A Lateral Control Assistant for the Dynamic Model of Vehicles Subject to State Constraints

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***Abstract*— An assistant control scheme for the dynamic model of a car to help the driver track a given reference or to keep the car in a given lane while making sure that all the system states satisfy pre-defined constraints is given. The assistant control algorithm is based on a hysteresis switch and the formal properties of the closed-loop system are studied via a Lyapunov-like analysis. Simulation results showing the effectiveness of the driving assistance system are presented.**

NOMENCLATURE

β	sideslip angle [rad]
$\dot{\psi}$	yaw rate [rad/s]
ψ_L	heading error, relative yaw angle [rad]
δ	actual steering angle [rad]
δ_d	steering angle at the column system [rad]
ρ	road curvature $[\mathrm{m}^{-1}]$
η_t	width of the tyre contact patch [m]
B_u	damping coefficient of the steering system $[\mathrm{Nm}/(\mathrm{rad} \times \mathrm{s})]$
$C_f(C_r)$	front (rear) tyre cornering stiffness [N/rad]
I_z	moment inertia of the car about the yaw-axis $[\mathrm{kg} \times \mathrm{m}^2]$
J_s	moment of inertia of the steering system $[\mathrm{kg} \times \mathrm{m}^2]$
k	the activation function
K_a	visual anticipatory control of the driver
K_c	proportional gain of the transfer function representing the compensatory steering control of the driver
$l_f(l_r)$	distances of the front (rear) tyres to the mass center [m]
m	mass of the car [kg]
R_s	reduction ratio of the steering system, *i.e.* $R_s = \delta_d/\delta$
T_a	assistance torque generated by the controller [Nm]
T_d	driver's torque [Nm]
$T_i(T_l)$	lag (lead) time constant of the transfer function representing the compensatory steering control of the driver
T_n	neuromuscular lag time constant of the driver
T_p	driver's preview time [s]
T_s	self-aligning moment of the steering system [Nm]
u_f	feedback control input to the system (6) [Nm]
v_x	longitudinal speed of the car [m/s]
$\bar{x}$	the vector denoting the bound of the state x
y_L	lateral deviation of the car [m]

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I. INTRODUCTION

More than one million people are killed by road traffic accidents and an additional 20 to 25 million are injured or disabled in one year all over the world [1]. Based on road crashes statistics, at least 40% of death are caused by improper turning of the steering wheel [2]. Therefore a lateral control support system has significant importance in improving safety.

Using the human-machine cooperation taxonomy introduced in [2], [3], on the basis of the increasing level of automation, all lateral assistance systems can be divided into four categories: Electronic Stability Control (ESC), Vision Enhancement Systems (VES), Lane Departure Warning Systems (LDWS) and Lane Keeping Assistance Systems (LKAS). ESC is designed to brake one or more wheels when the car is oversteered or understeered. This type of assistance is effective in crashes related to loss of control [4] and is able to reduce such type of crashes by 25% to 70%. Nevertheless it is activated only if the car starts to skid: it is designed to correct the driver's behaviour in emergency situations, rather than to prevent crashes. VES, as the name implies, provides additional information on the driving environment, such as road markings, beacon pointing to the tangent point at a bend and adaptive front-light systems, to enhance driver's vision in low-visibility circumstances, but does not provide any intervention [5], [6], [7]. Compared with the above two assistant systems and automatic driving systems, drivers trust more the latter two lateral control support system: LDWS and LKAS [8]. By using road markings, LDWS are able to inform the driver if the vehicle is close to the boundary of the lane, while LKAS aim to assist the driver continuously and physically act on the vehicle by steering. The co-action mode (LKAS) has been regarded as a good choice to physically help the driver rather than simply give warnings or attract the driver's attentions [2]. Researches on the driver's negative adaptation manner to the driving assistant systems have shown that it is important to make sure that the driver does not excessively rely on the assistant system [9]. Therefore, the lateral control assistance scheme presented in the paper is not always active and the driver should take effort in controlling the car.

