A framework for the optimization of terminal airspace operations in Multi-Airport Systems

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Abstract

Major cities like London, New York, and Tokyo are served by several airports, effectively creating a Multi-Airport System (MAS), or Metroplex. The operations of individual Metroplex airports are highly interdependent, rendering their efficient management rather difficult. This paper proposes a framework for the design of dynamic arrival and departure routes in MAS Terminal Maneuvering Areas, which fundamentally changes the operation in MAS airspaces for much improved efficiency when compared to the current situation. The framework consists of three components. The first presents a new procedure for characterizing dynamic arrival and departure routes based on the spatio-temporal distributions of flights. The second component is a novel Analytic Hierarchy Process (AHP) model for the prioritization of the dynamic routes, which takes into account a set of quantitative and qualitative attributes important for MAS operations. The third component is a priority-based method for the positioning of terminal waypoints as well as the design of three-dimensional, conflict-free terminal routes. Such a method accounts for the AHP-derived priorities while satisfying the minimal separation and aircraft maneuverability constraints. The developed framework is applied to a case study of the New York Metroplex, using aircraft trajectories during a heavy traffic period on typical day of operation in the New York Terminal Control Area in November 2011. The proposed framework is quantitatively assessed using the AirTOp fast-time simulation model. The results suggest significant improvements of the new design over the existing one, as measured by several key performance indicators such as travel distance, travel time, fuel burn, and controller workload. The operational feasibility of the framework is further validated qualitatively by subject matter experts from the Port Authority of New York and New Jersey, the operator of the New York Metroplex.

Keywords: multi-airport system; Metroplex; spatio-temporal clustering; analytic hierarchy process; 3-D route design

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1. Introduction

The continuous increase in air travel demand coupled with rapid urban growth during the past century have resulted in the emergence of secondary airports in the proximity of large metropolitan areas (Sidiropoulos et al., 2015). These are in addition to the primary airport serving a given area, and together this gives rise to a Multi-Airport System (MAS) or "Metroplex" (e.g. London, New York, Tokyo). Due to the spatial proximity of the Metroplex airports, their operations are interdependent and complex, rendering their efficient management a difficult challenge (Lall, 2018). In the absence of an effective centralized coordination of the operations, the prevailing practice in such systems is for Air Traffic Controllers (ATCos) to allocate traffic in an ad-hoc manner based on their experience. This results in a sub-optimal utilization of the potential capacity of the system (Clarke et al., 2010).

There has been a few attempts to optimize the operation of MAS Terminal Maneuvering Area (TMA), primarily through airspace redesign consultations [Los Angeles World Airports, 2014; NATS, 2013; New York Area Rroute Traffic Control Center (ARTCC), 2011]. However, such attempts often rely on ad-hoc measures tailored for specific airport systems; for example, Visual Flight Rules (VFR) corridors have been allocated in both the New York and Los Angeles basin Metroplexes. Unsurprisingly, while such measures may result in locally improved solutions for an individual airport, they are often sub-optimal for the system of airports, and pose numerous difficulties in their widespread deployment due to a lack of transferability. In view of this, a number of generic attempts have been made to improve Metroplex operations, including: improved aircraft scheduling, optimized capacity trade-offs and optimized infrastructure planning (McClain, 2013; Clarke et al., 2012; Atkins et al., 2011; Donaldson and Hansman, 2011; Ramanujam and Balakrishnan, 2009; de Neufville, 1995; Hansen, 1995; Hansen and Weidner, 1995; Hansen and Du, 1993). However, all these approaches are limited by the current terminal airspace structure, i.e. its geometry, which is given a priori. In contrast, it is our contention in this paper that the existing static airspace structures may be the main impediment for improving Metroplex operations. We demonstrate this by proposing a novel framework for the new design and planning of terminal airspace in the MAS, based on the *dynamic airspace configuration* (DAC) concept (Kopardekar et al., 2007), together with the improved design of conflict-free arrival and departure routes in order to increase operational efficiency in terms of the travel distance, travel time, fuel burnt, controller workload and cost.

This framework takes both a systematic and holistic approach for a paradigm-shifting design of MAS terminal airspaces, through the seamless integration of a:

- demand characterization method that identifies major traffic flow patterns during the operational horizon (24 hours), and derives dynamic routes accordingly (see Section 3);
- route prioritization framework that enables the decision maker to influence the design based on a set of demand characteristics essential for MAS operations (see Section 4.2); and
- 3-D routing problem for constructing the airspace structures (arrival and departure routes) in accordance to the dynamic routes (see Section 4.4).

This framework embraces a novel Concept of Operations (CONOPs), namely the *dynamic route* concept, which shifts from the traditional ad-hoc, First-Come-First-Serve (FCFS) service policy that handles aircraft individually, towards a strategic service policy based on the systematic assignment of aircraft to a set of dynamic routes. A dynamic route is defined to be a group of flights that share similar spatial and temporal characteristics. In addition, we let each dynamic route be associated with exactly one MAS airport and consist of either arrival or departure flights to/from that airport.

The proposed CONOPs recognize significant traffic flow patterns as they evolve in both space and time; and the designed routes are meant to dynamically accommodate such demand patterns. The dynamic route service policy allows the MAS operations to achieve a higher level of efficiency, which is demonstrated in this paper by the use of fast-time simulation modelling (see Section 6). Furthermore, it enhances the First-Come-First-Serve (FCFS) principle, which can be applied to existing flights already assigned to specific dynamic routes, thus promoting equity. In order to do this either, the air traffic controllers can either handle the established traffic on a FCFS basis, or schedule flights along each dynamic route in order to further optimize the operations. The dynamic route service policy can be applied to operations at either the strategic (a few days prior to flight) or pre-tactical (up to 3 hours prior to flight) levels. A hallmark of the research presented in this paper is the generic nature of the proposed methodology and its compatibility with subsequent, case-specific designs. The complete framework for developing and assessing the proposed concept is summarized in Figure 1.

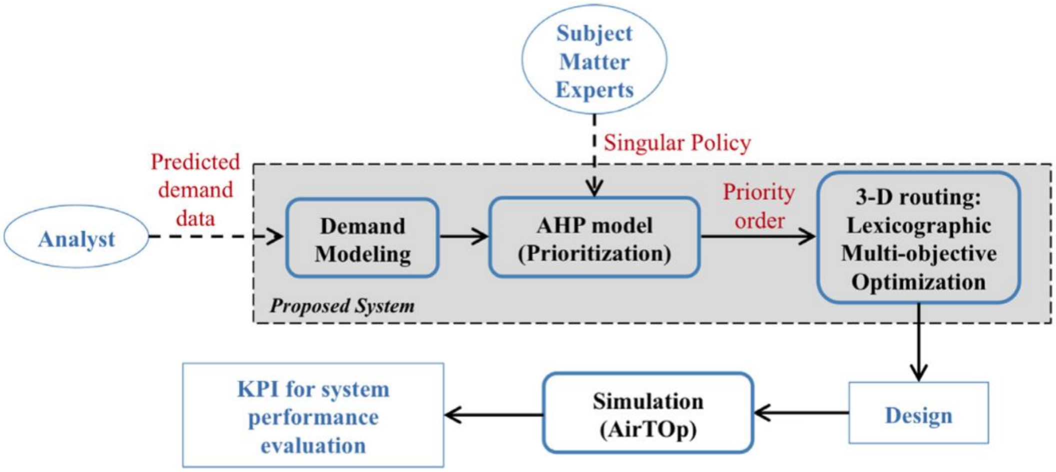

Figure 1. Framework for developing and assessing the proposed concept of operations.

Previous research typically overlooks the operational feasibility of the proposed solutions for a Metroplex. It is worth noting that the focus of the previous research is either on arrival or departure operations in isolation, or on the operation of a subset of the Metroplex airports irrespective of the others. Isolating a subset of operations fails to represent the complexity of the full-scale problem, and thus the feasibility and effectiveness of any proposed solutions may be compromised. This paper bridges this gap through a comprehensive assessment of the proposed framework using fast-time simulation modeling (FTS) (AirTOp soft) of the entire New York MAS, comprising of the airports of John F. Kennedy (JFK), LaGuardia (LGA), Newark Liberty International Airport (EWR) and Teterboro (TEB), arguably among the most complex Metroplex systems in the world in terms of the number and proximity of the airports, as well as its current airspace design. In addition, qualitative confirmation of the operational feasibility of the proposed framework is obtained through structured interviews with Subject Matter Experts (SMEs) from the Port Authority of New York and New Jersey (PANYNJ) and the Federal Aviation Administration (FAA).[1] FTS is a useful technique for rapidly

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[1] PANYNJ is responsible for the administration of the New York MAS airports; the FAA is responsible for the provision of air traffic control over the MAS.

evaluating the feasibility and efficiency of airspace design scenarios based on realistic and accurate inputs. In this paper, the current operational features in the New York MAS are accurately replicated in the FTS using the Performance Data Analysis and Reporting System (PDARS) data provided by the PANYNJ. The simulation results of the proposed design indicate a substantial improvement over the current MAS operations, as measured by several Key Performance Indicators (KPIs) including the total distance travelled, travel time, fuel burnt and air traffic controller workload (see Section 6 for details).

The main contributions made by this paper are elaborated as follows.

i. **Dynamic route design.** Based on the dynamic route concept, we provide a novel methodology for terminal airspace design in high-density terminal airspace that accounts for dynamic demand patterns as they evolve temporally and spatially. Our approach is novel also because it focuses on separation of flights at the route level (individual flights are assigned to specific optimal routes) rather than on a flight-by-flight basis. The latter has been the focus of most studies (e.g. Nikoleris, 2014) and is not a viable solution for high-density airspace in the authors' opinion. Moreover, the existing standards and regulations regarding terminal airspace design (Eurocontrol, 2013) fail to provide a design methodology specific to Metroplex systems or high-density terminal operations. The dynamic route concept aims directly to tackle this and, through this paper, outlines a complete and detailed framework for the design of such airspaces. Finally, while Standard Instrument Departure Routes (SIDs) and Standard Instrument Arrival Routes (STARs) in current operations are typically ignored due to their inefficient design and non-responsiveness to growing demand, to be replaced by vectoring (Interview, 2015), the dynamic route concept allows terminal airspace structures to be utilized in fundamentally new ways.

ii. **Multi-Objective Optimization (MOO)**. The 3-D routing of all the dynamic routes is solved by a Lexicographic Multi-Objective Optimization (LMOO) method. This method is novel as it is the first attempt to utilize a semi-qualitative method (i.e. the AHP) in order to analyze, decompose and synthesize the numerous criteria involved in real-world decision making that pertain to terminal airspace management. The resulting routing algorithm, by virtue of the AHP, is the first to provide feasible, efficient and up-to-date routes that can be used in real operations. To the best of our knowledge, this is the most comprehensive use of the AHP method applied to air traffic management (ATM) routing to date. Moreover, the AHP reduces a computationally intractable MOO problem into a sequence of routing problems that are much easier to solve, while accounting for all the operational considerations.

iii. **Real-world validation.** The effectiveness and real-world relevance of our methodology is demonstrated by the use of fast-time simulation (FTS) as an external method of validation. Real-world operations data was collected and processed to provide a realistic, quantifiable assessment of terminal airspace design for a typical operational period in the NY MAS. Comparison with the current situation proves our design to be superior by a significant margin, as measured by several KPIs. We also conduct a qualitative validation of our design, by inviting SMEs to examine the proposed routing structures as well as the feasibility of the entire design. Last but not least, our study covers the entire four-airport NY Metroplex, internalizing all the interdependencies among these airports and providing a holistic investigation of the airspace. This is in contrast with previous studies that use internal methods of validation applied only to a subset of the airports, or specific types of operations in isolation.

iv. **Generality and transferability.** The proposed methods are validated for the NY Metroplex, which is arguably the most complex terminal airspace in the world, in terms of the number of airports, airport proximity and runway geometry, traffic volumes, traffic mix, airspace user mix and the number of saturated airport runways. Our methodologies pertaining to demand characterization, AHP and 3-D routing do not rely on any assumptions that prevent them from being calibrated and applied to other airport systems. Although the framework is initially designed for Metroplex systems, it can be transferred to the cases of single airport systems with minimum modification, thereby providing an advanced method for terminal aircraft routing and airspace management.

The proposed design is aligned with the Single European Sky Air Traffic Management Research (SESAR) and Next Generation Air Transportation System (NextGen), as it proposes a centralized framework for design, control, and evaluation of the entire MAS, while allowing for variations in the control strategies via the AHP with up-to-date information. It also embraces the concepts of Dynamic Airspace Configuration (DAC), 4-D trajectories, Arrival Manager (AMAN) and Departure Manager (DMAN) systems, due to its dynamic route concept and compatibility with case-specific designs (e.g. see Section 6.3).

The rest of the paper is organized as follows. Section 2 offers some relevant background and literature review. Section 3 elaborates the procedure for route classification, which serves as a preliminary of the 3-D route design. Section 4 presents the detailed route design problem expressed as a terminal fix selection problem and a lexicographic multi-objective optimization problem using decision maker preference information from a novel AHP route prioritization model. A real-world case study of the proposed framework is outlined in Section 5 and the fast-time simulation-based validation is presented in Section 6. Finally, Section 7 provides some concluding remarks and several future research directions.

2. Background and literature

Multi-airport systems (MAS) are among the most significant regional scaling mechanisms, enabling the air transportation system to adapt and evolve to meet travel demand (Bonnefoy, 2008). The Joint Planning and Development Office (JPDO) provides a simple definition for Metroplex as: "a group of two or more adjacent airports whose arrival and departure operations are highly interdependent" (Atkins et al., 2011). There have been several definitions of MAS in the literature that utilize a variety of characteristics to describe their formation, including spatial proximity of the airports, traffic volumes (FAA, 2014; McClain, 2013; Bonnefoy, 2008; de Neufville and Odoni, 2003), and ownership of the airports (ACI, 2002). Atkins et al. (2011) provide a set of qualitative and quantitative definitions of Metroplex (e.g. observation-based, model-based and simulation-based) to better describe specific Metroplex phenomena. Most recently, Sun et al. (2017) develop a novel temporal metric to define a Multi Airport Region (MAR) based on the travel time required for passengers to commute between airports. The development of new airports (e.g. the future Beijing Daxing International airport serving the JingJinJi megaregion) and the continuous expansion of existing ones in major metropolitan areas (Li et al., 2017), as well as the increasing occurrence of Metroplex phenomena even in smaller airport systems (Atkins et al., 2011), all underline the significance of MAS as a key component of the modern ATM system.

The effective management of the MAS is a key requirement for the relevant stakeholders including the airports, Air Navigation Service Providers (ANSPs), air traffic controllers, and airspace users (especially the airlines). Currently, MAS operations are loosely coordinated (Clarke et al., 2010), resulting in many system inefficiencies that compromise the stakeholders' goals. The lack of an effective Airport Collaborative Decision Making (A-CDM) leads to non-cooperative gaming among MAS airports for the use of airspace resources (Sidiropoulos et al., 2015). The ANSPs are required to provide the necessary structures that ensure efficient services for both airports and airspace users. However, the lack of effective network planning results in stringent operational constraints on the latter. Airlines face the problem of minimizing their operational costs while providing satisfactory services whilst at the same time having to abide by the relevant regulations and other operational constraints. Under such a competitive operational environment, the burden of coordinating air traffic demand in the MAS is placed on the air traffic controllers, who, due to capacity constraints and increased workload, often resort to the vectoring of flights and the extensive use of holding stacks (mainly in Europe and, to a lesser extent, the US where ground-holding programs are typically employed).

