Partitioning of Mixed-mode Fracture in Adhesively-bonded Joints: Experimental Studies

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Abstract

The fracture resistance, \(G_c\), of adhesively bonded joints is known to vary significantly with loading mode and because joints are usually subject to mixed-mode loading in service, it is essential to be able to partition \(G_c\) into its I/II components so the appropriate failure criterion can be derived. Various partitioning schemes have been proposed in the literature but they give different results and which is most suitable has been an open question. In this work, joints were tested under a wide range of I/II loading ratios and the various partition schemes were assessed. The singular and global schemes were found to define the extremes, while Davidson’s non-singular field scheme represents an intermediate case. The semi-analytical scheme proposed by Conroy et al. exhibited a gradual transition from the singular to the global solutions and was shown to be the best approach.

Keywords: Mixed-mode Fracture; Fracture Mechanics, Debonding, R-curve tests, Adhesives

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NOMENCLATURE

**British Alphabet**

- $a$  
  Crack length
- $a_e$  
  Effective crack length
- $B$  
  Specimen width
- $C$  
  Specimen compliance
- $C_{\alpha}, C_L$  
  Crack and free length coefficients of the compliance function of beam-like geometries
- $E$  
  Young’s modulus
- $E_i$  
  Elastic modulus in the direction “$i$”
- $E_s$  
  Flexural modulus of the substrate material in the longitudinal direction
- $E'_I$  
  Equivalent elastic modulus for orthotropic materials in $I$-direction
- $E''_II$  
  Equivalent elastic modulus for orthotropic materials in $II$-direction
- $f$  
  Contribution of the singular solution to the mode mix (according to the SACA method)
- $F_v$  
  Correction factor for large displacements
- $G_c$  
  Critical energy release rate or fracture energy
- $G_i$  
  Energy release rate for mode “$i$”
- $G_{ic}$  
  Critical energy release rate or fracture energy for mode “$i$”
- $G_{ij}$  
  Shear modulus in the “$i$-$j$” plane
- $G_{II}/G$  
  Mixed-mode ratio in energy terms
- $h$  
  Laminate half thickness or substrate thickness
- $h_e$  
  Effective thickness in asymmetric test specimens
- $h_1, h_2$  
  Substrate thicknesses in asymmetric test specimens
- $L$  
  Free length
$l_{CZ}$  Cohesive zone length

$l'_{CZ,I}, l'_{CZ,II}$  Estimates of the mode I and II components of the cohesive zone length

$l_{nd}$  Normalised damage length

$M$  Correction factor for the analytical estimates of the cohesive zone length

$M_1, M_2$  Bending moments

$N_p$  Correction factor for the presence of loading blocks

$P$  Load

**Greek Alphabet**

$\alpha$  Thickness ratio for asymmetric test specimens

$\gamma$  Thickness ratio (Davidson mode decomposition theory)

$\Delta_{clamp}$  Clamp correction factor

$\nu$  Poisson’s ratio

$\nu_{ij}$  Poisson’s ratio corresponding to a contraction in the direction “$j$” when an extension is applied in the “$i$” direction

$\rho$  Dimensionless material parameter in Davidson’s crack tip element

$\sigma_{max,I}$  Normal cohesive stress

$\sigma_{max,II}$  Tangential cohesive stress

$\sigma_Y$  Yield stress

$\Phi(\alpha)$  Mixed mode ratio as a function of the thickness parameter

$\Omega$  Mixed mode parameter (Davidson’s mode partitioning method)

$\Omega_{NSF}$  Effective mixed mode parameter (Davidson CTE/NSF)

$\Omega_{SF}$  Effective mixed mode parameter (Davidson CTE/SF)
1 INTRODUCTION

Since the middle of the 1940s, when major advances in polymer science started to take place, adhesives have found increasing use in demanding engineering applications [1], since adhesive bonding can offer a better performance compared to more traditional joining methods. This superiority is particularly evident when joining carbon-fibre reinforced-plastics (CFRPs) due to (a) weight savings and (b) improved fatigue life arising from a more even stress distribution in the bonded region.

Now, the methodology of fracture mechanics has proven especially well-suited to (a) characterise and study the toughness and (b) predict the fatigue and durability performance of adhesive joints (e.g. [2-4]). Particularly encouraging results for quasi-static loading conditions have been obtained with the so-called ‘energy approach’, in which the applied energy-release rate, $G$, is calculated as a function of the external loads and then compared to the value of the adhesive fracture energy, $G_c$, so as to predict failure of the bonded joint. Combined with analytical models, or ever more sophisticated numerical methods, this approach has become very popular to predict the quasi-static properties of bonded joints [5,6] and is also finding extensive use in fatigue and durability studies [1,7-9]. Yet, despite the continuous developments in the fields of fracture and damage mechanics, the accuracy of this methodology relies ultimately on the experimental determination of the value of $G_c$ and having a proven and accurate scheme to partition the energy-release rate into its individual Mode I and Mode II components.

