Measurements of Optical Radiation from High-Intensity Laser-Plasma Interactions

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To my family.
The role of the author

This section outlines the role of the author in the work presented in this thesis. Experiments on high power laser systems necessarily involve a large team of people. The author was involved in the experimental planning, and in setting up and running the experimental diagnostics. The experimental data presented in chapters 5 and 6 were collected and analysed by the author. The model of chapter 3 on the emission of coherent transition radiation is the work of the author. The experimental data presented in chapter 7 were taken by the author. Most of the data analysis of chapter 7 has been performed by J. Schreiber. The analytical model and numerical work on the nonlinear pulse evolution is the work of the author. The particle-in-cell simulations presented in this thesis were made with the code OSIRIS and were all run and analysed by the author, unless otherwise cited. The OSIRIS code was developed by the OSIRIS consortium (UCLA/USC/IST).

Declaration

I hereby certify that the material of this thesis, which I now submit for the award of Doctor of Philosophy, is entirely my own work unless otherwise cited or acknowledged within the body of the text.

Signed,

Claudio Bellei
29th October 2009
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Abstract

This thesis presents experimental and theoretical results on the interaction of high-intensity lasers with solid and gaseous targets. All the measurements that are described belong to the optical region of the spectrum.

The interaction with solid targets has been investigated for two different intensity regimes. Intensities of up to $10^{21}$ W cm$^{-2}$ have been accessed on the VULCAN laser system at the Rutherford Appleton Laboratory whereas the JETI laser system at the Institut für Optik und Quantenelektronik in Jena allowed to reach intensities of up to $4 \times 10^{19}$ W cm$^{-2}$. For both regimes, the transport of relativistic electrons generated in the interactions has been investigated through measurements of the optical radiation emitted from the rear surface of the solid targets. Polarimetry and angular distribution measurements indicate that the radiation presents a high degree of polarisation and is non-isotropically emitted. It is, therefore, mainly attributed to transition radiation. A theoretical model has been developed in order to interpret and validate the experimental observations. As a result, for the high intensity regime variation of the signal strength of the transition radiation with respect to the direction of observation is attributed to the presence of $\mu$m-scale filaments.

The interaction with gaseous targets has been investigated at the Astra Gemini facility at the Rutherford Appleton Laboratory, for peak intensities of up to $3 \times 10^{19}$ W cm$^{-2}$ in a spot size of 20 $\mu$m FWHM. In this experiment the properties of the laser pulse were studied after interaction with the targets. For this purpose, a second harmonic generation FROG device was used. This allowed to determine both the pulse duration and the temporal phase of the pulse, giving an insight on the dependence of the pulse properties with respect to interaction length and electron number density. The experimental results show that the nonlinear evolution of the pulse can lead to compression from 45 fs before the interaction to a single pulse of below 20 fs duration, after propagating in the gaseous medium.
# Contents

List of Figures.  
Chapter 1. Introduction  
Thesis outline  
Chapter 2. Models relevant to high-intensity laser-plasma interactions  
  2.1. The framework of our experimental investigations  
    2.1.0.1. Propagation of electromagnetic waves in plasmas.  
    2.1.0.2. Laser intensity.  
    2.1.0.3. Pulse duration.  
  2.2. Absorption of electromagnetic waves in plasmas.  
    2.2.1. Inverse bremsstrahlung (collisional absorption)  
    2.2.2. Resonance absorption  
    2.2.3. Vacuum heating (Brunel heating)  
    2.2.4. jxB heating  
    2.2.5. Experimental results on laser absorption  
  2.3. Electron transport in overdense plasmas  
    2.3.1. Relevance to fast ignition  
    2.3.2. Relevance to ion acceleration  
    2.3.3. Fast electron propagation and Alfvén limit  
    2.3.4. More on the Alfvén limit  
    2.3.5. Electron scattering in plasmas  
    2.3.6. Beam instabilities  
  2.4. High-intensity laser interaction with underdense plasmas  
    2.4.1. Nonlinear pulse evolution in underdense plasmas  
      2.4.1.1. Propagation in a nonlinear crystal  
      2.4.1.2. Propagation in a partially ionised plasma  
      2.4.1.3. Propagation in a fully ionised plasma  
    2.4.2. Comoving frame and quasistatic approximation  
    2.4.3. Pulse distortion  
    2.4.4. Laser wakefield acceleration (LWFA)  

11  
21  
27  
29  
29  
29  
30  
31  
31  
31  
33  
35  
36  
37  
38  
38  
40  
41  
43  
44  
45  
46  
46  
47  
47  
47  
49  
50  
52
Chapter 3. Theory of Transition Radiation

3.1. The physics of transition radiation and the concept of formation zone
3.2. Properties of the transition radiation from a single charge
3.2.1. Energy and spectrum of the radiation
3.2.2. Angular distribution
3.2.3. Polarisation
3.3. Coherent Transition Radiation
3.4. Transition radiation in laser-solid experiments: a model
3.4.1. Physical explanation for the phase factor.
3.4.2. Analytical expression for the coherence function.
3.4.3. Useful formulas for the coherent radiation
3.4.4. Spectrum of the CTR
3.4.4.1. Broadening of the spectral lines - effect of the number of bunches.
3.4.4.2. Spectral decay of the radiation - effect of target thickness and fast electron temperature.
3.4.4.3. Differential energy VS target thickness.
3.4.5. Measuring the polarisation of transition radiation
3.4.6. Transition Radiation and PIC simulations

Chapter 4. Experimental methods

4.1. Diagnostic techniques
4.1.1. Polarisation measurements of optical radiation
4.1.2. Optical spectrometer
4.1.3. Electron spectrometer
4.1.4. Laser energy transmission
4.2. Pulse length measurements
4.2.1. Second harmonic generation
4.2.1.1. Basic equations in collinear geometry.
4.2.1.2. Phase matching condition.
4.2.1.3. Phase matching bandwidth.
4.2.1.4. Group velocity dispersion (GVD)
4.3. SHG Autocorrelation
4.4. SHG Frequency Resolved Optical Gating (FROG)
4.4.1. The GRENOUILLE
4.4.2. Pulse retrieval

Chapter 5. Angular distribution measurements of the optical rear emission

5.1. The JETI laser system
List of Figures

1.1 The stages required to achieve central ignition (top) or fast ignition (bottom). Adapted from Atzeni & Meyer-ter-Vehn [19]. 22
1.2 Conceptual design of the HiPER building. Copyright © 2009, HiPER Project. 24
1.3 Physical mechanism for the emission of transition radiation. 25
1.4 The ATLAS detector at the LHC. The inner detector is a transition radiation tracker (TRT). © The ATLAS experiment at CERN [30]. 26
1.5 a) In laser-solid interactions, coherent transition radiation arises when the electron bunches produced at the interaction region cross the rear surface of the target. Shown in figure are the bunch length $\delta t$ and the bunch delay $\Delta t$. b) In laser-gas experiments, emission of optical radiation can be induced by placing an external radiator in the direction of the laser beam. 27

2.1 Sketches of the propagation mechanisms for different plasma conditions. a) and b), overdense plasma; c) and d), underdense plasma. Note that the wavelength is longer in the plasma rather than in vacuum. This is due to the index of refraction in the plasma, $\eta < 1$. 32
2.2 Mechanism of resonance absorption. 34
2.3 2D3V Osiris PIC simulation of the interaction of a high-intensity laser pulse with a solid target, with initial density $n = 100n_{cr}$. a) Electron density in the simulation box $xy$. The laser is directed from the left of the box to the right. Due to the hole boring process, the laser pushes the critical surface inwards. Electron bunches are visible, separated by a characteristic wavelength which corresponds to half the laser wavelength. The presence of electron bunches is also clarified by the phase-space plot $x-p_x$ in b). 37
2.4 Ignition energy as a function of the fuel density at stagnation. The dot-dashed line assumes that it is always possible to have optimal range and beam radius, as in equation (2.9). The dashed line introduces a constraint to the beam radius, with no dependence on particle range. Finally, the solid curve has constraints on both particle range and beam radius. The parameter $f_R$ is defined in (2.10) and $\lambda_{ig}$ is the wavelength of the igniting beam. 39
2.5 Dependence of the quantity $X$ (related to the maximum obtainable ion energy) on $\tau_L/\tau_0$, as given in equation (2.12). \hfill 41

2.6 a) Schematic of the filamentation and two stream modes, showing the direction of the wave vector and of the electric field of the mode. The beam is supposed to be directed along the $z$ axis. Also shown are two sketches of the density profiles produced by the b) filamentation and c) two stream instabilities. \hfill 46

2.7 a) Density, b) intensity and c) frequency variations determine changes in the local group velocity. As a result, a pulse can be compressed from the back or from the front. \hfill 51

2.8 An example of a wakefield produced by a laser pulse with $a_0 = 3$ and $c\tau_L = \lambda_p$. These curves are obtained from the solution of the Poission’s equation (2.34). \hfill 52

3.1 Differential energy spectrum of the transition radiation for $\gamma = 1000$ and a single aluminium/vacuum interface. The plasma frequency for aluminium is $\hbar\omega_p = 32$ eV. \hfill 57

3.2 Angular distribution of the transition radiation radiation for a particle directed normal to the interface. \hfill 58

3.3 Angular distribution of the transition radiation as a function of the observation angle, for a) an electron traversing normal to the interface and b) at an angle $\psi = 40^\circ$. The curves are obtained for two values of the electron energy, $\gamma = 2$ and $\gamma = 26$. \hfill 59

3.4 a) Dipole radiation field induced by a driving field $E_i$. For a linear medium, the polarisation vector is parallel to $E_i$. b) Geometry and variables introduced for evaluation of the collective transition radiation field. c) Parameters relevant to the evaluation of the direction of oscillation of the transition radiation field, for an electron moving normal to the interface. The radiation is observed in the plane $xz$. \hfill 60

3.5 Polarisation properties of transition radiation. a) and b) For a charge crossing normal to the interface, the radiation is radially polarised. The magnetic field, not shown, would be azimuthal. c) In general, there is a component of the electric field parallel and normal to the radiation plane. \hfill 61

3.6 a) The geometrical parameters involved in the calculations of transition radiation. b) The generation of electron bunches from a laser-solid interaction. \hfill 63

3.7 Parameters used for the evaluation of the phase term for a single electron. \hfill 65

3.8 The coherent transition radiation emitted by an electron beam with a Gaussian transverse spatial profile is equivalent to the diffraction of a plane wave with
an aperture with a Gaussian transmission function. In the far field, the spatial
distribution of the radiation also exhibits a Gaussian profile, being maximum on
the symmetry axis.

3.9 a) A plot of the function \( f(\xi) \) for \( n_b = 3 \), showing the peaks at \( \xi = m\pi, m = 0, 1, 2, ... \) b) Variation of the number of bunches \( n_b \) as a function of the
normalized HWHM of the spectral lines, \( \xi_{1/2} \).

3.10 a) An example of the behaviour of the function \( |F_1(\tau)| \), given in equation (3.63),
for different temperatures \( u_t = 2 \) and \( u_t = 20 \). Parameters: \( \psi = 0^\circ, \phi = 90^\circ, \alpha = 0^\circ, \theta = 20^\circ \). b) Decay of the spectrum (dashed lines) for the same parameters. The peaks given by the term \( \sin^2(n_b\xi)/\sin^2(\xi) \) previously described are also given for completeness.

3.11 Differential energy \( dW/d\omega d\Omega \) VS target thickness for the case of an electron
directed towards target normal (\( \psi = 0^\circ \)) and an observation angle \( \theta = 20^\circ \).
The different colors refer to different fast electron temperatures. Figure b) is the same as a), but with a different horizontal axis in order to better show the behaviour for a target thickness below 100 \( \mu m \). This range incorporates most of the experimental values.

3.12 A lens system is viewing the target rear side in the direction of target normal,
while a second lens is viewing at an angle, in the horizontal plane. If an electron
producing transition radiation is directed towards the target normal, for the
first system there is no net polarisation; for the second system, the main net polarisation is horizontal.
In figure, the angles defining the position of the center of the lens are also shown,
that will be used for subsequent analysis. For example, for the lens viewing at
target normal it is \( \beta_1 = 0^\circ, \beta_2 = 0^\circ \).

3.13 A lens is viewing the target rear side along target normal (\( \beta_1 = 0^\circ, \beta_2 = 0^\circ \)).
If we allow the electrons to be directed at different angles in the \( yz \) plane
(\( \psi \in [-30^\circ, 30^\circ] \)), the resulting polarisation ratio varies as in b).

3.14 Same as for Figure 3.13, but for a lens positioned at 45° from target normal in the
plane \( yz (\beta_1 = 45^\circ, \beta_2 = 0^\circ) \). For electrons directed towards the imaging system
(\( \psi \approx 45^\circ \)) the ratio of the polarisations \( r \) is \( \approx 1 \).

