Carbon Fibre laminates with engineered fracture behaviour

by

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Abstract

A new bio-inspired microstructure design approach was developed to improve the translaminar toughness and damage tolerance of Carbon Fibre Reinforced Plastic (CFRP) structures. The microstructure designs take inspiration from the microstructures of biological composites by adopting the most important toughening mechanisms, and applying them to CFRP laminates. Carefully placed patterns of laser-engraved micro-cuts are inserted in the microstructure of the laminate during the manufacturing process. These micro-cuts change the crack propagation path during translaminar fracture, hence allowing to engineer the fracture behaviour of the composite.

The microstructure design approach led to remarkable improvements in the maximum tensile load (up to 189%) and translaminar work of fracture (up to 460%) during Compact Tension test for CFRP laminates with Cross-Ply and Quasi-Isotropic (QI) layups when compared with the corresponding un-modified laminates. Furthermore, a significant improvement in the damage resistance under indentation test was demonstrated for a QI laminate with engineered microstructure. These results demonstrate that microstructure design holds the potential to improve the damage tolerance of CFRP structures in industrially-relevant applications.

A semi-analytical Fibre Bundle Model (FBM) was developed to investigate the role of dynamic stress concentrations, and of fracture mechanics-driven failure, on the longitudinal tensile strength of fibre-reinforced composites. In particular, the investigation was focused on the size effect: a decrease in the bundle strength with an increase in the number of fibres. To the knowledge of the author, it is the first attempt in the literature to investigate these two physical mechanisms in a FBM. It was shown that, although the dynamic stress concentrations significantly decrease the predicted bundle strength, do not allow to predict the right trend of the size effect shown by the experimental results. On the contrary, including fracture mechanics-driven failure in the bundle simulation allowed to predict the right trend of the size effects on the bundle strength. These results suggest that fracture mechanics is a physical mechanism which might be necessary to consider to correctly predict the longitudinal tensile strength in large composite bundles.
Declaration of originality

The work presented hereafter is based on research carried out by the author at the Department of Aeronautics of Imperial College London and it is all the authors own work except where otherwise acknowledged. No part of the present work has been submitted elsewhere for another degree or qualification.

Gianmaria Bullegas

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Copyright declaration

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Dedication

To Laura and to my family
“Because their ideas of progress in the old world are contrary to our new habits of thoughts. Those fellows believe that one cannot become a general without having served first as an ensign; which is as much as to say that one can't point a gun without having first cast it oneself”

*Jules Verne,*

*From the Earth to the Moon.*
List of publications & dissemination

Parts of the work presented in this thesis have been disseminated through written publications, oral presentations as well as public outreach events which are listed below as of September 2017.

Peer-reviewed journal publications


Conference publications


Invited seminar presentations


Public outreach

Nomenclature

Abbreviations
CFRP  Carbon Fibre Reinforced Plastics
CNC  Computer Numerical Control
CP  Cross Ply
DIC  Digital Image Correlation
FBM  Fibre Bundle Model
FE  Finite Element
FFM  Finite Fracture Mechanics
FRP  Fibre Reinforced Plastics
HFBM  Hierarchical Fibre Bundle Model
QI  Quasi Isotropic
QSI  Quasi Static Indentation
SEM  Scanning Electron Microscopy
UD  Uni-Directional
WLT  Weakest Link Theory

Symbols
$\alpha$  Deflection angle
$\Delta$  Finite difference
$\ell$  bundle length
$\gamma$  Shape parameter of the analytical stress redistribution
$\xi$  Fraction of un-cut fibres along the array of micro-cuts
$\lambda_{\text{dyn}}$  Dynamic stress magnification factor
$\lambda_{\text{fm}}$  Fracture Mechanics sensitivity parameter
\( \lambda_{sl} \)  Shear lag factor

\( N \)  Set of elements in the bundle model

\( G_{II} \)  Mode II fracture toughness

\( G_{I} \)  Mode I fracture toughness

\( W \)  Translaminar work of fracture

\( \text{CoV} \)  Coefficient of variation

\( \nu_{12} \)  Major Poisson’s ratio of the composite

\( \phi \)  Diameter

\( \text{Pr}^{fa} \)  Probability of bundle failure

\( \text{Pr}^{po} \)  Probability of bundle pull-out

\( \rho \)  Shape parameter of Weibull distribution

\( \sigma \)  Longitudinal tensile stress in the bundle cross-section

\( \tau \)  Shear stress/strength

\( A \)  Area of the bundle cross-section

\( a_{ct} \)  Critical equivalent crack size

\( a_{eq} \)  Equivalent characteristic crack size

\( C \)  External perimeter of the bundle cross section

\( E_2 \)  Transverse modulus of the composite

\( E_1 \)  Longitudinal tensile modulus of the composite

\( G_{12} \)  Shear modulus of the composite

\( h \)  Crack deflection height

\( I_{ct} \)  Equivalent crack critical index

\( k \)  Stress concentration factor

\( l \)  Length in the bundle model

\( N \)  Number of Monte Carlo simulations

\( n \)  Number of elements in the bundle model

\( P \)  Load measured during the CT test

\( p \)  Space between to subsequent micro-cuts

\( R \)  Reverse factor for fibre element strength

\( r \)  Distance between fibre elements in the bundle cross section

\( S \)  Survival probability

\( s \)  Distance between micro-cuts along the fibre direction
$SD$  Standard deviation of bundle strength distribution
$t$  Ply thickness
$t^k$  Simulation time variable
$V_f$  Fibre volume fraction of the composite
$w$  Micro-cut width
$X$  Tensile strength
$X^{ij}$  Strength threshold for each fibre element
$X_m$  Average tensile strength for the bundle

**Subscripts and superscripts**

$0^o$  Refers to the $0^o$ plies
$90^o$  Refers to the $90^o$ plies
$\xi$  Refers to properties along the array of micro-cuts
$\infty$  Asymptotic value at the edges of the bundle
$b$  Refers to the bundle
$cl$  Refers to the cluster of broken fibres
$cr$  Refers to critical value for crack propagation
$cut$  Refers to the single micro-cut
dyn  Refers to dynamic effects
cut  Refers to the single fibre element in the bundle
eq$  Equivalent
$fa$  Refers to failed element in the bundle
$fm$  Refers to fracture mechanics effects
$f$  Refers to the single fibre
ini  Refers to fracture initiation (at the onset of non-linearity)
in  Refers to intact elements in the bundle
$L$  Linear stress distribution
$mat$  Refers to the matrix
$max$  Maximum value over the interval
$min$  Minimum value over the interval
$po$  Pull-out
$prop$  Refers to fracture propagation
rl Recovery length
sim Refers to simulation results
sl Shear lag
st Refers to elements that have reached the shear lag stress limit
uncut Refers to un-cut composite between to micro-cuts
U Uniform stress distribution
μ Friction property
i Index of fibres in the bundle
j Index of sections in the bundle
s Index of clusters in the bundle
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Chapter 1

Introduction

1.1 Motivation and Objective

Carbon Fibre Reinforced Plastics (CFRPs) can achieve remarkable mechanical properties in terms of specific strength and specific stiffness. Moreover, the properties in composite laminates can be tailored along given directions and optimized for the design of the structure. For these reasons, CFRPs are a material of choice for the latest generation of lightweight structures in industries such as transportation, energy and sporting goods.

One of the main drawbacks of CFRPs is their relative low damage tolerance. They are particularly sensitive to the presence of stress concentrations and mechanical defects which can create sharp cracks in the structure, leading to sudden and catastrophic failure [10–14]. Consider, for example, a typical CFRP system used in the aeronautic industry: the T300/907; the ratio between un-notched and notched strength, which is a measure of damage tolerance, can be as low as 50% [14].

A consequence of the low damage tolerance is that composite structures today are overweight and composite materials are still under-utilized, for example in the automotive industry. Even the Boeing 787, one of the most efficient aircraft flying today, is 6.4 t (10%) overweight [15]. The Boeing Company estimates that 20 kt of fuel (5 Mt of CO\textsubscript{2}) per year can be saved globally by structural weight reduction [16]. This calls for new technological developments to improve
toughness and damage resistance in CFRPs.

In particular, this thesis focuses on translaminar fracture, which involves through-the-thickness crack propagation along an in-plane direction in CFRP laminates under longitudinal tensile loading [17–20]. The energy per unit of projected area necessary to propagate this type of damage in the laminate is the translaminar work of fracture. Since the strength and modulus of CFRP laminates are strongly degraded by the translaminar failure of the load-aligned plies, increasing the translaminar work of fracture can lead to a significant improvement in the damage resistance and damage tolerance of CFRP structures.

Biological composites found in structural applications (bones, tooth enamel and sea shell), despite being made of relatively brittle building blocks, exhibit remarkable combinations of stiffness, strength and toughness, when compared with their constituent phases [21–25]. For instance, monolithic aragonite shows a work of fracture that is about 3000 times less than that of the composite shell (nacre) made of microscopic platelets of the same material [26].

Toughening and strengthening mechanisms in bio-composites are not completely understood [25, 27, 28]. Yet, there are some characteristic features that are common to most structural bio-composites and appear to be desirable to obtain high strength and toughness:

- discontinuous microstructure formed by hard and stiff mineral inclusions, capable of carrying load and guaranteeing high strength and modulus, staggered in a highly ordered structure and connected by a compliant matrix capable of transferring shear load and dissipating energy during deformation;

- hierarchical organization of the material from the microstructure to the macrostructure;

- use of crack deflection mechanisms to increase energy dissipation during fracture.

The key question addressed in this dissertation is the following: would it be possible to improve the toughness and damage resistance of CFRP laminates, while retaining strength, by engineering their microstructure to promote bio-inspired toughening mechanisms during fracture?
1.2 Structure of the Thesis

To answer the question posed in Section 1.1, a new microstructure design technique was developed. Patterns of micro-cuts perpendicular to the fibre direction are used to create CFRP laminates with a discontinuous, yet highly ordered, microstructure which guarantees efficient load transfer between the carbon fibre bundles. The microstructure design takes inspiration from biological composites by adopting the most important toughening mechanisms, and adapting them to the properties of the CFRP laminates.

Initially, patterns of micro-cuts have been inserted in the $0^\circ$ plies of a thin-ply CFRP laminate with Cross-Ply (CP) layup to promote the formation of hierarchical pull-out structures during crack propagation. An analytical model to predict the probability of bundle pull-out during transplaminar crack propagation was developed and validated through an experimental parametric study. The model was used to design three hierarchical patterns of micro-cuts and the patterns have been tested using Compact Tension (CT) specimens to measure the transplaminar work of fracture. Tensile tests were carried out on a laminate containing the best performing hierarchical pattern to measure the un-notched tensile strength. This work is presented in Chapter 2.

In Chapter 3, patterns of micro-cuts in the $0^\circ$ plies of thin-ply CP laminates were used to promote crack deflection, and the interaction of failure mechanisms between neighbouring plies with different fibre orientation. An analytical model based on Finite Fracture Mechanics (FFM) was developed to predict the maximum crack deflection height and was used to design three patterns of micro-cuts called Shark-Teeth for their characteristic shape; the patterns were tested again using CT specimens to measure the transplaminar work of fracture.

The experience gained in the previous two chapters was used to engineer the transplaminar fracture behaviour of Quasi-Isotropic (QI) laminates. Patterns of micro-cuts were inserted in the $0^\circ$ and $\pm45^\circ$ plies of the laminate to promote crack deflection, and increase energy dissipation mechanisms. The FFM model was validated through a parametric study and used to design six different microstructures, which were tested using CT specimens to measure the transplaminar work of fracture. The best performing microstructure design was applied to a QI laminate subject to an indentation test to improve its damage resistance. This part of the study, which
is presented in Chapter 4, is of particular importance given the wide industrial application of QI laminates.

Finally, in Chapter 5, a semi-analytical Fibre Bundle Model was developed to investigate the role of dynamic stress concentrations, and of fracture mechanics-driven growth of critical clusters of fibres, on the longitudinal tensile failure of fibre-reinforced composites. The model uses shear-lag to calculate the stress recovery along broken fibres, and an efficient field superposition method to calculate the stress concentration on the intact fibres. It can simulate the tensile failure of bundles up to 10000 fibres directly. The model predictions have been validated against experimental results for micro and macro composite bundles from the literature.

This modelling approach, in its current state, provides valuable insights on the role of dynamic stress concentrations and fracture mechanics on the longitudinal tensile failure of CFRPs. With further development, it holds the potential to become a more general and flexible tool for the design and optimization of engineered microstructures for fibrous composites.
Chapter 2

On the role of hierarchical microstructures in CFRP laminates with engineered fracture behaviour

2.1 Introduction

Visual examination of the translaminar fracture surface of CFRP laminates (fracture surface associated with the translaminar fracture) reveals a hierarchical organization of fibre pull-outs, where single fibres are pulled out of small bundles, which in turn are pulled out of larger bundles [18,29,30]. Most authors agree that the translaminar work of fracture in CFRPs is directly related to the energy dissipated by debonding and by friction during the formation of these hierarchical surface [19,31–34]. These results suggest that an increase in toughness can be achieved by controlling the morphology of these hierarchical surfaces and promoting the formation of longer bundle pull-outs. Analogously, biological composites with hierarchical microstructures often show remarkable values of toughness when compared to their constituent phases [35–37].

The insights from modelling and experimental results in CFRP [18, 19, 29, 30], and the example from natural composites [21–24, 35–38] indicate that there is a potential for CFRPs with an engineered hierarchical microstructure to overcome the classical dichotomy between
strength/stiffness and toughness.

This chapter explores the idea of using patterns of carefully-placed micro-cuts perpendicularly to the fibre direction in the 0° plies of a Cross-Ply (CP) CFRPs laminate to create a discontinuous, yet highly ordered, microstructure (see Fig. 2.1). The presence of the micro-cuts is expected to change the behaviour of the composite during translaminar crack propagation, promoting the formation of large pull-out structures and increasing energy dissipation.

The laminates tested for this study were manufactured using thin-ply unidirectional (UD) prepregs (from 30 to 50 μm in thickness). Thin-ply prepregs exhibit enhanced resistance against delamination, which results in higher un-notched tensile strength and fatigue strength when compared with standard-size prepregs [39, 40] (typically from 120 to 200 μm in thickness). Furthermore, the smaller ply thickness allows a more fine control over the microstructure, which is clearly beneficial in the context of the microstructure design technique developed in this work.

Also, both modelling and experimental results [19, 41] showed that the toughness decreases significantly with the ply thickness, thus the technique developed in this thesis to increase the work of fracture of CFRP laminates has the potential to be particularly beneficial for thin-ply laminates.

The aims of the present chapter are:

i to develop a manufacturing technique to create CFRP laminates with precisely-placed patterns of micro-cuts;

ii to design patterns of micro-cuts to cause crack deflection and promote formation of large bundle pull-outs; and

iii to validate the concept experimentally by measuring the translaminar work of fracture of CFRP laminates with hierarchical patterns of micro-cuts using Compact Tension specimens.
2.2 Materials, manufacturing and test methods

2.2.1 Material system used

The material system used in this work is a thin-ply UD carbon-epoxy prepreg, TR50s/K51, provided by Skyflex [1] in two grades (which corresponds to different ply thicknesses); individual fibre and laminate properties are reported in Tab. 2.1.

2.2.2 Specimens definition

The behaviour of thin-ply composites with patterns of micro-cuts during translaminar fracture propagation was studied using Compact Tension (CT) specimens (Fig. 2.2(a)). The specimens have a symmetric CP lay-up ([90, (0, 90)20]s) with forty 0° plies. The thinner prepreg material (grade A in Tab. 2.1) is used for the 0° plies, and the thicker prepreg (grade B in Tab. 2.1) is
Table 2.1: TR50s/K51 properties [1, 2]

<table>
<thead>
<tr>
<th>Single fibre properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fibre diameter [μm]</td>
<td>6.82</td>
</tr>
<tr>
<td>Longitudinal modulus $E^f$ [GPa]</td>
<td>240</td>
</tr>
<tr>
<td>Average tensile strength $X^f_m$ [GPa]</td>
<td>4.9</td>
</tr>
<tr>
<td>Coefficient of Variation of strength $\text{CoV}^f$</td>
<td>0.24 $^a$</td>
</tr>
<tr>
<td>Reference length $\ell_f$ [mm]</td>
<td>10</td>
</tr>
<tr>
<td>Fracture toughness $G^f$ [J/m$^2$]</td>
<td>7.4 $^b$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Matrix properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode-II toughness of the matrix $G^\text{mat}_{\text{II}}$ [kJ/m$^2$]</td>
<td>1</td>
</tr>
<tr>
<td>Matrix shear yielding $\tau_{\text{sl}}$ [MPa]</td>
<td>88.5</td>
</tr>
<tr>
<td>In-situ frictional stress $\tau_{\mu}$ [MPa]</td>
<td>10 $^c$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Laminate properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal ply thickness$^d$ [mm]</td>
<td>A: 0.03 B: 0.055</td>
</tr>
<tr>
<td>Fibre areal weight$^d$ [g/m$^2$]</td>
<td>A: 30 B: 50</td>
</tr>
<tr>
<td>Nominal fibre volume fraction [g/cm$^3$]</td>
<td>0.60</td>
</tr>
<tr>
<td>Cured resin density [g/cm$^3$]</td>
<td>1.20</td>
</tr>
<tr>
<td>Nominal fibre density [g/cm$^3$]</td>
<td>1.82</td>
</tr>
<tr>
<td>Nominal laminate density [g/cm$^3$]</td>
<td>1.60</td>
</tr>
<tr>
<td>Longitudinal CFRP modulus $E_1$ [GPa]</td>
<td>125.3</td>
</tr>
<tr>
<td>Transverse CFRP modulus $E_2$ [GPa]</td>
<td>8.4</td>
</tr>
<tr>
<td>Major CFRP Poisson’s ratio $\nu_{12}$ [GPa]</td>
<td>0.28</td>
</tr>
<tr>
<td>Shear modulus $G_{12}$ [GPa]</td>
<td>5.1</td>
</tr>
<tr>
<td>Intralaminar fracture toughness $G^9_{\text{II}}$ [kJ/m$^2$]</td>
<td>0.255 $^e$</td>
</tr>
</tbody>
</table>

$^a$This property was back-calculated using the HFBM [42] to fit the composite tensile strength.
$^b$As estimated by Honjo [43].
$^c$Nominal property for carbon-epoxy systems [34].
$^d$Two different grades (A and B) were used.
$^e$Measured by Teixeira et al. [41].

used for the 90° plies, to guarantee better isolation of the pull-outs formation mechanisms in the different 0° plies. Each 0° ply contains a pattern of micro-cuts aligned with the test section of the specimen as shown in Fig. 2.2(b). By having a different pattern in each 0° ply, while keeping a symmetric lay-up, it is possible to test up to 20 different patterns in each CT specimen.

The design of the CT specimens used in this work is similar to others reported in the literature [18, 29, 30, 41, 44–46], but with one important modification: the specimens contain a central notch which terminates with a blunted semicircular shape, instead of a sharp machined notch. This semicircular blunt notch overlaps with a 10 mm straight cut laser-engraved in each 0° ply before lamination, thus terminating in a 5 mm long laser-cut sharp notch which acts as
crack initiator during the tensile test. Since the pattern of micro-cuts and the straight cut are engraved simultaneously with the same laser process, this specimen design guarantees that the translaminar crack propagating from the notch will be perfectly aligned with the patterns of micro-cuts. Furthermore, the laser-cut notch is considerably sharper than those that can be achieved with other methods in the literature [47], as a tip radius of down to $\sim 7\mu m$ can be achieved with the machine used in this study.

### 2.2.3 Laminate manufacturing

#### 2.2.3.1 Laser cutting technique

Laser micro-milling was used to create the patterns of micro-cuts in the 0° plies via an engraving process. The laser machine (Oxford Lasers, Series A) uses a laser beam with a diameter ranging from 10 to $15\mu m$ in the focal region with a sub-micron precision in the positioning of each micro-cut.

Fig. 2.3 shows an example of a straight cut produced in a UD thin-ply prepreg (grade A in Tab. 2.1). It is possible to notice the presence of a small heat affected zone, which extends around the cut, where the superficial resin layer has been affected by the heat. It is not possible
to notice any evident defect in the un-cut portion of the fibres; nor any effect on the fibre-matrix interface in the heat affected zone.

2.2.3.2 Single-ply laser engraving

A 200 mm by 250 mm CP laminate plate ([90,(0,90)_{20}]) was manufactured using hand lay-up (Fig. 2.4). Before lamination, each 0\degree ply was separately machined and laser-engraved with 6 distinct patterns of micro-cuts, each at a different location (Fig. 2.4(b)). Each pattern is preceded by the initial 10 mm straight cut (laser-cut notch, Fig. 2.4(a)).

2.2.3.3 Laminae alignment method

Four alignment holes were also created in each ply by the laser at the same time as the patterns and the laser-cut notch. A fixture formed by a flat support plate and four metal pins was used during the lay-up process to align the holes in each ply. Since the pin-holes, patterns and laser-cut notch are realized with the precision of the laser, this technique ensures the alignment of the patterns in the final laminate.
Figure 2.4: Representation of the laser engraving process and lay-up process: (a) detail of the laser engraved pattern, (b) single ply, (c) lay-up process, (d) specimens cut out of the final laminate.

2.2.3.4 Water-jet cutting

After curing the laminate in an autoclave in accordance with the manufacturer specification [1], a CNC water-jet machine was used to cut the plate into the specimens geometry (Fig. 2.4(d)). The same pin-holes alignment system was used to guarantee the coincidence of the patterns of micro-cuts in the laminate plate with the test section of the corresponding CT specimen.
2.2.4 Test methods

The CT specimens were tested using an Instron load frame with a 10 kN load cell; each specimen was loaded under displacement control at a rate of 0.5 mm/min. A video strain gauge system (Imetron) was used to measure and record the relative displacement of two target points drawn on the surface of the specimens (Fig. 2.5). Using FE, it was demonstrated that the relative displacement of these two target points is practically equal to the relative displacement of the load application points, which would be more difficult to measure experimentally. Load measurements were recorded via the Instron load frame and synchronized with the relative displacement of the two target points measured by the video strain gage system.