The value of the adhesive fracture energy, $G_c$, is highly dependent on the mode of crack growth, which is the proportion of crack-opening and crack-sliding modes, described as crack growth under modes I (opening tensile) and II (in-plane shear) loading, respectively. Several partitioning schemes have been suggested in the literature, and whilst the majority give identical results when the crack is located symmetrically in the specimen, the mode-mix predictions from these different partitioning
schemes vary significantly for asymmetric joints [10,11]. Based on analytical solutions, numerical techniques, or a combination of the two, the partitioning schemes can typically be split into two groups according to [11,12]: termed ‘local’ and ‘global’ approach schemes. Those strategies belonging to the first category (e.g. [13-15]) assume the existence of a local stress-singularity at the crack tip. Davidson and his co-authors developed a crack tip element approach, assuming a singular field at the crack tip [16,17] and gained identical results to [13-15]. This approach is known as the crack-tip element singular-field (CTE/SF) partitioning scheme. However, the relatively large process zones at the crack tips reported for many composites and structural adhesives suggests that the fracture process is not always controlled by a stress-singularity at the crack tip. In such cases, the non-singular field version of the crack-tip element approach (CTE/NSF) may be more applicable [18,19]. Alternatively, when the singularity is totally obscured by localised damage, the global energy-balance approach scheme may be employed to partition the total energy-release rate into its individual components (e.g. [20-22]). However, the universal applicability of the global methods has also been called into question (e.g. [23]). Conroy et al. [24] have recently proposed a novel, semi-analytical scheme based on a cohesive zone model to compute the Mixed-mode partitioning ratio in beam-like geometries. This study has shown that the value of the mode-mixity has a unique relationship with the normalised damage length ahead of the crack and that the damage length can be defined in terms of the initially unknown values of applied energy-release rate. The normalised damage length was defined as the length of the cohesive zone divided by the smallest characteristic length, i.e. the thickness of the thinner substrate in beam specimens. An iterative process is required to estimate the length of the cohesive zone and the normalised value of mode-mixity, i.e. normalised with respect to the upper and lower bounds of the global and local solutions, is then calculated from the established relationship between that value and the normalised damage length. This approach was termed the ‘Semi-Analytical Cohesive Analysis (SACA)’ scheme and typically yielded mode-mixity values which were somewhere in between the global and local partitioning values.
The overall aim of the present paper is to explore these different partitioning schemes and identify the most applicable technique which can describe the experimental results for a tough aerospace epoxy-film adhesive. The adhesive is used in bonded joints, using CFRP substrates, which involve various mode-mixities, i.e. Mode I, Mode II and Mixed-mode tests, and the results are presented in Section 5 of this paper. The Mode I tests have been reported previously [25]. In Section 6 the experimental data is fitted according to the different partitioning schemes.

2 MATERIALS AND EXPERIMENTAL PROCEDURE

The adhesive used in this investigation is a rubber-toughened epoxy film adhesive, AF-163-2OST, (3M, UK), designed for both solid panel and honeycomb construction in aircraft applications [26]. Unidirectional CFRP laminates were used as substrates for the joints. Prepreg tape was used for the composite substrates: unidirectional 977-2/HTS (Cycom, UK). The properties of the materials are shown in Table 1.

Prior to bonding, the composite substrates were grit-blasted and dried in a fan-oven for 24 hours at 100°C. Two layers of the film adhesive were then stacked together and cured according to the manufacturer’s recommendations, using steel wires of diameter 0.4 mm to maintain a constant thickness of the adhesive layer. Each specimen incorporated an initial crack in the adhesive layer, which was formed by inserting a very thin film of poly(tetrafluoroethylene) [27]. Aluminium-alloy end-blocks were bonded to the CFRP substrates once the curing cycle was completed so that loads could be applied to the test specimen. Finally, after removing any excess adhesive and inspecting the adhesive bondline, the edges of the joints were coated with a thin layer of white paint to facilitate crack detection in the subsequent tests.

The double-cantilever beam (DCB) test was used to determine the pure Mode I values of the adhesive fracture energy, $G_{Ic}$, and these tests and results have been reported previously [25]. The end-loaded split (ELS) geometry was employed to determine the fracture energy for pure Mode II
loading for the joints, which yielded the Mode II value of the fracture energy, $G_{IIc}$. Three different test configurations were selected to characterize the Mixed-mode I/II fracture behaviour, namely: the asymmetric DCB test (ADCB) and both the symmetric and asymmetric versions of the fixed-ratio mixed Mode test (i.e. the FRMM and AFRMM specimens, respectively), see Figure 1. Two nominal thicknesses of CFRP substrates (i.e. 2 and 4mm) were combined to produce two different types of Mixed-mode joints: (a) Symmetric specimens were manufactured using 4mm thick CFRP beams; (b) asymmetric joints resulted from bonding together 2mm and 4mm thick CFRP substrates.

It is noteworthy that, regardless of the partitioning scheme considered, each of these asymmetric configurations yields a single Mixed-mode ratio when tested as ADCB specimens. In contrast, when tested as the AFRMM test (i.e. a single loaded arm) each specimen produces two different ratios, depending upon whether the thicker or the thinner arm is loaded. The thicknesses of the substrates, $h_1$ and $h_2$, are defined in Figure 1, and the thickness ratio is $\alpha (=h_2/h_1)$. The dimensions of the joints labelled in Figure 1 are shown in Table 2. The symmetric FRMM joint can be seen as a particular case of the AFRMM geometry in which $\alpha = 1$.

The specimens were tested quasi-statically in accordance with the relevant standards and protocols. While the international standards ISO 25217 [27] and ISO 15114 [28] were followed for the ADCB and ELS cases, respectively, the ESIS TC4 protocol [29] was taken as the reference for the FRMM and AFRMM joints. The load, $P$, and displacement, $\delta$, were recorded during the test, monitoring the crack length, $a$, at intervals with an optical travelling microscope. In order to obtain reproducible initial crack tip conditions (and hence comparable adhesive fracture energy measurements), the Mode II and Mixed-mode joints were firstly pre-cracked under Mode I loading to propagate the initial defect by a distance of approximately 5mm from the end of the PTFE insert film. The crack lengths, $a$, shown in Table 2 are the lengths after the pre-cracking has been conducted.
3 DATA REDUCTION SCHEME