The design of shared lateral controllers has become a growing field of interest for researchers and engineers. Several papers detailing the control schemes have been published. The paper [10] has presented a shared lateral controller of the vehicle based on a PI observer and a forcing controller which combines the driver input and the feedback control input, while the paper [11] has introduced a stochastic model predictive controller (MPC) to prevent the departure of the car from the lane in which the steering torque generated by the driver is regarded as a Gaussian disturbance. Adaptive Dynamic Programming (ADP) methods have been applied in the paper [12] to design the steering assistance control without knowledge of the vehicle model and the driver model. However, ADP requires large on-line computation, which is not cost-effective in application. This paper presents an assistant control design for the lateral dynamics of the vehicle based on the driver model given in [13] and provide rigorous mathematical proof for the stability and the "safety" of the closed-loop system. In addition, an analytical solution which needs little computation load is proposed to assist the driver in keeping the car in the lane.

The paper is organized as follows. Section II describes the model we study and formulates the assistant control problem. The design of the feedback controller and the mechanism to activate the feedback controller are given in Section III, in which formal properties of the closed-loop systems are presented. Section IV gives two case studies to illustrate how the assistant control algorithm works for constant or time-varying road curvatures. Finally, conclusions and suggestions for future work are presented in Section V.

II. SYSTEM MODELING AND PROBLEM STATEMENT

On the basis of the well-known bicycle model, the lateral dynamics of the car can be described by the equations [14]

$$\begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \delta, \tag{1}$$

where β, $\dot{\psi}$ and δ denotes the sideslip angle, the yaw rate and the actual steering angle, respectively, and

$$a_{11} = -\frac{2(C_f+C_r)}{mv_x}, \qquad a_{12} = -1 - \frac{2(C_f l_f - C_r l_r)}{mv_x^2},$$
$$a_{21} = -\frac{2(C_f l_f - C_r l_r)}{I_z}, \qquad a_{22} = -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x},$$
$$b_1 = \frac{2C_f}{mv_x}, \qquad b_2 = \frac{2C_f l_f}{I_z}.$$

Note that the description of all the parameters are given at the beginning of the paper.

In addition, the dynamics of the steering system is given by the equation

$$J_s \ddot{\delta}_d + B_u \dot{\delta}_d = T_d + T_a - T_s, \tag{2}$$

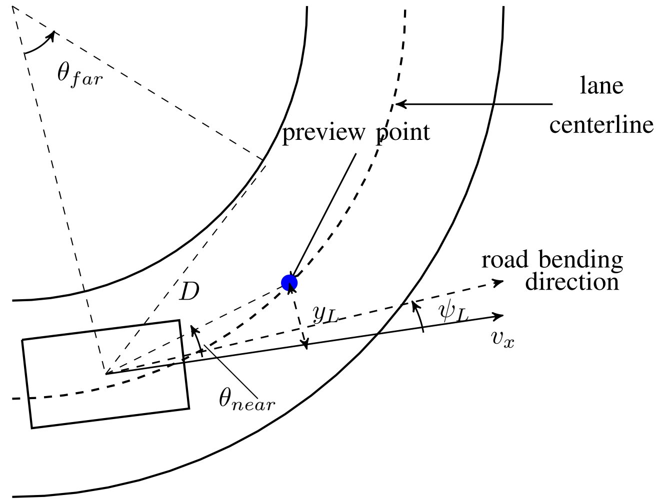

Fig. 1. Visual Angle Definitions in Lane Keeping Cases.

where $\delta_d = \delta R_s$ and the self-aligning moment T_s is calculated as

$$T_s = -\frac{2C_f\eta_t}{R_s}\beta - \frac{2C_f l_f \eta_t}{R_s v_x}\dot{\psi} + \frac{2C_f\eta_t}{R_s^2}\delta_d. \tag{3}$$

In lane keeping (trajectory tracking) cases, the dynamic relationship between the vehicle and the road central line (reference trajectory) can be described by the equations

$$\begin{aligned} \dot{y}_L &= v_x\beta + T_p v_x \dot{\psi} + v_x \psi_L, \\ \dot{\psi}_L &= \dot{\psi} - v_x \rho, \end{aligned} \tag{4}$$

where y_L and ψ_L are defined in Fig. 1.

