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Anti-de Sitter Black Holes in Gauged $N = 8$ Supergravity

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ABSTRACT

We present new anti-de Sitter black hole solutions of gauged $N = 8$, $SO(8)$ supergravity, which is the massless sector of the $AdS_4 \times S^7$ vacuum of M-theory. By focusing on the $U(1)^4$ Cartan subgroup, we find non-extremal 1, 2, 3 and 4 charge solutions. In the extremal limit, they may preserve up to 1/2, 1/4, 1/8 and 1/8 of the supersymmetry, respectively. In the limit of vanishing $SO(8)$ coupling constant, the solutions reduce to the familiar black holes of the $M_4 \times T^7$ vacuum, but have very different interpretation since there are no winding states on S^7 and no U-duality. In contrast to the T^7 compactification, moreover, we find no static multi-center solutions. Also in contrast, the S^7 fields appear "already dualized" so that the 4 charges may be all electric or all magnetic rather than 2 electric and 2 magnetic. Curiously, however, the magnetic solutions preserve no supersymmetries. We conjecture that a subset of the extreme electric black holes preserving 1/2 the supersymmetry may be identified with the S^7 Kaluza-Klein spectrum, with the non-abelian $SO(8)$ quantum numbers provided by the fermionic zero modes.

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1 Introduction

The correspondence between anti-de Sitter space and conformal field theories on its boundary [1, 2, 3, 4] has revived an interest in gauged extended supergravities which arise as the massless sector of the Kaluza-Klein compactifications of $D = 11$ supergravity, such as $AdS_4 \times S^7$ and $AdS_7 \times S^4$ or Type IIB supergravity, such as $AdS_5 \times S^5$ [5]. Gauged $N = 8$ $D = 4$ supergravity [6, 7], which is the massless sector of the S^7 compactification, has also featured in a recent cosmological context with attempts to reconcile an open universe with inflation [8, 9, 10, 11]. Although this Kaluza-Klein compactification was the subject of much investigation in the past [12], relatively little effort has been devoted to the issue of black hole solutions of the gauged $N = 8$ theory[3]. This is the subject of the present paper.

Although both are maximally symmetric, the S^7 compactification differs from the T^7 in several important respects. First of all, the global $SO(8)$ is promoted to a gauge symmetry. Secondly, the underlying supersymmetry algebra is no longer Poincare but rather AdS_4 and the Lagrangian has a non-vanishing cosmological constant Λ proportional to the square of the gauge coupling constant g:

$$G\Lambda \sim -g^2, \tag{1.1}$$

where G is Newton's constant. Consequently, we shall be seeking black hole solutions that are asymptotically AdS rather than Minkowski. We also face the difference that the gauge group is non-abelian. By focusing on the $U(1)^4$ Cartan subgroup, we find non-extremal 1, 2, 3 and 4 charge solutions. In the extremal limit they may preserve up to 1/2, 1/4, 1/8 and 1/8 of the supersymmetry, respectively. In the limit of vanishing $SO(8)$ coupling constant, the solutions reduce to the familiar black holes of the $M_4 \times T^7$ vacuum, but have very different interpretation since there are no winding states on S^7 and no U-duality. In contrast to the T^7 compactification, moreover, we find no static multi-center solutions. Also in contrast, the S^7 fields appear "already dualized" so that the 4 charges may be all electric or all magnetic rather than 2 electric and 2 magnetic. Curiously, however, the magnetic solutions preserve no supersymmetries.

Previous papers [16, 17, 18] have explored the possibility that the BPS spectrum of toroidally compactified string and M theory, and in particular the Kaluza-Klein modes, could be identified with extreme black hole solutions of the low-energy supergravity theory. It was found that this identification was consistent not only with the mass and charge spectrum [16] but also with the spins and supermultiplet structure [19, 20, 21] and with the dipole moments and gyromagnetic ratios [22]. In a similar spirit, we here conjecture that a subset of the AdS electric black hole solutions preserving half the supersymmetry may be identified with the S^7 Kaluza-Klein spectrum [23, 24, 12] with the non-abelian $SO(8)$ quantum numbers provided by the fermionic zero modes.

[3] BPS black holes arising in the $SU(2)\times SU(2)$ version of gauged $(N = 4, D = 4)$ supergravity, which is the massless sector of the $S^3 \times S^3$ compactification of $(N = 1, D = 10)$ supergravity, have recently been discussed in [13]. Our solutions will be significantly different from these. In particular ours are asymptotically AdS while those of [13] are asymptotically neither AdS nor Minkowski. Additionally, both BPS and non-BPS black hole solutions in gauged $(N = 2, D = 5)$ supergravity were examined in [14] and [15] respectively.

2 $N = 8$ gauged supergravity

We follow the conventions of [6, 7], and denote the fields of the massless $N = 8$ supergravity multiplet by $(e^\alpha_\mu, \psi^i_\mu, A^{[IJ]}_\mu, \chi^{[ijk]}, \mathcal{V}_{[ij]}{}^{[IJ]})$, where i, j are $SU(8)$ indices and I, J are $SO(8)$ indices. The 70 real scalar degrees of freedom are represented by the 56-bein

$$\mathcal{V} = \begin{pmatrix} u_{ij}{}^{IJ} & v_{ijKL} \\ v^{klIJ} & u^{kl}{}_{KL} \end{pmatrix}, \tag{2.1}$$

transforming under local $SU(8)$ and rigid E_7. In the gauged supergravity theory, the 28 vectors A^{IJ}_μ transform in the adjoint of $SO(8)$, with resulting non-abelian field strengths $F^{IJ}_{\mu\nu} = 2(\partial_{[\mu} A^{IJ}_{\nu]} - g A^{IK}_{[\mu} A^{KJ}_{\nu]})$. We also define the fully $SO(8) \times SU(8)$ covariant derivative as, for example, $D_\mu \varphi_i{}^I = \nabla_\mu \varphi_i{}^I - \frac{1}{2}\mathcal{B}^j_{\mu\, i}\varphi_j{}^I - g A^{IJ}_\mu \varphi_i{}^J$. Here $\mathcal{B}^i_{\mu\, j}$ is a composite $SU(8)$ connection, defined along with the scalar kinetic terms $\mathcal{A}^{ijkl}_\mu$ according to the condition

$$D_\mu \mathcal{V}\mathcal{V}^{-1} = -\frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & \mathcal{A}^{ijkl}_\mu \\ \mathcal{A}_{\mu\, ijkl} & 0 \end{pmatrix}. \tag{2.2}$$

Note that here D_μ is the fully covariant derivative, so that $\mathcal{B}^i_{\mu\, j}$ is defined indirectly by the vanishing of the diagonal blocks in (2.2).

