Vertical bargaining and countervailing power

Discussion paper 2010/10

June 2010
Vertical bargaining and countervailing power*

Alberto Iozzi  Tommaso Valletti
Università di Roma “Tor Vergata” Imperial College London,
alberto.iozzi@uniroma2.it Università di Roma “Tor Vergata” and CEPR
t.valletti@imperial.ac.uk
April 2010

Abstract

We study the existence of countervailing buyer power in a vertical industry where the input price is set via Nash bargainings between one upstream supplier and many differentiated but competing retailers. In case one bilateral bargaining fails, the supplier still has the ability to sell to the other retailers. We show that the capacity of these other retailers to react in the final market has a dramatic impact on the supplier’s outside options and, ultimately, on input prices and welfare. Under downstream quantity competition, we find either no or opposite support to the hypothesis of countervailing power on input prices, as the retail industry becomes more concentrated. With price competition, we find a case for countervailing power, but its existence depends on the degree of product differentiation and on the ability of competing retailers to react to a disagreement.

JEL Numbers: L50

Keywords: Countervailing buyer power, Nash bargaining.

*We thank Paul Dobson for very useful discussions. The usual disclaimers apply.
1 Introduction

The question of how input prices (including wages) are set is quite a subtle one, both in Industrial Organization and in Labor Economics. While in most retail markets consumers are atomistic and, thus, are reasonably modelled as price takers when patronizing a particular seller, it is less clear who has the ability to set the input price in vertically-related markets. In particular, in “tight” oligopolies with a few upstream firms and a few downstream firms, a framework of bilateral negotiations, with individually-negotiated input prices, seems to be quite appropriate.\footnote{Bilateral relations are typically explained by investments in specialized assets made by both parties that lock them into the relationship. Alternatively, a “market interface” model should be applicable when there are many upstream firms and input purchases are made at list prices. See Inderst (forthcoming) for a contrast between these two alternative ways of modeling vertical contracting.}

For example, food manufacturing has traditionally been highly concentrated, and today concentration is high also in retailing, and in the rise (Dobson and Waterson, 2007). With high concentration on both sides, the relationship between concentration, market power, and efficiency is complex. It has been disputed, since the influential 1952 book of J.K. Galbraith American Capitalism: The Concept of Countervailing Power, that consolidation in the downstream (retailing) sector may actually be beneficial to consumers as fewer, more powerful, buyers, could negotiate cheaper input prices with upstream suppliers, and final consumers could benefit to the extent that input price reductions were also passed through on to them.

In the presence of high concentration downstream, upstream firms may also respond by merging in order to increase their bargaining position. Horn and Wolinsky (1988, henceforth HW) develop a model of a downstream duopoly in which firms acquire inputs through bilateral monopoly relations with suppliers. They explicitly account for the fact that the terms on which inputs are sold are determined in bargaining between each firm and its supplier, using a Nash axiomatic approach to model the bargaining. They find that an upstream merger to monopoly is more profitable when downstream products are substitutes. Besides IO applications, the use of a Nash axiomatic approach has also been widely used in Labor Economics to study the wage determination between oligopolistic firms and unions (see, e.g., Davidson, 1988;
Dowrick, 1989). There, an industry-wide union corresponds to a single upstream firm in the IO applications, while firm-level unions are the counterpart to independent upstream firms.

The Nash solution to a bargaining problem involves the determination of payoffs for each party, together with a specification of the disagreement point in case negotiations break down (outside options). The modeling of the outside option is of course not an issue in all those cases where the bargaining parties are assumed to be locked into a single bilateral relationship: if a negotiation breaks down, the two parties have no alternative and the outside option is zero (see, for instance, Correa-López and Naylor, 2004; Symeonidis, 2008 and 2010; Naylor, 2002; Correa-López, 2007). In the case of input suppliers, this assumption is not very palatable: when a negotiation between a supplier and a retailer breaks down, the former may still obtain positive profits by selling the input to other retailers. The determination of these disagreement profits then involves the definition of what behavior the other downstream firms have in case of a disagreement.

The focus of this paper is on the role of the modeling choice for the behavior of rivals’ firms in case of disagreement in a Nash bargaining in vertically-related markets. This is a general methodological problem that we specifically apply to the determination of input prices. We revisit the countervailing buyer power problem posited by Galbraith, and show that the modeling choice of the outside options changes, sometimes dramatically, the way in which the downstream market structural parameters affect the input price and the market equilibrium.

HW themselves argue that there are at least two plausible specifications for analyzing the behavior of downstream firms in disagreement. In a first scenario, the breakdown of the negotiation between a retailer and the input supplier is observed by the rival downstream firms; they react and make, in the downstream market, optimal choices which take into account that there is now one less competing firm. In the sequel, we will label this case Reaction. Alternatively, the breakdown of the negotiation may not be observed by the rival downstream firms. Therefore, they keep making their optimal choices in the downstream market as if all competing firms were present. We will label this case No Reaction, which is the specific case explicitly chosen by HW.
In this paper, we introduce a simple model with a single upstream firm, and many downstream firms, and revisit the question of countervailing power. Do downstream structural parameters affect the input price? And how? We use a linear demand model with a varying number of competing and differentiated firms, under the different hypotheses of Reaction and No Reaction. We also conduct the analysis for both Bertrand and Cournot downstream competition. We find that countervailing power, interpreted as a more concentrated downstream market, may not be able to keep input prices low. In our framework, this actually never happens when downstream competition is in quantities, independently of the type of reaction to disagreement. An increase in downstream concentration may instead lower the input prices with Bertrand competition: this depends, however, on the type of reaction to breakdowns, and on the degree of product differentiation.

We also conduct other comparisons. We show how, under Cournot competition, the input price is always lower with No Reaction compared to Reaction, for any degree of product differentiation and for any number of firms. Intuitively, if other retail firms can observe and react to the disagreement of a rival, they will increase their quantities when realizing they have one rival less, which, in turn, improves the value of the supplier’s outside option and generates a higher input price in equilibrium. Under Bertrand competition, instead, the result is exactly reversed: when remaining firms observe a disagreement, they react by pushing up their prices and this worsens the supplier’s position in its outside option compared to No Reaction, resulting in a lower input price. Finally, under the hypothesis of Reaction, we find that the input price is always lower when downstream competition is in prices instead of quantities; this last result is completely reversed in the case of No Reaction.