3.15 Same as for Figure 3.14, but this time electrons do not move in the plane
\( yz (\alpha = 100^\circ \) instead of \( \alpha = 90^\circ \), see Figure 3.6a for an explanation of the
geometrical parameters used in this chapter). In this case, for electrons directed
towards the imaging system (\( \psi \approx 45^\circ \)) the ratio of the polarisations \( r \) is \( < 1 \).
3.16a) Ray tracing used to determine the spectral and angular distribution of the radiation. b) Spectral distribution of the radiation. 78

3.17 Angular distribution of the radiation emitted around the second harmonic of the laser frequency ($\lambda \in [488, 513]$ nm), showing the typical “two lobe” distribution of transition radiation. 78

4.1 Principle of operation of a Wollaston prism. 80

4.2 Calibration of the polarisation with a Wollaston prism. A He-Ne at 532 nm was sent through a sheet polariser and then through the optical beamline. In this configuration, the Wollaston prism allows both polarisations to be detected on the same CCD camera. a1) and a2) signal for sheet polariser with horizontal polarisation, for different integration times (as given in the figures). b1) and b2) signal for sheet polariser with vertical polarisation. 81

4.3 An example of a signal in a shot at the JETI laser in Jena. The signals of the two orthogonal polarisations were integrated at different cutoff intensities with respect to the maximum signal level: a) cutoff = 0.5, b) cutoff = 0.6; c) cutoff = 0.7, d) cutoff = 0.8. 81

4.4 Schematic of a Czerny-Turner spectrometer. 83

4.5 Sketch for non-collinear second harmonic generation. 85

4.6 a) Experimental set-up for the paper by Maker et al. [101] b) Intensity of the second harmonic as a function of the crystal angle, for the case of no pump depletion, eq. (4.82). 89

4.7 Principle of operation of a single shot SHG autocorrelator. 91

4.8 Schematic of a GRENOUILLE. In Figure, I. and IV. are cylindrical lenses, II. is a Fresnel bi-prism, III. is the nonlinear crystal, V. is a spatial filter (to block other beams other than the second harmonic) and VI. is a CCD camera. Adapted from reference [98]. 93

4.9 Example of a FROG retrieval from an experimental FROG trace. a) is the experimental trace and b) the retrieved trace, which is consistent with the pulse in c) and d) (temporal intensity and phase) or, equivalently, in e) and f) (spectral intensity and phase). 94

5.1 Focal spot images taken with an 8 bit CCD camera for two different filtering levels. 99

5.2 Experimental set-up. In figure, I. are Wollaston prisms, II. are interference filters at 400 nm and III. is a high-reflectivity filter at 800 nm. 100
5.3 Measured dose by the ionisation chamber as a function of laser energy, for a 2 \( \mu \)m, a), and 10 \( \mu \)m, b), Titanium foils.

5.4 Plot of the dose VS the corresponding energy of the optical radiation (at the second harmonic) for the 10 \( \mu \)m Ti foil, for the observation angle looking at 0° a) and laser axis b).

5.5 Correlation plots between the energy of the second harmonic, for the different imaging channels: a), b) and c) are plots for the 2 \( \mu \)m Ti foil; d) refers to shots with the 10 \( \mu \)m Ti foil.

5.6 Images at 0° for the case of a 10 \( \mu \)m Ti foil, for different interference filters placed in front of the CCD camera. The color scales are linear.

5.7 a), b) and c) Raw images for the three channels, for a shot with a 2 \( \mu \)m Ti target. a) laser direction (+45°), b) target normal (0°), c) −45° from target normal. d) An example of a result for 0° viewing angle, for a shot with a 10 \( \mu \)m Ti foil. We can appreciate the presence of an unpolarised background of radiation, on top of which is the radiation from the escaping fast electrons.

5.8 Plots of the polarisation ratios for 0° viewing angle, for different values of the laser energy. The laser was focused to a 2 \( \mu \)m Ti foil. a) and b) are the corresponding statistical averages evaluated according to expressions (5.89) and (5.90).

5.9 Plots of the polarization ratios for −45° viewing angle, for different values of the laser energy. The laser was focused to a 2 \( \mu \)m Ti foil. a) and b) are the corresponding statistical averages evaluated according to equations (5.89) and (5.90).

5.10 Theoretical ratio of horizontal/vertical polarisation for the observation angles at ±45° and different direction of electrons, for different electron temperatures (see chapter 3). In a), the range of angles defining the observation cone of the imaging system is highlighted. The behaviour of the curve around the observation cone is clarified in b).

5.11 Polarisation plot versus laser energy for the 2 \( \mu \)m foil and +45° observation angle.

5.12 Energy of the optical rear emission as a function of target thickness. The data correspond to statistical averages over, typically, 10 shots.

5.13 Energy of the optical rear emission as a function of \( I_A^2 \), for different parameters (observation angle, target thickness). The data correspond to statistical averages.
over, typically, 10 shots. The Beg’s and Wilks’ scaling laws are best fitted to the experimental data.

6.1 Experimental configurations. Experiments a) and b) were performed in TAP and c) in TAW. In a), I is a sheet polariser. In a) and c), II is an interference filter. Finally, in c), III is a Wollaston prism.

6.2 Optical spectra for different shots. Left column: results from experiment b) in Figure 6.1 (red spectrum: viewing angle at 55°; blue spectrum: viewing angle at 30°). Right column: experiment c) in Figure 6.1.

6.3 A laser pulse with a Gaussian spatial profile produces hole boring of the target front surface. Even assuming that the equi-phase surfaces are planes (i.e. the pulse is within its Rayleigh range) the deformation of the critical surface is such that, after some time, a fraction of the laser energy cannot be considered to be incident at normal anymore.

6.4 Optical images of target rear side for (a) 5, (b) 15, (c) 25 µm Au targets, for experiment a). (a) Left and central panes correspond to horizontal and vertical polarisation respectively, right pane is resulting polarisation map. (b) and (c) have been taken without a polariser. (d) Radius of emission evaluated at $\frac{1}{2}$, $\frac{1}{e}$ and $\frac{1}{e^2}$ of the maximum intensity.

6.5 An example of a 10 degrees wedge target. Image taken from an optical microscope, courtesy of C. Spindloe.

6.6 Sketches of the wedge targets used in experiment c), showing our definition of angle of the wedge $\alpha$ and angle of observation $\theta$.

6.7 Polarisation analysed OTR images; LHS horizontal polarisation, RHS vertical polarisation for a) 50 µm foil, b) 35° wedge, c) 10° wedge (here top is horizontal and bottom vertical polarisation). In c), the center is over-exposed to enhance target visibility.

6.8 Variation of polarisation with observation angle $\theta$ (or, equivalently, wedge angle $\alpha$). Experiment (black circles) and theoretical predictions for different directions of electron filaments $\delta$ (dashed lines). The shaded region is obtained assuming that the electrons are directed within the cone of the collection optics $\delta \in [25°, 42°]$.

6.9 Total normalised energy collected by the imaging system as a function of the angle of the electrons with respect to the front side target normal, for different electron temperatures (blue: 0.5 MeV, green: 2 MeV, red: 4.5 MeV, turquoise: 9.5 MeV). The two broken lines mark the range of angles included in the cone.
of the first lens. Each curve is re-scaled to its maximum value, in general more energy is radiated at higher temperatures.

6.1 Variation of signal intensity with observation angle $\theta$ (or, equivalently, wedge angle $\alpha$). Experiment (black circles) and theoretical fits (in arbitrary units) for different FWHM diameters of electron filaments (dashed lines).

6.1 Beam blooming (or better FWHM diameter of the electron beam) as a function of electron energy for an initially zero-radius beam that propagates through 50 $\mu$m of copper. The curves show Monte Carlo calculations, the analytical model as given by equation (6.95) and the same model where $\Lambda_s$ is varied by a factor of 10. Image courtesy of J. R. Davies.

6.1 Electron spectra for different wedge targets, measured by an electron spectrometer positioned on the laser axis (experiment c). Image courtesy of S. R. Nagel.

6.1 Number of electrons per unit time (i.e. current) as seen at the target rear side, after propagation through 50 $\mu$m of copper. The electron bunches are initialised as impulses at the target front side. Even with collisional effects the current is clearly modulated after propagation. Image courtesy of J. R. Davies.

7.1 Pulse evolution in the SVEA approximation, equations (7.105) and (7.100), for the case of $a_0 = 3$ and $k_0 = 100$. a1)-a4) Initial conditions ($\tau = 0$). b1)-b4) Pulse properties when $\tau = 80$. Note that figures a1) and a4) are identical to b1) and b4), which is due to the approximations made, while the instantaneous frequency of the pulse evolves from an unchirped pulse a3) to a positively chirped pulse b3).

7.2 Laser pulse and plasma properties for $k_0 = \sqrt{n_{cr}/n_e} = 25$ and after propagation through 1 cm of plasma, for a) $a_0 = 0.5$, b) $a_0 = 1.0$, c) $a_0 = 3.0$, d) $a_0 = 6.0$, e) $a_0 = 10.0$, e) $a_0 = 20.0$. The dashed line represents the original shape of the pulse envelope.

7.3 Results of 2D PIC simulations, for a laser pulse with $a_0 = 4.8$ propagating (from left to right) through a plasma with $n_e = 2 \times 10^{18}$ cm$^{-3}$. a) On-axis profile of the laser vector potential after 4.2 mm of propagation. b) On-axis electron energy spectrum after 9.6 mm of propagation. Image courtesy of S. P. D. Mangles, from reference [148].

7.4 Set-up of the experiment. The relevant diagnostics for our pulse measurements are the GRENOUILLE, optical spectrometer, probe (for the interferometric analysis of the plasma density) and the diode.

7.5 Schematic of the method used to determine the time direction of the reference pulse (left: early in time; right: late in time). From the FROG trace a) we retrieve
the pulse b) and we choose the time direction so as to give the correct chirp (positive). We then construct the Wigner transform c), which will be used as a reference for the other shots. In the Wigner transform we have also overlayed the trace of the instantaneous frequency (red curve).

7.6 FROG traces and Wigner transforms for three different laser pulses, starting (upper row) from the same reference shot in Figure 7.5. For the other shots, of the two possible Wigner transforms the one that presents the smallest change with respect to the previous shot is chosen (left: early in time; right: late in time).

7.7 FROG traces a1), b1), c1), d1) and corresponding Wigner transforms a2), b2), c2), d2) for different shots on a 5 mm gas-jet and varying electron density: a) vacuum shot, b) \( n_e = 2.3 \times 10^{18} \text{ W cm}^{-2} \), c) \( n_e = 3.8 \times 10^{18} \text{ W cm}^{-2} \), d) \( n_e = 4.7 \times 10^{18} \text{ W cm}^{-2} \).

7.8 Temporal intensity (upper row) and spectrum (lower row) for the same shots in Figure 7.7. In the spectral data, the black curve refers the spectrum retrieved from the FROG trace. The red broken curve refers to the spectrum measured from the independent optical spectrometer.

7.9 Pulse shapes for different interaction lengths: a) vacuum, b) 4 mm, c) 6 mm and d) 8.5 mm.

7.10 Dependence of the FWHM pulse duration with electron density a) and interaction length b). The broken line denotes the resolution limit of the GRENOUILLE device.

7.11 Energy transmission for the a) density and b) interaction length scans. The red and blue symbols are measurements inferred from the optical spectrometer and from the diode, respectively. The broken curves are constructed from equation (7.115), for different values of the fitting parameter \( k \).

7.12 Dependence of the pulse peak power with electron density a) and interaction length b).

8.1 Particle tracking in the 2D3V Osiris simulation. The laser, with an \( a_0 = 20 \) and a spot size of 5 \( \mu \text{m} \) FWHM, is incident (normal incidence) from the left of the simulation box to a 5 \( \mu \text{m} \) target with \( n_e = n_i = 100n_{cr} \). Resolution: 1.3 cells/plasma wavelength, 72 particles per cell (electrons+ions). The simulation was initialised with immobile ions and an electron temperature of 0.75 keV.

8.2 Electron density (top) and temperature (bottom) from LSP simulations, for a target with thickness of 50 \( \mu \text{m} \), \( Z=30 \) and \( \rho = 10 \text{ g cm}^{-3} \) and after 500 fs from the beginning of the interaction. The fast electron transport is investigated.
for different laser intensities at $10^{18}$ Wcm$^{-2}$, $10^{19}$ Wcm$^{-2}$ and $10^{20}$ Wcm$^{-2}$ (left-right). Image courtesy of R. G. Evans, from reference [136].

8.3 Wigner transform of the complex electric field for various propagation lengths in a simulation at $n_e = 1.75 \times 10^{21}$ cm$^{-3}$, for a pulse with a Gaussian waist of 21 µm and a peak normalised vector potential $a_0 = 2.75$. The laser envelope ($E^2$) is overlayed for clarity. Image courtesy of S. P. D. Mangles.
CHAPTER 1

Introduction

At the birth of the laser history, when Irnee D’Haenens$^1$ made the joke that the laser was “a solution looking for a problem” [1], it would have certainly been difficult to imagine that, among the uncountable applications, lasers would have allowed the pursuit of controlled thermonuclear fusion and the acceleration of electrons and ions with electric fields with several orders of magnitude better than conventional accelerators.

