Fretting Wear of Misaligned Spline Couplings

by

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Abstract

Spline couplings are able to accommodate some misalignment, although this may result in fretting wear of the teeth. This can reduce coupling life or cause complete tooth fracture. Computational and experimental methods have been used to investigate the problem for straight, involute splines.

Boundary element analyses of complete couplings have identified four contact regimes that may occur; full tooth contact may exist, teeth may contact at ends alone and teeth may fully separate during rotation. At large misalignments the coupling may topple leading to contact on both sides of the teeth. The regime is dependent on the ratio of misalignment to torque and on coupling geometry.

The various forms of pressure distribution over the tooth surface have been presented. The analyses have also revealed the relative tangential displacement between the teeth for a complete revolution. This slip path differs significantly from that predicted by a rigid body analysis. An indication of maximum wear depth has been calculated for each coupling, assuming Archard's wear law applies. The wear depth varies axially along the tooth, being greatest at the ends.

Several design and operating parameters thought to influence tooth wear have been investigated. The misalignment angle has the greatest effect on maximum wear depth whilst the torque has little influence. For a fixed diameter of coupling, wear appears to decrease for greater numbers of teeth. Minimising the diameter is the most effective way of designing for minimal wear. Equations have been presented in dimensionless form for parameters describing the wear, moment and pressure distribution.

Experimental tests of modified spline couplings have supported the computational results and shown that effective lubrication is critical. Further experimental tests have allowed the fretting process and wear particle movement to be viewed.

Software has been produced to simplify selection of the optimum spline geometry for an application.
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## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Pressure angle</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Coefficient of friction</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Misalignment angle</td>
</tr>
<tr>
<td>$\omega t$</td>
<td>Angular position of shafts</td>
</tr>
<tr>
<td>$b$</td>
<td>Facewidth</td>
</tr>
<tr>
<td>$b^*$</td>
<td>Dimensionless facewidth</td>
</tr>
<tr>
<td>$c$</td>
<td>Axial distance between centre of pressure and centre of facewidth</td>
</tr>
<tr>
<td>$C_{max}$</td>
<td>Pressure distribution skewness parameter</td>
</tr>
<tr>
<td>$D_b$</td>
<td>Base diameter</td>
</tr>
<tr>
<td>$D_p$</td>
<td>Pitch diameter</td>
</tr>
<tr>
<td>$E$</td>
<td>Elastic modulus</td>
</tr>
<tr>
<td>$E^*$</td>
<td>Dimensionless elasticity/torque parameter</td>
</tr>
<tr>
<td>$F_N$</td>
<td>Normal force acting on a surface</td>
</tr>
<tr>
<td>$F_T$</td>
<td>Tangential force acting on a surface</td>
</tr>
<tr>
<td>$k$</td>
<td>Wear coefficient</td>
</tr>
<tr>
<td>$m$</td>
<td>Module</td>
</tr>
<tr>
<td>$M$</td>
<td>Bending moment in direction of misalignment (about axis of misalignment)</td>
</tr>
<tr>
<td>$M_{perp}$</td>
<td>Bending moment about axis perpendicular to axis of misalignment</td>
</tr>
<tr>
<td>$p$</td>
<td>Contact pressure</td>
</tr>
<tr>
<td>$P_{max}/P_{av}$</td>
<td>Tooth load parameter (maximum tooth load / average tooth load)</td>
</tr>
<tr>
<td>$s$</td>
<td>Slip distance</td>
</tr>
<tr>
<td>$T$</td>
<td>Torque</td>
</tr>
<tr>
<td>$W$</td>
<td>Dimensional wear parameter</td>
</tr>
<tr>
<td>$x'$</td>
<td>Slip in direction of tooth profile</td>
</tr>
<tr>
<td>$y'$</td>
<td>Slip in axial direction</td>
</tr>
<tr>
<td>$Y$</td>
<td>Position on tooth in axial direction relative to centre of tooth</td>
</tr>
<tr>
<td>$z$</td>
<td>Number of teeth</td>
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</tbody>
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I

Introduction

1.1 Spline couplings

![Figure 1.1 - Form of a basic spline coupling](image)

A spline coupling is a device by which power can be transmitted between rotating machine elements through a series of protuberances, or teeth. Spline couplings consist of an externally toothed shaft section ("male") that is matched to an internally toothed shaft section ("female") – Figure 1.1. They are common components in rotating machinery, and are often used to transmit large torques. They can be used to couple two shafts, or to attach gear sections or other components to a shaft.

Spline couplings provide a relatively simple and inexpensive method of coupling shafts and can have a very high torque capacity with minimal weight. They offer a number of beneficial features including reversibility, self-alignment, ease of assembly and disassembly, the ability to operate at high speeds and the ability to accommodate some angular misalignment. Spline couplings do not require precise axial positioning of the coupled shafts; some couplings are designed to allow appreciable amounts of relative axial motion between the two components whilst the coupling is loaded and rotating – "plunge".

Because of their wide ranging abilities, spline couplings can be found in a variety of industries and applications, from automotive and aeronautical transmissions to simple household appliances.
The spline teeth could be of any appropriate shape, but the involute form is commonly used. Varying modules, heights and pressure angle are possible, giving a range of different tooth shapes (Section 2.4.2). The teeth may be axially straight or helical. Splines can be designed with a large clearance between the back (unloaded) faces of the teeth, or could have a minimal clearance or interference fit.

Spline couplings may be made from many materials, although steel is used for the majority of components transmitting a substantial torque. The teeth may also be coated or an alternative surface treatment applied. Lubrication (solid, grease or liquid) is generally used within the coupling to reduce tooth wear. For involute forms, male teeth can readily be made using gear cutting tools; female components are commonly formed through broaching.

Perhaps due to their simplicity, the design of spline couplings has traditionally been rather basic. The size (diameter) of the coupling is selected to limit the hoop stresses within the outer wall, and Shigley suggested “neither wear nor bending stress is important in the design of spline teeth” [1]. Typical stress analysis techniques used currently involve a rather simplistic model of the coupling and assumed tooth loads. The coupling is selected by the pitch diameter required to limit torsional stresses and, despite the variety of tooth forms available, little consideration may be given to selecting the most appropriate; the selection can be arbitrary or based on previous designs. Several design standards exist and offer a range of options for tooth size and form.

A number of possible failure modes exist, including: shearing of the externally toothed component wall, bursting of the wall of the internally toothed component, tooth shearing, fatigue fracture due to torque cycling, and excessive wear of the load bearing areas of the teeth. With increasing demands of transmission systems and stronger materials, tooth wear has become an important issue, particularly in the aviation industry.

Tooth wear can be caused by torque cycles leading to slip between the contacting teeth faces. Torque cycles can also lead to failure through fatigue. Spline couplings designed to operate with large amounts of axial movement between shafts are likely to experience heavy wear caused by this axial movement. Tooth wear may also be
caused by misalignment. A small amount of misalignment can be accommodated by spline couplings, but will result in slip between mating teeth as the shafts and coupling rotate, which may cause wear.

Tooth wear caused by misalignment alone can be extremely damaging and almost all couplings will have some degree of misalignment. This study is an investigation of wear caused by misalignment alone.

1.2 Fretting wear

Fretting is a term that is used to describe several wear and damage processes ("fretting corrosion", "fretting fatigue" and "fretting wear"). Fretting is caused by small amplitude oscillations between surfaces in contact. The definition of "small amplitude" is not precise but, typically, situations where the slip amplitudes are less than about 200 µm are described as fretting. This type of motion can lead directly to damage by material loss and/or crack propagation and is associated with other damaging events such as increased corrosion and fatigue crack initiation. Spline couplings operating with misalignment experience relative motion between the teeth with amplitudes of this order, hence wear of spline coupling teeth can be classified as fretting wear.

Fretting wear is usually associated with lower wear rates than reciprocating wear but, because of the areas in which it can occur, fretting can often be a cause of (sometimes catastrophic) failure. Besides loss of structural integrity by cracks and material loss, it can lead to failure by loss of function due to eradication of critical tolerances, seizure of machine parts due to wear debris, or loosening of fixings.

In some instances, the motion leading to the fretting is intentional, in an electrical switch or pivot for example. However, in many cases fretting occurs between surfaces that are designed to be fixed rigidly to one another with no relative motion, such as bolted joints and flanges. In these instances, mechanical vibrations from an external source can cause movement sufficient for fretting damage. Within spline couplings, the fretting motion is an unintended result of the rotation, caused by imperfect alignment.
In steel components, fretting can often be identified by the appearance of reddish-brown surfaces and powder – iron(III) oxide. Powdery oxide debris, which also occurs in fretting of other materials, can have an important role in fretting wear; with the limited movement between the surfaces, the wear debris is likely to remain within the contact area and so influence the contact loads and displacements. The oxidised debris has a greater volume than the iron from which it is formed, and so can cause seizure between components. Localised pits are sometimes found in fretted regions, which can also act as sites for crack initiation.

Two common approaches to reducing fretting damage can directly oppose one another. Reducing sliding amplitude would be a means of reducing wear. However, if this is achieved by an increase in friction (for example tightening bolts or removing lubrication), then the increased stresses promote wear. The opposite approach is to reduce friction (for example by lubrication). This, however, has the drawback that it can lead to greater slip amplitudes and so promote greater wear. Which of these methods is most suitable is likely to vary from one application to another and is reviewed in Section 2.1.5.

1.3 Research programme: background and objectives

Spline tooth wear is a problem for the aviation industry where safety and reliability are essential. Although not usually a cause of immediate failure, if tooth wear is observed during maintenance then it often necessitates replacement of the coupling. The benefit of reducing tooth wear would be to reduce the frequency and cost of replacement, as well as reducing the likelihood of catastrophic failure. An easy means of reducing tooth wear would clearly be advantageous for other industrial uses as well.

A recent collaborative research programme involving the DTI, Westland Helicopters, Rolls Royce, Computational Mechanics, Nottingham University and Imperial College, attempted to develop a methodology to reduce spline tooth wear. This is reviewed in Section 2.4.3. Although it did not produce any clear remedies to the problem, the work resulted in a better understanding of spline couplings and development of
boundary element software capable of analysing spline couplings. This allowed the current research programme to be undertaken.

In practice, the choice of spline coupling material may be limited and the method of lubrication may be restricted. Similarly, it may not be possible to reduce the degree of misalignment. However, a number of different spline coupling designs may be suitable for a given application and there is currently no method of determining which one should be selected. The choice is usually based on custom and practice. It may be the case that the wear for each design will vary, such that there will be an optimum design of spline. There may be restriction on the possible size (diameter) of the coupling due to strength requirements but, for example, should there be many small teeth or a few large teeth? What ratio of facewidth/diameter should be chosen?

The primary objective of this research is therefore to develop a means by which it is possible to determine the spline coupling geometry for which a given misalignment will cause the minimum damage for a particular application.

Four aspects were identified as necessary to achieve this objective. These were an understanding of the dynamic behaviour of spline coupling teeth when operating with misalignment; prediction of tooth contact loading and movement; assessment of spline tooth wear mechanisms; and establishing the function of wear debris within spline couplings.

The work carried out for this research consisted of three main tasks: experimental spline wear tests, a fretting visualisation study and computational analysis of spline couplings. These are reported in Chapters 3, 4 and 5 respectively. The experimental tests establish possible mechanisms for fretting wear of spline teeth and permit a basic comparison of different spline geometries and operating conditions. The computational part of the work shows the contact situation between spline teeth in greater detail.

For this research to help reduce wear in spline couplings, it is necessary to provide a method by which the findings can be used in practice. The development of a software tool that can be used to select the optimum spline coupling for a particular application constitutes the final segment of the research program and is detailed in Chapter 6.
This research is restricted to straight, involute tooth spline couplings. Although various other configurations could be analysed, this type of spline is the most common and of most interest for the aviation industry. The results for other types of spline couplings are likely to be similar in nature to those being studied.

Only wear due to misalignment has been considered. Wear in spline couplings operating with large scale axial movement (plunge) is likely to be of a different, and more severe, form.

Although the work is specifically for spline couplings, some aspects may be relevant to other instances of fretting wear. The fretting visualisation study should be applicable for fretting of large contact surfaces in general. The computational analysis techniques could also be developed for a similar study into fretting of other devices.

1.4 Misalignment of spline couplings

Before reporting the main tasks carried out for the study, it is necessary to characterise the form of misalignment covered. It is also useful to consider the practical origins of shaft misalignment, how this relates to misalignment of spline couplings, and examine aspects of spline coupling behaviour that are intuitively expected or can be analytically determined.

1.4.1 Types of misalignment

Shaft misalignment will occur when the fixings supporting the rotating shafts are such that the shafts are not both concentric and parallel. This is not necessarily caused by an error in assembly – the tolerances of location could be sufficient to allow misalignment capable of producing wear. Operating loads may also be large enough to cause shaft flexure and hence misalignment.

Whatever the cause, misalignment can generally be considered as being due to misalignment of shafts due to positioning of bearings or similar fixtures. Figure 1.2 illustrates different ways in which the coupling may be misaligned. The bearings may be positioned (or displaced due to loading) in a way that the shafts are parallel but not concentric ("parallel misalignment" [2]); not parallel, but concentric at the centre of the coupling ("angular misalignment"); or a combination of the two. In the latter case,
the misalignments may not be in the same plane and so further complexities are introduced. In a single plane, the combination of angular and parallel misalignment implies that the effective angular rotation is not about the centre of the coupling.

Figure 1.2 - Types of misalignment (exaggerated deformations)

Whatever the type of misalignment, the shafts will bend to a certain degree on assembling the coupling. This results in a bending moment being applied to the coupling, and possibly a net shear force. A misalignment can therefore also be considered as a bending moment applied to the coupling.

In this study, only an angular misalignment about the centre of the coupling is considered with no net shear force. Any shear force produced in the experimental spline wear tests would be minimal.

Examination of Figure 1.2 also reveals an issue that must be accounted for when quantifying a misalignment and when comparing results between experimental work, computational studies and practical applications. The inherent, or remote, misalignment of the shafts (shown by the red lines), is much larger than the misalignment occurring close to the coupling section (Figure 1.3). Some of the
remote shaft misalignment is accommodated by elastic shaft flexure, and some through both elastic and rigid-body displacements of the coupling components.

The positioning of the bearings and the length and stiffness of the shafts will affect the relationship between coupling and remote shaft misalignments, as will the effective stiffness of the coupling - itself dependent upon spline geometry.

The misalignment will also vary along the length of the coupling due to elastic tooth flexure, and so misalignment must be defined with reference to a fixed remote point. The misalignment angle at this point is used to quantify the misalignment of the system. It is equivalent to the misalignment of theoretically rigid supports at that point.

![Figure 1.3 - Comparison of remote misalignment and misalignment in vicinity of coupling](image)

1.4.2 Effects of misalignment

In a perfect spline coupling connecting perfectly aligned shafts under constant torque, there would be no slip between teeth and the pressure distribution would be identical for all teeth. As well as producing slip between teeth, misalignment will change the pressure distribution over the teeth. Intuitively, it is apparent that a misalignment will
cause contact to occur at the ends of the spline teeth. This is confirmed through the observation that worn splines usually show damage towards the ends of the teeth and has been analysed with a rigid body assumption by Buckingham [3].

A misalignment applied to a rigid spline would cause contact to occur only at the end extremities of the teeth. Elastic deformation will cause the contact to spread along the length of the teeth. As the shafts rotate, the direction of misalignment will vary relative to each tooth and so produce an oscillating pressure distribution.

The misalignment will also cause movement tangential to the tooth surface. An estimation of the slip due to misalignment can be obtained through a rigid body analysis by considering the displacement of teeth due to the imposed angle, and neglecting the component of displacement in the direction normal to the tooth surface. Varying the direction of misalignment allows a "rigid body slip path" to be produced. This is detailed in Appendix A. The slip paths are found to be described by the equations:

\[ x' = Y \theta \sin \omega t \]

\[ y' = \frac{1}{2}D_p \theta \sin (\omega t + \pi - \alpha) \]

where: 
- \( x' \) = slip in tooth profile direction; 
- \( y' \) = slip in axial direction; 
- \( Y \) = position in axial direction relative to centre of tooth; 
- \( \theta \) = misalignment angle (radians); 
- \( D_p \) = pitch diameter; 
- \( \alpha \) = pressure angle of spline; 
- \( \omega t \) = angular position of shafts

This produces a slip path in which the axial movement and the movement perpendicular to this are sinusoidal, and out of phase by \( \pi - \alpha \). This rigid body slip path is compared with computed results in Section 7.1.1.
2

Review of Fretting Wear, Spline Couplings and the Boundary Element Method

2.1 Fretting

2.1.1 Introduction and definitions

2.1.1.1 Variety of definitions

Fretting occurs when two surfaces in contact experience a small relative oscillatory motion. This is a rather incomplete definition since the term "small" is not quantified and absolute values for a limiting amplitude of motion vary within the literature. However, many examples of wear can be identified as fretting by certain characteristics, principally the presence of oxidised debris within the contact [4].

Within the literature, several terms are associated with fretting, including "fretting corrosion", "fretting wear", and "fretting fatigue". These terms are not uniquely defined and mean different things to different authors, the reasons for which become apparent when considering the wide variety of situations in which fretting may cause damage and the many forms of such damage.

The first complete book on fretting by Waterhouse entitled "Fretting Corrosion" [5] argued against some previous definitions and offered the following:

- Fretting corrosion – the forms of damage which arise when two surfaces in contact and nominally at rest with respect to each other, experience slight periodic relative movement or the type of fretting damage when the debris produced is a chemical reaction product between constituents of the surface and the environment.

- Fretting – the action which produces fretting damage or fretting corrosion.

- Fretting fatigue – the combined action of fretting and fatigue.
• Fretting damage and fretting wear – the results of fretting action.

The term fretting corrosion is less common in recent publications, but the terms fretting wear and fretting fatigue are still often used to describe or imply different situations. Vingsbo and Söderberg acknowledge that there is confusion in the literature regarding the distinction between the different types, making comparison between different reports difficult [6]. Before considering the reasons behind the definitions, it is necessary to review how fretting may occur, the forms it may take, and the type of damage it may cause.

2.1.1.2 Cyclic stress or cyclic motion

Fretting requires some form of relative movement between surfaces in contact. Often the motion is caused by the body of one surface moving relative to the body of the second surface (bulk movement). For a typical example, a box of tightly packed metal rods being transported may experience vibrations that cause the rods to shuffle back and forth.

Alternatively, the surface movement may be caused by differential strains between the surfaces of two bodies in contact. This occurs when a cyclic stress is applied to one of the bodies. For example, a metal strut may experience varying loads during its operation and thus expand and contract. If a second component is clamped onto that strut, this expansion and contraction will cause the relative tangential displacement between the surfaces of the strut and the second component.

In the second of these situations, at least one of the components is undergoing a cyclic stress associated with normal fatigue. This leads to the definition of fretting fatigue as fretting occurring between two components in which one experiences a bulk cyclic stress. The word bulk is necessary since a local cyclic stress occurs in the vicinity of the contact region for all cases of relative motion, which may lead to fatigue crack damage itself.

2.1.1.3 Stick and slip

The effects of friction on the surface movement have a significant effect on the nature of the fretting. In simplistic terms, if the imposed tangential force is higher than the product of friction coefficient and normal force, then surfaces will move relative to
one another (slip), whilst if it is lower then no relative motion will occur. The consequence of this condition is demonstrated in the case of a spherical contact.

Mindlin analysed the situation of two spheres in contact and subject to both a normal and a tangential force [7]. The pressure distribution due to the normal force is known from a Hertz analysis. Mindlin determined that the tangential stresses at the surface vary parabolically from a minimum at the centre to a (theoretically infinite) maximum at the outer edge. However, the maximum tangential traction is limited to the product of friction coefficient and normal force, hence the shear stress at the edges is reduced and slip occurs (Figure 2.1). An inner circle may exist in which the tangential force is insufficient to cause relative motion, and the surfaces remain together (“stick”), surrounded by an outer annulus in which the surfaces experience micro-slip. The sizes of the stick and slip regions are dependent on the values of normal and tangential force and friction coefficient. If the tangential force is large, then the entire contact may be in slip, referred to as “gross slip”. Following the analysis of Mindlin there should always be an outer annulus of slip [8], but this may be negligibly small, and such cases are referred to as “stick”. The intermediate case of some stick and slip is referred to in the literature as “partial slip” or “stick/slip”. The Mindlin analysis assumes elastic conditions and smooth surfaces, but can be qualitatively applied to real contacts [6].

![Mindlin analysis of spherical contact](image)

**Figure 2.1 - Mindlin analysis of spherical contact**
Many investigators have found that these different regimes affect the severity and type of damage that occurs. Partial slip has been identified by many as promoting crack initiation and propagation [6, 9], whereas gross slip is associated mostly with wear by material removal [6]. This is reviewed more in Section 2.1.2.4; the various meanings of fretting terms follow from these observations.

The known analytical solutions of stick and slip regimes for various contact conditions have been reviewed by Hills and Urriolagoitia Sosa, aimed at improving the understanding of fretting [10]. However, they acknowledge that the way in which this information can be used to design against fretting is still unclear.

2.1.1.4 Further definitions

Many authors use the terms fretting fatigue and fretting wear to distinguish between damage through crack initiation/propagation and material removal respectively [11]. Others refer to a partial slip regime as fretting fatigue and gross slip as fretting wear, e.g. [6, 12], presumably based on the most prominent form of damage of the two regimes. This is despite the fact that material removal and crack propagation may occur concurrently [13] and during either regime [14]. Bill defines fretting wear as material removal and fretting fatigue as a reduction in fatigue life due to surface damage from fretting [13].

Another common definition is fretting wear as the case of fretting action caused by bulk oscillatory movement of the bodies and fretting fatigue as the case of fretting action caused by cyclic loading of a component [15]. Sauger et al. [16] refer to this as fretting wear being caused by vibration and fretting fatigue by external loading. This may also lead to confusion as crack growth and material removal wear can occur under both types of loading [11].

Other definitions also exist. Koenen et al. [17] refer to fretting fatigue as occurring for amplitudes below 20 \( \mu \text{m} \) and fretting wear for greater amplitudes, perhaps because higher amplitudes favour gross slip whilst lower amplitudes favour partial slip. Szolwinski and Farris [18] restrict the term fretting itself to the cases of stick and partial slip, referring to other cases where gross slip occurs as reciprocating sliding.
2.1.1.5 Fretting or reciprocating sliding?

This returns to the question of how small an amplitude is necessary for the designation of fretting, rather than a more general case of sliding or reciprocating sliding wear. Various absolute values are given: 50 μm [13], 100 μm [19], 200 μm [20]; whilst Imai et al. conduct “fretting” experiments at 0.75 mm [21]. It seems apparent that an absolute value of sliding amplitude is not appropriate, but depends on various aspects of the situation under consideration. It may be determined by the observation that the specific wear rate at lower amplitudes is generally less than that for unidirectional or reciprocating sliding, and may increase with amplitude. One definition for the amplitude for transition from fretting to reciprocating sliding is therefore the amplitude at which the specific wear rate becomes independent of amplitude [6], i.e. when it begins to show the characteristics and mechanisms of unidirectional sliding.

As there is little movement between surfaces in fretting, wear particles removed from the bulk material are likely to remain within the area of contact. This debris retention is closely associated with fretting wear [22] and may be one of the physical attributes that distinguish fretting from general oscillatory sliding [23].

Tests carried out at 0.75 mm slip amplitude were claimed to show characteristics of fretting rather than sliding wear [21]. These tests were carried out with a large contact area (100 mm x 100 mm) and so the initial wear particles formed (perhaps as “ordinary” wear [21]) remained in the contact and fretting wear resulted. This geometry effect has important implications for laboratory testing which commonly uses small contacts.

The type of wear particles, or debris, produced under fretting action is a characteristic trait of fretting action. For steel components, fretting is most commonly identified by the appearance of a red-brown powder at the contact. This is an iron oxide (Fe₂O₃); oxidation of the surface, and more notably the debris, is significant in fretting and explains how the term “fretting corrosion” became used. However, fretting damage has been found to occur in an inert environment and for materials that do not oxidise and the term is now less common [22].
2.1.1.6 Summary of general fretting features and relevance for spline couplings

To prevent confusion between the various fretting terms, the following definitions are implied unless otherwise stated:

- Fretting or fretting action – relative oscillatory motion between two surfaces in contact (irrespective of its cause).

- Fretting wear or fretting damage – change in surface or surface conditions caused by fretting. Fretting wear usually refers to the case of oscillating movement without bulk cyclic stresses, i.e. not fretting fatigue.

- Fretting fatigue – the particular case of fretting in which the damage is influenced by a bulk cyclic stress in one component.

The terms material removal and crack initiation/propagation are used to distinguish the main forms of damage. Under fretting fatigue, the usual form of failure is through complete component fracture – a consequence of the bulk stresses.

This research is mostly concerned with material removal rather than the propagation of large cracks throughout the structure. For this reason, this chapter predominantly covers the more general case of fretting in which any bulk cyclic stresses do not significantly affect the contact stresses. However, cyclic stresses within the spline are possible due to tooth bending and so fretting fatigue is briefly reviewed in Section 2.1.6.

2.1.2 Fretting wear – regimes, stages, and experimental observations

2.1.2.1 Where and why

Fretting wear is a problem experienced in a large number of situations throughout many industries. Low identifies the many occurrences of fretting wear in power plant [24], and fretting wear is also a significant problem in aircraft components [25], nuclear reactors [26], electrical contacts [27], steel wire ropes [28] and even hip prostheses [29].
In attempting to understand and minimise or eliminate the problem, fretting wear has been studied in the laboratory for many years. From the 1960s to the early 1980s, fretting fatigue was of major interest and received much attention, but since that period, wear damage through material loss has become of more importance [19].

Investigations of fretting wear often use a test apparatus that applies an oscillating motion to one specimen, which is loaded against a stationary specimen. Most recent experimental studies (not including fretting fatigue) use a ball-on-flat (e.g. [30, 31]) or crossed-cylinder geometry (e.g. [14, 32]) as this provides an analytically determined contact condition, simple set-up and simple manufacture of specimens. Although these conditions allow fundamental studies of fretting wear, they may not be common in practical situations and care must be taken when relating the results to other contacts.

Fretting fatigue experiments, which are reported in Section 2.1.6, apply a cyclic stress to one specimen rather than a oscillatory movement, usually through a standard tension-compression fatigue test rig. These type of experiments are not necessarily appropriate for other types of fretting wear, but have often been used to study fretting without detailed consideration of the effect of the bulk stress, and alongside the former test method [33]. The use of this test method was criticised by Vincent et al. [11].

Unfortunately, the results of the many studies are not always in agreement, and are often contradictory [34]. This has already been seen in the various definitions, but it also concerns fundamental understanding of the problem. For example, Stowers and Rabinowicz note that there is disagreement as to whether the wear per unit sliding distance is greater, less than or identical to general sliding wear [35].

Rather than dismissing many conclusions reached by various authors, it is quite likely that the results depend on the precise details of the experimental conditions, including factors not originally accounted for. Beard listed over 50 parameters that could affect fretting [36]. The fact that wear results can vary by several orders of magnitude even under identical conditions [37], further complicates the matter.

One factor that is likely to play an important role in the variation of results is the effect of wear debris particles. It has been explained how these are trapped in the contact and will thus clearly affect the contact condition. Godet introduced an
approach to wear analysis ("The Third Body Concept") in which the action of the debris was the definitive issue for wear [38], and this is discussed in Section 2.3.1. The dwell time of a particle is the amount of time that the particle remains active within the contact area. This must be linked to the contact geometry (particles in larger contact areas have greater (average) distances to travel before escaping), and thus experimental contact geometry is a vital aspect of the experimental set-up.

2.1.2.2 Quantifying wear

Although fretting wear is a significant problem, the actual quantities of wear (in terms of material removed) are small, making accurate quantitative measurement difficult. The weight loss of a specimen can be a very small percentage, and oxidation of the surface can affect the results. Particle re-attachment can also lead to recordings of negative wear [39].

The normal displacement between the two contact specimens can be used to give a semi-quantitative assessment of wear, but results depend on particle movement. This technique does allow the wear to be continuously monitored during the experiment, which is not possible with many other methods. The normal approach could be converted to a wear volume, assuming a particular wear scar morphology [40], but still requires additional wear measurements and has limited accuracy.

A method called "Thin-Layer-Activation" (TLA) uses an irradiated specimen to measure very small quantities (as low as 5 ng) of wear and can also determine metal transfer [41]. It is, however, an expensive technique.

A comparison of TLA, normal approach, 3D surface topography and geometrical modelling is given in [39].

2.1.2.3 Fretting logs and fretting maps

Two useful tools for the study of fretting wear are fretting logs (or friction logs) and fretting maps. Fretting logs are based on dynamic measurements of tangential force and displacement amplitude. The tangential force is plotted against the corresponding displacement amplitude for a complete cycle. (Often the friction coefficient ($\mu$) replaces the tangential force ($F_T$), but these can be interchanged since $F_T = \mu F_N$ and
the normal force, \( F_N \), is invariably constant in these tests.) The resulting plot reveals the operating stick/slip regime.

![Figure 2.2 - Tangential force (T) v displacement (d) for conditions of stick (a), partial slip (b) and gross slip (c). (From [6])](image)

Figure 2.2 (from [6]) shows the three cases of stick, partial slip and gross slip. In the case of stick (a), the tangential force is proportional to the displacement indicating that all of the movement between the two specimens is accommodated elastically and there is no slip. For partial slip (b), a hysteresis loop is seen, indicating energy dissipation as areas of the contact experience micro-slip and so frictional energy loss. For gross slip (c), a wider loop is formed and a drop in tangential force occurs, followed by a period of movement with constant force. This shape reveals the greater energy dissipation. Since the kinetic friction is lower than the static friction, a drop in tangential force occurs once the stick region is eliminated. Note that the linear, elastic portion of the plot is identical for each situation.

Fretting logs are three-dimensional traces of these tangential force v. displacement plots, with the third axis showing number of cycles (or time). They reveal how the contact conditions and stick/slip regime vary with time.

Fretting maps were introduced by Vingsbo and Söderberg [6] as a means of characterising the operating regime and damage for two variables. It is obvious that the operating regime will depend upon the imposed displacement amplitude, but it will also depend on various other parameters. Figure 2.3 shows some examples of fretting maps produced from their experimental work.
These fretting maps are valid only for the conditions of the experimental tests in which they were obtained, i.e. same material, geometry, etc. The form of some of the maps can be deduced logically. Figure 2.3(a) shows the effect of normal load and displacement amplitude on regime. With increasing amplitude, the regime moves from stick through partial slip to gross slip as would be expected. With a higher normal load, the limiting value of tangential force is increased (since it depends on $\mu \times F_N$), and thus the amplitude for transition is increased. The shape of the fretting map for frequency cannot be deduced and is found experimentally.

This type of fretting map was named a "running condition fretting map" (RCFM) by Zhou et al. [42]. A "material response fretting map" (MRFM) used a similar method to indicate the primary means of damage for their fretting fatigue studies of aluminium – cracking, particle detachment or no degradation. The two types of map
could not be superimposed, demonstrating that the stick/slip regime does not uniquely determine the type of damage. A theoretical MRFM was developed for fatigue loading conditions by Fouvry et al. [15].

Fretting logs show that experiments may involve transitions from one stick/slip regime to another. Zhou and Vincent introduced a “Mixed Fretting Regime” (or MFR), which may occur when a change in stick/slip regime happens during an experiment [43]. It was claimed that gross slip does not immediately follow partial slip as suggested by others, but that the MFR occurs. It was found that the MFR is the most damaging in terms of fatigue crack formation.

However, the mixed fretting regime cannot be considered to be a regime as for stick, partial slip and gross slip, since these cover all permutations. It is perhaps more appropriate to consider the MFR as a particular change in conditions during fretting. This distinction is made in [9].

2.1.2.4 Experimental observations of stick and slip

The form of damage strongly depends on the type of regime. Although there are varied opinions on the mechanisms of wear, there is broad agreement regarding the forms of damage for each regime. These observations and mechanisms are based on unlubricated conditions.

Vingsbo and Söderberg discussed the damage and mechanisms through the different stick/slip regimes from experiments carried out using crossed steel cylinders in dry air [6]. In the case of stick, very limited surface damage was seen. A perfect spherical geometry would produce a single circular region of contact at the centre of the contact. For real surfaces, multiple contact points occur at the asperity peaks over a similar sized circular region. These localised contact points were identified in SEM micrographs. In the partial slip regime, an inner circle exists exhibiting the isolated asperity contact points of the stick regime with no significant wear. This is surrounded by an outer annulus of severe wear, with extensive crack formation. This corresponds to the slip annulus of the Mindlin analysis. In particular, small cracks were seen close to the stick region. This was identified as the region where stresses are highest (from the Mindlin analysis). Where cracks intersected, wear particles were produced.
In gross slip, the entire contact consisted of sliding wear marks in the direction of movement. Particles were generated from intersecting cracks, and fragmented through further fretting action. Extensive plastic shearing produced a scale-like appearance with delamination of particles, enhanced by oxidation, whilst crack propagation was less. This reduction in crack propagation could be attributed to various aspects of gross slip. The subsurface may be worn away faster than the crack front can propagate, thus removing the crack. The material removal also favours particle production, which can then reduce the shear stresses that propagate the cracks, and removes surface discontinuities [15].

The appearance of the wear scars (isolated asperity contacts in stick region, and outer regions of severe wear and crack formation) is common throughout many experimental studies (e.g. [12, 14]). Small, isolated “micropits” are also reported at low amplitudes [44]. On rough surfaces, wear marks were seen after a few cycles “over peaks of furrows” [45].

2.1.3 Mechanisms of fretting wear

Various mechanisms have been proposed for fretting wear. Hurricks reviewed many of these suggestions in 1970 [22]. Two stages of fretting were identified – an initial stage and a steady state. The steady state is a result of wear particles (generally oxidised) covering much of the contact area and has a reduced wear rate. Others described this as a three-stage process (e.g. [16, 46]), the intermediate stage being the process of formation or oxidation of wear particles.

Mechanisms reviewed by Hurricks involved “molecular attrition”, adhesion followed by extrusion and oxidation of material, scraping of oxide layer by asperities, abrasive action of oxide debris, plastic flow of contact points and oxidation of continuously removed layers.

Adhesion is the most commonly reported mechanism of wear in the initial stage. Vingsbo and Söderberg [6] identified the isolated contact points in stick as being caused by adhesion of the asperities. Although these asperities are plastically sheared during the fretting action, the plasticity is deemed negligible compared to the bulk elastic energy and thus the appearance of elastic stick in the fretting log prevails.
Adhesion was also proposed by Toth [34], who suggested that an increase in wear rate at larger amplitudes is caused by a greater probability of adhesion with asperities traversing a greater area. Hurricks concluded in his review of mechanisms that the initial stage of fretting involved the dispersal of any pre-existing oxide layer, and metal transfer through an adhesive process [22]. Wear particles formed were said to be subsequently oxidised and dispersed over the contact, resulting in the steady state of reduced wear. This steady-state wear was said to be through fatigue processes rather than abrasion.

Aldham et al. found that the initial stage of wear lasts up to 3000 cycles, and consists of plastic deformation and adhesion with metal transfer, but with little metal loss [46]. An intermediate stage was said to be the conversion of metal to oxide. In the steady stage, for which the wear rate is lower following the formation of oxide beds, occasional metal-metal contact was said to be necessary for the observed continuation of wear. Wear volume increases through a growth in wear scar area rather than depth [39, 46].

Ohmae and Tsukizoe [47] suggested a transition in mechanism at a slip amplitude of 70 μm. At larger amplitudes, plastic deformation of asperities and adhesion initially occur, followed by oxidation and abrasive wear in the steady state; at lower amplitudes, an oxidative wear mechanism was believed to occur.

The delamination mechanism proposed by Suh [48], has been suggested as a mechanism in fretting wear by some. Scanning-electron-microscopy of fretting wear scars revealed surface damage and plate-like wear particles indicative of delamination [49, 50]. Mechanisms involving scraping or breaking of asperities were said to be unacceptable given that wear is increased for smooth surfaces. Adhesion in the initial stages may occur, but subsequent material loss was said to be through delamination, following a fall in adhesion or development of slip regions [33]. Vingsbo and Söderberg claimed that the delamination is enhanced by oxidation [6].

The delamination theory was supported by Sproles and Duquette, who added that crack propagation occurs at several levels [51], rather than at a single depth of maximum stress originally suggested by Suh. However, Zhang et al. also found similar
debris and cracks at various levels, but argued that stripping of oxide layers occurs and not delamination [52].

Abrasion is sometimes mentioned as a mechanism (e.g. [17, 34, 47]) but Hurricks claimed that abrasion is only a minor factor in the fretting process [22].

2.1.3.1 Role of wear particles

The role of wear particles is particularly important and yet still uncertain. The action of wear particles forms the basis of the “Third-Body Approach” [38]. This section reviews the experimentally observed role of wear particles. The wear modelling aspects of the third body behaviour is reviewed in Section 2.3.1.

The effect of wear particles will almost certainly vary according to the precise contact conditions. There is much agreement that debris within the contact is beneficial, e.g. [40, 46], and accounts for the reduction of wear in the steady state, although some suggest that debris increases wear through abrasion [34]. Farrahi et al. refer to the debris as acting as “minute ball-bearings” and thus reducing friction and wear [53]. The fact that debris particles are (at least partly) responsible for a reduced wear rate is supported by the observation that an air flow across the contact which removes wear particles from the contact can lead to a ten-fold increase in wear [54].

Most experimental studies use steel components, which usually produces red-brown wear particles. These have been identified in many studies as being α-Fe₂O₃ [52, 55], possibly with a small amount of metallic Fe [53]. Initially the debris which escapes the contact may contain some metallic particles, but after further fretting action, and in the steady state of fretting, the debris generally consists of the oxidised particles [46].

Different shapes and sizes of particles have been seen. Thin plate-like debris is commonly seen, with a thickness of around 1-4 μm [50]. Powdery debris of various sizes is also produced (e.g. 15-20 μm [52], 0.1-1.0 μm [53]). Wear particles may separate the two surfaces and so reduce the wear [56], the shearing of the particles dissipating the energy. There are several ways in which particles can accommodate such movement; these “velocity accommodation mechanisms” are identified by Berthier et al. [57].
Tests by Lyons and Collins showed that the wear (measured as normal approach) was doubled by adding grooves to the contact surface [23]. This was attributed to wear particles being trapped in the grooves and so not acting to reduce wear.

This load carrying ability of wear particles is a key component of the third body approach (Section 2.3.1). However, Aldham et al. claim there was no evidence of oxide beds carrying load, and that the lower wear rate may be due to lower particle formation rate once the beds exist [46].

The wear particles were found to be beneficial by Colombié et al. due to a load carrying ability [45]. Removing the particles at intervals during fretting experiments was concluded to increase wear, whilst the addition of artificial wear particles (iron oxide particles obtained elsewhere) prior to the tests reduced wear. This conclusions was disputed by Pendlebury, who claimed that the statistical variation of results could not support it [58]. Based on new fretting tests and the results of Colombié, Pendlebury concluded that the debris escape (which is fundamental to the third-body wear theory) does not affect the wear significantly. In long-duration fretting tests, for which the contact resistance was measured, regular and intermittent breakdown of the oxide beds was recorded (seen as a reduction in contact resistance). It was suggested that the associated metal-metal contact was responsible for wear, even after a long period of running. The intermittent nature was proposed as an explanation for the statistical deviations in wear volumes.

Addition of oxide particles prior testing was also used by Iwabuchi [59]. The particles were sometimes found to be beneficial and sometimes hostile. Their role was found to be dependent on amplitude and load, such that a map of beneficial and hostile regions could be produced (Figure 2.4). The varying effect was claimed to be a result of competition between formation of a compacted oxide layer and abrasive action prior to formation.
Figure 2.4 - Role of oxide particles: effect of load and amplitudes [59]

× indicates hostile role; ○ indicates beneficial role.

Wear particles may be a cause of greater damage in some other circumstances. If the two fretted components are clamped together, the greater volume of the oxidised debris can increase the clamping pressure [49] and thus increase damage. If a build up of wear particles occurs at particular locations within the contact, rather than spread over the surfaces, the contact kinematics [60] and geometry [61] may be changed, possibly to a more damaging case.

The determination of mechanisms of wear in the steady state is further complicated by the fact that more than one mechanism may operate concurrently in the same contact [62]. The influence of the wear particles may even depend on the rigidity of the specimens and test rig [63].

The particles will obviously only affect the wear mechanism whilst they are in the contact and so particle escape and movement is important. Amongst the many relevant factors is the kinematics of the oscillating motion – the debris escape for a twisting motion of a ball on flat was found to be much more than for an oscillating rolling motion, as was the wear rate [56, 60].

2.1.3.2 Oxidation

Oxidation must be ruled out as the primary mechanism of wear in fretting [22] since wear can occur for materials that do not oxidise and in inert environments [64]. However, it undoubtedly can have a major influence on the mechanism and thus the
type and severity of wear which occurs. For high temperature fretting of steel, it may become the primary factor that controls wear [65].

A comprehensive study of oxidation in fretting is provided by Bill for titanium and other aluminium alloys by comparing results under nitrogen and dry air [66]. Greater wear volumes were recorded in air than in nitrogen, and the appearance of the wear scar varied. The role of oxidation was classified into four groups: non-protective for non-oxidative materials or environments; sequential stripping of oxide layer; interaction with fatigue cracks promoting propagation; and protection through formation of thick oxide film. Thus, oxidation processes may be beneficial, harmful or have no effect.

Oxidation is strongly dependent on temperature, but there is disagreement on the temperature rise that occurs in fretting [67]. The formation of Fe$_2$O$_3$ debris is said by many to indicate high temperatures, requiring fretting to generate large temperature rises at the contact.

