Compliant kagome lattice structures for generating in-plane waveforms

James Bird\textsuperscript{a,\ast}, Matthew Santer\textsuperscript{a}, Jonathan Morrison\textsuperscript{a}

\textsuperscript{a}Department of Aeronautics, Imperial College London
Prince Consort Road, London. SW7 2AZ, UK

Abstract

This paper details the design, manufacture and testing of an adaptive structure based on the kagome lattice geometry – a pattern with well documented interesting structural characteristics. The structure is used to produce in-plane travelling waves of variable length and speed in a flat surface. The geometry and dimensions, as well as the location and compliance of boundary conditions, were optimized numerically, and a pneumatically-actuated working demonstrator was manufactured. Static and dynamic photogrammetric and force measurements were taken. The structure was found to be capable of producing dynamic planar waveforms of variable wavelength with large strains. The lattice structure was then surfaced with a pre-tensioned membrane skin allowing these waveforms to be produced over a continuous plane.

Keywords: Adaptive structures, Kagome lattice, Optimization

2010 MSC: 74K10
1. Introduction

Compliant structures can offer elegant engineering solutions to problems which are challenging or expensive to tackle with traditional engineering components. The movement typically facilitated by bearings and sliders can be replicated with prescribed structural compliance in an otherwise rigid assembly. With no complex moving parts, compliant structures can be easier to maintain and manufacture than the mechanisms which they emulate. They also offer a solution when manufacturing small sizes or large quantities of joints is impractical, or the cost prohibitive.

The work presented in this paper details the design, manufacture and testing of a compliant structure based on the kagome geometry. This optimal structure is then used to support and drive a pre-tensioned membrane skin. The surface – and therefore also the structure – have been developed to generate exclusively in-plane dynamic waves of variable length and speed which represents a challenging structural requirement. Furthermore, these in-plane travelling waves have a specific application as a flow control device, where their influence on a turbulent boundary layer has been shown to dramatically reduce skin-friction drag [2].

The kagome geometry has been investigated extensively as an infinite planar

![Relaxed kagome lattice.](image1)

![Actuated kagome lattice.](image2)

Figure 1: The response to the kagome lattice to actuation. A bar of the structure, indicated by a dashed line, is replaced with an actuator and a displacement is imposed. The deformation, confined to a ‘corridor’ of the structure, was calculated with a linear finite element analysis [1].
lattice. In this configuration, it has been shown to possess some unique and attractive properties which lend it to forming the basis of an active structure. When a member of the frame is elongated, the structural deformation is confined to a region in line with the actuation [3], as illustrated in Figure 1. The basis of the present work is the successful exploitation of this corridor of actuation in a practical setting. This trait is useful as the deformations are confined to a specific region and are also far reaching, allowing a single actuator to influence a large yet bounded area of the lattice.

The response of the infinite planar structure to actuation has been investigated previously in both linear [3] and nonlinear [4] studies. In the linear case, the degree to which the actuation influences the structure is highly dependent on the stockiness of the members [3] – the more slender the structure, the further the imposed displacement propagates through the repetitive framework. As the structure becomes increasingly slender, its response becomes similar to its pin-jointed equivalent, with modes of kinematic indeterminacy being inherited as modes of bending deformation in the rigid jointed case [5].

When large displacements are considered, and geometric nonlinearities become relevant, the response of the infinite kagome lattice is not as useful. Infinite frameworks, or those which are very large relative to their internal dimensions, can be formally statically determinate in terms of Maxwell’s equation, but possess self-stress in practice.

In a large lattice frame with welded joints, the deformation imposed at the center will decay towards the perimeter [1], whereas if considered as a pin-jointed frame, the deformation will continue indefinitely. The zero displacement at the perimeter of the welded frame impacts the determinacy of the structure in practice – if considered in terms of a pin-jointed frame this decay in deformation is effectively a built-in boundary condition around the edge of the structure. The resulting modes of self-stress impinge upon the corridor mode seen in Figure 1, leading to the deformation being confined to the area local to the actuator, with much larger stresses for a given displacement being generated [5].
Evidently, the role of determinacy is crucial when considering a structure for actuation. The optimal case is a frame which is both statically and kinematically determinate. When these conditions are satisfied the structure can resist any external load, as it is kinematically determinate and therefore possesses no mechanisms, and can also be easily actuated as the static determinacy ensures that the length of any member can be altered without generating internal stress. Furthermore, the lack of self-stress ensures that if any mechanisms are created, they can move freely and are not restricted by higher-order tightening of the structure with deformation [6]. Although no infinite repetitive structure can be both statically and kinematically determinate [7], finite structures can be with the correct application of boundary conditions and patch-bars.

Active trusses produced by connecting two planar kagome lattices together, via a double or single tetrahedral mesh, have used this technique [8]. By breaking the mid-plane symmetry, and by adding patch-bars, a sandwich structure was created which possesses both static and kinematic determinacy. Finite element analysis demonstrated that the rigid-jointed structure based on this pin-jointed analysis was superior as an adaptive structure. Specific members were then replaced with linear actuators, and the fabricated structure was assessed and found capable of producing out-of-plane displacements while resisting external loading.