It is these applications that motivate much of the work done in the study of laser-plasma interactions, including the work presented in this thesis. More specifically, this work concerns high-intensity laser-plasma interactions. This terminology arises from the fact that, above a threshold usually quoted at $10^{18} \text{Wcm}^{-2}$, the quiver motion of the electrons under the influence of the laser field becomes relativistic. It is the advent of the chirped pulse amplification technique (CPA) in 1985 [2] that has allowed this regime to be reached. Today intensities in excess of $10^{20} \text{Wcm}^{-2}$ are possible in many laser systems around the world (RAL [3], Jena [4], Livermore [5], Los Alamos [6], Michigan [7], ...). At these intensities, all materials can be considered to be ionised in a few laser cycles and the laser mainly interacts with a plasma, a globally quasi-neutral system of electrons and ions.

When high-intensity lasers are focused against solid or gaseous targets, particles such as electrons, protons, neutrons, heavy ions, x-rays, $\gamma$-rays or positrons can be produced [8, 9, 10, 11, 12]. In particular, beams of relativistic electrons are routinely observed in laser-gas and laser-solid interactions.

Laser-gas interactions have proved to be the most interesting for electron acceleration. Indeed, the accelerating fields supported by plasma waves ($E \sim 100 \text{GV/m}$) can produce mono-energetic electron beams with energy spreads $\Delta E/E$ of a few percent [8, 13, 14], accelerating the electrons to GeV energies in a centimeter of propagation through a plasma [15, 16]. This must be compared to the accelerating fields of RF accelerators ($E \sim 100 \text{MV/m}$) that, for the same energy, would require a 10 m-long accelerating structure.

In laser-solid interactions, the quality of the electron beams, in terms of maximum energy and energy spread, is not appealing for table-top electron accelerators. In particular, the energy distribution is usually Maxwellian. However, these electron beams can be of

$^1$Irnee D’Haenens assisted Theodor Maiman at the Hughes Research Laboratory (Malibu, CA) in making the first laser in 1960.
interest as “energy carriers”, delivering energy from the interaction region to the plasma background through the course of their stopping distance. It is, essentially, this idea that has led Tabak et al. to the development of the fast ignition scheme to inertial confinement fusion (ICF) [17].

At the time of writing, it is a very exciting moment for ICF research. The indirect drive approach to ICF will be tested at the National Ignition Facility (NIF) in the US, that is due to start its ignition campaign later this year, and by the Laser Megajoule (LMJ) in France, expected to be operative within the next few years.

In the original approach to ICF (central ignition, Figure 1.1 top), a small (diameter of about 2 mm) spherical pellet made of an external ablator and of deuterium and tritium (DT) is uniformly illuminated by either laser pulses (direct-drive) or by x-rays (indirect-drive). In the latter case, the x-rays are produced by conversion of laser light in the walls of a holraum made of a high atomic number material [18, 19].

The radiation produces ablation of the surface of the pellet which drives a spherical implosion. As a result, part of the driver energy is transformed into kinetic energy of the imploding shell. At the end of the implosion (stagnation), this kinetic energy is then partially converted into internal (thermal) energy and a hot temperature DT plasma is formed (the hot spot). If the temperature, density and mass of the hot plasma are high enough, a self-sustained burning wave is produced and a sensible fraction (> 15-20 %) of the total DT atoms undergoes fusion reactions,

\[ \text{H}_2^2 + \text{H}_3^3 = \text{He}_4^4 + n + 17.6 \text{ MeV}. \]

**Figure 1.1.** The stages required to achieve central ignition (top) or fast ignition (bottom). Adapted from Atzeni & Meyer-ter-Vehn [19].
A figure of merit of an implosion is the gain, defined as the ratio of the total energy released from fusion reactions to the total laser energy, $G = E_f/E_L$. Fuel ignition requires that, at stagnation, the hot spot reaches an average temperature of 5 keV within a fuel areal density of about 0.3-0.4 g/cm$^2$, corresponding to the range of an $\alpha$ particle [17].

The 192 beams at the NIF should deliver up to 1.8 MJ of laser energy at the third harmonic of Nd:glass ($\lambda_{3\omega} = 351$ nm). The fusion of the DT atoms in the pellet should produce largely above 10 MJ of energy, with a gain $G > 10$ [20, 21].

A step towards a more efficient, thus cheaper, way to achieve fusion might be possible with the fast ignition approach to ICF previously mentioned [17]. In this scheme ignition is achieved in two stages: in a first stage the pellet is again compressed to high densities. However this time the energy delivered to the target is not sufficient for producing the burning wave. The “spark” that gives rise to the burning wave is in fact produced in a second stage, where an ultra-intense laser beam produces a beam of electrons to “ignite” the DT plasma (Figure 1.1 bottom). Alternative schemes suggest the use of laser-produced protons or heavy ions for ignition [22, 23], or of a secondary shock launched during the implosion of the capsule (shock ignition) [24].

The main advantage of the fast ignition concept is that the fuel assembly is much less sensitive to laser light nonuniformities and to the growth of hydrodynamic instabilities (in particular, the Rayleigh-Taylor instability), allowing for higher gain and lower driver energy. However, this is at the expense of uncertainties in the efficiency coupling from the second, ultra-intense beam, to the fuel. If this efficiency can reach 25%, an energy gain $G \approx 100$ could be achievable with 250 kJ of compression energy and 80 – 120 kJ of ignition energy [25, 26, 27].

This scheme will be pursued with the HiPER project (Figure 1.2), which has been included in the European Strategy Forum on Research Infrastructures (ESFRI) roadmap that lists 35 opportunities for major science facilities over the next 20 years.

In fast ignition, the ignition energy is crucially dependent on the conversion efficiency of the laser energy into hot electrons and on the temperature of the hot electron distribution. Recent Monte Carlo simulations show that the ignition energy can be well reproduced by $E_{ig} \approx \eta_h 22/[1 - \exp(-1.54/T_e)]$ kJ [27], where $\eta_h$ is the conversion efficiency into hot electrons and $T_e$ is the temperature of the distribution. This is true assuming an initially collimated beam of electrons impinging against the compressed DT core and that the energy coupling from the fast electrons to the hot plasma happens via Coulomb collisions or from excitation of collective modes (plasma waves). However, before interacting with the denser plasma the fast electron beam is produced at around the critical density.
In this case, electromagnetic instabilities could lead to beam filamentation and transport inhibition [28]. Such processes could significantly increase the required ignition energy. This explains why electron generation and transport from the critical surface to the dense core constitute the major issue of the fast ignition concept.

Some of the results of this thesis investigate the physics of fast electron transport by means of optical transition radiation, which is a process first studied with particle accelerators.

At this point it should not be surprising that there are similarities between the experimental techniques adopted in laser-plasma physics and those of high energy and nuclear physics. Electron spectrometers, Thomson parabolas, silicon diodes, ICT (Integrating Current Transformers), scintillators, photo-nuclear activation are some of the diagnostics and techniques typically used in laser-plasma experiments.

Transition radiation is produced when a charged particle passes through the interface of two media with different dielectric constant. The field of the particle excites the dipoles/free charges in the material which in turn radiate. The radiation is, in principle, emitted both in the forward and in the backward direction (Figure 1.3). An interesting property of transition radiation is that the amount of emitted radiation is strongly dependent on the Lorentz $\gamma$ factor of the particle, which is useful for particle identification when time-of-flight methods or Cherenkov radiators become inefficient. Cherenkov radiation has been discovered experimentally by Cherenkov in 1937 and explained theoretically by Frank and Tamm in the same year. It is produced when a charged particle travels through
a medium with a velocity $v$ larger than the phase velocity of the light in that medium, i.e. when $v \geq c/\eta$ ($\eta$ being the index of refraction); this defines a threshold velocity $\beta_{\text{th}} = v_{\text{th}}/c = 1/\eta$. For $\beta \geq \beta_{\text{th}}$, a coherent wavefront is then produced that moves at an angle $\theta$ from the particle’s trajectory that is given by $\cos \theta = 1/(\beta \eta)$. In Cherenkov detectors, particle identification can be achieved by using the property that Cherenkov radiation is only emitted above the threshold velocity. However, for ultra-relativistic particles transition radiation detectors are more efficient. For example, the ALICE and ATLAS (Figure 1.4) experiments at the LHC have transition radiation detectors that allow identification of electrons from the dominant background of more massive pions [29, 30].

Transition radiation has become a standard technique also in the accelerator community as a beam diagnostic. For this purpose, a foil is usually placed in the beamline at $45^\circ$ incidence and the radiation, collected at $90^\circ$, is then sent to a different set of diagnostics such as single or multiple shot autocorrelators, streak cameras or THz spectometers. From this diagnostic it is possible to determine the full 6-dimensional $(x, y, z, p_x, p_y, p_z)$ phase space informations on the electron bunch [31].

Transition radiation has been essentially used as a beam diagnostic for the investigations presented in this thesis. In a laser-solid interaction, transition radiation naturally occurs when the fast (relativistic) electrons produced at the interaction region cross the interface separating the target from the vacuum. The fast-varying oscillations of the laser field produce a modulated current of fast electrons, that is usually referred to as being composed of electron bunches, as in Figure 1.5a.

The properties of these fast electron beams are significantly different from those of particle accelerators. First, each bunch is of sub-fs duration [32], being smaller than the laser period, instead of several hundreds of fs. Secondly, the bunches are separated by
fs, instead of µs or above. As a result, in the case of laser-solid interactions coherent radiation can be emitted in the optical region. In fact, there are two levels of coherence: at wavelengths longer than the duration of a single bunch $\delta t$, each bunch can radiate coherently; at wavelengths longer than the delay between bunches $\Delta t$, there is coherence due to the envelope of the electron bunches (Figure 1.5a). This coherence effect happens already at optical wavelengths in the case of high-intensity laser-solid interactions. For accelerators, single-bunch coherence can be observed at the THz band ($\lambda \sim 10^{-4}$ m), while “envelope” coherence could, in principle, be observed at radio-frequencies, the threshold wavelength depending on the repetition rate of the system.

In the case of laser-gas interactions, in the wakefield acceleration scheme (LWFA) the duration of each electron bunch and their separation is expected to be of the order of a plasma wavelength which is dependent on electron density. However, considering that the plasma must be underdense, the plasma wavelength cannot be smaller than the laser wavelength ($\sim 1 \mu m$). Thus, coherent radiation should be manifest in the infrared region. Measurements performed at the Lawrence Berkeley National Laboratory have indeed confirmed that coherent transition radiation is produced at the gas-vacuum interface, in the THz region [33, 34, 35].

Measurements have also been made of the spectrum of the optical radiation emitted after placing a radiator (foil) in front of a gas-jet [36], as in Figure 1.5b. In this case, peaks in the spectra show the presence of coherent radiation also at optical wavelengths, suggesting the presence of a “fine” (sub-µm) structure of the electron bunch. The benefit

**Figure 1.4.** The ATLAS detector at the LHC. The inner detector is a transition radiation tracker (TRT). © The ATLAS experiment at CERN [30].
In laser-solid interactions, coherent transition radiation arises when the electron bunches produced at the interaction region cross the rear surface of the target. Shown in figure are the bunch length $\delta t$ and the bunch delay $\Delta t$. In laser-gas experiments, emission of optical radiation can be induced by placing an external radiator in the direction of the laser beam.

The use of the foil is two-fold: first, it blocks laser light from being collected in the optical system; second, it provides a sharp metal-vacuum interface where optical transition radiation can be more efficiently produced than at the interface between the gas-jet and the vacuum. In order to preserve this fine structure, the radiator has to be positioned close to the gas-jet exit; however, the minimum distance must be chosen so that the intensity of the transmitted laser pulse is below the ionisation threshold of the foil material.

This radiation could be sent to an autocorrelator to measure the duration of an electron bunch driven in a LWFA experiment. The long-term aim of measuring the electron bunch duration by means of transition radiation, not covered in this thesis, have triggered the experimental investigations presented in chapter 7. Instead of measuring the duration of the transition radiation signal, measurements have been performed on the temporal characteristics of relativistic ($a_0 \approx 3$) pulses propagating through gas-jets. This has allowed the first measurements, in this intensity regime, of both the envelope and phase of the transmitted beam. Such measurements reveal the complex nonlinear behaviour of high-intensity laser propagation in a plasma. The experience gained in using the techniques of short-pulse measurements could also, eventually, lead to measurements of the duration of the transition radiation signal in laser-solid interactions.

**Thesis outline**

This thesis is organised as follows:

**Chapter 2** provides a brief description of the relevant models of high-intensity laser-plasma interactions, with emphasis on the absorption mechanisms, hot electron transport and non-linear pulse evolution in an underdense plasma.
Chapter 3 introduces the theory of transition radiation, reviewing the main results for laser-solid interactions. Based on this, a model is developed that is used for interpretation of the experimental results described in chapters 5 and 6.

Chapter 4 describes the main experimental methods relevant to the subsequent chapters. Chapter 5 describes an experiment performed with the JETI Ti:sapphire laser system at the Institut für Optik und Quantenelektronik in Jena. This experiment was aimed at investigating the angular distribution properties of the optical radiation emitted from the rear side of solid targets, for intensities of up to $4 \times 10^{19}$ W cm$^{-2}$.