The maximum temperature has been analytically determined as being the same as for uni-directional sliding at the maximum speed, with only a small periodic variation due to the oscillation [68]. Since sliding velocities in fretting are small (e.g. slip amplitude of 20 $\mu$m at 50Hz has maximum speed of under 7 ms$^{-1}$), the temperature rise would not be large enough to significantly promote oxidation. An average temperature rise of 44°C was reported by Ghasemi et al. [69].

Mechanical factors, rather than a temperature rise, are recognised as a probable mechanism for the formation of Fe$_2$O$_3$ [45, 55]. The mechanical action of grinding and the crushing of the debris provide sufficient energy for oxidation to occur. A more elaborate explanation for oxidation of debris particles involves a complex exoelectron emission [69].

The debris may originally be formed as metallic particles, which are subsequently oxidised or may be detached from an upper oxidised layer of the surface. Sproles and Duquette claimed the latter was more likely [51]. Evidence to suggest the former includes the observation that differences in oxygen content occur in debris found within the contact and out of the contact [55]. The actual oxidation of the debris does not directly affect the wear rate [20].
Oxidation of the surface layer during fretting does appear to occur – Aldham et al. [46] identified scratches in the direction of fretting beneath a surface layer of oxide.

### 2.1.3.3 Subsurface damage

The subsurface damage is just as important as the surface degradation [52]. Zhang et al. found that the extent of subsurface damage increased with load and slip amplitude, and consisted of an upper layer of compacted oxide, a layer of severe plastic deformation in which the metal was comminuted and orientated in the direction of sliding, and a plastically deformed region.

Zhou et al. described a “Tibologically transformed structure” (TTS) occurring under fretting conditions, which is much harder than the bulk material [70]. This consists of a subsurface region (which can reach 50 µm depth after just 100 cycles) in which the bulk material is altered by the contact, and it is from this region that the particles become detached. They found that the TTS was not formed by high temperatures or oxidation, but by the mechanical energy of the motion causing plastic deformation and generating recrystallisation. An energy threshold (which can be determined by the energy dissipation shown in the tangential force/displacement plots) was necessary for the formation of the TTS, which usually occurred within the first few cycles. Regarding its influence on the mechanism, it was noted that the depth of the TTS layer remains constant in the steady stage of fretting – i.e. the rate of formation of TTS below the surface is equal to the rate of removal of particles from the top layer [16]. The stick/slip regions of fretting contacts are also evident in the formation of the TTS, which occurs initially at the outer slip annulus.

### 2.1.4 Fretting wear parameters and their effects

#### 2.1.4.1 Amplitude

Although many parameters will affect the stick/slip regime, varying the amplitude will have the most obvious and direct effect. The various other parameters will lead to changes in the transition amplitudes.

It can be difficult to accurately determine the transitions between regimes since the amplitudes are small, and movement of the test apparatus may account for some of the imposed amplitude. A “system free criteria”, in which the effect of apparatus is
accounted for has been developed [8]. Despite the complex behaviour, an attempt has been made to relate the stick/slip transition amplitudes to basic material properties [32].

For general sliding wear, the wear is usually found to be proportional to sliding distance, allowing a specific wear rate to be determined (wear per unit sliding distance). However, many experimental studies have found a variation in this specific wear rate for different amplitudes. Warburton claimed that this renders the concept of a specific wear rate obsolete for fretting [44].

Zhang et al. found a critical amplitude of 70 μm, below which little wear occurred [52]. Increasing the amplitude beyond this led to a rapid increase in wear, reaching a constant rate (per unit sliding distance) at 300 μm. A similar transition amplitude and wear rate variation was found by Ohmae and Tsukizoe [47]. A review of fretting by Waterhouse concluded that the specific wear rate increased with amplitude from 10 to 150 μm [49], whist Bill found an increasing specific wear rate up to 50 μm [13]. Ghasemi et al. also identified a non-constant specific wear rate – increasing the amplitude five fold (from 20 to 100 μm) increased the wear per cycle by a factor of 100 [69].

However, Stowers and Rabinowicz found that the specific wear rate was independent of amplitude and thus claimed the study of fretting is no more complex than that of sliding wear, and that Archard's wear law is valid [35]. They claimed that the reason for observed differences is entirely due to a loss of movement between the surfaces compared to the measured amplitude, i.e. the actual slip distance at the interface is lower than that measured.

The reason for a possible lower specific wear rate for smaller amplitudes is uncertain [19], and could be due to greater debris retention reducing metal contact and accommodating the movement. The greater debris retention for lower amplitudes was visibly observed in experiments by Baker and Olver [20]. Another possibility suggested is that a smaller area is traversed by the stresses and thus the dislocation density is reduced leading to less damage by delamination. There is also less chance of adhesion [46].
There is also some suggestion of a minimum amplitude for fretting, below which wear does not occur, possibly 0.5 μm [28] or 0.05 μm [71]. However, under such small amplitudes, it is difficult to be certain that the imposed displacement is not entirely taken up by elastic deflection of the specimen bulk or test rig – “false fretting” [57].

Since fretting fatigue is particularly damaging in the partial slip regime, the fretting fatigue life is minimum at the transition amplitude from partial slip to gross slip. The 5-30 μm slip range is reported as most damaging for fatigue by Yan [12].

A diagrammatic summary of the effect of amplitude is shown in Figure 2.5, taken from [6].

![Figure 2.5 - Effect of fretting amplitude (Δ) on specific wear rate and fatigue life (from [6])](image)

2.1.4.2 Load

Zhang et al. found an almost linear relationship between load and wear volume [52]. This is supported by the observation that the specific wear rate (which accounts for load) was constant for different loads [58]. Farrahi, however, found load to have little influence on resultant wear [53]. If no change in wear volume occurs, this does not necessarily mean no increase in damage – an increase in load can increase the volume of metal transferred between surfaces through adhesion [46].

The load will have a significant effect on the transition amplitudes for the stick/slip regimes, as identified in Figure 2.3. Bryggman and Söderberg [32] found a linear
increase in transition amplitude with load, although the transition to gross slip appeared to reach a limiting amplitude after which increasing load had little effect.

For low amplitude fretting in the stick regime, increasing the load did not affect the size of the wear scar area, but increased the size of the individual asperity contacts which had undergone plastic shearing [14].

One aspect of load that has not received much attention is dynamic loading. This is particularly relevant to spline couplings. Waterhouse stated that the hammering action produced if debris particles are regularly removed from a clamped contact can rapidly accelerate wear [49]. Dynamic loading was also found to immediately cause severe surface fatigue in pivots [72]. Tests by Pendlebury found that intermittent loading could increase wear [58]. When the contact surfaces were brought back together after separation, metal-metal contact was often recorded even when complete oxide bed separation had previously been present. Transverse displacement was one explanation for this, allowing metallic areas and asperity peaks to contact new locations.

2.1.4.3 Number of cycles

As already reviewed regarding the mixed fretting regime, the contact conditions might change during the course of fretting leading to changes of regime and wear rate. For a constant displacement amplitude, a fretting contact regime may change from stick to partial slip, or from partial slip to gross slip.

In the stick regime, the local asperities contacts may grow in size as they are plastically sheared, whilst any outer slip annulus grows and reduces the dimension of the stick region [12]. Thus the critical amplitudes for partial slip and gross slip gradually decrease, to a limiting value, with number of cycles.

Zhang et al. found a gradual reduction in wear rate with increasing number of cycles [52]. Lyons and Collins found that the normal approach of the two bodies under fretting action could be adequately described as being proportional to load and accumulated sliding distance, such that the number of cycles did not affect the rate of approach [23].
2.1.4.4 Materials

The form of the damage can vary slightly with differing materials. Yan found sliding wear grooves parallel to the direction of sliding for copper whereas a wavy appearance was seen in steel, with lines perpendicular to the direction of sliding [12]. The latter was attributed to the oxide layer acting as thixotropic viscous liquid that is pushed back and forth.

Harder materials are often believed to be better at limiting wear, although some have found increased hardness to have little or a detrimental effect [55]. An experimental study by Dong showed a change in wear scar appearance with hardness; harder steels showed abrasive scratches with more cracks and delamination whilst softer steels showed greater plastic deformation. The harder steels had the best overall wear performance [55].

Kayaba found the wear to be dependent on hardness only when there was no black Fe$_2$O$_3$ wear debris present [73].

The combination of materials will determine the strength of adhesion, and thus wear rate [33], particularly in the initial stages. Materials which are employed to resist corrosion can be particularly susceptible to fretting if the corrosion resistance is a result of an oxide layer [28]. Non-metals will also experience fretting wear, although the fretting wear of ceramics can differ significantly from that of metals [74].

2.1.4.5 Lubrication

Given the low sliding velocities in fretting, boundary lubrication is the most important aspect for lubrication [21, 30]. Additionally, the ability of an oil to prevent oxygen access is considered beneficial [21, 49], although Shima et al. claim that this has a detrimental effect by reducing the formation of protective oxide layers and oxidised debris [31]. The latter was supported by observations that, at low amplitudes with high viscosity oil, there was significant tearing and roughening of the surface and friction was greater than for a dry contact.

The viscosity of an oil affects both the boundary lubrication and oxygen shielding aspects. A higher viscosity reduces oxygen diffusion and thus may offer greater
protection through oxygen shielding, but also hinders penetration into the contact and thus reduces the effectiveness of the boundary lubrication [31].

A study of the respective roles of boundary lubrication properties and viscosity found that boundary lubrication properties were more influential than the viscosity [75]. For oils with good boundary lubrication properties, a low viscosity was best; for oils with poor boundary lubrication properties, high viscosities were better.

The displacement amplitude may alter the effect of a lubricant. Shima et al. found that the wear depended on the friction coefficient [31]. At larger amplitudes, oil reduced the friction and wear, but at lower amplitudes (around 5-10 μm), the friction was greater than for dry fretting.

A recent, comprehensive review of lubrication methods in fretting wear was provided by Zhou and Vincent, covering solid, liquid and grease lubricants [76]. The effects of lubrication on fretting wear were said to vary considerably and be quite complex. For example, the solid lubricant MoS₂ was reported to cause the wear rate to fluctuate. The initial benefit is lost as the lubricant degrades but, in doing so, lubricant particles are formed which improves wear temporarily before being subsequently ejected from the contact.

The retentive nature of fretting contacts has been discussed in relation to the prevention of debris escape. The closed contact also makes it difficult for lubricant to penetrate the contact. The geometry of the contact will influence this, and has been investigated by Imai et al. [21]. Large, flat specimens (100 mm x 100 mm) were used to represent typical fretting situations more accurately. The addition of a fine mesh of grooves allowed the lubricant to penetrate the contact more easily and led to a dramatic reduction in wear.

If the lubricant is circulated, it may wash away debris [77]. This may or may not be advantageous, depending on the role of the debris and the amount of protection offered by the lubricant.
2.1.4.6 Geometry

The geometry of a contact can alter the ease with which wear particles can escape, and thus change the wear rate [78]. For lubricated contacts, the geometry will also affect the ability of the oil to penetrate the contact as discussed in the previous section.

Another effect of geometry is proposed by Bryggman and Söderberg [32]. They suggest that the critical amplitude for stick/slip transitions is greater for point contacts than flat contacts, due to the size of individual asperity contacts. With a larger contact, the number of high asperities is increased and thus the size of each individual asperity is reduced and less able to accommodate elastic strain without slip.

A comparison between cylindrical pads and flat pads showed a difference of initial crack orientation for experiments carried out under fretting fatigue loading conditions [79].

2.1.4.7 Frequency

The effect of frequency is complex. Bryggman and Söderberg found that the stick/slip transition amplitudes initially increased with frequency, but increasing the frequency further resulted in a drop in critical amplitudes [32]. The complex behaviour was attributed to the two competing effects of increasing frequency. The greater temperatures produced would soften the material, whilst the increased strain rate would have the opposite affect. The frequency effect was particularly evident for specimens of niobium, for which material properties are particularly sensitive to strain rate [14].

For experimental studies, accelerated wear could be achieved by running at high frequencies. However, frequency can alter the wear over a given number of cycles [80], particularly for partial slip conditions [81], and so accelerated tests may not give representative results. This is particularly true if a change in stick/slip regime occurs.

2.1.4.8 Temperature (ambient)

The temperature can alter both the material properties and the role of oxidation. Although temperature rise due to fretting appears to be small in most cases, the ambient temperature will have an affect. It is generally found that increasing temperature reduces wear [49]. A rise in temperature from 20° to 300°C has produced
a ten-fold reduction in wear [82]. There may also be a transition temperature at which a rapid reduction in wear can occur due to a change in mechanism.

Aside from the oxidation of the surface, the oxidation of the debris will alter. Kayaba reports that the oxidation state of the steel debris is \( \alpha-\text{Fe}_2\text{O}_3 \) below 200°C, \( \text{Fe}_3\text{O}_4 \) for 200-570°C, and \( \text{FeO} \) above 570°C [82]. It is suggested that the transition in wear is due to \( \text{Fe}_3\text{O}_4 \) offering better protection than \( \text{Fe}_2\text{O}_3 \). Hurricks found a similar reduction in wear with temperature, although above 500°C, the wear rate began to increase once more [65]. This was attributed to the formation of \( \text{FeO} \) rather than \( \text{Fe}_3\text{O}_4 \).

Bill [66] found a marked increase in the fretting wear of titanium for increasing temperature up to 540°C. Increasing the temperature further led to increased oxidation, which produced a protective film, and thus the wear volume fell with increased temperature from the maximum at 540°C.

Fretting has also been studied at temperatures as low as 77K [17].

### 2.1.4.9 Environment

The environment can affect wear in several ways. Under most circumstances, a non-oxidising environment is beneficial [49, 66]. The environment can alter the adhesion between materials and thus initial wear rates [33]. Increasing humidity has been reported to reduce fretting wear [49]. Humidity (or moisture) may also affect the movement of debris [83]. Fretting wear rate may depend on ambient pressure [84] and may be changed by an airflow at the interface (through changes in wear particle movement) [54].

### 2.1.5 Fretting wear palliatives

If fretting wear is found to be a problem for an application, several remedial approaches may be taken: improve material properties, reduce slip or reduce friction. A contradictory aspect between the latter two methods was identified by Zhou and Vincent [76]. A reduced friction coefficient is beneficial, but this may result in increased slip, which is harmful. The choice of increasing friction coefficient or reducing friction coefficient was found to depend on geometry and stick/slip regime by Chivers and Gordelier [85] (although fretting-fatigue experiments were used).
2.1.5.1 Design

Careful design of a component may help reduce fretting damage. For example, an analysis of ball bearings showed that an increased groove radius could reduce fretting wear, although possibly introducing detrimental side effects [86].

Optimal design of pivots was investigated by Peterson [72], although the main conclusion was simply to design to minimise slip.

Other suggestions for reducing fretting wear include eliminating stress concentrations that cause slip, increasing pressure through a reduction of area, and using small non-metallic inserts (e.g. rubber or PTFE) to separate the surfaces [77]. Even inserting paper between the surfaces may reduce wear [5].

2.1.5.2 Materials and lubrication

The role of lubrication and materials was reviewed in Sections 2.1.4.4 and 2.1.4.5. Careful selection can help reduce fretting wear. It is also important to determine the regime and mechanism by which failure occurs to enable the correct selection of material [72]. This must be done with care. Meng and Ludema claimed that it is a common and erroneous assumption that if a particular wear mechanism can be identified in a system, then a material to resist that mechanism will help [87]. It was stressed that the choice of material itself brings the wear mechanism into the system.

The recent review of lubrication in fretting [76] has identified appropriate lubrication for different circumstances. An important conclusion was that solid lubricants are better for partial slip conditions and liquid lubricants are better for gross slip conditions. Sato et al. suggested combining solid and liquid lubricants [88] – solid lubricants were found to offer good protection initially but poor long term performance, whilst liquid lubricants were found to offer poor initial protection which later increased (perhaps through improved feeding following surface roughening).

2.1.5.3 Coatings and surface treatments

Bill investigated various treatments and coatings for steel, finding silver plate to be one of the most effective [83]. The effect of coatings may be down to the changes in hardness [78]. The effect of a coating depends on temperature [78], and humidity [83].
A more recent and comprehensive study of surface modifications was carried out by Fu [89], and a selection method was presented. The beneficial action of these methods was said to be due to residual compressive stresses, reduced friction, increased hardness, surface chemistry, or increased roughness. Fu also found that some techniques are beneficial for wear but may decrease fatigue life. Similarly, Carton et al. [90] found that a coating may lead to a transition from partial slip to gross slip and thus reduced cracking but increased wear.

The benefit of coatings may be short lived, as they will eventually wear off and the conditions revert to that of the bulk material [90, 91]. On electrical connectors, silver coatings were seen to flow and thin over time [92], reducing their effectiveness.

### 2.1.6 Fretting fatigue

Under conditions of fretting, the fatigue strength of a component may be reduced by 70-80% [77]. The fatigue endurance limit was also reduced by 60% under fretting conditions for experiments of Pape and Neu [79]. A comprehensive documentation of fretting fatigue has been provided by Waterhouse [93].

The mixed fretting regime and partial slip conditions are accepted to be most damaging for fatigue. The minimum fatigue life is likely to occur at the transition amplitude from partial slip to gross slip, when the cyclic shear stresses are greatest. Faanes analytically predicted the stick/slip region for a fretting contact, which varies with friction coefficient [94].

Hoeppner and Goss revealed that a critical threshold of fretting damage was necessary for a reduction in fatigue life [95]. If the fretting contact was removed from a test specimen before this damage, then no loss of fatigue life was seen. If the threshold damage had been reached, then removing the contact would not stop the reduction in fatigue life. This threshold has been investigated further [96], including a study of the effects of materials.

As a fatigue crack propagates further into the material, the tip moves away from the influence of the contact stresses and the bulk stresses act alone. The stress amplitude acting on the crack tip may then fall below that necessary for propagation so that the
The crack remains dormant [97]. The details of the fretting contact become less relevant once the crack has propagated a short distance [79].

The fretting contact will therefore most significantly affect the initiation of fatigue cracks rather than propagation. The initiation stage has been studied by Fellows et al. [98], and the shear stress amplitude was deemed to be a good prediction criteria.

Analysis of fretting fatigue crack propagation has been attempted using basic linear elastic fracture mechanics [18, 99, 100], and crack initiation criteria [15]. The local contact stresses and the direction of the crack, which is initially strongly influenced by these local stresses, must be taken into account for the initial stages of propagation [99]. The growth of the crack in the bulk material can be determined through normal fracture mechanics using the Paris equation, but the initiation accounts for most of the component life [100].

Methods of reducing fretting fatigue are similar to those for reducing other types of fretting damage, and so there are many different and sometimes contradictory suggestions. Bill found that changes that improved fretting wear, were generally also good for fretting fatigue [13], although a correlation was difficult in the case of coatings. Gordelier and Chivers [101] reviewed various different approaches to reducing fretting fatigue, and noted that greater wear can lead to the removal of wear cracks and actually improve fatigue life.

### 2.2 Visualisation of wear

The use of a transparent specimen to visualise a contact has been used by some investigators, particularly in respect of assessing the behaviour of wear particles [61]. Comparing results for transparent specimens and other materials could be hazardous given the variation in results seen throughout the study of fretting, but the use of sapphire and steel specimens has been found to produce the same type of debris and the same transition from fretting to reciprocating sliding as for identical conditions with steel-steel contacts [20].

Chalk and glass has also been used as an effective method of showing particle behaviour [45, 62, 102]. Particle migration was found to depend on location within contact, particle size and shape, mechanical parameters, and obstacles generated by
compacted particles [45]. Play and Godet observed two distinct regions in a sliding contact [102]. In the entry zone, asperity contact caused wear particle detachment; the quantity of wear particles increased further along the contact until, at a distinct boundary, a build up of particles generated a full third-body bed which separated the chalk and glass bulk surfaces.

Experiments of unidirectional sliding [103] showed a real contact area and a transitional contact area, in which the gap between surfaces was small enough for debris to plough the surface.

A glass section was used to study the particle behaviour for an oscillating bearing by Berthier and Play [61]. Large aggregates of debris were collected during the motion. The geometry of the contact was also altered by the debris movement, as debris gathered in distinct locations. An intermittent debris ejection from the centre of the contact was seen, with cracks formed near areas of particle build-up.

Steel on glass or sapphire specimens have also been used for fretting studies. A critical amplitude for gross slip of 3 μm was found for a steel ball fretting on a glass surface [104], below which debris was formed around the edge of the contact and cracks in the glass were seen, typical of partial slip. Baker and Olver investigated the effect of amplitude and lubrication on wear particle trapping within a fretting contact using a steel ball-sapphire disk arrangement [20].

Visualisation of a polymer-steel contact revealed how the kinematics of the motion may alter once wear had occurred [60]. Under an oscillating twisting action, the wear debris escaped rapidly from the contact around the entire periphery whereas, under an oscillating rolling motion, particle ejection occurred from the side and at a reduced rate.

2.3 Wear models

Wear models are required to explain how wear occurs and enable predictions of wear to be made. A key objective is to produce quantitative, predictive equations that can be used to forecast the amount of wear for a particular situation.
A detailed review of the many equations found in the literature was carried out by Meng and Ludema [87]. Over 300 equations were found in over 5000 papers, but no consistent or universal set of equations could be found which would be applicable for general use. There were over 100 different parameters used for general sliding wear, i.e. over 100 factors affecting wear, and over 600 different names for these parameters.

Equations were empirical or based on theoretical contact mechanics. The Archard wear equation was highlighted as a predictive wear equation. The Archard wear equation is:

\[ W = K_s \frac{P}{P_m} \]

where \( W \) is the wear volume; \( s \) is sliding distance; \( P \) is the applied load; \( P_m \) is the flow pressure (which is equivalent to hardness); and \( K \) is a constant. The value of \( K \) could be attributed to various factors, and is known as the wear coefficient. This must be determined from experimental wear tests. This equation is commonly used and can be found in various forms. Values of the wear coefficient for different tribological situations can be found in various handbooks.

The applicability of the Archard wear equation to fretting wear was discussed with reference to the sliding amplitude in Section 2.1.4.1. Lyons and Collins [23] investigated failure prediction methods for fretting and found that the normal approach in fretting could be modelled as proportional to normal load and accumulated sliding distance for any one test condition.

It may not be necessary to accurately quantify the wear predicted, but a common requirement is to obtain a parameter that can indicate the comparative wear for various conditions. This is required for the current study of spline wear. The product of shear stress and displacement amplitude has been used as a wear criteria to optimise the design of ball bearings [86], the results of which were experimentally confirmed. A shear stress x relative slip parameter was also proposed by Ruiz et al. for fretting-fatigue experiments [105]. These may be related to the Archard wear equation since the shear stress is linked to the normal load by the friction coefficient.
A distinction must also be made between wear (material damage) and durability (acceptable component life) [106]. The type of damage that can be allowed must also be considered for practical applications. For example, the formation of an oxide layer may assist in the reduction of wear, but the oxide prevents metal-metal contact in electrical contacts experiencing fretting and is therefore unacceptable [28].

Most wear laws/equations that are used for predicting wear, including the Archard equation, are based on simplistic two-body analyses. In reality, particles between the surfaces including those detached from the bulk due to earlier wear, affect the wear behaviour. Complete and rigorous modelling of fretting wear must account for all aspects of the wear process including oxidation, third body behaviour, the subsurface material transformation, and the basic mechanical aspects of material removal and cracks.

### 2.3.1 Third body approach

The importance of wear particles in fretting wear was briefly reviewed in Section 2.1.3.1. Once the extent of the particles becomes appreciable, the simplified two body asperity contact models used for many wear laws and proposed mechanisms is inappropriate. The action of the wear particles becomes a critical component of the contact and should be included in any theoretical analyses. This has been used by Godet and others for an approach to wear study in which the particle behaviour is the defining aspect of wear – the “Third Body Approach” [38]. (The two bodies in contact are the “first bodies”, and the interfacial layer of lubricants and detached wear particles is the “third body”.) A “triplet” which consists of the system or mechanism, the first and the third bodies must all be accounted for [106].

This third body approach is an attempt to provide a globally coherent, mechanically determinable method of approaching wear modelling [38]. It applies many of the same ideas as found in the thick-film oil lubrication theories that are commonly used. The third bodies have load carrying abilities, and the detachment and movement of these third bodies are critical [107]. Godet reported that third bodies have a load carrying capacity when there is sufficient quantity to give proper first body separation and that this requires that the valleys of the surface roughness (Abbot’s volume) are
filled. Visualisation of different systems was used to determine the various third body functions.

This third body approach distinguishes between wear and particle detachment [108]. Particle detachment generates third bodies, which continue to play a role in the contact. A particle was said to be a bona-fide wear particle only once it has escaped permanently from the contact region. The wear process could therefore be considered a particle source and sink flow problem [57], in which the source is particle detachment from first bodies and the sink is particle ejection. Berthier et al. claimed that, contrary to what is usually believed, wear is governed by the sink. This is because sufficient particle detachment will occur to generate the required amount of third body.

Fundamental to the third body approach is the idea of velocity accommodation mechanisms. The fretting movement between two contacting bodies can be accommodated by both the first and third bodies through various means, which are listed in [57]. These can be used to explain the various forms of surface damage that occur. The velocity accommodation mechanisms have also been shown experimentally [109].

Since the third body behaviour is a fundamental aspect of this approach, the movement of debris within and out of the contact is important. The methods of particle motion were discussed by Godet et al. [106]. The particle movement can depend on contact shape, machine dynamics, the particle rheology and the boundary conditions [109]. Unfortunately, modelling these from first principles would require many significant advances in fundamental knowledge of the wear process [106].

Analytical formation, readhesion and loss parameters for wear particles in fretting were suggested by Pendlebury based on a probabilistic approach [64]. The geometry and amplitude factors of debris trapping, which have been seen to be vital, were included.

The motion of wear particles in contact interfaces has also been the subject of analytical reasoning and computational simulation. The flow behaviour of particles, particularly solid lubricant particles, has been analysed and computer simulations performed [110-114]. These analyses, however, generally assume a relatively thick
bed of particles, as would occur in the steady state of fretting wear. These methods have not been reviewed further since accurate modelling of the initial stages of particle movement in contact with the first body surfaces is not available, and the effect of the particle behaviour on wear can not be determined. This is discussed further in Section 2.6.

2.4 Spline couplings

2.4.1 Couplings and misalignment

A range of flexible couplings exists which permit the connection of shafts with varying amounts of misalignment [2, 115]. These include gear couplings, universal joint couplings, quill shafts, and elastomeric couplings; each type has its own benefits and disadvantages. Although not necessarily designed for accommodating misalignment, spline couplings can operate satisfactorily with misaligned shafts and have numerous benefits of their own (Section 1.1). They are particularly effective for high power applications – Dudley claimed that a spline coupling with involute teeth will transmit more torque for its size than any other type of coupling [116].

Spline couplings have some similar features to gear couplings, in which two sets of gear teeth mesh with a common outer sleeve that connects the two shafts. Spline couplings are, however, simpler components and yet find a range of uses in even some of the most advanced industries – Kececioglu and Koharcheck reported that over 200 spline connections were used in some US Navy aircraft [117]. According to Ku and Valtierra, gear couplings may usually operate at much smaller angular misalignments and be more heavily loaded than spline couplings [118].

Neale et al. used a rigid body analysis to look at the contact and moments on a misaligned gear coupling [115]. The teeth that bear most of the load were shown to be at an angle, equal to the pressure angle, from the axis of misalignment. The frictional forces on these teeth were reported to be in the axial direction. The variation in sliding speed was also compared for teeth at 90° to one another. The maximum bending moment that could occur was formulated from this analysis.
Shaft misalignment causes a multitude of problems and is the subject of several volumes [2, 115]. Despite the advantages of proper shaft alignment, Piotrowski estimated that 99% of rotating machinery is misaligned [2]. This is not purely due to poor manufacturing or assembly – changes in shaft alignment occur during operation and as a result of external loads. Piotrowski also identified a difference between “shaft misalignment” and “coupling misalignment”; in this case, however, the difference between the two involved possible discrepancies between shaft centre and centre of rotation (i.e. two shafts may be apparently perfectly aligned, but rotate about different axes because of an incorrect bore or a bent shaft). The definition of misalignment given was “the deviation of relative shaft position from a collinear axis of rotation measured at the points of power transmission”. In spline couplings, this is difficult to quantify since the power transmission may occur along the entire length of the spline teeth. The role of shaft flexure was discussed as a separate issue.

2.4.2 Design of spline couplings

Both involute and straight splines can be found, although involute teeth appear to be most common. The involute profile is reported to be preferred because it provides self-centring capabilities and can be manufactured using gear cutting tools [119]. Several international and national standards exist which define sizes and forms of spline teeth. Some basic formulae and charts that allow selection of appropriate designs can be found in various handbooks. For example, for SAE straight-sided splines, the torque capacity (per inch facewidth) is given by $T = 1000NRh$, where $N$ is number of teeth, $R$ is mean radius and $h$ is tooth height [119].

The basic procedure for design of involute spline couplings was presented by Dudley [116, 120]. The size of the spline coupling for various applications could be determined from a graph of operating torque vs. required pitch diameter. For fixed splines, a facewidth of one third the pitch diameter was said to give the same shear strength for the teeth as for the shaft, but there was no real evidence or theory as to what would be the optimum facewidth for flexible couplings. The choice of number of teeth was based on cost and ease of manufacture – and was reported to have “no appreciable effect on tooth stress”. Five different tooth forms were compared (Figure 2.6), each of which have various benefits of alignment and manufacture. Three types
of fit (major diameter, side bearing, minor diameter) were also compared, the
differences again being the relative ease of manufacture and centring abilities.

Figure 2.6 - Common spline tooth forms [120]

An elementary stress analysis procedure was presented in [116] to assist in selecting
correct spline sizes. This was based on the shaft stresses of the male spline shaft
section, the shear and compressive stresses of the teeth and the bursting stress of the
female spline shaft section, together with some modification through the use of
nominal wear life, fatigue life and application factors. Spline stress calculations used
currently may be based on the same rudimentary analysis [121].

2.4.3 Wear of spline couplings

The effects of spline coupling misalignment were analysed by Buckingham in 1961
[3]. Both straight and crowned splines were considered. It was claimed that, with
misalignment, the contact shifts from full contact on all teeth to contact on two teeth
only. For rigid, uncrowned teeth, this would occur on the edges or corners alone.
Buckingham described how elasticity would bring more teeth in contact and lead to a
variation in load intensity from the edge of the tooth – this was investigated as being
equivalent to a wedge of varying thickness. The misalignment was said to cause a
clearance between teeth, which was calculated assuming rigid bodies. The minimum
backlash necessary to prevent “cramping” (back face tooth contact) could be obtained
from this. The separation (or not) of adjacent teeth would depend on the relative
magnitude of the elastic deflection and this clearance. This is demonstrated in the
discussion of Chapter 5.

Buckingham also provided a basic method of determining the maximum coupling
load, based on the wedge analysis, that involved summing the allowable loads for
each individual tooth. The tooth load was reported to go from a maximum to zero and
then back to the maximum. Finally, it was claimed that a small a number of teeth as possible should be used and the facewidth should be kept to a minimum.

Wear of spline teeth due to fretting action was experimentally studied by Ku, Weatherford and Valtierra [25, 118, 122, 123]. Wear was said to be inevitable in spline couplings if some misalignment is present [118].

The tests used a rig in which the inner spline was gyrated around at a known misalignment, whilst the coupling itself was non-rotating. Spline wear was recorded as the combined wear depth of both splines (based on a normal approach method) by monitoring movement of the load arm. Splines were of diameter 5/8 inch. and facewidth half-inch. The applied torque was 350 in-lb (approx. 40 Nm) and the speed was 4400 rpm.

For unlubricated splines, wear was seen to begin immediately and proceeded at a virtually constant rate [25]. The effect of lubrication with grease was highly dependent upon the type of grease used. In some circumstances, an initial period ("induction") of very light wear occurred, before a sudden change to the high wear rate of the unlubricated splines. For a different grease, the wear started immediately and occurred at a faster rate than the unlubricated condition. This was attributed to the grease trapping wear debris within the teeth contacts and causing accelerated abrasive wear. Roughening of the surface was found to prolong any induction period [122].

Solid lubricant films (MoS$_2$) gave no wear protection – wear rate was identical to unlubricated splines. Liquid lubrication consistently improved wear performance; differences between various oils were attributed to differences in boundary lubrication properties.

Silver coating gave little improvement in wear, although a 0.001 inch thick coating of electroless nickel was found to give a 25 fold improvement of wear life [118]. However, once these coatings were worn through, the wear proceeded at the rate of unprotected splines.

The observed effect of number of teeth is shown in Figure 2.7 [118]. The lower tooth numbers were said to "relieve the cramped contact condition, thereby reducing the tooth contact load and thus wear".
Interestingly, the wear of crowned splines was greater than for uncrowned splines [118]. This was explained as being caused by the more counterformal contact. The use of crowning for misaligned splines was, however, reported to generally reduce fatigue and tooth fracture. Ku and Valtierra also added that “tooth crowning tends to improve load sharing among the teeth, [while aggravating] the counterformal contact” and that it is a balance of the two that determines the outcome. In a discussion to the paper, Bowen, emphasised that the results of crowning are peculiar to the conditions tested and that, in general, crowning can be very beneficial in reducing tooth wear. A contact ratio and “critical misalignment angle necessary to obtain optimum lubricated wear conditions” were discussed and said to depend on the crown radius, pitch radius, misalignment angle, torque and stiffness.

Increasing misalignment angle between 0.17° and 0.34° was found to significantly increase wear — (0.0035 inch after 330 hours c.f. 0.012 inch wear after 50 hours respectively). The effect of misalignment angle was described as “overwhelming”.

A discussion by Grassi emphasised that the splines used in the experimental studies were small and that this must be taken into account when considering larger splines [118].

The effect of environment was looked at by Weatherford et al. [122]. Operating in nitrogen gave a reduced wear rate and a noticeable incubation period even without
lubricant. However, fracture of the spline shaft section occurred which was blamed on increased friction. The addition of oxygen and moisture was found to have complex results. Moisture alone and oxygen alone were detrimental, but the combination of moisture and oxygen was not as bad. Various different oils were also compared for their wear benefits. Antioxidants in particular were found to be helpful in improving the induction period.

Weatherford et al. also found that decreasing the misalignment amplitude led to reduced wear but caused fracture of the specimens [122]. A possible explanation for this was that the reduced relative motion reduced the exposure of the tooth surface to oxygen and thus increased friction, as for running in a nitrogen environment.

Mathematical models for distribution of wear life, based on the work of Ku, Valtierra and Weatherford, were developed by Kececioglu and Koharcheck to enable predictive evaluation of spline life [117]. This approach relies on many experimental tests with similar operating conditions to those of the intended application.

Newley claimed that abrasive action was the cause of wear of spline couplings, particularly three body abrasion [124]. The lubrication and oxidation behaviour was examined; it was reported that fretting wear rate was proportional to the activity of oxygen in the oil, and that a greater proportion of the harder \( \text{Fe}_2\text{O}_3 \) oxide was found in the debris when higher wear rates were observed.

A spline coupling study has been carried out more recently, involving Westland Helicopters, Rolls Royce, Computational Mechanics, Nottingham University and Imperial College, supported by the DTI [125]. The need for a method which could enable the development of spline couplings with greater torque capacity and lower wear was identified. A boundary element package was developed to enable contact analyses of three-dimensional objects with the complexity of spline couplings [126], and this was validated with photo-elastic studies of spline couplings [127].

A similar test rig to that used by Ku et al., albeit on a larger scale, was also used for further spline wear tests. Additional test were performed on a “flat-on-flat” test rig; this used contact specimens of size similar to that of a spline tooth and with an arced reciprocating motion, similar to what was expected to occur between spline teeth.
The study achieved several advances that enable better analysis of spline couplings, but did not develop a system or methodology which could be used to design splines.

Variations in torque and axial load are also a source of cyclic stresses, which can lead to fretting fatigue. A finite element (FE) analysis of a helical spline coupling was performed by Hyde et al. [128] – bending moments were not considered. To facilitate easy experimental testing of such conditions, a simplified test specimen that was representative of a spline tooth was required. The FE analysis was used to compare the slip and load conditions with various test specimen geometries that were designed to represent such a tooth. This specimen could then be used in standard fretting-fatigue type experiments. The specimen obtained could accurately represent the surface conditions, although there were differences in the subsurface stresses.

2.5 The boundary element method for elastic analysis

2.5.1 The boundary element method

The boundary element method is a numerical method that enables the determination of a variable within a domain, through solution of the governing partial differential equations over the boundary of the domain. The technique can be used for scalar potential variables such as temperature or vector quantities as in the case of elastic stress analysis, for which it is used here. In elastic analysis, the quantities are the tractions and displacements for which the governing equations are compatibility and equilibrium [129].

Many books are available which describe the theories behind the various methods in detail [129, 130]; this section reviews briefly the theories and application of the boundary method that are relevant to the elastic analysis of bodies as used in this computational study of spline couplings.
2.5.2 Fundamentals of the boundary element method for elastic analysis

Finite element analysis is a well-known and conventional tool used by engineers to analyse the elastic response of a body to imposed boundary conditions. The entire body volume is discretised into a number of elements, and the response of each element is approximated in a simplified way [129]. The boundary element method is less commonly used, but may offer some advantages in certain situations. For example, if there is a high volume to surface ratio, then the boundary element method may be a better technique [129]. In the boundary element method, only the surface of the body is discretised. Two types of boundary element method can be found: indirect and direct methods; the direct method is used in the software application selected and is reviewed in this section. In each case, an integral equation over the boundary is formed which must then be solved.

The direct method is based on the Maxwell-Betti reciprocal theorem which states that the sum of displacements due to a set of loads, A, multiplied by the loads of a second load case, B is equal to the sum of displacements due to load case B multiplied by the loads of load case A.

The method also relies on the knowledge of a “fundamental solution”, the solution of which is known. For the case of elastic stress analysis, the fundamental solution used is that of a point force somewhere in the component – the displacements and tractions that result from this point force are known [129 Appendix C]. The fundamental solution is substituted for a load case in the reciprocal theorem, yielding an integral equation in which the only unknown terms are on the boundary. (The full derivation of this equation can be found in [129] and [131].) This boundary integral equation is solved by discretising the boundary into a mesh of elements and using a quadratic (or other) approximation of the variables for each element. This produces a matrix of simultaneous equations, which can be solved using standard techniques, giving the solution to the problem over the boundary. The reciprocal theorem and fundamental solution can then be used to determine the results at any point within the body (“internal points”).
2.5.3 Contact analysis with the boundary element method

The software chosen to analyse the behaviour of spline couplings was BEASY [132]. This software was developed for contact analysis of spline couplings during the spline coupling study reviewed in Section 2.4.3 [125]. The software uses the boundary element method described in the previous section.

The use of the boundary element method for contact problems requires additional algorithms to describe and define the contact interface. The two main techniques are the penalty function method and the constraint method. The constraint method is used in the Beasy implementation of contact analysis [126] and is described here. This method allows node-to-surface configurations, but a node-to-node algorithm is employed in the version of Beasy used for the spline analyses. A pair of corresponding nodes defines each point of the contact interface, one node on each surface.

Each nodal pair can be out of contact (open), in contact and slipping (slip in frictional contacts, sliding in frictionless contacts), or a "sticking" contact. The constraints for displacement and traction boundary conditions are then appointed according to the contact type.

An iterative procedure is required to solve a contact problem [131]:

1. A standard boundary element analysis of the complete structure.
2. The contact configuration at each node is examined to determine if the type of contact has changed.
3. The contact configurations are changed if necessary and the process repeated with the new constrain equations. If no change in contact configuration, the final solution is reached.

To improve accuracy and convergence, the load can be applied as a series of increments (which in Beasy are predetermined and adjusted according to the rate of convergence [126]). For each incremental load, the iterative procedure is performed to obtain a solution.
The contact analysis routine was verified with an elastic punch analysis [126]. Comparisons of results with photo-elastic studies also showed that the software gave a good representation of the stress states within spline couplings [126, 127].

Further details of the boundary element method used are given in Section 5.2.1.

2.6 Discussion

2.6.1 Frettin

Fretting is a complex subject and is not yet fully understood. There is still some disagreement on the basic definitions and effects of various parameters. Beard claimed that fretting literature was "encumbered with misconceptions, half-truths, and sometimes with downright errors" [36]. Berthier et al. presented and attempted to clarify many of these misconceptions [133].

The mechanism of fretting wear appears to initially be caused by predominantly adhesive action. Particle formation can be through this adhesive action or delamination. A second stage of fretting, following particle bed formation, occurs with a lower wear rate, although the mechanism is unclear. In experimental studies of fretting it is usually sought to accelerate the wear. The frequency and amplitude dependency of fretting makes this difficult. An increase in load appears to be the best method of accelerating wear, although if this involves a change in contact regime then results may not be acceptable.

Many factors affect fretting wear, and the geometry appears to be highly influential. When investigating a particular situation in which fretting is a problem, it is therefore necessary to replicate the geometry, kinematics and contact conditions as closely as possible. In particular, the stick/slip regime should be identified.

2.6.2 Wear modelling

It is difficult to develop predictive and quantitative models of fretting wear [64]. The commonly used Archard wear equation still appears to be the most appropriate method. Although various other parameters, such as frequency, will alter the wear coefficient, the equation appears to offer the best evaluation of wear for similar
conditions. There is, however, some doubt as to whether a constant wear coefficient can be used for differing amplitudes. For this study, the quantification of wear is not necessary, although would be helpful. A comparative technique is of more importance. This comparative approach is also unaffected by the variation in wear rate with number of cycles reported to occur in fretting.

The third-body approach to wear appears to be a valuable method of modelling wear, but requires more fundamental research. It is also necessary to consider the failure mechanism for the application of interest. For this study of spline couplings, the operation of a coupling with a full oxide powder bed is not particularly relevant. For a complete oxide bed to be formed, significant wear must already have occurred – the aim is to prevent significant wear. The spline coupling would be likely to need replacing if wear had progressed to such a stage. Moreover, the oxide beds would be cleaned away on inspection. Thus the presence of wear particles cannot be relied upon to prevent further wear. The wear following formation of an oxide bed (the steady-state) is important, but it is the initial stage that is critical.