The so-called two-level time-delay based model can be used to model the driver [15]. The corresponding dynamics is expressed as

$$\begin{aligned} \dot{x}_d &= \begin{bmatrix} -\frac{1}{T_i} & 0 \\ \frac{K_c(T_l - T_i)}{T_i^2} & -\frac{1}{T_n} \end{bmatrix} x_d + \begin{bmatrix} 0 & 1 \\ K_a & -\frac{K_c T_l}{T_i} \end{bmatrix} u_d, \\ T_d &= \begin{bmatrix} 0 & \frac{1}{T_n} \end{bmatrix} x_d, \end{aligned} \tag{5}$$

where $x_d = [x_{d1}, x_{d2}]^T$, $u_d = [\theta_{far}, \theta_{near}]^T$. Note that x_{d1} and x_{d2} are two internal states of the driver model (5), while θ_{near} and θ_{far} are the two visual angles defined in Fig. 1. Note that all the parameters (*i.e.* K_a, K_c, T_i, T_l, T_n) related to the driver model (5) can be identified by typical system identification methods with real-time data collected from different maneuvers the driver does and typical values are given in [16].

To sum up, Fig. 2 shows the block diagram of the overall system, the dynamics of which can be described as

$$\dot{x} = Ax + B_1\rho + B_2 T_a, \tag{6}$$

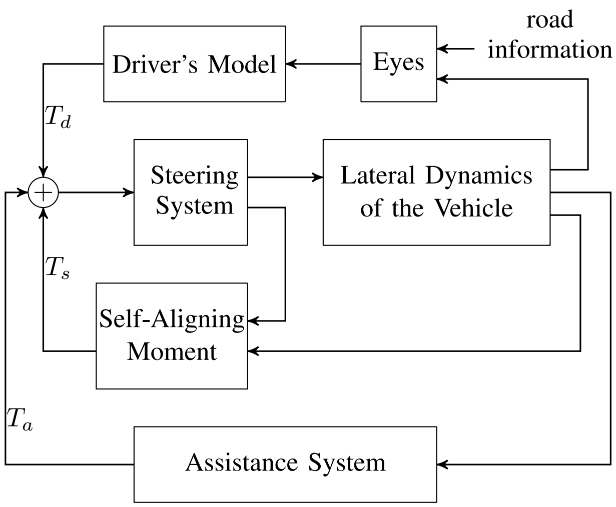

Fig. 2. Block Diagram of the System

where T_a and ρ are the control input and the exogenous input of the system, $x = [\beta, \dot{\psi}, \delta_d, \dot{\delta}_d, x_{d1}, x_{d2}, y_L, \psi_L]^T$ is the system state, and

$$A = \begin{bmatrix} a_{11} & a_{12} & b_1/R_s & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & b_2/R_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & a_{46} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 & a_{57} & 0 \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} & a_{67} & 0 \\ a_{71} & a_{72} & 0 & 0 & 0 & 0 & 0 & a_{78} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = [0, 0, 0, 0, 0, b_{61}, 0, b_{81}]^T, \quad B_2 = [0, 0, 0, b_{42}, 0, 0, 0, 0]^T,$$

with

$$\begin{aligned} a_{41} &= \tfrac{2C_f \eta_t}{R_s J_s}, & a_{42} &= \tfrac{2C_f l_f \eta_t}{R_s v_x J_s}, & a_{43} &= \tfrac{-2C_f \eta_t}{J_s R_s^2}, \\ a_{44} &= -\tfrac{B_u}{J_s}, & a_{46} &= \tfrac{1}{J_s T_n}, & a_{55} &= -\tfrac{1}{T_i}, \\ a_{57} &= \tfrac{1}{T_p v_x}, & a_{65} &= \tfrac{K_c(T_l - T_i)}{T_i^2}, & a_{66} &= -\tfrac{1}{T_n}, \\ a_{67} &= \tfrac{K_a}{T_p v_x}, & a_{71} &= v_x, & a_{72} &= v_x T_p, \\ a_{78} &= v_x, & b_{61} &= D K_a, & b_{81} &= -v_x, \end{aligned}$$
$$b_{42} = \tfrac{1}{J_s}.$$