Chapter 6 presents measurements of optical radiation on the Vulcan laser at the Rutherford Appleton Laboratory, for intensities ranging from $8 \times 10^{19}$ to $\sim 10^{21}$ W cm$^{-2}$, showing a clear signature of the presence of coherent transition radiation (CTR). These measurements provide the first direct evidence for the presence of recirculating currents in laser-solid interactions. Moreover, they allowed the indirect determination of the filament size of the fast electrons with unprecedented spatial resolution, below the optical resolution of the imaging system.

Chapter 7 describes frequency resolved optical gating (FROG) measurements of the nonlinear evolution of the Gemini laser at the Rutherford Appleton Laboratory, after interacting with supersonic helium gas-jets of different lengths and densities. Compression and spectral broadening of the pulse were observed and the dependence of these processes with plasma density and interaction length will be presented.

Chapter 8 provides the summary and conclusions of the work described in the thesis. Possible extensions into future research are discussed.
CHAPTER 2

Models relevant to high-intensity laser-plasma interactions

This chapter describes the theoretical models used to interpret and validate the experimental results in the field of laser-plasma interactions. The first section sets the boundaries of our experimental investigations: we deal in this thesis with the interaction of high intensity, short pulse lasers with overdense and underdense plasmas.

The second section of this chapter describes the mechanisms of laser energy absorption into a plasma, with emphasis on the interaction with overdense plasmas. This is still a debated issue. Qualitatively, numerical and experimental results agree on the fact that the absorption fraction increases with $I\lambda^2$, in the presence of a preplasma and at some oblique angle of incidence [37]. Understanding the absorption of the laser energy is fundamentally related to the determination of the coupling of the laser energy into fast electrons produced in these interactions. Section 2.3 explains how laser-beam coupling and electron transport are of crucial importance for determining the energy of the igniting beam in the fast ignition approach to inertial confinement fusion and is also relevant for the optimisation of ion acceleration.

Finally, the interaction with underdense plasmas will be described, with an emphasis on nonlinear laser pulse evolution and wakefield generation.

2.1. The framework of our experimental investigations

Given a set of experimental or theoretical results, the instinctive reaction of a laser-plasma physicist is to ask about the type of plasma (underdense/overdense), laser intensity and pulse duration. At the zeroth order, the physics of a laser-matter interaction can indeed be categorised according to these three parameters. It is useful to briefly discuss their importance. As a result, we will understand the fundamental difference between an underdense and an overdense plasma. Furthermore, we will be able to appreciate that, for the experimental conditions relevant to this thesis, the electron motion under the influence of the laser field is relativistic and that the hydrodynamic response of the medium can be neglected on the timescale of the laser pulse duration.

2.1.0.1. Propagation of electromagnetic waves in plasmas. The linear dispersion relation for a planar electromagnetic wave, with amplitude $E(z,t) = E_0 \exp[i(k_0 z - \omega t)]$ in
vacuum, normally incident onto an unmagnetised, collisionless plasma is given by

\[ c^2 k^2 = \omega_L^2 - \omega_p^2, \]  

(2.1)

where \( k \) is the wave number and \( \omega_L, \omega_p \) are the wave and plasma (angular) frequencies. From (2.1) it follows that when \( \omega_L < \omega_p \) the wave is reflected and exponentially decreases in amplitude inside the plasma. The condition \( \omega_L > \omega_p \) \( (\omega_L < \omega_p) \) can be alternatively formulated in terms of electron density \( n_e < n_{\text{cr}} \) \( (n_e > n_{\text{cr}}) \) and defines an underdense (overdense) plasma. The term \( n_{\text{cr}} \) is defined as the critical density.

Assuming \( \omega_L^2 \ll \omega_p^2 \) (usually fulfilled for solid targets), for an overdense plasma the distance at which the amplitude has decreased by a factor \( 1/e \) (collisionless skin depth) is easily found to be

\[ \delta_s = \frac{c}{(\omega_p^2 - \omega^2)^{1/2}} \simeq \frac{c}{\omega_p}. \]  

(2.2)

2.1.0.2. Laser intensity. Under the influence of an electromagnetic wave, a charged particle is set into an oscillatory motion. The quiver velocity that the particle acquires is, in the non-relativistic limit, proportional to the inverse of the particle’s mass. As a consequence, in a plasma consisting of electrons and ions it is usually a good approximation to consider that the electromagnetic field is only coupled to electrons.

As the electric field increases its magnitude, so does the quiver velocity. To find out at which laser intensity a classical (intended as non-relativistic) description of the electron motion certainly breaks down, we can set the electron quiver velocity to be greater than the speed of light. In other terms, we can set

\[ a_0 \geq 1, \]  

(2.3)

where \( a_0 \) is the normalized vector potential in vacuum. Indeed, in the non-relativistic case the condition (2.3) is just

\[ a_0 \simeq \frac{\nu_{\text{os}}}{c} = \frac{eE_L}{m_e\omega_L c} \geq 1. \]

This expression can be reformulated in terms of the product of the average laser intensity, \( I = 1/2 c\varepsilon_0 E_L^2 \), and the square of the laser wavelength,

\[ I\lambda^2 \geq 10^{18} \text{ W\mu m}^2\text{cm}^{-2}. \]

This condition will be fulfilled for all the experimental parameters described in this thesis, and is usually considered to define the regime of high–intensity laser-matter interaction. In this regime, the relativistic plasma frequency is

\[ \omega_p^{\text{rel}} = \frac{\omega_p}{\sqrt{I_L}}, \]  

(2.4)
where $\gamma_\perp$ is the relativistic factor associated with the electron quiver motion. For a linearly polarised wave it is $\gamma_\perp \simeq \left[1 + \frac{1}{2a_0^2}\right]^{1/2}$, while in the case of circular polarisation, $\gamma_\perp = \left[1 + a_0^2\right]^{1/2}$ [38].

An interesting consequence of (2.4) is that a high-intensity laser pulse can propagate above the critical density, up to $n = \gamma_\perp n_{cr}$ which is called the relativistic critical density.

2.1.0.3. Pulse duration. The ablation produced in a laser-solid interaction causes the material to expand at about the speed of sound [39],

$$c_s = \left(\frac{Zk_BT_e}{m_i}\right)^{1/2} \simeq 3.1 \times 10^7 \left(\frac{T_e}{\text{keV}}\right)^{1/2} \left(\frac{Z}{A}\right)^{1/2} \text{ cm s}^{-1}.$$  (2.5)

For a pulse with a duration $< 1$ ps, assuming $T_e = 1$ keV and $Z/A = 0.5$ the plasma would expand for at most $c_s\tau_L \simeq 0.2 \mu$m during the interaction. This is smaller than the laser wavelength and, as a result, for such short pulses the hydrodynamic motion can be neglected. This is the case relevant to the laser systems described in this thesis. Ion motion is, however, important at later times and results in the expansion of the target.

### 2.2. Absorption of electromagnetic waves in plasmas.

2.2.1. Inverse bremsstrahlung (collisional absorption). As we have seen, in the case of an overdense plasma Maxwell’s equations give, as a solution, that an evanescent wave is produced inside the plasma. One would be tempted to say that the evanescence of the wave results in the absorption of electromagnetic energy into internal energy of the plasma. Actually, what we are dealing with here is a collisionless plasma for which the resistivity is zero. In other terms, the assumptions made when deriving the dispersion relation (2.1) imply that the medium has infinite conductivity, it is a perfect conductor. In the evanescence region, the field is oscillating and thus produces an oscillating current of electrons (“surface” oscillation). However, for a pulse with a finite duration the electrons stop their quiver motion as soon as the laser field is over. In the absence of collisions, the energy is thus re-emitted in the form of photon energy and the laser pulse is perfectly reflected, with no losses. A similar argument applies for the case of an underdense plasma, so that there is no net transfer of energy from the field to the electrons within the assumptions made.

In order to allow the field to transfer energy to the plasma we can consider energy transfer through collisions, or we need to consider some other (nonlinear) mechanisms of absorption. The first case is the inverse-bremsstrahlung process for which, as we will see, the wave vector has a real and imaginary part for both underdense and overdense plasmas (Figure 2.1). To recover the dispersion relation for the collisional case, we first notice that, in writing the dispersion relation (2.1), it is assumed that the momentum equation
2. MODELS RELEVANT TO HIGH-INTENSITY LASER-PLASMA INTERACTIONS

Figure 2.1. Sketches of the propagation mechanisms for different plasma conditions. a) and b), overdense plasma; c) and d), underdense plasma. Note that the wavelength is longer in the plasma rather than in vacuum. This is due to the index of refraction in the plasma, \( \eta < 1 \).

for the electron fluid is given by

\[
m_e \frac{\partial v_e}{\partial t} = -eE. \tag{2.6}
\]

If the collisional term \(-m_e v_{ei} v_e\) is included on the RHS of (2.6), we allow the transfer of energy from the electrons to the ions (assumed at rest) at the collisional rate \( v_{ei} \). The contribution of electron-electron collisions to the absorption is usually neglected, because at a given point in space all the oscillating electrons respond in the same way to the oscillating field and no relative velocities are produced. However, for a hot plasma at relativistic temperatures two electrons at the same point but with different energies respond differently to the field. In this case, electron-electron collisions can contribute to the absorption [40].

Under the main hypothesis that the kinetic energy gained in the quiver motion is much smaller than the electron thermal energy (non-relativistic laser pulses) [40], the collision frequency \( v_{ei} \) is given by

\[
v_{ei} = \frac{2^{1/2} n_i Z^2 e^4 \ln \Lambda}{12 \pi^{3/2} e_0^2 m_e^{1/2} T_e^{3/2}} \approx 5 \times 10^{-17} Z^2 n_i \text{[cm}^{-3}] \frac{(T_e [eV])^{3/2}}{(T_e [eV])^{3/2}} \text{ s}^{-1}.
\]

For solid targets it can be reasonably assumed that \( n_e = Z n_i \sim 10^{24} \text{ cm}^{-3} \) so that \( Z^2 n_i \sim \)}
For plasma temperatures ranging between $10^{-2}$ and $10^4$ eV, the collision frequency can span several orders of magnitude, $\nu_{ei} = 10^3 - 10^{12}$ s$^{-1}$.

With the inclusion of the frictional term $-m_e v_{ei} v_e$ in (2.6), the new dispersion relation reads

$$c^2 k^2 = \omega_p^2 - \frac{\omega_p^2}{1 + i\nu_{ei}/\omega},$$

from which, for an overdense plasma with $v_{ei} \ll \omega_L \ll \omega_p$,

$$k \simeq i \frac{\omega_p}{c} \left| 1 + i \frac{v_{ei}}{\omega_L} \right|^{-1/2}.$$

The corresponding (collisional) skin depth is then

$$\delta_s = \frac{c}{\omega_p} \left| 1 + i \frac{v_{ei}}{\omega_L} \right|^{1/2}.$$

The collisional skin depth is larger than the corresponding collisionless value (2.2). However, optical frequencies are high enough ($\omega_L \sim 10^{15}$ s$^{-1}$) that this effect is negligible in laser-plasma interactions.

Well before relativistic effects come into play, collisional absorption becomes an inefficient absorption mechanism. In fact, when the intensity increases the corresponding plasma temperature rises, however collisions become less efficient in absorbing the laser energy, since $\nu_{ei} \propto T_{ei}^{-3/2}$. In addition, if the electron distribution is not Maxwellian, the collisional rate will be further decreased. In general, collisional absorption starts to become less effective for

$$I \lambda^2 \geq 10^{14} \text{Wcm}^{-2} \mu \text{m}^2.$$

Although this condition is largely satisfied for high-intensity laser-matter interactions, above this threshold a high absorption efficiency is still observed. Therefore, different mechanisms of absorption must be considered, that do not directly rely on collisions of electrons with ions and are often defined as collisionless. They represent the main absorption mechanisms relevant to our experimental conditions and will be the subject of the next sections.

### 2.2.2. Resonance absorption.

In discussing the propagation of an electromagnetic wave normally incident on a plasma slab we pointed out that, in the nonrelativistic case, the propagation continues only up to the critical density. When a plane wave is obliquely incident, at an angle $\theta$ from normal, on a solid target with a preformed plasma with a density gradient $\nabla n$, the wave follows a path according to the well known ray equation

$$\frac{d}{ds} \left( \eta \frac{dr}{ds} \right) = \nabla \eta,$$
where \( \mathbf{r} \) is the position of the ray, \( \eta \) is the index of refraction and \( s \) is a curvilinear coordinate along the ray path. This implies that, under the conditions of oblique incidence and in the presence of a density gradient, the wave cannot reach the critical density. In fact, the wave can only propagate up to the density \( n_{cr} \cos^2 \theta \), after which it exponentially decreases in amplitude just as we have previously described. This can be seen by conserving the component of the wave vector which is orthogonal to the density gradient between infinity (vacuum) and the turning point: \( \omega/c \sin \theta = \eta_{t.p.} \omega/c \), where \( \eta_{t.p.} \) is the index of refraction at the turning point, \( \eta_{t.p.} = (1 - \omega_p^2/\omega_{t.p.}^2)^{1/2} \).