Some computational simulation of particle behaviour is possible, but computational modelling of contacts on the molecular scale is “in its infancy” [4], and is not yet at a stage where it can successfully be utilised within this spline coupling research.

2.6.3 Spline couplings

The design of spline couplings is based on simplified analyses that do not allow full optimisation. The details of the tooth loads are not known which prohibits a more accurate determination of the stress state. Misalignment has been considered only with a rigid body approach and much simplified elastic representations. This can help explain some aspects of spline coupling behaviour but, again, does not allow optimisation or any assessment of wear.

Experimental wear tests have revealed some trends in spline coupling wear. Given the variation in wear results for fretting experiments, the results can only be used with confidence for similar sized coupling and operating conditions. In particular, the results for crowned splines have shown that generalisation would be inaccurate.
There remains a definite need to improve the knowledge of the workings of spline couplings and to develop methods to reduce the damage that occurs, both from fatigue caused by torque variations, and wear caused by misalignment.

2.6.4 Current research

This research is intended to establish, explain and predict the contact conditions that occur between spline coupling teeth. A more accurate and realistic analysis of spline couplings is required, beyond that of Buckingham [3] and Neale et al. [115]. This is provided by the boundary element method.

Research of fretting wear has produced many differences in reported behaviour. This makes predictions of fretting wear trends for spline couplings difficult and quantitative predictions of wear volume cannot be made. These differences also necessitate representative experimental tests, since comparisons between common laboratory tests and spline couplings are not sufficient.

This research does not attempt to add to the fundamental fretting studies, but instead apply the current knowledge to the contact of the spline coupling teeth. If improved methods of predicting fretting wear become available, the actual contact conditions of the spline teeth will still be required. Despite the problems of modelling fretting wear, and the lack of a full third body approach, the Archard wear equation can be used for an initial assessment of wear.

The fretting wear visualisation portion of this work is an acknowledgement that the wear particles produced will contribute to the wear mechanism. Replicating the motion of spline teeth, the results are used to judge if and how the specific motion of spline teeth affects the particle motion. Concurrently, the wear mechanism can be examined.
3
Experimental Study of Spline Couplings

3.1 Purpose of study

A controlled test programme of spline coupling wear is needed to determine the type of wear caused by misalignment, assess how the wear occurs and determine by which factors it is affected. This study is able to provide a qualitative and quantitative evaluation of spline coupling wear for various coupling designs and running conditions, and also show any other modes of failure which may occur. This can be used as a basis for suggesting possible wear mechanisms.

With the wear of spline couplings exhibiting the characteristics of fretting wear, it is necessary for the conditions of controlled experiments to match those of real-life situations as closely as possible, as explained in Section 2.1.4.6. The size of the contact area, the materials, loads and displacements must all replicate the conditions of real applications for the study to be a useful method of evaluation. For these reasons, typical laboratory fretting tests such as HFRR are not suitable. Instead, the tests carried out for this study use actual spline couplings, although modified in some respects.

In order to produce appreciable wear within a reasonable time period, it is necessary to accelerate the wear process. To achieve this, the spline couplings used for the tests do not have a full complement of teeth; only three teeth are present on the male component of the coupling. This results in an increased tooth load and thus an accelerated wear rate compared to that which would occur if all teeth were present.

The wear tests allow a quick evaluation of some design parameters and operating conditions. The wear tests are also needed in order to judge the effects of those parameters that cannot be considered by any computational or analytical methods being used in this research programme. Lubrication cannot accurately be evaluated computationally (merely by the specification of a coefficient of friction) but can be evaluated experimentally. This is an important aspect of the wear tests.
Similarly, wear debris particles are not modelled. The experimental study is needed in an attempt to determine whether, or how, the wear debris particles affect the wear mechanism and wear rate. The interaction of lubrication and debris particles is likely to be an important factor affecting wear of spline couplings.

The experimental spline coupling wear tests carried out for this research programme go some way to achieving all of the above. The main limitation is due to the lack of a full complement of teeth and is this is discussed in Section 3.4.

The experiments also allow an evaluation of the computational study. Computational analyses can be made of the wear tests and comparisons between the computational results and the wear test observations are of substantial benefit in judging how well the computational method can predict wear of spline couplings. This comparison is made in Sections 5.3.4 and 5.4.7.

3.2 Description of experiment

3.2.1 Test rig

The test rig used for the investigation was originally developed and used at Imperial College for its part of the DTI collaborative spline study [125]. The rig was used to evaluate the wear resistance of different coatings and surface treatments as well as different geometries and lubrication methods. Unfortunately, problems with the rig at the time meant that, although the different coatings could be evaluated, the wear pattern showed inconsistencies and the resulting data is unreliable. The rig is shown in Figure 3.1.
The rig simulates the motion that occurs due to a misalignment when the shafts rotate. In the test rig, the spline coupling does not rotate – instead the misalignment itself rotates around the coupling, generating the same form of motion. A schematic is shown in Figure 3.2.
The female component of the test specimen is bolted to the base of the rig. The male test specimen is attached to a gyrating conical shaft via a thin torsion plate through which the torque is applied. The torsion plate is loaded via cables and pulleys by two weight pans. The torsion plate is sufficiently flexible in the vertical plane to ensure that the variation in planar orientation caused by the misalignment is nullified and the torque on the coupling remains steady and constant. The upper end of the conical shaft is fitted to the centre of an offset disc by a ball bearing and a Y bearing (Figure 3.3). This disc is fitted within a fixed, upper disc such that the top of the gyrating arm is offset horizontally from the central spline coupling axis. This offset creates the desired angular misalignment between the male and female components of the spline coupling (Figure 3.4). The upper disc is rotated by an electric motor about an axis concentric with the spline coupling axis. As the disc rotates, the misalignment rotates around the static coupling producing the same motion between coupling teeth as would occur in a rotating coupling with static misalignment.
The torque can be changed by varying the load in the weight pan. The angle of misalignment can be varied by altering the magnitude of the offset at the top of the
gyrating arm. (The upper disc and lower disc are designed with non-concentric sections so that the offset can be changed by locating the discs at different angular positions.)

The entire coupling is encased in an oil container, allowing the coupling to be operated fully immersed in oil. Two methods of lubrication have been used in the test programme – "wetted", in which the spline teeth are smeared with oil before assembly and no additional oil is used, and "immersed", in which the coupling is completely filled with oil. In the case of immersed lubrication, the oil was continuously circulated through the coupling from inside, at low speed, to ensure that the teeth were submerged in oil at all times. The oil used in the tests was Castrol 5000, an ester based aeroengine oil to specification MIL-L-23699.

Previous tests carried out on the rig had shown a variation in wear pattern between individual teeth on the same spline. This was thought to be caused by an unintentional misalignment of the rig. For the rig to simulate rotating shafts with a constant misalignment, it is essential that the imposed misalignment remains constant as it rotates around the shaft – i.e. the centre of rotation of the upper disc must be concentric with the spline axis. On measuring this, the axes were found to be over 2 mm apart. This leads to a situation unrepresentative of any situation likely to be experienced by a real coupling. This "static" misalignment would, effectively, simulate a coupling in which one shaft was rotating around the other, as well as rotating in the usual sense. This misalignment was corrected and sections of the rig welded in place to prevent any further misalignment developing.

A second possible cause of static misalignment was also identified. The female spline is fixed to the base of the rig by eight bolts and it was these alone that located the coupling. However, the clearance boltholes in the test specimen do not locate the coupling with sufficient accuracy. This was remedied by the addition of a locating ring, which would fix outside the base of the female test specimen. Following these modifications, the axes of the coupling and disc rotation were found to be concentric to within 100 μm, corresponding to a static angular misalignment of less than 0.01°.
3.2.2 Instrumentation

Thermocouples are located at the top of the coupling to monitor the coupling temperature during operation, and near the motor to act as a safety cut-off in case of excessive loading. In addition to this, further instrumentation was added to monitor the forces generated and the tooth wear.

The angular misalignment of the coupling within the test rig results in a bending moment on the coupling. This bending moment was recorded by strain gauges on the conical shaft. This permits an estimate of the stiffness of the coupling by relating bending moment to angular displacement. For any given spline geometry, torque and misalignment angle, the bending moment will also be influenced by the friction between contacting teeth. Changes in bending moment observed during the test could reveal changes in friction coefficient, possibly caused by the existence of wear debris within the contact zone.

The bending moment at the top of the conical arm must be zero (or negligible) since the Y-bearing at the top of the arm allows rotation of the arm about this point. It was therefore possible to calibrate the strain gauges by monitoring the output for known loads applied at this end. The direction of the bending moment rotates with respect to the non-rotating conical arm as the misalignment direction changes. As the strain gauges only monitor the bending moment in one plane, the output of the strain gauges produces an alternating value. The bending moment output was therefore rectified before input to the chart recorder.

It has also been noted in some applications of spline couplings that an unexpected axial force may be generated. Strain gauges on the conical gyrating shaft allowed this axial force to be monitored during testing.

Visual observations of the worn spline teeth at the end of the test period provide qualitative results of wear and, to some extent, could be assessed to provide a quantitative evaluation of wear area. A more direct quantitative value for the wear depth was provided by LVDTs attached to the weight pans on either side of the torsion plate. As the teeth wear and material is removed, the male spline will rotate a small amount to maintain tooth contact. This motion is transferred to the torsion plate.
and results in a lowering of the weight pans that is monitored by the LVDTs. This provides a value for the “normal approach” of the spline teeth.

Some assumptions are made in the interpretation of the LVDT results. It is assumed that all teeth wear by the same amount and that the wear is such that the contacting teeth will be able to move towards each other. For example, even reasonably deep, small isolated pitted areas will not result in the teeth moving together if the surrounding area remains unworn. The material removed by the wear process may also remain in place, albeit detached from the bulk metal. In these cases, no wear will be recorded by the LVDTs as there is no approach of teeth. Alternatively, the wear debris may become oxidised and increase in volume, thereby pushing the teeth apart and producing an apparent negative wear. This could also occur if wear debris is dislodged from the location of origin and becomes attached to an unworn area. It is necessary to take these possibilities into account when interpreting the LVDT wear results.

### 3.2.3 Test specimens

The spline coupling test specimens used for the work have straight, involute spline teeth. They differ from normal splines in that the male component has only three teeth present, spaced equally around the circumference (only 18 and 30 tooth splines have been used). The male effectively has the remaining teeth cut away before use. This increases the load per tooth allowing an acceleration of the wear process at reasonable torque values. A male and female test spline is shown in Figure 3.5

![Figure 3.5 - Spline coupling test specimens](image-url)
The spline couplings are of a size comparable to many spline couplings in use in the aviation industry, and so meet the requirements of representative contact geometry for fretting wear tests.

A secondary benefit of the reduced number of male teeth is the possibility of re-using the female component. The female component is significantly more expensive than the male, but only three of the teeth are worn during each test. Changing the rotational position of the female for each test allowed different teeth to be brought in contact with the three male teeth. The test procedure does not cause any damage to the unworn teeth, and the temperatures experienced by the coupling are not high enough to affect the material; therefore, the re-use of the female does not affect the results.

The specimens used in this programme are made of hardened and tempered steel (4½% Ni-Cr-Mo 0.1%C). All couplings had a pitch circle diameter of 45 mm and a facewidth of 45 mm. Modules of 1.5 and 2.5 mm were available (which would represent 30 tooth and 18 tooth splines respectively).

### 3.2.4 Test procedure

Each test consisted of running the spline coupling for forty hours with the misalignment rotating at 720 rpm, producing approximately of 1 700 000 cycles. If the spline coupling appeared to fail within the forty hours (indicated by the LVDT measurements) the test was stopped immediately. Each test was carried out over a number of days, rather than forty continuous hours. The torque remained on the coupling to ensure that any wear debris was not dislodged after each portion of the test.

For each test, the temperatures, bending moment, axial force and LVDT readings were recorded on a chart recorder. As a precautionary measure, a cut-off was used to switch off the rig if the LVDT readings approached the limit of the LVDT range to prevent lost data. The cut-off would also activate if the temperature exceeded an unacceptable limit. At the end of each test the spline coupling was removed and the wear on the male teeth inspected.
3.2.5 Initial pre-test running of rig

Prior to the main series of tests, the rig and test procedure were verified in a number of short runs of the test rig. The tests previously carried out on the rig showed different wear patterns on each tooth. Following the modifications, the initial tests produced similar wear patterns on each tooth, confirming that the rig was operating as intended.

In the case of three-toothed couplings, the misalignment may be directed towards a tooth or a non-toothed section of the male component; this would not occur to such an extent if all teeth were present. The differences in bending moment as the misalignment moves between tooth and space could be felt by rotating the misalignment by hand. To produce consistent results between tests, it was necessary to ensure that the plane of measurement of bending moment was the same with respect to the three male teeth for each test.

The initial testing was also necessary to calibrate the rectified bending moment by simultaneously monitoring the direct output on an oscilloscope. The rectified output was set to record the maximum bending moment occurring during the rotation.

3.3 Results

A total of seven wear tests were carried out at various operating conditions and with different geometry splines. The conditions and spline geometry used are shown in Table 3.1.

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Module</th>
<th>Lubrication Method</th>
<th>Torque</th>
<th>Misalignment Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWT 1</td>
<td>2.5</td>
<td>wetted</td>
<td>935 Nm</td>
<td>0.30°</td>
</tr>
<tr>
<td>SWT 2</td>
<td>1.5</td>
<td>immersed</td>
<td>935 Nm</td>
<td>0.15°</td>
</tr>
<tr>
<td>SWT 3</td>
<td>1.5</td>
<td>wetted</td>
<td>935 Nm</td>
<td>0.30°</td>
</tr>
<tr>
<td>SWT 4</td>
<td>1.5</td>
<td>immersed</td>
<td>935 Nm</td>
<td>0.30°</td>
</tr>
<tr>
<td>SWT 5</td>
<td>1.5</td>
<td>wetted</td>
<td>935 Nm</td>
<td>0.15°</td>
</tr>
<tr>
<td>SWT 6</td>
<td>1.5</td>
<td>wetted</td>
<td>935 Nm</td>
<td>0.22°</td>
</tr>
<tr>
<td>SWT 7</td>
<td>2.5</td>
<td>wetted</td>
<td>750 Nm</td>
<td>0.30°</td>
</tr>
</tbody>
</table>

Table 3.1 - Spline wear test programme
3.3.1 Wear observations

3.3.1.1 SWT 1 (2.5 mm; wetted; 0.30°; 935 Nm)

Wear test SWT 1 was carried out at a nominal angular misalignment of 0.3°. However, following this test it was noticed that the rig design was such that the offset at the top of the gyrating conical shaft was not accurately fixed. The locating of the Y-bearing on the lower rotating disc was obtained solely by bolts, which allowed a small but significant amount of positional variation. The misalignment of this test can therefore only be deemed accurate to within 0.07°. Prior to subsequent tests, the exact offset at the top of the shaft was measured directly, and misalignment angles for subsequent tests are correct to 0.01°.

A distinctive wear pattern was produced on the male teeth (Figure 3.6). It showed a clear variation in wear severity over the facewidth of the teeth, with the upper end of the male teeth (as located in the test rig - towards the male shaft) showing the greatest amount of wear. The surface was very rough and had several brown areas indicating oxidation (Figure 3.7). Two of the teeth showed some larger pits (approx. 0.3 mm deep x 0.5 mm diameter) at the lower edge of the worn area, near the centre of the tooth. The middle of the tooth showed little wear over most of the tooth height, although there was a thin line of light pitting along the entire length of the tooth. Towards the lower end, approximately 7 mm from the end, the wear became more severe and spread out.
Figure 3.6 - Worn teeth of SWT I
Figure 3.7 - Enlarged view of upper end of teeth (SWT 1)
3.3.1.2 SWT 2 (1.5 mm; immersed; 0.15°; 935 Nm)

With good lubrication and a low misalignment angle, this wear test showed no appreciable wear on two of the teeth, with just slight damage of the surface at the upper end and near the tip. The third tooth showed only two slight bands of very lightly polished surface along the length of the tooth, with very light pitting towards the upper edge (Figure 3.8).

Figure 3.8 - Worn teeth of SWT 2
3.3.1.3 SWT 3 (1.5 mm; wetted; 0.30°; 935 Nm)

After approximately 32 hours of running, the LVDTs indicated an amount of wear greater than the thickness of the teeth. The test was stopped at this time and the teeth examined. The extensive wear and damage of the three male teeth from this test are shown in Figure 3.9 to Figure 3.12.

Figure 3.9 - Worn teeth of SWT 3 after cleaning

Figure 3.10 - Close up of tooth fracture zone, tooth 2
This wear test showed the most severe damage of all seven tests as would be expected from the high torque, poor lubrication and large misalignment angle. All three teeth had fractured near the centre of the tooth, and sections of the teeth had broken away and remained loosely attached by lubricant and debris. A crack had propagated around the entire circumference of the wall. This can be seen in Figure 3.13.
In addition to the several large fractured sections of tooth, there was a large amount of wear debris surrounding each tooth (Figure 3.14). Most of the material removal occurred at the upper end of the male teeth; the wear depth was over twice that of the lower end. At the tip of the upper ends, almost the entire thickness of the tooth had worn away (Figure 3.15). The surface at the worn end was very dark, indicating significant oxidation. The upper 5 mm (approx.) of the tooth showed some deep grooves. Below this point, the worn surface was relatively smooth up to the fractured
sections. The lower section showed a rough surface, with some oxidation and some metallic surface.

![Figure 3.15 - Worn profile of upper end of male tooth (SWT 3)](image)

### 3.3.1.4 SWT 4 (1.5 mm; immersed; 0.30°; 935 Nm)

SWT 4 showed a very light band of polished surface along the length of the teeth. Near the centre of the tooth facewidth, there was a small amount of light pitting and scratches for approximately 8 mm along the wear band.

![Figure 3.16 - Worn teeth of SWT 4](image)

### 3.3.1.5 SWT 5 (1.5 mm; wetted; 0.15°; 935 Nm)

Each tooth from SWT 5 showed two bands of light wear along the length of the teeth, similar to SWT 2 but more pronounced. The wear was greater at both the lower 5 mm
and, more noticeably, at the upper end. At the upper end, the wear also spread farther across the tooth profile, between the two wear bands. No wear debris remained attached to the teeth. The worn teeth are shown in Figure 3.17.

Figure 3.17 - Worn teeth of SWT 5

3.3.1.6 SWT 6 (1.5 mm; wetted; 0.22°; 935 Nm)

SWT 6 used an intermediate misalignment angle between that of SWT 4 (light wear) and that of SWT 3 (complete tooth failure). The worn teeth are shown in Figure 3.18.

The wear was mostly confined to the upper half of the teeth. The surface was rough and showed much oxidation and some pits towards the centre of the tooth. The transition from worn surface to unworn surface near the tooth centre clearly shows that the wear is most severe at the sides of the contact area (in the profile/height direction), particularly on inner diameter. Again, there was a small amount of lightly
worn surface at the lower 6-8 mm of the tooth. There was an appreciable amount of wear debris attached to the teeth, along the entire length of the spline (Figure 3.19).

Figure 3.18 - Worn teeth of SWT 6

Figure 3.19 - Wear debris of SWT 6
3.3.1.7 SWT 7 (2.5 mm; wetted; 0.30°; 750 Nm)

Test SWT 7 was carried out at a lower torque than the previous tests. There was, however, substantial wear and damage to the teeth (Figure 3.20). Unlike previous tests, the wear was quite uniform over the length of the teeth, with significant material loss over the entire contact area. All teeth showed some cracks near the centre of the facewidth, with a section of one tooth completely detached. The brown oxidised areas were more extensive at the upper and lower ends of the teeth. Quite early on during the running of the test, noticeable rattling could be heard from the coupling, and this continued sporadically throughout the test. The profile of the worn tooth viewed from below is shown in Figure 3.21, indicating the severity of the wear.

Figure 3.20 - Worn teeth of SWT 7
3.3.2 Quantitative wear evaluation

It was seen in Section 3.3.1 that the wear tests carried out with immersed lubrication showed virtually no wear after 40 hours. The LVDTs showed no significant movement for these tests (SWT 2 and SWT 4). For the wetted lubrication test with the low misalignment angle of 0.15°, (SWT 5) the LVDTs indicated a normal approach of just 13 μm at the pitch diameter after the forty hours of running. The wear evolution and the final wear value for the remaining tests are given in Figure 3.22 to Figure 3.25. The results are expressed as total movement of the male spline at the pitch circle diameter and give an indication of the total wear depth for both teeth.

For test SWT 3, in which a crack propagated around the entire wall circumference, the value of movement at the pitch diameter is greater than the combined wear of the two teeth. In the latter stages of the wear test, much of the rotational displacement of the male spline would have been due to significant twisting of the coupling, and so the values indicated are not comparable to results for the other wear tests.
3.3.3 Bending moment and axial force measurements

It was found that temperature variations of the conical shaft on which the strain gauges were attached caused an axial force to be recorded when none was present. It is thought that this was caused by the hollow shaft flexing outwards such that the outer surface showed a compressive strain, which was interpreted as an axial force. Because of the design of the rig and conical shaft, it was not possible to eradicate this problem entirely and so the sensitivity of the strain gauges measuring axial force was limited. The bending moment measurements were not affected as the configuration of strain gauges was such that any net axial force or compressive strain would be ignored.
All of the wear tests produced negligible axial force readings (actual readings were approximately those that would be expected due solely to the temperature effect). However, on one occasion during the initial pre-test running of the rig, a rapid increase in recorded axial force was seen. Concurrently, a large rattling noise from within the coupling could be heard. The axial force in this case was greater than would be expected from temperature effects and the speed at which it increased could not be explained by the slower rise in temperature. The coupling was running under wetted lubrication and, following application of further lubrication to the coupling whilst the rig was operating, the axial force rapidly decreased. Therefore, although no axial forces were recorded in the main series of tests, it is possible that, under certain conditions, a large axial force can be generated within the coupling, possibly aided by the presence of wear particles (indicated by the rattling noise). The temperature effect prevented any accurate measurement of the force generated.

Wear tests SWT 2, SWT 4 and SWT 5 showed a constant value of bending moment amplitude throughout the forty hours of the test. The values are given in Table 3.2. The bending moments reported are the maximum of the cyclic bending moment that is observed in the plane of measurement, and represents the bending moment in the direction of applied misalignment.

<table>
<thead>
<tr>
<th>Test</th>
<th>Bending moment amplitude / Nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWT 2</td>
<td>340</td>
</tr>
<tr>
<td>SWT 4</td>
<td>420</td>
</tr>
<tr>
<td>SWT 5</td>
<td>370</td>
</tr>
</tbody>
</table>

Table 3.2 - Bending moments for spline wear tests

The bending moments recorded for the remaining tests varied over the test period and are shown in Figure 3.26 to Figure 3.29.
3.3.4 Summary of results

A summary of the wear tests is given in Table 3.3.
<table>
<thead>
<tr>
<th>Test number</th>
<th>Module / mm</th>
<th>Lubrication method</th>
<th>Torque / Nm</th>
<th>Misalignment angle</th>
<th>Wear at PCD / µm</th>
<th>Bending moment / Nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWT 1</td>
<td>2.5</td>
<td>Wetted</td>
<td>935</td>
<td>0.30°</td>
<td>110</td>
<td>570 - 350</td>
</tr>
<tr>
<td>SWT 2</td>
<td>1.5</td>
<td>Immersed</td>
<td>935</td>
<td>0.15°</td>
<td>~0</td>
<td>340</td>
</tr>
<tr>
<td>SWT 3</td>
<td>1.5</td>
<td>Wetted</td>
<td>935</td>
<td>0.30°</td>
<td>&gt; 1500</td>
<td>560 - 380</td>
</tr>
<tr>
<td>SWT 4</td>
<td>1.5</td>
<td>Immersed</td>
<td>935</td>
<td>0.30°</td>
<td>~0</td>
<td>420</td>
</tr>
<tr>
<td>SWT 5</td>
<td>1.5</td>
<td>Wetted</td>
<td>935</td>
<td>0.15°</td>
<td>13</td>
<td>370</td>
</tr>
<tr>
<td>SWT 6</td>
<td>1.5</td>
<td>Wetted</td>
<td>935</td>
<td>0.22°</td>
<td>52</td>
<td>430 - 310</td>
</tr>
<tr>
<td>SWT 7</td>
<td>2.5</td>
<td>Wetted</td>
<td>750</td>
<td>0.30°</td>
<td>815</td>
<td>630 - 420</td>
</tr>
</tbody>
</table>

**Table 3.3 - Summary of wear test results**

### 3.4 Discussion

#### 3.4.1 General assessment of experimental tests

The importance of mimicking contact conditions exactly has been stressed. Although the size of the contact and the lubrication conditions will match those of a spline coupling in a real application, the use of only three teeth on the male component leads to some inconsistencies. Firstly, the tooth loads will be higher, necessary to accelerate the wear process but possibly affecting the outcome, particularly if a difference in stick/slip conditions exists. Secondly, and more significantly, the spread of load over different numbers of teeth is not replicated, and so the comparison between the 2.5 mm module and the 1.5 mm module spline will not accurately represent the comparison between 18 tooth and 30 tooth splines respectively.

The test rig relies on the tooth contacts to deflect the gyrator shaft, thereby creating the misalignment. This is not the same as in a real misaligned spline for which the shafts themselves are inherently at an angle. There may be a small difference in contact between the two methods; this can be realised by considering what would occur if there was a large clearance between the back faces and between the tips of the teeth, no torque and a small misalignment offset. Under such conditions, the external spline could move in a circle within the internal spline, without ever being misaligned angularly. With a large applied torque (as used in the tests), this effect should be negligible.
The misalignment angle of SWT 1 was not known to a high degree of accuracy due to the problem with the rig design explained in Section 3.3.1.1. The bending moment values for SWT 1 and SWT 3 (also carried out at 0.3° misalignment) were very similar. However, the bending moment for SWT 7 is approximately the same (or greater) than that for SWT 1, yet occurred with a lower applied torque. This would suggest that the misalignment angle for SWT 1 is somewhat less than the quoted 0.3°. It appears that the misalignment angle for SWT 1 was slightly below 0.3°, but not to the extent of ±0.07° possible due to the design problem.

The misalignment angle recorded for the tests was measured as the angle between the centre of the spline coupling and the top of the gyrating conical shaft. This is comparable to a remote shaft misalignment angle rather than the misalignment angle close to the coupling. Besides the usual shaft flexure, misalignment could also be accommodated through the fixtures of the male coupling to the torsion plate and shaft, and so it is difficult to predict the actual misalignment of the coupling. It was not possible to measure accurately the misalignment of the coupling specimen itself. This distinction is only of relevance for comparisons with computational work.

The recorded normal approach values do provide a method of quantifying wear, but there are some limitations. For example, the increase in wear from SWT 5 to SWT 6 (increase in misalignment angle from 0.15° to 0.22°) results in an increase in normal approach from 13 μm to 52 μm. The maximum wear depth does appear to have increased by a similar factor, but the wear area is also greatly increased. Similarly, the maximum wear depth of SWT 7 does not appear to be eight times that of SWT 1, but the whole tooth is worn, allowing a greater rotational movement of the male spline in the coupling.

The wear rate, indicated by the change in normal approach value, does not show a consistent pattern between tests. Test SWT 1 appears to show a slowdown in wear after an initial high rate of wear, as is often found in fretting (Section 2.1). The remaining tests appear to show an almost constant wear rate after an initial induction period (10 - 15 hours) of low wear rate, similar to that found by Weatherford et al. for test on lubricated splines [25].
The LVDT measurements may not provide a particularly good evaluation of wear, due to wear particles remaining trapped in the worn regions. This makes it extremely difficult to assess whether wear particles are playing a detrimental role, having no effect or a beneficial effect on the wear rate. A likely role of wear debris is to cause the initiation of pitting or cracks, which could lead to failure by a different mode, rather than giving a general increase in material removal. This aspect is investigated in Chapter 4.

The bending moment was found to decrease during some tests. This could be caused by wear debris lubricating the contacts and reducing friction. Alternatively, a change in tooth shape due to material removal at the ends, giving an almost “crowned” condition, could reduce the effective stiffness of the coupling and thus reduce the bending moment.

3.4.2 Effects of varying conditions and geometry

The assessment of lubrication conditions is an important objective of the experimental study, since no such assessment can be made using the computational techniques. The results show how an improvement in lubrication can lead to a massive improvement in wear life. Under the test conditions of SWT 3 and SWT 2 (0.3° misalignment, 935 Nm torque), the use of immersed lubrication with oil circulation rather than simple wetted lubrication meant the difference between a virtually unworn spline and teeth worn almost completely through, with complete tooth fracture.

The high dependence of wear upon misalignment angle is also demonstrated through tests SWT 5, SWT 6, and SWT 3, in which the misalignment angle was increased through 0.15°, 0.22° to 0.30°, whilst the corresponding wear depths recorded at the pitch circle increased from 13 μm, through 52 μm, to greater than 1500 μm. Such an increase in wear depth due to such a small change in misalignment angle is a notable result of the wear tests, and agrees with the findings of Ku [118].

Tests SWT 1 and SWT 3 were both conducted at the same operating conditions using splines of different module. The increase in module, and thus increase in tooth thickness, will alter the stiffness of the tooth which is likely to alter the pressure distribution. A secondary result of having thicker tooth is that a thicker tooth can
better cope with a given depth of wear. For the same wear depth, a smaller module spline would have lost a larger portion of its thickness and stiffness, which could lead to more slip (and hence even more wear) and a greater likelihood of fracture due to bending fatigue. This intuitive observation should not be overlooked.

In the case of SWT 1 and SWT 3, however, the wear depth shown on the 2.5 mm module coupling, if repeated on the smaller 1.5 mm module tooth, would not significantly affect its strength, and so it must be assumed that the increase in damage is due to fundamental differences in the contact behaviour of different sized teeth rather than just a worn tooth effect. It must be re-emphasised that these tests do not account for a change in contact behaviour that may occur as a result of different numbers of teeth.

The effect of wear debris cannot be fully ascertained from this set of experiments, but some basic observations on the movement of debris within the spline coupling can be made. Figure 3.14 and Figure 3.19 show the spread of debris around the tooth. The debris can spread over a wide area and across the whole length of the tooth with just wetted lubrication, even with the “stirring” motion of the test rig – centrifugal forces on a rotating coupling are likely to increase the spread of debris and is another discrepancy between the test rig simulation and actual splines.

Comparisons of wear severity between the different tests have shown many predictable outcomes with misalignment angle and lubrication. Tests SWT 1 and SWT 7 reveal an interesting, and perhaps unexpected, change in damage. Here, a reduction in torque led to an increase in wear damage and further fatigue damage. The maximum wear depth did not increase significantly, but the total wear (or area of wear) was greatly increased, with wear occurring across the tooth face rather than being restricted to the ends. The problem with the accuracy of the angle for SWT 1 hinders the interpretation of these results, but the presence of fatigue cracking was, in fact, predicted based on the computational results presented in Chapter 5.

3.4.3 Surface appearance and wear patterns

All of the significantly worn tooth areas show the same form of surface damage – a mottled, brown (oxidised) appearance with occasional longer grooves or valleys. The slip path predicted by the rigid body analysis gives an almost elliptical slip path which
would suggest a more evenly worn surface such as that seen, rather than a directional wear pattern. Grooves in the wear surface would be caused by wear debris particles of a large enough size being detached from one area and slowly being moved within the tooth contact. This type of behaviour is investigated visually in the following chapter. In areas of mild wear, the wear often takes the form of small pits, often along a single band of wear.

For teeth, and areas of teeth, on which there is no major wear damage, there may be bands of light wear, or a polishing effect, along the length of the facewidth. This is predominantly near the tip of the tooth and at points corresponding to the tip of the female tooth. This would coincide with the larger pressures expected at the tooth edges due to the discontinuity in profile, similar to a flat-edge contact. This increase in wear at the tips can best be seen at the transition from heavily worn to lightly worn areas of teeth, such as in Figure 3.6 and, in particular, Figure 3.18.

A clear pattern also emerges for the spread of wear in the axial direction. The teeth of tests SWT 1 and SWT 6 show a marked transition from worn to unworn areas, the end of the male tooth from which the shaft emerges (the upper end) being the worn end. Other tests, both with significant wear and low wear, exhibit their most severe damage at this end, although this distinction is not so obvious for the test at lower torque (SWT 7). Near the opposite (lower) end, the wear in some cases begins to reappear or increase in severity.

The only discrepancy in this wear positioning, is the appearance of some larger pits in tests SWT 1, SWT 2, and particularly SWT 4. The pits, which vary in size, are found near the centre of the tooth. The preference of crack formation under the partial slip regime was initially considered as an explanation for this. However, the slip distances here are not much lower than those at the ends of the teeth, and so it is unlikely that there will be a change from gross slip to partial slip along the tooth. The entrapment of wear debris or particle contaminant at this location is a possibility, due to a more linear reciprocating slip path (Section 5.3.1.3).

The tooth fractures of SWT 3 and SWT 7 occurred towards the middle of the teeth. The existence of a partial slip fretting regime may be assisting fatigue cracks formation, although it is doubtful that this is the cause for the reasons given above. A
more likely explanation is due to a change in pressure distribution regime as predicted computationally (Section 5.3.2.1). The complete fracture of teeth can occur in real-life applications, and can cause major failures of systems. The high tooth loads used in the experiments to accelerate wear would have greatly increased the chances of tooth fracture, and it is unlikely that such a fracture would have occurred under the same conditions if a full complement of teeth had been used.

3.5 Summary of spline wear tests

The spline wear tests have confirmed that the magnitude of misalignment in a spline coupling is critical to the tooth wear. Effective lubrication will also play an important role in reducing wear. The applied torque may affect the wear and failure of the spline teeth in a complex way – a reduction in torque does not necessarily lead to a reduction in damage.

The wear rate in many of the tests showed an initial period of low wear followed by either a sudden or a gradual change to a higher wear rate. One test, however, showed a gradual reduction in wear rate. Wear damage appears to initiate as small pits (as revealed in areas of mild wear). Further wear tends to produce a rough surface with significant oxidation of surface and debris. Tooth fracture from crack propagation was also a cause of failure in some tests, accelerated by the very high tooth loads. Some larger, isolated pits were formed, possibly through the action of trapped wear debris. This is one of the topics investigated in the following chapter.
4

Visualisation Study of Fretting

4.1 Purpose of study

As reported in the literature review, there are several aspects of fretting wear which are not completely understood, or for which contradicting observations exist. It is clear that the nature of fretting wear is dependent on the geometry of the contact, and that this could be a result of the retentive nature of the small amplitude motion which reduces dispersion of any wear particles from the contact zone. The spline wear tests reported in the previous section provided information on the resultant wear of a spline coupling operating after several hours, and an indication of the amount and spread of wear debris. However, it is not possible to see how the debris particles were generated or their behaviour during the test, nor determine the mechanism of wear. The purpose of the work reported in this section is to improve the knowledge of the fretting wear mechanisms occurring within spline couplings and the role of wear debris particles. This has been achieved through visually observing a representative contact as the fretting action occurs.

Many fretting tests used in laboratory situations use small contacts undergoing linear reciprocating slip of small amplitude. This is a poor representation of the contact conditions between spline coupling teeth, which is both non-linear and of large contact area. The High Frequency Reciprocating Rig (HFRR) has previously been used to study fretting and has been modified to allow visualisation of the process by Baker and Olver [20]. The contact area in these experiments had a radius of $31 \mu m$. The wear observed at the end of the tests was not sufficiently comparable with spline teeth wear. The small, localised pits sometimes seen on spline coupling teeth were of particular interest. This could not occur in HFRR experiments, where the size of the pits would be large compared to the contact area. The visualisation study carried out for this research required a different design of experiment and different test specimens.
The test rig used for this visualisation work was originally designed to mimic the motion that was believed to occur in spline couplings due to angular misalignment, and uses a reciprocating arc motion rather than linear. It had recently been used at Westland Helicopters Ltd to study various aspects of spline tooth wear using steel-on-steel test specimens [134, 135]. The tests had shown some wear features similar to those observed on spline coupling teeth, and which cannot be identified on the smaller test specimens commonly used. Figure 4.1 shows worn test specimens from two of these tests [135]. The bottom example clearly shows some large, localised wear pits, which are of greater size than the contact diameter of the HFRR tests.

![Worn test specimens from steel-steel tests using spline-tooth type test rig, from [135]](image)

Apart from the specific role of wear debris on the mechanism of fretting, the transport of the wear debris within and out of the contact area could provide information for obtaining methods of reducing wear in spline couplings. The production of wear debris is an inevitable fact of fretting wear, but once it is produced, it may or may not affect further wear. Once the debris has exited the contact, it can no longer act as a load-carrier or an abrasive third-body. If the role of third-bodies within spline couplings can be determined, it would be useful to understand how particles are eventually removed from the contact region, and what factors affect it.

The computational analysis section of the work reported in the following chapter does not take into account any aspect of wear particles; this experimental work could also provide further insight for comparing the computed data with observations from real applications.

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4.2 Description of experiment

The tests were carried out on the test rig used for the spline tooth simulation tests described above. The test specimens have a contact zone size of $2.5 \times 54$ mm which is comparable to the contact area of a single spline coupling tooth used in the experimental work and found in many aviation applications. A schematic of the rig is shown in Figure 4.2.

Figure 4.2 - Schematic of the one-tooth fretting rig

The test specimens are loaded together with a constant force by dead weights via a lever. The lower test specimen is then moved in a horizontal arc by the upper beam which pivots about a point close to the test specimens; this pivot would represent the centre of the coupling. A heated oil bath originally allowed the tests to be conducted with lubrication at elevated temperatures.

To enable visualisation of the contact, some modifications were required, and further modifications were made to improve the performance of the rig. Firstly, the upper test specimen was replaced with a sapphire block, which allowed the contact to be seen without separating the specimens. The work by Baker and Olver verified the suitability of a sapphire-steel contact for replacing steel-steel contacts for the purpose of visualisation of fretting wear [20]. An alternative upper specimen holder was designed and built, containing a window through which the specimens could be
observed. The lower specimen (mild steel) was housed in the original lower specimen holder.

The set-up of the test specimens and visualisation equipment is shown in Figure 4.3.

![Figure 4.3 - Set-up of test specimens and visualisation equipment](image)

Two magnifications were available, 2.5x and 10x, allowing visualisation of areas 2.11 × 1.58 mm and 620 × 465 μm respectively. The microscope could be moved over the entire contact area to allow different parts of the contact surface to be viewed during the tests, and to follow any debris movement.

The weight of the upper, reciprocating arm was reduced by drilling out several holes along the length and chamfering the edges to reduce the reciprocating mass. This improved stability of the rig and clarity of the images recorded. A final modification
to the rig, needed to enable the new upper specimen holder to be used, was the removal of the oil container and heaters – no tests were carried out with lubrication.

Although it was possible to adjust the slip amplitude, all tests reported here were carried out with a constant amplitude. The lower test specimen was moved about a pivot approximately 55 mm perpendicular to the centre of the specimen, through an angle of approximately 0.5°. The actual slip varies over the specimen surface, and the motion is not linear but arced. Towards the ends, the motion is predominantly perpendicular to the test piece; towards the middle, the motion is predominantly parallel. Apart from being similar to the movement of spline coupling teeth, this also allows different slip amplitudes to be seen during the same test. This theoretical motion is shown in Figure 4.4. The actual motion experienced by the specimen differed slightly from a pure arc and is reported in Sections 4.3 and 4.4.

![Figure 4.4 - Applied motion of lower specimen](image)

The tests performed were not as extensive as originally intended due to difficulties with the rig and visualisation equipment design. When the rig had been used previously (with steel-on-steel test pieces), difficulties in correctly aligning the specimens were reported. Although the upper arm is on a bearing that allows the lower test piece to self-align along its length, the alignment in the perpendicular direction is determined by the exact position of the upper specimen. The lower specimen moves on an arc about the pivot in the vertical direction (as well as the
horizontal plane of the slip motion), until it is forced against the upper specimen. It had been necessary to conduct many tests until the test pieces were brought into alignment by varying the amount of shim on the upper specimen holder. For these tests, a new upper specimen holder was required, and so an accurate alignment was not immediately possible. Alignment was aided by being able to view the test pieces as they were brought slowly into contact, but with lower normal loads than used for the steel-on-steel tests, the small misalignments that existed were more problematic and led to wear occurring predominantly on one side of the test piece. Although the pressures were not uniform or symmetric, the test piece did appear to be in full contact with no separation. The rig design is also such that, as the specimens wear, the misalignment will alter – the original steel-on-steel tests may not have been perfectly aligned throughout but instead have had an average misalignment of zero.

A comparison of the effects of differing loads was originally intended but was not possible due to repeated fracture of the sapphire specimen. The first tests were carried out with no additional weights on the load pan, producing a normal load of 1700 N (mean pressure of 12.5 MPa), and some were carried out at a normal load of approx. 2700 N (20 MPa). When a higher load was applied, the sapphire test piece/viewing window fractured shortly after starting the rig motor. The sapphire also fractured at the lower loads, following a long period of operation. This fracture is likely to be due to localised contact where debris particles are trapped within the contact, and made worse by the misalignment of the test pieces generating higher contact pressures than anticipated.

The frequency of motion was varied during the tests as deemed appropriate to generate maximum wear and enable the behaviour to be viewed with sufficient clarity. The microscope was also moved about the contact to identify areas of possible interest.

### 4.3 Results

Several interesting observations were made during the series of tests, including the transport and behaviour of wear debris and the generation of different forms of wear. Some of these can be seen in the video frame images reproduced in this section; the moving video images gave a better impression of the contact and allowed debris
movement to be seen. The observations are listed and explained in this section and discussed in Section 4.4. The main findings are summarised in Section 4.5.