The work explored in this paper involves actuating a compliant structure dynamically, taking advantage carefully designed geometrically nonlinear deformations to produce controllable travelling waves in a quasi-static manner. Others, however, have considered standing and travelling mechanical waves, which are of a different nature to the waves considered in this paper and travel acoustically through the material or lattice. Work has been undertaken exploring the acoustic properties of lattices numerically, through Bloch and finite element analysis [9], and also experimentally. Despite the different nature of the acoustic waves and the waves explored in this paper, similar experimental techniques have been employed. Laser doppler vibrometry [10] and the cross-correlation of
high speed photography [11] gave phased-reconstructed and time-resolved information, respectively, on how these different waves propagate through a finite lattice structure.

In this paper a structure based on the kagome lattice is designed to create in-plane deformations which allow the generation of travelling and standing waves, both by considering its determinacy and also through an energy based modal optimization routine. This structure is then characterised numerically and experimentally. A membrane surface was then added so continuous planar deformations could be generated. Through photogrammetric and force measurement the system was found to be capable of producing in-plane waveforms of large amplitude and frequency efficiently with minimal actuation force. The performance of the physical structure corroborates finite element predictions and ratifies the design methodology outlined.

2. Structural Design

The structure was designed in several stages. Initially a geometry was selected based on a pin-jointed frame model, and suitable boundary conditions were determined. This outline was then given a non-dimensional scale using a modal analysis [12]. This step ensured the structure rejected out-of-plane displacements, while deforming in the manner required. This design, material properties and external constraints were then combined to produce a dimensioned struc-

![Figure 2: An illustration of how the planar kagome lattice can be used to discretise waveforms of varying length.](image)

(a) Large wavelength  (b) Short wavelength
ture for manufacture. Compliant boundary conditions were added to further improve the performance of the system, with their characteristics determined by a short optimization routine.

2.1. Objectives

The goal of the system was to produce in-plane travelling waves of variable length by exciting adjacent corridors of deformation (as shown in Figure 1) with varying amplitudes of displacement, in the static case, or phase in the dynamic case. This principle is demonstrated in Figure 2 where two waves of different length are discretised in a linear finite element solution. All finite element results presented in this paper have been carried out using the commercial Abaqus FEA solver [13], with the structure discretised as a framework of shear deformable beam elements.

3. Geometric design

Optimal adaptive structures are both kinematically and statically determinate. To make them active, a member of the structure is then typically replaced with an actuator, allowing nodal displacements to be controlled. If the member removed is not replaced, a mode of kinematic indeterminacy is created. The approach employed in this paper involves creating inextensible mechanisms which mimic the displacements seen in Figure 2 and then constraining them with actuators, returning the structure to full determinacy.

The mechanisms and self-stress within the structure can be determined using a matrix analysis of the equilibrium matrix, as detailed by Pellegrino [6]. For the kagome pattern, the number of in-plane modes of kinematic indeterminacy, ignoring any rigid body modes, is equal to the number of straight lines which span the perimeter, visible in Figure 3 as the 16 groups of colinear members which are aligned so they start on the edge and traverse the entire structure. Each of these lines corresponds to a strip of tiled unit-cells, with the rotational
Figure 3: The sixteen modes of kinematic indeterminacy for a finite pin-jointed planar kagome lattice.

symmetry resulting in 3 mechanisms spaced at 120° intervals in each. Sixteen such modes are displayed for a free-floating structure in Figure 3. These modes are not unique, and are rather just one set of basis vectors which span the left-null-space of the equilibrium matrix for the geometry shown.

Boundary conditions can be applied to the structure in Figure 3 to remove certain modes. If modes a to d are desired, then built in boundary conditions can be applied to suppress the other inextensible mechanisms. The location of these fixed points needs to be chosen to maintain the static determinacy of the structure which ensures the desired mechanisms remain finite. A simple rubric to achieve this ensures no two boundary conditions (including actuation
Figure 4: A finite kagome lattice structure, with a fixed number of unit cells, parametrised with dimensions $d_1$–$d_3$. Points) lie on the same line of the lattice. Once the fixed boundary conditions are selected, actuation boundary conditions can then be applied to modes $a$ to $d$, creating an active structure that is kinematically and statically determinate.

4. Non-dimensional design

With the geometry and constraints selected, the internal proportions of the structure needed to be determined. The overall size of the structure in terms of number of connections was chosen and fixed. Figure 4 shows the whole structure, with the location of the boundary conditions shown with dots, and the actuation points shown with diamonds. The internal dimensions of the structure were parametrised by $d_1$–$d_3$. Arbitrary, but feasible, ratios for the slenderness, defined as $\frac{2\sqrt{3} d_3}{d_1}$, and the aspect ratio, defined as $\frac{d_1}{d_2}$, were set. The structure was then split into orthogonal modes of compliance using a singular value decomposition in a process detailed in [12]. The variables $d_1$–$d_3$ were optimized to enhance modes which resulted in a non-dimensional structure which had an overwhelming propensity to deform exclusively in-plane [12]. Parameters $d_1$–$d_3$ were adjusted to ensure the 6 lowest energy modes of deformation of the structure corresponded to modes which can be used to produce the corridor waveforms seen in Figures 1 and 2.