As discussed in Section I our driving assistant controller is not always active. Therefore the assistant torque T_a can be defined as

$$T_a(t) = k(t) u_f(t), \tag{7}$$

where u_f is the feedback controller we are going to design in Section III-A and $k \in \{0, 1\}$. Note that $k = 0$ and $k = 1$ indicate that the feedback assistance controller is not active and is active, respectively.

Suppose $\mathcal{S}_a \in \mathbb{R}^8$ is a given set describing the feasible state values of the system (6) and defined as

$$\mathcal{S}_a = \{x(t) \in \mathbb{R}^8 \mid x_i(t) \le \bar{x}_i, \forall t \ge 0, i \in \{1, 2, \dots, 8\}\}, \tag{8}$$

where $x_i(t)$ denotes the i^{th} element of the vector $x(t)$. The driving assistant control problem for the lateral dynamic model of the car can be formulated as follows.

Given the system (6)-(7) and a set of feasible states $\mathcal{S}_a$, find (if possible)

- a feedback controller u_f;
- an activation function $k(t)$;
- a safe set $\mathcal{R}_s$;

such that the closed-loop system with the assistant controller has the following properties.

P1) The system constraints (8) are satisfied all the time, *i.e.* $x(t) \in \mathcal{P}_a$ for all $t \ge 0$;

P2) $\lim_{t \to \infty} k(t) = 0$;

P3) $\lim_{t \to \infty} x(t) = x_r$, where x_r is the equilibrium point of the system (6) with $T_a = 0$.

III. DESIGN OF THE SHARED-CONTROL FOR THE REAR-WHEEL DRIVING CAR

This section provides a solution to the assistant driving control problem stated in Section II such that the system constraint[1] $x(t) \le \bar{x}$ are satisfied for all $t \ge 0$.

A. Design of the Feedback Controller

In this subsection we present a feedback controller such that the states of the closed-loop system (6)-(7) with $k = 1$ belong to the set $\mathcal{S}_a$ for all $t \ge 0$.

Suppose $x_r = [x_{r1}, x_{r2}, \dots, x_{r8}]^T$ denotes the trim state of the system (6) for a given road curvature ρ. Then

$$\dot{x}_r = A x_r + B_1 \rho = 0.$$

The first step to design the feedback controller is to remove the constraints. Define the new variable $z = [z_1, z_2, \dots, z_8]^T$ with

$$z_i = \log \frac{x_i - \bar{x}_i}{x_{ri} - \bar{x}_i}. \tag{9}$$

Using the variable z_i, the system (6) with the feedback controller can be rewritten as

$$\dot{z} = (M_e M_r)^{-1} \{A[M_e(x_r - \bar{x}) + \bar{x}] + B_1 \rho + B_2 u_f\}, \tag{10}$$

where $M_e = \operatorname{diag}(e^{z_1}, e^{z_2}, \dots, e^{z_8})$ and $M_r = \operatorname{diag}(x_{r1} - \bar{x}_1, x_{r2} - \bar{x}_2, \dots, x_{r8} - \bar{x}_8)$.

[1]Note that the inequality constraints are element-wise.

Assumption 1: For typical values of the vehicle parameters and the driver model parameters, there exists a symmetric positive definite matrix P such that

$$(M_r^{-1} A M_r)^T P + P M_r^{-1} A M_r = -Q,$$

where Q is a diagonal matrix with all diagonal entries positive.

Lemma 1: Consider the system (6) controlled by the feedback controller

$$u_f = -k_u B_2^T (M_r^{-1})^T M_e P z, \tag{11}$$

where $k_u > 0$ (*i.e.* $T_a(t) = u_f(t)$ for all $t \geq 0$). Then the closed-loop system has the following properties.

- $\lim_{t\to\infty} x(t) = x_r$;
- $x(t) \leq \bar{x}$ for all $t \geq 0$.