Images recorded at 10× magnification are identified as such in the caption; all other images were at 2.5× magnification. Most of the observations made in this section were found to occur on more than one occasion and are thus assumed representative of the type of behaviour expected. No discernible differences between the two different loads were found. Seven different test pieces were used and showed similar characteristics, although some showed greater wear on the upper edge and some on the lower edge, dependant on misalignment.

The slip amplitude could be observed directly from the movement of steel below the camera. The movement at some points was more elliptical than reciprocating and can be seen in some of the wear patterns. The movement was predominantly perpendicular to the length of the specimen (vertical in the images) at the ends of the contact and parallel to the length near the centre as expected (Figure 4.4). However, the amplitudes differed from that expected. The amplitude at the corners was approximately 525 μm whilst the amplitude at the centre of the contact was as low as 50 μm (approx.).

The first test was carried out with an old test specimen used in the previous lubricated steel-on-steel experiments. The test piece was lightly cleaned prior to use, but the surface remained dark and heavily oxidised. Figure 4.5 shows the bottom right corner of the test specimen. (The camera was set up such that the test specimen pivots about a point below the bottom edge.)

![Figure 4.5 - Previously worn test specimen](image-url)
Shortly after the motor was turned on, areas of dark oil appeared as the motion released oil and debris trapped at the surface. Although this was not intended, it did allow some additional observations regarding the behaviour of the oil.

Figure 4.6 - Oil spread during test

Figure 4.6 shows three images taken over a period of 30 seconds/1000 cycles and shows the oil gradually spreading over the contact surface from its original appearance to the right of the view. Eventually the entire surface was covered with the oil. The darkness of the oil-covered area prevented the surface or wear debris from being viewed and attests to the difficulty of investigating the effects of lubrication using this method. It may be possible to view the first stages of wear through clean oil, but once appreciable wear has occurred then oil contamination prevents further observation of the surface.

Figure 4.7 - Oil jets from side of contact

At the edges of the contact, the oil was forced out of the contact zone in small jets from a series of localised points along the side of the specimen. A jet of oil appeared to occur at random intervals as the steel specimen moved back (the edge moved inwards to the centre of the contact), and squeezed the oil film forcing small droplets to be released. On increasing the speed, the ejection of oil from the surface was increased, with an increasing number of initiation points. These two examples of oil
movement will affect the generation of wear and the movement of the smaller wear particles.

In the main series of tests, debris particles of different sizes were seen. Some larger particles had dimensions of approximately 250 µm; typical mid-sized particles had dimensions of around 50 µm. There was also a large quantity of very fine, powdery debris. This was too small to identify individual particles, but could be seen in clusters. Most of the wear debris had a dark appearance indicating oxidised material, but there were some metallic particles. The larger particles tended to be more metallic, with the fine powdery debris always appearing to be oxidised. Most of the mid-sized particles, which were scattered over much of the contact surface, were oxidised, although many that were seen attached to the sapphire outside the contacting region were metallic (Figure 4.8). Although no detailed count was made, it seemed that the large particles were found more frequently near the centre of the contact region where the slip amplitude is lower.

![Figure 4.8 - Large wear particles (10x)](image-url)
Figure 4.9 - Particles of various sizes scattered within contact

(10x)

The larger and mid-sized particles were found scattered remotely over the specimen surface. Figure 4.9 shows a rather densely covered area. There appeared to be slightly more large and mid-sized particles attached to the steel than the sapphire. The particles adhere to either surface rather than existing within the interface in the manner of an oil or solid lubricant.

Figure 4.10 - Wear particles adhering to sapphire and steel

(10x)

Figure 4.10 shows two particles of wear debris near the centre of the test piece. In the second image, the steel test piece has moved to the right. The right particle is adhering to the steel and this moves to the right with it; the left particle is adhering to the sapphire and thus does not move between the two images. On only one occasion did wear particles appear to be moving about within the contact without adhering to
either surface: a small group of particles appeared to jog about within a confined area of the contact, moving with the steel specimen. The particles appeared to be trapped in a valley between asperities whilst not adhering to the bottom.

At the edges of the contact is an area of sapphire that is alternately in and out of contact as the steel specimen moves. Large deposits of debris are found at the boundaries of this area; the debris that might normally rest in this region is swept back and forth by the edges of the steel specimen and eventually deposited at the boundaries. Once a debris particle is pushed beyond the region of contact, it remains stationary and this leads to the build up of debris. It is predominantly powdery debris that is deposited – many larger particles are broken up when brought in contact with the edge (shown later in Figure 4.16). A line of debris deposited outside the region of contact is shown in Figure 4.11. The dark line of debris matches the contour of the steel surface edge, and is located at the extremity of its motion. Similar lines of debris are seen in other images of this chapter.

![Debris contour along bottom edge](image)

**Figure 4.11 - Debris contour along bottom edge**

Powdered debris also builds up at the inner boundary of area swept by the contact edge. This is less pronounced than at the outer boundary since the debris may continue to move. An example of this is seen later in Figure 4.30. A similar effect was seen in the test carried out with the pre-worn specimen, where an area of moisture existed within the contact (Figure 4.12). It is assumed that the moisture was water
from the humid air, since the specimens were cleaned with toluene and wiped dry prior to use.

Figure 4.12 - Wear particle deposits around region of moisture

The two images are at the opposite ends of the slip motion. A semi-elliptical region of moisture can be seen which oscillated back and forth. There are some wear particles contained within the moisture, but there is also a deposit of powdery debris at the edges of the area covered by the moisture movement. This suggests that isolated regions of lubrication or moisture could possibly lead to concentrated regions of particle deposits which may then lead to localised wear damage.

Although adhering to one surface for long periods of time, the debris within the contact would also move around. The movement of the debris was neither consistent nor gradual. The particles rapidly jumped from one static (relative to a surface) location to another, where it might once again remain located for a long period of time. There was usually no obvious reason for a particle to suddenly move, and there was no general direction of movement. The oscillating frequency did not affect the movement of the particles – higher frequencies did not appear to promote movement. A sudden change in frequency, however, particularly an increase in speed, did regularly cause particles to move to a new location. On a number of occasions, once a particle had moved from one position, it would continue to move several times in quick succession before reaching a new “stable” position where it would once again remain for a longer period of time.

The movement of all debris particles observed occurred by the same process which involved transfer of the particle between the two surfaces. This process is shown schematically in Figure 4.13.
Figure 4.13 - Schematic diagram of particle transport through material transfer

A wear particle initially adhering to one surface (the lower surface in the above schematic) transferred to the opposite surface and thus moved relative to the original surface. At a different point in the slip motion, the particle then re-attached to the original surface at a new location. This process could then be repeated. The timing and position within the slip motion at which a transfer occurred varied, although was often at an extremity of the range of motion.

This process can also lead to the break-up of larger debris particles, which is shown in Figure 4.14
Figure 4.14 - Break-up of wear particle

Figure 4.14 shows a mid-sized wear particle being broken up over two cycles of motion. The red dot is a reference point on the steel specimen to distinguish the position within the cycle. In (a), the whole debris particle is intact and is adhering to and moving with the steel specimen. In the following upwards motion of the steel specimen, a section of the particle adhered to the sapphire and became detached, thus remaining in the same location, whilst the upper section of the particle remained adhering to the steel – (b) shows the steel in the high point of the cycle with the two separated parts of the debris circled.

On the following downwards motion of the specimen, the lower section of debris re-attached to the steel, so that both parts of the particle moved downward (c). On the subsequent upwards motion, parts of the debris again adhered to the sapphire and some to the steel causing further break up of the debris, which had now spread over a wider area (d). The particles then all adhered to the steel surface and moved downwards (e) until, at the end of the second cycle (f), the debris had broken into
small, indistinguishable particles. The original debris particle was previously in existence for many cycles, prior to these two cycles that caused its destruction. Thus, it does not seem to be a slow grinding of material that leads such particle break up, but a sudden mechanical tear.

Figure 4.15 shows a particle moving upwards and transferring from the steel specimen to the sapphire in one single cycle.

Figure 4.15 - Particle transfer between surfaces (10x)

NOTE: The dark mark on the right of the image is on the microscope lens and not on either test piece.
In the first three images, a dark wear particle can be seen coming into view attached to the steel specimen as it moves upwards. In the forth and fifth images, the steel continues to move upwards, but the particle has become attached to the sapphire and remains in place. In the remaining images, the steel test piece moves back down, without the particle attached, thus the particle has moved upwards and transferred to the opposite surface.

The particle continued to move upwards by this process and eventually came in contact with the edge of the test piece. At this point, it gradually became broken down with each contact with the edge, over a period of approximately three seconds or fourteen cycles. This is seen in Figure 4.16. The upper part of the particle is seen to be slowly broken away with each cycle. In the final image, there is no trace of the original particle.
Figure 4.16 - Break up of particle through contact with edge (10x)

The adhesive behaviour of the wear particles is demonstrated further by the following two sequences.
Figure 4.17 - Pivoting of particle and elliptical slip path (10x)

Figure 4.17 shows one cycle of movement. A long, horizontal piece of debris can be seen on the right, which is attached to the sapphire. The two particles on the left are attached to the steel, near an area of wear at the edge. Although difficult to ascertain from these still images, the largest piece of debris actually pivots back and forth about its right hand side – the particle is adhering to the sapphire at this point and is being moved back and forth by the steel below. It is also clear from this sequence of pictures that the slip path in this instance is more elliptical than a reciprocating arc. Shortly after these images were taken, this point of adherence could be seen through the complete rotation of the debris particle about its right hand side (Figure 4.18) over
a few cycles. The particle eventually transferred completely to the steel specimen. The particle was observed regularly during the test and did not appear to break apart.

Figure 4.18 - Particle rotation (10x)

Much of the powdered wear debris was found deposited off of the edge of the contact region and some was spread over the contact surface leading to a darkened image. There were also several areas within the contact region where a large mass of powdered debris existed, clustered together but not amalgamated into a single piece of debris. Large portions of the powder would move about together rather than flowing. However, it did appear that this type of debris might be moved within the contact by the general vibrations of the surfaces rather than just through transferring from one
surface to another. Figure 4.19 shows a mass of powdery debris near the lower edge of a worn test piece. In the first two images, the lower band of particles is adhering to the sapphire specimen, and a mass of powder to the right is adhering to the steel surface. In (e), a mass of powder has transferred to the steel. The powder spreads more easily and is formed into a vertical line of powder as more powder transfers, with the horizontal band of powder moving to the right. The mass of powder moved as a whole, rather than individual particles flowing within (compare the left hand side of the first and last images). After several hundred cycles, the mass of powder on the steel specimen broke up as the lower half transferred temporarily to the sapphire, and the process was repeated.
Figure 4.19 - Movement of fine, powdery particles

The interest in wear particles is mainly in regard to how it may affect the wear. No large pitting or significant damage was seen to be caused by wear debris in the few tests carried out. One piece of debris did have a minor affect on the bulk surface and this is seen in Figure 4.20.

Figure 4.20 - Particle movement and indentation (10x)

Figure 4.20 shows a large wear particle at the edge of the steel specimen towards the centre of the test piece. The first four images were taken over approximately a one-
minute interval with the rig operating at 2.5 Hz. The first image shows a long wear particle near the edge of the specimen, where it had been located for a relatively long period of time. The particle then moved from its position; in one cycle, it moved to the location shown in (b) and remained there for several cycles. In the original location of the particle, an impression had been left in the steel surface matching the shape of the particle. The lines of surface roughness can be seen through the indentation, suggesting that this piece of debris originated elsewhere.

In subsequent cycles, the particle moved further into the contact region. Thus, although originally created near the edge of the contact, which should facilitate the movement of the debris out of the contacting area, the particle moved further inwards where it could lead to more damage. The direction of movement of the particle depends solely upon the point in the cycle at which the transfer occurs, i.e. whether the steel is moving to the right or left/up or down.

The final image (e) shows the same area at the end of the test. The original damage caused by the particles can be seen clearly, but no further wear had occurred.

At various points over the contact surface, small scratches were seen on the steel test pieces which would have been caused by small, hard particles of wear debris. Many of the scratches were formed within the first few cycles of operation; in these cases, the wear debris could be the broken asperities of the steel removed upon initial contact and during the first movement. Scratches were seen up and down the edges of the specimens yet relatively few were observed within the contact; this may be due to the increased surface roughness seen at the edges following manufacture (therefore more asperities to be broken) or due to the increased pressures expected at the edges of the flat-on-flat contact.
Figure 4.21 - Scratch marks after short running with detached particle

Figure 4.21 shows some scratch marks caused within the first 25 cycles of operation. The inset shows a magnified view of the scratches to the right. Here a metallic piece of debris could be seen at the tip of one of the scratches (circled). It is not possible to determine whether the particle caused the scratches or if it is the material removed from within the scratch. The scratch marks are in the direction of motion (the image is from the top left of the specimen – scratches on the right hand side were reversed) and have an arced shape. They are typically of the same length as the slip amplitude at that point.

Figure 4.22 - Scratch formation

Figure 4.22 shows the beginning and end of another scratch generation. Here, the scratch rapidly progressed along the length of the specimen with each cycle causing a wide area of wear. In this instance a piece of debris must have moved rapidly along
the specimen with each cycle before eventually being ejected from the contact or adhering to one surface.

Figure 4.23 - Scratch in middle of contact, showing movement path

Figure 4.23 shows an almost elliptical scratch caused by debris within the contact. The scratch mark suggests a possible slip path between the steel and sapphire which differs from the expected reciprocating arc. In fact, the actual slip path (obtained by comparing the position of the specimen through consecutive frames of video) resembles the path shown in red. There is a slight deviation from the arc, which could be caused by some movement at the bearings. The scratch consists of several individual scratch paths each displaced a small distance from the next. Several elliptical shaped scratch marks could be seen at various locations over the specimen surface, each orientated in different directions as determined by the position on the specimen relative to the centre of rotation.
Figure 4.24 - Scratches and wear evolution

Figure 4.24 shows an image of the upper left edge of a specimen following approximately 10 and 400 cycles of operation. Several scratches can be seen following the initial few cycles. The same scratches can be seen in the later image with no additional scratches formed.

The later image also reveals a thin band of wear and debris along the edge of the contact. This band occurred along the length of the specimen, and the thickness varied in accordance with the varying slip amplitude experienced along the length of the specimen. The left hand third of the specimen (upper portion) is seen in Figure 4.25. Note the wear band in the top left corner spreading along the top edge.

Figure 4.25 - Wear band along upper edge of specimen

Figure 4.26 - Wear evolution, example 2
Figure 4.26 and Figure 4.27 show the evolution of wear on the upper edge of two test specimens over a period of time. In Figure 4.27, (a) is the test steel surface prior to running; (b) shows the specimen after a few cycles of operation in which several scratches were formed. After 300 cycles (c), more general surface wear can be seen and the scratches appear to have got darker which could be due to oxidised wear particles becoming trapped within the scratched area. (d) shows the same area after 2000 cycles and a wide band of general wear and oxidation can be seen at the edge, with the original scratches no longer visible.
Figure 4.28 - Wear evolution, example 4

Figure 4.28 is another series of images showing the evolution of wear. As the upper band of wear grows, the unworn area below gets notably darker as the number of fine wear particles within the contact increases.

The images of wear shown seen in Figure 4.21 to Figure 4.28 are at the top edge of the contact. Figure 4.11 is taken from the same test as Figure 4.21, and shows the bottom left edge of the contact. The wear is much higher on the upper edge due to misalignment that causes greater pressures on the upper edge. Nevertheless, the lower edge is slightly deformed along the tip, and the powdery debris is swept out from the contacting region to form a line along the edge contour, confirming that the entire specimen surface was in contact.
Wear also occurred on the bottom edge of a specimen when misaligned in the opposite direction. Figure 4.29 shows some wear marks at various points on the lower left part of a test piece. The wear occurred in bands along the direction of the roughness grooves. Each wear scar has a flat side along a crest line, and it was also observed that a wear scar may stop growing upon reaching a line. Thus, surface finish is likely to have a most significant effect on the wear mechanism.

Figure 4.29 - Wear scars on lower half of test specimen (10x)
Figure 4.30 shows the gradual increase in wear damage on the lower left edge of the test specimen. In the first images (a-e), a large elliptical scratch can be seen to the left of the image. The pits occurred along the entire length of the test piece – the scratch did not affect the results. In the later images, the view has moved to the right (by approximately one third of the image size).

The images show that the wear occurred initially as a series of very small pits that grew in size and number. Although the scratch did not affect the pit generation along the edge, the pitting contours around the scratch and penetrates farther up the contact than elsewhere. The material along the edge of the scratch may have been raised by deformations as the scratch was formed, hence the enhanced wear.

Over the 130 000 cycles of the images shown, the wear increases noticeably. In the first image, the wear is made of distinct, small (20 – 40 µm) pits. Over the period of testing, the pits grew in size and eventually merged and became indistinguishable as separate pits. Unlike the movement of debris, this process is gradual and no change can be seen in consecutive video frames. The pits predominantly run along a line of roughness. This is most visible in the first two images. From the final image, it also appears that a groove has prevented pits from growing upwards (note the flat edges on the upper row of pits).

As well as the growth of the original pits, more pits are formed higher up on the test piece. In (i), three new pits can be seen half way up the image, which were not present in image (g). In the intermediate picture, the first signs of pit formation can be seen.
The pit surfaces are mainly dark brown/black, through which some metallic areas can be seen. The dark appearance may be due to an oxidised surface, or may be caused by oxidised particles contained within the pits. The newly formed upper pit in (j) shows a distinct metallic area within, which was previously dark in image (i). In this case, oxidised wear debris was trapped within the pit. In (k), a dark mass of debris can be seen at the top centre of the image. This is material transferred from the top of the pit onto the sapphire – the pit appears slightly smaller than in (i).

As only a gradual change in pit shape could be seen, the material from pits must have been removed as relatively small particles. Figure 4.31 shows a mass of powdery debris (although rather difficult to identify as such from a static image). This was approximately 500 μm away from the area viewed in Figure 4.30, and it is likely to be material removed from these pits and similar pits in the vicinity.

Also notice the band of powdered wear particles within the contact (most notably in images d, f and g). This band is located at the end extremity of the motion and matches the contours of the pit edges. The powdered debris is swept inwards in the same way as for Figure 4.11.

![Figure 4.31 - Powdered debris in area close to pit formation](image)

Figure 4.32 shows a similar scratch at the bottom edge and wear pits along the bottom edge. Wear pits identical to these occurred along the entire length of the specimen.
Figure 4.32 - Additional scratch mark and wear pits (10x)

Figure 4.33 shows the wear on a typical specimen used for a steel on steel contact test carried out previous to the current set of tests and is presented to allow comparison with the results of the steel on sapphire tests presented here.

Figure 4.33 - Wear scars of specimen previously used for steel-steel contact

Figure 4.34 shows an area in the middle of the upper edge of a test piece. The images are consecutive frames (1/25\textsuperscript{th} second apart) and show the extremity of the motion as the contact stops moving in one direction and begins to move in the opposite. A small circular region can be seen to the middle right of the images on which a localised point contact occurred. It was on an area such as this that the sapphire was cracked during operation. The two images also show how the view could be altered by differing directions of tangential force. Along the edge of the contact to the left of the point contact, the image on the right is darker and appears to be worn or containing
debris, whereas the image on the right has a metallic appearance. This gives difficulties in comparing and assessing whether areas are worn or contain debris. In this instance, the spectrum of colours around the contact suggests a region of moisture, which may be causing the change in optical appearance.

![Image](image_url)

**Figure 4.34 - Point contact on asperity**

### 4.4 Discussion

This visualisation work carried out has been a basic study that has allowed some features of fretting wear to be seen that could not be demonstrated by other techniques. The tests did not cover as wide a range of operating conditions as may occur in practical spline coupling operations, being limited to one slip angle and two loads. The possibility of carrying out tests at much greater loads would have been beneficial. A greater load would have accelerated the wear process allowing a greater amount of data to be obtained, and would have allowed a useful comparison between different operating conditions.

The sapphire specimen used should have allowed higher loads given a more uniform pressure distribution without the point contacts and misalignment that occurred. The use of greater loads would have required more than just a thicker block of sapphire for the upper specimen. A greater thickness of sapphire would have reduced the optical clarity of the images and may have required higher quality components. The distance between microscope objective and contact surface would also have to increase leading to difficulties with the optical set-up (the microscope objective was very close to the viewing window in the current set-up). The rig design also made alignment of the two specimens problematic, and this was made worse by the low loads, which caused most wear to occur on only one edge.
The wear that has been observed does appear to be similar in some respects to that which occurred for the steel-on-steel tests of Olver et al [135]. However, the larger localised wear pits were not formed, possibly due to the low loads and misalignment. Higher loads would have led to wear occurring over the entire specimen and might have generated such pits – perhaps allowing the role of wear debris to be confirmed.

The observation that the majority of the large wear particles adhered to the steel rather than the sapphire may be due to the different materials, but there was sufficient debris adhering to the sapphire to suggest this was not a large effect. Much of the powdered debris was seen on the sapphire specimen. This is more likely to be because the entire steel specimen was permanently in contact and moved within the area of the sapphire specimen, i.e. the boundaries of the alternating contact/non-contact region were on the sapphire specimen alone. Additionally, small particles could become “lost” in the rough surface of the steel specimen.

The movement between the two specimens and the slip paths did vary slightly from the reciprocating arc expected, perhaps due to movement at the bearings at the end of each back and forth motion leading to a small transverse motion. The magnitude of the slip amplitude varied across the surface, but by a larger amount than expected given the difference in rotation radii. At the edges, the observed slip amplitude was around 500 μm perpendicular to the specimen whereas it was as low as 50 μm along the specimen at the centre. This implies significant elastic deformation of the steel specimen, and that movement in the perpendicular direction was easier than along the length of the specimen. The elastic deformation of the steel specimen was observed visually during a test, when the lower specimen holder was seen to move whilst part of the contact remained stationary – i.e. in stick.

The observations of the wear debris particles suggested that the particles do not behave as a fluid continuum of flowing particles that would be expected of many powders which act as lubricants. If more wear particles are formed, and a complete third-body bed is produced, the particle flow may be more continuum-like and similar to the simulations reported in Section 2.3.1.

The particles also adhere to each surface rather than moving about within the contact interface. The method by which they do move is through adhesion to the surfaces.
The movement of the particles does not show any trend or overall direction of movement but appears entirely random and dependent upon the precise timing of transfer. In practice, the sudden and random transfer of material is likely to be due to very small changes in the slip amplitude caused by slightly varying rig conditions or material wear leading to a change in contact conditions. This is supported by the observation that a change in operating speed often caused sudden particle movement. Such a change in slip path could lead to asperities and debris which previously approached one another, without contacting, finally moving into contact and causing material to transfer. Adhesion to the surface was also the cause of particle break-up.

Because of the light loads, it is possible that parts of the surface were not fully loaded and the interface gap was large enough to accommodate the wear particles. Greater loads may have resulted in greater particle-surface interaction, and possible debris compaction or grinding.

Figure 4.20 was the only occasion for which debris particles appeared to have a damaging effect upon the bulk metal. Greater normal loads may have produced more such occurrences. Whilst the large sized particles may cause damage, powdered debris could assist in lubricating the interface. The powdered debris observed in these tests did not appear to show such behaviour, but instead adhered to either surface as a cluster of particles that often behaved as a single entity.

The regular scratches seen over the tooth surface generally occurred at the start of the tests and made only a minor contribution to the overall wear. The pits seen in Figure 4.29 and Figure 4.30 were more significant; their generation was gradual and the method by which it occurred difficult to ascertain. The importance of the roughness is demonstrated by the fact that the pits occur in lines along the direction of machining. The gradual nature of their growth would be possible through the generation of very fine powdery wear particles that cannot be individually identified, possibly through an adhesive mechanism. The last pit to be created in Figure 4.30 appeared to contain a mass of such powdery debris within, which was eventually dispersed.

All of the wear seen in these tests has been material removal under gross slip. No areas of stick have been studied and no fatigue cracks generated. Therefore, the
observations cannot be considered as a complete example of the fretting wear phenomenon.

4.5 Summary of findings

Movement of the specimens was seen to squeeze the oil film leading to jets of oil escaping from the contact. This may carry smaller wear particles out of the contact. Patches of moisture or oil within the contact could cause wear particles to be "washed up" around the patch, creating localised build-ups of particles. Build up of debris also occurred at the extremities of motion on the larger, upper specimen.

Both metallic particles and oxidised particles were formed; metallic particles were generally larger than the oxidised particles. Fine, powdery particles (~1 μm) were common, as were mid-sized particles (~50 μm). There were fewer large particles, with sizes up to approximately 250 μm.

Large and mid-sized particles were generally isolated, adhering to either surface. Powdery debris was seen grouped together as well as spread over the contact surfaces. The powdery debris was seen to move en masse, rather than as a fluid-like continuum. The particles usually moved by transferring from one surface to the other.

Break up of particles occurred through adhesion to both surfaces, being torn apart rather than crushed and ground. The break-up of debris was enhanced when brought in contact with the rough and heavily loaded edges of the steel specimen.

Particle movement direction appeared to be random, depending upon the point in the cycle at which transfer occurred. It is likely that transfer occurs when the particle makes contact with a new asperity due to small changes in slip path caused by rig movement or wear, or a reduction in local interface gap caused by wear.

Surface scratches were generated in the first cycles of movement, predominantly at the edges of the contact. The scratches appeared to be caused by hard particles trapped in the contact, and could spread as the particle moved along the surface.

Fretting wear appeared to initiate as a series of small pits, which would gradually grow in area and eventually merge. The wear area was dependent on slip amplitude.
The wear scars were formed on crests of grooves for the rough surface, and pit growth appeared to arrest at furrows of grooves.

The observations were made with a light load and some misalignment between specimens. The steel surface was quite rough, showing distinct polishing/machining lines. The many reported experimental studies of fretting wear have shown many variations in results, hence the observations made here may only be found in similar contacts; other possible debris transport, break-up and wear processes cannot be ruled out.
5

Computational Modelling of Spline Couplings

5.1 Purpose of study

The experimental work carried out for this research programme, and reported in the previous two chapters, has provided insights into the fretting wear mechanism as well as giving indications of the type and severity of wear likely to occur for different spline couplings and operating conditions. However, two substantial requirements for achieving the objective of the project remain to be fulfilled. Neither of the two experimental methods provide any information about the actual contact conditions within real spline couplings during operation under misalignment – something that is essential for the understanding of wear of spline teeth and which can only be estimated using rigid body analysis. Furthermore, the limited number of spline wear tests does not provide sufficient information to enable a proper assessment of a full range of different coupling types. The shortcomings of using a three-toothed coupling also reduce the differentiation between spline coupling geometries. Computational analysis is used to overcome both of these deficiencies.

The computational analysis allows a spline coupling connecting two misaligned shafts to be modelled and produces the required contact information. The modelling makes it possible to see how the teeth move and what contact pressures exist between mating teeth as the shafts rotate. The slip values and pressure distributions produced by the analysis can be combined to generate an indication of wear for each part of each tooth and hence suggest a wear susceptibility for each model.

The analysis does not take into account any material loss due to wear or the effects of debris within the contact or lubrication (other than by a user defined friction coefficient). This has both beneficial and detrimental aspects. Although the analysis may not entirely represent the contact conditions of a worn spline or allow different lubrication methods to be assessed, it does permit the geometrical and operational
factors to be studied fundamentally, without the complications of lubrication action, manufacturing defects or random debris movements. The results therefore provide for a good comparison of the effects of operating and geometric variables alone.

The computational analysis allows a substantial number of different spline coupling geometries and operating conditions to be compared without the limitations of rig size, rig facilities and test specimen costs. The only limitations of the computational work are processing speed (and hence number of analyses) and hard drive space (model size). A large database of results for analysed couplings can be interpolated and extrapolated to provide estimated results for any other coupling and operating condition combination (within the limits of extrapolation) and negate the need for future lengthy analyses. This could then be used at the design stage rather than for analysing an existing spline. It is this facility that is needed for the eventual spline design software package.

5.2 Method of analysis

5.2.1 The boundary element method

The boundary element method is a means of determining the elasto-static response of a body to applied loads and displacements/stresses and strains. It is similar to the finite element method commonly used to analyse the behaviour of engineering components, differing in that only the surfaces of the body are explicitly defined and used in the computation (although it is possible to access information for positions within the body itself through the use of “internal points”). Boundary element modelling is, in some respects, faster and less resource intensive than the finite element equivalent. For the analysis of contacts, as required in this study, it is particularly suitable since only information on the surface behaviour (at the contacting body interfaces) is of interest. For these reasons, boundary element software was chosen as the computational method.

The general features of the boundary element method are dealt with in greater detail in Section 2.5. In summary, the boundary surface of a component is defined using a mesh of boundary elements. Boundary conditions of tractions, displacement, springs, etc. are defined to act on selected elements. The boundary element analysis then
computes the state of the component under the actions of the boundary conditions, providing stresses, tractions and the deformed shape of the component. This can be carried out in either two or three dimensions.

The boundary element method can also be extended to allow calculation of contact conditions. One contact analysis method uses a “node-to-node” constraint-based algorithm, and it is this method that is used for the spline coupling analyses. The node-to-node method is only suitable for conforming contacts with low displacements – spline couplings can therefore be modelled provided there is little axial movement. On each contact surface, there are a number of element nodes, for which there are corresponding nodes on the adjacent surface. Depending on the outcome of the analysis, each node is assigned a state, each of which is associated with compatibility and equilibrium conditions. These states are “stick” (nodes are coupled – displacements and tractions sum to zero), “open” (not in contact – tractions are zero), “slip” (normal displacements and tractions sum to zero, tangential traction = $\mu \times$ normal traction) or “sliding” (no friction, normal displacements and tractions sum to zero, tangential tractions are zero) [131].

The boundary element software package used is “Beasy”, a commercially available tool produced by Computational Mechanics, Southampton, UK [132]. Computational Mechanics were a partner in the DTI collaborative spline coupling research programme [125] and the Beasy software was further developed specifically for the analysis of spline coupling contacts. Various methods of validation for spline coupling analyses were carried out, as reported in [126] and summarised in Section 2.5. Beasy was therefore an obvious choice for this spline coupling analysis programme.

Beasy includes a pre- and post- processor (Beasy-IMS) as well as a boundary element solver. The models are created within the IMS, from which a data file is produced containing the essential information of the model to be passed to the solver. Three-dimensional models are created within Beasy by first defining the co-ordinates of appropriate points on the object’s surface. Surface lines are created by connecting these points and then surface patches are formed from three or more connecting lines. These surface patches define the geometry of the object being modelled. One or more “zones” can be defined from these patches, representing different parts of the object or
convenient modelling sections. Where the model consists of two objects in contact, each object would be a separate zone. A suitable element mesh is then applied to the surface patches, either automatically or by manually specifying the number of elements for each individual patch. It is these elements that define the object for the purpose of analysis.

All elements in three-dimensional analyses are either triangular or quadrilateral and have six or nine "mesh points" respectively. The mesh points define the geometry of the element, thus the geometry of each element is quadratic in each direction. During the calculation process, "nodes" are applied to each element. It is at the nodes that the problem variables (e.g. displacement) are computed. The number of nodes applied to each element can vary depending on the type of element that is selected by the user. The selection of element types is fundamental to the accuracy and the interpretation of results. For quadrilateral elements, there are four types of elements:

- **Constant** – one node alone is applied to element. Each problem variable is assumed constant over the element

- **Linear** – four nodes are applied to the element at the corners. The problem variables are assumed linear within the element

- **Quadratic** – nine nodes are applied to the element as for mesh points (at corners, sides and at centre). The problem variables can therefore vary quadratically in each direction.

- **Reduced quadratic** – eight nodes. As for quadratic, without central node.

The requisite material properties (e.g. elastic modulus and Poisson's ratio) of the object are entered into the model. Each zone can have a different value for each property, which is constant and isotropic.

The loads, displacements, stresses and strains, etc. applied to the object (the boundary conditions) are specified by selecting the appropriate type, the element or patch to which it applies and the direction and magnitude. A feature that is of particular use for the spline coupling analyses is “sequential loading”. A series of “load cases” can be defined, each specifying a set of boundary conditions, which are then solved sequentially. The first load case is solved in the normal manner, with the subsequent
load cases using the results of the previous load case as the initial condition to which the boundary conditions are applied. This is necessary in contact analyses to take account of the non-linearity of friction.

For contact interfaces between different objects in a model, a single patch (or patches) is used to define the edge of each object. An “initial gap” boundary condition specifies this as an interface between two bodies and defines the actual gap between the objects’ surfaces prior to loading, usually 0.00. Static and kinetic friction coefficients are also assigned. The number of “load steps” in which to apply the boundary conditions can be specified and the tolerance indicating convergence can be set. Alternative methods of creating contacts exist but are not used here, where the conformal nature of the tooth contacts is best suited to the method described.

5.2.2 The boundary element model

In order to make qualified appraisals of the results from the computational results, it is necessary to consider the model used. It is this that determines both the accuracy of the results computationally, and the relevance of the results to real-life cases. With the complexity involved in spline coupling geometries and with multiple contact areas, this becomes particularly important.

In some circumstances, a computational model of a spline can be simplified either by the use of cyclical symmetry, in which the coupling is represented by a small angular sector of the male and female components, or by modelling either a single tooth or all teeth on one component alone with an assumed pressure applied to the tooth contacts. This might be suitable for assessing the stresses within the roots of the teeth and shaft sections. The situations under investigation here, however, are not cyclically symmetrical because of the applied bending moment. Additionally, the exact contact pressures are not known but are to be determined. A full three-dimensional model of a complete, connected coupling was therefore required.

Two different forms of model have been used in this study – fully toothed models for investigation of real splines, and three toothed models for comparison with the experimental spline wear tests reported in Chapter 3. The time taken for an analysis to be carried out increases with model complexity; with the already complex nature of spline couplings, the details of the model were reduced as much as possible. The
results for the contacting areas of the teeth are those that can be used for evaluation of wear whereas the results for other parts of the coupling are not as significant for the purposes of this study. The model was designed with these considerations.

A geometric model for a fully toothed spline is shown in Figure 5.1. A simplistic spline arrangement is used. The externally toothed component (the upper component in the model) consists of a hollow shaft section on which straight involute spline teeth are formed on the lower part of the external surface. The internally toothed component is also a hollow shaft section on which spline teeth are formed on the upper part of the inner surface. The outer diameter of the male shaft is equal to the minor diameter of the external spline teeth; the inner diameter of the female shaft is equal to the major diameter of the internal spline teeth. The inner diameter of the upper shaft is variable, as is the outer diameter of the lower shaft. The ends of the teeth are straight and normal to the spline axis — no undercut, chamfer or fillet radii are used at the ends of the teeth as this would increase complexity with little or no effect on the contact regions. The male spline teeth of all models used have a smaller facewidth than the female teeth, and are centrally located.

![Side View (Wireframe)](image1)
![Top View (Solid)](image2)

**Figure 5.1 - Geometric model for a fully-toothed spline**

The profile of the contacting teeth will affect the distribution of pressure over the tooth surface and so the defining of this profile is critical. The profile of each tooth
includes a root section and an involute section. The profile of the involute section is an accurate representation of the involute form. The roots were represented by elliptical forms to match the tangent of the involute at the form diameter and the root circle.

For a single tooth pair, the contacting face consists of a contact patch, two involute patches either side of the contact patch for the area of the female teeth not in contact (due to its longer length), and a root section for each tooth. The geometry of the involute sections is defined by 13 points and the root sections by 6 points. For contacting tooth faces, the contact patch profile is defined by 11 points and the root patches are defined by 8 points (including some within the involute section). This is sufficient to accurately define the involute shape.

The arrangement of tooth form assumes contact on one face of the tooth only, as would be the case in most couplings. No contact can occur on the reverse face, but there is also no computational requirement that the volumes of the two components do not overlap outside of the contact zone, i.e. there is no part of the software that prevents reverse face overlap. This had to be checked manually for each analysis.

A geometric model for a three-toothed spline is shown in Figure 5.2. The model is similar to that of a fully toothed spline. The externally toothed spline has three teeth located at 120° to one another and a plain circular section connecting the root of each tooth. The internally toothed spline has six teeth, one on either side of the male teeth present. The section between toothed areas on this spline is of an increased diameter – it resembles an extended tooth of height one third of the main teeth. This increased thickness is necessary to represent the stiffness of the fully toothed internal splines used in the wear tests without the complexity introduced by many teeth.
The application of the computational mesh on the geometric surfaces is critical to both the analysis time and accuracy of results. Of the four possible element types, two are used on the model. The non-contacting elements are defined as reduced-quadratic elements, which represent the best compromise of accuracy and speed. The contacting elements are defined as linear elements. Although the computed values at the nodes are forced to change linearly, the geometry and mesh point values still vary quadratically. Computational Mechanics, authors of the Beasy software program, reported that reduced-quadratic elements were not particularly suitable for contacts, and suggested linear elements would be a better option.

The mesh used for non-contacting regions was determined by comparing run-times and results for various combinations. Some finely meshed analyses failed to complete due to lack of hard disk space, and analysis time increased rapidly with number of elements. The mesh used can be seen in Figure 5.3. This shows the 24 elements used at the top and bottom shaft cross-sections, and the elements around the circumference of the shafts, which are sufficient to represent the varying conditions. The roots of the teeth would constitute a large proportion of the number of elements in the model, and so a coarse mesh was used in this area. This implies that the stresses reported locally in the root region should not be interpreted as realistic of root stresses in actual couplings.

The meshing of the contact areas was more critical for both analysis time and accuracy. With the iterative method used for contacts, the time for completion is very
heavily dependent on contact mesh. With the discontinuous form at the edge of the
tooth contacts, a singularity occurs at the tooth edges, which would require a very high
mesh density in that area. Tests using different mesh densities did indeed show a
variation in maximum pressure (occurring at the tooth edge) as would be expected.
The final mesh density chosen for the models of the studies was determined by speed
and hardware resources. Accurately modelling the edges of the contact would be
impossible with current facilities, and the speed of the highest mesh possible was
inadequate. It was considered to be more useful to have comparative results between
many analyses, which would be made possible using the same mesh size for all
models. A two element (profile) by ten element (lead) mesh was used for the bulk of
the analyses.

The surface geometry definitions (which depend on number of defined points) and
boundary element definitions (quadratic variation for each element) are not identical
and can vary considerably at the tooth profiles. This is apparent in Figure 5.4. There
is a clear and significant variation in form at the roots, as much as 100 µm for a
typical coupling used in this study. The involute profiles are more closely followed

Figure 5.3 - Boundary element mesh
by the boundary elements, varying by less than 5 μm for the sizes of couplings analysed. Although there is a small discontinuity in form at the edges of the contacts, this is not likely to influence the contact results significantly given the coarse mesh. More importantly, a single contact patch defines both the male and female tooth face, thereby removing any possibility of pressure variations caused by deviation between the male and female tooth profiles, i.e. the contact surfaces are completely conformal and in contact.

![Geometry and Element Mesh](image)

**Figure 5.4 - Geometric surface and boundary element form of tooth profiles**

The meshing of the three-toothed models was similar to that of the fully toothed models except that the reduced complexity allowed for higher density meshes on the contact patches. Various mesh densities have been used.

Boundary conditions applied a torque and an angular misalignment to the coupling and defined the interface between the two zones making up the male and female splines. The misalignment was applied through the use of a displacement boundary condition on the upper and lower ends of the coupling. The bottom of the internally toothed component was fixed in the direction of the coupling axis, i.e. the displacement in the z-direction was specified as 0.0. No displacements were specified in the x- or y-directions so that the component was free to twist about the axis in
response to the torque and so that no net shear force would be included in the analysis. The angular misalignment was defined by displacements at the top of the externally toothed component. The magnitudes of displacements in the x-, y- and z-directions that would be produced by a rotation about a horizontal axis passing through the coupling axis were calculated. These were then applied to the model, thus producing the required angular rotation in a specified direction. The axis of rotation for the misalignment angle was chosen to be at the midpoint of the spline teeth to best represent a purely angular misalignment.

The torque was applied to the female component on the lower row of elements, through tractions applied tangentially to the outer surface. This differed from the arrangement in the spline test rig, but was necessary given that the male component cannot displace rotationally in response to the torque due to the fixed displacement applied to the top.

An initial gap between the contacting teeth was specified as zero for most analyses. For analysis of manufacturing errors and crowning, the value was altered between different teeth or varied between the individual mesh points of each tooth. A complete model of a fully toothed spline coupling is shown in Figure 5.5.

![Boundary element model with boundary conditions](image)

**Figure 5.5 - Boundary element model with boundary conditions**
All analyses required at least two sequential load cases; many used five or more. For each load case, the direction of misalignment was moved around the coupling to simulate the shafts rotating (as in the spline test rig). The misalignment was moved in the direction equivalent to that which would occur for the male component driving. This is the same as that of the spline test rig. The consequences of female or male component driving are briefly discussed in Section 7.1.1.

Because of the complex detail of spline couplings, some form of automatic model generation software was needed. Although some involute tooth generation tools exist, they were either not suitable for use with Beasy or produced sections that would still require a significant amount of additional modelling work. A complete model generation software tool was written to produce the spline coupling models. This also ensured consistent geometry definitions between models. The software is able to produce complete, assembled spline couplings based on user inputs of male and female tooth geometries, tooth length, shaft lengths and shaft diameters. A second software tool was written to apply the chosen mesh to any spline coupling produced with the model generation software. Again, this ensured consistent definitions and numbering of the elements. A third piece of software was used to calculate and apply the displacements needed on the male coupling to produce the angular misalignment, while a fourth software program applied the remaining boundary conditions and specified material properties.

These software tools all ensured consistent and accurate models, which would have been difficult or impossible to produce manually. Each program required the minimal number of user inputs to define the geometry or values for torque and misalignment and created a text file which could be imported by the Beasy IMS as automated commands.