During tensile testing, the stress concentration at the end of the semicircular notch caused the opening of the laser-cut notch, which then acted as crack initiator, as shown in Fig. 2.5. The test was stopped when the cross-head displacement reached 3 mm. At the end of each test, the specimen was wedged before unloading in order to prevent crushing of the fracture surface.
2.3 Bundle pull-out probability

2.3.1 Analytical model

Fig. 2.6 shows a bundle of length $\ell$ and width $w$, included in a ply of thickness $t$, formed as the result of the presence of a laser micro-cut in the proximity of an approaching translaminary crack. During crack propagation, the region near the crack tip experiences high tensile stress. The tensile stress is transferred to the bundle through shear stress at its four lateral interfaces.

The stress profile in the bundle can be calculated using a shear-lag analysis which assumes perfectly-plastic behaviour of the matrix and a uniform tensile stress in the bundle cross section [48, 49]. If the strength of the bundle is sufficient to withstand this tensile stress, the bundle will survive, causing debonding of the interfaces along the fibre direction and forming a pull-out (Fig. 2.1(b)). If the strength of the bundle is not sufficient to withstand the tensile stress, the bundle will break and the crack will propagate through its base (Fig. 2.1(c)).

Since the strength of a bundle of fibres is a stochastic variable [42,50–53], the event of pull-out is stochastic as well and the probability of bundle pull-out ($Pr^{po}$) can be defined as the complement to 1 of the probability of bundle failure ($Pr^{fa}$) under the given stress profile. The probability of bundle failure $Pr^{fa}$ was calculated using the Hierarchical Fibre Bundle Model (HFBM) developed by Pimenta and Pinho [42].

The shear-lag analysis used in this model assumes a uniform distribution of tensile stress across
the bundle section and a perfectly-plastic behaviour of the matrix. The fibre stress is nil at
the top of the bundle (i.e. location of the micro-cut) and varies linearly along the length of the
bundle:

\[
\sigma(x) = \frac{C_b \cdot \tau_{sl}}{A_f} \cdot x \quad \Rightarrow \quad \sigma_{max} = \frac{C_b \cdot \tau_{sl}}{A_f} \cdot \ell = \lambda_{sl} \cdot \ell, \tag{2.1}
\]

where \(C_b = 2 \cdot (w + t)\) is the external perimeter of the bundle, \(A_f = V_f \cdot w \cdot t\) is the total
cross-section area of the fibres in the bundle, \(\tau_{sl}\) is the shear strength of the matrix and \(V_f\) is
the fibre volume fraction.

The condition for the formation of a bundle pull-out is that the strength of the bundle has to
be sufficient to withstand the linear stress distribution described by Eq. (2.1). The strength of
a bundle is a stochastic variable, therefore the probability of bundle pull-out (for a bundle of
length \(\ell\), width \(w\) and thickness \(t\)) can be defined as

\[
Pr^{po}(\ell, w, t) = S^{L}_{n_f,\ell}(\sigma_{max}), \tag{2.2}
\]

where \(S^{L}_{n_f,\ell}(\sigma_{max})\) is the survival probability of a bundle of length \(\ell\) with \(n_f\) fibres under a linear
(superscript L) stress distribution and is a function of the maximum stress \(\sigma_{max}\). The number of
exposed fibres in the bundle can be calculated as \(n_f = \frac{4 \cdot A_f'}{\pi \phi_f} \), where \(\phi_f\) is the single fibre diameter,
and \(A_f' = V_f \cdot t \cdot \min\{t, w\}\). This assumes that, in the presence of a notch and a strong stress
gradient, the strength of the bundle is determined by the number of fibres contained in a square
bundle closer to the notch rather than by the number of fibres in the entire area of the bundle
(if \(w > t\)).

The survival probability \(S^{L}_{n_f,\ell}(\sigma_{max})\) can be calculated using the generalized WLT \cite{42} as

\[
\ln[S^{L}_{n_f,\ell}(\sigma_{max})] = \frac{1}{\sigma_{max}} \int_{0}^{\sigma_{max}} \frac{\ell}{\ell} \cdot \ln[S^{U}_{n_f,\ell'}(\sigma)]d\sigma, \tag{2.3}
\]

where \(S^{U}_{n_f,\ell'}(\sigma)\) is the survival probability of a bundle of reference length \(\ell'\) with \(n_f\) fibres
under uniform stress distribution (superscript U) and can either be calculated from constituent
properties using the HFBM developed by Pimenta and Pinho \cite{42} or measured experimentally.

By substituting Eq. (2.2) in Eq. (2.3) and after algebraic manipulation, it is possible to obtain
the following expression for the probability of bundle pull-out:

$$\ln[\Pr^{po}(\ell, w, t)] = \frac{1}{\ell_t \lambda_{al}} \int_{0}^{\sigma_{max}} \ln[\frac{S_{U,t}}{S_{U,t}^o}] d\sigma, \quad (2.4)$$

where $\lambda_{al}$ is defined in Eq. (2.1).

### 2.3.2 Experimental parametric study

An experimental parametric study was carried out to obtain a correlation between the geometrical parameters of the pattern of micro-cuts and the probability of formation of bundle pull-outs on the translaminar fracture surface of the CT specimens. The model defined in Section 2.3.1 was used to define the range of parameters over which the experimental parametric study would be conducted. Using the properties in Tab. 2.1, the pull-out probability $\Pr^{po}$ (Fig. 2.7) can be seen to change from 100% to 0% for bundle lengths between $0.2 \leq \ell \leq 0.8$ mm, regardless of $w$ and $p$, with reference to the pattern design in Fig. 2.8.

Each pattern tested was cut in a single ply, and consisted of a series of micro-cuts which repeats uniformly along the test section of the specimen (Fig. 2.8). Since the objective of this parametric study is to investigate the effects of each micro-cut geometry separately, the distance between the micro-cuts in a pattern is arbitrarily set to a large value ($p = 5 \cdot w$) to avoid interactions,
and the bundle thickness is fixed by the ply thickness. The geometry of the pattern is therefore defined by the two parameters $w$ and $\ell$, which determine the width and the length of the bundle pull-out, respectively.

Based on the results of the analytical model, five different values of the parameter $w$ were selected, ranging from 0.025 mm up to 0.5 mm. For each value of $w$, eight different values of the parameter $\ell$ were selected, ranging from 0.025 mm up to 2 mm (Fig. 2.7). Therefore, 40 different patterns were defined (Tab. 2.2).

Three CT specimens were used in this experimental study (SP1 to SP3 in Tab. 2.2); the dimensions and orientation of the CT specimens are described in Fig. 2.2. Each specimen is formed by a CP laminate with forty $0^\circ$ plies; therefore, it is possible to test 20 different patterns of micro-cuts disposed in symmetrical position across the lay-up in each specimen. The distribution of the 40 patterns in the three CT specimens is defined in Tab. 2.2.

### Table 2.2: Combinations of values of $w$ and $\ell$ used in the parametric study

<table>
<thead>
<tr>
<th>Patterns</th>
<th>$w$ [mm]</th>
<th>$\ell$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.025</td>
<td>0.025 0.05 0.075 0.1 0.125 0.15 0.175 0.2</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.375</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.1 0.2 0.3 0.4 0.4 0.5 0.55 0.6 0.65</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.2 0.4 0.6 0.7 0.8 0.9 1.0 1.1</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.25 0.5 0.75 1.0 1.25 1.5 1.75 2.0</td>
</tr>
<tr>
<td>Specimens</td>
<td>SP1 &amp; SP3</td>
<td>SP2</td>
</tr>
</tbody>
</table>
In order to assess possible interactions of patterns in neighbouring 0° plies, specimen SP1 and SP3 contain the same 20 patterns, but placed in a different position across the lay-up. In this way, an eventual dependency of the pull-out probability on the lay-up sequence could be identified. In specimen SP1, the length \( \ell \) of the pattern in a specific layup position is defined by following Tab. 2.2 row by row (from column 2 to 5), while the length \( \ell \) of the pattern in equivalent layup positions in specimens SP3 follows the order of Tab. 2.2 column by column (from column 2 to 5). The distribution of patterns in specimen SP2 also follows Tab. 2.2 row by row (from column 6 to 9). The corresponding value of \( w \) for every pattern is always defined in column 1 of Tab. 2.2.

### 2.3.3 Results of the parametric study

Fig. 2.9(a) shows the fracture surface of specimen SP1 away from the laser notch. It is possible to notice large bundle pull-outs corresponding to the positions of the micro-cuts in the 0° plies. These large bundle pull-outs are not disposed uniformly on the surface. In fact, there are 0° plies with a regular sequence of pull-outs, and others where the bundle pull-outs are more sparse or not present at all. The latter indicates bundle failure (Fig. 2.1(c)) for the particular pattern in that ply.

A statistical analysis of the fracture surfaces was performed in order to correlate the geometrical parameters of the patterns of micro-cuts and the bundle pull-out probability. For each pattern, the number of bundle pull-outs (as opposed to bundle failures) was counted based on the SEM observation of the entire fracture surface and divided by the total number of micro-cuts in the pattern to obtain the experimental pull-out probability. The results of this statistical analysis are shown in Fig. 2.10, and are compared against the initial modelling predictions.

Note that each 0° ply was engraved with a pattern of micro-cuts with specific values of \( w \) and \( \ell \). Each specimen contains the same pattern twice in symmetrical position across the lay-up. Therefore, each experimental symbol in Fig. 2.10 is the average of bundle pull-outs in two plies. Furthermore, specimen SP3 (hollow symbols) contained the same patterns as specimen SP1 (full symbols) but in different position across the lay-up.
Table 2.3: Geometrical parameters of the hierarchical patterns of micro-cuts

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Pattern</th>
<th>$w$</th>
<th>$p$</th>
<th>$\ell_1$</th>
<th>$\ell_2$</th>
<th>$\ell_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP4</td>
<td>Baseline</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SP5</td>
<td>H1</td>
<td>0.03</td>
<td>0.03</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SP6</td>
<td>H2</td>
<td>0.03</td>
<td>0.03</td>
<td>0.2</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>SP7</td>
<td>H3</td>
<td>0.03</td>
<td>0.03</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

2.4 CFRP composites with hierarchical microstructure

2.4.1 Hierarchical microstructure design

Following the successful generation of bundle pull-outs during the parametric study, the study proceeded to design micro-structural patterns aimed at achieving laminates with higher values of translaminar work of fracture. Fig. 2.11 shows the three pattern designs used in this study. These patterns exploit the idea of using a micro-cut to produce a large bundle pull-out and scale it hierarchically from 1 to 3 levels of hierarchy. Each pattern was used for all $0^\circ$ plies of a specific specimen.

The single micro-cut length $w$ was chosen equal to 0.03 mm for all patterns. An inter-space $p = 0.03$ mm was left between the cuts. The pull-out lengths for each hierarchical level were chosen considering the results of the pull-out probability model (which proved to correlate well with the experimental results in Fig. 2.10), in order to guarantee high probability of bundle pull-out (close to 100%) while dissipating a large amount of energy during the pull-out process. Tab. 2.3 shows the geometrical parameters of each pattern.

The translaminar work of fracture for the three different pattern designs was predicted considering the energy necessary to fracture the fibres in the inter-space during the bundle formation, plus the energy dissipated by debonding and friction during the bundle pull-out process. The analytical development of the model for a hierarchical structure of bundle pull-outs with $n$ levels of hierarchy is described in Appendix A.1. The values of translaminar work of fracture expected for the three patterns (H1, H2 and H3) as a function of the bundle length $\ell$ are shown in Fig. 2.12.
2.4.2 Translaminar work of fracture tests

A total of four CT specimens were used in this experimental study. One CT specimen (SP4) was defined without any pattern of micro-cuts so that it could be used as a baseline reference for the translaminar work of fracture of the material. The other three CT specimens (SP5-SP7) contain the 3 hierarchical pattern designs as defined in Tab. 2.3.

The dimensions, lay-up and materials used to manufacture the CT specimens are identical to those used in the parametric study in Section 2.2.2. In each specimen, all 0° plies were laser-engraved with the same pattern of micro-cuts. The pattern was repeated regularly along the entire length of the specimen and aligned with the test section. Accordingly, the experimental set-up and test procedure used for these tests were the same as those described in Section 2.2.4.

The modified compliance calibrated method [18,29,44] was used to calculate the work of fracture of the laminate for each specimen. The rule of mixtures was applied to calculate the work of fracture of the 0° plies, given the work of fracture of the laminate measured in the experiments and the intralaminar toughness of the 90° plies given in Tab. 2.1.

2.4.3 Results of work of fracture tests

Load vs. opening displacement (relative displacement of the two load application points) and translaminar work of fracture of the 0° plies vs crack length for the four specimens are shown in Fig. 2.13. The specimens with the hierarchical patterns of micro-cuts show a stable crack propagation behaviour and a significant increase in both the maximum load and translaminar work of fracture when compared with the baseline specimen.

Tab. 2.4 reports the maximum loads recorded during the tensile test for each specimen and the values of work of fracture for crack initiation and crack propagation. The latter are compared with the values predicted by the toughness model. The initiation work of fracture was defined at the maximum value of the load. The propagation work of fracture was defined as the average value once the toughness curve had levelled out and was fairly constant (steady-state). For the baseline material (without any pattern of micro-cuts), the measured value of initiation work of fracture is comparable to the non-linearity onset value measured by Teixeira et al [41], which
Table 2.4: Results of the CT test for the four specimens and comparison with modelling predictions of the translaminar work of fracture of the 0° plies. \( \Delta W_{0\phi} \) is the increase in propagation work of fracture with respect to the baseline material/specimen.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>( P_{\text{max}} ) (kN)</th>
<th>( W_{0\phi}^{\text{init}} ) (kJ/m²)</th>
<th>( W_{0\phi}^{\text{drop}} ) (kJ/m²)</th>
<th>( \Delta W_{0\phi} )</th>
<th>( W_{0\phi}^{\text{sim}} ) (kJ/m²)</th>
<th>Predictions error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.92</td>
<td>29.5</td>
<td>32.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>H1</td>
<td>0.89</td>
<td>31.0</td>
<td>37.0</td>
<td>+15</td>
<td>40.0</td>
<td>+8</td>
</tr>
<tr>
<td>H2</td>
<td>1.08</td>
<td>43.3</td>
<td>51.2</td>
<td>+60</td>
<td>53.3</td>
<td>+4</td>
</tr>
<tr>
<td>H3</td>
<td>1.58</td>
<td>101.1</td>
<td>-</td>
<td>&gt;214</td>
<td>165.0</td>
<td>-</td>
</tr>
</tbody>
</table>

makes sense given the flat R-curves obtained in this study, resultant from the very sharp initial notches produced with laser.

Fig. 2.14 shows a comparison of the entire translaminar fracture surface for the bottom halves of the four CT specimens. In the baseline material (Fig. 2.14(a)), the fracture surface of the 0° plies are characterized by single fibre or small bundle pull-outs with lengths up to 0.04 mm. On the contrary, patterns H1 and H2 (Fig. 2.14(b) and (c)) exhibit large bundle pull-outs in the 0° plies with very good alignment across the laminate. The bundle pull-outs repeat regularly over the entire test section of the specimen and the translaminar crack remained in the plane initially defined by the laser notch.

Finally, the fracture surface of pattern H3 (Fig. 2.14(d)) only shows two series of hierarchical pull-outs next to the laser-cut notch. In the rest of the fracture surface, the crack propagated in the region of the specimen above and below the patterns of micro-cut in anti-symmetrical positions in the anterior and posterior parts of the specimen. It is also possible to notice a significant amount of fibre bridging in the in the 90° plies.

### 2.5 Discussion

#### 2.5.1 Laser engraving technique

The laser engraving process used in this work created patterns of micro-cuts with typical cut thickness of 15\( \mu \)m. Although micro-cuts images obtained via optical microscope and SEM (Fig. 2.3) show the presence of a heat affected zone around the cut as a result of the laser-
cut process, no sign of damage to the fibres was noticeable; nor any effect on the fibre-matrix interface in the heat affected zone.

In conclusion, the laser engraving process allows very good precision in creating the micro-cuts and does not significantly damage the surrounding material. Therefore, it is suitable for creating CFRP laminates with hierarchical patterns of micro-cuts, or other micro-structural features.

2.5.2 Lay-up process

The fracture surfaces in Fig. 2.14 show excellent precision in the alignment of the patterns of micro-cuts and the laser-cut notch in each 0\(^\circ\) ply with the test section of the CT specimen. Therefore, the alignment technique proved successful. This was possible due to the simultaneous laser engraving of the patterns of micro-cuts, the laser-cut notch and the alignment holes, as well as the use of the corresponding alignment fixture during the lay-up of the panel and the water-jet cutting process.

2.5.3 Bundle pull-out probability

The results of the experimental parametric study in Fig. 2.10 clearly show a dependency of the pull-out probability on the bundle length for all patterns tested. Although differences in pull-out probability between corresponding patterns in specimens SP1 and SP3 are present (in particular for \(w = 0.1\,\text{mm}, 0.2\,\text{mm} \) and \(0.5\,\text{mm}\)), these are not deemed to be significant considering the steep gradient of the pull-out probability function in the corresponding parts of the graph. Therefore, these results suggest that the position of the pattern in the lay-up did not influence the pull-out probability.

Furthermore, there was no significant difference in the pull-out probability of bundles with different widths \(w\), and the experimental results correlate well with model predictions (which assume a square bundle of dimensions \(w = t = 0.03\,\text{mm}\)). This justifies the assumption made in the model (see Section 2.3.1) that, in the presence of a notch and a strong stress gradient, it is not the total area of the bundle that determines the probability of pull-out, but rather the area of the sub-bundle with area \(t^2\) that is closer to the crack tip. Since only one specimen was
tested for each configuration, these experimental results do not account for eventual variability of the manufacturing process.

The good agreement between the analytical pull-out probability model and the experimental parametric study, and the success of the hierarchical pattern designs, confirm that the model developed in this chapter is suitable to design hierarchical microstructures to exploit crack deflection in CFRP laminates. Furthermore, these results also indirectly confirm the accuracy of the HFBM [42] in predicting the failure probability of bundles of fibres with different sizes.

2.5.4 Hierarchical microstructure design

The translaminar fracture surfaces of specimens SP5 and SP6 shown in Fig. 2.14(b) and (c) demonstrate that the patterns H1 and H2 succeed in producing hierarchical bundles of microcuts over the entire fracture surface of the specimens when compared with the baseline material (specimen SP4). The formation of these hierarchical structures of pull-outs completely changed the mechanical response of the specimen, which went from unstable crack propagation (Baseline material in Fig. 2.13(a)) to stable crack propagation (Patterns H1 and H2 in Fig. 2.13(b) and (c)) with a correspondent increase in translaminar work of fracture.

The increase in work of fracture for patterns H1 and H2 is in good agreement with the prediction of the analytical model which accounts for the energy necessary to fracture the fibres in the inter-space between the cuts, plus the energy dissipated by debonding and friction during the bundle pull-out process. This shows that the increase in work of fracture measured during the tests is due to the additional energy dissipated during the formation of the hierarchical structures of pull-outs, and other failure mechanisms such as delamination did not play a relevant role. In conclusions, the hierarchical patterns H1 and H2 behaved as designed and were effective in increasing the translaminar work of fracture of the baseline material. Only one specimen was tested for each configuration; while this is sufficient to demonstrate the magnitude of the increase in work of fracture possible with this approach, more test repetitions would be needed to quantify in a statistically significant way the exact increase in work of fracture for any specific pattern.

Regarding pattern H3, the features of the fracture surface and the test data shown in Fig.
2.14(d) and Fig. 2.13 indicate the following sequence of events:

- the crack initially propagated on the plane defined by the laser-cut notch, producing the pull-out of two lines of hierarchical bundles and determining the rising effect in the toughness curve;

- the consequent increase in the load necessary to continue crack propagation led to tensile failure in the 90° plies and consequent propagation of the crack to a region of the specimen without pattern of micro-cuts;

- the lower translaminar work of fracture of the baseline material in this region of the specimen led to catastrophic failure of the specimen.

It is plausible that, if the pattern of micro-cuts were repeated in the entire specimen rather than just in the test section, the crack could have reached a steady state propagation state. Accordingly, the toughness curve would have reached a constant value corresponding to the work of fracture propagation value, but more tests would be necessary to demonstrate this conclusion. This explanation has two main implications:

- specimen SP7 showed a shortcoming in the design of the CT specimen, rather than a failure of the microstructure design defined by pattern H3;

- the maximum work of fracture values obtained from the test are for initiation and cannot be compared directly with the toughness model because the final propagation value for the pattern H3 is expected to be higher that the one measured.

Furthermore, the un-notched tensile strength of a quasi-isotropic laminate containing the pattern H3 micro-cuts design was measured experimentally and was found to be 12% lower than the strength of a laminate made of the baseline un-modified material (Appendix A.2). The same pattern led to a 70% increase in the maximum tensile load and 214% increase in the initiation value of translaminar work of fracture measured during the CT tests (Section 2.4.3). In conclusion, the microstructure design defined by pattern H3 achieved a significant increase of performances over the baseline material in a notch test scenario, while retaining most of the strength of the baseline material in a un-notched test scenario.
2.6 Conclusions

This chapter has investigated the concept of using patterns of micro-cuts perpendicular to the fibre direction to create large bundle pull-outs and therefore increasing the translaminar work of fracture of CFRP laminates. The following conclusions can be reached:

- it is possible, using hierarchical patterns of micro-cuts, to increase the translaminar work of fracture significantly, without compromising the un-notched tensile strength of the material. The 214% increase in initiation work of fracture achieved in this chapter does not appear to be close to the actual limit for this technique;

- the alignment methodology developed in this work enables precisions of the order of 1 μm in the positioning of the individual plies during lay-up;

- the notch tip manufacturing methodology developed in this work leads to the sharpest translaminar notches in the literature, with a tip radius of ~ 7 μm;

- the analytical model for predicting the probability of bundle pull-out was successful over a wide range of variation of microstructure parameters, and proved to be a useful tool in guiding the design of hierarchical patterns of micro-cuts to increase translaminar work of fracture.
Figure 2.9: SEM micrograph of fracture surfaces obtained during experimental parametric study: (a) fracture surface of specimen SP1 away from the laser notch. The $90^\circ$ plies are characterized by a relatively uniform fracture surface with few fibre fractures. The $0^\circ$ plies are characterized by large bundle pull-outs in correspondence to the laser micro-cuts; (b) magnification of a large bundle pull-out protruding from $0^\circ$ ply.
Figure 2.10: Experimental probability of bundle pull-out; the data points are obtained through statistical analysis of the fracture surface. The analytical probability of bundle pull-out for \( w = t \) is plotted for comparison.

