Rotational Energy Harvesting for Low Power Electronics

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Declaration

I herewith certify that all material in this dissertation which is not my own work has been properly acknowledged.

Hailing Fu
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Abstract

Energy harvesting is one of the key technologies for the realization of autonomous sensing. The aim of this thesis is to develop low-frequency broadband rotational energy harvesting solutions for self-powered sensing.

As an example of rotational energy harvesting, an airflow energy harvester using a miniaturized turbine and piezoelectric transduction was first introduced. Rotation was converted from airflow by the turbine, and a piezoelectric beam was actuated by the turbine rotor using magnetic plucking. Issues, including high cut-in speed and low output power at high rotational frequencies, were discovered. In order to decrease the cut-in speed, a self-regulating mechanism was proposed and integrated. The magnetic plucking strength can be passively adjusted according to the rotational speed. This self-regulating harvester exhibited a lower cut-in speed.

In order to understand the limited performance at high rotational frequencies and to optimize the design, a theoretical model was built. Different arrangements were investigated, showing that this harvester is ideal to operate at low excitation frequency far below the piezoelectric beam’s resonant frequency. Bistable behaviour was also employed and studied to enhance the energy harvesting capability over a wide bandwidth at low frequency. Then, a complete self-powered condition monitoring system, including a bistable frequency up-convert ing harvester, a power management circuit and a wireless sensor node, was studied and developed to implement the concept of self-powered sensing.

Finally, a fundamental study was conducted for three types of rotational energy harvesters, including electromagnetic, piezoelectric resonant, and piezoelectric non-resonant harvesters. Scaling laws for each type were established to study harvesters’ performance for different operating frequencies and device dimensions. This study provides a guideline for selection and design of rotational energy harvesters with specific requirements of device dimension and operating frequency.
Acknowledgements

Here is the place where I would like to thank all the people that helped me through my PhD study and those who shared the memorable, hard, exciting and great moments with me in this journey.

First of all, I would like to express my sincerest gratitude to my supervisor, Prof. Eric M. Yeatman. It has been my great honour to work with Eric. He is that kind of person who gives you full freedom on your research while giving you tremendous guidance when you are in doubt. Eric is very generous in offering help and support in different forms, from the summer school application, attending conferences to providing financial support for my PhD study. Being his student, I not only learnt how to be a qualified researcher, but also and more importantly, how to be a good person.

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Last but not least, I would like to thank my family and my girlfriend Miss Dan Gao for supporting me spiritually throughout this PhD study and my life in general. Thanks for being so patient and supportive.
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# List of Symbols

## Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Beam width ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>Initial angular position of the driving magnet</td>
<td>[$^\circ$]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Beam thickness ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>Geometrical factor</td>
<td>[-]</td>
</tr>
<tr>
<td>$\delta(x)$</td>
<td>Dirac delta Function</td>
<td>[-]</td>
</tr>
<tr>
<td>$\epsilon_{r,33}$</td>
<td>Piezoelectric relative permittivity constant</td>
<td>[-]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Structural damping coefficient</td>
<td>[Pa·s]</td>
</tr>
<tr>
<td>$\eta_r(t)$</td>
<td>Modal Mechanical Coordinate Expression</td>
<td>[-]</td>
</tr>
<tr>
<td>$\zeta_r$</td>
<td>Mechanical damping ratio of the $r$th mode</td>
<td>[-]</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>Initial position of the beam</td>
<td>[$^\circ$]</td>
</tr>
<tr>
<td>$\dot{\theta}_r$</td>
<td>Piezoelectric coupling term in physical coordinates</td>
<td>[-]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Air viscosity</td>
<td>[Pa·s]</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Magnetic constant</td>
<td>[m·kg·s$^{-2}$·A$^{-2}$]</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Total damping ratio of the beam</td>
<td>[-]</td>
</tr>
<tr>
<td>$\xi_m$</td>
<td>Total mechanical damping ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$\xi_{a,b}$</td>
<td>Air damping ratio of the beam</td>
<td>[-]</td>
</tr>
<tr>
<td>$\xi_{a,m}$</td>
<td>Air damping ratio of the mass</td>
<td>[-]</td>
</tr>
<tr>
<td>$\xi_{in}$</td>
<td>Internal structural damping ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Air density</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>Conductor resistivity</td>
<td>[Ω·m]</td>
</tr>
<tr>
<td>$\rho_{Cu}$</td>
<td>Conductor resistivity of copper</td>
<td>[Ω·m]</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Density of magnets</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Density of piezoelectric material</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Density of substrate material</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Maximum stress on the beam</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\sigma_t$</td>
<td>Tensile strength of the beam</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\upsilon(x, t)$</td>
<td>Displacement of the beam at position $x$ in the</td>
<td>[m]</td>
</tr>
</tbody>
</table>
length direction and time $t$

$\phi_y$ Function of dimensions and gaps of magnets [-]
$\phi_r(x)$ Mass-normalized eigenfunction [-]
$\Phi_0$ Magnetic flux linkage [V·s]
$\psi$ Dimension function for potential energy calculation [-]
$\omega$ Rotational frequency of external energy sources [Hz]
$\omega_b$ Oscillating frequency of the beam [Hz]
$\omega_n$ Resonant frequency of the spring [Hz]
$\omega_r$ Resonant frequency of the beam operating in the $r$th vibration mode [Hz]
$\omega_{re}$ Resonant frequency of the beam [Hz]
$\omega_{tr}$ Rotational frequency of turbine rotor [Hz]

### Latin Upper Case Letters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Length of the driving magnet</td>
<td>[m]</td>
</tr>
<tr>
<td>$A_0$</td>
<td>Amplitude of beam vibration</td>
<td>[m]</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Average area enclosed by coil</td>
<td>[m²]</td>
</tr>
<tr>
<td>$A_w$</td>
<td>Cross-section area of the wire</td>
<td>[m²]</td>
</tr>
<tr>
<td>$B$</td>
<td>Width of the driving magnet</td>
<td>[m]</td>
</tr>
<tr>
<td>$B_0$</td>
<td>Amplitude of the first model coordinate</td>
<td>[-]</td>
</tr>
<tr>
<td>$B_0M$</td>
<td>Maximum initial amplitude</td>
<td>[-]</td>
</tr>
<tr>
<td>$B_r$</td>
<td>Remnant flux density of magnets</td>
<td>[T]</td>
</tr>
<tr>
<td>$C$</td>
<td>Height of the driving magnet</td>
<td>[m]</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Capacitance of the storage capacitor</td>
<td>[F]</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Capacitance of the piezoelectric beam</td>
<td>[F]</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Equivalent capacitance of beam stiffness</td>
<td>[F]</td>
</tr>
<tr>
<td>$D_r$</td>
<td>Rotor Diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>$D_w$</td>
<td>Coil wire Diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>$\Delta E$</td>
<td>Variation of energy in a capacitor</td>
<td>[J]</td>
</tr>
<tr>
<td>$F_{dy}$</td>
<td>Magnetic force between driving and tip magnets</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{ft}$</td>
<td>Magnetic force between fixed and tip magnets</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{in}$</td>
<td>External forces on the beam</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{mag}$</td>
<td>Magnetic force along the $y$-axis</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{st}$</td>
<td>Force applied on the tip magnet by one source</td>
<td>[N]</td>
</tr>
</tbody>
</table>
(driving or tip magnet) along the y-axis

- \( F_{\text{tip}} \): Tip excitation force [N]
- \( F_{\text{tip}}^m \): Maximum tip excitation force [N]
- \( F_0 \): Amplitude of tip excitation force [N]
- \( G_r^g \): Gain factor between force and voltage [V/N]
- \( G_{\text{rc}} \): Gap between magnets and coils [m]
- \( I \): Moment of inertia [kg\( \cdot \)m²]
- \( J \): Magnet magnetization strength [T]
- \( J' \): Magnet magnetization strength [T]
- \( L \): Characteristic linear dimension [m]
- \( L_m \): Mean length of each turn [m]
- \( L_p \): Length of the piezoelectric beam [m]
- \( L_r \): Equivalent inductance of beam mass [H]
- \( L_w \): Length of the wire [m]
- \( N_e \): Number of exciters [-]
- \( N_p \): Number of pole pairs [-]
- \( N_{\text{pe}} \): Number of piezoelectric beams [-]
- \( N_t \): Number of turns for each spiral [-]
- \( P_{\text{avg}} \): Average input power to the piezoelectric beam [W]
- \( P_{\text{out avg}} \): Average output power of the harvester [W]
- \( P_{\text{max}} \): Theoretical maximum output power [W]
- \( \Delta P \): Potential depth [J]
- \( R_c \): Resistance of coils [Ω]
- \( R_l \): Resistive load [Ω]
- \( R_l^m \): Optimal load resistance [Ω]
- \( R_r \): Equivalent resistor of mechanical damping [Ω]
- \( TR_r \): Turns ratio of the \( r \)th transformer [-]
- \( U_e \): Beam elastic potential energy [J]
- \( U_{ij} \): Distance between corners in two magnets [m]
- \( U_m \): Beam magnetic potential energy [J]
- \( U_t \): Total beam potential energy [J]

along the x-axis

- \( V(t) \): Voltage on the resistive load [V]
- \( V_{kl} \): Distance between corners in two magnets [m]

along the y-axis

- \( V_r \): Equivalent input voltage [V]
\( V_{rms} \) \hspace{1cm} \text{Rms output voltage of electromagnetic generators [V]}

\( W_{pq} \) \hspace{1cm} \text{Distance between corners in two magnets along the z-axis [m]}

\( V \) \hspace{1cm} \text{Voltage [V]}

\( V_{pzt} \) \hspace{1cm} \text{Voltage on the piezoelectric beam [V]}

\( V_{ref} \) \hspace{1cm} \text{Voltage reference from LTC1540 [V]}

\( YI \) \hspace{1cm} \text{Bending Stiffness of the beam [N·m²]}

\( Y \) \hspace{1cm} \text{Young’s modulus of substrate [Pa]}

**Latin Lower Case Letters**

\( a \) \hspace{1cm} \text{Length of the tip magnet [m]}

\( b \) \hspace{1cm} \text{Width of the tip magnet [m]}

\( b_p \) \hspace{1cm} \text{Width of the piezoelectric beam [m]}

\( c \) \hspace{1cm} \text{Height of the tip magnet [m]}

\( c_a \) \hspace{1cm} \text{Viscous air damping coefficient [N·s/m²]}

\( c_s \) \hspace{1cm} \text{Strain rate damping coefficient [N·s/m²]}

\( d_{31} \) \hspace{1cm} \text{Piezoelectric constant [m/V]}

\( d_x \) \hspace{1cm} \text{Gap between two magnets along the x-axis [m]}

\( d_{x0} \) \hspace{1cm} \text{Initial gap between two magnets along the x-axis [m]}

\( d_y \) \hspace{1cm} \text{Gap between two magnets along the y-axis [m]}

\( d_z \) \hspace{1cm} \text{Gap between two magnets along the z-axis [m]}

\( d_{z0} \) \hspace{1cm} \text{Initial gap between two magnets along the z-axis [m]}

\( \Delta d_{tf} \) \hspace{1cm} \text{Initial offset between tip and fixed magnets in the beam bending direction [m]}

\( \bar{e}_{31} \) \hspace{1cm} \text{Piezoelectric stress constant [C/m²]}

\( f_e \) \hspace{1cm} \text{Input excitation frequency [Hz]}

\( f_i \) \hspace{1cm} \text{Frequency of the } i^{th} \text{ output voltage peak [Hz]}

\( f_{re} \) \hspace{1cm} \text{Resonant frequency of the beam [Hz]}

\( h \) \hspace{1cm} \text{Thickness of the beam [m]}

\( h_p \) \hspace{1cm} \text{Thickness of the piezoelectric layer [m]}

\( h_s \) \hspace{1cm} \text{Thickness of the substrate layer [m]}

\( h_{td} \) \hspace{1cm} \text{Gap between tip and driving magnets in beam width direction [m]}

\( h_{tf} \) \hspace{1cm} \text{Gap between fixed and tip magnets in beam [m]}
width direction

\( i_r(t) \) \hspace{1em} \text{Current in the circuit} \hspace{1em} [A]

\( k_c \) \hspace{1em} \text{Filling factor} \hspace{1em} [-]

\( k_{Cu} \) \hspace{1em} \text{Filling factor of copper} \hspace{1em} [-]

\( k_s \) \hspace{1em} \text{Spring constant} \hspace{1em} [N/m]

\( m \) \hspace{1em} \text{Mass per length of the piezoelectric beam} \hspace{1em} [kg/m]

\( m_b \) \hspace{1em} \text{Mass of the beam} \hspace{1em} [kg]

\( m_{pm} \) \hspace{1em} \text{Mass of the proof mass} \hspace{1em} [kg]

\( m_s \) \hspace{1em} \text{Mass of the spring} \hspace{1em} [kg]

\( m_{sm} \) \hspace{1em} \text{Mass of the sliding (driving) magnet} \hspace{1em} [kg]

\( q_r(t) \) \hspace{1em} \text{Charge in the circuit} \hspace{1em} [C]

\( r_i \) \hspace{1em} \text{Inner radius of coils} \hspace{1em} [m]

\( r_m \) \hspace{1em} \text{Rotational radius of the driving magnet} \hspace{1em} [m]

\( r_{m0} \) \hspace{1em} \text{Initial rotational radius of the driving magnet} \hspace{1em} [m]

\( r_o \) \hspace{1em} \text{Outer radius of coils} \hspace{1em} [m]

\( t \) \hspace{1em} \text{Thickness of coils} \hspace{1em} [m]

\( t_h \) \hspace{1em} \text{Thickness of the harvester} \hspace{1em} [m]

\( \dot{u} \) \hspace{1em} \text{Beam vibrating velocity} \hspace{1em} [m/s]

Acronyms

AC \hspace{1em} \text{Alternating Current}

ADC \hspace{1em} \text{Analog-to-Digital Converter}

BSN \hspace{1em} \text{Body Sensor Network}

DC \hspace{1em} \text{Direct Current}

EMREH \hspace{1em} \text{Electromagnetic Rotational Energy Harvester}

FEM \hspace{1em} \text{Finite Element Method}

IC \hspace{1em} \text{Integrated Circuit}

IDC \hspace{1em} \text{International Data Corporation}

IoT \hspace{1em} \text{Internet of Things}

MEMS \hspace{1em} \text{Microelectromechanical System}

NFC \hspace{1em} \text{Near-Field Communication}

PM \hspace{1em} \text{Permanent Magnet}

PM \hspace{1em} \text{Power Management Circuit}

PNRREH \hspace{1em} \text{Piezoelectric Non-Resonant Rotational Energy Harvester}
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRREH</td>
<td>Piezoelectric Resonant Rotational Energy Harvester</td>
</tr>
<tr>
<td>PZT</td>
<td>Lead Zirconate Titanate</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>SHM</td>
<td>Structure Health Monitoring</td>
</tr>
<tr>
<td>SSHI</td>
<td>Synchronized Switch Harvesting on Inductor</td>
</tr>
<tr>
<td>TPMS</td>
<td>Tire Pressure Monitoring System</td>
</tr>
<tr>
<td>WSN</td>
<td>Wireless Sensor Network</td>
</tr>
</tbody>
</table>
1 Introduction

This chapter aims to make a brief overview on the scope covered in this thesis. Research objectives and the structure of the thesis are also outlined. The detailed literature review will be presented in Chapter 2.

1.1 Energy Harvesting and the Internet of Things

In recent years, much attention has been drawn on the Internet of Things (IoT). IoT is a technology that enables objects embedded with sensors, actuators and network connectivity to collect and share data with others, and eventually to make intelligent decisions [1]. In June 2016, market research firm IDC predicted that IoT spending will reach $1.7 trillion in 2020, while Gartner forecasted that a hefty 21 billion IoT devices will be connected by that year [2], showing the profitable opportunities and significance of this technology to our life in the future.

In the IoT, wireless sensor nodes are generally included among the monitored objects. These nodes collect critical information that represents the status of those monitored objects. To collect the information, different types of sensor with varying functionalities are generally installed, such as those for temperature, humidity, strain distribution, and flow rate [3]. This massive information, also known as big data, enables the concept of pervasive and smart sensing, control and prediction of things. Examples of this technology include structural health monitoring (SHM) [4], body sensor networks (BSN) [5], and smart cities [6].

For the IoT, a key concern is the durability and independence of these sensing nodes when they are widely distributed or installed in some inaccessible locations. However, there is one significant bottleneck for the wide application of this technology: power supply. Normally, these nodes are powered by batteries which have limited lifetime and require regular replacement or recharging [7]. It is not a problem when the number in use is
modest, but when millions or billions of sensor nodes are installed, it will be a huge burden for battery replacement and disposal.

Thanks to the development of micro-electromechanical systems (MEMS), wireless communication and semiconductor technologies, the power consumption of wireless sensors has decreased to a large extent \[8\]. It provides an opportunity to power these sensor nodes using ambient passive energy sources, such as light, airflow, vibration or temperature gradients. Energy harvesting, a technique which harnesses waste energy from environments to power electronic devices, has drawn great research interest for a decade.

1.2 Rotational Energy Harvesting for Autonomous Sensing

Among all the energy sources available in the environment where wireless sensors are often installed, rotational energy is a widespread or easily acquirable from other energy sources, including fluid flow, machine motion, and ocean wave motions. This has been proven by large-scale power generation from wind turbines. Wind energy production is growing rapidly every year and has reached around 4% of worldwide electricity usage, 11.4% in the EU \[9\]. This also demonstrates the good establishment of this field.

However, for small-scale energy harvesting from rotation, the progress is still under-developed compared with large-scale wind power generation or small-scale energy harvesting from other sources, including vibration, solar and thermal energy \[10\]. Issues, including cut-in (start-up) speed requirement, device miniaturization and broadband operation, still remain \[11\], but the success of this technology will be beneficial to many autonomous sensing applications.

One specific application of this technology is in SHM. This is the process of monitoring and predicting the status of aerospace, civil, and mechanical infrastructure \[4\]. The aim of SHM is to enhance the safety and security, or to reduce the interruption and costs for maintenance, by autonomously monitoring the status of structures. In these applications, normally a huge number of sensor nodes are distributed in the structures, such as high-rise buildings, national bridges, and highways \[12\]–\[14\]. Rotational energy is often readily available or easily acquirable in these scenarios. Using rotational
energy in the ambient environment to power these electronics will enable the system to operate autonomously without considering the power-shortage issue.

Another potential application is in condition monitoring of moving objects, including vehicles, wildlife and even the human body. A tire pressure monitoring system (TPMS) is a good example in which rotational energy harvesting is beneficial. TPMS units are installed on car rims to regularly monitor tire pressure and send the collected data to the central control panel via wireless transmission [15]. This unit is a key component to maintain driving safety, but it also faces the issues of power supply. Using vibration or airflow induced by vehicle motion to power the TPMS units is a potential solution to avoid batteries. The same principle can be applied to condition monitoring of other critical components in moving objects, such as the condition of bearings in trains or the biomedical parameters of patients [5,16].

Considering the under-developed status of rotational energy harvesting and the broad potential applications, this research aims to develop effective rotational energy harvesting solutions for autonomous low-power sensing applications. Technologies to harness low-frequency and broadband rotational energy and to improve the conversion capability will be investigated and examined. Potential applications for the proposed harvesters will be explored.

1.3 Research Objectives

The aim of this research is to achieve effective broadband energy harvesting from low-frequency and random rotation for autonomous sensing devices in the IoT applications. For these applications, the requirements for rotational energy harvester design are significantly different from those for conventional dynamo design. Firstly, due to the dimension requirements of general IoT applications, rotational energy harvesters should be on the centimetre or millimetre scale. Miniaturization of traditional electromagnetic generators or using other conversion mechanisms needs to be considered to ensure the harvesters to be comparable in size to the batteries typically used. Different conversion mechanisms which are inefficient for large-scale power generation become available for small-scale rotational energy harvesting. Comparison of different mechanisms is necessary to understand their advantages and
The core objectives are summarised as follows:

- Review and summarize the state-of-the-art of rotational energy harvesting and identify the bottlenecks and drawbacks.
- Investigate and implement suitable strategies to improve the power generation capability and operating bandwidth.
- Develop an effective energy harvesting system for self-powered sensing applications.
- Evaluate the performance of different types of rotational energy harvesters in terms of device dimension and rotational frequency.

1.4 Thesis Structure

This thesis presents the work on piezoelectric energy harvesting from low-frequency random rotation for low-power electronics. The thesis is divided into 9 chapters. The outline for each chapter is as follows:

- Chapter 1 briefly introduces the background of this work and discusses the main objects and thesis contents.
- Chapter 2 provides a comprehensive literature review on energy harvesting technology. Energy harvesters collecting different energy sources and using different conversion mechanisms are investigated and compared. Special attention is paid to rotational energy harvesting. Challenges and issues are discussed. Strategies to address these issues are reviewed.
- Chapter 3 presents a rotational energy harvester from airflow using a miniaturized turbine and piezoelectric conversion. The design considerations and operating principles are discussed. The design was implemented and examined experimentally. Advantages of this harvester are presented, and potential improvements are identified.
- Chapter 4 introduces a self-regulation mechanism to reduce the cut-in (start-up) speed requirement. This mechanism can passively adjust
the magnetic plucking force on the transducer according to the rotational speed of the turbine's rotor. This design enables the harvester to start up at a much lower speed and to exhibit a wider operating speed range.

- Chapter 5 focuses on the theoretical study of piezoelectric beams under tip plucking. A distributed-parameter theoretical model was established to investigate the electromechanical behaviours for different harvester configurations. The presented analysis provides a general guideline for rotational energy harvesting using piezoelectric transduction and magnetic plucking.

- Chapter 6 includes the employment of bi-stability in the rotational energy harvester to enhance its performance at low frequency. The performance with bi-stability was examined theoretically and experimentally. This design improves the performance of the harvester over a wide operating frequency range.

- Chapter 7 presents a complete self-powered condition monitoring system which includes a bistable frequency up-converting harvester, a power management circuit and a wireless sensor node. Design consideration and operating principle were discussed. This work provides a practical solution for self-powered sensing.

- Chapter 8 investigates the performance of different types of rotational energy harvesters for different device dimensions and rotational frequencies. Theoretical models were built for different types. The scaling laws for each type were discovered.

- Chapter 9 summarises the achievements in this thesis and explores the future work.
2 Literature Review

This chapter provides a detailed review of the state-of-the-art in the field of energy harvesting. Considering that the aim of energy harvesting is to provide a self-reliant solution in terms of power supply for general IoT applications, wireless sensor nodes and their power requirements are investigated first, showing the target of power output required by energy harvesters. Then the available energy sources in the environment where sensor nodes are often installed are summarized, and their power densities for different scenarios are analysed.

After that, the fundamental principles and structures of energy harvesting technology are introduced, before the scope is narrowed down to rotational energy harvesting. Different harvester designs and conversion mechanisms are discussed. The advantages and drawbacks of these harvesters are analysed. Finally, strategies to overcome the current challenges and issues about rotational energy harvesting are discussed for further improvement.

2.1 Wireless Sensors and Power Requirements

The Internet of Things is a well-known technology as part of the fourth industrial revolution. Wireless sensor networks (WSN), as an enabler of the IoT applications, have drawn great attention. Many applications, such as structure health monitoring, smart agriculture and body sensor networks, become realizable due to the development of WSN technology [17]. As one of the critical components in these networks, wireless sensor nodes shoulder the duties of sensing, data acquisition, localized processing and wireless transmission [18].

Generally, these nodes are powered by batteries which have limited lifetime and are often bulky. Moreover, the replacement of batteries is costly and inconvenient, especially when sensors are installed in hazardous or inaccessible locations. Therefore, power supply is a key bottleneck for the
development of the IoT technology. Exploiting ambient passive energy to power these nodes is a promising solution. Energy harvesting, as a potential alternative to batteries, has drawn great attention in the last decade. Before investigating strategies for energy harvesting, it is necessary to clarify the power requirements for general wireless sensors.

The formation of a typical wireless sensor node is shown in Fig. 2.1. The whole system is divided into 4 subsystems according to their functionalities. The power requirement is estimated in this work using components or devices commercially available or provided in the literature. The total power requirement is estimated by summarizing the power consumption of each subsystem.

![Figure 2.1: Block diagram of a typical wireless sensor, adapted from [7].](image)

### 2.1.1 Sensing Subsystem

The elements contained in the sensing subsystem are generally sensors and an analog-to-digital conversion (ADC) module. It aims to collect physical information from the detected objects and convert the information into digital data. The consumed energy depends on the quality and type of sensors and also the required sampling rate, as when no data acquisition is needed, sensors are basically in an off-state, and no power is consumed. The typical types of sensors used in condition monitoring are thermal sensors, accelerometers, hygrometers and certain bio-sensors for heart rate or respiratory rate, etc. Examples collected from off-the-shelf products and literatures are shown in Table 2.1. The power requirements for active sensing are on the micro-watt level. When the duty cycle is considered, the average
power requirement can be lower.

<table>
<thead>
<tr>
<th>Sensors</th>
<th>Product Code</th>
<th>Manufacturer</th>
<th>Voltage (V)</th>
<th>Current (µA)</th>
<th>Bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>TMP103</td>
<td>TI</td>
<td>2</td>
<td>1.5</td>
<td>8</td>
</tr>
<tr>
<td>Humidity</td>
<td>HDC1000</td>
<td>TI</td>
<td>4</td>
<td>0.82/1.2</td>
<td>11</td>
</tr>
<tr>
<td>Vibration</td>
<td>BMA280</td>
<td>Bosch</td>
<td>2</td>
<td>6.5/130</td>
<td>14</td>
</tr>
<tr>
<td>Barometric Pressure</td>
<td>MPL3115A2</td>
<td>Freescale</td>
<td>2</td>
<td>40</td>
<td>20</td>
</tr>
</tbody>
</table>

* Two numbers represent the power requirements for different operating modes.

### 2.1.2 Controlling Subsystem

The controlling subsystem usually consists of a signal processor and a memory unit. The processor controls the activities of all sensors and implements some local signal processing. In addition, it also controls data communication by sending commands to the radio transceiver. The processor works in a low-power sleep (idle) mode when no processing and controlling is required. The power requirements of several commercial micro-controllers are summarized in Table 2.2. The power consumption can be estimated when the duty cycle and operating frequency are determined. For example, if the controller EFM32G200 operates at 7 MHz with a duty cycle of 5%, the average power consumption is 37.8 µW. This number can be even minimized by choosing a lower operating frequency or a lower duty cycle.