The important aspects of the model used that must be taken into account when assessing results are: root stresses do not accurately represent the local stress state; contact results that are sufficient to show general variations along the profile of the tooth and, more importantly, axially along the tooth; and the comparative nature of pressure values at the tooth edges rather than actual maximum values.
5.2.3 Verification

To confirm that the results produced in this computational study describe the conditions within real spline couplings adequately, several forms of verification have been carried out. The Beasy boundary element software has been verified with many different models by the creators, and is known to produce results of the accuracy expected. Indeed the use of the software for modelling spline couplings has been verified in detail during the DTI collaborative spline coupling work (reported in Sections 2.4 and 2.5). Further validation has been carried out for the models used for the current study, and some additional validation analyses have been made to check the software used during the creation of the models.

5.2.3.1 Hertz analysis

A simple Hertzian contact analysis was performed by using rectangular blocks loaded together with an initial gap that varied over the contact surface to produce a spherical contact boundary. The resultant pressure distribution is shown in Figure 5.6. Some negative values of pressure were obtained around the edge of the contact zone and have been removed from the graph. This negative pressure was also found to occur for the spline models and is explained in Section 5.3.1.1. The mesh used on the model was relatively coarse and so the contact area can only be determined to approximately 10 mm. Hertzian analysis predicts a contact diameter of 60.3 mm; the Beasy produced a contact of 70 mm, which could be a result of the 10 mm size mesh. The maximum Hertzian pressure is 98.4 MPa, and the computed maximum pressure is just under 110 MPa.
5.2.3.2 Verification of model creation software – torque application

To check the software used to produce the computational models, several different analyses were carried out. This included the method of torque application (this section), the displacement offsets used to apply the misalignment (Section 5.2.3.3), and the overall spline coupling model (Section 5.2.3.4).

The method of torque application was validated by applying a torque to a simple hollow shaft, using the same techniques and software used in the spline study. The shaft was of similar size and mesh to that of the coupling, and a torque of 2000 Nm was applied. The rotational twist of the shaft was obtained from displacements at the mesh points at the bottom end of the coupling and was found to be within 1.5% of the analytical solution for the torsional twist of the hollow shaft. Using the angular rotation results, the actual torque applied was calculated to be 1973 Nm (98.6% of the intended value).

5.2.3.3 Misalignment application

The software used to generate the angular misalignment is complex and validation of this program was required. This entailed using the software to apply various misalignments to a simple hollow shaft and monitoring the displacement produced. Additionally, the variation of stresses and displacements on the surface to which the boundary condition is applied was monitored to confirm that the misalignment is
consistent and that all mesh points are moved in the manner intended. Some results of this validation are shown in Figure 5.7.

Figure 5.7 - Validation of angular misalignment application

5.2.3.4 Spline coupling model

Although a complete analytical validation of the spline coupling model is not possible, simple checks on the model were made by viewing the deformed shape and through contour plots of results. Some of these are shown in Figure 5.8.

Figure 5.8 - Sample results of spline coupling analyses
Additional validation was possible by comparing the torque applied to a model with the torque transmitted through the teeth. A frictionless model was used to avoid any frictional component of torque and a torque of 500 Nm was applied. The torque transmitted through each tooth was calculated as: average pressure × tooth contact area × \( \frac{1}{2} \) pitch diameter × cos (pressure angle). The profile of the teeth was simplified as being flat rather than involute, and thus the tooth load is slightly reduced. The calculated torque for the validation analysis was between 472 and 483 Nm for each load case, around 96% of the nominal torque. The slight variation could be due to the profile simplification alone.

5.2.4 Design and operating parameter study

The research programme as a whole is centred on a study of the effects of different spline coupling parameters on the wear that occurs due to rotation under misalignment. These effects have been studied systematically by the designation of the main spline coupling and operating condition parameters. Analysis of many different computational models with varying parameters then allowed the consequential effects of each parameter to be judged. There are many factors that govern how the wear occurs and how much wear occurs, not all of which can be studied computationally. It would also not be feasible to investigate all of the parameters that can be studied given the computational costs. Some of the many factors thought likely to play a role in the wear process are given in Table 5.1, grouped into those that cannot be included in the computational model, those which can be included but are not investigated for the current study, and those which have been investigated.
<table>
<thead>
<tr>
<th>Not included in computational analysis</th>
<th>Not included in current investigation</th>
<th>Included in current investigation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Properties</td>
<td>Spline Geometry</td>
<td>Spline Geometry</td>
</tr>
<tr>
<td>surface treatments</td>
<td>tooth height proportion</td>
<td>number of teeth</td>
</tr>
<tr>
<td>surface finish</td>
<td>(only half height teeth)</td>
<td>module</td>
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<tr>
<td></td>
<td>tooth contact type (only single side</td>
<td>pressure angle</td>
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<tr>
<td></td>
<td>contact)</td>
<td>facewidth</td>
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<tr>
<td></td>
<td>tooth type (only straight, involute)</td>
<td>(all other specific spline</td>
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<tr>
<td></td>
<td>tooth thickness (varied only in</td>
<td>geometry parameters such as major</td>
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<tr>
<td></td>
<td>proportion to module)</td>
<td>and minor diameters are varied in</td>
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<tr>
<td></td>
<td>wall thickness (varied only in</td>
<td>proportion to the module)</td>
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<td></td>
<td>proportion to diameter)</td>
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<tr>
<td></td>
<td>engagement length and position (full</td>
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<td></td>
<td>length of male facewidth, located</td>
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<td>centrally)</td>
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<td>manufacturing errors</td>
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<td></td>
<td>tooth modifications</td>
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<td>(investigated separately)</td>
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<tr>
<td>Corrosion</td>
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<td>Third body effects</td>
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<td>Geometry modification</td>
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<td>following initial wear</td>
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<td>Rotational speed</td>
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<td>Material Properties</td>
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<td>Poisson’s ratio (fixed at 0.3)</td>
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<td></td>
<td>Operating Conditions</td>
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<td>torque</td>
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<td>misalignment angle</td>
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<td></td>
<td>friction coefficient</td>
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<td>Material Properties</td>
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<td>Elastic modulus</td>
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</table>

Table 5.1 - Factors affecting wear of spline coupling teeth
The parameters that are included in the study have been combined to form dimensionless groups. This simplifies the process of referring available analysis results to different sizes and conditions, as well as reducing the number of distinct parameters that needed to be varied during the study. The non-dimensional parameters used are listed in Table 5.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^* = \frac{E}{T}m^3$</td>
<td>Dimensionless elasticity/torque $(E = \text{elastic modulus}, T = \text{torque}, m = \text{module})$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Misalignment angle at 20 module (degrees)</td>
</tr>
<tr>
<td>$z$</td>
<td>Number of teeth</td>
</tr>
<tr>
<td>$b^* = \frac{b}{m}$</td>
<td>Dimensionless facewidth $(b = \text{facewidth of male component})$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Pressure angle</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Friction coefficient</td>
</tr>
</tbody>
</table>

**Table 5.2 - Non-dimensional parameters**

The actual spline geometry is derived from the $z$, $b^*$, $\alpha$ and $m$ parameters. The spline tooth diameters (minor, form and major) and the circular tooth thickness/space-width are fixed proportions of the module; the wall thicknesses are fixed proportions of the pitch diameter ($z \times m$); the female facewidth is a fixed proportion of the male facewidth. The formulae for these parameters are given in Table 5.3 and are based on the dimensions of a spline wear test specimen spline, which itself was designed to ANSI B92.2M-1980 30° Full Fillet Side Fit specifications.
<table>
<thead>
<tr>
<th>Dimension</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male major diameter</td>
<td>Pitch diameter + 0.800 × module</td>
</tr>
<tr>
<td>Male form diameter</td>
<td>Pitch diameter - 1.072 × module</td>
</tr>
<tr>
<td>Male minimum diameter</td>
<td>Pitch diameter - 1.934 × module</td>
</tr>
<tr>
<td>Circular tooth thickness</td>
<td>1.552 × module</td>
</tr>
<tr>
<td>Female major diameter</td>
<td>Pitch diameter + 1.882 × module</td>
</tr>
<tr>
<td>Female form diameter</td>
<td>Pitch diameter + 1.200 × module</td>
</tr>
<tr>
<td>Female minimum diameter</td>
<td>Pitch diameter - 0.436 × module</td>
</tr>
<tr>
<td>Circular spacewidth</td>
<td>1.584 × module</td>
</tr>
<tr>
<td>Female facewidth</td>
<td>1.333 × male facewidth</td>
</tr>
<tr>
<td>Male shaft bore diameter</td>
<td>0.444 × pitch diameter</td>
</tr>
<tr>
<td>Female shaft diameter</td>
<td>1.555 × pitch diameter</td>
</tr>
</tbody>
</table>

**Table 5.3 - Formulae for derived spline geometry parameters**

The importance of specifying a reference point for misalignment angle was explained in Section 1.4. The misalignment angle has been defined at 20 modules from the tooth ends. This definition is required to keep all lengths non-dimensional. However, the analyses were all carried out with shaft lengths of 50 mm, for which the applied misalignment is only identical to the defined misalignment for 2.5 mm module splines. It was originally believed that a 50 mm length would be sufficiently small to prevent any significant shaft flexure such that the applied misalignment would accurately represent the coupling misalignment. However, inspection of the results revealed that if the imposed shaft misalignment was used, then the results would be dependent on module. This is demonstrated in Table 5.4, which shows the results for analyses carried out with identical non-dimensional parameters, but for which the misalignment angle was specified at different locations. The meaning of the result parameters is explained in the next section, but the table shows values are not identical when the application distance is 50 mm, but are (virtually) identical when the application distance is $20 \times m$. 
<table>
<thead>
<tr>
<th>Module 2.5 mm</th>
<th>Module 3.5 mm</th>
<th>Module 5.0 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1° at 50 mm</td>
<td>0.1° at 50 mm</td>
<td>0.1° at 100 mm</td>
</tr>
<tr>
<td>(20 × m)</td>
<td>(14.3 × m)</td>
<td>(20 × m)</td>
</tr>
<tr>
<td>C_{max}</td>
<td>0.422</td>
<td>0.453</td>
</tr>
<tr>
<td>P_{max}/P_{av}</td>
<td>1.227</td>
<td>1.497</td>
</tr>
<tr>
<td>W/Em</td>
<td>37.594 × 10^{-6}</td>
<td>59.024 × 10^{-6}</td>
</tr>
<tr>
<td>M/T</td>
<td>0.567</td>
<td>0.580</td>
</tr>
</tbody>
</table>

Table 5.4 - Comparison of results for analyses with identical \( E^* \), \( z, b^*, \alpha, \) and \( \mu \) parameters using different modules and misalignment angle application positions

Since the imposed misalignment angle for each analysis was specified at the 50 mm shaft ends, it was necessary to convert this “shaft” misalignment angle to a misalignment angle as defined for the parameter study, \( \theta \). The bending moment of the coupling was used to calculate the angular deflection of the shafts, and the imposed misalignment angle was adjusted accordingly (see Appendix B for further details). The misalignment angles for the 2.5 mm module splines did not require adjustment, and the results for these analyses have been used for most of the quantitative data processing.

5.3 Results

5.3.1 Introduction to results – a typical spline coupling

The boundary element software produces results in three formats relevant to this work. The .sfx results files can be imported into the Beasy IMS and used to display several types of results (stresses, traction, displacements, etc.) as contour plots on the model surface or as graphs of values at selected mesh points, and provides a view of the model as deformed by the applied loads. This has been used to produce general views of the coupling for qualitative assessment and to check for reverse face overlap of spline teeth.
A second results file contains all basic results (locations and directional vectors for all mesh points, boundary conditions, tractions and displacement of all mesh points, etc.) in a text format. Although this file is very large (typically over 120 Mb) and is of little use in its basic format, it has been used to obtain quantitative results for bending moment calculations. The bending moment caused by the displacement misalignment cannot be obtained directly from Beasy. To calculate the bending moment, the vertical tractions on the lower end of the female spline were used. The bending moments for each individual mesh point in the lower end were combined to produce the bending moment on the coupling overall. This calculation was carried out automatically through additional software.

A third results file (contact report) contains details of the contact conditions for each load case of all elements and mesh points that form the interface between the contacting teeth. The file contains the original location of each mesh point, the displacement vector of the mesh point for both surfaces (i.e. the male spline tooth and female spline tooth movement), normal and tangential traction values between the two surfaces at the mesh point, and the slip at the mesh point (the relative displacement between surfaces resolved in the tangential direction). It is this file from which all of the contact results in this chapter are obtained.

The contact report file produced directly by the software contains the information in a format that is difficult to use. Software was written specifically for this study that is able to extract the pertinent information from the file and display it or reorganise it in a format more suitable for examination. The software also allowed easy manipulation of the results to provide extra information useful for the study.

The software requires an input file that specifies the mesh points identities for each tooth separately, and the position in which it exists relative to the tooth face in a grid format. With $2 \times 10$ elements for each tooth, and 9 mesh points for each element, each tooth contains $5 \times 21$ mesh points at which contact information is available.

The software is able to produce graphical images showing the value of a selected result type over each tooth contact surface and a comma separated values file for any selected result type for more quantitative evaluations of the contacts. Some of the result types available are used and explained below.
To help explain the type of results produced and give a basic insight into the contact behaviour of a spline coupling, a reference spline coupling has been selected and the results presented in Sections 5.3.1.1 to 5.3.1.5. Although no single spline coupling can realistically typify all possible variations, the coupling presented here shows the effects of misalignment clearly without exhibiting the extreme effects of an excessive misalignment angle which have been found to occur. The coupling has 18 teeth of module 2.5 mm giving a pitch diameter of 45 mm and a male tooth facewidth of 27 mm. The torque applied to the coupling is 500 Nm. The angular misalignment is 0.1° and is rotated around the coupling in five steps of 90° producing a complete revolution, with load case 1 and load case 5 being identical loading conditions (albeit with different load histories). A friction coefficient of 0.3 was used.

5.3.1.1 Pressure distributions

The clearest way to appreciate how the spline teeth contact is to look at the pressure distributions for each tooth pair contact. Figure 5.9 indicates how the images in this chapter are used to display these, and subsequent, results. Each rectangular patch represents the contact region of a tooth pair. The left hand side of each patch is towards the centre of the coupling radially, towards the root of the male spline teeth and at the tip of the female spline teeth. The right hand side is at the tip of the male spline teeth. The upper edge is at the end of the male spline teeth from which the shaft emerges – the top of the teeth as located in the model shown in Figure 5.3. The teeth are displayed in horizontal rows – each horizontal row showing the conditions for all teeth of a single spline coupling state. The first tooth is in the 0° direction.
Figure 5.9 - Explanatory diagram for graphical display of results

Figure 5.10 - Pressure distribution for typical spline coupling with rotating misalignment
The pressure distributions for the various stages of rotation of the typical spline coupling are shown in Figure 5.10. White areas (zero pressure) indicate areas of the teeth that are not in contact.

These results are not exactly as reported in the contact report file. At a few mesh points on the edge of the area actually in contact, some mesh points are reported with negative contact pressures. This was said to be due to extrapolation of the nodal results (used internally by the Beasy software) to mesh point results used by the contact report file. The negative contact pressures are not used within Beasy and so do not represent inaccuracies in the overall analysed model. They have been removed from this and subsequent pressure distribution plots, and the value of pressure at those points has been altered to zero.

To define some result parameters, it is necessary to look at some aspects of the pressure distributions. The region of pressure varies from tooth to tooth for each load case. The centre of pressure clearly varies from tooth to tooth, as does the total tooth load. The centre of pressure has been found to deviate axially from the centre of the tooth because of both torque and misalignment. The torque alone produces a constant offset of centre of pressure axially along the spline teeth while the misalignment causes an additional, varying component of offset of centre of pressure. This is demonstrated in Figure 5.11, showing the magnitude of tooth load and the distance of centre of pressure axially from the bottom of the spline tooth.
Figure 5.11 - Tooth load and centre of pressure for typical spline coupling at 90° stages of rotation
This observation has been used to provide two quantitative values to describe the pressure distribution. $C_{\text{max}}$ is defined as the maximum value for axial deviation of the centre of pressure from the centre of the spline tooth ($c$) as a proportion of the overall facewidth: $C_{\text{max}} = c / b$. A second parameter is the maximum tooth load divided by the average tooth load, $P_{\text{max}}/P_{\text{av}}$. These two parameters give an indication of how much the angular misalignment causes the tooth loads to vary and by how much the pressure distribution is skewed towards the ends of the teeth. They can be used to compare results for different analyses.

5.3.1.2 Gap between teeth

The initial gap between the two contacting faces is defined as zero. The previous section showed how parts of the teeth may separate during rotation. The gap between teeth is shown in Figure 5.12.

![Figure 5.12 - Gap between teeth of typical spline coupling](image_url)

5.3.1.3 Slip values

The slip values reported by Beasy are the magnitude of the tangential separation of mesh points of the two contacting surfaces that were initially together. It is important to realise that this is neither the total slip since the beginning of the application of load...
nor the slip between points since the previous load case. The distribution of slip value over the spline teeth as reported by Beasy is given for the 90° load case (load case 2) in Figure 5.13.

![Figure 5.13 - Slip values reported by Beasy for typical spline coupling, with misalignment in the 90° direction](image)

There are unexpected and unusual values for slip recorded at the corners of the teeth. The values here are not reasonable for the conditions being analysed. The error appeared to be within the Beasy software, and following discussions with the authors, this was found to be the case. Using the erroneous slip values would lead to large inaccuracies. To correct the results of the corner areas, the results given in the contact report file were modified by extrapolation of the results of mesh points known to be correct. The centre column of the upper and lower two rows of slip results was not affected by the errors and was used as the basis of the corrective modifications. The results for the outer column results were extrapolated from this centre column by matching the variation of slip in the profile direction of the third (and correct) row of results. As with the case of the negative pressures, the slip value errors were produced during interpolation of the nodal results to mesh point results, and hence the errors do not imply errors in the overall Beasy results. The corrected slip values are shown in Figure 5.14.

![Figure 5.14 - Corrected slip values for typical spline coupling, with misalignment in 90° direction](image)
A vector plot (Figure 5.15) of slip shows more clearly how various points of the tooth surface have moved relative to the adjacent point on the opposite surface, relative to the unloaded position.

Of perhaps more use in the understanding of wear, is how the teeth have moved relative to one another during each part of the rotation, i.e. the slip since the previous load case. This can be calculated from the basic results and is shown in Figure 5.16.
The vectors of slip can be combined through the sequential load cases to produce a plot of the slip path during a rotation of the spline. This is achieved through load cases 2 to 5 and returning to load case 2, and shows the path traced out by a fixed point on the female spline tooth on to the male tooth face. In rotating shafts, this path would be traced out continuously during the rotation. Load case 1 is not used for this purpose as it represents the movement from an unloaded position. The slip path obtained through sequential load cases is shown in Figure 5.17. The slip paths appear jagged, as only 90° steps are included; the actual slip path will, of course, be a smoothed version of this. The slip paths for each tooth should be, and are, similar as each tooth experiences the same contact conditions during a rotation of the coupling.

![Figure 5.17 - Slip path through load cases for typical spline teeth](image)

The similarity between load cases can be seen in all previous results. The patterns of pressure, gap, slip, etc., show almost identical forms for all of the load steps, but with the pattern being shifted through the teeth to remain consistent relative to the direction in which the misalignment is applied. This implies that load history does not have a significant impact. Load case 1 differs most from other load cases as it depicts the initial loading of the coupling rather than rotational movement. Most of the results
contained in the following section are based on a single load case, load case 2. The load history can become more influential with higher friction coefficients.

The similarity between load cases can also be taken advantage of in the generation of slip paths. Since the contact conditions for a particular tooth are dependent only on the direction of misalignment relative to that tooth and not on the load history, it is possible to determine the contact conditions through the rotation of the spline with the use of a single load case alone. Each tooth is used in turn to obtain the contact conditions of a tooth at a particular angle to the misalignment. This provides many more stages of information for a complete rotation, equal to the number of teeth on the coupling. The slip path obtained by this method ("through teeth") shows a smoother, more accurate form (Figure 5.18). The slip path should be the same irrespective of load case used.

![Figure 5.18 - Slip path obtained through teeth for typical spline coupling](image)

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The slip path shown here does not provide sufficient information to make conclusions about the slip occurring in spline couplings – it was seen previously that the teeth come in and out of contact at various locations over the tooth surface. Whilst the teeth are temporarily separated, the relative tangential slip will not lead to wear (assuming no wear debris or particles are in the contact). Figure 5.19 is a useful picture to explain this phenomenon; here, the darkness of the slip path is varied in proportion to the pressure occurring at that point on the tooth surface for the particular instant along the slip path. The darker lines show where, and in what direction, the most damaging slip is likely to occur.

Figure 5.19 - Slip path for typical spline coupling showing variation of pressure
5.3.1.4 Wear

As discussed in Section 2.3, the wear of a material is generally proportional to the load multiplied by slip distance. In the case of the closed loop slip path seen between the spline teeth, this can be restated as wear depth being proportional to the cyclic integral of pressure over slip path. This can be used to define a wear depth parameter of the form

\[ W = \int p \, ds \]

Thus, the wear depth at any point after a particular number of cycles will be proportional to \( W \). The detailed meaning of this wear depth parameter is discussed in Section 5.4.4. The slip paths can be calculated using the sequential load cases or through sequential teeth. The wear parameter of the typical spline coupling is shown using these two methods in Figure 5.20 and Figure 5.21 respectively. The wear parameter should be similar (or equal) whether calculated through teeth or through load cases, and irrespective of load case or tooth used.

![Figure 5.20 - Wear parameter for typical spline coupling, calculated through load cases](image-url)

18.9 MPa.mm
The wear depth parameter varies over the tooth surface, but is generally greatest at the corners of the contact area where pressures and slip values are highest. The maximum value of the wear depth parameter can be used to give an indication of the wear susceptibility for the coupling under the analysed loading conditions. The maximum value is divided by the product of elastic modulus and module to yield a non-dimensional wear parameter, \( W_{\text{max}}/E_m \), used in the non-dimensional parameter study.

### 5.3.1.5 Bending moment

Bending moment values have been calculated for each load case in directions parallel and perpendicular to the direction of applied misalignment. Additionally, any axial force produced by the loads has been calculated. The values for the typical spline coupling are shown in Table 5.5. Note that the moments in the direction perpendicular to the misalignment are non-zero.
Table 5.5 - Bending moments and axial force for typical spline coupling.

<table>
<thead>
<tr>
<th>Direction of Misalignment</th>
<th>Bending moment in direction of misalignment / Nmm</th>
<th>Bending moment perpendicular to direction of misalignment / Nmm</th>
<th>Axial Force / N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>2.82 x 10^5</td>
<td>-2.68 x 10^3</td>
<td>288</td>
</tr>
<tr>
<td>90°</td>
<td>2.84 x 10^5</td>
<td>8.61 x 10^3</td>
<td>464</td>
</tr>
<tr>
<td>180°</td>
<td>2.82 x 10^5</td>
<td>9.00 x 10^3</td>
<td>505</td>
</tr>
<tr>
<td>270°</td>
<td>2.84 x 10^5</td>
<td>8.59 x 10^3</td>
<td>522</td>
</tr>
<tr>
<td>360°</td>
<td>2.82 x 10^5</td>
<td>9.02 x 10^3</td>
<td>504</td>
</tr>
</tbody>
</table>

A positive value for perpendicular bending moment indicates a bending moment in advance of the rotating misalignment. (i.e. for the 90° load case, the direction of the perpendicular bending moment is in the 180° direction.)

The bending moments in the perpendicular direction are two orders of magnitude lower. However, later analyses showed the direction of this bending moment was not consistent between analyses – accuracy of results for this component is discussed in Section 5.4.5. The perpendicular bending moment for the first load case differs from the remainder since it represents a movement from an aligned position unlike the remainder.

The bending moment in the direction of misalignment (which is similar for all load cases) is converted to non-dimensional form using the applied torque, giving a further result parameter used for the parameter study – $M/T$. Similarly, the perpendicular moment is reported as $M_{perp}/T$.

5.3.1.6 Summary of results parameters

Five different quantitative non-dimensional results parameters are obtained from the computational analyses. A large amount of qualitative data is also provided by the
graphical images of pressure distributions, tooth gaps and slip paths. The non-dimensional results parameters are listed in Table 5.6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\text{max}}$</td>
<td>$= c / b$, where $c$ is maximum displacement of centre of pressure along facewidth (from centre of tooth) $b$ is facewidth</td>
</tr>
<tr>
<td>$P_{\text{max}} / P_{\text{av}}$</td>
<td>maximum tooth load / average tooth load</td>
</tr>
<tr>
<td>$W / E m$</td>
<td>$W$ is integral of contact pressure around slip path for one revolution</td>
</tr>
<tr>
<td>$M / T$</td>
<td>$M$ is bending moment in direction of misalignment $T$ is applied torque</td>
</tr>
<tr>
<td>$M_{\text{perp}} / T$</td>
<td>$M_{\text{perp}}$ is bending moment in direction perpendicular to misalignment</td>
</tr>
</tbody>
</table>

Table 5.6 - Non-dimensional result parameters

### 5.3.2 Basic design and operating parameters study

A total of 35 analyses have been carried out as part of the parameter study. The six non-dimensional parameters were all varied and several different modules of spline were used (the module is used to scale many of the dimensional parameters).

#### 5.3.2.1 Pressure distributions

The pressure distribution has been found to vary significantly between different analyses. For most of the analyses carried out, the dependence of the distribution on load history is negligible, i.e. the results for load cases 3 to 5 are similar to load case 2 with the tooth numbers displaced appropriately. The form of pressure distribution has been found to depend on the ratio of misalignment and torque, or, in dimensionless terms, $E^* \times \theta$. The pressure distribution for the 90° load case of selected analyses is given in Figure 5.22. The spline coupling geometry is identical for each analysis shown; only misalignment angle and torque are varied.
The pressure distributions have been classified into four distinct forms, or “contact regimes”:

**Uniform** – all teeth are in full contact. (a)

**Cyclic** – some teeth are not in full contact; all teeth remain partially in contact. Implies that each tooth will remain partially in contact at all times but an area of the tooth will separate from the mating tooth during rotation. The parameter study results show that one end (the upper end) may remain in contact at all times with the opposite end separating (b) or each end of the tooth may alternatively separate (c and d).

**Discontinuous** – one or more teeth are not in contact at all. Implies that each tooth will become fully unloaded during the rotation. (e)
Toppled – this term has been introduced to describe a condition in which contact on one side of the tooth alone cannot be maintained. This is explained in Section 5.4.2.2. As the model does not allow reverse face contact, this condition cannot be analysed correctly. It is identified in the Beasy results by a supposed net “negative tooth load” which is erroneously given as the result. With the computational model as it is, there is no equilibrium condition which can be obtained.

The summary table in Section 5.3.2.7 classifies the pressure distribution for each analysis.

For each analysis, it was necessary to check for any reverse face overlap by examining the deformed shape within the Beasy IMS. Reverse face overlap was observed in analyses PS2 and PS3, and both of these had already been rendered invalid due to a toppled regime. PS 5 showed a small amount of contact at the ends of two teeth, but no significant overlap, and so the results of this analysis have been included in the study. No other analyses showed reverse face overlap.

The variation in pressure distribution with the geometric parameters $b^*$, $z$ and $\alpha$ can be seen in Figure 5.23 to Figure 5.25. The effect of friction coefficient is shown in Figure 5.26.

![Figure 5.23 - Effect of dimensionless facewidth, $b^*$, on pressure distribution](image)

- a) $E^*=6.56; \theta=0.1; b^*=5.4; z=18; \alpha=30; \mu=0.3$
- b) $E^*=6.56; \theta=0.1; b^*=10.8; z=18; \alpha=30; \mu=0.3$
- c) $E^*=6.56; \theta=0.1; b^*=15.2; z=18; \alpha=30; \mu=0.3$
Figure 5.24 - Effect of number of teeth, z, on pressure distribution

a) \( E^* = 6.56; \theta = 0.1; b^* = 10.8; z = 18; \alpha = 30; \mu = 0.3 \)
b) \( E^* = 6.56; \theta = 0.1; b^* = 10.8; z = 21; \alpha = 30; \mu = 0.3 \)
c) \( E^* = 6.56; \theta = 0.1; b^* = 10.8; z = 24; \alpha = 30; \mu = 0.3 \)
d) \( E^* = 6.56; \theta = 0.1; b^* = 10.8; z = 27; \alpha = 30; \mu = 0.3 \)
Two dimensionless result parameters, $C_{\text{max}}$ and $P_{\text{max}}/P_{\text{av}}$, were introduced in Section 5.3.1 and provide a quantitative evaluation of the pressure distribution. The pressure
distributions presented in Figure 5.22 show an increasing “skewness” of pressure distribution; the corresponding values for $C_{\text{max}}$ and $P_{\text{max}}/P_{\text{av}}$ are given in Table 5.7.

<table>
<thead>
<tr>
<th></th>
<th>$C_{\text{max}}$</th>
<th>$P_{\text{max}}/P_{\text{av}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.215</td>
<td>1.028</td>
</tr>
<tr>
<td>b</td>
<td>0.375</td>
<td>1.105</td>
</tr>
<tr>
<td>c</td>
<td>0.422</td>
<td>1.227</td>
</tr>
<tr>
<td>d</td>
<td>0.422</td>
<td>1.229</td>
</tr>
<tr>
<td>e</td>
<td>0.500</td>
<td>1.618</td>
</tr>
</tbody>
</table>

**Table 5.7 - $C_{\text{max}}$ and $P_{\text{max}}/P_{\text{av}}$ parameters for Figure 5.22**

These parameters allow the effects of each input parameter to be directly compared. Couplings c) and d) of Figure 5.22 have almost identical pressure distribution forms (although the magnitudes of the pressure differ). It was noted that the value of $E^*\theta$ for these two analyses were identical, and it became apparent that the pressure distribution was dependent on the ratio of misalignment ($\theta$) and torque (a denominator in $E^*$). Contours of the parameters $C_{\text{max}}$ and $P_{\text{max}}/P_{\text{av}}$ for one particular coupling geometry (maintaining all four other dimensionless parameters constant) have been plotted on axes of $\theta$ and $E^*$ in Figure 5.27 and Figure 5.28. Note that $C_{\text{max}}$ can not exceed 0.5, but the spline coupling does not immediately topple once this value is reached. There is therefore a wide band of conditions which have a value of 0.5, as indicated by the arrow in Figure 5.27. This is discussed in Section 5.4.2
Figure 5.27 - Contours of $C_{\text{max}}$ for $\theta$ and $E^*$

\[ (\theta^* = 10.8; \ z = 18; \ \alpha = 30; \ \mu = 0.3) \]

Figure 5.28 - Contours of $P_{\text{max}}/P_{\text{av}}$ for $\theta$ and $E^*$

\[ (\theta^* = 10.8; \ z = 18; \ \alpha = 30; \ \mu = 0.3) \]
The variation of the parameters $C_{\text{max}}$ and $P_{\text{max}}/P_{\text{av}}$ with the geometry parameters $b^*$ and $z$ are given in Figure 5.29 to Figure 5.32. Two different conditions/geometries were analysed at both 20° and 30° pressure angles. The pressure angle had a negligible effect on $C_{\text{max}}$ for either case. Decreasing the pressure angle from 30° to 20° did, however, cause the value of $P_{\text{max}}/P_{\text{av}}$ to increase from 1.227 to 1.240 (18 tooth model) and from 1.497 to 1.559 (21 tooth model).

![Graph showing the effect of dimensionless facewidth, $b^*$, on $C_{\text{max}}$.](image)

Figure 5.29 – Effect of dimensionless facewidth, $b^*$, on $C_{\text{max}}$

Condition 1 - $E^*=3.28; \theta=0.1^\circ; z=18; \mu=0.3; \alpha=30^\circ$

Condition 2 - $E^*=6.56; \theta=0.1^\circ; z=18; \mu=0.3; \alpha=30^\circ$

Condition 3 - $E^*=3.28; \theta=0.111-0.116^\circ; z=18; \mu=0.3; \alpha=30^\circ$

Condition 4 - $E^*=3.28; \theta=0.213-0.229^\circ; z=18; \mu=0.3; \alpha=30^\circ$

Condition 5 - $E^*=6.56; \theta=0.111-0.107^\circ; z=18; \mu=0.3; \alpha=30^\circ$

NOTE: Due to the method of misalignment application, the values of $\theta$ vary slightly for conditions 3-5. The analyses have been grouped together to provide more comparative data. For the small range of $\theta$ covered, the effects of facewidth dominate.
Figure 5.30 - Effect of dimensionless facewidth, $b^*$, on $P_{\text{max}}/P_{\text{av}}$

Conditions as for Figure 5.29

Figure 5.31 - Effect of number of teeth, $z$, on $C_{\text{max}}$

$E^* = 3.28; \theta = 0.1^\circ; b^* = 10.8; \mu = 0.3; \alpha = 30^\circ;$
Figure 5.32 - Effect of number of teeth, $z$, on $P_{max}/P_{av}$

The friction coefficient also affects the pressure distribution significantly (Figure 5.33 and Figure 5.34)

Figure 5.33 - Effect of friction coefficient, $\mu$, on $C_{max}$

$(\theta=0.1; b^*=10.8; z=18; \alpha=30^\circ)$

Figure 5.34 - Effect of friction coefficient, $\mu$, on $P_{max}/P_{av}$
5.3.2.3 Stick / slip regime

The stick/slip behaviour of the contact is vital in assessing the type of fretting damage which may occur. The majority of the analyses carried out were reported as being in slip at all times. There were occasionally one or two mesh points reported as being in stick, which depended upon load case. These points tended to be at the points of high contact pressure rather than low slip, and were not always consistent throughout the load cases. These isolated points are unlikely to affect the nature of the wear. An example of this is shown in Figure 5.35c. Note that even with the lowest misalignment angle of just 0.01°, the tooth surface is in slip throughout.

![Figure 5.35 - Stick-slip behaviour for selected analyses with predominant slip](image)

a) slip throughout  
\[(E^* = 6.56; \theta = 0.2; b^* = 10.8; z = 18; \mu = 0.3; \alpha = 30^\circ)\]

b) slip throughout with low misalignment angle  
\[(E^* = 6.56; \theta = 0.01; b^* = 10.8; z = 18^\circ; \mu = 0.3; \alpha = 30)\]

c) gross slip with occasional points of isolated stick  
\[(E^* = 8.31; \theta = 0.1; b^* = 5.4; z = 18; \mu = 0.3; \alpha = 30^\circ)\]

With a higher value of torque and low misalignment angle, some areas of the tooth do begin to stick during the rotation. Increasing the coefficient of friction also results in more areas moving into the stick regime and an entire tooth may stick under certain conditions. Figure 5.36 shows such conditions.
Figure 5.36 - Stick-slip behaviour for conditions resulting in significant areas of stick

a) small stick area despite low friction coefficient

\[(E^* = 3.28; \theta = 0.1; b^* = 10.8; z = 18; \mu = 0.3; \alpha = 30^\circ)\]

b) larger areas of stick with increasing friction coefficient but lower torque

\[(E^* = 6.56; \theta = 0.1; b^* = 10.8; z = 18; \mu = 0.7; \alpha = 30^\circ)\]

c) full tooth in stick condition with high torque and friction coefficient

\[(E^* = 3.28; \theta = 0.1; b^* = 10.8; z = 18; \mu = 1.0; \alpha = 30^\circ)\]

The areas of stick are not entirely consistent through each load case (Figure 5.37). This is entirely reasonable for the first load case of each analysis, in which the male coupling is displaced from the unloaded, central position.
5.3.2.4 Slip paths

The slip path of the typical spline coupling presented in Section 5.3.1.3 is typical of many of the analyses. The changing directions of the loop axes are similar throughout, with the major axis approximately vertical towards the centre of the tooth, turning as would be expected closer to the tooth ends. The "openness" of the loop also generally increased further from the centre. A discontinuous "bend" in the loop is also common, usually on the upper half of the tooth.

The precise shape of the path loop (the openness, the angle of the major axes, the bend) does vary substantially for some of the analyses. The amplitude, or total slip distance, is affected by all the parameters, particularly tooth number (z) which implies a change in diameter. Although the amplitude changes, the shape of the slip path is not greatly affected by the diameter, either through increasing z or increasing module (for which the facewidth also increases proportionally). Figure 5.38 shows the slip path for such analyses of varying diameter, as calculated through sequential teeth.
As was the case with pressure distribution form, the slip path form appears to vary with the ratio of misalignment and torque ($E^*\theta$). The slip amplitude does increase with misalignment (Figure 5.39 a and b) – a higher torque does not, therefore, negate the increase in slip amplitude produced by the greater misalignment angle. If the increase in torque leads to areas moving into the stick regime, however, then a reduction of slip amplitude could occur. Increasing the torque, or decreasing the misalignment angle, tends to close the loop. This is most apparent in the upper half of Figure 5.39c.
Although the slip path does change in the above cases, it may have very little effect on the wear pattern. Since the tooth may not be loaded for much of the cycle, the slip path will only be loaded for approximately half of its length, and thus the shape of the path in the unloaded section will not be of great importance for wear itself. The shape may affect the movement of debris and the availability of lubricant in that area.

The pressure angle has been seen to have little effect on the pressure distribution. It also has only a small effect on the slip amplitude, although there is a noticeable change in the form of the slip path when changing the pressure angle from just 30° to 20° (Figure 5.40). With the lower pressure angle, the loop becomes much straighter and more "pointy". Additionally, for the 20° pressure angle, the slip path crosses over itself for the upper half of the tooth, forming a figure 8, rather than a loop.
The shape of the slip path is affected by the facewidth. Near the centre of the tooth, there is little change but with the greater extent of the larger facewidth, the upper and lower extremities experience vastly different slip path shapes (Figure 5.41).

The friction coefficient affects the slip amplitude and, to some extent, the slip path shape (Figure 5.42). If an increase in friction coefficient moves the coupling into a stick-slip regime, then the slip path shape and amplitude can change dramatically (Figure 5.43).

Figure 5.40 – Effect of pressure angle, $\alpha$, on slip path

a) $E^*=6.56; b^*=10.8; z=18; \alpha=30^\circ; \mu=0.3; \theta=0.1^\circ$

b) $E^*=6.56; b^*=10.8; z=18; \alpha=20^\circ; \mu=0.3; \theta=0.1^\circ$
Figure 5.41 - Effect of dimensionless facewidth, $b^*$, on slip path

a) $E^*=6.56; \theta=0.1^\circ; b^*=5.4; z=18; \alpha=30^\circ; \mu=0.3$

b) $E^*=6.56; \theta=0.1^\circ; b^*=10.8; z=18; \alpha=30^\circ; \mu=0.3$

c) $E^*=6.56; \theta=0.1^\circ; b^*=15.2; z=18; \alpha=30^\circ; \mu=0.3$
Figure 5.42 – Effect of friction coefficient, $\mu$, on slip path

a) $E^* = 6.56; \theta = 0.1^\circ; b^* = 10.8; z = 18; \alpha = 30; \mu = 0.3$

b) $E^* = 6.56; \theta = 0.1^\circ; b^* = 10.8; z = 18; \alpha = 30; \mu = 0.7$
Figure 5.43 - Slip path change for friction coefficient producing transition from gross slip to stick-slip

a) $E^* = 3.28$; $\theta = 0.1^\circ$; $b^* = 10.8$; $x = 18$; $\alpha = 30$; $\mu = 0.3$

b) $E^* = 3.28$; $\theta = 0.1^\circ$; $b^* = 10.8$; $x = 18$; $\alpha = 30$; $\mu = 1.0$

5.3.2.5 Wear

The wear depth parameter has been calculated for each analysis using the "through teeth" method. The wear depth parameter appears to be predominantly pressure driven, with the areas experiencing the highest pressures having the greater wear depth. Thus the distribution of wear parameter varies with the dimensionless parameters in a similar fashion to the pressure distribution. The wear distribution of a number of analyses is shown in Figure 5.44.
The maximum wear depth parameter is used to determine the severity of wear for each analysis. The value for the second load case is used. This maximum occurs at the upper left mesh point for all analyses. The analysis with a friction coefficient of 1.0, however, differs from all other analyses; for the third, forth and fifth load cases, the maximum wear depth parameter is not at the upper left hand corner, but the lower left hand corner. This is the only instance in which this behaviour has been found and the higher dependence on load history for greater friction is expected. The use of sequential teeth to represent the rotation of the coupling is therefore not entirely appropriate for this case. Using sequential load cases for this case gives a maximum dimensional wear depth parameter of 1.79 MPa.mm, whilst using the through teeth method gives 1.39 MPa.mm for the second load case and around 2.56 MPa.mm for load cases three to five. All other analyses for which a complete rotation was modelled result in similar values for wear parameter irrespective of calculation method or load case used.

The effect of the dimensionless parameters on the maximum wear depth parameters is fundamental to the objectives of the research. The effects of the design and operating parameters on the non-dimensional form of the wear parameter are presented in Figure 5.45 to Figure 5.47.
Figure 5.45 - Contours of $W/Em$ for $E^*$ and $\theta$

$\left( b^*=10.8; \ z=18; \ \alpha=30^\circ; \ \mu=0.3 \right)$

Figure 5.46 - Effect of dimensionless facewidth, $b^*$, on $W/Em$

Condition 1 - $E^*=3.28; \ \theta=0.1^\circ; \ z=18; \ \mu=0.3; \ \alpha=30^\circ;$

Condition 2 - $E^*=6.56; \ \theta=0.1^\circ; \ z=18; \ \mu=0.3; \ \alpha=30^\circ;$

Condition 3 - $E^*=3.28; \ \theta=0.111-0.116^\circ; \ z=18; \ \mu=0.3; \ \alpha=30^\circ;$

Condition 4 - $E^*=3.28; \ \theta=0.213-0.229^\circ; \ z=18; \ \mu=0.3; \ \alpha=30^\circ;$

Condition 5 - $E^*=6.56; \ \theta=0.111-0.107^\circ; \ z=18; \ \mu=0.3; \ \alpha=30^\circ;
Two conditions were analysed at pressure angles of both 20° and 30°. In both instances, reducing the pressure angle lead to a rise in maximum wear depth parameter of around 12% (Table 5.8).