The analysis of the experimental data was carried out using the ‘Corrected Beam Theory’ method with an ‘Effective Crack Length (CBTE)’ scheme [30]. The calculations shown below have been written in terms of the experimentally-measured compliance, $C$. In order to minimise the uncertainties arising from the visual determination of the crack length, this data reduction scheme uses experimental compliance data (corrected for the end-block effects via $N_v$, if necessary) to compute an effective crack length, $a_e$ [30]. The effective crack length is defined in terms of the compliance coefficients, $C_a$ and $C_L$. The values of these coefficients for the specimen geometries used here are shown in Table 3. The resulting value of $a_e$ is then used to determine the fracture energy, $G_c$, which is thus independent of the measured crack lengths. Thus, we have:

$$a_e = \left( \frac{1}{C_a} \left[ \left( 2BE_s h_e^3 \frac{C}{N_v} \right) - C_L (L + \Delta_{clamp})^3 \right] \right)^{\frac{1}{3}} \tag{1}$$

$$G_c = \frac{3P^2 C_a a_e^2}{4 B^2 E_s h_e^3} \cdot F_v \tag{2}$$

where the substrate dimensions and compliance terms are shown in Figure 1 and Table 3. The other terms are defined as:

$E_s$ = Flexural modulus of the substrate in the longitudinal direction;

$C$ = Specimen compliance;

$P$ = Load;

$\Delta_{clamp}$ = Clamp correction factor;

$F_v$ = Correction factor for large displacements;

$N_v$ = Correction factor for the presence of loading blocks;
\[ h_e = \text{effective height} \]

\[ h_e = h \] for the FRMM and ELS test specimens

\[ h_e = \frac{h_1 + h_2}{2} \] for the ADCB and AFRMM test specimens

This methodology was initially proposed for the analysis of the ELS test configurations [28, 30]. However, it offers significant advantages for analyses of these Mixed-mode tests, since the same scheme can be used for all test configurations. Further, this method overcomes the difficulties in defining the crack length from direct experimental observations, which can be particularly problematic for tough adhesives, such as that investigated in the present work, as the percentage of Mode II loading is increased.

4 MIXED-MODE PARTITIONING

The experimental results presented in this paper have been partitioned using the four different schemes described in Section 1, namely the global approach, both the singular (CTE/SF) and non-singular field (CTE/NSF) versions of Davidson’s local approach, and the semi-analytical cohesive analysis (SACA) approach.

The corresponding Mode I and II components \( G_{Ic}^{\text{mixed}} \) and \( G_{IIc}^{\text{mixed}} \), respectively and where \( G_c = G_{Ic}^{\text{mixed}} + G_{IIc}^{\text{mixed}} \) are then expressed as a mode-mixity in the form of:

\[
\frac{G_{II}}{G} = \Phi(\alpha) \rightarrow \begin{cases} 
G_{Ic}^{\text{mixed}} = G_{I/IIc} \cdot (1 - \Phi(\alpha)) \\
G_{IIc}^{\text{mixed}} = G_{I/IIc} \cdot \Phi(\alpha)
\end{cases}
\] (3)

where \( \Phi(\alpha) \) defines the Mixed-mode ratio, i.e. the Mode II component, \( G_{II} \), over the total energy-release rate, \( G \) (i.e. \( \Phi(\alpha) = G_{II}/G \)), and \( G_{I/IIc} \) is the fracture energy in mixed-mode. The value of \( \Phi(\alpha) \) depends on the type of test joint, the thickness ratio, \( \alpha \), and the partitioning scheme employed.
4.1 Partitioning via the global energy-balance scheme

Based upon simple beam theory, the global energy-balance approach proposed by Williams [20-22] predicts a Mixed-mode ratio independent of the crack length for the AFRMM test geometry:

\[ \Phi(\alpha)_{\text{global}}^{\text{AFRMM}} = \frac{3\alpha^4}{3\alpha^4 + (1 + \alpha)^2} \]

(4)

However, in the ADCB configuration, since both arms are loaded with transverse forces which generate equal but opposite bending moments at the crack tip \((M_2 = -M_1 = Pa)\), the global scheme proposed by Williams predicts pure Mode I conditions regardless of the thickness ratio \(\alpha\). Thus, for the ADCB test geometry:

\[ \Phi(\alpha)_{\text{global}}^{\text{ADCB}} = 0 \]

(5)

4.2 Partitioning via the singular and non-singular field versions of Davidson’s local schemes

The Mixed-mode ratio estimated by Davidson’s scheme for the AFRMM case is given by:

\[ \Phi(\alpha)_{\text{local}}^{\text{AFRMM}} = \frac{\alpha^3}{(1 + \alpha)^2[(1 + \alpha)^3 - \alpha^3]} \left[ 2\sqrt{3}\alpha \cos \Omega + \sqrt{\frac{(1 + \alpha^3)(1 + 3\alpha^2)}{\alpha^3(1 + \alpha)}} \sin(\Gamma + \Omega) \right]^2 \]

(6)

where:

\[ \sin \Gamma = \frac{\sqrt{3}(1 - \alpha^2)}{2\sqrt{(1 + \alpha)(1 + \alpha^3)}} \]

(7)

Equations (6) and (7) are valid for both the singular or non-singular field versions of the local analysis, and only require the appropriate definition of the Mixed-mode parameter, \(\Omega\). This parameter depends not only on \(\alpha\) but also on the conditions assumed at the crack tip; i.e. whether
the stress state at the crack tip is dominated by a singular or a non-singular field, leading to the CTE/SF and CTE/NSF versions of the analysis, respectively. Davidson and co-workers derived the expressions for $\Omega_{SF}$ and $\Omega_{NSF}$ [17-19]. For the singular field result, the value of $\Omega_{SF}$ can be obtained via linear interpolation of the following curves:

\[
\Omega_{SF} = \begin{cases} 
\Omega(\gamma, \rho = 1) = 23.529\gamma - 6.8406\gamma^3 + 1.0706\gamma^5 \\
\Omega(\gamma, \rho = 2) = 24.158\gamma - 7.9834\gamma^3 + 1.6055\gamma^5 \\
\Omega(\gamma, \rho = 4) = 24.375\gamma - 8.1897\gamma^3 + 1.6650\gamma^5 \\
\Omega(\gamma, \rho = 8) = 24.382\gamma - 8.2627\gamma^3 + 1.6924\gamma^5 
\end{cases}
\]  

