Additive Manufacturing of Locally Resonant Metamaterials

Irfan Mohammad Hussain Raza

Supervised by Lorenzo Iannucci and Paul T. Curtis

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Imperial College London
Department of Aeronautics

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Abstract

For many engineering applications, vibrations and sound can cause a multitude of issues. Several methods for damping out these vibrations are utilised today such as insulation foam used in walls or heavy granite bases for machinery and optical equipment. More recently, the development of acoustic and elastic metamaterials has shown the possibility to manipulate propagating waves in ways that were not previously possible, such as wave attenuation at an exponential rate.

Locally Resonant Metamaterials (LRMs) have been shown to attenuate waves with wavelengths two orders of magnitude greater than the lattice constant of the LRM, making them well suited for low frequency applications. They typically consist of a core, an elastic lining, and a matrix material. Much of the research into LRMs is modelling based, with fewer experimental results to correctly verify different designs. One reason for this is that the manufacture of LRMs can be difficult as they require multiple material properties, and design consistency, particularly as the LRM geometries become more complex.

Additive manufacturing (aka 3D printing) promises the ability to make complex shapes reliably and repeatedly. Hence, 3D printing techniques could be used to make LRMs. In this project, a custom built 3D printer is developed, which utilises different deposition techniques to allow it to manufacture an LRM. This facilitates the fabrication of more varied designs of metamaterial, which would have been too impractical otherwise to manufacture. The printer is fully customisable in LabVIEW and utilises a unique ‘point-cloud’ method to process part geometry.

More recently, active applications of LRMs have been explored to achieve behaviours that passive metamaterials cannot. One subset of active metamaterials is the growing field of metamaterial energy harvesting. This is the principle that
metamaterials can be used to convert vibrations and sound into another form of usable energy, such as electricity. Two concepts are introduced which use an LRM type design with a magnetic core. By the phenomenon of electromagnetic induction, as a propagating wave induces a periodic displacement on the core, a current is induced in adjacent wires which could potentially be stored for later use in an application.
Declaration of Originality

The work hereby presented is based on research carried out by the author at the Department of Aeronautics of Imperial College London, and is all the author’s work except where otherwise acknowledged. No part of the present work has been submitted elsewhere for another degree or qualification.
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Nomenclature

Symbols

\(A\) area
\(B\) bulk modulus or magnetic field strength
\(c\) speed of sound or speed of light
\(D\) transfer matrix
\(E\) Young’s modulus or electric flux
\(F\) force matrix
\(\vec{F}\) force vector
\(F\) input force
\(f\) frequency
\(G\) shear modulus
\(\vec{G}\) reciprocal lattice vector
\(h_i\) Hankel function
\(I\) identity matrix
\(I\) second moment of area
\(j_i\) spherical Bessel function
\(K\) stiffness matrix
\(\vec{k}\) wave vector
\(k\) wave number
\(k_i\) spring constants \(i = 1, 2, \ldots\)
\(L\) elasticity tensor
\(l\) length
\(M\) mass matrix
\(m_i\) mass \(i = 1, 2, \ldots\)
\(m_{st}\) total mass
\(m_{eff}\) effective mass
\(N\) octave smoothing iteration (see Appendix C.3)
\(n\) refraction index or numerical integer
\(p\) pressure
Symbols contd.

- $Q$ transformation matrix
- $qL$ non-dimensional wave number
- $r$ radius
- $\vec{r}$ position vector
- $t$ time
- $U$ displacement matrix
- $u_i$ displacement
- $\dot{u}_i$ velocity
- $\ddot{u}_i$ acceleration
- $V$ volume

Greek Symbols

- $\alpha$ real component of the wave number
- $\beta$ imaginary component of the wave number
- $\epsilon$ electrical permittivity
- $\gamma$ eigenvector
- $\kappa$ bulk modulus
- $\lambda$ Lame’s first parameter or temporal loss factor
- $\mu$ magnetic permeability or shear modulus
- $\Omega$ normalised frequency
- $\omega$ angular frequency
- $\omega_0$ natural frequency
- $\Phi$ eigenvectors or scalar potential
- $\Phi_B$ magnetic flux
- $\rho$ density
- $\nabla$ Laplace Operator
- $\tau$ time
- $\theta$ angle or mass ratio
- $\xi$ electromotive force
**Abbreviations**

3D  Three Dimensional  
3DP  3D Printing  
ADFM  Acoustic Double Fishnet Metamaterial  
AC  Alternating Current  
AM  Additive Manufacturing  
BD  Block Diagram  
CAD  Computer Aided Design  
CDWP  Continuous Direct Write Printing  
DAQ  Data Acquisition Device  
DDWP  Droplet Direct Write Printing  
EM  Electromagnetic  
FDM  Fused Deposition Modelling  
FDTD  Finite Difference Time Domain  
FEM  Finite Element Method  
HR  Helmholtz Resonator  
IBZ  Irreducible Brillouin Zone  
LRM  Locally Resonant Metamaterial  
MAM  Modal Analysis Method  
MST  Multiple Scattering Theory  
PC  Phononic Crystal  
PWE  Plane Wave Expansion  
PZT  Piezoelectric (generic)  
RUC  Representative Unit Cell  
SLA  Stereolithography  
SLM  Selective Laser Melt  
SLS  Selective Laser Sintering  
TGME  Triethylene Glycol Monomethyl Ether  
UI  User Interface
Chapter 1

Introduction

1.1. General Introduction of Metamaterials

From reducing the impact of vibrations on the fatigue life of a structure, to creating a more comfortable environment for users, the ability to attenuate elastic and acoustic waves has been on the wish list of scientists and engineers for many years. Traditionally, the method used to stop a propagating wave was to use an insulating material such as foam or a highly dense material, which could be heavy or occupy a large volume. These methods typically result in a linear decay of the amplitude of the propagating sound wave[32]. Metamaterials, which are fabricated periodic structures for the purpose of wave manipulation, have brought about the possibility of achieving wave propagation behaviour not observed in nature. One such behaviour is the exponential decay in a wave’s amplitude. This attenuation effect is created by the presence of ‘band gaps’, which are regions of frequencies where waves cannot propagate. These band gaps are formed by different processes depending on the type of metamaterial [17]. One key feature of these metamaterials is that they have been shown to achieve attenuation at an exponential rate, as opposed to a linear rate. This allows these metamaterials to potentially be smaller and lighter than the traditional insulating materials they replace. The initial research into the formation of band gaps focused on the electromagnetic wave spectrum [33], mainly in the microwave regime as the wavelengths were of the order of centimetres [34]. Therefore,
manufacturing of control devices was simpler, as opposed to higher frequencies which require smaller features. Further research focused on near infra-red attenuators with the eventual hope of making a metamaterial that operated in the visual frequency range. In the early 1990’s, it was shown that the principles which caused band gaps in the electromagnetic regime could also be applied to acoustic and elastic waves [35], facilitating the future concept of acoustic and elastic metamaterials.

The two main subsets in acoustic and elastic metamaterials are ‘Phononic Crystals’ (PCs) and ‘Locally Resonant Metamaterials’ (LRMs). PCs have been shown to attenuate waves utilising the Bragg scattering effect [36]. To achieve this phenomenon, a metamaterial requires periodic inclusions with a lattice constant (the length of one unit cell) similar to the wavelength of the incoming wave. For low frequency attenuation, where the wavelengths can range from centimetres to several metres, a PC would have to be correspondingly large as well, making it impractical for engineering applications. This constraint is one reason why the focus of research into PCs is mainly in the megahertz range [32].

LRMs on the other-hand have been shown to attenuate waves with wavelengths two orders of magnitude greater than the lattice constant of the metamaterial [3], making them well suited for low frequency vibration applications. LRM Is periodic media that typically utilise negative effective density to attenuate waves via a ‘mass-spring-mass’ system, rather than using the Bragg scattering effect. For this reason, research into LRM Is has focused mainly in the audible range of sound frequencies (20 Hz to 20 kHz). An early example of an LRM achieving a band gap was demonstrated experimentally by Liu et al. [3] where he was able to show partial attenuation of waves at 380 Hz.

The wave attenuation created by an LRM is dependent upon its natural frequency (as shall be described later). It is therefore possible to tailor the location and width of the frequency band gap of a metamaterial by altering its resonant frequency (i.e. altering the materials, geometry and/or the periodicity of the periodic structure). Several papers have been published describing methods to optimise a metamaterial to operate at a desired frequency bandwidth [4, 32, 37, 38].
An alternative use of metamaterials which is receiving increased attention is ‘energy harvesting’ [10]. This is the principle that metamaterials can be used to convert vibrations and sound into another form of usable energy, such as electricity. Energy harvesting is not limited to metamaterials, but there is growing interest in their use because of the unique properties in wave manipulation that metamaterials exhibit. As LRMs can operate at lower frequencies than other metamaterials, their use for vibrational wave conversion is of particular interest as it allows the possibility of capturing energy from machinery and vehicular vibration, which typically is between 10 Hz to 10 kHz [39]. This principle of converting vibrations into useful energy could be utilised in a number of different scenarios. For example, the vibrations in a car engine while stationary at traffic lights could be converted into electricity to help recharge the battery, and thus reducing the noise of the engine to nearby pedestrians and wildlife. One energy harvesting application using metamaterials was created by Carrara et al in 2013 [9]. Their design uses aluminium stubs in a parabolic arrangement to focus waves onto a piezoelectric ‘focal point’ which in turn generates electricity. This harvesting effect is observed at approximately 35 kHz and was able to generate a 2 V output from the piezoelectric patch, 4 times greater than harvesting energy with no aluminium stubs. At that frequency, it may not be applicable to industrial applications, but is a good demonstration of the potential use of energy harvesters.

The standard design of LRMs is typically a 3 component composite material consisting of a core, an elastic lining, and a matrix material. Manufacturing of LRMs, even small batches, remains an impracticality as the different designs require new tooling to make consistent parts with the correct periodicity [17]. Many of the papers describing how to optimise an LRM only present modelling work and do not show experimental verification. This impracticality of manufacturing LRMs is also one of the barriers for this technology being adopted into industrial applications.

Additive manufacturing, or 3D printing as it is more commonly known, is a field of research that has developed rapidly in recent years and promises many capabilities beyond those offered by traditional manufacturing methods. One such promise is the
ability to make complex shapes reliably and repeatedly. Additive manufacturing can be a potential solution to making LRMs faster than previously possible. However, 3D printers usually have only been capable of printing with one material. More recently, machines are now able to print with two or three materials. The methods used for these printers can limit the number of materials that can be used in a single print, while high initial hardware costs discourage experimentation with these printers to fabricate using new materials.

In this project, a custom made multi-material 3D printer is introduced that utilises different printing techniques to deposit metal alloy, rubber and plastic materials all within one object to allow the designs of LRMs to be explored. This printer is controlled by LabVIEW and therefore is fully customisable. The printer also uses a bespoke method of processing geometry known as the ‘Point-Cloud’ method, which facilitates the additional information required for the custom multi-material printer.

### 1.2. Thesis Outline

The following chapters in this thesis are outlined as follows:

- **Chapter 2:** A literature review is performed to investigate the various types of metamaterials that exist, and their benefits and deficiencies when designed for wave attenuating applications. The use of metamaterials for energy harvesting applications is also investigated. The four main metamaterial modelling approaches are examined and their advantages and disadvantages are assessed. Comment is made regarding the impact of viscoelastic effects upon LRM behaviour.

- **Chapter 3:** This chapter starts with a brief review of the various printing techniques used in additive manufacturing, and a critical assessment is made about their use in a multi-material application. The section then details the creation of the new multi-material printer, and describes its unique control program and geometry processing method.
1.3. Innovation and Novelty

- **Chapter 4:** This chapter describes in further detail the modelling process for a given LRM design, its fabrication and subsequent testing. Three different LRM designs are fabricated and acoustically tested using an impedance tube. The results from the testing are then analysed and critical comment is made about the various LRM designs and the quality of their manufacture.

- **Chapter 5:** This chapter introduces a novel energy harvesting concept and outlines its behaviour when tested, and a comparison is made to modelled behaviour.

- **Chapter 6:** This final chapter summarises the main findings of this thesis and outlines future work that can be performed to expand the fields of LRMs and multi-material additive manufacturing.

1.3. Innovation and Novelty

This thesis introduces three novel concepts which have not previously been published:

1. The development of a multi-material printer which is the first to utilise two different printing techniques that have not yet been used together in one print. The printer is also the first to successfully deposit a metal alloy, a rubber compound and a plastic component into one composite object. The geometry processing utilised by the printer is also a bespoke ‘Point-Cloud’ method that allows additional information to be supplied to the printer when compared to the traditional STL file and G-code format used for 3D printers.

2. The first time a 3 material component LRM has been fabricated entirely using additive manufacturing methods. Different LRM designs have been modelled, fabricated and tested. One of these LRM concepts has not been manufactured or tested previously, justifying the development of the multi-material 3D printer.
3. The creation of an energy harvesting metamaterial concept, which can be partially fabricated using additive manufacturing methods, is built and subsequently tested. These specific designs have not been studied previously. Conclusions are made regarding their future viability when compared to alternative metamaterial energy harvester concepts.

1.4. List of Publications

The research performed in this thesis has been disseminated through publications and oral/poster presentations, which are listed below.

Peer Reviewed Journal Publications


Conference Papers


Poster and Oral Presentations

- Defence and Security Doctoral Symposium 2016 (Shrivenham, UK) - Poster

- 17th European Conference on Composite Materials (Munich, Germany) - Oral
• Defence and Security Doctoral Symposium 2015 (Shrivenham, UK) - Poster
• 7th MAST Defence Materials Forum 2015, Defence and Security PhD Programme (Shrivenham, UK) - Oral/Poster
• 3rd International Conference on Phononic Crystals/Metamaterials, Phonon Transport and Phonon Coupling 2015 (Paris, France) - Poster
• 2013 AFOSR Annual Grantees’/Contractors’ Meeting for “Mechanics of Multifunctional Materials and Microsystems” (Washington DC, USA) - Oral
Chapter 2

Review of Acoustic and Elastic Metamaterials

2.1. Introduction

This chapter introduces the concept of acoustic and elastic metamaterials, how the subject originated, and the future applications of these composite materials. The various mechanisms that result in metamaterial behaviour are examined, and the different metamaterial designs that have been created are described in detail. The more recent focus in active and energy harvesting metamaterials is reviewed. These are metamaterials which can use some form of external control to improve a design's performance, or convert sound and vibration waves into electricity. The chapter proceeds to detail the current modelling techniques used to determine metamaterial behaviour, and a comparison between the various methods is performed. The current challenge in manufacturing metamaterials, specifically locally resonant metamaterials, is addressed and its effect upon experimentally validating different design concepts.
2.2. Electromagnetic Origins

Research into the field that is now known as acoustic and elastic metamaterials has been active since the early 1990s. However, the foundation of the subject originates with research into electromagnetic metamaterials, which began in the late 1980s. Much of the current electromagnetic research is based upon the fundamental Maxwell's equations, which were published in the 19th century. In this research, Maxwell [40] was able to unite all previous theories regarding electricity and magnetism into a consistent theory, by showing that electricity, magnetism and light are all forms of electromagnetic energy. This can be neatly summarised into 4 equations (where \( E \) is the electric flux and \( B \) is the magnetic flux):

\[
\nabla \cdot E = 0
\]

\[
\nabla \times E = \frac{1}{c^2} \frac{\partial E}{\partial t}
\]

\[
\nabla \cdot B = 0
\]

\[
\nabla \times B = -\frac{\partial B}{\partial t}
\]

The idea of a material that could artificially manipulate an electromagnetic wave travelling through it first originated in a paper published by Veselago [33] in 1968. Based upon the electromagnetic dispersion equation, derived from the Maxwell's equations, he noted that the only material parameters included in the dispersion equation are the electric permittivity (\( \varepsilon \)) and magnetic permeability (\( \mu \)). Veselago concluded that the path of propagation of an electromagnetic wave through a medium is governed by the electric permittivity and magnetic permeability, and no other material properties. In particular he made reference to the refractive index (\( n \)) equation,

\[
k^2 = \frac{\omega^2}{c^2} n^2
\]

\[
n = \sqrt{\varepsilon \mu}
\]
where \( k \) is the wave vector, \( \omega \) is the angular frequency and \( c \) is the speed of light. Through a complex mathematical proof, he was able to determine that if either the permittivity or the permeability were to be negative, then the propagation of an electromagnetic wave would cease. Furthermore, he was able to show that if both permittivity and permeability were negative at the same time, then the wave would still propagate but the medium would have an effective negative refractive index. Unfortunately this research did not progress as the need for such a capability did not exist at that time.

In the 1980s, Yablonovitch was one of a few scientists who further developed the concepts created by Veselago, and in 1987 published a paper \[41\] in which he mentioned that the use of a material that could block spontaneous electromagnetic emission would be hugely beneficial to semiconductors. He stated that by using a three dimensional periodic dielectric structure, a ‘phase slip’ is created, which results in the formation of a band gap. Yablonovitch’s paper was widely read and the concept of creating wave attenuating devices increased in popularity. In 1991, Yablonovitch \[42\] was able to demonstrate a microstructure that would produce a full band gap. The structure consisted of a periodic face centred cubic lattice with holes drilled in the centre of each face. Up to this point, attempts at creating an electromagnetic bang gap had resulted in pseudo gaps, rather than full band gaps. This paper expanded on the research carried out by Ho, Chan, and Soukoulis \[43\] who performed similar testing using a diamond crystal structure. In 1991 Genack and Garcia \[44\] achieved a similar result by using an arrangement of metallic and dielectric spheres. They exploited a new method of band gap creation in which incoming electromagnetic waves scatter, resulting in destructive-interference. The overall result was a transmission loss of the incoming wave.

At this time, the term ‘Photonic Crystal’ was used to describe devices that could manipulate electromagnetic waves. The word ‘photonics’ first appeared in the late 1960s, to describe a new research field where light was used to perform functions that traditionally fell within the domain of electronics, such as telecommunications and information processing.

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2.2. Electromagnetic Origins

Sigalas et al [45] produced a paper in 1992 in which they demonstrated that the metamaterial behaviour for electromagnetic waves could equally be applied to an elastic wave given, the electric permittivity and magnetic permeability terms were replaced with corresponding elastic based material properties. It was at this point that research into ‘Phononic Crystals’ commenced (see Section 2.3.1) and the research in acoustic/elastic metamaterials branched off from electromagnetic based research into its own field of study. However, advances in the electromagnetic domain continued to influence research in the acoustic/elastic domain.

In the following few years, the theory behind photonic band gaps was further investigated, exploring different geometries and fabrication processes. It was determined that the periodicity of these types of photonic crystal devices had to be equivalent to the wavelength of the incoming wave [46]. Experiments mainly took place within the microwave regime as the wavelengths were of the order of centimetres. At that time it was difficult and costly to fabricate devices to manipulate waves with shorter wavelengths. However, Krauss et al published a paper in 1996 [47] in which he stated that he had created a photonic crystal capable of manipulating waves in the infra-red region as opposed to the microwave region. At this time, no publications had been able to replicate the negative permittivity and permeability that Veselago had predicted. This was largely due to the materials and designs studied did not possess a large enough magnetic response to electromagnetic radiation to induce a negative magnetic permeability.

In 1999, Pendry, whilst working for GEC-Marconi, created a copper etched split-ring structure which, when exposed to radio signals, produced a previously unseen response [48]. The exposure to the radio signal induced an electric current in the copper wire. The corresponding magnetic field was aligned in the opposite direction to the applied magnetic field. This was the first time negative permeability had been achieved. His device was referred to as an ‘electromagnetic metamaterial’ as it was of a different design to the typical photonic crystal. As his metamaterial was based on a resonance system, the negative permeability was observed over a narrow frequency band. The idea of having one ring inside the other was to allow a large capacitance and hence lower the resonating frequency and concentrate the electrical...
field. This allows for the resonator to resonate at frequencies where the wavelength is much larger than the diameter of the outer circle.

(a) The typical design of a split-ring resonator [48]. (b) Split-ring resonators on a grid that achieved negative refraction [49].

Figure 2.1.: Early concepts in electromagnetic metamaterials.

Following the publication of Pendry's paper [48], Smith et al published their findings in which they managed to achieve negative refraction [49–51]. Their metamaterial design was an adaptation of Pendry's split ring resonator, applied to a grid like lattice. This was a milestone in electromagnetic metamaterial research, as negative refraction had never been achieved before in any form. Following this, research into electromagnetic metamaterials increased and potential applications for negative reactive media were envisioned.

2.3. Wave Attenuating Acoustic and Elastic Metamaterials

2.3.1. Phononic Crystals

Following on from the initial research into photonic crystals, researchers started to investigate if the process by which electromagnetic waves could be altered also applied to acoustic waves in a fluid and elastic waves in solid. In 1992, Sigalas and Economou published a paper [45] in which they analysed the propagation of an elastic or acoustic wave within an inhomogeneous system. The system they modelled consisted of identical spheres placed periodically within a homogeneous
medium. They noted that acoustic waves in a fluid were only longitudinal, whilst elastic waves within a solid could be longitudinal or transverse. Subscripts \( t \) and \( l \) refer to transverse and longitudinal respectively.

\[
\frac{\partial^2 \rho}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 \rho}{\partial t^2} = 0 \quad (2.7)
\]

\[
c_t = \sqrt{\frac{\mu}{\rho}} \quad (2.8)
\]

\[
c_l = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad (2.9)
\]

The acoustic wave equation (2.7) defines the propagation of an acoustic wave within a homogeneous material. It was noted that the speed of sound term \( c \), in the equation was dependent upon the materials density \( \rho \), the shear modulus \( \mu \), and Lamé’s first parameter \( \lambda \) which is a function of the shear modulus and the bulk modulus \( \kappa \). Hence, the only material parameters that alter a sound wave’s propagation is the material’s density, shear modulus, and Lamé’s first parameter. In a fluid, where the shear modulus is zero, only the bulk modulus and the density affect the propagation of the sound wave.

In 1993, Kushwaha et al [35] also published similar findings to Sigalas and Economou. Based on the photonic crystals concept that had been developed for the electromagnetic domain, Kushwaha described a periodic composite structure made from two materials with different elastic properties. He coined the term ‘Phononic Crystal’ (PC) to describe a 2-component elastic composite material, that could achieve an acoustic band gap in a similar method to a photonic crystal.

Kushwaha made reference to some of the achievements made in electromagnetic metamaterials and stated the same type of effects could be true for acoustic and elastic waves. He made particular note of the use of elastic wave attenuation that could help create a vibration-less environment to allow high quality mechanical processes or scientific testing to be carried out. Kushwaha had experimented by creating a face centred cubic PC, but was only able to create a pseudo-gap (i.e. no complete
2.3. Wave Attenuating Acoustic and Elastic Metamaterials

band gap was created for all points in the Irreducible Brillouin Zone, see Section 2.6).

In the following years, the research into acoustic domain wave attenuation was further developed by Kafesaki et al [52], who examined the effect of different inclusion shapes upon the attenuation performance. Much of the research in this period was dedicated to ways of predicting and modelling the acoustic band gap of a structure. During this time, it was generally agreed that the method by which attenuation occurred in a PC was via scattering of incoming waves, resulting in anti-phase reflections causing the waves to cancel out. More specifically, the dominant scattering phenomena was Bragg scattering [53, 54].

In physics, Bragg's law gives the angles for constructive and destructive scattering from a crystal lattice [55]. In the realm of acoustic and elastic wave attenuation, Bragg scattering occurs when waves are incident upon a PC with high impedance contrast between the matrix materials and the inclusion (i.e. scattering) material. Wave attenuation is achieved when the scattered waves interact with each other in a destructive manner (see Figure 2.2).

![Figure 2.2.: The Bragg scattering effect in a PC. The incoming waves reflected off the spherical inclusions result in re-radiated waves that can result in destructive interference.](Image source: [2])

There are various requirements for scattered waves to be constructive instead of destructive. The angle of the incoming sound wave must meet certain criteria to create a destructive interference, rather than a constructive interference (see Figure 2.2). This is a useful feature as it allows for the creation of ‘acoustic switches’ for certain frequencies. However, due to multiple wave scattering, the dominant effect is destructive interference. The more important requirement for Bragg scattering to
occur is the requirement that the periodicity of the inclusion (i.e. the lattice constant of the crystal) is of the same order as the wavelength of the incoming sound waves [36]. This means that for a PC to attenuate low frequency acoustic waves, which traditionally are the more difficult to attenuate, the size of the PC itself would be impractically large for many engineering purposes. This realisation would lead to a shift in methodology for many researchers attempting acoustic wave attenuation and wave manipulation.

For the ultrasonic frequency range, PCs remain very relevant with continued research in the area. Current research is focussed on active systems for ultrasonic wave control and attenuation, non-linear systems, and broad-band ultrasonic band gap creation [17, 32].

### 2.3.2. Locally Resonant Metamaterials

In the late 1990s, it was discovered that micro mechanical oscillators could achieve wave attenuation that did not use Bragg scattering. When a mechanical oscillator is driven through resonance, the phase delay of the response acceleration relative to the driving force changes from a small amount to nearly 180 degrees as the displacement amplitude becomes large. The large scattered and re-radiated fields interfere constructively or destructively or cause anomalous phase shifts when added to the incoming wave [56].

The idea of using resonance to achieve wave attenuation became increasingly popular as it overcame the need for a lattice constant equivalent to the wavelength of the sound wave, therefore allowing for practical sized wave attenuators that could be used to attenuate audible sound frequencies [57]. At the time, these became known as ‘acoustic metamaterials’, drawing a distinction between the different methods used to achieve wave attenuation. (Recently the term ‘acoustic metamaterial’ has been used to describe the overall field of sound wave manipulation using micro-engineered composites. However, there is continued debate as some researchers state that locally resonating metamaterials are a form of PC.)
The initial acoustic metamaterials that arose were binary composites, with one constituent material acting as the matrix material and one acting as the oscillator in a ‘mass-spring’ type system. The requirement for such localised resonance to occur was that the phase speed of the inclusion material was much lower than that of the matrix material. As such, the inclusion would oscillate at a resonant frequency and initiate a phase shift in the material.

This concept was taken further by the introduction of a three-component (ternary) unit cell metamaterial by Liu et al in 2000 [3]. The addition of a heavy resonator in the middle of each unit cell in metamaterial significantly reduced the resonant mode required for attenuation (see Section 2.3.3) compared to other acoustic metamaterials at the time. Liu built a metamaterial where each unit cell consisted of a lead sphere coated in a silicon rubber, all within an epoxy matrix. He made an array of 8x8x8 of these cells (see Figure 2.3).

Figure 2.3.: LRM design introduced by Liu et al. [3]. The LRM unit cell has a lead core, coated in silicone rubber and held by an epoxy matrix.

Liu modelled this metamaterial using the multiple scattering method which he had adapted to make it applicable to ternary acoustic metamaterials (see Section 2.6.2). Modelling and experimental values correlated and showed that this metamaterial had wave attenuating capabilities for frequencies around 380 Hz and 1350 Hz (specific values not mentioned in paper [3]), which represented a step change in performance from previous metamaterials as they operated in a much lower frequency range than previously possible. This also represented the first experimental evidence of a locally resonant structure demonstrating negative dynamic density [58].
2.3. Wave Attenuating Acoustic and Elastic Metamaterials

(see section 2.3.3 for the mathematical explanation of negative effective density).

The ternary resonator type setup, introduced by Liu, gained much attention due to its unique design and performance improvement, and was further developed. These types of metamaterials also became known as mass-in-mass type resonators or Locally Resonant Metamaterials (LRMs). It was discovered that as this attenuation mechanism was independent of order or periodicity, it was possible to create metamaterials with a high concentration of resonators, allowing them to couple strongly with each other to enhance their resonating capability [59].

Just like Bragg scattering, the frequency band gap created by the resonating units was very narrow and hence not ready for broadband frequency sound attenuation.

In 2003, Ho et al [60] published a paper stating that as the resonance frequency is dependent upon the unit cell geometry, they could simply vary the geometries of the unit cells and hence achieve wave attenuation over a range of frequencies. They managed to achieve an 11 dB increase in sound attenuation over the range 200-500 Hz, when compared to a control sample. Their setup had two LRMs of one resonant frequency in series, followed by another set of two LRMs with a different resonant frequency [60]. In 2003 Sheng et al [61] wrote that the maximum attenuation is achieved when the resonant mode equates to the movement of the dense spherical resonator. Higher frequency attenuation is achieved by resonant modes in the soft ‘spring’ rubber layer.

One of the most recent and well-read investigations into the design of LRMs was conducted by Krushynska et al. in 2014 [4]. In this study, Krushynska investigated several LRM designs with varying resonator and lattice shapes (see Figure 2.4). In each design iteration, conclusions were made on what design features would be optimal to produce the widest band gap (Note: the modeling was performed on 2D models with a plain-strain boundary condition applied in the perpendicular direction). The main conclusions were as follows:

1. For (in-plane) wave attenuation, a circular cross section produces a wider band gap than a square cross-section.
2. The optimal filling ratio (resonator and lining area compared to the total matrix area) for a circular cross-section LRM is 70% (a similar conclusion was found by Tan et al [38]).

3. A periodic lattice system is optimal when compared to randomly placed resonators in a matrix. Periodicity results in a coupling effect which enhances the LRM's wave attenuation.

Figure 2.4.: LRM design optimisation performed by Krushynska et al. [4]. a) The 2D square lattice LRMs modelled by Krushynska, with varying core and elastic lining shape. b) Graph showing the width of the band gaps achieved for each design of LRM with varying filling ratios. (‘Normalised Frequency’ refers to the frequency being analysed, divided by the frequency of the resonant mode equating to the displacement of the core.)

The design of LRMs has evolved from the classical design introduced by Liu et al. One such design is the stubbed plate which was introduced in 2011 by Assouar et al [62]. This design is meant to be added onto existing structures to allow elastic wave attenuation as opposed to acoustic wave attenuation (see Figure 2.5(a)). Another is the concept of a ‘mass-in-(mass-in-mass)’ system introduced by Tan et al [63]. This is a mechanical oscillator within another mechanical oscillator as shown in Figure 2.5(b). This set up has been shown to broaden the frequency band gap again by introducing different resonant frequencies to the system.
2.3. Wave Attenuating Acoustic and Elastic Metamaterials

(a) Stubbed Plate Design by Assour et al [62].

(b) ‘Mass-in(Mass-in-Mass)’ design proposed by Tan et al [63].

Figure 2.5.: Alternative proposed LRM designs.

Other researchers are investigating how to couple various attenuating methods like Bragg scattering and localised resonance into a single metamaterial in order to broaden the band gap generated [64–66].

2.3.3. Negative Effective Density

The concept of negative effective density was best demonstrated by Liu's ternary locally resonant metamaterial [3]. The principal of how this negative effective density is created was best explained later by Huang et al. in 2009 [67] when they showed that the design could be equated to a ‘mass-in-mass’ type system. As stated previously, when the input frequency is near the resonant frequency of a mechanical oscillator, the phase change is almost 180 degrees which results in destructive interference and wave attenuation occurs. The model of a mass-in-mass type resonator is shown in Figure 2.6(a).

By considering the equations of motion for the two masses we obtain:

\[
m_1 \ddot{u}_1 = F + k_2(u_2 - u_1) \tag{2.10}
\]

\[
m_2 \ddot{u}_2 = k_2(u_1 - u_2) \tag{2.11}
\]
2.3. Wave Attenuating Acoustic and Elastic Metamaterials

(a) Model used for derivatation of Negative Effective Density [38].

(b) Plot showing Effective Mass Vs. Normalised Frequency. Regions below zero effective mass equate to the location of a band gap.

Figure 2.6.: Negative effective density mechanism in LRM.

Where $F$ is the driving force, $u$ is the displacement of the respective masses, and $k$ is the corresponding spring constant. Assuming that the displacement of each mass follows harmonic wave behaviour, similar to that of the applied force (for an acoustic/elastic wave) we have:

$$F(t) = F_0 e^{-i\omega t}$$  \hspace{1cm} (2.12)

$$u_\gamma(x,t) = \hat{u}_\gamma e^{-i\omega t}$$  \hspace{1cm} (2.13)

Where $\omega$ is the frequency and $\gamma = 1, 2$ in this case. To obtain the effective mass behaviour of the microstructure, the following equation must be satisfied:

$$F(t) = -m_{eff} \omega^2 \hat{u}_1$$  \hspace{1cm} (2.14)

Solving the above equations, we obtain the effective mass of the microstructure:

$$m_{eff} = m_1 + \frac{m_2 k_2}{k_2 - m_2 \omega^2}$$  \hspace{1cm} (2.15)
Hence we can conclude that the effective mass, $m_{\text{eff}}$, is dependent upon frequency, $\omega$. By further manipulating the equation, the normalised effective mass becomes:

$$\frac{m_{\text{eff}}}{m_{\text{st}}} = 1 + \frac{\theta}{1 + \theta} \left[ \frac{(\omega/\omega_2)^2}{1 - (\omega/\omega_2)^2} \right]$$

(2.16)

$$m_{\text{st}} = m_1 + m_2$$

(2.17)

$$\theta = \frac{m_2}{m_1}$$

(2.18)

From equation 2.16 it is clear that the relation $\frac{m_{\text{eff}}}{m_{\text{st}}}$ versus $\frac{\omega}{\omega_2}$ is dependent on only a single parameter, $\theta$. The term $\frac{\omega}{\omega_2}$ represents the normalised operating frequency of the system. The plot in Figure 2.6(b) represents the normalised mass plotted against the normalised frequency. When $\frac{\omega}{\omega_2}$ nears 1, this represents the local resonance frequency of the internal mass ($m_2$) system. It is evident that a narrow band gap region exists when the resonance frequency is reached, as the effective mass becomes negative (see Figure 2.6(b)). This is negative effective density. It has been proven that this negative effective mass region corresponds to the band gap region of the dispersion curve when wave propagation is considered. This leads to the result where a wave cannot be transmitted through the structure when the frequency $\omega$ is near $\omega_2$ [68]. It can also be seen that as $\theta$ increases, the band gap region becomes broader [38].

This effective negative density interpreted physically means that the average momentum of the metamaterial is moving in the opposite direction to the wave propagation. This implies two points (see [61, 69]);

1. The core of each of the mechanical oscillators must be heavier than the corresponding matrix material for that unit cell.

2. The core must be moving near to 180 degrees out of phase with the rest of the matrix. Hence the clear benefit of having a metamaterial with a higher filling fraction.
2.3.4. Helmholtz Resonators

Helmholtz Resonators (HRs) consist of a cavity within a rigid material, connected to a fluid matrix through a much narrower neck. They are not as new a concept to science as PCs or LRM are, having been used by the ancient Greek and examples of which can be seen in many churches as devices to reduce reverberation [70]. A high pressure peak of a sound wave passing over the neck of the cavity causes the air in the neck to move in, causing the air within the cavity to be compressed. As the sound wave propagates and a low pressure node is reached, this compressed air now forces the air in the neck to move out again and the process repeats. This is analogous to a mass-spring system, where the volume of air in the neck of the cavity is a mass and the volume of air in the main body of the cavity is the spring (see Figure 2.7).

![Figure 2.7: Schematic of a Helmholtz Resonator and its equivalence to a mass-spring system](image1)

(a) A schematic of a typical Helmholtz Resonator and its equivalence to a mass-spring type system
(b) A one dimensional HR type metamaterial created by Fang et al [71].

Like the LRM type resonators, there is no requirement for a lattice constant in order to achieve wave attenuation and the size of the HRs can be sub-wavelength. One major benefit of using HRs for acoustic wave attenuation as opposed to elastic wave attenuation, is that there is no transfer of energy from a fluidic system to an elastic system as there is for LRMs. This reduces the occurrence of wave reflection at the fluid-solid boundary. This fact is a key consideration for the focus of current acoustic metamaterial research. Recent advances in the understanding of wave attenuation have led to renewed research in HRs as they have the ability to achieve a negative bulk modulus. One of the first papers to show that HRs could
achieve negative bulk modulus (see Section 2.3.5) was published by Fang et al [71] in 2006. In this paper he showed ultrasonic wave attenuation near 30 kHz. There are now numerous publications which demonstrate the attenuation effect that HRs have on acoustic waves [72–74]. In a similar manner to LRM s, current research with HRs is examining how to make the system ‘active’ (i.e. controllable) so that various performance characteristics can be improved, including band gap width and attenuation magnitude. De Bedout et al created a partially-active system as early as 1996 [75], which operated by varying the volume of the cavity of a resonator, and hence varying its natural frequency. However, this was prior to the discovery of the mechanism of a HR to achieve negative dynamic material properties. In terms of active metamaterials, there have been various designs by Baz et al [76, 77] and Reynolds [78] which utilise piezoelectric material to introduce control to HR-like metamaterials.

### 2.3.5. Negative Bulk Modulus

The bulk modulus of a material relates the elastic deformation of a material that leads to a change in volume. For a fluid medium, the volume of the fluid is reduced upon compression and vice versa. A material with a negative bulk modulus has the opposite effect of this whereby as a medium is compressed, the volume it occupies increases and contracts as the compression is reduced [71]. As HRs can be modelled as a mass-spring system, they have an associated resonance mode. When a sound wave travelling down a pipe meets a HR, the high pressure and low pressure nodes cause the mass of air in the HR to move in and out. If the frequency of the sound is near the resonant frequency of the HR, the mass of air in the neck moves 180° out of phase with the high and low pressure nodes of the sound wave in the pipe, causing the wave to attenuate. This is the effect of negative bulk modulus and is similar to the out-of phase motion achieved by LRM s exhibiting negative effective density (see Section 2.3.3).
2.3. Wave Attenuating Acoustic and Elastic Metamaterials

2.3.6. Double Fishnet Metamaterials

The idea of an acoustic double fishnet metamaterial (ADFM) was first pioneered in electromagnetic metamaterial research [79–81]. The structure of an ADFM consists of two periodically perforated plates, separated by a sub-wavelength gap (see Figure 2.8)[82]. When the holes in the two plates are aligned, the pair of aligned holes behave as a long pipe with a gap at its mid-length. Odd-order modes (pressure anti-node in the region of the gap) experience a frequency shift, due to volume flow leakage into the gap, whereas even-order modes (pressure node in the region of the gap) are largely unaffected [83, 84]. When the plates are misaligned the conditions for zero transmission change. There remains a pressure antinode at the boundary of a square unit cell located centrally over the input hole. However, for no sound to be transmitted, there must be a pressure node located at the output hole. In this case, there is no pressure variation at the output hole and therefore no coupling between the holes in the two plates, thus no transmission of sound [83].

One of the clear advantages of ADFMs is that they use one material only to manufacture a metamaterial, as the second component material is the fluidic medium the wave is propagating in, such as water or air. This also means there is less energy reflected as the waves pass from one medium to another, as is the case for PCs and LRMs. In addition to these advantages, the structure could potentially be load bearing, without affecting the overall performance of the ADFM, and so could readily be used in engineering applications.

The gap between plates can sometimes be filled with another constituent material.
to change attenuation performance. It has been shown that this type of metamaterial can also display negative effective bulk modulus [5]. More recently, designs of ADFMs have become more complicated. Once such design utilises small HRs within an ADFM structure to lower the attenuating frequency of the metamaterial by a factor greater than 2 [85]. There is also a strong focus on using ADFMs for acoustic evanescent waves enhancement [86, 87] (see Section 2.5.)

2.3.7. Comparison of the Varying Mechanisms

The varying types of acoustic and elastic metamaterials each have their benefits and deficiencies. Depending on their application, one metamaterial mechanism may be better suited than another (see Table 2.2). For example, there is a general consensus that for low frequency sound and vibration wave attenuation, PCs are not well suited as the wavelength requirement would make them impractically large for many applications [17, 32]. The same is true for ADFM type metamaterials. Hence for low frequency applications, there is much focus on the use of LRM s and HRs, where the structural size of the metamaterial has less impact on the operating frequency of the metamaterial.

