\frac{\partial v'}{\partial t} + \frac{\partial}{\partial s} (\bar{v}v') = - \left( \frac{1}{\rho} \frac{\partial p'}{\partial s} - \frac{\rho'}{\rho^2} \frac{\partial \bar{p}}{\partial s} \right)
\]

(7.3.16)

\[
\frac{\partial v'}{\partial t} - \left( 2 \frac{\bar{v}v'}{R_c} - \frac{\bar{v}^2 R_c'}{R_c^2} \right) = - \left( \frac{1}{\rho} \frac{\partial p'}{\partial n} - \frac{\rho'}{\rho^2} \frac{\partial \bar{p}}{\partial n} \right)
\]

(7.3.17)

where $'$ and $\bar{}$ denote respectively the steady-state and perturbation components of the flow variables. The dividing streamline is moved primarily by the motion of the fin from which it originates. Thus the normal velocity $v'_n$ is of the same order as $\partial r'/\partial t$, where $r'$ is the motion of the fin. But, for an oscillation at frequency $f$, $\partial / \partial t$ is of order $f$. Hence, $\partial v'_n / \partial t$ is of order of magnitude $f^2 r'$. Denoting by $U$ and $L$ a characteristic velocity and length respectively for the seal, and defining $f_b = U/L$, $\bar{v}/\bar{R}_c$ is of order of magnitude $f_b$. To determine the order of magnitude of the fluctuations $v'$ and $p'$, we will use the results of the theoretical analysis. Eq. (3.3.11) in the theoretical analysis indicates that $v'$ is of order of magnitude $p'/(\rho \bar{v})$; and Eq. (3.3.28) shows that $p'$ is of order of magnitude $\bar{p} a^2 (f_b / f_{ac}) (f / f_{ac}) r' / h$ at low frequency ratios and $\bar{p} a^2 c'/h$ at high frequency ratios. In terms of the characteristic length $L$, $p'$ is of order of magnitude $\bar{p} a^2 (f_b / f_{ac}) (f / f_{ac}) r'/L$ at low frequency ratios and $\bar{p} a^2 r'/L$ at high frequency ratios. It is reasonable to think that $R_c'$ is of order of magnitude $r'$. This gives the following orders of magnitude for the terms in Eq. (7.3.17) at low frequency ratios.

\[p_0 \text{ in Eq. (3.3.28) being the amplitude function of } p', c_{10} \text{ and } c_{20} \text{ the amplitude functions of the changes in clearance at the inlet and outlet } c'_1 \text{ and } c'_2 \text{ which are of order } r'.\]
frequency ratios:

\[
\frac{\partial v'_n}{\partial t} \sim f^2 r' \quad (7.3.18)
\]

\[
\frac{\bar{v}'}{R_c} \sim \frac{1}{M^2} \frac{f_b}{f_{ac}} \frac{f^2 r'}{f_{b'}^2} \quad (7.3.19)
\]

\[
\frac{\bar{\psi}^2 R'_c}{R_c^2} \sim f^2 b' r' \quad (7.3.20)
\]

\[
1 \frac{\partial p'}{\rho \partial n} \sim \frac{1}{M^2} \frac{f_b}{f_{ac}} \frac{f}{f_{ac}} \frac{f^2 r'}{f_{b'}^2} \quad (7.3.21)
\]

\[
\frac{\rho' \partial \bar{p}}{\rho^2 \partial n} \sim \frac{f_b}{f_{ac}} \frac{f}{f_{ac}} \frac{f^2 r'}{f_{b'}^2} \quad (7.3.22)
\]

and at high frequency ratios:

\[
\frac{\partial v'_n}{\partial t} \sim f^2 r' \quad (7.3.23)
\]

\[
\frac{\bar{v}'}{R_c} \sim \frac{1}{M^2} f_{b'}^2 r' \quad (7.3.24)
\]

\[
\frac{\bar{\psi}^2 R'_c}{R_c^2} \sim f^2 b' r' \quad (7.3.25)
\]

\[
1 \frac{\partial p'}{\rho \partial n} \sim \frac{1}{M^2} f_{b'}^2 r' \quad (7.3.26)
\]

\[
\frac{\rho' \partial \bar{p}}{\rho^2 \partial n} \sim f^2 b' r' \quad (7.3.27)
\]

For a representative labyrinth seal configuration:

\[
f \approx 10^3 \text{Hz} \quad (7.3.28)
\]

\[
U \approx 10^2 \text{m/s} \quad (7.3.29)
\]

\[
L \approx 10^{-3} \text{m} \quad (7.3.30)
\]

\[
f_b \approx 10^5 \text{Hz} \quad (7.3.31)
\]

\[
\frac{f}{f_{ac}} \approx 1 \quad (7.3.32)
\]

\[
M \approx 1 \quad (7.3.33)
\]

Therefore the term \(\frac{\partial v'_n}{\partial t}\) is negligible. The magnitude of the different terms in Eq. (7.3.17) has been checked with the aid of CFD simulation results on the single-cavity labyrinth seal studied in the preceding chapter. Two modes
of vibration are considered: a mode representative of HP support (HPS) and a mode representative of LP support (LPS). The time histories of \(\rho', v', v'_n, \partial p'/\partial n\) and \(R'_c\) are computed. The results are Fourier-transformed to evaluate the real and imaginary part of the perturbations in complex form \(q' = q_0 e^{i2\pi ft}\). The results are presented in Fig. 7.22. We see that the contribution of \(v'_n\) is negligible. The normal equation can then be rewritten, after division by \(\rho \frac{\rho^2}{R_c}\) on the left and right hand side:

\[
\frac{\rho'}{\rho} + 2\frac{v'}{v} - \frac{R'_c}{R_c} = \left(\frac{\partial p'}{\partial n}\right) / \left(\frac{\rho \frac{\rho^2}{R_c}\right)}
\]

We can see in Fig. 7.22 that \(\frac{R'_c}{R_c}\) and \(\frac{\partial p'}{\partial n}\) / \(\frac{\rho \frac{\rho^2}{R_c}\) are the dominant terms in this equation, although the term \(\frac{v'}{v}\) and particularly the term \(\frac{v'_n}{v}\) have a non negligible influence, more noticeable on the imaginary part.

In the present form, Eq. (7.3.34) is of little use to model the motion of the dividing streamline: we need a relation quantifying the motion of the dividing streamline in the \(z\) direction. In the following, we will derive an expression of the local motion of the dividing streamline \(z'(s)\) as a function of the change in curvature along the streamline \(R'_c\). We have:

\[
z(s_0, t) = z_0(t) + \int_{0}^{s_0} \frac{\partial z}{\partial s} (s, t) ds
\]

where \(z_0\) is the \(z\) coordinate at the fin tip leading edge from which the dividing streamline originates (chosen as origin of the \(s\)-coordinate). But \(\frac{\partial z}{\partial s} = s_z = -\sin(\alpha)\) where \(s_z\) is the component of \(s\) in the \(z\) direction, and \(\alpha\) is the angle between the local tangent and the axial direction (positive for negative slopes).

Between the time instant \(t\) and \(t + \delta t\), the radius of curvature at \(s\) and \(s + ds\) changes by an amount \(\delta R_c\) and \(\delta R_c(s + ds) = \delta R_c + \frac{\partial \delta R_c}{\partial s} ds\) respectively. This causes a change in the angle of the local tangent by an amount \(-\delta \alpha\) (Fig. 7.23). We have:
Figure 7.22: Single-cavity labyrinth seal - Budget along the dividing streamline of linearised normal momentum equation (Eq. (7.3.17)). Harmonic variation of the form $q' = q_0 e^{i(2\pi f t)}$ is assumed for the flow variables. The real and imaginary part of the contributions from $\rho'$, $v'$, $v'_n$, $\partial p'/\partial n$ and $R'_c$ to the momentum budget are plotted - Nominal conditions, Table 3.1.
7.3 Three-control-volume model

Figure 7.23: Change in flow angle between time instants $t$ and $t + \delta t$.

\[
\tan(-\delta \alpha) = \frac{\delta R_c(s + ds) - \delta R_c}{ds}
\]
\[
\delta \alpha = -\frac{\partial \delta R_c}{\partial s}
\]

since $\delta \alpha$ is small. Thus:

\[
\alpha' = -\frac{\partial R_c'}{\partial s}
\]

and:

\[
s_z + s_z' = -\sin(\bar{\alpha} + \alpha')
\]
\[
= -\left(\sin(\bar{\alpha}) \cos(\alpha') + \cos(\bar{\alpha}) \sin(\alpha')\right)
\]
\[
= -\sin(\bar{\alpha}) - \cos(\bar{\alpha}) \alpha'
\]
\[
s_z' = -\cos(\bar{\alpha}) \alpha'
\]