The issue of the modeling choice of the outside options has been somehow overlooked by the existing literature. The motivation is possibly to ascribe to HW themselves which, albeit perfectly correctly, state that the two types of retailers’ behavior in case of disagreement “do not have any qualitative effects on the points [they] make” (HW, p. 412). We will show that this statement cannot be generalized to many simple settings and that, on the contrary, the full consequences of the hypotheses used in subsequent analyses have not been fully comprehended.
This paper contributes directly to the understanding of countervailing buyer power, a phenomenon originally identified by Galbraith (1952). Formalizations are more recent though, starting with von Ungern-Sternberg (1996) in a Cournot model with Reaction, and Dobson and Waterson (1997) who study Bertrand competition with differentiated products and with Reaction. The methodological implications of our analysis ensure, however, that our paper is related to a much larger literature aimed at studying, in the context of vertically-related industries, things such as the incentives to merge, the effects of different bargaining structures, or the impact of input price discrimination. To give just a few examples, Milliou and Petrakis (2007) study the incentives for upstream mergers when firms can also choose the contract type (linear vs. two-part input prices) in a model in which the input price is set via a Nash bargaining with Reaction. Marshall and Merlo (2004) and Dobson (1994) analyze pattern bargaining in linear wages, both using the case of Reaction. Gal-Or and Dukes (2006) study merger incentives in the media industry, where media stations bargain with producers (linear) advertising rates, in a model with No Reaction. Dukes et al. (2006) show the effects of downstream cost reductions on upstream profits when linear transfer prices are bargained, with No Reaction to disagreements. Gal-Or (1997) studies the rationale for exclusionary contracts when health insurance companies and hospitals bargain over the reimbursement rate (i.e., the input price), with Reaction to negotiation breakdowns. O’Brien (forthcoming) employs a bargaining framework to study the effects of price discrimination over the linear input sold to competing downstream firms, and analyzes the role of outside options, though they are taken as exogenous and unrelated to the type of competition played by the downstream firms. All these works generalize along several dimensions the seminal contribution of HW, often not providing sufficient discussion of the role played by the modeling choice of the outside options. Moreover, some of these contributions contrast their findings directly with HW, despite solving, de facto, the case with Reaction, which makes some comparisons unwarranted.  

\[^2\text{See also Chen (2003).}\]

\[^3\text{Bilateral Nash bargaining models play an increasing role in empirical work too. Starting with Chipty and Snyder (1999), several papers have estimated models of input pricing (programs) in the}\]
This paper concentrates mainly on input prices, to show the dramatic impact that modeling reactions can have on equilibrium outcomes. As this is our main interest, we take as given several other key assumptions: we employ the Nash axiomatic approach, assume that agreed input contract terms are observed by all downstream firms when competing in the retail market, and concentrate on linear input prices. These assumptions all correspond to the case of HW and most of the ensuing literature cited above. They have important implications and we discuss each one in turn.

Nash axiomatic approach. As a consequence of this approach, a bargaining pair cannot write contracts specifying different terms in the event of a breakdown in rivals’ negotiations: this issue is investigated by Inderst and Wey (2003) and de Fontenay and Gans (2005), who study a sequence of bilateral negotiations. We employ the so-called “split-the-difference” rule which characterizes the asymmetric Nash bargaining solution, as this gives a role to outside options. It is well-known that the unique subgame-perfect equilibrium of a non-cooperative bargaining model with outside options and a risk of breakdown converges to the asymmetric Nash bargaining solution, when the time period between offers and counteroffers gets small. In fact, many of the papers with bilateral negotiations draw a connection with a non-cooperative bargaining, although it is fair to say that the informational assumptions to make this connection are often difficult to map to reality.

Observable contracts. If contracts were not observable to downstream competitors, then commitment problems would arise as the supplier’s contract terms to one firm would not affect the downstream rivals’ retail choices (McAfee and Schwartz, 1994; cable television industry. Crawford and Yurukoglu (2009) calibrate a model with linear input prices and Reaction to study what would happen if cable companies were to offer individual channels (à la carte) instead of bundles. Rennhoff and Serfes (2008) study a similar setting, though we could not retrieve the type of reaction they employ for disagreements. Grennan (2009) estimates a Nash bargaining model between hospitals and medical equipment suppliers. See also Ellison and Snyder (2010) for the case of pharmaceuticals.

4These bargainings give rise to the Shapley value. See also Björnerstedt and Stennek (2007).
5See Appendix B for more details. The advantage of using the strategic approach is that it explicitly delivers the disagreement points and bargaining power, and indicates how they depend on features of the underlying game.
The supplier’s opportunism problem in each bilateral contract would turn the supplier into his worst competitor, and the input price would be set at cost under “passive beliefs”. Downstream structural parameters would essentially play no role, and one could not address the question of countervailing power in a meaningful way. The assumption of observability of input contracts is quite appealing when studying the union-firm wage bargaining problem, as unions typically announce their deals with employers as soon as they are concluded. Instead, this is less likely to be so in the presence of hidden terms of trade between manufacturers and retailers.

**Linear input prices.** These are easier to justify when dealing with wage bargaining, though they can be found in many industries. A desirable property of linear input prices is that downstream (and, where present, upstream) conditions can affect input prices. Linear contracting is nevertheless a restrictive assumption. We should however note that, if observable non-linear contracts were set by a single upstream supplier, then they would completely eliminate intrabrand competition and always achieve the full monopoly outcome. If, instead, competing retailers make simultaneous take-it-or-leave-it offers to the same manufacturer, Marx and Shaffer (2007) find that upfront payments lead to exclusive dealing provisions, with only one retailer selling in equilibrium. In both cases, the question of countervailing power would again not be very meaningful. The case of Nash axiomatic bargaining when contracts are observable and nonlinear is yet to be examined in full by the literature.\(^8\)

The structure of the paper is as follows. We present the model in Section 2. In

---

6Observability here refers to contractual terms at the time of market competition, which is not to be confused with the *No Reaction* case in disagreement.

7See the discussion in Inderst and Valletti (2009), where it is argued that linear prices should be employed when preferential terms enhance a buyer’s competitive position in the downstream markets, which would not be case with two-part tariffs that would lead to an adjustment only in the fixed part of the tariff.

8Bargaining over observable nonlinear input prices is studied, among others, by O’Brien and Shaffer (2005), Antelo and Bru (2006), Milliou and Petrakis (2007), and Symeonidis (2008 and 2010). The opportunism problem is reintroduced when the supplier negotiates separately with a “non-orchestrated” number of retailers, when these bilateral negotiations result in binding contracts (i.e., they cannot be withdrawn later after other outcomes have been observed). These results depend delicately on the fact that contracts cannot be made contingent on market structure. See Miklos-Thal et al. (forthcoming).
Sections 3 and 4, we derive the equilibrium input price under the two hypotheses of Reaction and No Reaction, and for the two cases of downstream Cournot and Bertrand. Section 5 discusses and summarizes our results. In a separate Appendix, we generalize HW’s findings to a general number of firms, Bertrand competition, and different reactions in case of disagreement.

2 The model

We consider an industry in which a single upstream supplier sells an intermediate good to $N \geq 2$ downstream firms. Downstream firms use this input to produce differentiated goods and sell them to final consumers. The ratio of input to output is identical to all downstream firms, and is normalized to one. Each downstream firm $i$ pays a linear input price $w_i$ to the upstream supplier and does not incur any other cost. The costs of the single upstream supplier are normalized to zero.