Figure 2.11: Hierarchical patterns of micro-cuts (thick-lines), and the correspondent pull-out structures which are expected to form during translaminar fracture (thin-lines): (a) one level of hierarchy; (b) two levels of hierarchy; (c) three levels of hierarchy.
Figure 2.12: Translaminar work of fracture as function of the bundle length for the three hierarchical patterns as predicted by the present model.

Figure 2.13: Test data for the four CT specimens: one made of the baseline thin-ply composite without any modification and the other three containing hierarchical patterns of micro-cuts: (a) Load vs. opening displacement; (b) work of fracture of the 0° plies vs. crack length.
Figure 2.14: SEM micrograph of fracture surfaces obtained during toughness measurements for the hierarchical pattern: (a) baseline material without any micro-cut; (b) hierarchical micro-cuts with one level of hierarchy; (c) hierarchical micro-cuts with two levels of hierarchy; (d) hierarchical micro-cuts with three levels of hierarchy.
Chapter 3

On crack deflection and failure-mode interaction in CFRP laminates with engineered fracture behaviour

3.1 Introduction

In the previous chapter, thin-ply CFRP laminates with a hierarchical organization of the microstructure were developed by taking inspiration from the microstructure of biological composites. These laminates had a cross-ply lay-up in which the 0° plies were laser engraved with patterns of micro-cuts during the lamination process, while the 90° plies were left un-touched. The patterns of micro-cuts promoted the formation of hierarchical bundle pull-outs in the 0° plies, therefore increasing energy dissipation, and allowed to achieve up to 214% increase in the translaminar work of fracture of the 0° plies.

In one of the microstructure designs investigated in Chapter 2, the presence of the micro-cuts, in addition to promoting the formation of pull-outs in the 0° plies, also caused multiple splitting and tensile failure of the fibres in the 90° plies. The combination of these two failure mechanisms caused the crack to change its direction and to propagate to a region of the specimen without micro-cuts. These results suggest that patterns of micro-cuts can be used not just to promote
the formation of bundle pull-outs but also to cause interaction of the failure mechanisms between neighbouring plies and to promote crack deflection. This interaction effect, if suitably harnessed, can further increase energy dissipation and translaminar toughness of the laminate.

Crack deflection is a powerful toughening mechanism used in biological composite to smear and trap the crack front and achieve higher toughness [21–25, 38, 54]. Crack deflection is also used in engineering materials. Nanoparticles are used as inclusions in polymeric materials to deviate and branch the crack propagation pattern, therefore creating more complex fracture surfaces and increasing energy dissipation [55–57]. Mirkhalaf [58] demonstrated how crack deflection induced by the presence of a 3D array of laser engraved micro-cracks can be used to control the crack propagation pattern and significantly increase the toughness of glass.

In this chapter, crack deflection and the interaction of failure mechanisms between neighbouring plies with different fibre orientations in thin-ply laminates with cross-ply lay-up is investigated. This is achieved by using patterns of aligned micro-cuts in the $0^\circ$ plies to steer the crack from its original fracture plane and force it along a tortuous path, causing the interaction of failure mechanisms in the $0^\circ$ and $90^\circ$ plies. The materials and test methods used are detailed in Section 3.2, the microstructure designs are presented in Section 3.3, and the results are shown in Section 3.5. A discussion of the results and the following conclusions are presented in Section 3.6 and Section 3.7 respectively.

### 3.2 Materials and test methods

The material system used in this work is a thin-ply UD carbon-epoxy prepreg (TR50s/K51) provided by Skyflex [1]. The prepreg comes in two grades, which correspond to the two different ply thicknesses of 30 $\mu$m (grade A) and 50 $\mu$m (grade B). Individual fibres and laminate properties used in this work can be found in Tab. 3.1.

Compact Tension (CT) specimens have been used to study the behaviour of thin-ply laminates with patterns of micro-cuts during translaminar fracture propagation. The CT specimens design is the same described in Chapter 2: the specimens have a symmetric cross-ply lay-up ($[90, (0, 90)]_{20s}$) where a thinner prepreg material (30 $\mu$m) was used for the $0^\circ$ plies, and a rela-
Table 3.1: TR50s/K51 properties [1, 2]

<table>
<thead>
<tr>
<th>Single fibre properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fibre diameter [μm]</td>
<td>6.82</td>
</tr>
<tr>
<td>Fibre Fracture toughness $G_f$ [J/m$^2$]</td>
<td>7.4 $^a$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Matrix properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode-I fracture toughness $G_{\text{mat}}$ [kJ/m$^2$]</td>
<td>0.255 $^b$</td>
</tr>
<tr>
<td>Mode-II in-situ interfacial toughness $G_{\text{II}}$ [kJ/m$^2$]</td>
<td>1</td>
</tr>
<tr>
<td>Matrix shear yielding $\tau_s$ [MPa]</td>
<td>88.5</td>
</tr>
<tr>
<td>In-situ frictional stress $\tau_f$ [MPa]</td>
<td>10 $^c$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Laminate properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal ply thickness $^d$ [mm] A: 0.03 B: 0.055</td>
<td></td>
</tr>
<tr>
<td>Longitudinal CFRP modulus $E_1$ [GPa]</td>
<td>125.3</td>
</tr>
<tr>
<td>Transverse CFRP modulus $E_2$ [GPa]</td>
<td>8.4</td>
</tr>
<tr>
<td>Major CFRP Poisson’s ratio $\nu_{12}$ [GPa]</td>
<td>0.28</td>
</tr>
<tr>
<td>Shear modulus $G_{12}$ [GPa]</td>
<td>5.1</td>
</tr>
<tr>
<td>Translaminar work of fracture $G_p$ [kJ/m$^2$]</td>
<td>32.2 $^e$</td>
</tr>
</tbody>
</table>

$^a$As estimated by Honjo [43].
$^b$Measured by Teixeira et al. [41].
$^c$Nominal property for carbon-epoxy systems [34].
$^d$Two different grades (A and B) were used.
$^e$Measured in Chapter 2

A pattern of micro-cuts aligned with the test section of the specimen.

The CT tests were carried out using an Instron load frame with a 10 kN load cell; each specimen was loaded under displacement control at a rate of 0.5 mm/min. A video strain gauge system (Imetrum) was used to measure and record the relative displacement of the load application points in the specimens, and the modified compliance calibrated method [18, 20, 29, 44] was used to calculate the laminate work of fracture from the experimental load-displacement data.

3.3 Microstructure design

3.3.1 Finite Fracture Mechanics model for crack deflection

Fig. 3.1(a) shows a $0^\circ$ ply of thickness $t$ included in a cross-ply laminate with a translaminar crack propagating from left to right at the initial coordinate $y = 0$ in the local reference system. The ply has been engraved with an array of micro-cuts, each perpendicular to the fibre direction.
Figure 3.1: Finite Fracture Mechanics can be used to predict the crack propagation path in a $0^\circ$ ply with an array of pre-existing micro-cuts: (a) intact $0^\circ$ ply in a Cross-Ply laminate; (b) $0^\circ$ ply with array of micro-cuts; (c) the crack is deflected by the array of micro-cuts with deflection angle $\alpha$; (d) the crack jumps back to the original fracture plane.

(Fig. 3.1(b)). The geometrical parameters of the pattern of micro-cuts are the single cut length ($w$), the space between the micro-cuts ($p$), and the vertical distance between the micro-cuts ($s$).

Fig. 3.1(c) shows the crack deviating from its original plane and propagating along the array of micro-cuts; each micro-cut defines a bundle which pulls out of its neighbour. It is assumed that the propagation along the deflected direction happens in finite steps from one micro-cut to the next, progressively causing the debonding of a new portion of the $0^\circ$ ply from its neighbouring $90^\circ$ plies (Fig. 3.1(c)). $\Delta h = s$ and $\Delta x = w + p$ are the finite increments in the vertical and horizontal directions respectively, and they determine the crack deflection angle $\alpha$:

$$\alpha = \arctan\left(\frac{\Delta h}{\Delta x}\right). \quad (3.1)$$

At each step along the pattern of micro-cuts ($y = h$), the crack has two options: (i) it can
either jump back to the initial propagation plane \((y = 0)\), as shown in Fig. 3.1(c); (ii) or it can propagate to the next micro-cut and continue along the deflected direction \((y = h + \Delta h)\), as shown in Fig. 3.1(d). In the first case (i), the crack needs to cause an intralaminar split with the next bundle portion, and the translaminar failure of the fibres at the base of the bundle, which requires the finite amount of energy \(\Delta W_p\):

\[
\Delta W_p = t \cdot h \cdot G_{II}^{\text{mat}} + t \cdot \Delta x \cdot G_{I}^{\text{ply}},
\]  

(3.2)

where \(G_{II}^{\text{mat}}\) is the mode II toughness of the matrix, and \(G_{I}^{\text{ply}}\) is the propagation toughness at the base of the bundle.

In the second case (ii), the crack needs to propagate along the pattern of micro-cuts to the next bundle, and it needs to cause the debonding of the lateral surfaces of the bundle, which requires the finite amount of energy \(\Delta W_s\):

\[
\Delta W_s = t \cdot \Delta x \cdot G_{I}^{\xi} + 2 \cdot h \cdot \Delta x \cdot G_{II}^{\text{mat}} + 2 \cdot \Delta x \cdot \Delta h \cdot G_{II}^{\text{mat}} + t \cdot \Delta h \cdot G_{II}^{\text{mat}},
\]  

(3.3)

where \(G_{I}^{\xi}\) is the toughness along the pattern of micro-cuts, and can be calculated using the rule of mixture:

\[
G_{I}^{\xi} = \xi \cdot G_{I}^{\text{uncut}} + (1 - \xi) \cdot G_{I}^{\text{cut}},
\]  

(3.4)

where \(G_{I}^{\text{cut}}\) is the fracture toughness of the micro-cuts (which is typically filled with resin), \(G_{I}^{\text{uncut}}\) is the toughness of the un-cut portion of composite between two micro-cuts, and \(\xi\) is the fraction of un-cut fibres along the array of micro-cuts.

It was assumed that the condition for the crack to continue to propagate along the deflected direction is that the finite energy necessary to propagate the crack to the next micro-cut is less than the energy required to jump back to the original propagation plane. Therefore, by equating the two energies in Eq. (3.2) and Eq. (3.3), and solving for \(h\), it is possible to obtain the threshold value of crack deflection height \(h = h_{\text{thr}}\) for which the crack will stop following the array of micro-cuts:

\[
h_{\text{thr}} = \frac{t \cdot \Delta x \cdot (G_{I}^{\text{ply}} - \xi \cdot G_{I}^{\text{uncut}} - (1 - \xi) \cdot G_{I}^{\text{cut}}) - (2 \cdot \Delta x^2 + t \cdot \Delta x)}{(2 \cdot \Delta x - t) \cdot G_{II}^{\text{mat}}} \cdot \tan \alpha.
\]  

(3.5)
3.3.2 Envelope curves

Once the relationship between the local value of crack deflection height $h$ and the local value of the deflection angle for the pattern of micro-cuts $\alpha$ has been established in Eq. (3.5), it is possible to define a continuous function of the crack deflection height $h(x)$, where $x$ is the crack horizontal propagation coordinate (Fig. 3.1(a)), for which

$$\tan(\alpha) = \frac{dh}{dx}. \quad (3.6)$$

By substituting $h_{thr}$ with $h(x)$ and Eq. (3.6) in Eq. (3.5) it is possible to write the following differential problem:

$$\begin{aligned}
\frac{dh}{dx} + A \cdot h(x) &= B; \\
h(0) &= 0;
\end{aligned} \quad (3.7)$$

where

$$B = \frac{t \cdot \Delta x \cdot (G_{I}^{\text{ply}} - \xi \cdot G_{I}^{\text{uncut}} - (1 - \xi) \cdot G_{I}^{\text{cut}})}{(2 \cdot \Delta x^2 + t \cdot \Delta x) \cdot G_{II}^{\text{max}}}; \quad (3.8a)$$

$$A = \frac{2 \cdot \Delta x - t}{(2 \cdot \Delta x^2 + t \cdot \Delta x)}. \quad (3.8b)$$

The solution of the differential problem in Eq. (3.7) is

$$h(x) = \frac{B}{A} \cdot (1 - e^{-A \cdot x}), \quad (3.9)$$

and represents the maximum crack deflection height that is possible to achieve along the crack propagation coordinate $x$ and defines a boundary of applicability of the crack deflection technique described in this section.
Figure 3.2: Microstructure design concept: patterns of laser-engraved micro-cuts perpendicular to the fibre direction are inserted in the $0^\circ$ plies of the laminate to promote crack deflection.

Table 3.2: Geometrical parameters of three patterns of micro-cuts investigate in this study.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Pattern</th>
<th>$w$ [$\mu$m]</th>
<th>$p$ [$\mu$m]</th>
<th>$s$ [$\mu$m]</th>
<th>$\alpha$</th>
<th>$h_{\text{max}}$ [$\mu$m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT0</td>
<td>Baseline</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CT1</td>
<td>ST1</td>
<td>30</td>
<td>30</td>
<td>60</td>
<td>45.0°</td>
<td>600</td>
</tr>
<tr>
<td>CT2</td>
<td>ST2</td>
<td>30</td>
<td>30</td>
<td>90</td>
<td>56°</td>
<td>600</td>
</tr>
<tr>
<td>CT3</td>
<td>ST3</td>
<td>30</td>
<td>0</td>
<td>60</td>
<td>63°</td>
<td>900</td>
</tr>
</tbody>
</table>

3.4 Shark-teeth microstructure design

For this study, three different patterns of micro-cuts have been designed based on the concept shown in Fig. 3.2 and with the geometrical parameters shown in Tab. 3.2. All patterns have $w = 30\mu$m long micro-cuts perpendicular to the fibre direction; the space $p$ between the micro-cuts and the vertical distance between the micro-cuts $s$ were uniform within a pattern, but varied from pattern to another, which in turn imposed different deflection angles $\alpha$ for the different patterns. The deflection angle is the angle between the line defined by the array of micro-cuts and the original propagation direction of the crack (which in the current case is horizontal).

The microstructure design concept is based on the idea of using the micro-cuts in the $0^\circ$ plies to steer the incoming translaminar crack from its original propagation plane and cause crack deflection. When the deflected crack reaches the maximum height $h_{\text{max}}$, the direction of the pattern of micro-cuts is inverted, and the crack is deflected in the opposite direction. The full
pattern is defined following this rule periodically for the entire test section of the specimen. The maximum deflection height ($h_{\text{max}}$) for each pattern was decided upon following the model developed in Section 3.3.

3.5 Results

Four different microstructures were tested during this work using four different CT specimens (see Tab. 3.2). One specimen was manufactured without any pattern of micro-cuts and was used as baseline for the behaviour of the un-modified material, while the other three specimens contained the patterns of micro-cuts ST1 to ST3.

Fig. 3.3 shows the results of the CT tests for the four specimens. From the load vs. displacement plot in Fig. 3.3(a), it is possible to notice how the behaviour of the baseline material is characterized by large load drops which indicate unstable crack propagation. On the contrary, the specimens with the engineered microstructure show a smoother load curve, indicative of stable crack growth, with a substantial increase in the maximum load carried by the specimen during the test ($P_{\text{max}}$ increased by up to 68%).

Accordingly, the specimens with engineered microstructure show higher values of the laminate work of fracture.
work of fracture (Fig. 3.3(b)). The laminate work of fracture vs. opening displacement plots for
the engineered materials exhibit a strong R-curve effect in the initial portion of the graph up
to 7-8 mm of crack length, followed by a flatter region during steady state crack propagation,
which is used to define the final value of translaminar work of fracture of the laminate $G_{\text{II}}^{\text{lam}}$. A
summary of the main test results can be found in Tab. 3.3; it can be noticed that the increase
in the laminate work of fracture for the best performing microstructure (pattern ST3) with
engineered microstructure is 460% when compared with the baseline material.

Fig. 3.4(a) to (d) show the four different fracture surfaces obtained from the CT tests. By
comparing Fig. 3.4(b),(c) and (d) to Fig. 3.4(a), it is clear that all three patterns of micro-cuts
have been successful in promoting crack deflection during the fracture process. As a results of the
fracture deflection process, large triangular pull-outs formed in the 0° plies, giving the inspiration
for the name “shark-teeth” design. It is also possible to notice that, while the 90° ply of pattern
ST1 presents a single matrix fracture plane, the 90° plies in the specimens with the patterns ST2
and ST3 present multiple splits along the fibre direction and multiple fractures perpendicular
to the fibre direction.

Fig. 3.5(b) and (c) show an X-rays analysis of the fracture surfaces of the three specimens with
engineered microstructure. It is possible to clearly identify the profile of the “teeth” formed
by the patterns of micro-cuts, surrounded by a bright area which corresponds to the splits and
fractures in the 90° plies. The X-ray images show no signs of delamination between the plies
propagating from the fracture surface inside the specimen.

In the previous chapter, it was demonstrated that the contribution of large bundle pull-outs
in the 0° plies to the total translaminar work of fracture of the laminate can be accurately
estimated by considering the energy dissipated via debonding and friction during the pull-out
process. Using the same approach described in Appendix A.1, the contribution of the bundle
pull-outs in the 0° plies to the total work of fracture of the laminate was calculated and compared
to the work of fracture measured during the experiments for specimens CT1 to CT3 (Fig. 3.6).
Table 3.3: Summary of the results of the Compact Tension test for the four specimens. $\Delta P_{\text{max}}$ is the increase in maximum tensile load and $\Delta W_{\text{lam}}$ is the increase in work of fracture with respect to the baseline material.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>$P_{\text{max}}$ (kN)</th>
<th>$\Delta P_{\text{max}}$</th>
<th>$W_{\text{prop}}^{\text{lam}}$ (kJ/m$^2$)</th>
<th>$\Delta W_{\text{lam}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.99</td>
<td>-</td>
<td>12.6</td>
<td>-</td>
</tr>
<tr>
<td>ST1</td>
<td>1.49</td>
<td>+51%</td>
<td>50.5</td>
<td>+300%</td>
</tr>
<tr>
<td>ST2</td>
<td>1.61</td>
<td>+63%</td>
<td>58.1</td>
<td>+361%</td>
</tr>
<tr>
<td>ST3</td>
<td>1.66</td>
<td>+68%</td>
<td>70.6</td>
<td>+460%</td>
</tr>
</tbody>
</table>

3.6 Discussion

Visual examination of the fracture surfaces in Fig. 3.4(a) to (d) show that, for all three microstructure designs, the patterns of micro-cuts in the $0^\circ$ plies were effective in promoting crack deflection. Furthermore, the crack deflection in the $0^\circ$ caused multiple splits and fractures in the neighbouring $90^\circ$ plies, which are not present in the baseline material. These observations demonstrate the existence of an interaction between the translaminar failure mechanisms in neighbouring plies with different ply orientations.

Since the angles and pitches of the patterns of micro-cuts for the three microstructure designs were decided upon using the model developed in Section 3.3, and all three patterns were effective in promoting crack deflection, these experimental results support the use of this model for designing microstructures to increase the fracture toughness of carbon fibre laminates.

The three “shark-teeth” microstructure designs led, in all cases, to a marked increase in the laminate work of fracture, which then translated into a higher maximum tensile load under CT test. The increase in laminate work of fracture is mainly due to the extra dissipating mechanisms active in the specimens with the “shark teeth” designs. Since the X-ray analysis of the specimens (Fig. 3.5) did not show any sign of delamination between the plies, these mechanisms of energy dissipation must be the crack deflection in the $0^\circ$ plies and the formation of large splitting and fractures in the $90^\circ$ plies.

The energy dissipated via debonding and friction by the pull-outs in the $0^\circ$ plies for each pattern was calculated using the model presented in Appendix A.1, and compared with the total translaminar work of fracture of the laminate $W_{\text{lam}}^{\text{prop}}$ obtained from the CT tests. The compari-
son shows that the pull-outs in the $0^\circ$ plies account only for a portion of the total work of fracture of the laminate (Fig. 3.6). For pattern ST2, in particular, the contribution of the $90^\circ$ plies to the total work of fracture of the laminate was 55%, compared to the 45% contribution of the pull-outs in the $0^\circ$ plies (see Fig. 3.6). This comparison demonstrates the significant contribution of the failure mechanisms in the $90^\circ$ plies to the total work of fracture of the laminate.

3.7 Conclusions

The results presented in this chapter demonstrate that patterns of micro-cuts can be successfully used, in thin-ply CFRP laminates, to promote crack deflection in the $0^\circ$ plies and the interaction of failure mechanisms between neighbouring plies with different fibre orientation. These effects drastically increase the maximum tensile load during CT test by up to 68% and the laminate work of fracture by up to 460% during Compact Tension tests. The latter represents a significant improvements with respect to the 214% increase in toughness that was obtained using micro-cuts to promote the formation of pull-outs in the $0^\circ$ plies obtained in Chapter 2. In particular, the following conclusions can be reached:

- patterns of micro-cuts in the $0^\circ$ plies are successful in steering the crack from its original propagation direction and, hence, in causing crack deflection;
- crack deflection causes the formation of large pull-outs in the $0^\circ$ plies, thereby dissipating energy through debonding and friction, thus contributing to the increase in the work of fracture of the laminate;
- furthermore, crack deflection in the $0^\circ$ plies also causes an interaction with the failure mechanisms of the neighbouring $90^\circ$ plies; this leads to multiple splits and tensile failures in the $90^\circ$ plies, and considerably increases the contribution of the latter to the work of fracture of the laminate (up to 55% of the the total work of fracture of the laminate);
- the Finite Fracture Mechanics model developed in Section 3.3 can be used to design successful patterns of micro-cuts to increase toughness.
Figure 3.4: SEM micrographs of the fracture surfaces of the four Compact Tension specimens: (a) baseline material without any micro-cut; (b) laminate with micro-cuts pattern ST1; (c) micro-cuts pattern ST2; (d) micro-cuts pattern ST3.
Figure 3.5: X-ray images for the three Compact Tension specimens with engineered microstructure: (a) pattern ST1; (b) pattern ST2; (c) pattern ST3.

Figure 3.6: Comparison between the total translaminar work of fracture measured experimentally for the four CT specimens, and the energy dissipated by the pull-outs in the 0° plies for the three specimens with engineered microstructure (calculated with the model in Appendix A.1).
Chapter 4

Towards Quasi Isotropic laminates with engineered fracture behaviour for industrial applications

4.1 Introduction

In the previous chapter, following the inspiration from biological composites such as bones and shells [21–25,38,54,58], Cross-Ply (CP) ([90,0]n,90)s composite laminates with a hierarchical organization of the microstructure (called “shark-teeth” microstructure) were developed, and demonstrated a 460% increase in translaminar work of fracture of the laminate when compared with the un-modified baseline material.