With the aim of improving Metroplex operations, Ren et al. (2009) analyzed four major US Metroplex areas and identified a set of critical issues and operational interdependencies in these airspaces. The authors developed a framework in order to evaluate the impact of each issue based on a qualitative scale. This they complemented with a link-node queuing process model developed for the optimization of arrival flows. Despite the improved performance of the system in terms of delay, the authors' framework failed to provide a generic methodology as it relied on ad-hoc route geometry. Li et al. (2011) extended the previous model to test three different generic Metroplex layouts (tandem, parallel and crossing). For a given layout, they examined the impact of sharing terminal fixes and the system-wide scheduling of operations. Among the several operational scenarios examined, the decoupling of fixes and the temporal coordination between airports were deemed the preferred options in order to improve system efficiency. Atkins et al. (2011) developed a unified model to simultaneously optimize route assignment, sequencing, and scheduling for aircraft operations in a Metroplex. The resulting deterministic Mixed Integer Linear Program (MILP) problem was applied to a two-airport scenario and achieved a 13% increase in aircraft throughput. However, the scope of Atkins et al.'s (2011) is limited, as the routing method was developed for specific types of route interactions (e.g. two parallel routes) and the model was only implemented for a specific route interdependency between LGA and TEB airports. Moreover, the model stipulated a priori route geometry and assignment, thereby failing to accommodate more general demand patterns.

Another body of literature focuses on demand modeling for the analysis of Metroplex operations. Timar et al. (2010) developed a tool for analyzing the current and future traffic demand, which enables the generation and allocation of demand to terminal fixes for different Metroplex terminal airspace configurations. Their tool aims to assess the impact of directional traffic distribution on flight delays for both Atlanta Hartsfield-Jackson International (ATL) and Los Angeles International (LAX) airports. Their results reveal that flight delays are sensitive to fix loading, and that demand distribution has a significant impact on the delays at the fix, airport, and Metroplex levels. To optimize flight schedules, Timar et al. (2011) developed a model of high traffic density Metroplex airspace operations. This model effectively identified the most frequently used arrival and departure procedures and airways using trajectory-based modeling. The model was developed for the southern California Metroplex consisting of Los Angeles International (LAX), Burbank (BUR), Long Beach (LGB), Ontario (ONT), San Diego (SAN), and Santa Ana (SNA). Validation of this model was by simulations, which provided sufficient detail on the current route structure. While models of this type can be used as the basis for route scheduling algorithms, they are constrained by the existent airspace design and ATC actions.

Focusing on the temporal de-confliction of flights as means to improve Metroplex operations, Wieland et al. (2014) formulated an integrated arrival departure and surface scheduling system for Metroplex operations. The platform consisted of a set of optimization algorithms for sequencing, runway assignment and route allocation with the objectives of maximizing throughput, enhancing safety and minimizing the environmental impact. The model was applied to the NY Metroplex (only for JFK and LGA) for a high-traffic day before the Thanksgiving Holiday, when traffic is at its peak during the during. The model successfully reduced departure delays by 22%. However, the exclusion of EWR and TEB from the analysis is a potential drawback as acknowledged by the authors.

The following two Traffic Flow Management (TFM) initiatives for both single-airport and multi-airport terminal airspaces are mentioned here due to their widespread application in terminal airspaces in the U.S. NASA has developed the Traffic Management Advisor (TrMA)[2] (Nedell et al., 1990) and the Precision Departure Release Capability (PDRC) (Engelland et al., 2013) tools, for the

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[2] In the US, the Traffic Management Advisor is typically abbreviated as TMA. In this paper, we use TrMA to avoid confusion with Terminal Maneuvering Area (TMA).

scheduling of arrivals and departures, respectively. The primary function of the TrMA is the real-time scheduling of arriving traffic within approximately 200 NM from touchdown. The tool is superior to other AMAN systems, as the scheduler is complemented by a graphical user interface and by additional interactive tools that aid the adjustment of schedules. The PDRC, on the other hand, optimizes the departure schedule for take-off times with a high precision. For flights with PDRC, the schedule is automatically communicated to the Air Route Traffic Control Center (ARTCC), which, in turn, calculates the ascent trajectories from the take-off point to the merge point in the overhead flow. With the goal of improving flow efficiency and reducing controller workload, both the TrMA and the PDRC are deployed in several terminal areas in the U.S., including Metroplexes.

Given the significance of Metroplex systems for serving air traffic demand in large metropolitan regions (e.g. New York, Los Angeles, San Francisco), the FAA has initiated the Optimization of Airports and Procedures in Metroplex (OAPM) (FAA, 2014) task force for the improvement of operations. OAPM identified 21 sites as Metroplex areas throughout the US (FAA, 2014) and assigned dedicated study teams to explore the opportunities for improving their operations. However, OAPM has mainly focused on less busy Metroplex systems so far [e.g. Houston Metroplex comprising of one primary and one secondary airport (Bonnefoy, 2008)], implementing ad-hoc measures to improve operations without providing a general model for the holistic terminal airspace design.

In order to overcome the aforementioned limitations, as stated earlier, this paper proposes a novel concept of operations based on *Dynamic Airspace Configuration* (DAC) (Kopardekar et al., 2007). While most studies relating to dynamic airspace configuration focus on the en-route phase, i.e. dynamic sector configurations (Sergeeva et al., 2017; Zelinski and Lai, 2011; Bloem and Gupta, 2010; Gianazza, 2010; Klein et al., 2008; Martinez et al., 2007; Delahaye et al., 1998), only a few in the literature have adopted the concept of DAC for terminal area operations. In terminal airspace, the most widely studied DAC concept is the dynamic airspace sectorization, which concerns with the re-configuration of the sector boundaries within the terminal airspace and attempts to increase airspace capacity by re-distributing the workload of the ATCos (Kopardekar et al., 2009). This concept allows the airspace to adapt to the wind conditions that determine the runway configurations. Currently, however, dynamic sectorization is not supported by any decision support tools to assist in the selection of the optimal airspace configuration so as to reflect the projected traffic demand or weather conditions (Kopardekar et al., 2009). Rather, these are done in an ad-hoc manner based on prior experience, and inevitably lead to sub-optimal operations of the system, which affect en-route sectors surrounding the terminal airspace (Kopardekar et al., 2009).

To overcome this limitation, the transition towards a functional type of sectorization is made via the concept of tube design for terminal routes. Kopardekar et al. (2009) suggest that tube design can serve as dedicated conflict-free arrival and departure routes in terminal airspace, providing guidance between the Top Of Descent (TOD) / Top Of Climb (TOC) and the final approach fix/take-off for arrivals and departures, respectively. The configuration of these corridors should be dynamic and reflect the current traffic and weather conditions (e.g. avoidance of adverse weather) (Kopardekar et al., 2009). The conflict-free terminal routes alleviate the need for ATC to provide lateral and vertical separation between aircraft on different routes, and instead enable them to simply monitor and ensure that aircraft stay on their predefined flight path and maintain the required longitudinal separation between aircraft on the same route. This approach can effectively increase airspace capacity by overcoming the limitation imposed by controller workload.

The importance of DAC in the TMA has been acknowledged by both the NextGen and SESAR programs with the use of strategically de-conflicted trajectories as the preferred and most efficient solution for highly congested areas (Eurocontrol, 2008; Hahn et al., 2007). However, literature on the design of dynamic terminal routes is rather scarce. Chen et al. (2012) and Chen et al. (2013) proposed a model for the optimal routing in single-airport systems by combining the DAC with an extended terminal airspace design, which includes additional airspace, thereby providing flexibility to enable severe weather avoidance. Zhou et al. (2014) proposed a fast-marching method for sequentially designing the 3-D routes, where a metaheuristic method is employed to determine the optimal order by which the routes are designed. However, such a heuristic approach ignores operational considerations and constraints, thereby rendering the approach impractical. Pfeil (2011) developed a model for a single-airport TMA DAC under uncertain weather conditions. In this study, an A*-based routing algorithm for the design of conflict-free 3-D terminal routes was proposed with the objective of minimizing the total distance travelled by all the routes. These routes were solved sequentially for arrivals first and subsequently departures. Despite the promising experimental results, the ad-hoc priority order (arrivals over departures) did not allow for additional operational considerations to be incorporated (e.g. arrival/departure push strategies), which are critical in multi-airport environments.

Zou (2010) developed the “Tree-Based Route Planner (TBRP)” to produce flexible routing structures in terminal airspaces. The routing structures are modelled as 2-D weather-avoidance trees between an extended terminal airspace boundary (100-200 NM) and one or more arrival fixes of the current terminal airspace boundary (40-50 NM). The TBRP employs a dynamic programming approach for the generation of the weather-avoidance trees with the objectives of maximizing aircraft throughput and minimizing flow complexity. However, the model does not address the design of the route structure in the current terminal airspace (within the 40-50 NM radius). Choi et al. (2010) developed a heuristic scheduling algorithm to investigate the impact of ad-hoc arrival merge point configurations on arrival scheduling performance for an extended terminal airspace design (up to 200 NM). The model considers aircraft trajectory uncertainties under the assumption that they are propagated linearly with respect to the route length. However, Choi et al.’s (2010) model is developed only for the arrival merge points, and ignored both the departures and the remaining arrival route structures.

In the Metroplex environment, improved management of arrival flows through dedicated terminal fixes and the splitting of arrival flows was observed after a test at the southwest boundary of the New York TRACON (Hahn et al., 2007). The creation of the dedicated "DYLIN" EWR arrival route combined with relocating area navigation (RNAV)-capable satellite airport traffic to the published JAIKE arrival resulted in reduced controller workload. Dedicated de-conflicted terminal fixes, which can be considered a precursor to the conflict-free routes, are expected to be widely adopted in the NAS in order to handle the future increase of air traffic demand (Hahn et al., 2007).

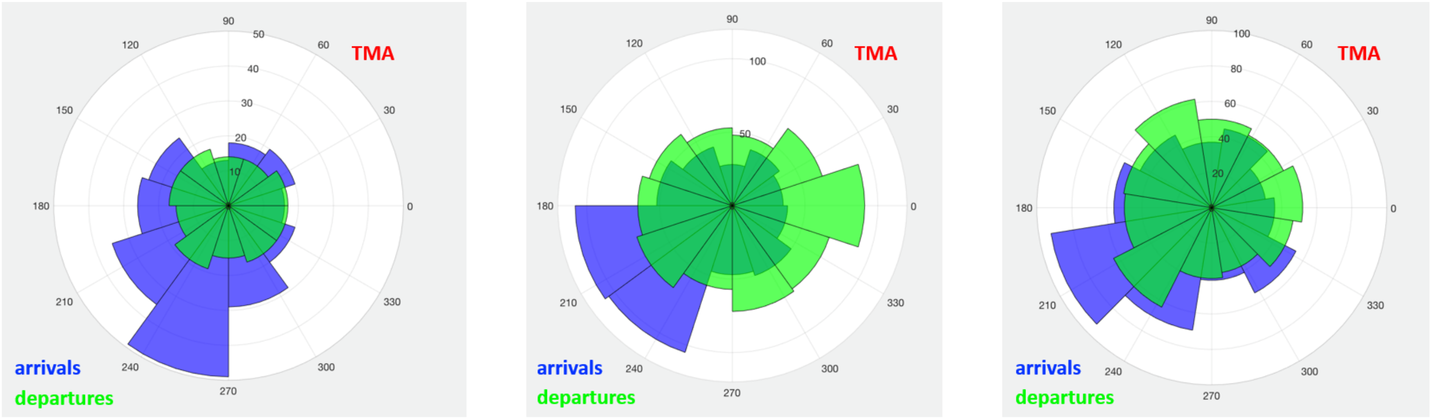
Studies concerned with the inevitable evolution of terminal airspace design to meet future air traffic demand suggest the concept of DAC as an adequate solution (FAA, 2013; Kopardekar et al., 2009; Eurocontrol, 2008; Hahn et al., 2007). In particular, Kopardekar et al. (2009) envisaged the future design of super-density Metroplex terminal airspaces enabled by advanced decision support tools considering the dynamic changes in air traffic flows. Kopardekar et al. (2009) and the FAA (2013) recommended that such a system should be supported by efficient RNAV routes, the provision of merge/diverge operations, extended terminal area airspace, expanded use of the 3 NM horizontal separation and the dynamic reconfiguration of sector boundaries.

This paper is significant in that it proposes a new framework for addressing the inefficiencies of Metroplex operations, by switching from the existent static route structure to the dynamic route service policy. To embrace such a concept, appropriate technical approaches have been introduced and implemented in every phase of the TMA design problem, including demand characterization, dynamic route formulation and prioritization, as well as 3-D routing within the TMA.

3. Demand characterization: Spatio-temporal clustering

A prerequisite for the application of a DAC solution is the accurate depiction of the traffic demand patterns both in space and time. In order to achieve this, the use of adequate and appropriate data is imperative. The data can be drawn from the latest Shared Business Trajectories (SBTs) and distributed via an advanced system [e.g. the Enhanced Traffic Management System (ETMS), Time Based Flow Management (TBFM) in the US]. Airlines use internal optimization systems to calculate the optimal routes of their flights in terms of the fuel consumption, cost and travelled distance (time). However, such optimization problems are constrained by numerous important factors associated with trajectory planning, such as airspace charges, punctuality, weather conditions, airspace configuration and route availability (Saturn project, 2015). The uncertainties associated with these factors can cause significant variability in the air traffic flows of the MAS and they contribute to the consensus regarding the inadequacy of current static airspace structures in serving dynamic air traffic demand. The International Civil Aviation Organization (ICAO) has taken a number of initiatives in an attempt to tackle this issue by revising the current static flight planning procedures (protocols) to provide more flexibility to the airspace users, while at the same time ensuring transparency to the other stakeholders (ICAO, 2015).

Figure 2 illustrates the directional distribution of arrival and departure flights in a few sampled time periods during a 24-hr horizon in the New York Metroplex TMA (Nov 2011). Here, the TMA is modeled as a cylinder with a radius of 150 km and height of 7 km centered at the centroid of the four NY airports. The Y-axis in the figure indicates the direction $(0^o - 360^o)$ at which the flights intercept the TMA boundary, and the X-axis indicates the time of entry/exit. The high concentration of flights observed at certain angles is indicative of the location of arrival or departure fixes. To derive useful information about significant traffic flow patterns and obtain the dynamic routes, a spatio-temporal clustering algorithm (Sidiropoulos et al., 2017) is used as described below.



06:00-11:00 11:00-16:00 19:00-22:00

Figure 2. Polar histograms of the variation of arrivals / departures geographical distribution in the TMA in example consecutive periods.