We assume a linear demand structure for the final good, where inverse demand for the generic downstream firm $i$, given its own output $q_i$ and output $q_j$ of each of its rivals, is given by

$$p_i = 1 - q_i - \gamma \sum_{j \neq i} q_j$$

for $i; j = 1; \ldots; N; i \neq j,$ (1)

whenever this is positive (Singh and Vives, 1984; Hächner, 2000). This inverse demand function is derived from the quasi-linear quadratic utility function of a representative consumer

$$U = \sum_i q_i - \frac{1}{2} \left( \sum_i q_i^2 + 2\gamma \sum_{j \neq i} q_i q_j \right) + I,$$

for $i, j = 1, \ldots, N; i \neq j,$ (2)

where $I$ is the consumption of other goods. The parameter $\gamma$ describes the degree of homogeneity between the goods produced by downstream firms. We restrict our attention to substitute goods and therefore let $\gamma \in [0, 1]$: when $\gamma = 1$, downstream goods are homogeneous, while when $\gamma = 0$ we have independent goods.

Inverting (1), it is also possible to obtain the system of linear direct demand functions. With $N$ goods sold in the final market, the demand for the generic firm $i$ is...
given by
\[
q_i = \frac{(1 - p_i) [1 + \gamma(N - 2)] - \gamma \sum_{j \neq i} (1 - p_j)}{(1 - \gamma)[\gamma(N - 1) + 1]} \quad \text{for } i, j = 1, \ldots, N; i \neq j, \quad (3)
\]
whenever this is positive.

Competition in the industry is described by a two-stage game as follows. At stage 1, the upstream firm negotiates separately with each downstream firm \(i\) the linear input price \(w_i\). (The non-cooperative Nash equilibrium of these bargainings is further discussed in the next section.) At stage 2, the downstream firms observe the outcomes of stage 1 and compete against each other, either in prices or in quantities, given the values of \(w_i\) from stage 1. We derive the pure strategy equilibrium of this game.

### 2.1 Bargaining

The \(N\) first-stage negotiations are conducted simultaneously so that, during bargaining, the firms’ negotiators treat the other input prices as given.\(^9\) Each bargaining is obtained using the two-person Nash solution. The outcome is then a set of input prices which represents a Nash equilibrium in the Nash bargainings.

More formally, denote by \(\pi_D^i(w_i, w_{-i})\) the profit in the last stage of downstream firm \(i\) and by \(\pi_U(w_i, w_{-i})\) the profit of the upstream firm, where \(w_i\) is the input price to firm \(i\) and \(w_{-i}\) is the \((N - 1)\)-dimensional vector of input prices to all the other downstream firms. Let also \(\pi^O\) be the disagreement payoff for the upstream firm. Since each downstream firm \(i\) has no alternative supplier, its disagreement payoff is simply zero. At stage 1, the upstream supplier and each downstream firm \(i\) form a separate bargaining unit and set \(w_i\) to maximize the following Nash product

\[
\max_{w_i} \Omega_i = [\pi_U(w_i, w_{-i}) - \pi^O]^{\beta}[\pi_D^i(w_i, w_{-i})]^{1-\beta} \quad \text{for } i = 1, \ldots, N, \quad (4)
\]

where \(\beta \in [0, 1]\) denotes the bargaining power of the upstream firm relative to that of the downstream firm. The FOC of this problem can be written as

\[
\frac{\beta \pi_D^i(w_i, w_{-i})}{1 - \beta \pi_U(w_i, w_{-i}) - \pi^O} = \frac{\partial \pi_D^i(w_i, w_{-i})}{\partial w_i} / \frac{\partial \pi_U(w_i, w_{-i})}{\partial w_i} \quad \text{for } i = 1, \ldots, N. \quad (5)
\]

\(^9\)For the upstream monopolist, this means that \(N\) separate negotiators are sent to conduct independent negotiations with each downstream firm.
Bargaining outcomes are observable by all, and the equilibrium of the game is found as the Nash solution to the $N$ separate bargaining problems. We concentrate only on symmetric equilibria.

The disagreement payoff of the upstream firm in (4) is crucial to our analysis and is worth some further discussion. In the event of an unsuccessful negotiation between the upstream supplier and firm $i$, the upstream firm can still sell to the remaining $N - 1$ downstream firms, and thus has an outside option equal to $\pi^O = \sum_{j \neq i} w_j \hat{q}_j$, where $\hat{q}_j$ is the quantity sold, in case of a disagreement, by each downstream firm $j$ different from $i$.

The breaking down of the negotiation between the upstream supplier and firm $i$ makes this firm unable to produce its good and sell it in the final market. This has two immediate consequences. In the first place, consumers are unable to buy good $i$ and the system of demand functions has to, therefore, be re-adjusted. In the system of inverse demand functions (1), the quantity demanded of good $i$ must be set equal to zero. The system of direct demands (3) has instead to be re-obtained by removing good $i$ from the consumer’s choice when inverting the system of inverse demand functions.

While this re-adjustment at the consumer level is uncontroversial, the other consequence of the breaking down of the negotiation for firm $i$ depends on the way the other downstream rivals react to the disagreement, which in turn hinges on their possibility of observing the negotiation breakdown. We assume two possible scenarios:

- **No reaction**: The breakdown of the negotiation between firm $i$ and the input supplier *is not observed* by the rival downstream firms. Therefore, they are not able to react and do not adjust their behavior to the absence of firm $i$ in the downstream market. On the contrary, all the rival downstream firms adopt their optimal strategic behavior (in prices or quantities) as if all $N$ firms were present in the downstream market. Formally, in the case of downstream Cournot competition, the outside option of the upstream firm is obtained by noting that $\hat{q}_j = \hat{q}_j^N(w^*)$ where $\hat{q}_j^N(w^*)$’s are the last-stage anticipated quantities in a $N$-firm equilibrium, calculated at the anticipated equilibrium input prices, and which are therefore independent from the currently negotiated $w_i$. In the case of Bertrand
competition, \( q_j \)'s are the quantities bought (after the consumer’s re-adjustment of her optimally-purchased basket) when firms still play the anticipated last stage retail prices in a \( N \)-firm equilibrium, as a function of the equilibrium input prices.

- **Reaction:** The breakdown of the negotiation between firm \( i \) and the input supplier is observed by the rival downstream firms. They react to this by adopting an optimal choice (in prices or quantities) which takes into account that only \( N-1 \) firms operate in the downstream market: the upstream provider’s outside option profits have to be calculated accordingly. Formally, in the case of Cournot competition downstream, \( q_j = \hat{q}^{N-1}_j(w^*) \), where \( \hat{q}^{N-1}_j(w^*) \)'s are the last-stage equilibrium quantities (at negotiated input prices) when \( N-1 \) firms compete.\(^{10}\) Under price competition, \( q_j \)'s are the quantities bought (after the consumer readjustment of her optimally-purchased basket) in a \((N-1)\)-firm Bertrand equilibrium in the final stage of the game, as a function of the negotiated input prices.