<table>
<thead>
<tr>
<th>Type</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC</td>
<td>Simple design and easy to manufacture</td>
<td>Impractical for low frequency applications</td>
</tr>
<tr>
<td></td>
<td>Attenuation in multiple directions</td>
<td></td>
</tr>
<tr>
<td>LRM</td>
<td>Ideal for low frequency applications</td>
<td>Complicated to manufacture</td>
</tr>
<tr>
<td></td>
<td>Attenuation in multiple directions</td>
<td></td>
</tr>
<tr>
<td>HR</td>
<td>Ideal for low frequency applications</td>
<td>Not suitable for elastic wave damping</td>
</tr>
<tr>
<td></td>
<td>Adapted for reducing acoustic noise in a fluid</td>
<td></td>
</tr>
<tr>
<td>ADFM</td>
<td>Simple design and easy to manufacture</td>
<td>Impractical for low frequency applications</td>
</tr>
<tr>
<td></td>
<td>Easily tuneable</td>
<td>Not suitable for elastic wave damping</td>
</tr>
</tbody>
</table>

When comparing the use of HRs vs LRM s, there are several factors to consider:
2.3. Wave Attenuating Acoustic and Elastic Metamaterials

- **Acoustic Attenuation or Vibration Attenuation** - HRs are well placed for acoustic attenuation of sound waves in a fluid medium. As the fluid medium interacts directly with the cavity and neck of the HRs, there is less reflected energy than associated with the acoustic attenuation with LRMs. To attenuate an acoustic wave in a fluid with an LRM, the energy of the wave first has to transfer from the fluid to the matrix material of the LRM, which can introduce reflected waves [32] which may not be desirable. However, it is still possible to achieve sound attenuation with an LRM. For vibrations in a structure, where waves propagate within a solid, then HRs cannot be used as they require a fluid structure interaction to create a negative bulk modulus effect [73]. However, LRMs are well placed as they can be attached directly to a structure and their mechanism for achieving negative effective density remains valid. Alternatively, LRMs with the setup as described by Liu et al [3] with the matrix material being epoxy, the LRM can instead be load bearing and replace part of the vibrating structure entirely.

- **Fluid Movement** - If the application requires the movement of fluid, then HRs are well adapted as they can be added to a channel of the moving fluid and offer no restrictions to flow. LRMs would need to be placed in the path of the channel for the attenuation effect to occur. Hence the LRM would require the introduction of some porosity to allow a fluid to move through it. This would reduce the filling ratio of the metamaterial and could potentially reduce the attenuation effect of the LRMs [4].

- **Ease of Manufacture** - The manufacture of LRMs is a complicated matter. Many LRMs manufactured for testing have been made by hand, and currently no research has been performed into the manufacture of LRMs. Alternative LRM designs have been created that make use of current manufacturing capability [19], but this can introduce limitations to the capability of the LRM. HRs are also potentially complicated to manufacture. Their design is a simple cavity and neck opening, but introducing these into an application requires high precision subtraction manufacturing techniques.
• **Weight Restrictions** - HRs are ideal for weight saving applications as they are simply cavities within a fluid canal. These cavities can reduce the weight of a structure, but could also introduce unwanted weaknesses and stress concentrations [17]. As LRM requires a dense core with a light matrix material, they could potentially add weight to a structure and so are not well suited for weight restrictive applications. However, if the LRM, which can be load bearing, were to replace part of the structure, then the weight penalty may be minimal and justified.

## 2.4. Active Metamaterials

The aforementioned metamaterials have all been passive type devices. There is a strong focus on the creation and application of active metamaterials. This is the concept of using some form of external control, be it electronic or magnetic, to either enhance the attenuation performance of a metamaterial, or to tune it inorder to operate at varying frequencies.

One of the earliest types of active metamaterial was a PC with a deformable structure created by Bertoldi and Boyce [6]. The shape of the elastomeric inclusions within this PC would deform when exposed to a critical external load (see Figure 2.9). The application of this load would result in the shifting of the observed band gaps from one frequency region to another, effectively making an ‘acoustic switch’. The elastic nature of the material allowed for multiple cycles of loading and unloading of the inclusions, and demonstrated the reversible and repeatable nature of this design. They highlighted potential applications in engineering whereby a metamaterial could be used to detect varying stresses experienced by a component by listening to a change in the acoustic noise output. Similarly, Wang et al [88] have produced a design of LRM that can alter its response due to a buckling type deformation of the ‘spring’ part of the metamaterial. Their design can alter its resonant modes under stress but also completely stop its metamaterial behaviour by buckling the spring strut in the metamaterial design.
2.4. Active Metamaterials

Another early example of an active type metamaterial was created by Akl and Baz, [76, 77] by attaching piezoelectric (PZT) patches to one wall in the cavity of a HR. By varying the applied voltage to the patch using an external ‘charge feedback control’ circuit, the stiffness of this patch could be changed. With this procedure, they were able to tune the bulk modulus of the metamaterial and subsequently alter its response to propagating waves, effectively tuning it to operate at varying frequencies.

In 2012 Casadei et al [7] introduced a two-dimensional PC plate with cylindrical scatterers, which was integrated with piezoelectric patches (see Figure 2.10). Depending on the inductance of the controlling circuit, the PC plate could achieve tuneable attenuation and negative group velocities in the range from 100-130 kHz. This was also an example of using an active metamaterial set-up to create a wave guide (i.e. altering the path of a wave to a new desired direction). This, as stated by Casadei [7], was effectively an acoustic logic gate.

Airoldi and Ruzzene[89] have created a wave guide type active metamaterial made from shunted piezoelectric patches attached to either side of a metal plate. The shunts form periodic resonant elements, but when the shunts are turned off, the behaviour of the plate is similar to a homogeneous metal plate again. Similar designs have also been produced by Badreddine Assouar and Oudich [62, 90].

The creation of magneto-elastic type materials (i.e. elastically compliant materials with nano or micro sized magnetic particles embedded within) has allowed the creation of magnetically controlled active metamaterials. An early design by Ro-

Figure 2.9.: A phononic crystal exhibiting a band gap region shift when axially loaded to a critical value [6].
2.4. Active Metamaterials

(b) Full wave-field measurements demonstrating the waveguide capability of Casadei’s design. The fringe plot shows resultant displacement.

Figure 2.10.: Acoustic waveguide created by Casadei et al. [7].

billard et al. [91] was introduced that used magneto-elastic materials in a PC type set up. When exposed to an external magnetic field, it would change the shape of the PC’s inclusions and shift the frequency band gap. Robillard noted that this metamaterial was completely wireless and hence offered a design advantage when compared to alternative active metamaterials.

The design was further developed by Bou Matar et al in 2012 [8]. The design required square rods of Terfenol-D material in an epoxy matrix with a square lattice. Depending upon the strength of the externally applied magnetic field, the shape of the rods would alter, and correspondingly the frequency band gap generated would change. The important difference when compared to Robillard’s design was the application of the magnetic field created a new band gap at higher frequencies, but left the original band gap relatively unchanged (see Figure 2.11).

In 2014, Chen et al. [92] explored the use of a magneto-elastic film with an aluminium ring to make an active acoustic metamaterial (see Figure 2.12(a)). In their set up, the elastomer film was exposed to high gradient axial magnetic fields provided by a cylindrical magnet. Dependent upon the distance from the magnetic source, the response of the elastomer to the magnetic field could induce a negative effective density below the resonant frequency for the structure in the absence of
2.4. Active Metamaterials

The formation of a new band gap (in red) by the application of an external magnetic field in a magnetoelastic type PC [8].

The use of simple magnetic based metamaterials has more recently been investigated in 2015 by Hobeck et al [93]. A metamaterial inspired vibration suppression system that contained a series of zig-zag cantilever beams was designed (see Figure 2.12(b)). A small magnet was placed at the end of each cantilever and another was placed in a fixed location directly opposite the cantilevers magnet with a polarity such that the magnets were opposed to one another. The purpose for doing this was that Hobeck theorised a highly non-linear response to incoming vibrations. They predicted a 41.0% suppression of vibration amplitudes using the cantilever alone. With the magnet attached, this figure rose to 84.5% [93]. This is not entirely an active system as no external control is utilised, but by varying the strength of the magnet they could obtain varying wave attenuation performance.

2.4.1. Energy Harvesting Metamaterials

A recent focus in metamaterial research has been the creation of energy harvesters. Energy harvesting is the principal of taking energy in the environment and converting it to some form of useful energy, typically electricity. This idea has existed for many years, for example, the water mills or wind turbines which convert kinetic energy to generate mechanical or electrical power.

The process of converting vibration waves to electrical energy can be electrome-
2.4. Active Metamaterials

(a) Magneto-elastic type LRM developed by Chen et al [92].
(b) Magnetic enhanced cantilever vibration suppression device [93].

Figure 2.12.: Magnetic based active metamaterials.

Mechanical (typically using piezoelectrics), electromagnetic or electrostatic [94, 95]. Piezoelectric crystals (PZT), when exposed to mechanical strain, create an electrical polarity that can be connected to an external circuit, measured and stored. To increase the voltage of the electrical polarity, thicker crystals can be used, or multi-layer designs can be created [96].

Electromagnetic based harvesters utilise a magnet and a coil to generate electricity (Faraday’s Law [97]). The magnet must move relative to the coil for electromagnetic induction to occur. To increase the output from an electromagnetic based design, the strength of the magnet used could be amplified, the number of turns of the coil can be increased, or the velocity of the magnet with respect to the coil can also be amplified. In electromagnetic based systems, the damping resulting from the electrical generation is proportional to the velocity of the magnet [96, 98].

Electrostatic based harvesters use the principle of capacitance on two electrically isolated plates. If these two plates have opposite charge and are placed perpendicular to each other, a force is exerted between the two. If the charge is kept constant, and the plates are moved relative to each other, a voltage is induced which can be measured and stored [96, 97].

The use of PZT elements for energy harvesting is generally favoured due to their simplicity to implement and their ability to generate voltages in the range that can be easily stored by capacitors. However, PZT elements can be brittle, and with low
output currents the overall power output can be low.

Electromagnetic harvesters are well established and can be used in various design configurations. They also offer the ability to have remote, or wireless energy harvesting and no direct wires are required on the magnet itself. In contrast to the PZT material, typically electromagnetic harvesters are associated with low voltage, high current output.

Electrostatic harvesters produce high voltage outputs, typically greater than 100V \cite{96}, but consequently have very low currents. Additionally, they require a constant connection to a power source to maintain the charge on the capacitive plates, making it unsuitable for certain applications.

There is growing interest in the use of metamaterials for energy harvesting because of the unique properties in wave manipulation that they exhibit. It has been shown that metamaterial based harvesters work optimally when the frequency of the harvested waves are close to the resonant frequency of the LRM \cite{94, 99, 100}. In the case of an LRM type metamaterial, the resonant frequency equates to the maximum displacement of the resonator, and hence the maximum potential energy to be harvested.

Some designs of metamaterials used in energy harvesting applications are similar to PC type designs. The resonant modes, where energy harvesting is optimal, is dependent upon the periodic structure of the harvester \cite{10, 101–103}.

In 2013, Carrara et al. \cite{9} used metallic stubs as scatterers in a parabolic arrangement to create a parabolic acoustic mirror (PAM). This PC type set up was used to reflect incoming waves onto a centralised point where a PZT patch was attached (see Figure 2.15). Carrara demonstrated that by having this centralised focal point, the vibration was greater than the equivalent point with no PAM around it and hence caused greater deformation of the PZT patch. The resulting voltage output was approximately 4 times as large when compared to the voltage measured with no PAM (see Figure 2.13(b)).

For PC type harvesters, the typical energy captured is from waves in the kilohertz to megahertz frequency range. Typically vibrations from machinery and vehicles
2.4. Active Metamaterials

(a) The parabolic arrangement of scatterers to focus waves.

(b) The voltage output of a PZT patch, with and without the PAM setup for a 55 kHz input wave.

Figure 2.13.: A phononic crystal type energy harvester [9].

vary within the 10 Hz to 10 kHz frequency range [39, 95].

To achieve wave attenuation at lower frequencies, the use of LRM type energy harvesters is favoured due to their ability to achieve resonant modes in the lower frequency range. This principle of converting low frequency vibrations into useful energy could be utilised in different scenarios. For example, the vibrations in a stationary car engine at traffic lights could be converted into electricity to help recharge the battery, reducing the noise of the engine to nearby pedestrians and wildlife. An additional potential application of energy harvesting metamaterials is to power micro-electromechanical devices, or ‘MEMs’ technology as it is more commonly known [104]. For example, this can include wearable technologies such as smart watches, which can be powered by the movement of an arm as you walk as opposed to a rechargeable battery [98].

Various designs for LRM based energy harvesters have been produced. In 2009 Gonella et al proposed a periodic cantilever honeycomb design [10], where the cantilevers were made from PZT material (see Figure 2.14). The design utilised the deformation of the honeycomb lattice upon application of a vibrational wave to deform the PZT cantilevers, and in doing so generated electricity. By varying the length of the cantilevers across the honeycomb lattice, Gonella was able to achieve resonant modes at various frequencies and hence a broad spectrum energy harvester.
was created. At peak output, the design was able to generate 0.769 W. The formation of the PZT cantilevers on the structure was designed such that all the load would go through the original honeycomb structure, and not through any of the PZT material itself. Gonella stated this capability meant it could be used in structurally sensitive locations such as aircraft frames, where structural integrity is a critical design factor.

![A piezoelectric cantilever honeycomb LRM design](image)

Figure 2.14.: A piezoelectric cantilever honeycomb LRM design as proposed by Gonella et al [10].

An alternative LRM type design was proposed by Ahmed et al [11, 12]. In this set up, a typical LRM with a heavy core, rubber coating, and a matrix material also had a disk of PZT material inserted into the rubber lining. With the mode 2 excitation (displacement of the heavy core) the resulting deformation of the piezoelectric patch generates a current that can be measured by an oscilloscope and eventually stored in a capacitor. When tested, the LRM design was able to achieve a 1.3 mW output at a resonant frequency of 420 Hz. This design has been further investigated recently in 2017 by Hu et al, who have attempted to optimise the design of a PZT based LRM energy harvester based on analytical models [105].

### 2.5. Alternative Applications for Metamaterials

As stated in Section 2.2, the foundations of metamaterial research originated in the electromagnetic wave regime. Many of the breakthroughs and novel concepts for the use of metamaterials originated from this sector and allowed similar developments for acoustic/elastic wave metamaterials. Two such breakthroughs have been the development of metamaterials for use as ‘perfect lenses’ and ‘cloaking’ [106].
2.5. Alternative Applications for Metamaterials

(a) Location of the PZT patch.

(b) Harvesting methodology for the LRM.

Figure 2.15.: The LRM type harvester concept proposed by Ahmed et al [11, 12].

Wave Focusing (Perfect Lens Concept)

In early 2000, following on from Smith’s paper where negative refraction had been achieved [50], Pendry wrote a landmark paper in which he stated that negative refraction could make a perfect lens [13].

Figure 2.16.: Pendry’s concept of a negative refractive material to create a perfect lens [13].

Pendry argued that optical lenses were unable to focus light onto an area smaller than a square wavelength of the light frequency used (known as the Rayleigh Diffraction limit [13]). For features which are smaller than the wavelength of the light, the reflected waves are known as ‘evanescent waves’ which can decay quickly, and therefore are not observed [13]. Whilst this is not a concern for most applications, it does limit the size of what can be observed with light in the visible spectrum. Electron microscopes exist which are able to resolve smaller features of an object.
than light microscopes can, but they require non-living samples. Pendry stated that via negative refraction, normal propagating light and evanescent waves from the object could pass through his perfect lens concept and contribute to the resolution of the final image (see Figure 2.16).

Although mathematically possible, no material existed that possessed suitable properties. Initially his paper [13] was not well received, but has now become one of his most well-known papers and indeed has been the reason why many scientists and engineers pursue this field of study today.

This has inspired the use of acoustic metamaterials to achieve a similar goal including the use of a PC made from tungsten carbide beads and water by Yang et al in 2004[107], and another PC made from stainless steel rods and air by Sukhovic et al.[108]. Since these early examples, further research into the application of acoustic metamaterials for the use of acoustic and elastic wave focusing have been explored [109–112].

**Cloaking**

In addition to the idea of a perfect lens, the realisation that negative refraction could be physically realised also allowed the possibility of an invisibility cloak [34, 113]. A cloak, in this sense of the word, is a device that can bend incoming light/electromagnetic waves around an object. These waves would then reform together on the opposite side of the object, in such a manner that the waves do not appear to have distorted. An observer in front of, or behind, the object would not be able to detect the object's presence from its interaction with the waves.

This has long been the dream of many science fiction writers, but now the idea of a working cloak did not seem so far-fetched. Various scientists such as Pendry et al [34] and Smith et al [51, 114], studied the concept of cloaking for years, with further development of the mathematical models, but no physical cloaks had been fabricated. It was not until 2006 that a working cloak had been developed by Schurig et al. [14].
2.5. Alternative Applications for Metamaterials

Figure 2.17.: The electromagnetic cloak created by Schurig et al.[14].

His experiment consisted of a metal cylinder placed in the field of propagating microwaves of a single frequency. Around this cylinder was a structure made from concentric circles of the split-ring resonator that had been used initially to demonstrate negative refraction, similar to what had been developed by Pendry[48]. The metamaterial was not a perfect cloak (there was some distortion), but it did show a reduced ‘shadow’ from the cylinder, reduced reflections from the cylinder, and the re-attachment of the propagating electromagnetic waves. This was enough to demonstrate that the device was indeed a cloak. In 2013, Landy et al [115], whilst working closely with Schurig and Smith, developed an alternative cloak design with better wave reattachment and reduced shadow than observed in 2006. To date, no research team has yet managed to create a cloaking device in the optical frequency regime.

Investigations into the creation of an acoustic cloak originated in 2006 [116–118]. Chen et al [119] proved mathematically that acoustic cloaking could be achieved. They showed that unlike metamaterials used to achieve wave attenuation, acoustic cloaking metamaterials must simultaneously display negative bulk modulus and negative density. The science of cloaking has progressed recently and the idea of transformation acoustics has arisen, based on the principle of ‘transformation optics’ as developed by Pendry et al [34]. Transformation acoustics is based on the principle of using metamaterials to transform the local spatial coordinate system.
around an object such that, from the phonon's point of view, its propagation is un-
 altered. From an external viewpoint, it travels in a curved path around the object 
[120, 121]. An experimental breakthrough came by Zhang et al [122], where they 
were able to bend ultrasonic sound waves around a circular obstacle and reform 
them on the opposite side. The result was not perfect as there was some distortion 
in the reformed waves, but the experiment displayed much better ‘cloaking’ than 
previous attempts. Alternative designs for an acoustic cloak have been proposed 
and experiments carried out [123, 124]. Effective acoustic cloaking remains a very 
relevant research goal and indeed is the subject of many recent papers [125–127].

2.6. Modelling of Metamaterials

Determining the wave attenuation characteristics of a metamaterial can be done 
in several ways, and is necessary if metamaterials are to see use in modern applica-
tions. The behavioural response of a metamaterial is typically determined from its 
band structure, also sometimes referred to as a frequency band diagram. The band 
structure diagram is a frequency ($f$) versus wave vector ($\vec{k}$) plot, analysed at certain 
points within a representative unit cell (RUC) of the periodic metamaterial. The 
wave vector, $k$, shows how waves of a certain frequency propagate through a RUC. 
The wave vector is composed of a real and imaginary part. The real part is known as 
the propagation constant, and it describes the phase shift of the propagating wave. 
The imaginary part of the wave vector describes the attenuation factor.

The use of a RUC for modelling the behaviour of a periodic media has been 
established as more efficient than modelling the metamaterial as a whole [129]. The 
unit cell of a periodic media is determined by its lattice structure. For this unit 
cell, a reciprocal lattice is defined, which is then used to generate the Irreducible 
Brillouin Zone (IBZ) for the periodic media. The IBZ is an important concept in 
the study of periodic structures. It can be shown that the properties and behaviours 
within the IBZ can be applied to all parts within the periodic media. Therefore, 
if the wave propagation behaviour is known within the IBZ, it is known across the
2.6. Modelling of Metamaterials

(a) IBZ for a Simple Cubic Lattice (image source [28])
(b) IBZ for a Simple Tetragonal Lattice (image source [28])
(c) IBZ for a Simple Hexagonal Lattice (image source [28])
(d) Image of a 2D PC type metamaterial and its corresponding IBZ shown in the inset. [128]
(e) An example band structure diagram. Based on the PC in Figure 2.18(d). The solid and dashed lines represent the modelled behaviour from the plane wave expansion and the finite element method respectively.

Figure 2.18.: Example IBZ for different lattice types and an example frequency band structure for a PC with a square a lattice. The band gap is shown as the grey region. Waves with frequencies within this region are attenuated.

metamaterial. The vertices of the IBZ are represented by letters. For a simple cubic lattice, the vertices are represented by \( \Gamma, X, M, \) and \( R \) (see Figure 2.18(a)).

Of the numerous methods that can be used to generate the band structure of a metamaterial, some are easier to implement than others. However, some allow more information about the metamaterial to be determined. In this section, a review is carried out of the different modelling techniques commonly used for metamaterial modelling. This includes the ‘Plane Wave Expansion’ (\( PWE \)) method, the ‘Multiple Scattering’ (\( MST \)) method, the ‘Finite Element Modelling’ (\( FEM \)) method, and the ‘Finite Difference Time Domain’ (\( FDTD \)) method.
2.6. Modelling of Metamaterials

2.6.1. Plane Wave Expansion Method

The Plane Wave Expansion (PWE) method is a numerical technique used to model the behaviour of metamaterials. Originally utilised to determine the band structure of photonic metamaterials in the electromagnetic regime, it has found wide applicability to acoustic and elastic metamaterials, although is mostly used for the modelling of PCs.

The PWE method states that system parameter functions (density and speed), and the wave functions in the wave equation can be expanded in a Fourier series [130]. The Bloch-Floquet theorem can then be applied to this expansion to model the metamaterial as a RUC. The starting wave equation is:

\[ \nabla \cdot \left[ \frac{1}{\rho(r)} \nabla p(r) \right] + \frac{\omega^2}{\rho(r)c^2(r)} p(r) = 0 \]  

(2.19)

where \( \vec{r} \) is the position vector, \( \rho(\vec{r}) \) is the mass density, \( c(\vec{r}) \) is the speed of sound, and \( p(\vec{r}) \) is the pressure field. It is possible to rewrite Eq. 2.19 with a definition of the scalar potential \( \Phi(\vec{r}, t) \) such that \( \rho \ddot{\vec{u}} = \nabla \Phi \) (\( \dddot{\vec{u}} \) is the displacement vector):

\[ \frac{1}{\rho c_l^2} \frac{\partial^2 \Phi}{\partial t^2} = \nabla \cdot (\rho^{-1} \nabla \Phi) \]  

(2.20)

where \( \frac{1}{\rho c_l^2} \) is the longitudinal elastic constant. Applying the Bloch-Floquet theorem and rearranging to determine the pressure field gives:

\[ p(\vec{r}) = e^{i(\vec{k} \cdot \vec{r} - \omega t)} \sum_{\vec{G}} \Phi_{\vec{k}}(\vec{G}) e^{i(\vec{G} \cdot \vec{r})} \]  

(2.21)

where \( \vec{k} \) is a wave vector, and \( \vec{G} \) is the reciprocal lattice vector and \( \omega \) is the angular frequency. The summation is made for all potential reciprocal vectors. As this is a periodic structure, some terms can be expanded by discrete waves:

\[ \frac{1}{\rho(r)} = \sum_{\vec{G}} \sigma(\vec{G}) e^{i(\vec{G} \cdot \vec{r})} \]  

(2.22)

\[ \frac{1}{\rho(r)c^2(r)} = \sum_{\vec{G}} \eta(\vec{G}) e^{i(\vec{G} \cdot \vec{r})} \]  

(2.23)
where $\sigma(\vec{G})$ and $\eta(\vec{G})$ can be determined from the inverse Fourier transform of $\rho(\vec{r})$ and $(\vec{r})$ respectively.

Substituting Eqs.2.21 - 2.23 into the original wave equation (Eq.2.19) gives:

$$- \sum_{\vec{g}} \left[ \sigma(\vec{G} - \vec{G'}) (\vec{k} + \vec{G}) \cdot (\vec{k} + \vec{G'}) - \eta(\vec{G} - \vec{G'}) \omega^2 \right] \sigma_{\vec{k}\vec{G}} = 0 \quad (2.24)$$

Using a number of $N$ Fourier components, an approximate $N \times N$ matrix is formed, for which $\Gamma$ can be determined:

$$\sum_{\vec{G'}} \Gamma_{\vec{G},\vec{G'}} \Phi_{\vec{k}}(\vec{G'}) = 0 \quad (2.25)$$

The determinant of this resulting matrix can then be determined to find the relationship between the frequency and wave vectors.

$$\det|\Gamma_{\vec{G},\vec{G}'}| = \det[\sigma(\vec{G} - \vec{G'}) (\vec{k} + \vec{G}) \cdot (\vec{k} + \vec{G'}) - \eta(\vec{G} - \vec{G'}) \omega^2] \sigma_{\vec{k}\vec{G}} = 0 \quad (2.26)$$

Once the above relationship is known, the frequency band structure can be generated. The PWE calculates angular frequency as a function of the wave vector ($\omega(\vec{k})$). Therefore, the imaginary part of the band structure is not able to be determined.

Certain terms in the above derivation will alter depending on the shape of RUC being analysed, and the shape of the inclusions within the RUC. The reciprocal lattice vectors $\vec{G}$ will change depending upon the lattice type and if the model is 2D or 3D. For example, to perform a 2D analysis on a square RUC, the vectors are defined as follows:

$$\vec{G} = \left( \frac{2\pi}{a} \right) (n_x + n_y) \quad (2.27)$$

where $a$ is the lattice constant. The density term $\rho(\vec{r})$ is calculated as a function of the position to correctly represent the shape and mass of the inclusions within the RUC.

The PWE method has been used in several various studies to generate the band structure of different metamaterial designs [35, 131-133]. As the PWE method uses Fourier expansions to describe the behaviour of a metamaterial, the number of
2.6. Modelling of Metamaterials

waves used in the expansion affects the accuracy of the result. As a result, the PWE method can be computationally expensive. In addition to this limitation, there are several scenarios in which the PWE method struggles to model the metamaterial correctly. The PWE method works well with circular/spherical inclusions, but numerical convergence issues arise if irregular shaped inclusions are modelled, and/or the filling fraction of the inclusion is very high or low [17, 134]. Additionally, if the metamaterial being modelled contains a fluid inclusion with a solid matrix, convergence issues arise [135]. However, the PWE method does work well if the inclusion is solid, and the matrix material is fluidic. This is one reason why the PWE method is favoured for modelling PCs in air.

2.6.2. Multiple Scattering Method

The Multiple Scattering Method (MST) method was first used for modelling acoustic metamaterials in the early 2000’s by various research groups [36, 54, 136]. As the dominant attenuation effect caused by a PC is Bragg scattering, the MST has found particular relevance for modelling of PCs (although it has been applied for the modelling of LRM too). The MST is an adaptation of the Korringa-Kohn-Rostoker method [17] that had been used extensively for electronic band structure calculations. The MST has seen use in many applications where the PWE method failed to converge, or the metamaterial being modelled contains some fluidic material that the PWE fails to model correctly.

Similar to PWE, the MST method solves the elastic wave equation (Eq. 2.28 for waves incident on the metamaterial and scattered waves. The form of the equations shown is the same as presented in [136]:

\[(\lambda 2\mu)\nabla(\nabla \cdot \vec{u}) - \mu \nabla \times \nabla \times \vec{u} + \rho \omega^2 \vec{u} = 0\]  

(2.28)

where \(\lambda\) and \(\mu\) represent the Lamé constants and \(\vec{u}\) is the displacement field. The MST operates by determining the scattered wave displacement field in relation to the incoming wave field. The method utilises the spherical Bessel and Hankel functions, and Mie elastic scattering solution to predict where these waves cancel out and
which waves progress to the second layer of the metamaterial. The general solution to Eq. 2.28 is:

\[ u(\vec{r}) = \sum_{lmn} [a_{lmn} J_{lmn}(\vec{r}) + b_{lmn} H_{lmn}(\vec{r})] \]  

(2.29)

\[ J \] and \( H \) are defined as follows:

\[ J_{lm1}(\vec{r}) = \frac{1}{\alpha} \nabla [j_l(\alpha r) Y_{lmn}(\vec{r})] \]  

(2.30)

\[ J_{lm2}(\vec{r}) = \frac{1}{l(l+1)} \nabla \times [\vec{r} j_l(\beta r) Y_{lm}(\vec{r})] \]  

(2.31)

\[ J_{lm2}(\vec{r}) = \frac{1}{l(l+1)\beta \alpha} \nabla \times \nabla \times [\vec{r} j_l(\beta r) Y_{lm}(\vec{r})] \]  

(2.32)

\[ H_{lm1}(\vec{r}) = \frac{1}{\alpha} \nabla [h_l(\alpha r) Y_{lmn}(\vec{r})] \]  

(2.33)

\[ H_{lm2}(\vec{r}) = \frac{1}{l(l+1)} \nabla \times [\vec{r} h_l(\beta r) Y_{lm}(\vec{r})] \]  

(2.34)

\[ J_{lm2}(\vec{r}) = \frac{1}{l(l+1)\beta \alpha} \nabla \times \nabla \times [\vec{r} h_l(\beta r) Y_{lm}(\vec{r})] \]  

(2.35)

where \( \alpha = \sqrt{\frac{\omega}{

\rho + 2\mu}, \beta = \sqrt{\frac{\omega}{

\rho + 2\mu}} \), \( j_l(x) \) is the spherical Bessel function of the first kind, and \( h_l(x) \) is spherical Hankel function of the first kind. The incident waves upon a scatterer can be separated into two parts; firstly the incident component, and secondly the sum of all the scattered components. These are shown in Eq. 2.36 and Eq. 2.37 respectively.

\[ u_{in}^{i(0)}(\vec{r}_i) = \sum_{lmn} \alpha_{lmn}^{i(0)} J_{lmn}^i(\vec{r}_i) \]  

(2.36)

\[ u_{in}(\vec{r}_i) - u_{in}^{i(0)}(\vec{r}_i) = \sum_{j \neq i} \sum_{l' m' n'} \beta_{l' m' n'} H_{l' m' n'} j(\vec{r}_i) \]  

(2.37)

where \( (\vec{r}_i) \) and \( (\vec{r}_j) \) represent the point in space measured from scatterers \( i \) and \( j \).

Based on these previous equations, the following relationship can be defined:

\[ H_{l' m' n'} j(\vec{r}_i + R_i - R_j) = \sum_{lmn} G_{l' m' n'}(R_i - R_j) J_{lmn}^i(\vec{r}_i) \]  

(2.38)
where $G$ is the vector structure constant. After the calculation of this constant [136], the following equation can be derived:

$$H_{lmn}^j(\tilde{r}_j) = \sum_{lmn} G_{lmn}^{ij} J_{lmn}(\tilde{r}_i)$$

(2.39)

The scattered displacement field can be calculated from the incident displacement field by the use of a scattering matrix:

$$B = TA$$

where $A = \{b_{lmn}\}$, $B = \{a_{lmn}\}$, and $T = t_{lmn}^{m'n'}$, which is the scattering matrix. The scattering matrix is derived from the Mie scattering solution, which is determined for the scatterer [136]. This leads to the following solution, which is the general form of the MST:

$$\sum_{j'm'n'} \left( \partial_{ij} \partial_{jl} \partial_{mn} \partial_{n'n'} - \sum_{l'm'n'n'} t_{l'm'n'n'}^{j'l'm'n'n'} G_{l'm'n'n}'^{ij} \right) a_{lmn}^{j'} = \alpha_{lmn}^{(0)}$$

(2.40)

The normal modes of the metamaterial response to an external perturbation can be found by solving the determinant of the general form of the MST:

$$\text{det} \left| \partial_{ij} \partial_{jl} \partial_{mn} \partial_{n'n'} - \sum_{l'm'n'n'} t_{l'm'n'n'}^{j'l'm'n'n'} G_{l'm'n'n}'^{ij} \right| = 0$$

(2.41)

Applying Bloch boundary conditions to Eq. 2.41 allows the solution to be true for a periodic system. This produces the following adapted equation:

$$\text{det} \left| \partial_{ij} \partial_{jl} \partial_{mn} \partial_{n'n'} - \sum_{l'm'n'n'} t_{l'm'n'n'}^{j'l'm'n'n'} G_{l'm'n'n}'^{ss} \right| (\tilde{k}) = 0$$

(2.42)

where $s$ and $s'$ refer to the scatterers within a RUC being modelled, and $\tilde{k}$ is the wave vector. From Eq. 2.42, the relation between the frequency and the wave vector can be plotted to generate the frequency band structure of the model.

One of the additional benefits of MST when compared to the PWE method is that it allows the calculation of the transmission and reflection coefficients, which can be useful when making comparisons with acoustic testing and theory. The
transmission coefficient is a measure of how much wave energy is transmitted through a material. The reflection coefficient is a measure of how much energy is reflected by a material. To calculate these coefficients, the MST is applied to a model with several layers of periodic scatterers (inclusions). Analysing the first layer, the transmission and reflection coefficients are calculated by determining the wave as it enters the layer, and then as it passes through to the start of the subsequent layer. The same coefficients are then calculated for a wave passing through two layers of scatterers. Both of these calculations generate transmission matrices and reflection matrices. By comparing these matrices, it is possible to determine the transmission and reflection coefficients for the PC.

A known limitation of the MST, in addition to its complexity and difficulty to implement correctly for different types of metamaterials, is that it works well for designs where the inclusions are smooth, regular shapes such as spheres or cylinders [134]. Non-regular inclusion shapes are more difficult to model.

2.6.3. Finite Element Modelling (Modal Analysis)

Finite element modelling (FEM) is a method of discretising a domain into several nodes and elements, which can be used to approximate a numerical solution to differential equations. It is widely used in many engineering sectors to solve various types of problems. The term, finite element modelling, is used for various forms of analysis. With regards to acoustic metamaterials, it refers to the method of generating the Mass (\(M\)) and Stiffness (\(K\)) matrices for a domain, upon which a modal analysis can be performed. There are also alternative FEM based modelling methods for metamaterials, such as the Finite Difference Time Domain analysis (see Section 2.6.4). Henceforth in this thesis, to avoid confusion, the following outlined method for generating the band structure will be referred to as the Modal Analysis Method (MAM).

The MAM technique is a type of numerical modelling that has been applied to metamaterials research and has proven to be easier to implement when compared to alternative methods, such as the MST method. The primary reason for this is the existence of commercial software packages that can readily generate the Mass ma-
2.6. Modelling of Metamaterials

trix ($\mathbf{M}$) and Stiffness matrix ($\mathbf{K}$) for simple and irregular metamaterial domains. Similar to the MST and PWE, the RUC of a metamaterial design is modelled with Bloch-Floquet boundary conditions applied. The MAM is a frequency based analysis and relates the frequency to the wave vectors in the RUC. The final result allows the frequency band structure to be generated for the metamaterial design.

The use of MAM to generate the band structure of a metamaterial begins with generating the geometry for the metamaterial design using CAD software. This geometry is then discretised into elements and nodes. Bloch wave boundary conditions (see Eqs. 2.43 and 2.44), are added to the nodes on the border of the RUC:

\[ \mathbf{U}(l) = \mathbf{U}(0)e^{i\mathbf{k} \cdot l} \quad (2.43) \]
\[ \mathbf{F}(l) = \mathbf{F}(0)e^{i\mathbf{k} \cdot l} \quad (2.44) \]
\[ |\mathbf{K} - \omega^2\mathbf{M}|\mathbf{U} = \mathbf{F} \quad (2.45) \]

where $\mathbf{k}$ is the wave vector and $l$ is the lattice constant of the periodic media [15]. Eq.2.45 is the general form of the modal analysis equation. Matrices ($\mathbf{U}$) and ($\mathbf{F}$) show the displacement and force on each node respectively.

Figure 2.19.: Application of Bloch boundary conditions [15]
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Applying the Bloch boundary conditions requires the displacement matrix \((U)\) and force matrix \((F)\) for the RUC to be reduced with a transformation matrix \((Q)\), which defines the boundary conditions. In Eq. 2.46, \((I)\) is the identity matrix and \((0)\) is a zero matrix.

\[
Q = \begin{bmatrix}
I & 0 \\
0 & I \\
e^{ikl} & 0
\end{bmatrix}
\]  
(2.46)

The reduced displacement matrix \((\tilde{U})\) and force matrix \((\tilde{F})\) are shown below.

\[
U = Q\tilde{U} \quad \text{(2.47)}
\]

\[
F = Q\tilde{F} \quad \text{(2.48)}
\]

For consistency in the matrices, \((U)\) and \((F)\) are arranged such that the corresponding displacement and forces for the independent boundary and internal nodes of the RUC are at the top of the matrices. The nodes with the applied Bloch boundary wave conditions occupy the bottom rows of the matrix.

Eq. 2.46 is multiplied by the Hermitian of \((Q)\) to ensure force continuity [137] (a Hermitian matrix is a complex square matrix that is equal to its own conjugate transpose):

\[
Q^H[K - \omega^2M]QU = Q^HQF \quad \text{(2.49)}
\]

where \(H\) denotes the Hermitian transpose of the matrix and \(\omega\) is the angular frequency.

Eq. 2.49 can be rearranged into the form of an eigenvalue problem by setting the right hand side of the equation to zero and simplifying (see Eq. 2.50). The band structure for the RUC of a metamaterial can then be found by setting the wave vector \((\vec{k})\) and solving for \(\omega^2\). The wave vector is set to correspond to points along the edge of the IBZ.

\[
[\tilde{K} - \omega^2\tilde{M}]\tilde{U} = \tilde{F} = 0 \quad \text{(2.50)}
\]
2.6. Modelling of Metamaterials

\[ \tilde{K} = Q^H K Q \]  
(2.51)

\[ \tilde{M} = Q^H M Q \]  
(2.52)

The main benefit of the MAM method is its relative ease in implementation for 2D and 3D models. This allows for more complex metamaterial designs to be characterised. There are many commercial software packages available to generate the stiffness and mass matrices. Subsequent calculations, as they are mainly array based, are programmable within a mathematical computing environment such as MATLAB. Similar to the PWE method, as MAM method determines angular frequency as a function of the wave vector \( \omega(k) \) in the eigenvalue problem, the imaginary part of the band structure is not able to be determined.

2.6.4. Finite Difference Time Domain Method

The Finite Difference Time Domain (FDTD) method is a popular method of modelling metamaterial behaviour. It has been used in several publications (see [138–140]), but unlike other methods such as the PWE or MAM methods, the FDTD performs calculations in the time domain. The method has proved popular for its ability to determine transmission coefficients as well as band structure for a given metamaterial design.

The FDTD method for metamaterial analysis starts with the elastic wave equation.

\[ \frac{\partial^2 \tilde{u}}{\partial t^2} = \frac{1}{\rho(x, y, z)} \bar{\nabla} \cdot \bar{\sigma} \]  
(2.53)

where \( t \) is time, \( \rho(x, y, z) \) is the mass density, \( \tilde{u}(x, y, z, t) \) is the displacement field, and \( \bar{\sigma} \) is the total stress tensor. If the wave propagating medium is a fluid, and the input wave is harmonic (\( p = p_0 e^{i\omega t} \)), the Helmholtz wave equation is used:

\[ \nabla \cdot \left( -\frac{1}{\rho_0} \nabla \rho_0 \right) - \frac{\omega^2 p_0}{\rho_0 c^2} = 0 \]  
(2.54)
2.6. Modelling of Metamaterials

where $p$ is the pressure and $c$ is the speed of sound for the medium.