This result was achieved by inserting patterns of carefully-designed micro-cuts in the 0° plies of the laminate before lamination. The patterns of micro-cuts caused crack deflection in the 0° plies during translaminar crack propagation, and promoted the interaction of failure mechanisms between neighbouring plies with different fibre orientation. These two effects are important sources of energy dissipation and were shown to be responsible for the overall increase in the translaminar work of fracture of the laminate.

The first two chapters initially focused on CP laminates because this layup offers a clean and
reliable test case to investigate fibre failure, explore new microstructure designs, and to validate modelling approaches. However, CP laminates have limited interest in the design of composite structures for practical applications.

In this chapter, the engineered microstructure concept is applied to improve the fracture performance of CFRP laminates with a Quasi-Isotropic (QI) layup ([45, 0, −45, 90]_ns), which are widely used in composite structures, and therefore of high practical interest. Due to major designing and experimental challenges related to the strong dependency of the crack deflection behaviour on the lay-up sequence, this represents a significant step-forward in the development of composite laminates with engineered fracture behaviour for practical applications.

The Finite Fracture Mechanics model developed in Section 3.3 is used to guide the microstructure design for QI laminates, and the limit of applicability of this model is explored through a parametric study. The increase in translaminar work of fracture of QI laminates with engineered microstructure is tested using Compact Tension (CT) specimens. The best performing microstructure design is then applied to a QI laminate subject to a quasi-static indentation test. The material, specimens designs and test methods used are detailed in Section 4.2, and the results are shown in Section 4.3. A discussion of the results and the following conclusions are presented in Section 4.4 and Section 4.5 respectively.

4.2 Experiments

4.2.1 Material and manufacturing

The material system used in this work is a thin-ply (20μm) UD carbon-epoxy prepreg (TR30/K51) manufactured by Skyflex [1]. The single fibre, matrix and prepreg properties can be found in Tab. 4.1.

All specimens were manufactured by hand lamination using the procedure developed in Chapter 2. Each single ply of the laminate was laser-engraved with patterns of micro-cuts before lamination using a micro-milling laser machine (Oxford Lasers, Series A). During lamination, a special alignment system was used to guarantee that the patterns of micro-cuts precisely
Table 4.1: TR30/K51 properties [1,2]

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single fibre properties</td>
<td></td>
</tr>
<tr>
<td>Fibre diameter [μm]</td>
<td>6.8</td>
</tr>
<tr>
<td>Fibre fracture toughness $G_f$ [J/m²]</td>
<td>7.4 $^a$</td>
</tr>
<tr>
<td>Matrix properties</td>
<td></td>
</tr>
<tr>
<td>Mode-I fracture toughness $G_{mat}^{I}$ [kJ/m²]</td>
<td>0.255 $^b$</td>
</tr>
<tr>
<td>Mode-II in-situ interfacial toughness $G_{II}^{mat}$ [kJ/m²]</td>
<td>1 $^c$</td>
</tr>
<tr>
<td>Matrix shear yielding $τ_{sl}$ [MPa]</td>
<td>88.5</td>
</tr>
<tr>
<td>In-situ frictional stress $τ_{f}$ [MPa]</td>
<td>10 $^d$</td>
</tr>
<tr>
<td>Laminate properties</td>
<td></td>
</tr>
<tr>
<td>Nominal ply thickness [mm]</td>
<td>0.02</td>
</tr>
<tr>
<td>Longitudinal CFRP modulus $E_1$ [GPa]</td>
<td>101.7 $^e$</td>
</tr>
<tr>
<td>Transverse CFRP modulus $E_2$ [GPa]</td>
<td>6$^e$</td>
</tr>
<tr>
<td>Major CFRP Poisson’s ratio $ν_{12}$ [GPa]</td>
<td>0.3$^e$</td>
</tr>
<tr>
<td>Shear modulus $G_{12}$ [GPa]</td>
<td>2.4$^e$</td>
</tr>
<tr>
<td>Translaminar work of fracture $G_{ply}^{I}$ [kJ/m²]</td>
<td>32.2 $^f$</td>
</tr>
</tbody>
</table>

$^a$As estimated by Honjo [43].
$^b$Measured by Teixeira et al. [41].
$^c$Nominal value for toughened epoxy resin system
$^d$Nominal property for carbon-epoxy systems [34].
$^e$Fuller and Wisnom [59].
$^f$Measured in Chapter 2

overlapped in the final laminate to form the designed microstructure.

After curing the laminate in an autoclave in accordance to the manufacturer specification [1], a CNC water-jet machine was used to cut the plate into the final specimens geometry. The same alignment system was used to guarantee the alignment of the patterns of micro-cuts in the laminate plate with the test section of the corresponding specimen.

4.2.2 Compact Tension test

CT tests were used to study the translaminar crack propagation behaviour in QI laminates with engineered microstructures. The specimen geometry and the test rig setup are shown in Fig. 4.1(a). This type of specimen design has been widely used in the literature [18, 29, 30, 41, 44–46].

Each specimen measured $65 \times 60$ mm and had a symmetric cross-ply lay-up ($[45,0,-45,90]_{22s}$) which corresponded to a nominal thickness of 3.52 mm. Each ply in the specimen contained a
pattern of micro-cuts aligned with the test section of the specimen as shown by the red chevron pattern in Fig. 4.1(a).

A total of seven CT specimens were tested: one specimen was manufactured without any pattern of micro-cuts to measure the behaviour of the baseline material, while the other six contained the patterns of micro-cuts shown in Fig. 4.2. The patterns of micro-cut in the different ply orientations are drawn according to the following colour code: 0° ply-cuts in red, +45° in blue, −45°. The 90° plies did not contain any micro-cut.

Pattern 1 (Fig. 4.2(a)) includes micro-cuts only in the 0° plies. The design concept followed was the same previously used for the Shark Teeth microstructure in CP laminates (Chapter 3). The maximum height of the pattern in the 0° direction was decided upon based on the maximum crack deflection height predicted by the Finite Fracture Mechanics criteria developed in Section 3.3, adjusted for the different ply thickness used in the present work (see Appendix C.1).

Pattern 2 (Fig. 4.2(b)) includes micro-cuts only in the ±45°. This pattern is formed by arrays of micro-cuts perpendicular to the fibre direction, and exploits the natural tendency of the ±45° plies to split along the fibre direction to promote the formation of tooth-like pull-out structures during the fracture process.

Pattern 3 (Fig. 4.2(c)) combines the two previous approaches by having patterns both in the 0° and in the ±45° plies. Again, the maximum height of the pattern in the 0° direction was decided upon based on the maximum crack deflection height predicted by the Finite Fracture Mechanics criteria, while the height of the patterns in the ±45° plies was set to about 60% of the height in the 0° plies. The peaks and lows of the patterns are offset by half a period in the horizontal direction to maximize the friction between neighbouring plies during the pull-out process.

Patterns 4 and 5 (Fig. 4.2(d) and (e)) are scaled-up versions of pattern 3, using scaling factors of 1.5 and 2 respectively. These patterns were designed to test the limit of applicability of the FFM model which was used to design pattern 3.

Finally, pattern 6 (Fig. 4.2(f)) uses the same micro-cuts design defined in pattern 1 for the 0° plies, but re-oriented in the local fibre orientation for the ±45° plies. The main characteristic of this pattern is that the basic shape of the tooth is the same in the three main load bearing
directions of the laminate.

In all patterns designs, the 90° plies did not include any micro-cuts because, in Chapter 3, the interaction of failure modes between the 0° plies with patterns of micro-cuts and the intact 90° plies proved extremely beneficial in increasing the total work of fracture of CP laminates.

The CT tests were carried out using an Instron load frame with a 10 kN load cell; each specimen was loaded under displacement control at a rate of 0.5 mm/min. The relative displacement of the load application points in the specimens was recorded using a DIC measurement system (Imetrum). Data reduction was performed to calculate the laminate work of fracture from the experimental load vs. displacement data using the modified compliance calibrated method [18, 29, 44].

4.2.3 Indentation test

Quasi-Static Indentation (QSI) tests were performed to measure the response of composite laminates with engineered microstructure under localized out-of-plane loading. The designs of the specimen and of the fixture used for this test are shown in Fig. 4.1(b).

Each specimen measured 60 × 60 mm and had a symmetric QI lay-up ([45, 0, −45, 90]_6s) which corresponds to a nominal thickness of 0.96 mm. Each ply in the specimen contained a pattern of micro-cuts aligned with the centre of the specimen as shown by the red chevron pattern in Fig. 4.1(b).

A total of three specimens were tested for this part of the study: one specimen was manufactured without any pattern of micro-cuts to measure the behaviour of the baseline material, while the other two contained pattern 6 as shown in Fig. 4.2(f) (but re-oriented along the two main specimen directions as shown in Fig. 4.1(b)). This pattern of micro-cuts was chosen because, as it will be shown in Section 4.3, it was the best performing during the CT tests.

The fixture design for the QSI test was based upon the ISO 6603-2 standard [60] for impact tests. The indenter head was hemispherical with a 20 mm diameter and was lubricated before each test to avoid friction with the surface of the specimens. The specimens were clamped on a support rig with an inner diameter of 40 mm.
The indentation tests were carried out using an Instron load frame with a 10 kN load cell at a strain rate of 1 mm/min. During the test, the indentation force was measured via the load cell, and the displacement of the indenter was measured using a DIC measurement system (Imetrum) at a position 20 mm from the contact point (Fig. 4.1(b)).

Data reduction was performed by integrating the force vs. displacement diagram to obtain the total energy dissipated by the specimen during the test. The integration was stopped when the residual indentation load fell below 0.1 kN. At this point, the specimen was considered to be completely broken and the residual load was due essentially to the friction between the specimen and the sides of the indenter.

4.3 Results

4.3.1 Compact Tension test results

Fig. 4.3 shows a comparison of the translaminar fracture surfaces for the seven CT specimens tested. In the baseline specimen without patterns of micro-cuts (Fig. 4.3(a)), the fracture surface of the 0° plies is characterized by single fibres or small bundle pull-outs with lengths up to 40 μm. On the contrary, the specimens which contained patterns 1 to 6 (Fig. 4.3(b) to (g)) exhibit large bundle pull-outs in the 0° and 45° plies, with very good alignment across the laminate. The bundle pull-outs repeat regularly over the entire test section of the specimen and the translaminar crack remained in the plane initially defined by the laser notch.

Load vs. opening displacement (relative displacement of the two load application points) and translaminar work of fracture of the laminate vs. crack length for the seven CT specimens are shown in Fig. 4.4. While the baseline material shows a discontinuous load vs. displacement curve, typical of an unstable crack propagation process, all the specimens with patterns of micro-cuts show a stable crack propagation behaviour. This behaviour was also confirmed by visual examination of the specimens during the test.

Tab. 4.2 details the maximum value of the load ($P_{\text{max}}$) recorded during the test for each specimen and the values of the work of fracture for crack initiation ($W_{\text{lam}}^{\text{init}}$) and propagation ($W_{\text{lam}}^{\text{prop}}$). The
Table 4.2: Test results for the seven CT specimens tested. For each specimen, the maximum value of the load recorded during the test ($P_{\text{max}}$), and the values of the work of fracture for initiation ($W_{\text{lam}}^\text{init}$) and propagation ($W_{\text{lam}}^\text{prop}$) obtained from the data reduction are shown. $\Delta W_{\text{lam}}$ is the increase in propagation work of fracture with respect to the baseline material/specimen, and $\Delta P_{\text{max}}$.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>$P_{\text{max}}$ (kN)</th>
<th>$W_{\text{lam}}^\text{init}$ (kJ/m$^2$)</th>
<th>$W_{\text{lam}}^\text{prop}$ (kJ/m$^2$)</th>
<th>$\Delta W_{\text{lam}}$</th>
<th>$\Delta P_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>1.39</td>
<td>11.5</td>
<td>11.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P1</td>
<td>1.67</td>
<td>22.5</td>
<td>24.5</td>
<td>+96</td>
<td>+20</td>
</tr>
<tr>
<td>P2</td>
<td>1.34</td>
<td>17.5</td>
<td>18.2</td>
<td>+52</td>
<td>-4</td>
</tr>
<tr>
<td>P3</td>
<td>1.39</td>
<td>21.3</td>
<td>22.1</td>
<td>+85</td>
<td>0</td>
</tr>
<tr>
<td>P4</td>
<td>1.48</td>
<td>19.7</td>
<td>22.3</td>
<td>+71</td>
<td>+6</td>
</tr>
<tr>
<td>P5</td>
<td>1.46</td>
<td>21.9</td>
<td>22.7</td>
<td>+90</td>
<td>+5</td>
</tr>
<tr>
<td>P6</td>
<td>1.77</td>
<td>33.3</td>
<td>35.7</td>
<td>+189</td>
<td>+27</td>
</tr>
</tbody>
</table>

The initiation work of fracture was defined at the maximum value of the load, while the propagation work of fracture was defined as the average of the energy dissipated per unit area once the R-curve had levelled out and was fairly constant (steady-state). For the best performing pattern design tested (pattern 6), an increase of 189% in the propagation work of fracture, and of 27% in the maximum tensile load have been measured.

4.3.2 Indentation test results

Fig. 4.5 shows a visual comparison of the damage areas created during the indentation test for the baseline material and for a representative specimen with the engineered microstructure. In the top part of the image, a schematic representation of the specimen is shown along with an x-ray image of the damage area. In the bottom part of the image, pictures of both faces of the tested specimens are shown. It is immediately clear that the presence of the patterns of micro-cut changed the damage morphology, causing the formation of four large translaminar cracks in the specimen with the engineered microstructure, as opposite to the three translaminar cracks in the specimen with the baseline material. The x-ray images show a slightly smaller damage area for the specimen with the baseline material, but there is not a significant different in the extent of delaminations for the two specimens.

Load vs. indenter displacement and energy dissipated vs. indenter displacement for the three specimens are shown in Fig. 4.6. In Fig. 4.6(a), the specimen with the baseline material shows
an unstable damage process: it is possible to identify three main load drops, followed by a rapid degradation of the stiffness of the specimen. On the contrary, the specimens with the engineered microstructure exhibit a stable failure process and a gradual increase in compliance. As a result, the specimens with the engineered microstructure have a significantly higher capacity to dissipate energy during the test, as shown in Fig. 4.6(b).

4.4 Discussion

4.4.1 Crack deflection and translaminar work of fracture

Fig. 4.3(b), (d) and (g) (which correspond to patterns 1, 3 and 6) show the presence of perfectly formed structures of pull-outs in the 0° plies of the specimens. These pull-outs in the 0° plies formed because the crack, during propagation, followed precisely the path defined by the patterns of micro-cuts in those plies. On the contrary, Fig. 4.3(e) and (f) (which correspond to patterns 4 and 5) show structures of pull-outs in the 0° plies which are only partially formed.

The latter is the result of the following process: in patterns 4 and 5, the crack initially followed the patterns of micro-cuts in the 0° plies and was deflected from its original propagation plane (i.e. along the initial laser notch). Once the deflected crack reached a distance of about 550 μm (on average) from the initial propagation plane, it stopped following the patterns of micro-cuts (up to the designed maximum height of the pattern), and jumped-back to the initial propagation plane, causing a split in the 0° ply along the fibre direction (Fig. 4.3(e) and (f)). The incomplete pull-out formation for patterns 4 and 5 resulted in a mechanical response during the CT test which was very similar to that of pattern 3 (with complete pull-out formation up to 480 μm), as shown in Fig. 4.4(c) and (d).

This behaviour is quantitatively consistent with the predictions of the Finite Fracture Mechanics (FFM) model shown in Appendix C.1, which predicts that the maximum crack deflection height achievable in the 0° plies for the current material system and ply thickness is about 500 μm. Since the formation of complete bundle pull-outs in the 0° plies is critical for a successfully engineered fracture behaviour with increased work of fracture of the laminate, the FFM model (Section 3.3) is clearly an important and useful tool in the design of microstructures for increasing fracture
toughness.

4.4.2 Interaction of failure mechanisms in neighbouring plies with different fibre orientation

From the experimental and modelling results in Chapter 3, it is known that the interaction of failure mechanisms adds an important contribution to the total translaminar work of fracture of the laminate. From the fracture surfaces of the specimens containing patterns 3 to 6 (Fig. 4.3(d) to (g)), it is possible to notice that the crack deflection in the load bearing plies (0° and ±45°) interacted with the damage formation in the 90° plies (which did not have any pattern of micro-cuts), causing the formation of large splits and fibre failure, thus increasing energy dissipation.

From the fracture surfaces of the specimens containing patterns 1 to 3 (Fig. 4.3(b), (c) and (d)), it is possible to investigate the interaction of failure mechanisms in the 0° and ±45° plies. The patterns of micro-cuts in the ±45° and in the 0° plies appear to act independently and form the same pull-out structures both when they are acting singularly (patterns 1 and 2), and in combination (pattern 3). Accordingly, they do not exhibit a positive interaction behaviour in relation to the mechanical properties. Both the work of fracture and the maximum tensile load under CT test of pattern 3 (which contains patterns in the 0° and ±45° plies) were lower than those of pattern 1 (with micro-cuts only in the 0° plies) and relatively similar to pattern 2 (with micro-cuts only in the ±45° plies), as shown in Fig. 4.4(a) and (b).

From the visual observation of fracture surfaces of the specimen containing pattern 6 (Fig. 4.3(g)), it is difficult to draw meaningful conclusions regarding the interaction of the patterns of micro-cuts in the 0° and ±45°. However, pattern 6 achieved significantly better performances in the CT test when compared with pattern 3 during the CT test (Fig. 4.4(e) and (f)), despite the fact that both patterns have the same micro-cuts design in the 0° plies (Fig. 4.4(e) and (f)). This could be due to a positive interaction of failure modes between the 0° and ±45° plies for this particular pattern.
4.4.3 Indentation test

The presence of the pattern of micro-cuts changes the mechanical response of the indentation specimens and their damage morphology. In the baseline specimen, there is a sharp drop after the peak load, and the specimen fails with three main crack fronts. In the specimens with the engineered microstructure, the failure process is more stable, without sharp load-drops, and the specimens formed four main crack fronts (clearly related to the pattern of micro-cuts) during the failure process.

The specimens with engineered microstructure also showed an increase in energy dissipation during the test, and failed with a significantly larger displacement to failure. The x-ray images do not show a significant difference in delamination between the two types of specimens, therefore it is reasonable to conclude that the increase in the energy dissipated during failure in the specimens with engineered microstructure was due both to the increased translaminar work of fracture produced by the patterns, and to the increased number of translaminar cracks induced by the design.

4.5 Conclusions

These are the main outcomes which emerge from the present study:

- carefully designed patterns of micro-cuts inserted in the microstructure of QI laminates are effective in increasing the resistance of the laminate during translaminar crack propagation. An increase of 27% in the maximum tensile load under CT test, and of 189% in the laminate work of fracture have been achieved experimentally for the best performing pattern design tested during this study;

- similarly, carefully-designed patterns of micro-cuts are effective in improving the damage resistance of QI laminates subject to indentation test. An increase of 40% in the maximum deflection to final failure, and of 43% in the total energy dissipated have been achieved experimentally;
In both tests, the design of the pattern of micro-cuts was guided by the results of a previously-developed Finite Fracture Mechanics model (Section 3.3). The success of these tests proves that this model is a valid tool to aid microstructure design.

In conclusion, the developed microstructure design concept, which uses patterns of micro-cuts to control crack formation, orientation and propagation, can be successfully used to increase damage resistance of Carbon Fibre laminates, in particular for thin-ply laminates which have an inherently low translaminar work of fracture. Since this was just a first attempt to apply microstructure design to QI laminates, it is reasonable to assume that this technique has the potential to increase the damage resistance of carbon fibre laminates even further.
Figure 4.1: Experiments design: on the left hand side it is possible to see the set-up of the loading fixtures, while on the right hand side the specimen’s dimensions are shown. (a) CT test design; (b) Quasi-Static Indentation test design.
Figure 4.2: Schematic representation of the 6 patterns of micro-cuts used for the CT test. Each specimen had a QI layup ([45, 0, −45, 90]_{22s}) with micro-cuts included in the 0° and ±45°; while the 90° plies did not contain any micro-cut. The patterns in each ply are represented according to the following colour code: 0° ply-cuts in red, +45° in blue, −45° in green.
Baseline (no cuts)

Pattern 1 (cuts in the 0° plies only)
Pattern 2
(cuts in the ±45° plies only)

Pattern 3
(cuts in the 0° and ±45° plies)
Pattern 4
(cuts in the 0° and ±45° plies)

Pattern 5
(cuts in the 0° and ±45° plies)
Figure 4.3: SEM micrographs of the fracture surfaces of the seven CT specimens tested. All specimens have been manufactured from a thin-ply (20 μm) CFRP prepreg and have a QI lay-up ([45, 0, −45, 90]_{2s}).
Figure 4.4: Effect of the different patterns on the load vs. displacement and work of fracture vs. displacement recorded during the CT tests
Figure 4.5: Comparison of the damage morphology for the indentation test specimens using visual examination and x-ray analysis.

Figure 4.6: Effect of the engineered microstructure on the quasi-static indentation response of CFRP laminates.
Chapter 5

On the roles of dynamic stress concentrations and fracture mechanics in the longitudinal tensile failure of fibre-reinforced composites

5.1 Introduction

The strength and stiffness of Fibre Reinforced Plastic (FRP) laminates is controlled, to a great extent, by the fibres in the load-aligned plies, thus fibre-dominated tensile failures (also known as translaminar failures) can lead to a significant drop in local stiffness, and can trigger catastrophic failure of an entire composite structure. Consequently, being able to accurately characterize and predict the longitudinal tensile strength of FRP plies is of great importance.

The longitudinal tensile strength of UD composites is characterized by strong size effects connected to both the length of the specimens and the total number of fibres [61–64]. Although manufacturing and testing artefacts may influence the results and interfere with measured size effects, most researchers agree that size effects in composites are due to the intrinsic properties and failure mechanisms of the material [61].
Figure 5.1: Size effects in the longitudinal tensile strength of UD carbon-epoxy composites. (a) Size effect in the strength of composite bundles of constant length but different filament counts [3]. (b) Size effect in the strength of scaled UD composites measured in tapered specimens especially designed to fail in the gauge section and avoid gripping effects [6].