<table>
<thead>
<tr>
<th>Microcontrollers</th>
<th>Manufacturer</th>
<th>Minimum Voltage (V)</th>
<th>Active current (µA/MHz)</th>
<th>Sleep current (µA)</th>
<th>Running frequency (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFM8SB1</td>
<td>Silicon Labs</td>
<td>1.8</td>
<td>150</td>
<td>0.05</td>
<td>24.5</td>
</tr>
<tr>
<td>EFM32G200</td>
<td>Silicon Labs</td>
<td>2</td>
<td>54</td>
<td>0.02</td>
<td>7</td>
</tr>
<tr>
<td>PIC12LF1571</td>
<td>Microchip</td>
<td>1.8</td>
<td>30</td>
<td>0.02</td>
<td>32</td>
</tr>
<tr>
<td>MSP432P401</td>
<td>TI</td>
<td>1.62</td>
<td>90</td>
<td>0.025</td>
<td>48&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>MC9S08MM128</td>
<td>Freescale</td>
<td>3</td>
<td>1000</td>
<td>0.5</td>
<td>24</td>
</tr>
</tbody>
</table>

<sup>a</sup> The running frequency is up to 48 MHz for this MCU.
2.1.3 Transmitting Subsystem

Wireless transmission is essential to avoid wiring problems for sensor nodes. This subsystem is used to transmit sensor data and to communicate with the whole sensor network. Several wireless communication technologies are widely utilized nowadays, such as ZigBee, Bluetooth and NFC. Commercial communicating modules with low-power consumption are summarized in Table 2.3. Compared with other subsystems, transceivers are the most power consuming module. The active power consumption of transceivers is on the 1-100 milliwatt level. Luckily, most of the time, this module stays in the sleep mode, and the average power consumption can be reduced to a reasonable value.

Table 2.3: Commercial radio transceivers and their operation parameters

<table>
<thead>
<tr>
<th>RF Transmitter</th>
<th>Manufacturer</th>
<th>Voltage supply (V)</th>
<th>Receiver active current (mA)</th>
<th>Transmitter active current (mA)</th>
<th>Sleep current (µA)</th>
<th>Frequency Band</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC1000 28</td>
<td>TI</td>
<td>2.1-3.6</td>
<td>7.4</td>
<td>10.4</td>
<td>0.2</td>
<td>Sub-1 GHz</td>
</tr>
<tr>
<td>CC2500 29</td>
<td>TI</td>
<td>1.8-3.6</td>
<td>13.3</td>
<td>21.2</td>
<td>0.4</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>DA14580 30</td>
<td>Dialog</td>
<td>0.9-3.45</td>
<td>3.7</td>
<td>3.4</td>
<td>0.6</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>MC13202 31</td>
<td>Frescale</td>
<td>2-3.4</td>
<td>37</td>
<td>30</td>
<td>0.2</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>Si4430 32</td>
<td>Silicon Labs</td>
<td>1.8-3.6</td>
<td>18.5</td>
<td>30</td>
<td>0.45</td>
<td>Sub-1 GHz</td>
</tr>
<tr>
<td>MRF89XA 33</td>
<td>Microchip</td>
<td>2.1-3.6</td>
<td>3</td>
<td>25</td>
<td>0.1</td>
<td>Sub-1 GHz</td>
</tr>
<tr>
<td>ADF7030-1 34</td>
<td>Analog</td>
<td>2.2-3.6</td>
<td>21.2</td>
<td>50</td>
<td>0.01</td>
<td>Sub-1 GHz</td>
</tr>
</tbody>
</table>

From the above discussions, it is clear that the controlling subsystem and the communicating subsystem consume more power in the system, but the required power can be reduced by setting a reasonable duty cycle for the whole system, which means these units need to be designed to operate in a low-power sleep mode for most of the time. If some of the components listed above are chosen, the average power consumption for a wireless sensor node with a relatively low duty cycle can be on the scale of 100’s of micro-watts. In fact, commercial wireless sensor nodes with low power consumption are available in the market. Nechibvute et al. 17 summarized the low-power sensor nodes, and a prediction of 100 µW power requirement was stated, which further testifies to the estimation above. Therefore, energy harvesters whose power generation capability is on the micro-watt level can be applied to power these wireless sensor nodes, allowing a sensor network to be fully autonomous.
2.2 Energy Sources and Harvesting Systems

2.2.1 Energy Sources

There are plenty of passive and wasted energy sources in the environment where wireless sensors are often installed, including thermal gradients, vibrations, fluid flow, and solar, etc. Hence, it is of great importance to analyse their power density, limitations and potential applications in order to appropriately collect them with suitable conversion mechanisms and architectures. Comprehensive surveys and reviews for energy harvesting from different sources have been carried out by Roundy et al. [35], Mitcheson et al. [36], Daqaq et al. [37] and Lu et al. [38].

Table 2.4 summaries the general energy sources with details about their power density and characteristics. The ambient energy sources are prevalent, and their power density seems promising, but there are quite many limitations for each source, which impede the realization of battery-less wireless sensor nodes [39].

<table>
<thead>
<tr>
<th>Energy source</th>
<th>Character</th>
<th>Power density</th>
<th>Comments &amp; Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radio frequency</td>
<td>Widely spread</td>
<td>&lt;1 µW/cm²</td>
<td>Power density is relatively low, although it is widespread in environments.</td>
</tr>
<tr>
<td>Light</td>
<td>Outdoor</td>
<td>0.1 W/cm²</td>
<td>Solar energy harvesting is well developed for large scale, but not for small-scale.</td>
</tr>
<tr>
<td>Light</td>
<td>Indoor</td>
<td>100 µW/cm²</td>
<td></td>
</tr>
<tr>
<td>Thermal</td>
<td>Human</td>
<td>60 µW/cm²</td>
<td>Temperature difference is required to generate electricity.</td>
</tr>
<tr>
<td>Thermal</td>
<td>Industrial</td>
<td>10 mW/cm²</td>
<td></td>
</tr>
<tr>
<td>Airflow</td>
<td>Widespread and random</td>
<td>1 mW/cm²</td>
<td>Achieved in a micro turbine at 0.5 litres/s. Airflow is generally random and low-speed.</td>
</tr>
<tr>
<td>Vibration</td>
<td>Hz-human, kHz-machines</td>
<td>4 µW/cm², 800 µW/cm²</td>
<td>Easy to harvest, but the density highly depends on extraction.</td>
</tr>
</tbody>
</table>

Among these sources, vibration energy has drawn great attention and has been extensively studied for a decade. The progress and research topics on vibration energy harvesting are summarized in several comprehensive review papers, such as vibration energy harvesting from human motion and machine operation in [36, 41], nonlinear mechanisms for broadband energy harvesting in [42, 43] and functional materials for energy conversion in [44].

However, as another type of kinetic energy, rotational energy harvesting is less investigated for power supply, but has unique dynamics that would be beneficial to some applications, including tire pressure sensing [45, 46].
condition monitoring of rotating machines \[47, 48\] and fluid flow energy harvesting using miniature turbines \[49\]. Rotational energy generation is a well-developed technology for large-scale power generation, including wind turbines or hydroelectric power plants, but for small-scale rotational energy harvesting, the available power is limited. Moreover, the rotational energy or rotation converted from other sources often exhibits the characteristics of low and random frequencies, which makes energy harvesting from this source challenging.

2.2.2 Energy Harvesting Systems

Before introducing the state-of-the-art of rotational energy harvesting, the configuration of general energy harvesting systems is discussed first. Energy harvesting aims at converting ambient energy into usable electricity to power wireless sensors. Therefore, different functionalities, including energy conversion and power management, are needed.

For a typical energy harvesting system, it can be separated into several units according to its functions, as shown in Fig. 2.2. Initially, energy is captured from ambient sources by energy harvesters via certain mechanisms, such as a direct force from foot strike \[50\], or inertial movement of a proof mass from vibrations \[51\], or rotational motion of miniature air turbines powered by airflow \[52\]. Then the energy captured by the harvester can be converted into electricity via several conversion mechanisms including electrostatic, piezoelectric, triboelectric and electromagnetic.

![Figure 2.2: Composition of a typical energy harvesting system. Other harvesting systems with simpler or more complex configurations also exit.](image)

However, the generated electricity at this stage is generally in an alternating form which is unable to be used for powering electronics. Current
rectification is required to covert the voltage into a DC form. A full-bridge rectifier in parallel with a subsequent storage capacitor is the general rectification circuit, but it has a limited power conversion capability due to the clamped voltage of the storage capacitor. Synchronized switch harvesting on inductor (SSHI) [53] and pre-charging circuits [54] have been developed for piezoelectric devices to enhance the performance of the rectification circuits.

Still, the voltage on the storage capacitor is varying and unsuitable for subsequent loads. Voltage regulation circuits, such as DC-DC converters, are necessary to regulate the voltage in a stable range satisfying the requirement of subsequent loads. Examples of self-powered energy harvesting systems have been reported in the literature, including a self-powered wireless alarm microsystem powered by a vibration energy harvester [55], a human fitness tracker powered by a footstep energy harvester [56], and a self-powered wireless sensor node powered by an aerosol-deposited PZT flexible energy harvester [57].

2.3 Rotational Energy Harvesting: Classification

As a class of kinetic energy, rotational motion is readily available in our daily life, ranging from domestic apparatus to industrial machines. Electromagnetic transduction driven by turbines [58] is a common example of generating electricity from rotation, and this paradigm is ubiquitous in macro-scale generation systems to convert energy sources such as fluid flow, fossil fuels and nuclear power into electricity.

However, for small-scale (e.g. <1 W) power generation, electromagnetic transduction is not as dominant as than in macro-scale power generation due to the dimension and operating frequency scaling effects. A range of mechanisms, including harvesters based on electrostatic, piezoelectric and triboelectric conversion, become comparable to electromagnetic harvesters. In this section, the state-of-the-art of rotational energy harvesting is discussed.
2.3.1 Inertial Harvesters for Varying Motion

The great majority of large scale electrical generation uses a continuously rotating electromagnetic machine for transduction, in which power is extracted from the relative motion between a stationary part, anchored in place, and a rotating part coupled to a source of motion such as a turbine. In miniature energy harvesting, the source of motion (the “host”, e.g. the human body or vibrating machinery) is typically much larger than the intended generator, and so the need to attach the harvester to both the moving body and a stationary base, or to two locations moving relative to each other (e.g. upper and lower leg [59]), places severe constraints on both the degree of miniaturisation and on usable locations.

Instead, if the motion is reciprocating, an inertial mass can be used in place of the fixed anchor, and this has been the basis for a very wide range of harvesters. Although most of these have proof masses that move rectilinearly [36], rotating proof masses are also possible. Pillatsch et al. developed a rotational energy harvester for wearable devices, as shown in Fig. 2.3 [60]. Rotational motion was induced by human linear motion using an eccentric semi-circular mass pivoting around a central axis. A piezoelectric beam was clamped on the harvester’s frame on the human body and plucked by the rotating mass in a non-continuous fashion. This kind of harvester has the advantage of having no fixed end stops, and can be driven by both rotating and rectilinear motion.

![Figure 2.3: Piezoelectric non-resonant rotational energy harvesters for wearable devices from Pillatsch et al. [60]](image)

While rotating inertial harvesters can be driven by rotating host motion,
this motion must be reciprocating, or at least varying in some way, such as in speed or orientation. Manla et al. developed a non-contact piezoelectric rotational energy harvester using the centrifugal force as the impacting force, as shown in Fig. 2.4 [61]. The harvester used magnetic forces to levitate a center magnet for the purpose of applying axial load on the piezoelectric devices (Fig. 2.4(a)). The center magnet was driven by the centrifugal force on this magnet by rotation. In order to use the centrifugal force generate relative motion of the middle magnet, the device was attached to the wheel using a host that kept it in a horizontal position during rotation, as shown in Fig. 2.4(b). In this case the centrifugal force on both the magnets and the piezoelectric elements was sinusoidal. The requirement of constant horizontal position added complexity and limitation to this device.

2.3.2 Continuous Rotating Harvesters

Continuous rotation with constant speed and the generated centrifugal force cannot be used to drive a truly inertial harvester (i.e. one in which the transducer acts on relative motion between the moving host and an inertial mass). However, centrifugal force can be used for other purposes, such as frequency tuning, or position adjusting, rather than direct energy harvesting. Gu and Livermore demonstrated a passive self-tuning rotational energy harvester for variable speed situations, as shown in Fig. 2.5 [62]. The harvester rotated in the vertical plane and was comprised of two beams: a relatively...
rigid piezoelectric beam for power generation and a flexible driving beam with a tip mass mounted at the free end. The mass impacted the generating beam repeatedly due to the beam deformation caused by the gravity. Centrifugal force on the tip mass from the rotation adjusted the resonant frequency of the flexible driving beam. The natural frequency tracked the rotation frequency over a range.

Figure 2.5: Schematic of the rotational energy harvester using centrifugal force for frequency tuning and gravity for beam impact from [62].

In certain situations, gravity can provide the counter-force for energy harvesting instead of inertial force, since its direction will vary with respect to the centrifugal force on an eccentric mass. Thus the same structure can be used as that of a rotating inertial harvester, although with the restriction that the plane of motion must have a vertical component. Roundy and Tola developed a rotational energy harvester for tire pressure monitoring applications using the gravitational field [63]. An actuator ball was installed inside a curved track on the vehicle rim. The relative motion between the ball and the rim was created by the gravitational field. The ball rolls back and forth along the track with regard to the influence of gravity, impacting two piezoelectric beams fixed alongside the track once per cycle. Similarly, Toh et al. presented a continuous rotational electromagnetic energy harvester using gravitational torque [64]. An off-centred mass was attached on a conventional DC generator to create gravitational torque. This torque generated a relative motion between the generator and its rotating host, and then electric energy was collected from the relative motion.
2.3.3 Micro Fluid-Flow Turbines

A final implementation of a miniature rotating harvester is one that uses the motion of fluid, typically air, past a body to which the harvester can be attached. An example is the micro-wind turbine. Perez \textit{et al.} presented a centimetre-scale electrostatic wind turbine for airflow energy harvesting, as shown in Fig. 2.6 [65]. The flow motion was converted to the rotation of turbine blades, and electrets were mounted on the edges of blades as rotating components. The stationary electrodes were fabricated on the turbine housing. The relative motion in this case was induced by airflow, and electrical power was collected via the variation of the capacitance between the rotating electrets and the static electrodes. Fu \textit{et al.} developed a footstep energy harvester using heel-strike induced rotation [66]. An air bladder was embedded into a shoe cushion to create airflow, and the rotational motion was generated by a miniature turbine with a DC generator attached.

![Electret-based electrostatic airflow energy harvester using a miniaturized turbine](image)

Figure 2.6: Electret-based electrostatic airflow energy harvester using a miniaturized turbine [65].

Based on the above discussion, we can identify three classes of rotating miniature motion energy harvesters: inertial devices driven by varying (usually reciprocating) motion; continuously rotating devices using gravity as the counter force; and fluid flow turbines. The choice among these types of harvesters depends mainly on practical constraints in particular applications.

\footnote{The author was involved in this work as the first contributor in a project between our group and Prof Kim’s group at MIT. This work is not included in the following chapters in this thesis.}

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2.4 Rotational Energy Harvesters: Transduction

In terms of energy transduction for rotational energy harvesting, the dominant mechanisms are piezoelectric, electromagnetic and electrostatic. Their pros and cons are extensively investigated in vibration energy harvesting. Electrostatic conversion requires initial charges on the electrodes. Electret-based harvesters provide a solution, in which the material is only charged once during fabrication with a high voltage, but the capability is still limited by the magnitude of capacitance that can be achieved and by the charge density of electrets. Electromagnetic transduction uses the relative motion between coils and magnets. This method is widely used in power generation, such as conventional DC generators, but for small scale devices, the performance is constrained by the number of coils, the magnetic flux gradient and the relative velocity between the components. The piezoelectric approach is a favourable conversion mechanism for its simplicity in structure and high output voltage. However, the brittle characteristic of this material is detrimental to its long-term stability. Additional considerations are required to protect it from degradation. In this section, rotational energy harvesters using different transduction mechanisms are reviewed.

2.4.1 Electromagnetic Harvesters

Electromagnetic transduction is a dominant conversion mechanism in rotational energy generation on large scale. However, its performance is largely limited on small scale due to the size scaling effect. In order to compensate the losses by dimension shrinking, electromagnetic rotational energy harvesters normally operate at high rotational frequencies. A typical example is the electrical generator in kinetic wristwatches, which was first proposed in a patent in 1976 by Berney from EPFL. In this patent, Berney introduced a motion generator using electromagnetic transduction for time regulation. The generated power was used to precisely regulate the watch movement. This idea was further developed in a later patent by Hayakawa from Seiko Epson Corporation in 1991. This design has been successfully implemented in Seiko kinetic wristwatches as shown in Fig. 2.7. The rotor generates rotation from human activities using an off-center rotor, and the rotational motion is then amplified (up to ∼100,000
rpm \( \text{rpm} [73] \) by gear trains before the motion is used to generate electricity by a micro-electromagnetic generator. The generated electricity is then utilized for powering an IC unit to compare the frequency difference between the watch movement and a quartz oscillator. Finally, a geomagnetic braking subsystem, which is also powered by the motion harvester, is used to precisely regulate watch movement.

Figure 2.7: Schematic of Seiko spring drive technology, showing the operating principle of human motion energy harvesting and watch movement regulation (from Seiko Watch Corporation website).

Kinetron is an innovative organization specialized in micro kinetic systems, especially in micro electromagnetic energy harvester design including micro water turbines, micro generators and motors [74]. These devices are designed for applications including watches, mobile phones, pedal illumination systems and other consuming products. In watch applications, the claw-pole generator MG4.0 is the most commonly used design. A 14-pole \( \text{Sm}_2\text{Co}_{17} \) magnet and 1140 windings at a resistance of 320 \( \Omega \) are integrated. The typical output power is 10 mW at a rotational speed of 15,000 rpm with the dimensions of \( \varnothing 4.0 \times 2.2 \text{ mm} \).

Donelan \textit{et al.} developed a biomechanical energy harvester generating electricity from walking with minimal user effort, as shown in Fig. 2.8 [75]. The harvester used the relative motion between upper and lower leg to generate rotation. After being amplified by a gear train, the rotation energy was converted into electricity by an electromagnetic generator. This novelty of this design is that unlike other human-motion-based harvesters which use positive muscle work, this device engaged power generation at the end of
the swing phase of walking, assisting deceleration of the joint. However, this
harvester is inconvenient for wearing due to its large dimensions. Similarly,
Xie and Cai developed an in-shoe electromagnetic harvester for motion energy harvesting [76]. The foot-strike motion was amplified by trapezoidal
sliders, and then the linear motion was converted into rotation by a gear
train. A micro-electromagnetic generator was adopted as the transducer.
1.2 W of output power was extracted from walking at 4 km/h.

Figure 2.8: Biomechanical energy harvester. (A) The device worn on each
leg. (B) The design of the harvester. (C) The schematic diagram
showing its operation principle (from [75]).

These devices are successful applications of electromagnetic rotational
energy harvesters for wearable devices. However, the high transmission-
ratio and friction-free gear trains are costly and complicated in fabrication,
which is impractical in many scenarios.

In the literature, the high rotational frequency is also achieved using other
methods in some particular applications. Researchers from MIT conducted
an extensive gas turbine program 10 years ago, in which they developed
millimeter-scale MEMS gas turbine generators aiming to produce 10-50 W
of power from a less than one cubic centimeter device [77,78]. The generator
was designed to operate at 1,000,000 rpm, and the rotation was provided
by a gas turbine with compressor and burner on a single chip. However,
no experimental results with a whole assembled device were reported in the
literature.

Other researchers focus more on the micro-turbogenerator subsystem design using external flow energy. Peirs et al. developed a axial micro-turbine with a rotor diameter of 10 mm \cite{79}. The harvester operated at 160,000 rpm with 16 W of electrical output power. Holmes et al. demonstrated an axial-flux permanent magnet electromagnetic generator with a diameter of 7.5 mm \cite{80}. The device was tested at 30,000 rpm with an output power of 1.1 mW. Arnold et al. presented a detailed design optimization and characterization process for a micro-PM generator \cite{81}. Different fabrication methods for generator components were discussed, and the presented device with a diameter of 10 mm produced 8 W of DC power at a rotational speed of 305,000 rpm. Raisigel et al. reported an 8 mm planar generator consisting of a permanent magnet rotor cut out of bulk SmCo5 or NdFeB and a silicon stator with electroplated planar coils \cite{82}. The harvester generated 5 W at 380,000 rpm and 14.6 mW at 58,000 rpm.

These harvesters seem to be promising in energy harvesting at small scale (mW to W). However, the extremely high rotational frequency is inaccessible without high transmission-ratio gear trains or high-speed or high-pressure fluid flows. These conditions are inaccessible in many practical applications.

Explorations have also been conducted by researchers to adopt the electromagnetic transduction in low rotational frequency situations. Howey et al. developed a centimeter-scale (\( \varnothing 32 \) mm) wind turbine for airflow energy harvesting in pipes and ducts, as shown in Fig. 2.9 \cite{83}. Different fabrication methods, including traditional machining for turbine casing, rapid prototyping for supporting structures and flexible printed circuit board technology for coils and the generator stator, were employed. The device can operate at air speeds down to 3 m/s and generated 4.3 mW at 10 m/s (corresponding to 4,000 rpm of the rotor rotating frequency). This design can be used for powering electronics in wireless sensor networks or even be used as a flow-meter with frequency as the output. Similarly, Zhao et al. developed two air-driven planar magnetic generators with an even lower cut-in speed (1-2 m/s) \cite{84}. Generator 1 with the volume of 0.38 cm\(^3\) harvested 521 mW at 32,000 rpm, whereas Generator 2 with the volume of 0.82 cm\(^3\) collected 39.5 mW at 6,200 rpm. However, the corresponding airflow speeds were not mentioned for these rotational frequencies. In comparison, Kishore et al.
developed an airflow turbine with relatively large dimensions (the chord length is 75 mm) for wireless and portable devices with the cut-in speed of 1.7 m/s \[85\]. The coils used in this device was wire-wound, and the device was capable of producing 9.8 W at wind speed of 10 m/s (corresponding to 1400 rpm). Although the power generated is huge, the dimensions of this device limit its application in many cases where device dimension is a big consideration.

Romero-Ramirez from Michigan Tech studied energy harvesting from human motion \[86\]. Micro-rotational electromagnetic generators were developed using MEMS technologies. Multiple pole pairs and increased coil turns were adopted to omit the use of gear trains. Several prototypes were designed and fabricated. For the sixth prototype, the generator had a volume of 2 cm\(^3\) (casing is not included.). 20 pole pairs were formed by placing NdFeB magnet pieces in a rotating disk. A 4-turn and 4-layer coil with internal resistance of 20 Ω was integrated. A maximum power of 234 µW (137 mV\(_{\text{rms}}\)) was measured when walking at 1.34 m/s with the device amounted on the ankle vertically.

More recently, Kim et al. developed a rotary electromagnetic generator for medical implants \[87\]. The harvester used blood flow and pressure to generate rotational motion, and the motion was converted into electricity by a generator composed of a tiny rotor with magnets and external coils with ferrite cores. The device collected a peak output power of 3.4 mW.
at a blood pressure of 54.75 mmHg and blood flow of 2.68 L/min (Cor-
responding to 350 rpm). In [88], Joseph et al. presented the design and
implementation of rotational energy harvesting for biomedical applications
of Lab-on-a-Disc scenarios. Two types of rotational energy harvesters using
piezoelectric and electromagnetic transduction were developed. The elec-
tromagnetic harvester produced an electrical power of 125 mW at 800 rpm.
However, the dimension of the devices is not clearly depicted.

In summary, electromagnetic conversion has also been widely studied for
rotational energy harvesting, but there are quite a few issues.

- Many designs operate at relatively high rotational speeds (e.g. > 10,000
  rpm) which are inaccessible for many practical applications.

- Gear trains are required in some designs in order to amplify the low-
  frequency rotation, but it would be costly for manufacturing precision
gears for energy harvesting devices.

- Due to the limited dimension of devices (i.e. limited coil turns) and
  magnetic flux density, low output voltage is common for electromagnetic
  harvesters. This low voltage causes many challenges for the
  subsequent power management.

2.4.2 Piezoelectric Resonant Harvesters

Piezoelectric conversion has been widely used in vibration energy harvesting
due to its simplicity and compatibility with micro-fabrication technology,
but for rotational energy harvesting, piezoelectric conversion is less devel-
oped. Initially, researchers used rotational motion to induce an excitation
force close to the sinusoidal form on the beam. These devices operate as
resonant energy harvesters which have high output power at or close to
resonance, but limited output when operating at non-resonant conditions.
Priya et al. demonstrated a piezoelectric windmill for airflow energy har-
esting [89]. In this design, airflow energy was first converted into rotation
by a windmill. An oscillating force was applied on the piezoelectric beams
using direct impact generated by the stoppers on the rotating host. A pro-
totype with 12 bimorphs generated 10.2 mW at the oscillating frequency of
6 Hz (the dimensions of each bimorph were 60 × 20 × 0.6 mm³.). Gu and
Livermore developed a passive self-tuning piezoelectric energy harvester for
rotational applications\textsuperscript{62,90}. As mentioned earlier in Fig. 2.5, the harvester was mounted on a rotational host and comprised of two beam: a relatively rigid piezoelectric beam and a flexible driving beam. The driving beam impacted the piezoelectric beam by deflection of the driving beam under gravitational force. The harvester exhibited a wide bandwidth at low frequency.

In these designs, the excitation is from direct impact on the piezoelectric transducers, which is detrimental to the reliability of the system. In order to avoid the direct impact, other excitation mechanisms have been investigated. Khameneifar \textit{et al.} used the gravitational force of the tip mass on a piezoelectric beam to induce beam vibration from rotation\textsuperscript{91}. The beam was mounted on a rotational host, whose rotational axis is perpendicular to the direction of gravity. A maximum power of 6.4 mW at 22 Hz was extracted by a $50.8 \times 38.1 \times 0.13$ mm$^3$ transducer. Similarly, Sadeghi \textit{et al.} presented a two-degree-of-freedom energy harvester for broadband rotational energy harvesting\textsuperscript{92}. The harvester is a bimorph piezoelectric beam mounted on a flexible substructure. The fundamental frequency (8.8 Hz) and the second natural frequency (10.5 Hz) are close to each other, creating a wide operating bandwidth for this harvester.

Apart from gravitational force, another excitation mechanism that is widely used in piezoelectric rotational energy harvesters is magnetic plucking. One successful implementation is from Karami \textit{et al.} as shown in Fig. 2.10\textsuperscript{93}. In this work, a piezoelectric rotary transducer was integrated into a miniature wind turbine. Magnetic plucking was achieved by the magnetic force between magnets on piezoelectric beams’ free ends and turbine blades. The magnetization direction for the magnets on the turbine blades was alternating, creating a sinusoidal excitation force on the beams. 5 mW was extracted by the harvester at 200 rpm with the device dimension of $80 \times 80 \times 175$ mm$^3$. Similar designs with different considerations, including turbine types\textsuperscript{94}, magnet arrangements\textsuperscript{95}, optimal operating conditions\textsuperscript{96} and cut-in speed\textsuperscript{97}, were carried out by researchers from different institutes.

Due to the resonant nature of these harvesters, they can only operate efficiently when the excitation frequency is the same or close to the transducers’ resonant frequency. Therefore, these harvesters are normally limited to situations where the input excitation frequencies are constant or vary in
2.4.3 Piezoelectric Non-Resonant Harvesters

In many applications, including rotating machinery and vehicle wheels, the rotational frequency is not constant. Therefore, harvesters with wide bandwidth are necessary. For electromagnetic harvesters, the output power is proportional to the rotational frequency squared \[80\]. Bandwidth is not a problem for these harvesters. However, for vibration-based piezoelectric rotational energy harvesters, the requirement to operate at the transducers’ resonant frequency is a hindrance for broadband operation. Different designs have been proposed to overcome the bandwidth issue.