In this case, if a component of the electric field vector \( \mathbf{E} \) is in the direction of \( \nabla n \) (the case of a \( p \)-polarized pulse), the evanescent parallel component of the laser field can drive plasma oscillations in the direction of the density gradient (Figure 2.2). This process is particularly efficient where the natural frequency of the plasma oscillations \( \omega_p \) resonates with the laser frequency \( \omega_L \), i.e. where \( \omega_p = \omega_L \). In this region a highly nonlinear plasma wave can be produced, and collisions or particle trapping and wave-breaking can damp the amplitude of the oscillations thus absorbing wave energy [42]. This effect was first studied in the context of microwave propagation in the ionosphere, and then applied to laser-plasma interactions by Freidberg et al. in 1972 [43]. A number of later publications have investigated this process more in detail, for values of \( I \lambda^2 \leq 10^{17} \text{ Wcm}^{-2} \text{ m}^{-2} \) [44, 45, 46].

There is an optimum angle of incidence at which resonance absorption occurs. On the one hand, the tunneling distance between the turning point at which \( n = n_{cr} \cos^2 \theta \) and the resonance point at which \( n = n_{cr} \) increases with the angle of incidence. On the other hand, the component of the driving field \( E \sin \theta \) parallel to the density gradient increases with \( \theta \). Freidberg et al. find that maximum absorption, of about 50\%, occurs for an angle of incidence \( \theta \approx 23^\circ \). This value is also quoted in subsequent publications.

The energy transferred to the Langmuir wave ultimately leads to the production of a hot tail of electrons. Wilks and Krueer [42] write the temperature of the hot electron
component that is due to resonance absorption as

\[ T_{\text{hot}} \simeq 10 \left[ T_{\text{keV}} I_{15} \lambda_{\mu m}^2 \right]^{1/3} \text{keV} , \]

where \( T_{\text{keV}} \) is the electron background temperature. Strictly speaking, this result is only valid for long pulses \((c\tau_p \gg \lambda_p)\) but a qualitative agreement is expected for the case of ultra-short, relativistically intense pulses.

Results from PIC simulations show that the electrons accelerated by this process are mainly directed from regions of high density to low density [42], because the phase velocity of the electrostatic wave is directed towards the low-density regions [44].

Two- and three-dimensional effects can significantly affect the amount of absorption due to resonance absorption. The formation of a rippling of the electron critical surface when the intensity of an ultrashort laser pulse exceeds \(10^{19} \text{W} \mu \text{m} \text{cm}^{-2}\), due to a Rayleigh-Taylor-like instability, can in fact provide additional regions where \( \mathbf{E} \cdot \nabla n \neq 0 \), allowing for resonance absorption even at normal incidence or for an s-polarized laser pulse [42].

2.2.3. Vacuum heating (Brunel heating). Resonance absorption is not an efficient absorption mechanism when the density gradient is steep, i.e. when the amplitude of oscillation of the electrons exceeds the density scale length \( L, v_{\text{os}}/\omega_L > L \), where \( v_{\text{os}} = eE_L/m_e \omega_L \). In this case the plasma cannot support Langmuir waves. However, another efficient absorption mechanism can take place when the wave is \( p \)-polarized, as first discussed by F. Brunel in 1987 [47].

A simple model, based on a capacitor approximation, was used by Brunel to explain this mechanism. Consider a \( p \)-polarized laser pulse, with amplitude \( E_0 \), incident at an angle \( \theta \) on a relativistically overdense solid, with a steep-density profile. Assuming perfect reflection of the laser pulse, a standing wave is set up outside the target, with a longitudinal driving field that oscillates as \( 2E_0 \sin \theta \sin(\omega_L t) \). This component of the driving field can pull the electrons in the first layers of the target out into the vacuum. In principle, electrons can be also accelerated towards the target. However, the most energetic electrons that are produced with this mechanism are those that are accelerated into the vacuum. It is these electrons that interact with the vacuum field and are accelerated to a speed of \( 2v_{\text{os}} \sin \theta \) (in the non-relativistic limit) before re-entering the solid.

To a good approximation, these energetic electrons can be considered to be “lost” to the solid because inside the target the field is evanescent so that their energy can only be partially given back to the field. As a result, electrons are directly accelerated in the direction of the density gradient (target normal).

It is important to point out that the efficiency in energy absorption via vacuum heating is angular dependent, just as for the case of resonance absorption. The incident angle for
which the absorption efficiency is maximum is, in this case, $> 60^\circ$ and depends on the field amplitude and the reflectivity of the solid.

### 2.2.4. jxB heating

In the non-relativistic case, the ponderomotive force of a linearly polarised laser pulse acting on an element of electron fluid is given by

$$ f_p = -\frac{\varepsilon_0 \omega_p^2}{4 \omega_L^2} [1 - \cos(2\omega_L t)] \nabla E_L^2. $$

This expression is composed of a secular term which pushes the electrons out of the high-intensity region, steepening the electron density, and a time dependent component which makes the electrons oscillate at twice the laser frequency. Again, due to the evanescence of the wave inside the solid, this force will be the greatest in the first layers of the target.

The physical picture is, therefore, very similar to the vacuum heating mechanism. The result is that electrons oscillating at twice the laser frequency can gain energy and be accelerated in the laser direction. Electrons accelerated via this process present two main differences with respect to the electrons accelerated in the case of vacuum heating: for $j \times B$ heating, electrons are expected to be accelerated towards the laser direction, in bunches separated by half the laser wavelength (Figure 2.3); in the vacuum heating case, electrons are expected to be accelerated in the direction of target normal, in bunches separated by the laser wavelength.

In Figure 2.3a the effect of the hole boring process can also be appreciated: the radiation pressure exerted on the electrons is coupled to the ions via an electric field set-up by the space charge field. As a result, the laser acts like a piston and pushes the critical surface in the laser propagation direction. The dependence of the laser intensity on radius produces a crater inside the target, since the intensity is maximum at the symmetry axis.

Incidentally, the oscillating component in (2.8) vanishes for a circularly polarized laser beam. This realisation has triggered the idea that, by using circularly polarised laser pulses, it is possible to hinder electron heating, enough to allow the radiation pressure regime for ion acceleration to be reached, with intensities that are available on current laser systems ($I \lesssim 10^{21}\text{W cm}^{-2}$) [48, 49, 50, 51]. Using the simplest picture of a single fluid model, this can be explained by the fact that, in the radiation pressure regime, the driving force that is due to the radiation pressure $|\nabla p_L|$ must exceed the contribution due to the heating of the electron population $|\nabla p_e|$. In this way the target can be accelerated as a whole, with a very efficient scaling for the maximum ion energy that is $\propto (I_L \tau_L)^\alpha$, where $\alpha = 2$ in the non-relativistic limit and $\alpha = 1/3$ in the ultra-relativistic limit [52]. This must be compared to the energy efficiencies for TNSA [53] and shock acceleration [54] that both scale as $I_L^{1/2}$ and for which it is not clear if the relativistic regime for ion acceleration will be accessible.
2.2. ABSORPTION OF ELECTROMAGNETIC WAVES IN PLASMAS.

2.2.5. Experimental results on laser absorption. The most complete investigation on total laser absorption at relativistic intensities has been published by Ping et al. [55], for intensities ranging from $10^{17}$ W cm$^{-2}$ to $10^{20}$ W cm$^{-2}$. In this investigation, the absorption of the $p-$polarised laser pulse was measured at $6^\circ$ (almost normal incidence) and $45^\circ$ incidence. The authors find that the absorption is enhanced for the highest intensities, reaching $80\% - 90\%$ for the case of $45^\circ$ incidence and $60\%$ for near-normal incidence. Different targets were used, namely Al foils with thickness $1.5 - 100 \mu m$ and $400 \mu m$-thick Si plates. However, no significant dependence of the absorption on target thickness and material was observed.

The energy absorbed is mostly converted into energy of the fast electrons and ions. Published results on the absorption into fast electrons point to an absorption fraction into fast electrons between $10\%$ and $40\%$, for different laser systems and incidence angle [37]. As these values are significantly below the total absorption fractions measured by Ping et al. [55], we must conclude that a significant fraction of the laser energy is converted into ion (plasma) expansion [37] and into heating of the plasma.

An efficient absorption into fast electrons is important for the fast ignition approach to inertial confinement fusion, as will be clear in the next section. The importance of fast electron transport for ion acceleration will also be discussed in the following.

**Figure 2.3.** 2D3V Osiris PIC simulation of the interaction of a high-intensity laser pulse with a solid target, with initial density $n = 100n_{cr}$. a) Electron density in the simulation box $xy$. The laser is directed from the left of the box to the right. Due to the hole boring process, the laser pushes the critical surface inwards. Electron bunches are visible, separated by a characteristic wavelength which corresponds to half the laser wavelength. The presence of electron bunches is also clarified by the phase-space plot $x-p_x$ in b).
2. MODELS RELEVANT TO HIGH-INTENSITY LASER-PLASMA INTERACTIONS

2.3. Electron transport in overdense plasmas

2.3.1. Relevance to fast ignition. Since the original paper on fast ignition by Tabak et al. [17] it was recognised that the success of the fast ignition approach to ICF was crucially dependent on the coupling efficiency of the igniting beam to the fast (or suprathermal) electrons and on the coupling of the electron energy to the compressed core. As the laser energy is mainly absorbed at the critical density, this is where the majority of the fast electrons are produced. From this region the fast electrons have to be transported through the plasma up to the core. Tabak et al. proposed to use two beams in order, first, to push the critical surface closer to the core and then, with a second beam, to produce the fast electrons. Currently the idea is to use just one beam for both processes (hole-boring fast ignition) or to use a cone attached to the pellet in order to provide an open region for the igniting beam (cone-guided fast ignition) [56].

Besides the uncertainties involved in the transport of the fast electrons to the core, in 1999 Atzeni has shown with a series of two-dimensional hydrodynamic simulations that the minimum ignition energy and intensity of the igniting particle beam scale as

\[
E_{\text{ig}} = 100 \left( \frac{\rho}{100 \text{ g cm}^{-3}} \right)^{-1.85} \text{kJ},
\]

\[
I_{\text{ig}} = 2.4 \times 10^{19} \left( \frac{\rho}{100 \text{ g cm}^{-3}} \right)^{0.95} \text{Wcm}^{-2},
\]

for optimal values for the duration and radius of the igniting beam,

\[
t_{p} = 54 \left( \frac{\rho}{100 \text{ g cm}^{-3}} \right)^{-1.85} \text{ps},
\]

\[
r_{b} = 60 \left( \frac{\rho}{100 \text{ g cm}^{-3}} \right)^{-0.97} \text{\mu m}.
\]

These simulations were obtained assuming a beam of unspecified particles with straight trajectory and a range \(0.3 \leq R \leq 1.2 \text{ g cm}^{-2}\) impinging onto a precompressed sphere of DT with uniform density \(\rho\). A typical figure for the compressed density of the fuel is \(\rho = 300 \text{ g cm}^{-3}\), for which \(E_{\text{ig}} = 13 \text{ kJ}, I_{\text{ig}} = 7 \times 10^{19} \text{ Wcm}^{-2}, t_{p} = 7 \text{ ps and } r_{b} = 21 \text{ \mu m}\). For a beam of electrons with energy \(E_{\text{el}} = 1 \text{ MeV}\), these parameters correspond to a current and density of current of, respectively,

\[
J = \frac{eE_{\text{ig}}}{t_{p}E_{\text{el}}} \approx 2 \times 10^{9} \text{ A},
\]

\[
j = \frac{eE_{\text{ig}}}{t_{p}E_{\text{el}} \pi r_{b}^{2}} \approx 1 \times 10^{14} \text{ A cm}^{-2}.
\]
2.3. ELECTRON TRANSPORT IN OVERDENSE PLASMAS

More recent calculations of the minimum ignition energy [25, 57] have included the effects of non-optimal particle range \((R > 1.2 \text{ g cm}^{-2})\) and of the beam radius. In [25], the particle range was written as

\[
R = 0.6f_R T_{\text{hot}} \text{ g cm}^{-2},
\]  

(2.10)
in which \(T_{\text{hot}}\) is the energy of the (monoenergetic) particles and \(f_R\) is a parameter describing uncertainties in the determination of the electron stopping power in a plasma. The energy \(T_{\text{hot}}\) was further specified as a function of the intensity of the igniting laser beam and of its wavelength, using the scaling

\[
T_{\text{hot}} = 0.272(I_{18} \lambda_{\text{um}}^2)^{1/2} \text{ MeV}.
\]  

(2.11)

As a result, this analysis clarifies that the range of the fast particles and the effects of the focusing conditions can significantly affect the ignition energy (Figure 2.4). Shortening of the particle range \((f_R < 1)\) could decrease the required ignition energy. Recent calculations, that only consider scattering effects, set the value of \(f_R\) to \(\simeq 0.93\) for electrons of 1 MeV being stopped in a DT plasma at 300 g cm\(^{-3}\) [58]. Beneficial effects are also gained if a scaling that gives a lower \(T_{\text{hot}}\) is found to be more appropriate, for example Beg’s
scaling [59, 60]

\[ T_{\text{hot}} = 0.215 (I_{18} \lambda^2_{\mu m})^{1/3} \text{ MeV}. \]

Current estimates for the HiPER project are that the igniting beam should have an intensity in excess of \(10^{20} \text{ W cm}^{-2}\) and being frequency-doubled or tripled from the fundamental of the Nd:glass laser at \(\lambda_L = 1.06 \mu m\).