Using the preceding results, we obtain for the perturbation of the location of the streamline at $s = s_0$:

\[
z'(s_0, t) = z_0(t) + \int_0^{s_0} \cos(\bar{\alpha}) \frac{\partial R_c'}{\partial s} ds
\]
\( \frac{\partial R'_c}{\partial s} \) can be evaluated from Eq. (7.3.34):

\[
\frac{\partial R'_c}{\partial s} = R_c \left[ \frac{\partial}{\partial s} \left( \frac{\rho'}{\bar{\rho}} \right) + 2 \frac{\partial}{\partial s} \left( \frac{\nu'}{\bar{v}} \right) - \frac{\partial}{\partial s} \left[ \left( \frac{\partial p'}{\partial n} \right) / \left( \frac{\nu^2}{R_c} \right) \right] \right] \\
+ \left[ \frac{\rho'}{\bar{\rho}} + 2 \frac{\nu'}{\bar{v}} - \frac{\left( \frac{\partial p'}{\partial n} \right)}{\left( \frac{\nu^2}{R_c} \right)} \right] \frac{\partial R'_c}{\partial s}
\]  
(7.3.44)

Combining Eq. (7.3.43) and (7.3.44), we obtain a relation between the motion of the streamline in the z direction, the motion of the fin leading edge and the perturbations in density, velocity and normal pressure gradient along the streamline. The coefficients in this relation, \( \cos(\alpha), \bar{R}_c, \bar{\rho}, \bar{v}, \frac{\partial}{\partial s} \), depend on the steady-state of the flow along the streamline and the shape of the dividing streamline. The gradients of the perturbations along the streamline \( \frac{\partial}{\partial s} \) might depend on the particular mode shape. For modelling purposes, we can look for a linear relationship of the form:

\[
\frac{z'_m}{c} = \alpha_c \frac{z'_0}{c} + \alpha_\rho \frac{\rho'}{\bar{\rho}_2} + \alpha_v \frac{(\nu^2)'}{\bar{\nu}_2^2} + \alpha_p \frac{\frac{\partial p'}{\partial n}}{\bar{\rho}_2 \bar{\nu}_2^2 / R_c}
\]  
(7.3.45)

where \( z'_m \) is the average motion of the dividing streamline, \( z'_0 \) is the motion of the entrance fin leading edge, \( \rho' \) and \( (\nu^2)' \) are the fluctuations in density and velocity (squared) on the dividing streamline, \( \frac{\partial p'}{\partial n} \) is the fluctuation in normal pressure gradient on the streamline. \( \bar{\rho}_2, \bar{\nu}_2 \) are the average velocity and density in CV2. Simulation results show that a reasonable value for the characteristic radius of curvature \( R_c \) used to non-dimensionalise \( \frac{\partial p'}{\partial n} \) is \( R_c = 10L_c \) where \( L_c \) is the length of the cavity. We need to evaluate the fluctuations along the streamline from the values of the fluctuations in the through-flow and cavity vortex control volumes:

\[
\rho' = \beta_2 \rho'_2 + \beta_3 \rho'_3
\]
\[
(\nu^2)' = \beta_2 (\nu^2)'_2 + \beta_3 (\nu^2)'_3
\]
\[
\frac{\partial p'}{\partial n} = \frac{\beta_2 \rho'_2 - \beta_3 \rho'_3}{\delta_{SL}}
\]

The values of the constants \( \alpha \) and \( \beta \) are adjusted with the aid of CFD results.
This leads to the following complex equation for the perturbation of CV2 height \( h' = -z'_m \) (using the fact that \( z'_m = -c'_A \)):

\[
    h' = \frac{\alpha_p c}{\rho_2} (\beta_2 p'_2 + \beta_3 p'_3) + 2\frac{\alpha_v c}{u_2} \beta_2 u'_2 + \frac{\alpha_p c}{\rho_2 u_2^2 \delta/R_c} (\beta_2 p'_2 - \beta_3 p'_3) = \alpha_p c'_A
\]  

(7.3.46)

This equation completes the system given by Eq. (7.3.9) to (7.3.11) which can now be solved.

### 7.3.5 Correction of CV3 modelling

**Introduction**

In the development of the 3 CV model, we had assumed that CV3 behaved as a closed cavity. The validity of this assumption has been checked with the aid of CFD simulation results. To this end, the motion of the dividing streamline in the simulations is computed. This motion is imposed for the upper boundary of CV3 and the resulting perturbations are computed based on Eq. (7.3.11). The results obtained are presented in Table 7.1. We see that the results obtained with the model do not agree with CFD results. Further analysis of CFD simulations showed that CV3 is opened during part of the vibration because the dividing streamline does not impinge onto the next fin. This allows some air to escape from the cavity. This is illustrated in Fig. 7.24 for a HPS mode. Due to the radial location of the outlet fin relative to the inlet fin during vibration, the dividing streamline passes above the outlet fin. The mass flow rate escaping

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>CFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho'/\rho_{ref} )</td>
<td>0.51</td>
<td>0.07</td>
</tr>
<tr>
<td>( p'/p_{ref} )</td>
<td>0.72</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 7.1: Single-cavity labyrinth seal - Modulus of first harmonic of unsteady density and pressure in cavity predicted by model with closed CV3 for a HPS mode \( (x_p/p = -1, f/f_{ac} = 1) \) - Nominal conditions, Table 3.1.
Figure 7.24: Single-cavity labyrinth seal - Location of dividing streamline during vibration for a HPS mode \((x_p/p = -1, f/f_{ac} = 1)\). The dividing streamline does not impinge on the outlet fin - Nominal conditions, Table 3.1.
from the cavity during vibration is presented in Fig. 7.25. The variation is not

harmonic. For modelling purposes, we need to replace this non-harmonic signal by a harmonic one. The evaluation of this mass flow rate from CFD results is difficult due to precision issues, for instance concerning the location of the dividing streamline. Fig. 7.26 shows the location of the dividing streamline in the steady-flow determined with the visualisation software Tecplot. This streamline does not touch the second fin which violates the mass flow conservation. Some results obtained while imposing the (slightly corrected) escape flow characteristics extracted from CFD flutter results are presented in Table 7.2. We see that taking into account this escape flow improves our results. Consequently, this escape flow should be included in the model.
7.3 Three-control-volume model

Figure 7.26: Location of dividing streamline above second fin determined with the visualisation software Tecplot from steady-state CFD results. This streamline does not touch the second fin which violates the mass flow conservation - Nominal conditions, Table 3.1.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>CFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho'/\rho_{ref}$</td>
<td>0.062</td>
<td>0.07</td>
</tr>
<tr>
<td>$p'/p_{ref}$</td>
<td>0.087</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 7.2: Single-cavity labyrinth seal - Modulus of first harmonic of unsteady density and pressure in cavity predicted by model with open CV3 for a HPS mode ($x_p/p = -1$, $f/f_{ac} = 1$) - Nominal conditions, Table 3.1.
Modelling of CV3 escape flux

The escape flux can be modelled by introducing a clearance below the streamline in the model. The average and fluctuating part of this clearance can be evaluated using CFD simulations. The computed first harmonic of the unsteady clearance gives an evaluation of the fluctuating part. This evaluation need to be corrected due to precision errors (for example concerning the location of the dividing streamline) but also because the escape flux is non-harmonic. Moreover, since during part of the vibration, the dividing streamline loses contact with the outlet fin, the unsteady outlet clearance should be replaced by the unsteady height above the streamline in the evaluation of the flux out of CV2. Since the clearance below the dividing streamline remains a small fraction of the clearance variation, this correction can be neglected. The escape fluxes are expressed as follows:

\[
\frac{F_0}{F} = \frac{c_0}{\bar{c}}
\]  

(7.3.47)

where \( \bar{F} \) represents the steady-state value of the flux of mass, momentum or energy at the entrance of the exit fin, \( F_0 \) the escape flux and \( c_0 \) the unsteady gap between the dividing streamline and the exit fin tip. This gap is modelled as follows:

\[
c_0 = z_{SL20} - z_{F20}
\]  