Frequency tuning is an attractive method to enhance the bandwidth. This mechanism has been widely investigated in vibration energy harvesting \[98, 99\]. The resonant frequency of harvesters can be tuned to the operating frequency using active \[100\], passive \[101\] or semi-passive mechanisms \[102\]. In rotational energy harvesting, frequency tuning was also achieved by using centrifugal force to passively adjust the stiffness of rotating beams with a proof mass \[90, 103\]. However, this mechanism requires the piezoelectric beam to be rotatable, which creates difficulties for power transfer from rotating beams to static electronics. In addition, in order to realize frequency matching between beam resonant frequency and device operat-
ing frequency over a wide range, precise structural design is indispensable, which would increase the expense.

Compared to frequency tuning, frequency up-conversion is a relatively undemanding solution to achieve broadband operation. In this mechanism, piezoelectric beams are normally fixed on a static host, and plucked by mechanical impacts [104, 105] or magnetic force [106] from rotating parts at an operating frequency well below the resonant frequency of the beams. Therefore, the beams can vibrate freely at resonance after plucking in each plucking cycle. Pozzi et al. developed a knee-joint energy harvester to harness human motion [107]. The harvester used the relative motion between upper and lower leg to generate rotation. The piezoelectric beams (the dimensions for each beam are $31.8 \times 12.7 \times 0.38 \text{ mm}^3$) on the inner hub fixed to the upper leg were plucked by the plectra on the outer ring mounted on the lower leg. An output power of 2.06 mW was extracted at 1 Hz walking speed. Also using mechanical impact, Yang et al. presented a rotational piezoelectric wind energy harvester, as shown in Fig. 2.11 [108]. 12 piezoelectric beams were installed at the circumference of the rotating fan, forming a polygon shape inside the fan. Seven elastic balls were placed freely in the polygon structure as beam plectra. When the fan rotated, the elastic balls impacted the beams due to the inertial force or gravity. The dimensions of the piezoelectric beam were $47 \times 20 \times 0.5 \text{ mm}^3$, and the optimal output power was $613 \mu\text{W}$ at a rotational speed of 3.3 Hz. In [109], Wei and Duan developed a piezoelectric rotary energy harvester for low and highly variable rotatory motion. The harvester consisted of an 8-sided polygon gear as beam plectra and 8 piezoelectric beams. The free vibration was eliminated by the polygon gear, and beams vibrated with regard to the profile of the gear. However, in these harvesters the mechanical impacts are detrimental to the piezoelectric materials. Degradation of these materials would be a big concern for the durability of the harvesters.

In order to avoid the direct impact on the piezoelectric materials, magnetic plucking has been adopted in many rotational energy harvesters. Pilletsch et al. developed a scalable piezoelectric rotary harvester for human body excitation [110]. A cylindrical rotary proof mass was adopted to actuate an array of 8 piezoelectric beams via magnetic attraction. The dimensions of each beam are $72 \times 5 \times 0.5 \text{ mm}^3$. At a frequency of 2 Hz, a maximal power output of 2.1 mW was achieved. However, the proof mass rolled on
two linear guiding rails, which limited the travel range of the rotary motion. In a later version, they used an eccentric semi-circular mass to replace the cylindrical rotary proof mass \cite{111}. The harvester collected a peak output power of 43 µW at 2 Hz and 20 m/s$^2$ with the beam dimensions of 19.5 × 1 × 0.37 mm$^3$, when the rotor went into a continuous rotation. Inspired by this design, the magnetic plucking mechanism was applied into many other rotational scenarios, including miniature airflow turbines \cite{112,113}, rotating systems \cite{106} and wearable devices \cite{114}.

Recently, non-linear behaviour has also been investigated and adopted in rotational energy harvesting combined with frequency up-conversion to enhance the performance over a broader bandwidth. Tang \textit{et al.} presented a rotational energy harvester for tire pressure monitoring systems (TPMS) \cite{115}. The harvester consisted of a driving magnet sliding in a tube mounted on a wheel rim and two piezoelectric beams with tip magnets at the free end. In this device, the nonlinear behaviour was applied to the driving magnet by adding two repulsive magnets at each end of the tube. This magnetic support eliminated mechanical collision and improved the overall efficiency of the harvester. Similarly, Roundy and Tola also developed a rotational energy harvester for TPMS applications \cite{63}. Instead of using magnets as end stoppers, bistable restoring springs were adopted. The nonlinear behaviour enhanced the performance at low frequency.

For piezoelectric non-resonant harvesters, they operate effectively for excitation frequencies in the non-resonant range. They are ideal for harvesting
2.4.4 Other Types of Harvesters

Apart from electromagnetic and piezoelectric transduction, there are other conversion mechanisms, including electrostatic and triboelectric. Due to the development of electret fabrication, the external power supply is not necessary for electrostatic conversion. Therefore, this conversion mechanism is becoming popular again for energy harvesting. However, this mechanism is not yet widely adopted in rotational energy harvesting, although according to the current available reported devices [65, 116], this technology is quite promising due to its simplicity and good performance. Limitations for electrostatic harvesters could be the charge density of electrets and capacitance that can be achieved.

Triboelectric conversion is another possible option for rotational energy harvesting. A few devices have been developed from Prof Wang’s group at Georgia Tech. [117–119]. These devices demonstrated possible solutions for kinetic energy harvesting. However, more consideration should be given on high output impedance, input requirements, mechanical reliability and output stability.

Combining different conversion mechanisms, similar to hybrid cars, is another option to enhance the performance. Different designs have been reported, such as [120] and [121]. These designs improved the performance of the system, but it might introduce additional difficulties for subsequent power management circuits.

2.5 Characteristics of Rotation Sources

The source of rotational energy is also critical for energy harvester design, as the concerns are different for different cases. Compared to the amount of energy collected by harvesters, the rotational energy source can be divided into two categories - unlimited and limited. For unlimited sources, like rotating wheels and machines, the energy available from the host is tremendous compared with the energy collected by the harvesters, and the rotational motion of the host is not significantly affected by the harvesters, so the main considerations are the amount of energy that can be collected from broadband rotational energy at low frequencies.
the source \[64\], the operating bandwidth \[63\], and the compactness of the device \[60\]. For limited sources, such as miniature airflow turbines, the energy stored by the rotating host is modest and the harvesters should exploit the energy to the largest extent. Therefore, the harvesters have an impact on the rotational motion of the rotating host. The efficiency \[122\]-\[123\], the start-up requirements \[124\] and the operating bandwidth \[125\] are the main consideration in this case.

2.6 Conclusions

This chapter reviews the progress of energy harvesting technology for wireless sensor nodes. The power requirements of sensor nodes are investigated and estimated. Then, available ambient sources and the characteristics are compared and reviewed. A review of recent progress in rotational energy harvesting is provided afterwards. Advantages and issues of different technologies are analysed showing the bottlenecks and potential breakthroughs for rotational energy harvesting. This chapter provides guidelines for the rotational energy harvester designs in this research.
3 Energy Harvesting from Flow-Induced Rotation

This chapter presents the design and experimental implementation of a rotational energy harvester using a miniaturized turbine and piezoelectric conversion. Rotation is induced by a miniature turbine from airflow. The turbine’s transduction is achieved by magnetic plucking of a piezoelectric beam by the turbine rotor. The magnetic coupling is formed by two magnets on the beam’s free end and on the rotor. Frequency up-conversion is realized by the magnetic excitation, allowing the rotor to rotate at a low frequency while the beam can vibrate at its resonant frequency after each plucking. The operating range of the device is, therefore, expanded by this mechanism. Two arrangements of magnetic orientation have been investigated, showing that the repulsive arrangement has higher output power. A prototype was built and tested in a wind tunnel. This prototype verified the concept of rotational energy harvesting using a miniaturized turbine and piezoelectric transduction. Issues were discovered from this prototype, including the high cut-in speed and low output power at high frequencies.

3.1 Design and Operating Principle

A rotational energy harvester using a miniaturized turbine and piezoelectric conversion was developed for airflow energy harvesting. The schematic is presented in Fig. 3.1 and 3.2. The turbine rotor is supported by a rolling bearing mounted on a shaft that is fixed on the rear casing. The rear casing is stationary with the front casing enclosing the turbine rotor. A slot is designed to hold the piezoelectric beam using a screw on the rear casing. Two magnets, which form the magnetic coupling, are installed in the harvester: one on the piezoelectric beam’s free end (tip magnet in Fig. 3.1) and the other on the rotor (primary magnet in Fig. 3.1).
Airflow, which enters the turbine radially and exits axially, activates the turbine rotor. The rotor plucks the piezoelectric beam once per rotation cycle by magnetic coupling. Frequency up-conversion is implemented by the magnetic excitation, allowing the rotor to rotate at low frequencies while the beam can keep operating at its resonant frequency, enabling the device to have a wide operating range. In addition, the non-contact excitation method avoids the mechanical impact on the piezoelectric beam, enhancing the device reliability. As this study focuses on the combination study of miniaturized turbines and piezoelectric transduction, turbine design and optimization was not conducted. Future work on turbine design
and performance optimization could be carried out to improve the system performance.

### 3.2 Optimization of Magnetic Coupling

The magnetic coupling is a critical design factor, as the power output of the beam and the start-up airflow speed are directly determined by the magnetic coupling force between the magnets. In order to understand the influence of magnetic coupling on the performance of the piezoelectric beam, an equivalent experimental set-up was built as shown in Fig. 3.3(a).

![Figure 3.3: Magnet arrangement optimization: (a) Experimental Set-up and (b) Magnetisation direction of magnets (attractive arrangement).](image)

The piezoelectric beam was installed on an adjustable platform so that the gaps between the two magnets in 3 axes can be controlled accurately by the micrometers on the platform. A DC motor was used as an analogy to the rotating turbine. A plate with the same diameter as the turbine rotor was mounted on the motor’s shaft with a magnet fixed on it. The magnetisation direction is in the vertical direction as shown in Figure 3.3(b). The sizes of these components are listed in Table 3.1. The piezoelectric beam is a customized bimorph cantilever beam. The material model is M1100 from Johnson Matthey Piezo Products.

Two arrangements of the magnetic orientation were investigated for different rotational frequencies of the primary magnet. The repulsive arrangement has better performance at high frequencies with larger power output (Fig. 3.4(a)). One speculative reason is that for the attractive configuration, the beam follows the motion of the primary magnet when the rotational fre-
Table 3.1: Dimensions of components.

<table>
<thead>
<tr>
<th>Item</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Magnet</td>
<td>5 mm × 4 mm × 1 mm</td>
</tr>
<tr>
<td>Tip Magnet</td>
<td>1 mm × 1 mm × 1 mm</td>
</tr>
<tr>
<td>Piezoelectric Beam</td>
<td>26.5 mm × 1.5 mm × 0.3 mm</td>
</tr>
</tbody>
</table>

frequency is low, whereas when the frequency reaches a certain level, the inertia of the vibrating beam keeps the beam moving in the same direction for a while even when the magnetic force is opposite. In this case, negative work could be done on the piezoelectric beam, and the output from the beam decreases. This phenomenon happens at high frequencies for the attractive configuration. The repulsive configuration avoids this conflict between the beam motion and the magnetic force, and maintains the positive work on the beam. Therefore, there is a difference for the RMS output voltage at high frequencies.

The influence of the vertical gap between the magnets is also illustrated in Figure 3.4(b), which shows the variation of output voltage against the vertical gap. A 2 mm gap was chosen for the final design as a trade-off between the start-up airflow speed and the output power.

Figure 3.4: Experiment results: (a) RMS output voltage for attractive and repulsive arrangements of magnets versus rotational speed, and (b) influence of the vertical gap between magnets on output voltage of the beam.
3.3 Prototype and Experimental Set-Up

Rapid prototyping was used to fabricate the turbine rotor and casing. Other components were purchased as off-the-shelf products, including cuboidal magnets, piezoelectric beams and rolling bearings. A prototype was assembled as shown in Fig. 3.5(a). The overall dimensions are 47 mm $\times$ 42 mm $\times$ 25 mm.

![Prototype and experimental set-up](image)

Figure 3.5: Prototype (a) and experimental set-up (b). The test section is shown in an enlarged figure within the red lines in (b).

The harvester was tested in the wind tunnel shown in Figure 3.5(b). This wind tunnel is an open loop wind tunnel with continuously variable airspeed control. It is comprised of four major components: contraction, test section, diffuser and fan. Airflow enters the tunnel from the contraction section and exits from the diffuser. The performance of the harvester was measured in the test section, whose dimensions are 100 mm $\times$ 85 mm $\times$ 85 mm. A pitot tube used to measure the airflow speed was installed in parallel with the harvester in the test section, and the reading of airflow speed was received from a micro manometer. The airflow speed measured in the test section is not a free-stream speed because of the limited size of the section.

3.4 Results

Fig. 3.6(a) shows the successful implementation of the concept of frequency up-conversion. The turbine rotor rotates at 11 Hz which can be derived from the excitation frequency of the beam. The beam operates in a freedamped-vibration form at its resonant frequency, i.e. 133 Hz after each
plucking. The vibration frequency is unaffected by the excitation frequency due to the plucking behaviour.

Figure 3.6: Output voltage of the harvester with a 1.8 MΩ resistor: (a) at 2.7 m/s (Rotation speed: 11 Hz) and (b) at 3.61 m/s (Rotation speed: 40 Hz).

The device has also been tested with different loads and airflow speeds. Fig. 3.7(a) illustrates the peak and average output power against differing load resistances at 2.7 m/s. A peak power output of 159 µW was obtained with a 270 kΩ load. The fluctuation of output power is caused by the instability of the shaft and the variation of airflow speed. Fig. 3.7(b) depicts the rotational frequency of the turbine rotor and the output peak power against airflow speed. The device started operating at 3.5 m/s and kept rotating before the airflow speed fell below 1.84 m/s. The start-up airflow speed is quite high compared to the operating range of the harvester. Additional design is necessary to decrease the start-up airflow speed in order to extend the operating range. The cut-in speed is not a free-stream speed because of the small size of the wind tunnel, and the free-stream cut-in speed would be higher.

In addition, the output power at high airflow speeds decreases sharply due to the high plucking frequency which makes the output of the piezoelectric beam between two excitation cycles overlap with each other, as illustrated in Figure 3.6(b). The output voltage is undermined due to the cancelling effect of the overlapping. In order to maintain the output power at high airflow speeds, the beam oscillation needs to be damped quicker to avoid this cancelling effect. Therefore, larger electrical damping would be beneficial. Solutions to broaden the operating range will be investigated in the following chapters.
3.5 Conclusions

This chapter introduces a rotational energy harvester using a miniaturized turbine and piezoelectric conversion for airflow energy harvesting. Magnetic excitation was employed to pluck the beam without physical contact, enhancing the device reliability. Frequency up-conversion was implemented using the magnetic plucking method, which allows the beam to operate at resonance with any low-frequency and random input excitation.

Two magnet arrangements were investigated, showing that the repulsive arrangement had a better performance at high airflow speeds. The influence of the vertical gap between magnets was also examined. The peak voltage output had a sharp drop with the increase of the vertical gap, which means the torque needed for start-up decreased as well.

A prototype was fabricated and tested in a wind tunnel. A peak output power of 159 $\mu$W was obtained with a 270 k$\Omega$ load at 2.7 m/s. The device started working at 3.5 m/s and kept operating when the airflow speed fell to 1.84 m/s. Issues were discovered for improvement, including the high start-up airflow speed and the limited power output at high excitation frequencies.

The novelty of this research lies in the employment of piezoelectric conversion using beam plucking in a miniaturized turbine for airflow induced rotation energy harvesting. Issues were identified for further improvement.
4 Self-Regulation Mechanism for Low Cut-In Speed

This chapter presents the design and demonstration of a self-regulating rotational harvester for airflow energy harvesting. A self-regulation mechanism was proposed and implemented in a miniaturized turbine to control the plucking force for a piezoelectric beam according to the rotational frequency of the turbine rotor. This mechanism was realized by a micro-planar spring fabricated from titanium foil using laser machining. This mechanism automatically changes the relative position of the magnets for different rotational speeds, making the coupling weak at low rotational frequencies and strong at high frequencies. Hence, the device can start up at a low airflow speed and the output power can be ensured when the airflow speed is high.

A theoretical model was established to analyse the turbine’s performance and advantages, and to optimize its design parameters. A prototype was fabricated and tested in a wind tunnel. The start-up airflow speed was 2.34 m/s, showing a 30% improvement against a harvester without this mechanism.

4.1 Introduction

For airflow energy harvesters, there is a threshold airflow speed for the harvester to start up. This speed is named the cut-in speed. In order to enable the harvesters to operate at lower speeds, researchers have investigated and adopted different mechanisms. Howey et al. employed high-quality jewel bearings to minimize the friction [52]. Kishore et al. adopted large-size blades to generate a higher driving torque at low speeds [126], which is undesirable for small-scale devices.

A feasible way to decrease the start-up speed and to maintain the output power at high speeds is to alter the magnetic coupling automatically along
with the airflow speed. Self-regulating mechanisms, as a potential solution, have been used in many energy harvesting devices \[127\]–\[131\]. Lallart et al. developed a broadband vibration energy harvester using additional sensing and actuating components to adjust the resonant frequency of the beam along with the excitation frequency \[129\]. The bandwidth of the structure improved significantly, but the generated power was partially consumed by these components. Miller et al. built a self-tuning vibration energy harvester by mounting a sliding proof mass on a clamped-clamped beam \[130\]. The position of the mass was adjusted automatically with respect to the vibration frequency, extending the bandwidth of the device. Gu et al. designed a rotational self-regulating energy harvester by adjusting the beam’s tensile stress generated by the centrifugal force \[131\]. The stress adjusted the stiffness and the resonant frequency of the beam, enabling the resonant frequency to be identical to the excitation frequency in a wide operating range. In this chapter, a self-regulating airflow (rotational) energy harvester was designed and is presented to achieve a low cut-in speed.

### 4.2 Self-Regulating Rotational Energy Harvester

The schematic of the rotational harvester for airflow energy harvesting and the self-regulating mechanism are illustrated in Fig. 4.1 and Fig. 4.2. The turbine has six inlets arranged on the lateral sides of the hexagonal prism casing, allowing the device to operate with airflow from any direction. The turbine’s transduction is achieved by non-contact plucking of a piezoelectric cantilever by the magnetic force which is formed by magnets on the turbine rotor and the cantilever free end. The cantilever mounted on the turbine casing is plucked once per motion cycle and operates in a free vibration form after each plucking.

The main resistance impeding the turbine to start up is the magnetic force imposed on the turbine rotor. Once the turbine rotates, the driving torque and the inertia of the turbine rotor can easily conquer the magnetic resistance. In order to decrease the strength of the magnetic coupling in the static situation and to intensify the strength immediately after the turbine starts up to enhance the power output, a self-regulating mechanism was designed to dynamically adjust the relative radial gap between these magnets in response to airflow speed.
Figure 4.1: Design of the micro turbine with piezoelectric conversion, showing the implementation of the self-regulating mechanism.

The main components to realize the mechanism are a micro-spring and two guide rails. A magnet is mounted on the guide rails with two sliders which can only move along the rails. These rails ensure that the magnet does not have relative tangential movement to the turbine rotor. The spring is rigidly connected with the magnet as shown in Fig. 4.1. The length of the spring is passively controlled by the centrifugal force of the rotating magnet with respect to different rotational speeds. Hence, the gap between the magnets is passively adjusted in response to the airflow (rotational) speed.

4.3 Modelling of Self-Regulation Mechanism

The schematic of the self-regulating system is shown in Fig. 4.3(a). When there is no rotation or the airflow is too weak to start up the turbine, the spring is unstretched. The gap between the magnets is correspondingly
large, ensuring that the coupling is weak enough for the turbine to start up easily. When the airflow speed is increased and the rotational speed of the rotor rises, the centrifugal force generated by the rotating magnet increases. As the spring is stretched, the gap decreases, intensifying the magnetic coupling and improving the output power.

Figure 4.3: (a) Schematic of the self-regulating turbine and (b) magnetic coupling configuration.

Assuming that the turbine is activated by airflow and operates at a rotational frequency of $\omega_{tr}$, the gaps between magnets in 3 axes are

\[
\begin{align*}
  d_x &= r_{m0} + d_{x0} - r_m \cos(\omega_{tr} t - \alpha_{m0}), \\
  d_y &= r_m \sin(\omega_{tr} t - \alpha_{m0}) - v(L, t), \\
  d_z &= d_{z0},
\end{align*}
\]  

(4.1a, 4.1b, 4.1c)

where $d_{x0}$ and $d_{z0}$ are the initial gaps along the x-axis and z-axis when the rotor is static, with the sliding magnet’s angular position $\alpha_{m0} = 0$, $v(L, t)$ is the tip displacement of the piezoelectric beam in the beam bending direction and, $r_m$ is the rotational radius of the sliding magnet which is given by

\[
  r_m = \frac{k_s r_{m0}}{k_s - m_{sm} \omega_{tr}^2},
\]

(4.2)

where $k_s$ is the spring constant, $r_{m0}$ is the initial radius of the sliding magnet and $m_{sm}$ is the mass of the sliding magnet.

The magnetic coupling force, $F_{mag}$, in the beam vibration direction can be calculated numerically using the theory developed by Akoun and Yonnet [132]. The configuration of the magnet coupling is illustrated in Fig. 4.3(b). The magnetic force between two cuboidal magnets in the beam vibration...
direction can be calculated using

$$F_{mag}^y = \frac{J \cdot J'}{4 \pi \mu_0} \sum_{i=0}^1 \sum_{j=0}^1 \sum_{k=0}^1 \sum_{l=0}^1 \sum_{p=0}^1 \sum_{q=0}^1 (-1)^{i+j+k+l+p+q} \cdot \phi_y, \quad (4.3)$$

where $J$ and $J'$ are the magnetization of magnets, $\mu_0$ is the magnetic constant, and $\phi_y$ is a function of the magnet dimensions and their gaps in 3 axes. The function is given by

$$\phi_y = \frac{1}{2} (U_{ij}^2 - W_{pq}^2) \ln(r - V_{kl}) + U_{ij} V_{kl} \ln(r - U_{ij})$$

$$+ U_{ij} W_{pq} \tan^{-1} \left( \frac{U_{ij} V_{kl}}{r W_{pq}} \right) + \frac{1}{2} r V_{kl}, \quad (4.4)$$

where

$$U_{ij} = d_x + (-1)^i A - (-1)^i a,$$  \hspace{1cm} (4.5a)

$$V_{kl} = d_y + (-1)^j B - (-1)^j b,$$  \hspace{1cm} (4.5b)

$$W_{pq} = d_z + (-1)^k C - (-1)^k c,$$  \hspace{1cm} (4.5c)

$$r = \sqrt{U_{ij}^2 + V_{kl}^2 + W_{pq}^2}. \quad (4.5d)$$

These lengths, $U_{ij}$, $V_{kl}$ and $W_{pq}$, correspond to the distance between the cube corners and their projections on the axes. The parameters $i$, $j$, $k$, $l$, $p$ and $q$, are equal to 0 or 1 according to the specific corner. $A$, $B$, $C$, $a$, $b$ and $c$ are the dimensions of magnets, as shown in Fig. 4.3(b).

The magnetic force on the tip magnet excites the piezoelectric beam once per cycle. Erturk and Inman established a comprehensive theoretical model for the beam operating with base excitation \[133\]. By adapting this theory to tip force excitation, the mechanical equation describing the motion of the beam is given as follows:

$$YI \frac{\partial^4 v(x, t)}{\partial x^4} + c_s I \frac{\partial^5 v(x, t)}{\partial t \partial x^4} + c_a \frac{\partial v(x, t)}{\partial t} + m \frac{\partial^2 v(x, t)}{\partial t^2}$$

$$- \frac{\partial_t V(t)}{r} \left[ \frac{d\delta(x)}{dx} - \frac{d\delta(x - L_p)}{dx} \right] = F_{mag}^y(x) \delta(x - L_p), \quad (4.6)$$

where $YI$ is the bending stiffness, $v(x, t)$ is the transverse deformation of the beam, $c_s I$ is the internal damping, $c_a$ is the viscous deformation damping,
$m$ is the mass per unit length of the beam, $\delta_x$ is the Dirac delta function, $\vartheta_r$ is the piezoelectric coupling term in physical coordinates, $L_p$ is the length of the beam and $V(t)$ is the voltage across a resistive load $R_l$.

Using the method of separation of variables, the beam displacement $\nu(x,t)$ can be expressed as a convergent series of the eigenfunctions:

$$\nu(x,t) = \sum_{r=1}^{\infty} \phi_r(x) \eta_r(t),$$

where $\phi_r(x)$ and $\eta_r(t)$ are the mass normalized eigenfunction and the modal mechanical coordinate of the cantilever beam with respect to its $r$th mode shape. The mechanical equation can be further reduced to the modal coordinate by substituting Eq. (4.7) into Eq. (4.6), multiplying $\phi_r(x)$ and integrating over the beam length:

$$\frac{d^2\eta_r(t)}{dt^2} + 2\zeta_r \omega_r \frac{d\eta_r(t)}{dt} + \omega_r^2 \eta_r(t) - \vartheta_r V(t) = \phi_r(L_p)F_{\text{mag}}(t),$$

where $\zeta_r$ is the modal damping ratio and $\omega_r$ is the effective undamped modal frequency of the $r$th mode shape. The modal damping ratio $\zeta_r$ are normally calculated using the logarithmic decrement method from experimentally measured beam displacement curve. The detailed deduction process from Eq. (4.6) to Eq. (4.8) can be found in the book written by Erturk and Inman [134] or Pillatsch’s thesis [135].

The corresponding electrical equation of a bimorph piezoelectric beam in series connection with a resistive load $R_l$ is

$$\frac{C_p}{2} \frac{dV(t)}{dt} + \frac{V(t)}{R_l} + \sum_{r=1}^{\infty} \vartheta_r \frac{d\eta_r(t)}{dt} = 0,$$

where $C_p$ is the inherent capacitance of the piezoelectric beam.

The performance of the piezoelectric turbine was analysed numerically using the above analysis in Matlab/Simulink. This work is based on the model developed by Pillatsch [135], in which a piezoelectric beam under magnetic plucking at the tip was model using the distributed parameter model. Adaptations and extensions have been made in terms of using a more precise
magnetic force model and the consideration of the self-regulation mechanism. A design was completed with the parameters listed in Table 4.1. The piezoelectric beam is a customized bimorph cantilever beam. The material model is M1100 from Johnson Matthey Piezo Products.