Of course, the total energy of the igniting laser beam must also account for the coupling efficiency into the generation of a forward current of electrons, \(E_{\text{ig}}^{\text{total}} = \eta_f E_{\text{ig}}\). From all these considerations, it is clear that the energy of the igniting beam is dependent on some critical parameters such as coupling efficiency and beam transport.

2.3.2. Relevance to ion acceleration. The modelling of target normal sheath acceleration (TNSA) has seen a large number of publications in the last few years, since the first models based on a self-similar expansion of the plasma into vacuum [61, 53]. The TNSA mechanism is strictly related to the generation of fast electrons at the target front surface. These fast electrons lead to the formation of an electron cloud at the rear side of the target, and a corresponding electric field. The positive charges (ions) in the first atomic layers feel the longitudinal electric field and are thus accelerated. In a first stage, the accelerated ions are only a small fraction of the total number of electrons that are present in the cloud. In this stage, ion acceleration can be treated by considering only test particles in the presence of the stationary sheath field. In a second stage, the plasma expands as a whole. However, the maximum energy of the ions is given in the first stage [62]. This greatly simplifies the analysis, although a detailed description of the ion spectrum must also account for the second stage of the acceleration mechanism.

Besides plasma expansion models, electrostatic models have also been used to predict ion energies [62, 63, 64]. With the exception of the model proposed by Schreiber et al. [64], all the published models are one-dimensional, but it is generally possible to include the dependence of the maximum ion energy on the divergence of the fast electron beam [65]. Schreiber’s model is based on the solution of the electrostatic potential for a disc of radius \(R\), which is related to the transverse dimensions of the fast electron beam at the target rear side. This approach allows a closed solution in two dimensions to be obtained; it gives an electrostatic potential that is bound throughout all the space, in contrast with most of the other models. For a laser pulse with duration \(\tau_L\), Schreiber et al. find that the maximum energy that an ion can gain in the sheath potential is given by the implicit equation

\[ \frac{\tau_L}{\tau_0} = X \left( 1 + \frac{1}{2} \frac{1}{1 - X^2} \right) + \frac{1}{4} \ln \frac{1 + X}{1 - X}, \]

(2.12)
2.3. ELECTRON TRANSPORT IN OVERDENSE PLASMAS

where $\tau_0$ is the ratio of the fast electron beam radius $R$ and the maximum velocity that an ion can gain in the potential well, for an infinite acceleration time, $\tau_0 = R/v_\infty$. Similarly, $X$ is related to the maximum energy $E_m$ that an ion can gain in the acceleration time $\tau_L$ and to the theoretical maximum ion energy $E_\infty$, for infinitely long acceleration, $X = (E_m/E_\infty)^{1/2}$. The quantity $E_m$ is the experimental maximum ion energy, that should be maximised. The quantity $E_\infty$ can be written as

$$E_\infty = 2Zm_e c^2 \left( \eta_f P_L / P_R \right)^{1/2} ,$$

where $P_R = m_e c^2 / r_e = 8.71 \text{ GW}$ ($r_e$ is the classical electron radius), $P_L$ is the laser power and $\eta_f$ is the absorption efficiency into hot electrons.

For what concerns the present thesis, we are mostly interested in the expression (2.12) because it predicts that higher energy ions are achievable for smaller source sizes. This is most clear in Figure 2.5, after noting that the quantity $X$ is an increasing function of $\tau_L/\tau_0$ and remembering that $\tau_0$ is proportional to the radius $R$ of the sheath field at the target rear side. Hence, understanding and controlling the divergence and transport of the fast electrons could prove important for the optimisation of ion acceleration.

2.3.3. Fast electron propagation and Alfvén limit. When a laser produces a current density of fast electrons $j_f$ in a solid target, the initial space-charge field produced by the beam is opposed by a return current $j_b$ from the background plasma that, with good approximation, restores the quasi-neutrality condition [66, 67]

$$j_f + j_b = j_{\text{net}} \approx 0 .$$

The return current can be actually provided by a displacement current as well. Due to the inertia of the plasma electrons, at the beginning of the propagation it is indeed the
2. MODELS RELEVANT TO HIGH-INTENSITY LASER-PLASMA INTERACTIONS

Displacement current that neutralises the beam current. At this early stage, and in a 1D situation, it is in fact \( \partial E / \partial t = -c^2 j_f \). After a time of the order of the reciprocal of the plasma frequency, the background plasma can then provide a return current [68]. The plasma return current is set by an electric field that can be calculated according to Ohm’s law as \( E = \eta j_b \), where \( \eta \) is the plasma resistivity.

During the transport process, a magnetic field grows in time according to

\[
\frac{\partial B}{\partial t} = -\nabla \times E = -\nabla \times (\eta j_b) \approx \nabla \times (\eta j_f) .
\]

The magnitude of the forward current allowed to propagate is strongly dependent on the resistivity of the plasma. This is all contained in the previous equation, however a clearer picture can be gained from a simple estimate, as done by Davies [68, 69, 70]. Assuming a constant resistivity and a constant beam current, the magnetic field grows as \( B \sim \eta j_f t / \pi R^3 \), where \( j_f = 2\pi R^2 j_f \) is the total beam current and \( R \) is the beam radius for a cylindrical beam. On the other hand, the magnetic field can also be calculated from Ampere’s law as \( B = \mu_0 j_{\text{net}} / 2\pi R \). Therefore we conclude that

\[
J_{\text{net}} \sim 2 \frac{t}{t_D} j_f ,
\]

in which \( t_D = \mu_0 R^2 / \eta \) is called the current decay time. The relation (2.14) is only valid for \( t \ll t_D \). The physics contained in this simple relation is, however, important: it states that the net current (beam current + return current) increases with time. Because we have assumed that the beam current is constant, this must be due to the fact that the return current is decreasing with time. This decrease of the return current with time is, in principle, justified by the presence of collisions and by the fact that the return current tends to separate from the beam current due to mutual repulsion (the latter effect is, however, not included in the previous derivation) [69].

When magnetic diffusion is also included in the calculations, the current decay time gives an estimate of the time required to reach the state \( J_{\text{net}} = J_f \) (instead of \( J_{\text{net}} = 2 j_f \) which is non physical as it must be \( J_{\text{net}} \leq J_f \)). At this time the magnitude of the return current is negligible compared to the beam current, and the condition for current neutrality (2.13) is not valid anymore.

However, long before arriving at this stage, the net forward current is limited by the Alfvén current \( J_A \). From equation (2.14), the time required to reach the state \( J_{\text{net}} = J_A \), called the magnetic inhibition time \( t_I \), can be estimated as [70, 68]

\[
t_I \sim \frac{t_D}{2 j_f / J_A} .
\]

For times \( t > t_I \), the beam current is limited by \( J_A \). The smaller the plasma resistivity, the longer is \( t_I \) and, therefore, the time for which a forward current greater than \( J_A \) can be
To conclude this paragraph, we stress that these simple estimates are only useful for the purpose of gaining a physical picture of the processes involved. More realistic models, that can only be numerically solved, show a more complicated physics. For example, if the plasma resistivity is allowed to vary then the magnetic field does not grow linearly in time anymore but saturates at a certain magnitude. This means that, in reality, a return current can always be provided by the plasma background [71].

2.3.4. More on the Alfvén limit. It is useful to estimate the particle number density of the fast electrons produced in a laser-solid interaction, with respect to the number density of the plasma background. Assuming that a fraction \( \eta_f \) of the laser energy \( E_L \) is converted into a current of fast electrons with energy \( T_e \), into a volume \( \pi r_L^2 c \tau_L \), the fast electron density is

\[
n_f \sim \frac{\eta_f E_L}{T_e \pi r_L^2 c \tau_L} = \frac{\eta_f I_L}{c T_e} \approx 10^{21} \eta_f I_{18}^{2/3} \lambda_{\mu m}^{-2/3} \text{ cm}^{-3},
\]

where we have used \( T_e \approx 215 (I_{18} \lambda_{\mu m}^2)^{1/3} \text{ keV} [59, 60] \).

Assuming \( \eta_f = 0.5 \), for a pulse with an intensity of \( 10^{21} \text{ W cm}^{-2} \) and \( \lambda = 1 \mu m \), it is \( n_f \sim 5 \times 10^{22} \text{ cm}^{-3} \). Considering that, in a solid-density plasma, it is \( n_b \sim 10^{24} \text{ cm}^{-3} \), it is a reasonable assumption that the electron beam is diluted with respect to the background plasma (\( n_f \ll n_b \)). However, in case of a large preplasma the beam density can be comparable to the background density.

A close analogy, first studied by Hannes Alfvén in 1939 [72], is the motion of cosmic rays in the interstellar medium. In this case it is also possible to assume that the beam is diluted. As a result, with a good approximation the fast particles are not subject to electric field (any space-charge field is suppressed by the presence of the background plasma). This is equivalent to assuming that the background behaves as a good conductor. In an electro-static analogy of two charged particles in a plasma, if the distance between the two charged particles is much greater than the Debye length (i.e. the charged particles are diluted in the plasma)

\[
\lambda_D = \sqrt{\frac{n_e T_e}{m_e \varepsilon_0}}
\]

the two particles will not influence each other, their Coulomb field being effectively screened by the background plasma.

In his original work, Alfvén considers a source of cosmic rays (mainly protons) that propagates from a plane situated at \( z = 0 \). The circuit can be thought to be closed by, for example, an identical beam of opposite charges (“negative rays” in the words of Alfvén) directed towards negative values of \( z \).
Even though the electric field can be neglected, the beam will still be subject to the self-generated magnetic field. This field can deviate the particles towards the axis. In fact, for a given current density $J_f$, above a certain radius the particles are directed backwards by the magnetic field. This radius defines a maximum current that can propagate in the forward direction. This is the Alfvén current $I_A$, which has a magnitude

$$I_A \sim 17\beta\gamma \text{kA}.$$  

2.3.5. Electron scattering in plasmas. In recent years a number of authors [73, 74, 58, 27] have tackled the problem of multiple scattering in plasmas. These works are all relevant to the physics of laser-solid interactions and fast ignition. Of course, in a real scenario self-generated electromagnetic fields should also be taken into account. However, it is currently believed that, at above solid density, the main contribution to the stopping of the electrons is due to binary collisions which rapidly damp the growth of electromagnetic modes [28].

Some inaccuracies have been found in references [73] and [74] by the authors of the successive works. Therefore, in the following we will base our discussion on the results of Solodov&Betti and Atzeni&Schiavi&Davies [58, 27].

Considering only Coulomb interactions, a fast electron in a background plasma can lose its energy by either binary collisions or by exciting collective effects (plasma oscillations). For impact parameters which are smaller than the plasma Debye length, collisions are essentially binary. However, at larger distances the collective response of the plasma leads to the excitation of plasma oscillations. In fact, in a plasma a beam of electrons can even produce a wakefield. This is at the basis of the energy doubling of the 42 GeV electron beam at the Stanford Linear Accelerator [75].

A fast electron can excite plasma oscillations in the neighborhood of its path, up to a distance [76, 58]

$$d \sim \frac{v}{v_{th}} \lambda_D = \frac{v}{\omega_p} = 17\beta \left( \frac{n_e}{10^{23}\text{cm}^{-3}} \right)^{-1/2} \text{nm},$$

where $v$ is the velocity of the electron and $v_{th}$ the thermal velocity and it is assumed that $v > v_{th}$ (otherwise plasma oscillations cannot be excited). Clearly, just as the pump laser that excites plasma waves loses its energy as it produces plasma waves, when charged particles produce plasma oscillations they lose energy and this is the origin for the stopping power induced by collective effects.