<table>
<thead>
<tr>
<th>Condition</th>
<th>$W/Em$ for $\alpha=30^\circ$</th>
<th>$W/Em$ for $\alpha=20^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $E^<em>=6.56; b^</em>=10.8;$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z=18; \mu=0.3; \theta=0.1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37.6 x $10^{-6}$</td>
<td>42.9 x $10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>2. $E^<em>=6.56; b^</em>=10.8;$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z=21; \mu=0.3; \theta=0.1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>59.0 x $10^{-6}$</td>
<td>65.9 x $10^{-6}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.8 - Effect of $\alpha$ on wear depth parameter, $W/Em$

When considering the effect of friction coefficient on wear depth parameter, it is important to recall that the wear parameter is solely the integral of pressure over the slip path, not an accurate expression of the actual wear depth. The implications of this are dealt with in 5.4.4, but Figure 5.48 shows how increasing the friction coefficient may actually lower the wear depth parameter.
5.3.2.6 Bending moments

The bending moment in the direction of the applied misalignment, and in the vertical plane perpendicular to this, has been calculated for each analysis. In the direction of misalignment, the bending moment for each load case of an analysis generally varies by no more than around 3%. In the perpendicular direction, the bending moments ($M_{p_{\text{perp}}}$) calculated for load cases 2 to 4 vary by a great deal more than this, sometimes 75% or more. The values in the perpendicular direction are typically some orders of magnitude smaller, and thus computational errors will be more significant. The perpendicular bending moment for load case 1 is not consistent with the following values, and in some cases occurs in the opposite direction to load case 5 (for which the misalignment is applied in the same direction). This might be explained by the fact that, for the first load case, the coupling is moved from the unloaded central position, whereas for the subsequent load cases the movement is from an alternative position. This section uses the second load case for all analyses to provide some consistency.

For a given misalignment angle, the value of bending moment in the direction of misalignment depends upon the effective stiffness of the coupling. This moment, and hence stiffness, is not only a function of the spline geometry, but also of the applied torque and, indeed, the magnitude of the misalignment. Contours of $M/T$ are plotted for $E^* \nu \theta$ in Figure 5.49, and once more, the $E^* \theta$ dependency appears to hold true.
Figure 5.49 - Contours of $M/T$ for $E^*$ and $\theta$

($b^*=10.8; \ z=18; \ \alpha=30; \ \mu=0.3$)

The effect of the facewidth and tooth number parameters is shown in Figure 5.50 and Figure 5.51.
Decreasing the pressure angle from 30° to 20° led to a reduction in bending moment of around 10%. Increasing friction coefficient had the effect of increasing stiffness and thus bending moment (Figure 5.52).
The bending moment in the perpendicular direction shows a much less consistent behaviour with each parameter. The variation of $M_{\text{perp}}/T$ with the dimensionless operating and design parameters is shown in Figure 5.53 to Figure 5.56 and Table 5.9. The magnitude of the perpendicular bending moment is, for almost all analyses, less than 5% of the main bending moment.

Figure 5.52 - Effect of friction coefficient, $\mu$, on $M/T$

$E^*=3.28$

$E^*=6.56$

$(\theta=0.1; b^*=10.8; z=18; \alpha=30^\circ)$

Figure 5.53 - Contours of $M_{\text{perp}}/T$ for $E^*$ and $\theta$

$(b^*=10.8; z=18; \alpha=30; \mu=0.3)$
Figure 5.54 - Effect of dimensionless facewidth, $b^*$, on $M_{\text{perp}}/T$

Condition 1 - $E^*=3.28; \theta=0.1^\circ; z=18; \mu=0.3; \alpha=30^\circ$
Condition 2 - $E^*=6.56; \theta=0.1^\circ; z=18; \mu=0.3; \alpha=30^\circ$
Condition 3 - $E^*=3.28; \theta=0.111-0.116^\circ; z=18; \mu=0.3; \alpha=30^\circ$
Condition 4 - $E^*=3.28; \theta=0.213-0.229^\circ; z=18; \mu=0.3; \alpha=30^\circ$
Condition 5 - $E^*=6.56; \theta=0.111-0.107^\circ; z=18; \mu=0.3; \alpha=30^\circ$

Figure 5.55 - Effect of number of teeth, $z$, on $M_{\text{perp}}/T$

$E^*=3.28; \theta=0.1; b^*=10.8; \mu=0.3; \alpha=30^\circ$
### Table 5.9 - Effect of pressure angle, $\alpha$, on $M_{\text{perp}}/T$

<table>
<thead>
<tr>
<th>Condition 1.</th>
<th>$M_{\text{perp}}/T$ for $\alpha=30^\circ$</th>
<th>$M_{\text{perp}}/T$ for $\alpha=20^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^<em>=6.56; \beta^</em>=10.8; z=18; \mu=0.3; \theta=0.1$</td>
<td>$17.2 \times 10^{-3}$</td>
<td>$28.9 \times 10^{-3}$</td>
</tr>
<tr>
<td>Condition 2.</td>
<td>$24.0 \times 10^{-3}$</td>
<td>$24.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>$E^<em>=6.56; \beta^</em>=10.8; z=21; \mu=0.3; \theta=0.1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Figure 5.56 - Effect of friction coefficient, $\mu$, on $M_{\text{perp}}/T$

$(\theta=0.1; \beta^*=10.8; z=18; \alpha=30)$

#### 5.3.2.7 Summary of results

The details of all analyses are shown in Table 5.10, together with the result parameters, the pressure distribution form and the stick/slip regime.
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5.3.3 Further design and manufacturing considerations

One analysis has been carried out to look at the effects of crowning, and a second analysis has investigated the effects of a manufacturing error – a pitch error of a single tooth. Both analyses used the typical spline coupling geometry and conditions reported in Section 5.3.1.

5.3.3.1 Crowning

A crowning in the lead direction (along the axis of the coupling) was applied to each tooth of the coupling. The crowning was applied symmetrically, and the radius of crowning was 3.6 m, giving an initial gap of 25 μm at each end of the tooth. The crowning is applied by varying the specified initial gap between the mating teeth for each of the mesh points.

The pressure distribution (Figure 5.57), slip paths (Figure 5.58), and wear parameter (Figure 5.59) for both the crowned and non-crowned couplings are shown below. The operating conditions for each analysis were the same (torque of 500 Nm, misalignment of 0.1°, friction coefficient of 0.3).

![Pressure distribution for coupling](image)

**Figure 5.57 - Pressure distribution for coupling a) with crowning and b) without crowning**
Figure 5.58 - Slip paths for coupling a) with crowning and b) without crowning

Figure 5.59 - Wear parameter distribution for coupling a) with crowning and b) without crowning

The crowning has resulted in the contact occurring towards the centre of the tooth rather than at the ends and a reduction in wear parameter of about 50%. The bending moment generated also fell with $M/T$ for the crowned spline being 0.33 compared to 0.57.
5.3.3.2 Pitch error

The effect of pitch error will depend on which teeth are incorrectly spaced, and by how much. For example, if every second tooth is 20 \(\mu\)m advanced, then the contact may occur on these teeth alone, i.e. only 50\% of teeth in contact. If one side of the coupling has all teeth advanced by 20 \(\mu\)m, then some teeth on the opposite side must still come into contact. Accumulative pitch error further complicates the generalisation of pitch error.

The analysis carried out here was of a single tooth advanced by 25 \(\mu\)m. This was achieved by specifying an initial gap of 0.0 \(\mu\)m for a single tooth, and 25 \(\mu\)m for the remaining teeth. The coupling geometry and operating conditions are as for the crowning analysis. The contact conditions for each tooth will depend on the relative position of the tooth error in addition to the misalignment; therefore, unlike all previous analyses, the form of the pressure distribution differs for each load case. The pressure distribution is shown in Figure 5.60. Despite the higher contact pressures, all teeth remain in the slip regime. Note that the two teeth either side of tooth 6 are permanently out of contact.
Figure 5.60 - Pressure distribution for coupling with pitch error on tooth 6

The wear parameter for each tooth is shown in Figure 5.61. This is calculated through sequential load cases. The analysis with no pitch error gave a maximum wear parameter of 18.9 MPa.mm using this method.

Figure 5.61 - Wear parameter distribution for coupling with pitch error on tooth 6

The slip path for each tooth varies, although the form is generally the same as for without the pitch error for most teeth. For the advanced tooth, the slip path appears to be a reciprocating arc, rather than a loop, i.e. the loop is fully closed, particularly over the upper half of the tooth.
5.3.4 Spline wear test analyses

An analysis was carried out for each spline wear test conducted. Two significant simplifications were necessary:

A friction coefficient had to be assumed for each analysis, most used 0.3. Comparisons of different methods of lubrication cannot be made based on results for different friction coefficients alone – the wear parameter assumes a particular wear coefficient that will change with lubrication method.

The misalignment angle of the coupling is assumed to be that of the complete shaft arrangement, i.e. the shaft is assumed not to bend. Although this will not be the case, it is not possible to accurately predict what proportion of misalignment is carried through the coupling and what proportion through the shaft.

Initially, each analysis was run with friction coefficient of 0.3 and with a tooth contact mesh of 2 x 10 elements, as with the parameter study analyses. Given that only three teeth were present, it is not possible to use sequential teeth as a method of obtaining results. The results will also be highly dependent on misalignment direction, which may act towards a tooth or a space. Thus a complete revolution was analysed with 9 load steps of 45°, and all results used sequential load cases. Additionally, one load case is not sufficient to represent the typical pressure distribution – the pressure distribution plots in this section show each load case for a single tooth. The contact was reported to be in slip for all analyses, although there were some isolated points of stick as found in the parameter study.

The pressure distributions for some of the analyses, using $\mu = 0.3$, are shown in Figure 5.62.
The slip path during the rotation for each tooth shows some unusual characteristics not seen in the parameter study. The slip path is not smooth, but there are several sharp vertices. Also, the slip path for each tooth can differ. This suggests that the number of load cases is not sufficient to accurately predict the slip path, although a less smooth slip path would be expected due to the tooth-space difference.

The wear parameter distribution and maximum wear parameter are shown in Figure 5.63 together with a picture of a worn tooth from the corresponding wear test(s).
W max = 314 MPa.mm
a) SWT 1
0.3°, 935 Nm
2.5 mm module
Wetted

W max = 319 MPa.mm
b) SWT 5 and 2
0.15°, 935 Nm
1.5 mm module
Wetted and Immersed

W max = 795 MPa.mm
c) SWT 3 and 4
0.3°, 935 Nm
1.5 mm module
Wetted and Immersed

W max = 464 MPa.mm
d) SWT 6
0.22°, 935 Nm
1.5 mm module
Wetted

W max = 321 MPa.mm
e) SWT 7
0.3°, 750 Nm
2.5 mm module
Wetted

Figure 5.63 - Computed wear parameter (using $\mu=0.3$) and actual worn tooth for spline wear tests
The lubrication effects are not included within the computational wear parameter and so analyses correspond to both immersed and wetted tests for similar conditions. The friction coefficient, however, is likely to vary between the wetted and immersed lubrication tests. Figure 5.64 shows the effect of friction coefficient on the computational results. This does not permit comparison between the wear parameter values for wetted and immersed tests (since the excluded wear coefficient is dramatically changed), but does give an indication of the likely change in wear pattern (noticeable at the lower end).

\[ \mu = 0.3 \quad \text{SWT 2} \quad \text{Immersed} \]

\[ \mu = 0.6 \quad \text{SWT 5} \quad \text{Wetted} \]

**Figure 5.64 - Effect of friction coefficient on wear parameter distribution**
The bending moment for each analysis and the bending moment recorded during the experimental test are shown in Table 5.11.

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Bending Moment from Test / Nm</th>
<th>Bending Moment from Analysis / Nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWT 1</td>
<td>570 - 350</td>
<td>682</td>
</tr>
<tr>
<td>SWT 2</td>
<td>340 (immersed)</td>
<td>522 ($\mu = 0.3$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>615 ($\mu = 0.6$)</td>
</tr>
<tr>
<td>SWT 3</td>
<td>560 - 380</td>
<td>725</td>
</tr>
<tr>
<td>SWT 4</td>
<td>420</td>
<td>725</td>
</tr>
<tr>
<td>SWT 5</td>
<td>370 (wetted)</td>
<td>522 ($\mu = 0.3$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>615 ($\mu = 0.6$)</td>
</tr>
<tr>
<td>SWT 6</td>
<td>430 - 310</td>
<td>678</td>
</tr>
<tr>
<td>SWT 7</td>
<td>630 - 420</td>
<td>595</td>
</tr>
</tbody>
</table>

Table 5.11 - Computed and actual bending moments for spline wear tests

For the tests in which the bending moment changed through the duration, it is most appropriate to compare the initial value, for which the tooth remains in its original state as used for the computation. The computed bending moment is significantly greater than that recorded for the test for most analyses. This is discussed in Section 5.4.7.

5.4 Discussion

5.4.1 General assessment of computational method

The computational modelling had two primary purposes: obtaining detailed information of the contact between spline teeth for a coupling operating under misalignment and providing a means by which different designs of couplings could be quickly and easily compared for wear susceptibility. The work has indeed shown how the contacting teeth move about and their loading pattern. Although some of the results are intuitive (such as contact occurring at ends of misaligned spline teeth), the
work has explicitly identified different types of contact regimes, and the conditions for which the different regimes will occur. The second objective, which is of immediate practical benefit, has been achieved through the formulation of equations based on the dimensionless parameters and these are presented in Sections 5.4.2 to 5.4.5.

Since this is a model of a real system, it is necessary both to review the accuracy of the results and how well the model represents the actual system. Several aspects can lead to differences between the computational model and a real coupling:

Material removal. After a short period of time, any wear that has occurred will lead to a variation in tooth form which will, in turn, alter the contact behaviour. This is not included in the computational model, which assumes perfect, unworn teeth. The modelling therefore will give a better indication of the initial behaviour of a coupling, but may not reveal wear which occurs at a later stage. For example, some spline tests showed wear over the whole tooth width for which the computational analysis predicted contact at the ends alone. Material removal at the ends of the teeth would eventually lead to contact and wear occurring at the centre, which would not have been identified computationally.

Lubrication effects. It has already been seen that increasing the friction coefficient can lead to a reduction in the wear parameter obtained for a model and this could lead to the wrong assumption that removing lubrication might be beneficial. This is not a problem with the modelling, but one of possible misinterpretation. The lubrication could, however, alter the actual contact behaviour; the cyclically moving contact region seen in some of the analyses is, in effect, a rocking motion, which could lead to a hydrodynamic lubrication film. This could affect the distribution of load over the tooth surface. This is only relevant for fully lubricated contacts, and it is unlikely that it would change the general form of the contact behaviour or the ranking of different couplings.

Wear debris. The spline wear tests showed some wear features that are likely to be caused by wear particle action; these features cannot be identified computationally. Like many analytical approaches to wear, the modelling assumes a two-body behaviour for what may often be a third-body problem. With fundamental wear studies, this is particularly problematic as the wear mechanism may be highly
dependent on the third body. However, the modelling of the spline couplings is on a macro-scale rather than looking at the contact interface on a micro-scale, and so this does not significantly affect the findings. An investigation of the transport of wear particles was one objective of this study; the slip information can help in an estimation, but the fretting visualisation studies have shown how complex and unpredictable this subject might be.

Model type. The above imperfections of the modelling have been due to the limitations of current software technology. The design of the model itself is just as important. The application of a misalignment could have been achieved in a number of ways, each with their own benefits and problems. The method used has the benefit of preventing any shear force acting on the system. This would add an extra parameter to the problem and would necessitate a great deal more analyses. There could be a shear force acting on an actual coupling, which may or may not depend on and/or generate the angular misalignment. However, for most spline coupling set-ups, this is likely to be a minor contributor to the contact behaviour. The method used also has the benefit that a precise misalignment angle is known which can be related to practical situations.

The problems associated with this method are that the distance at which the misalignment is applied will affect the actual coupling misalignment. In addition, the axis of misalignment (the horizontal axis about which the male coupling is displaced to give the misalignment angle) is set, arbitrarily, to half way along the spline teeth. The choice for position of this axis should not affect results greatly – the most likely consequence is a net horizontal force on the female coupling, which will be negated (as for shear forces) by movement of the female coupling which is free to float in the horizontal plane.

Load steps and model geometry. The use of more than four load steps to represent a full rotation would have been preferred, but would have greatly increased the solution time and hardware resources required. Fortunately, the ability to use sequential teeth instead of sequential load cases for some of the analyses, made possible by the insensitivity of most results to load history, has avoided any real need for more load cases. To improve speed, the fine details of the spline, such as chamfering of the
ends of the teeth, and accurate representation of the roots, have been disregarded. Given the coarseness of the mesh used, this will have no effect on the contact results.

The potential accuracy of the boundary element method has proved to be very high in previous benchmark analyses carried out by the software authors. The accuracy in this study is limited by the mesh used. The number of elements on the contact region has the most influence on the results of this study. The mesh used is too coarse to provide a very detailed pressure distribution in the profile direction or actual pressure values at precise locations, but it does show up the fact that the pressures are greater at the tips. In the axis of the coupling, the ten elements provide a good variation of pressure along the facewidth that is sufficient to show the variations between different contact regimes. A higher resolution of contact mesh was not feasible, and would not necessarily have provided any more information that would be useful for this study. This is particularly true at the edges of the contact where a singularity exists – increasing the mesh density would have led to ever increasing pressures. The high pressures would cause plasticity in a real coupling which is not included in the boundary element software. The actual values for contact pressures are averaged over the element regions and so must be thought of as a comparative guide between models. Consequentially, all results must be considered as averaged over a small area around the mesh point.

The use of non-dimensional parameters within the study should facilitate comparisons of spline couplings of different sizes (modules). Each analysis model was originally created with misalignments specified at the ends of 50 mm shafts, which is equal to the defined misalignment angle only for 2.5 mm module splines. For other modules, the misalignment angle has been calculated from the imposed shaft misalignment angle, the reported bending moment and the shaft flexure. However, to improve accuracy, the parameter equations presented in the following sections are optimised for the 2.5 mm module analyses.

5.4.2 Pressure distributions

5.4.2.1 General characteristics of pressure distributions

The pressure distribution plots presented in this chapter provide the best way of visualising how the teeth contact. Different forms of pressure distributions have been
identified, but all show some similar characteristics. Some of these can be identified in the pressure distribution of a fully aligned coupling.

Figure 5.65 - Pressure distribution for coupling with no misalignment

Figure 5.65 shows the pressure distribution for a coupling with no misalignment. The pressure distribution is not uniform over the tooth surface but rises at the edges of the contact. This is due to discontinuity of the surface and is similar to a flat-edged contact. The pressure distribution is also non-symmetrical about the tooth centre. This is explained by differences in torsional stiffness between the two components. The male component, which is smaller and therefore of lower torsional stiffness, will be deflected a greater amount by the applied torque than the female component resulting in a reduction in pressure at the end away from the shaft (the lower end). A similar effect occurs in the profile direction, thus creating a higher pressure at the inner diameter edge of the contact. These features can also be seen when a misalignment is applied and the skewness in pressure distribution due to torque is responsible for the lower end separating before (and by a greater amount than) the upper end in all cases.

5.4.2.2 **Analysis of effects of misalignment and torque**

When a misalignment is applied to the coupling, the pressure distribution varies from tooth to tooth. Four different pressure or contact regimes were defined in Section 5.3.2.1, and show distinctive features not seen in an aligned coupling. The regimes are defined in relation to the tooth separation that may occur. With or without this separation, it is clear that the pressure distribution is skewed further to the ends of the teeth than by torque alone. For some teeth, it is skewed further towards the upper end, and for some it becomes skewed towards the lower end. Separation may occur on parts of teeth (cyclic regime), on whole teeth (discontinuous regime), or not at all (uniform regime). The reasons for this behaviour and the various contact regimes can be discussed in terms of a simple rigid body analysis.
The pressure distribution for any given tooth depends upon its position relative to the direction of misalignment. A much-simplified assessment of the effect of misalignment can be made by considering a rigid spline coupling, similar to the analysis performed by Buckingham [3]. Figure 5.66 shows a coupling viewed from above. In the diagram, the female component remains upright, and an angular misalignment about the centre of the coupling is applied to the male component such that the top moves to the right. The misalignment will cause some teeth to partially "overlap" the mating tooth and partially slip tangentially to the mating tooth. Where teeth overlap, a higher pressure will be generated within a real coupling – the greater the overlap, the greater the increase in pressure. This analysis allows the general form of the misaligned pressure distribution to be forecast. The contacting surface of the tooth at an angle of \((90 - \alpha)\) to the direction of misalignment, where \(\alpha\) is the pressure angle, is approximately perpendicular to the direction of misalignment (it is perpendicular only at the pitch diameter due to the involute tooth shape). Because of the misalignment, the top half of the male tooth will "overlap" the female tooth, and the lower half will move apart. Similarly, at the tooth diametrically opposite, the lower half will overlap. Because the misalignment direction is perpendicular to the tooth surfaces, the amount of overlap will be greater for these teeth than any other, and hence these two teeth experience the greatest loads. This was reported by Neale et al. for misaligned gear couplings [115].

In contrast, the teeth at right-angles to these will show virtually no overlap. This is shown by the arrows of Figure 5.66. The intermediate teeth will vary between the two extremes. The pressure distribution of Figure 5.10 shows how this is manifested in a real coupling – the tooth at \(90-\alpha\) degrees to the misalignment (tooth number 4 at the 0° and 360° stages) shows the most skewed pressure distribution and highest pressures, and the teeth at 90° to these show a more symmetrical and uniform distribution (albeit virtually zero). In all cases, the tooth at "90-\(\alpha\)" shows the greatest pressure and most skewed distribution as the motion of the tooth is fully normal to the tooth surface.
The teeth cannot, of course, overlap and so only the ends of the two teeth will be in contact for a rigid coupling, explaining the separation for the opposite ends of the teeth. With elasticity, the teeth will deflect somewhat allowing the contact to spread further along the tooth. Dependent upon the amount of elasticity and the angle of misalignment, the midpoint or mean tooth position for one tooth surface may have moved away from the mating tooth. Buckingham explained how this clearance was dependent on the elasticity, tooth load and misalignment [3]. This motion causes the male component to displace rotationally with respect to the female component as the teeth are effectively “jacked apart”. This causes all teeth to move apart by a small amount and explains how full tooth separation may occur in intermediate teeth.

With a higher misalignment angle, there will be a greater chance of tooth separation and greater jacking apart of teeth, moving the regime towards a more cyclic/discontinuous mode. A higher torque (lower $E^*$) will force the teeth together more, and the greater elastic deformation will allow more of the tooth to remain in contact, and so move the coupling towards a more uniform contact regime, as would reducing stiffness. This gives rise to the $E^*\theta$ relationship for contact regime seen in Figure 5.22 and Figure 5.27.

An alternative way of considering the contact regime refers to the bending moment that occurs along with misalignment. It was seen in Section 1.4 that a misalignment angle generates a bending moment (and vice versa). Figure 5.67 shows two teeth of a spline coupling, with the contact pressures represented by a total tooth load and the...
centre of pressure. (More teeth merely complicate the analysis although the same observations can be made.) A frictionless contact is assumed. A similar analysis of the forces and moments on a gear coupling is given in [115] with frictional contact.

Figure 5.67 - Two tooth spline for bending moment analysis

With no misalignment/bending moment, the forces act at the same location at, or more accurately near, the centre of the tooth (all locations and distances are taken in the axial/facewidth direction). This results in no net bending moment in the axial direction. If a bending moment is applied in the direction shown, then the tooth pressures must alter to balance this moment if the coupling is to remain in equilibrium. The centre of pressure for each tooth moves away from the centres until the applied bending moment, \( M \), is balance by the tooth moment, \( 2 \times F_N \times c \). With a full complement of teeth, the centre of pressure will vary for each tooth as found computationally (Figure 5.11). The value of \( c \) for the teeth shown, which are at 90-\( \alpha \) and 270-\( \alpha \) degrees to the misalignment, usually represents the value of \( C_{\text{max}} \) used to quantify the pressure distribution. The value of the tooth loads, \( F_N \), is fixed by the applied torque since Torque = \( F_N \times \text{base diameter, } D_b \). Therefore for an increase in applied moment, the value of \( c \) must increase such that

\[
M = 2 \times F_N \times c
\]
This expresses quantitatively the relationship between $C_{\text{max}}$, or pressure distribution, and ratio of misalignment to torque ($E^*\theta$). This analysis can also demonstrate the fourth contact regime, "toppled". This regime could not be identified through the computational analysis, since the model is inappropriate for this type of coupling contact. From Figure 5.67 and the equations above, it becomes apparent that there is a limiting bending moment that can be applied with the existing free body diagram. The centres of pressure for the teeth can only move so far – up to the ends of the facewidth. Therefore the maximum bending moment which can be balanced through the tooth loads is given by $M = F_N \times b$, or $M = T \times b / D_b$. If a larger moment is applied, then the coupling will no longer be in equilibrium and will "topple" over. In practice, other parts of the coupling will then come into contact and eventually bring the coupling back into equilibrium. The bending moment at which toppling occurs is dependent on torque which, for most applications, is likely to be sufficient to prevent toppling. The toppling effect can be felt in an unloaded coupling when it is held and a torque and misalignment applied by hand. If the backlash is minimal, then reverse face contact may occur before toppling and thus prevent a sudden toppling movement. Reverse face overlap has not been investigated in either the computational analyses or this rigid body simplification.

It was shown above that the pressure distribution is skewed by torque alone due to differences in stiffness, even with no misalignment. The minimum value of $C_{\text{max}}$ is therefore non-zero. For the typical spline coupling geometry of this study, the value of $C_{\text{max}}$ for the non-aligned coupling was 0.164 and was independent of torque. This will affect the lower left section of Figure 5.27, for which no results are presented.

The maximum values of pressure reported by the computational analysis vary with $E^*$ and $\theta$ as would be expected: decreasing $E^*$ leads to an increase in maximum pressure and increasing $\theta$ leads to a rapid increase in maximum pressure. It is the maximum value of pressure that affects the wear parameter rather than the pressure regime, but it must be remembered that the computed pressure is effectively an average pressure over a small elemental area of the tooth surface.
5.4.2.3 The $C_{\text{max}}$ and $P_{\text{max}}/P_{\text{av}}$ parameters

The use of the $C_{\text{max}}$ parameter to describe the pressure distribution has the explicit derivation above, and comparing regimes with differing and identical values of $C_{\text{max}}$ has shown that this is a good method of quantifying the contact regime. The parameter does, however, have a maximum value of 0.5. This value does not necessarily indicate the onset of toppling and although it generally indicates a discontinuous regime, it could still occur within the cyclic regime. A discontinuous regime could also have a $C_{\text{max}}$ value of less than 0.5 (e.g. PS24). This is explained with reference to Figure 5.68, which shows possible contact configurations.

![Figure 5.68 - Contact regimes showing change in $C_{\text{max}}$ location](image)

- Position and tooth of $C_{\text{max}}$

**Figure 5.68 – Contact regimes showing change in $C_{\text{max}}$ location**

a) Cyclic with $C_{\text{max}} < 0.5$, on tooth 8  
b) Cyclic with $C_{\text{max}} = 0.5$, on tooth 13  
c) Discontinuous with $C_{\text{max}} = 0.5$, tooth 13 (tooth 3 separated)  
d) Discontinuous with $C_{\text{max}} = 0.5$, on tooth 12  
e) Discontinuous, with $C_{\text{max}} < 0.5$, on tooth 12
For most pressure distributions (Figure 5.68a), the tooth on which the maximum deviation of centre of pressure ($C_{\text{max}}$) occurs is the most heavily loaded, which is at 90-\(\alpha\) degrees to the misalignment (tooth 8). However, just before a tooth fully separates (Figure 5.68b), $C_{\text{max}}$ will occur on the least loaded tooth, which is only just in contact at the tips. If only the very end of the tooth contacts, the value of $C_{\text{max}}$ will be 0.5, but all teeth are in contact and so the regime is not discontinuous. (For the boundary element results, a $C_{\text{max}}$ value of 0.5 implies that only the top row of mesh points is in contact.) Increasing the misalignment further will remove all contact from that tooth (making the centre of pressure deviation for that tooth zero), and the value of $C_{\text{max}}$ is obtained from the adjacent tooth (Figure 5.68c). Therefore, once a value of 0.5 is reached, changes in pressure distribution cannot be identified through $C_{\text{max}}$ – it is the tooth on which this occurs that changes (Figure 5.68c, d). Figure 5.27 shows a wide band of operating conditions for which $C_{\text{max}}= 0.5$ for this reason. Depending on the precise area of contact, a $C_{\text{max}}$ value of less than 0.5 could occur for a discontinuous regime if the contact extends beyond the very tip of the tooth adjacent to the unloaded tooth (Figure 5.68e).

Whereas the change in $C_{\text{max}}$ can be directly related to the changes in pressure distribution as explained above, the changes of the $P_{\text{max}}/P_{\text{av}}$ parameter cannot be related in such a straightforward manner. This parameter is therefore not such a precise indication of the contact regime and may be dependent on the fine details of the pressure distribution (a possible explanation for Figure 5.30 showing a different trend for one set of conditions). This parameter does allow a prediction of maximum tooth load, which may be useful for a more considered stress analysis of a spline tooth.

### 5.4.2.4 Effects of spline geometry and friction

The pressure distribution changes with the geometry of the coupling and the coefficient of friction. Figure 5.29 shows that increasing facewidth, $b^*$, moves the pressure distribution towards the discontinuous regime. When considering the reason for this, it is necessary to relate the facewidth, the misalignment angle and the bending moment. Although the total length over which the misalignment angle is applied is increased, the normal movement of the tooth ends for the given misalignment is also increased such that increasing facewidth leads to an increase in bending moment.
Since the average tooth load is not affected, the deviation of centre of pressure must increase to balance this increase in bending moment. However, this does not in itself imply an increase in $C_{\text{max}}$, or change in pressure distribution, since this parameter is defined with respect to the facewidth and both the deviation of centre of pressure and facewidth have increased.

Since it is always the ends of the teeth that contact initially, it might be anticipated that the contact area at the ends of the teeth remains the same for different values of facewidth. (This would result in an increase in $C_{\text{max}}$ but no change in pressure regime.) This is not the case – the area of contact varies with facewidth but shows no consistent trend.

The effect of a change in facewidth is a complex interaction of misalignment angle, bending moment and geometry but the boundary element results reveal that an increase in facewidth does lead to a more cyclic or discontinuous regime. However, although the separation is increased for a larger facewidth, the maximum pressure decreases. This is possibly due to an increase in tooth flexure, which results in a decrease in angularity at the ends of the teeth where the maximum pressure is found.

Increasing the number of teeth also leads to a more cyclic/discontinuous regime. An increase in the number of teeth, $z$, implies an increase in pitch diameter for constant module, and so a reduction in total tooth load for a given torque. Since the value of $F$ has decreased (Figure 5.67), the value of $C_{\text{max}}$ must increase for a given bending moment. In this case, the interchanging of bending moment and misalignment angle is appropriate since $M/T$ is largely independent of $z$ (Figure 5.51). The maximum pressure increases slightly for increasing $z$.

The pressure angle has only a small influence on the pressure regime and $C_{\text{max}}$ parameters for the 20° and 30° cases looked at. It has been seen that the actual tooth which experiences the greatest load is dependent on $\alpha$, but for 18 tooth splines in which the teeth are spaced at 20°, no significant change can be observed for a 10° change in $\alpha$. A higher friction coefficient leads to a more uniform/less cyclic pressure distribution. Until now, friction has been ignored and it has been assumed that the bending moment is transmitted entirely by the contact pressures. Figure 5.66 shows that the teeth at 90° to the most heavily loaded will slip tangentially, thus creating a
tangential traction. This force also contributes to the overall bending moment. If the friction coefficient is raised, this contribution is increased and the contact pressures do not need to generate such a high bending moment. However, the bending moment itself also increases with friction coefficient for a given misalignment – the net effect is a less cyclic/more uniform pressure distribution as witnessed in Figure 5.33.

5.4.2.5 Equations for dimensionless result parameters

It is possible to formulate equations for the parameters $C_{\text{max}}$ and $P_{\text{max}}/P_{\text{av}}$ using the results of the parameter study. This makes it possible to predict the pressure distribution based on the known geometry and operating conditions for any spline coupling without the need for time consuming computational analysis. The equations also highlight the trends for the input parameters described above. The equations have been optimised for the 2.5 mm module analyses as described in Section 5.4.1 but should be equally valid for any module if the correct value for misalignment angle is used. The equations and the standard deviation with the parameter study results are:

$$C_{\text{max}} = 0.5 - 30 / (58 + 0.01 E^* \theta b^* z^3) \quad (\text{s.d.} = 4\%)$$

$$P_{\text{max}}/P_{\text{av}} = 1 + 0.45 E^* \theta \quad (\text{s.d.} = 10\%)$$

($\theta$ for all parameter study equations is in degrees)

5.4.2.6 Stick and slip regime

The differentiation between partial slip and gross slip regimes is important for fretting wear assessment. In the typical reciprocating rig laboratory test, partial slip occurs when a portion of the contact remains in stick throughout the motion whilst gross slip occurs when all parts of the contact slip at some time. For every spline coupling analysis, even those which show some areas of stick during the rotation, all parts of the tooth slip at some point and so should be described as gross slip. Areas reported to be in stick by Beasy imply that that area has remained in stick for a significant portion of the rotation. Significant areas of stick may lead to increased damage over a period of time, with the boundary between the stick and slip areas being most susceptible to damage (Section 2.1). This contradicts the wear parameter assessment, which is lowered by periods of stick.
5.4.2.7 Effects of pressure regime and tooth separation

Although the type of pressure distribution does not directly affect the wear parameter, it could play a role in other aspects. The oscillating movement has already been described as being a rocking motion on each tooth that could generate a beneficial hydrodynamic lubrication film. The oscillating contact could also have a detrimental effect on the life of the coupling by increasing fatigue damage. Increasing misalignment angle will lead to a change in pressure distribution and an increase in cyclic stresses and thus fatigue. For a constant misalignment angle, decreasing the torque will also lead to a change in pressure distribution but it is not immediately clear whether the actual oscillating stresses will be affected. Figure 5.69 shows the oscillating components of pressure distribution for analyses PS6 and PS8, for which the geometry and misalignment angle were identical but in which the torque (500 Nm for PS6; 1000 Nm for PS8) and thus contact regime ($C_{max}=0.422$ for PS6; 0.375 for PS8 – see Figure 5.22) varied. These results are obtained by subtracting the pressure distribution for an aligned coupling from the misaligned model.

![Figure 5.69 - Oscillating components of pressure distribution](image)

It is clear that the area affected by the oscillating stresses will be greater for PS6 (lower torque) than for PS8, so that there is a greater area for possible fatigue crack growth with a more cyclic regime. The actual amplitude of bending moment at the centre of the tooth due to the tooth loads did not show much dependence on torque – it increased by less than 10% for the larger torque. The dynamic effect of separation, with the mating teeth continually hitting against each other at opposite sides, may add to the amplitude of stress. This is likely to be particularly true for a discontinuous regime. This behaviour suggests a possible increase in fatigue damage with high
misalignments for lower values of torque, as seen in the spline wear tests. The occurrence of dynamic loading has been found to be particularly damaging in several previous studies of fretting (Section 2.1.4.2).

The occurrence of separation could also significantly affect the movement of wear debris within the contact zone, perhaps significantly increasing the rate of material removal from the contact. The maximum gap between the teeth was as much as 67 μm in some of the analyses. This would be sufficient to allow many of the particles to be washed away by a lubricant or through the tooth movement. What affect this has on wear rate is still unclear.

5.4.3 Slip paths

The general form of the slip paths is similar to that predicted by a rigid body analysis (see Section 7.1.1). The angle of the primary axis depends on the position relative to the axis of rotation; the angles of the slip path diagrams vary with number of teeth (pitch diameter) and facewidth accordingly. The slip paths are not symmetrical about the centre due to the skewness of the pressure distribution caused by torque, so that the paths on the lighter loaded lower half are more open than those on the top half. The slip paths at some locations on the tooth also show a sharp bend for certain conditions and some areas experience a slip path that crosses over itself to form a figure 8 rather than a loop. These are due to the varying pressures and the effects of elasticity.

Increasing the misalignment angle gives a proportionate increase in slip amplitude, as does increasing the pitch diameter through an increase in number of teeth or module. Increasing the torque can lead to a small reduction in slip amplitude; this is partly due to an increase in friction reducing the amount of slip but not entirely. An additional frictionless analysis (not included in the parameter study results) has shown a small decrease in slip with a doubling of torque suggesting that the increased elastic deflection alone will lead to a small reduction in slip amplitude. Note that the slip amplitude is caused solely by the rotating misalignment and so is not affected by the initial slip from the application of the torque.

Figure 5.19, showing the pressure variation through the slip path, is perhaps the best way in which the local behaviour of the spline tooth interfaces can be visualised. This
shows the directions of the heavily loaded slip, which could be compared to wear seen on real spline couplings. It also shows that the behaviour of any part of the tooth contact is unlike any laboratory tests used regularly for fretting studies. Under the conditions represented in Figure 5.19, the motion is, in some ways, closer to one of unidirectional sliding. Even without separation, the motion is not pure reciprocating sliding due to the transverse movement. This highlights the difficulty in relating laboratory tests to spline coupling behaviour.

### 5.4.4 Wear

The wear parameter used for this study is based on the Archard wear equation, which states that the wear rate per unit sliding distance is:

\[
\text{Wear rate} = K \times \frac{\text{Normal Load}}{\text{Material Hardness}} \quad \text{or} \quad \text{Wear rate} = k \times \text{Normal Load}
\]

where \(K\) is an empirically determined wear coefficient, and \(k\) is \(K/\text{Hardness}\). Thus the wear volume can be expressed as:

\[
\text{Wear volume} = k \times \text{Normal Load} \times \text{Slip Distance}
\]

For the purposes of this study, in which the load varies over the contacting surface and with sliding distance, this equation can be re-written to provide the wear depth at a point on the surface through one complete revolution. This gives:

\[
\text{Wear depth} = k \int p \, ds
\]

where \(p\) is the pressure and \(s\) is the distance along the slip path. The dimensional wear parameter defined in Section 5.3.1.4 is:

\[
W = \int p \, ds
\]

and is therefore proportional to the wear depth provided the value of \(k\) remains constant. This assumption prevents any direct comparison for different lubrication methods, friction coefficient, or materials unless the coefficient is accounted for. Since the pressure is an average pressure over a small elemental area represented by a
single mesh point, the wear depth must also be considered an average wear depth over that area.

The value of $k$ has been found to increase with slip amplitude for fretting wear (Section 2.1.4.1). This is not included in the wear parameter, and is a possible source of error. A difficulty with including the slip amplitude within the wear parameter is in determining the appropriate slip amplitude to use, given the complex and unloading nature of the slip path. There is also some disagreement as to how the wear coefficient is affected by the slip amplitude.

The friction coefficient dependency of the wear parameter is predominantly due to the reduction in slip path. As a friction coefficient variation is inevitably accompanied by a change in surface conditions, and hence a change in wear coefficient, no meaningful comparison of results for different friction coefficients can be made. Clearly, reducing lubrication does not decrease wear. However, the results are fundamentally accurate. If the friction was increased sufficiently then no slip and no wear would take place, although more serious damage would almost certainly be caused.

Within the parameter study, an alternative, dimensionless wear parameter is defined as $\frac{W}{Em}$. For the purposes of formulating equations and interpolating results, it is the maximum value of this dimensionless parameter that is used; to obtain a value proportional to maximum wear depth, the value of $W$ should be used. Accordingly, both forms of wear parameter are used throughout. It may be appropriate to consider $\frac{W}{Em}$ when comparing different modules, as a specific wear depth may be less cause for concern on larger teeth.

The maximum wear depth is critical for assessing the life of a spline, but the total wear volume might also be a factor. Figure 5.44, showing the value of wear parameter over the tooth surface for different analyses, demonstrates how different areas of the tooth may wear depending on the design and operating conditions. The variation between a) and b) is of particular interest – the torque of case b) is twice that of a). The maximum wear depth has not changed but the total wear volume has increased. This can be identified by the darker density over the middle of the tooth. The area of wear is related to the contact regime, since separation prevents wear.
The effects of the various input parameters are derived from how they alter the pressure and slip path. The misalignment angle has a predictably strong influence on the maximum wear depth parameter, increasing both the slip amplitude and the maximum pressure. Figure 5.45 clearly shows this relationship. It also shows how the torque has little effect on the wear parameter – the maximum pressure may increase slightly but the slip decreases slightly. The total wear predicted does increase as explained above for differing regimes.

The behaviour of the wear parameter with changing facewidth may not be intuitive, and differs from that predicted from a rigid body analysis. The wear parameter is consistently greatest at the ends of the teeth; a larger facewidth increases the slip amplitude most at these ends and so it might be expected that the wear parameter would increase as a result. In fact, the wear parameter decreases with increasing facewidth. The increase in facewidth also acts to decrease the maximum pressure (explained previously) and it is this that dominates the results. The larger facewidth also causes a more cyclic regime and thus the high pressures act for a smaller proportion of the slip path, also acting to reduce the wear parameter.

The facewidth values used in the parameter study were of similar magnitude to the pitch diameter; a much longer facewidth would contribute more to the generation of slip (see rigid body analysis). The wear parameter might therefore approach a minimum or begin to rise with further increases in facewidth since the maximum pressure cannot continue to fall indefinitely. This behaviour was not observed for the range of parameters analysed in this study.