The first 6 linear modes of compliance are shown in Figure 5. The desired corridor deformations required to discretise the travelling waves can be produced.
Figure 5: The six largest modes of compliance for the structure in Figure 4 with aspect ratio 160 are shown.

with a weighted sum of these modes. The weightings were found by looking at the displacements at a node in each corridor in the modes of compliance, and solving to find weightings which resulted in displacement at only one node. The weights are displayed normalised in Table 1 and the resulting sum of the weighted modes is shown in Figure 6. As expected, the symmetry in the modes, as well as in the corridors of actuation is reflected in the repeated values in Table 1. Ultimately, a structure with an aspect ratio of $\approx 160$ and a slenderness ratio of $\approx 11$ was found to be suitable for producing the required waveforms.

<table>
<thead>
<tr>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Mode 5</th>
<th>Mode 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corridor 1 weights</td>
<td>$-0.466$</td>
<td>$0.594$</td>
<td>$0.499$</td>
<td>$0.367$</td>
<td>$0.185$</td>
</tr>
<tr>
<td>Corridor 2 weights</td>
<td>$0.374$</td>
<td>$-0.335$</td>
<td>$0.159$</td>
<td>$0.444$</td>
<td>$0.579$</td>
</tr>
<tr>
<td>Corridor 3 weights</td>
<td>$-0.353$</td>
<td>$0.152$</td>
<td>$-0.473$</td>
<td>$-0.414$</td>
<td>$0.381$</td>
</tr>
<tr>
<td>Corridor 4 weights</td>
<td>$0.353$</td>
<td>$0.152$</td>
<td>$0.473$</td>
<td>$-0.414$</td>
<td>$-0.381$</td>
</tr>
<tr>
<td>Corridor 5 weights</td>
<td>$-0.374$</td>
<td>$-0.335$</td>
<td>$-0.159$</td>
<td>$0.444$</td>
<td>$-0.579$</td>
</tr>
<tr>
<td>Corridor 6 weights</td>
<td>$0.466$</td>
<td>$0.594$</td>
<td>$-0.499$</td>
<td>$0.367$</td>
<td>$-0.185$</td>
</tr>
</tbody>
</table>

Table 1: Weightings of the modes in Figure 5 required to produce the corridor modes in Figure 6.
As the purpose of the structure is to create dynamic waveforms of large amplitude, it must be fabricated from a material that possesses excellent fatigue resistance and a large yield strain. Fully hardened AISI 301 grade stainless steel was chosen for these characteristics [14]. It was also selected for its excellent corrosion resistance and ready availability in the form of packing shim, precisely manufactured with thickness upwards of 3 \( \mu m \).

The design objective was to produce a structure capable of generating the largest wavenumbers, as illustrated in Figure\[2\] while achieving the largest deformation amplitude.

Maximising the in-plane displacements was achieved by minimising dimension \( d_2 \), to 30 \( \mu m \), limited by a combination of practical and out-of-plane loading constraints. The other dimensions were subsequently determined by the ratios calculated in the previous section section and their values are displayed in Table\[2\]

The structure with the dimensions in Table\[2\] and boundary conditions as in Figure\[4\] was then modelled undergoing a peak actuation displacement of 4 mm.
Table 2: Dimensions of the fabricated lattice structure, where \( d_1 \) is the out-of-plane thickness, \( d_2 \) is the in-plane thickness and \( d_3 \) is the length of a single member, as illustrated in Figure 4.

After a convergence study, the structure was discretised with 10 Timoshenko shear deformable beam elements per bar of the lattice. The geometrically linear and non-linear solutions were found using ABAQUS \(^{13}\) and are shown in Figure 7, with all of the boundary conditions imposed by ensuring displacement continuity.

The nonlinear analysis demonstrates that the choice of boundary conditions based on the linear pin-jointed analysis do not facilitate the large deformations as expected. While their selection might prevent first-order stiffening of the structure, the strains are of a sufficient magnitude that higher-order effects cause the structure to stiffen. This is highlighted by the area indicated by the ellipse in Figure 7b. As the structure is forced to the side by the actuation occurring at the diamond, the deformation results in large areas of the structure being pulled inwards, towards the corridor being actuated. As the structure is forced to the side with a large amplitude, material is pulled in via axial stresses in the members to replace it. This energy intensive effect is clearly detrimental to the

![Figure 7: A comparison between the geometrically linear and nonlinear response of the structure to a 25\% actuation strain. The actuation point is indicated with a diamond.](image)
performance of the active structure. To overcome this, and to alleviate the axial stresses, the rigid boundary conditions were replaced to allow compliance.

6. Design of compliant boundary conditions

The compliant boundary conditions were modelled as two springs acting at the boundary conditions and actuation points. Although the optimum degree of stiffness is likely to vary slightly at each location, the compliance was characterised by just two values $k_1$ and $k_2$ to simplify the optimization process and also to ease manufacture. The model is shown as a schematic diagram in Figure 8.

Any torsional stiffness imposed by the boundary condition was neglected, as the in-plane bending of the thin kagome structure will dominate over any prescribed stiffness. Enough translational compliance at the boundary condition was needed to allow the structure to move normal to the direction of actuation, while also being stiff enough to provide suitable support to oppose the actuation force. The compliant boundary conditions were modelled as springs connected to the structure in ABAQUS [13]. The values of $k_1$ and $k_2$ were then altered with the same peak displacement of 4 mm, and a geometrically nonlinear finite element solution was found. As there were only two variables, and a

<table>
<thead>
<tr>
<th>$k_1$</th>
<th>$k_2$</th>
<th>Thickness</th>
<th>Width</th>
<th>Height</th>
<th>Young’s Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 N m$^{-1}$</td>
<td>$3.5 \times 10^9$ N m$^{-1}$</td>
<td>50 µm</td>
<td>6.35 mm</td>
<td>12 mm</td>
<td>186 GPa</td>
</tr>
</tbody>
</table>

Table 3: Physical properties of the optimized leaf springs.
physical appreciation of the waveforms produced was needed, the values of $k_1$ and $k_2$ were determined manually. A leaf spring was then designed from the same grade of steel as the structure. Its dimensions are detailed in Table 3.