(7.3.48)

where \( z_{SL20} \) is the motion of the dividing streamline above the exit fin and \( z_{F20} \) is the motion of the exit fin. The motion of the dividing streamline above the exit fin is written as a linear combination of the average motion of the dividing streamline in the cavity \( z_{m0} \) and the motion of the exit fin.

\[
z_{SL20} = \lambda_1 z_{m0} + \lambda_2 z_{F20}
\]  

(7.3.49)

\( \alpha, \lambda_1 \) and \( \lambda_2 \) are parameters of the model.
7.3.6 Numerical fluxes

The inviscid flux vector $F$ at the inlet and outlet of the first and second control volume are computed using the Roe approximate Riemann solver [25]. With this numerical scheme, the fluxes are properly upwinded according to the direction of propagation of information. Using this flux allows the perturbations to be correctly propagated at the interface. In this formulation, the fluxes at an interface are written:

$$F = \frac{F(Q_L) + F(Q_R)}{2} - \frac{1}{2} |A_I| (Q_R - Q_L) \quad (7.3.50)$$

where $F(Q_L)$ and $F(Q_R)$ are the inviscid fluxes evaluated using the state on the left and on the right of the interface respectively, and $A_I$ is the flux Jacobian evaluated at Roe’s average state. The perturbed flux $F'$ is written:

$$F' = \frac{A(Q_L) + |A_I| Q'_L}{2} + \frac{A(Q_R) - |A_I| Q'_R}{2} \quad (7.3.51)$$

where $A(Q_L)$ and $A(Q_R)$ are the Jacobians of $F(Q_L)$ and $F(Q_R)$ respectively.

7.3.7 Boundary conditions

The number of boundary conditions to be imposed at the inlet and outlet depends on the local value of the Mach number. There are two types of boundary conditions:

- physical boundary conditions are associated with waves in the flow propagating from the boundaries toward the interior of the computational domain; the information propagated by these waves must be known at the boundaries;

- numerical boundary conditions are associated with waves propagating from the interior of the domain toward the boundaries; the information
propagated by these waves influence the values of the flow quantities at the boundaries; thus some values at the boundaries cannot be imposed and a numerical scheme must be devised at the boundaries to determine them.

In labyrinth seals the inflow is subsonic so we have to impose three physical boundary conditions and we need one numerical boundary condition. The outlet flow can be subsonic or supersonic. If the outlet flow is subsonic, we need one physical boundary condition and three numerical boundary conditions. If it is supersonic, no physical boundary condition can be imposed and all the boundary conditions are numerical. Different forms of physical and numerical boundary conditions are possible. Here a linearised form of the Riemann invariants boundary conditions is used. In the present context, the four Riemann invariants are:

\[ w_1 = \frac{p}{\rho^\gamma} \]  \hspace{1cm} (7.3.52)
\[ w_2 = v \]  \hspace{1cm} (7.3.53)
\[ w_3 = u + \frac{2a}{\gamma - 1} \]  \hspace{1cm} (7.3.54)
\[ w_4 = u - \frac{2a}{\gamma - 1} \]  \hspace{1cm} (7.3.55)

Their linearised form is:

\[ w'_1 = \bar{w}_1 \left( \frac{p'}{\bar{\rho}} - \gamma \frac{\rho'}{\bar{\rho}} \right) \]  \hspace{1cm} (7.3.56)
\[ w'_2 = v' \]  \hspace{1cm} (7.3.57)
\[ w'_3 = u' + \frac{\bar{a}}{\gamma - 1} \left( \frac{p'}{\bar{\rho}} - \frac{\rho'}{\bar{\rho}} \right) \]  \hspace{1cm} (7.3.58)
\[ w'_4 = u' - \frac{\bar{a}}{\gamma - 1} \left( \frac{p'}{\bar{\rho}} - \frac{\rho'}{\bar{\rho}} \right) \]  \hspace{1cm} (7.3.59)

Using the subscript \( I \) to denote imposed values, 1 for values at the inlet boundary and 2 for values in the CV adjacent to the inlet boundary, the boundary
conditions at the inlet can be written:

\[ w'_{11} = w'_{I} \]  (7.3.60)
\[ w'_{21} = w'_{2I} \]  (7.3.61)
\[ w'_{31} = w'_{3I} \]  (7.3.62)
\[ w'_{41} = w'_{42} \]  (7.3.63)

Using the subscript \( n \) for values at the outlet boundary and \( n - 1 \) for values in the CV adjacent to the outlet boundary, the boundary conditions at a subsonic outlet can be written:

\[ w'_{1n} = w'_{1n-1} \]  (7.3.64)
\[ w'_{2n} = w'_{2n-1} \]  (7.3.65)
\[ w'_{3n} = w'_{3n-1} \]  (7.3.66)
\[ w'_{4n} = w'_{4I} \]  (7.3.67)

At a supersonic outlet, the last boundary condition is replaced by \( w'_{4n} = w'_{4n-1} \).

For the imposed values of the perturbations at the inlet and outlet, we assume that the inlet and outlet flow are unperturbed, thus \( w'_{iI} = 0 \) for \( i = 1, \ldots, 4 \).

7.3.8 Solution procedure

We look for a travelling wave solution of the form \( Q = Q_0 e^{i(2\pi ft - ky)} \), where \( f \) is the oscillation frequency and \( k \) the wave number. The partial derivatives in Eq. (7.3.9) to (7.3.11) can then be replaced by:

\[ \frac{\partial}{\partial t} = i2\pi f \]  (7.3.68)
\[ \frac{\partial}{\partial y} = -ik \]  (7.3.69)
\[ \frac{\partial}{\partial x} = ik \]  (7.3.70)
After substitution, we obtain a system of algebraic equations. The equations are solved in terms of the primitive variables \( U' = (\rho', u', v', p') \). We use the Jacobian of the transformation between conservative and primitive variables to obtain equations for \( U' \):

\[
Q' = MU'
\]

\[
M = \begin{bmatrix}
1 & 0 & 0 & 0 \\
\bar{u} & \bar{\rho} & 0 & 0 \\
\bar{v} & 0 & \bar{\rho} & 0 \\
\frac{\bar{q}^2}{2} & \bar{\rho}\bar{u} & \bar{\rho}\bar{v} & \frac{1}{\gamma - 1}
\end{bmatrix}
\]

where \( \bar{q}^2 = \bar{u}^2 + \bar{v}^2 \) is the square of the magnitude of the velocity.

### 7.3.9 Results

In this section, results obtained with the analytical model are compared to CFD results. The test cases considered are the single-cavity labyrinth seal and the 4-fin labyrinth seal studied in the previous chapter. The comparison is made in terms of the aerodynamic work. To this end, the amplitude of motion is set to 1% of the clearance in the model as in CFD simulations. The steady-state value in the control volumes of the analytical model are obtained by averaging CFD steady-state results over the relevant areas.

**Single-cavity labyrinth**

Results concerning the influence of the pivot point location at three frequency ratios are presented in Fig. 7.27. There is a reasonably good agreement between the 3 CV model and CFD results at the two lower frequency ratios. At a frequency ratio of 1.5, the 3 CV model tends to overestimate the aerodynamic work.
7.3 Three-control-volume model

Figure 7.27: Single-cavity labyrinth seal - Influence of pivot point position $x_p/p$ on aerodynamic work $w_{aero}$ - Nominal conditions, Table 3.1.