At mention in the Introduction, both these approaches have been used extensively in the literature. The two-stage approach implies that, in both cases of Reaction and of No Reaction, agreed input prices are observed at stage 2, when firms compete (either in prices or in quantities). What differs, between the two cases, is only if the possible disagreement of one downstream firm could be observed by the other firms, therefore without changing input prices, which would involve instead some sequential game that we must avoid with a reduced two-stage approach. This is internally consistent if disagreements are permanent.\(^{11}\)

\(^{10}\)In disagreement, firm \( i \) does not produce anything, and the vector of equilibrium input prices \( w^* \) for the rivals does not include \( w_i^* \). In the literature, this case is sometimes referred to by setting \( w_i \) to infinity. However, since we are always very clear on the type of reaction to disagreement, we slightly abuse the notation by using \( w^* \) in both cases.

\(^{11}\)An alternative interpretation possibly arises if disagreements were temporary. In this case, a disagreement effectively implies that the disagreeing firm just “arrives a bit late” in the retail market, acting there as a Stackelberg follower, while the other \( N-1 \) firms are Stackelberg leaders. This is another instance where the difference between price and quantity competition becomes apparent. In a quantity-setting game, a temporary disagreement would be detrimental to the disagreeing firm, since the other retailers would have a first-mover advantage in making their strategic choices. But in a
In the following sections, we characterize the equilibrium of our market game under the two different hypotheses of Reaction/No Reaction when determining outside options, and under two different modes of competition, Cournot and Bertrand. Thus we consider four possible scenarios.

3 Cournot competition

We start the analysis with the case of downstream Cournot competition. Each retailer sets its final quantity to maximize

$$D_i = (p_i - w_i)q_i.$$ 

In case of $N$ firms operating in the downstream industry, by solving the system of FOCs of these problems, we obtain the second-stage subgame equilibrium quantities

$$\hat{q}_i^N(w_i, w_{-i}) = \frac{(1 - w_i)[\gamma(N - 2) + 2] - \gamma \sum_{j \neq i}(1 - w_j)}{2 - \gamma}(N - 1) + 2), \text{ for } i, j = 1, \ldots, N; i \neq j. \tag{6}$$

These quantities determine the agreement payoffs of the downstream retailers and of the upstream supplier in the first stage of the game, which are respectively given by

$$D_i(w_i, w_{-i}) = \left(\hat{q}_i^N(w_i, w_{-i})\right)^2 \text{ and } U_i(w_i, w_{-i}) = \sum_{i} w_i \hat{q}_i^N(w_i, w_{-i}).$$

3.1 No reaction

In case of disagreement, retailer $i$ cannot sell anything, thus $q_i = 0$, but the other retailers do not readjust their expected Nash-Cournot quantities. Hence the other $N - 1$ firms would still be selling $\hat{q}_j = \hat{q}_j^N(w^*)$, which denotes the second stage anticipated equilibrium quantity when the input prices are set at their equilibrium level. The outside option for the upstream monopolist is then $\pi_O = \sum_{i \neq i} w_j \hat{q}_j^N(w^*)$.

To solve for the equilibrium input price, we can use directly (5) and the hypothesis of symmetry, which allows us to write $w_i = w_j$. In a symmetric equilibrium, we have that

$$\hat{q}_i^N = \frac{1 - w_i}{2 + \gamma(N - 1)}; \text{ we also have simplified expressions for the firms’ profits, } \pi_i^D = \left(\hat{q}_i^N\right)^2, \pi_i^U = Nw_i\hat{q}_i^N \text{ and } \pi_i^O = (N - 1)w_i\hat{q}_i^N,$$

so that we can write $\pi_i^U - \pi_i^O = w_i\hat{q}_i^N$. In eq. (5), we can then write the LHS (ignoring the first ratio in $\beta$’s) as

$$\frac{2(1 - w_i)[2 + \gamma(N - 2)]}{(1 - 2w_i)(2 - \gamma)[2 + \gamma(N - 1)]},$$

which is decreasing in $w_i$; similarly the RHS could be written as

$$\frac{2(1 - w_i)[2 + \gamma(N - 2)]}{(1 - 2w_i)(2 - \gamma)[2 + \gamma(N - 1)]},$$

price-setting game the reverse would apply, as there would be a first-mover advantage.
increasing in \( w_i \). The equilibrium input price results in
\[
\tilde{w}_{CR}^{NR} = \frac{\beta}{2} \left( 1 + \gamma \frac{(1-\beta)(N-1)}{2} \right).
\] (7)

This result is formally expressed in the following Proposition:

**Proposition 1** When downstream firms compete in quantities, and there is No Reaction to negotiations breakdowns, the input price is given by (7) and it is decreasing both in \( N \) and in \( \gamma \), for all \( \beta \in [0,1] \).

Therefore, in the current case with downstream Cournot competition, we do not support the idea that higher concentration downstream exerts countervailing buyer power and pushes down the input price. On the contrary, the input price is lower the higher the number of downstream firms, and it goes down to zero, for any degree of bargaining power and product differentiation, as \( N \to \infty \).

### 3.2 Reaction

In case of Reaction, the payoffs of both the upstream and the downstream firms in the event of successful bargaining are identical to the previous case. The difference concerns the quantities sold by the remaining firms in case the upstream monopolist is in disagreement with firm \( i \) and its resulting payoffs. Clearly, also in this case, \( \tilde{q}_i = 0 \), but the other firms now re-adjust their quantities in the downstream market as they anticipate that firm \( i \) produces nothing. In particular, the quantities \( \tilde{q}_j \) they would be selling are the second stage equilibrium quantities in an industry with only \( N-1 \) firms, that is,
\[
\tilde{q}_j^{N-1}(w^*) = \frac{(1-w_j^*)(\gamma(N-3) + 2) - \gamma \sum_{k \neq i,j} (1-w_k^*)}{(2-\gamma)(\gamma(N-2) + 2)}.
\] (8)

Notice that \( \tilde{q}_j^{N-1} \) does not depend on \( w_i \). In a symmetric equilibrium where \( w_i = w_j \), the wholesale price is given by the following expression
\[
\tilde{w}_{CR}^R = \frac{\beta}{2},
\] (9)
so that we can state the following Proposition:  

**Proposition 2** *When downstream firms compete in quantities, and there is Reaction to negotiations breakdowns, the input price is given by (9) and it is independent of $N$ and $\gamma$, for all values of $\beta$.*

In other words, this price depends only on bargaining power $\beta$, while downstream structural parameters such as the number of competing firms or the degree of product differentiation play no role.  