The structural finite element solution with the added compliance is displayed in Figure 9, indicating the extent of the improvement.

When considering ways to manufacture the structure, it became apparent producing continuous strips which span the width of the lattice would not be possible with the available equipment. To allow for this, the horizontal straight lines traversing the span of the structure, comprising many co-linear individual members between nodes, were split into three and were then displaced together by a single coupled actuator. As the deformation created by the actuation decays into the structure [1], this approach had the added benefit of increasing the average displacement over the surface.

In order to test the performance of the new design, the most arduous forcing situation was imposed. Alternate rows were driven in opposite directions. Figure 9 illustrates the geometrically linear response of the structure with built-in boundary conditions, the geometrically nonlinear deformations when the boundaries are fixed, and the response of the structure when the boundary conditions are compliant.

The linear deformations are to be expected, and match the sorts of deformations seen previously [3]. When the structure is supported rigidly, it is forced to undergo stretching dominated deformation. This can be seen in Figure 9b as members of the lattice are pulled straight under actuation. The sinusoidal

### Table 4: Boundary reaction force and corresponding von Mises stress in the structure with boundary conditions (BCs) as in Figure 9.

<table>
<thead>
<tr>
<th>Boundary conditions</th>
<th>Max. reaction force</th>
<th>Max. von Mises stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear, built in BCs</td>
<td>0.7638 N</td>
<td>224.4 MPa</td>
</tr>
<tr>
<td>Nonlinear, built in BCs</td>
<td>912.9 N</td>
<td>3.823 GPa</td>
</tr>
<tr>
<td>Nonlinear, flexible BCs</td>
<td>0.6893 N</td>
<td>313.1 MPa</td>
</tr>
</tbody>
</table>
displacements seen in Figure 9a give way to a less ordered deformation pattern when the nonlinearities are taken into account. When the boundary conditions are given the compliance calculated previously, the deformation, as displayed in Figure 9c compares far better with the linear calculation, indicating that by allowing the structure to move inwards, bending dominated deformation can occur. This inward movement is clearly captured as the corridors are no
longer centred on the relaxed structure, with the effect increasingly pronounced with distance from the middle of the lattice. With carefully selected compliant boundary conditions it is possible to achieve the large displacements required, without imposing large energy intensive axial stretching of the members. Another benefit of the compliant boundary conditions is the increased independence of the corridors, as demonstrated experimentally in Figure 18. Crucially, this linear behaviour allows the flexibly held structure to discretise waveforms of varying length.

The increased performance of the structure is illustrated in Table 4. By having compliant boundary conditions, both the stresses within the structure and the peak reaction force required are reduced by orders of magnitude. With the design determined, and shown to perform well numerically, a physical demonstrator was then produced.

7. Manufacture of the kagome lattice structure

The high aspect ratio and slender members meant machining the lattice with conventional methods or out of a single piece was not currently possible. Instead, it was assembled from interlocking thin strips, as demonstrated in Figure 10.

Figure 10: A diagram illustrating the method of assembling the kagome lattice. Notches are cut in strips of the structure, which are then slotted together and adhered using cyanoacrylate.
Figure 11: The assembled kagome structure, manufactured from 301 fully-hard stainless steel with the joints fixed with cyanoacrylate glue.

The strips, A, B and C were laser cut from sheets of packing shim using an A-355 micro-machining laser, manufactured by Oxford Lasers. The machine’s high accuracy of ±2 µm ensured that the assembled structure was highly regular and planar.

The fabricated strips were then assembled by hand as in Figure 10 to create the kagome lattice. The slot thickness $s_{oe}$ is $\sqrt{3}d_2 = 52.0\mu$m. When all the strips were in place, the structure was measured and inspected, and all the connections between the strips were fixed using a drop of cyanoacrylate.

Figure 11 shows the assembled structure from two different viewpoints, illustrating the high aspect ratio and the regular nature achieved. This lattice was then mounted via leaf springs to a rigid base, or to the actuators which drive it, as shown in Figure 12. All connecting blocks were produced using a Connex
ObJet 350 rapid-prototyping machine. Figure 13 shows how these blocks were used to support the leaf spring and the kagome structure.

Each corridor is driven by a single actuator which is connected to the kagome lattice structure in three places via a CFRP rod and the leaf springs. Each driving rod is connected to a pneumatic actuator, a double acting CJ1B4-10U4

Figure 12: A CAD model of a single actuated module, showing the kagome lattice, actuators and supporting structure.
air-cylinder manufactured by SMC, mounted on the rigid base.

The manufactured structure and sub-structure are displayed in Figure 14. The pneumatic actuators are connected via tubing to VQ110 solenoid valves, also manufactured by SMC. For convenience, and to minimise the low-pass filtering effect of long lengths of pneumatic tubing, these valves are held in two manifolds and mounted on either side of the structure under the base. The assembled structure with the actuators, connectors and leaf springs is displayed in Figure 14. The system was then tested to assess its performance as an adaptive structure.