While the complete gauged $N = 8$ Lagrangian is rather involved [7], the bosonic part is fairly standard, and may be written as

$$\mathcal{L} = \frac{1}{2\kappa^2}\sqrt{-g}\left[R - \frac{1}{2\cdot 4!}\mathcal{A}^{ijkl}_\mu \mathcal{A}^\mu_{ijkl} - \frac{1}{4}(F^+_{\mu\nu\, IJ}(2S^{IJ,KL} - \delta^{IJ}_{KL})F^{+\,\mu\nu}{}_{KL} + \text{h.c.}) - V\right], \tag{2.3}$$

where $S^{IJ,KL}$ is defined in terms of the scalars through the condition $(u^{ij}{}_{IJ} + v^{ijIJ})S^{IJ,KL} = u^{ij}{}_{KL}$, and $F^+_{\mu\nu}$ is the self-dual part of $F_{\mu\nu}$. Finally, the potential arises from the $SO(8)$ gauging, and is given by

$$V = -2g^2[\tfrac{3}{4}|A_1{}^{ij}|^2 - \tfrac{1}{24}|A_{2\,i}{}^{jkl}|^2], \tag{2.4}$$

where $A^{ij}_1 = \frac{4}{21}T_k{}^{ikj}$ and $A_{2\,i}{}^{jkl} = -\frac{4}{3}T_i{}^{[jkl]}$ and $T_i{}^{jkl}$ is the T-tensor of [7]:

$$T_i^{jkl} = (u^{kl}{}_{IJ} + v^{klIJ})(u_{im}{}^{JK}u^{jm}{}_{KI} - v_{imJK}v^{jmKI}). \tag{2.5}$$

In a purely bosonic background, the supersymmetry transformations of the fermions are given by

$$\begin{aligned} \tfrac{1}{2}\delta\psi^i_\mu &= D_\mu\epsilon^i + \tfrac{1}{\sqrt{2}}[\tfrac{1}{4}\overline{F}^{-\,ij}_{\nu\lambda}\gamma^{\nu\lambda} - gA^{ij}_1]\gamma_\mu\epsilon_j, \\ \delta\chi^{ijk} &= -\tfrac{1}{2}\mathcal{A}^{ijkl}_\mu\gamma^\mu\epsilon_l + [\tfrac{3}{2}\gamma^{\mu\nu}\overline{F}^{-\,[ij}_{\mu\nu}\delta^{k]}_l - 2gA_{2\,l}{}^{ijk}]\epsilon^l, \end{aligned} \tag{2.6}$$

where $D_\mu\epsilon^i = \nabla_\mu\epsilon^i + \frac{1}{2}\mathcal{B}^i_{\mu\, j}\epsilon^j$. Note that the gauge fields enter the supersymmetry transformations with scalar factors, since (to lowest order) $\overline{F}_{\mu\nu}$ is defined through $F_{\mu\nu}{}^{IJ} = (u_{ij}{}^{IJ} + v_{ijIJ})\overline{F}_{\mu\nu}{}^{ij}$.

In the case of ungauged $N = 8$ supergravity, black holes are completely characterized by 28 electric and 28 magnetic charges under the $U(1)$ gauge fields. In the present case, however, the gauge group is non-abelian, and hence the situation is less clear. In order to proceed, we work in an abelian truncation of the gauged $N = 8$ theory by focusing only on the $U(1)^4$ Cartan subgroup of $SO(8)$. In particular, we choose the Cartan generators to correspond to adjacent index pairs:

$$\{A_\mu^{12}, \quad A_\mu^{34}, \quad A_\mu^{56}, \quad A_\mu^{78}\}, \tag{2.7}$$

and set the remaining gauge fields to zero. Note that while in principle it is important to check that this provides a consistent truncation, in practice as long as the supersymmetry variations (at least partially) vanish the state is essentially ensured to be BPS.

For the scalars, we work in symmetric gauge [25, 26] where the 56-bein may be written as

$$\mathcal{V} = \exp\left\{-\frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & \phi_{ijkl} \\ \phi^{mnpq} & 0 \end{pmatrix}\right\}, \tag{2.8}$$

with ϕ^{ijkl} self-dual. Let us denote $SO(8)$ index pairs $\{12, 34, 56, 78\}$ by (α) where $\alpha = 1, \ldots, 4$. Specializing to real scalars, we are lead to the following ansatz:

$$\phi^{ijkl} = \phi_{ijkl} = \sqrt{2}[\phi^{(12)}(\epsilon^{(12)} + \epsilon^{(34)}) + \phi^{(13)}(\epsilon^{(13)} + \epsilon^{(24)}) + \phi^{(14)}(\epsilon^{(14)} + \epsilon^{(23)})]_{ijkl}, \tag{2.9}$$

where numbers in parentheses correspond to appropriate index pairs so that *e.g.* $\epsilon_{ijkl}^{(13)} = \pm 1$ whenever $\{i, j, k, l\}$ corresponds to a permutation of $\{1, 2, 5, 6\}$. This ansatz is of course self-dual by construction. Thus we have reduced the original 70 (real) scalar degrees of freedom to just three in this specialization. For this case, using the definition (2.2), we find that the $SU(8)$ connection and scalar kinetic terms become[4]

$$\mathcal{B}_\mu{}^i{}_j = -2gA_\mu^{ij}, \qquad \mathcal{A}_\mu{}^{ijkl} = \partial_\mu \phi^{ijkl}. \tag{2.10}$$

When restricted to the abelian $U(1)$ gauge fields, and with the scalar ansatz (2.9), the bosonic lagrangian, (2.3), may be rewritten as

$$\begin{aligned} \mathcal{L} \;=\; & \frac{1}{2\kappa^2}\sqrt{-g}\Big[R - \tfrac{1}{2}\left((\partial_\mu\phi^{(12)})^2 + (\partial_\mu\phi^{(13)})^2 + (\partial_\mu\phi^{(14)})^2\right) - V \\ & -2\left(e^{-\lambda_1}(F_{\mu\nu}^{(1)})^2 + e^{-\lambda_2}(F_{\mu\nu}^{(2)})^2 + e^{-\lambda_3}(F_{\mu\nu}^{(3)})^2 + e^{-\lambda_4}(F_{\mu\nu}^{(4)})^2\right)\Big], \end{aligned} \tag{2.11}$$

where the scalar combinations $\{\lambda\}$ are given by

$$\begin{aligned} \lambda_1 &= -\phi^{(12)} - \phi^{(13)} - \phi^{(14)}, \\ \lambda_2 &= -\phi^{(12)} + \phi^{(13)} + \phi^{(14)}, \\ \lambda_3 &= \phi^{(12)} - \phi^{(13)} + \phi^{(14)}, \\ \lambda_4 &= \phi^{(12)} + \phi^{(13)} - \phi^{(14)}, \end{aligned} \tag{2.12}$$

and the scalar potential is

$$V = -4g^2\left(\cosh\phi^{(12)} + \cosh\phi^{(13)} + \cosh\phi^{(14)}\right). \tag{2.13}$$

[4]Note that $SU(8)$ and $SO(8)$ indices are indistinguishable here. This is a consequence of specializing to a particular gauge choice for the scalars.

Note that the $\{\lambda\}$ are not all independent as $\lambda_1+\lambda_2+\lambda_3+\lambda_4=0$. The $U(1)$ gauge fields in (2.11) are essentially the $SO(8)$ triality rotated combinations

$$\begin{pmatrix} F^{(1)}_{\mu\nu} \\ F^{(2)}_{\mu\nu} \\ F^{(3)}_{\mu\nu} \\ F^{(4)}_{\mu\nu} \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} F^{12}_{\mu\nu} \\ F^{34}_{\mu\nu} \\ F^{56}_{\mu\nu} \\ F^{78}_{\mu\nu} \end{pmatrix} \equiv \frac{1}{2}\Omega\begin{pmatrix} F^{12}_{\mu\nu} \\ F^{34}_{\mu\nu} \\ F^{56}_{\mu\nu} \\ F^{78}_{\mu\nu} \end{pmatrix}. \tag{2.14}$$

For later convenience we have defined the matrix Ω, which satisfies $\Omega=\Omega^T$ and $\Omega^2=I$.