Table 4.1: Design parameters of the piezoelectric harvester.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_p \times b_p )</td>
<td>Beam size</td>
<td>26.5 mm \times 1.5 mm</td>
</tr>
<tr>
<td>( h_p )</td>
<td>Thickness of piezo layer</td>
<td>0.1 mm</td>
</tr>
<tr>
<td>( h_s )</td>
<td>Thickness of substrate</td>
<td>0.1 mm</td>
</tr>
<tr>
<td>( r_m )</td>
<td>Magnet rotation radius</td>
<td>12 mm</td>
</tr>
<tr>
<td>( a \times b \times c )</td>
<td>Driving magnet size</td>
<td>0.5 \times 0.5 \times 0.5 mm³</td>
</tr>
<tr>
<td>( A \times B \times C )</td>
<td>Tip magnet size</td>
<td>0.5 \times 0.5 \times 0.5 mm³</td>
</tr>
<tr>
<td>( d_{z0} )</td>
<td>Initial gap in z-axis</td>
<td>3.2 mm</td>
</tr>
<tr>
<td>( J )</td>
<td>Magnetization of magnets</td>
<td>1.17 T</td>
</tr>
<tr>
<td>( \rho_m )</td>
<td>Density of magnets</td>
<td>7400 kg/m³</td>
</tr>
<tr>
<td>( \tilde{e}_{31} )</td>
<td>Piezoelectric constant</td>
<td>(-22.2) V \cdot m/N</td>
</tr>
<tr>
<td>( d_{31} )</td>
<td>Piezoelectric charge constant</td>
<td>(-315) m/V</td>
</tr>
<tr>
<td>( \rho_p )</td>
<td>Density of piezoelectric material</td>
<td>7700 kg/m³</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>Density of substrate material</td>
<td>1500 kg/m³</td>
</tr>
<tr>
<td>( Y )</td>
<td>Young’s modulus of substrate</td>
<td>140 GPa</td>
</tr>
<tr>
<td>( \epsilon_{r33} )</td>
<td>Relative permittivity constant</td>
<td>4500</td>
</tr>
</tbody>
</table>

Fig. 4.4 illustrates the regulating behaviour and the operating principle of the harvester based on the above equations and parameters. The system has a weak magnetic coupling at low frequencies (13.8 Hz in Fig. 4.4(a)), which enables the system to start up at low airflow speeds. The coupling is enhanced at high frequencies (33 Hz) by the centrifugal force of the sliding magnet, allowing the output power to be improved.

The spring constant is a critical parameter determining the self-regulating behaviour. In order to decrease the start-up airflow speed and to intensify the magnetic coupling quickly after start-up, the behaviour was investigated for different spring constants \( k_s \). The initial length, \( r_{m0} \), of the spring is 8.2 mm, and the initial gap \( d_{z0} \) of the magnet in the x-axis direction is 3.8 mm. The regulating behaviour initiates when the turbine starts operating and terminates at the maximum magnetic coupling position \((r_m = r_{m0} + d_{z0})\) by a mechanical stopper on the spring.

As illustrated in Fig. 4.5, the magnetic coupling after the regulating stage
Figure 4.4: Simulated Results: Gap between the magnets, magnetic force and output voltage of the piezoelectric turbine. (a) operating at 13.8 Hz and (b) operating at 33 Hz. \( k_s = 8.5 \text{ N/m} \)

Figure 4.5: Simulated self-regulating behaviour for different spring constants. (a) Spring length versus turbine rotational frequency and (b) peak magnetic force in the beam bending direction versus rotational frequency.

is 6 times higher than that in the initial stage, which indicates the effect of the regulating mechanism. The range of the self-regulating behaviour depends on the spring constant, extending with increasing spring constant. In this mechanism, the regulating range should be as narrow as possible in order to enhance the output power as soon as the rotor starts up. The spring constant, therefore, should be low enough to fulfil the requirement.
4.4 Design and Fabrication of Micro-Planar Spring

In order to design a spring with a low spring constant, different shapes of springs are first investigated using FEM. Four types of springs with uniform dimensions were simulated, including square-shape, v-shape, sine-shape and u-shape. As illustrated in Table 4.2, the u-shape spring has the lowest spring constant among the four types, but further consideration is needed to decrease the spring constant.

Table 4.2: Different spring shapes and corresponding simulated spring constant with uniform dimension.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Spring Constant (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square shape</td>
<td>0.590</td>
</tr>
<tr>
<td>V-shape</td>
<td>0.580</td>
</tr>
<tr>
<td>Sine shape</td>
<td>0.583</td>
</tr>
<tr>
<td>U-shape</td>
<td>0.311</td>
</tr>
</tbody>
</table>

Different structural parameters of the u-shape spring were considered, including spring turns, the length and width of each turn and the width of the spring beam. The model of the u-shape spring is shown in Fig. 4.6. Considering the limited space on the turbine rotor, the operating direction of the spring is designed to be perpendicular to its length direction. The simulated results are illustrated in Fig. 4.7. The number of turns has a significant impact on the constant, decreasing the spring constant with in-
creasing turns. The optimized spring has the spring constant of 1.58 N/m, as shown in Fig. 4.7(d). The maximum stress from the simulation for this specific design is 595 MPa. The spring material should have a larger elastic limit to maintain the elastic deformation. The width of the spring beam is 100 µm and the width and length of each turn are 10 mm and 0.7 mm respectively.

![Figure 4.7](image)

Figure 4.7: Simulated spring constant and maximum stress of the u-shape spring against different parameters. (a) Spring turns, (b) length of each turn, (c) width of each turn and (d) width of the spring beam.

For micro-spring manufacturing, the general method is silicon-based fabrication [136]. However, the material that can be used for this method is limited, and the spring generally cannot produce a large distortion [137]. Laser machining, as an alternative solution, is capable of cutting materials in a wide range [138]. To achieve the ultra-low spring constant and large distortion, titanium foil is chosen for its advantage in flexibility and high elastic limit (786 – 910 MPa [139]). The fabricated spring is shown in Fig. 4.8(a). In order to measure the spring constant, an experimental set-up was built as shown in Fig. 4.8(b). The spring is fixed on a spring holder which is rigidly mounted on a testing bench. A laser sensor is used to measure the displacement of the spring caused by the vertical vibration.
The spring constant was measured based on the impulse excitation technique \cite{140}. A vertical impulse excitation was imposed on the testing bench close to the spring. The vibration data of the spring were collected from the sensor. The results are shown in Fig. 4.9. The vertical vibration mode of the spring is excited by the impulse input and its resonant frequency in the vertical direction is 7.62 Hz. The spring constant can be calculated using

$$k_s = m_s \omega_n^2,$$

(4.10)

where $m_s$ is the mass of the spring and $\omega_n$ is the resonant frequency. The spring constant is 0.78 N/m based on the above equation.

Figure 4.9: Spring vibration, showing its resonant frequency in vertical orientation.
4.5 Results and Discussion

4.5.1 Testing of Self-Regulation

In order to examine the effect of the self-regulation, a simplified equivalent experimental set-up was built in the first step, as illustrated in Fig. 4.10. A DC-motor was employed as an analogy to the turbine rotor. The regulating mechanism was implemented on a rotating disk with the same diameter as the turbine rotor. The plate was mounted on the motor’s shaft rigidly. A piezoelectric beam was installed on an adjustable platform with micrometers controlling the position of the beam in 3 axes accurately. The vertical gap between magnets was 3 mm, and the initial radial position of the sliding magnet was 5 mm. The cantilever was connected with a 100 kΩ resistor where the output voltage was measured.

![Figure 4.10: Equivalent experimental set-up to examine the self-regulating mechanism.](image)

The prototype was tested for different rotational frequencies. The output voltage at 11 Hz and 21 Hz is presented in Fig. 4.11, showing the variation of the amplitude of the output voltage caused by the changing magnetic coupling. The peak output voltage is magnified 6.44 times from 0.41 V at 11 Hz to 2.64 V at 21 Hz, which implies the intensification of the magnetic coupling with the stretch of the planar spring caused by the increased rotational frequency.

In order to visualize the deformation of the micro-spring, a fast VGA
camera Pike F-032C (208 fps) was employed to record the regulating effect against the rotational frequency of the disk. The result is shown in Fig. 4.12. The spring is unstretched initially and reaches its maximum displacement, 5 mm, at 17.9 Hz. The difference between the theoretical analysis in Fig. 4.5 and the experimental results is caused by the fabrication inaccuracy and friction in the device.

Figure 4.12: Deformation of the micro-spring for different frequencies with the envelope of the edges indicated. (a) Static, (b) 11.1 Hz, (c) 14.2 Hz and (d) 17.9 Hz.

Fig. 4.13 illustrates the peak and RMS output voltage versus the rota-
tional frequency. The regulating behaviour happens from 10 Hz to 18 Hz. The output voltage at low excitation frequency (<10 Hz) is negligible, which means the energy converted by the piezoelectric beam is marginal. Hence, the only energy consumption at low frequency is caused by the friction in the system. The system, therefore, is much easier to start up. The output voltage is improved at high frequencies (>18 Hz), which ensures the output power of the device after start-up.

![Figure 4.13: Peak and RMS output voltage of the beam against the rotational frequency of the rotating disk. The load connected was 100 kΩ.](image)

### 4.5.2 Turbine Implementation

The regulating mechanism was then implemented in a rotational energy harvester using a miniaturized turbine, as shown in Fig. 4.14(a). The turbine rotor and casing were built from Verowhite Plus material using the 3D printer Stratasys Objet 500 Connex 3. The assembled prototype is presented in Fig. 4.14(b). The overall dimensions of the piezoelectric turbine are $37 \text{ mm} \times 18 \text{ mm}$.

The prototype was tested in a miniature wind tunnel. Its schematic is shown in Fig. 4.15. The turbine was tested versus load resistance and airflow speed respectively. The optimal load resistance of the device is 100 kΩ, as shown in Fig. 4.16. The maximum peak power output at 3.94 m/s airflow speed is 705 µW.

In order to validate the self-regulating mechanism, the device was tested
Figure 4.14: Prototype. (a) Implementation of the self-regulating mechanism and (b) miniaturized piezoelectric turbine.

Figure 4.15: Schematic of the experimental set-up.

Figure 4.16: Peak and average output power of the harvester versus load resistance at 3.94 m/s airflow speed.

at different airflow speeds. Fig. 4.17 depicts the peak output power and the rotational frequency of the turbine rotor against airflow speed. The start-up airflow speed is 2.34 m/s, and the regulating behaviour terminates at 4.21 m/s, where a 742 µW peak output power was measured with a 100 kΩ load. The falling-off of output power at high airflow speeds is a common
phenomenon in piezoelectric turbines [11, 14]. However, this design has successfully addressed the issue by adopting a stopper on the spring. Additional energy generated by the increased airflow speed is dissipated by the friction between the stopper and the turbine shaft. Hence, the rotational frequency of the turbine rotor remain unchanged above 4 m/s airflow speed, as shown in Fig. 4.17. The operating range is further increased by the passive speed-controlling technique. However, this direct impact on the spring is detrimental to the device durability.

Figure 4.17: Peak average output power and rotational frequency of the harvester with a 100 kΩ load versus airflow speed.

The device without the self-regulating mechanism was also tested. The start-up airflow speed was 3.35 m/s. It indicates that the self-regulating mechanism has reduced the start-up airflow speed to 30%.

4.6 Performance Comparison

In order to illustrate the power generation capability, typical airflow energy harvesters in the literature are summarized, as shown in Table 4.3. Compared to other airflow energy harvesters, the harvester in this work shows improved performance in power density and start-up airflow speed. The design from Howey et al. [52] has the highest power density; however, the structure is complex and the cost for fabrication is likely to be much higher. In addition, the power density using electromagnetic conversion is in the same order as the density using piezoelectric conversion in this chapter. The power density in this paper is 4.48 μW/cm³ which can be further improved.
by integrating multiple piezoelectric beams and optimizing the structure of the device. The design from Kishore et al.\textsuperscript{126} has the lowest start-up speed, but the size is quite large, and the power density is the lowest. The start-up speed of the device in this chapter can be decreased by minimizing the frictions, such as using better bearings or reducing the frictions between rails and sliders for the self-regulating mechanism. In addition, it is worth mentioning that the operating speed range is also a critical consideration for airflow (rotational) energy harvesting, as in many applications airflow (rotational) speeds vary in a wide range. In summary, three critical challenges for airflow (rotational) energy harvesting are power density (energy conversion efficiency), cut-in speed and wide operation range.

<table>
<thead>
<tr>
<th>Device</th>
<th>Transduction</th>
<th>Dimension [mm]</th>
<th>Power Density\textsuperscript{b}</th>
<th>Start-Up Speed</th>
<th>Broadband Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Myers et al.\textsuperscript{142}</td>
<td>T &amp; PE</td>
<td>96 × 254 × 127</td>
<td>1.6</td>
<td>2.41 m/s</td>
<td>Yes</td>
</tr>
<tr>
<td>The author\textsuperscript{11}</td>
<td>T &amp; PE</td>
<td>47 × 42 × 25</td>
<td>2.5</td>
<td>3.5 m/s\textsuperscript{d}</td>
<td>No</td>
</tr>
<tr>
<td>Howey et al.\textsuperscript{52}</td>
<td>T &amp; EM</td>
<td>32 × 50</td>
<td>7.4</td>
<td>3 m/s</td>
<td>Yes</td>
</tr>
<tr>
<td>Kishore et al.\textsuperscript{126}</td>
<td>T &amp; PE</td>
<td>100 × 80 × 65</td>
<td>0.87\textsuperscript{c}</td>
<td>1.1 m/s</td>
<td>No</td>
</tr>
<tr>
<td>Karami et al.\textsuperscript{141}</td>
<td>T &amp; PE</td>
<td>80 × 80 × 175</td>
<td>0.45</td>
<td>3.0 m/s</td>
<td>No</td>
</tr>
<tr>
<td>Kwon et al.\textsuperscript{143}</td>
<td>PE</td>
<td>100 × 60 × 30</td>
<td>1.4</td>
<td>3.6 m/s</td>
<td>No</td>
</tr>
<tr>
<td>Zhu et al.\textsuperscript{144}</td>
<td>PE</td>
<td>141 × 100 × 55</td>
<td>1.7</td>
<td>2.5 m/s\textsuperscript{d}</td>
<td>No</td>
</tr>
<tr>
<td>Perez et al.\textsuperscript{65}</td>
<td>T &amp; ES</td>
<td>40 × 10</td>
<td>44</td>
<td>1.5 m/s</td>
<td>Yes</td>
</tr>
<tr>
<td>The author\textsuperscript{113}</td>
<td>T &amp; PE</td>
<td>37 × 18</td>
<td>4.5</td>
<td>2.34 m/s\textsuperscript{d}</td>
<td>Yes</td>
</tr>
</tbody>
</table>

\textsuperscript{a} T - Turbine, PE - Piezoelectric, ES - Electrostatic and EM - Electromagnetic.
\textsuperscript{b} The air speed is 4 m/s and the unit is µW/cm\textsuperscript{3}.
\textsuperscript{c} The air speed is 1.9 m/s. The power density is lower at 4m/s.
\textsuperscript{d} The test was conducted in a wind tunnel with limited testing cross-section. The airflow condition was not equivalent to the free stream condition.

### 4.7 Conclusions

In conclusion, a self-regulating rotational harvester using a miniaturized turbine for airflow energy harvesting is introduced in this chapter. In order to reduce the cut-in speed, a passive self-regulating mechanism was proposed by dynamically adjusting the magnetic coupling between the two magnets on the turbine rotor and the piezoelectric beam using a micro spring. The device starts up at a low airflow speed, and the output power is ensured when the airflow speed is high. A theoretical model was established to analyse the harvester’s performance and advantages, and to optimize its...
A micro-planar spring with ultra-low spring constant 0.78 N/m was designed with the optimized shape and structural parameters. The spring was fabricated from titanium foil using laser cutting with a spring beam width of 100 µm. The spring was then installed onto a rotating plate on a DC motor to examine the effect of the regulating mechanism.

Eventually, the regulating mechanism was achieved in a piezoelectric turbine with the overall size of 37 mm×18 mm. The cut-in speed is 2.34 m/s, showing a 30% improvement against a non-regulated turbine. A peak output power of 742 µW was measured at 4.21 m/s with a 100 kΩ load.

In this design, the limited power issue at high rotational frequency is addressed using a mechanical stopper on the spring. Although it solved this issue, the mechanical interaction between the spring and the stopper will affect the reliability of the system. Other mechanisms to broaden the operating speed range at high speed need to be adopted.

The contribution of this chapter is the development of the self-regulation mechanism for reducing the start-up speed requirement of airflow energy harvesters or harvesters with limited input excitation force. The design and fabrication of the micro-planar spring also provide a guidance for future development of micro-springs and self-regulating systems.
5 Rotational Energy Harvester under Tip Plucking

Starting from this chapter, the research focuses on the rotational piezoelectric transduction part. Mechanisms will be investigated to enhance the power harvesting capability of the piezoelectric transduction from rotation. External rotation is assumed to be available and unlimited compared to the converted energy. A controllable stepper motor will be used to provide the input in the following experiments in order to achieve precise control of the rotational motion.

A piezoelectric transducer under tip excitation using magnetic plucking is theoretically analysed and experimentally verified to understand the electromechanical behaviour of this type of harvester and to optimize the conversion capability. Different structure configurations were investigated in both the time and frequency domains. This piezoelectric harvester exhibits a wide bandwidth at low frequency, and is also feasible to be adapted to many rotational energy harvesting scenarios, such as energy harvesting from human motion, rotating machines or fluid flow.

5.1 Design and Operating Principle

The schematic of the rotational energy harvester is shown in Fig. 5.1. This design is similar to that presented in Fig. 4.3(a). However, the self-regulation mechanism is not considered in this design. Also, different from the previous airflow energy harvesters where rotation is provided by turbines, in this study, the rotational energy is assumed to be already available from external energy sources.

In this design, a non-contact magnetic plucking mechanism is achieved. The beam is contactlessly plucked once per cycle by the magnetic force provided by the driving magnet. This mechanism avoids the risk of degradation
Figure 5.1: Schematic of the rotational energy harvester.

of piezoelectric materials caused by direct impacts [108, 145]. Considering the brittle characteristic of piezoelectric materials, the contactless plucking mechanism enhances the reliability of the device.

For conventional inertial piezoelectric harvesters, most have been resonant narrowband devices, often operating at high frequency (100’s of Hz to kHz) [146, 147]; such devices are not suitable to practical low-frequency broadband applications. The limitation is a direct consequence of the low electromechanical coupling coefficients of piezoelectric transducers. Their output impedances are dominated by the internal capacitance that cannot be tuned out with an inductor of realistic size and efficiency. As a result the electrical circuit cannot effectively damp the mechanical oscillation to achieve a low Q. This explains why piezoelectric transducers normally have a narrow bandwidth at high frequency. Frequency up-conversion, a mechanism allowing the piezoelectric transducers to operate at their resonant frequency from any low-frequency input motion, addresses the narrow operating bandwidth issue.

5.2 Theoretical Modelling and Analysis

5.2.1 Modelling of the Piezoelectric Beam

Lumped parameter modelling and distributed parameter modelling are two dominant analytical modelling methods for the dynamic analysis of piezoelectric beams. The lumped parameter model describes the dynamics of a certain point by calculating the equivalent mass, stiffness and damping of a beam to the concerned point [148, 149]. The process of equivalence limits the model to a single-degree-of-freedom system, and a correction factor is
required when the tip mass is considered at the beam’s free end [134]. Compared with the lumped parameter model, the distributed parameter model has higher accuracy by avoiding the process of equivalence and by considering the structure as a series of infinitely small elements in a continuous fashion.

In this study, the distributed parameter model is used according to the work established by Erturk and Inman [134]. Theories about the magnetic modelling and piezoelectric conversion have been discussed in Section [4.3]. For reading convenience, key equations are rewritten in this section. The analysis is based on the case of a bimorph connected in series with a load resistance, as shown in Fig. 5.2.

The dynamic equation of the piezoelectric beam under the magnetic plucking at the free end can be written as

\[
Y \frac{\partial^4 v(x,t)}{\partial x^4} + c_s \frac{\partial^5 v(x,t)}{\partial x \partial t^2} + c_a \frac{\partial v(x,t)}{\partial t} + m \frac{\partial^2 v(x,t)}{\partial t^2} - \vartheta r v(t) \left[ \frac{d\delta(x)}{dx} - \frac{d\delta(x - L)}{dx} \right] = F_{mag}(t) \delta(x - L),
\]

\[(5.1)\]
The electromechanical behaviour can be calculated using the above equations. These equations were numerically solved in Matlab/Simulink.

5.2.2 Arrangement of Magnets and Piezoelectric Beam

In order to maximize the input power from the revolving host to the piezoelectric cantilever, the arrangements of magnetic coupling and the beam orientation were investigated in terms of the beam bending direction, the magnet rotation direction, the magnetization direction and the initial relative position of the two magnets. Four potentially achievable arrangements were chosen as shown in Fig. 5.3. In all cases, the cantilever is oriented along the x-axis direction and the beam bending direction is along the y-axis. There are other possible configurations, but they are either equivalent or inferior compared with the illustrated four configurations. The details of the configurations are described below.

(a) The beam bending direction (y-axis) is parallel to the magnetic rotation direction (y-axis) and perpendicular to the magnetization direction (z-axis). The magnet rotation direction is perpendicular to the magnetization direction. The plane composed by the centroids of the two magnets and the magnet rotation direction is parallel to the y-z plane.

(b) The beam bending direction (y-axis) is perpendicular to the magnet rotation direction (x-axis) and the magnetization direction (z-axis). The magnet rotation direction is perpendicular to the magnetization direction. The plane composed by the centroids of the two magnets and the magnet rotation direction is the x-y plane.

(c) The beam bending direction (y-axis) is parallel to the magnet rotation direction (y-axis) and the magnetization direction (y-axis). The magnet rotation direction is in parallel with the magnetization direction. The plane composed by the centroids of the two magnets and the magnet rotation direction is parallel to the y-z plane.
Figure 5.3: Arrangements of the magnetic coupling in terms of magnetization direction, beam bending direction and magnetic moving direction.

(d) The beam bending direction (y-axis) is perpendicular to the magnet rotation direction (x-axis) and parallel to the magnetization direction (y-axis). The magnet rotation direction is parallel to the magnetization direction. The plane composed by the centroids of two magnets and the magnet rotation direction is the x-y plane.

The magnetic force for each configuration was calculated using the theories provided in Section 4.3. The design parameters of the harvester are the same as those in Table 4.1. In order to validate the theoretical model, the finite element method (FEM) was also adopted to compare with the theoretical results.

The results for specific configurations are listed in Table 5.1. The theoretical results are close to the results from FEM with errors less than 3%. This verifies the validity of the theoretical model for further analysing the performance of different arrangements.

The magnetic force in the beam bending direction against the angular
Table 5.1: Peak magnetic force with different arrangements using theoretical and simulation methods

<table>
<thead>
<tr>
<th>No.</th>
<th>Theory [mN]</th>
<th>FEM [mN]</th>
<th>Error</th>
<th>Initial Gaps (dx, dy, dz)</th>
<th>Beam Bending Direction</th>
<th>Magnetization Direction</th>
<th>Rotation Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>7.24</td>
<td>7.20</td>
<td>0.6%</td>
<td>(0, 0, 5 mm)</td>
<td>Y - Axis</td>
<td>Z - Axis</td>
<td>Y - Axis</td>
</tr>
<tr>
<td>(b)</td>
<td>7.73</td>
<td>7.51</td>
<td>2.9%</td>
<td>(0, 5 mm, 0)</td>
<td>Y - Axis</td>
<td>Z - Axis</td>
<td>X - Axis</td>
</tr>
<tr>
<td>(c)</td>
<td>5.12</td>
<td>5.24</td>
<td>2.3%</td>
<td>(0, 5 mm, 0)</td>
<td>Z - Axis</td>
<td>Y - Axis</td>
<td>Z - Axis</td>
</tr>
<tr>
<td>(d)</td>
<td>15.45</td>
<td>15.38</td>
<td>0.5%</td>
<td>(0, 0, 5 mm)</td>
<td>Z - Axis</td>
<td>Y - Axis</td>
<td>X - Axis</td>
</tr>
</tbody>
</table>

Position of the rotating magnet for different arrangements are illustrated in Fig. 5.4. For arrangement (a) and (c), the direction of magnetic force is alternating during each excitation cycle, but the direction for arrangement (b) and (d) is almost uniform during the whole excitation period. The magnetic force for arrangement (d) is much higher than that for other arrangements, which shows the potential option for the rotational energy harvester.

Figure 5.4: Theoretically achieved magnetic force in the beam bending direction against angular position of the rotating magnet for different arrangements.

The average output power of the piezoelectric transducer is determined by
the input power rather than the input magnetic force. In order to determine the best arrangement, the input power is calculated using the magnetic force and the vibration velocity of the piezoelectric beam. The average input power is given as follows:

$$P_{avg} = \frac{\int_{t_0}^{t_1} F_{mag}^i \cdot \dot{\nu}(L, t) dt}{t_1 - t_0},$$

(5.3)

where $F_{mag}^i$ is the magnetic force in the beam bending direction ($i$-axis, $i = x, y, z$) and $\dot{\nu}(L_p, t)$ is the transverse velocity of the piezoelectric beam’s free end. $t_0$ and $t_1$ are the time considered for averaging. The duration should be long enough to alleviate the influence of the variation of the input power during one cycle. In this calculation, one full rotation cycle in the steady state is considered for averaging.

Fig. 5.5 shows the beam tip velocity, the magnetic force in the beam bending direction and the input power for different magnetic field arrangements with the excitation frequency of 30 Hz. As illustrated, the input power is determined by both the magnetic force and the beam tip velocity. When their phases are the same, the work is done on the piezoelectric beam. However, the energy is extracted from the piezoelectric beam by the rotating magnet, when the magnetic force direction is opposite to the direction of beam tip velocity.

The phase difference between the magnetic force and the beam tip velocity is mainly caused by the different operating frequencies of the vibrational beam and the driving magnet. In this design, the beam is under a pulse excitation by the magnetic force, and oscillates in a damped-free-vibration form, in which the operating frequency of the beam is its natural frequency. The frequency variation of the driving magnet generates fluctuation of the phase difference, leading to variation of the input energy per excitation cycle. Hence, although the first arrangement in Fig. 5.5(a) has a higher input power at 30 Hz, it is worth investigating the dependence of the input power on frequency in detail.

First, the achievable energy by the piezoelectric beam per excitation cycle is calculated against rotational frequency of the driving magnet, as shown in Fig. 5.6. The harvester has a wide bandwidth compared to conventional resonant vibration energy harvesters. Meanwhile, the input energy has significant variations with regard to different excitation frequencies for different
Figure 5.5: Beam tip velocity, magnetic force in the beam bending direction and beam input power for different arrangements, operating at 30 Hz. (a), (b), (c) and (d) are corresponding to the arrangements in Fig. 5.3.
arrangements. The general trend is upward with ripples when the frequency is high. At low frequencies (below 30 Hz), arrangement (a) has the highest input energy; at high frequencies (above 30 Hz), arrangement (d) is the best.

The variation of input energy for different arrangements is mainly the consequence of the interaction between the magnetic force and the beam velocity. Take arrangement (a) and (d) as examples. For arrangement (a), the magnetic force (as shown in Fig. 5.5(a)) is alternative in one cycle. Therefore, at low frequencies, the plucking effect (the catch-and-release process) is quite significant, and the frequency up-conversion mechanism starts to be effective at low frequencies. However, for arrangement (d), the magnetic force is unidirectional, and its magnitude is much higher with the same relative positions. At low frequencies, the beam motion is determined by the form of the magnetic force. The plucking effect is limited. Only when the plucking frequency is high enough, can the beam plucking effect be effective. This is indicated by the relatively low input energy at low frequencies for arrangement (d), although the magnetic force is larger for arrangement (d). The performance for arrangement (d) becomes superior at high frequencies due to the plucking effect and the stronger magnetic force.