7.1. Structural static measurements

Both static and dynamic photogrammetric measurements were then taken of the structure, as well as recording the actuator supply pressure via a Honeywell NBP series pressure transducer, and the applied actuation force with the ATI load cell. A schematic diagram of the salient apparatus is shown in Figure 15.

The static displacement measurements were taken using a Nikon D5200 DLSR camera. The camera trigger was controlled via a National Instruments USB-
Figure 14: A manufactured actuated module, without the solenoid valves, pneumatic tubing and control board.

6212 DAQ. This device also recorded the pressure supplied to the actuators. The acquisition was synchronised with the force measurements such that the mean force and pressure could be determined over the exposure of the photograph.

Figure 15: An illustration of the force and photogrammetric measurement arrangement.

PSV-500-3D Vibrometer

19
The analysis of the photographs was carried out using functions within the Computer Vision and Image Processing MATLAB toolboxes [15].

The camera was calibrated using a chequerboard image to remove lens distortion, and a mapping between the coordinate system of the structure and the camera was found. A transformation allowed the tracking of objects in the photographs in pixels to be related to in-plane displacements in millimetres. Figure 16 shows the original photographs of the chequerboard and structure undergoing action, and the same photographs after being transformed such that the coordinate basis vectors defined by the chequerboard are orthogonal, and clearly describe the deformation of the structure. The coordinate system is illustrated by the red arrows.

The structure itself, a tracking dot mounted on the structure, and stationary tracking dots mounted on the apparatus were coloured red, blue and green.
respectively to match the Bayer filter on the camera to facilitate their selection in post processing. Each of the six actuators corresponding to each of the six driven corridors in the module was removed in turn and attached to the load cell. The displacement was found by tracking the center of a blue dot adhered to the corridor in question. The location of the circle, and hence the deformation, was determined with a Hough transform. Tracking of red dots on the apparatus ensured no relative motion between the camera and the experiment during the test.

The resulting data is shown in Figure 17a where the spanwise force is plotted against the spanwise displacement for the six corridors, numbered in the order they occur in positive $x$, in Figure 16c. Figure 17b shows the reaction force on actuated node for a given displacement, when the same lattice is modelled with a geometrically nonlinear finite element scheme in ABAQUS [13]. The symmetry of the structure results in only three datasets in the numerical study. Figure 17a illustrates how the corridors on the edge of the lattice are significantly easier to actuate — this is reflected in the finite element analysis. Their additional compliance is due to their freedom to deform independently of the rest of the

![Figure 17a](image1.png)
![Figure 17b](image2.png)

(a) Experimental actuation distance against force in $z$ for the six corridors.
(b) Geometrically nonlinear finite element calculation of displacement against force.

Figure 17: A comparison between experimental and numerical response of the structure.
structure.

The finite element analysis under predicts the stiffness of the structure for two main reasons. The beam mode employed assumes the members of the structure meet at a point, whereas, in practice the adhesive used to secure the connections takes up a finite amount of space. This reduces the slenderness of the structure as the members of the structure become effectively shorter. This effect is more pronounced at the boundary conditions where plastic blocks grip the frame. These connectors have to be robust enough to repeatedly withstand the actuation forces resulting in their increased thickness and encroachment on the frame. Second, so that important features could be extracted from the photographs, the entire structure and surroundings was spray-painted matt black. While every effort was made to keep the coat as thin as possible, it is likely to have a non-negligible effect on the stiffness of the whole structure, especially considering the members are only 30 $\mu m$ in thickness, and the cubic-proportional stiffening effect of adding material thickness.

Although the magnitude of the stiffness is not the same for the numerical and physical models, the trends between the two are similar. The response of the corridors on the edge of the structure, 1 and 6, is more linear than the corridors within the lattice. This is to be expected as the higher-order effects come from the need of the structure to deform elsewhere, normal to the actuation direction, as illustrated in Figure [6b]. The edge corridors can achieve large deformations in the direction they are forced, and bend normal to that direction without affecting the rest of the structure. When the displacements become very large, there is insufficient material within the corridor to support the displacement, and the actuator starts pulling on the rest of the structure. This occurs at about 5 mm in Figure [17b]. The nonlinear behaviour of the two models for the inner corridors, 2–5, is similar with stiffness increasing with displacement.

The disparity between the finite element model and the physical module is to be expected, and is explained, but is not ultimately very important. The key result is that, even though the structure was manufactured by hand with meth-
Figure 18: Colour-selected photographs of the structure under action, thresholded so only the structure is visible. Each row corresponds to a separate corridor experiencing actuation, with force increasing from left to right.
ods which intrinsically produce imperfections, the stiffness of the structure is still low enough that it can produce sizeable deformations with the actuators selected based on the numerical modelling. The manufactured structure responds broadly in the same manner as the perfect numerical model. The actuation stiffness is not the only consideration, the fidelity of the deformations is important as it is required to efficiently reproduce the travelling waves. The deformations can be extracted from the photographs used to track the dot to find the displacements plotted in Figure 17a.