3.1 Temporal clustering

In order to detect the point in time when there is a significant change in the temporal distribution of flights in the terminal area, the following procedure is followed. We begin by partitioning the spatial domain (i.e. 0-360° on the TMA boundary) into discrete number of zones $z_k \in Z$, and the time horizon into discrete time steps $t_i \in T$. We also denote by $N_{i,k}$ the number of flights in zone z_k during time interval t_i.

First, we define $\text{Change}_{i,k}$ to be a binary variable indicating whether a significant temporal change in demand is detected for zone z_k from time t_{i-1} to t_i; that is,

$$\text{Change}_{i,k} = \begin{cases} 1, & \left|N_{i,k} - N_{i-1,k}\right| \geq T_1 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Here, T_1 is a user-defined threshold. Then we count the total number of zones where a change in demand is detected, and compare it with another threshold T_2:

$$\text{Change}_{i} = \begin{cases} 1, & \sum_k \text{Change}_{i,k} \geq T_2 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where Change_i indicates whether the demand pattern in the entire MAS TMA changes significantly from time t_{i-1} to t_i. The threshold values can be determined statistically for a given number of spatial and temporal steps by analyzing the distribution of the demand changes from historical data, and subsequently setting appropriate critical values. They can be also determined via an optimization procedure that maximizes the performance of the resulting clustering, while taking into account uncertainties in the dynamic traffic demands (Sidiropoulos et al., 2017).

Naturally, t_{i-1} and t_i are assigned to the same temporal cluster if $\text{Change}_i = 0$, meaning that the demand patters in these two periods are deemed similar. The procedure for creating the temporal clusters is summarized below.

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a.  Initially, define each time period $t_i \in T$ to be a temporal cluster.
b.  For $i = 2, 3, ...$, assign $t_i$ to the temporal cluster containing $t_{i-1}$ if $\text{Change}_i = 0$
c.  Return the final temporal clusters when b is checked for every $i$.
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3.2 Spatial clustering

Section 3.1 provides a method of partitioning the time horizon into a number of disjoint temporal clusters, each one corresponding to a unique demand pattern. Further *spatial clustering* is needed in order to derive the *dynamic routes* for each airport and operational type. The flights that belong to each temporal cluster are segregated according to the following characteristics: (1) type of operation (arrival/departure); (2) relevant MAS airport; and (3) direction (angle) of entry/exit.

The flights within each temporal cluster can be initially grouped into "arrival" or "departure" for each of the MAS airports. Subsequently, the flights within each resulting group (e.g. JFK-arrivals, EWR-departures, etc.) are further clustered according to the direction in which they enter or exit the TMA using a *K*-means spatial clustering algorithm. The *K*-means algorithm is a center-based algorithm and an appropriate choice for this application, as the resulting cluster center marks the intercept of the dynamic route with the TMA boundary (i.e. the unimpeded terminal waypoint for that cluster). The optimal number of clusters K is selected using the *gap criterion* heuristic (Tibshirani, Walther & Hastie, 2001). Figure 3 illustrates the spatial clustering of flights on the TMA boundary; we note that routes R_4 and R_5 have a conflict, as the minimum distance required does not separate their corresponding fixes.

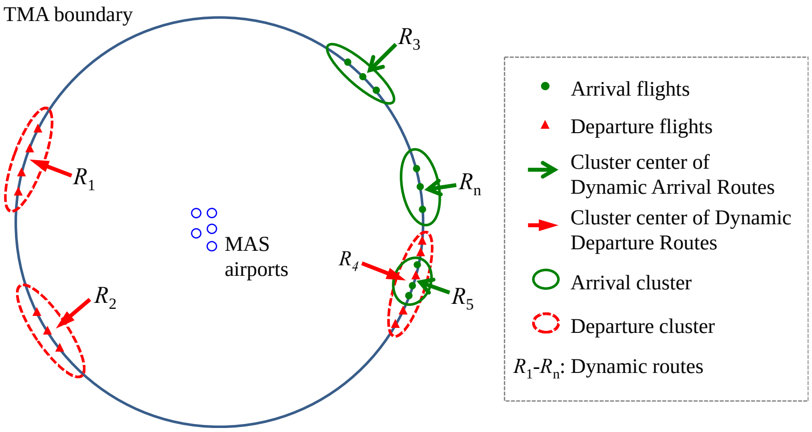

Figure 3. Dynamic routes for arrivals and departures (Sidiropoulos et al,, 2017).

The geometry of the TMA allows for only a maximum number of clusters corresponding to the maximum number of terminal waypoints $N_{max} = \frac{2\pi R}{D_{min}}$, rounded to the nearest integer below. Here, R is the TMA radius and D_{min} is the minimum separation distance between two adjacent terminal fixes. This constraint is taken into account in the clustering algorithm.

There are other ways to define the dynamic routes based on clustering, such as an integrated spatio-temporal 4-D clustering. However, the rationale for separated spatial and temporal clusters is to allow decoupled design/operation periods, such that each dynamic route is associated with exactly one well-defined time period. This is important as it provides flexibility for the decision maker in order to invoke different control strategies at different time periods of the day with clearly defined boundaries. On the other hand, an integrated 4-D clustering results in spatial and temporal coupling that leads to non-separable operation by either time or space. This would make the proposed framework less flexible and unlikely to be adopted in real-world operations.

The proposed spatio-temporal clustering approach can be applied to any given operational horizon. It is computationally efficient and can be applied to a variety of data sets with different levels of spatial and temporal resolution. Note that the choice of the threshold values T_1 and T_2 has an impact on the clustering results and, subsequently, on the 3-D routing problem. In fact, a detailed sensitivity analysis, as well as an optimization problem for the selection of the most appropriate threshold values, has been presented in Sidiropoulos et al. (2017). In that study, different clustering results were evaluated using the total sum of the distances of the individual flights from the centers of their assigned clusters. Moreover, spatio-temporal clustering in the presence of demand uncertainty is also addressed based on robust optimization.

4. Three-dimensional routing within the terminal area

In the case of MAS terminal airspace design, arrival and departure routing falls under the category of the many-to-many shortest path problem, where many terminal waypoints need to be connected to multiple airports (runways) in the most efficient manner. However, given the design objectives set out in this paper (i.e. all routes should be conflict-free to ensure safety and reduce controller workload), the many-to-many routing problem has certain limitations. Specifically, solving for all the routes simultaneously could:

- Lead to poor route geometries, as routes that need to use the same airspace would have to make additional horizontal and vertical turns to satisfy the conflict-free constraint. This reduces the smoothness of the routes and may result in operational infeasibility;
- Significantly increase computational complexity and resource requirements; and
- Ignore information on a decision maker's preference or a priori requirements.

To overcome these limitations, the routing problem is formulated as a sequence of constrained shortest path problem based on route priorities, to be derived from several qualitative and quantitative characteristics. In turn, the main benefits of this lexicographic optimization approach include the following:

- The sequentially solved one-to-one constrained shortest path problem yields much more smooth route geometries with fewer horizontal and vertical turns compared to solving all the routes simultaneously. This is desirable from a route feasibility perspective as well as from the need to reduce air traffic controllers' workload.
- The lexicographic solution procedure is computationally tractable, as it suffices to solve a set of one-to-one constrained shortest path problems. Computational efficiency becomes increasingly pronounced when the number of dynamic routes increases.
- The lexicographic approach requires a ranking of the dynamic routes, which naturally caters to the need to reflect the decision maker's preferred routes. This is achieved via the analytic hierarchy process model.

We start with a formal formulation of the multi-objective optimal routing problem in Section 4.1.

4.1 *Analytical formulation of the optimal routing problem*

The TMA route design problem consists of two parts: (i) a terminal fix selection problem on the TMA boundary (as motivated by the potential conflict among dynamic routes illustrated in Figure 3), and (ii) a 3-D routing problem within the TMA. The purpose of such a design is to serve the demands along the dynamic routes in the most cost-effective way. These two sub-problems are detailed below.

4.1.1 Terminal fix selection problem formulation

The terminal fix selection problem, which precedes the 3-D routing problem, can be formulated as follows. We let ξ_i be the location on the TMA boundary that corresponds to the i-th dynamic route (e.g. the cluster centers in Figure 3). Due to potential conflict among different dynamic routes, a subset of these routes needs to be relocated along the TMA boundary. Denoting the relocated fix for the i-th dynamic route by $\bar{\xi}_i$, the terminal fix selection problem aims to solve:

$$\min_{\bar{\xi}_i}\left[dist(\xi_1,\bar{\xi}_1), dist(\xi_2,\bar{\xi}_2), \dots, dist(\xi_i,\bar{\xi}_i), \dots, dist(\xi_n,\bar{\xi}_n)\right]^T \in \mathbb{R}^n \\ \text{Subject to } dist(\bar{\xi}_i,\bar{\xi}_j) \geq \Delta,\ \forall i \neq j \qquad (3)$$

where $dist(\cdot,\cdot)$ denotes the distance between two fixes on the TMA boundary, Δ is the minimum separation. The meaning of Eqn (3) is quite clear: in relocating the fixes, we aim to minimize their displacements (in the interest of minimum flying distance), while maintaining the minimum separation among all the relocated fixes.

Remark. Formulation of Eqn (3) applies to any TMA boundary geometry and the distance measure between two fixes. In the simplified case of a circular boundary with radius R, the location of the fixes can be parameterized by the angle θ based on polar coordinates, and the distance between two fixes θ_1 and θ_2 reduces to $|\theta_1 - \theta_2|R$.

4.1.2 3-D routing problem formulation

For the 3-D routing problem, the objective is to minimize the travel distances[3] of all the dynamic routes within the TMA. There are multiple ways of mathematically representing a route in the 3-D Euclidean space, e.g. using Cartesian or polar coordinates. In view of the flying and maneuvering characteristics of the aircraft, we propose a novel representation illustrated in Figure 4.

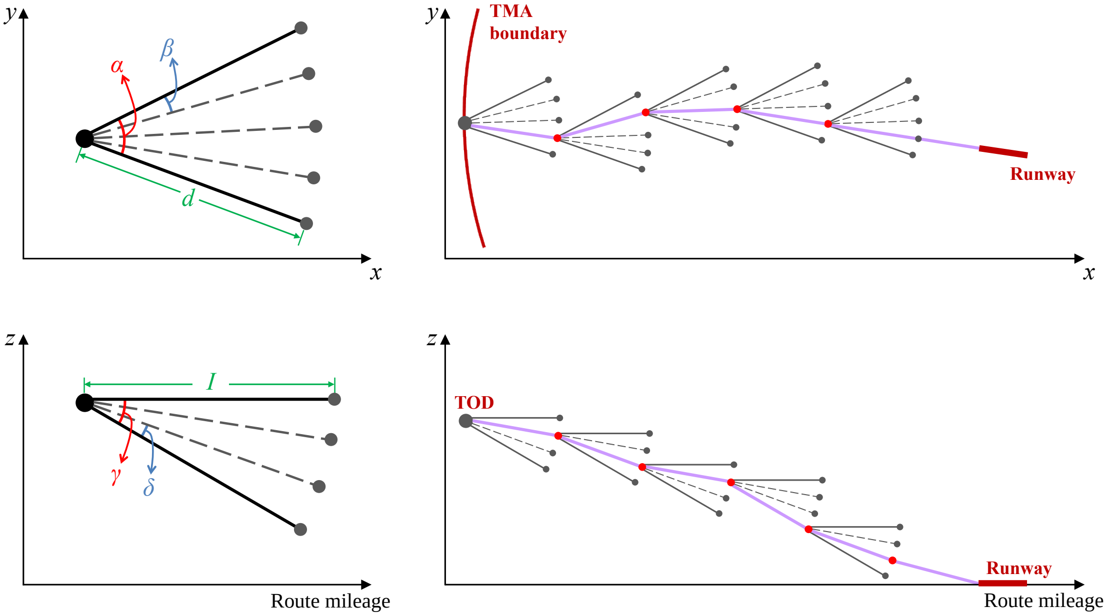

Figure 4. Representation of 3-D route (arrival) in the TMA.

In Figure 4, the dynamic route (for arrival) consists of a sequence of points connecting the fix on the TMA boundary to the runway. At each intermediate point, the next point along the route is defined via a horizontal and vertical change in the heading with a discrete number of angles (with increments β and δ, respectively) and the fixed track distance (d and I, respectively). Notice that the ranges of the angles (i.e. α and γ) depend on the maneuvering capabilities of the aircraft, altitude/speed and type of operation (arrival/departure) associated with the dynamic route. Similar representation can be defined for departure routes.

For a given dynamic route, we let H and V be the discrete sets of horizontal and vertical turning angles, respectively, and let N be the number of intermediate points along the route. Then the 3-D dynamic route r can be uniquely represented as a sequence of segments:

$$r = (z_1, z_2, \dots, z_n) \in \mathbb{R}^{2\times n} \quad \text{where } z_i = (h_i, v_i)^T \in H \times V \tag{4}$$

We further define the distance between two routes $dist(r_1, r_2)$ to be the minimum Euclidean distance between any two points from the two routes. Moreover, for any external area ω represented as a subset of the 3-dimensional Euclidean space, we similarly define its distance from a route $dist(r, \omega)$. Then, the optimization problem for the 3-D routing can be formulated as:

$$\begin{gathered}\min[F(r_1), F(r_2), \dots, F(r_n)] \in \mathbb{R}^n \\ \text{Subject to } dist(r_i, r_j) \geq \Delta,\ \forall 1 \leq i \neq j \leq n,\ \ dist(r_i, \omega) \geq \Delta,\ \forall 1 \leq i \leq n, \forall \omega \in \Omega\end{gathered} \tag{5}$$

where $F(r_i)$ denotes the travel distance of the i-th dynamic route, Ω is the set of external areas to be avoided by the dynamic routes (e.g. no-fly zones, high terrain, noise and emission sensitive areas, and convective weather).

The Multi-Objective Optimization (MOO) problem (4)-(5) explicitly takes into consideration the following operational constraints:

1. Horizontal and vertical separation between any two dynamic routes for safety reasons and for embracing the conflict-free concept;

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[3] Here, although the travel distance is used for simplicity, as it is a typical objective used by the airlines, it is also directly related to more sophisticated objectives, such as flying time and fuel burnt.

2. External spatial constraints such as airspace restrictions and areas influenced by convective weather; and
3. Aircraft performance (maneuverability capabilities), which precludes unrealistic flight maneuvers following certain route design.

To reduce the complexity of the MOO problem in the search for a satisfactory solution, we adopt a Lexicographic Multi-Objective Optimization (LMOO) approach by solving the individual objectives in (5) sequentially following an *a priori* preference order, to be derived in Section 4.2 using a novel AHP route prioritization model. Previous research has followed a similar approach (Zhou, 2014; Pfeil, 2011) for single-airport terminal routing, but selecting an arbitrary preference order and failing to account for the full range of operational characteristics encountered in a Metroplex.