Several comments are in order here. The first is that, save for the potential, (2.13), and numerical factors in the definition of the scalars, the truncated bosonic action (2.11) is identical to that of a closed string compactified on T^2 with "diagonal" scalars[5]. While on the one hand this may not be too surprising, since supersymmetry must necessarily constrain the couplings between the scalars and vectors, on the other hand it is somewhat remarkable since the four $U(1)$ fields have rather different interpretations in the two cases: as the Cartan generators of $SO(8)$ for the gauged supergravity, and as two Kaluza-Klein and two winding gauge fields for the T^2 compactification. Following up on this correspondence, the second point is that we have *a priori* constrained the three scalars $\phi^{(12)}$, $\phi^{(13)}$ and $\phi^{(14)}$ to be real. We believe that allowing the scalars to be complex in (2.9) would in fact lead to a complete correspondence between (2.11) and the general T^2 compactified effective lagrangian with three complex scalars. Nevertheless, this brings up the issue that the additional scalar degrees of freedom do play a role in terms of giving rise to additional conditions on the bosonic solutions in order to maintain a consistent truncation. Finally, it is important to realize that when $g\neq 0$ the potential (2.13) may not be ignored, and fixes the asymptotic scalar values to vanish, $\phi^{(12)}_\infty=\phi^{(13)}_\infty=\phi^{(14)}_\infty=0$, with corresponding negative energy density $V_\infty=-12g^2$.

A slight notational complication arises in expressing the supersymmetry variations (2.6) in terms of explicit field components (as opposed to fully $SO(8)$ invariant quantities). As above, we denote $SO(8)$ index pairs $\{12,34,56,78\}$ by (α) where $\alpha=1,\ldots,4$, in which case the single $SO(8)$ index $i=1,\ldots,8$ may be replaced by the combination $i_{(\alpha)}$ with the latter i taking on either 1 or 2 corresponding to the first or the second of the pair (α). In this case the gravitino variation may be written as

$$\begin{aligned} \delta\psi^{i_{(\alpha)}}_\mu &= \nabla_\mu\epsilon^{i_{(\alpha)}} - 2g\Omega_{\alpha\beta}A^{(\beta)}_\mu\epsilon_{ij}\epsilon^{j_{(\alpha)}} + \frac{g}{4\sqrt{2}}\left(e^{\lambda_1/2}+e^{\lambda_2/2}+e^{\lambda_3/2}+e^{\lambda_4/2}\right)\gamma_\mu\epsilon_{i_{(\alpha)}} \\ &\quad + \frac{1}{2\sqrt{2}}\Omega_{\alpha\beta}e^{-\lambda_\beta/2}F^{(\beta)}_{\nu\lambda}\gamma^{\nu\lambda}\gamma_\mu\epsilon^{ij}\epsilon_{j_{(\alpha)}}, \end{aligned} \tag{2.15}$$

where the sum over β is implied. For the spin-1/2 fermions and the Cartan ansatz, we find immediately that $\delta\chi^{ijk}$ vanishes unless exactly two indices belong in the same pair (α). Writing the first two indices as paired, we find

$$\delta\chi^{(\alpha)i_{(\beta)}} = -\tfrac{1}{\sqrt{2}}\gamma^\mu\partial_\mu\phi^{(\alpha\beta)}\epsilon^{ij}\epsilon_{j_{(\beta)}} - g\Sigma_{\alpha\beta\gamma}\Omega_{\gamma\delta}e^{\lambda_\delta/2}\epsilon_{ij}\epsilon^{j_{(\beta)}} + \Omega_{\alpha\delta}e^{-\lambda_\delta/2}F^{(\delta)}_{\mu\nu}\gamma^{\mu\nu}\epsilon^{i_{(\beta)}}, \tag{2.16}$$

[5]Note that to make the actual correspondence, two of the gauge fields (say $F^{(2)}$ and $F^{(4)}$) need to be dualized to provide a consistent identification with the string dilaton (in this case $\phi^{(13)}$). Of course the choice of fields to dualize is only determined up to string-string-string triality [19].

provided $(\alpha) \neq (\beta)$. The tensor $\Sigma_{\alpha\beta\gamma}$ selects out a particular (γ) depending on $(\alpha\beta)$, and is defined by

$$\Sigma_{\alpha\beta\gamma} = \begin{cases} |\epsilon_{\alpha\beta\gamma}|, & \text{for } \alpha, \beta \neq 1 \\ \delta_{\beta\gamma}, & \text{for } \alpha = 1 \\ \delta_{\alpha\gamma}, & \text{for } \beta = 1. \end{cases} \tag{2.17}$$

3 The $a = \sqrt{3}$ black hole in gauged supergravity

In the previous section we have described a truncation of gauged $N = 8$ supergravity to its abelian $U(1)^4$ sector. The resulting lagrangian and corresponding fermion supersymmetry may be further simplified by focusing on single-charge black hole solutions. In particular, we take $\phi^{(12)} = \phi^{(13)} = \phi^{(14)} \equiv \phi$ with corresponding $F^{(1)}_{\mu\nu} \equiv F_{\mu\nu}$ non-vanishing. The resulting lagrangian then becomes

$$\mathcal{L} = \frac{1}{2\kappa^2}\sqrt{-g}\left[R - \frac{3}{2}\partial_\mu\phi\partial^\mu\phi - 2e^{3\phi}F_{\mu\nu}F^{\mu\nu} + 12g^2\cosh\phi\right] + \ldots, \tag{3.1}$$

where the dots refer to fields that will be set equal to zero in our solution. Note that a scaling $\phi \to \phi/\sqrt{3}$ may be performed to normalize the scalar kinetic term canonically, which also demonstrates the correspondence to the conventional $a = \sqrt{3}$ definition, where a is the scalar-Maxwell parameter appearing in $e^{a\phi}F_{\mu\nu}F^{\mu\nu}$.

For the single-charge solution, it is straightforward to see that the supersymmetry transformations, (2.15) and (2.16), essentially take on multiple identical copies, one for each different (α) value. It is thus sufficient to focus on a single supersymmetry parameter, *e.g.* $\epsilon^{i_{(1)}}$, instead of the more general $\epsilon^{i_{(\alpha)}}$. Note that this also demonstrates how such a solution can be interpreted in an $N = 2$ or $N = 4$ context. The resulting supersymmetry variations are

$$\begin{aligned} \tfrac{1}{2}\delta\psi^i_\mu &= \nabla_\mu\epsilon^i - gA_\mu\epsilon_{ij}\epsilon^j + \frac{g}{4\sqrt{2}}\left(e^{-3\phi/2} + 3e^{\phi/2}\right) + \frac{1}{4\sqrt{2}}e^{3\phi/2}F_{\nu\lambda}\gamma^{\nu\lambda}\gamma_\mu\epsilon^{ij}\epsilon_j, \\ \delta\chi^i &= -\frac{1}{\sqrt{2}}\gamma^\mu\partial_\mu\phi\epsilon^{ij}\epsilon_j - \frac{g}{2}\left(e^{-3\phi/2} - e^{\phi/2}\right)\epsilon_{ij}\epsilon^j + \frac{1}{2}e^{3\phi/2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon^i, \end{aligned} \tag{3.2}$$

where we have dropped unnecessary $SO(8)$ index-pair symbols.