As the aspect ratio of the structure is very large, and the structure very slender, it was not possible to extract a perfect two-dimensional mapping of the structure as it deformed. Instead, one face of the lattice was coloured green, which extended \(\approx 1\) mm down the edge of the members of the structure. The camera was then inclined slightly to the vertical so that the green edge could be resolved. The same transformation was applied to the images as in Figures 16b and 16c. The green edges were extracted from the images by subtracting the blue and red portions, and applying a threshold to the image. Any spurious artefacts introduced by the post-processing were then removed manually. The resulting structural deformations at three actuation forces for each corridor are displayed in Figure 18.

Each row in Figure 18 corresponds to a different corridor of the structure. It was not possible to control the force applied to the structure with any precision as the regulator controlling the pressure had considerable backlash. This was further compounded by the high levels of friction in the actuators. This is evident in large amount of hysteresis observed when force is applied to the structure, and when the supply pressure is removed, displayed in Figure 19. The alignment of the air-cylinder to the structure had a significant impact on friction and the amount of hysteresis present.

The displacement in Figure 18 qualitatively match the geometrically nonlinear deformations seen in Figure 9. The displacements are always constrained to the corridor, even when the displacements are large, an important indication that...
the structure is performing in the desirable bending dominated way.

7.2. Structural dynamics measurements

The analysis carried out so far has been concerned with the static response of the structure, with the assumption that as long as no resonant modes are excited, it can be treated as quasi-static. However, the frequency response of the system still needs to be determined if dynamic waveforms are to be produced. This analysis was carried out by driving one of the center corridors of the structure at frequencies ranging from 0 to 50 Hz. The velocity of the response was measured with a Polytech PSV-500-3D-M vibrometer system, focused on small strip of card mounted on a node of the corridor. As with the static measurements, simultaneous force and pressure data was recorded.

The actuator pressure on either side of the piston was chosen so that equal force was applied in both directions of the stroke. The power spectral density of the structural response for the range of driving frequencies is plotted in Figure 20.

As expected, there are peaks at the forcing frequencies, and smaller peaks corresponding to harmonics. The amplitude of the structural response increases with frequency to about 27 Hz, where it flattens off before attenuating rapidly above $\approx 37$ Hz.
Figure 20: The frequency response of the structure for a range of forcing frequencies.

Figure 20 does not have any peaks which correspond to a resonant mode being excited. This is important as it means the quasi-static model used to design the structure is suitable – the forcing does not cause the structure to vibrate unexpectedly. The target static waveforms in Figure 2 should occur when the forcing is dynamic.

Each of the peaks at in Figure 20 consist of two peaks separated by $\approx 1$ Hz. This may also be the result of the nonlinear structural behaviour, or the discontinuous nature of the forcing introducing another frequency – the piston is controlled in a binary manner, even if the structural response is broadly sinusoidal. The velocity time-series is displayed in Figure 21 when the structure is actuated at 40 Hz, illustrating both the sinusoidal response as well as the repeatable motion of the structure. The data in Figure 21 represents a phase average of 10 measurements of the structure to reduce the noise from interrupted laser return signal, each sampled at 12.8 kHz for a duration of 10 s, with the signal to the actuator acting as a reference signal.

As can be seen from Figure 20, most of the energy is at the forcing frequency, and so the RMS of the whole time-series, for both the velocity and the force, was taken as a measure of the performance of the system. As the frequency in-
creases, the displacement, calculated from the integral of the velocity, decreases. The higher the frequency the actuator, the less time there is to fill the chamber with high-pressure air, resulting in lower actuation pressures and consequently a smaller displacement against the stiffness of the structure. The reduction in displacement was initially attributed to an increase in inertial forces at higher frequencies, however the dynamic stiffness suggests it is the delay in the pneumatics that is detrimental to the performance. This is evident in the reduction in the force provided by the air-cylinder with increasing frequency in Figure 23.

At low frequencies the forcing takes the form of a square-wave as the supply pressures can accelerate the actuator to its full travel and back in less time than the time period of the forcing, leading to the large displacements up to $\approx 6\,\text{Hz}$ in Figure 22.

In order to assess the structure, rather than the actuator, the force can be normalised by the displacement to give the dynamic stiffness. This measure, illustrated in Figure 24, indicates that at frequencies below $\approx 40\,\text{Hz}$ the stiffness of the structure dominates, with a response that is independent of frequency. Above 40 Hz the dynamic stiffness increases, indicating that inertial forces are now playing a dominating role.

The dynamic response of the structure measured in these tests is as expected. However, it was observed that the experimental set-up resulted in sub-optimal performance of the system. The results obtained indicate the structure will not perform at frequencies over 50 Hz. However, when with the actuators are

![Figure 21: The velocity measured using the vibrometer in the middle of one of the central 'corridors' of the structure, when forced at 40 Hz.](image)
held rigidly, the system can be observed oscillating comfortably at frequencies in excess of 60 Hz. This is due to some unavoidable experimental design considerations. The pneumatic tubing that supplies the actuators must be long and slack enough that it imparts negligible forces on the load-cell. This increase in length inevitably acted as a low-pass filter, increasing the latency of the forcing and dampening the higher frequencies. Furthermore, the actuator was not attached rigidly to the base, rather to the load-cell by means of an arm and lockable ball joints. While this allowed any of the six actuators to be connected to the load-cell easily, it also resulted in a small amount of displacement and rotation through the deformation of the measurement system. The friction in the actuators is highly dependent on their alignment, and this, combined with
some flexibility in their restraint, significantly reduces the performance of the system.