The longitudinal tensile failure of FRPs is governed by the formation of clusters of broken fibres which, once reaching a critical size, can evolve catastrophically. Since the nucleation of these clusters is usually triggered by the presence of defects in the fibres (weak fibres), a larger composite structure (with more and/or longer fibres than a smaller one) will be more likely to have more defects, which will make the formation of a critical cluster of broken fibres more likely. Fig. 5.1 shows two examples of size effects on the longitudinal tensile failure of UD carbon-epoxy specimens [3, 6].

Size effects pose a challenge for the design of large composite structures based on experimental data measured from small coupons, and quantifying them through predictive models has been the subject of a recent blind benchmark exercise [65] where most models failed to predict size effects and formation of clusters of broken fibres accurately.

Predicting the longitudinal tensile strength of FRP bundles accurately at the macro-scale requires the ability to take into account at least these three micro-mechanical effects: the stochastic variability of the single fibre strength; the stress recovery inside the broken fibres due to the shear stress in the matrix and the stress redistribution on the intact fibres; and the damage evolution during the failure process. To this end, several Fibre Bundle Models (FBMs) have been developed in the literature.
FBMs typically consider a parallel array of fibres with stochastic strength, loaded remotely under tension (bundle stress $\sigma_\infty$ and strain $\varepsilon_\infty$) [3, 8, 42, 50, 51, 66–76]. Once the weakest fibre fails, this generates a stress concentration in the neighbouring intact fibres, potentially leading to their failure. Remote tensile stresses or strains are progressively increased until all fibres are broken, or until the composite cannot withstand further load increments. In general, the ultimate strength of the bundle is a stochastic variable which has to be characterized statistically.

The problem of calculating the probability distributions of bundle strength can be approached analytically, or through Monte Carlo simulations. Analytical FBMs are typically classified depending on the load sharing in the neighbourhood of a fibre break: (i) Global (or Equal) Load Sharing (GLS or ELS) consider the same stress concentration on all non-broken fibres [50, 51, 66]; (ii) Local Load Sharing (LLS) assume that the closest neighbours to the broken fibre undergo higher stress concentrations than the more distant ones [67–69]. Pimenta and Pinho [42] recently proposed an analytical hierarchical scaling law for the strength of composite fibre bundles which has been extensively validated against experimental results [65], and predicts full strength distributions of bundles with millions of fibres in less than 1 second.

Monte Carlo FBMs proposed in the literature can be broadly classified into two categories depending on the method used to calculate the stress field around broken fibres [77]:

- Finite Element (FE) methods, which use full-field FE solvers [8, 68–73] or simplified spring-based models [3, 74, 75, 78] to calculate the stress field in the bundle. Full-field FE models can be further subdivided in single-scale models, which simulate the stress field in the entire bundle at each step of the Monte-Carlo simulations process [8, 68–70]; and two-scales models, which use FE to calculate the deterministic response of Unit Cells (UCs) with different numbers of fibre breaks, and then use those responses in Monte-Carlo simulations [71–73]. A main drawback with FE Monte-Carlo simulations is computational efficiency, as a very fine mesh of fibre elements and a large number of simulations are required to achieve representative results.

- Combined field-superposition methods, which calculate deterministic stress fields near single-fibre breaks at a first stage, and then use a superposition method to include those fields in the failure simulations of fibre bundles with multiple breaks. The literature on this
type of simulation is extensive, and most of the earlier models [79–83] considered analytical solutions for stress fields. More recently, Swolfs et al. [76] used FE simulations of UD composites (with realistic fibre packings) to calibrate an analytical stress redistribution function. However most of these models struggle to capture how stress concentrations and recovery lengths are affected by interacting clusters of broken fibres [8].

Despite attempting to simulate the micro-mechanical evolution of damage during the composite tensile failure, most Monte Carlo FMBs in the literature tend to overestimate the final bundle strength when compared with experimental data, and to underestimate the decrease in strength with the increasing number of fibres in the bundle (size effect) [3, 74–76].

Bazant [84] showed that final failure of a composite structure is governed by the composite strength for small-scale components, and by the composite fracture toughness for large-scale components. Pimenta et al and Henry et al [85, 86] applied this concept to predict tensile failure of aligned discontinuous composites using a non-linear fracture mechanics criterion, which combines strength- and toughness-dominated failure modes. However, most FBM s which use high-fidelity representation of the failure process only consider fibre stress overload (i.e strength of materials approach) as the bundle failure criterion, and do not include fracture mechanics based failure criteria for the growth of larger clusters of broken fibres. This is an important observation as there is growing experimental evidence which suggests that unstable failure of a carbon fibre/polymer matrix bundle occurs when a cluster of approximately 14 or more broken
fibres is formed [7,87] (Fig. 5.2).

Furthermore, fibre failure is a dynamic process, resulting in a change in the stress field over time, before it finally dampens out to the static level. Dynamic stress concentrations can be significantly higher than static ones, as shown by modelling results [67,88–91]. Nevertheless, this effect is ignored and only static equilibrium stress states are considered in all state-of-arts FBMs [9].

This chapter aims to investigate the role of dynamic stress concentrations, and of fracture mechanics-driven growth of clusters of broken fibres, in the longitudinal tensile failure. For this, an efficient Monte Carlo FBM with a semi-analytical field superposition method to calculate stress concentrations around clusters of broken fibres was developed. The stress redistribution for single broken fibres and clusters has been validated against analytical and FE results from the literature [8]; the method is also able to capture analytically the dependency of the stress recovery length on the cluster size.

The Monte Carlo simulation process was optimized using statistical analysis to allow the direct simulations of large bundle sizes. Using this technique, it was possible to explore the size effects for large composite bundles by direct simulation, without relying on analytical extrapolation of the simulated results. Finally, this allowed us to investigate for the first time the effects of dynamic stress concentration and toughness dominated failure on the composite strength distribution and on the related size effects for large composite bundles.

This chapter is organised as follows. Section 5.2 explains the baseline model and its variants (including the model geometry, the steps involved in the simulation of the damage and how the data is post-processed), while the algorithms required for the numerical implementation are presented in Section 5.2.5. Section 5.3 contains an overview of the numerical results and a comparison between experimental and predicted strength distributions both for micro and macro-bundles. Finally, Section 5.4 draws the main conclusions.
5.2 Model development

5.2.1 Introduction

This section describes in detail the development of the baseline version of the model (referred to as model BA) which considers static equilibrium stress states, and uses strength of materials as the only failure criterion. Two model variants including dynamic effects and fracture mechanics effects are presented in Sections 5.2.3 and 5.2.4 respectively. To achieve a reliable comparison of the results, all the models share the same common structure described once for model BA in the next section.

5.2.2 Baseline model

Model BA simulates the failure of a bundle of parallel fibres under longitudinal tensile loading and provides the statistical strength distribution for the bundle. The simulation strategy can be broken down into three main components:

- Model definition: the numerical, geometrical and mechanical properties are defined, and a stochastic strength distribution is assigned to the fibres. This is described in Section 5.2.2.1.

- Failure simulation: an asymptotic stress \( \sigma_\infty(t^k) \) which is a function of the time variable \( t^k \) is applied to the bundle to drive the failure process. When a fibre element fails as a consequence of this load, the broken fibre is unable to carry the asymptotic stress, which needs to be redistributed on the surrounding fibres. The stress redistribution is computed in three steps: (i) calculation of the shear-lag stress along the broken fibre, (ii) redistribution of the load on the surviving intact fibres, and (iii) verifying whether new fibre elements break due to stress overload and updating \( \sigma_\infty(t^k) \) to advance the simulation. Steps (i), (ii) and (iii) are discussed in Sections 5.2.2.2, 5.2.2.3 and 5.2.2.4, respectively. This procedure is repeated until final failure of the bundle (the definition of the bundle failure changes depending on the failure criterion applied, as described in the following sections).
Post-processing: at the end of the failure simulation, the final value of the bundle strength $X_b$ is obtained, being equal to the maximum of $\sigma_{\infty}(t^k)$ during the simulation. Then, the process is repeated over different realizations of the initial stochastic assignment of the fibre strength in a Monte Carlo simulation, and the parameters of the bundle strength distribution are extracted. This simulation is optimized as described in Section 5.2.2.5 to achieve computational efficiency while ensuring the reliability and validity of the output.

5.2.2.1 Bundle geometry and fibre strength

Fig. 5.3 shows a bundle of fibres of length $l_{\text{sim}}$ and diameter $\phi_l$, with fibre volume fraction $V_f$ and inter-fibre spacing $s$ in the $x$ and $y$ direction. The current version of the model assumes that fibres are packed in a square arrangement, as it has been established that the differences due to fibre arrangements are remarkably small [82,92].

Each fibre in the bundle is subdivided into smaller fibre elements of size $l_{el}$, therefore creating $j$ cross sections with $j = 1, ..., n_{sec}$ in the bundle along the direction $z$, each one containing fibre elements $i$ with $i = 1, ..., n_f$. The indices $i,j$ therefore define each of the $n_{el} = n_f \times n_{sec}$ fibre elements in the bundle. The bundle is loaded in tension by the asymptotic stress $\sigma_{\infty}(t^k)$ applied to the fibres extremities.

To model the stochastic variability of the fibre strength, a strength value $X_{ij}$ is assigned to each fibre element following a Weibull distribution [93]

$$F_{el}(X_{ij}) = 1 - \exp\left(\frac{X_{ij}}{X_{l_{el}0}}\right)^m,$$

where $X_{l_{el}0}$ is the scale parameter of the strength distribution for a fibre element of length $l_{el}$, and can be determined using the Weakest Link Theory (WLT) [77,94]:

$$X_{l_{el}0} = X_{l_{r}0} \left(\frac{t_r}{l_{el}}\right)^{1/m},$$

where $X_{l_{r}0}$ and $m$ are the scale and shape parameters respectively for a single fibre with reference length $l_r$. The Weibull distribution is widely adopted in the literature [8,19,42,74–76,95–99], considering the generally good correlation with single fibre tests [4,5]. However, other authors
have proposed alternative distributions, e.g. Bimodal Weibull or Weibull of Weibull distributions [3, 100–102].

![Diagram of model geometry](image)

Figure 5.3: Description of the model geometry.

### 5.2.2.2 Shear-lag stress and recovery length

At each step $t^k$ of the simulation, it is possible to define the set $N_{fa}$ of failed elements in the bundle. For each fibre $i$ with at least one failed element $(ij)_{fa}$, the longitudinal stress in the failed element goes to zero ($\sigma^{(ij)_{fa}} = 0$), but it is recovered in the rest of the fibre due to the shear stress transmitted by the matrix via a shear-lag mechanism [48, 49, 51, 103, 104].

Assuming that the axial load is only carried by the fibres, and the matrix is loaded in shear to the yielding stress $\tau_{sl}$ (perfectly plastic behaviour), the shear-lag stress limit $\sigma^{(ij)(i)_{fa}}_{sl}$ (maximum stress level allowed by the shear-lag stress recovery) for each element $(ij)$ in fibre $i$ due to the failed element $(ij)_{fa}$ can be calculated applying force equilibrium:

$$\sigma^{(ij)(i)_{fa}}_{sl} = \sum_{i(j-(j)_{fa})} C^{ij}_{sl} \cdot \frac{\tau_{sl}}{A_f} \cdot l_{el},$$

(5.3)

where $C^{ij}_{sl}$ is the shear-lag boundary, which is the contour over which the shear stress is transmitted for each fibre element, and which might not be constant along the length of fibre $i$, as explained below.

In the case of two or more failed elements in the same fibre, the shear-lag stress limit for each

---

1 For the brevity and clarity of notation, the dependency from the time variable $t^k$ will be omitted through the rest of this sections.
element in the fibre is the smallest one of those relative to the different breaks. Hence,

\[
\sigma_{sl}^{ij} = \begin{cases} 
\infty & \text{if } \not\exists (ij)_{fa} \in \text{fibre } i; \\
\min_{(ij)_{fa}} \left( \sigma_{sl}^{(ij)(ij)_{fa}} \right) & \text{if } \exists (ij)_{fa} \in \text{fibre } i.
\end{cases}
\] (5.4)

The portion of the broken fibre \(i\) where \(\sigma_{sl}^{ij} \leq \sigma_\infty\) is the recovery length \(l_{rl}^{(ij)_{fa}}\) associated with the failed fibre element \((ij)_{fa}\). The elements \((ij)_{st}\) within a recovery length are considered to be saturated elements, since they reached the shear-lag limit.

When the recovery lengths associated with two or more failed elements belonging to neighbouring fibres overlap, the load transmission among the fibres is impeded, and consequently then the elements inside this region are considered to be part of the same cluster. This consideration modifies the shear-lag boundary \(C_{sl}^{ij}\) for the involved fibres, thus varying the shear-lag stress and the recovery lengths (this effect is presented in Fig. 5.4). Therefore, the calculation of the shear-lag profile needs to be performed iteratively. Both the calculation of the shear-lag boundaries \(C_{sl}^{ij}\) and the iterative procedure are described in Appendix D.1.

### 5.2.2.3 Stress redistribution

Once the shear-lag stress limit is defined for each fibre, the final stress state in the bundle is computed by redistributing the loss of stress \((\sigma_\infty - \sigma_{sl}^{(ij)_{st}})\) from the broken fibres to the remaining intact fibres. At each bundle cross-section \(j\), it is possible to define the set \(N_{st}^j\) of the saturated elements \((ij)_{st}\) that reached the shear-lag limit \((\sigma_{sl}^{(ij)_{st}} \leq \sigma_\infty)\), and the set \(N_{in}^j\) of the intact elements \((ij)_{in}\) (for which \(\sigma_{sl}^{(ij)_{in}} > \sigma_\infty\)).

An analytical power law is used to efficiently compute the stress redistribution [8]. The additional stress \(\Delta\sigma^{(ij)_{st}(ij)_{st}}\) redistributed on the intact element \((ij)_{in} \in N_{in}^j\) as a result of the stress loss on element \((ij)_{st} \in N_{st}^j\) has the following expression:

\[
\Delta\sigma^{(ij)_{st}(ij)_{st}} = \Phi^{(ij)_{st}} \cdot \left( \frac{r^{(ij)_{st}(ij)_{st}}}{s} \right)^{-\gamma},
\] (5.5)

where \(r^{(ij)_{st}(ij)_{st}}\) is the distance between fibre elements in the cross section and is normalized by
Figure 5.4: Shear-lag mechanism: cluster size as a function of the position along the $z$ direction and the available neighbours to transmit the load (indicated by red arrows); and shear-lag stress profiles for intact (fibres 1 and 4) and broken fibres (fibres 3 and 4) for a discretization with $n_{el} = 40$ elements.
fibre spacing \( s \) (hereafter indicated with \( r^{(ij)}_{m} \)), and \( \gamma \) is the parameter which controls the shape of the stress redistribution function. The variable \( \Phi^{(ij)}_{m} \) is calculated by imposing force equilibrium to the bundle cross-section:

\[
\sigma_{\infty} - \sigma^{(ij)}_{sl} = \sum_{N^{ij}_{m}}^{N^{ij}_{m}} \Phi^{(ij)}_{m} \cdot \left( F^{(ij)}_{m} \right) \gamma \Rightarrow \Phi^{(ij)}_{m} = \frac{\sigma_{\infty} - \sigma^{(ij)}_{sl}}{F^{(ij)}_{m} \cdot (F^{(ij)}_{m})^{\gamma}}.
\] (5.6)

The final stress field at each bundle cross-section is computed applying the principle of superposition of effects to the stress concentration generated by each saturated element:

\[
\sigma^{ij} = \begin{cases} 
\sigma^{ij}_{sl} & \text{if } ij \in N^{ij}_{sl}, \\
\sigma_{\infty} + \sum_{N^{ij}_{m}}^{N^{ij}_{m}} \Delta \sigma^{(ij)}_{m} \text{if } ij \in N^{ij}_{m}. 
\end{cases}
\] (5.7)

As a consequence of the stress concentration, it may happen that new elements in the section reach their shear-lag limit \( (\sigma_{\infty} + \sum_{N^{ij}_{m}}^{N^{ij}_{m}} \Delta \sigma^{(ij)}_{m} > \sigma^{ij}_{sl}) \), and then the excess stress has to be further redistributed. This is achieved by re-calculating the stress field using Eqs. (5.5) to (5.7) iteratively, but starting each iteration with the previous field \( \sigma^{ij} \) instead of \( \sigma_{\infty} \).

The stress field for fibres 1-4 of Fig. 5.4 is displayed in Fig. 5.5, using \( \gamma = 2 \). The appropriate value for this parameter was determined via a comparison with data of stress concentrations in bundles with clusters of broken fibres generated via FE simulations from St-Pierre et al [8] (Fig. 5.6). The value of \( \gamma = 2 \) is shown to capture very well the stress concentration factor for clusters of various sizes and is used in all the simulations throughout the document.

### 5.2.2.4 Bundle failure simulation process

The bundle is loaded by imposing \( \sigma_{\infty}(t^k) \) until final failure. As initial condition to start the simulation \( (t^k = 0) \), the bundle is loaded to \( \sigma_{\infty}(t^k = 0) = \min(X^{ij}) \) to break the weakest fibre element. The stress dropped by the broken elements is then redistributed over the intact elements following the procedure described in Sections 5.2.2.2 and 5.2.2.3, and the new stress state in the bundle is calculated while keeping the value of the asymptotic stress constant. At
this point, a reserve factor is calculated for each fibre element in the bundle:

\[
R_{ij}^{(t_k)} = \frac{X_{ij}}{\sigma_{ij}^{(t_k)}}. \quad (5.8)
\]

Two different scenarios are possible depending on whether all elements are able to withstand the current stress level \( R^{ij}(t^k) > 1 \quad \forall \ (ij) \), or whether some elements are standing a stress level over their assigned strength \( \exists \ (ij) : \ R^{ij}(t^k) \leq 1 \). In the first scenario, the asymptotic stress is increased to break a new element in the bundle:

\[
\sigma_{\infty}(t^{k+1}) = R_{\text{min}}(t^k) \cdot \sigma_{\infty}(t^k), \quad (5.9)
\]

where \( R_{\text{min}}(t^k) = \min \{ R^{ij}(t^k) \} \) is the minimum reserve factor of all the elements in the bundle.

In the second scenario, one or more fibre elements are due to fail under the current stress state.
Figure 5.6: Distribution of the stress concentration factor $k$ in the cross-section of a bundle of $n_f = 900$ fibres with a cluster of broken fibres in the centre. Geometry of the bundle and material properties are defined in Table 1 of St-Pierre et al [8]. (a) $k$ as a function of the normalized distance $\bar{r}$ from an individual broken fibre calculated by the current model for different values of the parameter $\gamma$, and the corresponding results from [8]. (b) $k$ as a function of the normalized distance $\bar{r}$ to the centre of a cluster of size $n_b$ calculated by the current model with $\gamma = 2$ validated against FE results from [8].

Two different approaches have been considered: the first is (i) a single breaking approach, where the stress level is updated according to Eq. (5.9) thus failing only the element with the lowest reserve factor. Fig. 5.7(a) shows the variation of $\sigma_\infty(t^k)$ during the damage simulation for a square bundle with 16 fibres. The plain blue regions indicate the steps with fibre elements failing under the stress concentration; in these cases, the applied stress decreases from step $t^k$ to step $t^{k+1}$.

The alternative is (ii) a multiple breaking approach where all elements which have $R_{ij} \leq 1$ are failed in the same time step, while the applied stress is maintained constant $\sigma_\infty(t^{k+1}) = \sigma_\infty(t^k)$. The stress is redistributed again in the subsequent step of the simulation, potentially causing more breaks. The process is repeated until all elements in the bundle can withstand the current level of stress, then the asymptotic stress is raised again following Eq. (5.9), or the final failure of the bundle is reached (Fig. 5.7(b)).

In Fig. 5.7 the values of strength predicted by both models coincide because the scenario where the single and multiple breaking approaches take place appears after reaching the maximum. However, this is not always the case. No strong evidence was found to decide a priori which approach is the most suitable between (i) and (ii). The single breaking approach appears to
be the most accurate because it can account for the fact that, in reality, no two elements fail exactly at the same time, and each failed element changes the state of stress in the bundle. The multiple breaks approach is probably less accurate but is widely used for other statistical strength models in the literature [3, 8, 19, 42, 71, 72, 74–76, 83, 95–99], and allows for significantly faster simulations given that the stress state does not have to be re-computed for each failed element. Both approaches are considered in this work, and the differences they introduce in the simulations will be discussed in Section 5.3.1.

5.2.2.5 Monte Carlo stopping criterion

The selection of an appropriate number of Monte Carlo simulations is critical to ensure the accuracy of the model predictions. Most models in the literature use a fixed number of simulations for all bundles sizes, despite the fact that both experiments and models show a decrease in the bundle strength variability when increasing the number of fibres in the bundle [4, 5, 42]. In this work, a bundle-size variable number of simulations was implemented as a way to increase computational efficiency, while keeping the accuracy of the results constant across all bundle sizes.

Given a sample with $N$ simulations, and following the Central Limit Theorem, it is possible to
calculate the confidence interval at 95% as

\[ w_{IC}^{95\%} = \left[ \bar{X}_b - t(N - 1, 0.025) \cdot \frac{SD_b}{\sqrt{N}}, \bar{X}_b + t(N - 1, 0.025) \cdot \frac{SD_b}{\sqrt{N}} \right], \tag{5.10} \]

where \( t(N - 1, 0.025) \) is the value of the Student’s-t distribution with \( N - 1 \) degrees of freedom for a cumulative probability of 97.5%. The mean and standard deviation \( SD_b \) of the sample are used as estimators of the equivalent normal distribution parameters.

Fig. 5.8 shows the flowchart of the implementation of this criterion in the model. The number of Monte Carlo simulations performed is the minimum that assures that the width of this interval predicts the mean strength with a maximum accepted error \( E_{\bar{X}} \). A value of \( E_{\bar{X}} = \pm 1\% \) is used for all the simulations in the document (see Appendix D.3 for more details on the choice of the maximum accepted error). Additionally, a minimum number of Monte Carlo simulations \( N_{\min} = 12 \) is set to assure that the initial estimation of the mean and standard deviation are statistically meaningful.

### 5.2.3 Dynamic effects model

In this section, the first model variant (hereafter designated as model DE) is implemented to investigate the effects of dynamic stress concentrations on the bundle failure process and final strength.