Given the sorted set of objectives $\{F(r_k), k = 1, \dots, n\}$ in a decreasing order of importance, the LMOO solves the following sub-problems sequentially:

$$\left.\begin{array}{c}\min F(r_k) \\ \text{s.t.}\ \ dist(r_i, r_k) \geq \Delta,\ \forall 1 \leq i < k,\ \ dist(r_k, \omega) \geq \Delta,\ \forall \omega \in \Omega\end{array}\right\} \text{Sub-problem } k, \qquad k = 1,2, \dots, n \tag{6}$$

In other words, the LMOO solves for the most important route first, then uses it as an additional constraint when solving for the second important route, and this procedure carries on for the rest of the routes. In this process the routes are designed sequentially and the feasible set is constantly updated and becomes more stringent.

Following such a LMOO approach, the rest of Section 4 is organized as follows. Section 4.2 derives the route priorities based on the AHP and several qualitative and quantitative characteristics. Section 4.3 details the solution procedure for the terminal fix selection problem based on the derived priorities. Finally, Section 4.4 carries out the LMOO procedure in the design of the 3-D routes within the TMA using a modified A* algorithm.

4.2 Route prioritization based on Analytic Hierarchy Process

To rank the dynamic routes while taking into account a range of qualitative and quantitative characteristics of Metroplex operations, we propose an enhanced Analytic Hierarchy Process (AHP) model in this section. The AHP is a multi-criteria decision making technique that seeks to decompose a complex decision into a series of pair-wise comparisons between criteria that are considered important to achieve a specific goal (Saaty, 1990). It can incorporate both quantitative and qualitative information into the decision-making process, while allowing the weights or priorities of relevant factors to be derived from expert knowledge. The AHP is a powerful tool and an adequate approach for enhancing the decision-making process in MAS; however, to date it has had very limited application in ATM and, to the authors' knowledge, has not been used for terminal airspace routing. There have been numerous prior applications of the AHP in ATM. Asfe et al. (2014) used the AHP to combine and rank the key empirical factors that influence air traffic delays. Lintner et al. (2009) developed the Aerospace Performance Factor (APF) based on the AHP for the assessment of safety performance using a range of localized safety measures, such as reported safety incidents, which were in turn weighted by experts and normalized against system operations. DiGravio et al. (2014) used AHP to develop reactive and proactive indicators to predict future safety critical situations to aid decision-making. Finally, AHP has been used in design problems such as routing (Sattayaprasert et al., 2008), the testing of new ATM CONOPS (Castelli & Pellegrini, 2011) and ATM en-route airspace structure design (Cong et al., 2011). The proposed AHP model employed in this paper is illustrated in Figure 5 and elaborated below.

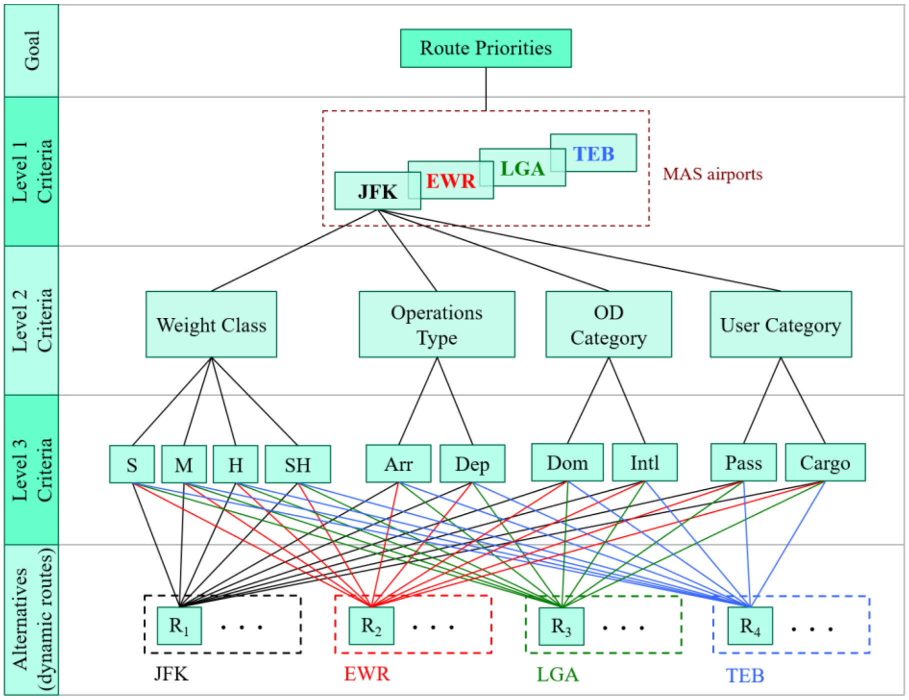

Figure 5. Illustration of the AHP model (New York Metroplex example).

The AHP can be described by the following steps:

1. Define the objective of the decision process;
2. Decompose the objective into elements, based on their common characteristics; and
3. Organize the elements into levels according to their relative importance in a top-down scheme.

The nodes in Figure 5 represent the criteria or the alternatives (i.e. dynamic routes), while the links indicate their relationships. The highest level in the hierarchy is the goal, which is to prioritize and rank the dynamic routes. At the first level, the routes are assessed solely based on their relevant MAS airports.[4] The second level consists of criteria describing the aircraft mix. In particular, "Weight class" is based on the ICAO standard and refers to the number of small (S), large (L), heavy (H), and super-heavy (SH) aircraft on each route. "Operations Type" refers to the number of arrivals or departures on a given route. The "OD category" distinguishes domestic and international flights, while the "User category" identifies the number of passenger and cargo aircraft. Finally, all the dynamic routes are identified at the bottom level. To use the AHP model, the decision maker needs to pairwise compare the elements at each level with respect to their parent node. The AHP model proposed here has been validated by SMEs from the PANYNJ using the Saaty scale (Saaty, 1990).

The developed AHP model combines both the qualitative and quantitative characteristics that affect operations in MAS. The decision maker is called upon to conduct pairwise comparisons for all the different hierarchies in the model. Once the comparisons are complete, the quantitative characteristics of the elements that comprise the alternatives (the "dynamic routes") are obtained. The relative weights (eigenvectors) of the examined factors are then calculated based on the eigenvalue approach. To articulate this procedure mathematically, we let matrix A be the pointwise reciprocal of the pairwise comparison matrix. We consider the following eigenvalue problem:

$$Aw = \lambda_{max} w, \quad e^T w = 1 \tag{7}$$

where w is the vector of ratio-scale priority weights, λ_{max} is the principal eigenvalue of the matrix A and e^T is a unit row vector (Saaty, 1990). Consequently, the decision maker's judgments need to be evaluated for their consistency. The consistency of the matrix A is addressed by calculating the Consistency Index (CI):

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{8}$$

where n is the order of the matrix. CI is then used to generate the Consistency Ratio $CR = CI/RI$, where RI is the Random Index generated by Saaty for a very large sample of randomly populated matrices. In order for the assessment to be complete, the CR should be no greater than 0.10 (both CI and CR are unitless). Otherwise, the decision makers should revisit their assessment to improve consistency (Saaty, 1990).

The weights derived for each of the criteria levels (levels 1 to 3 in Figure 5) are used to calculate the priorities of the dynamic routes using the referenced AHP method (Schoner and Wedley, 1989) to account for the proportionality between the criteria and their

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[4] Specific airports may be deemed of higher importance than others. Alternatively, the number of origin/destination pairs that each airport serves may be used for this assessment.

relevant options. The proportionality condition is enforced by the inclusion of a constant K to account for this:

$$x_k = K \cdot q_k \cdot \Sigma_h T_{h,k} \tag{9}$$

where

x_k is the importance of criterion k with respect to the objective
q_k is a scale factor which converts measurement on criterion k to units of the objective
$T_{h,k}$ is the absolute measurement of option h on criterion k

The referenced AHP is selected in this paper as it accounts for proportionality effects under different criteria. The final priorities are calculated as a weighted sum based on the derived weights for different criteria at various levels in the hierarchy. The detailed mathematical derivation of the priorities is outlined in Appendix A for the clarity of our presentation.

The hierarchy structured in this model is developed on the basis of the literature relating to airport and MAS operations; and the subsequent criteria are validated by seven SMEs. These SMEs represent different stakeholders including air traffic controllers, air navigation service providers and airport authorities (e.g. the PANYNJ, the NY airports and the FAA) with more than 20 years of experience in air traffic control facilities and airport operations. The validation process was based on questionnaires and semi-structured face-to-face interviews. The questionnaires addressed various issues of Metroplex operations and their structure aimed to enable the SMEs to identify inefficiencies in Metroplex systems. Conducting the interviews face-to-face enabled the interviewees to highlight additional areas important to MAS operations there were not covered in the initial questionnaires. The identified characteristics were combined into the AHP model presented in this paper. The SMEs assessed the structure of the proposed AHP model for:

1) Completeness of the hierarchy, hierarchy levels, and criteria (elements) to capture the several operational characteristics that currently affect management decisions; and
2) Representativeness of the hierarchy order and AHP structure to reflect current decision-making order of importance.

After the validation of the AHP structure, the SMEs were asked to conduct pairwise comparisons of different levels of the hierarchy for a given test period (see the Appendix B for some examples of the AHP pairwise comparison). The AHP results for the NY case study are presented in Section 5.2.

The proposed prioritization approach also accounts for the competition between different airports for airspace capacity, as the airport-specific objectives are factored into the AHP model and are directly associated with the dynamic demand characteristics at the corresponding level of the hierarchy. This yields a centralized assignment while allowing for particular push strategies to be implemented for each airport..

4.3 Priority-based terminal fix selection model

This section determines the points of entry/exit on the TMA boundary for the dynamic routes, by resolving the potential conflicts (e.g. see Figure 3) according to the derived priorities. In other words, we aim to assign each dynamic route to a unique terminal fix on the boundary while satisfying the minimum separation constraint; and we do so by minimizing the displacements between the initial entry/exit points (given by the cluster centers as shown in Figure 3) and the final fixes.

We begin by describing the procedure for resolving two conflicting dynamic routes (see Figure 6 for an illustration). When the distance between their entry/exit points is less than the minimum separation, the two dynamic routes need to be kept apart from each other with their precise displacements dependent on their relative priorities. Specifically, let p_1 and p_2 be the priorities of routes 1 and 2, respectively, $d_{1,2} (< \Delta)$ be the distance between their current entry/exit points where Δ denotes the minimum separation. The inverse distance weighted method is used to calculate the displacements of the two routes:

$$\Delta x_1 = \frac{\Delta - d_{1,2}}{1 + \frac{p_1}{p_2}}, \qquad \Delta x_2 = \frac{\Delta - d_{1,2}}{1 + \frac{p_2}{p_1}} \tag{10}$$

Obviously, the route with higher priority has smaller displacement. Figure 6 explains the aforementioned procedure intuitively.

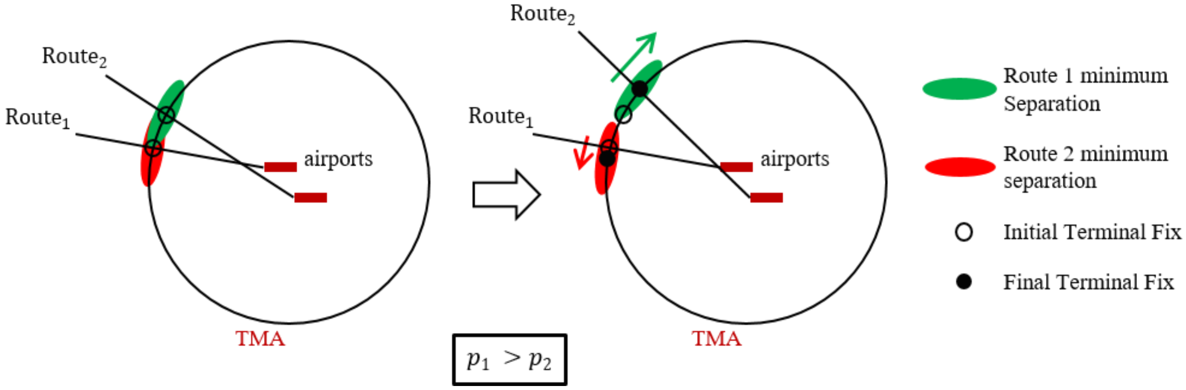


Figure 6. Priority-based re-positioning of dynamic routes. The terminal fix corresponding to route with higher priority (p_1) has smaller displacement compared to the terminal fix corresponding to route with lower priority (p_2).

In order to ensure that the minimum separation is met for any neighboring fixes on the boundary of the TMA, an iterative rearrangement method is developed as follows. We examine all the points in sequence in a rearrangement cycle, repositioning a pair according to (10) whenever the minimum separation is violated. After all the pairs of fixes on the rearrangement cycle are visited, we repeat this procedure as long as at least one change has been made within the previous cycle, i.e. we stop immediately after a full rearrangement cycle runs without reallocating any of the fixes.

Clearly, this formulation poses a straightforward limitation to the maximum number of terminal fixes, beyond which the problem becomes infeasible. Thus, appropriate constraints are applied to both the minimum separation distance and the maximum number of terminal fixes.

4.4 Lexicographic multi-objective optimization for 3-D routing

The priority-based terminal fix selection model presented above assigns the dynamic routes to their optimal entry/exit locations at the TMA boundary. The next step is to design the actual 3-D paths that aircraft follow within the terminal area. This design has to fulfill the following criteria: enable strategic de-confliction between different routes, minimize deviation from the shortest paths, accommodate additional traffic flow management initiatives, and provide feasible solutions that can be integrated into the current ATM system structure while enabling the future ATM system concepts outlined in SESAR and NextGEN (FAA, 2013). The 3-D conflict-free routes are designed according to the LMOO approach. The constrained routing problem for each dynamic route is solved using a modified A* algorithm, which uses a parameterized state space search scheme to account for different aircraft maneuvering capabilities and internal/external constraints.

The design of the flight paths for arrival and departure operations can be formulated as a state space search problem, with the location of the aircraft being represented by a state in the graph and the arrival or departure path represented by a tree in the same graph $G(N, E)$ where E is the set of edges representing the flight paths (see Figure 4). The following parameters are essential to fully describe the state of the aircraft:

- The current azimuth or bearing: the angle from the north that reveals the current direction of movement
- The current zenith angle: the vertical angle between the vertical and the current direction of movement
- The current 3-D coordinates: expressed in latitude, longitude and altitude

The successor operator determines the transition of an aircraft from a given node to a successor node. Such a transition depends on the current direction of movement (horizontally and vertically) and can be expressed in terms of increments of the horizontal and vertical turning angles with a maximum given by the current state of the aircraft. To reflect the operational characteristics of the aircraft, the successor nodes are parameterized by the incremental and maximum horizontal and vertical angles for traversing a given track distance; see Figure 4 for an illustration.