Using the well known $a = \sqrt{3}$ black hole solution as a guide, we now consider a spherically symmetric electric black hole ansatz. Before describing the black hole in gauged supergravity, we first observe that when $g = 0$ the above lagrangian (3.1) admits an ordinary $a = \sqrt{3}$ (electric) black hole solution [16]:

$$ds^2 = -H^{-1/2}dt^2 + H^{1/2}(dr^2 + r^2d\Omega^2), \tag{3.3}$$

where $H = 1 + Q/r$, and with the scalar and gauge field given (in the above normalization) by

$$e^{2(\phi-\phi_\infty)} = H, \qquad A_0 = \frac{\eta}{2\sqrt{2}}e^{-3\phi_\infty/2}H^{-1} \tag{3.4}$$

($\eta = \pm 1$ sets the actual sign of the charge). Furthermore, we recall that it was demonstrated in [27, 14, 15] that extreme black hole solutions generally have simple extensions to the case of non-zero gauging, and in fact generally retain their supersymmetry properties. Following [14, 15], we thus take the metric ansatz

$$ds^2 = -e^{2A} f dt^2 + e^{-2A}(\frac{dr^2}{f} + r^2 d\Omega^2), \tag{3.5}$$

corresponding to an AdS generalization of (3.3). Note in particular that the vacuum AdS solution is given by the choice $f = 1 + 2g^2r^2$ with $A = 0$.

With the metric (3.5) the gauge equation of motion becomes $\partial_r(e^{-2A+3\phi}r^2\partial_r A_0) = 0$ and is unaffected by both g and the function f. This suggests that the harmonic function ansatz, (3.4), simply carries over to the $g \neq 0$ case, with the only additional constraint that $\phi_\infty = 0$. Turning to the supersymmetry variation, $\delta\chi^i$, we find, using (3.4), that

$$\delta\chi^i = -\frac{1}{2\sqrt{2}}\frac{\partial_r H}{H}\epsilon^{ij}\gamma^r\left[\epsilon_j + f^{-1/2}(\eta\gamma_{\overline{0}}\epsilon_{jk}\epsilon^k + \sqrt{2}grH^{1/2}\gamma_r\epsilon^j)\right], \tag{3.6}$$

so that the natural half-supersymmetry projection is given by

$$P_\eta^{ij} = \tfrac{1}{2}[\delta^{ij} + f^{-1/2}(\eta\gamma_{\overline{0}}\epsilon_{ij} + \sqrt{2}grH^{1/2}\gamma_{\overline{r}}\delta^{ij})] \tag{3.7}$$

(acting on real spinors $\epsilon^i = \epsilon_i$) provided $f = 1 + 2g^2r^2H$. Using this expression for f, it is now straightforward to check that all bosonic equations of motion arising from (3.1) are satisfied. To summarize, the single-charge black hole solution in gauged supergravity is given by

$$\begin{aligned} ds^2 &= -H^{-1/2} f dt^2 + H^{1/2}(\frac{dr^2}{f} + r^2 d\Omega^2), \\ e^{2\phi} &= H, \qquad A_0 = \frac{\eta}{2\sqrt{2}}H^{-1}, \end{aligned} \tag{3.8}$$

where

$$H = 1 + \frac{Q}{r}, \qquad f = 1 + 2g^2r^2H. \tag{3.9}$$

We now turn to an examination of the supersymmetry properties of this solution. In addition to $\delta\chi^i$ given above:

$$\delta\chi^i = \frac{Q}{\sqrt{2}r^2}H^{-1}\epsilon^{ij}\gamma^r P_\eta\epsilon_j, \tag{3.10}$$

the gravitino variations in the background (3.8) are given by

$$\begin{aligned} \tfrac{1}{2}\delta\psi_0^i &= \partial_0\epsilon^i + \frac{g\eta}{2\sqrt{2}}\epsilon_{ij}\epsilon^j + \frac{g}{\sqrt{2}}H^{-1}f^{1/2}(1+H)\gamma_{\overline{0}}P_\eta\epsilon_i + \frac{Q}{4r^2}H^{-3/2}f\gamma_{\overline{0r}}P_\eta\epsilon^i, \\ \tfrac{1}{2}\delta\psi_r^i &= (\partial_r - \frac{Q}{8r^2}H^{-1})\epsilon^i + \frac{g}{2\sqrt{2}}H^{-1/2}f^{-1/2}(1+H)\gamma_{\overline{r}}\epsilon_i + \frac{Q}{4r^2}H^{-1}P_\eta\epsilon^i, \\ \tfrac{1}{2}\delta\psi_\theta^i &= \partial_\theta\epsilon^i - \frac{\eta}{2}\gamma_{\overline{0\theta r}}\epsilon_{ij}\epsilon_j + (1 - \frac{Q}{4r}H^{-1})f^{1/2}\gamma_{\overline{\theta r}}P_\eta\epsilon^i, \\ \tfrac{1}{2}\delta\psi_\phi^i &= \partial_\phi\epsilon^i - \frac{\eta}{2}\sin\theta\gamma_{\overline{0\phi r}}\epsilon_{ij}\epsilon_j + \frac{1}{2}\cos\theta\gamma_{\overline{\phi\theta}}\epsilon^i + (1 - \frac{Q}{4r}H^{-1})f^{1/2}\sin\theta\gamma_{\overline{\phi r}}P_\eta\epsilon^i. \end{aligned} \tag{3.11}$$

For Killing spinors, $P_\eta \epsilon_i = 0$, we may follow the construction of [27, 14, 15], to obtain

$$\epsilon = e^{-\frac{g\eta t}{2\sqrt{2}}\epsilon_{ij}} H^{-1/8} \left[\sqrt{f^{1/2}+1} - \sqrt{f^{1/2}-1}\gamma_{\overline{r}}\right] e^{-\frac{1}{2}\gamma_{\overline{\theta r}}} e^{-\frac{1}{2}\gamma_{\overline{\phi\theta}}}(1-\eta\gamma_{\overline{0}}\epsilon^{ij})\epsilon_0, \tag{3.12}$$

so that this solution in fact preserves exactly half of the supersymmetries. Note that substituting $\epsilon_0 = \frac{1}{\sqrt{2}}(1-\gamma_{\overline{r}})\tilde{\epsilon}_0$ and using the identity $\sqrt{f^{1/2}+1} \pm \sqrt{f^{1/2}-1} = \sqrt{2(f^{1/2} \pm (f-1)^{1/2})}$ indicates that (3.12) may be rewritten equivalently as

$$\epsilon = e^{-\frac{g\eta t}{2\sqrt{2}}\epsilon_{ij}} H^{-1/8} \left[\sqrt{f^{1/2}+(f-1)^{1/2}} - \eta\sqrt{f^{1/2}-(f-1)^{1/2}}\gamma_{\overline{0}}\epsilon^{ij}\right] e^{\frac{\eta}{2}\gamma_{\overline{0\theta r}}\epsilon_{ij}} e^{-\frac{1}{2}\gamma_{\overline{\phi\theta}}}(1-\gamma_{\overline{r}})\tilde{\epsilon}_0, \tag{3.13}$$

which is the form that appears in [27]. Although we have focused on $N = 8$ in the present case, this Killing spinor construction is general, and also applies in the $N = 2$ and $N = 4$ truncations of the full $N = 8$ theory.