For the simplicity of developing the FDTD methodology, only the elastic wave is analysed. The model is discretised into a series of nodes and elements, as done in the MAM method, and a probing signal is sent into the model from a user defined input location. The FDTD method involves solving the elastic wave equation for the stress and displacements at each node, with small increases in time as it uses an explicit solving method.

For a three dimensional material, the central difference scheme (second order accurate) is utilised to generate the following approximations of the derivatives:

$$
D_1[G(i, j, k, l)] = \frac{1}{\Delta x_1} \left[ G(i + \frac{1}{2}, j, k, l) - G(i - \frac{1}{2}, j, k, l) \right] \quad (2.55)
$$

$$
D_2[G(i, j, k, l)] = \frac{1}{\Delta x_2} \left[ G(i, j + \frac{1}{2}, k, l) - G(i, j - \frac{1}{2}, k, l) \right] \quad (2.56)
$$

$$
D_3[G(i, j, k, l)] = \frac{1}{\Delta x_3} \left[ G(i, j, k + \frac{1}{2}, l) - G(i, j, k - \frac{1}{2}, l) \right] \quad (2.57)
$$

$$
\frac{\partial^2}{\partial t^2} [G(i, j, k, l)] = \frac{1}{(\Delta t)^2} \left[ G(i, j, k, l + \frac{1}{2}) - 2G(i, j, k, l) + G(i, j, k, l - \frac{1}{2}) \right] \quad (2.58)
$$

where $G$ is the medium being modelled, and $D$ is the derivative with respect to space. These equations can be substituted back into Eq.(2.53) to determine the displacements for the next time step. This is an iterative procedure which allows the wave equation to be solved numerically.

Periodic boundary conditions need to be applied to the side of the model parallel to the direction of the wave's propagation. In addition to this, absorbing boundary conditions need to be added to the ends of the model representing the inlet and outlet of the perturbation wave. This is to stop the reflected wave affecting the results from the model. The result of this numerical calculation is a time dependant displacement field for the metamaterial model. The transmission and absorption coefficients can be determined by comparing the results from the metamaterial model to an equivalent model with a homogeneous medium. Fourier transforms are taken of the two resultant time domain signals to generate frequency domain signals. The
ratio of the metamaterial's signal to the homogeneous material's signal produces the frequency dependant transmission coefficient.

To determine the band structure for the metamaterial using the FDTD method, the displacement field and the stress tensor must satisfy Bloch's theorem:

\[ \tilde{u}(\tilde{r}, t) = e^{i\vec{k}.\tilde{r}} \tilde{U}(\tilde{r}, t) \] (2.59)

\[ \tilde{\sigma}(\tilde{r}, t) = e^{i\vec{k}.\tilde{r}} \tilde{\Sigma}(\tilde{r}, t) \] (2.60)

where \( \tilde{r}(x, y, z) \) is the position vector and \( \vec{k}(k_x, k_y, k_z) \) is the Bloch wave vector. \( \tilde{U}(\tilde{r}, t) \) and \( \tilde{\Sigma}(\tilde{r}, t) \) are periodic spatial functions satisfying the Bloch requirement that \( \tilde{U}(\tilde{r} + \vec{l}) = \tilde{U}(\tilde{r}) \) and \( \tilde{\Sigma}(\tilde{r} + \vec{l}) = \tilde{\Sigma}(\tilde{r}) \), where \( \vec{l} \) is the lattice constant of the periodic media.

Eq. 2.59 and 2.60 are inserted into Eq. 2.53 to give:

\[ \frac{\partial^2 \tilde{U}}{\partial t^2} = \frac{1}{\rho(\tilde{r})} i\vec{k}.\tilde{\Sigma}(\tilde{r}, t) \] (2.61)

To solve Eq. 2.61, one specifies a 3D wave vector \( \vec{k}(k_x, k_y, k_z) \) along the principal direction of the IBZ. The equation is then solved at the discretised points in the RUC for both space and time. A Fourier transform is taken of the resultant time domain signal. The peaks in the frequency domain signal represent the eigenfrequencies of the normal modes of the system for the given wave vector, \( \vec{k} \).

The FDTD allows for additional information about a propagating wave to be determined, including the transmission coefficient. It is capable of determining the band structure for a metamaterial design, and can cope with non regular geometry. Being in the time domain and using an explicit solver, means that this method can be computationally very expensive, particularity if a very fine mesh is used for the model, and a small time step is chosen. With the availability of parallel computing however, this factor is increasingly less of a limitation.
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Finite Element Modelling (Time Domain Analysis)

The FDTD analysis can also be utilised with a FEM solver in the time domain. The FDTD method discretises into points and the differential equations are approximated at each point. The FEM discretises a model into volumes, or elements, and approximates the solution to differential equations element by element. The method of generating the transmission and reflection coefficients using this method are the same as the FDTD method. The main advantage of the FEM method is that it can better represent the geometry of a metamaterial than a FDTD. This is particularly true for complex metamaterial designs. It can also be easier to implement with the prevalence of modern FEM software.

Table 2.2.: Summary of the main advantages and disadvantages of the main metamaterial modelling techniques.

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWE</td>
<td>Well established</td>
<td>Can be computationally expensive</td>
</tr>
<tr>
<td></td>
<td>Works well for fluid matrix mediums</td>
<td>Model geometry affects convergence</td>
</tr>
<tr>
<td></td>
<td>Calculates wave profile data</td>
<td>Does not generate wave profile data</td>
</tr>
<tr>
<td>MST</td>
<td>Well established</td>
<td>Complicated to implement</td>
</tr>
<tr>
<td></td>
<td>No convergence issues</td>
<td>Use for LRM and HR modelling is limited</td>
</tr>
<tr>
<td></td>
<td>Calculates wave profile data</td>
<td>Requires smooth inclusion geometry</td>
</tr>
<tr>
<td>MAM</td>
<td>Increasing popular method</td>
<td>Does not generate wave profile data</td>
</tr>
<tr>
<td></td>
<td>Can model complicated geometry</td>
<td>Can be computationally expensive</td>
</tr>
<tr>
<td></td>
<td>Easy to implement</td>
<td></td>
</tr>
<tr>
<td>FDTD</td>
<td>Well established</td>
<td>Complicated to implement</td>
</tr>
<tr>
<td></td>
<td>Time domain analysis gives additional data</td>
<td>Can be computationally expensive</td>
</tr>
<tr>
<td></td>
<td>Fourier transform generates frequency band structure</td>
<td></td>
</tr>
</tbody>
</table>

2.6.5. Non-Linear Effects

Depending on the type of metamaterial being analysed, to improve the accuracy of modelling, non-linear effects need to be taken into account. This mainly focuses on damping arising from the constituent materials in a metamaterial. For example, LRM s typically have rubber as one of their component materials. Rubber is a viscoelastic material and susceptible to hysteresis with large deformation or cyclic loading. If hysteresis is present, this can result in changes to the frequency band structure of a model. Hence these types of non-linear effects have been widely stud-
2.6. Modelling of Metamaterials

ied, and are still being researched today [4, 93, 141, 142]. For periodic metamaterial applications, there are two considerations; 1) structural damping 2) viscous or viscoelastic damping.

Considering structural damping, which arises from normal material dissipative properties, the normal application of the Bloch boundary conditions on a RUC model assumes that there is no dissipation of wave energy with time:

\[ U(l) = U(0)e^{i(kl-\omega t)} \]  

Mukherjee et al [143] showed that the above application of Bloch's theorem needs to be adapted to include the inherent structural damping observed in all periodic structures. Hussein et al performed the calculations to adapt the theory to include the application of a temporal loss factor, (\( \lambda \))[144].

\[ U(t) = U(0)e^{i(kl)}e^{\lambda t} \]  

The incorporation of this temporal factor produced results more consistent with tests of free vibration analysis on periodic structures [144].

Considering the viscoelastic behaviour in a structure, an early model was developed by Boltzmann [145]. The model was based on the theory that the damping force arising from the viscoelasticity of a material depends upon the history of its motion. This could be computed using the integral:

\[ V(t) = \int_0^t \mu e^{-\mu(t-\tau)}\dot{U}(\tau)d\tau \]  

where \( V \) is an internal variable, \( \mu \) is the shear Lamé constant, \( t \) is total time, \( \tau \) is time, and \( \dot{U} \) is a the velocity [16, 17]. Developing this equation to be applied to a viscoelastic type metamaterial has been shown by Hussein et al in 2013 [16]. The process and notation outlined in Hussein's paper is followed here. (The fully worked derivation of this model is performed in [16, 17, 146].) Eq.(2.64) can be modified by
2.6. Modelling of Metamaterials

using the Leibniz integral rule to the form:

\[ \ddot{V}(t) + \mu V(t) = \mu \dot{V}(t) \] (2.65)

The system of equations for a viscoelastically damped 1D system is:

\[ M_r \dddot{U} + \beta C_r(\kappa) \dot{V} + \omega^2 K_r(\kappa) U = 0 \] (2.66)

where \( M_r, K_r, C_r \) refer to the mass, stiffness and damping matrices respectively, \( \kappa \) is the wave number, and \( \beta \) is a measure of damping. Substituting Eq. (2.65) into Eq. (2.66) gives

\[ M_r \dddot{U} + \beta C_r(\kappa)[\dddot{U} - \frac{1}{\mu} \dot{V}] + \omega^2 K_r(\kappa) U = 0 \] (2.67)

Eq.(2.67) can then be made into an eigenvalue problem by pre-multiplying by \( C_r \) and dividing by \( \mu^2 \), which gives:

\[
\begin{pmatrix}
0 & M_r & 0 \\
M_r & \beta C_r(\kappa) & -\frac{2}{\mu} C_r(\kappa) \\
0 & -\frac{2}{\mu} C_r(\kappa) & \frac{2}{\mu^2} C_r(\kappa)
\end{pmatrix}
\gamma +
\begin{bmatrix}
-M_r & 0 & 0 \\
0 & \omega^2 K_r(\kappa) & 0 \\
0 & 0 & \frac{2}{\mu} C_r(\kappa)
\end{bmatrix}
\begin{bmatrix}
\dddot{U} \\
\ddot{U} \\
\dot{U}
\end{bmatrix} = 0
\] (2.68)

Obtaining the 6 eigenvalues, \( \gamma \), sets of roots appearing as complex conjugates can be extracted which physically represent the modes of damped wave propagation.

One of the conclusions from this is that as \( \mu \) increases, the more viscous behaviour is observed (i.e. damping is based more on the velocity of the material component).

For lower values of \( \mu \), the more viscoelastic behaviour is observed (i.e. the more the damping is dependent upon the history of movement for the component).

When analysing the effect of this viscoelasticity upon the band structure of a metamaterial, the dominant effect is the increasing limit of the upper bound of the band gap, with no effect upon the bottom bound \([16, 146]\) (see Figure 2.20). Additionally, this effect increases with wave number \( \kappa \).

This is a desirable behaviour for most wave attenuation applications and hence viscoelastic type LRMs are well suited for this purpose. Another important conclusion
based on this result is that the lower bound of the band gap can be determined from simpler zero-damping models, as the lower bound of the band gap is typically what dictates the design of an LRM [4].

![Figure 2.20.](image)

Figure 2.20.: Variation in the upper boundary of the band gap for an LRM with varying damping levels ($\frac{\beta}{\omega}$) [16, 17].

### 2.7. Manufacture of Metamaterials

As the field of acoustic and elastic metamaterials is relatively young, it has yet to be adapted into mainstream industrial applications. Much of the research into metamaterials has been theory orientated. Experimental verifications of the various designs that have been created are less published. One likely reason is the difficulty in making these metamaterials because of their small and composite nature. Experimental set ups that are created are often made by hand, hence small variations in a design can be costly and time consuming to manufacture and test.

With recent advances in the development of 3D printing, different types of metamaterial can now be printed and tested. One example was created by Reynolds et al in 2013 [78], in which he used a laser sintering type 3D printer (see Section 3.2.4) to make an object with several HRs embedded within it (see Figure 2.21(a)). The material used was ‘Polyamide 6’ and Reynolds concluded the quality of the
finished part was good. In another example, an acoustic cloak was fabricated in 2013 by Sanchis et al [147], with the use of a fused deposition type printer (see Section 3.2.3). The design included a spherical ball surrounded by concentric rings, all made from the same unspecified plastic material (see Figure 2.21(b)). A partial one dimensional cloaking affect was observed at 8.55 kHz. In both these examples, the geometry is complex and would have been difficult to fabricate using traditional manufacturing methods. These are both good examples of the advantages that additive manufacturing can have in the study of metamaterials.

As demonstrated, current commercial 3D printers work well for certain designs of metamaterial like HRs. This is not true for LRM, as they typically have 3 different material components and with nested objects (i.e. one shape inside another), making it difficult to fill a material component later in a lost-wax casting type procedure. Additionally, there are no current commercial 3D printers that can print with three different types of material with suitable properties to make an LRM.

Previously LRM designs have been fabricated using traditional techniques, which can be labour intensive. As such, the likelihood of industrial adoption of LRM is reduced, as automated manufacturing processes are strongly favoured over labour based manufacturing processes. There is a strong desire for an effective way to make
LRMs, as many see acoustic and elastic metamaterials now reaching a transition point from design to realisation with real world applications [17, 32, 39].

Some attempts to fabricate an LRM using additive manufacturing methods do exist. As stated, due to the limitations of 3D printers, the designs of LRMs have been altered to meet the printing capability. In 2016, Qureshi et al produced a cantilevered LRM design [18] that was manufactured using a droplet direct write type printer (see Section 3.2.5). The cantilevered design was made from one plastic-like material called VeroWhite [18]. The matrix component of the LRM was a long square rod with periodic thin branches along its length which represent the elastic component. At the tip of each branch was a mass of the plastic material, which represents the resonator (core) of the LRM (See Figure 2.22). The frequency band gap was not calculated for the design, but numerical analysis showed the resonant mode equating to the oscillation of the resonator occurring at 277 Hz. As the design utilised only one material, any tuning of the design is purely geometry based. Additionally, in this current set up, the attenuation would only be observed in one direction, whereas a classical LRM design can attenuate in all three spatial directions.

![Image of printed LRM design and numerical analysis model](image-url)

(a) Printed LRM design by Qureshi et al. (b) The numerical analysis model and the resulting resonant mode at 277Hz.

Figure 2.22.: A one-material cantilevered LRM design made using a 3D printer [18].

Another recent example of making an LRM using additive manufacturing was demonstrated by Matlack et al in 2015 [19]. In this paper, Matlack highlighted the lack of ability to fabricate LRMs effectively because of current limitations with 3D
printing. As such, they designed an LRM to be made using a FDM printer (see section 3.2.3) where the design of the LRM was adapted to meet the capability of the printer. The matrix and elastic components of the LRM were made of the same material but their geometry was altered to provide the required stiffness for each component (see Figure 2.23). The heavy core was a dense steel block that was manually placed into the partially 3D printed model. To do this, the manufacturing process had to be paused, and then continued once the blocks were in place. The ability to pause a printer is not common, and the printer they used was relatively new and included this feature. This partially printed method of making the LRM has sped up its manufacture compared to traditional methods, yet it is still not ideal as it requires a manual step and the design variability is limited.

This lack of manufacturing capability for LRMs is a hindrance to their further research and adoption by industry. In the following section, a custom 3D multi-material printer is made such that a typical 3 material LRM can be manufactured using additive manufacturing techniques. With this device, rather than altering an

Figure 2.23.: A partially printed LRM as designed by Matlack et al. [19].
LRM design to match manufacturing capability, the manufacturing capability has been advanced to provide more options for new LRM designs.

2.8. Chapter Conclusions

In this chapter, a review of the various types of acoustic and elastic metamaterials has been performed. The subject's origins in electromagnetic research was investigated, and how that research has influenced similar discoveries in the acoustic and elastic wave regime. The creation of LRMs has been commented upon, and the mechanism by which they can achieve negative effective density was also shown. A comparison of the different metamaterial designs was also performed, and ultimately conclusions are made as to why LRMs are best suited for low frequency wave attenuation applications, for both acoustic and elastic waves.

The current focus on active and energy harvesting metamaterials was assessed, and various designs that achieve these capabilities were described. The modelling techniques used for metamaterials research were shown, and the mathematical proofs for their use were outlined. A comparison was made of these methods and their suitability for modelling LRMs was assessed. The effect of viscoelasticity in a metamaterial was shown to increase the band gap of an LRM design, which is a desirable effect for wave attenuation applications.

Finally, the manufacture of metamaterials, specifically LRMs, was highlighted as one of the challenges in current metamaterials research. Its impact upon the adoption of the technology by industry, and experimentally verifying different LRM designs was shown. Various attempts at creating metamaterials through additive manufacturing techniques were investigated. This confirmed that some LRM designs are altered to meet current manufacturing capability, when ideally, the manufacturing capability should be advanced to facilitate the manufacture of optimal LRM designs.
Chapter 3

Additive Manufacturing Printer

3.1. Introduction

Locally Resonant Metamaterials (LRMs) are composite structures that can be difficult to manufacture. As a result, many publications regarding LRM design provide modelling results without experimental verification [4, 37, 148]. LRMs have been built using additive manufacturing (AM) techniques. However, their designs have been altered to match the AM capability and not vice-versa [18, 19].

In this chapter, a review of current additive manufacturing capabilities is performed and their suitability for multi-material applications is commented upon. Following this review, details are presented regarding a bespoke 3D printer that was designed and built to manufacture LRMs. The printer utilises both Continuous Direct Write Printing (CDWP) and Droplet Direct Write Printing (DDWP) techniques to fabricate an object containing three different material components.

The chapter proceeds to describe the novel control program of the printer, and the novel geometry processing method utilised. Finally, details are given for the different initial tests performed on the printer to verify its capabilities. A proof of concept design is fabricated, which is the first printed material to contain a metal alloy, a rubber compound, and a UV cure resin, all within one object.
3.2. Review of Additive Manufacturing Techniques and Capabilities

The concept of building an object layer by layer has existed for several decades using techniques such as ‘Rapid Manufacturing’, ‘Solid Freeform Fabrication’, and ‘Digital Direct Manufacturing’. These methods have now been amalgamated into the umbrella term of ‘Additive Manufacturing’ (AM), which is now the recognised term for this field of study. Its origins can be dated back to 1984, when Charles Hull invented Stereolithography [149] and the `.STL’ file format, which would become the most common type of 3D printing file format. Objects made by this and other similar processes started to appear in the early 1990s. Due to robust patents filed for the concept of AM and environmental conditions required for 3DP, the development of the field was limited. Many of these patents began to expire in the early 2000s, which allowed for increased development in the field and has facilitated the rapid development of advanced printing techniques.

In 2005, the creation of the RepRap initiative [150] made AM open source, leading to its increased adoption and use by members of the public, in addition to industry or research institutions. Now AM, or 3D Printing (3DP) as it is now more commonly known, has developed rapidly due to the increase in awareness of its potential capabilities, which has led to continuing research funding. It is increasingly being utilised in industry to facilitate the quick and efficient manufacture of objects and products with design complexity, and indeed has been highlighted as a major technology disruptor for traditional manufacturing systems in the future [151, 152]. It is also widely being integrated with other advanced technologies to offer capabilities previously unachievable, such as flying drones with 3D printing capabilities on-board [153]. To conclude, it is well placed as a technique to manufacture LRM.
3.2. Review of Additive Manufacturing Techniques and Capabilities

3.2.1. Basic Concepts

There are several different methods of fabricating an object using AM. Whilst these methods have their respective advantages and disadvantages, the basic process of fabricating an object is the same and is defined below:

1. An object is designed using Computer Aided Design (CAD) software.
2. The CAD model is converted to the STL format via a tessellation process [154] (see Section 3.5) within the CAD program.
3. The STL file is sent to the printer which ‘slices’ the file into cross-sections at different heights (i.e. the cross-sections are in the horizontal plane).
4. The printer fabricates the object layer by layer, according to the slices created in step 3.
5. Post-processing steps, such as sanding or polishing, are performed on the printed object to achieve the final desired part.

Step 4 in the above process is where the printers vary the most as they utilise different techniques to deposit, and then cure a material to fabricate an object. Each of these techniques have their advantages and disadvantages and some can be used with certain types of materials only.

In the following section, each of the major types of printing techniques is examined, and specific mention is made about the viability of each method for printing in multiple materials. It should be noted that various adaptations of the highlighted techniques exist, but for the purposes of this review, only the principal printing techniques are explored.

3.2.2. Stereolithography

Stereolithography (SLA) printing involves the use of a light source, which focuses onto a vat containing a photopolymer liquid (see Figure 3.1). Regions which are exposed to the light will harden [154]. The printing bed sits just below the surface of the liquid. A laser will then outline the first slice of the object which will form the
first layer. The printing bed will then be lowered further into the vat containing the photopolymer liquid. This adds a small amount of material onto the object which can then be hardened with laser light as it forms the second slice. This process repeats until the full object has been made. Once printed, the object needs to be separated from the remaining photopolymer liquid, and cleaned before its intended use.

As the laser width directly correlates to the accuracy of the printed part, a high resolution component is possible. Additionally, the surface finish is sufficiently good to reduce the need for post-processing and machining. Depending on the material used, the handling of the materials is generally considered to be safe so minimal protective equipment is advised (i.e. gloves should be worn, but face masks are not required).

Figure 3.1.: SLA method (courtesy of Custompartnet Inc. [20]). The red arrows indicate the moving components of the printer during the manufacture of an object.

The light curable photopolymers used in this process can be stiff and lightweight. This printing method was used extensively for prototyping of parts, which could then be tested and examined before the final component was fabricated using traditional techniques. One of the main drawbacks of this printing technique is the time taken to produce an object. Using a laser with a smaller diameter beam can create
high resolution parts, but takes an extended amount of time to scan the entire area of one slice. The SLA process speed has been increased by the introduction of light projection, whereby an entire layer of the material is cured at once by light projecting through a mask [155]. More recently, in 2015, a photopolymer printing process very similar to SLA called ‘Continuous Liquid Interface Production’ was developed [156]. By altering the rate at which oxidation occurs in the liquid, printing times were decreased by 25x-100x, depending upon the shape of the part printed.

Despite these recent advances, the viability of using SLA or similar techniques for multi-material printing is limited. The main reason for this is the limited availability of different light curable photopolymer resins. Additionally, many SLA photopolymers exhibit similar material and mechanical properties when cured [31]. As the object is made in a container filled with the building material, the switch to using a different material would require time to drain the vat and then refill it with the new material. Alternatively, the printing bed can be moved to a different vat, containing a new material, before the printing process could continue. However, this could increase the risk of material cross contamination, and also the possibility of the object experiencing unwanted disruption during the building material switching process.

3.2.3. Fused Deposition Modelling

Fused Deposition Modelling (FDM) involves a wire of material, usually a plastic polymer, which is softened via heat, and then extruded through a nozzle onto a printing bed (see Figure 3.2). The nozzle follows a path laying down the material, which then hardens by cooling. Once one layer is complete, the nozzle moves to the next layer to deposit the next slice in the digital file, and repeats the process until the object is built.

FDM has become the most common type of printing available, due in part to the low number of patents on this technology. Additionally, the cost for FDM printers and materials is relatively cheap, hence these printers have been widely adopted for use in art, decoration, prototyping and for general hobbyists. These printers are
3.2. Review of Additive Manufacturing Techniques and Capabilities

Figure 3.2.: FDM method (courtesy of Custompartnet Inc. [20]). The red arrows indicate the moving components of the printer during the manufacture of an object.

widely available for purchase and they have helped fuel the hype and desire for 3DP from the general public.

The materials that can be deposited using FDM techniques are constantly growing [31, 157, 158]. As such, FDM printing is increasingly being adopted for industrial uses. Historically, these printers were associated with fragile objects, as the adhesion between different layers of the printed part could be weak. However, this has become less of an issue as nozzle design and printing materials have advanced. The resolution of the printed parts was also a factor as the thickness of the wire used directly affected the surface finish of the part, and typically FDM printed parts would have a ‘stair-case’ effect where ridges are noticeable. Additionally, the rate of cooling for parts with varying thickness could introduce unwanted deformations. The tool path chosen could also introduces anisotropic properties [159] to the final printed part. Users had to manually ensure the part was positioned correctly in the printer’s program to allow correct deposition and strength properties. This introduces unwanted delays, so more advanced FDM printers use software to allow the customisation of the nozzle path. For parts with highly curved sides, gaps between adjacent depositions could introduce unwanted porosity into the object, weakening the structure and making its use limited for certain applications.
3.2. Review of Additive Manufacturing Techniques and Capabilities

For multi-material applications, FDM has not proved as viable as alternative printing techniques due to the issues associated with FDM printers described above. Printers with multiple nozzles do allow the deposition of different plastic polymers such as PLA or ABS. More recently materials such as wood and nylon have been deposited using this method, but not in multi-material applications [31].

3.2.4. Selective Laser Printing

Selective laser printing is similar to SLA, whereby a laser projects onto a powdered material to sinter or melt it together. New powder is added by a large roller sweeping over the object and ensuring a uniform layer of unheated particles rests atop the already printed object. This process repeats until the object has been fully fabricated (see Figure 3.3). Once the printing process is complete, the part is submerged under unused powdered material, which must then be removed to retrieve the printed object. The surface of the part may have a rough texture, and hence requires machining or polishing to achieve a smooth finish. Additionally, the printed parts need to go through a heating and cooling cycle to relieve latent stresses introduced during the manufacturing process. Selective Laser Sintering (SLS) was the first form of selective laser printing. The limited power of lasers during the development of SLS meant that for metal powder, only partial melting occurred, resulting in particles sintering together. For certain applications, this porosity may be a desirable feature. For example, porosity can allow for lighter structures and the flow of fluids through an object. Modern SLS printers can vary their laser power to control the level of porosity in a component [160].

With the development of more powerful lasers, Selective Laser Melting (SLM) became possible and fully solid metal objects could be fabricated [161, 162]. For structural applications, porosity limits the component’s strength and damage tolerance, due to the increase of stress concentrations. The high power lasers used in SLM completely liquefy the local powdered material, which then cools to form solid objects with no porosity. The capabilities of these technologies are currently being explored extensively.
3.2. Review of Additive Manufacturing Techniques and Capabilities

Figure 3.3.: SLM method (courtesy of Custompartnet Inc. [20]). The red arrows indicate the moving components of the printer during the manufacture of an object.

SLS and SLM are most commonly used for printing metallic structures, and can deposit many types of metals including stainless steel, gold, silver, and titanium [163, 164], and non-metals such as Nylon [20]. One of the limiting factors with SLS and SLM is the quantity of unused powder deposited during the printing process. For many metallic powders, once they have been exposed to a heating cycle, they cannot be reused, as it may impact the quality of future printed components. The consequence of this is that material wastage using these methods can be high if only a small number of items are being built. There are current investigations into the number of times unused powder can be recycled before the quality of the printed component is compromised [23].

The utility of SLS or SLM for multi-material printing is limited as the methods are better suited for powder printing material, which can be costly to make, and limits the number of materials that can be used. Additionally, the process to remove one powder and introduce another can be labour intensive and the risk of material cross-contamination is high. The fine particles also present a health and safety risk, thus SLS and SLM operators must use specialist safety equipment when using these machines.

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3.2. Review of Additive Manufacturing Techniques and Capabilities

3.2.5. Droplet Direct Write Printing

Droplet Direct Write Printing (DDWP), also known as inkjet or Drop-on-Demand (DoD) printing, initially found use in desktop paper printers, before laser printing was created. The concept works by building an object droplet by droplet from a material that usually hardens through solvent evaporation or UV light exposure [31] (see Figure 3.4). The droplets are formed by the controlled squeezing of a cylinder containing the material. This compression wave travels down the cylinder and results in a droplet of the material to be formed. The squeezing is performed typically by a piezoelectric case on the cylinder, but the droplet formation can also be performed by the rapid heating of the material causing a bubble to form. This bubble formation produces an ejection of some material from the cylinder to equalise the pressure [165]. The droplet diameter typically varies between 10 microns to 150 microns. This results in 3D objects that can have a very smooth surface finish [166]. This method is also capable of printing a variety of materials including ceramics, polymers, elastomers, and certain metals [167]. DDWP requires the printing material in the cylinder to be of a relatively low viscosity. To achieve this requirement, the material can be heated as this reduces the viscosity level. The material can also be in a solvent form, which will then evaporate upon deposition, or can be cured in an oven. These solvent based materials tend to utilise quick drying solvents, which can cause a high occurrence of nozzle blockages. As such, many commercial DDWP printers utilise routine purging of the nozzles with high pressure pneumatic force to clear the nozzle to ensure no blockages build up. This purge material collects in a separate reservoir which needs to be emptied periodically. UV light curing is also a widely used method to cure a material due to its rapid hardening, therefore reducing print times.

As the droplets formed are very small, printing a large object with an inkjet device can take a long time. To mitigate this, most commercial DDWP printing devices use print heads with a series of multiple nozzles, which are all connected to the same reservoir. The print head then scans over the print bed in one direction
3.2. Review of Additive Manufacturing Techniques and Capabilities

3.2.6. Continuous Direct Write Printing

Continuous Direct Write Printing (CDWP) is very similar to the DDWP method, the main difference being that CDWP requires extrusion of a material through a nozzle and not the formation of a droplet. The material is extruded by the movement of a plunger, which is typically controlled by a pneumatic force (see Figure 3.5). This plunger can also be threaded and connected to a servo motor to extrude and deposits the material as needed to form the object. Curing takes place and the process repeats. There is limited movement in the perpendicular direction during printing.

DDWP printing is recognised as being one of the best methods to use for multi-material printing. This is because the materials that are being printed are self-contained in separate reservoirs, and then only the material that will be used for the object is deposited onto the printing bed, hence the “direct write” nomenclature. Additionally, the range of materials that can be used with DDWP covers many types of materials, and the list is increasing. For these reasons, DDWP techniques have already been employed with commercial multi-material printers (see Sec 3.3).

Figure 3.4.: DDWP Method (courtesy of Custompartnet Inc. [20]). The red arrows indicate the moving components of the printer during the manufacture of an object. (Not all DDWP printers use a levelling blade.)
3.2. Review of Additive Manufacturing Techniques and Capabilities

Figure 3.5.: a) A CDWP nozzle and material deposition method. b) A DDWP nozzle and material deposition method.

high viscous materials. The material is held in a reservoir, typically a syringe, with the nozzle attached directly at the end. Depending on the size of the nozzle and the pneumatic force applied, the rate of material deposition is easily controllable. Hence, large nozzles with a high pressure can be used to print large objects relatively quickly, or a small nozzle with a low pressure can be used to make small components with a high resolution. As the material is deposited in continuous lines, it is typical for the final printed objects to have a striation effect, similar to the ‘stair-case’ effect seen with FDM printing. Unlike DDWP, there is no viscosity limit for printing and this is useful as it allows a greater range of materials to be deposited than could be deposited with DDWP [31, 166]. Recently, the use of CDWP printing to deposit an epoxy with short fibres was successfully introduced by Compton et al [168]. This demonstrates a clear advantage of the CDWP method to print with pre-mixed particulate material, a process difficult to replicate with other printing techniques.

CDWP is also well suited for multi-material printing because of the range of materials that can be deposited and the risk of material contamination is low.

3.2.7. Binder Jetting

Binder Jetting is a process of building parts which in concept is similar to SLS or SLM, except instead of a laser, an inkjet nozzle deposits a binding agent onto the printing bed containing a thin layer of powdered print material (see Figure 3.6). Objects can be fabricated from materials including stainless steel, ABS plastic and
glass ceramics [31]. The inkjet nozzle deposits binding material in the regions of the printing bed that correspond to the location of the part. Once one layer has been bound, a roller deposits a new layer of powder ready for the binding agent to be applied [169]. This process repeats until the part is formed. Once completed, the part needs to be removed from the residual powder on the printing bed, similar to SLS and SLM. This is usually achieved with a hand held vacuum. The powder itself is very fine and can be harmful if breathed in, as the powders can cause irritation to the lungs. Hence there are many health and safety processes that must be in place before using one of these printers. Once the part is separated from the unused material, it is cured in an oven to allow the powder to fuse together. This method of heating once the whole part has been bound together reduces the occurrence of residual stress that is typically observed with the SLS or SLM method. The surface finish of the final cured component is typically rough and hence additional post-processing of some form is needed, such as machining and polishing, depending on design requirements.

The use of Binder Jetting for multi-material printing can be cumbersome as the particles of powder can be difficult to replace with other powders. This raises the risk of material cross-contamination [31]. Investigations into mixing steel and carbon powders to achieve variable strength stainless steel have been performed with some positive results [170]. The consensus for this print method is that it is well suited for single material printing, but is of limited applicability for multi-material printing.

3.2.8. Merits and Limitations of Additive Manufacturing

The field of Additive Manufacturing is rapidly growing and it offers many advantages when compared to traditional manufacturing technologies. The main advantages for AM techniques are as follows:

1. Traditional subtractive methods of manufacture (i.e. cutting material away) can produce large amounts of material waste, especially if the final object to be made has very irregular geometry. With increasing environmental concerns, and the raw materials cost being a major expense for many industries,
3.2. Review of Additive Manufacturing Techniques and Capabilities

Figure 3.6.: Binder Jetting method (courtesy of CustomPartnet Inc. [20]). The red arrows indicate the moving components of the printer during the manufacture of an object.

AM techniques offer the ability to make objects using the minimal amount of material required. This is especially true for parts made from expensive raw materials, such as titanium, and is the reason why many industries are looking at AM to reduce costs.

2. The act of fabricating an object layer by layer allows for much more complicated designs to be made, which would have been impractical, or impossible, if attempted to manufactured using traditional techniques. Indeed, many items today are not optimal for their purpose as manufacturing methods have limited the shape of the item. This is not a factor for AM which is producing components that look vastly different to their traditionally fabricated counterparts.

3. The cost for tooling of a part can be very expensive, hence for small batch numbers, AM is comparatively cheaper. This is a key factor for why AM has been widely adopted in the design and prototyping industries where several unique designs need to be made and tested before final approval. This also offers increased production flexibility which is ideal for low capacity industries, such as the aviation and space sectors.
3.2. Review of Additive Manufacturing Techniques and Capabilities

There are also associated disadvantages of AM that limit the adoption of AM for industrial use.

1. The list of available materials that can be deposited using an AM technique is much smaller when compared to the list of materials that can be fabricated using traditional methods [31]. This is a key decision factor for companies when choosing to adopt AM based methods.

2. The ‘cost per unit’ is not as scalable as with traditional methods. For high output industries such as the automotive or consumer goods industries, it still remains too expensive to use AM for mass production. Although this trend is changing, with many forecasting the adoption of 3DP, its rapid growth in terms of research and development should see it become much more cost effective in the future.

3. The hardware costs for many printers is still relatively high and materials that can be used on printers are usually proprietary, and hence can be prohibitively expensive. In addition, many materials have expiry dates associated with them which can cause printer blockages if used outside of their shelf life.

4. The time to make one component can also be a large hindrance. The process of manufacturing a part layer by layer requires small amounts of material to be added at a time and then cured. This could be very prohibitive for the production of large objects, where traditional methods can be much more time effective once established. Again, this is a diminishing trend with recent advances such as ‘continuous liquid interface’ printing (see Sect. 3.2.2).

Despite these limitations, the general market trend shows strong growth for this sector and many industries are working to adapt to the opportunities and changes in business that AM can offer [152, 171, 172]. For example, companies like UPS and FedEx in the logistics industry, are planning to incorporate AM into their services. These companies can foresee a point in the near future where items are made locally, on demand, by an AM printer. The need for mass production of parts in places like
3.3. Existing Multi-Material Printers

China, which are stored in warehouses and then shipped across the world, is a diminishing business model for many industries.

Table 3.1.: Summary of the advantages and disadvantages of the main AM techniques and their viability for multi-material applications [31].

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SLA</strong> (Section 3.2.2)</td>
<td>Good resolution (μm scale)</td>
<td>Low material variability</td>
</tr>
<tr>
<td></td>
<td>Low safety risk</td>
<td>High material contamination risk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Porosity risk</td>
</tr>
<tr>
<td><strong>FDM</strong> (Section 3.2.3)</td>
<td>Fair resolution (mm scale)</td>
<td>Low material variability</td>
</tr>
<tr>
<td></td>
<td>Inexpensive</td>
<td>Bonding strength risk</td>
</tr>
<tr>
<td></td>
<td>Low safety risk</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Potentially quick print time</td>
<td></td>
</tr>
<tr>
<td><strong>SLS and SLM</strong> (Section 3.2.4)</td>
<td>Good Resolution (particle dependant)</td>
<td>Expensive to purchase/use</td>
</tr>
<tr>
<td></td>
<td>Can print high melt materials</td>
<td>High safety risk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>High material contamination risk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Residual stress likely</td>
</tr>
<tr>
<td><strong>DDWP and CDWP</strong> (Sections 3.2.5 &amp; 3.2.6)</td>
<td>Good resolution (μm scale)</td>
<td>Print time can be long</td>
</tr>
<tr>
<td></td>
<td>Good material selection</td>
<td>Clogging risk</td>
</tr>
<tr>
<td></td>
<td>Low safety risk</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low material contamination risk</td>
<td>Viscosity requirements</td>
</tr>
<tr>
<td><strong>Binder Jetting</strong> (Section 3.2.7)</td>
<td>Good resolution (particle dependant)</td>
<td>High material contamination risk</td>
</tr>
<tr>
<td></td>
<td>Can print high melt materials</td>
<td>High safety risk</td>
</tr>
<tr>
<td></td>
<td>Low temperature process</td>
<td>Expensive to use/run</td>
</tr>
<tr>
<td></td>
<td>Potentially quick print time</td>
<td>Porosity Risk</td>
</tr>
</tbody>
</table>

3.3. Existing Multi-Material Printers

There are several major companies that produce additive manufacturing printers for commercial use including EOS, 3DSysystems, and Stratasys [31]. The majority of printers produced by these companies are designed to print with one material only and use only one printing technique.

In terms of multi-material printers, the Connex range of printers from Stratasys are the industry benchmark (see Figure 3.7(a)). Their first Connex printer, released in 2006, used the DDWP method to deposit two different photopolymer materials within one print. The types of materials that could be printed included epoxy-like resins (e.g. VeroClear) and rubber-like materials (e.g. TangoBlackPlus). These
materials are made by Stratasys, specifically for use on the Connex printer. These materials are deposited using printheads with multiple nozzles, hence the Connex printer is able to mitigate the long building times associated with DDWP. Additionally, all the materials deposited are UV curable, which is a fast method of curing. Despite the capability of this printer, the high initial hardware costs (typically greater than £200k) and the inability to use the printer to deposit non propriety materials make its use for experimentation limited.

![Stratasys Connex 350 multi-material printer](image1.png)

![A multi-material brush printer](image2.png)

Figure 3.7.: Established commercial printer for multi-material applications [21].

With regards to non-commercial printers designed for research, various approaches have been taken. A multi-material SLA printer was designed by Choi et al in 2011, which successfully produced objects made from different photopolymer liquids [22]. The printer included 4 vats with different materials, and several objects were successfully made (see Figure 3.8). However, as the printing bed moved from one vat to another, material cross contamination was an issue, despite the introduction of cleaning processes. The materials used were all photopolymer resins of different colours, and this printer was mainly used as a demonstrator of the technology. As is typical with SLA, the resolution of the final parts was good. Features of the order of hundreds of microns were fabricated. This set up demonstrated a viable multi-material printing capability, however, the lack of material availability/variability that can be utilised with SLA is a limitation for its suitability for multi-material
3.3. Existing Multi-Material Printers

applications.