The relative importance of collective effects for the stopping power with respect to binary collisions can be estimated by using the results in [58]. The stopping power due to binary collisions is written as
\[
\left( \frac{dE}{ds} \right)_b = -2\pi r_0^2 m_e c^2 n_e \frac{1}{\beta^2} \left[ \ln \left( \frac{m_e^2 c^2 (\gamma - 1) \lambda_D^2}{2\hbar^2} \right) + 1 + \frac{1}{8} \left( \frac{\gamma - 1}{\gamma} \right)^2 - \left( \frac{2\gamma - 1}{\gamma^2} \right) \ln 2 \right].
\]

This formula comes from integrating the Möller cross section for electron-electron collisions. The stopping power due to collective effects (plasma waves) is, instead,

\[
\left( \frac{dE}{ds} \right)_{p.w.} = -2\pi r_0^2 m_e c^2 n_e \frac{1}{\beta^2} \ln \left( \frac{\beta c}{\omega_p \lambda_D} \right)^2.
\]

The ratio \( r \) between these two terms is thus

\[
r = \frac{(dE/ds)_{p.w.}}{(dE/ds)_b} = \frac{2\ln \left( \frac{\beta c}{\omega_p \lambda_D} \right)}{\ln \left( \frac{m_e^2 c^2 (\gamma - 1) \lambda_D^2}{2\hbar^2} \right) + 1 + \frac{1}{8} \left( \frac{\gamma - 1}{\gamma} \right)^2 - \left( \frac{2\gamma - 1}{\gamma^2} \right) \ln 2}.
\]

The relative importance of these two terms is weakly dependent on the electron energy \( \gamma \) and is of the order of 10\%, so that collective effects are not negligible. For example, for an electron with \( \gamma = 1.4 - 100 \) (\( E_{\text{kin}} = 0.25 - 51 \text{ MeV} \)) travelling through a plasma with \( T_e = 5 \text{ keV} \) and \( n_e = 7 \times 10^{25} \text{ cm}^{-3} \) (typical parameters in a fast ignition scenario for \( \rho = 300 \text{ g cm}^{-3} \)) we find \( r = 0.08 - 0.09 \). For parameters relevant to laser-solid interactions this parameter is slightly higher, mainly because of the lower temperature (i.e. smaller Debye length) and lower density (i.e. smaller \( \omega_p \)). As an example, for a fully ionised copper plasma (\( Z = 29 \)) for which \( n_e = 2.4 \times 10^{24} \text{ cm}^{-3} \) and assuming a temperature \( T_e = 100 \text{ eV} \), the ratio between the collective and binary stopping power is \( r = 0.15 - 0.17 \) for the same energy range (\( \gamma = 1.4 - 100 \)).

It is not surprising that the effects of binary collisions in (2.16) only include electron-electron collisions and not electron-ion collisions. In fact, for the purpose of evaluating the stopping power, electron-ion collisions can be considered to be an elastic process. However, electron-ion collisions are important in order to evaluate the scattering of electrons in plasmas. Parameters such as penetration depth, range straggling and beam blooming must therefore include all the informations in terms of electron-electron, electron-ion collisions and collective effects. For reference, we define here the penetration depth and range straggling as the mean distance \( \langle x \rangle \) and corresponding standard deviation \( S = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \) for an electron in its original direction of motion. Also, we define beam blooming as the standard deviation for the electron distribution in the transverse direction, \( B = \sqrt{\langle y^2 \rangle} \) [74]. Beam blooming describes the diffusion of the beam in the transverse direction, assuming that initially the particles have all the same direction.

**2.3.6. Beam instabilities.** When collisions are neglected, as it is usually the case for particle-in-cell simulations, condition (2.13) can be satisfied for any value of the fast elec-
tron current \( j_f \). However, the propagation of a beam of charged particles in a collisionless plasma is subject to several instabilities. These processes still constitute a very active area of research in plasma physics, with applications also in space plasma physics.

It is now recognised that filamentation (often called Weibel), two stream (Figure 2.6) or an oblique-mode instability can occur when an electron beam propagates through a collisionless plasma [77]. The filamentation instability is due to the growth of unstable electromagnetic modes and leads to the break-up of the beam into filaments with a size of the order of the collisionless plasma skin depth \( c/\omega_p \), which for a solid-density plasma is \( \sim 10^{-3} - 10^{-2} \ \mu\text{m} \). The two stream instability is an electrostatic instability, where the wave and electric field vectors are parallel to the beam propagation direction.

When collisions are included, resistive filamentation can occur. This is an extension of the filamentation instability in the presence of a collisional return current [78]. The presence of density modulations in the plasma background can also lead to strong filamentation of the fast electron current [79].

On a macroscopic level, the electron filaments can be subject to envelope instabilities such as beam hosing or beam sausaging [78].

2.4. High-intensity laser interaction with underdense plasmas

2.4.1. Nonlinear pulse evolution in underdense plasmas. The evolution of a laser pulse propagating in a medium, whether this be a nonlinear crystal or a partially or fully
ionised plasma, must obey Maxwell’s equations. We write here the relevant equations in their macroscopic form:

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \]
\[ \nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}. \]

For an unmagnetised and nondispersive medium, the electric displacement \( \mathbf{D} \) and magnetic field \( \mathbf{H} \) can be rewritten in the usual form,

\[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \]
\[ \mathbf{H} = \frac{\mathbf{B}}{\mu_0}, \]

so that the previous equations become

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2.17) \]
\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \frac{\partial \mathbf{P}}{\partial t}. \quad (2.18) \]

Taking the curl of (2.17) and substituting (2.18), we finally get

\[ \nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial \mathbf{j}}{\partial t} + \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}, \quad (2.19) \]

in which we have neglected a term containing \( \nabla (\nabla \cdot \mathbf{E}) \). Equation (2.19) is very general, and it can be simplified in the more specific cases of propagation in 1) a nonlinear crystal, 2) a partially ionised gas and 3) a fully ionised gas.

2.4.1.1. Propagation in a nonlinear crystal. If we want to describe the nonlinear propagation of a pulse in a crystal for harmonic generation, equation (2.19) can be simplified to

\[ \nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}. \quad (2.20) \]

This is the result of the fact that in a dielectric no free currents are present, implying that \( \mathbf{j} = 0 \).

2.4.1.2. Propagation in a partially ionised plasma. In a partially ionised plasma the full equation (2.19) should be solved, since both bound electrons and free currents are present,

\[ \nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial \mathbf{j}}{\partial t} + \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}. \quad (2.21) \]

2.4.1.3. Propagation in a fully ionised plasma. In this case the polarisation vector \( \mathbf{P} \) vanishes, as no bound electrons are present, and the resulting equation is

\[ \nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial \mathbf{j}}{\partial t}. \quad (2.22) \]
This equation is actually not ideal for studying the propagation of a laser pulse in a plasma, because of the presence of the time derivative of the current, $\partial j / \partial t$. We can indeed do a better job if we introduce the electrodynamic potentials $\Phi$ and $A$ such that

$$E = -\nabla \Phi - \frac{\partial A}{\partial t}, \quad (2.23)$$

$$B = \nabla \times A. \quad (2.24)$$

After substituting (2.24) into (2.18), using (2.23) and choosing a Coulomb gauge, $\nabla \cdot A = 0$, the new wave equation reads

$$\nabla^2 A - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = -\mu_0 j + \frac{1}{c^2} \frac{\partial (\nabla \Phi)}{\partial t}. \quad (2.25)$$

After defining the normalised units,

$$a = \frac{e}{m_e c} A, \quad (2.26)$$

$$\phi = \frac{e}{m_e c^2} \Phi, \quad (2.27)$$

and rewriting the current term as $j = ne c \beta$, equation (2.25) can be rewritten in the equivalent form

$$\nabla^2 a - \frac{1}{c^2} \frac{\partial^2 a}{\partial t^2} = -k_p^2 \frac{n}{n_0} \beta + \frac{1}{c} \frac{\partial (\nabla \phi)}{\partial t}, \quad (2.28)$$

in which $k_p = \omega_p / c = (e^2 n_0 / m_e e_0)^{1/2} / c$ and $n_0$ is the density of the unperturbed background plasma.

In a 1D situation, the transverse derivative of any quantity is null. If we define $z$ as the direction of propagation of the wave, from the Coulomb gauge $\nabla \cdot a = 0$ we find that the longitudinal component of the vector potential $a_z = 0$, so that only the transverse component $a_\perp$ matters. This implies that the wave equation needs only to be specified for the transverse component. Neglecting the last term, equation (4.80) can be thus rewritten as

$$\frac{\partial^2 a_\perp}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 a_\perp}{\partial t^2} = -k_p^2 \frac{n}{n_0} \beta_\perp. \quad (2.29)$$

The 2 scalar equations described by (2.29) contain 5 unknowns ($a_\perp, n, \beta_\perp$) so other equations are required to close the system. The other equations that are needed are Gauss’
law, the fluid continuity equation and the momentum equation,

\[
\frac{\partial^2 \Phi}{\partial z^2} = k_p^2 \left( \frac{n}{n_0} - 1 \right), \quad \text{Gauss’ law} \tag{2.30}
\]

\[
\frac{\partial n}{\partial t} = -c \frac{\partial}{\partial z} (n \beta_z) , \quad \text{continuity equation} \tag{2.31}
\]

\[
\frac{d}{dt} (\gamma u) = \frac{\partial \Phi}{\partial x} - c \frac{\partial a^2}{2\gamma \partial z} , \quad \text{momentum equation} \tag{2.32}
\]

2.4.2. Comoving frame and quasistatic approximation. It is convenient to perform
the following variable transformations in the wave equation (2.29): \( \xi = z - c \beta t, \quad \tau = t \).
In this way, from the chain rule it follows that \( \partial/\partial z = \partial/\partial \xi \) and \( \partial/\partial t = \partial/\partial \tau - c \beta \partial/\partial \xi \).
In practice, this variable transformation corresponds to having a coordinate system that
moves at speed \( \beta \). If \( \beta \) corresponds to the group velocity \( \beta_g \), this coordinate system
travels at the speed of the pulse and is called comoving frame. It is the mathematical
equivalent of having an infinite amount of cameras in the laboratory frame, that are able
to take a snapshot of the pulse and plasma characteristics when the pulse is in the field of
view of the camera. So, all the cameras are still in the laboratory frame and no relativistic
Lorentz boosts are required.

The transformation into the comoving frame is a standard technique in PIC simulations,
because it allows one to maintain the computational box of a size of about the laser
pulse envelope. In terms of the evolution equations previously discussed, it is useful in or-
der to simplify (quasistatic approximation) the system of equations (2.29), (2.30), (2.31),
(2.32). The physical origin of the quasistatic approximation follows from the following
consideration: in the comoving frame, we expect the fields \( a_\perp \) and \( \phi \) and the plasma
quantities \( n, \beta_z, \gamma \) to change on a time scale that is long compared to a plasma period. If
the laser pulse is shorter than this characteristic time of evolution, a quasistatic condition
can be assumed. In this case, the five equations for the nonlinear pulse evolution can be
approximated to two equations only.

Different results for the evolution equations can be found in the literature, after passing
to the comoving frame and using the quasistatic approximation [80, 81, 82, 83, 84]. The
earliest paper, by Sprangle et al., considered a frame moving at \( \beta_t = 1 \). This is correct,
because there is no constraint in the change of variables \( z, t \rightarrow \xi, \tau \). However, the frame
where the quasistatic approximation works best is the one that moves at the group velocity
of the pulse, \( \beta_g \).
Due to its simplicity, we prefer here to quote the result of Sprangle et al. [80]

\[
\frac{2}{c} \frac{\partial^2 a}{\partial \xi \partial \tau} - \frac{1}{c^2} \frac{\partial^2 a}{\partial \tau^2} = k_p^2 \frac{a}{1 + \phi} ,
\]

(2.33)

\[
\frac{\partial^2 \phi}{\partial \xi^2} = \frac{k_p^2}{2} \left[ \frac{1 + a^2}{(1 + \phi)^2} - 1 \right].
\]

(2.34)

This is valid for underdense plasmas for which \( \beta_g \approx 1 \). Considering that, from the standard dispersion relation, it is \( \beta_g^2 = 1 - \omega_p^2/\omega_0^2 \), this condition will be true for \( \omega_p \ll \omega_0 \), or \( n_0 \ll n_{cr} \). This set of equations will be used in Chapter 7 to understand the evolution of the envelope and phase of the pulse, for interpretation of the experimental results.

2.4.3. **Pulse distortion.** The interaction of a laser pulse in a plasma results in a feedback between the pulse and the plasma, as it is clear from equations (2.33) and (2.34).

In a 1D scenario and for short pulses \( c \tau_L \lesssim \lambda_p \), the most notable effects in the pulse characteristics are pulse compression and frequency variations. Pulse compression can be explained in an intuitive way with the argument given by Mori [85]. If two photons are initially separated by a distance \( L \), in the comoving frame the distance between them evolves according to

\[
\frac{1}{L} \frac{\partial L}{\partial \tau} = -\frac{1}{c} \frac{\partial v_g}{\partial \xi},
\]

(2.35)

where \( v_g \) is the local group velocity. Thus, if \( \partial v_g/\partial \xi > 0 \) the pulse compresses. In a collisionless, unmagnetised plasma the group velocity of a linearly polarised laser pulse is

\[
v_g = c(1 - \omega_p^2/\gamma_\perp \omega_0^2)^{1/2},
\]

(2.36)

where \( \gamma_\perp = (1 + a_0^2/2)^{1/2} \) for linear polarisation.

Expressing the index of refraction as \( \eta = (1 - \omega_p^2/\gamma_\perp \omega_0^2)^{1/2} \), a variation of the group velocity as in (2.36) is related to a variation in the index of refraction via \( v_g \simeq c \eta \) so that “group velocity” or “index of refraction” are two equivalent expressions. We stress that this is only true for a plasma, as in general the index of refraction is only related to the phase velocity.

The expression for the group velocity as given in (2.36) is *local* because all the quantities are functions of the comoving coordinate \( \xi \) (and of \( \tau \)): \( \omega_p = \omega_p(\xi, \tau), \gamma_\perp = \gamma_\perp(\xi, \tau), \omega_0 = \omega_0(\xi, \tau) \). Therefore, we conclude that variations of the group velocity can arise from local changes in density (through \( \omega_p \)), laser amplitude (through \( \gamma_\perp \)) and local frequency of the pulse (through \( \omega_0 \)).