(8)

where $\gamma = \log_{10}(\alpha)$ and $\rho$ is a function of the elastic properties which is equal to 1 for isotropic materials and defined as follows for the orthotropic case:

\[
\rho = \frac{1}{2} \sqrt{\frac{E_1}{E_2}} \left( \frac{1}{G_{12}} - \frac{2v_{12}}{E_2} \right)
\]

(9)

The “effective” values of the Mixed-mode parameter, $\Omega_{NSF}$, derived by Davidson et al. [18, 19] for the non-singular field solution using equations (8) are given by:

\[
\Omega_{NSF} = \begin{cases} 
-24.0 & \text{for: } \gamma < -0.468 \\
60.409\gamma - 41.738\gamma^3 & \text{for: } -0.468 < \gamma < 0.468 \\
24.0 & \text{for: } \gamma > 0.468 
\end{cases}
\]

(10)

For the ADCB test geometry, the application of Davidson’s theory results in a Mixed-mode ratio which varies with $\alpha$ and the conditions assumed at the crack tip:

\[
\Phi(\alpha)_{\text{local}}^{\text{ADCB}} = \sin^2(\Gamma + \Omega)
\]

(11)

where: $\Omega = \Omega_{SF}$ or $\Omega_{NSF}$ depending on the assumed conditions.
4.3 Partitioning using the SACA scheme

The SACA partitioning scheme recently presented by Conroy et al. [24] assumes that the Mixed-mode ratio for a particular geometry is somewhere between the global and local solutions described above:

\[
\frac{G_{II}}{G} = f \cdot \left( \frac{G_{II}}{G} \right)_{\text{singular}} + (1 - f) \cdot \left( \frac{G_{II}}{G} \right)_{\text{global}}
\]  

(12)

where \( f \) defines the degree of singularity.

Furthermore, employing numerical techniques based on a cohesive zone model, these authors have reported that a unique relationship exists between the length of the fully developed failure process zone (FPZ) and the contributions of the local and global solutions to the mode mix:

\[
f = 0.9682 e^{-0.24 l_{nd}} + 0.0983 e^{-0.02 l_{nd}}
\]  

(13)

where \( l_{nd} \) is the normalised cohesive length, i.e. the length of the cohesive zone, \( l_{CZ} \), divided by the thickness of the thinner substrate. However, prior knowledge of the fracture criterion for the material of interest is required to estimate the size of the FPZ, as it has been shown to be both geometry and material dependent, as well as a function of the actual Mixed-mode ratio. To circumvent this issue, the SACA scheme employs an iterative process in which the length of the FPZ (i.e. \( l_{CZ} \)) is calculated as an average of the lengths corresponding to the pure opening and shear cases (\( l'_{CZ,I} \) and \( l'_{CZ,II} \), respectively) using the Mixed-mode ratio \( (G_{II}/G) \) as the weighting function:

\[
l_{CZ} \approx M \left[ \left( 1 - \frac{G_{II}}{G} \right) l'_{CZ,I} + \left( \frac{G_{II}}{G} \right) l'_{CZ,II} \right]
\]  

(14)

where \( M \) depends on the cohesive zone model and the loading configuration, and the analytical expressions proposed by Bao and Suo for slender bodies [31] are used to compute the values of \( l'_{CZ,I} \) and \( l'_{CZ,II} \):
\[ l'_{CZ,II} = \left( E'_I \frac{G_{IC}}{(\sigma_{max,I})^2} \right) \frac{1}{4} \left( \frac{h_1 + h_2}{2} \right)^3 \]  

\[ l'_{CZ,II} = \left( E'_II \frac{G_{IIc}}{(\sigma_{max,II})^2} \right) \frac{1}{2} \left( \frac{h_1 + h_2}{2} \right)^{\frac{1}{2}} \]  

(15)  

(16)

where \( E'_I \) and \( E'_II \) are the equivalent elastic moduli for orthotropic materials in the orthotropic directions. The constants in Eq 13 are ‘semi-empirical’ fits from the Conroy et al [24]. These were deduced from the normalised mode mixity vs. normalised damage length for a wide range of mixed-mode geometries. Their accuracy was fully evaluated in ref [24].

The SACA analysis yields mode-mixity values which are somewhere between the global and local partitions. Very encouraging results have been obtained with this technique, which has been shown to explain some of the contradicting data previously reported in the literature by allowing a smooth transition from the singular to the global solution as the size of the damage zone increases.

4.4 Comparisons of the local and global approach schemes

Figure 2 illustrates the mode-mix predicted for the ADCB (Fig 2a) and AFRMM (Fig. 2b) tests by the two local schemes attributed to Davidson [17,18] and the global approach scheme presented by Williams [20-22] as a function of the relative thickness of the substrates, \( \alpha \). The value plotted is the Mode-mix ratio, \( G_{II}/G \), as defined in Equation (3). This has a value of zero for pure Mode I. As the value of the shear Mode II component increases, then the value of the Mixed-mode ratio increases, and pure Mode II occurs for \( G_{II}/G=1 \). The partitions based on the SACA scheme [24] are always between the global and local schemes (see equation 12), but the results from the SACA partitioning scheme are highly dependent on the values of fracture toughness in Mode I and II.