The baseline model, as almost the totality of the Fibre Bundle models in the literature, only considers the static equilibrium stress field when simulating damage. In reality, when a fibre
fails, the stored elastic energy is released in form of a dynamic stress wave. This wave propagates throughout the intact fibres during a short time interval of duration $\Delta t_{\text{dyn}}^k$ (hereafter transient interval), causing a dynamic stress concentration, and then is dampened by the material and the stress field reverts to the static equilibrium one (Fig. 5.9).

When incorporating dynamic effects, the model definition and post-processing are the same used for the baseline model as described in Sections 5.2.2.1 and 5.2.2.5; whereas the failure simulation procedure is modified. The damage in the bundle can progress either through (i) fibre failure due to the dynamic stress concentration during the transient interval $\Delta t_{\text{dyn}}^k$, or (ii) fibre failure due the rise of the (quasi-static) asymptotic stress $\sigma_\infty(t^k)$. Since the characteristic time of dampening for the dynamic effects (transient interval) is much shorter that the time-scale required for varying the asymptotic stress, it is assumed that dynamic failure occurs under constant remote stress.

In the first time scale (dynamic time scale), associated with the damage created during the dynamic wave propagation, the dynamic stress field is computed as described in Section 5.2.3.1 with the damage simulation approach described in Section 5.2.3.2. In the second time scale (static time scale), the dynamic effect have already disappeared and the stress field reverts to the static solution described in Section 5.2.2.3.

### 5.2.3.1 Dynamic stress field

Fig. 5.9 shows schematically the stress evolution in a fibre during the transient interval [9]. This model does not seek to simulate the stress evolution during the entire transient interval. Instead, an upper boundary of the dynamic stress will be estimated, and used for our model. In this way, comparing our static and dynamic predictions will reveal an upper-bound for the role of dynamic effects.

Considering that dynamic effects act by increasing the stress concentration on the intact fibres during the transient interval, it is assumed that the dynamic stress field can be computed from
the static one \((\sigma^{ij})\) as

\[
\sigma^{ij} = \begin{cases} 
\sigma_{st}^{ij} & \text{if } ij \in N_{st}^j, \\
\sigma_\infty + \lambda_{\text{dyn}} \cdot \sum_{N_{st}}^j \Delta \sigma^{(ij)_{\text{in}}(ij)_{\text{at}}} & \text{if } ij \in N_{\text{in}}^j.
\end{cases}
\] (5.11)

where \(\lambda_{\text{dyn}}\) is the dynamic magnification factor.

Dynamic stress concentrations have been reported to range between 160\% and 200\% of the corresponding static ones depending on the material properties and fibre packing [9, 67, 88–91]. The theoretical maximum dynamic magnification factor for a spring-mass system without damping subject to a step load is \(\lambda_{\text{dyn}} = 2\). This value will be used for all fibre elements in this work, so as to obtain an upper-bound for the role of dynamic effects.

### 5.2.3.2 Damage simulation process

In Model DE, the simulation is also initiated by making \(\sigma_\infty(t^0 = 0) = \min \left(X^{ij}\right)\). The static stress concentration is computed using the same procedure as described in the baseline model; then, the dynamic stress field is calculated with Eq. (5.11).

If the dynamic stress concentration causes some elements in the bundle to stand a stress over their assigned strength threshold, the first element in the wave path (closest element to the previous point of failure) is the first one to fail, and it releases another dynamic wave which, in turn, can cause more element failures. Since each fibre failure causes a variation of the dynamic
stress levels, a single breaking approach is adopted and the dynamic stress field is re-calculated after each step. During this process, which happens in the dynamic time scale, the asymptotic stress remains constant:

\[ \sigma_\infty(t^{k+1}) = \sigma_\infty(t^k). \]  

When the dynamic stress concentration does not cause any new fibre element failure, the algorithm reverts to the static time scale, the stress state reverts to the static one and the reserve factor is calculated (Eqs.(5.8) and (5.9)). The simulation continues by updating the asymptotic stress to break a new element:

\[ \sigma_\infty(t^{k+1}) = R_{\text{min}}(t^k) \cdot \sigma_\infty(t^k). \]  

### 5.2.4 Fracture mechanics model

During the bundle failure process, clusters of broken fibres may form due to initial fibre failures driven by strength-of-materials, and may start acting as cracks in the material (Fig. 5.2). If the energy release rate associated with these clusters/cracks is higher than the corresponding fracture toughness of the material, they can trigger catastrophic failure. In this section, a model that accounts for fracture mechanics driven failure from clusters of broken fibres is proposed, and will be referred to as model FM hereafter.

Again, the structure of model FM is quite similar to the baseline model: the calculation of the shear-lag stresses, the stress redistribution and the damage simulation process are carried out in the same fashion, accordingly to Sections 5.2.2.2, 5.2.2.3 and 5.2.2.4 respectively. However, the size of each equivalent crack is monitored during the failure simulation, introducing an additional step at each iteration. Hence, if any cluster is greater than the critical cluster size (which is determined using the fracture mechanics criterion described in Section 5.2.4.1), it is assumed to cause catastrophic failure of the bundle and \( \sigma_\infty(t^k) \) is taken as the bundle strength (Section 5.2.4.1). If no cluster ever reaches critical conditions, the simulation follows the same procedure as in the baseline model.
5.2.4.1 Critical cluster size

At each step $t^k$ of the simulation\(^2\), for each failed fibre element $(ij)_k$, it is possible to identify a portion of the fibre $i$ where $\sigma_{kl}^i \leq \sigma_{\infty}$: this is the recovery length $l_{rl}^{(i)}$ associated with the failed fibre element $(ij)_k$. When the recovery lengths associated with two or more failed elements belonging to neighbouring fibres overlap (see Fig. 5.4(a)), all the fibre elements inside the recovery lengths for each fibre are considered to be part of the same cluster. Thus, it is possible to define $n_{cl}$ sets $N_{cl}^s$ of fibre elements in the bundle (with $s = 1, ..., n_{cl}$) which correspond to the different clusters. The size of each cluster is defined by the number of fibre involved $n_{sf}^s$.

In each bundle cross-section $j$, it is possible to define different sets $N_{cl}^sj$ of neighbouring fibre elements associated with the cluster $s$ in section $j$. Each set $N_{cl}^sj$ contains $n_{cl}^sj$ elements $(ij)_s$ and can be idealised as an equivalent translaminar crack with equivalent characteristic crack size (Fig. 5.10):

$$a_{cl}^{sj} = \sqrt{\frac{4 \cdot n_{cl}^sj \cdot A_f}{\pi \cdot V_f}}. \quad (5.14)$$

In order to determine the conditions for critical propagation of this equivalent crack, an analogy with the problem of a flat penny-shaped crack in a isotropic cylinder is used (Appendix D.2). Following the analogy, the critical cluster size for the equivalent crack will be inversely proportional to the square of the equivalent stress $\sigma_{eq}$:

$$a_{cl}^{sj} = \frac{\lambda_{lm}}{\sigma_{eq}^{sj}}, \quad (5.15)$$

where $\lambda_{lm}$ is treated as a free parameter of the model.

The fracture toughness failure criterion described above is implemented in the bundle failure simulation as follows. At each step of the simulation, the equivalent cracks (Fig. 5.10(c)) for each cluster of broken fibres are identified at each bundle cross-section and the crack criticality index is calculated

$$I_{cl}^{sj} = \frac{a_{cl}^{sj}}{a_{cl}^{eq}}. \quad (5.16)$$

If $I_{cl}^{sj} \geq 1$ for any equivalent crack, the bundle is considered failed and the remote stress $\sigma_{\infty}(t^k)$

---

\(^2\)For the brevity and clarity of notation, the dependency from the time variable $t^k$ will be omitted through Section 5.2.4.1
at that step is taken as the final bundle strength.

Figure 5.10: Fracture mechanics in the model. (a) Composite bundle loaded in tension with a cluster of broken fibres (highlighted in orange). (b) Detail of the pull-out phenomenon: to create the crack on the equivalent cross section $j$, apart from breaking the fibres it is necessary to pull a certain fibre length (red cylinders) out of the matrix, acting against $\tau_f$. (c) Equivalent crack and definition of the equivalent crack size.

5.2.5 Model flowcharts

In order to analyse all the features described above, six different models were created. The first four correspond to different versions of model BA, implementing different strategies in the damage simulation or for the calculation of the shear-lag stress limit. Models DE and FM are the model variants implementing dynamic effects and fracture mechanics, respectively. The differences between the models are summarized in Table 5.1.

5.3 Results and discussion

This section compares the results provided by all the model variants described in Section 5.2 and validates the predicted strength distributions versus other modelling approaches from the
Start of the failure simulation

- Bundle geometry
  - \( a_0, \Phi, h_0, V_1 \)

- Numerical parameters
  - \( h_0, \tau, \lambda_{dyn}, \lambda_{stat} \)

- Mechanical properties
  - \( x^{(t)}, m, r_i, t_{ij} \)

Define input properties and assign random strength \( X^{(t)} \) to each element with a randomly generated number \( \xi^{(t)} \):

\[
X^{(t)} = X^{(0)} + \frac{E}{h_i} \ln(\xi^{(t)})
\]

Set \( \sigma_{stat}(t^*) = 0 \) to break the weakest element \((i)_{stat}\) in the bundle:

\[
X^{(0)}_{stat} = \min(X^{(t)}) \quad \sigma_{stat}(t^*) = X^{(0)}_{stat} \quad \sigma^{(t)} = \begin{cases} 
X^{(0)}_{stat} & \text{if } (i) \notin N_{stat} \\
0 & \text{if } (i) \in N_{stat}
\end{cases}
\]

For each broken fibre \( i \), calculate the shear-lag boundaries for each element \((j)\) without computing clusters and estimate the shear-lag stress:

\[
C_{ij}^{(t)} = \frac{C_j}{h_i} \quad \sigma_{ij}^{(0)}(t^*) = \sum_{(k)_{stat}} \frac{C_{jk}}{h_j} \frac{r_{ij}}{h_{ij}}
\]

For each cross-section \( j \), find the saturated elements to locate the clusters and update the boundaries of all the \( \sigma_{ij}^{(t)} \) elements of each cluster \( x \):

\[
\delta_{ij}^{(t)} = \begin{cases} 
1 & \text{if } \sigma_{ij}^{(t)} \leq \sigma_{stat}(t^*) \\
0 & \text{if } \sigma_{ij}^{(t)} > \sigma_{stat}(t^*)
\end{cases}
\]

For each broken fibre \( i \), re-calculate the shear-lag boundaries for each element \((j)\) considering clusters and re-calculate the shear-lag stress:

\[
C_{ij}^{(t)} = \begin{cases} 
\frac{C_j}{h_i} \quad \text{if } (i) \notin N^j_{ij} & \sigma_{ij}^{(0)}(t^*) = \sum_{(k)_{stat}} \frac{C_{jk}}{h_j} \frac{r_{ij}}{h_{ij}} \leq \delta_{ij}^{(t)} \\
0 & \text{if } (i) \notin N^j_{ij}
\end{cases}
\]

For each cross-section \( j \), calculate the static stress field and the dynamic stress field, and the corresponding reserve factors for each element \((j)\):

\[
\sigma^{(t)} = \begin{cases} 
\sigma^{(t)} + \sum_{(k)_{stat}} \Delta \sigma^{(0)}(t)_{stat} \Delta h_{stat} & \text{if } (i) \notin N^j \quad R^{(t)} = \frac{X^{(t)}}{\sigma^{(t)}} \\
\sigma^{(t)} + \lambda_{stat} \sum_{(k)_{stat}} \Delta \sigma^{(0)}(t)_{stat} \Delta h_{stat} & \text{if } (i) \notin N^j \quad R^{(t)}_{dyn} = \frac{X^{(t)}}{\sigma^{(t)}}
\end{cases}
\]

Find the element \((i)_{stat}^{(t)}\) with \( R^{(t)}_{stat} < 1 \) closest to the last failed element \((i)_{stat}^{(t)} = (i)_{stat}\) break it and update \( \sigma_{stat}(t^*)\):

\[
\sigma_{stat}^{(t)} = \min(\sigma_{stat}^{(t)} - \sigma_{stat}^{(t)}) \quad \text{if } (i)_{stat}^{(t)} \in N_{stat} \\
\sigma_{stat}^{(t)} = 1 \\
\text{if } (i)_{stat}^{(t)} \notin N_{stat}
\]

Find the element \((i)_{dyn}^{(t)}\) with the lowest \( R^{(t)}_{dyn} \), break it and update \( \sigma_{stat}(t^*)\):

\[
R^{(t)}_{dyn} = \min(R^{(t)}_{dyn}) \\
(\text{if } (i)_{dyn}^{(t)} \in N_{stat}) \\
\sigma_{stat}^{(t)} = R^{(t)}_{dyn} \quad \sigma_{stat}^{(t)} \]

End of the failure simulation

\( X_0 = \max(\sigma_{stat}) \)

Yes

\[ 3. \quad \sigma_{ij}^{(t)} > \sigma_{ij}^{(t)} \text{ } \forall (i) \] ?

No

\[ \text{Is } \sigma_{ij}^{(t)} \leq \sigma_{ij}^{(t)} \text{ } \forall (i) \] ?

Yes

Is any \( R^{(t)}_{dyn} \leq 1 ? \)

No

Yes

(b)
Figure 5.11: FBM models flowcharts. (a) Model BA with three model sub-variants. (b) Model DE. (c) model FM.
Table 5.1: Models developed and algorithm differences between them.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Model family</th>
<th>Damage simulation</th>
<th>σ_{sl} profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA.1</td>
<td>Baseline</td>
<td>Single breaking</td>
<td>Exact</td>
</tr>
<tr>
<td>BA.2</td>
<td>Baseline</td>
<td>Multiple breaking</td>
<td>Exact</td>
</tr>
<tr>
<td>BA.3</td>
<td>Baseline</td>
<td>Multiple breaking</td>
<td>Simplified</td>
</tr>
<tr>
<td>BA.4</td>
<td>Baseline</td>
<td>Multiple breaking</td>
<td>Simplified</td>
</tr>
<tr>
<td>DE</td>
<td>Dynamic effects</td>
<td>Single breaking</td>
<td>Simplified</td>
</tr>
<tr>
<td>FM</td>
<td>Fracture mechanics</td>
<td>Multiple breaking</td>
<td>Simplified</td>
</tr>
</tbody>
</table>

literature, and experimental data. Prior to the comparison, a numerical convergence study was carried out to select the appropriate numerical parameters, as discussed in Appendix D.3.

When presenting results for long bundles, a model with a representative simulation length is firstly simulated and then the results are scaled to the desired bundle length \( l_b \) using WLT (see details in Appendix D.4). This strategy is used to reduce the computational time in the simulations. No scaling is applied to the number of fibres in the bundle, which means that the actually desired number of fibres is simulated directly to obtain all the results.

5.3.1 Baseline model

5.3.1.1 Comparison of baseline models versions

Fig. 5.12 shows the comparison between the three versions of the baseline model. Bundle sizes up to 1600 fibres have been simulated. The nominal input properties for the fibres and resin were obtained from Okabe and Takeda [3] and are listed in Table 5.2. In general, all the model versions predict a constantly increasing bundle strength with the number of fibre in the bundle. Also, the variability decreases substantially with bundle size, thus the selection of a bundle-size variable number of Monte Carlo realizations is proven to be a very effective approach. The number of required simulations to keep the same confidence level drops with the bundle size, allowing for great savings in total computational time.

Fig. 5.12(a) compares the predicted bundle strength for models BA.1 (single breaking approach) and BA.2 (multiple breaking approach). The two solutions are in almost exact agreement, suggesting that both approaches are equivalent with respect to the predicted bundle strength.
However, Fig. 5.12(b) shows that model BA.2 allows a slight improvement in terms of computational efficiency because the number of steps necessary for the simulations reduces as more fibre elements fail at each step. Therefore, this model seems more suitable to simulate large bundle sizes.

Fig. 5.12(c) compares strength predictions for model BA.2 (exact shear-lag stress profile), model BA.3 (simplified shear-lag stress profile) and model BA.4 \( (C_{ij}^{sl} = C_f \text{ for all fibre elements}) \), corresponding to neglecting completely the effects of the clusters of broken fibres on the recovery length. Model BA.3 overestimates the mean and slightly reduces variability with respect to Model BA.2. This result is expected because the simplified shear-lag stress profile may underestimate the recovery lengths, thus reducing the region of influence of the stress concentration. In comparison, the difference between model BA.4 and model BA.2 is more significant (in general, larger than 10%). This means that model BA.3 captures most of the effect of clusters on the recovery length, with a comparatively small error in the strength prediction.

The use of the simplified shear-lag stress in model BA.3 allows a strong improvement of the computational cost (Fig. 5.12(d)), with a reduction of the average time per simulation by more than one order of magnitude in comparison with model BA.2. Given this consideration, and the small error in the prediction of the bundle strength, model BA.3 appears to be the most appropriate for conducting simulations on large bundle sizes, and will be used hereafter as the baseline model.

Table 5.2: Nominal input properties. Geometrical and mechanical properties are obtained from [3] and correspond to UD T800H/3631 composites (Toray).

<table>
<thead>
<tr>
<th>Bundle geometry</th>
<th>Numerical parameters</th>
<th>Mechanical properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_f )</td>
<td>( l_b )</td>
<td>Packing</td>
</tr>
<tr>
<td>[( \mu m )]</td>
<td>[mm]</td>
<td>[-]</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>Square</td>
</tr>
</tbody>
</table>

5.3.1.2 Validation of modelling approach for large bundles

This section compares strength predictions from model BA.3 to those from the 3D shear-lag model developed by Okabe and Takeda [74], which uses a shear and tension springs lattice to
Figure 5.12: Numerical results of the model BA. (a) Predicted strength and coefficient of variation for models BA.1 and BA.2. (b) Computational cost for models BA.1 and BA.2. (c) Predicted strength and coefficient of variation for models BA.2 (exact stress recovery), BA.3 (simplified stress recovery) and BA.4 (no dependency from cluster size) (d) Computational cost for models BA.2 and BA.3.

calculate the stress state in a fibre bundle with broken fibres. The 3D shear-lag model considers arrays of parallel fibres, each subdivided in smaller fibre elements, and uses a Weibull of Weibull (WOW) distribution to assign strength values to the individual fibre elements [3]. The simulation process, besides being strain driven instead of stress driven, is similar to the one used in this work, and Monte Carlo simulations are used to generate statistical strength distributions for the bundle strength. In order to obtain a fair comparison, WOW was implemented in model BA.3 using the same formulation given in Okabe and Takeda [3] as described in Appendix D.5.

Fig. 5.13 compares the strength distributions predicted by model BA.3 and the 3D shear-lag model [74] for two different bundle sizes: \( n_f = 324 \) and \( n_f = 1024 \) (the bundle length is \( l_{\text{sim}} = 0.8 \)
mm in both cases). The input parameters used for model BA.3 are given in Table 5.2, apart for the input properties of the WOW distribution which are the same given in [3] ($X_0^{rl} = 3740$ MPa, $\rho_1 = 5.7$ and $\rho_2 = 5.4$ for $l_r = 50$ mm).

The results of model BA.3 exhibit an excellent correlation with the 3D shear-lag model results; the differences in the average bundle strengths for bundles of 324 and 1024 fibres are $-2.9\%$ and $-2.1\%$, respectively. This good agreement validates the use of the analytical stress redistribution, and the failure algorithm used in this work. However, the analytical stress redistribution proposed is computationally inexpensive in comparison to solving numerically a large system of equations. This should allow the current modelling approach to be used for larger bundles, as will be shown in Section 5.3.4.1.

![Figure 5.13: Comparison between strength distributions generated with model BA.3 and the 3D shear-lag model from Okabe and Takeda [3]. (a) Predicted strength distribution for a bundle with $n_f = 324$. (b) Predicted strength distribution for a bundle with $n_f = 1024$.](image)

### 5.3.2 Dynamic effects

In this section, model DE is compared with the baseline model BA.3. The comparison was performed with $\lambda_{dyn} = 2.0$ for all the elements in the bundle, which corresponds to the maximum theoretical dynamic stress concentration for a step function loading without damping. This case represents an upper-bound for the maximum intensity of the dynamic effects, and is meant to provide a theoretical lower-bound for the strength predictions.

Fig. 5.14(a)-(b) shows the comparison between models DE and BA.3 for the nominal input
properties (Table 5.2). The difference in the predicted bundle strength is around 10%, but the increase in strength with the number of fibres remains. Also, variability is higher in model DE, thus increasing the number of Monte Carlo realizations and the computational time.

Fig. 5.14(c)-(d) compares the maximum cluster size at bundle failure for both limit cases $\lambda_{\text{dyn}} = 2.0$ (maximum dynamic effects) and model BA.3 (no dynamic effects) directly for the simulated bundle. In Fig. 5.14(c), when introducing dynamic effects, the clusters do not change their maximum size (measured in number of failed fibre elements contained), but the average distance between consecutive breaks during the simulation is smaller (Fig. 5.14(d)), suggesting that dynamic effects lead to clusters that are more coplanar. The formation of coplanar clusters during tensile tests is confirmed by computer tomography experiments [9] (see also Fig. 5.2), and is a feature that models which neglect dynamic stress concentration struggle to represent correctly. Therefore, the lower bundle strength predicted with including dynamic effects appears to be the result of the higher stress concentration and easier damage localization around clusters of broken fibres introduced by the dynamic effects.

5.3.3 Fracture mechanics

This section presents the strength predictions for model FM, which features a fracture mechanics failure criterion. A parametric study was carried out analyzing different values of $\lambda_{\text{fm}}$ as a free parameter of the model which accounts for the approximate nature of the fracture mechanics theory implemented in the failure criterion, and for the uncertainty on the material property properties to be used for this type of model. The geometrical parameters of the model and the fibre properties are the same used in the previous sections and are shown in Table 5.2; while the properties used to implement the fracture mechanics criterion and the values of $\lambda_{\text{fm}}$ considered in the parametric study are listed in Table 5.3.