B. Shared Control Theorem

Based on the value of the Lyapunov function

$$L = z^T P z, \tag{12}$$

the state space of the system (10) can be divided into three parts, the safe set $\mathcal{R}_s$, the hysteresis set $\mathcal{R}_h$ and the dangerous set $\mathcal{R}_d$. Given the matrix P, the definition of the three sets are given by the equations

$$\begin{aligned} \mathcal{R}_s &= \left\{ z \in \mathbb{R}^8 : z^T P z \leq b_2 \right\}, \\ \mathcal{R}_h &= \left\{ z \in \mathbb{R}^8 : b_2 < z^T P z < b_1 \right\}, \\ \mathcal{R}_d &= \left\{ z \in \mathbb{R}^8 : z^T P z \geq b_1 \right\}, \end{aligned} \tag{13}$$

where $b_1 > b_2 > 0$ are user-selected constants.

The feedback activation function k can then be defined as

$$k(t) = \begin{cases} 0, & z \in \mathcal{R}_s, \\ l, & z \in \mathcal{R}_h, \\ 1, & z \in \mathcal{R}_d, \end{cases} \tag{14}$$

where

$$l = \begin{cases} 0, & \text{if } z \text{ enters } \mathcal{R}_h \text{ from } \mathcal{R}_s, \\ 1, & \text{if } z \text{ enters } \mathcal{R}_h \text{ from } \mathcal{R}_d. \end{cases}$$

According to the definitions of T_a and the activation function k given in (7) and (14), respectively, the assistant controller is not active if the system state is close to its reference values (*i.e.* $z \in \mathcal{R}_s$) and is active only if the system state is sufficiently far away from the reference (*i.e.* $z \in \mathcal{R}_d$). The hysteresis switch l is used to reduce oscillations.

Theorem 1: Consider the system composed of the combination of the lateral dynamic model of a car and a two-level time-delay based driver model (6) with an assistant controller (7)-(11)-(14). Assume that the constraints (8) are satisfied at $t = 0$. In addition, Assumption 1 holds. Then there exists a symmetric positive definite matrix P such that the system in closed-loop with the assistant controller has the following properties.

1) The constraints (8) are satisfied for all $t \geq 0$.
2) The assistant controller is not active all the time and is turned off automatically if the system states are "close to" the reference values.
3) $\lim_{t\to\infty} T_a(t) = 0$, $\lim_{t\to\infty} x(t) = x_r$.
4) The assistant control input T_a is bounded.

Proof: To begin with we prove claim 1) by contradiction. Suppose that there exists $\tilde{t} > 0$ and $i \in \{1, 2, \ldots, 8\}$ such that $x_i(\tilde{t}) > \bar{x}_i$. Due to the continuity of the system states, there exists $0 < t_0 < \tilde{t}$ such that $x_i(\tilde{t}) = \bar{x}_i$. Therefore, $z_i(t_0) = +\infty$, $z(t_0) \in \mathcal{R}_d$ and the assistant controller is active, which contradicts the fact that the system constrains are satisfied if the feedback controller is active.

Claim 2) is a consequence of the definition of the activation function k given in (14).

Similarly to the proof of Theorem 1 given in [17] and [18], consider the Lyapunov function candidate given by (12). It is continuous and non-increasing by Lemma 1. In addition, $\dot{L} = 0$ if and only if $x = x_r$. Therefore claim 3) is a consequence of the general results in [19] and of the facts that x_r is the equilibrium point for the systems (6) with and without assistant control T_a. In addition, the states of the closed-loop system (6)-(7)-(11)-(14) converge to x_r, indicating that $\lim_{t\to\infty} z(t) = 0$. In other words, $\lim_{t\to\infty} z(t) \in \mathcal{R}_s$, yielding $\lim_{t\to\infty} k(t) = 0$. Hence, claim 3) holds.