As commented above, all the partitioning schemes agree for the various test specimens for the symmetric cases (i.e. \( \alpha = 1 \)), whereas significant discrepancies arise for the unbalanced,
asymmetric, specimens. For example, considering the global approach scheme, for the symmetric ADCB specimen, i.e. for the DCB specimen, the ratio is zero, indicating pure Mode I. For the symmetric AFRMM specimen, i.e. for the FRMM specimen, the ratio of $G_{II}/G$ is 3/7. The global approach also attributes all ADCB test configurations to be pure Mode I. If the local approaches to partitioning are accepted to be correct, the Mixed-mode ratios achievable with the ADCB specimen may vary but are still limited. For the AFRMM test, the differences between the schemes increase notably with the degree of the asymmetry, affecting not only the specific values of $G_{II}/G$ estimated for a particular value of $\alpha$ but also the Mixed-mode range covered by each test configuration. According to the Williams’ global partitioning scheme, virtually any ratio could be achieved with AFRMM joints. On the other hand, Davidson’s schemes predict a narrower range for this type of test specimen, particularly if the singular field solution is considered.

5 EXPERIMENTAL RESULTS

5.1 Results of Mode I tests

The Mode I results for DCB test specimens made with either metallic or composite substrates, and bonded with the epoxy-film adhesive as used for the other modes reported in the present paper, have been reported in [32]. The value of $G_{Ic}$ obtained for cohesive fracture using the CBTE analysis method was essentially independent of which substrate material was used. The average value measured was $G_{Ic}=3032 \pm 179 \text{ J/m}^2$. The R-curves were essentially flat, with $G_{Ic}$ being independent of crack length.

5.2 Results of Mode II tests

ELS specimens were employed to investigate the fracture behaviour of the adhesive under in-plane shear loading conditions. The clamping fixture was calibrated using the inverse ELS test as advised in [28]. The value obtained for the clamp correction was 17.8mm and was used in the CBTE analysis of the experimental data.
The load-displacement traces deviated from linearity at approximately the point of apparent crack initiation. Following crack initiation, defined using the 5% increase in compliance criterion, extensive crack tip damage in the form of micro-cracks occurred together with crack growth. During this phase, the slope of the curve decreased progressively but the load continued to rise and did not reach a steady-state.

After unloading, the specimens were broken apart in Mode I to expose the fracture surfaces. Visual inspection confirmed that the failure had been cohesive, in the adhesive layer. However, the Mode II region exhibited a very characteristic roughness (i.e. “hackle” marks) associated with the formation of numerous micro-cracks at approximately 45° inclination ahead of the main crack tip.

The Mode II fracture energy, $G_{IIc}$, increased progressively as the crack propagated through the test specimen, without reaching a stable plateau, i.e. a steadily rising R-curve was observed, see Figure 3. The formation of the micro-cracks and the extensive damage that accumulated ahead of the continuously growing crack significantly hindered the definition of the true crack length. This problem, exacerbated by the considerable bending of the substrates, led to very few data points being measured and considerable scatter in the results. The uncertainties in the crack length measurements highlighted the advantages of the CBTE method, see Section 3. The difficulties in distinguishing between true crack growth and the damage accumulated ahead of the tip made the identification of the initiation point particularly difficult. The MAX/5% criterion for the onset of crack growth produced the most consistent results, giving an average initiation value of $G_{IIc}=3240\pm1668$ J/m².

Even though these tests did not allow the determination of a plateau value for $G_{IIc}$, there is clear evidence to confirm that the values of Mode II fracture energy of this adhesive, for both initiation and propagation, are significantly higher than the corresponding Mode I values. In view of the shape of the resistance curve shown in Figure 3, a plateau value may be indicated above about 15,000 J/m². Such values are at least a five-fold increase with respect to the Mode I values.
Examination of the experimental results can attribute these relatively high values to the failure mechanisms in Mode II: namely, (a) the formation of multiple micro-cracks and their subsequent coalescence and (b) the friction between the fracture surfaces. Such mechanisms can account for the additional energy absorption as well as for the rising R-curves obtained in these tests [33].

5.3 Results of Mixed-mode I/II tests

Four different test geometries were used for these tests (see Figure 1 and Table 2): ADCB joints with substrate thicknesses of 2mm and 4mm; FRMM joints with a substrate thickness of 4 mm; and AFRMM joints with substrate thicknesses of 2 mm and 4 mm with either the thinner or thicker substrate loaded (i.e. termed AFRMM 2mm and AFRMM 4mm, respectively). As described below, difficulties were encountered in extracting ‘plateau’ results for $G_c$, particularly for the AFRMM 4mm joint with the highest Mode II component. It should be noted that the results used were all from specimens which showed cohesive failure within the adhesive layer. Also, for the AFRMM 4mm specimen, the results were extracted just prior to the fast crack growth into the substrates, as described below.

The load-displacement curves for the four types of specimen were examined. Using the definition of a 5% compliance increase as the crack initiation criterion, the load continued to rise after crack initiation before reaching a maximum load for all the test geometries. Two different types of behaviour were then observed: in the ADCB joints, the symmetric FRMM specimens and the AFRMM 2mm joints in which the 2mm arm was loaded (i.e. $\alpha = 0.5$), the load decreased progressively as the crack grew in a stable manner. In these cases, the corresponding unloading traces returned to the origin indicating that the substrates had not experienced permanent deformation. On the other hand, in the asymmetric joints loaded via the 4 mm arm (i.e. $\alpha = 2$) the response became unstable soon after the inflexion point, with the crack propagating rapidly to the clamping point and the load dropping abruptly.
Subsequent inspection of the fracture surfaces revealed that those $P-\delta$ traces exhibiting a stable crack propagation region corresponded to joints that had failed cohesively in the adhesive layer. Conversely, sudden failure was often associated with a change in the type of failure from cohesive in the adhesive layer to interlaminar in the CFRP substrates. It is considered that the increase in the Mode II contribution induced by the geometrical asymmetry led to a failure mechanism which involved the formation of shear micro-cracks, the subsequent coalescence of which gradually drove the crack propagation path closer to the interface and increased the likelihood of interlaminar failure. This type of failure was particularly evident in the AFRMM joints where the load was applied via the thicker substrate. Thus, this problem appears to be more severe under Mixed-mode loading, than under pure shear loading, because only one arm is loaded in the AFRMM specimens.