Fig. 7.28 shows results on the influence of the frequency ratio for a HPS mode ($x_p/p = -1$) and a LPS mode ($x_p/p = 1$). The tendency of the 3 CV model to overestimate the aerodynamic work at high frequency ratio is clearly visible. The analytical model is able to predict the correct stability but the magnitude of the aerodynamic work is overestimated. Fig. 7.29 and 7.30 compare the magnitude and phase of the unsteady pressure predicted by the 3 CV model and computed with CFD as a function of the frequency ratio for a LPS mode ($x_p/p = 1$). There are some important discrepancies on the magnitude. These discrepancies become more severe as the frequency ratio is increased. This explains the discrepancies on the aerodynamic work at high frequency ratios. It has been shown in Section 5.4 that the first cavity has a singular behaviour because of the importance of the vena contracta at the entrance fin. In the analytical model, all fin tips are treated in the same manner. This could account for the discrepancies observed on this single-cavity case.
Figure 7.28: Single-cavity labyrinth seal - Influence of frequency ratio $f/f_{ac}$ on aerodynamic work $w_{aero}$ for a HPS mode ($x_p/p = -1$) and a LPS mode ($x_p/p = 1$) - Nominal conditions, Table 3.1.
7.3 Three-control-volume model

Results on the influence of the pressure ratio are presented in Fig. 7.31. For most cases, the predictions of the analytical model agree qualitatively with CFD results. The model is able to predict the change of stability at low pressure ratio for the HPS mode at $f/f_{ac} = 1$. At $f/f_{ac} = 1.5$, the curve of aerodynamic work given by the analytical model is similar in shape but shifted towards positive values of the work; consequently the analytical model predicts a change of stability at a lower pressure ratio. For the LPS mode at $f/f_{ac} = 0.5$, there are some important discrepancies at low pressure ratio. These discrepancies can be caused either by an error on the magnitude or on the phase of the unsteady pressure. The magnitude and phase of the unsteady pressure computed by CFD and the analytical model for this mode are presented in Fig. 7.32. The magnitude of the unsteady pressure is in reasonable agreement between the model and CFD. The phase is well predicted by the analytical model at the pressure ratios above 1.4. Below this pressure ratio, CFD simulations predict a sharp
7.3 Three-control-volume model

Figure 7.30: Single-cavity labyrinth seal - Influence of frequency ratio $f/f_{ac}$ on unsteady pressure phase for a LPS mode ($x_p/p = 1$) - Nominal conditions, Table 3.1.

increase of the phase; this sharp increase is not reproduced by the analytical model.

There are some important discrepancies at the lowest frequency ratio for the HPS mode. The magnitude and phase of the unsteady pressure computed by CFD and the analytical model for this mode are presented in Fig. 7.33. At high pressure ratio, the magnitude is reasonably well predicted by the analytical. However, there are 5 degrees of difference on the phase. This difference is enough to cause the observed discrepancies on the aerodynamic work.

The model performs well at resonance contrary to the single-control-volume model used for the theoretical analysis in Chapter 3 and the two-control-volume model of the previous section. It was suspected then that the large error at resonance was due to the crude modelling of the fin tip flow. We see that by using a control volume at the fin tips, we improve significantly the predictions.
Figure 7.31: Single-cavity labyrinth seal - Influence of pressure ratio $\Pi$ on aerodynamic work $w_{aero}$ for a HPS mode ($x_p/p = -1$) and a LPS mode ($x_p/p = 1$) - Nominal conditions, Table 3.1, except for pressure ratio.
Figure 7.32: Single-cavity labyrinth seal - Influence of pressure ratio $\Pi$ on unsteady pressure magnitude and phase for a LPS mode ($x_p/p = 1, f/f_{ac} = 0.5$) - Nominal conditions, Table 3.1, except for pressure ratio.
Figure 7.33: Single-cavity labyrinth seal - Influence of pressure ratio $\Pi$ on unsteady pressure magnitude and phase for a HPS mode ($x_p/p = -1, f/f_{ac} = 0.5$) - Nominal conditions, Table 3.1, except for pressure ratio.
Four-fin labyrinth

Plots of the aerodynamic work as a function of the pivot point location and the frequency ratio are produced for the 4-fin labyrinth seal in Fig. 7.34 and Fig. 7.35. Here the compliance with CFD results is good.

Fig. 7.36 compares the phase distribution in the labyrinth predicted by CFD and obtained with the two-control-volume and three-control-volume models for three locations of the pivot point. The frequency ratio is equal to one as for Fig. 7.34. The three-control-volume model is able to reproduce the significant change of phase between adjacent cavities contrary to the two-control-volume model; this results in phase predictions closer to CFD.

The stability of a LPS mode \( \left( \frac{z_p}{p} = 1.5 \right) \) at the actual natural frequencies of the modes of nodal diameter -5 to 5 (found in [16]) is investigated in Fig. 7.37. The model predicts a strong instability at ND-3. In test, high amplitude vibrations were recorded at the same nodal diameter number (di Mare et al. [16]). Application of Abbott’s frequency ratio criterion also indicates the instability of this mode (Fig. 7.38). Indeed, here the seal is supported on the low-pressure side and, according to Abbott’s criterion, instability should occur when the mechanical frequency is lower than the acoustic frequency. For ND-3, the frequency ratio is much lower than one which is consistent with the instability.

7.4 Summary

A two-control-volume and a three-control-volume analytical model for labyrinth seal flutter have been presented in this chapter. The agreement between the two-control-volume model and CFD results is reasonable on a single-cavity labyrinth seal case. On a 4-fin labyrinth seal case, the two-control-volume model predicts the correct evolution of the aerodynamic damping with the position of the pivot point; however, it predicts an unstable range of pivot point locations not ob-
Figure 7.34: 4-fin labyrinth seal - Influence of pivot point location $x_p/p$ on aerodynamic work $w_{aero} - f/f_{ac} = 1$ - Nominal conditions, Table 5.1.

Figure 7.35: 4-fin labyrinth seal - Influence of frequency ratio $f/f_{ac}$ on aerodynamic work $w_{aero}$ for a LPS mode ($x_p/p = 1.5$) - Nominal conditions, Table 5.1.
Figure 7.36: 4-fin labyrinth seal - Unsteady pressure phase distribution in the labyrinth for three pivot locations - $f/f_{oc} = 1$ - Nominal conditions, Table 5.1.

Figure 7.37: 4-fin labyrinth seal - Logarithmic decrement $\delta$ for seal modes of nodal diameter $ND = -5$ to $ND = 5$ - Nominal conditions, Table 5.1, $x_p/p = 1.5$, frequency of each mode taken from di Mare et al. [16].
Figure 7.38: 4-fin labyrinth seal - Abbott’s frequency ratio criterion - Nominal conditions, Table 5.1, $x_p/p = 1.5$, frequency of each mode taken from di Mare et al. [16].
served in CFD results. Further analysis has shown that the two-control-volume model predicts a phase difference between adjacent cavities much smaller than CFD. The discrepancies have been attributed to the modelling of the flow at the fin tips. To alleviate this issue, a three-control-volume has been developed. This model adds a control volume for the fin tip and solves the axial momentum equation for the fin tip control volume and the through-flow control volume. A modelling of the motion of the dividing streamline based on the linearised Euler equations in streamline coordinates has also been proposed for this model. On the single-cavity case, the model gives reasonably good results at frequency ratios lower than one but tends to overestimate the aerodynamic work at higher frequency ratios. A possible cause could be the lack of modelling of the vena contracta at the first fin tip. The results on the 4-fin labyrinth are in good agreement with CFD results. The three-control-volume model is able to reproduce the significant change of phase between adjacent cavities contrary to the two-control-volume model. When investigating the influence of the nodal-diameter number on the 4-fin case, the three-control-volume model predicts a strong instability at ND-3; the same mode was identified as the cause of highest measured flutter amplitude in engine test. The present chapter has presented a prediction tool for seal flutter that could be used routinely during seal design due to low computational cost. In the next chapter, we propose an aeroelastic design procedure for labyrinth seals based on the findings of the previous chapter and the tools developed in this chapter.
Chapter 8

Aeroelastic design methods

8.1 Methods

From the results of the previous chapters, two aeroelastic design methods can be proposed. The first one is an extension of Abbott's stability criterion. Abbott's stability criterion can be represented by the stability map shown in Fig. 8.1a. It was shown by Ziegler [54] and Schmidt [45] that the support side on the abscissa should be replaced by the location of the virtual pivot point of the vibration mode. Moreover, the results presented in Section 6.2 show that the location of the pivot point affects significantly the flutter boundary: as the pivot point is put further away from the centre of the labyrinth, the stable range of frequency ratio increases (see Fig. 6.6). When the pivot point is close to the centre of the labyrinth, the change of stability occurs at a frequency ratio around 1, although the exact value is configuration dependent (see Fig. 6.11). Taking this into account, we propose to replace the stability map derived using Abbott's stability criterion by the one in Fig. 8.1b. The advantage of this new stability map is that it shows that one can increase the stability by putting the virtual pivot point further away from the centre whether this pivot point is on the high-pressure side or low-pressure side. By designing the seal so that the
8.1 Methods

8.1a: Stability map according to Abbott's stability criterion.

8.1b: Stability map derived from present study.

Figure 8.1: Stability maps.
virtual pivot point is far enough from the centre, only modes with very high or very low frequency ratios will be potentially dangerous. The exact slope of the flutter boundary seems, however, to be case dependent; in our present study we obtain an average slope of -0.1.