When written in the form (3.12), the above Killing spinors resemble a supersymmetry projected version of the corresponding Killing spinors in pure Anti-de Sitter space[6]:

$$\epsilon(AdS) = \left[\sqrt{f^{1/2}+1} - \sqrt{f^{1/2}-1}\gamma_{\overline{r}}\right] e^{-\frac{1}{2}\gamma_{\overline{\theta r}}} e^{-\frac{1}{2}\gamma_{\overline{\phi\theta}}} e^{-\frac{gt}{\sqrt{2}}\gamma_{\overline{0}}}\epsilon_0. \tag{3.14}$$

Additionally, the Killing spinors may be contrasted with those arising in the ungauged theory. Taking $g \to 0$, we find

$$\epsilon(g=0) = H^{-1/8} e^{-\frac{1}{2}\gamma_{\overline{\theta r}}} e^{-\frac{1}{2}\gamma_{\overline{\phi\theta}}}(1-\eta\gamma_{\overline{0}}\epsilon^{ij})\epsilon_0, \tag{3.15}$$

which satisfies the well known Killing spinor condition $P^0_\eta \epsilon(g=0) \equiv \frac{1}{2}(1+\eta\gamma_{\overline{0}}\epsilon^{ij})\epsilon(g=0) = 0$. An added consequence of this simple structure in the $g \to 0$ case is that the fermion zero modes are easily constructed solely by changing the sign of the projection in (3.15). Such zero modes are immediately orthogonal to the Killing spinors and furthermore satisfy the supergauge condition $\gamma^\mu \delta\psi^i_\mu = 0$ [28, 29]. Unfortunately this situation is not as clear when $g \neq 0$; this is mainly due to complications arising from the nature of the projection (3.7) in the gauged supergravity. For this reason, although it is manifest that this black hole preserves exactly half of the supersymmetries, its supermultiplet structure arising from the fermion zero mode construction [29, 22] is less well understood. On the other hand, the fact that it preserves half the supersymmetry presumably means that it belongs to the short maximum spin 2 supermultiplet.

4 Multiple charge black holes

Returning to the complete simplified $N = 8$ lagrangian (2.11) we note that, in the absence of the scalar potential, this admits well known supersymmetric black hole solutions with up to four charges. Since the single charge solution has a straightforward generalization for $g \neq 0$, as shown above, one may wonder whether this is also true for the four charge solution. A

[6] Some of the difference in the t-dependent terms may be eliminated by a suitable gauge transformation. This issue also arises in the next section when considering the multiple charge black hole.

careful examination of the equations of motion arising from (2.11) shows that this is in fact the case. In contrast to the usual form of the action arising from T^2 compactification of the closed string (which may be written in either S, T or U form [19]), in which a dilaton scalar is singled out, the Lagrangian (2.11) treats all three scalars and four gauge fields symmetrically. In practice, this indicates that we are interested in a four electric charge black hole solution. We find

$$\begin{aligned}
ds^2 &= -(H_1H_2H_3H_4)^{-1/2} f dt^2 + (H_1H_2H_3H_4)^{1/2}(\frac{dr^2}{f} + r^2 d\Omega^2),\\
e^{2\phi^{(12)}} &= \frac{H_1H_2}{H_3H_4}, \qquad e^{2\phi^{(13)}} = \frac{H_1H_3}{H_2H_4}, \qquad e^{2\phi^{(14)}} = \frac{H_1H_4}{H_2H_3},\\
A_0^{(\alpha)} &= \frac{\eta_\alpha}{2\sqrt{2}} H_\alpha^{-1},
\end{aligned} \tag{4.1}$$

where

$$H_\alpha = 1 + \frac{Q_\alpha}{r}, \qquad f = 1 + 2g^2 r^2 (H_1H_2H_3H_4). \tag{4.2}$$

From the decomposition of the $N = 8$ spinor parameter ϵ_i into the four $\epsilon_{i_{(\alpha)}}$ where $\alpha = 1, \ldots, 4$, we see that the $N = 8$ supersymmetry variations, (2.15) and (2.16), break up into four sets, involving separate $\pm$ signs in the combination of the field strengths. Focusing on a single set of variations, we find

$$\begin{aligned}
\tfrac{1}{2}\delta\psi_\mu^{i_{(1)}} &= \nabla_\mu \epsilon^{i_{(1)}} - g(A_\mu^{(1)} + A_\mu^{(2)} + A_\mu^{(3)} + A_\mu^{(4)})\epsilon_{ij}\epsilon^{j_{(1)}}\\
&\quad + \frac{g}{4\sqrt{2}}\left(e^{\lambda_1/2} + e^{\lambda_2/2} + e^{\lambda_3/2} + e^{\lambda_4/2}\right)\gamma_\mu \epsilon_{i_{(1)}}\\
&\quad + \frac{1}{4\sqrt{2}}\left(e^{-\lambda_1/2}F_{\nu\lambda}^{(1)} + e^{-\lambda_2/2}F_{\nu\lambda}^{(2)} + e^{-\lambda_3/2}F_{\nu\lambda}^{(3)} + e^{-\lambda_4/2}F_{\nu\lambda}^{(4)}\right)\gamma^{\nu\lambda}\gamma_\mu \epsilon^{ij}\epsilon_{j_{(1)}},\\
\delta(2\chi^{(3)i_{(1)}}) &= -\sqrt{2}\gamma^\mu \partial_\mu \phi^{(13)}\epsilon^{ij}\epsilon_{j_{(1)}} - g\left((e^{\lambda_1/2} + e^{\lambda_3/2}) - (e^{\lambda_2/2} + e^{\lambda_4/2})\right)\epsilon_{ij}\epsilon^{j_{(1)}}\\
&\quad + \left((e^{-\lambda_1/2}F_{\mu\nu}^{(1)} + e^{-\lambda_3/2}F_{\mu\nu}^{(3)}) - (e^{-\lambda_2/2}F_{\mu\nu}^{(2)} + e^{-\lambda_4/2}F_{\mu\nu}^{(4)})\right)\gamma^{\mu\nu}\epsilon^{i_{(1)}},\\
\delta(\chi^{(2)i_{(1)}} + \chi^{(4)i_{(1)}}) &= -\tfrac{1}{\sqrt{2}}\gamma^\mu\partial_\mu(\phi^{(12)} + \phi^{(14)})\epsilon^{ij}\epsilon_{j_{(1)}} - g\left(e^{\lambda_1/2} - e^{\lambda_3/2}\right)\epsilon_{ij}\epsilon^{j_{(1)}}\\
&\quad + \left(e^{-\lambda_1/2}F_{\mu\nu}^{(1)} - e^{-\lambda_3/2}F_{\mu\nu}^{(3)}\right)\gamma^{\mu\nu}\epsilon^{i_{(1)}},\\
\delta(\chi^{(2)i_{(1)}} - \chi^{(4)i_{(1)}}) &= -\tfrac{1}{\sqrt{2}}\gamma^\mu\partial_\mu(\phi^{(12)} - \phi^{(14)})\epsilon^{ij}\epsilon_{j_{(1)}} - g\left(e^{\lambda_2/2} - e^{\lambda_4/2}\right)\epsilon_{ij}\epsilon^{j_{(1)}}\\
&\quad + \left(e^{-\lambda_2/2}F_{\mu\nu}^{(2)} - e^{-\lambda_4/2}F_{\mu\nu}^{(4)}\right)\gamma^{\mu\nu}\epsilon^{i_{(1)}}.
\end{aligned} \tag{4.3}$$