Fig. 5.15(a) compares the predicted strength and variability between Model FM and BA.3 for different values of $\lambda_{\text{fm}}$. It is observed that, when a fracture mechanics failure criterion is considered, the predicted bundle strength decreases significantly. Furthermore, considering fracture mechanics can change the overall size effects for bundles with low values of $\lambda_{\text{fm}}$; strength presents a maximum for medium-size bundles (under 100 fibres) and then decreases with the
Figure 5.14: Numerical results of model DE (implementation of dynamic effects). (a) Predicted strength and coefficient of variation for models DE and BA.3. (b) Computational cost for models DE and BA.3. (c) Average maximum cluster size for the minimum and maximum intensity of dynamic effects. (d) Average distance between consecutive breaks during the simulation process for the minimum and maximum intensity of dynamic effects.

bundle size. Finally, considering fracture mechanics does not cause a significant change in bundle variability, the number of Monte Carlo simulation considered is similar to that for Model BA.3. Fig. 5.15(b) shows the average critical cluster size as a function of the number of fibres in the bundle for different values of $\lambda_{\text{fm}}$, directly for the simulated bundle. Critical cluster sizes predicted by model FM are much smaller than the maximum sizes given by models BA and also for model DE (Fig. 5.15(b)). Additionally, the critical cluster size appears to reach an horizontal asymptote, as opposed to the continuous growth shown by models BA and DE (Fig. 5.14(c)). Although direct comparison with experimental results is not possible at this point, this result is in line with experimental evidence, which has not reported clusters greater than
14 fibres, even for large bundles [9, 65]. Other simulation models in the literature which do not consider fracture mechanics also tend to severely overestimate the critical cluster size [65].

Figure 5.15: Numerical results of model FM (implementation of fracture mechanics for different values of \( \lambda \)). (a) Predicted strength and coefficient of variation for models FM and BA.3. (b) Average maximum cluster size.

Table 5.3: Composite mechanical properties and values of \( \lambda \) for the parametric study with Model FM.

<table>
<thead>
<tr>
<th>Composite mechanical properties</th>
<th>Parametric study</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E ) ([\text{MPa}])</td>
<td>( \nu ) ([-\text{]})</td>
</tr>
<tr>
<td>120</td>
<td>0.28</td>
</tr>
</tbody>
</table>

5.3.4 Validation against experimental results for micro-bundles

Fig. 5.16 shows a comparison between experimental strength data for composite micro-bundles and the strength distributions predicted by models BA.3 and DE. Fibre properties were taken from the experiments of Beyerlein and Phoenix [4] and Kazanci [5] and are listed in Table 5.4. Two different matrices were used in the experiments: matrix (I) is a low modulus, flexible epoxy, consisting of a blend of 50% DER 221 and 50% DER 732 with DEH 26 as curing agent; while matrix (II) is a high modulus, stiff epoxy, with a 100% of resin DER 331 with the same curing agent. The matrix yield stresses were obtained from Netravali et al [105]; they are \( \tau_{\text{sl}} = 3.96 \) MPa and \( \tau_{\text{sl}} = 41.67 \) MPa for matrices (I) and (II), respectively.
Model FM is not included in this comparison because the effect of fracture mechanics starts to become significant for large bundles (Fig. 5.15(a)), and also because it would have been necessary to assume a value of $\lambda_{fm}$. When applicable, strength predictions for the same material system obtained via Monte Carlo simulation of a full-field FE bundle model from St-Pierre et al [8] are included for comparison.

Fig. 5.16(a)-(d) shows the comparison with four-fibre bundles for two different bundle lengths: 10 mm and 200 mm. All results were obtained using $l_b = 10$ mm in the model, which allowed direct comparison with experiments for 10 mm long bundles; the comparison with 200 mm long bundles was performed by scaling the modelling results using WLT (see Appendix D.4). For the low-strength matrix (I) the bundle strength distributions (obtained by the present model and by the experiments) correlate very well and the predicted mean values do not deviate more than 3.3% (Table 5.5).

For the higher-strength matrix (II), predictions for models BA.3 and DE appear to overestimate slightly the experimental strength, and are very close to the results of the FE simulations from St-Pierre et al [8]. It should be noted that Netravali et al [105] have reported the occurrence of debonding at the fibre matrix interface during single fibre fragmentation tests with epoxy (I), while no debonding was observed for the flexible epoxy (II). Debonding at the fibre matrix interface results in a longer recovery length and this may lower the bundle strength (see Fig. 5.12(c)). This consideration may explain why both modelling results tend to overestimate the experimental results.

Fig. 5.16(e)-(f) shows the comparison with experimental results for $l_b = 10$ mm micro-bundles with 7 fibres. The simulation results were obtained by linear interpolation from square bundles of 4 and 9 fibres. In this case, predicted strength distributions for matrix (I) deviate slightly from the experimental data, while the agreement with matrix (II) is better.

### 5.3.4.1 Comparison with macro-bundles

Fig. 5.17 compares experimental data for the average strength of composite bundles ranging from one thousand up to one million fibres obtained from Okabe and Takeda [3] with the predictions of the current model. Results were obtained with properties defined in Table 5.2 by simulating
Figure 5.16: Experimental validation with micro-bundles of models BA.3 and DE. (a) 4 fibres, epoxy (I), \( l_b = 10 \text{ mm} \). (b) 4 fibres, epoxy (II), \( l_b = 10 \text{ mm} \). (c) 4 fibres, epoxy (I), \( l_b = 200 \text{ mm} \). (d) 4 fibres, epoxy (II), \( l_b = 200 \text{ mm} \). (e) 7 fibres, epoxy (I), \( l_b = 10 \text{ mm} \). (f) 7 fibres, epoxy (II), \( l_b = 10 \text{ mm} \).
Table 5.4: Fibre properties and bundle geometry for the comparison with experimental data in micro-bundles. Obtained from [4,5].

<table>
<thead>
<tr>
<th>Reference</th>
<th>Type</th>
<th>$C_f$ [m]</th>
<th>$X_b^I$ [MPa]</th>
<th>$m$ [-]</th>
<th>$l_b$ [mm]</th>
<th>$l_{bb}$ Packing</th>
<th>$V_f$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beyerlein and Phoenix (1997)</td>
<td>AS4</td>
<td>6.85</td>
<td>4493</td>
<td>4.8</td>
<td>10</td>
<td>10/200 Square</td>
<td>0.70</td>
</tr>
<tr>
<td>Kazanci (2004)</td>
<td>IM6</td>
<td>5.63</td>
<td>5283</td>
<td>5.4</td>
<td>10</td>
<td>10 Hexagonal</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Table 5.5: Statistical mean values and coefficient of variations predicted by models BA.3 and DE and compared with experimental data. in Fig. 5.16. Values in parenthesis indicate error with respect to experiments.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Experiments</th>
<th>Model BA.3</th>
<th>Model DE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{X}_b$ [MPa]</td>
<td>$CoV_b$ [%]</td>
<td>$\bar{X}_b$ [MPa]</td>
</tr>
<tr>
<td>5.16a</td>
<td>3553</td>
<td>15.3</td>
<td>3605 (+1.5%)</td>
</tr>
<tr>
<td>5.16(b)</td>
<td>3868</td>
<td>9.9</td>
<td>4359 (+12.7%)</td>
</tr>
<tr>
<td>5.16(c)</td>
<td>2587</td>
<td>10.1</td>
<td>2710 (+8.8%)</td>
</tr>
<tr>
<td>5.16(d)</td>
<td>3049</td>
<td>9.4</td>
<td>3422 (+12.2%)</td>
</tr>
<tr>
<td>5.16(e)</td>
<td>4661</td>
<td>10.0</td>
<td>4220 (−9.5%)</td>
</tr>
<tr>
<td>5.16(f)</td>
<td>5193</td>
<td>11.0</td>
<td>5169 (−0.5%)</td>
</tr>
</tbody>
</table>

$l_{sim} =$ 1mm bundles and scaling the results to $l_b = 10$mm with WLT.

Fig. 5.17(a) shows the results for models BA.3 and DE. Both models significantly overestimate the strength of large bundles. Furthermore, the size effect (decrease in strength with the number of fibres in the bundle) observed in the range of the experiments cannot be reproduced by any of the two models, which both exhibit a positive trend for the bundle strength. These results suggest that the strength of large composite bundles, and in particular the size effects, cannot be correctly predicted considering only strength of materials (even in the case of dynamic stress concentration) as a failure theory.

Fig. 5.17(b) compares the predicted average strength from Model FM with the experimental data. A parametric study was carried out by varying the value of $\lambda_{fm}$ (see figure legend). The strength predictions from model FM are in the same range of the experimental results.
Furthermore, the reduction in the predicted mean strength with the bundle size is compatible with the trend of the experimental data, as shown by the trend lines fitted to the computational results. Note that, although the predictions depend on $\lambda_{nm}$ (which is treated as a free parameter of the model at this point), the model predicts the correct size effect (reduction of strength with increase in the number of fibres) for all values of $\lambda_{nm}$ considered. This suggests that failure in large composite bundles may be driven by fracture mechanics, and that size effects are not reproducible via direct simulation only considering strength of materials.

![Figure 5.17: Experimental validation with micro-bundles of models BA.3, DE and FM. (a) Results for models BA.3 and DE. (b) Results for model FM (with red colour indicating best agreement).](image)

### 5.4 Conclusions

A family of semi-analytical fibre bundle models was developed to efficiently simulate the longitudinal tensile failure of large composite bundles. A field superposition method was used by all models to calculate the stress concentrations around clusters of broken fibres, and has been validated against analytical and FE results from the literature. To the knowledge of the author, this is the first time that a field superposition method is able to capture the effect of clusters of broken fibres on the stress recovery length. Additionally, a method with a bundle-size dependent variable number of Monte Carlo simulations was implemented to improve the computational efficiency, hence allowing the direct simulation of bundle sizes up to thousands of fibres.

A baseline model (model BA) was developed to reproduce the key features involved in the bundle
tensile failure process: variability of single fibre strength and stress concentration around broken fibres. Four sub-variants of the baseline model were created to investigate different aspects of the simulation algorithm. Model BA.1 features an exact calculation of the stress recovery length for clusters of broken fibres, and uses a non-simultaneous single fibre failure simulation algorithm. Model BA.2 uses a multiple broken fibre simulation algorithm instead, while model BA.3 implements an approximate calculation of the stress recovery length for clusters of broken fibres and model BA.4 does not account for the effects of the size of the cluster of broken fibres on the recovery length.

All model BA variants assume strength of materials as the only bundle failure criterion, and only consider static equilibrium stress states during the simulation. Model DE was developed in order to investigate the effects of dynamic stress concentration on the bundle failure process, while model FM was developed to investigate the effects of including fracture mechanics based failure criteria in the simulation process. To the author knowledge, this is the first time in the literature that dynamic effects and Fracture Mechanics are investigated through direct simulation in a fibre bundle model.

The following conclusions have been reached during the present study:

a) On model BA variants:

- the non-simultaneous fibre failure simulation strategy implemented in Model BA.1 and the multiple fibre failure simulation strategy implemented in model BA.2 are substantially equivalent with regards to the strength prediction, although model BA.2 allows a slight reduction in computational time;

- the comparison between model BA.2, BA.3 and BA.4 demonstrates that the influence of clusters of broken fibres on the stress recovery length has an effect on the final bundle strength (reducing it by about 10% when compared to model BA.4). This effect is captured fully by model BA.2 and with a 5% error by model BA.3. Considering the strong advantage of model BA.3 in terms of computational efficiency, it appears the most appropriate to conduct simulations on large bundle sizes.

b) Models BA.3 and DE show a generally good correlation with experimental strength distributions for micro-bundles of 4 and 7 fibres with two different resin types. They also show good
agreement with Monte Carlo simulations carried out using a full FE bundle model [8].

c) Model DE, which shares the same basic algorithm as model BA.3 but including the effects of dynamic stress concentrations, shows a maximum reduction possible of about 10% in the predicted bundle strength when compared with model BA.3.

d) Both models BA and DE predict an increase in strength with the bundle size and severely overestimate experimental results, even considering the worst case scenario for dynamic effects in model DE. These results suggest that there are important aspects of the physics of the problem that are not considered in models BA and DE.

e) Model FM, which shares the same basic algorithm as model BA.3 but includes a fracture mechanics failure criterion, predicts lower bundle strengths and, most importantly, a negative trend for the strength of large bundles in agreement with experimental results. It also predicts a smaller critical cluster size which stays rather constant even for large bundles, in agreement with experimental evidence [7,87]. These results suggest that fracture mechanics is a physical mechanism which might be necessary to consider to correctly predict the longitudinal tensile strength in large composite bundles. This is arguably the most important outcome of the work presented in this chapter.
Chapter 6

Conclusions

6.1 Summary

In this dissertation, a new bio-inspired microstructure design approach was developed to create CFRP laminates with engineered fracture behaviour, with the final goal of improving toughness and damage tolerance in CFRP structures. The microstructure designs take inspiration from the microstructures of biological composites by adopting the most important toughening mechanisms and applying them to CFRP laminates. By modifying the microstructure, it is possible to control the crack propagation path, hence to engineer the fracture behaviour of the composite. Below are the most important results achieved during the work.

- Carefully placed patterns of micro-cuts have been inserted in the $0^\circ$ plies of a CFRP laminate with Cross-Ply (CP) layup to promote the formation of hierarchical pull-out structures during translaminar crack propagation. An analytical model to predict the probability of bundle pull-out formation was developed and validated through an experimental parametric study. The model was used to design three hierarchical patterns of micro-cuts which have been tested using Compact Tension (CT) specimens. The three patterns behaved in accordance with the model predictions and, for the best design, achieved an increase of 70% in the maximum tensile load during a CT test, and 214% in the translaminar work of fracture of the $0^\circ$ plies when compared with the baseline material. By contrast, the
specimens with the best performing hierarchical patterns of micro-cuts showed only a 12% reduction in un-notched strength when subject to a standard tensile test. These results demonstrate that it is possible, using hierarchical patterns of micro-cuts, to increase the translaminar work of fracture, without compromising the un-notched tensile strength of the material.

- Patterns of micro-cuts have been inserted in the 0° plies of a CP CFRP laminate to promote crack deflection, and the interaction of failure mechanisms between neighbouring plies with different fibre orientation. An analytical model based on Finite Fracture Mechanics (FFM) was developed to predict the maximum crack deflection height and was used to design the patterns of micro-cuts. The technique allowed to achieve a 68% increase in the maximum tensile load during a CT test, and a 460% increase in the laminate translaminar work of fracture, for the best performing design. Using a specifically developed work of fracture model, it was demonstrated that a significant part of the increase in the work of fracture is due to the interaction of fracture mechanisms between contiguous plies with different orientation. Therefore, this interaction of fracture mechanisms is an important aspect to consider in the design of effective microstructures to improve damage tolerance.

- The experience gained in the first two studies was used to design an engineered microstructure for Quasi-Isotropic (QI) CFRP laminates. Patterns of micro-cuts have been inserted in the 0° and ±45° plies of the laminate to promote crack deflection, and increase energy dissipation through the interaction of fracture mechanisms between contiguous plies with different orientation. This technique allowed to achieve a 27% increase in the QI laminate maximum tensile load during a CT test, and a 189% increase in the translaminar work of fracture during Compact Tension tests. Furthermore, microstructure design was used to improve the damage resistance of a QI laminate subject to indentation test, and an increase of 43% in the total energy dissipated, and of 40% in maximum deflection at complete failure was achieved. Given the industrial relevance of QI laminates, these results demonstrate that microstructure design can be used to improve the damage tolerance of CFRP structures in practical applications.

- A semi-analytical Fibre Bundle Model (FBM) was developed to investigate the role of dynamic stress concentrations, and of fracture mechanics-driven growth of critical clusters of
fibres, on longitudinal tensile failure of fibre-reinforced composites. The model uses shear lag to calculate the stress recovery along broken fibres, and an efficient field superposition method to calculate the stress concentration on the intact fibres, which has been validated against analytical and FE results from the literature [8]. The baseline version of the model uses static equilibrium stress states, and considers strength of materials as the only failure criterion which can drive bundle failure. Two model variants have been developed which include dynamics stress concentrations and a fracture mechanics failure criterion respectively. To the knowledge of the author, it is the first attempt in the literature to investigate these two physical mechanisms in a FBM by direct simulation of large composite bundles.

It was shown that, although the dynamic stress concentrations significantly decrease the predicted bundle strength, do not allow to predict the right trend of the size effect shown by the experimental results. On the contrary, including fracture mechanics-driven failure in the bundle simulation allowed to predict the right trend of the size effects on the bundle strength. These results suggest that fracture mechanics is a physical mechanism which might be necessary to consider to correctly predict the longitudinal tensile strength in large composite bundles.

In conclusion, this work gives a positive answer to the main research question set in the introduction of this dissertation: it is possible to significantly improve the toughness and damage resistance of CFRP laminates using microstructure design to engineer their fracture behaviour. These results could lead to more efficient composite structures, and a wider adoption of composite materials in the industry.

6.2 Future Work

The microstructure design technique developed in this work proved successful in increasing toughness and damage resistance of Carbon Fibre laminates. However, this was just a first attempt, and it is reasonable to assume that this technique has the potential to increase the performance of CFRP laminates considerably further.

It should be noted that important aspects related to the use of patterns of micro-cuts to engineer
the fracture behaviour of CFRP laminates still need to be investigated in depth. The effect of
the ply thickness on crack deflection caused by patterns of micro-cuts can be validated though
an experimental parametric study. The effects of different combinations of fibres and matrix
systems on crack deflection and bundle pull-out formation can be investigated experimentally
as well. A comprehensive model to describe the interaction of failure mechanisms between
neighbouring plies with different fibre orientation during trans laminar crack propagation still
has to be developed.

Further toughness increase could be achieved through systematic optimization of the microstruc-
ture design using high-fidelity modelling tools. The Fibre Bundle Model developed in Chapter 5,
onec t expanded to include the effects of fibre matrix debonding and the resulting work of friction
during fibre bundle pull-out, could be used to predict the trans laminar work of fracture during
longitudinal tensile failure. Since the model allows a fine control of the bundle microstructure
and the direct simulation of bundles up to tens of thousands of fibres, it could be used to actively
design the microstructure of the composite and to investigate the effects of particular patterns
of micro-cuts on the final tensile strength and trans laminar work of fracture.

Microstructure design was successfully applied to improve the damage resistance of CFPR lam-
inates subject to indentation load. It is reasonable to expect that this technique would increase
energy dissipation during impact events as well, and thus improve the compression-after-impact
performance of laminate structures. It is also expected that this microstructure design concept
can be applied to release stress concentration around rivet holes or skin stiffeners, and to prevent
catastrophic propagation of trans laminar cracks. Furthermore, a more progressive failure of the
substrates could avoid damage localization in bonded joints.

As a closing remark, it should be highlighted that, as proven in Chapter 2, there is a small
trade-off between un-notched tensile strength and toughness for CFRP laminates when patterns
of micro-cuts are used to engineer the fracture behaviour. Similar trade-offs are likely to exist for
the un-notched fatigue and compressive strength. However, it is important to highlight that this
technique, rather than being intended as a way for creating a new all-purpose type of material,
should be a tool to be integrated in the composite-structure design process. Microstructure
design can be used to tailor the properties of the material in specific parts of the structure to
fit the local design requirements, therefore creating a process where the design of the structure and the point-by-point design of the microstructure of the material are complementary and are carried out simultaneously.
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Appendices
Appendix A

A.1 Translaminar fracture toughness model

Fig. A.1 shows a schematic representation of a two levels hierarchical bundle being pulled out of the fracture surface. $C[i]$ and $A[i]$ are the perimeter and area of the cross section of the level-$i$ pull-out respectively, and $\ell[i]$ is the level-$i$ pull-out length. $P(x)$ is the pull-out force which is equal to the total friction force acting on the lateral surface of the bundle and is a function of the pull-out coordinate $x$. The expression for $P(x)$ changes during the pull-out process, depending on which surfaces of the bundle are in contact with the rest of the ply. Assuming that $\ell_1 \leq \ell_2$, it is possible to write $P(x)$ as

$$P(x) =\begin{cases} P_a(x) &= \tau_u \cdot [2 \cdot B_1 \cdot \ell_1 + C_2 \cdot (\ell_2 - x) + 2 \cdot t \cdot (\ell_1 - x)], & \text{if } x \leq \ell_1; \\ P_b(x) &= \tau_u \cdot [2 \cdot B_1 \cdot \ell_1 + C_2 \cdot (\ell_2 - x)], & \text{if } \ell_1 < x \leq \ell_2; \\ P_c(x) &= \tau_u \cdot 2 \cdot B_1 \cdot (\ell_1 + \ell_2 - x), & \text{if } \ell_2 < x \leq \ell_1 + \ell_2; \\ 0, & \text{if } x > \ell_1 + \ell_2; \end{cases}$$

\hspace{1.5cm} (A.1)

where $\tau_u$ is the in-situ frictional stress of the matrix as given in Tab. 2.1, and

$$\begin{align*}
B_1 &= (w + p); \\
C_1 &= 2 \cdot (B_1 + t); \\
A_1 &= B_1 \cdot t;
\end{align*}$$

and

$$\begin{align*}
B_2 &= 3 \cdot (w + p); \\
C_2 &= 2 \cdot (B_2 + t); \\
A_2 &= B_2 \cdot t.
\end{align*}$$

\hspace{1.5cm} (A.2)

Analogous expressions could be derived for the case of $\ell_1 \geq \ell_2$. 

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Figure A.1: Schematic representation of a bundle pull-out for hierarchical pattern with two levels of hierarchy; the graph on the right side shows the force $P$ necessary to pull the bundle out of the fracture surface.