With the selected feedback controller (11) the value of the Lyapunov function (12) is not increasing. This leads to the result that z is bounded and u_f is bounded. Therefore, claim 4) is a direct consequence of the definition of T_a. ■

Remark 1: The proposed assistant controller is applicable to shared-control systems. In other words, the driver should be involved in the control loop and a physical warning should be given by the assistant controller if the driver leaves his/her hands off the steering wheel or the driver does not provide any steering torque while he/she should do so for a certain amount of time.

Remark 2: To achieve lane changing the driver needs to either turn off the assistant controller, or use path planning algorithms in the outer loop to generate a feasible trajectory in which case ρ is no longer the real road curvature, but the curvature of the planned trajectory.

IV. CASE STUDIES

This section discusses two case studies: a circular motion of constant speed and a lane keeping maneuver with varying

TABLE I

VEHICLE PARAMETERS

η	0.15	B_u	2.5	I_z	1500
C_f	170390	C_r	195940	J_s	0.05
l_f	1.48	l_r	1.12	m	1625
R_s	12				

TABLE II

DRIVER MODEL PARAMETERS

T_i	0.14	T_l	1.16	T_n	0.11
K_a	56.98	K_c	36.13	T_p	2

road curvature. In both cases we see clearly that with the help of the assistant controller the lateral deviation is much improved, indicating the effectiveness of the lateral assistant control of the car. The parameter values are given in Tables I and II.

A. Circular Motion with Constant Speed

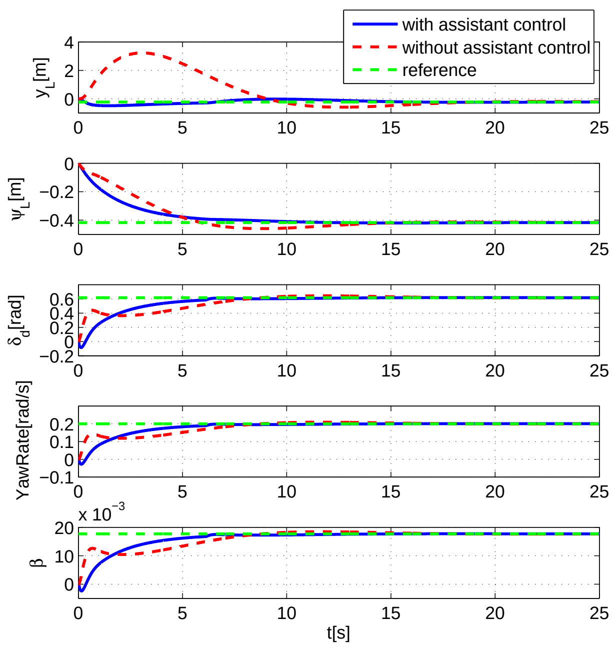

Fig. 3. Time histories of the lateral deviation y_L, the relative yaw angle ψ_L, the steering angle δ_d, the yaw rate and the side-slip angle β for the system (6) with assistant control (blue, solid) and without assistant control (red, dashed). The green, dotted line indicates the reference time history for the corresponding variables.

Consider the lateral dynamic model of a car with the two-level time-delay based driver model described by the equation (6). Assume that the car aims to keep in its lane which is a circle with radius equals to 50 m, *i.e.* $\rho = 1/50$. We also assume that the constraint for the lane keeping case is given as

$$[\beta, \dot{\psi}, \delta_d, y_L, \psi_L]^T \leq [0.05, 0.3, 1, 0.5, 0.5]^T. \tag{15}$$

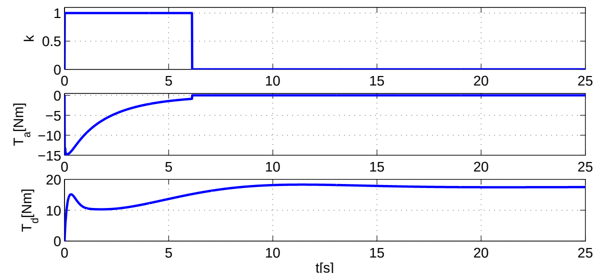

Fig. 4. Time histories of the variable k, T_a and T_d for the system (6) with assistant control.