A variation of this design with five vats was demonstrated by Bartolo et al [173], although the presence of material cross contamination was greater with this device, due to a less effective cleaning stage.

![Printer Set-up.](image1)

(a) Printer Set-up.

![Examples parts made. Each colour is a new material.](image2)

(b) Examples parts made. Each colour is a new material.

Figure 3.8.: Multi-Material Stereolithography type printer [22]

The desire for multi-material metal printing stems from the need to have a component that gradually changes from one type of metal to another to achieve different mechanical properties. There is growing demand for this, especially from the space sector and the manufacturing of satellites. The ability to make unique, lightweight structures with good thermal properties is highly desirable for a space environment. As such, the SLM type multi-material printers are receiving keen commercial interest. A demonstrator two-material SLM printer was developed in 2017 by Demir et al [23] (see Figure 3.9). This printer had specialised components to allow two powders to be deposited at the same time or individually, depending on the application’s needs. The two materials trialled were iron and aluminium. The beam width of the laser was 212 μm, which allowed for small block features to be manufactured showing a gradual change from iron to aluminium. The final components had a rough surface feature, but as the authors explained, this was a technology demon-
3.3. Existing Multi-Material Printers

(a) Printer schematic.  (b) Example parts made showing the material variability with height.

Figure 3.9.: Multi-Material SLM printer made by Demir et al [23].

strator, and they recognise the further work needed to achieve the target quality that commercial SLM machines can achieve.

Perhaps the most accomplished multi-material printer was developed by Shim et al. in 2012 [24] (see Figure 3.10). This printer utilised six CDWP nozzles to allow the printing of biostructures. The authors noted the complexity of making certain biostructures at that time, and highlighted additive manufacturing as the best method to generate the complex multi-material geometry. Two of the CDWP nozzles were capable of heating up to 150°C to allow a certain biopolymers to be extruded. The remaining 4 nozzles were at room temperature to allow the deposition of hydrogels [24]. The printer heads were all attached to independent Z linear stages, which were placed on a XY gantry stage. The performance of the printer was evaluated and it performed well for its given purpose. The printer initially was used to create simple three-material composite objects as a demonstration, but eventually was used to produce a more complex bone shape that could be used to help bone regeneration.

One of the most recent advances in multi-material printing is the creation of ‘Project Escher’ by the CAD software company AutoDesk [25]. This is software which can be used to help build very large objects by splitting a large task into several smaller ones and distributing commands simultaneously to several different
3.3. Existing Multi-Material Printers

nozzles. This is intended to facilitate the making of large 3D printed objects, and to reduce the time taken to fabricate an object. As a demonstrator of the software, a unique set up was created with 5 different FDM type nozzles. They were used in parallel to build several components ranging from 1 meter to 2 meters in length (see Figure 3.11). This multi-head set up is also a great advancement for multi-material printing as each FDM nozzle can print using its own wire/material.

Figure 3.10.: Six-Nozzle CDWP printer created by Shim et al [24].

Figure 3.11.: Multi-nozzle printer created for ‘Project Escher’ by AutoDesk [25].
Table 3.2.: Summary of current multi-material printers.

<table>
<thead>
<tr>
<th>Printer</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Connex</strong> <em>(DDWP Printer)</em> [21]</td>
<td>Good resolution (µm scale)</td>
<td>Exclusive material list</td>
</tr>
<tr>
<td></td>
<td>Commercially used</td>
<td>Materials are expensive to acquire</td>
</tr>
<tr>
<td></td>
<td>2 material capability</td>
<td>Materials have a short shelf life</td>
</tr>
<tr>
<td></td>
<td>Good variance of material properties</td>
<td>High hardware costs</td>
</tr>
<tr>
<td><strong>Choi, 2011</strong> <em>(SLA Printer)</em> [22]</td>
<td>Good resolution (µm scale)</td>
<td>Low Material Variability</td>
</tr>
<tr>
<td></td>
<td>Four material printer</td>
<td>Material cross-contamination risk</td>
</tr>
<tr>
<td><strong>Demir, 2017</strong> <em>(SLM Printer)</em> [23]</td>
<td>Highly sought after capability</td>
<td>Material cross-contamination risk</td>
</tr>
<tr>
<td></td>
<td>2 material printer</td>
<td>Fair resolution (mm scale)</td>
</tr>
<tr>
<td></td>
<td>Material gradation achieved</td>
<td></td>
</tr>
<tr>
<td><strong>Shim, 2012</strong> <em>(CDWP Printer)</em> [24]</td>
<td>Good resolution (µm scale)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6 CDWP nozzles utilised</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Heated nozzle capability</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low material cross-contamination risk</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Good material variability</td>
<td></td>
</tr>
<tr>
<td><strong>Project Escher, 2017</strong> <em>(FDM Printer)</em> [25]</td>
<td>Targeted towards industry</td>
<td>Low material variability</td>
</tr>
<tr>
<td></td>
<td>Large component capability</td>
<td>Not suitable for small objects</td>
</tr>
<tr>
<td></td>
<td>Technology demonstrator</td>
<td>Fair resolution (mm scale)</td>
</tr>
</tbody>
</table>

3.4. Custom Printer Design and Set-up

This section introduces a custom built multi-material 3D printer that is capable of depositing three different materials during a single print. The printer utilises both DDWP and CDWP methods. This is because both methods show promising multi-material printing capability and are comparatively easier to implement than alternative printing methods (see Table 3.2).

The printer itself is fully controlled via LabVIEW [174]. LabVIEW stands for ‘Laboratory Virtual Instrument Engineering Workbench’ and is a visual programming language. It is well established and is used in many engineering applications. One of the key advantages of using LabVIEW is that it can easily integrate with several pieces of hardware and communicate with them simultaneously. Additionally, its visual interface allows easy understanding of complex programs, and facilitates a program's wider use. Having the control program operate in the LabVIEW environment allows all aspects of the printing process to be adjusted (see Section 3.6).
In addition to this custom program, a bespoke geometry processing method was devised which utilises a point-cloud method (see Section 3.5).

The main design objectives for this printer were as follows:

- The capability to deposit 3 different types of material, including a plastic polymer type material, a rubber-like material, and a high density material for the matrix, elastic coating and core of the LRM respectively.
- To be able to deposit all three materials in one single print.
- To process STL files to facilitate the wider use of the printer.
- To control the program in LabVIEW, to allow greater control and customisability of the printer’s actions.

Given these design specifications, a novel printer design was designed and built.

The printer was designed to deposit a range of different types of material including resins, rubber and low melting point alloys. Of the various printing capabilities, direct write methods have proven some of the most adept for multi-material printing [31, 175] (see Table 3.2), largely due to their lower risk of material cross contamination and large range of printable materials. As such, there are three different nozzles on the printer; two DDWP nozzles and one CDWP (see Sections 3.2.5 and 3.2.6.)

The inkjet nozzles were purchased from MicroFab [176] and are of a piezoelectric (PZT) design. A small glass tube is covered in a PZT lining which can constrict and expand as a voltage is applied. The voltage is applied as a waveform and can be sinusoidal, square or triangular (see Figure 3.12). This action squeezes the material inside the glass tube and creates a pressure wave which propagates through the material. One end of the glass tube narrows and ends in an orifice, where a droplet is formed. This droplet lands on the printing bed and forms part of the printed material (see Figure 3.13).

Each DDWP nozzle on the printer has an orifice diameter of 80 µm. The recommended maximum viscosity of the material being deposited is 20 mPas. Material
3.4. Custom Printer Design and Set-up

Figure 3.12.: Example sinusoidal and square waveforms that were used to deposit Isopropyl alcohol during nozzle cleaning. Both signals have a pulse width of 6.2 ms, a maximum amplitude of 20V, and frequency of 81 Hz. The negative applied voltage can be used to mitigate the formation of satellite drops.

![Sinusoidal and square waveforms](image)

Figure 3.13.: DDWP Nozzles on the multi-material printer.

(a) A PZT type inkjet nozzle (b) Exploded view of the PolymerJet™ system which can heat up to 240°C [176].

Figure 3.13.: DDWP Nozzles on the multi-material printer.

with viscosities higher than this can still be jetted, but it becomes increasingly difficult to create consistent droplets.

To increase the potential range of materials that can be printed, one of these inkjets is part of a PolymerJet™ system which allows the printing material to be heated up to 240°C. This reduces the viscosity of a material in order to bring it into the acceptable range for inkjet printing. The second inkjet nozzle does not heat up, and prints at room temperature. The nozzles are connected to a signal generator
which can send different signals to each device, allowing either a single material to be printed at a time or both materials simultaneously. The signal required to make a droplet is material dependent and is triggered externally by the control program (see Section 3.6). In addition, the nozzles are connected to a compressed air supply and a vacuum supply to ensure the correct pressure for printing. The air supply to the nozzle must be sufficient to fill the void created by the jetted droplets. If the back pressure is too high, then satellite droplets may also be created, which results in a lower quality print. Alternatively, if the pressure is too low, a vacuum is created during the formation of the droplet, which can prevent it leaving the nozzle.

The CDWP nozzle is part of the EFD Nordson Ultimus™ V system. A compressed air supply is connected to the Ultimus™ V control box, which feeds a controlled air pressure to a 10 cc syringe containing the print material. The material is extruded through the nozzle at the end of the syringe by a plastic plunger. There are various nozzle tips that can be purchased to fit onto the end of the syringe, but only 250 μm and 100 μm nozzle tips were used. The Ultimus™ V system applies a small vacuum after each deposition to ensure correct deposition amounts. The nozzle is held at room temperature, but is not subject to the viscosity requirements of a DDWP nozzle. The control box that operates the CDWP nozzle can be triggered manually via a foot pedal on the ground or electronically via the printer control program.

With these DDWP and CDWP nozzles, a wide range of materials can be deposited, hence making it suitable for the manufacture of LRM and other future applications.

The movement of the nozzles is carried out using Newmark Systems XYZ linear stages. The X and Y stages are both NLE 150 mm stages. The Z stage uses the NLS4 150 mm linear. The XY stages can be operated independently and have a quoted accuracy of ±3μm and a repeatability of ±0.5μm. The Z stage has a quoted accuracy of ±60μm and a repeatability of ±10μm. The stages use stepper motors for the movement and are linear encoded to ensure correct positioning. The three nozzles are attached to the Z stage which moves in a perpendicular direction to the
print bed (aka ‘print platen’). The Z stage has a slightly higher carrying capacity and will not suddenly drop the nozzles under gravity, if there is a power failure. The print bed is attached to the X and Y stages. This set up ensures the nozzles are only moving up and down and as such reduces their probability of dislodging. The nozzles are placed in a line with a large gap separating the heated DDWP nozzle to mitigate the conduction of heat to the adjacent nozzles. The item being fabricated moves under the appropriate nozzle and as the material layers build up, the height of the nozzles are increased relative to the print bed. The acceleration, deceleration, and velocity of the stages are controlled by the user and are set prior to printing. The printer is placed upon an optical table in order to reduce unwanted vibrations during the print. A silent air compressor provides the pneumatic pressure that needs to be applied to the three different nozzles. Glass microscope slides are attached to an unheated print bed and the printer deposits onto these glass substrates. A heated print bed can be attached, and is useful when depositing with heated materials directly onto a substrate. The heated bed reduces any deformation due to the rapid cooling of a hot material on a cold printing bed. The full printing set up, can be seen in Figure 3.14.

3.5. Geometry Processing

For a part to be made using AM techniques, it must first be designed using CAD software. There are many commercial software packages available to do this, including ‘AutoCAD’, ‘Solidworks’, and ‘CREO’. Once the part is designed, the standard method to process CAD geometry to make it suitable for a 3D printer, is to convert it into a ‘STL’ file. An STL file format tessellates the surface of a CAD model into a series of triangular facets and vertices. As such, the conversion process is seen as an approximation of the surface of the CAD part. Typically, for highly curved components, the accuracy of the geometry approximation increases as the number of facets and vertices increase. In Figure 3.15, a sphere is represented with different resolutions of facets and vertices. The low resolution STL would not be suitable for any printing method.
3.5. Geometry Processing

(a) Printer Setup

(b) Nozzle arrangement with 2 DDWP nozzles and 1 CDWP nozzle

(c) Schematic of the multi-material printer.

Figure 3.14.: Custom made multi-material printer overview.
The resultant object would not resemble a sphere. The medium resolution STL could be used for most printers, as long as the geometry approximation (i.e. distance between the CAD surface and the representative STL surface) is within design tolerances. The high resolution STL would contain redundant information given the resolution of most printers. This file could be used on DDWP printers given their high resolution capability.

The required number of facets is dependent upon the printing method used and the resolution of the nozzle. For large straight surfaces, the STL typically represents this as a few facets, each with a large surface area.

Figure 3.15.: STL file resolution: 1) Original CAD file 2) Low resolution 3) Medium resolution - suitable for most printers. 4) High resolution - suitable for DDWP type printers. [26]

An STL file can be in Binary, or ASCII format. The ASCII file displays the CAD data in a legible format, but does take up more memory than a Binary STL file format [1]. The ASCII format labels the XYZ coordinates for 3 points, which correspond to the vertices of a triangular facet. A ‘facet normal’ vector is also written, which corresponds to the direction normal to the facet surface. This normal vector is important as it points away from the object, and so the program can determine where to print and where not to print. This information then repeats for every facet used in the CAD conversion process.
3.5. Geometry Processing

The STL file format was created in the 1980s, and with time other formats have been developed including ‘STEP’, ‘DXF’, and ‘3DS’ [178]. However, each of these listed file formats have not been widely adopted for the following reasons:

- The STEP file format includes the use of solids, wire-frames, and boolean modelling to represent a 3D material. By many in the AM community, it is seen as unnecessarily complex and hence has not been adopted widely [178].

- The DXF file format utilises triangular meshes, similar to the STL format, however, it is a specialised format for 2D drawings. When applied to 3D models, the resultant file can be comparatively large and require extra processing time. Additionally, the 3D represented model typically requires manual altering if the geometry varies greatly between subsequent drawings. [178].

- The 3DS file format is well suited for 3D structures and is used widely in animation applications. It uses a mesh structure to represent a geometry, and includes information regarding colour and texture. However, it also includes information such as lighting, and contrast. This additional information makes the file very large and hence its low utilisation in in the AM sector [178].

The STL remains the established format for 3D printers and is widely used still. Despite its established status, there are some disadvantages to using the STL format:
3.5. Geometry Processing

1. **Redundant Information** - The vertices information is provided for each facet in an STL file. However, each vertex is also a point on at least 2 adjacent facets as well, and so the coordinate point is repeated at least three times. This is redundant information, and for large parts with many facets, this redundant information can increase processing times.

2. **Surface Definition** - The STL file format only defines the surface of a part. For the vast majority of printers, this information is all that is needed to print a component. For more advanced printers, such as multi-material printers, more information is required to describe how a part should be made. This desired information includes:
   - Material to be deposited (i.e. nozzle selection),
   - Cure method,
   - Nozzle Acceleration, Deceleration, and Speed,
   - Level of Porosity.

As AM printers have increased in capability, the limitations of the STL format are increasingly evident. Alternative formats are not only desired, they are needed [179].

To account for the need to have added functionality with multi-material deposition, a solution was found by creating a separate MATLAB program that can be incorporated into the control program for the printer, and output specific print commands when provided with an STL file. The ‘Point-Cloud’ method was created to allow additional information to be added to the geometry of an object, before being printed.

### 3.5.1. The Point-Cloud Method

The Point-Cloud method is an algorithm written in MATLAB that allows the geometry of a component to be represented as a series of points, called a ‘point-cloud’. This process of creating a point-cloud representation of a part is similar to the process of voxelization [180]. Voxelization is a method of representing geometry
as a series of fixed volumes.

By having points instead of volumes representing a component, it is easier to define coordinates that a printer can use to determine where to deposit material. The steps performed to convert an STL file into the Point-Cloud format are shown below:

1. The MATLAB script reads in an STL file from the single material part and assigns all the vertices and corresponding facets into separate arrays.

2. The maximum and minimum XYZ values are determined using searching functions in MATLAB.

3. Once these values are obtained, a grid of points is created \((\mathbf{r})\) which is based on a user defined resolution. (Typically this resolution is determined by the type of nozzle being used.)

4. For each facet, the centre-point, \(r_0\), is found and these points are stored in an array.

\[
\mathbf{r}_0 = \frac{\mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3}{3}
\]  

(3.1)
5. The unit normal vector for each facet, $\hat{V}_N$, is also calculated and stored in a separate array.

$$\hat{V}_N = \frac{(\hat{V}_2 - \hat{V}_1) \times (\hat{V}_3 - \hat{V}_1)}{|(\hat{V}_2 - \hat{V}_1) \times (\hat{V}_3 - \hat{V}_1)|}$$ \hspace{1cm} (3.2)

6. An algorithm is set-up to cycle through each point in the grid. Its nearest facet is determined by using Pythagoras’s theorem to calculate the distance from the grid point, $\bar{r}$, to the different facet centres, $\bar{r}_0$. Once the nearest facet has been identified, it is calculated if the grid point is inside the object or not using vector geometry.

$$\hat{V}_N \cdot (\bar{r} - \bar{r}_0) < 0 \quad (3.3)$$

$$\hat{V}_N \cdot (\bar{r} - \bar{r}_0) = 0 \quad (3.4)$$

$$\hat{V}_N \cdot (\bar{r} - \bar{r}_0) > 0 \quad (3.5)$$

If Eq. 3.3 is true, the point $\bar{r}$ is inside the part and retained.

If Eq 3.4 is true, the point $\bar{r}$ is on the surface of the part and retained.

If Eq 3.5 is true, the point $\bar{r}$ is outside the part and deleted.

7. The remaining grid points show a geometrical representation of the part in point form (i.e. a point-cloud). At this stage, the point-cloud data is separated into various ‘slices’. With each of these slices, the points on the edge of the slice are identified as these represent points that the printer will print to and from.

8. These points are collated and rearranged in such a manner that reading the array in a sequential manner will create a path for the nozzle to build the slice.

$$X_{min} \rightarrow X_{max}, Y_0; X_{max} \rightarrow X_{min}, Y_1; X_{min} \rightarrow X_{max}, Y_2; \ldots$$

The file produced is called a ‘coordinate file’. This process repeats until the slice is complete, and then repeats for the next slice, until the nozzle path for the full object has been created. (Note: The nozzle path creation process out-
lined above can be altered such that printing the nozzle follows an alternating $X$ and $Y$ direction path on sequential slices.)

Displaying the geometry as a point-cloud allows additional information to be supplied at each point in the point-cloud system (see Figure 3.18). This facilitates the printer to vary what material is used, how it is deposited, or any further information required to create the final component. The amount of information that can

Figure 3.18.: Various stages of the Point-Cloud method. The Stanford Bunny is used as the original CAD file to demonstrate the process on irregular geometry.
be added is not fixed. Therefore, as printer capabilities increase, this methodology can still be applied.

To print with multiple materials, the printer control program not only requires the coordinates to print to, but also requires instructions on which nozzle to use. To add this information in, the CAD part must first be separated into the various different material components. Each component is processed as separate STL files in steps 1 to 8 and then merged to create the final coordinate file. This merging process also allows the user to specify which nozzle to use with the different material component part.

This is not an ideal set-up, but attempts to automate the separation of a multi-material STL did not prove successful. Given time constraints, it was decided that the manual process of separating the STLs into their material components would be more time effective. The Connex printer (described in Section 3.3) control program is able to process multi-material STLs by requesting the user to manually select which material is used for each component in their program user interface. This was not replicable within the LabVIEW environment used for the printer's control program (see Section 3.6). The LabVIEW program can display the STL files but is unable to allow the user to manipulate the file further, once uploaded.

The process of merging the separate material coordinate files is carried out in a separate MATLAB function, and the process is as follows:

1. The user uploads up to 3 different coordinate files, one for each nozzle being used.

2. The program separates the files into their separate X, Y, and Z coordinates and assigns these into an array, one for each nozzle.

3. Each array has an additional column added which assigns the number (1, 2, or 3) to each row in the column, corresponding to which nozzle will be used.

4. An algorithm then searches for the smallest Z values in all three arrays. Once the smallest Z values are found, the corresponding rows are copied into a final array. The algorithm then searches for the next smallest Z values and repeats
until all rows from the three individual nozzle arrays have been placed and sorted into the final array. The format of the final coordinate file is as follows:

\[
N_1, X, Y, Z_0 \\
\vdots \\
N_2, X, Y, Z_0 \\
\vdots \\
N_3, X, Y, Z_0 \\
\vdots \\
N_1, X, Y, Z_1 \\
\vdots \\
N_2, X, Y, Z_1 \\
\vdots \\
N_3, X, Y, Z_1 \\
\vdots \\
\text{etc.}
\]

It should be noted that the final coordinate file is a list that is read sequentially by the control program. Each line is read as an array, allowing additional information to be added as required. The amount of information that can be added is not limited, hence the method is viable for the inclusion of additional printing capabilities in the future.

The time taken to process the geometry for an STL file was examined, and it is dependent upon the number of points in the user defined grid, and the number of facets in the STL file (see Figures 3.19 and 3.20). Analyses were carried out to assess the effect on the runtime of the program, as both factors varied.

To assess the effect of the number of facets, a cube was created in the CAD software CREO [181]. This cube was then converted to five different STL files, each with a different maximum chord length for a facet. This meant that for a given shape, the number of facets that are used to represent the shape will vary. These STL files were then processed, keeping the number of grid points constant (see Figure 3.19). Additionally, the same cube was processed again, this time keeping the number of
facets constant, and varying the number of grid points (see Figure 3.20). These programs were run on a computer with a 3.4 Ghz i7 processor with 16 GB of RAM.

![Figure 3.19.: Effect on program runtime with increasing number of facets in a STL file.](image1)

![Figure 3.20.: Effect on MATLAB program runtime with increasing number of grid points used in the geometry processing algorithm.](image2)

What is evident from Figures 3.19 and 3.20 is that the time taken for the program to run has a linear relationship with the number of facets, and the number of grid points being processed. This is an important result as there are scenarios where the number of facets required do effect the accuracy of the coordinate file.

It was observed that when the original CAD file has large flat surfaces, the default STL conversion created very large facets in order to tessellate this area. However, this corresponds with less facet centre points against which the Point-Cloud algorithm can use to compute which grid points are in the object and which are outside
(see Section 3.5.1). The resulting point-cloud model can vary greatly from the original CAD shape. To mitigate this, additional facet centre points need to be added, and this can be achieved by increasing the number of facets used to represent the shape. (See Figure 3.21)

![Figure 3.21: Effects of the number of Facets in a STL file processed using the Point-Cloud method.](image)

Figure 3.21 confirms that increasing the number of facets increases the accuracy of the model. This scenario is not seen when the CAD file has irregular geometry with many curves and vertices. The default STL conversion typically represents this irregular geometry with many facets, and hence works well with the geometry processing algorithm (see Figure 3.18).
3.6. Control Program

A LabVIEW based program was created to control the 3D printer and allows for full control of the printing process. This was possible because each variable in the printing process can be set as a constant value, or modified by the user within the LabVIEW environment.

The LabVIEW software package was chosen to make the program due to its ability to easily connect with external hardware, and also its pictorial method of writing would allow for easier understanding of the printer for new users. This pictorial method is an alternative interface for the C/C++ language upon which LabVIEW is based. It also allows the user to create a user interface (UI) for ease of use, and again allows the program to be used more intuitively. The control program was written in a ‘state space’ format to allow additional features to be added to the program without affecting the original functionality.

3.6.1. State Space Programming

The program was created such that each function is in a different state, which is called upon as needed. This is known as ‘state-space programming’, which means that only one action can happen at any one time, and a new action cannot start until the previous one has finished. This allows additional functionality to be added to the program without affecting the original program. It also prevents the user accidentally starting two processes unknowingly, and potentially causing harm to the printer. This design philosophy is also known as ‘bottom-up’ programming where all the actionable states of the program are controlled by a top-level state, which performs no actions itself. The alternative program set up is called ‘series programming’ where all functions follow the previous actions. This requires less user control, and so is favoured for fully automated systems, but also raises the risk of incorrect use of the printer which could damage the printer or user.
3.6. Control Program

Prior to starting the program, the user specifies several printing parameters which include:

- Communications settings for the CDWP and DDWP nozzle control boxes. Each control box connects to the computer via a RS-232 serial input cable.
3.6. Control Program

These have an associated ‘COM’ channel (1, 2, or 3), and a ‘Baud Rate’ which defines the rate of data transmission (19200 bits/second is always used).

- The print settings for each nozzle (dependant on material being deposited). These are the deposition signals for the DDWP nozzles, and the pneumatic pressure for both the DDWP and CDWP nozzle.
- The speed, acceleration and deceleration for each XYZ stage, defined in steps/second (see Section 3.7.1).

Once the print settings have been selected, the program initiates and checks the communication between the various control boxes and the control program. If communication is established, the user defined print settings are sent to the relevant box, ready for deposition.

Upon completion of this state, the program goes into the system idle state, where the program remains in idle mode until the user requests an action or process to start. This is achieved by using a LabVIEW ‘event structure’ option. An event structure tells the program to continually loop in idle mode, until the user selects an action, at which point the program proceeds to carry out that action.

The four states that can be chosen when the program is in the idle state are (see Figure 3.23):

1. **Stage Movement State** - This state allows the user to enter commands to move the XYZ robotic stages manually. This is helpful when initially setting up a printing process, to ensure the nozzles are in the correct location. The code used is known as ‘Galil code’ and was supplied with the robotic stages when they were purchased (see Section 3.7.1). Once the movements have finished, the program returns to the waiting/idle state.

2. **Shape Creator State** - There are several pre-defined shapes built into the program, such as the ability to make a single line, a circle, or to build a cylinder. These are included to allow the user to experiment with a new material to see how it deposits (i.e. Does the material spread? Is the material prone to clogging? etc.) and hence find the optimal settings such as nozzle speed,
droplet frequency, and pneumatic pressure. If this state is chosen the Object Printing State is also executed before finally returning to the waiting/idle state.

3. STL Upload State - This state allows the user to upload an STL file and view it for inspection prior to printing. The process that allows the STL file to be printed is discussed in Section 3.5. If this state is chosen, the Object Printing State is also executed, before finally returning to the waiting/idle state.

4. Coordinate File Upload State - One of the stages in processing an STL file is the creation of a separate coordinate file containing all the information required to print a multi-material part (see Section 3.5). If the user possesses a pre-processed STL file, they can upload the corresponding coordinate file into the control program to speed up the printing process. If this state is chosen, the Object Printing State is also executed before finally returning to the waiting/idle state.

3.6.2. Object Printing State

The object printing state is where the physical process of manufacturing a design is performed. An overview of the process is shown in Figure 3.24.

Prior to any print, a coordinate file needs to be created (see section 3.5). The file contains a series of coordinates and a nozzle number which the program uses to determine several print parameters including:

- which nozzle to use,
- the direction and distance of the line to be deposited,
- the material deposition time,
- when to move the printing bed to allow the deposited material to be cured (if UV curing is required),
when not to deposit any material, but proceed to the next coordinate.

The speed of each nozzle is assigned before initiating the program, but can be altered during a print if desired. As shown in Section 3.5.1, the format of the final coordinate file is as follows:

\[ N_1, X_1, Y_1, Z_1 \]
\[ N_2, X_2, Y_2, Z_2 \]
\[ N_3, X_3, Y_3, Z_3 \]
\[ \vdots \]

where \( N \) is the nozzle to be used and \( X, Y, Z \) are coordinates. The units of these coordinates are in microns (\( \mu \text{m} \)).

A sub-program is used to calculate the difference between consecutive lines in the coordinate file, resulting in a separate ‘print array’ in the following format:

\[ N_1, \Delta X_1, \Delta Y_1, \Delta Z_1 \]
\[ N_2, \Delta X_2, \Delta Y_2, \Delta Z_2 \]
where, $\Delta X_1 = X_2 - X_1$, $\Delta X_2 = X_3 - X_2$ (Same for $\Delta Y$, and $\Delta Z$ too).

Once this array is created, the program detects if the Z value has changed. If it has, the program sends commands to the XYZ stage controller to move the Z stage by $\Delta Z$ microns. If there is no change, the programs proceeds to the next evaluation stage. This stage is designed to detect if the nozzle being used has changed. If the nozzle has changed, commands are sent to the XYZ controller to move the partially printed object in the X direction until it is under the correct nozzle. Once this process is complete, the program proceeds to evaluate the $\Delta X$ and $\Delta Y$ values to determine in what direction to move and the distance it should travel, which is calculated as:

$$D_n = \sqrt{\Delta X_n^2 + \Delta Y_n^2}$$

$$T_n = D_n / S_n$$

Once $D$ is known, and its corresponding direction vector and deposition time are known, the commands for the XYZ stage controller and the corresponding nozzle controller are created. Finally, both commands are executed at the same time so that the nozzle moves and deposits material simultaneously. Once the deposition is complete, the program then returns to the print array and processes the next line until there are no more lines.

3.6.3. User Interface

For the printer control program, a simple UI was created to make the program more user friendly (see Figure 3.25). The UI shows the connection settings for each nozzle, as well as position data. Various tasks that can be performed are shown in a tab structure to simplify the screen. A tab structure is simply a display tool to show only the relevant controls and information for the task being performed. All other controls are hidden until required. This ensures that all information the user
wishes to see during the program's operation is visible on one screen.
A program in LabVIEW has two components: the user interface (UI), and the block
diagram (BD). The BD uses ‘wires’ to connect various processes and sub-programs
represented as blocks. If any of these blocks require a user input, or are required to
display information, these blocks link to the UI. Depending upon the block, the UI
link can be a graph, switch, table or variable control slide. This simplifies the user
interface and makes it more suitable for wider use. The BD itself can appear very
complicated. An overview of the BD for the control program is shown in Figure
### 3.6. Control Program

#### Operations
- Tab structure access for various options:
  - Manual entry for XYZ stage movement,
  - Pre-defined shape printing selection,
  - STL file upload.
  - Printing process start

#### Connections and Setup
- Connection settings are defined for the different control boxes.
- Used only during programme initiation.

#### Hardware Status
- Errors in any of the control box connection settings are highlighted here.
- Current state space is shown.
- Unit converter between mm and stepper motor steps.

---

**Figure 3.25.:** LabVIEW based printer control program user interface.
3.6 Control Program

Program Initiation -
- Communications with the various hardware devices is defined

State Space Program -
- This is the state structure of program. The window name (*) changes as different tasks are selected.
- The 'Object Printing State is currently shown'. Additional states can be seen in the appendix.
- When one state finishes, the program goes to the idle state, and awaits further commands.

Error Identification -
- Errors that cause the program to terminate are sent here to the user interface.

Figure 3.26.: Control program block diagram overview. Block diagrams of individual states can be seen in Appendix B.3.
LabVIEW has a series of built in controls to stop, pause, or even slow a program at any point during its operation to give the user full control. The slow feature also shows what the program is doing in the block diagram which aides with debugging. At any point in the printer control program's operation, the user can pause or abort the current action.

3.7. Initial Tests and Prints

3.7.1. Robotic XYZ Stage Controller Input

The XYZ linear stage robotic controller from Newmark Systems is connected to the computer running the control program via an RS-232 serial port. As such, the connection settings need to be chosen each time the control program initiates. The controller for the XYZ stages uses ‘Galil code’ as its command language. Galil is an ASCII based language where commands are prefaced by the command type, and then followed by three numeric values to specify the magnitude of the movement or command in each axis, X,Y and Z respectively, as demonstrated in Table 3.3.

Table 3.3.: Common Galil Commands. Note: The comma is used as a delimiter for X, Y, and Z directions. A, B, C are used to refer to the X stage, Y stage and Z stage respectively.

<table>
<thead>
<tr>
<th>Command</th>
<th>Galil Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Move Absolute</td>
<td>PA X,Y,Z</td>
</tr>
<tr>
<td>Move Relative</td>
<td>PR X,Y,Z</td>
</tr>
<tr>
<td>Set Speed</td>
<td>SP X,Y,Z</td>
</tr>
<tr>
<td>Set Acceleration</td>
<td>AC X,Y,Z</td>
</tr>
<tr>
<td>Set Deceleration</td>
<td>DC X,Y,Z</td>
</tr>
<tr>
<td>Current Coordinates</td>
<td>TP A,B,C</td>
</tr>
<tr>
<td>Execute Commands</td>
<td>BG A,B,C</td>
</tr>
</tbody>
</table>

There are two methods to move a linear stage. The ‘Move Absolute’ command and the ‘Move Relative’ command. The absolute command moves a stage with respect to a coordinate point in the global coordinate system. The relative command moves
a stage relative to its current position (i.e. its current location is always considered the origin).

As the linear stages use stepper motors, the units used in the commands are ‘steps’. For the X and Y stages, 1 step is equivalent to $1.0 \times 10^{-4}$ mm as they are metric based controllers. The Z stage is imperial based and so each step is equivalent to $1.27 \times 10^{-4}$ (10,000 steps is equivalent to half an inch). Commands can be sent individually, or collated together. The controller stores commands until they are executed or the controller is switched off, or reset. The ‘execute a command’ code, $(BG\ A,B,C)$, can allow all three axes to execute their commands at the same time, or each can be executed individually by deleting $A$, $B$, or $C$ as appropriate.

The robotic controllers have a quoted accuracy from the suppliers of 3 $\mu$m and a repeatability of 0.5 $\mu$m from the supplier. However, these values are not true for certain instances, as speed, distance moved, and acceleration and deceleration rates all have an effect upon the precision of the linear stages. A test was performed to ascertain the accuracy of the X linear stage for small displacements. The test was performed by moving the X linear stage at a constant speed of 10 mm/s. The distance that stage was moved was varied and the actual final location was compared with the desired placement.

![Expected Displacement vs Relative Error](image)

**Figure 3.27.**: Relative error in displacement with constant velocity of 10 mm/s. Error bars show the variance in results from multiple tests.

From Figure 3.27, it is clear to see that, as the requested displacement decreases, the percentage error increases. Multiple tests confirmed these findings. This shows that there is an associated error when performing small displacements. As the dis-
placement exceeds 100\(\mu\)m the percentage drops to less than 1%.

To determine the effect of speed, acceleration and deceleration of small displacements, another test was performed. For this second test, a constant displacement of 0.01 mm was requested, but the speed, acceleration and deceleration were all set at the same value, and then varied between each test.

![Graph showing relative error in displacement with varying velocity, acceleration and deceleration](image)

Figure 3.28.: Relative error in displacement with varying velocity, acceleration and deceleration. Error bars show the variance in results from multiple tests.

As can be seen from Figure 3.28, the accuracy of placement of the linear stage decreases as its speed increases, along with increasing acceleration and deceleration. This suggests that future designs which are printed with small features must be deposited at low speeds to give accurate placements of the material. This implies that the printer may need to vary its deposition rate and movement speed depending on the size of the feature being made.

With the Point-Cloud method of geometry processing, these small features can have extra information added to their corresponding coordinate points to instruct the printer to alter its settings accordingly. As such, this printer remains a viable option for printing of small objects and features.

### 3.7.2. Initial Prints with the CDWP Nozzle and Verification of the Printing Process

Various components on the printer were manufactured in house and some were purchased from external suppliers (see Appendix B). The manufactured components
include the nozzle bridge, which is the cross-beam which holds the nozzles and the vertical robotic stage. In addition, the nozzle holders were manufactured to ensure correct placement on the printer. The various components of the printer that were purchased, arrived at different points in the project. Initially, just the linear robotic stages and the CDWP nozzle were purchased. Therefore much of the early verification of the printing method and procedures were performed with the CDWP nozzle.

The CDWP nozzle is an ‘Ultimus™ V’ system, as supplied by EFD Nordson. It is a pneumatic dispenser which can be controlled by LabVIEW via an RS-232 serial port. The control box for the CDWP nozzle is connected to a gas supply which provides the pneumatic force to move a plunger in a syringe which deposits the material. There is a large variety of nozzles that can be connected to the syringe. In this project, only a 250μm nozzle and 100μm nozzle were used. When the Ultimus™ V device is first initialised by the printer control program, the user specifies the pneumatic pressure to supply to the syringe. The pressure forces a plunger into the syringe to extrude material. A small vacuum can also be applied after each extrusion to ensure there is no accidental deposition of material. This is especially useful for low viscosity materials. When the program executes, the control program sends a numerical value to the printer. This represents the time period which the control box should apply a pneumatic force on the plunger, and is measured in milliseconds. In these initial tests, a valve purge compound made by EFD Nordson, ‘Petrolatum’, was used as the deposition material as its gel-like consistency maintained its shape when deposited, and was inexpensive to use.

The first tests investigated the deposition rate of the CDWP and the speed of the nozzle. As the nozzle is moving, the pneumatic force on the plunger determines the rate that material is being deposited. If this deposition rate is high, but the nozzle speed is too slow, the resulting line can be very thick and a wavy appearance can be observed. If the nozzle speed is too high, a very thin line can be created. What can also happen, is that the material agglomerates into small beads of material, giving the line a dotted appearance, which was observed when using the purge material.
When the material deposition rate and nozzle speed are at their correct setting, a uniform line is printed (see Figure 3.29).

![Image of printed lines]

**Figure 3.29:** The top image shows the resulting wavy line when a material is deposited with a pneumatic pressure of 15 psi and a nozzle speed of 5 mm/s. The bottom figure shows the resulting uniform line achieved by increasing the nozzle speed to 6 mm/s. For both line deposits, a 100 μm nozzle was used.

The first shapes created were a series of circles and triangles. The circle was made using one of the pre-defined shapes in the ‘Shape Creator State’ (see Figure 3.23). The triangle was made using a pre-made coordinate file with just 3 coordinates in it. A 250μm nozzle was used during these initial tests.

From Figure 3.30(a), it is clear to see from the top set of printed circles, that the formation of the shape is accurate for large diameters. As this diameter decreases, the accuracy of the circle decreases. The visible gaps are due to a mismatch between the acceleration and deceleration rates, in comparison to the speed of the nozzle. Initially the acceleration and deceleration were set to high values as it was believed this would allow the nozzle to reach its deposition speed more quickly and sustained for longer, in doing so creating a better print. However, this results in a time delay from when the nozzle moves to when the plunger extruded material, and hence the gaps present. By setting the acceleration and deceleration to the same numerical value as the speed, these gaps disappeared and a higher quality print was achieved.
3.7. Initial Tests and Prints

(a) Initial circular designs printed. The set of circles on the bottom show the prints with the speed and acceleration corrected.

(b) Initial triangular designs printed to verify the use of a coordinate file for printing. The set of triangles on the right show the prints with the speed and acceleration corrected.

Figure 3.30.: Initial tests to verify the use of the custom multi-material printer.

This effect was also apparent when printing small triangles (see Figure 3.30(b)). The gaps were visible at the interface between consecutive lines. Setting the acceleration and deceleration to the same numerical value as the speed corrected this effect and higher quality prints were achieved. The type of material deposited will influence the speed, acceleration, and deceleration values required. Before any printing is undertaken with a new material, trials need to be performed to determine these values. The ability to customise the control program in LabVIEW means that setting these values is a simple process, and gives greater flexibility to the materials used with the custom multi-material printer.