Consequently, these factors, combined with the fact that the stiffest of the corridors in the static tests was examined, means the dynamic results are conservative in nature. They give an insight into the general performance of the actuator-structure system, while slightly underestimating its true capability.

In order to turn the structure into a system for generating planar continuous waveforms, a membrane skin was attached to the surface. This is described in the following section.

8. Manufacture and testing of the compliant surface

The lattice was covered by a membrane skin which was manufactured from Ecoflex 00-50 platinum-catalysed silicone rubber. It was chosen primarily for its ability to withstand large strains of \( \approx 950\% \) before failure, and its durability. The two-part mix was combined, and a known volume was decanted and measured using a syringe. It was then poured evenly over a smooth bounded area to create a skin of a known thickness. The skin needed to be as thin as possible to minimise both the inertial and viscoelastic forces it imposes on the structure, as well as any in-plane forces from stretching caused by the structural deformations. The thinnest skin which could be reliably produced had a thickness of 0.35 mm.

In order to prevent wrinkling of the surface under actuation and to limit out-of-plane deformations, the skin was pretensioned by 30% in both directions before first attaching to a frame, and then to the structure. The skin needed to be raised off the frame slightly to prevent the sharp edges of the structure creating holes. The pretension meant any discontinuities in the surface lead to a rapid failure. Circular spacers, laser-cut from card, were adhered to nodes along the actuated corridors using cyanoacrylate. This method protected the surface and also had the added benefit allowing the skin to be attached and moved only at
locations where the displacements were required. The skin was then attached to the frame by encapsulating the card discs on the structure with silicone.

The design of the module is shown in Figure 25 in an exploded view. The finished assembled module is shown in Figure 26.
9. Performance of the dynamic compliant surface

The objective was to create a dynamic surface capable of producing exclusively
in-plane waveforms of variable length. To assess the performance a measure
of both the in-plane and out-of-plane displacements was necessary. Although
it was established that the structure deforms almost exclusively in-plane, the
pre-tensioned skin spans across the members of the structure, and is therefore
only kept flat through first-order stiffness. Vibrometry measurements of the
surface, as well as high and low speed photogrammetry, were undertaken. The
experimental set-up is illustrated in Figure 27.
9.1. Static surface measurements

The first important consideration is that the corridors of actuation present in the structural analysis and visible in Figure 18 are not affected by the addition of the membrane skin. The surface was speckled with titanium dioxide and graphite power and photographed before and after a single corridor was actuated. Digital image correlation (DIC) was then performed over a range of actuation forces using ‘Ncorr v1.2’ an open source 2D DIC software package implemented in MATLAB [16]. The integration window was set at a radius of 30 pixels, chosen by decreasing the size of the window until noise was introduced. As the strains are large, a step-analysis was used, whereby the actuation displacement was increased gradually and intermediate photographs were used to calculate the final deformation of the surface. The peak displacements and strains calculated are shown in Figure 28 overlaying a photograph of the original undeformed surface.

Figure 28a shows the surface displacement in across the page. It is clear that the skin deforms only in the region corresponding to the actuated corridor of the structure. This indicates that the skin is suitably compliant and that it does not influence the manner in which the underlying structure behaves. Also visible in the deformation are the three portions of the structure. The $X$ displacement is shown in Figure 28b. The sinusoidal nature of the structural deformation produces alternating displacements, however the magnitude of these is less than the displacement in the direction of the actuation. Above and below the actuated region, the skin and structure can be seen being pulled inwards. This is the result of the leaf spring supports allowing the structure to move slightly normal to the direction of forcing. Each of the three sections per corridor are actuated in the centre, but the way they propagate displacements is not symmetric – the side which is ‘pushed’ deforms slightly less than the side which is ‘pulled’. This manifests as a spike in axial strain in the $Z$ direction between the two sections, visible in Figure 28c. Also clear is the slight compression of the skin either side of the deformed structure. If the skin was not pre-tensioned, this would
result in buckling of the surface. The region of deformation confined to the corridor visible in Figure 28a produces the shear strain visible in Figure 28d as the centre of the corridor moves while the edges are fixed in place by the adjacent corridors. The strain, $\epsilon_{xx}$, in Figure 28c forms a chequered pattern as the displacements in Figure 28b impinge on the static structure above and below. Again, the regions of negative, or compressive strain, would have likely led to the very thin surface bucking if it was not under pre-tension.

The static performance of the surface mirrors the way the structure deforms,
indicating that the addition of the thin and soft skin has a negligible impact. However, for the surface to create travelling waves, a dynamic assessment of its performance is needed.

10. Dynamic surface measurements

A Polytech PSV-500-3D-M vibrometer was directed at a point on the surface, via a front-surface mirror, as illustrated in Figure 27. The measurement point was located in the centre of the surface directly above the structure, to ensure it was at an anti-node of the waveform, and to give a good representation of the whole system. The surface was dusted in titanium dioxide to increase the strength of the laser return signal. It was then forced pneumatically from 0 Hz to 50 Hz, and the velocity recorded. The RMS of the velocity for the same of forcing frequencies is shown in Figure 29. The velocity increases in a linear fashion until the forcing frequency reaches \( \approx 43 \text{ Hz} \) and then rapidly decreases. The drop off occurs at a higher frequency than when the structure was tested alone, as in Figure 20. This is likely to be due to changes in the experimental set-up rather than any benefits caused by the addition of the skin, which would be expected to reduce its performance through an increase in mass and stiffness. The actuators were mounted to a fixed base, and the pneumatic tubing connecting them to the solenoid valves was kept to an absolute minimum.