Our finding is in stark contrast with the result of von Ungern-Sternberg (1996), derived in a setting similar to this Section, but with homogeneous goods. In his paper, the equilibrium input price (using our notation) is $w^* = \frac{\beta}{2 + \frac{\beta}{N + 1}}$, which is increasing in the number of downstream firms. His result is obtained using equation (5), but setting the RHS equal to $-1$, as it would happen in the case of a negotiation between the upstream firm and only one retailer. This hypothesis is, however, inconsistent with having also assumed the existence of an outside option for the upstream firm. This inconsistency results in an equilibrium expression of the input price which has the strange property to be equal to 1, when all the bargaining power is with the upstream firm ($\beta = 1$). This input price would choke-off demand completely, and no quantity would be sold. In our model, instead, with a fully-specified game, when $\beta = 1$ the input price is equal to $\frac{1}{2}$, i.e., the (linear) monopoly input price.

---

12 As with Proposition 1, the equilibrium values of the input prices and the comparative statics results in Propositions 2-5 come from a simple direct application of (5), the use of the symmetry assumption, and elementary algebraic manipulations. Proofs are therefore omitted.

13 An input price independent of the degree of product differentiation and the number of downstream firms is not a novelty in the literature. Milliou and Petrakis (2007) obtain the same result in a setting with two downstream firms. Similarly, in a model of wage bargaining between unions and oligopolistic firms, Dowrick (1989) finds that the degree of competitiveness in the retail market (expressed by a conjectural variation parameter) does not affect the wage level. All these results extend to the case of a bargaining over the input price, the finding that the input price set by the upstream firm as a TIOLI offer is invariant with respect to downstream market structure when the final demand function show constant elasticity of slope demand (Greenhut and Ohta, 1976). For further discussion, see Section 5.
4 Bertrand competition

We follow the very same framework introduced above, with the only difference that now downstream firms compete in prices. Each retailer sets its final price to maximize

\[ D_i = (p_i - w_i)q_i, \]

where \( q_i \) is given by eq. (3). In case of all \( N \) firms operating in the downstream market, after solving the system of FOCs with respect to prices, we obtain

\[
\hat{p}_i^N(w_i, w_{-i}) = \frac{(1 - \gamma)[2 + \gamma(2N - 3)] + [1 + \gamma(N - 2)][2 + \gamma(N - 2)]w_i + \gamma \sum_{j \neq i} w_j}{[2 + \gamma(2N - 3)][2 + \gamma(N - 3)]}.
\]

for \( i, j = 1, \ldots, N; j \neq k \). Let \( \hat{p}_i^N(w_i, w_{-i}) \) be the \( N \)-dimensional vector of such equilibrium prices. The resulting quantities that are demanded downstream and eventually supplied by the upstream firm are obtained by substituting all \( \hat{p}_i(.)'s \) back into (3), so that the output of the generic firm \( i \) is given by \( q_i(\hat{p}_i^N(w_i, w_{-i})) \). The agreement payoffs of the upstream supplier and downstream firm \( i \) can also be determined respectively as \( U_i(w_i, w_{-i}) = \sum_i w_i q_i(\hat{p}_i^N(w_i, w_{-i})) \) and \( D_i(w_i, w_{-i}) = [\hat{p}_i(w_i, w_{-i}) - w_i]q_i(\hat{p}_i^N(w_i, w_{-i})) \).

In case of disagreement between one retailer and the upstream supplier, only \( N - 1 \) remaining firms operate in the final market. Therefore, the system of demand functions is not (3) anymore, but the one derived from the maximization of the consumer’s utility defined only over the \( N - 1 \) remaining goods. Formally, this is given by

\[
q_j = \frac{(1 - p_j)[\gamma(N - 3)] - \gamma \sum_{k \neq j}(1 - p_k)}{(1 - \gamma)[\gamma(N - 3) + 1]} \quad \text{for } j = 1, \ldots, N - 1; j \neq k. \tag{10}
\]

4.1 No reaction

In disagreement, \( q_i = 0 \), and the other firms do not readjust their expected Nash-Bertrand prices of the last stage. The outside option of the upstream supplier can then be calculated plugging into (10) the anticipated Bertrand equilibrium prices \( \hat{p}_i(.) \), calculated at the anticipated equilibrium input prices \( w^\ast \).

The input price is obtained as the outcome of the bargaining problem as in (5), resulting in

\[
w^\ast_{NR} = \frac{\beta}{2\left(1 - \frac{(1 - \beta)(N - 1)(1 + \gamma(N - 1))}{(1 + \gamma(N - 2))(2 + \gamma(2N - 3))}\right)}, \tag{11}
\]
From (11), it follows that

**Proposition 3** When downstream firms compete in prices, and there is No Reaction to negotiations breakdowns, the input price is given by (11) and it is non-monotonic in $N$ and decreasing in $\gamma$, for all $\beta \in [0,1]$.

In particular, we have that, for low (respectively, high) enough values of $\gamma$, the input price is always increasing (respectively, decreasing) in $N$. When $\gamma$ is in a mid-range, the input price is first increasing and then decreasing in the number of downstream firms. This is depicted in Panel a) of Fig. 1, where the equilibrium input price is plotted against the number of downstream firms for different values of $\gamma$. The Figure illustrates that the role of downstream concentration to exert countervailing buyer power is limited to the case of downstream markets where retailers enjoy a sufficiently high market power, either because products are sufficiently differentiated or because competitors are relatively few.\(^{14}\)

\(^{14}\) A closer inspection of (11) reveals that $\partial w^\text{NR}_B / \partial N > 0$ when $\gamma < 2/3$, for all values of $N$. 

---

*Figure 1 - Equilibrium input price with Bertrand competition ($\beta = \frac{1}{2}$)*
4.2 Reaction

The payoffs of both the upstream and the downstream firms in the event of successful bargaining are identical to the previous case. Therefore, it only remains to be discussed what happens in disagreement. The remaining \( N - 1 \) downstream firms take into account the presence of one less competitor and choose their equilibrium prices, facing demand as given in (10). Plugging these prices back into (10), the quantity supplied by each downstream firm in case of disagreement can be computed.\(^{15}\) The equilibrium input price is given by

\[
 w^{R}_{B} = \frac{\beta}{2 \left( 1 + \gamma \frac{(1-\beta)(N-1)(1+\gamma(N-1)][1+\gamma(N-3)]}{(1+\gamma(N-2)][1+\gamma(2N-3)][1+\gamma(N-4)])} \right)}.
\]  

(12)

From (12), we can state

**Proposition 4** When downstream firms compete in prices, and there is Reaction to negotiations breakdowns, the input price is given by (12) and it is non-monotonic in \( N \) and increasing in \( \gamma \), for all \( \beta \in [0,1] \).