The choice of the particular linear combinations of the spin-1/2 supersymmetry variations used above is motivated by the correspondence to the $N = 4$ theory arising from T^2 compactification:

$$\begin{aligned}
2\chi^{(3)} &\longleftrightarrow \lambda \text{ (dilatino)}\\
\chi^{(2)} \pm \chi^{(4)} &\longleftrightarrow \tilde{\chi}^{1,2} \text{ (gauginos)}\\
\phi^{(13)} &\longleftrightarrow \eta \text{ (dilaton)}\\
\phi^{(12)}, \phi^{(14)} &\longleftrightarrow \rho, \sigma \text{ (internal } T^2 \text{ metric).}
\end{aligned} \tag{4.4}$$

For the four-charge solution (4.1), we find that preserving supersymmetry (in the $\epsilon_{i_{(1)}}$ sector) demands a particular choice of signs for the charges, $\eta_1 = \eta_2 = \eta_3 = \eta_4$ $(= \eta)$, in which case we find

$$
\begin{aligned}
\delta(2\chi^{(3)i_{(1)}}) &= -\sqrt{2}\epsilon^{ij}\gamma^r\partial_r \log\frac{H_1H_3}{H_2H_4}P_\eta\epsilon_{j_{(1)}},\\
\delta(\chi^{(2)i_{(1)}}+\chi^{(4)i_{(1)}}) &= -\sqrt{2}\epsilon^{ij}\gamma^r\partial_r \log\frac{H_1}{H_3}P_\eta\epsilon_{j_{(1)}},\\
\delta(\chi^{(2)i_{(1)}}-\chi^{(4)i_{(1)}}) &= -\sqrt{2}\epsilon^{ij}\gamma^r\partial_r \log\frac{H_2}{H_4}P_\eta\epsilon_{j_{(1)}},
\end{aligned}
\tag{4.5}
$$

where

$$
P_\eta^{ij} = \tfrac{1}{2}[\delta^{ij} + f^{-1/2}(\eta\gamma_{\overline{0}}\epsilon_{ij} + \sqrt{2}gr\mathcal{H}^{1/2}\gamma_{\overline{r}}\delta^{ij})],
\tag{4.6}
$$

with $\mathcal{H} = H_1H_2H_3H_4$. For the four charge solution the gravitino variations are

$$
\begin{aligned}
\tfrac{1}{2}\delta\psi_0^{i_{(1)}} &= \partial_0\epsilon^{i_{(1)}} - \frac{g\eta}{\sqrt{2}}\epsilon_{ij}\epsilon^{j_{(1)}} + \frac{g}{\sqrt{2}}f^{1/2}(2+r\partial_r\log\mathcal{H})\gamma_{\overline{0}}P_\eta\epsilon_{i_{(1)}} - \frac{1}{4}\mathcal{H}^{-1/2}f\partial_r\log\mathcal{H}\gamma_{\overline{0r}}P_\eta\epsilon^{i_{(1)}},\\
\tfrac{1}{2}\delta\psi_r^{i_{(1)}} &= (\partial_r + \frac{1}{8}\partial_r\log\mathcal{H})\epsilon^{i_{(1)}} + \frac{g}{2\sqrt{2}}\mathcal{H}^{1/2}f^{-1/2}(2+r\partial_r\log\mathcal{H})\gamma_{\overline{r}}\epsilon_{i_{(1)}} - \frac{1}{4}\partial_r\log\mathcal{H}P_\eta\epsilon^{i_{(1)}},\\
\tfrac{1}{2}\delta\psi_\theta^{i_{(1)}} &= \partial_\theta\epsilon^{i_{(1)}} - \frac{\eta}{2}\gamma_{\overline{0\theta r}}\epsilon_{ij}\epsilon_{j_{(1)}} + \frac{1}{4}f^{1/2}(4+r\partial_r\log\mathcal{H})\gamma_{\overline{\theta r}}P_\eta\epsilon^{i_{(1)}},\\
\tfrac{1}{2}\delta\psi_\phi^{i_{(1)}} &= \partial_\phi\epsilon^{i_{(1)}} - \frac{\eta}{2}\sin\theta\gamma_{\overline{0\phi r}}\epsilon_{ij}\epsilon_{j_{(1)}} + \frac{1}{2}\cos\theta\gamma_{\overline{\phi\theta}}\epsilon^{i_{(1)}} + \frac{1}{4}f^{1/2}(4+r\partial_r\log\mathcal{H})\sin\theta\gamma_{\overline{\phi r}}P_\eta\epsilon^{i_{(1)}}.
\end{aligned}
\tag{4.7}
$$

In certain cases we have used the identity $r\partial_r H_\alpha = 1 - H_\alpha$ [which holds for H_α given in (4.2)] when combining some of the factors in $\frac{1}{2}\delta\psi_\mu^{i_{(1)}}$ to form the half-supersymmetry projection terms.

We see that the Killing spinor equations, $\frac{1}{2}\delta\psi_\mu^{i_{(1)}} = 0$ with $P_\eta\epsilon_{i_{(1)}} = 0$, are practically identical with those that arise from the single charge case, (3.11). Thus the Killing spinors are similar to those of (3.12) and have the form

$$
\epsilon^{(1)} = e^{\frac{g\eta t}{\sqrt{2}}\epsilon_{ij}}\mathcal{H}^{-1/8}\left[\sqrt{f^{1/2}+1} - \sqrt{f^{1/2}-1}\gamma_{\overline{r}}\right]e^{\frac{\eta}{2}\gamma_{\overline{0\theta r}}\epsilon^{ij}}e^{-\frac{1}{2}\gamma_{\overline{\phi\theta}}}(1-\eta\gamma_{\overline{0}}\epsilon^{ij})\epsilon_0^{(1)}.
\tag{4.8}
$$

Until now we have only considered the first out of four sets of $N = 8$ supersymmetries, namely those parametrized by $\epsilon_{i_{(1)}}$. Naturally the form of the other three sets of variations are constrained by $N = 8$ supersymmetry, and differ only by the relative choices of signs between the four Cartan gauge fields. Preservation of (half) supersymmetry in each of the four sectors demands the following sign choices:

$$
\begin{aligned}
1:&\quad \eta_1 = \eta_2 = \eta_3 = \eta_4\\
2:&\quad \eta_1 = \eta_2 = -\eta_3 = -\eta_4\\
3:&\quad \eta_1 = -\eta_2 = \eta_3 = -\eta_4\\
4:&\quad \eta_1 = -\eta_2 = -\eta_3 = \eta_4.
\end{aligned}
\tag{4.9}
$$

It is not coincidental that these signs match those of Ω defined in (2.14). Because of the necessary difference in signs above, this indicates that in general, when all four charges are active, supersymmetry cannot be partially preserved in all sectors simultaneously. For one through four active charges, we find overall that 1/2, 1/4, 1/8 and 1/8 of the $N = 8$ supersymmetry can be preserved, in complete agreement with standard results [30, 31]. When all charges are equal, the solution may be obtained from a single scalar, single Maxwell field truncation with scalar-Maxwell parameter $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$ just as in the case of non-gauged supergravity [16, 20, 21, 19]. Of course, one can also choose the charges so that fewer or even no supersymmetries are preserved, even though the black holes are still extremal [16].