![Figure 29: RMS of the spanwise velocity, when the surface is driven at a range of frequencies.](image-url)
Figure 30: Power spectral density (PSD) of the velocity magnitude at a point on the surface, when the surface is actuated at a range of frequencies.

The full frequency response of the surface is shown in terms of power spectral density (PSD) in Figure 30 for each of the forcing frequencies. As expected, there is a peak at the forcing frequencies which increases in magnitude with frequency until $\approx 43$ Hz, as in Figure 29. Similarly, there are clear harmonics being excited with a lower amplitude. There is also energy at a higher frequencies $\approx 160$ Hz which is present regardless of the forcing frequency. When the surface is actuated, the whole mounting section vibrates which induces vibrations in the front surface mirror. This in turn is likely to lead to spurious velocity measurements from the changing angles of the optical path, which explains this noise.

To test the degree of out-of-plane deformation, a worst case scenario was created whereby the surface was actuated at 40 Hz with maximum force, discretising a wave of zero wavenumber, by actuating all of the actuators together in-phase.

The measurement points, and coordinate system are shown from the view of the vibrometer in Figure 31a. The Polytech PSV-500-3D-M vibrometer was then used to measure the three components of velocity of the surface, from which a map of RMS of out-of-plane displacement was produced, as displayed.
in Figure 31b. The surface displacements are very limited in the regions where it sits over the structure, but grow to a peak RMS displacement of 1.2 mm in the region between the corridors: this is still relatively small considering the minimum wavelength of the structure $\approx 55$ mm.

The individual corridors of the structure can be actuated sequentially to discretise different waveforms. In order to assess how well the surface performs, three different waveforms were discretised at 25 Hz and the deformations were captured with a Phantom v641 high-speed camera sampling at 1 kHz. Graphite particles were added to the surface, and using Dantec DynamicStudio the velocities of the surface was calculated through cross-correlation of the images. The spanwise velocity fields were then phase-averaged to produce the images in Figure 32.

Each column of Figure 32 corresponds to a different waveform, and each row is a different step in time. The first is a standing wave, of wavelength $\approx 55$ mm. The middle is a wall oscillation, which has an infinite wavelength, and the last is a travelling wave with a wavelength of 100 mm. The underlying geometry has been superimposed, and the color indicates velocity in m s$^{-1}$.

Figure 31: The measurement locations and the out-of-plane displacements of the actuated surface, measured with a vibrometer as illustrated in Figure 27.
Figure 32: The phase averaged spanwise velocity of a section of the active surface generating a standing wave, column 1, a wall oscillation, column 2, and a travelling wave, column 3. The colour indicates velocity in m s$^{-1}$. The position of the kagome lattice sub-structure has been superimposed. The waves of various length have been produced by adjusting the phase of actuation between the corridors.
The standing wave case shows alternate corridors of the structure driven in opposite directions. In the wall oscillation case, all the corridors are driven simultaneously, and in the travelling wave case they are driven with a phase delay. This phase delay $\tau_\theta$ is a function of wavelength of the travelling wave $\lambda$, and the spacing of the bars of the lattice $d_3$ and can be calculated in seconds as

$$\tau_\theta = \frac{\sqrt{3}d_3}{\lambda f}$$

where $f$ is the forcing frequency. For the results displayed in Figure 32 for the travelling wave case, the phase delay is 11.1 ms.

When the system is producing a wall oscillation, the whole surface moves as one, as illustrated by the large block of colour which its not defined by the underlying geometry of the structure. This extreme condition suggests that the surface is able to generate waves in a somewhat continuous manner – as long as the wavelength is greater than the spacing of the corridors in the structure, the surface will be able to reproduce it. The surface is not limited to wavelengths which are multiples of the corridor spacing, rather waves can span the lines of the structure. This is reinforced with the travelling wave case, where the crest of the velocity wave can clearly be seen moving across regions of the surface not defined by the structure. The standing wave case shows the smallest wavelengths possible – set by the spacing of the structure – while the wall oscillation shows a waveforms of infinite length. The travelling wave sits between these two cases and is illustrated in Figure 32 by a wave-front moving from left to right, indicating that the structure based on the kagome lattice and combined with a pre-tensioned surface can successfully produce in-plane travelling waves in a continuous surface.

11. Conclusions

The favourable attributes of the kagome geometry observed in the linear response of the infinite planar structure have been successfully exploited in a
practical setting. By applying a novel methodology using the corridors of actuation, coupled with carefully selected and designed compliant boundary conditions, the attractive linear qualities were realised even when the displacements were very large. Dynamic deformations were also produced, where adjacent corridors of the structure were driven with a phase difference, producing waveforms of varying length and speed.

A thin membrane skin was added to the structure so that these displacements could be transferred to deformations in a flat surface, allowing the in-plane waveforms to be produced in a structurally supported continuum.

In this way, a novel compliant adaptive surface was created and measured, via the development of challenging measurement techniques, to be capable of generating controllable travelling waves, demonstrating that the kagome lattice and its interesting structural properties are not only of academic curiosity, but can also have practical application.

12. Acknowledgements

The authors would like to gratefully acknowledge Airbus Group Innovations who funded the work presented in this paper, under agreement IW202838.
References