Simulation results are shown in Figs. 3 and 4, from which we see that the assistant controller is active at the beginning and is turned off automatically at $t = 6.12s$ since the states of the system are not far away from their reference values. Therefore, the value of T_a jumps from $-0.92Nm$ to $0Nm$ at $t = 6.12s$. With the help of the assistant controller, the lateral deviation is much improved, *i.e.* the maximum lateral deviation with T_a is less than 15% of that without T_a. In addition, all the system states have less oscillations.

B. Lane Keeping Maneuver with Varying Road Curvature

Consider again the lateral dynamic model of a car with the simplified driver model described by the equation (6). Suppose the road curvature is no longer a constant value. Instead, $\rho(t)$ is defined as

$$\rho(t) = 0.02 \sin(0.02t).$$

We also assume that the constraints are the same as those given in (15).

Simulation results are illustrated in Figs. 5 and 6. Unlike what is shown in Fig. 3, the time histories of the state variables (except for y_L) do not have much difference between the cases with and without T_a because the initial state value is the same as its reference value. However, the lateral deviation still has much improvement with the developed feedback controller. Moreover, Fig. 6 shows that T_a is turned on and off periodically because the road curvature is a periodic function.

V. CONCLUSIONS

The paper has presented a solution to the assistant driving control problem for the lateral dynamic model of a car with a simplified two-level time-delay based driver model. The assistance is activated only if there is sufficient state deviation from their reference values and is turned off automatically if the system state are close to the trim state. With the help of the assistant control the lateral deviation of the system is much improved and all the system constraints are satisfied for all times. Future work will focus on the assistant control for the nonlinear model of the lateral dynamics of the car.

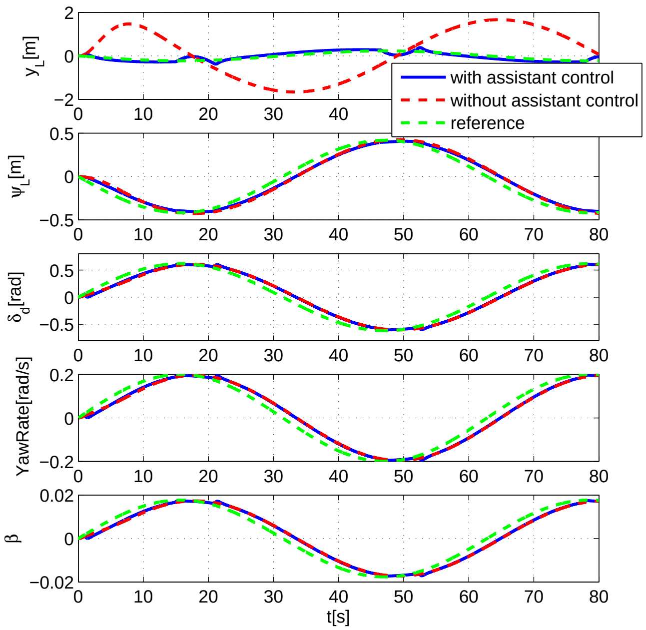

Fig. 5. Time histories of the lateral deviation y_L, the relative yaw angle ψ_L, the steering angle δ_d, the yaw rate and the side-slip angle β for the system (6) with assistant control (blue, solid) and without assistant control (red, dashed). The green, dotted line indicates the reference time history for the corresponding variables.

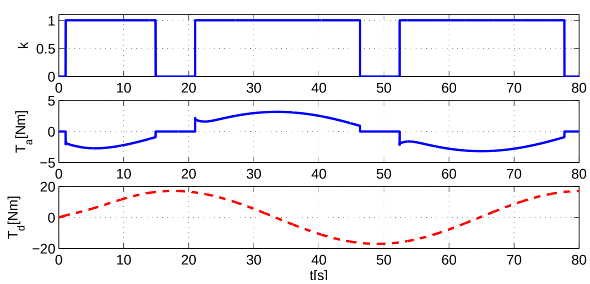

Fig. 6. Time histories of the variable k, T_a and T_d for the system (6) with assistant control.

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