The increase in maximum wear parameter with number of teeth shown in Figure 5.47 could be entirely due to the accompanying increase in pitch diameter and hence slip. The wear parameter does not increase that dramatically since the average tooth load is reduced for a given torque, lowering the maximum pressure. Additionally, the change in pressure distribution regime means that a lower proportion of the slip path occurs during contact. Of more interest to designers is whether more teeth of small module leads to a lower wear parameter than fewer teeth of large module for a coupling of fixed diameter. This can be judged via the formulation of an equation for $W_{Em}$ based on the parameter study results.
The results from the study suggest the following equation for approximating the wear parameter, $W/Em$, for couplings in the range of conditions studied:

$$W/Em = 5.8 \times 10^{-4} \times z \times \theta^{1.9} / b^{0.5}$$

The equation is valid for $\mu = 0.3$, with a standard deviation of 17%.

The equation indicates the dependency of wear on misalignment angle, facewidth and number of teeth and the lack of dependency on torque – the effect of $E^*$ is negligible. To determine the optimum number of teeth for a fixed diameter, the equation can be rewritten as:

$$\frac{W}{Em} = 5.8 \times 10^{-4} \times z \theta^{1.9} \frac{m^{0.5}}{b^{0.5}}$$

But the pitch diameter, $D_p$, is given by:

$$D_p = z m$$

Therefore:

$$\frac{W}{m} = A \frac{D_p}{m} m^{0.5}$$

$$W = A \times D_p \times m^{0.5}$$

where $A = 5.8 \times 10^{-4} \times \theta^{1.9} E/b^{0.5}$

Thus, for a constant pitch diameter, $W$ is proportional to the square root of the module. This suggests that it is preferable to use many small teeth rather than fewer large teeth. However, due to the required definition of misalignment angle, the distance over which the misalignment is applied varies with module. Thus for a particular remote shaft misalignment, the value of $\theta$ may not be constant if the module is changed. These effects are unlikely to change the outcome, but are included in the design software reported in the following chapter.

Only four different values of $z$ have been used in the parameter study, making a more accurate evaluation of the $z$ parameter exponent difficult. It also limits the range of
tooth numbers that can be analysed sensibly using the above equations. Computational models for greater numbers of teeth are more complex, requiring additional computational resources and significantly increasing time for solution.

The effect of pressure angle on the wear parameter is not included in the equation since only two analyses were carried out with a pressure angle other than 30° – the 20° pressure angle gave an increase of wear parameter of approximately 10%.

5.4.5 Bending moment

The bending moment gives an indication of the effective stiffness of the coupling itself; a stiffer coupling will generate a higher bending moment for a given misalignment angle. Section 5.4.2.2 explained the relationship between misalignment and bending moment, and the trends shown for $M/T$ follow from this. Section 5.4.2 also revealed how the bending moment will be related to the $C_{\text{max}}$ parameter, and this is used to formulate an equation for $M/T$:

$$M/T = 3.13 \left( C_{\text{max}} - 0.18 \right)^{0.9} b^{0.36} / z^{0.47}; \quad \text{(standard deviation of 9\% for } \mu=0.3)$$

The bending moments reported here are calculated from the normal tractions on the lower surface of the female coupling. The model has 24 elements on this face, which is adequate for the contact analysis, but which provides a limited number of discrete data points from which the bending moment could be calculated. Nevertheless, verification of the software gave only a 1\% error for a simplified model in which the bending moment was known.

The bending moment results produced for the direction perpendicular to the direction of misalignment show an erratic behaviour. No consistent trend can be identified. The directions (signs) of the bending moment occasionally differed from one analysis to another. Since the magnitude of the moment is significantly lower than that in the misalignment direction (typically less than 5\%), computational errors will be more significant. (The magnitude of the error is similar to that in the direction of misalignment but is therefore a higher percentage error.) The existence of a moment in this direction is not intuitive – there is no applied movement in this direction. Frictional forces appear to generate a large proportion of this moment. A frictionless
analysis resulted in a much lower perpendicular bending moment, possibly within the computational accuracy of the results.

5.4.6 Crowning and pitch error

The application of a lead crown has been seen to reduce the wear parameter, and this agrees with performance of crowned couplings in practice. The area of contact is moved closer to the centre of the facewidth and the pressure distribution is less cyclic. The slip amplitude at the centre of the coupling is not dissimilar to that at the ends and so is not responsible for the large reduction in wear parameter. For the conditions of the crowned analysis, the tooth loads are spread more evenly over all teeth leading to a reduction in maximum pressure. Additionally, the crowning allows the coupling to rock more freely without generating high forces – i.e. the effective stiffness has decreased, leading to a reduction in bending moment. These changes in contact conditions result in the reduced wear parameter. The displacement of centre of pressure along the facewidth has also been reduced, lowering the alternating stresses caused by the cyclic pressures, which will also reduce the probability of fatigue damage.

For the particular crowned analysis shown in Figure 5.57, the contact area remains rather localised and does not spread over the entire facewidth. If the contact had spread over the majority of the tooth surface, a further reduction in maximum pressure might have occurred. This could be achieved with a reduction in crowned radius. Therefore, it is likely that there will be an optimum value of crown radius, which will give the lowest wear parameter.

The presence of a pitch error in the analysis of Section 5.3.3.2 has lead to a threefold increase in wear parameter. The pitch error in this analysis was quite severe (25 \( \mu \text{m} \)) and most manufactured splines will have better quality control than this. However, the change in pressure distribution and wear parameter is significant. If the tooth had been retreated 25 \( \mu \text{m} \), rather than advanced, then the change in wear parameter would not have been so great – the increase in pressure would have been spread over the remaining 17 teeth. Real splines will show some teeth ahead of their theoretical location and some behind, and the extent of the pitch error will vary. This will lead to different behaviours, depending on the precise nature of the pitch errors. A detailed
study of pitch error would be extremely time consuming due to the various permutations. However, it is important to appreciate that the existence of pitch error will, in all cases, lead to an increase in maximum wear parameter, and differing wear parameters for each tooth with the possibility of some teeth experiencing no contact and no wear.

5.4.7 Comparison of results with spline wear test observations

The wear parameter distribution calculated for the computational analyses show a good correlation with the general form of the wear observed (Figure 5.63). The wear is always greatest at the upper left edge of the tooth as predicted, and the upper part of the tooth always shows more wear than the lower half as predicted. For some wear tests, the increase in wear towards the tips of the teeth can also be seen (Figure 5.63 d). A region of lower wear towards the middle of the tooth corresponds in general to a region of low wear parameter, and an increase in wear at the lower edge often occurs as the computational results suggest.

Comparing immersed and wetted lubrication tests with the computational results is difficult, not only because of the changing wear coefficient, but because little wear is observed on the immersed test with which to make the comparison. Figure 5.64 shows how the wear distribution (rather than actual wear depth) may change with a variation in friction coefficient, and suggests that with immersed lubrication the wear may spread more evenly over the tooth surface. The remaining wear comparisons are based on the results of the wetted lubrication wear test.

Although the form of the wear pattern is correct, the actual positions of high and low wear do not always correspond accurately. In addition, the computed bending moment results are generally higher than those recorded for the test. This suggests that the actual misalignment angle experienced by the coupling was lower than the value used in the analysis; this was expected to occur to some extent since the misalignment angle was measured over the length of the shaft and the shaft flexure was neglected. The wear observed on the teeth more closely resemble an analysis carried out at a lower misalignment angle (compare the wear parameter of Figure 5.63d at 0.22° misalignment with the worn tooth of Figure 5.63c, at a nominal 0.30°). It is difficult to use the bending moment values to ascertain the correct value of
misalignment since this is dependent on the coefficient of friction. It was originally hoped to be able to determine the actual coefficient of friction from these analyses, but with the high dependency on misalignment angle and the uncertainty of the applied misalignment angle, this is not possible.

Another reason for discrepancy between computational and experimental results is the duration of the experimental wear tests and changing conditions with material removal. Figure 5.63e predicts virtually no wear at the middle of the tooth, whilst there clearly is significant wear. As discussed in Section 5.4.1, the computational results are for a perfect spline with no existing wear; the initial wear may have occurred as predicted leading to a change in tooth shape which would bring the centre of the tooth into contact. At this point the centre of the tooth would begin to wear, i.e. the material removal acts to bring unworn areas of tooth into contact. Areas reported to be out of contact may also experience wear if debris particles of sufficient size become trapped between the two teeth.

The maximum wear parameter ranks the splines of same module in a similar ratio as might be predicted subjectively. However, when comparing between different modules (Figure 5.63 b and e), the wear parameter does not distinguish between subjectively high and low wear. This may be due to the different module splines having different effective stiffness, so that the actual misalignment near the coupling differs between tests using differing modules (i.e. $\theta$ is altered). The normal approach wear values from the experimental tests and the maximum wear parameter do not provide a representative comparison since the normal approach value depends on total wear volume, and on the location and extent of the worn areas, in addition to the maximum wear depth.

The maximum pressures predicted in the computational analysis of the spline wear test are extremely high and would lead to significant plasticity at the tooth edges. The computational analysis maintains a discontinuous sharp edge at the end of the tooth, which generates the high pressures; the actual spline would rapidly develop a change in edge radius leading to a reduced and sustainable maximum pressure. The pressure distributions of Figure 5.62 also reveal the problem of using only three teeth for the wear tests discussed in Chapter 3 – even with the high misalignment angles it is impossible for a tooth to become fully unloaded as might happen in an actual
coupling. The pressure distributions do show some highly cyclic regimes, however, which might be responsible for the fatigue damage seen on SWT 3 and SWT 7.
6
Spline Design Software

6.1 Requirements and features of software

The work reported in the previous sections has given a valuable insight into the behaviour of spline couplings with misalignment. It provides a means by which spline couplings can be designed to maximise the life due to wear – the overall objective of the research. To enable designers to make use of the knowledge gained, the results of the study was to be incorporated in a software tool that can automatically tell the designer which spline coupling would be the optimum choice for a particular application. The details of the application could be entered into the software and the optimum spline coupling, in terms of wear life due to misalignment, predicted.

The wear depends on both operating conditions and spline geometry and so for different sets of operating conditions, different spline geometries would be preferred. The choice of spline coupling geometry is also limited by strength requirements; a basic stress analysis based was to be included within the software, based on the method of Dudley [116]. The stress analysis was carried out according to the requirements of GKN Westland Helicopters Ltd [121].

The selected spline coupling must also meet any geometric constraints imposed by the application for which it is intended. For example, the optimum spline for wear may be too large or too small for the actual application. Hence geometric constraints must be taken into account when selecting the spline coupling.

Although the parameter study on which the software is based allowed any combination of module, tooth number, facewidth and pressure angle, for practical purposes the spline coupling is likely to be selected from an existing national or international design standard. Selecting a geometry for the spline coupling outside of these standards may lead to an improved wear life but would lead to greatly increased manufacturing costs and possible incompatibility with previously used couplings.
Ideally, the software would be able to predict the actual wear life of a coupling operating under given conditions, but the current state of wear modelling is unable to provide this information. However, the results from the parameter study do provide a quantitative wear evaluation that can indicate the likely proportional variation in wear life between different couplings operating under the same conditions. In addition, the useful wear life of the coupling depends on factors not included in the parameter study (Table 5.1). Some of these would be incorporated into the wear coefficient of the Archard wear equation used for the wear parameter. These factors need to be taken into account separately and cannot be quantitatively predicted.

The actual angular misalignment likely to exist within the coupling for an application would be unknown in most circumstances. The maximum misalignment between the shafts may be known at a reference point, based on location tolerances of bearings. This is not necessarily the same as the misalignment angle defined for the parameter study, for the reasons discussed in Section 1.4. The relationship between this misalignment and the known shaft misalignment depends on the relative stiffness of the shaft sections and coupling. The relationship between shaft misalignment and coupling misalignment can be calculated using the results of the parameter study. This was to be used by the software to predict the misalignment for a coupling based on the shaft dimensions and known shaft misalignment.

Aside from selecting the optimum spline coupling for wear performance and strength factors, there is further information that can be provided by the parameter study which may be useful for the designer and which could be produced by the software. This includes the contact regime, the slip path lengths and the tooth gaps.

In summary, the software requirements were:

- The software must determine the optimum spline coupling for wear based on the knowledge gained in the parameter study.

- The software should allow a designer with limited spline coupling knowledge to select the appropriate coupling design. It should be easy to use.

- The software should be a design tool rather than an analysis tool. The software should allow a designer to decide quickly which geometry should be used rather
than through the iterative process of selecting a design for later analysis and redesign.

- The software must include basic stress analysis features to limit the geometries from which the coupling is selected to those which meet the required strength. It should be possible to print or save the stress calculations for verification.

- The software must allow the user to apply geometric constraints in order to ensure that the design produced by the software will be suitable for the particular application. This includes ensuring the coupling will be small enough to fit within the available space, large enough to fit on any fixed size shafts or match up with other components, or fall within the choice of splines that can be manufactured or obtained by the user.

- The software could further optimise geometry by reducing size to the minimum necessary for the required strength.

- The software should calculate the angular misalignment likely to occur at the 20 module reference location, based on shaft dimensions, stiffness and remote misalignments. These values are to be used to calculate the wear parameter of the selected coupling.

- The software should allow the user to observe the effects of deviating from the optimum design. For example, how much more wear could be expected if a different facewidth was used for ease of manufacture.

### 6.2 Software outline

A basic software program has been produced which meets the requirements listed in Section 6.1. This section briefly describes its design. A graphical outline is given in Figure 6.1.

The software initially uses a series of dialog boxes to obtain the information required from the user. All possible spline geometries from within the selected design standards are then examined individually. If the basic geometry meets the applied constraints, then a simple stress analysis determines the maximum inner diameter (to
give minimum weight) for the external spline shaft section, based on the minimum stress reserve factor specified by the user. The software then determines the minimum facewidth for the spline teeth, again based on a simple stress analysis. A specific facewidth or minimum facewidth can alternatively be specified by the user. Stress analysis is then used to determine the minimum outer diameter for the shaft section of the female component. If, at any stage, strength requirements or geometry constraints cannot be met then the software proceeds to the next choice of spline geometry.

The facewidth is selected to be as short as possible for strength requirements. Inspection of the wear parameter equation reveals that increasing facewidth would reduce the maximum wear parameter. The option of increasing facewidth is available to the designer.

Once a suitable geometry has been obtained, the appropriate misalignment angle is calculated based on the shaft misalignment provided by the user. The wear parameter is then calculated using the equation presented in Section 5.4.4.

Every available geometry is checked, and a record of the coupling with the lowest three wear parameters is made. The coupling with the shortest tooth facewidth and smallest diameter are also stored for comparison. If the smallest diameter or facewidth is possible for more than one coupling, then that with the lowest wear parameter is selected.

Information on the selected couplings is then displayed, allowing the user to compare the wear parameter of the different designs, and select the most appropriate.
Does geometry for selected coupling meet geometric constraints?

Yes

No

Are there more spline coupling geometries available for examination?

No

Display Results

Yes

Select next coupling / next design standard

Select first coupling of first design standard

Obtain spline geometry for selected coupling using internal subroutine or datafile

Does geometry for selected coupling meet geometric constraints?

Yes

Determine maximum male component shaft inner diameter for required strength

Determine minimum facewidth for required strength

Determine minimum female component shaft outer diameter for required strength

No

Does calculated geometry meet geometric constraints?

Yes

If predicting coupling misalignment based on shaft misalignment, calculate misalignment angle for selected coupling

Calculate wear parameter for selected coupling

No

If wear parameter is lower than the current optimum spline, set current spline as optimum?
6.3 Example of software in use

This section provides an example of how the software can be used to select a spline coupling for an application. It also demonstrates the benefits that might result from selecting a coupling using this technique.

In this example, a coupling is required to link two shafts that transmit power over a distance of three metres. The coupling is situated 0.8 m from one end. The outer diameter of the coupling must fit within a casing of internal diameter 100 mm. The minor diameter of the externally toothed component must be greater than 50 mm to enable fixing to a second component. The tooth length must be less than 50 mm.

The average torque transmitted during operation is 1000 Nm, and the maximum torque encountered is 1800 Nm. The stress analysis is to assume a total bending moment of 205 Nm, a shear force of 1000 N and an axial load of 2000 N, based on measurements of an existing coupling.

The shafts are held in place by a series of bearings, for which the proscribed locating tolerances would allow a misalignment of 2° over the three-metre span. The longer shaft has an internal diameter of 40 mm and an external diameter of 70 mm. The shorter shaft has an outer diameter of 70 mm and an inner diameter of 25 mm.
6.3.1 Input application details

The first stage of the software is to input the application details and design restrictions. These are entered via a series of dialog boxes.

![Design Standard Selection](image)

**Figure 6.2 - Design standard selection box**

The user first selects which design standards the couplings may be taken from (Figure 6.2). In this case, a company design standard “MDS 1057” is selected. Only two design standards are shown, but further design standards can be added to the software.
The main constraints on overall size are then entered (Figure 6.3). Here, the maximum facewidth of 50 mm and maximum overall diameter of 100 mm is specified. A minimum facewidth of 10 mm is also specified to prevent unrealistic spline teeth.
A second set of possible geometric constraints is provided (Figure 6.4). These enable a specific range of spline couplings to be selected. In this case, only a minimum minor diameter for the externally toothed component is selected.
Figure 6.5 - Material and operating conditions

The operating conditions and material properties are entered in the next dialog box (Figure 6.5). The known loads are entered, and a minimum reserve factor of 2.5 has been selected. The material properties used for the stress analysis section can be entered directly or a known material selected from a list – here a carburising steel material has been selected.
Information for the prediction of misalignment angle is then entered (Figure 6.6). The shaft dimensions and a misalignment angle to a remote point can be entered and the misalignment (as defined for the parameter study, i.e. at 20 modules from the tooth ends) predicted. Alternatively, a standard “coupling” misalignment angle can be entered which is then used for each spline.
6.3.2 Results

Figure 6.7 - Results of software calculations

The results for the optimum spline are presented (Figure 6.7). The user can select to display information on any of the three optimum designs and the smallest facewidth and diameter designs. The design, module, number of teeth, pressure angle, facewidth and diameters of the coupling are presented. The misalignment angle calculated for the coupling and the maximum wear depth parameter are given (wear assessment) and the stress analysis results are presented in terms of minimum reserve factors for the shaft sections and the teeth. The details of the stress analysis routines can be viewed or saved as a text file for verification. A log file containing all user inputs and results is also saved. Note that there are several inactive options on the right of the dialog box, which can be used for future development of the software.
The results are given in Table 6.1, together with the ratio of wear to that of the optimum spline.

<table>
<thead>
<tr>
<th>Spline</th>
<th>Module / mm</th>
<th>No. of Teeth</th>
<th>Outer shaft diameter / mm</th>
<th>Face-width / mm</th>
<th>Effective angle / deg.</th>
<th>Wear parameter / MPa.mm</th>
<th>Wear ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum 1</td>
<td>1.667</td>
<td>22</td>
<td>47.5</td>
<td>24.5</td>
<td>1.74</td>
<td>3340</td>
<td>1.0</td>
</tr>
<tr>
<td>Optimum 2</td>
<td>1.667</td>
<td>25</td>
<td>51.4</td>
<td>19.1</td>
<td>1.75</td>
<td>4350</td>
<td>1.30</td>
</tr>
<tr>
<td>Optimum 3</td>
<td>1.667</td>
<td>28</td>
<td>56.3</td>
<td>15.3</td>
<td>1.77</td>
<td>5530</td>
<td>1.66</td>
</tr>
<tr>
<td>Small Diam.</td>
<td>1.667</td>
<td>22</td>
<td>47.5</td>
<td>24.5</td>
<td>1.74</td>
<td>3340</td>
<td>1.0</td>
</tr>
<tr>
<td>Small Fcw.</td>
<td>2.500</td>
<td>24</td>
<td>69.6</td>
<td>10.0</td>
<td>2.0</td>
<td>13600</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Table 6.1 - Spline coupling designs selected by software

6.4 Discussion

The optimum spline for wear calculated by the software is the spline with the smallest diameter that meets geometry and strength requirements. This will generally be the case, but the software allows a good comparison of different designs and allows the user to assess the effects of deviating from this design.

It is interesting to compare the best three splines. The second best spline has a wear parameter 30% higher than the optimum and the third best spline has a wear parameter 66% higher. Assuming all other factors are equal, the wear depth should vary by the same proportions, i.e. selecting the optimum spline can decrease the maximum wear by 40% compared to the third best spline. There are numerous other designs of spline that might have been chosen, and these would be expected to show even greater wear.

The coupling with the smallest facewidth (the minimum 10.0 mm specified) is expected to exhibit wear four times greater than the optimum. Notice that the coupling misalignment angle in this case is the full 2.0° compared to 1.75° for the other designs, i.e. the majority of the misalignment is carried through the coupling with little elastic shaft flexure.
The spline coupling that was ranked forth had a module of 2.5 mm and 16 teeth (pitch diameter of 40 mm compared to 36.7 mm for the optimum spline). The wear parameter for this spline was 6440 MPa.mm, which is almost twice that of the optimum spline despite a change in pitch diameter of just 8.25%. The misalignment angle (as defined for the parameter study) for this coupling was calculated to be the full 2.0°, explaining the difference between the two results. This also demonstrates the need to account for the differing effects of shaft flexure and the use of the correct misalignment angle.

The optimum spline suggested by the software relies on the maximum wear depth parameter being a suitable criteria for assessing spline tooth wear and the equation presented in the previous chapter being sufficient to predict this parameter. The software uses these purely computational results to predict wear. The comparisons between the spline wear tests and computational results of wear parameter in the previous chapter showed that the maximum wear parameter alone cannot depict the tooth wear in its entirety, particularly over longer periods of time. Further validation of the software using actual wear data of fully-toothed spline couplings is required before the results of the software can be used with confidence. The wear assessment produced by the software is a first attempt at quantifying differences in wear life between various couplings and is a marked improvement over many current design approaches in which fretting wear is neglected. Other failure modes (such as bending fatigue) must be considered separately.
7
Discussion

7.1 Analysis and behaviour of spline couplings with misalignment

7.1.1 Rigid body analysis

The computational analysis of spline couplings operating with a misalignment between the coupled shafts has revealed the complex loading and movement of the mating teeth. In some respects, a spline coupling can be considered as a rigid mechanism, allowing simple analysis of the static and dynamic behaviour. This has been used for previous analyses of couplings and does allow some features to be predicted, such as the occurrence of the “toppling” regime. However, unlike many mechanisms that are successfully analysed using such an approach, spline couplings may experience elastic behaviour that significantly alters their operation and affects the conclusions obtained by such methods.

A Cardan or Hooke universal joint is a coupling that allows a torque to be transmitted through an angle, typically permitting greater misalignment angles than can be accommodated by spline couplings. The angle is accommodated predominantly through rigid motion, as the two shafts pivot around about a central block, and so can readily be analysed as a simple rigid mechanism. With the lack of significant elastic deflection, the misalignment of the coupling can accurately be defined as the remote angle between the shafts, removing one difficulty present in spline couplings. As with spline couplings, the universal joint experiences a bending moment in the direction of misalignment known as a secondary couple [136]. Although cyclically changing during the rotation, this bending moment can be determined through the rigid body analysis and is merely dependent on the applied torque and the misalignment angle [136].
A rigid body analysis of spline couplings requires some additional simplifications and assumptions. At the extreme loading condition, it is possible to assume that the torque is transmitted through two diametrically opposite teeth. This type of analysis was shown for gear couplings (which share the same form as spline couplings) by Neale et al. [115], and was used for the spline coupling representation in Section 5.4.2.2. This analysis is particularly useful in analysing the moments generated in the coupling.

The maximum bending moment predicted by the rigid body model is $Tb/(D_p\cos\alpha)$ for a frictionless contact. For the spline geometry of section 5.1, also used for Figure 5.27, this gives a value for $M/T$ of 0.69. This compares well with the maximum bending moments obtained in the tests for such spline couplings close to the toppling regime (0.66). If the parameter equation for $M/T$ is used with a $C_{max}$ value of 0.5 (which for the case of two tooth contact represents the onset of toppling), the value of $M/T$ obtained is 0.68. This parameter equation was based solely on a regression formula for the computational results, and confirms the maximum bending moment obtained. There is, however, a discrepancy between the two methods. The parametric equation for the case of $C_{max}=0.5$ can be rewritten as:

$$M/T = 3.13 \times (0.5 - 0.18)^{0.9} \times \frac{b^{0.36}}{m^{0.36}} \times \frac{m^{0.47}}{D_p^{0.47}}$$

This implies that the maximum bending moment is proportional to $b^{0.36}/D_p^{0.47}$ whereas the rigid body analysis suggests that the maximum bending moment is proportional to $b/D_p$. There were few data points for varying $b^*$ or $z$ which could have led to inaccurate exponents for these parameters; this would be noticeable for conditions outside of the range studied. The bending moment for multiple-tooth contact will include contributions from each tooth. The extent to which this occurs will itself be dependent on $b$ and $D_p$ and the operating conditions. For the multiple-tooth contact, $M/T$ is not necessarily proportional to $b/D_p$, which is an alternative possible explanation for the discrepancy in coefficients (i.e. the exponents of $b^*$ and $z$ in the equation should be functions of $E^*$ and $\theta$).

Neale et al. also included friction in the analysis, again only for the two heavily loaded teeth. The frictional force was assumed to be in the axial direction and, for the case of male gear driving, directed towards the centre of the tooth from the edge contact on
the male tooth. The frictional force thus added to the bending moment in the misalignment direction.

The analysis of Neale produced the overall maximum bending moment that could occur. The frictional moment could instead be resolved into a component in the direction of misalignment and a component perpendicular to this (corresponding to the $M_{\text{perp}}$ of the boundary element models). This gives a maximum value for $M/T$ as:

$$
\frac{M}{T} = \frac{b}{d_p \cos \alpha} + \mu \tan \alpha
$$

and, in the perpendicular direction:

$$
\frac{M}{T} = \mu
$$

For the reference spline coupling, the maximum value for bending moment/torque with friction included is 0.87. This is significantly greater than that of the computational analyses. In the perpendicular direction, the maximum value for bending moment/torque is 0.3, which is also significantly greater than that of the analyses.

The frictional force appears to contribute less to the moment than would be predicted from the two-tooth rigid body analysis. For multiple tooth contact, the frictional forces will be distributed over several teeth. The moment arm for most of these teeth will be less than that of the heavily loaded teeth, thus reducing the total moment. If the teeth in contact spread beyond the axis line, the frictional forces of these teeth will also act against the moment produced by the adjacent teeth.

The frictional forces were assumed to act fully in the axial direction. The slip paths produced from the computational analyses suggest that this assumption may be rather inaccurate. Therefore, the accuracy of this rigid body analysis is reduced by the assumed slip direction and the assumption of two-tooth contact.

The computational analysis also suffers to some extent from an incorrect slip path. With the rotation modelled using steps of 90°, the slip path is jagged and formed of four straight lines as shown in 5.3.1.3. Although the slip path can be improved using
sequential teeth, the frictional forces acting on the coupling, as calculated by the software, will act along these lines of slip rather than the tangent to the actual slip path. This is a source of error in the computational analyses. The frictional forces appear to be less significant than the contact pressures, and so this error is likely to be small.

The direction of the frictional force also depends on the direction of rotation. If the rotation of a real coupling is reversed then contact will occur on the opposite sides of the teeth, and the situation will be similar to the original case. Reversing the rotation of misalignment direction whilst keeping the same side of the tooth in contact is equivalent to switching from male component driving to female component driving. Under such a change, the direction of the slip will be reversed and the frictional forces of the rigid body analysis will act in the opposite sense. For example, the maximum value of \( M/T \) will instead be \( b / (D_p \cos \alpha) - \mu \tan \alpha \).

All analyses in this study have been equivalent to the male component driving. The importance of direction of slip was seen to some extent in the difference between load cases 2-4 and load case 1, in which the motion was from a fully aligned central position. An investigation of the differences that may exist between the cases of male and female component driving is one topic for further study listed in Section 7.4.2 and may require additional load steps to overcome the problem of a simplified slip path mentioned above. A single analysis, using the same 90° change in misalignment direction, but with a revered rotation has confirmed that differences can occur. The wear parameter increased by 12% for the case of female component driving. With the limited quantitative accuracy of the results, particularly with the 90° load steps, this is relatively small, but may suggest that the male component driving is the preferred set up.

The analysis of the forces acting on the coupling has considered contact on one side of the tooth only. If toppling occurs, or if the amount of backlash is less than the displacement of the teeth, then contact will occur on the back face of the tooth or between other areas of the coupling. The maximum bending moment calculated above is therefore only the maximum bending moment for single sided contact. Greater misalignment angles will continue to increase the bending moment acting on the coupling and thus the shafts and support bearings.
The rigid body analysis of forces acting on the coupling using the two tooth simplification allows the most extreme loading condition to be examined, but the normal situation of multiple tooth contact cannot be analysed in the same way. With a misalignment applied, the tooth loads are not identical; the rigid body analysis cannot resolve the manner in which the torque is distributed through the various tooth contact forces. This prevents an accurate assessment of the intermediate conditions between a fully aligned and (almost) toppled coupling.

The rigid body analysis can also be used to predict the slip path between the mating teeth. This is achieved by calculating the rigid body displacement of the male coupling due to a misalignment about the centre of the coupling. The tangential component of displacement for each point on the tooth is obtained for a full rotation of the direction of misalignment to produce the slip path for that point. The details of the calculation are given in Appendix A. The displacement is sinusoidal in both the radial and axial directions, with a phase difference that depends on the pressure angle. The peak-to-peak distance of the slip path in the axial direction is approximately equal to the product of pitch radius and misalignment angle (in radians); in the radial direction it is approximately equal to the product of misalignment angle and axial distance from the centre of the tooth. The slip paths have been calculated for various tooth locations of the reference spline coupling and are compared to the slip path obtained from the boundary element modelling (for a frictionless analysis) in Figure 7.1.
Figure 7.1 - Slip paths for rigid body and boundary element analyses
The slip paths produced by the two methods show a good correlation in their variation over the tooth surface. The rigid body analysis overestimates the slip amplitude; the elasticity of the coupling reduces the actual slip distance. The rigid body analysis also simplifies the shape of the slip path and does not produce the crossover that may occur. The rigid body analysis generates slip paths that are symmetrical about the centre of the coupling, unlike the actual slip paths produced by the boundary element results.

Whilst the effects of elasticity prevent an accurate rigid body analysis, the rigid body movement also prevents the coupling from being analysed as a purely elastic component. The bending moment and angular deflection of the shaft section can be related by simple elastic theory. However, the relationship between coupling misalignment and bending moment is more complex; the effective stiffness of the coupling is a function of the geometry, applied torque and misalignment angle itself. The relationship between bending moment and misalignment angle is required to relate the defined coupling misalignment angle to the remote misalignment or actual bending moment of a real application. For the design software reported in Chapter 6, an iterative solution procedure is used to determine the actual bending moment and defined misalignment angle for a reference remote misalignment angle, assuming rigid supports at the reference points.

In practice, it will usually be difficult to predict or obtain the actual forces and displacements acting on the shaft-coupling system, and hence the actual misalignment angle may not be known. The ranking of the different coupling geometries in terms of wear parameter is not affected, but an accurate misalignment angle is required if the contact regime is to be determined.

### 7.1.2 Contact regimes

The boundary element analyses have revealed different forms of contact that may exist between the tooth pairs. These were classified in Section 5.3.2.1, according to the areas of separation. The geometry affects which regime occurs for a particular set of loading conditions, but it is the relationship between elasticity, torque and misalignment that produces the different types of contact. With a rigid component, misalignment would cause contact to occur only on the edges of the two heavily
loaded teeth. This tilting of the teeth causes a circumferential displacement that causes other teeth to move apart, as described by Neale et al. [115]. With elasticity, the teeth will deflect bringing more areas of the teeth in contact, and reducing the overall circumferential displacement. Buckingham modelled this behaviour as an elastic wedge [3], but a full computational analysis is needed to accurately predict the separation and contact pressures.

The pressure distribution form for a coupling was shown to be directly related to the misalignment angle, the elastic modulus and inversely to the applied torque. If the ratio of these parameters is kept constant, then the normalised pressure distribution remains identical. This could allow the tooth loads for any condition to be accurately predicted without need for a full computational analysis. For this study, only the $P_{\text{max}}/P_{\text{av}}$ and $C_{\text{max}}$ parameters have been correlated to the input parameters; these allow a qualitative estimation of the contact regime and the distribution of tooth loads.

The contact regime may be as important as the wear parameter for assessing the coupling durability. For a constant maximum wear depth parameter, a uniform regime may give a greater total wear volume, as shown in Figure 5.44. Nevertheless, the intermittent loading of a non-uniform contact regime may lead to more severe wear. This aspect is not included within the Archard wear equation used for the wear parameter. The contact regime must therefore be considered as a separate factor when assessing a particular coupling.

The contact regime may also influence other forms of damage, particularly fatigue failure due to tooth bending. Figure 5.69 showed that, although the maximum bending stresses may not be altered, the area affected by a high alternating stress is greater for a more cyclic regime. Again, the dynamic loading may further increase the stresses that occur and promote fatigue failure.

The separation of teeth under non-uniform regimes may have other consequences that could be beneficial in some circumstances. The removal of debris from the contact is likely to be increased when separation occurs. Perhaps more significantly, the separation will allow any lubricant to penetrate the contact region. The rocking action may even generate create a hydrodynamic lubrication effect. Neale et al. suggest that, for a gear coupling, perfect alignment should be avoided as it would hinder lubrication
of the contact area [115]. A minimum misalignment of approx. 0.05° was suggested. For a typical spline coupling with uncrowned teeth, a much greater angle of misalignment would be required to significantly improve lubricant penetration. The beneficial and detrimental effects of the various regimes are an area for further investigation.

7.1.3 Dimensionless analysis

A computational analysis was used to overcome some of the deficiencies of the rigid body analysis, but other errors may be introduced (Section 5.4.1). Although the mesh used for the analyses would not be sufficient for a full and accurate stress analysis of the two components, it was sufficient to reveal the general behaviour of the coupling. The dimensionless parameter study has also produced valuable information for relating the results obtained for the limited number of analyses to a variety of geometries and operating conditions, although extending the results much beyond the range of parameters used may increase errors.

For the analyses performed, four equations have been produced to predict the pressure distribution, tooth load, bending moment and wear depth parameters. The equations for the first three parameters show a standard deviation of between 4% and 10%. With the mesh being optimised for speed rather than accuracy, this is a reasonable value, and includes the computational errors as well as the correlation errors. An improvement on these values could be obtained by increasing the complexity of the equation and/or the number of significant figures for the equation constants. However, with the limited number of data points and the computational errors themselves, there is little justification for making such improvements.

The wear parameter equation gives a standard deviation of 17%. The greater error for this parameter could be expected, since it is strongly dependent on the maximum pressure value. The maximum pressure value occurs at the edge of the contact region where a singularity exists, and the computational errors are expected to be greater.
7.2 Experimental methods

The spline coupling wear tests reported in Chapter 3 were valuable for demonstrating the various failure modes and the form of wear damage on the teeth, particularly with reference to the computational analyses of the wear test models. However, the lack of a full complement of teeth severely reduced the similarity between the experimental conditions and real spline couplings. The difficulty of accelerating fretting wear tests was regularly acknowledged in the literature reviewed in Chapter 2, and there is no simple solution that can be applied in this case. A full complement of teeth would better represent a real coupling but would require a proportionately greater torque to accelerate the wear process. This would result in an unrealistic contact regime and, once more, an unrepresentative fretting test.

The fretting visualisation work provided some general observations of fretting wear and debris movement, and some observations agreed well with previously reported fretting wear studies. As discussed in the introduction to Chapter 4, the test rig was originally conceived to mimic spline tooth behaviour. The computational analysis results have shown that the slip path between spline teeth is rather more elaborate than that of the test rig. Additionally, the constant load of the visualisation test rig differs significantly from the load behaviour of spline teeth. These factors reduce the value of the tests for simulating spline coupling study.

The probabilistic/statistical nature of wear and particle transport was demonstrated in the visualisation tests. This has been recognised as important for many years – with wear coefficients being related to the probability of asperity junction fracture. This complicates the task of assessing visualisation work. The many factors which affect fretting wear also hinder visual investigations of the process.

Many studies of particle behaviour have involved a full layer of particles with complete, or almost complete, separation of the bulk surfaces. The visualisation study in this case did not include a full bed of particles. Such a contact may be more difficult to analyse, but will be the general case of initial wear, and may be of more interest for many applications. The lack of fluid-like powder flow under such situations was observed in the tests, with particles instead moving en masse and whilst attached to one of the first body surfaces.
7.3 Spline coupling design considerations

When designing a spline coupling for an application or evaluating an existing design, it is necessary to distinguish between the different wear and failure modes that may occur for the application, and how much wear can be tolerated. This study has been concerned with wear due to misalignment and originated from the need to minimise the problem for aviation applications. In these circumstances, only small amounts of wear can be tolerated for safety reasons. It is for this situation that the boundary element modelling best represents the actual conditions, and the wear parameter results should provide a good evaluation of the coupling.

The computational results suggest that the angle of misalignment is critical, the wear depth being approximately proportional to the square of the angle. A change in regime caused by increase in misalignment angle adds to the need to reduce the misalignment of a coupling. The experimental results confirm this, and also show the importance of the lubrication method.

This leads to the conclusion that, for any application, changes in the design and specification of the fixtures and lubrication could be of much greater benefit than changes in coupling design. Although the design of these aspects may be restricted, and the lubrication and alignment may be ordinarily designed as best as allowable within these restrictions, it is suggested that every effort is made to ensure optimum alignment and good lubrication, perhaps reconsidering the imposed restrictions.

The experimental results confirm that, of the spline geometry options, the pitch diameter has the biggest influence on the wear, with increasing pitch diameter increasing wear despite the lower average tooth loads. Only four analyses are available to examine the effects of pressure angle, and suggest that a 30° pressure angle is preferable to one of 20°.

Larger facewidths appear to improve the wear behaviour within the range of conditions studied; outside of this range, the behaviour may differ. The range of facewidth studied is approximately 0.3 to 0.9 \times \text{pitch diameter}. In most circumstances, the facewidth may be selected through strength and weight considerations rather than for a possible improvement in wear. An increase in
facewidth parameter $b^*$, implies a change of position for the misalignment application. The effect that this has on the wear parameter for varying facewidth can be accounted for by the spline design software reported in Chapter 6.

The spline design software offers the best method for optimising the coupling geometry. Most couplings will be designed from an existing design standard, which limits the choice of coupling design and thus the amount of optimisation. With these restrictions, the smallest pitch diameter that can be allowed will usually show the minimum wear. This suggests that the choice of strength reserve factor will control the actual wear, and should be kept to as low a value as acceptable for the application.

The design software assumes a pure angular misalignment. With this assumption, for constant shaft lengths, a relatively stiff coupling will experience the smallest "coupling" misalignment angle, with more of the angular misalignment accommodated through shaft flexure. As discussed in Section 7.1.1, the shaft supports are not rigid and the relationship between bending moment and misalignment angle will also be affected by these supports. A stiffer coupling may put greater stresses on the supports and thus increase the remote misalignment. In the terminology of the boundary element analysis, the boundary conditions provided by the shaft supports need to be defined and included in the analysis of the spline couplings. For most usual spline coupling set-ups, these effects are likely to be small. Shear forces on the coupling, particularly for parallel misalignment, are also affected by the type of boundary conditions for the shaft supports.

Although the torque does affect the overall volume of wear, the maximum wear depth is unaffected by the torque according to the parameter study results. The contact regime may even be more damaging for lower torques. In many circumstances, a spline couplings may be operated at torques significantly below the maximum design torque for long periods. It might reasonably be assumed that such operation will not contribute to the wear of the coupling; the results of this study suggest that this is not the case.

Besides tooth wear, tooth bending fatigue caused by misalignment or fatigue caused by torque cycles may also limit the life of a spline coupling. Preventing a strongly cyclic contact regime should help reduce tooth bending fatigue failure. Reducing the
damage caused by torque cycles requires a reduction in the maximum stresses and has not been considered in this study. The results obtained may allow for a better estimation of the maximum tooth loads, which may assist in predicting the fatigue life of a misalignment coupling.

Crowning and manufacturing errors have been examined only briefly in this study but are important aspects of spline design. The crowning analysis appeared to use a relatively large crown radius, causing a highly counterformal contact, yet still produced an improvement in wear parameter. Applying a crown to the tooth should in most circumstances improve the wear life of the coupling, although the experiments of Ku et al. [118] showed that this is not always the case. A pitch error is likely to always produce a greater maximum wear depth than a perfectly manufactured spline, even if the greater wear is on one tooth alone.

7.4 Further work

This study could be developed in four ways. Improvements in the computational analysis could be made, including a greater number of analyses for the parameter study. Areas of spline design excluded from the parameter study could also be analysed. The experimental work could be expanded upon and improved. Lastly, the results of the study could be used for other investigations of spline couplings.

The results of any further computational work (Section 7.4.1 and Section 7.4.2) could be incorporated into the spline design software produced during this research programme. The software could also be developed to allow the expected pressure distribution or slip paths to be presented to the user. The possibility for further developments was incorporated into the software; the result screen (Figure 6.7) includes options for a number of features which may be implemented following additional work.

7.4.1 Further computational analyses for parameter study

The parameter study was designed to enable different spline coupling geometries and operating conditions to be assessed quickly and easily without the need to perform a boundary element analysis for each coupling. The equations presented in Section 5.4
are the result of this study. To improve the correlation of the results, further analyses could be carried out using the same basic design of models. This would give a greater number of data points for correlation. In addition, the range of each parameter could be expanded; for example, the effects of a longer facewidth could be examined.

With faster computer processors, greater memory capacities and improved versions of the boundary element software, it may now be possible to carry out additional analyses with a greater mesh density to improve the accuracy of the raw computational results.