The ripples of the input energy at high frequencies are caused by the interaction between the beam vibration generated by two excitations, as shown in Fig. 5.7. When the excitation frequency is low (20 Hz in Fig. 5.7(a)), the beam has already rung down before the second excitation comes, but at high frequency (60 Hz in Fig. 5.7(b)), the beam is still vibrating when the second excitation comes. When the beam vibrating direction and the magnetic rotation direction are in the same direction, the input energy is enhanced, but when their motions are opposite, the input energy is decreased. 30 Hz (i.e. $T \approx 33$ ms) corresponds to the duration of the ring-down process.

In order to determine the best arrangement of the magnetic coupling for the piezoelectric rotational energy harvester, the average input power is calculated, as shown in Fig. 5.8. Arrangement (a) has the highest average input power at frequencies below 30 Hz; the average input power of arrangement (d) surpasses that of arrangement (a) when the frequency is over 30 Hz.

In terms of the magnetic force, it can be attractive or repulsive. Different directions of the force also affect the dynamics of the piezoelectric beam and the performance of power extraction. In the following analysis, different directions of the magnetic force for arrangement (a) and arrangement (d) are considered. Fig. 5.9 illustrates the average input power to the piezoelectric beam against rotational frequency for different configurations. For arrange-
Figure 5.6: Input energy to the piezoelectric beam per excitation cycle for different arrangements of magnetic coupling.

Figure 5.7: Magnetic force and beam tip displacement of arrangement (d) at different excitation frequencies, showing the interaction between the beam vibrations between two excitation cycles. (a) 20 Hz and (b) 60 Hz.

ment (d), when the magnetic force is attractive, the input power is always much higher than that of other configurations.

However, there is a drawback to this configuration. As shown in Fig. 5.3(d), when the magnetic force is attractive, the beam is pulled towards the driving magnet. Then the gap between the magnets is reduced. As a result, the attractive force is stronger. In some case, there is a risk of collision between the driving magnet and the tip magnet, which damages the reliability
Figure 5.8: Average input power to the piezoelectric beam for different arrangements of magnetic coupling versus frequency of the rotating magnet.

Figure 5.9: Average input power for the attractive and the repulsive orientations for arrangement (a) and (d).
of the system. Therefore, some protection mechanisms should be designed before using this arrangement, which might increase the complexity of the harvester. For arrangement (a), the possibility of collision is eliminated by maintaining the piezoelectric beam and the driving magnet moving in two parallel planes.

5.2.3 Electrical Behaviour of the Harvester

The mechanical strain in the cantilever beam is converted to electrical energy by the piezoelectric effect. The conversion is considered theoretically in Eq. 5.1 and Eq. 5.2. Using the above theories, the electrical behaviour of the harvester is studied for arrangement (a)-repulsive. Fig. 5.10 shows the output voltage of the beam on a 100 kΩ load under magnetic plucking. The rotational frequency of the driving magnet is 20 Hz (50 ms), whereas the beam vibrates at approximately 300 Hz (3.35 ms) which is its natural frequency ($f_n$). Frequency up-conversion is, therefore, achieved by this plucking mechanism, allowing the device to harvest low speed rotational energy with a broad bandwidth.

![Output voltage of the piezoelectric beam under magnetic plucking](image)

Figure 5.10: Output voltage of the piezoelectric beam under magnetic plucking, showing the implementation of frequency up-conversion.

Another special behaviour of the harvester is the ring-down pattern. For the first four peaks (shown in the red circle in Fig. 5.10) after the initial plucking, they are not typical damped free vibrations, as the driving magnet still affects the tip magnet during this period. The input power of the driving magnet varies with time, as shown in the figure. After that period,
there is no interaction between the magnets, and the vibration is then in a damped-free-vibration form.

In order to understand the behaviour of the piezoelectric beam under different rotational frequencies, a frequency sweep is presented in Fig. 5.11. This figure shows the instantaneous, RMS output voltage and output power versus rotational frequency of the driving magnet. The rotational frequency increases with a constant acceleration. The acceleration is small enough to ensure the accuracy of the response for specific frequencies.

Figure 5.11: Instantaneous and RMS output voltage on a 100 kΩ load against rotational frequency of driving magnet for arrangement (a)-repulsive.

Fluctuation of the output voltage is also exhibited in this figure. The reason has been shown in Fig. 5.7. There is a relationship between the frequency of each peak and the resonant frequency \( f_{rc} \) of the piezoelectric beam. For ease of discussion, the peaks from right to left in Fig. 5.11 are called the first peak, the second peak, etc. The frequency of the \( i^{th} \) peak can be expressed as

\[
f_i = \frac{f_{rc}}{n - i}, \tag{5.4}
\]

where \( n \) is the number of driving magnets on the revolving host.

The output voltage at the natural frequency is not the highest, as shown
in Fig. 5.11. This is caused by the complicated form of the magnetic force for arrangement (a) (shown in Fig. 5.4). In addition, the RMS output voltage and power at low frequencies is reasonably high, which shows the device has a broad bandwidth at low frequencies.

5.3 Experimental Validation

5.3.1 Experimental Set-Up

In order to verify the accuracy of the theoretical model, an experimental validation was carried out. The experimental set-up is illustrated in Fig. 5.12. The piezoelectric beam was clamped on a beam holder at one end. The holder was installed on an adjustable platform which is capable of precisely controlling the position of the piezoelectric beam in three directions. A high speed stepper motor (Phidgets 3303) with a step angle of 1.8° was placed underneath the beam with a revolving plate mounting on the motor’s shaft. The motor was driven by a bipolar motor control circuit (Phidgets 1067) which has a position resolution of 1/16 step. The acceleration is programmable to achieve any desired rotational speed. The maximum achievable speed is 4800 rpm (or 80 Hz) which is not usually attainable using stepper motors. A magnet was mounted on the revolving plate as the driving magnet. Magnetic plucking is formed by the driving magnet and the tip magnet at the beam’s free end. The relative position

Figure 5.12: Experimental set-up of the rotational energy harvester for the arrangement of (a)-repulsive.
of the magnets is regulated by the adjustable platform. A laser displacement sensor was also adopted in the experiment to measure the vibration of the piezoelectric beam. The output of the piezoelectric beam was directly connected with a 100 kΩ resistive load. In this experiment, arrangement (a)-repulsive was used due to its advantage of high output power and no potential risk of collision between the magnets. The parameters of the harvester are the same as those listed in Table 4.1.

In this experiment, due to the high driving torque of the stepper motor, the rotor’s motion is not affected by the interaction of the magnets. Therefore, the instantaneous velocity of the rotor is unaffected by the magnetic force, whereas in some cases, such as miniature turbines, the torque provided by the rotating host is limited. In these cases, the instantaneous velocity of the rotor is affected by the subsequent transduction methods, and the rotational velocity is not constant within one rotation.

5.3.2 Results and Discussion

Fig. 5.13 shows the theoretical and experimental tip displacement and output voltage of the rotational energy harvester. In this comparison, the stepper motor rotates at 30 Hz and the resonant frequency \( f_n \) of the piezoelectric beam is about 300 Hz. It is shown that the tip displacement and voltage fit well between the theoretical and the experimental results in terms of amplitude, frequency and damping. The mismatching at the beginning of each plucking could be caused by the phase difference generated in the experiment. Overall, this comparison indicates the capability of the theoretical model in analysing the performance of the harvester.

Then, a frequency sweep analysis was conducted to examine the behaviour of the harvester for different rotational frequencies. In the experiment, the motor rotated from static to 55 Hz with a constant acceleration. The instantaneous output voltage and RMS voltage and power against rotational frequency is illustrated in Fig. 5.14. Fluctuation of the output voltage can be observed as well from both the output voltage and power in the experimental results. The frequencies of the peaks from right to left are 50 Hz, 42.5 Hz, 37.5 Hz, etc. They are \( 1/n \) of the natural frequency of the beam (n=6,7,8,...). The result verifies the conclusion made in Eq. 5.4. From the RMS output voltage, it is seen that the system has a wide bandwidth at low frequency
Figure 5.13: Comparison of the electromechanical behaviour of the harvester acquired in theory and in experiment, showing the accuracy of the theoretical model. (a) Comparison of the tip displacement and (b) comparison of the output voltage.

with high power output. The RMS output power is higher than 15 µW between 15 Hz to 40 Hz.

5.4 Conclusions

In this chapter, a methodology for low-speed broadband rotational energy harvesting is presented. Frequency up-conversion, a mechanism to convert low rotational frequency to high vibration frequency, was implemented by
magnetic plucking of the piezoelectric beam. Two magnets, installed on a revolving host and the free end of the piezoelectric beam, were employed in the harvester to facilitate the magnetic plucking. The non-contact plucking mechanism improved the reliability of the system.

A theoretical model was established based on the distributed-parameter method to analyse and to optimize the harvester. Different configurations in terms of beam bending direction, magnetization, magnet rotation direction and initial relative position of the driving magnet were considered. The arrangement of (a)-repulsive was chosen as the best due to its merits of high output power and no potential collision. The electrical behaviour of the harvesters was also examined. The analysis shows that the harvester has a wide bandwidth at low rotational speeds, and the ripples at high frequencies are related to the beam’s natural frequency.

An experimental validation was carried out to verify the theoretical model. The experimental results fit well with the theoretical analysis. The harvester had more than 15 μW output power in the rotational frequency range from 15 Hz to 40 Hz. This rotational harvester has the advantage of compactness, simplicity in structure, high reliability (due to the non-contact plucking)
and wide operating bandwidth at low frequency. The power fluctuation at high frequencies is caused by the interference between the vibrating beam and the coming driving magnet. This issue can be alleviated by damping the beam oscillation more quickly in each plucking cycle. Solutions will be explored in the following chapters.

The main contributions of this chapter include the performance comparison of different harvester configurations and the study of the electromechanical dynamics of this frequency up-converting harvester in the time and frequency domains. This work is beneficial to understand the dynamics of frequency up-converting harvesters and also provides a guideline for harvester design. Power fluctuation at high rotational frequencies is explained in detail, indicating that improved electrical damping is a potential solution.
6 Broadband Energy Harvesting using Bi-Stability

This chapter reports the design and implementation of a bistable piezoelectric energy harvester to alleviate the power fluctuation and to enhance the output over a wide bandwidth. The bistable behaviour was realized by introducing a fixed magnet above the tip magnet at the cantilever’s free end. The repulsive magnetic force between these magnets creates two equilibrium positions for the piezoelectric beam. The harvester was designed to operate in the high energy orbit (double-well oscillation mode) to extract more energy from the rotational energy source. Harvesters with and without bi-stability were compared experimentally, showing the difference of power extraction on both the output power and bandwidth. The proposed method partially addresses the issues of power fluctuation at high rotational frequency, and enhances the output over a wide bandwidth.

6.1 Introduction

In order to broaden the operating bandwidth, nonlinear behaviour has been intentionally introduced in energy harvesters and intensively studied in the last decade. Harvesters using monostable and bistable characteristics have been developed to enhance the energy harvesting performance over a wide bandwidth [150]. Hardening and softening effects are two typical operating modes for monostable energy harvesters (nonlinear harvesters with one potential well). These modes can be achieved by applying tensile or compressive pre-loads in the axial direction of cantilever structures [151] to extend the operating frequency bandwidth close to the resonant frequency of its linear counterpart. For monostable harvesters, there are two steady state solutions, namely high-energy state and low-energy state (a well-known character of the forced-Duffing’s equation) [152]. It is critical
to design the harvester to operate in the high-energy state to improve the conversion capability.

Bi-stability is another non-linear behaviour that has been investigated intensively. Double potential wells are normally introduced in these devices using pre-loaded forces [153], magnetic levitation [154] or even residual stress generated during the fabrication process [155]. When these devices oscillate between the wells (snap-through effect), improved vibration amplitude and velocity can be achieved. Therefore, the output power is enhanced over a broader frequency bandwidth. However, the performance of these harvesters is generally input excitation-related [156,157]. These devices can operate in a single-well (intra-well) mode or double-well (inter-well) mode according to the excitation level. The performance can be inferior to their linear counterpart when operating in the single-well mode [158].

In this section, bi-stability and frequency up-conversion are combined in a rotational energy harvester to achieve enhanced performance over a wide operating bandwidth at low frequency. A theoretical model was established to study the electromechanical dynamics of this harvester. Key design factors to maintain the harvester operating at the high energy orbit were investigated. Experimental studies were also conducted to verify the design and theoretical study. This design provides a practical solution for low-frequency broadband rotational energy harvesting.

### 6.2 Harvester Design and Operating Principle

The schematic of the bistable rotational energy harvester is illustrated in Fig. 6.1. Rotational motion can be acquired from external rotating sources such as air turbines, vehicle wheels or rotating machinery.

In this design, in order to enhance the performance of the harvester over a wider bandwidth, a fixed magnet is mounted above the tip magnet, as shown in Fig. 6.1. This magnet and the tip magnet create a repulsive magnetic force which introduces two stable positions (equilibrium states) for the beam, as illustrated in Fig. 6.1. The beam stays in either Stable Position 1 or Stable Position 2, when the system is static. Therefore, in dynamic conditions, there are different oscillating modes, including single-well vibration and double-well vibration in periodic or chaotic manners. The stored energy in the beam varies significantly for different oscillating modes.
For the periodic double-well mode, vibration amplitude and velocity are the highest among these three modes. More power can be generated by this motion. Hence, the bistable mechanism allows the frequency up-converting harvester to generate more power at low rotational frequency.

![Schematic diagram of rotational energy harvester using frequency up-conversion and bistability.](image)

Figure 6.1: Schematic diagram of rotational energy harvester using frequency up-conversion and bistability. The transparent bent beams represent two stable positions of the beam. The red and blue colours in the magnets indicate the magnetization direction.

6.3 Theoretical Modelling and Analysis

The combination of bistability and frequency up-conversion makes the dynamics of the harvester more complicated. Therefore, it is necessary to build a theoretical model to study the electromechanical dynamics. Moreover, in order to enhance the performance of the harvester, periodic double-well vibration is required to generate a large vibration amplitude and velocity. In that sense, theoretical modelling is indispensable for the harvester design.

Fig. 6.2 depicts the force diagram of the system. Basically, there are two external magnetic forces ($F_{dt}$ and $F_{ft}$) applied on the piezoelectric beam by the driving magnet and fixed magnet respectively. Fig. 6.2(b) shows the decomposition of external forces in each axis. In the z-axis, the forces are in opposite directions. Due to the similar magnet arrangements and dimensions, the resultant force in this direction is negligible. In the x-axis,
these magnets are well aligned initially when the beam is in the original shape \((d_{x0} = 0)\). Due to the small deformation of the cantilever beam, the limited gap generated by beam bending in the x-axis is not enough to introduce a significant force component in this direction. Therefore, in this analysis, the external forces \((F_{dt}^{y} \text{ and } F_{ft}^{y})\) in the y-axis (beam bending direction) are the main consideration.

Figure 6.2: Force diagram of the bistable energy harvester. (a) Interaction between the tip magnet and fixed magnet. The force between them is repulsive. Dimensions and relative positions are illustrated for modelling. (b) Force diagram of the tip magnet enforced by the driving magnet and fixed magnet \((F_{dt}^{y} \text{ means the force applied by the driving magnet on the tip magnet along the y-axis.})\).

For the bistable beam in Fig. 6.1 the potential energy stored is the sum of the elastic potential energy \((U_e)\) of the beam and the magnetic potential energy \((U_m)\) of the tip magnet. Therefore,

\[
U_t = U_m + U_e \tag{6.1}
\]

In order to calculate the magnetic potential energy stored in the magnets, the theoretical model proposed by Akoun and Yonnet is adopted \[132\]. The potential energy stored by the tip magnet can be expressed as

\[
U_m = \frac{j}{4\pi \mu_0} \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} \sum_{l=0}^{1} \sum_{p=0}^{1} \sum_{q=0}^{1} (-1)^{i+j+k+l+p+q} \psi(U_{ij}, V_{kl}, W_{pq}, r), \tag{6.2}
\]
where $J$ and $J'$ are the magnetization of the magnets, $\mu_0$ is the magnetic constant, and $\psi$ is a function of the magnet dimensions and their relative positions in 3 axes. The function $\psi$ is given by

$$\psi = \frac{U}{2} (V^2 - W^2) \ln(r - u) + \frac{r}{6} (U^2 + V^2 - 2W^2) + \frac{V}{2} (U^2 - W^2) \ln(r - V) + UVW \tan^{-1}\left(\frac{UV}{rW}\right).$$  \hspace{1cm} (6.3)$$

where $U, V, W$ and $r$ were defined in Eq. 4.5(a) - (d).

The magnetic force between two magnets in the $y$-axis can thus be calculated using

$$F_{st}^y = \frac{J \cdot J'}{4\pi\mu_0} \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} \sum_{l=0}^{1} \sum_{p=0}^{1} \sum_{q=0}^{1} (-1)^{i+j+k+l+p+q} \cdot \phi_y,$$  \hspace{1cm} (6.4)$$

where $F_{st}^y$ means the force applied on the tip magnet by one source (driving magnet or tip magnet) in the $y$-axis, and $\phi_y$ is the dimension function which can be expressed as Eq. 4.4.

The elastic potential of the piezoelectric beam under bending motion can be given by

$$U_e = \frac{1}{2} YI \int_0^{L_p} \left(\frac{\partial^2 v(x,t)}{\partial x^2}\right)^2 dx,$$  \hspace{1cm} (6.5)$$

where $YI$ is the bending stiffness of the beam, and $v(x,t)$ is the transverse deformation of the beam. In order to acquire the deformation of the beam, a theoretical model for analyzing its dynamics is necessary. Erturk and Inman developed an accurate model for piezoelectric beams under base vibration using a distributed-parameter method [134]. In this chapter, adaptation has been made for the bistable rotational energy harvester model. The magnetic forces, $F_{st}^y$ and $F_{ft}^y$, are considered as external forces applied at the free end of the piezoelectric beam, and the tip magnet is regarded as a proof mass.

Then, the dynamics of the bistable piezoelectric beam under tip excitation can be written as

$$YI \frac{\partial^4 v(x,t)}{\partial x^4} + c_s I \frac{\partial^5 v(x,t)}{\partial x^5} + c_d \frac{\partial v(x,t)}{\partial t} + m \frac{\partial^2 v(x,t)}{\partial t^2} - \vartheta_r V(t) \left[\frac{d\delta(x)}{dx} - \frac{d\delta(x - L_p)}{dx}\right] = \left[F_{st}^y(t) - F_{ft}^y(t)\right] \delta(x - L_p),$$  \hspace{1cm} (6.6)$$

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and
\[
\frac{C_p}{2} \frac{dV(t)}{dt} + \frac{V(t)}{R_l} + \bar{e}_{31} h b \int_0^{L_p} \frac{\partial^3 v(x,t)}{\partial x^3 \partial t} dx = 0.
\] (6.7)

The average output power of the harvester on a resistive load \((R_l)\) is given by
\[
P_{\text{avg}} = \frac{\omega}{2\pi} \int_{t_0}^{t_0 + \frac{2\pi}{\omega_{tr}}} \frac{V^2(t)}{R_l} dt,
\] (6.8)

where \(\omega_{tr}\) is the rotational frequency of the driving magnet.

### 6.4 Dynamics of Bistable Harvester

The theoretical equations in Section 6.3 were modelled in Matlab/Simulink. A set of parameters is given in Table 6.1 for numerically investigating the dynamics of the harvester. The material properties are the same as those in Chapter 4, and the differences are the beam length and the dimensions of the fixed and driving magnets. The potential energy, operating modes, beam vibration displacement, output voltage and output power are discussed in this section.

#### Table 6.1: Parameters for piezoelectric rotational bistable energy harvesters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value in Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_p)</td>
<td>Length of beam</td>
<td>33.5 mm</td>
</tr>
<tr>
<td>(b_p)</td>
<td>Width of beam</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>(h_p)</td>
<td>Thickness of piezoelectric layer</td>
<td>0.1 mm</td>
</tr>
<tr>
<td>(h_s)</td>
<td>Thickness of substrate layer</td>
<td>0.1 mm</td>
</tr>
<tr>
<td>(a \times b \times c)</td>
<td>Dimension of tip magnet</td>
<td>0.5 \times 0.5 \times 0.5 mm</td>
</tr>
<tr>
<td>(A \times B \times C)</td>
<td>Dimension of fixed &amp; driving magnet</td>
<td>0.75 \times 0.75 \times 0.75 mm</td>
</tr>
<tr>
<td>(B_r)</td>
<td>Remnant flux density of magnets</td>
<td>1.17 T</td>
</tr>
<tr>
<td>(\rho_p)</td>
<td>Density of piezoelectric material</td>
<td>7700 kg/m(^3)</td>
</tr>
<tr>
<td>(r_m)</td>
<td>Rotating radius of driving magnet</td>
<td>12 mm</td>
</tr>
<tr>
<td>(\rho_s)</td>
<td>Density of substrate material</td>
<td>1500 kg/m(^3)</td>
</tr>
<tr>
<td>(\bar{e}_{31})</td>
<td>Piezoelectric constant</td>
<td>(-22.2 \text{ V} \cdot \text{m/N})</td>
</tr>
<tr>
<td>(d_{31})</td>
<td>Piezoelectric charge constant</td>
<td>(-315 \times 10^{-12} \text{ m/V})</td>
</tr>
<tr>
<td>(\bar{e}_{r33})</td>
<td>Piezoelectric relative dielectric constant</td>
<td>4500</td>
</tr>
<tr>
<td>(Y_s)</td>
<td>Young’s modulus of substrate</td>
<td>140 GPa</td>
</tr>
</tbody>
</table>

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6.4.1 Potential Energy of Bistable Harvester

For potential energy analysis, only the elastic energy of the beam and the magnetic potential energy between the tip magnet and the fixed magnet matter. Therefore, the driving magnet is not considered in this analysis. The potential energy of the harvester is depicted in Fig. 6.3. As illustrated in this figure, there are two potential wells for this harvester as a result of the combination of the elastic potential energy and the magnetic potential energy. Therefore, there are two equilibrium positions. The potential depth $\Delta P$ determines the input energy required to enable the beam to vibrate between the potential wells.

![Figure 6.3: Potential energy of the bistable rotational energy harvester. The gap between the fixed magnet and the tip magnet $h_{fit}$ is 2.5 mm. There is no gap initially between the fixed magnet and the tip magnet in the beam bending direction when the beam is not bent.](image)

In addition, from this figure, we can also find that for the bistable harvester, the required input energy for the beam to reach a specific deflection level is lower than that of the same harvester without bi-stability. For example, in order to get a beam displacement of 2.5 mm, the energy required for a normal piezoelectric beam ($\Delta P_1$) is much larger than that ($\Delta P_2$) of a bistable beam, as shown in Fig. 6.3. This also explains that for the same input, bistable energy harvesters can achieve a larger beam displacement.
and harvest more power when periodically vibrating between two potential wells.

For the two potential wells, the potential depth ($\Delta P$) is normally the same (i.e. the wells are symmetrical), but asymmetrical shapes can be achieved by adjusting the magnetic potential. Fig. 6.4(a) shows the method to do this. An initial offset ($\Delta d_{tf}$) in the beam bending direction is set between the fixed magnet and tip magnet. This initial offset changes the equilibrium positions of the beams, resulting in the variation of the potential well shapes. Fig. 6.4(d) depicts this variation for different values of offsets. The potential depths change for each well: one becomes deeper; the other shallower. Overall, the required input energy increases in order to maintain the system operating between two wells, which is a disadvantage to the system, especially for inertial energy harvesting, where the driving force direction is alternating, and lower potential barriers are desirable.

![Diagram of asymmetrical potential wells for the harvester](image)

**Figure 6.4:** Introduction of asymmetrical potential wells for the harvester. (a) Front view of the beam, tip magnet and fixed magnet arrangement. (b) Top view of the arrangement (with no offset between the tip magnet and fixed magnet in the beam bending direction.). (c) Top view with an offset ($\Delta d_{tf}$). (d) Potential energy of the harvester with different offsets in the beam bending direction.

However, in rotational energy harvesting, the driving force on the tip magnet is always in the same direction for each plucking when rotational motion is continuous. Therefore, a fixed beam initiation position is benefi-
cial to stabilize the output performance of the harvester. The asymmetric potential shape is important to increase the probability of the system to settle into the ideal potential after each excitation.

6.4.2 Operation Mode of the Bistable Harvester

For bistable energy harvesters, there are different operating modes. Considering the relationship between the input energy and the potential barrier, bistable harvesters can vibrate in one potential well, vibrate between two wells periodically, or vibrate chaotically between the single-well mode and the double-well mode.

Fig. 6.5 shows the beam tip displacement and velocity vs. displacement phase trajectory plots with an overlapped Poincaré map for different operation modes. The Poincaré map is plotted by indicating the beam’s velocity and displacement at each of a set of specific time points in each excitation cycle. This indicates the stability of the oscillation mode.

When the input energy is not enough to conquer the potential barrier, as shown in Fig. 6.5(a) and (d), the beam is confined in a single well. The beam displacement and velocity are limited. When the input is close to the potential barrier, the system vibrates randomly between the single-well mode and double-well mode, as shown in Fig. 6.5(b) and (e). The vibration displacement and velocity increase to some extent, but the performance is unstable and unrepeatable, as shown in Fig. 6.5(b) and the Poincaré map in Fig. 6.5(e). The system can also operate in the double-well mode periodically when the input energy is enough to penetrate the barrier for each excitation cycle (Fig. 6.5(c) and (f)). In this case, the performance is stable, and the duration for the beam to vibrate in the double-well mode is enhanced.

In addition, the mechanism of frequency up-conversion is also clearly illustrated in the displacement curves in Fig. 6.5(a), (b) and (c). The excitation frequency is 10 Hz for the driving magnet, but the beam is vibrating at 136 Hz. For each excitation cycle, the beam is plucked by the magnetic force first, then oscillates at resonance before the second excitation approaches. For the case of periodic double-well vibration, in each excitation cycle, the beam first oscillates between two wells for a few cycles. Then the beam settles into one well and operates in the single-well mode. The cycles of
Figure 6.5: Beam tip displacement in time domain ((a), (b) and (c)) and velocity vs. displacement phase trajectory plots with an overlapped Poincaré map as black circles ((d), (e) and (f)) for three operating modes of the rotational bistable energy harvester. (a) and (d) Single-well vibration. (b) and (e) Double-well chaotic vibration. (c) and (f) Double-well periodic vibration. In this calculation, the input force is the same. The magnetic potential energy is adjusted to get different operating modes.
double-well vibration can be adjusted by changing the potential energy of the tip magnet or the input energy from the driving magnet.

For asymmetrical potential well arrangements, the dynamics of the system vary significantly. Fig. 6.6 depicts the dynamics of harvesters with different offsets between the tip magnet and fixed magnet. The initiation or termination position for each cycle can be well controlled by setting appropriate offset values. This initial position also affects the input energy, and then the output power (as indicated in Fig. 6.6(e) and (f)) of the harvester, but still in both cases, the beam operates in the double-well mode.

With the variation of the offset from 0.5 mm to -0.2 mm, the initial position of the beam changes from one well to the other, as shown in Fig. 6.6(c) and (d). It is evident that during this transition, more complicated dynamic behaviour exists. It is worth studying the output of the harvester for different offsets and also different gaps ($h_{tf}$) between the fixed magnet and tip magnet in the z-axis (Fig. 6.1).