3.7.3. Initial Prints with the DDWP Nozzle

Of the two DDWP nozzles attached to the printer, one is part of a PolymerJet™ system which allows it to heat up to 240°C. Both nozzles are connected to a pneu-
matic control box, which regulates the pressure in each reservoir for each nozzle. The pressure required is determined by the rate of material deposition. This ensures that as the material leaves the reservoir, no vacuum is formed which can alter the rate of droplet deposition, affecting the quality of the final print. This control box also houses two temperature regulators connected to heating elements in the PolymerJet™ nozzle.

The other variable for both nozzles is the electrical signal supplied. Both nozzles are piezoelectric ink jet devices, which means that an electrical signal is sent to the nozzle, which causes a piezoelectric coating on the glass tube of the nozzle to squeeze and expand depending on the sign of voltage applied. The amplitude of the voltage varies the force applied (maximum voltage of 120 V can be applied). This force creates a pressure wave in the glass tube which results in the formation of a droplet, which is then jetted to form part of the printed object.

The electrical signals are controlled by a separate control box which connects to the computer via an RS-232 serial cable. These signal waveforms can be either sine, square, or triangular. All aspects of the wave are set including the pulse length, the amplitude, the median value, and the frequency (if not limited due to pulse length). Unlike the Ultimus™ V control box, the DDWP nozzle signal control box is not controlled directly via LabVIEW. Instead, the LabVIEW control box connects to an external National Instruments I/O USB-6009 device which can generate analogue signals. The USB device has 2 outputs, each connecting to a separate terminal on the DDWP signal control box. These are trigger terminals (one per nozzle), which can signal the control box to start depositing material. These signals are set on the computer using a software package provided by the DDWP nozzle supplier.

To verify that the triggering mechanism worked, and that the correct signal was being supplied, the LabVIEW control program was run to print a generic triangle. The cable which normally connects the signal control to a DDWP nozzle was rerouted to an oscilloscope. The output of the oscilloscope was used to verify that the amplitude, frequency and waveform were all as selected by the signal control box, and that these signal were being sent when triggered by the USB device. The same test was performed on the second DDWP nozzle and trigger terminal. In all cases,
the output was as desired and hence this validated this set-up as suitable for actual use when depositing material with DDWP nozzle.

To verify that a droplet could be made as these signals were sent, the PolymerJet™ system DDWP nozzle was set up to deposit distilled water (nozzle was unheated). The manufacturer of the DDWP nozzles provided the deposition settings required to make correct droplets (see Table 3.4). These setting were entered into the DDWP control program. The supplier did not state the level of pressure that was required when printing, and so several attempts to deposit the distilled water with the DDWP nozzle were made at varying pressures.

After several trials, with a back pressure of only 1 psi, droplets of distilled water were formed by the DDWP nozzle (see Figure3.32). These droplets were created consistently and repeatedly. This remained true when the DDWP control box was triggered by the LabVIEW based printer control program.

The amplitude of the applied voltage was also varied in a series of experiments, and it was observed that raising the voltage too high would lead to satellite formation as Figure 3.31(d) shows. Although in this image the droplets look similarly sized, typically they can vary and sometimes spread as they descend. As such, satellite formation should be avoided as it can lead to lower resolution prints.

To verify that the heating process on the PolymerJet™ system worked as intended, a DDWP nozzle was fitted and paraffin wax was deposited. The paraffin wax is a solid at room temperature, but has a melting point of 68°C. The wax pellets were placed in the reservoir and then the PolymerJet™ nozzle's heating elements were set to 80°C and turned on. Heating of the nozzle and material took approximately 4 minutes. Settings for printing with wax were not provided by the supplier. Hence, a series of trials were performed to determine the correct settings and nozzle speed. Initially the camera was used to observe the nozzle tip as the wax was depositing to see if droplets were being formed correctly. The nozzle was then programmed to move as the nozzle was depositing to see the resultant line. One of these early deposited lines is shown in Figure 3.33(a). The settings for this
3.7. Initial Tests and Prints

(a) A new DDWP nozzle tip with an 80 μm orifice.
(b) Pressure set at 50 psi, Purge of DDWP nozzle. Used to clear a nozzle.
(c) Pressure of 20 psi when depositing distilled water. No droplet formation, water stream agglomerates.
(d) Deposition of distilled water with high voltage amplitude signal applied, resulting in formation of an undesired satellite droplet (top droplet).

Figure 3.31.: Various examples of undesired material deposition due to incorrect print settings. These images were taken using a 150 mm focal length CCD camera placed adjacent to the nozzle. The back light is provided by a strobe LED controlled by the inkjet control program.

deposition had a 90 V signal amplitude and a nozzle speed of 2.5 mm/s. It is evident in this figure that the satellite drops formed some distance from the intended line. This highlighted the need to determine the correct print settings, as satellite formation could result in lower quality printed parts. It is also important to note that the wax droplet did not amalgamate well when on the printing bed, showing that the droplets cooled quickly upon deposition. Again this could be mitigated by
3.8. Multi-Material Printing for LRMs

Figure 3.32.: Consecutive droplets created by a DDWP nozzle. These images were taken using a 150 mm focal length CCD camera placed adjacent to the nozzle. The back light is provided by a strobe LED set to flash at 250 Hz.

The use of a heated printing bed. The correct settings were found (shown in Table 3.4) to deposit a clear line of wax dots successfully (see Figure 3.33(b)).

Table 3.4.: Correct print settings for print trials with a DDWP nozzle on the PolymerJet™ system.

<table>
<thead>
<tr>
<th>Print Setting</th>
<th>Distilled Water</th>
<th>Paraffin Wax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>21.5</td>
<td>80.0</td>
</tr>
<tr>
<td>Waveform</td>
<td>Square</td>
<td>Square</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>250</td>
<td>100</td>
</tr>
<tr>
<td>Pulse Length (μs)</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Pulse Amplitude (V)</td>
<td>42</td>
<td>60</td>
</tr>
<tr>
<td>Back Pressure (PSi)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Nozzle Speed (mm/s)</td>
<td>N/A</td>
<td>20</td>
</tr>
</tbody>
</table>

3.8. Multi-Material Printing for LRMs

After verifying the use of each nozzle, the control program, and the geometry processing, the next stage was to trial different materials that could be used to make an LRM. As stated in Section 2.3.3, an LRM typically consists of a core,
3.8. Multi-Material Printing for LRM

(a) Deposit with pulse amplitude increased to 90V and line speed reduced to 2.5 mm/s. Note the satellite droplets highlighted in red.

(b) Deposited wax droplets with settings shown in Table 3.4.

Figure 3.33.: Initial material deposits using paraffin wax to test a DDWP nozzle and the PolymerJet™ system which is coated by an elastic lining, and held in place by a matrix material. The optimal design for an LRM should have a high density material for the core and, a low density but stiff matrix material [4].

When choosing which materials to use for the LRM, a series of different materials were initially tested on the printer. This was to determine their ability to be deposited consistently, as well as their final printed quality. Several factors were considered including:

- the material's ability to deform once deposited,
- the rate at which it could be deposited,
- the curing methods,
- the material's mechanical properties and suitability for use in an LRM.

The DDWP nozzles on the printer can only print materials with a low viscosity (20 mPas or less), but the benefit is that it is possible to print a component with a good final resolution (features printed can be as small as 80 μm). However, the
time taken to print using these nozzles can be long as each droplet has a typical volume of only 2.145 nl.

There is less restriction on a material’s viscosity when deposited with the CDWP nozzle, as a higher viscosity material will simply require a greater pneumatic pressure to extrude it. Another benefit of the CDWP nozzle is that it has the ability to deposit a relatively large amount of material in a short space of time, but the final resolution of the printed component may be lower.

### 3.8.1. Matrix Material

For the CDWP nozzle, various UV curable resins were trialled, including ‘VeroClear®’, ‘Silfort UVHC 3000LS®’, and ‘Permabond UV625®’. All three materials were initially chosen due to their ability to be deposited at room temperature with a CDWP nozzle.

The Silfort UVHC 3000LS® is a UV curable hard-coat material from Momentive. It is designed to be deposited by inkjet nozzles onto plastic components, such as headlights, to protect them from weathering. Some settings for printing with the material were provided by Momentive (25 V amplitude with a 60 μm inkjet nozzle orifice). Initial trials printing with the material showed some success, with droplets agglomerating to form smooth patches of the material. However, it become apparent the deposited droplets were very brittle, and if attempted to be removed from the glass substrate, would simply crumble. Trials were made printing onto Kapton tape, which typically allows a printed component to be removed easily. However, the printed objects still remained too brittle and could not be handled. This material was deemed unsuitable for use as the matrix component of an LRM. No material characterisation tests were performed on this material once it was decided it was unsuitable for use.

VeroClear® is commonly used as an epoxy-like material, which is light weight and stiff. It can be deposited by both DDWP and CDWP nozzles. When exposed to UV light, it cured quickly and could be handled immediately. This appeared to be
the ideal material to deposit with. This material is commercially available for use on a Connex printer [21], and its material properties are published [182]. It has a Young’s modulus of $7.6 \times 10^2$ MPa, and a density of $1189 \, kg/m^3$. Initial attempts to deposit the material using a DDWP nozzle were unsuccessful. Inconsistent droplets were formed and there was a high occurrence of satellite formation. When the manufacturer of the material was contacted, they stated that VeroClear® is to be used exclusively on the commercial Connex printers (see Section 3.3), and would not provide any assistance with determining the printing settings. The material was trial printed using the CDWP nozzle with a 100 μm nozzle tip. However, due to its low viscosity (required for DDWP printing), the deposit material would run-off and not maintain its deposited shape. When curing the material, a single layer would already be deformed by the point it was exposed to UV light. Therefore, this material was deemed unsuitable for use as the matrix component of an LRM.

The Connex printers are able to rapidly expose the deposited VeroClear® to UV light once jetted, significantly reducing the occurrence of material run-off. This is achieved by using a sweeping UV light which immediately follows the nozzle as it translates across the object being built.

Permabond UV625® is a UV curable adhesive. It is designed to adhere to glass or plastics, upon exposure to UV light. The properties of this material are shown in Table 3.5. It is widely used for repair of components due to its durability and quick curing property. Its viscosity is too high to be used on a DDWP nozzle, so trials were performed using the CDWP nozzle. Its gel-like consistency maintained its deposited shape well when printed on Kapton tape attached to a glass microscope slide. The Permabond material was deposited with a height of 150 μm and a width of approximately 450 μm using the 250 μm nozzle diameter, a back pressure of 5 PSI, and a nozzle speed of 2 mm/s. Upon exposure to 365nm wavelength UV light (supplied by a UV torch), it cured as a stiff, durable material which could be handled without breaking. The material cured in approximately 5 seconds. When added upon already-cured UV625 material, it agglomerated well and did not leave gaps in the interstitial space between layers. This meant the material could be
built up vertically, making it suitable to use as the matrix material for the LRM.

### 3.8.2. Elastic Material

The elastic component of an LRM is typically rubber. Initial trials of printing rubber used the CDWP nozzle. The material deposited was Momentive™ ‘TSE397 Silicone’ and this deposited well and cured at room temperature by evaporation of its solvent. It has a Young’s modulus of 0.12 MPa, and a density of 1300 kg/m³. Its semi-flowable consistency meant it had too high a viscosity to use with a DDWP nozzle. As the CDWP ideally was to be used to deposit the matrix material, an alternative material had to be sourced, which could be deposited using DDWP methods.

Recent research had shown it was possible to print a rubber compound using ‘Litex T71S20®’ with DDWP nozzles [183]. More importantly, it was shown that the viscosity was low enough to be printed at room temperature and heating was not required (material properties are shown in Table 3.5). Therefore, it was suitable to use on the room temperature DDWP nozzle of the printer with its current configuration. Litex T71S20® is a carboxylated styrene-butadiene rubber which cures by the evaporation of its solvent at room temperature. It is a latex type rubber in a colloidal dispersion, with a 52% weight content and a viscosity of 12 mPas [183]. This material was successfully deposited when mixed with 1 g of triethylene glycol monomethyl ether (TGME) for every 10 g of Litex T71S20®. The TGME is used to delay the drying time of the rubber to stop the material clogging the nozzle, and to reduce the chance of agglomeration of the colloidal solution. Initial attempts at printing the rubber compound with the TGME added proved successful. A 26 V pulse wave was used for the inkjet nozzle signal [183]. However, it was observed that the curing times of the rubber (at room temperature) with the TGME added significantly increased when compared to deposited samples of rubber with no TGME added. A 1 mm diameter droplet of the rubber with TGME required in excess of 20 minutes to cure, when dropped onto a glass substrate. The original Litex material,
3.8. Multi-Material Printing for LRMs

with no TGME added, took appropriately 2 minutes to cure. The long cure times associated with the TGME material would make the rubber compound unsuitable for making the LRM as it would drastically increase the print times required to make a full LRM object.

It was decided to print with the Litex T71S20® material without being mixed with the TGME. The material viscosity of 12 mPas was well within the viscosity requirements of the DDWP nozzle, and so would be suitable for use. This reduced the curing times between each deposited layer to a matter of a few minutes rather than 20-30 minutes.

In instances where the nozzle was blocked, the printing process was paused, and a small amount of toluene was applied to the tip of the nozzle. This was followed by short purging of the material through the nozzle, and the print was then continued. The toluene was recommended by Lukic et al [183] as it causes the rubber to expand and soften to allow it to be forced out more easily with air pressure.

At the end of a printing process, the DDWP nozzle had to be purged and back flushed 2-3 times with the toluene material, and then repeated again with isopropyl alcohol (also known as IPA solution). This process was found to be sufficient to stop the nozzles becoming clogged between prints.

No other elastomer type materials were trialled using this DDWP nozzle.

3.8.3. Core Material

The final material to be chosen was for the core. As the printer has a heated inkjet nozzle capable of reaching 240°C, the ability of using low-melting point alloys became possible. When various solders were examined and compared to other polymers and ceramics which could be printed with this nozzle, it was clear that solder alloys had a significantly higher density (approximately 7 g/cm³), making them more suitable for an LRM application. As demonstrated in Section 2.3.3, the optimal LRM design will have a heavy core and light, stiff matrix material. A solder with the composition Sn96.5Ag3Cu0.5 was chosen as it has a melting point of 217°C, well within the range of the PolymerJet™ system DDWP nozzle. The viscosity of 2 mPas[184] was low enough to be deposited. The solder was first pre-heated to remove any flux and
then cooled. Once the flux had been removed, the solder alloy can be melted in the reservoir of the nozzle. It then passes through a 7\(\mu\)m stainless steel filter, and finally into the inkjet nozzle. Once deposited, the solder solidifies upon cooling.

An issue arose when printing with the PolymerJet\textsuperscript{TM} system device, whereby the droplets of solder cooled too quickly once jetted, resulting in a printed object with a consistency more similar to a powder rather than a solid object (a similar effect was seen when depositing with paraffin wax).

Different printing set-ups were explored, which included moving the nozzle closer to the substrate to reduce the time taken for the droplets to reach the printed object. A shield was also trialled whereby aluminium foil was placed around the nozzle to restrict air flow and reduce the rate of cooling of the droplets, but this did not achieve its desired result.

In an attempt to create larger droplets, a blocked nozzle had its tip cut off and then placed in the printer to trial printing with the solder. It was not possible to create the droplets as intended, but it was observed that large droplets of solder formed at the tip of the blocked nozzle due to a low vacuum pressure in the nozzle’s reservoir combined with the effect of gravity acting upon the solder. When these larger droplets fell, they remained molten for a short time after reaching the glass substrate. It was noticed that the shape formed by these droplets on the glass substrate was consistent and repeatable. When several of these cooled droplets were weighed on a scientific scale, they consistently weighed 1.21 g, which confirmed that these droplets had a consistent volume. This hybrid approach to depositing the molten solder proved a viable deposition technique due to its repeatability, and hence could be used to make an LRM.

Once the materials had been chosen, a simple design was created to verify that all three materials could be printed in one process. The design consisted of a Permabond 1 cm \(\times\) 1 cm UV625\textsuperscript{®} square with a with a central solder droplet (measuring 4 mm in diameter), and Litex T71S20\textsuperscript{®} in the interstitial space. As this was a relatively simple shape, the coordinate file was compiled manually, rather than generated from a CAD model. The Permabond square was deposited first with a 500 \(\mu\)m nozzle.
3.8. Multi-Material Printing for LRM

Table 3.5.: Summary of mechanical properties and print settings for the chosen materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Permabond UV625®</th>
<th>Litex T71S20®</th>
<th>Sn96.5Ag3 Cu0.5 Solder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nozzle Type</td>
<td>CDWP</td>
<td>DDWP</td>
<td>Heated DDWP</td>
</tr>
<tr>
<td>Metamaterial Component</td>
<td>Matrix</td>
<td>Coating</td>
<td>Core</td>
</tr>
<tr>
<td>Cure Method</td>
<td>UV cure</td>
<td>Solvent evaporation</td>
<td>Cooling</td>
</tr>
<tr>
<td>Nozzle Settings:</td>
<td>5 PSI pressure</td>
<td>+/- 32V Sine Waveform</td>
<td>225°C temperature</td>
</tr>
<tr>
<td></td>
<td>250 μm nozzle diameter</td>
<td>1.0 kHz frequency</td>
<td></td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>1190</td>
<td>1150</td>
<td>7365</td>
</tr>
<tr>
<td>Young’s Modulus (MPa)</td>
<td>1.3x10³</td>
<td>0.495</td>
<td>5.0x10⁴</td>
</tr>
<tr>
<td>Poisson Ratio</td>
<td>0.35</td>
<td>0.47</td>
<td>0.36</td>
</tr>
</tbody>
</table>

...tip, moving in an anticlockwise direction along the edge of the square. The solder droplet was then deposited in the centre of the square and allowed to cool. The Litex rubber was then deposited in the interstitial space between the square and the droplet. The nozzle speed was set 2 mm/s to allow the rubber to fill the area. Two passes with the rubber DDWP nozzle were required to fill the area. Once this rubber had cured, the fabrication of the composite object was complete (see Figure 3.34).

Figure 3.34.: A 3 component composite material printed to verify the custom printer could deposit 3 different materials utilising both CDWP and DDWP printing techniques.
This represents the first three component composite material to be printed with different direct-write methods, containing a metal, rubber, and plastic material. This also demonstrated the viability of using this custom printing set up to create a full LRM design.

Unfortunately, no 3D validation of the printing process with all the nozzles was performed prior to attempting the manufacture the LRM itself. This was a decision partly based on time constraints, but also the ability to make a full LRM would act as the validation of this printer to make composite 3D objects.

3.9. Chapter Conclusions

In this chapter, a review has been performed on the current capabilities of AM techniques and their viability for multi-material applications. Details are then provided regarding the design of a custom made 3D printer which utilises both CDWP and DDWP methods. This printer uses a bespoke control program in the LabVIEW environment, which facilitates its easy use, and high customisability.

Using MATLAB, an alternative geometry processing method is outlined, called the ‘Point-Cloud’ method. This method allows additional information about a design to be defined. This capability is not possible with the STL format, which is the standard format used in the AM sector. Additionally, the amount of information that can be added is not limited, hence the method is scalable for future developments and capabilities in AM printing. Further development is required to allow the method to automatically detect different materials within a single STL file, in order to avoid the user manually selecting which material belongs to which component in a multi-material design.

Various materials were tested on the printer to allow consistent and repeatable manufacture of components. The final materials chosen were Permabond UV625® , Litex T71S20®, and Sn96.5Ag3 Cu0.5 solder. A simple design was made and successfully fabricated using the printer. This is the first printer to successfully print with a metal alloy, a rubber compound, and a UV curable resin, all within one object. The use of direct write printing techniques facilitates the use of a wider range of printing...
3.9. Chapter Conclusions

materials than alternative printing methods.
A hybrid approach to depositing the metal alloy was employed. This was due to
the rapid cooling of the solder droplets failing to agglomerate once on the substrate.
This produced a metal alloy powder, rather than a solid object. The large droplet
deposition under gravity did produce consistent droplets, but its viability for use
in different scenarios is questionable. As future work, alternative materials can be
trialled in the PolymerJet\textsuperscript{TM} inkjet nozzle to allow for normal PZT based inkjet
droplet creation (i.e. the formation of a droplet by the constriction of a glass tube
by a PZT sleeve).
Chapter 4

LRM Design and Manufacture

4.1. Introduction

In this chapter, the modelling, manufacture, and testing of three different LRM designs is described. The modelling process used in this project to design an LRM is presented, including an optimisation procedure using the FEM software, LS-Dyna [27]. Following the creation of three different LRM designs, the custom built multi-material 3D printer is utilised to manufacture these LRMs. Once fabricated, the LRMs are tested using an acoustic impedance tube. The chapter aims to verify the use of the custom made 3D printer as a viable option for making LRMs, and hence helping the adoption of LRM technology for future applications.

4.2. Modelling for an LRM

As stated in Section 2.6, there are several methods available to model the behaviour of a metamaterial including the PWE, MST, MAM, FDTD, and FEM (time domain analysis) methods. All of these methods are able to generate the frequency band structure for a metamaterial.

It was decided to use the MAM method and a time domain FEM analysis to model an LRM (see Section 2.6.4), as the use of the advanced FEM software, LS-Dyna,
was possible for this project. LS-Dyna is a commercial FEM code that is well known and established for its time domain non-linear modelling capability [27].

Prior to the generating the band structure, a modal analysis was performed to determine the resonant modes of an LRM that correspond to the attenuation of a wave (see Section 2.3.3).

4.2.1. Resonant Modes and the Optimisation Process

For LRMs, the wave attenuation mechanism depends on the natural frequency of the structure. The lower limit of the frequency band gap can be identified by a modal analysis of an LRM's RUC [4]. The procedure to perform a modal analysis is as follows:

1. **LS-Ingrid:** The geometry and mesh for the part is created using the text based mesh generator LS-Ingrid. The input file is an ‘ing’ format which is purely text based, and hence no user interaction is required to generate the mesh once the ‘ing’ file has been defined. Once the preliminary model is made, the item is saved as a `.keyword` file. At this stage, the keyword contains the geometry, mesh and the part numbers for the model.

2. **LS-Prepost:** The keyword file is accessed by LS-Prepost. Periodic boundary conditions are applied to the sides of RUC and a modal analysis is selected. Once these additions have been made to the keyword file, it is saved.

3. **LS-Dyna:** The model is ready to be processed at this stage. The program LS-Dyna is used to perform the modal analysis by solving the following equation:

$$([K] - \omega^2[M])\Phi = 0$$

where $[M]$ and $[K]$ are the mass matrix and the stiffness matrix of the LRM respectively, and where $\omega$ are the eigenvalues and $\Phi$ are the eigenvectors.

For a classical LRM design with approximately 3500 elements, this analysis takes 10 seconds to complete (using an Intel i7 3.4 GHz CPU, with 16 GB of RAM). There are various types of output file depending on the type of analysis
4.2. Modelling for an LRM

performed. For a modal analysis, the output is called a ‘d3eigv’ binary plot file.

4. LS-Prepost: The ‘d3eigv’ plot file is accessed in LS-Prepost and allows the user to view the results in a variety of ways. One way is to observe the resultant displacement during of the resonant modes as an animated fringe plot (see Figure 4.1 for examples of fringe plots). This is a quick and visual method to determine the behaviour of the model at various resonant modes.

As shown in Section 2.3.3, for a classical LRM design, the attenuation effect is observed most prominently in the vibration mode which equates to the displacement of the central dense core. Additional attenuation frequencies are observed in later vibration modes which correspond to oscillations in the elastic lining of the LRM, however the dominant attenuation effect is observed with the oscillation of the dense core [37, 185]. Figure 4.1 shows the various resonant modes resulting from a modal analysis for an arbitrary LRM.

It was observed that modes 1-3 resulted in a rotation of the spherical resonator about the X, Y, and Z axes respectively (see Figure 4.1). Mode 4 resulted in the horizontal displacement of the spherical resonator and mode 5 and 6 were normal displacement to the mode 4 vibration. Modes 1-3 all have the same numerical value, as do modes 4-6, assuming the RUC of the model is symmetrical about all three axes. All subsequent modes resulted in excitation modes in the soft rubber layer. For this arbitrary model, a total of 50 eigenvalues were requested, and the analysis took approximately 10 seconds (using an Intel i7 3.4 GHz CPU, with 16 GB of RAM). It should be noted that in 2D modelling cases, as observed in numerous publications, only mode 1 equates to the rotation of the core, and mode 2 and 3 equate to the displacement of the core. Hence why there is some disparity between literature which quote mode 2 or mode 4 resonant frequencies as the attenuating frequencies.

For engineering applications, there will be a desire to shift the mode 4 natural frequency of an LRM as close as possible to the desired attenuation frequency. There
Figure 4.1.: Resonant vibration modes in an LRM. The images shown are cross-sections of the model. Periodic boundary conditions have been applied to all sides of the RUC. The mesh density is equivalent to 12 elements per wavelength (fine mesh) and the analysis was performed in LS-Dyna [27] (materials used are shown in Table 4.1).

There are 2 simple ways to achieve a shift in the mode 4 vibration frequency: 1) alter the material properties, 2) alter the geometry.

As the materials for the printer had already been chosen, the only option was to alter the geometry of the LRM. To manually alter the geometry of an LRM and perform a modal analysis normally is a simple process. However, doing so iteratively with
small changes can become tedious. To do this in a time efficient way, an optimisation procedure was set up to vary the geometry of the LRM and assess its mode 4 resonant frequency.

The optimisation procedure was introduced using LS-Opt, an optimisation software that couples with LS-Ingrid, LS-Prepost and LS-Dyna (see Figure 4.2). The optimisation procedure is designed to be run on the High Performance Computing (HPC) cluster at Imperial College London. An optimisation procedure was also devised to run the model entirely on one desktop computer if the HPC was not available. However this occupied the user's computer throughout the duration of the optimisation and therefore, purely for convenience, the HPC option was used. The desktop method is shown in Appendix A.

The method to implement the optimisation procedure is straightforward once set up. The main consideration are the input files, which have to be properly parameterised. The process of parameterising a variable is where the numerical value of the variable in the input file is replaced by a bracketed word (e.g. \〚Parameter1〛).

This is then recognised as a variable in LS-Opt which can replace this word with a numerical value for the analysis.
The numerical value inserted by LS-Opt can be determined by one of several optimisation algorithms. The simplest of these is a ‘space filling’ algorithm which takes a user defined maximum and minimum value and LS-Opt equally distributes values within these two points. A domain reduction option can be added such that after each modelling iteration, the difference between the maximum and minimum value is reduced. More complicated algorithms such as the Monte-Carlo and Koshal methods can also be employed.

The space filling algorithm is used for the optimisation of the LRM. The equally distributed result points show clearly the effect of geometry on the resultant behaviour of the model, which is useful information in the early stages of LRM design.

If the material and geometry of a part were being optimised, then both the ‘keyword’ and the ‘ing’ files would need to be parameterised. As the optimisation of the LRM only requires the geometry to be optimised, only the ‘ing’ file used with LS-Ingrid is parameterised. The variables which are parameterised are the radius of the core and the thickness of the elastic lining (see Figure 4.3(a)). The output from LS-Ingrid is a preliminary keyword file containing the meshed model and the part numbers of the model.

A secondary keyword contains all the additional information required for the modal analysis including material definitions, material assignments, and boundary conditions. This secondary keyword also contains a command to include the first preliminary keyword so that they are read as one file.

Once the file is returned from LS-Prepost, LS-Opt collates the preliminary and secondary keyword files into a subdirectory. These files are then submitted to LS-Dyna for the modal analysis to be executed. The ‘d3eigv’ file which contains the results from this analysis are stored by LS-Opt in sub folders that can be accessed by the user.

Once the overall process is complete, the final output is an ‘lsopt-db’ file which allows all the results to be viewed in both graphical and text based formats.
4.3. Generation of the Frequency Band Structure

In the early stages of this project, when this optimising procedure was being established, a 10 kHz attenuating LRM was being conceived. This LRM also had a spherical core and cubic lattice design. Using the matrix and material definitions as in Liu's LRM [3] (see Table 4.2) and tin as the core, the design was optimised for a 10 kHz mode 4 resonating frequency. The results from LS-Opt can be seen in Figure 4.3. This is shown as an example of the typical results obtained from this optimising procedure. The two variables of the optimisation were ‘rad1’ and ‘rad2’ corresponding to the radius (in millimetres) of the resonator and the outer edge of the rubber lining respectively. The lattice constant was not optimised in this set up.

![Figure 4.3](image)

(a) Parameterised variables for optimising an LRM
(b) Optimisation results: Green nodes are viable options given user constraints. The units are in $kHz$ for the frequency and $mm$ for the radii.

Figure 4.3.: Optimisation results for a 10 kHz attenuating LRM.

4.3. Generation of the Frequency Band Structure

Two methods were employed to generate the frequency band structure of a given LRM design; 1) the FEM (time domain) method and 2) the MAM method.

4.3.1. FEM (Time Domain) Method

Modelling using an FEM time domain analysis can be beneficial when compared to alternative methods such as the MAM and PWE, as it is able to generate wave
profile data in addition to the frequency band structure. The wave profile data allows the user to determine certain characteristics about the wave, such as the sound transmission loss.

Early in the project, it was envisioned that a LRM would be made that could attenuate a 10 kHz wave. As such, a design was created that had a mode 4 vibration at 9.95 kHz. The geometry and materials used for this design is shown in Table 4.1. The geometry was determined using an early implementation of the optimisation process outlined in Section 4.2.1.

Table 4.1.: Geometry and material properties of an LRM modelled using the FEM (time domain) analysis.

<table>
<thead>
<tr>
<th>Metamaterial Component</th>
<th>Material</th>
<th>Coating</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix</td>
<td>VeroClear</td>
<td>Silicone Rubber</td>
<td>Tin</td>
</tr>
<tr>
<td>Coating</td>
<td>1.0 mm RUC length</td>
<td>125 μ thick lining</td>
<td>500 μm diameter</td>
</tr>
<tr>
<td>Core</td>
<td>1189</td>
<td>1300</td>
<td>7365</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>7.6x10²</td>
<td>0.118</td>
<td>5.0x10⁴</td>
</tr>
<tr>
<td>Young’s Modulus (MPa)</td>
<td>0.35</td>
<td>0.47</td>
<td>0.36</td>
</tr>
<tr>
<td>Poisson Ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A time domain, linear elastic model was created (see Figure 4.4(a)). This type of analysis solves the wave equation (see Eq. 2.53 in Section 2.6.4). As the model was in LS-Dyna, the non-linear viscoelastic effects could be modelled once the viscoelastic properties of the materials had been determined. These properties can be determined from a dynamic mass analysis which can determine properties such as the shear modulus. The materials used in a LS-Dyna model are defined by individual keywords. The materials used in the FEM model are defined by linear elastic keywords, which require the density, Young’s modulus and Poisson ratio. The viscoelastic cards require the bulk modulus and the shear modulus at a short time interval and a long time interval. Initially, a linear elastic model was used.

The model had 5 LRMs in series in a horizontal placement, with a time dependant force applied on the left of the model, and an outlet zone on the far right of the model (see Figure 4.4(a)). At the end of the outlet zone was a non-reflecting boundary condition to prevent propagating waves reflecting back from the end of the model.
The model was set up as 2D, as making it 3D would have been computationally expensive. In previous studies [4, 185], it has been shown that 2D modelling with a plane-strain boundary condition applied could be used as a good approximate of a full 3D metamaterial. As such, the model was made from shell elements upon which a plane strain boundary condition could be applied.

A time dependant force of 0.005 N was applied to the left edge of the model, with a frequency of 9.95 kHz, equating to the mode 2 vibration frequency of the LRM (determined from the initial modal analysis on a single RUC of the LRM). Two points were selected to observe the displacement history; one on the matrix material and one on its corresponding core. The time history of this displacement is shown in Figure 4.4(c).

What this result shows is that there are distinct periods of time when the core and matrix are moving out of phase with each other, as the LRM responds to an input force at its resonant frequency. This is clear metamaterial behaviour. What is also apparent is that this out of phase movement is periodic and not constant with time.

One issue arose when using this model regarding the non-reflecting boundary (see Figure 4.4(a)). LS-Dyna allows the user to define nodes in a 2D model as ‘non-reflecting’, so that an incoming wave is not reflected. However, a reflection wave was observed in several model set ups. Alternative non-reflecting boundaries were trialled, included extending the outlet zone further and increasing element size to try and create numerical dispersion of the reflected waves, which did not prove successful. With the presence of this reflected wave, the generation of a frequency band structure would not be accurate for the LRM design.

At this point, it was decided to focus on using the MAM method to generate the frequency band structure of the model. It was known that it would be easier to implement and generate the band structure, when compared to the FEM time domain method.

The inability to use the time domain model would not be too much of a hindrance.
4.3. Generation of the Frequency Band Structure

(a) A 2D FEM time domain analysis model. The 9.95 kHz input force of 0.005 N is applied on the left of the model, whilst a non-reflecting boundary (shown in blue) is on the right edge of the model.

(b) Displacement vectors on the LRM at peak anti-phase movement. Fringe plot shows the displacement in meter units.

(c) Time vs Displacement plot for points on the matrix and core (shown in Figure 4.4(a)). The red shaded areas show the core oscillating approximately 180° out of phase with the matrix. This is the wave attenuating metamaterial effect.

Figure 4.4.: FEM (time domain) modelling on an LRM with a target attenuation at 10 kHz.
to the project. The main purpose of the modelling was to allow the design of LRM.
If an LRM was designed, manufactured and tested, the location of the measured
band gap in relation to the model could be used to verify that the manufacturing
process was valid. This methodology was used during the rest of the project.

4.3.2. MAM method

Following the optimisation process outlined in Section 4.2.1, the final geometrical
features of an LRM’s RUC are fixed. The next stage in the modelling process is to
determine the frequency band structure of the LRM. From the various modelling
methodologies available (see Section 2.6), the MAM method is used widely due to its
relatively simplicity of implementation and its ability to handle more complicated
geometries and mediums (see Section 2.6.4). The process for creating the band
structure diagram of a RUC is determined as follows:

1. **Keyword File to NASTRAN File** - The keyword file is the same as the
   model used in the optimisation procedure, except the parameterised values
   are replaced with their final values. It contains the node locations (i.e. the
   mesh), the materials and the material assignments. The keyword also contains
   the boundary conditions and analysis type, but these are disregarded in the
   next stage of this process. At this stage, it is recommended to run the modal
   analysis one more time to verify that the correct mode 4 vibration has been
   achieved. Once the model is final, it is exported as a NASTRAN file into
   ABAQUS, a commercial FEM software.

2. **Mass and Stiffness Matrices Generation** - The NASTRAN file is opened
   in ABAQUS. The model is converted into a ‘Job’ file and run with the analysis
   step set to ‘Substructure Generation’. This generates the mass and stiffness
   matrix of the LRM RUC in their full format. The final text file contains the
   full format matrices written in a matrix with only 4 columns at a time. Hence,
   the resulting text files are very large (typically 1-1.5 gigabytes).
4.3. Generation of the Frequency Band Structure

\[
\begin{bmatrix}
C_1 & C_2 & C_3 & C_4 \\
R_1 & \#_1 & \#_1 & \#_1 & \#_1 \\
\vdots & \vdots & \vdots & \vdots \\
R_{\text{end}} & \#_{\text{end}} & \#_{\text{end}} & \#_{\text{end}} & \#_{\text{end}} \\
C_5 & C_6 & C_7 & C_8 \\
R_1 & \#_1 & \#_1 & \#_1 & \#_1 \\
\vdots & \vdots & \vdots & \vdots \\
R_{\text{end}} & \#_{\text{end}} & \#_{\text{end}} & \#_{\text{end}} & \#_{\text{end}} \\
\text{etc.} & \text{etc.} & \text{etc.} & \text{etc.} & \text{etc.}
\end{bmatrix}
\]

In LS-Dyna, it is possible to generate the mass and stiffness matrices as output files which are in the Harwell-Boeing file format. The Harwell-Boeing format is a method of representing sparse matrices. Early iterations trying to convert the Harwell-Boeing formatted mass and stiffness matrices into their full format failed. It was decided to use the FEM software ABAQUS instead, as it automatically outputs the full matrices.

3. **Periodic Boundary Conditions** - The boundary conditions for the model are manually defined in an Excel worksheet. As this is an RUC type model, periodic boundary conditions are applied where there are dependent and independent nodes. The behaviour at a dependent node is influenced by the behaviour at its corresponding independent node on the opposite side of the RUC. In addition, the unit wave vector describing the propagation from the independent node to its corresponding dependent node is also written. This has to be true for all directions X, Y, and Z. The Excel worksheet is read by a MATLAB, prior to executing the MAM model.

For reference, the diagram explaining the application of the periodic boundary conditions from Section 2.6.4 is repeated in Figure 4.5.

4. **Wave Vector Assignment** - The next stage in this modelling process is to assign the wave vectors. It is these wave vectors that will be used to generate
4.3. Generation of the Frequency Band Structure

the points at which the resonant modes are assessed in an RUC. The wave vectors are set such that they outline the border of the Irreducible Brillouin Zone (IBZ) of the lattice structure of the LRM (see Section 2.6.4). Many metamaterials use a simple cubic lattice system or a simple hexagonal lattice system. Figure 4.6 shows a simple cubic lattice with the IBZ for the structure highlighted as a red line. The vectors are set such that they follow the path: \( \Gamma \rightarrow X \rightarrow M \rightarrow \Gamma \rightarrow R \rightarrow M \rightarrow X \rightarrow R \).

Additional IBZs for different lattice types are shown in Figure 2.18. These wave vectors are assigned directly in the MATLAB program used for the generation of the band structure and hence it is important to ensure the correct paths are chosen prior to executing the modelling. Each wave vector is then split into several points which the program will process.

5. MATLAB Program- Once the Mass and Stiffness matrices are created, the boundary conditions have been correctly assigned, and the wave vector paths have been defined, the generation of the frequency band structure can begin. The mathematics of how this is done are described in section 2.6.4. The run times for the model can be long and are dependent upon the number of nodes in the model and the accuracy of the final band diagram. For an RUC with approximately 3500 elements and 200 analysis points along the edge of the
4.4. Verification of Modelling Approach

To verify the correct implementation of the modelling technique, a previously published LRM design was modelled. The design chosen was the classic LRM design as first created by Liu et al. [3] (see Figure 2.3). The properties for this LRM's RUC are shown in Table 4.2.

Table 4.2.: Geometry and material properties of an LRM used for verification [3].

<table>
<thead>
<tr>
<th>Metamaterial Component</th>
<th>Matrix</th>
<th>Coating</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Epoxy</td>
<td>Silicone Rubber</td>
<td>Lead</td>
</tr>
<tr>
<td>Component Geometry</td>
<td>15.5 mm RUC length</td>
<td>2.5 mm thick lining</td>
<td>5 mm diameter</td>
</tr>
<tr>
<td>Density (kg/m$^3$)</td>
<td>1190</td>
<td>1150</td>
<td>7365</td>
</tr>
<tr>
<td>Young's Modulus (MPa)</td>
<td>1.3x10$^3$</td>
<td>0.495</td>
<td>5.0x10$^4$</td>
</tr>
<tr>
<td>Poisson Ratio</td>
<td>0.35</td>
<td>0.47</td>
<td>0.36</td>
</tr>
</tbody>
</table>
4.4. Verification of Modelling Approach

No geometry optimisation was performed as it was already known:

- a spherical lead core with a 5 mm diameter,
- a 2.5 mm thick silicone rubber lining surrounds the core,
- an epoxy cubic RUC with lengths of 15.5 mm.