7.4.2 Spline design options

The spline geometry used in this study was a basic design, consisting of straight involute teeth protruding from a hollow shaft with the male teeth centrally located relative to the female teeth. Other types of spline teeth and coupling arrangements are possible. Table 5.1 listed a number of spline design options that were not included in the parameter study, but which could be included in further analyses. For each additional option, the number of computational analyses required for a full parameter study increases rapidly.

Possible areas for further investigation are:

- **Different tooth heights** – the major and minor spline diameters can be varied independently to give different tooth sizes. The parameter study kept the actual tooth height proportional to the module.

- **Tooth shapes** – non-involute tooth profiles could be used. Flat teeth could be used at pressure angle equivalent to the involute teeth. Alternatively, parallel-sided flat teeth may be used. Changes in tooth shape may lead to appreciable changes in slip path.

- **Tooth thickness** – the tooth thickness for the parameter study was a fixed proportion of the module, but could be varied giving an additional design parameter. This may affect the stiffness of the coupling, but is unlikely to alter the pressure distribution, slip path or wear significantly.
- **Wall thickness** – the wall thickness will affect the torsional and bending stiffness of the coupling and thus the pressure distribution and misalignment angle. The spline models used in this study had teeth protruding from the shaft section. It is also possible to cut the teeth from the shaft; e.g. the externally toothed shaft section diameter may be equal to the major diameter of the teeth rather than the minor diameter.

- **Helical splines** – spline teeth that form a helix around the shaft are commonly used in some industries, and could be modelled using the same techniques.

- **Tooth corrections** – crowning was only briefly examined, and was not included in the parameter study. The crown radius represents an additional design parameter that could be investigated. Helical lead correction is another tooth modification that may be used to reduce the asymmetry of the loading due to the torque. These areas would be particularly beneficial to spline designers, allowing the optimal crown radius and lead correction to be calculated.

- **Facewidth** – the ratio of male facewidth to female facewidth was 3:4 for each analyses. The effects of varying this ratio could be investigated. Different forms of tooth engagement are also possible; for the parameter study, the male teeth were centrally located and thus fully engaged. The effects of having both the female and male teeth only partially engaged (i.e. only the ends of the teeth mesh) could be investigated.

- **Reversed direction (Female driving)** – all analyses were carried out with the misalignment direction rotating in the same direction equivalent to the male driving. With the female component driving, the slip path will be reversed, affecting the frictional forces. These effects could be investigated preferably using smaller load steps between misalignment positions.

- **Reverse face contact/varying backlash** – for the toppling regime, reverse face contact will always occur. Depending on the amount of backlash (i.e. ratio of circular tooth thickness to circular space width), reverse face contact may occur for any of the contact regimes. In the parameter study, if reverse face contact occurred, the results would be rendered invalid. Changes to the model
design could allow reverse face contact to be included, but would greatly increase solution time.

- **Manufacturing errors** – varying amounts of pitch error and cumulative pitch error could be investigated. A comprehensive study of pitch error, using the same non-dimensional approach as the parameter study, would involve several additional input parameters relating to the severity of error and the number and position of teeth on which the errors occur. Unlike other forms of tooth modification, each analysis would require a full rotation (i.e. 5 load steps), since each tooth experiences a different loading and the rotation cannot be represented by the sequential teeth. Other forms of manufacturing error could also be analysed by varying the initial gap applied to each tooth contact.

- **Loading** – a shear force applied to the coupling in addition to or instead of the misalignment angle could allow the effects of different types of misalignment to be examined.

### 7.4.3 Improvements to experimental work

Development of the visualisation work may substantially improve the understanding of the fretting wear behaviour of spline couplings. The current study did show the fretting wear process, but was not pursued due to the differences between the test rig behaviour and that found to occur in spline couplings.

To ensure that the visualisation work accurately represents the spline coupling behaviour, the test rig needs to be significantly modified or a new test rig designed. The test rig should:

- replicate the approximate shape of the slip path between the spline couplings revealed in the computational study. It is unlikely that the slip paths can be replicated accurately over the entire tooth surface, as this would require the same elastic deformation. However, an improvement on the reciprocating arc should be possible

- allow dynamic loading and specimen separation. A tilting/rocking motion on one specimen could be used to simulate the varying pressure distribution
experienced by each tooth and the separation that occurs for the non-uniform contact regime. This is particularly important to investigate the increase in wear that is expected to occur due dynamic loading and the consequences of the different contact regimes.

Improvements to the test rig are also required to ensure proper and consistent alignment and to allow greater normal loads. This will also require changes to the visualisation equipment to permit greater working distances for the microscope objective to contact interface.

The difficulties of conducting representative laboratory fretting tests at accelerated rates have been discussed. The three-tooth spline tests were possibly the best method of accelerating spline tooth wear, but the associated problems have been noted. It would be extremely beneficial to obtain wear data for fully toothed spline couplings, particularly of the same design and operating with the same conditions as the parameter study analyses. This would allow a much better evaluation of the computational wear parameter results and may even allow the computational results to be converted to actual wear depths. Such wear tests would also require accurate determination of the misalignment angle as defined in the parameter study.

7.4.4 Developments using results of study

The results obtained from this study could enable other aspects of spline coupling performance to be investigated in new ways.

The contact loads on the spline teeth can now be approximated with greater confidence and detail than previously possible. Using the equations for pressure distribution and tooth load parameters, the tooth contact loads and pressure distributions can be determined for any spline geometry, torque and misalignment (within the range of parameters investigated). The known contact pressures can then be applied to a boundary element or finite element model of either the male or female component of the coupling. This removes the lengthy contact analysis portion of the analysis and allows a much faster solution. Also, with only one component modelled and no contact patches, the complexity of the problem is reduced and a more detailed model can be used. This technique would allow a more accurate calculation of the material stresses. This could be used for a more accurate prediction of fatigue life and
improvements in coupling design to minimise stresses. With a more accurate
calculation of maximum material stress, it may be possible to reduce the strength
reserve factor and therefore use smaller diameter couplings. This would assist in
reducing wear.

The separation of the spline teeth can be obtained from the computational results. A
theoretical investigation of the hydrodynamic action caused by the rocking motion of
the teeth could be investigated using existing hydrodynamic lubrication laws.
Conclusions

The tooth contact of involute spline couplings operating with misalignment has been investigated using computational and experimental methods, with particular regard to the tooth wear. The tooth wear shows the characteristics of fretting wear; the wear mechanisms and particle behaviour for contacts similar to spline teeth have also been investigated.

The boundary element method was used to model the rotation of a misaligned coupling. The tooth contacts were analysed using the node-to-node constraint-based algorithm of the boundary element software, from which the contact pressures and relative displacements of the teeth have been obtained.

The misalignment was applied to the coupling as a fixed angular displacement about the centre of the coupling and was rotated in steps of 90°, in the direction equivalent to the male component driving. The resultant bending moment was also calculated, in both the direction of misalignment and perpendicular to this.

A number of relevant spline geometry and operating variables were specified and six dimensionless parameters defined, which have formed the basis of a parameter study that permits a wide variety of couplings to be assessed. A difference between remote shaft misalignment and misalignment at the coupling has been recognised and a consistent misalignment angle definition – the misalignment at 20 modules from the teeth ends – has been used.

Previous rigid body analyses have indicated that contact may occur at the ends of teeth alone. This has been shown and expanded upon by the computationally obtained pressure distributions. Four distinct contact regimes have been described; the regime is likely to strongly influence both tooth wear and fatigue damage. In increasing order of misalignment, the regimes are:

- uniform (full tooth contact on all teeth)
- cyclic (either one or both ends of teeth separate during rotation)
• discontinuous (contact on tooth ends alone with teeth fully separating during the rotation)
• toppled (misalignment too large for tooth contact to be maintained on single face of tooth alone)

The regime that occurs depends upon spline geometry, material elasticity, torque and misalignment. For a given spline geometry, the contact regime has been shown to be entirely dependent on the ratio of misalignment angle to torque. Increases in tooth number (at constant tooth module) or facewidth tend to alter the regime in the manner of a greater misalignment.

The pressure distributions have shown that, with or without misalignment, greater pressures occur at the edges of the tooth contact, and are highest at the inner diameter and at the end from which the male shaft emerges due to a difference in stiffness between the male and female component.

The relative tangential displacement (slip) between the teeth has been obtained from the computational results and the slip path for a complete rotation produced. The slip paths differ from those calculated using a rigid body analysis – the elasticity reducing the amount of slip. The slip paths vary over the tooth surface in a similar manner to that predicted, but show greater variations in shape. The actual slip paths are typically closed loops, although in some cases crossover. The slip paths are non-symmetrical, and often come to a point at one end. The shape and size depends upon the spline geometry and operating conditions; a constant ratio of misalignment angle to torque leads to an identical slip path shape (as for contact regime), although the size increases with misalignment angle. The variation in slip path shape and slip distance has been presented for a number of conditions.

The pressure distributions and slip distances have been combined to generate a wear depth parameter. This parameter, based on the Archard wear equation, is \( W = \int p \, ds \)
and, for constant wear coefficient, should be proportional to the wear depth at the point computed. The value of \( W \) varies over the tooth surface and the maximum value has been obtained for each analysis. The maximum wear parameter always occurs at the inner diameter of the tooth contact, at the end of the tooth from which the male shaft emerges, i.e. at the point of maximum pressure.
Two dimensionless parameters have also been defined to describe the pressure distribution. \( C_{\text{max}} \) is the maximum axial deviation of centre of pressure from the tooth centre, divided by facewidth; \( P_{\text{max}}/P_{\text{av}} \) is the ratio of maximum tooth load to average tooth load. Regression formulae for these three parameters and the bending moment have been formed from the computational parameter study results:

\[
C_{\text{max}} = 0.5 - 30 / (58 + 0.01 E^* \theta b^* z^3)
\]

\[
P_{\text{max}}/P_{\text{av}} = 1 + 0.45 E^* \theta
\]

\[
W/Em = 5.8 \times 10^{-4} \times z \times \theta^{1.9} / b^{0.5}
\]

\[
M/T = 3.13 (C_{\text{max}} - 0.18)^{0.9} b^{0.36} / z^{0.47}
\]

The accuracy of these regression formulae varies from 3% to 17% and reflects the relatively coarse computational mesh used. The equations reveal the influence of the various design and operating condition variables.

The wear parameter has been used to rank different spline geometries for wear susceptibility. The misalignment angle is critical – wear depth being proportional to the square of the misalignment angle. However, torque does not significantly affect the maximum wear depth (although may affect total tooth wear). The results also suggest that a large a facewidth as possible should be used to minimise wear and that, for a constant pitch diameter of spline, as many teeth as possible should be used (i.e. a finer pitch/smaller module). A software program has been produced that uses the parameter study results to predict an optimum spline for reducing wear. The software gives a quantitative value to describe the maximum wear depth; this is, however, based on purely computational results and should be validated with fully-toothed splines in practical applications before being relied upon as a definitive spline design tool. It does, however, give a first attempt at evaluating the differences between spline geometries for tooth fretting wear.

Experimental tests using modified spline couplings have supported the conclusions that misalignment angle has a substantial impact on wear, and that torque is less influential for maximum wear depth. The influence of contact regime on fatigue and wear also agrees with tests conducted at different values of torque. The tooth wear from many of the experimental tests showed a consistent variation in severity along the axial direction, with tooth wear being greatest at the upper end (where the male
shaft emerges), reduced damage at the centre, and a small increase at the lower end. This wear variation matched the distribution of wear parameter over the tooth surface.

A computational analysis of crowned spline teeth has shown that the crowning acts to reduce the oscillation of contact pressure and reduce the effective stiffness of the coupling. Even with a larger than optimum crowning radius that produced a highly counterformal contact, the wear parameter was significantly improved. An analysis of a 45 mm diameter coupling with a positive adjacent pitch error of 25 μm resulted in an increase in maximum wear parameter of 160%, and some teeth showing no contact and thus no wear.

A second set of experimental tests, with a contact of similar size to the spline teeth, used sapphire and steel specimens to enable the contact to be visualised during the fretting process. The work has shown that the wear may occur initially by the formation of tiny pits, which gradually grow in size and number. The pits may form along the crests of ridges of a rough surface. Wear particles in the initial stages of wear were formed in a range of sizes and adhered to either surface, without rolling or shearing to accommodate the relative displacement between specimens. Particles were followed to examine the particle transport that occurs under spline coupling-like slip; particle movement occurred when a particle transferred from one surface to another, at a seemingly random point in the oscillating movement cycle. Slight changes in contact conditions, caused by the test rig operation or specimen wear, that bring the particles into contact with new asperities is suggested as the reason for sudden particle transfer. Powdery debris was seen to come together and move en-masse rather than forming load carrying beds or acting as a low-shear stress interfacial layer. The normal loads for these tests were relatively low – higher contact pressures may cause the particle behaviour to differ.

This research has revealed the complex dynamic loading and slip of spline teeth that results from the combination of rigid body movement and elastic displacements. This knowledge can be used to improve the wear life of spline couplings operating with misalignment.
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Appendix A

Rigid body slip path calculations

The slip path is obtained by calculating the rigid body movement of a point on a tooth, $P$, due to a rotation about an axis. The axis is rotated around 360° to obtain the full slip path.

A flat tooth profile is assumed, rather than involute, to simplify the problem. For convenience, the coordinates are chosen so that the tooth is at $\alpha$ to the $x$ axis. This implies that movement in the $y$ direction is normal to the tooth surface and so does not contribute to the slip path.

Global coordinates are defined as $x,y,z$, with origin at the centre of the coupling.

Tooth coordinates are defined as $X,Y$ with the origin at the intersection of the pitch diameter of the tooth and the midpoint of the facewidth.

Global Coordinates $(x,y,z)$

![Global coordinates for rigid body calculations](image)

Figure A.1 – Global coordinates for rigid body calculations

$R$ is the pitch radius of the spline
The coupling is rotated through an angle $\theta$, about an axis along the vector $\mathbf{u}$, which is at an angle $\alpha t$ to the $x$ axis.

Tooth Coordinates $(X,Y)$

\[
\begin{align*}
\mathbf{u} &= \begin{bmatrix} \cos \alpha t \\ \sin \alpha t \\ 0 \end{bmatrix} \\
\mathbf{OP} &= \begin{bmatrix} R \cos \alpha + X \\ R \sin \alpha \\ Y \end{bmatrix}
\end{align*}
\]

From Faux and Pratt [137] p 73, the rotation of a point with vector $\mathbf{r}$, through an angle of $\theta$ about a vector through the origin with components $(u_1,u_2,u_3)$ is $\mathbf{r}'$ where:

\[
\mathbf{A} = \begin{bmatrix}
    u_1^2 + \cos \theta (1 - u_1^2) & u_1 u_2 (1 - \cos \theta) - u_3 \sin \theta & u_2 (1 - \cos \theta) + u_3 \sin \theta \\
    u_1 u_2 (1 - \cos \theta) + u_3 \sin \theta & u_2^2 + \cos \theta (1 - u_2^2) & u_3 (1 - \cos \theta) - u_2 \sin \theta \\
    u_3 u_1 (1 - \cos \theta) - u_2 \sin \theta & u_3 u_2 (1 - \cos \theta) + u_1 \sin \theta & u_3^2 + \cos \theta (1 - u_3^2)
\end{bmatrix}
\]

For rotation about the vector $\mathbf{u}$ as given above, this becomes:

\[
\mathbf{A} = \begin{bmatrix}
    \cos^2 \alpha t + \cos \theta (1 - \cos^2 \alpha t) & \cos \alpha t \sin \alpha t (1 - \cos \theta) & \sin \alpha t \sin \theta \\
    \cos \alpha t \sin \alpha t (1 - \cos \theta) & \sin^2 \alpha t + \cos \theta (1 - \sin^2 \alpha t) & -\cos \alpha t \sin \theta \\
    -\sin \alpha t \sin \theta & -\cos \alpha t \sin \theta & \cos \theta
\end{bmatrix}
\]

Figure A.2 – Tooth coordinates for rigid body calculations
But \((1-\cos^2 \omega t) = \sin^2 \omega t; (1-\sin^2 \omega t) = \cos^2 \omega t\)

Therefore, point \(P\) moves to point \(P'\) where \(OP'\) is given by:

\[
OP' = \begin{bmatrix}
(R \cos \alpha + X) \cos^2 \omega t + (R \cos \alpha + X) \cos \theta \sin^2 \omega t \\
+ R \sin \alpha \cos \omega t \sin \omega t (1 - \cos \theta) + Y \sin \omega t \sin \theta \\
\end{bmatrix}
\]

The movement of point \(P\) is the vector \(PP' = OP' - OP\)

\[
PP' = \begin{bmatrix}
(R \cos \alpha + X) \cos \omega t \sin \omega t (1 - \cos \theta) + R \sin \alpha (\sin^2 \omega t + \cos \theta \cos^2 \omega t) \\
- Y \cos \omega t \sin \theta - R \sin \alpha \\
- (R \cos \alpha + X) \sin \omega t \sin \theta + R \sin \alpha \cos \omega t \sin \theta + Y \cos \theta - Y \\
\end{bmatrix}
\]

The slip in \(X,Y\) coordinates is the \(x\) and \(z\) components of this vector. This gives:

**Slip in \(X\) direction**

\[
= (R \cos \alpha + X)(\cos^2 \omega t - 1 + \cos \theta \sin^2 \omega t) + R \sin \alpha \cos \omega t \sin \omega t (1 - \cos \theta) + Y \sin \omega t \sin \theta
\]

\[
= (R \cos \alpha + X)(1 - \sin^2 \omega t - 1 + \cos \theta \sin^2 \omega t) + R \sin \alpha \cos \omega t \sin \omega t (1 - \cos \theta) + Y \sin \omega t \sin \theta
\]

\[
= (R \cos \alpha + X)(\cos \theta - 1) \sin^2 \omega t + R \sin \alpha \cos \omega t \sin \omega t (1 - \cos \theta) + Y \sin \omega t \sin \theta
\]

**Slip in \(Y\) direction**

\[
= Y(\cos \theta - 1) + R \sin \alpha \cos \omega t \sin \omega t - (R \cos \alpha + X) \sin \omega t \sin \theta
\]

These equations have been used to calculate the rigid body slip path for comparison with computational results.
These equations can be simplified for small $\theta$, putting $\sin\theta = \theta$, $\cos\theta = 1$

Slip in $X$ direction
$$= Y\theta \sin\omega t$$

Slip in $Y$ direction
$$= R\theta \sin\alpha \cos\omega t - (R\cos\alpha + X)\theta \sin\omega t$$

...but for large spline $X << R$, therefore...

$$\approx R\theta \sin\alpha \cos\omega t - (R\cos\alpha)\theta \sin\omega t$$

$$\approx R\theta (\sin\alpha \cos\omega t - \sin\omega t \cos\alpha)$$

$$\approx R\theta (\sin(\alpha - \omega t) - \sin(\omega t - \alpha))/2$$

$$\approx R\theta (-\sin(\omega t - \alpha) - \sin(\omega t + \alpha))/2$$

$$\approx -R\theta \sin(\omega t - \alpha)$$

$$\approx R\theta \sin(\omega t + \pi - \alpha)$$

Thus the amplitude of the slip path in the $X$ direction is $Y\theta$, and the amplitude of the slip path in the $Y$ direction is approximately $R\theta$. The motion is sinusoidal in both directions, with the $X$ and $Y$ components being $(\pi - \alpha)$ out of phase.
Appendix B

Calculation of defined misalignment angle, $\theta$, for an applied remote misalignment

$m$ = spline module

$\theta$ = misalignment at $20m$ (as defined)

$\psi$ = misalignment at 50 mm (as applied)

$D_{io}$ = outer diameter of internal shaft

$D_{ii}$ = inner diameter of internal shaft

$= $ internal spline major diameter

$D_{Eo}$ = outer diameter of external shaft

$= $ external spline minor diameter

$D_{Ei}$ = inner diameter of external shaft

$M$ = bending moment due to misalignment

Figure B.1 – Misalignment angles for computational modelling and parameter study definition

In the boundary element analyses, the misalignment angle is applied at a distance of 50 mm from the end of the coupling. For the correct definition of $\theta$, the misalignment at $20m$ is required.
From the computational analysis, the bending moment of the coupling is known. No shear forces act on the system, therefore the bending moment is constant along the shaft. To obtain the correct misalignment, $\theta$, the deflection of the shaft sections between the points of application and the reference locations must be accounted for.

If the module is greater than 2.5 mm (i.e. $20 \times m$ is greater than 50 mm), then the correct misalignment is obtained by adding the deflected angle of the sections of shaft beyond the 50 mm points to the applied misalignment. For splines with a module less than 2.5 mm, the correct misalignment is obtained by subtracting the deflected angle of shaft sections beyond the $20m$ points.

The second moments of area are:

$$I_E = \frac{\pi}{64} (D_{ho}^4 - D_{ei}^4) \quad I_I = \frac{\pi}{64} (D_{io}^4 - D_{hi}^4)$$

The length of shaft section for which the shaft flexure angle is required is:

$$L = 20m - 50 \quad (m > 2.5)$$

$$L = 50 - 20m \quad (m < 2.5)$$

The end slope of a cantilever beam of elasticity $E$ with an applied bending moment is given by:

$$\text{End slope} = \frac{ML}{EI}$$

Therefore the angle at the end of the shaft is:

$$\Phi = \tan^{-1} \left( \frac{ML}{EI} \right)$$

This angle is calculated for both the external and internal shaft sections. The required misalignment angle, $\theta$, is then calculated as:

$$\theta = \psi - \Phi_E - \Phi_I \quad (m > 2.5)$$

$$\theta = \psi + \Phi_E + \Phi_I \quad (m < 2.5)$$
Regimes of Contact in Spline Couplings

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1 Introduction

Spline couplings allow shafts to be easily dismantled while providing a high torque capacity for minimum size. In addition, they can allow relative axial motion between the coupled shafts. For these reasons splines are one of the most commonly used elements in rotating machinery.

The size of the spline is usually determined by the diameter needed to limit the shear stress in the externally toothed ("male") coupling and by the wall thickness needed to limit the hoop tensile stress in the internally toothed ("female"). Consequently, little attention has traditionally been given to the details of the contact between the male and female teeth and a variety of tooth forms are available [1].

However, relative slip, of small amplitude can occur between the contacting tooth surfaces and this can give rise to fretting damage which may limit the life of the coupling and has the potential to compromise integrity. This has been a particular concern in aircraft mechanical systems and has been studied experimentally for many years [2,3]. Slip may be a consequence of elastic deformation in response to variations of torque [4] but is greatly increased by the presence of angular misalignment between the coupled shafts and the consequent bending moment. Couplings designed to accommodate relative axial motion ("plunge") are generally subjected to slip of much larger amplitude and are not considered here.

Recently, with the advent of numerical methods of contact analysis, it has become possible to determine the pressure and slip acting between the contacting spline teeth analytically. For example Leen and co-workers [5] have found the distribution of pressure and slip in an involute spline using the finite element method with stick-slip friction. The results of such analysis are potentially useful because fretting wear is highly dependent upon slip and pressure history. However, even with modern computing power, the analysis of even a single geometry of coupling is slow and complex. In addition, singularities of pressure may be present which make interpretation of the results difficult given that the mesh density is usually at a premium.

In the present paper, the results of a study of a generic involute spline coupling are presented as a function of a number of dimensionless quantities which control the contact behavior. The analysis was conducted using a commercial package based on the boundary integral element method.

The boundary element method is described in [6]. It entails the formation of a boundary integral equation which describes the traction and displacement conditions for the body under analysis. Dividing the surface into boundary elements, an equation describing the unknown displacement and traction values is obtained for each node. This matrix is then solved for the specified boundary conditions resulting in a complete solution for tractions and displacements over the body surface.

The results show a number of new or unreported phenomena and show distinct regimes of pressure and slip. The dimensional analysis allows the regimes to be determined for a wide range of geometry and operating conditions.

2 Method of Analysis

2.1 Relationship Between Misalignment and Bending Moment. Figure 1 shows a schematic view of a spline coupling attached to a flexible shaft subjected to an angular misalignment. The imposed elastic bending of the shaft causes the spline to be subjected to a moment. However, the magnitude of the moment depends on the terminal angle $\theta$ of the shaft which may differ from the remotely imposed angle due to the internal angular elastic deflection $\theta$ of the coupling which in turn depends itself on the bending moment.

If the spline is also subjected to appreciable torque then contact will occur on one side of the male and female teeth. The bending moment causes the center of pressure on the teeth to be displaced axially in opposite directions on each side of the coupling. When the shaft rotates this gives rise to a cyclic variation of pressure.

However, the center of pressure cannot be displaced beyond the ends of the teeth and the sum of the tooth forces is just dependent on the torque. This means that, for a given torque, a maximum bending moment can be applied, beyond which the spline cannot be in equilibrium with normal tooth forces restricted to one side of the teeth. Higher imposed bending moments will cause the spline to displace discontinuously through its internal clearance leading to contact on the reverse side of the teeth. This behavior ("toppling") is familiar from handling of spline couplings during assembly when torques are absent or very low.

In practice, contact on the reverse face may also occur if the
internal clearance is insufficient to accommodate the local elastic deformations. Friction in the teeth will affect this behavior but is unlikely to change its nature.

In the analysis that follows, we have assumed contact on only one face of each tooth, but have checked for reverse face overlap. This assumption is not valid for an imposed bending moment greater than a limiting value for "toppling," which depends on the applied torque.

The model couplings are subjected to a fixed torque and to a misalignment angle, fixed in magnitude at a specified axial location, but rotated progressively to simulate shaft rotation. The angle should not be confused with any remotely imposed angle which is likely to be considerably greater due to shaft flexure.

2.2 Dimensionless Groups. The splines analyzed here have involute profiles and a tooth height approximately equal to the module. The pressure angle, torque, elastic modulus, facewidth, tooth number, module, limiting coefficient of friction and misalignment angle have been varied while the tooth form and relation rim thickness have been held constant and only identical materials with Poisson’s ratio of 0.3 have been considered. Dimensionless groups describing the coupling are given in Table 1.

All other spline geometries such as tooth height, tooth thickness are adjusted proportionally to the module, and are based on the geometry of an experimental test spline. Details are in the appendix. The female facewidth is proportional to the male facewidth and the male shaft bore diameter and female shaft diameter are kept proportional to the pitch diameter.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Definition</th>
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<tr>
<td>$E^*$</td>
<td>Dimensionless Elasticity / Torque</td>
<td>$Em^* / T$</td>
</tr>
<tr>
<td>$b^*$</td>
<td>Dimensionless Facewidth</td>
<td>$b / m$</td>
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<tr>
<td>$\varepsilon$</td>
<td>Number of Teeth</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Pressure Angle</td>
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</tr>
<tr>
<td>$\mu$</td>
<td>Friction Coefficient</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>Misalignment Angle</td>
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</tr>
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</table>

2.3 Analysis.

2.3.1 Generation of Spline Geometry and BE Mesh. The spline geometry and meshing for each spline coupling were generated using software written especially for this study. The procedure used was first to calculate the co-ordinates of appropriate points on the coupling surface and then to form lines by connecting these points. Surface patches were then created by specifying four lines to form the edge of the patch. Taken together, the patches defined the boundary of the coupling. The boundary element mesh was then applied to the surface patches. The involute profile of the teeth was initially represented by 12 points through which a surface line was fitted. However, when the boundary element mesh was applied to the tooth surface, the involute profile was fitted with two quadratic boundary elements, hence the involute profile was only modelled as two quadratic lines. This leads to a small error in the involute shape, typically less than 1 $\mu$m for the 2.5 mm module cases studied. In addition, the conforming surfaces of the contacting male and female teeth were constructed using the same model profile, with the initial gap specified as zero. This ensured that any deviation from the involute profile did not lead to erroneous pressure variations.

2.3.2 Solution Method. The analyses were carried out using the boundary element software package BEASY (Computational Mechanics, Southampton, UK). The boundary element method explicitly describes the surface displacements and tractions at the surface nodes which is suitable for analysis of conformal contacts. The package uses a node to node contact algorithm to compute the contact interface conditions. Friction is also modelled, giving constraints for open, slipping, sticking and sliding (no friction) nodes. For contact analyses an iterative solution procedure is required; a boundary element analysis is performed on the contacting bodies; the contact configuration is modified according to this solution, and thus nodal constraints are modified. This is repeated until the contact configuration converges.

In the present analysis, a series of different load conditions were applied sequentially, necessary to accommodate the non-linearity of the frictional boundary conditions. A series of loads representing a complete rotation of the spline couplings was analyzed, providing slip distances and stick/slip transition information.

With the inclusion of a non-symmetrical bending moment and torque applied to the coupling, a complete three-dimensional model was necessary and no simplification from cyclic symmetry was possible. The tractions and displacements at the contact interface were of most importance, with the spline root stresses of less significance for the purposes of this study. The boundary element mesh was therefore constructed to maximize accuracy at the contact faces whilst reducing complexity at teeth roots in order to minimize solution time.

As discussed in section 2.1, the spline shafts affect how any misalignment will be passed through the spline teeth. The shafts for the boundary element model were set at 50 mm length and with diameters proportional to the tooth module to provide consistency through the various designs. This is sufficiently close that the additional bending deflections in the shaft were small compared to those in the coupling for the range studied so that the specified angle describes the true coupling misalignment with sufficient accuracy.

In each case, the coupling was loaded by a fixed torque applied to the lower outside edge of the female spline. The angular misalignment was produced by displacements specified at the upper end of the male corresponding to the required angular misalignment measured from a horizontal axis through the center of the coupling, whilst maintaining the lower end of the female spline shaft horizontal. The angular direction of the imposed misalignment was then changed in four steps of 90 deg leading to five sequentially applied load conditions corresponding to a complete...
Tooth 1

Contact mesh 2×10 elements on each tooth

360 deg rotation of the shaft. In addition, crowning and other tooth modifications have been incorporated by modifying the initial gap between the contact elements.

A typical three dimensional mesh of the entire coupling is shown in Fig. 2(b). Each contacting tooth surface was discretised as 2×10 elements, leading to 5×21=105 surface nodes (Fig. 2(c)). Each run took approximately 4–5 days on a personal computer having 1 Gb of random access memory and a Pentium III processor operating at 800 MHz.

3 Results

3.1 Introduction. A total of 14 conditions of geometry, loading and misalignment have been analyzed using the boundary element method. The results of interest for wear analysis are the contact pressures and slip amplitudes.

Distributions of pressure over the surface of each tooth pair in the coupling are illustrated in Figs. 3, 4, 6, 7, and 10. An explanation of the plots is given in Fig. 3. The left hand side of each plot represents the inner diameter of the contact patch (at the female minor diameter, toward the root of the male) with the right hand side representing the tip of the male tooth. The upper end of each plot is the part of the area of contact nearest to where the male shaft emerges from the female.

The distribution of pressure at any instant varies from one tooth pair to the next, and each tooth experiences a cyclically varying pressure distribution as the coupling rotates. This can be seen in Fig. 4, where the pressure distributions within the tooth contacts are plotted for each tooth and each of the five stages (angles) of rotation.

As the shaft rotates, the pressure distribution for the tooth at a given angle to the direction of misalignment remains similar. For this reason, a single misalignment direction can be taken as representative of the contact condition for the spline geometry and operating condition. The 90 deg rotation stage has been used to illustrate subsequent results.

The variation in pressure distribution and tooth load can also be seen in Fig. 5, which shows the center of pressure and load per tooth for the 90 deg rotation stage.

3.2 Effect of Operating Conditions on Pressure Distributions. Changes in operation conditions affect how the pressure distribution varies from tooth to tooth. With no misalignment present the pressure distribution would be identical from tooth to tooth. As the degree of misalignment increases, the center of pressure fluctuates from top to bottom with increasing amplitude. The applied torque also affects the center of pressure and variations of tooth load. This is shown in Fig. 6.

3.3 Effect of Spline Geometry on Pressure Distributions. The effect of spline geometry is of greatest interest for determining a preferred coupling for an application. Each dimensionless parameter has been varied and the resulting pressure distributions are given in Fig. 7.

3.4 Friction Regime and Relative Slip. Figure 8 shows the friction conditions for two cases where stick was encountered within the contact ("stick-slip regime"). All the other runs resulted in all elements being in the slip or open conditions ("gross slip regime").

The slip amplitude is known to play an important role in fretting wear. However, in the present instance, the relative slip between the contacting teeth describes a closed loop of varying shape. Typical shapes are illustrated in Fig. 9 for the conditions of Fig. 4. The main figure, left shows the trajectories of relative displacement of the contacting surfaces for a range of points on the tooth. The insets, right, show the heavily loaded part of the path for two selected points on the tooth. This is discussed further in section 4.

3.5 Tooth Crowning. Tooth crowning is regularly used to alleviate the effects of misalignment. Here, this has been investi-
gated by introducing a radius of relative curvature in the axial (lead) direction to the spline whose pressure distribution (when uncrowned) is shown in Fig. 4. The details of the geometry for this analysis are given in Table 3. The pressure distribution of a spline coupling with lead crowning is shown in Fig. 10. The effect of tooth crowning is to reduce the displacement of the center of pressure from the center of the tooth, hence reducing the flexural stiffness of the coupling.

3.6 Summary of Results. The results are summarized in Table 2. The regimes of pressure ("toppled," uniform, cyclic and discontinuous) are indicated. Here, "cyclic" indicates that part of a tooth surface becomes unloaded whereas "discontinuous" indicates that the tooth load declines to zero during some part of the rotation. Also indicated is the friction regime which was "gross slip" (all nodes in slip or open condition) except for the two
Fig. 7 Tooth pressure distributions for a range of parameters showing the effects of geometric effects and of coefficient of friction: (a) $E^* = 6.56$, $b^* = 10.8$, $z = 18$, $\alpha = 30$ deg, $\mu = 0.3$, $\theta = 0.1$ deg; (b) $E^* = 6.56$, $b^* = 5.4$, $z = 18$, $\alpha = 30$ deg, $\mu = 0.3$, $\theta = 0.1$ deg; (c) $E^* = 3.28$, $b^* = 10.8$, $z = 18$, $\alpha = 30$ deg, $\mu = 1.0$, $\theta = 0.1$ deg; and (d) $E^* = 6.56$, $b^* = 10.8$, $z = 27$, $\alpha = 30$ deg, $\mu = 0.3$, $\theta = 0.1$ deg.

Fig. 8 Local friction conditions showing stick slip: (a) $E^* = 6.56$, $b^* = 10.8$, $z = 18$, $\alpha = 30$ deg, $\mu = 0.3$, $\theta = 0.01$ deg. and (b) $E^* = 3.28$, $b^* = 10.8$, $z = 18$, $\alpha = 20$ deg, $\mu = 1.0$, $\theta = 0.1$ deg.

Table 2 Summary of results

<table>
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<tr>
<th>$E^*$</th>
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<th>$z$</th>
<th>$\alpha$ (degrees)</th>
<th>$\mu$</th>
<th>$\theta$ (degrees)</th>
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<th>Friction regime</th>
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<th>$C_{max}$</th>
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<td>-</td>
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<td>0.30</td>
<td>Topped</td>
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<td>-</td>
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<td>0.10</td>
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<td>0.333</td>
<td>(3)</td>
<td>0.57</td>
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Table 3: Analysis of coupling with lead crown

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<th>OPERATING CONDITIONS</th>
<th>RESULTS OF ANALYSIS</th>
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<td>Number of Teeth</td>
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<td>Torque</td>
</tr>
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<td>Module</td>
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<tr>
<td>Pressure Angle</td>
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<td>Male Facewidth</td>
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<td>Lead Crown Radius</td>
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</table>

<table>
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<th>PRESSURE REGIME</th>
<th>FRICTION REGIME</th>
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<th>(W / E m)</th>
<th>(M / T)</th>
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<tr>
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<td>Gross Slip</td>
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</tbody>
</table>

Fig. 9 Shape of the path of slip over tooth surface. The main figure, left shows the trajectories of relative displacement of the contacting surfaces for a range of points on the tooth. The orientation of the diagram is the same as for the pressure distributions (Fig. 3) with the axis of the coupling to the left. The insets, right, show the heavily loaded part of the path for two selected points on the tooth.

"stick-slip" cases illustrated in Fig. 8. It appears that stick can occur if the friction is high or if the imposed tangential displacement is low in comparison to the elasticity of the bodies.

Also included are the maximum relative axial displacement of the center of pressure, \(C_{max}\), from the tooth center, the maximum relative tooth load, \(P_{max} / P_m\), the ratio of bending moment to applied torque, \(M / T\), and the wear depth parameter, \(W\). These quantities are discussed in section 4. The numerical results were not valid for the "toppled" regime due to reverse face contact, as explained in section 2.

4 Discussion

4.1 Distribution of Pressure. The overall form distribution of pressure is similar to that found in a misaligned elastic punch as pointed out by Hyde et al. [3]. The following more detailed features are evident.

There is a concentration of pressure at the periphery of the area of contact due to the discontinuous geometry. In addition, the pressures are generally higher at the root of the male than at the tip and at the inner end of the male coupling than at the extremity within the female due to the inequality of local stiffness.

During shaft rotation, both the center of pressure and the tooth load oscillate with the maximum value occurring when the tooth makes an angle of \((\pi/2 - \alpha)\) to the direction of the applied misalignment when the tooth surface is most nearly normal to the direction of the imposed approach or recess. A remarkable feature is the decline of total tooth load at intermediate positions, leading in some instances to the complete unloading of each tooth twice per revolution of the shaft. (Fig. 6(b)). This occurs at positions \(\theta = -\alpha\), \((\pi - \alpha)\) and is due to the contact forces on the loaded teeth causing a relative rotational displacement of the toothed portions of the coupling.

The skewness of the pressure distribution is indicated by the parameter \(C\), defined as:

\[
C = \frac{d_p}{b}
\]

where \(d_p\) is the distance of axial displacement of the center of pressure from the center of the facewidth, and \(b\) is the facewidth itself. The parameter exhibits a steady part, due to torque and a variable part, due to misalignment.

The maximum value of \(C\) for each condition is included in the summary of results, Table 2.

For most of the results presented here, all the contacting elements were in the slip or open states (gross slip regime). A notable exception was the case illustrated in Figs. 7(c) and 8(a) where the coefficient of friction was 1.0. Here it is evident that the noncontacting regions of the teeth are reduced in area and the mean pressure is lower than for the corresponding low friction case. The bending moment (Table 2 last column) is also significantly affected by the coefficient of friction.

4.2 Slip Path. The slip path (Fig. 9) is similar to the relative surface tangential displacements that would occur in rigid bodies subjected to the same misalignment, which have a harmonic form. However, it is evident that the elastic deformations present here have distorted the slip path somewhat. Moreover, it is evident that both the pressure and the slip vary continuously during the rotational cycle. This has the effect that much of the relative displacement occurs at low or zero pressure. The phase of the two parameters is such that, near the tooth ends, the heavily loaded part of the slip cycle is in one particular direction, with the return portion being unloaded. The effective slip distance is accordingly only about one half of the total displacement distance. This is likely to be a significant difference between spline teeth and typical laboratory fretting tests which are often carried out at constant load, such as those conducted by one of the present authors [7].

A rational indication of local wear severity, related to the Archard wear equation [8], is the value of the parameter, \(W = \gamma d s d s\) where \(s\) is the displacement along the slip path. The maximum value of \(W / E m\) is given, for each condition studied, in Table 2 and may be interpreted as the highest average wear depth over any of the contact elements, for a unit wear coefficient. (The wear parameter in this form is not, of course a good indication of relative wear performance for different lubrication states where the wear coefficient might be expected to vary.) The effect of the crowning in reducing the wear depth parameter is evident and arises because the high pressure regions have been both reduced in magnitude and displaced into the region of lower slip. In addi-
Fig. 10 Tooth pressure distributions for spline with lead crowning. Conditions as described in Table 3, (as for Figs. 5 and 6(a), except for crowning radius of 3.6 m).

tion, it is noted that increasing the torque with the same misalignment has only a minor effect on the wear depth parameter because the improvement in the distribution of slip and pressure offsets the effect of the greater tooth loads. This means that misalignment angle is much more significant to wear than is the applied torque, as found in experiments by Ku [1]. In practice, of course, higher torques sometimes cause greater misalignments.

5 Conclusions

1. The evolution of slip and pressure between the teeth of spline couplings subjected to a steady torque and a rotating misalignment has been studied, using the boundary integral element method. The method incorporated stick-slip friction in the area of contact.

2. The results show that the center of pressure oscillates along the tooth length during rotation of the misaligned coupling. The total tooth load also varies in a cyclic fashion. In some cases the teeth become unloaded during part of their rotation.

3. The results are presented in non-dimensional form so that pressure distribution and wear parameters for a wide range of couplings may be estimated.

4. The cycle of slip and pressure are related in such a way that the contact has a unidirectional character with the high pressures occurring over around one half of the complete cycle. This may be a significant difference between spline teeth and typical laboratory fretting tests which are often conducted at constant load.

5. The predicted wear depth was found to be more dependent on misalignment than on torque and was significantly reduced by the adoption of a lead crown on the spline teeth.

Acknowledgment

The financial support of the Engineering and Physical Sciences Research Council and of GKN Westland Helicopters Ltd is gratefully acknowledged.

Appendix

Generic Spline Geometry.

Male form diameter = Pitch diameter + 1.072 × module
Male minimum diameter = Pitch diameter + 1.934 × module
Circular tooth thickness = 1.552 × module
Female major diameter = Pitch diameter + 1.882 × module
Female form diameter = Pitch diameter + 1.200 × module
Female minor diameter = Pitch diameter + 0.436 × module
Circular spacewidth = 1.584 × module
Female facewidth = 1.333 × male facewidth
Male shaft bore diameter = 0.444 × pitch diameter
Female shaft diameter = 1.555 × pitch diameter
Relative axial location: The male teeth are placed centrally with respect to the female.

The dimensions were based on ANSI B92.2M-1980 30 deg Full Fit Side Fit specifications. All analyses assume identical elastic properties for male and female splines, μ = 0.3.

References