In the case of ungauged supergravity, on the basis of these mass and charge assignments, it was further suggested [16, 20, 21, 19] that we interpret these four values of a as 1-, 2-, 3- and 4-particle *bound states* with zero binding energy. For example, the Reissner-Nordstrom ($a = 0$) black hole combines four ($a = \sqrt{3}$) black holes: an electric Kaluza-Klein black hole, a magnetic Kaluza-Klein black hole, an electric winding black hole and a magnetic winding black hole. This zero-binding-energy bound-state conjecture can, in fact, be verified in the classical black hole picture by finding explicit 4-centered black hole solutions which coincide with the $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$ solutions as we bring $1, 2, 3, 4$ centers together and take the remaining $3, 2, 1, 0$ centers out to infinity [20]. Such a construction is possible because of the appearance of four independent harmonic functions [32]. Moreover, this provides a novel realization of the *no-force* condition in that the charge carried by each black hole corresponds to a different $U(1)$. Thus the gravitational attraction cannot be cancelled by an electromagnetic repulsion but rather by a subtle repulsion due to scalar exchange. This phenomenon was also observed in [33]. In the above, for purposes of illustration, the special case has been chosen where all non-zero charges are equal to unity but it is easily generalized to the case of different electric charges Q_1, P_2, Q_3, P_4 where the interpretation is that of a $(Q_1 + P_2 + Q_3 + P_4)$-particle bound state with zero binding energy [34].

It is tempting to generalize this bound state picture to the black holes of the gauged supergravity discussed in the present paper. One interesting difference from the non-gauged supergravity case, however, is that we find no static multi-center solutions. While the present solution is again based on four harmonic functions, (4.2), they are however not strictly independent as they must all share the same center[7]. This is to be expected on physical grounds: the presence of a negative cosmological constant ensures that only single center solutions will be static.

5 The non-extremal solution

While we are mainly interested in the properties of supersymmetric black holes, we note that there is a straightforward generalization of the above solutions to the non-extremal case. In the ungauged case, the extremal solution can be "blackened" by the incorporation of a universal function $f = 1 - k/r$ modifying the standard p-brane metric of the form [35]

$$ds^2 = -e^{2A} f dt^2 + e^{2B}(\frac{dr^2}{f} + r^2 d\Omega^2). \tag{5.1}$$

[7]Examination of the equations of motion indicates that the obstruction to finding multi-center solutions arises not from the gauge equation, but rather from the scalar and Einstein equations.

Subsequently, it was shown in [15] that this prescription generalizes in the straightforward manner to AdS black hole solutions as well. In particular, note that the four-charge black hole metric of (4.1) has the identical form as the non-extremal metric (5.1) and hence appears compatible with the "blackening" procedure.

As the manipulations of the equations of motion arising from (2.11) are not particularly illuminating, we only present the result here. The essential feature of the non-extremal black hole solution is a merging of the AdS function $f = 1 + 2g^2r^2(H_1H_2H_3H_4)$ of (4.2) with the non-extremal function $f = 1 - k/r$ to arrive at

$$f = 1 - \frac{k}{r} + 2g^2r^2(H_1H_2H_3H_4). \tag{5.2}$$

In addition, there is a charge rescaling so that the physical electric charges are no longer related to the mass. Introducing μ_α ($\alpha = 1, \ldots, 4$) to parametrize the four charges, we may write

$$H_\alpha = 1 + \frac{k\sinh^2\mu_\alpha}{r}, \qquad A_0^{(\alpha)} = \frac{\eta_\alpha}{2\sqrt{2}}\coth\mu_\alpha H_\alpha^{-1}, \tag{5.3}$$

so that

$$F_{0r}^{(\alpha)} = -\frac{\eta_\alpha}{2\sqrt{2}}H_\alpha^{-2}\frac{k\cosh\mu_\alpha\sinh\mu_\alpha}{r^2}. \tag{5.4}$$

Note that the extremal limit is approached by letting $k \to 0$ and $\mu_\alpha \to \infty$ with $Q_\alpha \equiv k\sinh^2\mu_\alpha$ fixed.

Spacetime properties of the above black holes depend on the number of active charges ($n = 0, \ldots, 4$). For $n = 0$ the solution reduces to the Schwarzschild-anti-de Sitter black hole with a single horizon protecting the singularity at $r = 0$. On the other hand, the BPS solutions ($k = 0$) for $n = 1, 2, 3$ all have singular horizons at $r = 0$ (with zero area) as appropriate for extremal black holes. Somewhat surprisingly, though, the four-charge BPS black hole has no horizon, and so strictly speaking is a naked singularity[8]. In all cases, the existence of a regular horizon demands $k > k_{min}$ where k_{min} is a function of the active charges. This is in fact similar to the five dimensional case considered in [14, 15].

6 Magnetic black holes

We have seen that the $N = 8$ gauged supergravity naturally admits a four-electric-charge black hole solution. In fact it turns out that this solution is easily generalized to give magnetically charged black holes; although the full theory involves non-abelian $SO(8)$ gauge fields, the $U(1)^4$ truncation of (2.11) gives rise to bosonic equations of motion that are symmetric under the electric-magnetic duality

$$F^{(\alpha)} \to e^{-\lambda_\alpha} * F^{(\alpha)}, \qquad \lambda_\alpha \to -\lambda_\alpha. \tag{6.1}$$

[8]For four identical charges, the solution (4.1) reduces to the Reissner-Nordstrom-anti-de Sitter black hole whose properties were studied in [27]. That Einstein-Maxwell theory with a cosmological constant is a consistent truncation of $D = 11$ supergravity, and hence that $D = 11$ supergravity has the Reissner-Nordstrom-anti-de Sitter black hole as a solution, has been known for some time [36, 37].