This design's geometry was recreated in LS-Ingrid and then transferred into LS-Prepost. The periodic boundary conditions were applied using LS-Prepost's inbuilt keyword. Once the correct materials were assigned to the corresponding parts, a simple modal analysis performed. The result of the modal analysis showed the following results:

- Mode 1-3 = 260.7 Hz
- Mode 4-6 = 389.9 Hz
- Mode 7 = 877.1 Hz

In Liu's paper, using the MST method, he calculated the mode 2 vibration at approximately 400 Hz and the band gap was determined to range from 380 Hz to 570 Hz (specific values are not stated, these figures were determined from the frequency band diagram in [3]). Comparing the mode 4 vibrations, there was good correlation with the results.

The next stage of the modelling was to utilise the MAM model to determine the frequency band structure. The process outlined in Section 4.3 was used. The MATLAB model performed its function well and after a 2 days of running the process was complete. The frequency band structure generated can be seen in Figure 4.7(a).

The band structure generated from Liu's design showed a clear band gap formation in the range 390 Hz to 542 Hz. This correlated well with the band gap reported in Liu's paper. The lower limit was within a 10 Hz range between the two results. However, the upper limit of the band gap differed by approximately 30 Hz from Liu's predicted band gap. The reason for this discrepancy was concluded to be due to the different modelling techniques used.
4.4. Verification of Modelling Approach

(a) Frequency band structure of model. Grey region shows the modelled band gap between 390 Hz to 542 Hz. IBZ is shown in the inset.

(b) Mesh on Liu verification model. lattice constant is 1.55 cm.

Figure 4.7.: Results of model verification when performed on Liu's design [3].

The density of the mesh used in the input keyword file may also be a significant factor. There have been a number of investigations into the effect of mesh density upon final results [39, 134]. In both studies listed, a quoted mesh density requirement of 5 elements per wavelength at the maximum frequency being analysed is quoted as the requirements for a fine mesh. Anything less than this requirement can be considered a coarse mesh. In consultation with LSTC, the developers of LS-Dyna, a 6 element per wavelength (at maximum frequency) requirement was suggested
4.5. Testing using an Impedance Tube

for acoustic modelling. Using this principle, all models submitted to MATLAB for frequency band calculations met this requirement.

The mesh used in the verification modelling of Liu's design had a density of 12 elements per wavelength at 10 kHz, well within the modelled frequency range and mesh density requirements (see Figure 4.7(b)). As such, a coarse mesh can be ruled out as the source of the discrepancies in the upper boundary of the band gap. The band gap predicted using the MAM method within MATLAB was more conservative than Liu's MST model. Therefore, the process outlined in 4.3 was considered valid. This is especially important as the maximum attenuation effect is observed at the lower bound of the frequency band gap, and hence this will be the focus of operation and design for most potential applications.

As shown in Section 2.6.5, the viscoelastic effect upon an LRM is to increase the location of the upper limit of the band gap, with the lower limit remaining the same. Initially this was considered as one of the reasons for the discrepancy as the MAM method used is based on linear dynamics. However, Liu's MST model does not include viscoelastic behaviour either, and hence non-linear effects are not considered a suitable explanation.

For the purpose of designing an LRM such that the lower bound of the band gap is in the desired location, this method of modelling is valid. Therefore, it can be used to corroborate the testing of LRMs made on the custom 3D printer.

4.5. Testing using an Impedance Tube

There are various ways of testing acoustic and elastic metamaterials. The most common and well established method is the use of an impedance tube. This is an acoustic testing device that subjects a test sample to sound waves of varying frequency. The sound pressure levels are measured at various points in the tube, which can then produce results describing the acoustic behaviour of the test sample. The design of the tube is important as it will directly affect the range of frequencies that can be tested. The design of an impedance tube is dictated by the international standards, ISO 10534-2 [186].
4.5. Testing using an Impedance Tube

A ‘two-microphone’ set up will have a loudspeaker at one end of the tube and the test sample at the opposite end. At specific locations along the tube, there will be two microphones that measure the sound. Based on the measured sound amplitude at these points and the input frequency of sound, the absorption coefficient and the reflection coefficient of the test sample can be determined.

A ‘4 microphone set up’ will have two additional microphones on the opposite side to the test sample, again at specific locations dictated by [186]. With these additional microphones, the ‘Sound Transmission Loss’ can also be determined using a method created by Song and Bolton [187], which requires the formation of the ‘transmission matrix’. The sound transmission loss is a measure of the amount of energy that is lost in a wave as it passes through a sample. This variable is used in the acoustic testing of metamaterials.

The method to determine the transmission matrix requires the formation of plane waves which propagate in the tube and impact normally onto the surface of the test sample. Some of the sound energy is reflected, absorbed, or transmitted through the material. The sound that is transmitted through the material continues to propagate until it reaches the end of the tube where there is either a rigid end, an open end, or an anechoic termination (this will be explained later in this section). A typical transmission loss type impedance tube is shown in Figure 4.8.

![Figure 4.8.: A transmission-loss type impedance tube [29].](image-url)
The pressure levels at each of the microphones can be determined from the following equations [188]:

\[ P_1 = A e^{i(\omega t - kx_1)} + B e^{i(\omega t + kx_1)} \] (4.2)

\[ P_2 = A e^{i(\omega t + kx_2)} + B e^{i(\omega t - kx_2)} \] (4.3)

\[ P_3 = C e^{i(\omega t - kx_3)} + D e^{i(\omega t + kx_3)} \] (4.4)

\[ P_4 = C e^{i(\omega t + kx_4)} + D e^{i(\omega t - kx_4)} \] (4.5)

where \( A \) and \( B \) are the forward and reflected complex coefficients of sound pressure amplitude at the front of the test sample respectively. \( C \) and \( D \) are the forward and reflected complex coefficients of sound pressure amplitude behind the test sample respectively. \( k \) is the wave number and \( x_i \) are the distances from the front of the test sample to each microphones respectively.

These above equations can be rearranged as follows:

\[ A = \frac{i(P_1 e^{ikx_2} - P_2 e^{ikx_1})}{2sinkx_1 - x_2} \] (4.6)

\[ B = \frac{i(P_2 e^{-ikx_1} - P_1 e^{ikx_2})}{2sinkx_1 - x_2} \] (4.7)

\[ C = \frac{i(P_3 e^{ikx_4} - P_4 e^{ikx_3})}{2sinkx_3 - x_4} \] (4.8)

\[ D = \frac{i(P_4 e^{-ikx_3} - P_3 e^{ikx_4})}{2sinkx_3 - x_4} \] (4.9)

With these variables now calculable, the frequency dependent transmission matrix can be created:

\[
\begin{bmatrix}
A \\
B
\end{bmatrix} = \begin{bmatrix}
\alpha & \beta \\
\gamma & \delta
\end{bmatrix} \begin{bmatrix}
C \\
D
\end{bmatrix}
\]
4.5. Testing using an Impedance Tube

From the above transmission matrix, $\alpha$ is the main term of interest as it is used to directly calculate the sound transmission loss, which is determined as follows:

$$TL = -20\log(\alpha)$$  \hspace{1cm} (4.10)

The term $\alpha$ can be determined as follows:

$$\alpha = \frac{(A_1D_2 - A_2D_1)}{(C_1D_2 - C_2D_1)}$$  \hspace{1cm} (4.11)

where $A_1, C_1, D_1$ are the wave amplitudes with a rigid termination on the tube, and $A_2, C_2, D_2$ are the wave amplitudes when the impedance tube has an open termination. This is why the test is performed twice, once with a rigid end and once without.

4.5.1. Custom Impedance Tube and Control Program

A custom made impedance tube was made in accordance with international standards ISO 10534-2 [186]. Early in the project, a 10 kHz attenuating metamaterial was targeted. As such, this was the capability goal for the impedance tube when it was designed. This upper limit of the testing range directly determines the inner diameter of the tube. As the frequency increases, the diameter of the tube decreases, which limits the size of the sample that can be tested. As such, many commercial tubes have an upper frequency testing limit of 6 to 7 kHz. Given the 10 kHz target frequency, a custom built impedance tube was designed according to international design standards [186] (see Figure 4.9). The impedance tube would be a ‘4 microphone’ design and hence be able to determine the sound transmission loss. This was key for measuring the performance of metamaterials. The main geometrical features of the tube are shown in Table 4.3.

The tests are performed according to international standards [186]. Initially, the rigid stop is attached to the end of tube. The test is then repeated with the tube end left open. This is known as the ‘two load’ method [188].
4.5. Testing using an Impedance Tube

Table 4.3.: Impedance Tube Properties. (Microphone locations are given in reference to Figure 4.8.)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Range</td>
<td>2050 Hz - 10,500 Hz</td>
</tr>
<tr>
<td>Inner Tube Diameter</td>
<td>18 mm</td>
</tr>
<tr>
<td>Max Specimen Thickness</td>
<td>10 mm</td>
</tr>
<tr>
<td>Max Speaker Sound Pressure</td>
<td>71 dB</td>
</tr>
<tr>
<td>Microphone Locations</td>
<td>$x_1 = 5.6 \text{ mm}$, $x_2 = 4.2 \text{ mm}$</td>
</tr>
<tr>
<td></td>
<td>$x_3 = 5.2 \text{ mm}$, $x_4 = 6.6 \text{ mm}$</td>
</tr>
</tbody>
</table>

A ‘one load’ method exits which requires an anechoic termination at the end of the tube. Acoustic foam was purchased and tested but there was noticeable sound leakage across the testing frequency spectrum. Obtaining a truly anechoic material was difficult to achieve in the small space available in the tube. Therefore, the two load method was used.

![Custom made transmission-loss type impedance tube.](image1)

![LabVIEW based control program for the impedance tube. The magnitude and phase are recorded during the testing (see Appendix C).](image2)

Figure 4.9.: Custom made impedance tube for LRM acoustic tests.

The tube itself was made from aluminium and the different components of the tube were attached to each other by M4 screws. The sound was provided by a 16 mm diameter 700-20000 Hz speaker which covered the majority of the cross-sectional area of the inner tube. The sound pressure is recorded by pre-amplified
4.5. Testing using an Impedance Tube

1/4 inch YOGA condenser microphones, which are attached to an NI Systems Data Acquisition device (DAQ).

The impedance tube is controlled by a custom made LabVIEW program which automates the testing process (see Appendix C). Some user interaction is required to switch the type of tube termination during the second phase of testing, and to select the ‘second testing’ option in the program to allow the results to be written to an alternative location. The program communicates with the DAQ via a USB cable and based on the incoming signals from each microphone, determines the magnitude and phase in the frequency domain. Based on these results, the complex pressures (See Eqs. 4.2-4.5) can be measured. Following the calculations stated in Section 4.5, the frequency dependent sound transmission loss can be generated for the sample being tested.

The acoustic properties determined are specific to the test specimen. As the maximum specimen size is smaller for this impedance tube, when compared to commercial impedance tubes, it was not possible to verify directly the results from this tube to the results of a previously run experiment. However, the tube does meet the requirements for testing as outlined in the testing standards [186]. Initial tests were performed on aluminium discs to verify the results obtained were repeatable, which they were. However the results from these tests were not saved.
4.6. LRM Design 1

With the modelling, manufacturing and testing methods established, the ability to design a LRM and test it was possible. The first LRM design chosen was a variation of Liu's design with a simple cubic lattice and a spherical core. The materials that could be used had already been chosen based on the capability of the additive manufacturing facility (see Chapter 3). Due to the hybrid approach of depositing the solder material, it was decided that the core of the proposed LRM would be equivalent to one drop. Trials were performed where these solder droplets were deposited into recessed surfaces, and the final cooled droplet formed into a spherical droplet of solder with a 3.4 mm diameter. This proved ideal for use as a spherical core for an LRM but it did imply that alternative core shapes would be more difficult to fabricate. Based on findings from Krushynska et al [4] the optimal core shape for wave attenuation in a three-material component LRM is spherical, and hence it is unlikely that there will be many applications for non-spherical core LRMss (see Section 2.3).

4.6.1. Modelled Behaviour

The first LRM design made was similar to Liu's design and had a spherical core coated in a rubber of constant thickness, and held in place within a simple cubic type matrix. An LS-Ingrid model was created which contained a 3.4 mm diameter core, and a 0.25 mm thick rubber coating in a cubic lattice with a 5 mm lattice constant. A single modal analysis was run to verify that there were no meshing issues or program violations. The modal analysis was completed as expected. Following this the various input folders were parameterised so that the optimisation process could be completed, as outlined in Section 4.3.

The only variable in the design that was parameterised was the radius controlling the thickness of the elastic lining (equivalent to ‘rad2’ in Figure 4.3). The length of the RUC in the model was held at 5 mm as this would allow 3 resonators to fit in series and another 3 resonators in parallel in the sample holder for the impedance tube. Reducing the length of the RUC would reduce the strength of the final printed
material.

A target attenuation of 3550 Hz was assigned as the performance target for this LRM design. This was exactly 1500 Hz higher than the lower limit of the impedance tube, therefore well within its capability for testing. The LS-Ingrid input file was parameterised and the corresponding script files were generated to allow the geometry optimisation to take place. The resulting output from the optimisation can be seen in Figure 4.10.

![Figure 4.10: Optimisation of an LRM design with a target 3550 Hz attenuation frequency performed in LS-Opt. This graph shows the resultant mode 4 vibration frequency as the radius of the elastic lining varies. SI units shown.](image)

The optimisation was configured to show any design which had a mode 4 vibration in the frequency range 3500-3600 Hz as ‘feasible’. Configurations with results outside this range are shown in red and deemed ‘infeasible’. Figure 4.10 shows there were several configuration that were classed as feasible. The design with a mode 4 frequency closest to 3550 Hz was chosen. This design had an elastic lining of 0.55 mm thickness (the final geometry chosen for the LRM design is shown in Table 4.4). The mode 4 frequency with this geometry is 3571 Hz.
4.6. LRM Design 1

Table 4.4.: Material Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Permabond UV625®</th>
<th>Litex T71S20®</th>
<th>Sn96.5Ag3 Cu0.5 Solder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nozzle Type</td>
<td>Continuous Direct Write</td>
<td>Inkjet</td>
<td>Heated Inkjet</td>
</tr>
<tr>
<td>Metamaterial Component</td>
<td>Matrix</td>
<td>Coating</td>
<td>Core</td>
</tr>
<tr>
<td>Required Dimensions</td>
<td>5.0 mm length</td>
<td>0.55 mm thickness</td>
<td>3.4 mm diameter</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>1190</td>
<td>1150</td>
<td>7365</td>
</tr>
<tr>
<td>Young’s Modulus (MPa)</td>
<td>1.3x10³</td>
<td>0.495</td>
<td>5.0x10⁴</td>
</tr>
<tr>
<td>Poisson Ratio</td>
<td>0.35</td>
<td>0.47</td>
<td>0.36</td>
</tr>
</tbody>
</table>

With the geometry of the LRM’s RUC determined, the design was input into the MATLAB program to determine the frequency band structure of the diagram, as outlined in Section 4.3.

![Figure 4.11.: Frequency band structure of model ‘LRM Design 1’. Grey region shows the modelled band gap between 3490 Hz to 4815 Hz. IBZ for this cubic RUC is shown in the inset.](image-url)

4.6.2. Manufacture

An object was designed which contained 3 LRM unit cells in series. This was the maximum number of LRM that could be printed that would still fit within the impedance tube for acoustic testing (see Figure 4.12(b)). Once the CAD file was
4.6. LRM Design 1

generated, the geometry was processed for printing as outlined in Section 3.5. This process was straightforward as the printer settings had already been determined during the testing phase of the printer and so setting the resolution for the geometry processing was simple to do.

The composite object was printed by first depositing the Permabond UV625® matrix material, up to half the height of the unit cell using the CDWP nozzle. This was then followed by depositing the Litex T71S20® rubber using the room temperature DDWP nozzle. The print settings used for these nozzles can be seen in Table 4.5. Once the rubber had cured, the molten Sn96.5Ag3 Cu0.5 solder droplet were deposited into the cavities and allowed to cool using the heated DDWP nozzle. During this cooling process, all the nozzles on the printer were briefly purged and wiped to ensure no blockages, and then the print continued.

The rubber was then printed on top of the core after it had cooled to create the rest of the coating part of the metamaterial. Fortunately, the gap between the printing surface and the DDWP nozzles was greater than the radius of the Solder core which now protruded above the printing surface.

For the CDWP nozzle which deposits the remaining matrix material, this protruding core did present an issue as the CDWP nozzle needed to be very close (ideally within 100 μm) to the printing surface. As this nozzle moves along it would eventually hit the solder core and bend the nozzle and hence damage the print.

To avert this issue, a special MATLAB based program was created that introduced commands to the printer to raise the CDWP nozzle by 5 mm at the point where it would contact the core. The nozzle would then move laterally across the core and lower itself back onto the printing surface to continue the print.

This process added extra time to the print. The time taken to make one of these composite metamaterials was just over 5 hours. The long print time was not only due to the time taken to deposit the various materials, but time had to be allocated to allow the various materials to cure, to perform purges and cleaning of the nozzle tips to prevent clogging, and to allow the CDWP nozzle to rise and descend as it moved along the core. The process of printing from point-to-point, as required by the point-cloud method, inherently increases the time taken to print due to the
increased number of accelerations and deceleration cycles. This is one of the limita-
tions of using this printing method. The final printed LRM composite objects can
be seen in Figure 4.12.

Table 4.5.: Printer settings for each nozzle.

<table>
<thead>
<tr>
<th>Material</th>
<th>Permabond UV625®</th>
<th>Litex T71S20®</th>
<th>Sn96.5Ag3 Cu0.5 Solder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nozzle Type</td>
<td>Continuous Direct Write</td>
<td>Inkjet</td>
<td>Heated Inkjet</td>
</tr>
<tr>
<td>Nozzle Speed (mm/s)</td>
<td>2.0</td>
<td>1.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Nozzle Settings:</td>
<td>5 PSI pressure</td>
<td>+/- 32V Sine Waveform</td>
<td>225°C temperature</td>
</tr>
<tr>
<td></td>
<td>250 μm nozzle diameter</td>
<td>1.0 kHz frequency</td>
<td></td>
</tr>
</tbody>
</table>

(a) Cross section of LRM Design 1 CAD file used for printing the composite object. (3 LRM unit cells).

(b) Two sets of printed LRM ready for acoustic testing. (c) The internal structure of the LRM object. See Figure 4.13 for dimensions.

Figure 4.12.: Original CAD image and corresponding printed metamaterials.
Figure 4.13.: Alternative view of the object Fig. 4.12(c) with measured dimensions shown. This object was printed showing the different material at each point in the object. The left inclusion only has the Permabond UV625\textsuperscript{®} matrix material. The centre inclusion shows the Litex T71S20\textsuperscript{®} rubber printed as well, and the right inclusion shows the solder core deposited. The Permabond UV625\textsuperscript{®} and Litex T71S20\textsuperscript{®} materials both cure with a clear appearance.

**Printing Permabond UV625\textsuperscript{®}**

Several interesting observations were made during the printing process. The Permabond UV625\textsuperscript{®} UV curable adhesive has a gel-like consistency and maintained its deposited shape well when printed on a glass substrate until it could be cured. With the 250 \(\mu\text{m}\) nozzle diameter, a back pressure of 5 PSI, and a nozzle speed of 2 \(\text{mm/s}\), the Permabond material was deposited with a height of 150 \(\mu\text{m}\) and a width of approximately 450 \(\mu\text{m}\). However, there was a noticeable bumpy appearance at the ends of the printed object. It is believed that this is due to excess material coming through the nozzle as it paused between depositing different lines of material. This issue could potentially be resolved by increasing the vacuum pressure applied at the end of each deposit, and reducing the time taken for the nozzle to move from one line to the next. A smaller nozzle tip could also be used to allow for smaller feature sizes. A 100 \(\mu\text{m}\) nozzle was tried which produced a line with a much thinner profile, but resulted in significantly longer printing times. This was too impractical when attempting to print larger objects.
4.6 LRM Design 1

**Printing Litex T71S20®**

The room temperature inkjet nozzle deposited the Litex T71S20® rubber. This was a material which, when mixed with a small amount of triethylene glycol monomethyl ether (TGME), had successfully been printed using an inkjet method [183].

The TGME is normally used to delay the drying time of the rubber to stop the material getting clogged in the nozzle, and hence to reduce the chance of agglomeration of the colloidal solution. However, it was noticed that the curing times (at room temperature) of the rubber with the added TGME did significantly increase when compared to deposited samples of rubber with no TGME added. As such, it was decided to print with the Litex T71S20® material with no TGME mixed in. This reduced the curing times between each deposited layer to a matter of a few minutes rather than 20-30 minutes. In instances where the nozzle was blocked, the printing process was paused, a small amount of toluene was applied to the tip of the nozzle, followed by short purging of the material through the nozzle, and the print was then continued. The toluene causes the rubber to expand and soften, allowing it to be forced out easier with air pressure. This presented an issue as it required supervision of the nozzle as the rubber was being deposited.

The time taken to print can be adjusted using the settings in the LabVIEW program, and so it is hoped that by speeding up the nozzle speed, and reducing the delays between depositing each line of the rubber, the need to unblock this nozzle will not be a problem.

**Printing SAC Solder**

The SAC solder was deposited using the Polymer Jet™ nozzle. The system’s nozzle and reservoir were heated to 225°C which melts the solder. This process typically took 20 minutes. The molten solder was passed through a 7μm stainless steel filter to remove any particulates or debris in the reservoir. The effect of gravity on the solder caused it to collate at the end of an inkjet's nozzle tip. When enough solder had collected, it would drop into the recessed cavity in the printed object, forming the solder core.
4.6. LRM Design 1

The hybrid approach of depositing the solder via the formation of large droplets did allow for the creation of the solid spherical core. But there is doubt this method could be used if the cavity to be filled was of a more complex geometry. For example, if the cavity was cuboid rather than spherical, it would have to be assessed if the molten solder would fill the entire space, or if voids would be formed near the corners of the cuboid.

When the boundary between the rubber and matrix parts of the printed LRM were examined under a microscope, they showed good adhesion between the two with no air gaps visible. This was consistent with prior printing trials of the two materials. The boundary between the rubber and the solder showed less adhesion. Again, no air gaps were visible under microscope but little physical effort was required to separate the core from the lining. This clearly limits the use of using solder and the rubber material in applications with different geometries given the different material components can be easily separated. For use in LRMs, it is suitable as the solder is encapsulated in the rubber.

Taking an overview of the process, the printer proved capable in fabricating the composite material designed, and confirmed the functionality of the control program and geometry processing methods. The build process took approximately 5 hours to print the block of 3 unit cell LRMs. This is a long time to print a relatively small object. As there are different cure methods for each of the materials, the print process had to be paused to allow the rubber and solder to dry and cool respectively. The use of the point-cloud method means there are additional acceleration and deceleration cycles during the print. The UV curing method was the quickest, taking a matter of seconds to harden the Permabond adhesive. Many commercial multi-material printers adopt a purely UV cure method for this exact reason, but the drawback is that it does limit what materials can be used.

The final printed composite object does resemble the original proposed design well. The small discrepancies in the geometry of the printed part were within design tolerances, giving credibility to the use of the printer for future use with different materials and geometries. More importantly, it does show that the direct write
techniques used are capable and effective for multi-material printing. This presents a promising option for the development of more capable multi-material printers in the future.

4.6.3. Testing

To test the printed material, an additional set of 3 LRM's in series was printed using the same process as previously mentioned. Once this was fabricated, the 2 printed composite objects were set in a circular mould with the same dimension as the test sample holder. Permabond UV625® was then added to create a disc composite material containing 6 LRM unit cells. An additional disc of the Permabond material was also fabricated, without resonators, and this would be used as a test control to directly observe the metamaterial effect achieved by the LRM's. The composite disc and the control disc were placed in the sample holders and acoustically tested in accordance to industry standards [186]. The results from this testing were processed to determine the sound transmission loss of the test samples (see Section 4.5). The acoustic test on the metamaterial disc was performed twice. The results of these tests can be seen in Figure 4.14. It should be noted that this figure has had 1/30 octave smoothing [189] applied to show the trend in the results. The MATLAB based function which performs this smoothing can be seen in the Appendix C.4. The un-smoothed results can be seen in Appendix C.4. Due to the presence of noise in the output signal in the sound transmission loss curve, it was difficult to distinguish between the separate results, hence the need for smoothing.

4.6.4. Discussion

Figure 4.14 shows a clear increase in the sound transmission loss between the approximate frequencies of 3500 Hz to 4080 Hz, which is indicative of metamaterial behaviour. There is a clear difference in the sound transmission loss between the Permabond control disc and the metamaterial disc in the region indicated by the modelling. When the LRM is tested a second time, a similar result is observed, giving confidence to the results. This is the first three-material component LRM to
Figure 4.14.: Acoustic testing on LRM Design 1 (circular cross-section). The observed band gap is between 3500 Hz to 4080 Hz approximately. The control disc is made from Permabond UV625®.

be entirely fabricated using additive manufacturing methods.

The location of the lower limit of this band gap is approximately at the predicted location, confirming the modal analysis used was accurate. From Section 4.6.1, the original LS-Opt optimisation identified a mode 4 frequency of 3571 Hz. The MAM analysis showing the band gap originating at 3490 Hz. The observed result of approximately 3520 Hz is within 1.44% and 0.8% error respectively which is a good agreement.

The modelled upper limit of the band gap was identified to be at 4815 Hz. The observed upper limit is approximately 4080 Hz. This is a much larger error of 18%. It was foreseen that the attenuation performance would decrease near the upper limit of the band gap, however this large difference was not expected. One of the causes of this large discrepancy could be due to the low number of resonators used. As only 6 LRMs were in the test sample, the metamaterial was not truly periodic. The RUC model used is based on an infinite array of LRMs, and hence this could be the cause of the discrepancy in the results.
The peak transmission loss occurs at approximately 3700 Hz, and not at the lower limit of the frequency band gap where maximum attenuation occurs. One cause of this could be the mode 4 vibration frequencies differing between the LRMs due to slight variations in the geometry of the LRMs. The peak attenuation was within 4% of the intended location. Comparing the difference in sound transmission between the control disc and the metamaterial at this point shows a difference of 6 dB. Again, this is can be considered a good result.

In conclusion, the metamaterial’s performance closely resembled the modelled behaviour. The width of the band gap was smaller than expected, but its location was accurate. It is thought that small variations in the geometry of the each resonator is the cause for a shift in the peak transmission loss, although this is within 4% of the intended location.

The most important conclusion to draw is that the LRM does work and is within reasonable accuracy of the modelled behaviour. This validates the use of the 3D printer to manufacture such LRMs, and marks the first time a three material component LRM has been entirely fabricated using additive manufacturing methods.

4.7. LRM Design 2

Various different designs of LRMs exist as shown in Section 2.3.2. One design, as proposed by Hirsekorn [37], sees a small variation to the classical LRM design as created in Section 4.6.1. This variation requires the core of an LRM to be an ellipsoid, whilst maintaining the outer edge of the elastic lining as circular. The principal of this variation is to alter the mode 5 or 6 vibration frequency to be different from mode 4, and hence different resonant modes that can result in wave attenuation. By tuning these vibrations modes, the band gap generated by each mode could overlap and give an overall wider band gap than could be achieved using a classical spherical LRM design. In Hirsekorn's paper, although the design was modelled, an experimental configuration was not tested.

It was decided that Hirsekorn's design would form the basis of a second LRM that
can be designed and printed. Due to the hybrid deposition technique of solder droplets, the core would remain spherical and the elastic lining would now become an ellipsoid. This design would still achieve the main requirement of having two different wave attenuating modes. The direction of the major axis would be 45° to the horizontal to mimic Hirsekorn’s design. The minor axis, of course, would be perpendicular to this (see Figure 4.15(a)).

4.7.1. Modelled Behaviour

A model was created in LS-Ingrid and LS-Prepost as outlined in Section 4.2.1. An initial modal analysis was performed to verify that it worked correctly. In this initial modal analysis, the core diameter remained 3.4 mm, and the length of the unit cell remained 5 mm. Once that had been completed, the input files were parameterised and two target frequencies were set, 3550 Hz and 3900 Hz. The 3900 Hz target was deemed sufficiently far away from the initial target that, overall, a broader band gap would be observed.

The optimisation therefore included two output responses and two variables that could be altered, namely the radius of the major axis and the radius of the minor axis in the ellipsoid. The output for one of the target frequencies is shown in Figure 4.15.

![Figure 4.15.: Optimisation for LRM Design 2](image)

(a) Parameterised variables for LRM Design 2. (b) Optimisation results for one target frequency. Any variables coloured green are deemed feasible for use.
From the optimisation, an LRM with an oval coating with a 2.4 mm major axis radius and a 2.21 mm minor axis radius, achieved a mode 4 vibration at 3512 Hz and a mode 5 resonance at 3894 Hz. These were deemed suitably close enough to the target frequencies and hence this LRM geometry would be used in the band structure modelling.

With the geometry of the LRM's RUC determined, the design was then input into the MATLAB program to determine the frequency band structure of the diagram, as outlined in Section 4.3.

Figure 4.16.: Frequency band structure of model 'LRM Design 2'. Grey region shows the modelled band gap between 3830 Hz to 5070 Hz. The IBZ for this cubic RUC is shown in the inset.

Figure 4.16 shows there are several features which show discrepancies between the initial modal analysis and the frequency band diagram. The first is the location of the lower limit of the band gap. The optimisation showed a mode 4 vibration at 3512 Hz and a mode 5 resonance at 3894 Hz. The band structure shows the lower bound of the band gap at 3830 Hz. The reason for this discrepancy is unknown. One hypothesis is that the resonant mode oscillations occur in a perpendicular manner and therefore do not contribute to each other's wave attenuation mechanism. Hence in the model, as mode 4 oscillations occur, there can be eigenmodes in a di-
rection parallel to the mode 5 oscillation, which would allow waves up to the mode 5 frequency to propagate. Looking at the upper boundary, the total band gap is approximately 1200 Hz wide, less than in LRM Design 1. As the wave attenuation from mode 4 ceases, it is expected that the attenuation from mode 5 would extend the band gap. However, the model may allow eigenmodes in the direction parallel to mode 4 to propagate, therefore stopping the band gap at this location.

By building and testing this LRM, a clearer picture of the metamaterial behaviour will become apparent.

4.7.2. Manufacture

An object was designed which contained 3 LRM unit cells in series with this oval type design. The materials used were the same as shown in Table 4.5 in Section 4.6.1. Once the CAD file was generated, the geometry was processed for printing as outlined in Section 3.5. Again, the composite was printed in a similar manner to the first printer LRM design, by first depositing the Permabond UV625® matrix material up to half the height of the unit cell using the CDWP nozzle. This was then followed by depositing the Litex T71S20® rubber using the room temperature DDWP nozzle. Once the rubber had cured, the molten Sn96.5Ag3Cu0.5 solder droplets were deposited into the cavities using the heated DDWP nozzle and allowed to cool. During this cooling process, all the nozzles on the printer were briefly purged and wiped to ensure no blockages. Once the process had reached this stage, the print tray was removed so that the printed object could be imaged, to allow the internal structure of the part to be viewed and measured (see Figures 4.17 and 4.18).

The build tray was then re-attached to the printer (located at 3 points to ensure the same location) and the print process continued. The rubber was then printed on top of the core, when cooled, to create the rest of the coating part of the metamaterial. Once the rubber had been deposited and cured, the CDWP was used to deposit the rest of the matrix material until the component was complete. Again, the print command file had to be altered to allow the CDWP nozzle to rise and descend to avoid hitting the printed object during its print. The time taken to make one of these composite metamaterials was approximately 5 hours. Once the full LRM was
made, it was repeated to allow the next 3 LRM object to be printed so that testing could be performed.

![Cross section of LRM Design 2 CAD file used for printing the composite object. (3 LRM unit cells).](image1)

![Partially printed LRM. Units shown were measured by a Vernier calliper. Due to poor image quality, the outline of the different components have been added.](image2)

Figure 4.17.: Original CAD image and corresponding printed metamaterials.

The images taken during the mid-stage in the first printing batch of this LRM design were not processed until the full design had been made. It was not noticed that the image in Figure 4.18(a) was out of focus. Red lines have been added to this image in Figure 4.18(a), to show the edge of the different regions in the composite object. Additionally, both the Permabond and the Litex material cure with a clear appearance, and hence the yellow Kapton tape on the print bed gives the image a yellowish appearance. The dimensions shown were measurements taken using
4.7. LRM Design 2

a Vernier calliper. Alternative images of the metamaterial can be seen in Figure 4.18.

![Alternative view of the partially printed LRM shown in Figure 4.18(a).](image1)

![Fully printed LRM Design 2 (three LRM in series).](image2)

Figure 4.18.: Alternative views of LRM Design 2.

4.7.3. Testing

Two sets of printed metamaterials were placed in a circular mould to allow a test sample to be created, as described in Section 4.6.1. The composite disc and the control disc were placed in the sample holders and acoustically tested in accordance to industry standards [186]. The results from this testing were processed to determine the sound transmission loss of the test samples (see Section 4.5). The acoustic test on the metamaterial disc was performed twice. The results of these tests can be seen in Figure 5.14. It should be noted that this figure has had 1/30 octave smoothing applied to show the trend in the results. The un-smoothed results can be seen in Appendix C.4. Due to the presence of noise in the output signal in the sound transmission loss curve, it was difficult to distinguish between the separate results, hence the need for smoothing.
4.7. LRM Design 2

Figure 4.19.: Acoustic testing on LRM Design 2 (oval cross-section). The observed band gap is between 3700 Hz to 4200 Hz approximately. The control disc is made from Permabond UV625®.

4.7.4. Discussion

Figure 5.14 again shows that the printed LRMs exhibit clear metamaterial behaviour. This gives further confidence in the use of the custom 3D printer for this application. The observed band gap starts at approximately 3700 Hz, and ceases just under 4200 Hz. This was an interesting result as it was above the LS-Opt determined mode 4 vibration of 3512 Hz (5.1% error) and below the band gap generated lower bound, located at 3830 Hz (3.5% error).

As observed with the LRM Design 1, the upper bound of the observed band gap was well below the modelled upper limit. This can be attributed to the low number of resonators in the sample. The total width of the observed gap was approximately 500 Hz. This was slightly lower than the band gap width for the first LRM design tested, which had a width of 580 Hz. Additionally, the maximum sound transmission loss of 4.8 dB occurred at approximately 3850 Hz. This again can be attributed to small variances in the geometry of the LRM. There was low variation between
the two tests performed on the metamaterial across the testing spectrum. The maximum difference between the two test results was 0.8 dB at 4090 Hz. Regarding this design, it was inconclusive if the oval LRM set up produces a wider band gap than the classical circular LRM. However, given that there is a reduction in the maximum sound transmission loss, for specific wave attenuation applications, the classical circular LRM set up may be favoured.

Regarding the manufacturing of the metamaterial, the quality of the final component was suitable to make the LRMs. The features, although difficult to capture via camera, were close to the original CAD design. This gives further confidence in the use of the custom made multi-material printer for this application.

4.8. LRM Design 3

In 2012, Yamada et al created an LRM design which utilised a ‘level-set’ topology optimisation algorithm to generate an LRM design they stated could produce a wide band gap [148, 190]. The optimisation was based on a model of four resonators in series. The resulting design had a spherical core and rectangular unit cell, but the elastic lining of the LRM was non-uniform and extended in 4 directions which are symmetrical in both the X and Y axis (See Figure 4.20). This design is referred to as the ‘double-oval’ set up for the remainder of this section. The design used the same materials for each component as Liu's classical design from 2000 [3]. The optimisation was set to increase the band gap of the resulting frequency band diagram, and not to increase the amount of wave attenuation achieved. Little further information was provided about the model. No final geometry was stated other than the image shown in the paper, and no frequency band structure was generated to determine if the modelled width was greater or not. Only one subsequent paper was published by the research group regarding the use of their optimisation for an LRM design with 3 material components [191]. In this recently published paper, the double-oval is not referred to, and instead an LRM with rubber struts is studied.
This presented a clear opportunity to build and test the design, to characterise its metamaterial behaviour. This is also justification for why the easy manufacture of LRMs is needed. This design, due its relative complexity, has not been fabricated before, and is unlikely to be manufactured. As such experimental verification could not be performed, hindering the adoption of the design and technology, despite its potential applicability.

This band gap optimised design is the basis for a third LRM design to be fabricated using the additive manufacturing device, and tested on the acoustic impedance tube.

### 4.8.1. Modelled Behaviour

Yamada's optimisation was based on a 2D model and resulted in a more complex configuration than a simple spherical or elliptical design. The resultant design resembled a 'double-oval' set up. In correspondence with the author, a 3D version of the model was never created. For this design to be printed and tested, a 3D model had to be interpreted. The resulting design obeyed the angles achieved in Yamada's optimisation and the length and width of the oval components. The optimisation process as used in the previous two LRM designs was not employed as the geometry was being based on a pre-existing model. As it was already known that a core with
a diameter of 3.4 mm could be made, the resulting lengths and widths of the elastic lining and matrix were adjusted to be in correct proportions with the core. The angle of the major axis of these ovals is 33° from the horizontal plane. The major axis length of each oval is 8.2 mm, and the minor axis length is 4 mm. This design was programmed into LS-Ingrid as two oval type structures around a spherical core. Due to the extended length of the elastic component of the LRM, the classical cubic matrix was changed to a rectangular matrix. The external dimensions of this RUC were 7.5 mm x 6 mm x 6 mm. The cross-section of this LRM design can be seen in Figure 4.20(b).

A modal analysis was performed and the resulting mode 5 vibration was 2734 Hz. The mode 4 vibration was perpendicular to the waves intended propagation, and hence would not result in wave attenuation. This attenuating mode was much lower than the previously made LRMs. At the resonant frequency, it is still within the frequency capability of the impedance tube and therefore is suitable to be manufactured and tested.

As the RUC for this design is not a simple cubic set up, a simple tetragonal lattice design IBZ was used to generate the frequency band structure (see Figure 4.21).

Figure 4.21.: Frequency band structure of model ‘LRM Design 3’. Grey region shows the modelled band gap between 2662 Hz to 3564 Hz. The IBZ for this cubic RUC is shown in the inset.
From Figure 4.21, the modelled band gap originates at approximately 2660 Hz and ceases at 3560 Hz. The width of this band gap is 900 Hz, which is smaller than the band gap measured for both previously modelled LRMs. This modelling seemed to suggest that the optimised design would not achieve its design purpose of widening the band gap when compared to a classical circular LRM set up. Despite this, the LRM would still be made and tested.

4.8.2. Manufacture

An object was designed which contained 2 LRM unit cells in series with this double-oval type design. The materials used are the same as shown in Table 4.5 in Section 4.6.1. Once the CAD file was generated, the geometry was processed for printing as outlined in Section 3.5. Again, the composite was printed in a similar manner to LRM design 1, by first depositing the Perabond UV625® matrix material, up to half the height of the unit cell using the CDWP nozzle. This was then followed by depositing the Litex T71S20® rubber using the room temperature DDWP nozzle. Once the rubber had cured, the molten Sn96.5Ag3 Cu0.5 solder droplet were deposited into the cavities and allowed to cool using the heated DDWP nozzle. During this cooling process, all the nozzles on the printer were briefly purged and wiped to ensure no blockages.

Once the process had reached this stage, the print tray was removed so that the printed object could be imaged, to allow the internal structure of the part to be viewed and measured (see Figures 4.22 and 4.23).

For this design, it was noticed that the solder droplets formed were slightly more oval shaped than in LRM Design 1 and 2. When measured, this diameter was 190 \( \mu m \) larger than the designed diameter. One explanation of this could be that the droplet became oval shaped due to the greater amount of rubber around the core. The cavity into which the droplets were deposited may not have been as deep or as spherical as intended. Therefore, this could have affected how the droplets solidified. Despite this, as the droplets made are of constant volume, and mass, there should not be a significant change in the frequency of the mode 5 vibration. As such, testing can still occur.
Once the print bed was re-attached, the print continued in the same manner for the previously described LRM designs. The time taken to make one of these composite metamaterials was approximately 7 hours. This extra time was largely due to the increased use of the room temperature DDWP nozzle and the extra nozzle purges and cleaning required. Once the full LRM was made, it was repeated to fabricate another 2 LRM composite objects, to allow testing to be performed.