Following the ESIS TC4 protocol for the FRMM tests [29], the clamping fixture was calibrated using joints specifically manufactured for the inverse tests without any PTFE insert. The value of the correction factor arising from the clamping condition increased with the total thickness of the joints, showing evidence of a relationship between the stiffness of the clamping arrangement and that of the specimen clamped.

Figure 4 shows the crack growth versus the length of the propagating crack, i.e. the R-curves, that were recorded for various types of Mixed-mode test specimens. The data presented arises only from valid, cohesive crack growth. After a first part of the curve in which the fracture energy increased with the crack length, the curves reached a stable region where the value of $G_c$ was almost constant. The extent of the rising section appears to increase with the Mode II component and the plateau value was attained approximately after 5mm of crack propagation in the ADCB joints and the AFRMM specimens loaded via the thinner arm. Further crack growth was required to attain the plateau value for higher Mixed-mode ratios, i.e. $\approx$10-15mm in the FRMM specimen and even more when the thicker arm of the AFRMM was loaded.
Figure 5 illustrates the initiation (MAX/5%) and propagation values of $G_c$ obtained for the AFRMM specimens. These results clearly show that the total fracture energy, $G_c$, that was measured for the propagation plateau values steadily increased with an increasing Mode II component. As discussed above, this was typically accompanied by a change in the failure mechanism, which moved progressively towards that observed in pure Mode II. Comparing initiation and propagation values, it is clear that the differences in these values increases as the Mode II component increases.

The differences between the results obtained with the symmetric and asymmetric DCB joints are clearly inconsistent with the global partitioning scheme. According to this theory, the ADCB specimens would produce pure opening Mode I and therefore should yield the same value of total fracture energy, $G_c$, for the DCB and all the ADCB specimens. Instead, the values of $G_c$ from the ADCB specimens were notably higher than the value obtained with the symmetric DCB joints. This limitation has been reported elsewhere (e.g. [23]), suggesting that the Williams’ global partitioning scheme represents a lower bound.

6 THE MODE PARTITIONING SCHEMES

The plateau values of $G_c$ measured from the various test specimens were next employed to evaluate the various mode partitioning schemes. The values used in the calculations are summarised in Table 4.

6.1 Williams global partitioning scheme

Figure 6 shows the results obtained using the Williams’ global partitioning scheme. The plot of $G_{Ic}^{\text{mixed}}$ versus $G_{IIc}^{\text{mixed}}$ exhibits two distinct regions: an initial section where the value of $G_{Ic}^{\text{mixed}}$ appears to increase very rapidly from about 2700 J/m$^2$ to a maximum value of approximately 4300 J/m$^2$. This was followed by a progressive decrease of $G_{Ic}^{\text{mixed}}$ with an increasing Mode II component, $G_{IIc}^{\text{mixed}}$. The shape of the plot in Figure 6 allows extrapolation of a value for $G_{IIc}$ of approximately 17,800 J/m$^2$. This value is consistent with the R-curves measured in the ELS tests.
(see Figure 3). This value is a six-fold increase compared to the Mode I fracture energy and results in a $G_{IIc}/G_{Ic}$ ratio slightly larger, but still comparable, to those reported for similar rubber-toughened epoxy adhesives [e.g. 30].

### 6.2 Davidson’s Partitioning Schemes

In contrast with the response obtained with the Williams partitioning scheme, the plots of $G_{Ic}^{mixed}$ versus $G_{IIc}^{mixed}$ corresponding to the singular and non-singular field versions of Davidson’s schemes did not decrease with $G_{IIc}^{mixed}$ after the initial rising section, see Figure 7. Instead, the Mode I component of the fracture energy appeared to either (a) stabilise somewhere between 3500 and 4500 J/m$^2$ for the CTE/NSF or (b) increase linearly with the shear contribution in the CTE/SF case. Now, the shapes of these data curves made the extrapolation of $G_{IIc}$ impossible. Further, the shapes of these curves do not appear to be physically meaningful. Bearing this points in mind, the value of $G_{IIc}$ determined from the Williams solution (i.e. 17,800 Jm$^{-2}$), which is in the experimental range indicated by the ELS tests, was used for the further calculations below.

### 6.3 The SACA Partitioning Scheme

As discussed in section 5, the SACA partition relies on an iterative process to estimate the cohesive length, which requires the definition of a set of cohesive parameters to characterise the fracture process (i.e. $G_{Ic}$, $G_{IIc}$, $\sigma_{max,I}$ and $\sigma_{max,II}$).

The Mixed-mode ratios obtained with the SACA scheme for the ADCB and AFRMM joints are shown in Figure 8. For comparison, the solutions obtained with the schemes proposed by Williams and Davidson are also shown on these plots. For the ADCB joints, the SACA solutions are in agreement with the Davidson singular field solution. It should be noted that agreement with the Williams global scheme only occurs for the symmetric joint. For the AFRMM joints, the SACA solutions agree very well with those predicted by the Davidson CTE/NSF scheme for $\alpha \approx 1$ and $\alpha < 1$ (i.e. when $h_2 < h_1$). As the shear component increases ($\alpha > 1$) the SACA solution approaches the
Williams’ global solution. This transition from the local to the global solution is consistent with the variation of the semi-analytical estimates of the cohesive length. As shown in Figure 9, the development of longer process zones in the joints, as a consequence of the higher Mode II components, is accompanied by a decrease in the importance of the singularity, represented here by the parameter $f$ defined in equation (13). This observation correlates very well with the experimental observations, as the micro-cracking phenomenon described in section 6.2 was significantly less evident in the ADCB joints than in the AFRMM specimens.