The resulting four magnetic charge solution has the form

$$
\begin{aligned}
&ds^2 = -(H_1H_2H_3H_4)^{-1/2} f dt^2 + (H_1H_2H_3H_4)^{1/2}(\frac{dr^2}{f} + r^2 d\Omega^2),\\
&e^{2\phi^{(12)}} = \frac{H_3H_4}{H_1H_2}, \qquad e^{2\phi^{(13)}} = \frac{H_2H_4}{H_1H_3}, \qquad e^{2\phi^{(14)}} = \frac{H_2H_3}{H_1H_4},\\
&H_\alpha = 1 + \frac{k\sinh^2\mu_\alpha}{r}, \qquad f = 1 - \frac{k}{r} + 2g^2r^2(H_1H_2H_3H_4),\\
&F^{(\alpha)}_{\theta\phi} = \frac{\eta_\alpha}{2\sqrt{2}} k\cosh\mu_\alpha \sinh\mu_\alpha \sin\theta.
\end{aligned}
\tag{6.2}
$$

While the extremal limit is once again reached by taking $k \to 0$ and $\mu_\alpha \to \infty$ with $P_\alpha \equiv k\sinh^2\mu_\alpha$ fixed, the resulting extremal black hole is in fact not supersymmetric whenever $g \neq 0$! In the case of the magnetic Reissner-Nordstrom black hole, this phenomenon was previously found in [27]. (Note, however, that it is possible to obtain magnetic black holes that do preserve some supersymmetry if one allows for event horizons with non-spherical topologies [38]) To see that (6.2) admits no Killing spinors, we note that while the scalar potential (2.13) is symmetric under $\phi^{(\alpha\beta)} \to -\phi^{(\alpha\beta)}$, the scalar related terms in the supersymmetry variations (2.15) and (2.16) are not. In particular, focusing on $\delta\chi$, we find for example

$$
\begin{aligned}
\delta(2\chi^{(3)i_{(1)}}) \;=\; & \frac{1}{\sqrt{2}}\epsilon^{ij}\gamma^r \Big\{ \partial_r \log\frac{H_1H_3}{H_2H_4}[\delta^{jk} - i\eta f^{-1/2}\gamma_{\overline{0}}\gamma^5\epsilon_{jk}] \\
& + \partial_r((H_1+H_3) - (H_2+H_4))[\sqrt{2}grf^{-1/2}\gamma_{\overline{r}}\delta^{jk}] \Big\} \epsilon_{k_{(1)}}
\end{aligned}
\tag{6.3}
$$

(where $\eta \equiv \eta_1 = \eta_2 = \eta_3 = \eta_4$), indicating explicitly that the g-dependent term on the last line has a different structure than the others. This is in contrast with (4.5) for the electric solution where all terms combine to give the projection operator (4.6). Additionally, note that the matrices $[i\gamma_{\overline{0}}\gamma^5\epsilon_{ij}]$ and $[\gamma_{\overline{r}}\delta^{ij}]$ now commute, while previously, for the electric black hole, they had anticommuted in the absence of γ^5.

For both of the above reasons, we see that whenever $g \neq 0$ none of the supersymmetry variations vanish, and hence the magnetic solution is non-BPS (regardless of the choice of signs of the magnetic charges). In the $g \to 0$ limit, on the other hand, the last line of (6.3) drops out, and we are left with

$$
\delta(2\chi^{3i_{(1)}}) = \sqrt{2}\epsilon^{ij}\gamma^r\partial_r \log\frac{H_1H_3}{H_2H_4}\tilde{P}_\eta \epsilon_{j_{(1)}},
\tag{6.4}
$$

where $\tilde{P}^{ij}_\eta = \frac{1}{2}[\delta^{ij} - i\eta\gamma_{\overline{0}}\gamma^5\epsilon_{ij}]$ is the projection appropriate to a magnetically charged solution. Thus, in the absence of gauging, there is a direct correspondence between the supersymmetry properties of the electric and the magnetic black holes. However the gauged supergravity theory (in the abelian truncation) is apparently no longer invariant under electric-magnetic duality.

7 Kaluza-Klein states as black holes

For *ungauged* $N = 8$ supergravity, the supersymmetry algebra admits 4 central charges Z_1, Z_2, Z_3, Z_4. States fall into 5 categories according as they are annihilated by $4 \geq q \geq 0$ supersymmetry generators. q also counts the number of Z's that obey the bound $M = Z_{max}$. Non-rotating black holes (in the sense of vanishing bosonic Kerr angular momentum L) belong to superspin $L = 0$ supermultiplets [21, 19]. Starting with a spin $J = 0$ member, the rest of the black hole multiplet may then be filled out using the fermionic zero-modes [29, 22]. The spin will run from $J = 0$ up to $J = (8 - q)/2$. For *gauged* supergravity, the algebra is different with no central charges but the same multiplet shortening phenomenon still occurs [39, 12]. So we can be confident that the above black holes preserving $4, 2, 1, 0$ supersymmetries will belong to supermultiplets with maximum spins $2, 3, 7/2, 4$ [9]. Unfortunately, as far as we know, the analogue of the $M = Z_{max}$ condition has never been been spelled out in the literature. It is presumably some relation between the AdS quantum numbers (E_0, s) and the $SO(8)$ Casimirs.

It seems entirely consistent, therefore, to identify a subset of the maximum spin 2 black hole supermultiplets with the S^7 Kaluza-Klein spectrum, in analogy with the black hole Kaluza-Klein correspondence of ungauged supergravity [16, 17]. The subset in question will correspond to electric black holes whose mass is quantized in units of the inverse S^7 radius. However, this raises the puzzle of how the black holes carrying only $U(1)$ charges can be identified with the Kaluza-Klein particles carrying non-trivial $SO(8)$ representations. Although we have not demonstrated this explicitly, it seems reasonable to suppose that it is the fermion zero modes that provide the non-trivial $SO(8)$ quantum numbers just as they provide the non-trivial spin. The fact that these nonabelian charges arise from *fermionic hair* also nicely circumvents the usual no-hair theorems of classical relativity. In this connection, it would be interesting to repeat the gyromagnetic ratio calculations of [22] and verify that the fermionic hair again yields a gyromagnetic ratio equal to 1, as demanded by Kaluza-Klein reasoning.

It is furthermore tempting, in analogy with the ungauged case, to identify the 2, 3 and 4 charge solutions as 2, 3 and 4-particle bound states of the singly charged solution [19, 20]. However, although the quantum number assignments are consistent with this, we do not have multi-center solutions in the AdS case. Such a bound state interpretation would, of course, lead to states of arbitrarily high spin.

Another difference between the S^7 and the T^7 compactifications is that the $g \to 0$ limit of the gauged supergravity does not directly coincide with the massless sector of the T^7 compactification. They differ by various dualizations. Thus it was possible, for example, to find 4-charge solutions with all charges electric as opposed to the 2-electric and 2-magnetic charges of the ungauged theory. Moreover, whereas 2 charges are Kaluza-Klein modes and 2 are winding modes in the Type IIA string theory context, there is no T or U-duality associated with the S^7 compactification.

One might also generalize the purely electric and purely magnetic solutions of this paper to dyonic black hole solutions of gauged $N = 8$ supergravity. Neither magnetic nor dyonic black holes have any Kaluza-Klein interpretation and do not appear in the spectrum of the

[9]The maximum spin 5/2 solutions are (mysteriously?) absent just as for ungauged supergravity.

S^7 compactification of $D = 11$ supergravity. It would be interesting to provide their M-theory interpretation and to determine their role in the AdS/CFT correspondence. Might they be related to a Goddard-Nuyts-Olive [40] non-abelian duality, for example? Since, in the $g \to 0$ limit, we recover the black holes which were previously identified with the T^7 spectrum, moreover, it seems possible that M-theory can interpolate between the two topologies. Perhaps the $D = 11$ supermembrane, which interpolates between $AdS^7 \times S^7$ and flat spacetime [41, 42, 43], plays an important part in this.

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