Our parametric study has also shown that, at low Mach number, labyrinth seals were free from flutter. Thus, this parameter should be added to our aeroelastic design procedure. The effect of the air swirl can be taken into account in the frequency ratio. Large changes in clearance can occur during engine operation. Since no general conclusion can be made concerning the effect of the clearance, especially when the changes in clearance are large, it is advisable to carry the aeroelastic stability analysis for different values of the clearance.

Taking these findings into account, we propose the following aeroelastic design procedure:

1. check the value of the Mach number; if lower than 0.3, then no flutter issues are to be expected;

2. check the value of the axial location of the virtual pivot point $|x_p|/p$ for the natural modes of the structure; if this value exceeds ten or so, it is unlikely that the mode will lead to flutter instability;

3. if the two previous checks fail, plotting the mode location in a stability map such as the one in Fig. 8.1b, can indicate if the mode can lead to stability issues; if the mode is in or close to the unstable zones, flutter might occurs at this mode; use of CFD can give a definite answer;

4. repeat the procedure for different values of the clearance.

By using this procedure, one can limit the CFD calculations to the potentially unstable modes thus reducing the computational cost.

The second aeroelastic design method consists in using the analytical model presented in the previous chapter. The use of these two methods will be illustrated
8.2 Determination of pivot point location

Consider a seal member whose motion is a rotation of angle $\alpha$ about a pivot point $P(x_p, r_p)$. Its displacement vector $\delta x$ is given by:

$$\delta x = (\cos \alpha - 1)(x - x_p) - \sin \alpha (r - r_p)$$
$$\delta r = \sin \alpha (x - x_p) + (\cos \alpha - 1)(r - r_p)$$

(8.2.1)

Given a seal mode shape, and imposing an amplitude of modal displacement $x_0$, we can determine the angle $\alpha$ by selecting a line in the seal member and measuring its rotation (see Fig. 4.2a in Section 4.2). Then, using the displacement vector of some reference point on this line, we can compute the position of the point $P$ with Eq. (8.2.1).

8.3 Application

The geometry of the seal is shown in Fig. 8.2. The nominal conditions for this case are given in Table 8.1. The concern for this datum seal are the vibration modes of the stator. It is supported at the low-pressure side. The steady-state flow has been computed with CFD. The Mach number contours in the labyrinth are shown in Fig. 8.3. The Mach number of the through-flow is around 0.5. This is above the limit set in the design procedure described above (0.3), so we

<table>
<thead>
<tr>
<th>Nominal clearance $c$</th>
<th>0.42 mm</th>
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</thead>
<tbody>
<tr>
<td>Pitch $p/c$</td>
<td>4.7</td>
</tr>
<tr>
<td>Height of cavity $h/c$</td>
<td>4.7</td>
</tr>
<tr>
<td>Pressure ratio $H$</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Table 8.1: Nominal conditions for datum labyrinth seal case.
8.3 Application

Figure 8.2: Datum seal geometry. The rotating part is shown in red, the stationary part in green. The concern for this seal are the vibration modes of the stator.

Figure 8.3: Datum seal: Mach number contours in the labyrinth. The Mach number of the through-flow is around 0.5, which is high enough for instability to occur. Nominal conditions, Table 8.1.
estimates that instability can occur. The axial location of the pivot point for different modes of interest is shown in Fig. 8.4. The location of the pivot point moves from the LP side to the HP side between the nodal diameter numbers 4 and 5.

Figure 8.4: Datum seal: location of pivot point for modes of nodal diameter number 2 to 10. The location of the pivot point moves from the LP side to the HP side between the nodal diameter numbers 4 and 5. This is due to the fact that there is an inflection point in the mode shape near the junction between the arm and the labyrinth platform at the seal exit. As a consequence, the variations of the clearances at the fin tips is as if the seal were supported at the HP side. This can be seen in Fig. 8.5 for mode ND5. The minimum distance between the pivot point and the seal centre is $|x_p|/p = 38$ for mode ND8. At such a large distance, only relatively high frequency ratios for HP support and relatively low frequency ratios for LP support will be unstable according to the typical stability maps presented in Section 6.2. Modes ND-10 to ND10 are plotted on such a stability map in Fig. 8.6. None of the modes is in the unstable region. Consequently, we expect no stability issue with this seal. Fig. 8.7 shows the logarithmic decrement computed with the 3CV model. It is
Figure 8.5: Datum seal: mode shape for mode of nodal diameter number $ND = 5$. There is an inflection point in the mode shape near the junction between the arm and the labyrinth platform at the seal exit.
Figure 8.6: Datum seal: location of mode of nodal diameter number $ND = -10$ to $ND = 10$ on stability map. The numbers next to the red crosses indicates the nodal diameter number of the modes. All modes are outside of the unstable zone in green. Nominal conditions, Table 8.1.
Figure 8.7: Datum seal: logarithmic decrement \( \delta \) computed with 3CV model for modes of nodal diameter number \( ND = -10 \) to \( ND = 10 \). All modes are predicted to be stable \((\delta > 0)\). Nominal conditions, Table 8.1.

8.3 Application
positive for all modes as expected. Fig. 8.8 shows the logarithmic decrement computed with CFD. Surprisingly ND-4 and ND4 are unstable. This instability was confirmed by engine test. Given the configuration, it was surmised that the instability came from the LP cavity. Indeed the seal stator has a long arm which constitutes the upper wall of the LP cavity. Due to the length of this arm, a significant amount of work can be done in the LP cavity during vibration. An attempt was made to include the LP cavity in the model. It is modelled as a cavity of rectangular cross section. Given the complexity of the LP cavity, it is difficult to define the length and height of this cross section. In Fig. 8.9, results are presented for two different cavity lengths, $L_1$ and $L_2$, which are shown in Fig. 8.2, and compared to CFD results. The results given by the model without LP cavity are also shown. The model with LP cavity is able to predict the instability of ND4 and also ND-4 depending on the chosen cavity length.
Figure 8.9: Datum seal: comparison of logarithmic decrement $\delta$ computed with 3CV model including LP cavity and computed with CFD. Results are presented for two different cavity lengths, $L_1$ and $L_2$, which are shown in Fig. 8.2. The 3CV model including the LP cavity is able to predict the instability of the modes of nodal diameter number $ND = 4$ and also $ND = -4$ depending on the chosen cavity length. Nominal conditions, Table 8.1.
length. Unfortunately, it predicts that all nodal diameters above 3 are unstable. It also predicts a peak in damping for ND-3. This is due to an acoustic resonance phenomenon in the LP cavity. Indeed, taking into account the average value of the swirl in the LP cavity, the mechanical-to-acoustic frequency ratio is close to 1 for ND-3 as shown in Fig. 8.10. This resonance is not observed in CFD simulations. Nor is it observed with the second cavity length. Closer analysis of the results obtained with the second cavity length shows that there is a peak in aerodynamic work in the LP cavity for ND-3; however, for ND-3, the work in the inter-fin cavities opposes the work in the LP cavity which explains the absence of peak in the logarithmic decrement. A more accurate modelling of the LP cavity would be required to be able to reproduce CFD results. Moreover, the work done on the other side of the stator arm by the HP cavities is not modelled.

The results obtained with the model including the LP cavity seem to validate
that the instability is due to the LP cavity. Thus, checks on potential instabilities due to HP and LP cavities should be included in the aeroelastic design criteria. These cavities can be analysed using the same criteria as for seal inter-fin cavities. A stability map is produced for the LP cavity in Fig. 8.11. Modes

Figure 8.11: Datum seal: location of mode of nodal diameter number $ND = -10$ to $ND = 10$ on stability map of LP cavity. The numbers next to the red crosses indicates the nodal diameter number of the modes. Modes of nodal diameter number $ND \geq 3$ as well as $ND = -4, -5, -6$ are in the unstable region for the LP cavity in green. Nominal conditions, Table 8.1.