### 6.4.3 Power Output Analysis

The output power with different gaps in the z-axis and offsets in the y-axis is calculated using Eq. (6.8). Fig. 6.7 illustrates the variation of the output power for a given input force ($h_{td} = 3.2$ mm) and different fixed-tip magnet arrangements. The transition of the oscillation modes from single-well vibration to periodic double-well vibration can be identified. From this figure, the best operation region for this given input can be also determined ($3.2$ mm $< h_{tf} < 4$ mm and $-0.5$ mm $< \Delta d_{tf} < 1$ mm). The maximum output ($157 \mu$W) happens at $h_{tf} = 2.6$ mm and $\Delta d_{tf} = 0.4$ mm, but the performance in that local region varies significantly.

It is evident that the input force also affects the dynamics of the harvester. By changing the relative gap ($h_{td}$ in Fig. 6.1), the input force can be adjusted. The variation of the output power as a function of the driving force (determined by $h_{td}$) and force for bistability (determined by $h_{tf}$) is illustrated in Fig. 6.8. The variation of beam oscillation modes is indicated from the changes of the output power for different combinations of force from the driving magnet and force from the fixed magnet. The output is limited when the gap $h_{tf}$ is below 2.5 mm. The beam operates in the single-well mode in this range. Then, the output experiences a chaotic increase when the gap $h_{tf}$ is in the range of 2.5 mm to 3 mm, and the beam vibrates between single-well and double-well mode chaotically. When the gap $h_{tf}$ is larger than 3 mm, the performance is more stable and enhanced. In this range, the double-well vibration is in operation. It is worth mentioning that the operating frequency in this calculation is fixed (8 Hz). This frequency
affects the input energy of the piezoelectric beam. Therefore, it also affects the threshold for the beam to operate in the double-well mode. Therefore, an appropriate margin for the gap $h_{tf}$ compared to the threshold for the double-well operation is required when the harvester is operating with varying excitation frequencies.

Certainly, this transition is also relevant to the input driving force, which provides the energy to conquer the potential barrier generated by the fixed magnet. As shown in Fig. 6.8, a larger driving force (by reducing the gap $h_{td}$ between the driving and fixed magnet) is desirable to achieve higher

![Figure 6.6: Beam displacement, velocity-displacement phase trajectory and output voltage for bistable energy harvesters with asymmetrical potential wells.](image)

(a), (c) and (e) are for $\Delta d_{tf} = 0.5$ mm. (b), (d) and (e) are for $\Delta d_{tf} = -0.2$ mm.
output power, but the elastic limit of the material should be considered to ensure the reliability of the system. If the stress generated on the beam by the driving force is close to or exceeds the elastic limit, the piezoelectric material faces a high possibility of degradation [159]. This is the reason why 3 mm is chosen as the minimum gap ($h_{td}^\text{min}$) between the driving magnet and the tip magnet, by considering the stress generated by the driving magnetic force (determined by $h_{td}$).

In order to illustrate the advantage of a bistable mechanism in this rotational energy harvester, a comparison of output power between harvesters with (red surface) and without (blue surface) bi-stability is provided in Fig 6.8 as well. A distinct enhancement can be observed in this figure when the bi-stable harvester operates in the high energy orbit (double-well) for the same input driving force. This enhancement can be explained by the potential barrier reduction by the bistable mechanism. As shown in Fig. 6.3 to achieve a beam displacement of 2.5 mm, the potential barrier for a well-designed bistable beam is much lower than that of a beam without
Figure 6.8: Output power of the harvester for different driving forces (determined by the gap $h_{td}$ between the tip and driving magnet) and forces for bi-stability (determined by the gap between the tip and fixed magnet). The surface in red shows the output of the harvester with bi-stability, and the surface in blue is the output of harvester without bi-stability. This blue surface is extended from a single curve without the fixed magnet (without $h_{tf}$), providing a reference for comparison. The calculation is performed at 8 Hz.

Another consideration for energy harvester design is the operating bandwidth. As in many practical applications, the excitation (rotational) frequency in the ambient environment is not constant, such as the rotational frequency of moving vehicles. Therefore, it is desirable to maintain high output power over a broad bandwidth. Fig. 6.9 provides a comparison of the output power between a bistable harvester and a harvester without bi-stability (without the fixed magnet in Fig. 6.1). The output power from the bi-stable harvester is much higher than the harvester without bi-stability when the driving frequency is lower than 10 Hz, over the whole range of the gap between the driving and fixed magnet. When the frequency is higher than 10 Hz, the bistable harvester is inferior, but the output power for both harvesters is sensitive to the input driving frequency, and varies significantly. The reason the bistable harvester has a significant power decrease at high frequency is that the phase mismatching between the driving magnetic force and the motion direction of the vibrating beam becomes...
severe at high frequency. This mismatching affects the instantaneous input power to the piezoelectric beam. At a certain point (e.g. 10 Hz), the input power is insufficient to conquer the potential barriers of the bistable beam. The beam, as a result, operates in the single-well mode with low output power. Therefore, for this type of harvester, it is desirable to operate in the low-frequency range (e.g. $f < f_{re}/10$), and the bistable mechanism is beneficial to enhance the performance of the harvester at low frequency.

### 6.5 Experimental Validation

#### 6.5.1 Experimental Set-Up

An experimental set-up was built in order to implement the design and to verify the theoretical analysis, as shown in Fig. 6.10. The set-up is similar to that in Fig. 5.12. The different is that a fixed magnet was placed above the tip magnet on another positioning stage for introducing bi-stability.
Figure 6.10: Experimental setup for the bistable frequency up-converting rotational energy harvester.

6.5.2 Results and Discussion

Fig. 6.11 illustrates the dynamics of the harvester operating at different oscillation modes, which verify the theoretical analysis. Five operating modes were studied in the experiment, including double-well vibration with varying initiation or termination position (positive or negative well, (a), (f) and (k)), double-well vibration with fixed initiation position either in the negative well ((b), (g) and (i)) or in the positive well ((c), (h) and (m)), and single-well vibration within the negative well ((d), (i) and (n)) or the positive well ((e), (j) and (o)). The dynamics of the harvester vary significantly for different modes. When the harvester operates in the double-well mode, it has a high energy orbit ((f), (g) and (h)), which indicates that the output is higher. However, when the harvester operates in the single-well mode, the vibration displacement and output voltage are limited. Therefore, a reasonable relationship between the input force and the height of the potential barrier should be chosen to achieve effective harvesting. In addition, from the displacement-velocity portrait (f), (g) and (h), we can conclude that a more stable performance can be realized when the double-well vibration has a fixed initiation position (negative or positive well).

In order to identify the optimal load resistance, load resistors with different values were directly connected to the harvesters with and without bi-stability. The RMS output voltage and power are illustrated in Fig. 6.12.
Figure 6.11: Experimental results: Dynamics of different operating modes of the bi-stable frequency up-converting rotational energy harvester, including beam tip displacement (a)-(e), displacement-velocity phase portrait (f)-(j) and output voltage on a 150 kΩ resistive load (k)-(o), with columns from left to right corresponding to $\Delta d_{tf} = 0, -0.1, 0.1, -0.3$ and 0.3 mm respectively. The different operating modes were achieved by adjusting the relative offset $\Delta d_{tf}$ as shown in Fig. 6.4.
Figure 6.12: RMS output voltage and power against load resistance for harvesters with and without bi-stability. The driving frequency was 8 HZ for all tests. The driving force was the same ($h_{id} = 3.2$ mm). The bistable beam was oscillating in the double-well mode ($h_{tf} = 3.2$ mm and $\Delta d_{tf} = 0.1$ mm).

The optimal load resistance (129 kΩ) was the same for harvester with and without bi-stability. Fig. 6.13 provides a comparison of the tip displacement and output voltage between a bistable energy harvester operating in the double-well mode with a fixed initiation position and a harvester without bi-stability. The operating principle of the double-well mode is clearly demonstrated in Fig. 6.13(a). In one excitation cycle, the harvester first vibrates between two potential wells for a few cycles, and then settles in one stable position. In this case, the harvester always terminates in the negative potential well, and the next excitation cycle also initiates from this well. Therefore, this mode is called the double-well vibration with a fixed initiation or termination position. In addition, the concept of frequency up-conversion is also implemented as shown in these figures. The harvester is driven at 8 Hz, but the cantilever beam always operates at its resonance.

One significant phenomenon we can discover in this comparison is that the harvester with bistable mechanism can achieve a higher output voltage and damping coefficient by operating between two wells and then settling in one fixed potential well. From the beam displacement and output voltage, the values in the beginning of each excitation are much larger for the bistable harvester than those for the harvester without bi-stability, resulting in an improved performance. At the end of each excitation, the displacement and output voltage from the bistable beam are much lower than those from the harvester without bi-stability. Therefore, the beam in the bi-stable harvester is more likely to be static before the forthcoming excitation force appears, avoiding the interference between the vibrating beam and the coming driving magnetic force, which might affect the performance. This interference can already be observed in the harvester without bi-stability in Fig. 6.13(c).
and (d). If the driving frequency is higher, this interference will be more severe. In addition, the improved output voltage is also beneficial to the rectification process if the power management circuit is considered. The improved voltage allows more charge to flow through the bridge rectifiers, so that more power can be stored by the power management circuit.

The enhancement of the bistable harvester can be contributed by the improved input energy due to bi-stability or the improved conversion efficiency of the piezoelectric transducer also due to bi-stability. In order to study the contribution, a theoretical study was conducted using the same operating conditions as those in Fig. 6.13. The comparison is illustrated in Fig. 6.14. From the instantaneous input power curve (e) and (f)), the performance for the bi-stable harvester has a larger input energy in each excitation cycle (32.1 µJ VS. 21.5 µJ), which means this harvester is capable of capturing more power from the environment with the same plucking force. The output energy in each excitation cycle for the bistable harvester is also larger (12.5 µJ VS. 7.1 µJ). The conversion efficiencies for harvesters with and without bi-stability can be calculated using the input and output power of the piezoelectric transducer. The efficiencies are 38.9% and 33.6% for har-

![Figure 6.13: Comparison of the tip displacement and output voltage between a bistable beam operating in the double-well mode ((a) and (b)) and a harvester without bi-stability ((c) and (d)). The driving frequency was 8 Hz. The load resistance was 129 kΩ.](image)
vesters with and without bi-stability. For the output power, the bistable harvester has a 76.1% enhancement. From this analysis, we can conclude that the improvement is mainly contributed by the increased energy from the bistable harvester operating in the double-well mode. The conversion efficiency has a marginal increase compared to the enhancement of input power.

In order to examine the performance of the bistable harvester over a broad bandwidth, a frequency sweep test was conducted, as shown in Fig. 6.15. The advantage of the bistable harvester operating in the double-well mode is evident over a broad band from 1 to 11 Hz. At some frequencies the output is
two times higher than that from a harvester without bi-stability, such as the output at 10 Hz. The general trend in this figure fits well with the theoretical analysis shown in Fig. 6.9. The differences in amplitude and frequency are caused by inaccurate structural and material parameters in the theoretical calculation compared to the practical harvester. It is worth mentioning that the bistable harvester should be designed to operate in the double-well mode in order to improve the performance, because when it operates in the single-well mode, the performance is inferior to the performance of the harvester without bi-stability, as shown in the comparison in Fig. 6.15.

![Figure 6.15: Output power versus driving frequency for bistable harvester operating in single-well and double-well mode and harvester without bi-stability. For all tests, the driving force was the same \( h_d = 3.2 \text{ mm} \). For the double-well mode, \( h_{tf} = 3.2 \text{ mm} \) and \( \Delta d_{tf} = 0.1 \text{ mm} \). For single-well, \( h_{tf} = 2.7 \text{ mm} \).](image)

When the frequency is higher than 11 Hz, the performance of the bistable harvester operating in the double-well mode experiences a significant variation and decrease. The reason is that the interference between the vibrating beam and the subsequent magnetic force has a stronger effect on the instantaneous input power into the piezoelectric beam. At a certain frequency, e.g. 10 Hz, the input energy is lower than the potential barriers of the bistable harvester. The harvester, as a consequence, operates in the single-well mode with limited output power. However, this power fluctuation issue also exists in harvesters operating in other modes or without bi-stability, which
means for frequency up-converting harvesters, the driving frequency should be much lower (e.g. $f < f_r/10$) than the beam’s resonant frequency in order to avoid interference between excitation cycles. Increasing the electrical damping further to damp the beam vibration more quickly is another solution to further extend the operating bandwidth.

6.6 Conclusions

In this chapter, a frequency up-converting rotational bistable energy harvester is studied theoretically and experimentally. Frequency up-conversion and bistable mechanisms are integrated in this rotational harvester to achieve effective energy harvesting at low frequency over a broad bandwidth. A theoretical model was established based on distributed-parameter modelling to study the electromechanical dynamics of the harvester.

Different operating modes were analysed, showing a significant variation of performance. Among them, periodic double-well vibration shows a high energy orbit. In order to design the harvester to operate in this mode, different parameters including the relative position of the magnets, the driving force and driving frequency were investigated, providing comprehensive understanding of the behaviour of this type of harvester. Asymmetric potential wells were achieved by adjusting the magnetic potential. This well shape provides a simple way to control the initiation (termination) position of the beam in each excitation cycle, enabling the harvester to have a stable output and well-controlled behaviour. The asymmetric potential well is beneficial for cases similar to the continuous rotation scenarios, where the excitation force on the harvester is always in the same direction.

An experimental validation was conducted to verify the theoretical analysis. A close match was achieved between them. Different operating modes for the bistable harvester were also observed in the experimental study. A comparison between the bistable harvester operating in the double-well mode and a harvester without bi-stability was provided, showing the improved output power and damping coefficient of the bistable harvester. The harvester was finally tested in the frequency domain, and the dominant performance of the bistable harvester operating in the double-well at low frequency (1-11 Hz) mode was verified. The frequency up-converting rotational bistable energy harvester presented in this chapter is ideal for low-
frequency rotational energy harvesting for low-power sensing applications, including tire pressure sensing, condition monitoring of trains and health tracking of human beings. This study also provides a comprehensive guidance for rotational energy harvester design using frequency up-conversion and bi-stability.
7 A Self-Powered Condition Monitoring System

In this chapter, a complete self-powered condition monitoring system which includes a bistable frequency up-converting harvester, a power management circuit and a wireless sensor node is presented. Design considerations and operating principles are discussed.

7.1 Introduction

Wireless sensor networks (WSNs) are envisioned to be an important enabling technology for intelligent sensing and condition monitoring applications. Initiated from military applications such as battlefield surveillance, WSNs are nowadays widely used in many industrial and consumer applications, including body sensor networks, structural health monitoring and environmental condition sensing [161,162]. However, there are still many challenges, such as cost, data security, and longevity, that impede their development [163,164]. Among these challenges, power supply is one of the main issues limiting the longevity of sensor nodes, as most of the sensor nodes are powered by batteries which require regular replacement or recharging [165]. With the decrease of power consumption of electronics, it is possible to address this issue by employing ambient energy sources, such as vibration, airflow and heat, as power supply.

Energy harvesting, as a way to convert ambient energy sources into electricity, has become a potential alternative to batteries. However, compared to the power requirement of general sensor nodes, the power generating capability is still insufficient, especially when harvesters collect energy from random and low-frequency motions [150]. Efforts have been made in both structure and power management circuit (PMC) design to improve the performance.
For structure design, piezoelectric material coated on a cantilever beam is the common design for kinetic energy harvesting, but this type of harvester faces the issues of narrow bandwidth and high resonant frequency (100’s of Hz) [160], which is detrimental to harvest kinetic energy with low and random frequencies. Mechanisms including frequency tuning [166], non-linear dynamics [167] and frequency up-conversion [60] have been investigated in order to broaden the operating bandwidth and to lower the excitation frequency requirement. In Chapter 6, a bistable frequency up-converting harvester has already been developed. This harvester exhibits an increased power generation capability over a wide bandwidth at low frequency.

In terms of PMC design, a full-bridge rectifier in parallel with a storage capacitor is the standard PMC for piezoelectric energy harvesters [168]. It has the advantage of simplicity and low cost, but this circuit has high energy losses caused by the clamped voltage ($V_c$) of the storage capacitor. Power generated by piezoelectric transducers cannot be stored when the output voltage is lower than $V_c$. Synchronized switch harvesting on inductor (SSHI) is a technique developed by Guyomar et al [169]. This technique achieves an enhanced harvesting capability compared to conventional full-bridge rectification circuits, effectively by achieving stronger electrical damping. These circuits normally operate with inertial energy harvesters under base excitation [170, 171]. Such harvesters typically operate with broadband, stochastic motion [172]; consequently, appropriate switching instances are difficult to determine reliably. The variation in width and amplitude of peaks, and the presence of abundant local extrema, leads to unnecessary switching events, which consume net power, as well as missed switching events, counteracting the advantage of the SSHI circuits [173]. Conversely, frequency up-converting harvesters, which have uniform, single-frequency output waveforms even for stochastic input motion, are ideal for SSHI circuits. Furthermore, the high electrical damping provided by SSHI is particularly advantageous for frequency up-converting harvesters, as it also alleviates the above-mentioned power fluctuation issue of frequency up-converting harvesters, since the beam vibration decays more rapidly.

Still, it is worth mentioning that for kinetic energy harvesters, it is necessary to power electronics in a low duty-cycle and intermittent mode, due to the gap between the power requirement of electronics and the power generation capabilities of harvesters. Therefore, super-capacitors or rechargeable
batteries are required to store the continuously collected energy and to pro-
vide sufficient energy for sensor nodes to operate intermittently [174]. Power
regulation circuits are also compulsory to stabilize the output voltage of en-
ergy harvesting systems in an acceptable range for general electronics.

In this chapter, a complete energy harvesting solution for wireless sensor
nodes in environmental condition monitoring is presented. Bi-stability and a
SSHI interface circuit are integrated in a frequency up-converting harvester.
This harvester is capable of harvesting energy effectively over a wide band-
width at low frequencies. A power regulation circuit is designed to stabilize
the output voltage at 3.3 V. The generated electricity is finally used to fully
power a wireless sensor node for environmental condition monitoring.

7.2 System Architecture and Design

Consideration

The aim of this study is to develop a complete low-frequency broadband
energy harvesting system for wireless sensor nodes in condition monitor-
ing applications. The harvester should be able to efficiently convert low-
frequency random motion energy (vibration, airflow, etc.) into electricity,
and the power management circuit needs to store sufficient energy in the
power storage at a required voltage level for subsequent loads. The general
architecture and design consideration for this energy harvesting system are
presented in Fig. 7.1. Four main sections, including energy sources, energy
conversion, energy storage and low-power applications, are considered in
this design.

For energy sources, different types of kinetic energy sources such as vibra-
tion, fluid flow and rotation are considered as the potential energy sources
for the proposed harvester. In practice, these energy sources often exhibit
the characteristics of low and random frequencies, such as human and ve-

cicle motions. In this study, the bistable frequency up-converting harvester
presented in Chapter 6 is adopted to harvest energy from low-frequency
random motions.

The output voltage from a piezoelectric harvester is in an AC form, so
a rectification circuit is required. A SSHI interface circuit is adopted in
the power management circuit to improve the conversion capability. After
rectification, the power is stored in a supercapacitor. A chargeable battery is also an option, but for WSNs, especially transceivers, they normally have a high operation current. Considering the current supplying capability and the device dimensions, a supercapacitor is more desirable. However, the self-discharging issue of supercapacitors needs to be considered to maintain the efficiency. Low self-discharging rate and short charging duration are necessary. The voltage on the capacitor varies in a wide range. A voltage stabilization circuit is designed to regulate the output voltage at a constant value (3.3 V).

Many self-powered sensing applications, including wearable or implantable devices, condition monitoring of machinery and safety and security monitoring in buildings, can be enabled by this energy harvesting system. However, due to the limited power generated by the harvester, this system is ideal for low-power electronics operating in an intermittent mode.

In the following sections, detailed design for each module will be discussed.

### 7.3 Bistable Frequency Up-Converting Harvester

#### 7.3.1 Harvester Design

In order to effectively harness low-frequency and random kinetic energy, an adapted bistable frequency up-converting harvester from Chapter 6 is presented in Fig. 7.2. The piezoelectric beam is fixed on a host at one end, and a tip magnet is mounted at the beam’s free end. A driving magnet is moving...
under the tip magnet along the trajectory denoted by the dashed lines. This driving magnet can be mounted on a pendulum for human motion energy harvesting [60] or on a turbine motor for flow energy harvesting [113]. As long as the driving magnet is in motion, the magnetic force between the driving magnet and tip magnet will pluck the piezoelectric beam. Different energy sources, including vibration, fluid flow and rotation, can be adopted to provide the impetus for the driving magnet.

![Figure 7.2: Schematic of the bistable frequency-up converting piezoelectric energy harvester. The dashed lines indicate the motion trajectory of the driving magnet.](image)

### 7.3.2 Equivalent Circuit

In order to simulate the performance of the harvester combined with its power management circuit, an equivalent circuit model is necessary. The theoretical model for this harvester has been established in Chapter 6. For convenience of explanation, the key equations are reported in this chapter again. For piezoelectric beams under magnetic plucking, the electromechanical dynamics can be described by

\[
\frac{d^2\eta_r(t)}{dt^2} + 2\zeta_r\omega_r \frac{d\eta_r(t)}{dt} + \omega_r^2 \eta_r(t) - \vartheta_r V(t) = \phi_r(L_p) \left[ F_y^{[F]}(t) - F_y^{[f]}(t) \right],
\]

(7.1)
Based on the above equations, an equivalent circuit is built as shown in Fig. 7.3. The basic idea is to transfer equations (7.1) and (7.2) to the electrical model using Kirchhoff’s voltage and current laws [176]. Different vibration modes are considered in the equivalent model to achieve a more precise model when multi-vibration modes are dominant. For each vibration mode, the mechanical module and electrical module are coupled by an ideal transformer, whose turns ratio $TR_r$ is determined by $\vartheta_r$ in Eq. (7.1). The electrical components, $R_r$, $L_r$, and $\frac{1}{C_r}(r = 1, 2, 3)$, represent the mechanical damping ratio, the mass and stiffness of the beam, respectively. The applied forces on the beam are equivalent to a voltage source with a gain factor of $G_r$. In the electrical module, $C_p$ is the clamped capacitance of the piezoelectric beam, and the load represents subsequent load circuits. The components in the equivalent circuit and their corresponding expressions in the theoretical
model are listed in Table 7.1.

Using the above theories and the parameters listed in Table 6.1, the theoretical model and the equivalent circuit are calculated numerically in Matlab and simulated in Simulink respectively. A comparison between these results is provided in Fig. 7.4. These results have a close match in terms of frequency and amplitude. Therefore, this equivalent circuit is reliable to be used in the following circuit design, whereas the theoretical model cannot be directly applied in electrical circuits.

![Figure 7.4: Comparison of the numerical results between the theoretical model (a) and the equivalent circuit (b). The voltage was measured across a 120 kΩ resistor directly connected to the piezoelectric beam.](image)

### Table 7.1: Components and Their Expression in the Equivalent Circuit.

<table>
<thead>
<tr>
<th>Electrical Equivalent</th>
<th>Description</th>
<th>Mechanical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q_r(t))</td>
<td>Charge in the circuit</td>
<td>(\eta_r(t))</td>
</tr>
<tr>
<td>(i_r(t))</td>
<td>Current in the circuit</td>
<td>(d\eta_r(t)/dt)</td>
</tr>
<tr>
<td>(F_{in})</td>
<td>External forces on the beam</td>
<td>(F_{d} - F_{ft})</td>
</tr>
<tr>
<td>(G_r)</td>
<td>Gain factor between force and voltage</td>
<td>(\phi_r(L))</td>
</tr>
<tr>
<td>(V_r)</td>
<td>Equivalent input voltage</td>
<td>(G_r \cdot F_{in})</td>
</tr>
<tr>
<td>(R_r)</td>
<td>Equivalent mechanical Damping</td>
<td>(2\zeta_\omega_r)</td>
</tr>
<tr>
<td>(C_r)</td>
<td>Equivalent stiffness of the beam</td>
<td>(1/\omega_r^2)</td>
</tr>
<tr>
<td>(L_r)</td>
<td>Equivalent mass of the beam</td>
<td>(1)</td>
</tr>
<tr>
<td>(C_p)</td>
<td>Clamped capacitance of the beam</td>
<td>(C_p)</td>
</tr>
<tr>
<td>(TR_r)</td>
<td>Turns ratio of Transformer</td>
<td>(\delta_r)</td>
</tr>
</tbody>
</table>
7.4 Power Management Circuit Design

The PMC is designed to convert the AC output from the piezoelectric beam into a stabilized DC form with a constant voltage suitable for subsequent electronics. Fig. 7.5 provides a block diagram showing the functionalities of this PMC. Two main sub-circuits, namely the rectification & storage circuit and voltage stabilization circuit, are included. For the convenience of explanation, the equivalent circuit in Fig. 7.3 is plotted as a piezoelectric harvester block (as shown in Fig. 7.5) in the following sections.

Figure 7.5: General block diagram of the power management circuit (PMC), showing the function of each unit.

7.4.1 Rectification & Storage Circuit

This circuit converts the AC output from the piezoelectric beam and stores the energy in a supercapacitor. This function can be easily realized by a full-bridge rectifier in parallel with a capacitor, but a simple bridge rectifier circuit faces many energy losses due to the clamped voltage by the storage capacitor [170]. The SSHI circuit minimizes those losses. A simplified circuit and typical waveforms of a SSHI circuit are shown in Fig. 7.6. This circuit detects the voltage peaks. When peaks occur, the inductor $L$ is connected, and the voltage on the beam is reversed. By doing so, the voltage $V$ is always in phase with the beam vibrating velocity $\dot{u}$. More power can be extracted from the harvester ($P_{out} \propto \int V \dot{u} dt$).

Many self-powered SSHI circuits have been developed in the literature. In this chapter, a classic self-powered SSHI circuit from [177] is adopted, as shown in Fig. 7.7. Component parameters were redesigned for this piezoelec-
According on the functions, this circuit is divided into four blocks, namely equivalent piezoelectric model, positive and negative peak detection (PKD) circuits, voltage inversion circuit and rectification & storage circuit. The PKD circuits detect the voltage peaks and generate switching signals by comparing the output voltage of the piezoelectric beam and the reference voltages over capacitor $C_2$ and $C_3$. The switching signals are used to control the operation of the inductor in the voltage inversion circuit. The voltage on the beam is reversed when voltage peaks occur. Then, the power is rectified and stored.

Figure 7.7: Schematics of the parallel SSHI circuit from [177]. The circuit is divided into blocks based on their functions. The equivalent circuit of the piezoelectric beam in Fig. 7.3 is simplified as a PZT block in this circuit for convenience. The diodes are all with the model name of 1N4148. T1 and T3 - 2N3096. T2 and T4 - 2N3094. The storage capacitor is 50 µF here for quick charging. This value will be changed according to the power requirement of subsequent loads.
by a bridge rectifier and stored in a supercapacitor ($C_4$). Its capacitance should be large enough to maintain the operation of subsequent load circuits for a desired duration. The choice of capacitance will be discussed in the sensing application part.

The simulated results of the circuit are depicted in Fig. 7.8. Fig. 7.8(a) shows the open circuit voltage of the piezoelectric beam (with circuit in Fig. 7.7(a) only), and Fig. 7.8(b) is the output voltage of the beam with all the circuits in Fig. 7.7. Fig. 7.8(c) and (d) illustrate the current flowing through Transistors $T_2$ and $T_3$, respectively. Switching events are clearly indicated from the current variations. The operating principle of the SSHI circuit is also illustrated in this figure. The switching events are slightly behind the open-circuit voltage peaks in Fig. 7.8(a). This is caused by the time delay in the PKD circuits.