The Proposition illustrates that, with a sufficiently large degree of product differentiation (low values of \( \gamma \)), there is no countervailing buyer power when \( N \) is small. On the other hand, when products are sufficiently homogenous, the equilibrium input price \( w^{NR}_{B} \) increases with \( N \). This is also illustrated in Panel b) of Fig. 1 which shows that downstream concentration exerts countervailing buyer power only when downstream firms enjoy sufficiently low market power, either because products are sufficiently homogeneous or because firms are relatively numerous.\(^{16}\)

5 Discussion

Having obtained all the expressions for the input price under different modes of downstream competition and different reactions to breakdowns, in this Section we sum up

\(^{15}\)This is essentially the case solved by Dobson and Waterson (1997), the only difference being that we allow for generic values of bargaining power \( \beta \), while they deal with the case \( \beta = \frac{1}{2} \). We refer the reader to this paper for a more detailed discussion of this case.

\(^{16}\)A closer inspection of (12) reveals that \( \frac{\partial w^{R}_{B}}{\partial N} > 0 \) when \( \gamma > 0.424 \), for all values of \( N \). This threshold value is reduced to 0.358 when taking into account that \( N \) can take only integer values.
and discuss our findings.

We first provide a graphical illustration of the results, by plotting the values of the LHS and the RHS of equation (5), whose intersection determines the equilibrium input price. For simplicity, in this graphical treatment, we always assume equal bargaining power of the parties (i.e., $\beta = \frac{1}{2}$). It is important to recall that the bargaining solution has the property that a party becomes relatively stronger the higher is the value of its outside option and the more costly are its concessions (i.e. the adverse changes in the variable bargained upon). The LHS of (5) is the ratio between the profit levels of the two bargaining parties, net of the value of the outside option, whenever it exists. In all cases, this ratio is always decreasing in the equilibrium input price, $w^*$, because the downstream (upstream, respectively) net agreement profits are decreasing (increasing, respectively) in $w^*$. Notice that, for a given mode of downstream competition, the LHS of (5) changes with Reaction or with No Reaction, as the supplier’s profit in case of disagreement, $\pi^O$, differs under the two types of reactions.

The RHS of (5) is the ratio of the marginal effects of a change in $w_i$ on the firms’ profits. The latter can also be seen as the ratio of concession costs. For the buyer, a concession (an agreement to pay a higher input price) weakens its competitive position.
in the downstream market relative to rivals. For the seller, a concession is an agreement to accept a lower price. The behavior of this ratio with respect to $w^*$ is perhaps less intuitive, though it reflects the rather general property that the concession cost for a downstream firm relative to that of the seller, is higher the higher is the general level of input prices $w^*$ (and, thus, the smaller is the equilibrium quantity produced by the buyer). The RHS does not change with the type of reaction but only with the mode of competition: under Cournot competition, it shifts upwards as the degree of product differentiation decreases, while the shift is downwards under price competition.

More in details, in Panel a) of Fig. 2, we illustrate the case of downstream quantity competition. We keep the number of firms fixed, and plot the LHS and the RHS of (5) for the two cases of Reaction and No Reaction and for different values of $\gamma$. The downward sloping lines are the LHS of (5) in case of Reaction and No Reaction: the dashed lines are obtained when $\gamma = 0.8$ and the solid line, identical under Reaction and No Reaction, when $\gamma = 0$. The upward sloping lines are the RHS of (5), always identical under Reaction and No Reaction for any value of $\gamma$: as before, we use a dashed line for the case of $\gamma = 0.8$ and a solid line when $\gamma = 0$. The intersections of the relevant lines for the same value of $\gamma$ determine the equilibrium value of the input price. More in general, we plot (in bold black) the locus of the intersections of the two curves for all possible values of $\gamma$.

To understand the figure, start from the decreasing solid line: this is the LHS of (5), both for the cases of Reaction and No Reaction when $\gamma = 0$. The two are identical since the products are now fully independent and the downstream firms always choose their monopoly output. When we allow $\gamma$ to vary, the plots of the LHS of (5) change with or without reaction to the disagreement, because of the different profits obtained by the upstream firm in case of disagreement. These are higher under Reaction because, in case of disagreement, the remaining downstream firms observe that there is one less downstream competitor and offer a quantity larger than in the case of No Reaction. The resulting larger aggregate quantity benefits the upstream firm which makes a higher profit, for a given level of the input price. Therefore, for each strictly positive value of $\gamma$, the LHS of (5) with Reaction always lies above the corresponding line in case of No Reaction. We plot in the same graph also the RHS of (5), which we recall
to be identical for both cases of Reaction and No Reaction: the solid line is for $\gamma = 0$ and the dotted line, lying north-west to the former, is for $\gamma = 0.8$. These positions mainly depend on the the concession cost of the downstream firm, which increases with $\gamma$: under Cournot competition (and, more in general, with strategic substitutes), an increase in the input price has a negative direct effect on profits (due to the higher cost) and also an equally negative strategic effect, in that it worsens the competitive position of the firm relative to its rival. This latter strategic effect is stronger the more homogeneous the products are.

Panel a) of Fig. 2 allows us to study how the equilibrium input price varies with $\gamma$. In case of No Reaction, $w^*$ clearly always decreases as $\gamma$ increases. On the other hand, with Reaction, any change in $\gamma$ induces an equal upwards shift on the LHS and RHS of (5): the ratio of the levels and of the marginal effects is equally affected by the differentiation parameter and therefore $w^*$ is independent with respect to $\gamma$.

The case of Bertrand competition is shown in Panel b). Notice again that, for $\gamma = 0$, each downstream firm is a local monopolist and the mode of downstream competition does not affect the equilibrium: this implies that Panel b) coincides with Panel a) so that the solid lines are the same in the two panels. The two panels, however, show two remarkable differences. In the first place, the LHS of (5) with No Reaction always lies above the corresponding line in case of Reaction for each strictly positive $\gamma$. This is because the supplier’s profits in case of disagreement are always lower under Reaction. In this case, the remaining downstream firms observe that there is one less downstream competitor and set a price higher than in the case of No Reaction. Even after the consumers’ readjustment, these higher prices result in lower aggregate quantity, which therefore reduces the value of the upstream firm’s outside option, for a given level of input prices. The second difference is that now the RHS of (5) when $\gamma$ is positive lies south-east to the same line when $\gamma = 0$. As in the case of Cournot competition, this is again mostly driven by the concessions costs for the downstream firm, i.e., the numerator of the RHS of (5). However, contrary to Cournot, the negative direct effect on profits of an increase of $w^*$ is now compensated by a positive strategic effect due to the softening of competition as firms compete in strategic complements. Overall, this Panel shows clearly that, in the case of Reaction, $w^*$ decreases with $\gamma$,
while the opposite behavior occurs in the case of No Reaction.\footnote{The picture obtained by fixing γ and letting N vary is not provided here to save on space. In this case, the behavior of the RHS of (5) with respect to N is non-monotonic. On the one hand, this non-monotonic behavior explains the non-monotonicity of the equilibrium input price with respect to N described in Propositions 3 and 4. On the other hand, it generates a figure which is quite hard to read.}

We now turn to a more formal pair-wise comparison of the levels of the equilibrium input prices. We compare them in two ways: for a given mode of competition, between the two types of reaction to breakdowns; and for a given type of reaction, across the different modes of downstream competition. This is illustrated in the following Proposition.