$ND$ and above are in the unstable region for the LP cavity. This explains the instability predicted by the analytical model for these modes. Modes $ND$-4, $ND$-5 and $ND$-6 are also in the unstable region. Only mode $ND$-4 is predicted to be unstable by the analytical model. For mode $ND$-5 and $ND$-6, the stable contribution of the labyrinth overcomes the unstable contribution of the LP cavity.
8.4 Conclusions

Two new aeroelastic design methods for labyrinth seals have been applied to an industrial seal case. The two methods give consistent results, both predicting stability of the configuration investigated when the contribution of the LP cavity is ignored, and indicating a possible instability when it is taken into account. The validation case chosen illustrates the importance of taking into account the eventual contribution of the HP and LP cavities when assessing aeroelastic stability. By amending the analytical model to include a basic representation of the LP cavity, we were able to predict the instabilities predicted using a CFD model. However, some important discrepancies remain. These discrepancies could be solved by a better modelling of the HP and LP cavities in the analytical model.
Chapter 9

Conclusions and future work

9.1 Conclusions

The aim of this research project was twofold:

- increase the knowledge on seal flutter (clarify the influence of flow and geometric parameters on stability);
- develop new methods to increase the speed of seal flutter analyses.

To this end, the following approach has been used:

- a theoretical analysis and parametric CFD study have been carried out to investigate the influence of different parameters on stability;
- a bulk-flow modelling approach has been investigated to increase the speed of seal flutter analyses.

The main conclusions of these investigations are summarized below.
9.1 Conclusions

9.1.1 Theoretical analysis and parametric CFD study

Influence of pivot point location

The range of stable frequencies increases when the pivot point is put further away from the labyrinth: this implies unconditional stability when the pivot point is at infinity upstream or downstream (radial translation).

Influence of pressure ratio/Mach number

There is no instability at low pressure ratio/Mach number

Influence of swirl velocity

The aerodynamic work depends on the mechanical-to-acoustic frequency ratio $f_m/f_{ac}$ where $f_m$ is the frequency of vibration evaluated in the frame of reference of the swirling fluid.

Influence of clearance

The clearance affects stability firstly due to its influence on the mean flow. Moreover, a stabilising effect of an increase in clearance is predicted by the theoretical study; this is not fully confirmed by CFD results because increasing the clearance leads to an increase of the Mach number of the mean flow which can be destabilising.

Influence of pitch

Similarly to the clearance, the pitch affects stability firstly due to its influence on the mean flow. The pitch also affects the unsteady pressure phase difference between adjacent cavities, this phase increasing with an increase in pitch.

Influence of cavity height

Like the other geometric parameters, the cavity height affects stability due to influence the on mean flow. Moreover, theoretical and CFD results show that decreasing the cavity height has a stabilising effect.
9.1.2 Bulk-flow modelling

The investigation of the bulk-flow modelling approach for seal flutter has lead to the following conclusions:

- CFD results show that it is more appropriate to use one control volume for the through-flow and one for the vortex flow rather than a single control volume for the entire inter-fin cavity;

- bulk-flow modelling is able to reproduce qualitatively the influence of different parameters on stability;

- with a control volume at the fin tips, it is possible to reproduce the sharp change of phase observed in CFD between adjacent cavities and thus improve the predictions;

- quantitative discrepancies are observed on the first seal cavity: one possible cause could the lack of specific treatment for the first fin tip where there is a large flow separation (vena contracta);

- the application to an actual seal design has shown that there was a need to include the contributions of the HP and LP cavities in the analysis.

9.2 Future work

In this thesis, the flutter mechanisms has been investigated in straight-through labyrinth seals. Stepped steals are widely used in aero engine and it would be interesting to extend the investigation to these seals. In stepped seals, the through-flow pattern is more complex. This represents a modelling challenge for models of the bulk-flow type. Moreover, all the labyrinth seals investigated in this thesis had a uniform clearance distribution. In actual seals, non-uniform clearance can occur due to wear. Lewis et al. [31] showed that it was possible to stabilise a seal configuration while preserving its leakage performances.
by adopting a non-uniform clearance distribution (converging in their case). Further investigation would be needed to determine if some general aeroelastic design rule concerning the axial distribution of the clearance can be found.

As observed in Chapter 8, the contribution of the high-pressure and low-pressure cavities can be important for some configurations. Bulk-flow models are attractive because they increase significantly the speed of the analysis. However, as long as these models do not include the contribution of the high-pressure and low-pressure cavities, their reliability is limited. Some effort is needed to develop a modelling of these cavities which can have a complex shape.

Other effects have been neglected in this thesis because it was believed that their influence was of second order for seal flutter. Among these, it would be interesting to investigate how the presence of honeycombs on the seal platform affects the flutter characteristics. It has been shown that these honeycombs affect significantly both the leakage characteristics and the flow pattern (Schramm et al. [46]).

The results of the analytical models also show that a linearised approach is sufficient for seal flutter predictions. As a trade-off between full 3D non linear CFD calculations and bulk-flow models, one could consider 2D harmonic models where the geometry of the seal and the steady-state flow are accurately represented and a travelling wave solution of the form $U(x, r)e^{i(2\pi f t - n\theta)}$ is sought for the unsteady flow.

Finally, experimental data would be needed to validate the results of CFD simulations and bulk-flow model analyses.
Appendices
Appendix A

Three-control-volume model: escape flux out of CV3

A.1 Determination of escape flux

A.1.1 Direct method

The flow escaping the control volume below the dividing streamline during vibration can be determined directly by computing the flux of mass, momentum and energy between the dividing streamline and the outlet fin tip. This method is, however, subject to potentially large errors since the mesh above the fin tip is not fine enough to allow an accurate determination of the position of the dividing streamline at this location. Moreover, the fluxes determined by this method are non-harmonic and thus present modelling issues.
A.1.2 Harmonic balance

An alternative is to write the conservation of mass, momentum and energy in the cavity vortex control volume in harmonic form and to solve for the escape fluxes. This method requires the knowledge of the location of the dividing streamline in the cavity area at each time instant. We can expect the determination of this location to be less prone to errors. The harmonic balance for the cavity vortex control volume, taking into account the escape flux, can be written:

\[ i\omega (\rho S)_0 - ik (\rho u S)_0 = -F_0^o \]  
\[ i\omega (\rho u S)_0 - ik (\rho u v S)_0 = -F_0^{pu} + f_{wall,0}^x + f_{stream,0}^x \]  
\[ i\omega (\rho v S)_0 - ik (\rho v^2 S)_0 - ikS_0 = -F_0^{qv} \]  
\[ i\omega (\rho ES)_0 - ik (\rho E v S)_0 - ikS (pv)_0 = -F_0^{pE} + w_{wall,0} + w_{stream,0} \]

where \( k \) is the wavenumber, \( S \) is the area of CV3 and \( F_0 \) is the flux escaping from the gap below the dividing streamline (in complex form):

\[
F = \begin{bmatrix}
\int \rho udz \\
\int (\rho u^2 + p)dz \\
\int \rho v dz \\
\int \rho udHdz
\end{bmatrix}
\]

\( f_{wall,0}^x \) and \( f_{stream,0}^x \) are the axial components of the forces exerted on the fluid by the cavity wall and along the dividing streamline respectively. Similarly, \( w_{wall,0}^x \) and \( w_{stream,0}^x \) are the works done by the cavity wall and the dividing streamline.

A.1.3 Results

The list of the cases investigated is given in Table A.1. Before using the harmonic balance to compute the escape fluxes below the dividing streamline, we checked
A.1 Determination of escape flux

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Single-cavity labyrinth HPS mode $f/f_{ac} = 1.0$ - Amplitude of motion 20% of clearance</td>
</tr>
<tr>
<td>Case 2</td>
<td>Single-cavity labyrinth HPS mode $f/f_{ac} = 1.0$ - Amplitude of motion 1% of clearance</td>
</tr>
<tr>
<td>Case 3</td>
<td>4-fin labyrinth LPS mode $f/f_{ac} = 1.0$ first cavity</td>
</tr>
<tr>
<td>Case 4</td>
<td>4-fin labyrinth LPS mode $f/f_{ac} = 1.0$ second cavity</td>
</tr>
<tr>
<td>Case 5</td>
<td>Single-cavity labyrinth HPS mode $f/f_{ac} = 0.5$</td>
</tr>
</tbody>
</table>

Table A.1: Case definition.

<table>
<thead>
<tr>
<th></th>
<th>$\rho$</th>
<th>$\rho u$</th>
<th>$\rho v$</th>
<th>$\rho E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface average</td>
<td>7.5%</td>
<td>3.9%</td>
<td>10%</td>
<td>11%</td>
</tr>
<tr>
<td>Mass average</td>
<td>8.1%</td>
<td>4.0%</td>
<td>10%</td>
<td>13%</td>
</tr>
<tr>
<td>Mass-flow average</td>
<td>92%</td>
<td>18%</td>
<td>75%</td>
<td>101%</td>
</tr>
<tr>
<td>Conservative variables average</td>
<td>8.3%</td>
<td>4.0%</td>
<td>8.9%</td>
<td>8.6%</td>
</tr>
</tbody>
</table>

Table A.2: Harmonic balance for entire cavity: influence of averaging method. Case 1, Table A.1.