As shown in Figure 4, for a given aircraft location, the number of nodes indicating its next potential location can be determined based on the parameters used for the successor operator (e.g. $\alpha, \beta, \gamma, \delta, d, I$). These parameters depend on the type of aircraft and are expressed by the corresponding ascent and descent rates that can be found in the Base of Aircraft Data (BADA) (Nuic, 2010). Moreover, aircraft performance varies depending on the (1) altitude/speed and (2) the type of operation (e.g. arrival/departure). In order to capture the aircraft maneuverability as realistically as possible, from a mesoscopic perspective (in contrast to microscopic perspective, which employs a mass-based model for aircraft dynamics), four different successor operator functions are developed (designated as *StateSpaceSearch functions* ranging *I-IV*, see Table 1). The successor nodes are generated according to the corresponding function depending on the status of the current parent node. This allows, for example, steeper horizontal maneuvers in the proximity of the runway, which is typical of actual operations for the design of the final approach legs (arrivals) and initial climb legs (departures).

4.4.1 Heuristic function and node expansion scheme selection

The three-dimensional path that each dynamic route follows is designed based on the following state space search scheme. Nodes are expanded by combining the state space search strategy described above with an A* cost heuristic algorithm with the objective of minimizing the total distance travelled by each dynamic route. The A* algorithm examines nodes only in the direction of interest, which, for the design of 3-D arrival and departure routes, is obtained using the 3-D Euclidean distance.

The A* routing algorithm uses two lists for storing the nodes that are generated and expanded during the state space search: the OPEN and CLOSED lists. The former stores the available nodes that are sorted in an ascending order of their evaluation function value f_i. The A* always expands the state space search starting from the top of this list (e.g. selecting the node with the lower evaluation function value). Whilst the OPEN list only includes nodes that can be expanded, all the other nodes that have already been generated or cannot be used for expansion are stored in the CLOSED list. This list contains all the nodes that comprise the currently generated path and all other nodes that have greater evaluation function values after each expansion step are discarded.

4.4.2 Initialization

The design of the arrival and departure routes should comply with the current runway configuration. For this, the line segment that connects the initial two stages (nodes) created by the state space search scheme (Section 4.4.1) needs to be in alignment with the corresponding runway direction of movement (arrival/departure). This is achieved by providing, as input for the design of each route, these initial stages expressed via their 3-D coordinates depending on the airport the route flies to/from, its assigned runway and its type of operation. These nodes correspond to the actual location of the final approach fix for arrivals and initial climb waypoint for departures. The nodes are then expanded employing the state space search scheme proposed in Section 4.4.1.

For the design of both arrival and departure routes, the state space search initiates from the runway and moves towards the TMA boundary. This direction of search is preferred to its converse due to the following:

- It allows for the alignment of the routes with the runway directions.
- It reduces topological complexity of the routing structure, as routes are naturally directed to their corresponding terminal waypoints, which are already distributed to the terminal boundary in accordance with the selected terminal separation standards. The benefit of this strategy becomes more evident once a few routes are designed and the available airspace for the design of new routes is effectively reduced due to the conflict-free constraints.
- Designing routes by initiating the search from the terminal airspace boundary toward the runways has been found to result in additional horizontal and vertical turns per route hence reducing their smoothness and leading to additional airspace complexity (based on extensive trial-and-error experiments).

We describe below the different types of constraints that have been formulated into the routing problem below.

4.4.3 Constraints

Constraints are modeled as geometric objects (i.e. location and dimension) and stored in the CLOSED list. There are two main types of constraints: (1) those imposed by the already established routes, which are expressed via the horizontal and vertical separation; and (2) external spatial constraints, which can be conceptually expressed by typical geometric objects (e.g. cylinder) and represent airspace restrictions (e.g. no-fly zones around and above tall buildings, high terrain, noise and emission sensitive areas), areas influenced by convective weather, etc. The conflict-free concept of the route design is ensured at each node expansion step by calculating the horizontal and vertical distances between the location of each examined node and the location of the already established nodes or external objects. Nodes that fail to meet the required minimal separation are invalidated. The minimum separation distance requirements can be varied according to the Required Navigation Performance (RNP) specification. In the application reported in this paper the RNP-1 standards are selected with the horizontal and vertical separation being 3 NM and 1,000 ft, respectively. In the case of external constraints modeled by geometric shapes, e.g. a cylinder, the cylinder radius is used instead of the separation minima.

An additional operational requirement concerns with the modeling of the merge (diverge) movement towards (away from) the runway for arrivals (departures). This is enabled by excluding, from the constraint list, the first few nodes of each route in the vicinity of the runway (recall that the route design starts with the runway). The number of nodes to be excluded is user-defined, and depends on the parameters used in the state space search scheme.

The algorithm is further enhanced to handle situations where the state space search scheme cannot obtain valid nodes for route expansion. This could arise in bottleneck airspace where the demand from several routes exceeds the limited supply especially with stringent external constraints. In order to avoid potential route blockage, the algorithm *backtracks* a predefined number of nodes (selected by the user) and recommences the state space search in a horizontal direction outside the blocked area. The blocked area is modeled as a circle (see Figure 7) and the nodes are then expanded to the direction of interest indicated by the A* heuristic.

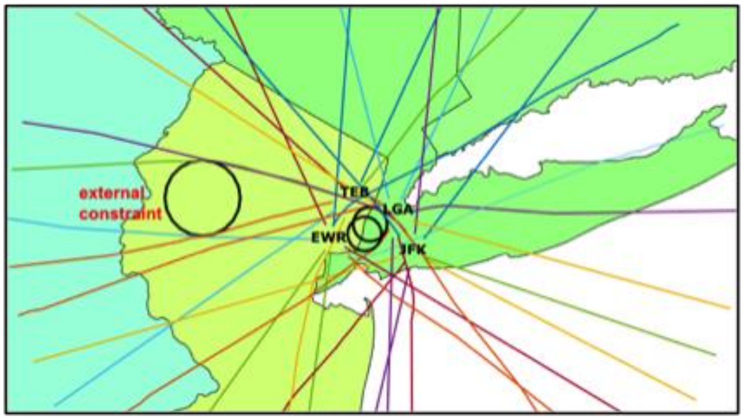


Figure 7. Modeling external constraints (shown as the black circles).

4.4.4 *Termination of a single route*

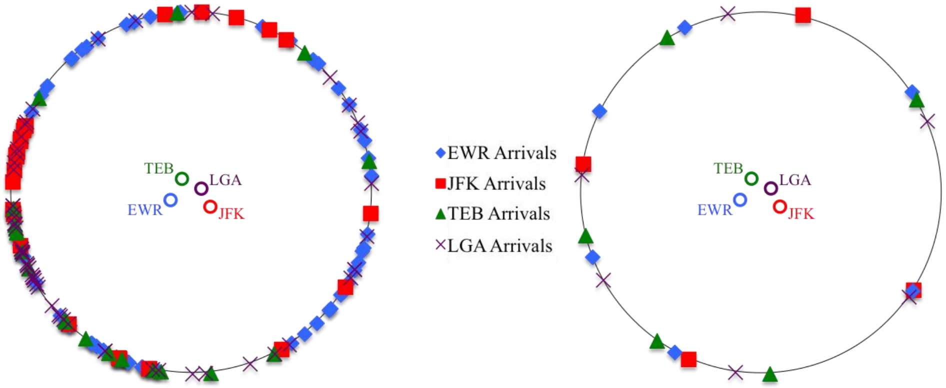
The calculation of each dynamic route terminates when a node reaches the target terminal waypoint of the current route. This is determined by comparing the distance from the current node to the destination with a user-defined distance threshold (3 km used in this paper, again to reflect real world operations).

5. New York Metroplex case study

5.1 Data description

The data required for the application of the framework include the scheduled (or historical) trajectories expressed as the 4-D coordinates for all flights for a given operational horizon, which can be drawn from the filed business trajectories. The data used for this NY Metroplex case study consist of aircraft trajectories during a typical day of operation in the NY TRACON in November 2011.[5] The trajectories were derived from the Performance Data Analysis and Reporting System (PDARS) and were provided by the PANYNJ. The time horizon of the case study is 10:30-13:15, which is characterized by heavy air traffic demand in the study area.

The PDARS data contain the actual aircraft trajectories, and are thus biased due to ATC actions. We pre-processed the data to remove such bias and reproduced the requested filed business trajectories. In order to do this, we assumed that all the airlines wish to fly the shortest path for any flight from their origin to their destination. The demand for an individual flight is expressed using the 4-D coordinates that mark the intersection of the great circle connecting the origin-destination pair and the small circle that represents the TMA boundary. The left hand side of Figure 8 shows such intersections for all the arrivals during 10:30-13:15 [as per the temporal clustering described in Section 3.1; also see Sidiropoulos et al. (2017) for more details], while the right hand side shows the centers of the spatial clusters (see Section 3.2 for the details of spatial clustering).



TMA entry points (arrivals)

Arrival fixes (cluster centers)

Figure 8. Entry points (left) and their cluster centers (right) for the period 10:30-13:15.

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[5] This day is selected based on the criteria that the Performance Data Analysis and Reporting System (PDARS) temporo-spatial distribution of flights is typical, and that the operation is free of unexpected events such as extreme weather and runway closure. This has been confirmed by the SMEs.

5.2 AHP modeling results and sensitivity analysis

The AHP model described in Section 4.2 is applied here to derive the priorities of the dynamic routes for the period 10:30-13:15. For this application, all the airports are considered of equal priority by the SMEs. The pairwise comparison assessments by the SMEs are checked for consistency with all the Consistency Ratios lower than 0.10. The final priorities for the dynamic routes are presented in Figure 9.

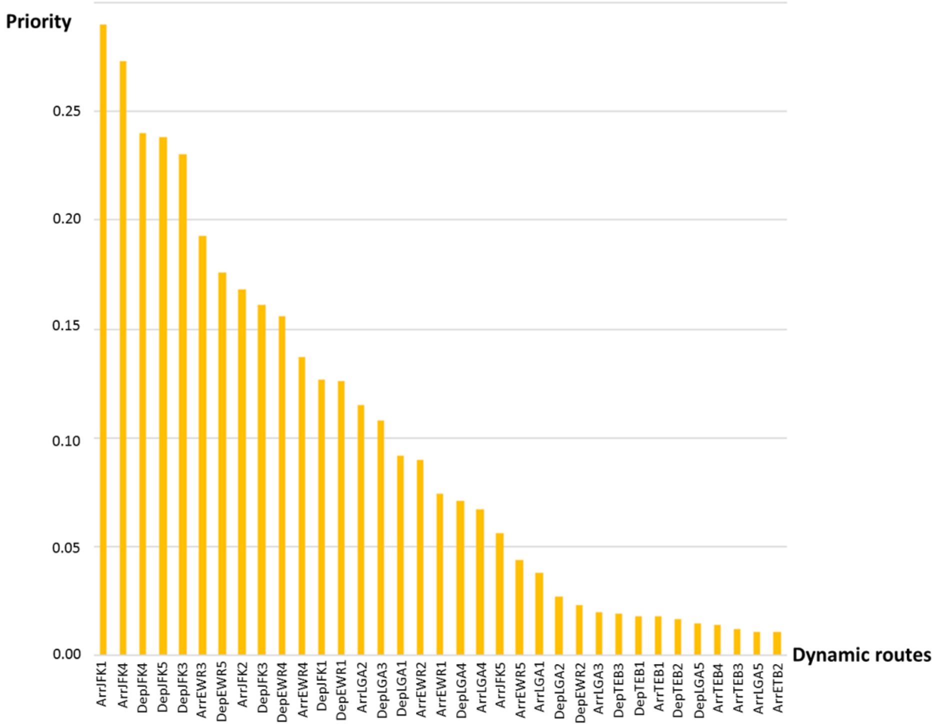


Figure 9. Final AHP route priorities (priorities before normalization - derived using Transparent Choice software).

The priorities of the dynamic routes obtained from the proposed AHP model play a vital role in the subsequent steps leading to the 3-D route design. The quantified relative importance of a given criterion at a certain level relative to the others at the same level is determined subjectively by the SMEs using the Saaty scale (Saaty, 1990). Some common issues that may arise when using the AHP are the following (Schoner and Wedley, 1989):

(1) ambiguity in the meaning of the relative importance of one criterion as compared to another; and
(2) rank reversal.

Regarding (1), Schoner and Wedley (1989) state that "*the AHP requires a specific meaning for the relative importance of criteria to yield the same answer as a direct solution*". This is achieved in the referenced AHP by means of a scaling factor imposed at each level to reflect the number of the available options at the immediate lower level (Schoner and Wedley, 1989). Regarding (2), rank reversal refers to the change in the final priority order by adding/removing options. From a methodological perspective, rank reversal is eliminated when using the referenced AHP (Schoner and Wedley, 1989) if the weights change according to Eqn (9).

However, rank reversal may also occur when the SMEs' pairwise comparisons (higher level criteria) or absolute measurement inputs (lower level criteria) are varied. This needs to be thoroughly understood by conducting sensitivity analysis with respect to the variation of the input. There are several methods for sensitivity analysis of the AHP-derived priorities (Ishizaka and Lusti, 2006; Leonelli, 2012), with the most popular being *numerical incremental analysis* [others include probabilistic simulations and mathematical modeling (Leonelli, 2012)], in which the weights are perturbed to yield a new set of priorities. These perturbations are applied to one parameter at a time and the corresponding effects on the local or global priority of the alternatives are graphically illustrated.

To ensure that the solution is well-defined and robust, numerical sensitivity analysis of the referenced AHP was conducted. This was done in this study by using *Transparent Choice* (Transparent Choice, 2016) for all the examined criteria. Figure 10 presents one instance of the sensitivity analysis for the EWR dynamic routes when the weight of *heavy A/C type* is perturbed. The vertical red line indicates the current criterion selection (34%) and the sloped lines indicate how the global priorities of the relevant dynamic routes may be varied with such weight. In this figure, for a rank reversal to occur, the weight should be increased to approximately 68%.