**Proposition 5** For all strictly positive values of β and γ, and for all values of $N \geq 2$, we have that

- $w_{NR}^C < w_{R}^C$ and $w_{R}^B < w_{NR}^B$;
- $w_{R}^B < w_{NR}^C$ and $w_{NR}^C < w_{NR}^B$.

It is possible to give an interpretation of the first line of inequalities in Proposition 5 by looking at Fig. 2. As already discussed above, with quantity competition (Panel a)), the LHS with Reaction always lies above its counterpart in case of No Reaction, while the RHS is the same. Clearly, this motivates the ranking of the equilibrium input prices, with and without reaction. A similar argument justifies the opposite results for the case of price competition (Panel b)).

A similar argument explains also the second line of inequalities in Proposition 5. First, we re-emphasize that both supplier’s and retailer’s profits are identical under quantity and price competition when $\gamma = 0$. Therefore, the solid lines when $\gamma = 0$ are identical in both Panels, and the equilibrium input prices are identical under Bertrand and Cournot competition: $w^*|_{\gamma=0} \equiv w^{\text{mon}} = 0.25$. As clearly shown in Fig. 2 by the bold lines, in case of No Reaction, $w^*$ decreases below $w^{\text{mon}}$ as $\gamma$ increases under Cournot competition (Panel a)), while the opposite holds in the case of Bertrand competition (Panel b)). An equal but opposite argument holds for the case of Reaction,
with the only difference that under Cournot the input price is always equal to \( w^{\text{mon}} \), while under Bertrand competition \( w^* < w^{\text{mon}} \).

Although we concentrated our analysis on input prices, we can also derive the following welfare result almost immediately:

**Proposition 6** Let \( W_{m}^{t} \) be the equilibrium welfare value when \( m = B, C \) is the mode of competition and \( t = R, NR \) is the type of reaction to disagreement. For all strictly positive values of \( \beta \) and \( \gamma \), and for all values of \( N \geq 2 \), we have that

- \( W_{C}^{R} < W_{C}^{NR} \) and \( W_{B}^{NR} < W_{B}^{R} \);
- \( W_{C}^{R} < W_{B}^{R} \) and \( W_{B}^{NR} < W_{C}^{NR} \).

Identical results hold for consumer surplus.

**Proof.** In a symmetric equilibrium when \( q^* \) denotes the equilibrium quantity sold by each downstream firm in the final market and \( p^* \) its equilibrium price, using (1) and (2) we can write consumer surplus and total welfare as

\[
CS(q^*) = U(q^*) - Np^*q^* = \frac{N(q^*)^2}{2}(1 + \gamma(N - 1));
\]

\[
W(q^*) = U(q^*) - Np^*q^* + N(p^* - w^*)q^* + Nw^*q^*
= N[q^* - \frac{(q^*)^2}{2}(1 + \gamma(N - 1))].
\]

Since \( CS(q^*) \) and \( W(q^*) \) are both increasing in \( q \) in the relevant range, and \( q^* \) is decreasing in \( w^* \), our claim follows directly from Proposition 5. \( \blacksquare \)

Proposition 6 is the immediate counterpart of Proposition 5. It shows that the ranking between the different cases analyzed, presented in Proposition 5 as a function of input prices, can be applied, when completely reversed, to the case of the welfare ranking. Proposition 6 does not provide additional comparative statics exercises on \( CS \) and \( W \), which could be carried out with respect to the structural parameters \( N \) and \( \gamma \) in the various cases. Though feasible, we chose not to do so in order to keep the paper short. We notice that these additional comparative statics are not necessarily equivalent to those on \( w^* \), as the structural parameters also enter directly the expressions for \( W \) and \( CS \).
6 Conclusions

This paper has shown that the original idea of Galbraith (1952) that countervailing power could keep input prices low is not a general result. In our framework, this can happen only with Bertrand competition, and only if the degree of product differentiation falls in some interval. Even in this case, the relevant interval is affected by the type of reaction expected in case of a negotiation breakdown.

Our main interest in this paper was to show how the role of modelling reactions to disagreements is sometimes not fully appreciated by the applied IO/Labor literature, despite having quite crucial implications on outcomes. We believe that there is not a superior or more realistic modeling disagreement choice: it will depend on circumstances. For instance, in the example of grocery stores and retailing, if negotiations fail, this will probably not be observed immediately by competing products (while customers will not find the product available on the shelves). In this case, rival products will not re-adjust their strategic choices under disagreement. In other circumstances, rivals would be able to react. A notable example could the bargaining of landing fees between airports and airline carriers: in disagreement, a flight will not be available, and since this is likely to be visible (because of the change in timetable over the internet where bookings can be made), rival airlines will realize they face less competition and react accordingly.

In this paper we chose countervailing power and the impact on input prices, though several further questions could be re-assessed using our methodological approach. We are not, however, saying that one should not expect robust results to arise, simply by changing the mode of competition or the type of reaction. The answer depends of course on the particular question asked. In the Appendix, we revisit HW and show that their central result is indeed very robust, as it arises independently from the type of reaction or from the mode of competition.

References


### Appendix A

In this Appendix we revisit HW’s main result, namely that an upstream merger always increases the input price to downstream firms. HW analyze the case of two downstream
Cournot firms and two upstream firms that may merge. Without an upstream merger, each upstream firm can supply only one specific downstream firm; with an upstream merger, the outside option of the upstream firm in disagreement is modeled as with No Reaction.