The energy necessary for the bundle formation is the sum of the energy dissipated by debonding and friction during the pull-out process (subscripts ‘deb’ and ‘fric’). These two components can be calculated as

$$
\begin{align*}
W_{\text{deb}} &= (C_1 \cdot \ell_1 + C_2 \cdot \ell_2) \cdot G_{\text{mat}}^\text{II}, \\
W_{\text{fric}} &= \int_0^{\ell_1+\ell_2} P \cdot dx = \int_0^{\ell_1} P_a \cdot dx + \int_{\ell_1}^{\ell_2} P_b \cdot dx + \int_{\ell_2}^{\ell_1+\ell_2} P_c \cdot dx \\
&= \tau_{\mu} \cdot (\frac{1}{2} \cdot C_1 \cdot \ell_1^2 + \frac{1}{2} \cdot C_2 \cdot \ell_2^2 + 2 \cdot B_1 \cdot \ell_1 \cdot \ell_2),
\end{align*}
\tag{A.3}
$$

where $G_{\text{mat}}^\text{II}$ is the in-situ interfacial toughness of the matrix given in Tab. 2.1. The translaminar work of fracture for the ply ($W_{\text{lam}}^\text{sim}$) can then be calculated as

$$
W_{\text{lam}}^\text{sim} = \frac{W_{\text{deb}} + W_{\text{fric}}}{A_2}. 
\tag{A.4}
$$
It is possible to extend the same approach to a bundle pull-out with \([n]\) levels of hierarchy:

\[
\begin{align*}
W_{\text{deb}} &= \sum_{i=1}^{n} (C_i \cdot \ell_i) \cdot G_{\text{mat}}^\Pi; \\
W_{\text{fric}} &= \tau_{\mu} \cdot \left( \frac{1}{2} \cdot \sum_{i=1}^{n} C_i \cdot \ell_i^2 + 2 \cdot \sum_{i=1}^{n} B_i \cdot \ell_i \cdot (\sum_{k=i+1}^{n} \ell_k) \right),
\end{align*}
\]

where

\[
\begin{align*}
B_i &= (2 \cdot i - 1) \cdot (w + p); \\
C_i &= 2 \cdot (B_i + t); \\
A_i &= B_i \cdot t.
\end{align*}
\]

Accordingly, the translaminar work of fracture for \([n]\) hierarchical levels can be written as

\[
W_{\text{sim}}^{\text{lam}} = \frac{W_{\text{deb}} + W_{\text{fric}}}{A_i}.
\]

### A.2 Un-notched tensile tests

Tensile tests were carried out to measure the un-notched tensile strength of thin-ply laminates with hierarchical patterns of micro-cuts. The specimens configuration is shown in Fig. A.2(a).

The specimens had a quasi-isotropic lay-up (lay-up sequence \([45, 90, -45, 0]_{5s}\)), and were manufactured from the grade A Skyflex UD prepreg (Tab. 2.1) using the same laser engraving and lay-up procedures described in Section 2.2.3. Glass fibre tabs were glued to the specimens to reduce gripping damage.

A total of ten specimens were used for this study: five specimens were manufactured using the baseline material without any pattern of micro-cuts; while the other five were manufactured using the modified material with the pattern H3 (Fig. 2.11(c)). For the five specimens with the pattern of micro-cuts, each ply in the laminate was laser engraved before lamination in a region correspondent to the central portion of the the specimen (pattern region in Fig. A.2(a)). The pattern of micro-cuts was oriented perpendicularly to the fibre direction in each ply and was repeated over the entire region with a period of 0.9 mm between each row of micro-cuts.

The specimens were tested using an Instron load frame equipped with a 50 kN load cell. The
Figure A.2: (a) Un-notched tensile specimen geometry (nominal dimensions in mm) with schematic representation of the pattern of micro-cuts H3; the pattern is always perpendicular to the fibre direction and is repeated uniformly over the entire pattern region for each ply in the laminate. (b) Average tensile strength and standard deviation for baseline material and material with pattern H3 (five specimens were tested for each configuration).

tests were carried out in displacement control (5 mm/min) and were interrupted after complete failure of the specimen.

The average strengths and standard deviations for the two specimen configurations tested can be found in Fig. A.2(b). The specimens with hierarchical patterns of micro-cuts show only a 12% reduction in un-notched strength and a significantly lower standard deviation when compared with the baseline material. Note that the 12% decrease in un-notched strength was determined by the same pattern of micro-cuts that led to a 70% increase in the maximum tensile load measured during the CT tests (Section 2.4.3).
Appendix B

B.1 Shark Teeth micro-structures design

B.1.1 Microstructure design

The curves defined by Eq. (3.9) are plotted in Fig. B.1 using the material properties in Tab. 3.1 for different values of the fraction of un-cut fibres $\xi$ and the horizontal step between bundles $\Delta x$. Fig. B.1 shows how the 3 “shark-teeth” patterns fit under the boundary defined by the curves.

B.1.2 Definition of material properties for the FMC model

The values of $G_{I}^{\text{ply}}$, $G_{I}^{\text{cut}}$, $G_{I}^{\text{uncut}}$, and $G_{II}^{\text{mat}}$ required for the model developed in Section 3.3 are difficult to measure experimentally, because they are in-situ properties. Therefore, they were decided upon based on equivalent specimen-level properties obtained from the literature.

Fig. B.2(a) shows an example of a large bundle pull-out caused by the presence of a laser micro-cut in the $0^\circ$ ply. The appearance of the fracture surfaces at the base of the pull-out is qualitatively similar to that of the $0^\circ$ plies in the baseline material without any micro-cuts. For this reason, the propagation toughness $G_{I}^{\text{ply}}$ was taken to be equal to the translaminar fracture toughness of the baseline material $G_{I}^{\text{ply}}$.

Fig. B.2(b) shows the fracture surface at the top of the bundle pull-out. The area of the laser-cut (in red in Fig. B.2(b)) can be recognized because the fibre ends are more blunt. It is assumed that this area was filled with resin during the curing process, thus the toughness of the laser-cut $G_{I}^{\text{mat}}$ is taken equal to the mode I toughness of the matrix $G_{I}^{\text{mat}}$. 

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Surrounding the laser cut area, there are two areas of translaminar fibre failure. The fibres in these areas failed on the same plane without significant pull-out, thus the fracture toughness of the un-cut fibres $G_{\text{I}}^{\text{uncut}}$ was calculated as the weighted average of the mode I toughness of the matrix $G_{\text{I}}^{\text{mat}}$ and of the fibres $G_{f}^{\text{I}}$ using the rule of mixture:

$$G_{\text{I}}^{\text{uncut}} = V_f \cdot G_{f}^{\text{I}} + (1 - V_f) \cdot G_{\text{I}}^{\text{mat}}.$$  \hspace{1cm} (B.1)

where $V_f$ is the fibre volume fraction of the composite. The values of $G_{f}^{\text{I}}$, $G_{\text{I}}^{\text{mat}}$, $G_{\text{II}}^{\text{mat}}$, $G_{\text{I}}^{\text{ply}}$, $V_f$ can be found in Tab. 3.1.
Figure B.2: SEM micrographs of translaminar fracture surface with large bundle pull-out caused by the presence of laser micro-cut in the 0° ply: (a) translaminar fracture surface of the 0° ply at the base of the bundle pull-out; (b) laser-cut area at the top of the bundle pull-out.
Appendix C

C.1 Microstructure design

The Finite Fracture Mechanics model developed in Section 3.3 was used to guide the design of the microstructure for this study. The function $h(x)$ in Eq. (3.9) can be used to determine an envelope curve (shown in Fig. C.1) for the current material systems and micro-cuts geometry, which represents a boundary of applicability of the crack deflection technique and which was used to design the patterns of micro-cuts for the $0^\circ$ plies of the microstructures used in this study. The values of $G_f^I$, $G_f^{mat}$, $G_h^{mat}$, $G_f^{ply}$, $V^f$ required to draw the $e$ curve can be found in Tab. 4.1. Tab. 4.1 details the resulting geometrical parameters of each pattern.

Table C.1: Geometrical parameters of the patterns of micro-cuts in the $0^\circ$ plies. $\Delta h$ and $\Delta x$ are the finite increments of the pattern of micro-cuts in the vertical and horizontal directions respectively, $\xi$ is the fraction of uncut fibres across the array of micro-cuts and $h_{max}$ is the maximum crack deflection height.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>$\Delta x$ (mm)</th>
<th>$\Delta h$ (mm)</th>
<th>$\xi$</th>
<th>$h_{max}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
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<td>0.03</td>
<td>0</td>
<td>0.48</td>
</tr>
<tr>
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<td>0.03</td>
<td>0</td>
<td>0.72</td>
</tr>
<tr>
<td>5</td>
<td>0.03</td>
<td>0.03</td>
<td>0</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Figure C.1: Envelope of applicability of the crack deflection concept and micro-cuts design, for the $0^\circ$ plies.
Appendix D

D.1 Shear-lag boundary calculation for clusters of broken fibres

At each bundle cross-section $j$, using the same definition of a cluster given in Section 5.2.4.1, it is possible to define the set $N^{sj}_{cl}$ of neighbouring fibre elements associated with the cluster $s$ in section $j$ (Fig. D.1(a)). Each set $N^{sj}_{cl}$ contains $n^{sj}_{cl}$ elements $(ij)^s_{cl}$ sharing the same shear-lag boundary:

$$C^{sj}_{cl} = \frac{C_f}{b_n} \sum_{N^{sj}_{cl}} n^{(ij)^s_{cl}}_{eln},$$

where $C^{sj}_{cl}$ is the shear-lag boundary of the cluster $s$ in section $j$, $n^{(ij)^s_{cl}}_{eln}$ is the number of intact elements surrounding each element $(ij)^s_{cl}$ (of a maximum of $n^{(ij)^s_{cl}}_{cl}$ neighbours), and $b_n$ is the base number of neighbours considered in the arrangement (or maximum number of surrounding elements to transmit the stress), which was taken equal to 4 for square fibre arrangement. Since it is assumed that all the elements in the same cluster have the same shear-lag stress profile, the portion of the shear-lag boundary corresponding to each element can be calculated as:

$$C^{(ij)^s_{cl}}_{sl} = \frac{C^{sj}_{cl}}{n^{(ij)^s_{cl}}_{cl}}.$$  

In the case of a single failed element surrounded by all intact elements, Eqs. (D.1) and (D.2) lead to $C^{(ij)^s_{cl}}_{sl} = C^{sj}_{cl}$ and $n^{(ij)^s_{cl}}_{eln} = n^{(ij)^s_{cl}}_{cl}$. This corresponds to a portion of the fibre circumference, depending on where the element is located in the arrangement (Fig. D.1(b)). This is also applicable for the non-saturated elements.
The calculation of the shear-lag stress boundaries $C_{sl}^{ij}$ is performed for all the elements in the bundle, applying these calculations for each cross section $j$:

$$C_{sl}^{ij} = \begin{cases} C_{sl}^{(ij)_1} & \text{if } (ij) \in N_{cl}^{s}, \\ \frac{C_{l}}{6n_{el}} \cdot n_{el}^{ij} & \text{if } (ij) \notin N_{cl}^{s} \forall s, j. \end{cases}$$

(D.3)

Since $C_{sl}^{ij}$ depends on the number of fibre elements belonging to the same cluster, and this number depends on the recovery lengths, hence on $C_{sl}^{ij}$, the problem of calculating the shear-lag stress profile for each fibre in the bundle has to be solved through an iterative procedure. The stress profile is firstly estimated assuming that all the neighbours are intact (this means $n_{cl_{in}}^{(ij)_0} = n_{el}^{(ij)_1}$ except for the fibres which belong to the perimeter of the arrangement, for which the effective contour is a portion of $C_l$). This solution is displayed in Fig. D.1(c) (dashed black line).

Then, the interacting regions are computed (patterned blue regions in Fig. D.1(c)) to identify regions with clusters, and the shear-lag boundaries are re-calculated applying again Eqs. (D.1) to (D.3) at each bundle cross-section. Using this implementation, the stress profile is re-calculated (red line in Fig. D.1(c)).

Due to the change in the recovery lengths, the interacting regions have been modified (the new interacting areas are the plain light blue regions in Fig. D.1(c)), and the computations have to be repeated iteratively until convergence to determine the exact solution (hereinafter exact shear-lag stress profile), which occurs when the interacting regions are constant (Fig. D.1(d), which corresponds to the solution given in Fig. 5.4).

Considering that the iterative process can be computationally expensive, and given the exploratory scope of this model, a first approximation for the stress profiles (hereinafter simplified shear-lag stress profile) can be obtained by computing the cluster sizes only once (Fig. D.1(c)), which is stopping the iterative process at the first step.
Figure D.1: Clusters computation and calculation of the shear-lag profile for the bundle and stress state presented in Fig. 5.4. (a) Calculation of the shear-lag boundaries in clusters of broken fibres. (b) Calculation of the shear-lag boundary on individual broken fibres. (c) Simplified shear-lag stress profile (solution when stopping after the first iterative step). (d) Exact shear-lag stress profile (solution when completing the iterative process).

D.2 Critical size for penny-shaped crack in isotropic infinite body according to LEFM and analogy with critical cluster of broken fibres in a composite bundle

D.2.1 Critical size for penny-shaped crack in isotropic infinite body according to LEFM

For a penny-shaped crack in a isotropic infinite cylinder (Fig. D.2), the mode I stress intensity factor $K_1$ according to Linear Elastic Fracture Mechanics (LEFM) [106–108] is

$$K_1 = 2 \cdot \left( \sigma_{th} - \sigma_b \right) \cdot \frac{\sqrt{a}}{\pi}$$  \hspace{1cm} (D.4)
where \( \sigma_\infty \) is the asymptotic stress, \( \sigma_b \) is the bridging stress acting on the free surfaces of the crack (see Fig. D.2(b)) and \( a \) is the characteristic crack length. The energy release rate is obtained directly from the stress intensity factor, and from the properties of the material:

\[
G_I = K_I^2 \cdot \left( \frac{1 - \nu^2}{E} \right),
\]

where \( E \) is the Young’s modulus and \( \nu \) the Poisson’s ratio. When this energy exceeds the mode I fracture toughness of the material \( G_{Ic} \), the crack becomes critical. Thus the critical crack size for a penny-shaped crack in an infinite body can be calculated combining Eq. (D.4) and Eq. (D.5):

\[
a_{cr} = \frac{G_{Ic} \cdot \pi \cdot E}{4 \cdot (\sigma_\infty - \sigma_b)^2 \cdot (1 - \nu^2)} = \frac{\lambda_{fm}}{(\sigma_\infty - \sigma_b)^2},
\]

where the parameter \( \lambda_{fm} \) accounts for the geometry of the problem and the mechanical properties of the materials:

\[
\lambda_{fm} = \frac{G_{Ic} \cdot \pi \cdot E}{4 \cdot (1 - \nu^2)}.
\]

### D.2.2 Analogy with critical cluster of broken fibres in a composite bundle

For the case of a cluster of broken fibres in a composite bundle, by introducing the analogy with the case of the penny-shaped crack just described, it is possible to define the critical size for an equivalent crack formed by the cluster of broken fibres \( s \) in the bundle section \( j \) as:

\[
a_{cr}^{sj} = \frac{\lambda_{fm}}{(\sigma_{eq}^{sj})^2},
\]

where \( \lambda_{fm} \) is treated as a free parameter of the model.

The equivalent stress \( \sigma_{eq}^{sj} \) can be calculated considering that (i) the axial load is not carried by the entire cross-section but only by the fibres, and (ii) the actual locations of the fibre breaks are not in the same plane, thus stress can still be transferred between the two faces of the equivalent crack due to the friction between the fibres:

\[
\sigma_{eq}^{sj} = V \cdot (\sigma_\infty - \sigma_{po}^{sj}),
\]
Figure D.2: Fracture mechanics for a penny-shaped crack in an isotropic cylinder. (a) Circular crack inside an infinite cylinder. (b) Detail of the crack showing the bridging stress $\sigma_b$ acting in the crack surfaces.

where $\sigma_{po}^{s_j}$ is the average pull-out stress due to the friction between the fibre pull-outs and can be calculated as

$$\sigma_{po}^{s_j} = \frac{\sum \tau_{bi} \cdot C_i \cdot |z^{(i)}_{cl} - z^{(i)}_{fn}|}{n_{el1} \cdot A_f},$$

where $\tau_{bi}$ is the frictional stress acting on the lateral surface of each fibre pull-out (Fig. 5.10(b)) and $|z^{(i)}_{cl} - z^{(i)}_{fn}|$ is the pull-out length (i.e. distance between the element belonging to the equivalent crack in section $j$ and the closest failed element along the same fibre $i$ belonging to the cluster). The pull-out stress decreases the energy available for crack propagation, and reflects the fact that clusters of broken fibres which are almost coplanar are more likely to become critical than clusters which are more dispersed.
D.3 Numerical convergence study

This section summarizes the convergence study conducted to select the appropriate numerical parameters for the simulations. A bundle length \( l_b = 1 \text{ mm} \) was simulated with the nominal geometrical and mechanical properties listed in Table 5.2.

Fig. D.3(a) shows the predicted mean strength and variability for different levels of bundle discretization (changing the element length \( l_e \)). The model appears to reach convergence for the range of element sizes simulated, although both the strength and variability curves show small residual fluctuations. This effect is attributed to the stochastic nature of the simulation, rather than to the discretization level and therefore is more likely to be affected by the required precision of the Monte Carlo simulation, which is discussed below.

Fig. D.3(b) shows how the number of Monte Carlo simulation was roughly constant for all the element sizes, and the computational cost is a direct function of the average time per simulation, which increases by 20 times when varying between \( u = 0.05 \text{ mm} \) and \( u = 0.001 \text{ mm} \). A element size \( u = 0.005 \text{ mm} \) is used for all the simulations in Section 5.3 to assure a good compromise between precision and computational efficiency.

Fig. D.3(c)-(d) show the results of varying the size of the confidence interval for the mean strength \( \bar{X}_b \) in Eq. (5.10), while keeping the level of discretization constant (\( u = 0.005 \text{ mm} \)). Decreasing the size of the confidence interval decreases the random fluctuation in the results, particularly for the coefficient of variation, but also produces a strong increase in the required number of Monte Carlo simulations (Fig. D.3(d)). A minimum precision of 1% appears adequate to produce reliable results with good performances regarding computational time, and was used for all the simulations in Section 5.3.

D.4 Length scaling

To improve computational efficiency, strength results for large fibre bundles (true length \( l_b \)) are obtained by simulating a bundle model with the same number of fibres, and a representative bundle length \( l_s < l_b \). The simulation results are then scaled to the true bundle length using
Figure D.3: Numerical convergence study. (a) Predicted strength and coefficient of variation for different discretization levels (changing the element size $l_e$). (b) Computational cost for different discretization levels. (c) Predicted strength and coefficient of variation for different precisions of the interval of confidence $\hat{X}_b$ (variation of the maximum error $E_X$ allowed). (d) Computational cost for different precisions of the interval of confidence of $\hat{X}_b$.

The scaling is performed applying the Weakest Link Theory (WLT) [77, 94], which states that a chain of length $l_n$ composed by $n$ elements of length $l_0$ survives under a remote stress $\sigma_\infty$ only if each of the elements survives under $\sigma_\infty$. Hence, the failure probability $F_b(\sigma_\infty)$ (or strength cumulative distribution function) for a bundle of length $l_b$ follows

$$F_b(\sigma_\infty) = 1 - \left[1 - F_s(\sigma_\infty)\right]^{l_b/l_0}.$$ 

(D.11)
where $F_s(\sigma_\infty)$ is the failure probability for a bundle of length $l_s$ under the stress $\sigma_\infty$, which can be obtained directly from the bundle simulation.

Inside the general framework of the WLT, two different scaling methods are implemented: (i) scaling the strength distribution directly, or (ii) scaling the statistical parameters of the distribution. In case (i), being $X_s^1 \leq X_s^2 \leq \ldots \leq X_s^N \ldots \leq X_s^{N_{MC}}$ the predicted bundle strengths corresponding to each one of the $N_{MC}$ Monte Carlo simulations for a bundle of length $l_s$, the value of the failure probability is assigned to each strength value following

$$F_s^{N_{MC}} = (N - 1)/N_{MC},$$

then $F_b^{N_{MC}}$ is calculated applying Eq. (D.11).

In case (ii), the predicted strength distribution for each simulated bundle with length $l_s$ is fitted using a Weibull distribution with parameters $m^s$ and $X_0^s$ that correspond to the mean bundle strength $X_b^s$ and standard deviation $SD_b^s$. These parameters are then scaled to $l_b$ using WLT:

$$m^b = m^s$$

$$X_0^b = X_0^s \left( \frac{l_b}{l_s} \right)^{1/m^s}$$

Since, in general, the accuracy of the Monte Carlo simulation tends to converge much faster for the mean strength than for the standard deviation, using the latter to perform the scaling can introduce artificial noise in the scaled results for the mean strength. For these reasons the curve $SD_b^s(n_l)$ is fitted with a power law $a \cdot (n_l)^b$ before applying the scaling to smooth random fluctuations related to the convergence of the Monte Carlo simulations.

Fig. D.4 shows an overview of the different scaling approaches. Fig. D.4(a) presents the strength mean and variability for different simulation lengths $l_s$, while Fig. D.4(b)-(d) show the same results scaled to $l_b = 5$ mm using methods (i), (ii) and (ii) without fitting the standard deviation respectively, and compared with the non-scaled solution ($l_b = l_s$).

Scaling the entire distribution (Fig. D.4(b)) tends to overestimate the strength; this is attributed to the fact that the use of Eq. (D.11) causes loss of accuracy in the lower tail of the distribution.
On the contrary, scaling the statistical parameters seems to lead to a more precise prediction of the mean strength, especially if the standard deviation is fitted (Fig. D.4(c)). Throughout this document, scaling technique (ii) will be applied if the entire strength distribution is not required.

Finally, it is important to observe that, for bundles length of $l_b = 5$ mm, both the simulated results and the scaled results predict an upward trend of the mean bundle strength with the bundle size. Thus, this is not an artefact of the scaling technique but a true result of the model.

![Figure D.4: Length scaling. (a) Predicted strength and variability for different simulations lengths. (b) Results for $l_b = 5$ mm scaling the entire distribution. (c) Results for $l_b = 5$ mm scaling the statistical parameters with fitting the standard deviation. (d) Results for $l_b = 5$ mm scaling the statistical parameters without fitting the standard deviation.](image)
D.5 Implementation of Weibull of Weibull distribution for the fibre elements strength

To model the stochastic variability of the fibre strength following a Weibull of Weibull distribution [3], a random strength $X^{ij}$ is assigned to each fibre element in the bundle following

$$F_{el}(X^{ij}) = 1 - \exp \left( - \frac{X^{ij}}{X_{el,0}} \right)^{\rho_1}, \quad (D.14)$$

where $\rho_1$ and $X_{el,0}^{i}$ are the shape and scale parameters, respectively, for the strength distribution of an element of length $l_{el}$ which belongs to fibre $i$. The scale parameter $X_{el,0}^{i}$ changes from fibre to fibre in the bundle following a second Weibull distribution

$$F_{el}(X_{el,0}^{i}) = 1 - \exp \left[ - \frac{l_{el}}{l_{r}} \left( \frac{X_{el,0}^{i}}{X_{el,0}^{i}} \right)^{\rho_2} \right], \quad (D.15)$$

where $X_{el,0}^{i}$ and $\rho_2$ are the scale and shape parameters, respectively, for a fibre of reference length $l_{r}$.