7 DISCUSSION

7.1 Mode I results

The Mode I experimental and numerical results have been presented and compared in our previous paper [25]. Good agreement between the experimental and predicted load-displacement curves was obtained. The experimental values of $G_{IC}$ could be reliably extracted from the R-curves since they showed a distinct plateau region, with value of $G_{IC} = 3032 \pm 179 \text{ J/m}^2$ being measured. In contrast, the trends shown from both the global scheme (Figure 6) and the CTE/SF and CTE/NSF schemes (Figure 7) suggest a somewhat higher value of $G_{IC}$ of up to about 4000 J/m$^2$.

7.2 Mode II results

The value of $G_{IIc}$ could not be reliably extracted from the R-curve for the ELS specimen since no plateau region was found (see Figure 3). Thus, the value of $G_{IIc}$ for such tough adhesives cannot be found from an ELS specimen of this dimension. A lower bound for the value of $G_{IIc}$ of 15,000 J/m$^2$ was extracted from the shape of the experimental R-curve. Further, in general agreement with this value, the trend from the global partitioning scheme extrapolated to a value of about 17,800 J/m$^2$. 
7.3 Mixed-mode results

These results were obtained using both ADCB and AFRMM joints. Loading for the ADCB joints was applied on both arms simultaneously and almost steady-state crack growth was obtained. However, the value of \( G_c \) extracted was significantly higher than the Mode I value, leading to the conclusion that the global partitioning scheme represents a lower bound value.

The FRMM specimens and the AFRMM joints loaded via the 2mm arm attained stable crack propagation. However, similarly to the ELS specimen, the AFRMM joints loaded via the 4mm arm did not reach steady-state crack growth. Experimental observations indicated that this could lead to complications in the failure path of the crack, as the greater Mode II component appeared to drive the crack towards the composite substrates and the extensive damage that accumulated ahead of the crack tip (in the form of micro-cracks) reached the end-clamp before the end of the test. As a result, this test geometry does not seem suitable for such a tough adhesive.

The initiation and propagation values of \( G_c \) for the AFRMM specimens are summarised in Figure 5. The propagation values increase with increasing Mode II contribution, as expected. However, the initiation values are almost constant and show no dependence on mode-mixity. Further, the initiation value is about 4000 J/m\(^2\) and this is in the range postulated for the \( G_{ic} \) extracted from the global, CTE/SF and CTE/NSF partitioning schemes.

7.4 Evaluation of the mode partitioning schemes

The values of Mixed-mode ratio for the symmetric specimens are in agreement for the four different schemes of partitioning investigated (see Figure 8). However, these four different schemes yield different values of the Mixed-mode ratio either side of the symmetric point (i.e. \( \alpha=1 \)). The results obtained with the SACA scheme for the ADCB joints are close to the singular field version of the local method, the CTE/SF scheme. For the AFRMM joints loaded via the thinner arm (i.e. \( \alpha < 1 \)
the SACA scheme is close to the non-singular field version, i.e. the CTE/NSF scheme, approaching the Williams’ global approach solution when the thicker arm is loaded (i.e. $\alpha > 1$).

The results for all four schemes of partitioning are summarised in Figure 10, where the total fracture energy, $G_c$, is plotted as a function of the value of the Mixed-mode ratio, $G_{II}/G$. This plot in Figure 10, together with results shown in Figure 6, clearly shows that the SACA partitioning scheme demonstrates three distinct, and important, advantages over the other partitioning schemes that have been considered in the present paper. Namely, the SACA scheme (a) correctly considers the ADCB test specimen as being a Mixed-mode test, (b) allows for a smooth transition for all the data points between the global and local solutions and (c) in this latter respect, also smoothly captures the results from the pure Mode II ELS test specimen where $G_{II}/G=1$. It should be noted that, even allowing for the typical scatter observed in the experimental results, none of the other partitioning schemes meet all of these required criteria. The SACA partitioning method therefore appears to render the most physically sensible partitioning results. The premise used in this partitioning scheme, that the size of the cohesive zone affects the fracture process, is an important feature of this method, which is not included in other partitioning approaches.

8 CONCLUSIONS

Measurements of the adhesive fracture energy, $G_c$, under various modes of loading, including Mixed-mode loading, have been used to evaluate four different mode-partitioning schemes, namely: the Williams’ global scheme, both the singular and non-singular versions of Davidson’s partitioning schemes (i.e. the CTE/SF and CTE/NSF methods, respectively), and the semi-analytical (SACA) scheme recently proposed by Conroy et al. Further, the corrected beam theory with effective crack length approach (CBTE) was found to be the only tractable data reduction method across the whole Mode I-II envelope for the tough adhesive investigated.
Failure was generally cohesive with the crack propagating through the adhesive layer in a stable manner. However, extensive damage in the form of micro-crack development ahead of the crack tip was observed as the proportion of Mode II loading was increased. These micro-cracks gradually drove the propagation path closer to the adhesive/substrate interface, leaving traces of interlaminar, unstable fracture for the AFRMM joints loaded via the thicker arm.

The resistance, R-curves, exhibited the typical rising effect, the magnitude of which increased with the Mixed-mode ratio. With the exception of the two geometries with the highest shear components, the value of $G_c$ reached a relatively stable plateau, which increased with the value of the Mode II component.