**Figure 7.8:** Simulated results: (a) open circuit voltage of the piezoelectric beam without any load circuit, (b) output voltage of the beam with all the circuits in Fig. 7.7, (c) and (d) current passing through $T2$ and $T3$ respectively and (e) voltage accumulated on the storage capacitor. The beam is plucked at 6 Hz.

Fig. 7.8(e) shows the voltage on the storage capacitor $C_4$ for circuits with SSHI. An appropriate voltage range should be determined for the storage capacitor in order to select the capacitance and to provide sufficient power
for subsequent loads. Three factors are considered to determine the voltage range.

- **Effective operation of the SSHI circuit.** It is shown in Fig. 7.8(b) that only when the voltage on the beam reaches a certain threshold does the SSHI circuit start to operate, and the maximum voltage is limited by the clamped voltage across the capacitor. Therefore, in order to maintain efficient operation, the clamped voltage should be high enough.

- **Capacitance.** For WSNs, total dimensions are also a big consideration. Therefore, small capacitance is beneficial to reduce the volume. However, sufficient energy is also important for subsequent applications. Increasing the operating voltage range is a solution to satisfy both requirements ($\Delta E = \frac{1}{2} C \cdot (V_1^2 - V_2^2)$).

- **Power consumption.** The clamped voltage also determines the power consumed by the internal components in the SSHI circuit. In order to maintain charging efficiency, reduced upper clamped voltage is desirable.

Considering these factors, the operating voltage range of the storage capacitor is pre-set as 3-7.5 V. Therefore, the capacitance of the storage capacitor can be determined when the power consumption of the load circuits is confirmed.

### 7.4.2 Voltage Stabilization Circuit

In order to control the clamped voltage in this voltage range, a voltage comparison circuit is necessary to manage the charging and discharging phase. Meanwhile, this voltage range is normally unacceptable for electronics, as the maximum input voltage is normally around 3.3 V. Therefore, a DC-DC step-down converter is also necessary to stabilize the output voltage.

The voltage range is controlled by a comparator with voltage dividing and comparison circuits, as shown in Fig. 7.9. The comparator LTC1540 has the characteristics of nano-power consumption and a fixed internal voltage reference ($V_{ref} = 1.18$ V). The comparator operates from a single 2 V to 11 V supply. Using this reference and the voltage dividers, the upper and lower limits are determined by control signals SW1 and SW2 respectively.

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The upper and lower limits are 7.6 V and 2.9 V. When the voltage on the capacitor (Outp) reaches the upper limit (7.6 V), the subsequent circuit is enabled by SW1 (SW1 > 1.18 V). The output of the comparator Enable is high (Enable = Outp, the power supply of the comparator), and the NPN transistor Q1 is enabled, and the power on the capacitor is transmitted to the DC-DC converter. In order to maintain the output Enable to be high before the voltage on the storage capacitor reaches the lower limit (2.9 V). The Enable signal is also connected to a voltage divider, and the control signal SW2 generating from Enable, is merged with SW1 after passing through one diode (D2 and D1) for each signal. This combined signal keeps the Enable signal in the high state, before Outp is lower than 2.9 V. As soon as the voltage on the storage capacitor (Outp) decreases to the lower limit (2.9 V), Enable is low, and the subsequent circuit is disabled. The voltage on the capacitor starts to increase again.

Figure 7.9: Realization of voltage stabilization in LTSpice. Voltage comparison and conversion are realized in this circuit.
The DC-DC converter used is LTC3388-3 which has an input voltage range of 2.7 - 20 V. Different output voltages are provided, including 2.8 V, 3.0 V, 3.3 V and 5.0 V. A transistor Q1 in Fig. 7.9 is adopted to control the input. The controlling signal is the output Enable from the comparator. The output voltage is fixed at 3.3 V in this design.

The operating principle of the voltage stabilization circuit is illustrated in the simulated results in Fig. 7.10. The switching signal Enable is high when the voltage on the capacitor is higher than the upper limit ($SW_1 > V_{ref}$). The energy stored on the capacitor is then regulated by the converter and consumed by the subsequent load. The switching signal Enable remains high until the voltage reaches the lower limit ($SW_2 < V_{ref}$). Then the converter and load circuit are isolated, and the storage capacitor starts to accumulate energy again. Its capacitance can be determined by the power consumption of the subsequent loads.

Figure 7.10: Simulated results of the voltage stabilization circuit, showing its operating principle. The figures on the right side depict the switching process for load operation.

### 7.5 Wireless Sensing Applications

With the energy harvested from the ambient environment, wireless sensor nodes can be fully self-powered. However, the power balance between the energy generation capability of harvesters and power consumption of
electronics needs to be calculated to maintain the normal operation of the system. In this work, a wireless sensor node (TelosB) for environmental condition monitoring applications is adopted as the load. Humidity, temperature and light sensors are embedded in this node. The IEEE 802.15.4 2.4 GHz Wireless Module is used for wireless communication. The integrated on-board antenna covers a range of 50 m indoors / 125 m range outdoors.

The power consumption of this node was measured, as shown in Fig. [7.11]. The peak power (data transmission) is about 35 mW, and the duration for data transmission on the millisecond level. When the micro-controller is in the active mode, and the data transmission is conducted at 0.5 Hz, the average power consumption is 6.2 mW. Therefore, the energy dissipated by the sensor node in each data transmission cycle is about 12.4 mJ. In order to ensure the successful data collection and transmission, the energy stored in the supercapacitor is designed to be enough for three data transmission cycles. Based on the power consumption of the sensor node and the operating voltage range of the storage capacitor, a 2.2 mF capacitor is chosen as the storage capacitor. The energy stored is around 52 mJ ($\Delta E = 1/2 \cdot C_s \cdot (V_1^2 - V_0^2)$), when the capacitor is charged from 3 V to 7.5 V. This amount of energy stored is enough for the node to complete three data transmission cycles at 0.5 Hz.

7.6 Experimental Validation

7.6.1 Experimental Set-Up

The whole system was implemented experimentally, as shown in Fig. [7.12]. The implementation of the bistable frequency up-converting harvester is illustrated in Fig. [7.12(a)]. A stepper motor was employed to provide external kinetic energy sources. In practical applications, other designs, such as miniature turbine, pendulum structures or even vibrating magnets can also be adopted to provide energy sources. The repulsive force between the tip and fixed magnets introduces two equilibrium states for the beam. Linear positioning stages are used to adjust the forces among these magnets to maintain the high-energy-orbit vibration.

The power management circuit was implemented on a PCB board, as
Figure 7.11: Measured power consumption of the sensor node TelosB. The data transmission frequency is 0.5 Hz and the micro-controller is on the active mode. The receiver is activated at 2 Hz.

Figure 7.12: Experimental set-up. (a) bistable frequency up-converting energy harvester and (b) power management circuit and a wireless sensor node TelosB.

shown in Fig 7.12(b). The dimensions are 4.5 cm × 6.5 cm. The output of the piezoelectric beam is connected to the input of the circuit. The regulated output is connected to the wireless sensor node in Fig. 7.12(b). The main components used in Fig. 7.7 and 7.12 are listed in Table 7.2. A 2.2 mF storage capacitor was adopted to accumulate sufficient energy for the sensor node.
Table 7.2: Main Components in the Power Management Circuit

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1 - D8</td>
<td>diodes</td>
<td>1N4148W</td>
</tr>
<tr>
<td>R1 &amp; R2</td>
<td>Resistors</td>
<td>4.7 MΩ</td>
</tr>
<tr>
<td>C1 &amp; C3</td>
<td>Capacitors</td>
<td>150 pF</td>
</tr>
<tr>
<td>Q1 &amp; Q3</td>
<td>PNP Transistors</td>
<td>MMBT3906</td>
</tr>
<tr>
<td>Q2 &amp; Q4</td>
<td>NPN Transistors</td>
<td>MMBT3904</td>
</tr>
<tr>
<td>L1 &amp; R3</td>
<td>Inductor &amp; its internal resistance</td>
<td>47 nH &amp; 600 mΩ</td>
</tr>
<tr>
<td>C4</td>
<td>Storage Capacitor</td>
<td>2.2 mF</td>
</tr>
</tbody>
</table>

7.6.2 Experimental Results

Fig. 7.13 provides a comparison of output voltage and tip displacement between a bistable frequency-up converting harvester with the SSHI circuit and the harvester without bi-stability and connected to a full-bridge rectifier. First of all, the mechanism of frequency up-conversion is illustrated in both configurations. The excitation frequency is low, whereas the beam oscillates freely at resonance (much higher than the excitation frequency) after each plucking.

The bistable behaviour is depicted in Fig. 7.13(c). For each plucking cycle, the beam vibrates in the double-well mode for a few cycle and then settles into the single-well mode. The double-well mode provides an improved displacement and output voltage. The performance of the SSHI circuit is also compared with a bridge rectifier, as shown in Fig. 7.13(b) and (d). For the SSHI circuit, the voltage reverses at voltage extrema, but the reversed voltage $V_r$ is lower than the extremum $V_m$ due to the power losses of the voltage inversion circuit. This can be minimized by improving the quality factor of this circuit.

For both configurations, the input force and frequency are the same, and only a capacitor is connected in the circuits for power storage (no resistive load). When the capacitor is fully charged, the output voltage of the bistable harvester with the SSHI circuit is much higher (4.3 V vs. 2.8 V), showing the advantage in power conversion.

The harvester was then connected to the power management circuit and the wireless sensor node. Fig. 7.14 depicts the charging and discharging cycles of the supercapacitor. The upper and lower voltage limits are 7.3 V
Figure 7.13: Comparison of output voltage ((b) and (d)) and beam tip displacement ((a) and (c)) between a piezoelectric beam without bi-stability and connected to a full-bridge rectifier (upper - blue) and a beam with bistability and connected to the SSHI circuit (lower - red). The energy was stored in a capacitor, and no additional load was connected. The capacitor was fully charged during this test. The input driving force and frequency (6 Hz) were the same for both tests.

and 3.2 V respectively. For each charging cycle, it takes 30 minutes, and the energy stored is enough for the sensor node to sense environmental parameters and to transmit three data packages during the discharging period. This result also verifies the function of the voltage divider and comparison circuit design.

For the charging cycle, the discharging rate of the storage capacitor during this period is also an important factor. If the discharging rate is comparable or larger than the charging rate, the circuit would be inefficient or non-functional. Fig. 7.15 shows the discharging curve of the storage capacitor. According to this curve, the discharging constant $\tau$ can be calculated using $V_c = V_0 \cdot e^{-t/\tau}$, and the value is 5.5 hours which is much larger than the duration of the charging cycle.
Figure 7.14: Charging and discharging phases of the storage capacitor for energy harvesting and supplying the wireless sensor node.

Figure 7.15: Discharging curve of the storage capacitor with the voltage stabilization circuit and the sensor node connected.

Fig. 7.16 illustrates the variation of the voltage across the storage capacitor and the sensor node during load operation (the discharging period). The load operation lasted for more than 7 seconds. Three data packages were transmitted by the sensor node successfully. The different slopes of the discharging curve ($V_{\text{Capacitor}}$) represent different operating modes of the sensor node, including idle and data collection and transmission modes. The voltage applied on the sensor node $V_{\text{Sensor Node}}$ maintains at 3.3 V during these three data transmission cycles.

Fig. 7.17 depicts the environmental condition data collected from a self-powered sensor node. A 1 mF capacitor was adopted to reduce the duration of the charging cycle. Data packages were successfully collected by the receiving sensor node approximately every 10 minutes. This result further demonstrates the capability of this self-powered system in condition monitoring.
Figure 7.16: Voltage variation on the storage capacitor and sensor node during the capacitor discharging period. The different background colors indicate the different operating modes of the sensor node.

Figure 7.17: Lab environmental conditions monitored by the self-powered sensor node. The harvester was operated in a frequency range between 10 and 13 Hz. The storage capacitor used was a 1 mF capacitor which can store enough energy to complete one data package transmission.

7.7 Conclusion

In this chapter, an integrated kinetic energy harvesting system is introduced for wireless sensor networks in environmental condition monitoring. This system includes a piezoelectric energy harvester, a power management circuit (PMC) and a wireless sensor node for environmental condition monitoring. Bi-stability and frequency up-conversion are combined in this
harvester, allowing it to effectively collect energy from low-frequency and random motions. In the PMC, a SSHI circuit was adopted to enhance the power conversion capability. In order to provide a stabilized voltage source for the sensor node, a voltage stabilization circuit was designed to regulate a wide-range input (3.2 V - 7.3 V) to a constant output (3.3 V). The generated energy is capable of fully powering this sensor node to communicate with other nodes at a rate of 1 data package per 10 minutes. This presented energy harvesting system provides a practical solution for self-powered sensing system design.
8 Classification and Theoretical Comparison of Rotational Energy Harvesters

In this chapter, three types of transduction systems for small scale rotating generators (energy harvesters) are compared, namely electromagnetic (EMREHs), piezoelectric resonant (PRREHs) and piezoelectric non-resonant rotational energy harvesters (PNRREHs). Theoretical models are established for each type to estimate the maximum achievable output power as a function of device dimension and operating frequency, in order to establish scaling laws for these systems. The EMREHs have a strong scaling effect on device dimension (as \(L^5\)) and operating frequency (as \(\omega^2\)), whereas the PNRREHs are less so (\(L^{2.5}\omega^{0.5}\)). The PRREHs are narrowband and ideal for cases where the driving frequency is constant. Current challenges and constraints for the different types are also discussed. This study provides a guideline for selection and design of rotational energy harvesters with specific requirements in terms of device dimension and operating frequency range.

8.1 Introduction

Electricity plays an indispensable role for almost every application in our daily life, ranging from domestic apparatus to industrial machines. Generally, electrical power is provided by rotating generators using electromagnetic transduction driven by turbines [58]. This paradigm is ubiquitous in macro-scale generation systems to convert energy sources such as fluid flow, fossil fuels and nuclear power into electricity.

However, for small-scale (e.g. <1 W) power generation for low power electronics, electromagnetic transduction is not the only solution, and a range of
conversion methods, including piezoelectric, electrostatic and triboelectric conversion become available. While rotating energy harvesters have been demonstrated using a number of transduction methods, there are currently no established principles by which to select the most suitable method in a particular application. This chapter presents a theoretical comparison of three key types of rotational energy harvester. The theoretical achievable output powers are analysed and compared for different harvester dimensions, and operating frequencies. Scaling effects on the different types are examined using the theoretical models. The results illustrate the differing sensitivity of harvester types to dimensions and operating frequency, and thus provide selection guidance for specific applications, as well as an indication of possible technological trends in this field. Current challenges and constraints are also discussed.

8.2 Classifications and Theoretical Analysis

In conventional macroscale power generation, electrical generators are overwhelmingly based on electromagnetic transduction. Other conversion mechanisms, including electrostatic, piezoelectric and triboelectric, are either impractical or inefficient. However, at small scales, e.g. centimeter or millimeter scale, the performance of electromagnetic generators are comparable to that of other mechanisms due to the dimension scaling effect.

Operating frequency has another influence on achievable performance. For large-scale rotational generators, gear trains are normally adopted to amplify the operating frequency, but for small-scale energy harvesters, gear trains may not be practical in some cases due to the fabrication difficulty or the cost of precise and high transmission-ratio gear trains. Many rotation sources in the environment, such as micro-wind turbine, vehicle wheels and rotating machines, have relatively low operating frequencies. For example, for a tire pressure sensor application, the speed of 500 rpm (~8 Hz) corresponds to a vehicle speed of about 50-60 km/h (dependent on tire diameter).

As a result of the limited dimensions and the low-operating frequency in some cases, other conversion mechanisms, such as piezoelectric, electrostatic and triboelectric, which exhibit more favourable scaling behaviours at small dimensions and low frequencies, become available. The focus of this work is
on the comparison of piezoelectric and electromagnetic conversion mechanisms. Based on the conversion mechanism and frequency dependence, rotational energy harvesters can be broadly classified into three categories: electromagnetic (EMREH), piezoelectric resonant (PRREH) and piezoelectric non-resonant rotational energy harvesters (PNRREH), as shown in Fig. 8.1.

For EMREHs, the rotating magnet leads to the variation of magnetic flux linkage in static coils, which generates electricity. For PRREHs, piezoelectric beams are excited by beam plectra, or exciters. The excitation force is in an equivalent form of a sinusoidal wave, and the beams generally operate at their vibrational resonances. For PNRREHs, the beam excitation mechanisms are the same as those for PRREHs, but the excitation frequency of the rotor is much lower than the resonant frequency of the piezoelectric beams, so although the plucked beams then oscillate resonantly, the system operates in a non-resonant mode. In order to compare performance, theoretical models for each type are established.

### 8.2.1 Electromagnetic Rotational Energy Harvesters

For EMREHs, there are generally two types of structures in terms of magnetic flux orientation, i.e. radial-flux and axial flux structures, as shown in Fig. 8.2. Radial-flux structures are widely adopted in macro-scale power generation due to the high design flexibility and high conversion performance. The geometries where a permanent-magnet rotor rotates inside stationary armature windings is considered; for simplicity the generators
with the permanent magnets in the stator, or with field windings rather than permanent magnets are excluded, as the scaling laws for these are similar. When device dimensions reduce to millimetre scale or below, radial-flux structures become undesirable due to limited magnet size, fabrication difficulty of coils with large coil area and coil turns and also structure compactness.

Figure 8.2: General electromagnetic generator designs: (a) radial-flux generator and (b) axial-flux generator.

On the other hand, axial-flux structures have a good compatibility with micro-fabrication. The design and fabrication of coils and permanent magnets are relatively easy and flexible to achieve high flux linkage in coils and compactness in structure. Therefore, this design is widely used in applications where the device dimensions are on the centimeter or millimeter scale, and power density is the priority in design [178]. As shown in Fig. 8.2(b), this type of harvester has a high diameter-thickness aspect ratio (the “pancake” geometry). Both the rotor (permanent magnets) and stator (coils) are capable of utilizing the whole diameter of the design.

For axial-flux generators, there are plenty of possible design configurations in terms of coil shape, coil layers and rotor-stator number, as shown in Fig. 8.3. For coil fabrication, wire winding is the typical method for conventional macro-scale electric generators. For micro-scale harvesters, micro-fabrication technologies, including silicon-based fabrication and laser cutting, are available for producing micro-planar coils, as depicted in Fig. 8.3. Multi-layer coils can be fabricated to improve the output. Harvesters with one rotor and two stators are also possible for axial-flux generators (Fig. 8.3(c)).

For electromagnetic harvester design, several key concerns are necessary to be addressed before electromagnetic harvesters can operate effectively.
Figure 8.3: Design variables and possible configurations for axial-flux EM-REHs: (a) planar coil design and key parameters, (b) multiple layer coil design (c) harvester with one rotor and two stators.

- Absolute value of magnetic flux density $B$ and flux gradient. High $B$ values require a high residual flux density of the magnets and a narrow gap between magnets and coils. The high magnetic flux gradient is required in order to achieve a high flux density variation ($dB/dt$) for a specific rotational frequency.

- Coil area and number of coil turns. Axial-flux structures allow the coil to be planar, and a large portion of the cross-sectional area of devices can be filled, as shown in Fig. 8.2(b). Different technologies, including traditional wire winding and micro-fabrication technologies, are available for fabrication. Methods to increase the number of turns for each layer are critical to improve the performance. Multi-layer coils can be integrated, but the attenuation of magnetic field caused by the increase of coil thickness (Fig. 8.3(b)) should be considered.

- Output voltage. For electromagnetic generators at low frequencies, one significant issue is the low output voltage. Beeby et al. studied the effect of coil diameter and coil turns on output power and output voltage for vibrating harvesters [179]. For a certain coil volume, the increase of coil turns by decreasing coil diameter augments the output voltage, but the output power is essentially the same because of the increased impedance. Therefore, by optimizing the coil diameter and coil turns, the output voltage can be improved. Also, gear trains can be adopted in some cases, such as wristwatches, to improve the operating frequency and the output voltage.
In order to understand the power generation capability of electromagnetic harvesters considering these key design limitations, a theoretical study is established, based on the design in Fig. 8.2(b) and 8.3. Harvesters with one rotor and one stator are adopted in this analysis. For the topology introduced above, the maximum flux linkage is

$$\Phi_0 = \beta_0 \cdot N_p \cdot N_t \cdot B_r \cdot A_c, \quad (8.1)$$

where $B_r$ is the remnant flux density of the permanent magnets, $\beta_0$ is a geometrical factor ($<1$) determined by coil-magnet arrangements, $A_c$ is the average area enclosed by each coil, $N_t$ is the number of turns for each spiral and $N_p$ is the number of pole pairs.

Assuming the flux linkage $\Phi_0$ varies with time sinusoidally, the rms output voltage can then be written as

$$V_{\text{rms}} = \frac{1}{\sqrt{2}} N_p \cdot \omega \cdot \Phi_0 = \beta_0 \cdot \sqrt{2} N_p^2 \cdot N_t \cdot B_r \cdot A_c, \quad (8.2)$$

where $\omega$ is the angular rotation speed of the rotor. The theoretically achievable output power is then given by

$$P_{\text{max}} = \frac{V_{\text{rms}}^2}{4R_c}, \quad (8.3)$$

where $R_c$ is the internal resistance of the coils. Given the low speed typical in these applications, the coil inductance can be neglected. The coil resistance can be calculated as

$$R_c = \rho_c \cdot \frac{L_w}{A_w} = \frac{\pi \rho_c k_c (r_o^2 - r_i^2) t}{A_w^2}, \quad (8.4)$$

where $\rho_c$ is the conductor resistivity, $L_w$ is the total length of the wire, $A_w$ is the cross section area of the wire, $r_o$ and $r_i$ are the inner and outer radius of the coils, $t$ is the thickness of the coils, and $k_c$ is the fill factor. In wire wound transformers, copper fill factors in the range of 0.5-0.6 can be achieved [181]. For micro-fabricated coils, the fill factor will be lower, because the spacing needed for each turn is larger than that for wire-winding coils.
The number of turns can be estimated using

$$N_t = \frac{L_w}{L_m} = \frac{k_c(r_o - r_i)t}{A_w},$$

(8.5)

where $L_m$ is the mean length of each turn. This equation is derived based on the theory in [180].

8.2.2 Piezoelectric Resonant Harvesters

Piezoelectric transduction has been extensively applied in vibration energy harvesting. These harvesters normally operate with base excitation and use the inertial force created by a tip proof mass to introduce beam vibration. However, in the rotating situation, the motion from the host is generally low-frequency continuous rotation. The variation of acceleration is not as significant as that in the vibrating case. Piezoelectric beams cannot be excited by inertial force from rotation. Therefore, other mechanisms using tip excitation, such as magnetic plucking [158] or direct impact [109], are employed. The tip force generated from rotation plucks the piezoelectric beams, and electricity is collected from their subsequent oscillation.

Considering the frequency response of harvesters, they can be classified into two categories: resonant and non-resonant, as shown in Fig. 8.1(b) and (c). For piezoelectric resonant harvesters, the excitation force is generally in a harmonic form and multi-exciter designs are designed on the rotating host to amplify the effective rotational frequency. The plucking frequency generally matches the resonant frequency of the piezoelectric beams. Therefore, this type of harvester is generally suitable for conditions where the rotational speed is constant or stable.

Two designs for possible implementation are shown in Fig. 8.4, using excitation by direct impact and magnetic plucking respectively. For the magnetic plucking method, the plucking force might be affected by environmental magnetic field or ambient ferromagnetic materials. Material selection should be considered in order to stabilize the performance. For the direct impact method, the constant and repetitive impact on the typically fragile, ceramic piezoelectric material is likely to affect the system reliability. In addition, there are several practical limits for these harvesters as well.

- Maximum excitation force. This force is limited by the elastic limit
of piezoelectric materials on the beam. Using materials with higher elastic limit or tapered beams could improve the maximum allowable force.

- Material fatigue [159]. Piezoelectric materials are brittle. Long-term operation is an important consideration in design. A safety factor between the maximum stress on the beam and the elastic limit should be considered.

- Number of piezoelectric beams. The power generating capability depends on the number of beams, but the space inside the device determines the maximum number. Another consideration is the need for rectification circuits for each beam which will increase system complexity.

In order to evaluate the performance of PRREHs, a theoretical model is built. Assuming the tip excitation force is in harmonic form, it can be described as

$$F_{\text{tip}} = F_0 \sin(\omega t),$$

where $F_0$ is the amplitude and $\omega$ is the operating frequency. The amplitude $F_0$ is constrained by the maximum stress, i.e. tensile strength ($\sigma_t$), that the piezoelectric beam can withstand.

According to distributed-parameter theory, the dynamic equation of piezoelectric beams is [113,133]

$$\frac{d^2 \eta_r(t)}{dt^2} + 2\zeta_r \omega_r \frac{d\eta_r(t)}{dt} + \omega_r^2 \eta_r(t) - \beta_r V(t) = F_{\text{tip}}(t) \delta(x - L_p),$$

(8.7)
where $\eta_r(t)$ is the model mechanical coordinate expression of the $r^{th}$ vibration mode, $\zeta_r$ is the modal damping ratio, $\omega_r$ is the effective undamped modal frequency, $\delta_x$ is the Dirac delta function, $\vartheta_r$ is the piezoelectric coupling term in physical coordinates, $L_p$ is the length of the beam and $V(t)$ is the voltage across a resistive load $R_l$.

For cantilever beams, the maximum stress $\sigma_m$ happens at the fixed end of the beam on the surface farthest from the neutral axis. The stress should be kept lower than the tensile strength $\sigma_T$ of the material to maintain the reliability of the device. The maximum stress can be calculated from

$$\sigma_m = \frac{Y \cdot h}{2\rho} < \sigma_T,$$

(8.8)

where $\rho$ is the curvature of the bended beam. The radius of curvature $\rho$ can be expressed as

$$\frac{1}{\rho} = \frac{\partial^2 v(x, t)}{\partial x^2},$$

(8.9)

The maximum tip force $F_{\text{tip}}^m$ can be then acquired using the above equations. For piezoelectric transducers, a simple equivalent electric circuit is a current source in parallel with the internal capacitance. The electrical equation of piezoelectric beams can then be written as

$$C_p \frac{dV(t)}{dt} + \frac{V(t)}{R_l} + \bar{\varepsilon}_{31} h_t b \int_0^{L_p} \frac{\partial^3 v(x, t)}{\partial x^2 \partial t} dx = 0,$$

(8.10)

where $C_p$ is the internal capacitance of the piezoelectric material, $\bar{\varepsilon}_{31}$ is the piezoelectric constant, $h_t$ is $(h_p + h_s)/2$, $b$ is the width of the beam and $L_p$ is the length of the beam.

The maximum achievable average output power can be then calculated using

$$P_{\text{max}} = N_{pe} \frac{\omega}{2\pi} \int_{t_0}^{t_0 + \frac{2\pi}{\omega}} \frac{V^2(t)}{R_l} dt,$$

(8.11)

where $N_{pe}$ is the number of piezoelectric beams. The number of beams is determined by the beam configuration and the maximum bending displacement. If the rotation frequency from the host is much lower than the resonant frequency of the beam, configurations that allow the beam to be longer should be adopted, and the number of exciters should be increased to ensure the harvester operates at resonance.
8.2.3 Piezoelectric Non-Resonant Harvesters

For rotational energy sources, the operating frequency is often varying. In many cases, such as car wheels and miniature air turbines, PRREHs are not suitable due to their narrow bandwidth. Therefore, non-resonant harvesters with wide operating bandwidth are desirable. The common topology for PNRREHs is illustrated in Fig. 8.1(c).