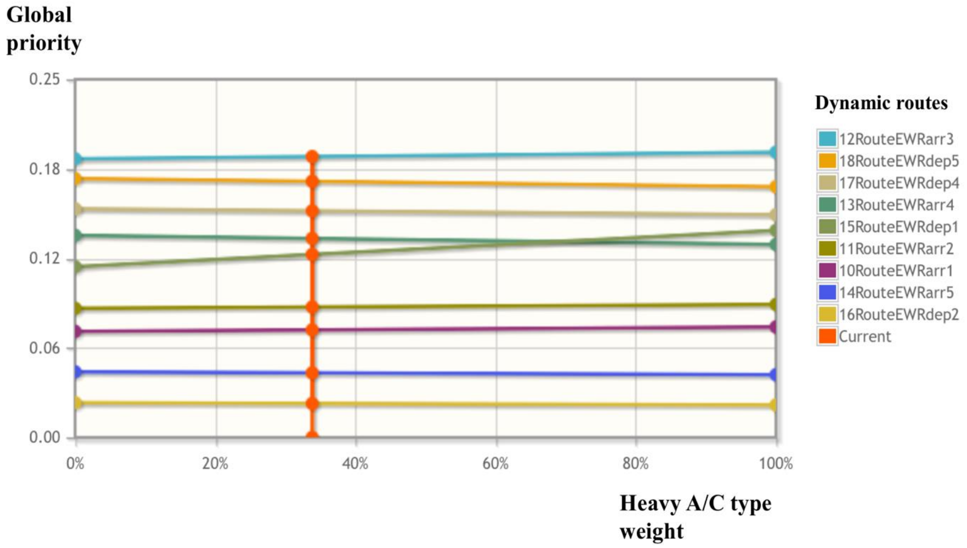


Figure 10. Sensitivity analysis for EWR heavy A/C type criterion (derived using Transparent Choice software)

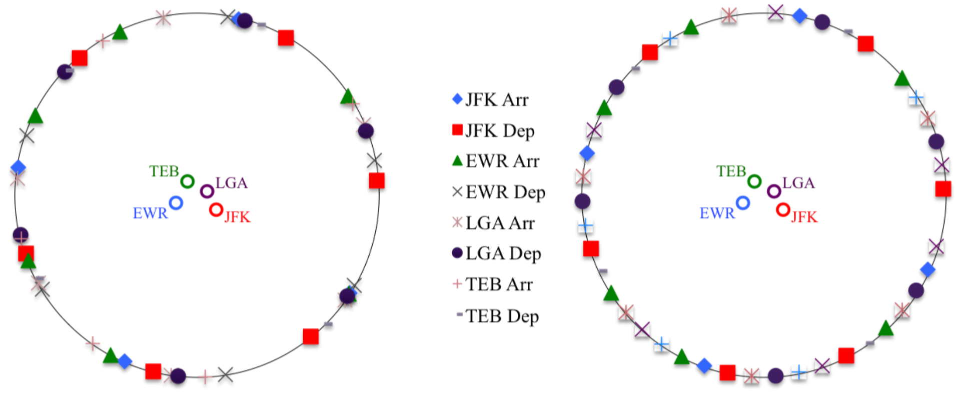
It is of crucial importance to identify the circumstances that might cause rank reversal. There are two sources of uncertainty. The first is an error in the judgment of the decision maker during the pairwise comparison process. This can be readily accounted for by means of the consistency assessment outlined in Section 4.2. The second source of uncertainty is the variability of the quantitative information regarding the various demand characteristics at the lower level of the hierarchy, which can be due to uncertainty in the realization of demand. Such uncertainty can be addressed by the distributionally robust optimization (DRO) approach proposed in Sidiropoulos et al. (2017). Furthermore, since such uncertainty affects only the lower-level elements, it has little impact on the overall priority, which is primarily determined by the weight derived by the pairwise comparisons. Thus, testing the pairwise comparisons for consistency using the consistency index, along with the DRO framework for deriving the dynamic routes can enhance the robustness of the final dynamic route priorities.

From Figure 9 it is evident that a rank reversal between a highly ranked alternative (e.g. ARRJFK1) and one which is lower-ranked (ARRTEB2) is highly unlikely. The routes that may be subject to a local rank reversal are those lower-ranked (e.g. ARRTEB3, ARRTEB2). Whether rank reversal can occur depends, quantitatively, on the consistency of the SME inputs measured by the consistency ratio (CR); see Section 4.2. Taking CR=0.10 as a benchmark, we are able to calculate the corresponding maximum changes in the route priorities using Eqn (11), which is ± 0.001. Perturbing the priority values of all the routes shown in Figure 9 by up to 0.001, we are able to enumerate all the scenarios of rank reversal or tie, of which there are 29. However, we note that this is a rather pessimistic representation of the rank reversal, as the actual CR calculated from the SME inputs in our case is 0.04, which is less than the benchmark value (0.10). In fact, applying CR=0.04 only yields three scenarios of rank reversal/tie. In other words, given the CR of 0.040, all possible perturbations of the route priorities yield three scenarios of rank reversal/tie, which are concerned with the last eight routes in Figure 9. This concludes a comprehensive sensitivity analysis of the AHP.

In Section 5.5, we extend the sensitivity analysis to the 3-D route design by investigating how the resulting route structures are affected by the 29 instances of rank reversal. As we shall see later, such effects are negligible.

5.3 Priority-based selection of terminal fixes for the NY Metroplex

For the selection of the terminal fixes (see Section 4.3), the minimum separation distance at the TMA boundary is $\Delta = 20$ km, which is greater than the absolute minimum required by the current standards i.e. 5NM or 9.26 km (ICAO, 2015). Normally, the optimal separation is jointly determined by the spatial configuration of the TMA and the number of dynamic routes, which in this case leads to the selected number of such fixes after several trial-and-error experiments. The additional margin between neighboring terminal fixes leads to a better initial distribution of the dynamic routes and results in fewer conflicts in 3-D routing. Figure 11 presents the result of the priority-based terminal fix selection model.



Initial terminal fixes (cluster centers)

Re-arranged (conflict-free) fixes

Figure 11. Initial unimpeded terminal fixes (left) and final terminal fixes (right) for the period 10:30-13:15.

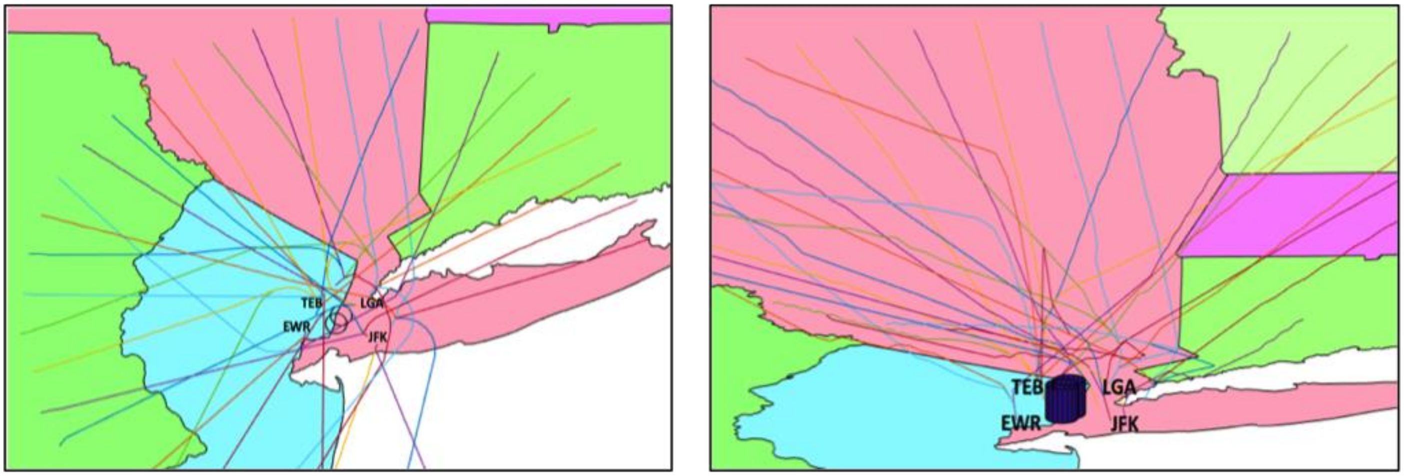
5.4 Three-dimensional routing results

Four different state space search schemes are utilized with the parameters presented in Table 1.

Table 1. Parameters used in the state space search schemes

State space search scheme	Traversal step (km)	Horizontal angle increment (deg)	Max number of horizontal increments	Vertical angle increment (deg)	Max number of vertical increments
I (arrivals)	3	2	6	1	3
II (arrivals)	3	2	5	1	3
III (departures)	3	2	6	1	4
IV (departures)	3	2	5	1	4

Figure 12 illustrates the dynamic routes for the NY Metroplex airports (JFK, EWR, LGA, TEB) obtained from the 3-D routing schema. It can be observed that most dynamic routes follow close-to-straight curves with little turning. In addition, two external constraints were added for this particular application to account for the no-fly zones around the World Trade Center and the Empire State Building. The no-fly zones are approximated as two cylinders centered at each site with a radius of 2.5 NM and altitude of 3,000 ft. The no-fly zones are indicated by the two black circles in the center of Figure 12(a) and by the dark-blue cylinders in Figure 12(b).



(a)

(b)

Figure 12. (a) Plane-view of resulting route structure for period 10:30-13:15. (b) 3-D view of resulting route structure for period 10:30-13:15 (the altitudes are exaggerated by 100 times).

5.5 *Sensitivity analysis of the routing solutions*

From a practical point of view, it is desirable to ensure that the 3-D routing result is robust against potential rank reversal in the dynamic routes, which is caused by reasonable perturbations in the AHP-related parameters. This section fulfills this purpose.

As mentioned in Section 5.2, the priority order, which effectively acts as an input variable to calculate the routing structure, is robust with very limited rank reversal as allowed for by the consistency ratio (CR). To further analyze the sensitivity of the routing structures, we follow a similar approach in Section 5.2 by taking the benchmark CR (0.10) and the CR from our case study (0.04). For CR=0.10, we enumerate all the 29 instances of rank reversal or tie; for CR=0.04, we enumerate all the 3 instances of rank reversal or tie. We then calculate the corresponding route structures through the 3-D route design framework.

A design KPI was selected to evaluate and compare the performance of the 29 different routing structures. This was chosen to be the total distance travelled by all the flights on their corresponding dynamic routes, as it not only corresponds to the original multi-objective optimization problem (5), but is also a direct indicator of route efficiency in the TMA. The tornado graph in Figure 13 illustrates the relative difference (in percentage) in the design KPIs for all the 29 scenarios when CR=0.10, and all the 3 scenarios (S5, S11, S18) when CR=0.04. It can be seen that the maximum variations in the KPI are approximately 0.22% (CR=0.10) and 0.03% (CR=0.04), which are negligible. Figure 14 presents the three new routing structures resulting from rank reversal/tie for CR=0.04. It is shown that the changes in the route structure are minor and are concerned with low-ranked dynamic routes with a very few flights in the selected test period. This provides strong evidence that the routing structure is robust against perturbations in the route priorities and potential rank reversal.

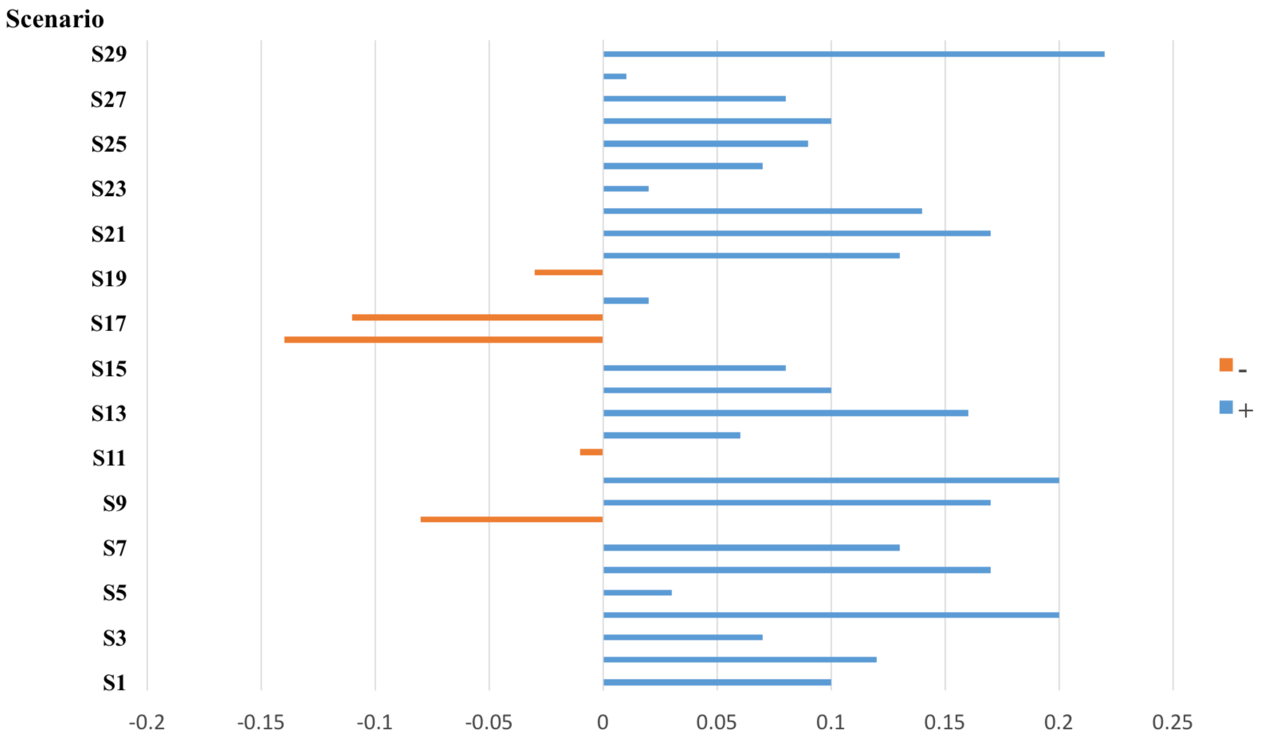


Figure 13. The 29 scenarios of rank reversal identified from the sensitivity analysis, and the corresponding changes in the total distance travelled by all the flights compared to the base scenario.

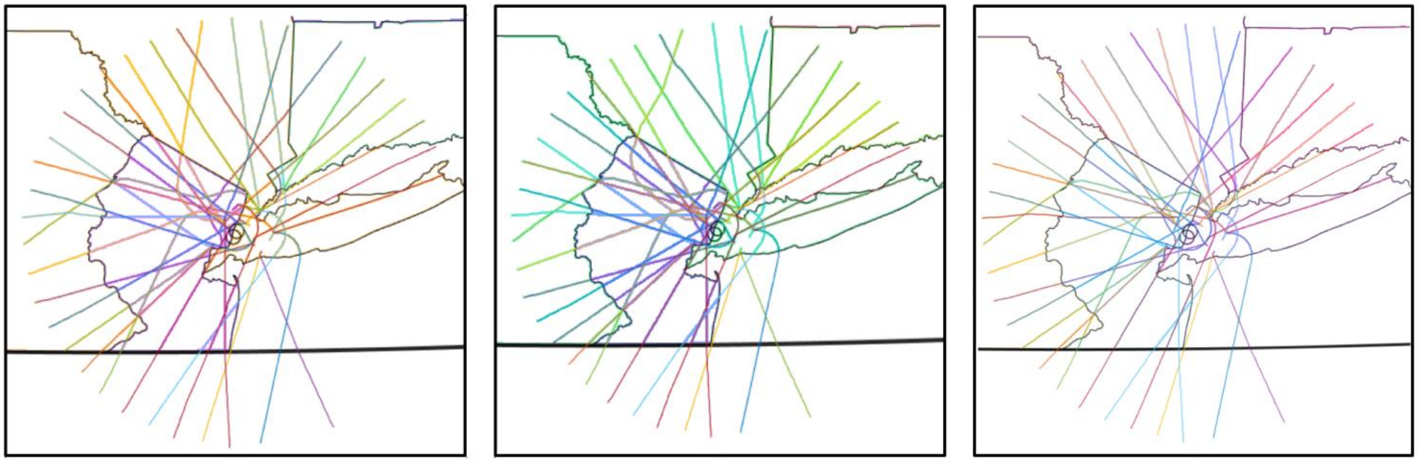

Scenario 1
Scenario 2
Scenario 3

Figure 14 The three routing structures resulting from rank reversal/tie for CR=0.04, based on the sensitivity analysis

5.6 Computational performance

In this section, we present a comprehensive evaluation of the computational performance of the proposed method. Figure 15 illustrates the computational performance of the routing algorithm, on a Matlab platform run on a standard laptop with 16 GB RAM and a 2.7 GHz Intel Core i7 processor. The figure shows the average computational time (over 100 independent runs) taken for the algorithm to solve the routing problem for a given number of dynamic routes. In addition to the number of routes, the computational performance is also dependent on the geometry of the problem, e.g. size of the TMA area, relative locations of the airports, orientation of the runways, and number of no-fly zones. Figure 15 also shows that the computational time increases super-linearly with respect to the number of routes. This is because as more routes are included, the constraint set for solving the next route becomes increasingly stringent and complex, which requires more computational time. Finally, given the fact that the algorithm has been tested for one of the most complex Metroplex systems in the world with satisfactory computational efficiency, the routing algorithm is suitable for implementation in other Metroplexes.