The harmonic balance for the entire cavity to assess the precision of the method. The influence of the choice of averaging is presented in Table A.2 for case 1. The best results for mass conservation are obtained with surface-averaging. The error in harmonic balance for the entire cavity is shown in Table A.3. The error is computed as follow for the mass conservation:

$$\text{Error} = \frac{|i \omega (\rho S)_0 - i k (\rho u S)_0 + F_{0u}|}{\max(|i \omega (\rho S)_0|, |k (\rho u S)_0|, |F_{0u}|)}$$  \hspace{1cm} (A.1.6)

The errors for the other conservation equations are computed in similar manner. The error in the harmonic balance of mass is around 3% for case 2 to 4. The higher errors for case 1 and 5 could be due to a lack of convergence of the inner iterations (iterations in pseudo-time steps) for these two cases.

The escape flux computed both with the direct and indirect methods are pre-

<table>
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<tr>
<th>Case</th>
<th>$\rho$</th>
<th>$\rho u$</th>
<th>$\rho v$</th>
<th>$\rho E$</th>
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<td>Case 1</td>
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<td>Case 2</td>
<td>3.5%</td>
<td>1.6%</td>
<td>11%</td>
<td>11%</td>
</tr>
<tr>
<td>Case 3</td>
<td>2.9%</td>
<td>9.5%</td>
<td>9.8%</td>
<td>3.8%</td>
</tr>
<tr>
<td>Case 4</td>
<td>2.7%</td>
<td>5.2%</td>
<td>11%</td>
<td>2.7%</td>
</tr>
<tr>
<td>Case 5</td>
<td>4.6%</td>
<td>2.0%</td>
<td>16%</td>
<td>5.6%</td>
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</tbody>
</table>

Table A.3: Harmonic balance for entire cavity using surface-averaging for cases 1 to 5, Table A.1.
sented in Table A.4. There are large discrepancies between the two methods. If

<table>
<thead>
<tr>
<th>Case</th>
<th>Flux</th>
<th>Indirect method</th>
<th>Direct method</th>
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</thead>
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<tr>
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<td>Re</td>
<td>Im</td>
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<td></td>
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<td>Case 4</td>
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<td>-1.2E+02</td>
</tr>
</tbody>
</table>

Table A.4: Harmonic balance: computed cavity escape flux for cases 1 to 5, Table A.1.

we plot the variation of the escape mass flow rate amplitude with the frequency ratio (Fig. A.1), we see that they both methods predict the same trend even if the magnitude is different. Figures A.2 and A.3 show the influence of the frequency ratio and the location of the pivot point on the magnitude of the escape mass flow rate for the single-cavity case. Figure A.2 shows the influence of the frequency ratio on the magnitude of the escape mass flow rate for a HPS mode ($x_p/p = -1$) and a LPS mode ($x_p/p = 1$). Figure A.3 shows the influence of the location of the pivot point. This magnitude has a minimum at a frequency ratio of 1 for both LPS and HPS. It was shown in the theoretical study that, when the frequency ratio is equal to one, the rate of change of the mass in a
A.1 Determination of escape flux

Figure A.1: Influence of frequency ratio on escape mass flow rate with direct and indirect method. Case 1, Table A.1.

Figure A.2: Influence of frequency ratio on escape mass flow rate for a HPS mode \( (x_p/p = -1) \) and a LPS mode \( (x_p/p = 1) \). Case 1, Table A.1.
tangential section is balanced by the tangential mass flow rate (cf. Eq. (3.3.37) in Section 3.3.2). Thus there would be no need for an escape flux. The observed minimum would be in agreement with this theory. Concerning the influence of the location of the pivot point, there is a maximum when the pivot point is in the middle of the cavity. In the simulations, the magnitude of the angle of rotation of the seal is chosen so that, for all modes, the magnitude of the change in clearance $c_{0 \max} = \max(c_{10}, c_{20})$ is the same. This leads to different angular magnitude for different locations of the pivot point. This angular magnitude is given:

$$\alpha_0 = \frac{c_{0 \max}}{|x| + \frac{\bar{p}}{2}}$$

(A.1.7)

And the magnitude of the difference in clearances at the inlet and outlet is:

$$|c_{10} - c_{20}| = p\alpha_0 = \frac{c_{0 \max}}{|x| + \frac{1}{2}}$$

(A.1.8)
The curve $|c_{10} - c_{20}|/c$ is shown in Fig. A.3. The escape flow is obviously linked to the difference in clearances. It is easy to see that the gap between the dividing streamline and the outlet fin will be of the same order as $|c_{10} - c_{20}|$.

We also notice that the distribution is not symmetric: the magnitude of the escape mass flow rate is higher when the pivot point is on the HP side than when it is on the LP side. The magnitude of the escape flow depends on the magnitude of the gap between the dividing streamline and the exit fin. When the pivot point is on the HP side, the motion of the entrance fin and thus of the dividing streamline is small and most of the motion occurs at the exit fin. The gap will have the same order of magnitude as the exit fin motion. When the pivot point is on the LP side, the largest motion occurs at the entrance fin and the motion of the exit fin is small. Thus the magnitude of the gap will depend on the magnitude of motion of the dividing streamline. But the magnitude of the motion of the dividing streamline will be different from the magnitude of motion of the entrance fin. Indeed, as explained in Section 5.4, the dividing streamline has a motion relative to the entrance fin tip. This motion is in opposite direction to the motion of the fin. Consequently the magnitude of motion of the dividing streamline will be smaller than the magnitude of motion of the entrance fin. This explains why the escape mass flow is smaller for LP side support than for HP side support.

The escape mass flow rate is expected to depend on the unsteady gap below the dividing streamline on one hand and the state upstream and downstream of this gap on the other hand. Indeed for the mass flow rate, we have:

$$m_0 = \bar{\rho} u_{c0} + (\rho u)_0 \bar{c}$$  \hspace{1cm} (A.1.9)

Analysis of the results shows that the second term is negligible and the escape mass flow rate is linearly related to unsteady gap below the dividing streamline as can be seen in Fig. A.4. Consequently, the escape mass flow rate can be
A.1 Determination of escape flux

Figure A.4: Escape mass flow rate $m_0$ vs gap below dividing streamline $c_0$ for a HPS mode ($x_p/p = -1$) and a LPS mode ($x_p/p = 1$). Case 1, Table A.1.
A.2 Analysis of local motion of dividing streamline

expressed as follows:

\[
\frac{m_0}{\bar{m}} = \alpha \frac{c_0}{\bar{c}} \quad (A.1.10)
\]

This constant is virtually independent of the location of the pivot point and of the frequency ratio. It might, however, depend on the operating point. The modelling of the escape flow is thus equivalent to the modelling of the unsteady gap below the dividing streamline.

In the following, the displacements will be referred to in a frame of reference moving with the leading edge of the inlet fin from which the dividing streamline originates. The escape gap depends on the motion of the dividing streamline above the outlet fin and the motion of outlet fin tip. The latter is determined by the mode shape. Analysis of simulation results shows that the former is a quite complicated matter. The motion of the dividing streamline will be analysed in the next section.

A.2 Analysis of local motion of dividing streamline

To understand the motion of the dividing streamline during vibration, we first need to explain its shape (Fig. A.5). Above the inlet fin, we have a sudden contraction ending at the vena contracta. The section at the vena contracta is a fraction of the clearance (typical values of the ratio are between 0.6 and 0.9). The vena contracta is followed by the initial expansion region. Then there is a recontraction due to the impingement of the cavity vortex on the dividing streamline. This vortex entrains the streamline down the cavity near the exit fin. After impingement on the exit fin, the flow recontracts to pass over the exit fin. We can thus distinguish four main regions on the dividing streamline:

- the contraction region up to the vena contracta controlled by the inlet clearance;
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- the initial expansion controlled by the characteristic of the shear layer;
- the region after this initial expansion and up to the impingement on the exit fin controlled by the interaction with the cavity vortex flow;
- the recontraction controlled by the interaction with the exit fin.