The results obtained with the Williams’ global partitioning scheme allowed an extrapolation of the value of $G_{IIc}$, i.e. $G_{IIc} \approx 17,800$ J/m2. The pure Mode II tests indicated that this value of $G_{IIc}$ is realistic, so this value was used in the assessment of the other partitioning methods. The global and singular schemes have been found to define the extreme cases. The SACA partitioning scheme employs a numerical technique, based on a cohesive zone model, to define the length of the cohesive or damage zone ahead of the crack tip. The results of Davidson’s non-singular partitioning schemes appear to yield suitable predictions only for intermediate sizes of the cohesive zone length. For relatively large cohesive zone lengths, that is for the higher shear loading tests, the results from Davidson’s partitioning schemes do not appear to be physically meaningful. The results obtained with the SACA scheme exhibited a gradual transition from the singular to the global solutions as the shear contribution increased. These results are consistent with the development of longer damage zones, as was indeed observed experimentally for relatively high Mixed-mode ratios. Further, this partitioning scheme circumvents the inconsistencies of the Williams’ global partitioning method, as was revealed by the ADCB test results.

Overall, the SACA partitioning scheme has proven fundamentally superior to the traditional partitioning schemes for Mixed-mode loading, as it accounts for the influence of both material
properties and test geometry on the size of the cohesive length and thus on the mode-mixity of loading the adhesively-bonded joints.

ACKNOWLEDGEMENTS

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9 REFERENCES


26. 3M™ Scotch-Weld™ Structural Adhesive Film AF163-2. 2009 November 2009]; Available from: [http://multimedia.3m.com](http://multimedia.3m.com).


### Table 1. Typical elastic properties of the materials

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_{11}$ (GPa)</th>
<th>$E$ (GPa)</th>
<th>$E_{22}$ (GPa)</th>
<th>$G_{12}$ (GPa)</th>
<th>$\nu_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRP UD 977-2/HTS</td>
<td>142.0</td>
<td>1.11</td>
<td>8.72</td>
<td>4.63</td>
<td>0.25</td>
</tr>
<tr>
<td>Adhesive AF-163-2OST</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2. Dimensions of the specimens (see Figure 1)

<table>
<thead>
<tr>
<th>Name</th>
<th>$L$ (mm)</th>
<th>$a$ (mm)</th>
<th>$h_1$ (mm) (nominal)</th>
<th>$h_2$ (mm) (nominal)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADCB</td>
<td>180</td>
<td>40-70</td>
<td>4</td>
<td>2</td>
<td>0.5</td>
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<tr>
<td>FRMM</td>
<td>130</td>
<td>77.5</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>AFRMM 2mm</td>
<td>120</td>
<td>45</td>
<td>4</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>AFRMM 4mm</td>
<td>110/120</td>
<td>75</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>ELS</td>
<td>130</td>
<td>91.5</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 3. Values of Compliance Coefficients, $C_{\alpha}$ and $C_{L}$, for the ADCB, AFRMM and ELS test geometries

<table>
<thead>
<tr>
<th></th>
<th>ADCB</th>
<th>AFRMM</th>
<th>ELS</th>
</tr>
</thead>
</table>
| $C_{\alpha}$ | \[
\frac{(1 + \alpha)^3(1 + \alpha^3)}{\alpha^3}
\] | \[
\frac{(1 + \alpha)^3 - \alpha^3}{\alpha^3}
\] | 3   |
| $C_{L}$  | 0                                                                   | 1                                                                   | 1   |

Table 4. Average propagation (average plateau) values of $G_c$ used in mode partitioning

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$G_c$ (J/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCB [25]</td>
<td>3032</td>
</tr>
<tr>
<td>ELS *</td>
<td>17800</td>
</tr>
<tr>
<td>ADCB</td>
<td>3973</td>
</tr>
<tr>
<td>FRMM</td>
<td>5851</td>
</tr>
<tr>
<td>AFRMM 4mm $^+$</td>
<td>11324</td>
</tr>
<tr>
<td>AFRMM 2mm</td>
<td>4363</td>
</tr>
</tbody>
</table>

* Estimated value (see section 6.1)

$^+$ Maximum value obtained in tests
Figure 1. Schematic representation of the (a) ADCB; (b) ELS; and (c) AFRMM test specimens and their corresponding loading configurations (note that the width of the joints, $B$, was 20mm in all cases).
Figure 2. Mixed-mode ratio predicted by the local and global schemes as a function of the thickness ratio $\alpha$ for (a) the ADCB and (b) the AFRMM test specimens.
Figure 3. Typical rising R-curve obtained for an ELS specimen. (The dashed line is a smooth line, best-fit, to the experimental data.)

Figure 4. Typical resistance curves for various types of Mixed-mode joints.
Figure 5. Initiation (MAX/5%) and propagation (average plateau) values of $G_c$ obtained for the AFRMM specimens.

Figure 6. Experimental propagation (plateau) results partitioned using the global energy-balance scheme proposed by Williams.
Figure 7. Experimental propagation (plateau) results obtained using (a) the singular field (i.e. CTE/SF) and (b) non-singular field (CTE/NSF) versions of the local partitioning schemes proposed by Davidson.
Figure 8. Mixed-mode ratios predicted by the SACA scheme compared with the Global and Local Partitioning schemes for (a) the ADCB and (b) AFRMM joints.
Figure 9. Variation of the singularity ratio \((f)\) and the Mixed-mode ratio \((G_{II}/G)\) with the length of the failure process zone (i.e. \(l_{CZ}\)) predicted by the SACA scheme for the ADCB and AFRMM 2mm joints.

Figure 10. Comparison of the failure locus corresponding to the SACA scheme with those obtained with the Williams’ partitioning scheme and both versions of Davidson’s partitioning schemes.