Figure 4.22.: Original CAD image and corresponding printed metamaterials.
4.8. LRM Design 3

4.8.3. Testing

Two sets of printed metamaterials were placed in a circular mould to allow a test sample to be created (see Figure 4.23(b)). The composite disc and the control disc were placed in the sample holders and acoustically tested in accordance with industry standards [186]. The results were processed to determine the sound transmission loss of the test samples (see Section 4.5). The results of these tests can be seen in Figure 5.17. It should be noted that this figure has had 1/30 octave smoothing applied to show the trend in the results. The un-smoothed results can be seen in Appendix C.4. Due to the presence of noise in the output signal in the sound transmission loss curve, it was difficult to distinguish between the separate results, hence the need for smoothing.

Figure 4.23.: Alternative views of LRM Design 2.

(a) Alternative view of the partially printed LRM.  (b) Test sample containing 4 resonators of the double-oval configuration.
4.8. LRM Design 3

Figure 4.24.: Acoustic testing on LRM Design 3, (double-oval cross-section). The observed band gap is between 2670 Hz to 3040 Hz approximately. The control disc is made from Permabond UV625®.

4.8.4. Discussion

Figure 5.17 shows clear metamaterial behaviour, as did the corresponding results for LRM Designs 1 and 2. The band gap is observed in the frequency range 2670 Hz to 3040 Hz. The lower bound is 73 Hz (2.3% error) from the mode 4 vibration determined from LS-Dyna, and 10 Hz different from the MAM method modelling. This band gap has a 370 Hz width, which is smaller than LRM Designs 1 and 2. The resonant mode is approximately 800 Hz lower, and as the attenuation frequency is lowered, it can be expected that the maximum band width attenuated also lowers. Hence, this may be the reason for the reduced band gap width.

The maximum sound transmission loss of this tested sample was only 2.5 dB above the control sample at a frequency of 2770 Hz. It is difficult to draw clear conclusions from this analysis, regarding this double-oval design proposed by Yamada et al. The reasons for this are as follows:

1. The printed object is an interpretation of a 2D model.
2. Due to sample size constraints, only four resonators were inside the test sample, and hence a larger test sample with more resonators may well have shown improved results.

3. The printing process replicated the intended geometry fairly well, however the slight oval shape of the resonator may have altered the performance of the test sample.

Regardless of this individual result, this does confirm the intended purpose of the printer, to be able to manufacture these rarer LRM designs to allow their performance to be determined in experimental conditions.

4.9. Chapter Conclusions

In this Chapter, the creation of the custom built multi-material 3D printer has allowed the successful fabrication of three different LRM designs. This represents the first time an LRM has been fabricated entirely using additive manufacturing techniques.

Each design was modelled using a modal analysis based in LS-Dyna. This incorporated an optimisation procedure using LS-Opt, which obtained the required geometry to achieve a desired mode 4 vibration frequency (corresponds to the start of the frequency band gap).

Once the geometry was finalised, the design was modelled using the MAM method, based in MATLAB. This design process proved straight forward in generating the frequency band structure for a particular LRM configuration.

Each of the different LRM designs were fabricated on the multi-material 3D printer. Good accuracy was obtained between the printed components and their original CAD models. The deposition of Permabond UV625® using the CDWP nozzle did leave some striation effects in the final appearance of the matrix, but this was not significant enough to greatly alter the final geometry of the component. The material itself deposited consistently and maintained its shape well before it could be cured with UV light.
The Litex T71S20® material also deposited consistently using the room temperature DDWP nozzle. The formation of 80 μm diameter droplets using this nozzle allowed the rubber printed components to have a good final resolution. The cure method for this material was the evaporation of its solvent. Consequently, if the material was unused for an extended period of time, there was the likelihood the rubber at the nozzle tip would harden, causing a blockage. This was mitigated by using regular purges of the material during a print, and a combination of toluene and IPA alcohol to remove rubber from blocked nozzles.

The core material for each LRM was made from Sn96.5Ag3Cu0.5 solder, which was deposited using the Polymer Jet™ DDWP nozzle on the printer. A hybrid approach of depositing solder worked well to form spherical cores for the LRMs. The cores made for LRM Design 3 did exhibit a slight oval shape. It is thought this may be due to the cavities into which these droplets were deposited were not entirely spherical. Despite this, the droplets made had a consistent volume and did allow the creation of these LRMs. It is not expected that this hybrid deposition method can be used to make cores of irregular geometries with the printer’s current configuration. An alternative core material would have to be found that can be deposited using the normal DDWP method and cure as a solid object. This could allow a more complex core shape to be fabricated, facilitating more varied LRM designs to be manufactured.

These LRM designs were then tested in a custom built acoustic impedance tube, which allowed the frequency dependent sound transmission loss to be determined for each LRM. The three different designs that were fabricated, along with their modelled and experimental results, are shown in Table 4.6.

The location of the lower limit of the modelled band gap, when compared to the experimental location, was generally accurate across all three designs. The location of the upper limit varied greatly, which is due to the low number of resonators within the tests samples. This also affected the maximum sound transmission loss recorded for all three designs.
Table 4.6.: Summary of modelling and experimental data for LRM Designs 1, 2 and 3.

<table>
<thead>
<tr>
<th>Metamaterial</th>
<th>Design 1</th>
<th>Design 2</th>
<th>Design 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross Section</td>
<td>Circular</td>
<td>Oval</td>
<td>Double-Oval</td>
</tr>
<tr>
<td>Modelled Band Gap (Hz)</td>
<td>3490 - 4815</td>
<td>3830 - 5070</td>
<td>2662 - 3564</td>
</tr>
<tr>
<td>Experimental Band Gap (Hz)</td>
<td>3500 - 4080</td>
<td>3700 - 4200</td>
<td>2670 - 3040</td>
</tr>
</tbody>
</table>

Only LRM designs 1 and 2 can be compared directly, as they both had 6 resonators within the test sample. It was observed that LRM Design 1, with the classical circular cross-section, had a larger band gap and a higher maximum sound transmission loss compared to LRM Design 2.

The main conclusion from this chapter is that the custom 3D printer achieved its design purpose. It has allowed the manufacture and subsequent experimentation of different LRM designs. This is the first time a 3 material component LRM has been entirely manufactured using additive manufacturing techniques.
Chapter 5

Energy Harvesting LRM.s

5.1. Introduction

This chapter introduces two energy harvesting concepts that are based on an LRM type configuration. This involves a matrix material, an elastic lining, and a heavy core. In both designs the cores are magnetic, and the energy harvesting is based upon electromagnetic induction. This chapter will outline the modelling approach used for these concepts, and show the results of experiments performed to test these designs. These designs are intended as a ‘proof of principle’ to understand their limitations and manufacturability. They should not be considered as final designs that will be ready for use in applications.

5.2. Energy Harvesting Approach

Energy harvesting (introduced in Section 2.4.1) is the principle of converting elastic or acoustic vibrations from the environment into a useful form of energy, like electricity. This can be achieved by incorporating electrostatic, piezoelectric, or electromagnetic principles into the design of an object [94, 95]. Each method has their advantages and disadvantages (see Section 2.4.1). Electrostatic designs require an initial electrical current to charge a capacitor, but can achieve a high voltage output. Piezoelectric (PZT) devices are widely utilised, but
5.2. Energy Harvesting Approach

are associated with low current output. Electromagnetic systems are associated with low voltage output, but can be wireless and hence viable for applications where a wired connection is not suitable.

There is growing interest in the use of metamaterials for energy harvesting because of the unique properties in wave manipulation that they exhibit. Most noise and vibration from cars or industrial applications lie within the low frequency regime 100-10000 Hz [39, 95]. PC type harvesters require geometries of comparable size to the wavelengths of the frequencies they manipulate. Hence for low audible frequencies, the size of a PC energy harvester needed could be impractically large.

It has been shown that metamaterial based harvesters work optimally when the frequency of the harvested waves are close to a resonant frequency [94, 99, 100]. In the case of an LRM type metamaterial, this resonant frequency equates to the maximum oscillation of the core.

As an electrostatic energy harvester requires an initial power supply, the concept of an LRM harvester focused on either PZT or electromagnetic methods. The first PZT type LRM was created by Ahmed et al [11, 12] in 2015. In this set up, a typical LRM with a heavy core, an elastic component, and a matrix material also had a disk of PZT material inserted into the rubber lining (see Figure 5.1).

![Figure 5.1.: The LRM type harvester concept proposed by Ahmed et al [11, 12], which uses a PZT patch to generate a voltage.](image)

With the mode 4 excitation (displacement of the heavy core) the resulting deformation of the PZT patch generated a voltage that was measured by an oscilloscope.
and eventually stored in a capacitor. When tested, the LRM design was able to achieve a 1.3 mW output per resonator at a resonant frequency of 420 Hz. This design has been further investigated recently in 2017 by Hu et al, who have attempted to optimise the design of a PZT based LRM energy harvester based on analytical models [105].

As a PZT based LRM had already been developed, it was decided that an LRM based upon an electromagnetic energy harvesting concept would be investigated. This project has focused on the manufacturing of LRMs using additive manufacturing methods. For the creation of energy harvesting LRMs, the utilisation of 3D printing methods would again be a design goal.

Initially, a simple variation to the classical LRM design was conceived that could potentially be used to make an electromagnetic energy harvester. By replacing the core of an LRM with a magnetic material, a current could be induced in adjacent wires, as the core oscillates at its resonant frequency. This charge could then be stored in a capacitor for later use. A diagram explaining this concept can be seen in Figure 5.2.

The mechanism of how a moving magnet can induce a voltage in a wire is based upon Faraday’s law of electromagnetic induction [193]. The simple form of Faraday’s law is:

$$\xi = -n \frac{d\Phi_B}{dt}$$  \hspace{1cm} (5.1)

where $\xi$ is the electromotive force, $\Phi_B$ is the magnetic flux, and $n$ is the number of loops of wire (according to Lenz's law [193]). Faraday's law states that as a conducting wire loop is moved or deformed in a magnetic field, an electromotive force is produced to oppose the movement of the loop. If the loop is cut and a voltmeter attached across the ends of this open circuit, a voltage would be recorded as the wire continues to move in relation to the magnetic field. The voltage on that wire as it moves through the magnetic field can be related to its velocity in the
5.2. Energy Harvesting Approach

(a) Electromagnetic energy harvesting LRM concept. An oscillating core could induce a current in an adjacent wire.

(b) Fleming’s Right Hand Rule [192], shows the direction of current flow in a wire moving though an magnetic field.

Figure 5.2.: Energy harvesting LRM concept.

following equation [193]:

\[ V = \bar{v} \bar{B} L \sin(\theta) \]  

(5.2)

where \( V \) is the voltage across the wire, \( \bar{v} \) is the velocity of the wire in relation to magnetic field, \( \bar{B} \) is the strength of the magnetic field vector, and \( \theta \) is the angle of the wire in relation to the magnetic field. If the magnetic field and wire are perpendicular, then \( \sin(\theta) = 1 \). Decreasing this angle reduces the level of voltage induced in the wire.

For an LRM with a magnetic core, if the magnetic field could be arranged such that it is perpendicular to the vector of its displacement, then a wire around the LRM should experience a voltage as it moves. This underpins the concept of an
5.3. Initial Design Concept

The initial design of the energy harvesting concept was based on a spherical core, coated in an elastic lining, and held in place by a matrix material, similar to LRM Design 1. The materials chosen for the elastic lining and the matrix material would be the same as used in the previously described passive LRMs (see Section 4.6.2). The solder alloy that was used as the core material in those designs was not magnetic and hence could not be used for this energy harvesting concept. An alternative magnetic based material that could be deposited using a DDWP nozzle had to be sourced.

In 2014, research by Song et al, [194] had successfully utilised an inkjet nozzle to deposit ferromagnetic fluid (aka ferrofluid). This is a colloidal solution with particles of ferromagnetic iron in an ink carrier fluid. The particles are typically coated with a surfactant to mitigate the magnetic particles agglomerating together. The magnetic particles are so small (typically nanoparticle size) that their individual magnetic attraction is too weak to overcome the repelling Van der Waal's forces of the surfactant. Hence the ink remains a colloidal solution.

Song et al were able to align the particles during deposition by using an external magnetic field as the droplets were deposited. Once the ink had dried, the particles maintained their deposited alignment. In doing so, they effectively printed small bar magnets in different shapes, including a square (5 × 5 mm) and a circle (radius of 3.5 mm).

By depositing ferrofluid in the manner demonstrated by Song et al, it was believed that a 3D magnetic sphere could be built using the custom 3D printer (see Chapter
3). Droplets of ink with ferromagnetic iron could be repeatedly deposited until the final core had a high percentage by weight of iron in the core. A ferrofluid was sourced (from MagnaMetals UK) that contained a high solid content (approximate 36% by volume) and a low quoted viscosity of 6 mPas. The quoted average ferromagnetic particle size was 10 nm. These material properties were all within the requirements for DDWP printing, and hence a printing trial was conducted.

A 5 ml sample of the material was placed in the reservoir of the room temperature DDWP nozzle. No print settings were stated in Song's paper [194], therefore several printing trials were performed.

A systematic approach was adopted by setting DDWP nozzle variables, such as the wave frequency, amplitude and pressure, at different intervals, then viewing the resultant droplets on camera. The images were checked for droplet consistency and satellite droplets. Initially the nozzle frequency was set at 750 Hz, and the amplitude was set at 80 V. At these settings, there was a large number of satellite droplets that formed. The wave amplitude for the DDWP nozzle was lowered until no more satellite droplets were observed. The pressure in the reservoir was then increased until there was consistent droplets formation, which was steady with time. In all trials, no blockages were recorded. The final settings used are shown in Table 5.1. An image of the typical droplet formed using these settings is shown in Figure 5.3.

Table 5.1.: Deposition settings for ferrofluid with a DDWP nozzle.

<table>
<thead>
<tr>
<th>Material</th>
<th>Temperature (°C)</th>
<th>Waveform</th>
<th>Frequency (Hz)</th>
<th>Amplitude (V)</th>
<th>Pressure (Psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ferrofluid</td>
<td>Room Temp</td>
<td>Sinusoidal</td>
<td>750</td>
<td>65</td>
<td>14</td>
</tr>
</tbody>
</table>

These deposition trials succeeded in determining the correct print settings needed for this material. The next stage was to characterise the material so that a design could be formulated. Prior to determining the material properties, the strength of the magnetic field in the ferrofluid was evaluated.

For a bar magnet, the magnetic field strength can be measured using a magnetic
5.3. Initial Design Concept

Figure 5.3.: Consistent deposition of ferrofluid with a DDWP nozzle.

field sensor and the equation [195]:

\[ B_{\text{axis}} = \frac{\mu_0 2\mu}{4\pi d^3} \]  \hspace{1cm} (5.3)

where \( B_{\text{axis}} \), is the measured strength of the magnetic field along the axis of a bar magnet, \( \mu_0 \) is the permeability constant \((4\pi \times 10^{-7})\), \( d \) is the distance from the centre of the dipole of the magnet to the sensor, and \( \mu \) is the magnetic moment. It is that is a property of the magnet itself and directly relates to the magnetic field strength [195].

Unfortunately, a magnetic field sensor was not available during this evaluation process. Prior to purchasing a sensor, a simple magnetic strength evaluation experiment was designed, which could be used to help determine the strength of the ferrofluid material.

A coil with approximately 200 loops of copper wire was connected to an oscilloscope. Small amounts of the ferrofluid were then dropped through the centre of the coil to see if there was an output registered on the oscilloscope. Referring to Eq. 5.4, if the output voltage, the number of loops in the coil, and the time taken for the drop to travel through the coil is known, the magnetic flux \((\Phi_B)\) can be approximated. The magnetic flux is a function of the area of the coil and the strength of the magnetic field.

Small amounts of the ferrofluid were dropped through the centre of the coil (2 cm inner diameter) to see if there was an output registered on the oscilloscope. The droplets were made by a pipette and were approximately 4 mm in diameter. It should
be noted that this experiment was not intended to determine the final magnetic field strength of the ferrofluid. It was more a way to approximate the magnitude of the strength to give an early indication of the types of voltages that could be expected in an energy harvesting LRM.

After several attempts, no output was observed. The oscilloscope was initially set to trigger once a signal greater than 20 mV was recorded. The noise level on the oscilloscope itself was approximately 10 mV and hence 20 mV was considered a low value. When no output was recorded, the trigger level was lowered to 15 mV, and then 12 mV. Again, at each of these intervals, no output signal was recorded. The trigger level was lowered to 11 mV but it was observed that the oscilloscope would trigger itself due to noise in the display signal.

To see if the system was connected correctly, a neodymium magnet, supplied with the ferrofluid, was dropped through the coil. A clear signal was observed each time the neodymium magnet was trialled of approximately 40-60 mV. The profile of this signal was a classical sinusoidal peak and trough, equating to the magnet moving closer to the coil, and then leaving the coil.

Neodymium based magnets are capable of producing much stronger magnetic fields than ferromagnetic based magnets with comparable magnet sizes. The stronger the magnetic field, the more energy could be harvested. It was therefore decided that for this LRM type energy harvester, the core should be made from neodymium.

With the revised approach, an issue arose regarding the potential fabrication process of an LRM with a neodymium core. No publications have shown neodymium being deposited using an additive manufacturing technology. Indeed neodymium based liquid inks do not exist as the attraction between particles is too strong and overcome the Van der Waal’s forces of the surfactants. This results in a non-colloidal solution with the magnetic particles clumped together.

As the custom made printer had already shown its capability in making passive LRM with solder alloy cores, the decision was made to focus on creating an electro-
magnetic based energy harvesting LRM, rather than an alternative energy harvesting design that could be printed.

5.4. The Chosen Energy Harvesting LRM Design.

As the original concept of a 3D printable ferromagnetic based energy harvesting LRM was no longer viable, a design based upon a neodymium core was created. The following concept was devised which would use a cylindrical neodymium bar magnet as the core. There are three directions in which this core could oscillate. These will be referred to X, Y, Z; where X is the horizontal displacement, Y is the vertical displacement, and the Z direction corresponds to the axial direction of the core (see Figure 5.4).

Figure 5.4.: Concept of an electromagnetic based energy harvesting LRM. The red lines represent conducting wires.

With this configuration shown in Figure 5.4, the matrix could readily attach to a vibrating structure. In-plane waves would be attenuated near the resonant mode equating to the displacement of the core in the X and Y directions. For out of plane waves, where the core would move in the Z direction, the manner in which the matrix could attach to a structure is more complicated, as space is required either end of the core to allow it to displace. This would limit how many layers of resonators could be utilised in an application as space would be required in between each layer to allow for the core displacement.

For these reasons, only in-plane vibrations were investigated initially. As there is
symmetry about the X and Y axis, only vibration in the Y direction was considered, as the results should be the same for vibrations occurring in the X direction.

Neodymium bar magnets are magnetised using a powerful electromagnetic coil, and can come in a range of strengths, rated from ‘N35’ increasing to ‘N52’. These stronger magnets typically imply that the size of the magnet is also larger. A trade-off was sought between the magnet size and strength. Various cylindrical magnets were considered for use, but the final chosen magnet had the highest magnetic strength to volume ratio with a magnet field strength of N42, and an 8 mm diameter by 12 mm length.

Having a cylindrical core allowed the matrix and elastic lining part of the LRM to still be manufactured using a 3D printer, and the magnetic core could be placed manually. With a core this large, and the need to only print an elastic and matrix material, it was decided to use a Connex multi-material printer, which can deposit a hard plastic material (VeroWhite®) and also a rubber material (TangoBlack+®) simultaneously [21]. The Connex printer, with its multi-nozzle DDWP set up, was capable of producing a high quality component in a significantly smaller amount of time than could be achieved by the custom made printer (see Section 3.8).

The two printer materials are commercially available, and their material properties have been tested and published [182]. The material properties for the final material components in the energy harvesting LRM can be seen in Table 5.2.

<table>
<thead>
<tr>
<th>Metamaterial Component</th>
<th>Matrix</th>
<th>Coating</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>16 mm RUC length</td>
<td>1.8 mm thick</td>
<td>8 mm diameter</td>
</tr>
<tr>
<td>Material Density (kg/m³)</td>
<td>VeroWhite® [182]</td>
<td>TangoBlack+® [182]</td>
<td>Neodymium</td>
</tr>
<tr>
<td></td>
<td>1190</td>
<td>1120</td>
<td>7010</td>
</tr>
<tr>
<td>Young's Modulus (MPa)</td>
<td>7.6x10²</td>
<td>0.19</td>
<td>4.1x10⁵</td>
</tr>
<tr>
<td>Poisson Ratio</td>
<td>0.35</td>
<td>0.47</td>
<td>0.28</td>
</tr>
</tbody>
</table>

With the materials for the energy harvesting LRM chosen, a model was created in LS-Ingrid to determine which resonant mode equated to the Y directional displace-
ment of the core (see Section 4.2.1). Initially, arbitrary dimensions for the unit cell length and the radius of the elastic lining were chosen (a circular elastic cross-section was used, similar to LRM Design 1). The dimensions of the core were fixed. An initial modal analysis was run to determine if the model was set up correctly. The results showed that mode 5 equates to the Y directional displacement of the core. For this configuration, the mode 5 frequency was 695 Hz. As no target frequency had been set, no further optimisation procedure (as demonstrated in Section 4.2.1) was started. It was decided to maintain the geometry from this initial analysis as this design was meant to be an indication of the viability of an electromagnetic energy harvesting LRM. The relevant geometry for each LRM component can be seen in Table 5.2.

5.5. Harvester Design 1

With the dimensions of the energy harvesting LRM's set, a design containing 5 resonators in series was created (see Figure 5.5).

![Figure 5.5.](image)

Figure 5.5.: The first energy harvester concept. The red line and arrows show the direction and location of the wire coil. The ‘N’ shows the polarity of the neodymium magnets. The total size of the metamaterial is 80x16x12 mm$^3$.

A 3D linear elastic FEM (time domain) model was created in LS-Dyna to show the displacement of the core with time. A 695 Hz harmonic Y directional displacement was applied to the bottom surface of the model to simulate how the metamaterial would be tested. No other boundary conditions were applied. With this model, the velocity of the cores could be determined. The fringe plot from the results of the modelling is shown in Figure 5.6(a).
5.5. Harvester Design 1

(a) Fringe plot showing the resultant velocity of the energy harvesting design at resonance. Velocity shown is in m/s.

(b) Y velocity of the first core with time.

Figure 5.6.: Time domain linear elastic model.

This resultant core velocity could then be applied to Eq. 5.2 to determine the voltage generated from this set up. This model was uncoupled between the linear elastic response of the input displacement, and the induced electromagnetic force experienced by the generation of a voltage in an adjacent wire. The model was made in LS-Dyna, which has an electromagnetic solver which is used typically for solving eddy currents in metallic plates for shape forming applications. Attempts to couple the linear elastic response of the model with resultant electromagnetic field resulted in several ‘error terminations’ (i.e. the model ceases). Due to time restrictions, it was decided to maintain using the linear elastic model, which would be suitable to give an approximation of the core velocity, and hence a magnitude of the induced voltage that could be expected.

In addition to the velocity, for the induced voltage to be determined, the length of wire exposed to the magnetic field had to be determined, as well as the strength of the magnetic field. Per resonator, the length of wire exposed was 16 mm. The number of loops of wire must also be taken into account. An assumption of 50 loops
5.5. Harvester Design 1

was made.

Eq.5.2, which relates the velocity, magnetic field strength and induced voltage is based on the assumptions of a uniform magnetic field. For this size of magnet, the magnetic field is not uniform along the length (Z direction) and hence the equation was adapted to take account of this:

$$V = \int_{z=0}^{z=12} \vec{v}L(z)\vec{B}(z)\sin(\theta(z)).dz \quad (5.4)$$

The strength of the magnetic field was supplied by measurements from the magnet supplier (the image of the resultant fridge plot can be seen in Appendix D). At the distance from the magnet, where the wires would be located, the magnetic field varied from 0.0675 Tesla to 0.0807 Tesla. The angles of the magnetic field with respect to the displacement vector during resonance were also determined (see Appendix D).

Having derived this information for this design, the data was input into Eq.5.4, and the resultant max voltage output per LRM during resonance was 0.0431 V. For the total 5 resonator design, the expected output was approximately 0.215 V. The power that could be generated with this maximum voltage would have to be determined once the resistance of the metamaterial was measured. This was a lower modelled output than expected, but was typical of electromagnetic based energy harvesting. To give context, on the oscilloscope, the noise when the device was unconnected was approximately 10 mV. Electromagnetic energy harvesters that can achieve high power outputs are able to increase the maximum velocity of the magnet in relation to the wire, increase the magnetic field strength of the magnet, and/or increase the number of loops used in an energy harvesting design. Despite this expected low output, the design was still manufactured so that an experimental test could be performed.

The design was created in the CAD software package CREO. The model was then exported as an STL file to the Connex printer’s control program. Once the correct materials had been assigned, the print was started and took approximately 4 hours (other items were being made simultaneously). The resultant object made by
the Connex printer had a very good resolution and consistency, and resembled the original CAD geometry well. An additional matrix and elastic component object was fabricated as a back-up, and both printed objects resembled each other well. With regards to the manufacturability of these LRMs, with the Connex printer, no significant issues were apparent. Once the part had been fabricated and cleaned, the cores were placed in the model. The cores were positioned with minimal effort, and once in place, did not dislodge when the metamaterial was manually shaken.

![Energy harvesting LRM design 1](image)

Figure 5.7.: Energy harvesting LRM design 1. The plastic bracket is used as a specimen holder during testing.

Insulated copper wire was then looped around the edge of the metamaterial as shown in Figure 5.7. A total of 53 loops of wire were placed on the object. To ensure that the wire was complete and no breakages had occurred during the winding process, the two ends of the wire were connected to a multimeter and the resultant voltage was zero, confirming the wire was intact. To be able to determine the final power, the resistance in the metamaterial was also measured ($Power(W) = \frac{V^2}{R}$), and the value was 93.1 $\Omega$.

Once the winding was complete, the end of the wires were soldered to thicker wires to allow them to be connected to an oscilloscope for testing. The resulting energy harvesting design concept can be seen in Figure 5.7. With the modelling and manufacturing of the concept complete, the testing could be performed.
5.5.1. Testing Apparatus

A simple experimental set up was devised which tested the energy harvesting capability of this design. The set up required the use of a vibration generator, connected to a signal generator, and a separate power supply. Attached to the oscillating node of the vibration generator was the metamaterial itself. A special holder was created to hold the metamaterial in place while it oscillated vertically during testing. The wires from this metamaterial would then be attached to the input of an oscilloscope to allow the metamaterial response to be observed. The experimental apparatus can be seen in Figure 5.8.

The signal generator was used to define the frequency of the displacement. The amplitude of the displacement was controlled by the power output from the external power supply. As the metamaterial oscillates due to the displacement created by the vibration generator, the induced voltage in the wires of the metamaterial are measured by the oscilloscope. These measurements can then be exported via USB to be processed on a computer. The input vibration frequency was set to sweep over a range of values, to allow the location of the peak voltage production to be located, which should correspond to the resonant frequency of the metamaterial.

Figure 5.8.: The experimental set up for energy harvesting testing. The vibration generator creates an approximate 500 µm sinusoidal displacement in the vertical direction (Y-axis).
5.5.2. Testing

The energy harvesting design was placed in the testing set up as shown in Figure 5.8. The oscilloscope was connected and tested to ensure it was reading the voltage as intended. An initial test was performed where the metamaterial was held in place, but the vibration generator was disconnected from it. The vibration generator was then turned on, sweeping through the testing frequencies. This was to verify whether the generator itself would influence the results observed on the oscilloscope. At full power (5 W), the oscilloscope did show a small variance in the results. An observable wave of approximately 4 mV was seen in the output from the oscilloscope. To counter this, the output power on the power supply was reduced until there were no more waves observed across the testing frequencies. With the metamaterial removed, this equated to an approximate 500 \( \mu \text{m} \) sinusoidal displacement in the Y direction. An alternative way to mitigate this risk is to increase the distance between the vibration generator and the test sample, as the electromagnetic field strength reduces with distance. This can be achieved using a longer connection rod. However, this can introduce unwanted vibration modes, affecting the results.

Once the output power had been set, the test then proceeded with a frequency sweep from 500 Hz to 900 Hz. The sweep would move in steps of 0.5 Hz every second. The maximum voltage measured during each iteration was recorded and the output is shown in Figure 5.9.

The results showed that for the range 500-900 Hz, no observable voltage increase can be seen. The output signal is in the range expected by noise created by the oscilloscope device. The tests was repeated twice but the same result was observed (hence only the first test is shown).

5.5.3. Discussion

Despite the clear vibration of the metamaterial observed during testing, the output did not show any energy generation across the range of frequencies being tested. This harvester design did not work as intended. In Figure 5.9, the testing signal was the
Figure 5.9.: Maximum recorded voltage for Harvester Design 1.

same as the noise signal generated by the oscilloscope under testing conditions. The observed signal is the maximum recorded voltage at each frequency increment (0.5 Hz). The average noise in the signal is 0.01 V. The test was repeated several times but in each case a similar result was obtained. Before and after each of these tests, the connections between the different devices were checked and the metamaterial's connections to the oscilloscope were also checked. Additionally, the wire loop around the metamaterial was checked with a multimeter to check the wire was still intact, which it was.

The modelled maximum voltage was 0.215 V. With the measured resistance, and the modelled maximum voltage, the expected power output was 0.49 mW. However, as no observable voltage was measured, it is not possible to determine the power output of the device.

It became apparent that one potential cause of this lack of energy harvesting may be due to the orientation of the magnets in the metamaterial. For bar magnets, there is always a north and a south pole. A north pole is attracted to a south pole and vice versa. Conversely, two north poles strongly repel each other, as is the same for two south poles. As all the magnets in series had the same polarity orientation,
adjacent magnets repelled each other, and hence restricted the ability of the magnets to oscillate in the LRM. When stationary, this effect was not visible. However as the cores displaced during testing, the effect of this repelling force was more apparent. This repelling force becomes stronger as the magnets move closer to each other, and hence the cores of the LRM s are forced into a similar configuration as shown in Figure 5.10. Due to these additional forces, the cores could not oscillate in unison, therefore severely limiting any energy harvesting capability for this design.

Figure 5.10.: Magnets with the same polarity may limit the ability to oscillate in the LRM (the south poles would be at the rear of the image shown). The red arrows show the repelling force acting upon each pair of magnets.

5.6. Harvester Design 2

Due to the lack of energy harvesting observed in the first harvester design, a variation was made whereby alternate cores had their magnetic polarity reversed. In doing so, the cores would now be attracted to each other (see Figure 5.11). In this set up, the magnetic attraction between the cores acted to maintain them in a straight line arrangement. This allowed them to oscillate in unison, the same as the mode 5 resonant frequency required for optimal energy harvesting.

With this new configuration, the wire looping used in the previous design was not usable as the magnets oscillating in unison would induce currents of opposing direction in wires on the same side of the metamaterial (see Figure 5.2(b)). As such, the wire looping had to be rearranged to take account of the different core
5.6. Harvester Design 2

Figure 5.11.: Magnets with alternating polarity are attracted to each other and hence aid their oscillation in unison. The blue arrows show the attractive forces acting upon each magnet.

Figure 5.12.: The second harvester concept. The red and blue line shows how the wire is arranged in the metamaterial.

orientations. A ‘weaving’ type arrangement was devised that would alternate sides of the object as the wire was wound around the LRM (see Figure 5.12).

Gaps in the interstitial space between adjacent unit cells in the metamaterial had to be created to allow the wires to pass through. Hence, a new matrix design was created in CREO to include these gaps.

As with the first harvester design, the matrix and elastic lining components were made on the Connex 3D printer, with the cores inserted afterwards. The quality of the print was again good and the cores fit well into the partially 3D printed metamaterial.

With the harvester fabricated, copper wire was weaved through the metamaterial. A total of 26 loops was possible before occupying the available volume in the interstitial space. Initially, it was planned to have sufficient space for 40 loops, but over-lapping wires required more space than anticipated. To ensure that the wire was complete and no breakages had occurred during the weaving process, the two ends of the wire were connected to a multimeter and the resultant voltage was zero, confirming the wire was intact. The resistance of this wire was measured and the value was
5.6. Harvester Design 2

65.2 Ω. The ends of the copper wire were soldered to thicker wire to allow them to be connected to an oscilloscope for testing. The resulting energy harvesting design concept can be seen in Figure 5.13.

![Energy harvesting LRM design 2. The plastic bracket is used as a specimen holder during testing.](image)

Using Eq. 5.2, and the revised number of wire loops in the metamaterial, the maximum voltage output per resonator was 0.0313 V. For the total 5 resonator design, the expected output was approximately 0.157 V. The low number of loops was a key reason for the lower modelled energy output when compared to the first harvester design. With the modelled maximum voltage, and the measured resistance of the metamaterial, the expected power output was $3.8 \times 10^{-4}$ W. Despite this expected low power output, the design was tested experimentally to determine how much energy could be harvested.

5.6.1. Testing

The energy harvesting metamaterial was placed in the testing set up as shown in Figure 5.8. The oscilloscope was connected and tested to ensure it was reading the voltage as intended. The same power output used in testing the first energy
5.6. Harvester Design 2

A harvesting design was used here.

The test then proceeded with a frequency sweep from 500 Hz to 900 Hz. The sweep increased in steps of 0.5 Hz every second. The maximum voltage measured during each iteration was recorded and the output is shown in Figure 5.14. The test was repeated to confirm the result obtained.

![Graph showing maximum recorded voltage for Harvester Design 2 (Tests 1 and 2).](image)

Figure 5.14.: Maximum recorded voltage for Harvester Design 2 (Tests 1 and 2).

5.6.2. Discussion

This second harvester design did show a slight increase in voltage output when compared to the first harvester design. Referring to the tests results shown in Figure 5.14, this increase was observed at 750 Hz, which was above the 695 Hz predicted mode 5 vibration frequency (7.9% relative error). However, the main concern regarding this result was the low maximum recorded voltage in the region where energy harvesting is optimal. The average maximum voltage of 0.018 V, is just above the noise level of the oscilloscope. At this point, the power output of the model was 4.9x10^{-6} W. This was a very low power output and substantially lower than the modelled output.

Whilst trying to determine the cause for this lack of power output, it was noticed that there was a design flaw in the way the wires had been wound around and
through the metamaterial. This flaw was as a result of a misunderstanding of how the magnetic field moves in relation to wires on the opposite side of the metamaterial. Initially, it was thought that as the core oscillates, one wire is moving closer to the magnet, whilst the wire on the opposite side moves away from the magnet. Therefore, as the motion is opposite, yet the direction of the magnetic field is the same, the direction of current also be opposite (see Figure 5.2). However, in terms of the motion of the wire in relation to the magnetic field, both wires on opposite sides move in the same direction, and hence the current flow in both wires points in the same direction. As these wires are connected in a loop, this results in the currents cancelling each other out. To help understand this point, please refer to Figure 5.15.

The observed increase in voltage output from the second harvester design could be explained by the motion of the core, once it has reached past the mid-point in its oscillation. The core would then be slightly closer to one wire when compared to the wire on the opposite side of the metamaterial. Hence the closer wire will experience a stronger magnetic field compared to the other. The resultant induced current will
also be larger, and so there is no complete cancellation of current.

When assessed against the FEM model and the magnetic field fringe plot (see Figures 5.6 and D.1 respectively) for the second harvest design, the difference in the magnetic field strength in both wires was determined when the core had displaced 60 μm in the Y direction. The velocity of the core at that point is 0.21 m/s. Using these values in Eq. 5.4 produces a voltage output for 5 resonators of 0.023 V. This is much closer to the experimental output of the second harvester design.

Due to this design flaw, two possible options were available to reconfigure this investigation:

1. Re-wire the metamaterial so that all the copper wire is on one side of the metamaterial and hence the induced current and voltage no longer cancels out.

2. Oscillate the metamaterial in the out of plane mode (Z axis direction), similar to the classical magnet through a coil experiment. In this configuration, the coil wiring can be still be kept as is.

Due to the time limitations on the project, the quickest option to reconfigure this investigation was to choose the second method and oscillate the metamaterial in the out of plane mode (Z direction). Rewiring the metamaterial would be too complicated as each loop would have to have the return wire far away from the metamaterial, making the design much more complex than originally intended.

5.7. Z Direction Oscillation with Harvester Design 1

Due to the wire location issues stated in Section 5.6.2, the metamaterial would now be oscillated in the Z direction (see Figure 5.12). This is the classic orientation for a magnet in a coil. The reason why this oscillation direction was not trialled before was that it would not have an attenuation effect on in plane vibrations in the metamaterial, and logistically could be difficult to attach to a vibrating structure.
As two metamaterial designs had been fabricated already, one would be used to evaluate the energy harvesting capability. It was decided to use the original Harvester Design 1 as the magnet polarity orientation would no longer have an effect on the oscillation of the cores. Additionally, the first LRM design had more wire loops compared to the second, and hence increased any potential energy harvesting capability.

Based on the modal analysis, the resonant mode that equated to the oscillation of the cores in the Z direction was mode 1, at a frequency of 278 Hz. The time domain FEM model used in Section 5.4 was adapted to have a sinusoidal displacement of 278 Hz applied to the metamaterial in the Z direction. The resultant Z velocity of the core in relation to time can be seen in Figure 5.16.

![Figure 5.16.: The Z velocity of the first core with time. The maximum modelled velocity is 0.35 m/s.](image)

Using this information, including the magnetic field strength at the wire location (See Appendix D.1), and the number of loops in the design (53), Eq. 5.4 can be solved to give an approximate value for the max voltage that can be generated. This value per resonator is 0.0341 V. For the total 5 resonator design, the total modelled output is approximately 170 mV. At this voltage output, the maximum power that can be generated is 3.1x10^-4 W.

**5.7.1. Testing**

Despite this lower power output, an experiment was performed to determine the actual max voltage output in the system. The experimental set up was the same as
shown in Figure 5.8, except the specimen holder was now perpendicular to its original orientation to allow oscillations in the Z direction, relative to the metamaterial. As the mode 1 vibration was 278 Hz, the signal generator was programmed to sweep from 175 Hz to 450 Hz in increments of 0.5 Hz every second. The power output of the vibration generator was set such that it no longer interfered with the observable result on the oscilloscope (as described in Section 5.5.1). Two tests were performed and the results of these tests can be seen in Figure 5.17.

![Graph showing maximum voltage vs. frequency for Harvester Design 1 with Z direction oscillation]

Figure 5.17.: Maximum recorded voltage for Harvester Design 1 with oscillations in the Z direction (Tests 1 and 2).

### 5.7.2. Discussion

From Figure 5.17, there is a clear spike in both result plots, which confirms that the energy harvesting mechanism is working with the wiring configuration in the first harvester design. The observed frequency of 311 Hz at which the spike occurs, corresponds to the mode 1 vibration in the system. This is within a 12% relative error of the modelled resonant frequency of 278 Hz. The amplitude of the spike itself is 0.052 V in Test 1 and 0.051 V in Test 2. This is a significantly larger voltage output than achieved by the in-plane-oscillation of the second harvester design. However, it is still approximately a third of the estimated value (0.170 V) based upon the modelling of the metamaterial. At this voltage output, the maximum power that
can be generated using this metamaterial is $2.9 \times 10^{-5}$ W (based on the measured resistance of 93.2 $\Omega$.)