These different characteristics are clearly visible on the motion of the dividing streamline during vibration (Fig. A.6) Analysis shows that the initial shear layer zone does not change during the motion of the streamline. Thus the motion of the streamline can be considered the sum of three contribution:

- a change in the height of the contraction above the inlet fin due to a change in inlet clearance which can be modelled using an unsteady flow coefficient;
- a motion in the cavity area due the change in pressure in the cavity vortex
A.2 Analysis of local motion of dividing streamline

and through-flow areas (modelled by Eq. (7.3.46) governing the motion of the dividing streamline between CV2 and CV3);

- a motion above the exit fin influenced both by the motion of the dividing streamline near this fin and the interaction with the exit fin; this can be modelled by writing that the motion above the exit fin is a linear combination of the motion of the dividing streamline near the exit fin and the motion of the exit fin.

Thus we use the following procedure:

1. determine the location of the dividing streamline at the vena contracta $r_{S2}$, just before impingement $r_{S3}$, above the leading edge of the exit fin $r_{S4}$ and average motion in the cavity $r_{m}$, at each time instant;

2. determine the unsteady flow coefficient $\alpha = (r_{stator} - r_{S2}) / (r_{stator} - r_{F1})$, where $r_{F1}$ is the location of the inlet fin leading edge.
A.2 Analysis of local motion of dividing streamline

3. determine the mean motion of the streamline relative to the vena contracta point \( r_{mr} = r_m - r_{S2} \) and Fourier transform;

4. \( r_{S3r0} = r_{S30} - r_{S20} = \lambda r_{mr0} \); determine \( \lambda \);

5. Fourier transform \( r_{S3} \) and \( r_{S4} \); determine \( \lambda_1 \) and \( \lambda_2 \) such that \( r_{S40} = \lambda_1 r_{S30} + \lambda_2 r_{F20} \), where \( r_{F2} \) is the location of the outlet fin leading edge.

We can find \( \lambda_1 \) and \( \lambda_2 \) using Cramer’s rule.

\[
\begin{align*}
\Re(r_{S40}) &= \lambda_1 \Re(r_{S30}) + \lambda_2 \Re(r_{F20}) \quad (A.2.1) \\
\Im(r_{S40}) &= \lambda_1 \Im(r_{S30}) + \lambda_2 \Im(r_{F20}) \quad (A.2.2) \\
\lambda_1 &= \frac{\Re(r_{S40}) \Im(r_{F20}) - \Im(r_{S40}) \Re(r_{F20})}{\Re(r_{S30}) \Im(r_{F20}) - \Im(r_{S30}) \Re(r_{F20})} \quad (A.2.3) \\
\lambda_2 &= \frac{\Re(r_{S30}) \Im(r_{S40}) - \Im(r_{S30}) \Re(r_{S40})}{\Re(r_{S30}) \Im(r_{F20}) - \Im(r_{S30}) \Re(r_{F20})} \quad (A.2.4)
\end{align*}
\]

Figure A.7 shows the flow coefficient obtained using this procedure as a function of the ratio \( c/t \), where \( c \) is the (unsteady) clearance and \( t \) the tip thickness for HPS and LPS. Clearly, the computed flow coefficient is not only a function of the clearance as expected. This means that the motion of the streamline at the vena contracta is already influenced by the pressure in the cavity. The average value of the flow coefficient is 0.8. We can assume this value to be constant. Using this assumption, we can compute the contribution of the change of clearance to the motion of the dividing streamline \( r_{S20} = \alpha r_{F10} \). We then remove this contribution to the mean motion of the streamline to compute \( \lambda \).

\[ r_{mr0} = r_{m0} - \alpha r_{F10} \quad (A.2.5) \]

Moreover, if we compare the motion at S3 to the mean motion of the streamline for HPS and LPS (Fig. A.8), we see that there are not simply related. This indicates that the motion at S3 is already influenced by the motion of the outlet fin. Thus we choose to replace \( r_{S3} \) by \( r_m \) in the formula for the motion \( r_{S4} \), \( r_{S40} = \lambda_1 r_{m0} + \lambda_2 r_{F20} \). The latter method gives unrealistic values for \( \lambda_1 \) and
A.2 Analysis of local motion of dividing streamline

Figure A.7: Flow coefficient vs clearance for a HPS mode \(x_p/p = -1\) and a LPS mode \(x_p/p = 1\). Case 1, Table A.1.

Figure A.8: Comparison between average motion of streamline and motion at S3 for a HPS mode P1 \((x_p/p = -1)\) and a LPS mode P5 \((x_p/p = 1)\). Case 1, Table A.1.
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\( \lambda_2 \) due to a slight phase difference between \( r_{S40} \) on one hand and \( r_{m0} \) and \( r_{F20} \) which are relatively in phase on the other hand. To solve this problem, we use several simulations at the same time and solve for \( \lambda_1 \) and \( \lambda_2 \) using a least square method. We obtain \( \lambda_1 \approx 0.429855 \) and \( \lambda_2 \approx 0.545032 \). The results of the fitting is presented in Fig. A.9. The values of these constant are adjusted in the actual model to better fit CFD results.

Figure A.9: Comparison between correlated and actual motion at S4 for a HPS mode \( (x_p/p = -1) \) and a LPS mode \( (x_p/p = 1) \). Case 1, Table A.1.
Appendix B

Mode shapes

In the labyrinth area of seals, the mode shape can be approximated by a rotation about a pivot point. This fact has been exploited in the present thesis to facilitate the study of the influence of the mode shape on stability. The mode shape is constructed artificially in two steps. Having chosen a location for the pivot point $P(x_p, r_p)$, a sensible angle of rotation $\alpha$ is chosen. Typically the angle is chosen such as to induce a change of a few percents of the clearance. For a point $M(x, r)$ in an axial-radial plane, the displacement vector $\delta x$ is given by:

$$
\delta x_x = (\cos \alpha - 1) (x - x_p) - \sin \alpha (r - r_p)
$$

$$
\delta x_r = \sin \alpha (x - x_p) + (\cos \alpha - 1) (r - r_p)
$$

The nodal pattern is then imposed:

$$
\delta x (x, r, \theta) = \delta \cos (n \theta)
$$

where $\theta$ is the azimuthal coordinate and $n$ the nodal diameter number.

The CFD code used for flutter simulations assumes that the modes are mass-normalised to compute the logarithmic decrement. Indeed the logarithmic decre-
ment $\delta$ is given by the following formula:

$$\delta = -\frac{w}{2E_k}$$

$$E_k = \frac{1}{2}v^2$$

where $w$ is the aerodynamic work, $E_k$ the kinetic energy of the structure and $v$ the modal velocity. In the general case, the kinetic energy of the structure is:

$$E_k = \frac{1}{2}v^2\Phi^T\overrightarrow{M}\Phi$$

When the modes are mass-normalised $\Phi^T\overrightarrow{M}\Phi = 1$ and we recover the formula used to compute the logarithmic decrement. To emulate the mass-normalisation process in our study, we assume that $\overrightarrow{M}$ is the identity matrix; we define the norm of a mode shape as $m = \Phi^T\Phi$, and the mode shapes are divided by $\sqrt{m}$ to obtain pseudo mass-normalised mode shapes.
Appendix C

Dependence of magnitude of unsteady pressure on pivot point location for single-control-volume seal model of Section 3.3

From Eq. (3.5.5), we have:

\[ |p_0| \sim |\alpha_0| \sqrt{\left( \frac{x_p}{p} \right)^2 + \left( \frac{1}{2\pi f*T_b} \right)^2} \]  

(C.0.1)

the coefficient of proportionality being independent of \( x_p \). The amplitude of motion is chosen so that the maximum change in clearance is 1%. When \( x_p < 0 \) (respectively \( x_p > 0 \)), the maximum change in clearance occurs at the exit
(respectively inlet). Thus when \( x_p < 0 \), we have from Eq. (3.5.2):

\[
c_{20} = 0.01 c
\]

\[
\alpha_0 = \frac{c_{20}}{P/2 - x_p} = \frac{0.01 c}{P/2 - x_p}
\]

and when \( x_p > 0 \):

\[
c_{10} = 0.01 c
\]

\[
\alpha_0 = \frac{c_{10}}{-P/2 - x_p} = \frac{0.01 c}{-P/2 - x_p}
\]

In both cases:

\[
|\alpha_0| = \frac{0.01 c}{P/2 + |x_p|}
\]

which, after insertion in Eq. (C.0.1), leads to Eq. (3.3.30).
Bibliography