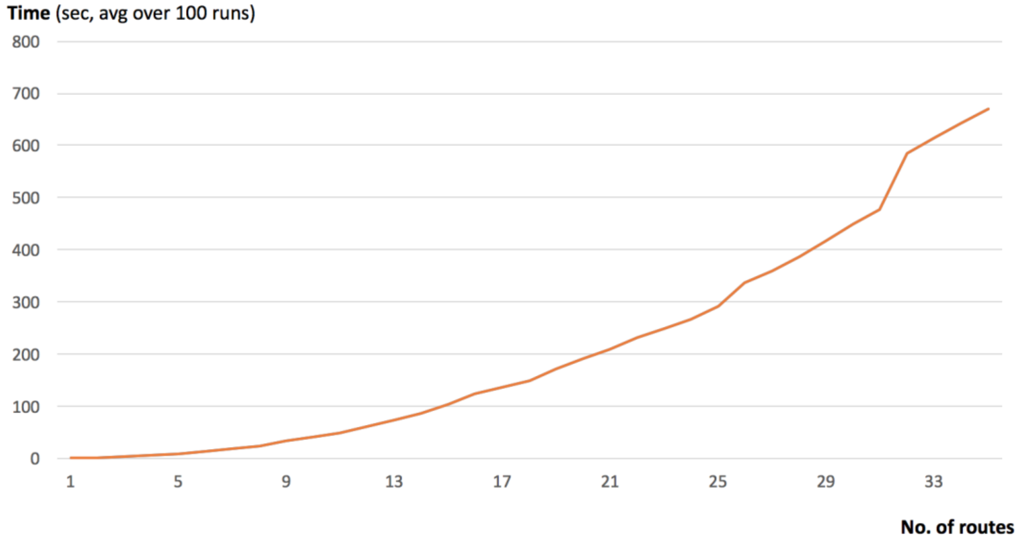


Figure 15 Computational performance of TMArouter as a function of the number of routes solved over time (sec). The displayed data represent average solution times over 100 runs of the algorithm.

The proposed framework, including demand characterization, dynamic route prioritization and 3-D route design, is suitable for decision making at a pre-tactical (24 hours in advance) and operational level (2-3 hours in advance). The input of the framework includes static information (e.g. TMA geometry, external constraints) and dynamic information (e.g. air traffic demand). Given the traffic demand, the subsequent demand characterization (i.e. S-T clustering) and route prioritization (i.e. AHP) can be easily performed with negligible computational time. The 3-D route design takes approximately 11 minutes in our case study. This makes the proposed framework entirely feasible on a pre-tactical decision-making level, where traffic demand data (forecast) are collected at the end of the day for the operation in the next 24-hr horizon. It can also enable near real-time operation on a rolling horizon basis, where air traffic demand forecast with 2-3 hours look-ahead-time can be used to implement the route design, which is performed at the beginning of each time epoch (with a length of 2-3 hours). The feasibility and efficacy of the rolling horizon approach is fully demonstrated in Sidiropoulos et al. (2017).

6. Simulation validation

Fast time simulation represents an essential aspect of any attempt to assess the impacts of airspace changes in a systematic manner (Majumdar et al. 2005). AirTOp, a fast time simulation model (AirTOp Soft S.A.), is used to validate the resulting design from the proposed method. The main objective of the simulation experiment is to compare the performances of the system under the *new operational scenario* designed by our method, in contrast to the baseline of the *current operational scenario*. In particular, the routing results for the period 10:30-13:15 presented in Section 5.4 are compared with the actual routing results for the same set of flights, as depicted in the PDARS data.

6.1 Current scenario

In order to develop the current scenario, an existing model for the NY Metroplex (produced by AirTOp) is used as the baseline model and adjusted to cover the major features of the current operations. This was done through extensive consultation with the SMEs from the PANYNJ (ex NY TRACON air traffic controllers and airspace planners) and incorporation of information from the Aeronautical Information Publication (AIP) of the four main Metroplex airports. Such information includes:

- Airspace: location of fixes; Standard Instrument Departure Routes (SIDs) / Standard Instrument Arrival Routes (STARs); holding patterns; vectoring areas for arrivals; horizontal separation standards (3 NM); and vertical separation standards (1,000 ft)
- Airport: runway orientations; runway geometric characteristics; runway dependencies (between the same runway or dependent runways of the same or neighboring airports); use of an operational arrival gap between dependent runways (to allow for departures to take-off between arrivals); and separation standards for take-off (based on wake turbulence category).

6.2 New scenario

The derived dynamic routes are established as standard arrival and departure routes, and the flights are assigned to their corresponding routes. While we consider the same horizontal and vertical separation standards and airport set-up as in the Current Scenario, we use no existing SIDs and STARs, holding stacks or vectoring areas from the Current Scenario. The final approach segment of each arrival route is complemented by the design of an ad-hoc vectoring area, to allow for ATC vectoring in the absence of an AMAN system. The same group of SMEs from the PANYNJ checked the geometric characteristics of such vectoring areas.

An AMAN system attempts to separate arrival traffic, and can be applied in the en-route part of the journey, or within the TMA, before the flights reach the final approach. Within the AirTOp simulation engine, a built-in AMAN system is implemented for some tests pertaining to both Current and New Scenarios. The purpose is to demonstrate the compatibility of the proposed route design and the existing AMAN system, as well as its full potential to improve the efficiency of the TMA operation. The AMAN within the AirTop simulation has been verified in numerous studies.[6]

6.3 Set-up of experiments and simulation results

The same set of flights are used in the simulations of both the Current and the New Scenarios in order to compare the system performances under the same demand input. Several variations of the base simulation scenarios are tested for a more comprehensive analysis as summarized in Table 2.

Table 2. Simulation scenarios (* based on empirical data rather than simulation).

Scenario	**Operations under**		
	Current design (a)	**New design (b)**	**Reference data (c)**
1	Current 70 NM	New 70 NM	-
2	Current 70 NM - AMAN	New 70 NM - AMAN	-
3	Current 150 NM	New 150 NM	Ref 150 NM *

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[6] Based upon interviews with AirTop Simulation Engineers, Brussels, 22/3/2016.

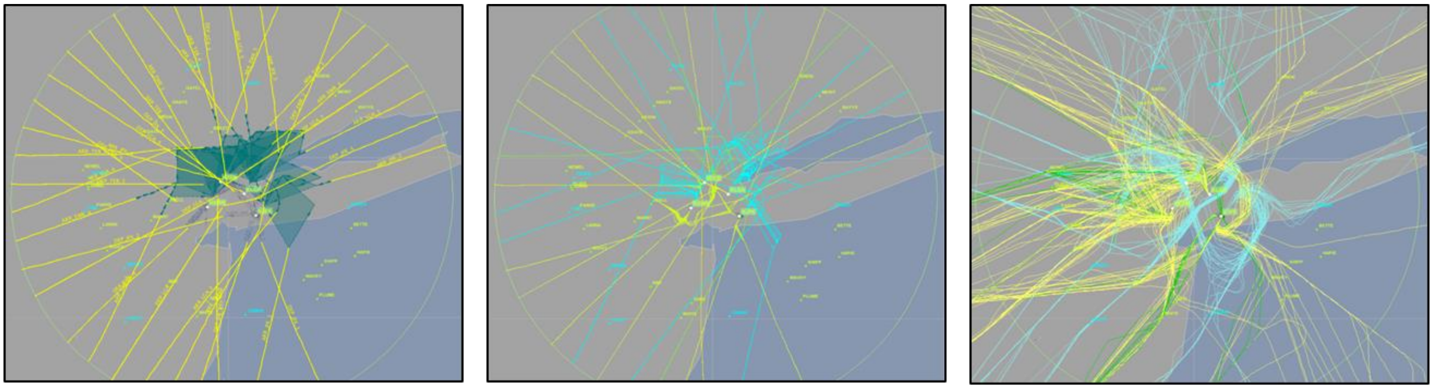
4	Current 200 NM	New 200NM	Ref 200 NM *
5	Current with increased demand	New with increased demand	-
6	Current with increased demand - AMAN	New with increased demand - AMAN	-
7	Current with increased demand	New with increased demand - AMAN	-

Under Scenario 1, the performance of the design is tested for a TMA radius of 70 NM around the geographical Metroplex centroid. Scenario 2 adds an AMAN system to both the current and new designs. Scenarios 3 and 4 test the extended impact of the terminal area design for a radius of 150 NM and 200 NM, respectively, for both the current and new designs; comparison with the reference PDARs data is also available. The reference data are not derived from the computer simulation but instead based on the empirical data relevant to the current operations. Finally, under Scenarios 5, 6 and 7, an increased demand profile is considered for both the current and new scenarios. The increased demand scenario is developed to test whether the airspace design is capable of handling demand surges or future demand growth. The demand is increased as follows. The demand for each dynamic route is increased proportionally to its contribution to the overall airport demand; this is to preserve the ranking obtained from the AHP, and thus preserve the same routing solution. On each individual route, cloned flights are generated by randomly sampling from the set of existing flights, and the timing of each clone flight is derived following a normal distribution within a range of 3 minutes around the initial flight generator. The total number of cloned flights and their distribution among the different routes reflect a hypothetical future demand scenario. In addition, the cloning process on each individual route ensures that no artificial conflicts are generated during the process (cloned traffic is not generated in temporal proximity of the base traffic sample).

The implementation of the current and new scenarios in AirTOp is illustrated in Figure 16. In particular, the newly designed dynamic routes in Figure 16(a) are represented by the yellow lines, while the final approach segments for the arrivals are complemented by vectoring areas, shown in pale green. The vectoring areas are designed on the basis of the following principles:

- The longer side of the vectoring area follows the initial dynamic route solution;
- The inner side of the vectoring area follows the most direct path to the final approach fix; and
- Overlapping vectoring areas should be separated by 1,000 ft.

Even though this study designs the vectoring areas in an ad-hoc fashion, in a more generic case, airport authorities may design them in advance and the routing solution can be calibrated to match them exactly.



(a) (b) (c)

Figure 16. Implementation of current and new designs in AirTOp. (a) Designed dynamic routes with vectoring areas. (b) Flight tracks under the new scenario (1-b in Table 2) in the simulation. (c) Flight tracks under the current scenario (1-a in Table 2) in the simulation.

The actual flight tracks of the new and current scenarios in the simulation are shown in Figure 16(b) and (c), respectively. The new scenario yields a much more organized and efficient structure of the arrival and departure paths compared to the current scenario, where the effect of extensive vectoring can be easily observed by the increased spread of the simulated routes.

In order to further quantify the efficiency of the different route designs, the following KPIs are calculated for each scenario.

a) Total distance travelled;
b) Total flight duration;
c) Total fuel burn;
d) Total work duration: the total sum of the time required for a controller to resolve a single conflict event; and
e) Total BMT delay: total delay for all the flights, measured as the difference between the actual time of arrival and the Business Management Trajectory scheduled time of arrival over specific fixes.

For d), AirTOp uses the DFS (Deutsche Flugsicherung GmbH, the German ANSP) event-based model for the assessment of controller workload. Under this model, each event that requires the attention/action of a controller is assigned a time value depending on its criticality; we refer the reader to Majumdar et al. (2005) for further details. Even though modeling controller workload is not the focus of this paper, the conflict-free concept embraced by the proposed route design is expected to lead to significant reductions on the controllers' workload, given that conflict resolution comprises the predominant part of the air traffic controller workload. This indeed is corroborated by the simulation results (see Table 3 and Table 4 for the current and increased demand scenarios, respectively).

Table 3. Comparison results in Scenarios for current demand scenario (traffic sample 1), 10:30-13:15.

Scenario		**Scenario description**	**Total distance (NM)**	**Total flight duration (hrs)**	**Total fuel burn (kg)**	**Total work duration (hrs)**	**Total BMT delay (hrs)**
1		Current 70 NM	45403.5	147:47:52	328783.43	12:59:00	25:30:04
		New 70 NM	39369.1	131:34:35	295416.75	09:41:32	17:30:32
		(New-Current) %	-13.29	-10.98	-10.15	-25.35	-31.40
2		Current 70 NM - AMAN	45937.6	149:59:34	331336.68	12:59:37	28:09:25
		New 70 NM - AMAN	38961.6	132:21:11	297300.72	09:47	20:43:58
		(New-Current) %	-5.19	-11.76	-10.27	-24.71	-26.37
3	3a	Current 150 NM	78312	234:40:42	524430.78	-	-
		New 150 NM	71219	214:12:20	448838.88	-	-
		(New-Current) %	-9.06	-8.72	-14.41	-	-
	3b	Ref Current 150 NM	77755.1	238:03:14	-	-	-
		New 150 NM	71219	214:12:20	448838.88	-	-
		(New-Current) %	-8.41	-10.02	-	-	-
4	4a	Current 200 NM	100488.1	296:15:56	662326.99	-	-
		New 200 NM	94468.8	274:28:10	584156.32	-	-
		(New-Current) %	-5.99	-7.36	-11.80	-	-
	4b	Ref Current 200 NM	99509.2	296:54:49	-	-	-
		New 200 NM	94468.8	274:28:10	584156.32		-
		(New-Current) %	-5.07	-7.56	-	-	-

Table 4. Comparison scenarios for increased demand (traffic sample 2), 11:00-12:30.

Scenario	**Scenario description**	**Total distance (NM)**	**Total flight duration (hrs)**	**Total fuel burn (kg)**	**Total work duration (hrs)**
5	Peak Current 70 NM	36739.9	124:28:22	236786.86	14:39:46
	Peak New 70 NM	31669.5	111:36:34	210684.67	10:47:23
	(New-Current) %	-13.80	-10.33	-11.02	-26.41
6	Peak Current 70 NM - AMAN	36002.2	122:11:04	235117.22	13:32:04
	Peak New 70 NM - AMAN	29195.6	100:56:58	203965.31	07:49:05
	(New-Current) %	-18.9	-17.2	-13.25	-42.22
7	Peak Current 70 NM	36739.9	124:28:22	236786.86	14:39:46
	Peak New 70 NM - AMAN	29195.6	100:56:58	203965.31	07:49:05
	(New-Current) %	-20.53	-18.90	-13.86	-46.68

It can be observed from Table 3 and Table 4 that the new design has superior performance compared to the current scenario, as it results in significant reductions for all the examined KPIs. When an AMAN system is applied on the arrival routes, the system performance is further improved, as the required vectoring by controllers is minimized, leading to shorter distances travelled, additional flight duration savings and lower controller workload. Overall, significant reductions are achieved for the total distance travelled, the total flight duration in the TMA, total fuel burn and total controller workload. In particular, the decrease in controllers' workload could be attributed to the fact that under the proposed concept of operations, the available airspace capacity is utilized in a more efficient manner, and is no longer strictly limited by the controller workload as at present. Finally, the comparisons between the current and new scenarios, with an extended radius of 150 NM and 200 NM, show that better management of operations within the TRACON could lead to additional improvement in the KPIs.