We generalize HW’s findings to a general number of firms, Bertrand competition, and different reactions in case of disagreement. In the absence of an upstream merger, we maintain HW’s hypothesis that each downstream firm deals with an independent upstream firm. Thus we modify our analysis to allow for the existence of \( N \) independent upstream firms. For both types of downstream competition, we first characterize the equilibrium input prices in the case of \( N \) independent suppliers.\(^{18}\) We then compare it with the ones obtained in the case of an upstream merger: when there is a single upstream supplier, the analysis corresponds to the one we have conducted in Sections 3 and 4.\(^{19}\)

Let us start with downstream Cournot competition. The second stage is the same as the one described in Section 3. In the first stage, each firm is in a bilateral monopoly relation with an independent supplier, and the Nash bargain problem with zero outside options gives

\[
I\_C = \frac{\beta}{2 + \frac{\beta}{(2 - \beta)(N - 1)}}.
\]

where the superscript \( I \) is a mnemonic for ‘independent’ upstream firms. This input price clearly simplifies to HW’s solution \( I\_C = \frac{\beta}{2} \) for \( \beta = \frac{1}{2} \) (see eq. (6) in HW). Comparing this with the input price obtained with a single merged upstream firm, respectively for the case of Reaction (7) and No Reaction (9), it is established immediately that

\[ I\_C < I\_R \text{ and } I\_C < I\_N. \]

We turn now to the case of downstream Bertrand competition, where the second stage is the same as the one studied in Section 4. In the first stage, with \( N \) independent upstream firms, the Nash bargain problem with zero outside options gives

\[
I\_B = \frac{\beta}{2 + \frac{\beta}{(2 - \beta)(N - 1)(1 + \gamma N - 2\gamma)}}.
\]

Comparing this with the input price obtained with a single merged upstream firm, respectively for the case of Reaction (7) and No Reaction (9), simple algebra allows us to find that

\[ I\_B < I\_R \text{ and } I\_B < I\_N. \]

Thus we have confirmed the robustness of the results of HW obtained for the special case of \( N = 2 \) and \( \beta = \frac{1}{2} \). This is formally stated in the following proposition:

\(^{18}\) As a downstream firm and its upstream supplier are locked into bilateral relations when they bargain, their outside options are zero, and in this case we should not worry about the type of reaction to disagreements.

\(^{19}\) As in HW, we consider an upstream merger to monopoly. While this may be rarely observed in practice, it can be a more realistic assumption in international mergers when upstream suppliers belong to geographically different markets, or when describing a union-firm wage bargaining problem.
Proposition 7 For any value of \( N \geq 2, \beta \in [0,1], \) and \( \gamma \in [0,1], \) an upstream merger to monopoly always increases the input price, independently from the type of downstream competition (quantity vs price) and from downstream reactions in case of a disagreement during bargaining (Reaction vs No Reaction).

Appendix B

In this Appendix we replicate the relationship between the Nash axiomatic approach we have employed, and a strategic representation of a bargaining game. Imagine two players are trying to divide a pie of size 1. Let \( w \) be the share going to agent 1, and \( 1 - w \) the share going to player 2. The player who makes the offer is determined randomly each period, and let it be agent \( i \) with probability \( p_i > 0, \) and \( p_1 + p_2 = 1. \) The payoffs from an agreement at date \( t \) are given by \( \delta \pi_i(w) \), where the functions \( \pi_i(w) \) are well behaved. The length of a period is a variable, given by \( \Delta, \) so that \( t = 0, \Delta, 2\Delta, ..., \) and the discount factor is written as \( \delta_i = \frac{1}{1 + r_i \Delta}. \) where \( r_i \) is the discount rate of player \( i. \) We allow for the possibility of exogenous breakdowns in the negotiations. Let \( \lambda_i \) be the Poisson arrival rate with which \( i \) believes an exogenous breakdown will occur; i.e., \( \lambda_i \Delta \) is the probability a breakdown will occur in a small interval of time of length \( \Delta. \) When a breakdown occurs, the game ends, and let \( b_i \) be the utility of player \( i \) in this event. The reservation values satisfy the following recursive relations:

\[
\begin{align*}
\pi_1(w_1) &= \frac{1}{1 + r_1 \Delta} [\lambda_1 \Delta b_1 + (1 - \lambda_1 \Delta)(p_1 \pi_1(w_2) + p_2 \pi_1(w_1))], \quad (15) \\
\pi_2(w_2) &= \frac{1}{1 + r_2 \Delta} [\lambda_2 \Delta b_2 + (1 - \lambda_2 \Delta)(p_1 \pi_2(w_2) + p_2 \pi_2(w_1))], \quad (16)
\end{align*}
\]

such that in any subgame, player 1 accepts any offer \( w \geq w_1 \) and player 2 accepts any offer \( w \leq w_2, \) and each player always proposes the reservation value of the other agent. For example, the first equation says that agent 1 is indifferent between accepting \( w_1 \) and rejecting it for a chance at a counteroffer after waiting \( \Delta, \) which brings the possibility of a breakdown.

Let \( \bar{w} = p_1 w_1 + p_2 w_2 \) be the average offer, which is arbitrarily close to both \( w_1 \) and \( w_2 \) for small \( \Delta. \) Consider a first-order Taylor’s approximation of (15) and (16) around \( w: \)

\[
\begin{align*}
\pi_1(w) + (w_1 - w) \pi_1'(w) &= \frac{1}{1 + r_1 \Delta} [\lambda_1 \Delta b_1 + (1 - \lambda_1 \Delta) \pi_1(w)] + o(\Delta), \quad (17) \\
\pi_2(w) + (w_2 - w) \pi_2'(w) &= \frac{1}{1 + r_2 \Delta} [\lambda_2 \Delta b_2 + (1 - \lambda_2 \Delta) \pi_2(w)] + o(\Delta), \quad (18)
\end{align*}
\]

where \( o(\Delta) \) is a function with the property that \( o(\Delta)/\Delta \to 0 \) as \( \Delta \to 0. \) If we multiply (17) and (18) respectively by \( (1 + r_1 \Delta)(1 + r_2 \Delta) \pi_2' \) and \( (1 + r_1 \Delta)(1 + r_2 \Delta) \pi_1' \), add the equations and simplify, we arrive at

\[
(1 + r_1 \Delta)p_1[\pi_2(w)(r_2 + \lambda_2) - \lambda_2 b_2] \pi_1'(q) + (1 + r_2 \Delta)p_2[\pi_1(w)(r_1 + \lambda_1) - \lambda_1 b_1] \pi_2'(w) = o(\Delta)/\Delta.
\]
As $\Delta \to 0$, this tends to

$$\frac{p_1}{r_1 + \lambda_1} \pi_2(w) - \frac{\lambda \lambda_2}{r_1 + \lambda_2} \pi_1(w) = - \frac{\pi_2'(w)}{\pi_1'(w)}.$$ 

To conclude, $w$ solves a generalized Nash problem like (5) in the main text, where the outside options are given by $\pi_i^O = \frac{\lambda \lambda_1}{r_i + \lambda_2}$ and the bargaining power of agent 1 is given by $\beta = \frac{p_1(\lambda_2 + \lambda_1)}{p_2(\lambda_2 + \lambda_1)}$. One interpretation is that $\pi_i^O$ is the utility that player $i$ can get by delaying the agreement forever, appropriately discounted. As $r_i \to 0$ expressions simplify to $\pi_i^O = b_i$ and $\beta = \frac{p_1 \lambda_2}{p_1 \lambda_2 + p_2 \lambda_1}$ and the disagreement points are the exogenous payoffs in the event of a breakdown (this is the case discussed in Binmore et al., 1986).