This disparity between the experimental and the modelled voltage outputs is most likely due to the electromotive force effect not being included in the model. Evidently, the velocity of the core is much lower than modelled, and hence the lower voltage output. This confirms that the model used is not suitable for this type of energy harvesting LRM. In the future, a multi-physics, coupled model between the wave equation and electromagnetic induction needs to be created. This future model should take account of any viscoelastic effects that may generate during the oscillation of a core in an LRM.

With regards to the future viability of this design, it is clear that the movement of the core in relation to the wire is not significant enough to allow higher voltages to be generated. The out of plane displacement is limited by the thickness of the elastic lining. Additionally, the dimensions of the matrices means the wires are located far from the magnetic core and hence the magnetic field strength is much weaker. However, reducing the size of the matrix will also reduce the strength of the metamaterial when being displaced. As such, if this configuration was to be employed again in the future, some geometry optimisation goals would be:

- Increase the thickness of the elastic lining to allow the core to achieve greater displacement amplitudes.
- Reduce the lengths of the matrix material such that the wires can be closer to the magnetic field. This would have to be modelled to ensure the metamaterial retains enough robustness when undergoing vibration.

The power output of 0.029 mW for this configuration is much lower when compared to PZT based LRMs [11, 12] that have achieved a 1.3 mW power output per resonator. Even with an optimised design, it is hard to imagine that the power generated could increase enough to become comparable to PZT based systems. Additionally, the perceived benefits of this electromagnetic approach, where the metamaterial itself could potentially be wireless and fabricated entirely from additive manufacturing methods, were not achievable in this design configuration.
5.8. Chapter Conclusions

In Chapter 5, an attempt was made to design and fabricate an electromagnetic based energy harvesting LRM. The initial concept of using the custom made 3D printer to deposit ferrofluid did not prove to be viable due to the low magnetic strength of the ferrofluid. No other magnetic inks were available that could be deposited using DDWP methods. An alternative design using neodymium magnets was devised, but this required that only the matrix and elastic components of the LRM were 3D printed. The core had to be put in place manually.

From the two neodymium based designs created, the first configuration did not show any energy generation. This is believed to be due to the orientation of the magnets in the metamaterial itself. The repelling force between adjacent cores limited their movement and hence no voltage was recorded above the noise level of the testing apparatus.

In the second design, alternating the polarity of the cores changed the repelling force to an attractive force. This would allow the cores to oscillate in unison, which is optimal for energy harvesting. However, testing of this design showed only a small voltage recorded above noise level of approximately 0.01 V. The calculated power output from this design was 0.0049 mW.

It was later discovered that a design flaw was the cause of the low power output in the second harvester design, and most likely partially responsible for the lack of power output observed with the first harvester design. The way in which the metamaterials had been wired had resulted in the induced currents partially cancelling each other out (see Figure 5.15).

Given the time constraints, rather than attempting to create a new wiring pattern for the metamaterial, the out of plane oscillations were investigated instead. This allowed the already fabricated Harvester Design 1 to be utilised, and still allow the energy harvesting capability of a magnetic LRM to be evaluated. In this orientation, the magnetic polarity would have little effect upon the harvesting capability. A spike in the maximum voltage recorded was observed in multiple tests where the
metamaterial was displaced at 278 Hz. The maximum voltage was 0.052 V, which equated to a power output 0.029 mW. This output is much lower than equivalent PZT based energy harvesting LRMs [11, 12], which have achieved a power output of 1.3 mW per resonator at 420 Hz oscillation.

Additionally, the perceived benefits of this electromagnetic approach, where the metamaterial itself could potentially be wireless and fabricated entirely from additive manufacturing methods, were not achievable in this design configuration. What is evident from the whole process of making and testing these energy harvesting LRMs is that the model used was not fit for purpose, and highlights the need to have a fully working multi-physics model. Once achieved, a more suitable design could be configured and tested. Based on the experience during this testing, any future configuration should have the following geometry optimisation goals:

- Increase the thickness of the elastic lining to allow the core to achieve greater displacement amplitudes.
- Reduce the lengths of the matrix material such that the wires can be closer to the magnetic field.
Chapter 6

Conclusions and Future Work

6.1. Summary and Conclusions

In Chapter 2, a review of the various types of acoustic and elastic metamaterials was performed. The subject's origins in electromagnetic research was investigated, and how that research has influenced similar discoveries in the acoustic and elastic wave regime. The creation of locally resonant metamaterials (LRMs) has been commented upon, and the mechanism by which they can achieve negative effective density was also shown. A comparison of the different metamaterial designs was performed, and ultimately conclusions are made as to why LRMs are best suited for low frequency wave attenuation applications, for both acoustic and elastic waves. Specifically, LRMs do not have a geometry requirement in relation to the frequency of waves attenuated, and hence can be much smaller than alternative metamaterial designs such as phononic crystals. The current focus on active and energy harvesting metamaterials was assessed, and various designs that achieve these capabilities were described.

The main modelling techniques used for metamaterials research were shown. A comparison was made of these methods and their suitability for modelling LRMs was assessed. The effect of viscoelasticity in a metamaterial was observed to increase the band gap of an LRM design, which is a desirable effect for wave attenuation applications.
Finally, the manufacture of metamaterials, specifically LRM s, was highlighted as one of the challenges in current metamaterials research. Its impact upon the adoption of the technology by industry, and experimentally verifying different LRM designs was postulated. Various attempts at creating metamaterials through additive manufacturing (AM) techniques were investigated. This confirmed that LRM designs are altered to meet current manufacturing capability, when ideally the manufacturing capability should be advanced to facilitate the manufacture of optimal LRM designs.

In Chapter 3, a review was performed on the current capabilities of AM techniques and their viability for multi-material applications. Following this review, details are provided regarding the design of a custom made 3D printer which utilises both continuous direct write (CDWP) and droplet direct write (DDWP) printing methods. This printer uses a unique control program in the LabVIEW environment, which facilitates its easy use, and high customisability.

Using MATLAB, an alternative geometry processing method is outlined, called the ‘Point-Cloud’ method. This method allows additional information about a design to be defined. This capability is not possible with the STL format, which is the standard format used in the AM sector. Additionally, the amount of information that can be added is not limited, hence the method is scalable for future developments and capabilities in AM printing. Further development is required to allow the method to automatically detect different materials within a single STL file, in order to avoid the user manually selecting which material belongs to which component in a composite material design.

Various materials were tested on the printer to allow consistent and repeatable manufacture of components. The final materials chosen were Permabond UV625®, Litex T71S20®, and Sn96.5Ag3 Cu0.5 solder. A simple 2D design was created and successfully fabricated using the printer. This is the first printer to deposit a metal alloy, a rubber compound, and a UV curable resin all within one object. The use of direct write printing techniques facilitates the use of a wider range of printing materials than alternative printing methods.

A hybrid approach to depositing solder alloy was employed. This was due to the rapid cooling of the solder droplets failing to agglomerate once on the substrate.
The result was a metal alloy with the consistency of a powder, rather than a solid object. The large droplet deposition under gravity did produce consistent droplets, but its viability for use in different scenarios is questionable.

In Chapter 4, the creation of the custom built multi-material 3D printer has allowed the successful fabrication of three different LRM designs. This represents the first time an LRM has been fabricated entirely using additive manufacturing techniques. Each design was modelled using a modal analysis based in LS-Dyna. This incorporated an optimisation procedure, using LS-Opt, to obtain the required geometry to achieve a desired mode 4 vibration frequency (corresponding to the start of the frequency band gap). Once the geometry was finalised, the design was modelled using the modal analysis method (MAM), based in MATLAB. This allowed the frequency band structure for a given design to be determined. Once modelled, each design was fabricated on the custom made multi-material printer.

Good accuracy was obtained between the printed components and their original CAD models. The deposition of Permabond UV625\textsuperscript{®} using the CDWP nozzle did leave some striation effects in the final appearance of the matrix, but this was not significant enough to greatly alter the final geometry of the component. The material itself deposited consistently and maintained its shape well before it could be cured with UV light.

The Litex T71S20\textsuperscript{®} material also deposited consistently using the room temperature DDWP nozzle. The formation of 80 \( \mu \)m diameter droplets using this nozzle allowed the rubber printed components to have a good final resolution. The cure method for this material was the evaporation of its solvent. Consequently, if the material was unused for an extended period of time, there was the likelihood the rubber at the nozzle tip would harden, causing a blockage. This was mitigated by using regular purges of the material during a print, and a combination of toluene and IPA alcohol to remove rubber from blocked nozzles.

The core material for each LRM was made from Sn96.5Ag3Cu0.5 solder, which was deposited using the Polymer Jet\textsuperscript{TM} DDWP nozzle on the printer. The hybrid
approach of depositing solder, by the formation of large droplets from a broken nozzle tip, worked well to form spherical cores for the LRM.s. The cores made for LRM Design 3 did exhibit a slight oval shape. It is thought this may be due to the cavities into which these droplets were deposited, were not entirely spherical. Despite this, the droplets had a consistent volume and did allow the creation of these LRM.s. It is not expected that this hybrid deposition method can be used to make cores of irregular geometries with the printer’s current configuration. An alternative core material would have to be found that can be deposited using the normal DDWP method and cure as a solid object. This could allow a more complex core shape to be fabricated, facilitating more varied LRM designs to be manufactured.

These LRM designs were tested in a custom built acoustic impedance tube, which allowed the frequency dependent sound transmission loss to be determined for each LRM. The three different designs that were fabricated, along with their modelled and experimental results, are shown in Table 6.1.

Table 6.1.: Summary of modelling and experimental data for LRM Designs 1, 2 and 3.

<table>
<thead>
<tr>
<th>Metamaterial</th>
<th>Design 1</th>
<th>Design 2</th>
<th>Design 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross Section</td>
<td>Circular</td>
<td>Oval</td>
<td>Double-Oval</td>
</tr>
<tr>
<td>Modelled Band Gap (Hz)</td>
<td>3490 - 4815</td>
<td>3830 - 5070</td>
<td>2662 - 3564</td>
</tr>
<tr>
<td>Experimental Band Gap (Hz)</td>
<td>3500 - 4080</td>
<td>3700 - 4200</td>
<td>2670 - 3040</td>
</tr>
</tbody>
</table>

The location of the lower limit of the modelled band gap, when compared to the experimental location, was generally accurate across all three designs. The location of the upper limit varied greatly, which is due to the low number of resonators within the tests samples. This also affected the maximum sound transmission loss recorded for all three designs.

Only LRM designs 1 and 2 can be compared directly, as they both had 6 resonators within the test sample. It was observed that LRM Design 1, with the classical circular cross-section, had a larger band gap and a higher maximum sound transmission
loss compared to LRM Design 2.

The main conclusion from Chapter 4 is that the custom 3D printer achieved its design purpose. It has allowed the manufacture, and subsequent experimentation of different LRM designs. This is the first time a three material component LRM has been entirely manufactured using additive manufacturing techniques.

In Chapter 5, an attempt was made to design and fabricate an LRM based energy harvester. The initial concept of using the custom made 3D printer to deposit ferrofluid did not prove to be viable due to the low magnetic strength of the ferrofluid. No other magnetic inks were available that could be deposited using DDWP methods. An alternative design using neodymium magnets was devised, but this required that only the matrix and elastic components of the LRM were 3D printed. The core had to be put in place manually.

From the two neodymium based designs created, the first configuration did not show any energy generation. This is believed to be due to the orientation of the magnets in the metamaterial itself. The repelling force between adjacent cores limited their movement and hence no voltage was recorded above the noise level of the testing apparatus.

In the second design, alternating the polarity of the cores changed the repelling force to an attractive force. This would allow the cores to oscillate in unison, which is optimal for energy harvesting. However, testing of this design showed only a small voltage recorded above noise level of approximately 0.01 V. The calculated power output from this design was 0.0049 mW.

It was later discovered that a design flaw was the cause of the low power output in the second harvester design, and most likely partially responsible for the lack of power output observed with the first harvester design. The way in which the metamaterials had been wired had resulted in the induced currents partially cancelling each other out (see Figure 5.15).

Given the time constraints, rather than attempting to create a new wiring pattern for the metamaterial, the out of plane oscillations were investigated instead. This allowed the already fabricated Harvester Design 1 to be utilised, and still
allow the energy harvesting capability of a magnetic LRM to be evaluated. In this orientation, the magnetic polarity would have little effect upon the harvesting capability.

A spike in the maximum voltage recorded was observed in multiple tests where the metamaterial was displaced at 278 Hz. The maximum voltage was 0.052 V, which equated to a power output 0.029 mW. This output is much lower than equivalent PZT based energy harvesting LRMs [11, 12], which have achieved a power output of 1.3 mW per resonator at 420 Hz oscillation.

Additionally, the perceived benefits of this electromagnetic approach, where the metamaterial itself could potentially be wireless and fabricated entirely from additive manufacturing methods, were not achievable in this design configuration.

In summary, the focus of this PhD thesis has been the additive manufacturing of LRMs. Having successfully designed and built a novel multi-material 3D printer, three different LRM designs were fabricated and tested successfully. This is the first time an LRM has been made entirely using additive manufacturing methods. In its current form, the printer can be used to further study various LRM designs, and allow experimental validation of these designs to occur. As the scale of additive manufacturing increases, and is further integrated into current manufacturing processes, this research will aide the adoption of LRMs into future industrial applications.

6.2. Suggestions for Future Work

Several suggestions are proposed for future work related to the investigations carried out in this thesis. The key contribution of this project has been the creation and successful use of a novel multi-material 3D printer. This multi-material capability is of keen interest to current research and industrial applications, particularly in the bioengineering sector [24, 31]. Given the unique capabilities of this printer in its current form, it would be interesting to investigate how it could be utilised to fabricate items for bioengineering applications, such as stents or micro-implants [196].
The LabVIEW environment in which the control program was made facilitates the high level of customisability of the printer. As such, it would be useful to investigate how the production of LRMs could be increased using AM techniques. One simple change could be to replace one of the DDWP nozzles with a multi-nozzle DDWP print head assembly, as used on commercial DDWP printers [21]. This would mitigate one of the main limitations associated with this printer, which is the long print times. A multi-nozzle assembly would allow a greater amount of material to be deposited in a given period of time, therefore potentially decreasing the time required to make an LRM.

The Point-Cloud method could also be adapted to take account of a new nozzle’s deposition requirements, such as the nozzle resolution or temperature requirements. Any additional information that is required could also be added into the program. Regarding the Point-Cloud method, it has been shown to be scalable and allow information to be added as needed. In its ASCII text format, it can also be transferred between different technologies easily. The inability of the method to detect different material components in a composite STL file is a limitation. If this functionality could be added, it would be a promising format to use in 3D printers in the future.

Regarding the LRMs that were made on the multi-material printer, the hybrid approach to depositing the solder core introduced some limitations in the design of LRMs that could be fabricated. Alternative materials that could be deposited by the heated DDWP should be investigated, as new materials for direct write printing is a strong focus for current researchers [31, 167]. The main requirements for any new material, is that it has a high density, and can cure as a solid object when deposited using a DDWP nozzle.

Furthermore, the impedance tube used during testing could be redesigned to allow a larger specimen to be tested. The limited volume available meant that the LRMs tested were never truly periodic and hence the full band gap width was never observed. A larger sample volume could allow the effects of stacked resonators to be investigated [17]. This is the principle that each successive LRM is of a slight design variation, therefore each will have a different resonant frequency. Stacking these
LRMs one after another could allow an overall broad spectrum attenuation to be observed.

In Chapter 5, the use of electromagnetic energy harvesting for LRMs in the classical configuration (matrix, lining, and core) did not prove a viable option for generating electricity due to the very low power output observed. This was in part due to the limited displacement of the core. Alternative designs could be explored, but not in the matrix, elastic, core configuration. Any new configuration should have an emphasis on the following design features:

- A strong neodymium magnet as the resonator in the metamaterial.
- A large displacement of the magnetic resonator. This would allow a greater magnetic flux change in adjacent wires, resulting in a larger induced voltage.
- A minimal distance between an oscillating magnet and adjacent wires in which the voltage is induced. This would allow a stronger magnetic field around the wires.

Additionally, the modelling process for an electromagnetic energy harvesting LRM should have the elastic response of the oscillator coupled with the electromotive force. This would produce a more realistic displacement and velocity profile of the core, and allow a more accurate prediction of the potential energy that can be generated for a given metamaterial design.
Bibliography


[24] Jin-Hyung Shim, Jung-Seob Lee, Jong Young Kim, and Dong-Woo Cho. Bioprinting of a mechanically enhanced three-dimensional dual cell-laden construct for osteochondral tissue engineering using a multi-head tissue/organ...


Appendix A

Acoustic Modelling

A.1. LS-Opt Optimisation Scripts

This is the LS-Opt input file to run a geometry optimisation, as used for LRM Design 1.

$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
Command file "commeta"
$Generated using LS-OPT Version 4.2
$ "Geometry optimization"
$ Author "IR"
$ Created on Tue May 03 14:18:22 2016
$ solvers 1
$ responses 1
$ $ NO HISTORIES ARE DEFINED
$ $ $ DESIGN VARIABLES
$ $ variables 2
 Variable 'R1' .00165
   Lower bound variable 'R1' .00165
   Upper bound variable 'R1' .00185
 Variable 'R2' .0019
   Lower bound variable 'R2' .0019
   Upper bound variable 'R2' .0024

$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$ OPTIMIZATION METHOD
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

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Optimization Method SRSM

SOLUTION "1"

DEFINITION OF SOLVER "1"

solver dyna960 '1'
solver command "C:\LSDYNA\program\ls-dyna_smp_d_R700_winx64_ifort101.exe"
solver input file "MetaA.k"
solver check output on
solver compress d3plot off

------ Pre-processor ------
prepro lsprepost
prepro command "F:\Models\LS-PrePost\4.3-x64\lsprepost4.3_x64.exe"
prepro input file "LSprepostingrid5a.cfile"

------ Post-processor ------

------ Metamodeling ------
solver order RBF
solver experiment design space_filling
solver number experiments 10
solver update doe
solver alternate experiment 1

------ Job information ------
solver concurrent jobs 1

RESPONSES FOR SOLVER "1"

response 'FREQUENCY1' 1 0 "DynaFreq 4 FREQ"

NO OBJECTIVES DEFINED

CONSTRAINT DEFINITIONS

constraints 1
constraint 'FREQUENCY1'
  lower bound constraint 'FREQUENCY1' 3600
  upper bound constraint 'FREQUENCY1' 3500
$ \text{PARAMETERS FOR METAMODEL OPTIMIZATION}$

$\text{Metamodel Optimization Strategy SEQUENTIAL}$

$\text{iterate param design 0.01}$
$\text{iterate param objective 0.01}$
$\text{iterate param stoppingtype and}$
$\text{iterate param response 1}$

$\text{OPTIMIZATION ALGORITHM}$

$\text{Optimization Algorithm hybrid simulated annealing}$
$\text{Use GSA}$
$\text{Set GSA Resolution 10000}$

$\text{JOB INFO}$

$\text{iterate 4}$
$\text{STOP}$
Appendix B

Multi-Material Printer

B.1. Components

The following 3D printer components were purchased from external suppliers:

From Newmark Systems:
- NLE-150-A - NLE Series Precision Linear Stage (XY Axis)
- NLS4-6-25-1 - NLS4 Series Precision Linear Stage (Z axis)
- NSC-G3 - 3 Axes Stepper Motor Controller

From EFD Nordson:
- 7012589 EFD Ultimus V dispenser system (0-100psi)
- EFD Optimum Syringe Barrel and Piston Kit (30pcs)
- EFD Optimum Tip Cap (50pcs)
- EFD Precision Dispense Tips (100 and 250 μm)

From Microfab:
- CT-MC3-04 Multichannel JetDrive III controller
- CT-PT-4 - Multichannel Pressure Controller
- PH-04a - PolymerJet High Temperature Printhead.
- MJ-SF-04 - High temperature microdispensing device (80μm) (x2)
- T-01 - Temperature Controller (for PH-04a) (x2)
- CM-VSU-03 - Basic Optics System (150mm)

B.2. Geometry Processing Matlab Script

```matlab
%OPEN ASCII STL FILE
file= 'EXAMPLE.STL';
```
% Preallocating Array Sizes
Nr = numel(textread(file,'%1c%*[^-\n]')); % Number of Rows in the file
Facets= fix((Nr-1)/7); % Number of Facets
Vertices= zeros(Facets,3);% Matrix allocation

%Algorithm to find the vertices
% The units of the STL file should be Microns (1x10^-6 m)
count3=0;
while feof(fid)==0 %while there are lines to read...
    NUM = fgetl(fid); %read the next line
    LINEa = sscanf(NUM,'%s %*s %*s %*s'); % find the word where vertex should be
    LINEb= 'vertex';
    if strcmpi(LINEa,LINEb)>0 % is L2 the same as L3
        count3=count3+1;
        T = sscanf(NUM, '%*s %f %f %f');
        Vertices(count3,1)= T(1);
        Vertices(count3,2)= T(2);
        Vertices(count3,3)= T(3);
    end
end
fclose(fid);

% Identifying the minimum XYZ values, This allows us the set the object to
% start at a location of 0,0,0, (if needed)
xmin = min(Vertices(:,1));
ymin = min(Vertices(:,2));
zmin = min(Vertices(:,3));
Vertices(:,1)=Vertices(:,1)-xmin ;
Vertices(:,2)=Vertices(:,2)-ymin ;
Vertices(:,3)=Vertices(:,3)-zmin ;
%Commented out for when using this program for multi material printing!!

% If reoriantation is needed
%Rord= V(:,2);
%V(:,2)=V(:,3);
%V(:,3) = Rord;
%V(:,1)=-1*V(:,1);
%V(:,2)=-1*V(:,2);
%V(:,3)=-1*V(:,3);

%Creating the Faces Matrix
F= zeros(Facets,3);
L6=1;
for b=1:Facets
    F(b,1)= L6;
B.2. Geometry Processing Matlab Script

```matlab
F(b,2) = L6+1;
F(b,3) = L6+2;
L6 = L6+3;
end

% Drawing the Object
figure;
grid on;
P = patch('faces',F,'Vertices',Vertices);
set(P,'FaceColor','b','FaceAlpha',0.4);
set(P,'EdgeColor','k','LineWidth',0.25);
axis equal;
axis vis3d;
title('STL File Object');
xlabel('x') % x-axis label
ylabel('y') % y-axis label
zlabel('z') % z-axis label
rotate3d on;

%% Creating the Grid limits
xmin = min(Vertices(:,1));
ymin = min(Vertices(:,2));
zmin = min(Vertices(:,3));
xmax = max(Vertices(:,1));
ymax = max(Vertices(:,2));
zmax = max(Vertices(:,3));

% Calculating the grid
INTx = xmax-xmin;
INTy = ymax-ymin;
INTz = zmax-zmin;
Nx = round(INTx/25); % Saying how many points will be on the grid
Ny = round(INTy/455); % 450 MICRON WIDTH (250 NOZZLE)
Nz = round(INTz/150); % 150 MICRON HEIGHT (250 NOZZLE)
xa = linspace(xmin,xmax,Nx);
ya = linspace(ymin,ymax,Ny);
za = linspace(zmin,zmax,Nz);
DELTAx = INTx/Nx; % Required for Scaling later on
DELTAy = INTy/Ny;
DELTAz = INTz/Nz;

% Defining any offsets_units are counts
offsetA = 0; % THIS IS THE OFFSET ASSOCIATED WITH THE DRAGGING NOZZLE
offcure = 250000; % THE OFFSET REQUIRED UV CURING

% Compute the Facet Normals and the Centres
```

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Fnormal = zeros(3, Facets);
Fcentre = zeros(3, Facets);

for nf = 1:Facets;
  % Calculate Vertices
  vec1 = [Vertices(F(nf,1),1); Vertices(F(nf,1),2); Vertices(F(nf,1),3)];
  vec2 = [Vertices(F(nf,2),1); Vertices(F(nf,2),2); Vertices(F(nf,2),3)];
  vec3 = [Vertices(F(nf,3),1); Vertices(F(nf,3),2); Vertices(F(nf,3),3)];

  % Calculate Facet Normal
  p1 = vec2 - vec1;
  p2 = vec3 - vec1;
  fnorm = cross(p2, p1);
  fnorm = fnorm / norm(fnorm);
  Fnormal(:, nf) = fnorm;

  % Calculate Position
  Fcentre(:, nf) = (vec1 + vec2 + vec3) / 3;
end

% Creating a file to have all the part points.
fileID = fopen('PointCloudXYZB.txt', 'w');
% Fill Grid
PCX = zeros(Nx*Ny*Nz, 1);
PCY = zeros(Nx*Ny*Nz, 1);
PCZ = zeros(Nx*Ny*Nz, 1);
count = 1;
decount = 1;
for nz = 1:Nz
  for ny = 1:Ny
    for nx = 1:Nx
      % Get Point
      p = [xa(nx); ya(ny); za(nz)];

      % Find Closest Facet***************
      dV = [Fcentre(1,:) - p(1); Fcentre(2,:) - p(2); Fcentre(3,:) - p(3)];
      dv = sum(abs(dV).^2);
      [v, ind] = min(dv);
      ind = ind(1);

      % Add point if inside object
      a = dot(Fnormal(:, ind), p - Fcentre(:, ind));
      ER1 = (a <= 0);

      if ER1 == 0
        fprintf(fileID, ' %6.2f %6.2f %6.2f\n', nx, ny, nz);
        PCX(count) = nx;
        count = count + 1;
      end
    end
  end
end
B.2. Geometry Processing Matlab Script

```matlab
    PCY(count)=ny;
    PCZ(count)=nz;
    count=count+1;
    else
        decount=decount+1;
    end
    ER1=0;
end
end
fclose(fileID);

%For the Point cloud plot
q1=find(PCX==0);
q2=find(PCX==0);
q3=find(PCZ==0);
PCX(q1)=[];
PCY(q2)=[];
PCZ(q3)=[];
C=zeros(size(PCX,1),3);
C(:,1)=PCX+(xmin/DELTAx);
C(:,2)=PCY+(ymin/DELTAy);
C(:,3)=PCZ+(zmin/DELTAz);
PCX=(PCX*DELTAx)+xmin;  % for correct scaling so the final plot has correct
% coordinates linked to above commented out section.
PCY=(PCY*DELTAy)+ymin;
PCZ=(PCZ*DELTAz)+zmin;
figure;
scatter3(PCX,PCY,PCZ,5,'b');
title('PointCloud of Object');
xlabel('x')  % x-axis label
ylabel('y')  % y-axis label
zlabel('z')  % z-axis label
axis equal;
rotate3d on
% Use this section if you have an variable height substrate.
%BaseX=PCX
%BaseY=PCY
%BaseZ=PCZ
%save base BaseX BaseY BaseZ
% The following is the create the Coordinate file for the Printer to use
% The various xmin,ymin,zmin are introduced to show the correct coordinates
% 10 and 7.8474 are to convert to units used by the printer.
zmaxb= max(C(:,3));
zminb= min(C(:,3));
fileID = fopen('ShellXYZB.txt','w');% The is the coordinate file
```
for z1=zminb:1:zmaxb;
    [ind]=find(C(:,3)==z1); % Finds at where in matrix C are equal to z1
    D=C(ind,:); % New matrix which points from one Z plane
    a= unique(C(ind,2));
    a2=size(a,1);
    pass=1; % used for print direction

    for a3=1:a2
        y1=a(a3);
        evenTest=mod(pass/2,1); % used for print direction (A-B, or B-A)
        [ind2]=find(D(:,2)==y1);
        E=D(ind2,:);
        xminb= min(E(:,1));
        xmaxb= max(E(:,1));
        %XA-odd
        if evenTest==0 % If line is odd, print in xmin to xmax direction
            fprintf(fileID,'%2.2f,%2.2f,%2.2f ...
                
',fix(((xminb*DELTAx))*10),fix(((y1*DELTAy))*10),... 
                fix(((z1*DELTAz))*-7.874)); ... 
            % This is always the first line to be printed
            for x1=xminb:1:xmaxb-1; ...
                % This for loop is to identify any gaps ...
                % between a max and min X value for constant Y,Z values
                [ind3]=find(E(:,1)==x1);
                if E(ind3+1,1)- E(ind3,1)>1
                    fprintf(fileID,'%2.2f,%2.2f,%2.2f 

',fix(((E(ind3,1)*DELTAx))*10),fix(((y1*DELTAy)... 
                        )*10),fix(((z1*DELTAz))*-7.874));
                    fprintf(fileID,'%2.2f,%2.2f,%2.2f

',((E(ind3+1,1)*DELTAx))*10)-offsetA,fix(((y1*DELTAy)... 
                        )*10),fix(((z1*DELTAz))*-7.874)-1);% no print
                else
                    end
            end
        end
        if y1==a(a2) % last line to printed on that Z plane.
            fprintf(fileID,'%2.2f,%2.2f,%2.2f 

',fix(((max(E(:,...
                        1))*DELTAx))*10),fix(((y1*DELTAy)*10),fix(((z1*DELTAz)... 
                        -*7.874));
            fprintf(fileID,'%2.2f,%2.2f,%2.2f

',((max(E(:,...
                        1))*DELTAx))*10)+5000,fix(((y1*DELTAy)*10+5000,... 
                        ((z1*DELTAz))*-7.874)-1);% no print
            fprintf(fileID,'%2.2f,%2.2f,%2.2f

',((max(E(:,1)... 
                        )*DELTAx))*10)-offcure,fix(((y1*DELTAy)*10)+5000,... 
                        fix(((z1*DELTAz))*-7.874)-2);% Adds an additional...
            extended point for UV torch

        end
    end
end

for a3=a2
    scanf a3(a3);
    evenTest=mod(pass/2,1); % used for print direction (A-B, or B-A)
    [ind2]=find(D(:,2)==y1);
    E=D(ind2,:);
    xminb= min(E(:,1));
    xmaxb= max(E(:,1));
    %XA-odd
    if evenTest==0 % If line is odd, print in xmin to xmax direction
        fprintf(fileID,'%2.2f,%2.2f,%2.2f 

',fix(((xminb*DELTAx))*10),fix(((y1*DELTAy)*10),... 
                fix(((z1*DELTAz))*-7.874)); ... 
        % This is always the first line to be printed
        for x1=xminb:1:xmaxb-1; ...
            % This for loop is to identify any gaps ...
            % between a max and min X value for constant Y,Z values
            [ind3]=find(E(:,1)==x1);
            if E(ind3+1,1)- E(ind3,1)>1
                fprintf(fileID,'%2.2f,%2.2f,%2.2f

',fix(((E(ind3,1)*DELTAx))*10),fix(((y1*DELTAy)... 
                        )*10),fix(((z1*DELTAz))*-7.874));
                fprintf(fileID,'%2.2f,%2.2f,%2.2f

',((E(ind3+1,1)*DELTAx))*10)-offsetA,fix(((y1*DELTAy)... 
                        )*10),fix(((z1*DELTAz))*-7.874)-1);% no print
            else
                end
        end
    end
    if y1==a(a2) % last line to printed on that Z plane.
        fprintf(fileID,'%2.2f,%2.2f,%2.2f 

',fix(((max(E(:,...
                        1))*DELTAx))*10),fix(((y1*DELTAy)*10),fix(((z1*DELTAz)... 
                        -*7.874));
        fprintf(fileID,'%2.2f,%2.2f,%2.2f

',((max(E(:,...
                        1))*DELTAx))*10)+5000,fix(((y1*DELTAy)*10+5000,... 
                        ((z1*DELTAz))*-7.874)-1);% no print
        fprintf(fileID,'%2.2f,%2.2f,%2.2f

',((max(E(:,1)... 
                        )*DELTAx))*10)-offcure,fix(((y1*DELTAy)*10)+5000,... 
                        fix(((z1*DELTAz))*-7.874)-2);% Adds an additional...
        extended point for UV torch

    end
end
else
    fprintf(fileID,'\%2.2f,\%2.2f,\%2.2f\r\n',fix(((max...(E(:,1))*DELTAx)*10),fix(((y1*DELTAy)*10),... fix(((z1*DELTAz)*-7.874));
end

%XB -even
else % If line is even, print in xmax to xmin direction
    fprintf(fileID,'\%2.2f,\%2.2f,\%2.2f\r
\n',fix(((xmaxb*DELTAx))*10),fix(((y1*DELTAy)... )*10),fix(((z1*DELTAz)*-7.874));
    for x1=xmaxb:-1:xminb+1; %This for loop is to identify... any gaps between a max and min X value for constant Y,Z values [ind3]=find(E(:,1)==x1);
        if E(ind3,1)– E(ind3-1,1)>1
            fprintf(fileID,'\%2.2f,\%2.2f,\%2.2f\r... \n',fix(((E(ind3,1)*DELTAx))*10),fix(((y1*DELTAy)... )*10),fix(((z1*DELTAz)*-7.874));
            fprintf(fileID,'\%2.2f,\%2.2f,\%2.2f\r... \n',fix(((E(ind3-1,1)*DELTAx))*10)... +offsetA,fix(((y1*DELTAy))*10),fix(((z1*DELTAz)... )*-7.874)-1); % no print
        else
            end
    end

if y1==a(a2) % last line to printed on that Z plane.
    fprintf(fileID,'\%2.2f,\%2.2f,\%2.2f\r\n',fix(((min...(E(:,1))*DELTAx))*10),fix(((y1*DELTAy)*10),fix((... (z1*DELTAz)*-7.874));
    fprintf(fileID,'\%2.2f,\%2.2f,\%2.2f\r\n',fix(((min...(E(:,1))*DELTAx))*10)+offcure,fix(((y1*DELTAy)... )*10),fix(((z1*DELTAz)*-7.874)-1); % Adds an ... additional extended point for UV torch
else
    fprintf(fileID,'\%2.2f,\%2.2f,\%2.2f\r\n',fix(((min(E(:,1)... )*DELTAx))*10),fix(((y1*DELTAy)*10),fix(((z1*DELTAz)... )*-7.874));
    end
end
pass=pass+1;
end
fclose(fileID);
B.3. Control Program Block Diagrams

Figure B.1.: LabVIEW block diagram for the printer control program. This image shows the interface with MATLAB. This is just one part of the full program, an overview of which is shown in Figure 3.26.
Figure B.2.: LabVIEW block diagram for the printer control program. This image shows pre-defined process to make a circle. This is just one part of the full program, an overview of which is shown in Figure 3.26.
Figure B.3.: LabVIEW block diagram for the printer control program. This image shows how the program commands the inkjet device to deposit material. This is just one part of the full program, an overview of which is shown in Figure 3.26.
Appendix C

Impedance Tube

C.1. Engineering Drawings

Figure C.1.: Impedance tube engineering drawing, showing the location of the microphones and the tube diameter.
Figure C.2.: Impeance tube assembly. The blue end of the tube represents the speaker.
C.2. Impedance Tube Control Program

Output Signal:
Control the output volume and frequency.

Amplitude:
The frequency dependent amplitude is shown here.

Phase:
The frequency dependent phase is shown here.

Input Signal:
The time domain signal from all 4 microphones is shown here.

Figure C.3.: LabVIEW based impedance tube control program user interface.
C.2. Impedance Tube Control Program

**Program Initiation:**
Connections to the NI DAQ and speaker are initialised.

**Main Program:**
Output signal is created.

**Audio Processing:**
The signals from all 4 microphones is processed here.

**Program Output:**
Results are written to an Excel spreadsheet.

Figure C.4.: LabVIEW based impedance tube control program block diagram.
C.3. 1/N Octave Smoothing Function

This smoothing function applies 1/\textit{NOCT} -octave smoothing to a frequency spectrum. This function was sourced from the MATLAB File Exchange. The Program was written by Christopher Hummersone at the University of Surrey [189].

\begin{verbatim}
function x_oct = smooth_spectrum(X,f,Noct)
%SMOOTH_SPECTRUM Apply 1/N-octave smoothing to a frequency spectrum
% https://uk.mathworks.com/matlabcentral/fileexchange/55161-1-n-octave-smoothing
% X_OCT = SMOOTH_SPECTRUM(X,F,NOCT) applies 1/NOCT-octave smoothing to
% the frequency spectrum contained in vector X sampled at frequencies
% in vector F. X can be a log-, magnitude-, or power-spectrum. Setting
% Noct to 0 results in no smoothing.
%
% Algorithm
%
% The function calculates the i-th smoothed spectral coefficient X_OCT(i)
% as the sum of the windowed spectrum. The window is a Gaussian whose
% centre frequency is F(i), and whose standard deviation is proportional
% to F(i)/NOCT.
%
% Copyright 2015 University of Surrey.
%
% ==-----------------------------
% Last changed: $Date: 2016-02-02 12:10:40 +0000 (Tue, 02 Feb 2016) $
% Last committed: $Revision: 456 $
% Last changed by: $Author: ch0022 $
% ==-----------------------------
%
%% Input checking
%
assert(isvector(X),'X must be a vector.');
assert(isvector(f),'F must be a vector.');
assert(isscalar(Noct),'NOCT must be a scalar.');
assert(isreal(X),'X must be real.');
assert(all(f>=0),'F must contain positive values.');
assert(Noct>=0,'NOCT must be greater than or equal to 0.');
assert(isequal(size(X),size(f)),'X and F must be the same size.');

%% Smoothing
%
% calculates a Gaussian function for each frequency, deriving a
% bandwidth for that frequency

x_oct = X; % initial spectrum
if Noct > 0 % don't bother if no smoothing
    for i = find(f>0,1,'first'):length(f)
    end
end
\end{verbatim}
C.4. Pre-smoothed Acoustic Results

```matlab
function g = gauss_f(f_x,F,Noct)
% GAUSS_F calculate frequency-domain Gaussian with unity gain
% % G = GAUSS_F(F_X,F,NOCT) calculates a frequency-domain Gaussian function
% % for frequencies F_X, with centre frequency F and bandwidth F/NOCT.

sigma = (F/Noct)/pi; % standard deviation
g = exp(-(f_x-F).^2./(2.*(sigma^2)))); % Gaussian
g = g./sum(g); % normalise magnitude
end
```

C.4. Pre-smoothed Acoustic Results
C.4. Pre-smoothed Acoustic Results

Figure C.5.: LRM Design 1.

(a) Test 1.

(b) Test 2.
C.4. Pre-smoothed Acoustic Results

(a) Test 1.

(b) Test 2.

Figure C.6.: LRM Design 2.
Figure C.7.: LRM Design 3.
Appendix D

Energy Harvesting

D.1. Magnetic Field Fringe Plot

Figure D.1.: Visualisation of the magnetic field as provided by magnet suppliers [30]. The dotted lines show the location of the rubber boundary, and the matrix boundary.
### D.1. Magnetic Field Fringe Plot

#### D.1.1. Magnetic Field Strength

Table D.1.: Magnetic field strengths at wire locations, and corresponding angles from the X axis. (1 Tesla is equivalent to 10000 Gauss).

<table>
<thead>
<tr>
<th>Point (X,Y)</th>
<th>Field Strength (Gauss)</th>
<th>Angle (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8,0)</td>
<td>675</td>
<td>90</td>
</tr>
<tr>
<td>(8,1)</td>
<td>685</td>
<td>76</td>
</tr>
<tr>
<td>(8,2)</td>
<td>710</td>
<td>63</td>
</tr>
<tr>
<td>(8,3)</td>
<td>747</td>
<td>51</td>
</tr>
<tr>
<td>(8,4)</td>
<td>778</td>
<td>39</td>
</tr>
<tr>
<td>(8,5)</td>
<td>808</td>
<td>26</td>
</tr>
<tr>
<td>(8,6)</td>
<td>802</td>
<td>14</td>
</tr>
</tbody>
</table>