A comparison between different TMA sizes reveals that a larger TMA implies less saving (in terms of percentage, although the absolute saving may increase as shown in Table 3). This is because most of the inefficiencies, including flight conflict and excessive controller intervention, occur within a certain range from the airport. Therefore, the proposed framework, aiming at reducing conflict and increasing operational efficiency, tends to be most effective in highly congested terminal areas, rather than for other phases of a flight (such as cruising), where fewer conflicts occur.

Of course, these results should be interpreted with caution and used only as indicators for further research, as there are additional factors that characterize this airspace that are not exhaustively considered in the modeling/simulation (e.g. letters of agreement, airspace boundaries).

Finally, the quantification of controller workload in the fast-time simulation is based on a relatively simple metric, which is the total sum of time required to resolve a single conflict event. We note that this is not the only metric for controller workload. In the authors' opinion, a more realistic and accurate assessment of the proposed design on ATCo workload requires real-time, human-in-the-loop simulation instead of fast-time simulation. This is beyond the scope of this paper; but we are working with the PANYNJ to plan this type of experiments in the future. Even with the relatively limited and simplistic metrics of workload, the conclusion still stands that the ATCo workload can be reduced due to the much less conflict achieved by the new design. Future study will be conducted to quantify such workload reduction more accurately.

7. Conclusions

As air traffic grows, MAS in the World's major cities are increasingly crucial in ensuring that such traffic can be adequately catered for without accompanying negative consequences. Yet it is in the very nature of the spatial proximity amongst the airports comprising a MAS, making their operations interdependent, to pose considerable challenges to their efficient management. In particular, the absence of any effective centralized coordination for such operations means that currently, air traffic controllers use their experience to manage traffic in such systems in an ad-hoc manner. It's no surprise therefore that the operations of such systems is far from optimal, leading to many negative consequences, e.g. poor usage of the potential capacity of the system, longer routings for aircraft.

Previous attempts to tackle this issue have focused on routing algorithms under the assumption that the design of the terminal airspace of the MAS is fixed. The problem with this approach is that not only does it lead to, at best, piecemeal improvements in the operations of a MAS, it also fails to take into account the major step changes proposed in air traffic management in the USA and in Europe, such as dynamic airspace configuration (DAC).

The framework outlined in this paper not only provides a novel routing algorithm for such a MAS, it also simultaneously tackles the terminal airspace design of the MAS. In this respect it accounts for both strategic and tactical operations in a MAS, which is in stark contrast to previous research in this domain.

It is important to note that the key innovation made in this paper concerns with the dynamic airspace configuration and the notion of dynamic route structure; a series of methodologies are developed along the way for demand characterization, dynamic route formulation and prioritization, as well as 3-D routing within the TMA, all aimed at the ultimate goal of improving design and flow efficiencies and reducing controller workload. The number of airports involved in this process is not essential for the application of the methodology. In other words, the same framework can be equally applied to single- or two-airport systems. The only difference is that the level of congestion reduction might vary, depending on the existing static route structure and level of traffic demand. However, these quantitative differences do not diminish the scientific value and applicability of our work as a novel solution for the design of terminal airspace in general.

In our current framework, including the fast-time simulation, we only consider fixed runway configurations. However, the framework is generic and can accommodate any configuration by simply altering the coordinates and runway directions for the 3-D route design. One could even consider runway configuration as an internal control variable of the integrated runway-configuration-3D-routing problem.

While the modeling of uncertain air traffic demand is out of scope of this paper, it can be addressed with minor modifications to the proposed framework as follows. The distribution of flight delay (or error in traffic demand prediction) can be obtained from historical data. The variations in the demand can be characterized using robust clustering method (Sidiropoulos et al., 2017), which allows the traffic demand uncertainties to be factored into the TMA design, such that the proposed framework can be applied on a pre-tactical level (with a 24-hr forecast) or operational level (with a 2-hr forecast). Notice that the level of uncertainties in the demand depends on the scope of the prediction (i.e. pre-tactical or operational).

Finally, this paper tests the proposed framework in arguably the world's most complex and busiest MAS, the New York Metroplex, by a hybrid combination of qualitative methods, i.e. the use of subject matter experts, and quantitative methods, i.e. the use of a high fidelity simulation model of the Metroplex. The results are highly promising and the implications of this are manifold. A major implication is that the framework outlined provides the basis for the design of airspace in any MAS system around the World, and thereby fulfills an objective of the ICAO. Therefore, the authors recommend the implementation of this framework by both the airport operators of a Metroplex system and by the ANSPs responsible for the design of the TMAs of single- or multi-airport systems, where the radius of the TMA boundaries should be determined case by case, to maximize the efficacy of the design.

Acknowledgments

The authors would like to thank the SMEs for their invaluable input in the identification of the characteristics used in the development of the AHP model in this study, as well as the validation of the simulation results. The Lloyd's Register Foundation sponsored part of this research.

Appendix A. Details of the AHP model

Based on Figure 5, the route priorities are calculated as:

$$p_i^a = f_i^a \cdot Air_i^a \cdot \left(\begin{array}{c} WC(S \cdot s_i^a + L \cdot l_i^a + H \cdot h_i^a + SH \cdot sh_i^a) \\ +OPS^a(ARR^a \cdot arr_i^a + DEP^a \cdot dep_i^a) \\ +OD^a(DOM^a \cdot dom_i^a + INT^a \cdot int_i^a) \\ +USER^a(PASS^a \cdot pass_i^a + CAR^a \cdot car_i^a) \end{array} \right) \tag{11}$$

subject to: $\sum_{a=1}^{A} \sum_{i=1}^{n^a} p_i^a = 1$

where

p_i^a: the priority of route *i* of airport a,
WC: AHP-derived weight for the 'weight class' attribute, normalized for the number of options of the weight class criterion within the 3nd level criteria,
S: AHP-derived weight for small aircraft,
s_i^a: the number of small aircraft on route *i* of airport a over the total number of small aircraft associated with airport a,
L: AHP-derived weight for large aircraft,
l_i^a: the number of large aircraft on route *i* of airport *a* over the total number of large aircraft associated with *a*,
H: AHP-derived weight for heavy aircraft,
h_i^a: the number of heavy aircraft on route *i* of airport *a* over the total number of heavy aircraft associated with *a*,
SH: AHP-derived weight for super heavy aircraft,
sh_i^a: the number of super heavy aircraft on route *i* of airport *a* over the total number of super heavy aircraft associated with *a*,
OPS^a: AHP-derived weight for the 'operations type' attribute of airport *a*, normalized for the number of options of the operations type criterion within the 3nd level criteria
ARR^a: AHP-derived weight for arrivals of airport *a*,
arr_i^a: the number of arrival aircraft on route *i* of airport *a* over the total number of arrival aircraft associated with *a*,
DEP^a: AHP-derived weight for departures of airport *a*,
dep_i^a: the number of departure aircraft on route *i* of airport *a* over the total number of departure aircraft associated with *a*,
OD^a: AHP-derived weight for origin-destination type, normalized for the number of options of the origin-destination criterion within the 3nd level criteria of airport *a*,
DOM^a: AHP-derived weight for domestic flights of airport *a*
dom_i^a: the number of domestic aircraft, over the total number of domestic aircraft
INT^a: AHP-derived weight for international flights of airport *a*
int_i^a: the number of international aircraft, over the total number of international aircraft
$USER^a$: AHP-derived weight for the 'user type' attribute of airport *a*, normalized for the number of options of the user type criterion within the 3nd level criteria
$PASS^a$: AHP-derived weight for passenger aircraft of airport *a*
$pass_i^a$: the number of passenger aircraft, over the total number of passenger aircraft
CAR^a: AHP-derived weight for cargo aircraft of airport *a*
car_i^a: the number of cargo aircraft, over the total number of cargo aircraft

The set of alternatives (dynamic routes) is located at the bottom level of the hierarchy (see Figure 5). Since the objective of the prioritization model is to assign the priorities to each individual route, a proportionality factor $f_i^a = \frac{n_i^a}{\sum_{a' \in M} \sum_{j=1}^{N^{a'}} n_j^{a'}}$ needs to be included in the 2nd level criteria to adjust the *per route* priorities to be analogous to the number of routes per airport. For this, n_i^a denotes the number of flights in routes i to/from airport a, N^a denotes the number of routes associated with airport a, and M is the set of MAS airports.

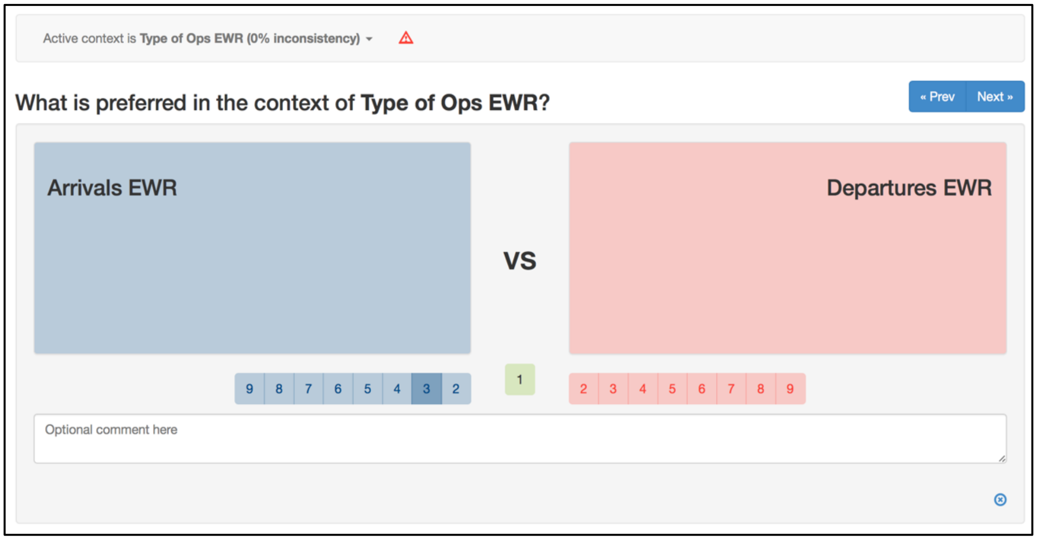
Appendix B. Details of the SME inputs to the AHP model

The hierarchy structured in this model is developed based on the literature on airport and MAS operations; and the subsequent criteria are validated by 7 Subject Matter Experts (SMEs). Some basic information on these SMEs are:

1. Program manager PANYNJ, ex-TRACON controller, computer science background with over 30 years of experience
2. Director of aviation environment and sustainability, aviation regional planning supervisor, PANYNJ, over 30 years of experience
3. Senior airport planner, over 10 years of experience, PANYNJ
4. Director at PANYNJ aviation department, manager of operations at EWR airport, over 30 years of experience
5. JFK, EWR, LGA, TEB tower managers, over 30 years of experience
6. Managing director United Airlines with over 30 years of experience.

Regarding the formulation of the AHP model: the SMEs were first interviewed individually or in small groups to assess the AHP structure. The model structure was revised to capture adequately the current state of affairs based on their assessments. This procedure was iterated until they unanimously agreed on the final AHP model structure presented here.

The validation of the AHP model was done by one SME, in the role of the TRACON manager. In general, the AHP model is formulated by combining the interests of the different stakeholders (airports) in terms of their preferred management strategies. The AHP structure based on the experience of the SMEs presents an objective assessment of the relative importance of each criterion. When it comes to the actual validation, the dominating factor that influences the priorities is air traffic demand. The management strategies of each airport are accounted for in this assessment. For example, both JFK and EWR are operating with a departure push strategy in the period 08:00-10:00, while the arrival / departure ratio for LGA and TEB would be 50 / 50 for this period. This information is known to the SMEs from a given Metroplex, and is directly derived by the demand data (in particular the arrival / departure slot allocation). Thus, it would make any difference to have more than one SME for the purpose of validation. Two examples of the AHP pairwise comparisons as conducted in Transparent Choice software are illustrated in Figure 17 and Figure 18.



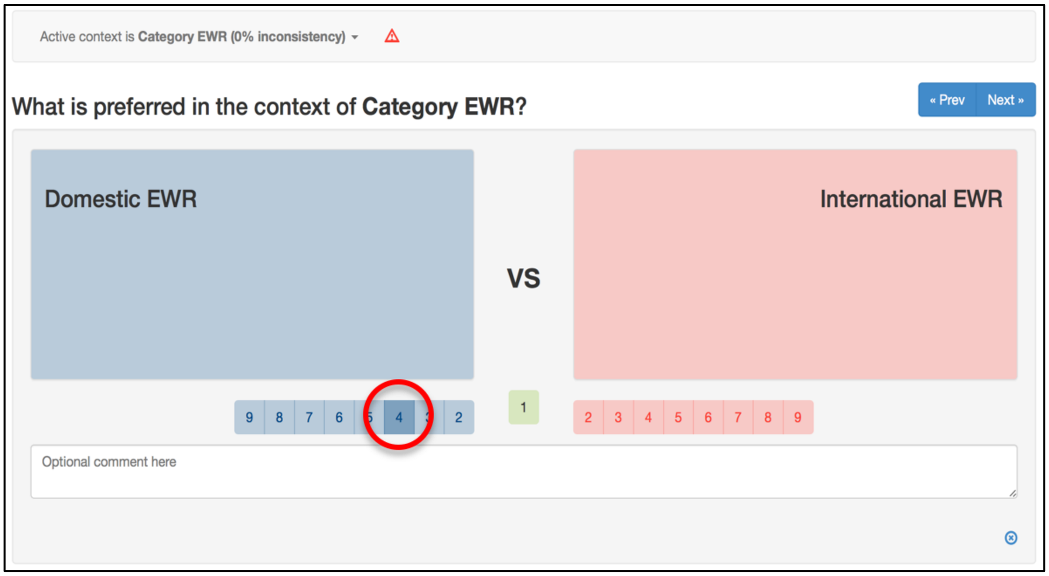
Active context is **Type of Ops EWR (0% inconsistency)**

What is preferred in the context of **Type of Ops EWR?**

Arrivals EWR VS Departures EWR

Optional comment here

Figure 17. SME pairwise comparison for EWR type of operation criterion, Transparent Choice software



Active context is **Category EWR (0% inconsistency)**

What is preferred in the context of **Category EWR?**

Figure 18. SME pairwise comparison for EWR category criterion, Transparent Choice software

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