Compared with resonant harvesters, the number of exciters in non-resonant harvesters is reduced, so that the excitation frequency \( f_e \) is much lower than the resonant frequency \( f_{re} \) of the piezoelectric beams, e.g. \( f_{re} > 10 f_e \). The operating principle is indicated in Fig. 8.5. The beam is first plucked by an exciter on the low frequency rotor, and then vibrates freely at its resonance. The oscillation has decayed before the next excitation comes. This mechanism is called frequency up-conversion, by which the low frequency rotation is converted to high frequency vibration.

![Figure 8.5: Displacement of exciter in the beam length direction and beam vibration, showing the operating principle of non-resonant harvesters.](image)

Another advantage of this mechanism is that the rotational motion and beam vibration are decoupled by the transient plucking for each excitation cycle. The kinetic energy is transferred to the beams during plucking. Once energy is stored in the beam deformation, the beam is released, and the stored energy is gradually converted into electricity by the electrical damping. Therefore, non-resonant harvesters show a wide operating bandwidth.

In order to evaluate performance for different scales and operating conditions, the theoretically achievable output power is estimated. As illustrated in Fig. 8.5, the vibration pattern of the beam in each excitation cycle is the
under-damped free vibration. The transverse displacement of the beam at the free end \((x = L)\) is then of the form

\[
v(L, t) = A_0 e^{-\xi \omega_{re} t} \cos \left( \omega \cdot \text{mod}(t, \frac{2\pi}{\omega}) - \theta_0 \right), \tag{8.12}
\]

in which \(\omega_{re}\) and \(\xi\) are the resonant frequency and the total damping ratio of the beam respectively, \(\theta_0\) is the initial position, and \(\text{mod}(x_1, x_2)\) is the function for the modulo operation.

Considering that the excitation frequency is much lower than the resonant frequency of the beam, the first vibration mode is, then, the dominant component in the beam vibration. Therefore, in the following calculation, only the first vibration mode is considered. The modal mechanical coordinate expression \(\eta_1(t)\) can then be written as

\[
\eta_1(t) = B_0 e^{-\xi \omega_{re} t} \cos \left( \omega \cdot \text{mod}(t, \frac{2\pi}{\omega}) - \theta_0 \right) \leq B_0, \tag{8.13}
\]

where \(B_0\) is the initial amplitude of the first model mechanical coordinate. From Eq. 8.8 and Eq. 8.9, the maximum stress in non-resonant harvesters can be rewritten as

\[
\sigma_m = Y \cdot h \frac{d\phi_1^2(x)}{dx^2} \Bigg|_{x=0} \eta_1(t) \leq \sigma_T \tag{8.14}
\]

The maximum initial amplitude of the model coordinate \(\eta_1(t)\) can be denoted by

\[
B_{0m} = \frac{2\sigma_T}{Y \cdot h} \left( \frac{d\phi_1^2(x)}{dx^2} \Bigg|_{x=0} \right)^{-1}. \tag{8.15}
\]

Therefore, the maximum vibrating condition for the piezoelectric beam is determined. The electromechanical dynamics of the non-resonant harvester can, then, be achieved by solving Eq. 8.10 and Eq. 8.12. The maximum achievable output power can be calculated from Eq. 8.11.

### 8.3 Scaling Effects for Different Classes

The achievable performance of rotational energy harvesters is strongly affected by device dimensions and by rotational frequency of the sources. In order to understand the performance variations of different harvester
classes, these scaling effects are examined. Considering that the aim of energy harvesting is to provide an alternative to conventional batteries, the dimensions of harvesters should be comparable to the size of the batteries they aim to replace. Therefore, the scaling analysis is focused on the millimeter to centimeter scale. In addition, as the aim of this research is for low-frequency rotation sources, the rotational frequency is confined below 20 Hz (1200 rpm).

### 8.3.1 Electromagnetic Harvesters

For the scaling analysis, the relative ratios of dimensions are retained. However, there are several exceptions. The cross sectional area $A_w$ of the coil wire and the remnant flux density $B_r$ of the permanent magnets are assumed to be constant. The minimum achievable wire diameter is a limiting factor in terms of fabrication. In order to achieve more coil turns, small wire diameter should be adopted for all the dimensions (5-20 mm). For flux density, if the magnet material and the relative gap between magnets and coil are constant, $B_r$ is constant as well. The scaling effect can be examined based on the theoretical model build in Section 8.2.1.

According to Eq. (8.2)-(8.5), the basic scaling law can be established. Assuming the number of pole pairs ($N_p$) and the geometrical factor, $\beta$, are the same for different scales, only $A_c$, $r_o$ and $r_i$ are affected by scaling. Based on the above assumptions, the scaling law can be built as

\[ N_t \propto L^2, V_{rms} \propto L^4 \omega, R_c \propto L^3 \text{ and } P_{\text{max}} \propto L^5 \omega^2, \]  

(8.16)

where $L$ is the characteristic linear dimension and $\omega$ is the rotational frequency. This scaling law shows how the possible output power is affected by both the size of the device and the operating frequency. The size scale clearly has a more significant impact ($L^5$). Since the scaling is more rapid than $L^3$, even the power density ($P/L^3$) is dropping rapidly with decreasing size.

The validity of this scaling law can be examined by comparison to the theory provided by Trimmer [183]. In this paper, he derives that the magnetic force is proportional to $L^3$ between a wire with constant current and a permanent magnet, and the output power is proportional to the magnetic force ($P = F \cdot v = F \cdot \omega r$). In our case, the current is not constant; from
Eq. 8.16, the current is proportional to \( L \omega \), and \( r \) is proportional to \( L \). Therefore, if the varied current is applied to Trimmer’s theory, the same scaling law with dimension is obtained.

In order to quantitatively study the scaling behaviour of EMREHs, a set of parameter values is given in Table 8.1. The rotor diameter \( D_r \) is the characteristic dimension and other dimensions are dependent on this parameter. The diameter varies from 5 mm to 20 mm. The maximum achievable output power is estimated using Eq. (8.1) to (8.5). The result is illustrated in Fig. 8.6. The scaling effect is clearly illustrated. As the harvester dimension decreases from 20 mm to 10 mm, the output power drops from 220 \( \mu \)W to 7 \( \mu \)W at 5 Hz. The same trend happens for the variation of the rotational frequency. The output power is 1.26 mW at 12 Hz for harvester diameter \( \geq \)20 mm, while the output power at 6 Hz is only 315 \( \mu \)W.

![Table 8.1: Parameters for electromagnetic rotational energy harvesters](image)

8.3.2 Piezoelectric Resonant Harvesters

For piezoelectric harvesters, the electromechanical behaviour is more complex than that of electromagnetic harvesters. There is no explicit solution for the output power as a function of device dimension and operating frequency. In order to study the scaling effects, the performance is first calculated using several specific sets of harvester parameters. The scale law is then extracted from the calculated output. These multiple data sets are used to guarantee the universality of the scaling law.
Before studying the scaling law, one parameter that we need to consider first is the mechanical damping ratio of the beam. The scaling effect of dimension on cantilever mechanical damping should be examined in the first place. This parameter is normally acquired from experiments. However, it is impossible to test the values for all scales.

The scaling effect on damping ratio has been studied by several researchers [184, 185]. There are normally four factors contributing to damping forces, including air damping, squeeze effect, internal structural friction and support loss, but the dominant factors at micro-scale are the air damping and internal structural damping [185]. The total mechanical damping ratio can be written as [184]:

$$\xi_m = \xi_{a,b} + \xi_{a,m} + \xi_{in}, \quad (8.17)$$

where $\xi_{a,b}$ and $\xi_{a,m}$ are the air damping ratio of the beam and proof mass respectively and $\xi_{in}$ is the internal structural damping ratio. For this type of harvester, there is typically no proof mass for the direct-impact excitation method (Fig. 8.4(a)), and the tip magnet can be regarded as the proof mass (Fig. 8.4(b)). The governing equations for these factors are shown below:

$$\xi_{a,b} = \frac{(3\mu \pi b_p + 0.75\pi \sqrt{2\rho_b \mu \omega b_p^2})}{2 \rho_a h b_p^2 \omega \epsilon}, \quad (8.18)$$
\[ \xi_a = \frac{(3\mu \pi a + 0.75\pi \sqrt{2\rho_a \mu \omega_b \omega_a^2})}{2\rho_a \theta_a^2 \omega_r e} \]  

and

\[ \xi_{in} = \frac{m_b}{m_b + m_{pm}} \cdot \frac{\eta}{2}, \]

where \( \mu \) is the viscosity of air \((1.846 \times 10^{-5} \text{ Pa} \cdot \text{s})\), \( b \) is the beam width, \( d \) is the width of the proof mass, \( \rho_a \) is the density of air, \( \eta \) is the structural damping coefficient determined by material properties, \( a \), \( b \) and \( c \) are the dimensions of the proof mass, and \( m_b \) and \( m_{pm} \) are the mass of beam and proof mass respectively.

From the above equation, the scaling law for mechanical damping is not directly achievable. In order to study the scaling effect, a common piezoelectric ceramic beam, M1100 Johnson Matthey, is chosen to quantitatively study this effect. Main structural and material parameters for the model are summarized in Table 8.2. In this table, we assume the beam dimension scales with the diameter of the harvester. Different relative relations (\( \alpha \) and \( \beta \)) will be studied to ensure the scaling law to be generic.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value in Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_r )</td>
<td>Rotor diameter</td>
<td>( 5 - 20 \text{ mm} )</td>
</tr>
<tr>
<td>( L_p )</td>
<td>Length of beam</td>
<td>( D_r )</td>
</tr>
<tr>
<td>( b_p )</td>
<td>Width of beam</td>
<td>( \alpha \cdot L )</td>
</tr>
<tr>
<td>( h_p )</td>
<td>Thickness of piezoelectric beam</td>
<td>( \beta \cdot L )</td>
</tr>
<tr>
<td>( a \times b \times c )</td>
<td>Dimension of proof mass</td>
<td>( (\alpha \times \alpha \times \alpha) \cdot L )</td>
</tr>
<tr>
<td>( \rho_p )</td>
<td>Density of piezoelectric material</td>
<td>7700 kg/m(^3)</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>Density of substrate material</td>
<td>1500 kg/m(^3)</td>
</tr>
<tr>
<td>( \varepsilon_{31} )</td>
<td>Piezoelectric constant</td>
<td>(-22.2 \text{ V} \cdot \text{m/N} )</td>
</tr>
<tr>
<td>( d_{31} )</td>
<td>Piezoelectric charge constant</td>
<td>(-315 \times 10^{-12} \text{ m/V} )</td>
</tr>
<tr>
<td>( \varepsilon_{r33} )</td>
<td>Piezoelectric relative dielectric constant</td>
<td>4500</td>
</tr>
<tr>
<td>( Y )</td>
<td>Young’s modulus of substrate</td>
<td>140 GPa</td>
</tr>
<tr>
<td>( \sigma_T )</td>
<td>Tensile Strength</td>
<td>80 MPa</td>
</tr>
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</table>

(A) resonant

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value in Calculation</th>
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<tbody>
<tr>
<td>( N_e )</td>
<td>Number of exciters</td>
<td>Variable</td>
</tr>
<tr>
<td>( N_{pe} )</td>
<td>Number of beams</td>
<td>1</td>
</tr>
</tbody>
</table>

(B) non-resonant

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value in Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_e )</td>
<td>Number of exciters</td>
<td>2</td>
</tr>
<tr>
<td>( N_{pe} )</td>
<td>Number of beams</td>
<td>1</td>
</tr>
</tbody>
</table>
As the viscosity of material is constant for the same material operating in the same conditions, the value can be calculated from one test of the mechanical damping coefficient, and then applied to other scales. From [186], a damping ratio of 0.0175 was measured. According to Eq. (8.17)-(8.20), the structural damping coefficient is 0.0355 Pa·s for a bimorph beam with carbon-fiber as the substrate (M1100).

Figure 8.7: Scaling effect of harvester dimension on mechanical damping ratio. (a) Variation of damping factors against dimension for a particular beam dimension ratio $\alpha = 0.8$ and $\beta = 0.06$ and (b) Scaling effect on total mechanical damping ratio for different beam relative dimension ratios.

Based on the above equations and structural parameters, the scaling effect on mechanical damping is calculated and shown in Fig. 8.7. As shown in Fig. 8.7(a), the internal structural damping is constant and is the dominant factor in the dimension (5-20 mm) that we focus on. The air damping is much weaker and increases with the decrease of beam dimension. The total damping remains almost constant in this dimension range, and therefore this scaling effect can be ignored. In Fig. 8.7(b), different relative ratios for beam dimension are considered in order to demonstrate the universality of the scaling effect on mechanical damping in this range. The steady state of total mechanical damping in this dimension range (5-20 mm) is still valid.

Due to the complexity of the governing equations of piezoelectric resonant rotational harvesters, the scaling law for the power generation capability is not explicitly indicated in the equations. In order to acquire the scaling information, the performance of the harvesters for different scales and rotational frequencies is quantitatively investigated first. The calculation is based on several specific data sets, as listed in Table 8.2 from common
Figure 8.8: Scaling effect of harvester dimension on (a) output power and (b) resonant frequency. The markers in (b) and (c) are results calculated using different beam relative dimensions, and the solid lines indicate the scaling laws for output power and resonant frequency.

Figure 8.9: Output power and scaling laws of PNRREHs: (a) Output power (for $\alpha = 0.06$, $\beta = 0.015$) versus harvester diameter and rotational frequency, (b) scaling effect of harvester dimension ($\alpha = 0.06$, $\beta = 0.015$) and (c) scaling effect of rotational frequency ($\alpha = 0.06$, $\beta = 0.015$).
piezoelectric beams with typical ceramic materials. The power output for \( \alpha = 0.06 \) and \( \beta = 0.015 \) is illustrated in Fig. 8.8(a). It is evident that resonant rotational energy harvesters have a narrow bandwidth and so should operate in conditions where rotational frequency is constant or varies in a narrow range.

For PRREHs, the resonant frequency is determined by the dimension of the transducers when the material parameters are fixed. Therefore, the rotational frequency is also a dependent variable of the harvester dimension. In order to examine the scaling effect on both the resonant frequency and the output power, these values are selected from Fig. 8.8(a) and depicted in Fig. 8.8(b) and (c). Different relative dimension ratios are investigated to examine the universality of the scaling law. Based on the calculation, the scaling laws for PRREHs can be expressed as

\[
\omega_{re} \propto L^{-1} \quad \text{and} \quad P_{\text{max}} \propto L^2,
\]

where \( L \) is the characteristic linear dimension (\( D_r \) in this case).

Compared to EMREHs (\( P_{\text{max}} \propto L^5 \)), PRREHs are much less affected by the scaling effect of harvester dimension, to the extent that power density actually increases with decreasing size. However, the bandwidth of PRREHs is limited by the operating requirement of resonant vibration. In addition, in order to match the resonant frequency, multiple driving magnets are required to amplify the low rotational frequency. The implementation of driving magnets at small scale (e.g. \( \mu \text{m} \) scale) is difficult, especially when the resonant frequency (\( \omega_{re} \propto L^{-1} \)) is high.

### 8.3.3 Piezoelectric Non-Resonant Harvesters

For PNRREHs, the excitation frequency and the resonant frequency of the beams are decoupled. Therefore, the frequency bandwidth is much wider, and the rotational frequency is not determined by the harvester dimension. The performance of PNRREHs is studied for different dimensions and rotational frequencies. The structural parameters used for performance evaluation are the same as those used for resonant harvesters with difference in the number of beam exciters. The output power as a function of harvester dimension and rotational frequency is illustrated in Fig. 8.9(a). The variation of output power has a similar trend to that of EMREHs. In order to understand the difference of the impact of the operation frequency and device dimension variation on the output power, the scaling law for output power of PNRREHs are studied for different harvester dimensions and rotational frequencies.
Figure 8.10: Output power versus rotational frequency and harvester dimension for different relative dimension ratios of piezoelectric beams.

Fig. 8.9(b) depicts the output power of PNRREHs as a function of harvester diameter. The output powers for different rotational frequencies are illustrated. The power is proportional to the harvester diameter \( (D_r) \) to the power of 2.5. Output power for different harvester diameters as a function of rotational frequency is shown in Fig. 8.9(c). The power is proportional to the rotational frequency to the power of 0.5. Different relative beam dimension ratios are investigated to ensure the stability of the scaling law on the output of PNRREHs, as illustrated in Fig. 8.10. Logarithmic coordinates are adopted for each axis, and the output power for different beam relative dimension ratios is parallel to each other, so the stability of the scaling law is verified. Based on the above analysis, the scaling law for PNRREHs is

\[
P_{\text{max}} \propto \omega^{0.5} L^{2.5}.
\] (8.22)

Hereto, the scaling effects of harvester dimension and rotational frequency on the output power of three types of rotational energy harvesters are discussed, as summarized in Table 8.3. These scaling laws can be used to estimate the performance of the same harvester design on different dimension and operating frequency scales. In addition, from the scaling laws we can find that the performance of EMREHs is more sensitive to harvester di-
Table 8.3: Scaling effect of harvester dimension and rotational frequency on the output power of three types of rotational energy harvesters.

<table>
<thead>
<tr>
<th>Power Output</th>
<th>Harvester Dimension (L)</th>
<th>Rotational Frequency (ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{EMREH}$</td>
<td>$L^5$</td>
<td>$ω^2$</td>
</tr>
<tr>
<td>$P_{PRREH}^a$</td>
<td>$L^2$</td>
<td>$-$</td>
</tr>
<tr>
<td>$P_{PNRREH}$</td>
<td>$L^{2.5}$</td>
<td>$ω^{0.5}$</td>
</tr>
</tbody>
</table>

$^a$ PRREHs are normally designed to operate at resonance, and $ω_n$ is proportional to $L^{-1}$.

The performance of the three types of rotational energy harvesters is compared in Fig. 8.11. Due to the specific design parameters used for each type, the figure only shows the general trend of performance variations for each type. Given that there are different designs and material properties available, it is impossible to give an absolute boundary for determining the best design for a particular region.

In Fig. 8.11, the performance of three different types of harvesters are combined and compared. Due to the fact that the calculation is conducted using specific design parameters for each type, the figure only shows the general trend of performance variations for each type. In addition, given that there are different designs and material properties available, it is impossible to give an absolute boundary for determining the best design for a particular region.

However, we can still find some useful guidelines for other researchers for rotational energy harvester design. PRREHs have the dominant performance over a narrow bandwidth at resonance. When harvester dimension and rotational frequency increase, the performance of EMREHs increases dramatically. This also explains the overwhelming application of electromagnetic induction at large scale. However, their performance decreases rapidly with the decrease of dimension and frequency. Certainly, in some cases, gear trains can be adopted to amplify the low frequency, and the performance of EMREHs can be maintained, but the cost and complicity
by adding gear trains should be evaluated. Compared to EMREHs with no gear trains, the performance of PNRREHs has a significant advantage for low-frequency and small-scale conditions. This type of harvesters also exhibits a wide bandwidth.

![Graph showing performance comparison of three types of harvesters](image)

**Figure 8.11:** Performance comparison of three types of harvesters for different harvester dimensions and rotational frequencies.

For the specific data set used in Fig. 8.11, the advantage of PNRREHs becomes significant when the device diameter is less than 10 mm and the rotational frequency is below 10 Hz, and the output power is generally below 1 mW. This also indicates that for rotational energy harvesting at small scale and low frequency, PNRREHs are desirable.

### 8.4 Conclusion

In this chapter, a theoretical comparison of different configurations of power rotational generators is presented for low operating frequency and small scales (millimeter to centimeter range). Three types of rotational energy harvesters, namely electromagnetic, piezoelectric resonant and piezoelectric non-resonant, are discussed and compared theoretically for different dimensions and operating frequencies. The power generation capability is studied
using specific design parameters to illustrate the general variations of performance for different dimension and operating frequency.

The scaling laws for device dimension and operating frequency on harvester performance are investigated and summarized for these harvesters based on theoretical analysis. Electromagnetic rotational energy harvesters are strongly affected by the scaling effect \( P \propto L^5\omega^2 \) compared to the others, whereas piezoelectric non-resonant rotational energy harvesters are relatively insensitive to dimension and frequency variation \( P \propto L^{2.5}\omega^{0.5} \). It is worth noting that the scaling laws for piezoelectric harvesters are valid when the harvester diameter is larger than 5 mm (because of the assumption of the constant mechanical damping ratio).

Based on the comparison of output power for the three types of harvester, the ideal operating zone for each type is identified. EMREHs are desirable to operate at high operating frequency at macro-scale (> 10 mm & > 10 Hz). PNRREHs are ideal for micro-scale cases operating at low frequency (< 10 mm & < 10 Hz). RRREHs are designed to operate at resonance. Therefore, the operating frequency should be constant or stable, and the harvester can be designed and optimized according to the frequency and dimension requirements. For EMREHs, gear trains can be adopted in some cases to mitigate the frequency scaling effect, but system complicity and fabrication cost should be considered.

This study provides a guideline for selection and design of rotational energy harvesters with specific device dimension and operating frequency requirements. The proposed scaling law offers a handy tool to estimate harvester performance for different dimension and operating frequency scales.
9 Conclusions and Future Work

The final chapter summarises the results and contributions of this research and highlights the original contributions and novelties. Potential development and future work are discussed based on the current progress. A list of publications that have arisen from this project is presented.

9.1 Conclusions

This research focuses on rotational energy harvesting for low-power electronics in IoT applications. The investigation started with a comprehensive review of energy harvesting technology, especially focused on rotational energy harvesting. Then, as an example of rotational energy harvesting, an airflow energy harvester using miniaturized turbine and piezoelectric transduction was developed with issues and potential advancement identified. Strategies to reduce the cut-in speed were studied and implemented to enable the harvester to start up at low airflow speed.

Later, the research scope was focused on the rotational transduction part in order to understand the harvester’s electromechanical dynamics. Different harvester configurations were studied theoretically to compare the performance variation for different excitation frequencies and to identify the best harvester design. Bi-stability was then integrated into a frequency up-converting harvester to enhance the power generation capability over a wide bandwidth at low-frequency. A theoretical model was established to understand the operation and to determine the high energy-orbit vibration. After that, an integrated energy harvesting system, including a bistable frequency up-converting harvester and a power management circuit, was built to fully power a wireless sensor node for environmental condition monitoring.

Finally, a fundamental classification and a performance evaluation of rotational energy harvesters were conducted, providing a general tool for re-
searchers to determine the best design for a specific dimension and operating frequency requirement. According to the contents in this thesis, the contributions are summarized as follows:

- Integration of miniaturized turbine and piezoelectric conversion using beam plucking. The dynamics of this combination were studied. Issues, including the high cut-in speed and power fluctuation in high input excitation frequency, were identified for further improvement.

- Self-regulation mechanism. A self-regulating method using the centrifugal force generated by the rotation of a turbine rotor was proposed and implemented to passively control the magnetic plucking strength of the piezoelectric beam according to the rotational frequency. A micro-planar spring was designed and fabricated to achieve the position control of the driving magnet. This mechanism allows the harvester to start up with a low plucking strength at low excitation frequency and to exhibit an increased plucking strength (output power) at high excitation frequency.

- Harvester configuration optimization. Different configurations in terms of magnetic polarization direction, beam orientation, driving magnet motion direction, and relative position of driving and tip magnets were considered and examined in a theoretical study to understand their electromechanical behaviours and to optimize the output power. The power fluctuation issue at high excitation frequencies was explained from this study.

- Introduction of bi-stability. A bi-stable frequency up-converting harvester was proposed and studied. Bi-stable behaviours allow the harvester to reduce the power fluctuation over a wide bandwidth at low frequency by enabling the harvester to operate in the high energy orbit. The theoretical study also provides an in-depth understanding of bistable frequency up-converting harvesters.

- SSHI circuit. Power fluctuation is still a limitation for bistable frequency up-converting harvesters when the excitation frequency is higher. The integration of the SSHI circuit enhances the electrical damping and alleviates the fluctuation further. Also, the better power gener-
ation capability of SSHI circuits enables the harvesting system to be more effective.

- Performance evaluation. Three types of rotational energy harvesters, including electromagnetic, piezoelectric resonant and piezoelectric non-resonant rotational harvesters were studied theoretically to evaluate their performance for different device dimensions and operating frequencies. Scaling laws were established, showing the different influence of device dimension and operating frequency on the performance. This study provides a guideline for selection and design of rotational energy harvesters with specific requirements on device dimension and operating frequency.

### 9.2 Future Work

In this project, different designs and mechanisms, including a miniaturized turbine, beam plucking, self-regulation, piezoelectric conversion, bi-stability and a SSHI circuit, are studied extensively and intensively in both theoretical and experimental studies for effective rotational energy harvesting from low-frequency and random motions. However, there are still many aspects to investigate to further enhance the power generation capability and also to make the harvesting system commercially available.

1. Micro-turbine dynamics. For the airflow energy harvesters, the work in this thesis is mainly focused on the piezoelectric transduction part. The dynamics of the turbine under plucking motion have not been investigated and optimized. A study including turbine dynamics will be beneficial to understand the system behaviour and finally to enable better transducer design.

2. Limited input energy sources. In the rotational energy harvester design, the input energy source is assumed to be infinite, and the motion is unaffected by the piezoelectric plucking motion. However, in many applications such as miniaturize turbines and human motion, the power provided by the rotational hosts is limited. Therefore, the host motion can also be affected (the same as the cogging effect in electromagnetic generators.). Studying the dynamics of harvesters with limited input energy sources will provide a better understanding and guidance for energy harvester design in
these particular scenarios.

3. Multi-plucking motions. For the harvesters presented in this thesis, there are just one driving magnet and one tip magnet (one piezoelectric beam) for transduction. Multi-driving and tip magnets will provide more plucking motions in each cycle. This has the potential to enhance the output and to broaden the operating bandwidth. Also, these multi-plucking motions will complicate the dynamics of the system. A study for multi-plucking motions has the possibility of creating a more effective energy harvesting device.

4. SSHI circuit design. A SSHI circuit was adopted for managing the output from a bistable frequency up-converting harvester. Enhancement has been achieved in an experiment, but a theoretical study and optimization will be necessary to understand the dynamics of the system and to further enhance the performance. Meanwhile, the parameters of electronic components, such as the capacitance and operating voltage range of the storage capacitor, can be further optimized.

9.3 Publications

Here are the publications that have arisen from this research.

Journal Publications


- H. Fu and E. M. Yeatman, Effective Kinetic Energy Harvesting using Bi-Stability and a Synchronized Switch Circuit with Frequency Up-


Conference Publications


References


[47] Y.-J. Wang, C.-D. Chen, and C.-K. Sung, “System design of a weighted-pendulum-type electromagnetic generator for harvesting en-


[56] H. Fu, K. Cao, R. Xu, M. A. Bhouri, R. Martínez-Botas, S.-G. Kim, and E. M. Yeatman, “Footstep energy harvesting using heel strike-induced airflow for human activity sensing,” in *Wearable and Im-


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