In the non-relativistic case, it is possible to linearise (2.36) thus separating the effect of each of these terms. Denoting with \( \delta n \) and \( \delta \omega_0 \) the density and frequency perturbations, respectively, (2.36) can be rewritten as [85]
2.4. HIGH-INTENSITY LASER INTERACTION WITH UNDERDENSE PLASMAS

\[
v_g = c \left[ 1 - \frac{1}{2} \frac{\omega_p^2}{\omega_0^2} \left( 1 + \frac{\delta n}{n} - \frac{\langle a^2 \rangle}{4} - 2 \frac{\delta \omega_0}{\omega_0} \right) \right]. \tag{2.37}
\]

We first discuss the effect of the density perturbation. We note that in the interaction of a high-intensity short pulse with an underdense plasma the laser pulse is in a depleted region of electrons. This is due to the ponderomotive force, that pushes electrons out of the region of high-intensity producing longitudinal plasma waves in its wake. The dependence of the group velocity on the density perturbation, as in (2.37), reveals that the highest value of the group velocity is in the region where the density is the smallest, as it would be expected. Figure 2.7a presents a typical density distribution \( n_e(\xi) \), consistent with Poisson’s equation. This is obtained by solving (2.34) for a pulse with \( c \tau_L = \lambda_p \). As it can be observed, the density has a minimum at the back of the pulse. Hence, it is expected that the pulse compresses. As this happens, the density modulation is enhanced.

Regarding the effect of intensity, \( \partial a/\partial \xi > 0 \) at the back of the pulse and \( \partial a/\partial \xi < 0 \) at the front. According to (2.35), the effect is to compress the pulse at the front and stretch it at the back (2.7b).

The evolution of the instantaneous frequency of the pulse in the wakefield leads to a red-shift at the front (\( \delta \omega_0 < 0 \)) and a blue-shift at the back (\( \delta \omega_0 > 0 \)) [86]. Therefore, the pulse acquires a positive chirp and even in this case self-compresses (Figure 2.7c).
We further add that in a 2 or 3-D scenario the laser field can also be subject to self-focusing and can be guided over longer distances compared to its Rayleigh length in vacuum. This is due to the dependence of the index of refraction with the radius \( r \), \( \eta = (1 - n_c(r)/\gamma_\perp(r)n_{cr})^{1/2} \), which exhibits a maximum on the axis of propagation of the laser field \( (r = 0) \). In fact, it is along this direction that the laser intensity is maximised; hence, along this direction the electron density \( n_e(r) \) shows a minimum while \( \gamma_\perp(r) \) exhibits a maximum. The effects of the radial dependence of \( n_e \) and \( \gamma_\perp \) on the process of self-focusing are usually denoted as ponderomotive self-focusing and relativistic self-focusing, respectively.

2.4.4. Laser wakefield acceleration (LWFA). As we have discussed, the laser pulse evolution leads to the formation of plasma waves trailing the pulse itself. Regardless of the pulse length, as the laser envelope travels at the group velocity \( v_g \simeq \left(1 - \omega_p^2/\gamma_\perp \omega_\perp^2\right)^{1/2} \), it excites plasma waves with a characteristic wavelength of oscillation, the plasma wavelength \( \lambda_p \). In fact, if two fluid elements are separated by a distance \( \xi \) in the comoving frame, they will be set into motion with a delay \( \Delta t = \xi/v_g \). As the natural frequency of oscillation is given by \( \omega_p \), a delay \( \Delta t \) in time corresponds to a phase difference \( \Delta \phi = \omega_p \xi/v_g \). Setting \( \Delta \phi = 2\pi \), and solving for \( \xi \), the plasma wavelength is found to be \( \lambda_p = 2\pi v_g/\omega_p \simeq 2\pi c/\omega_p \).

Physically, these oscillations are set by the laser ponderomotive force (2.8): the front of the pulse pushes electrons in the forward direction and the back in the opposite direction. A resonant condition exists when the pulse length is about half of a plasma wavelength, \( c\tau_L \simeq \lambda_p/2 \).
Due to the space charge, the plasma wave produced with this mechanism has associated an electric field with a phase velocity given by the group velocity of the laser pulse, \( v_{ph} = v_g \). The sign of the electric field changes over \( \sim \) half a plasma wavelength and is suitable for accelerating negative charges in the direction of the laser beam over half of the plasma wave; the other half of the plasma wave would accelerate the particles in the opposite direction (Figure 2.8). This has led Tajima and Dawson [87] to the realisation that, if an electron is trapped in the electric field of the plasma wave, it can gain relativistic energies before outrunning the wave. The maximum energy that can be gained by a trapped electron occurs for acceleration over half a plasma wavelength, in which case the electron is only subject to an accelerating field. In a 3D nonlinear scenario and for \( a_0 > 2 \), the maximum energy that an electron can gain in the wakefield corresponds to [88]

\[
\gamma_{\text{max}} \approx \frac{2}{3} n_{cr} a_0.
\]

From this it is apparent that low density plasmas are the most interesting for electron acceleration to high energies. For example, for an underdense plasma with \( n = 10^{-3} n_{cr} \) and a pump strength of \( a_0 = 3 \), it is \( E_{\text{kin}}^{\text{max}} = (\gamma_{\text{max}} - 1) m_e c^2 = 1 \text{ GeV} \). This acceleration happens over the distance required to the electron to travel over half a plasma wavelength (in the comoving frame), called the dephasing length. This occurs because the group velocity of the laser pulse is smaller than the speed of light, while the electron can reach \( \beta \rightarrow 1 \) while being accelerated in the wake. In the same nonlinear 3D blowout regime, an expression for the dephasing length is [88]

\[
L_d = \frac{2}{3\pi} \frac{n_{cr}}{n} \lambda_p \sqrt{a_0}.
\]

If the length of the accelerating distance is longer than the dephasing length, the electron is decelerated in the other half of the plasma wave, and the resulting energy is lower than the one given in (2.38). It is also clear from (2.39) that although higher energies can be achieved with low density plasmas, this is at the expense of a longer accelerating length. This is due to the fact that the magnitude of the accelerating electric field decreases with decreasing plasma density. An expression for the maximum amplitude of the accelerating field is

\[
E_{\text{max}} = \frac{1}{2} \sqrt{a_0} E_0,
\]

where \( E_0 = m_e c \omega_p / e \).
CHAPTER 3

Theory of Transition Radiation

In the previous chapter we described some of the laser absorption mechanisms that lead to acceleration of electrons to relativistic energies (fast electrons). Depending on their energy, these electrons can be stopped inside the material or reach the rear surface of the target, and can emit radiation in different forms: bremsstrahlung, fluorescence (such as $K_{\alpha}$), Cherenkov, synchrotron and transition radiation. The characterisation of this radiation can therefore give information on the fast electrons that produced it.

This chapter presents the theory of transition radiation together with a model developed to interpret the experimental data presented in chapters 4 and 5.

3.1. The physics of transition radiation and the concept of formation zone

It is well known that Maxwell’s equations predict that a charge moving in free space at constant speed and along a straight path does not radiate. When the charge moves in a material medium, instead of the vacuum, radiation can be emitted in the form of Cherenkov radiation. If the medium has electric properties that vary in space or in time another type of radiation can be emitted; this is called transition radiation. The simplest problem of this type is the passage of a charge across a boundary between two media with different dielectric constant; this is also the problem that we are mainly interested in, particularly for a charge that crosses the interface between a plasma and the vacuum. Physically, transition radiation is produced by the polarisation field induced by the charge passing through the material. The field generated by the particle excites the dipoles/free charges in the material which in turn radiate.

It is often said that transition radiation is produced when the particle crosses the interface separating the two media. To be more precise, transition radiation is formed within a certain region close to the interface and along the direction of propagation of the electron. This region is called zone of formation of the transition radiation. Typically one considers the case of a particle traversing normal to the interface. In this case it is natural to speak about a formation length in the direction of the particle. A qualitative discussion of the subject is found in J.D. Jackson, "Classical electrodynamics" [76].
A general expression for the formation length for a medium with relative dielectric constant \(\varepsilon_r\) is \([89]\)

\[
L_f = \frac{c}{\omega} \left| \frac{\beta}{1 - \beta\varepsilon_r^{1/2} \cos \theta} \right| \tag{3.40}
\]

where \(\omega\) is the angular frequency of the emitted radiation and \(\theta\) the angle of emission (between the observer and the direction of motion of the particle). As there are two media involved, there is a formation zone for each of the media. For a given energy of the particle and angle of emission, the formation length only depends on the relative dielectric constant \(\varepsilon_r = 1 - \omega_p^2/\omega^2\) (true for a metal or a plasma) and is maximised in vacuum where \(\varepsilon_r = 1\). In the case of a metal, it is possible to consider \(\varepsilon_r \simeq 1\) for wavelengths in the x-ray region \((\hbar \omega > 1 \text{ keV})\), for which \(\omega \gg \omega_p\). In fact, for a typical metal it is \(\hbar \omega_p \sim 10 \text{ eV} \ll 1 \text{ keV}\) \([89]\).

From qualitative arguments, for normal incidence and at relativistic energies, Jackson finds that the radiation is only appreciable provided that

\[
\eta(\omega) \gamma \theta \leq 1,
\]

where \(\eta(\omega)\) is the index of refraction at the frequency \(\omega\). Therefore, the radiation is confined in the cone \(\gamma \theta \lesssim 1\).

In the case of \(\varepsilon_r \simeq 1\) and considering that the radiation is maximised for \(\theta \simeq 1/\gamma\) it is

\[
L_f \simeq \frac{c}{\omega} \gamma^2.
\]

For example, for a 0.5 \(\mu\)m wavelength and a 10 MeV electron, \(L_f \simeq 35 \mu\)m in vacuum.

For a good conductor (\(|\varepsilon_r| \gg 1\)) such as a plasma or a metal when \(\omega \ll \omega_p\), since the medium is overdense at the wavelengths considered \((\omega \ll \omega_p)\) we expect the formation length in the medium to be of the order of the skin depth; in other words, electrons beyond the skin depth should not contribute to the emission of transition radiation. In fact equation (3.40) is consistent with this, as when \(|\varepsilon_r| \gg 1\) equation (3.40) becomes

\[
L_f \simeq \frac{c}{\omega_p \cos \theta}.
\]

The factor \(1/\cos \theta\) accounts for the different observation angles. For \(\theta \simeq 0\), we indeed recover the expression for the skin depth \(L_f = c/\omega_p\).

Transversely, the formation zone extends over distances \(T_f \sim \gamma \lambda\) (see, for example, \([90]\)). For example, for \(\lambda = 0.5 \mu\)m and and a 10 MeV electron, \(T_f \simeq 10 \mu\)m.

The concept of formation zone serves to define when transition radiation is efficiently produced. The emission is most efficient for a “sharp” gradient of the index of refraction at the interface. If its scale length is longer than the formation length \(L_f\) or if the transverse dimensions of the interface are smaller than \(T_f\), a decrease in the efficiency
of emission is expected. Usually the transverse dimensions of the interface are not important as they are typically bigger than $T_f$. However, the longitudinal dimensions are probably the reason for the short duration of the CTR signal observed in experiments of laser-plasma interactions [32, 91], as before (in the presence of a prepulse) and during the interaction a density gradient can form at the target rear side.

The first experimental evidence for the effects of the formation zone in the emission of transition radiation was presented by Yuan et al. in 1970 [89]. These effects were investigated with a multi-GeV positron beam incident on uniformly spaced aluminium foils; the length of the formation zone in Al was investigated by varying the thickness of the foils: when the thickness was smaller than the expected formation length, the signal of the transition radiation rapidly dropped. Similarly, the formation zone in air (that can be considered as vacuum) was investigated by varying the interspace between the foils.

3.2. Properties of the transition radiation from a single charge

3.2.1. Energy and spectrum of the radiation. In the case of highly relativistic electrons traversing normal to a single interface that separates a medium with the vacuum, simple expressions have been found for the differential and total radiation intensities [92]
3. THEORY OF TRANSITION RADIATION

\[
\frac{d^2W}{d\theta d\omega} = \frac{2\alpha \hbar \theta^3}{\pi} \left(\frac{1}{1/\gamma^2 + \theta^2 + \omega_p^2/\omega^2} - \frac{1}{1/\gamma^2 + \theta^2}\right)^2,
\]

\[
\frac{dW}{d\omega} = \frac{\alpha \hbar}{\pi} \left[\frac{\omega_p^2 + 2\omega^2/\gamma^2}{\omega_p^2} \ln \left(1 + \frac{\gamma^2 \omega_p^2}{\omega^2}\right) - 1\right],
\]

\[
W = \frac{\alpha \hbar}{3} \omega_p \gamma,
\]

where \( W \) is the energy emitted and \( \alpha \) is the fine structure constant, \( \alpha = e^2/(4\pi \varepsilon_0 \hbar c) \). Notably, the expression for the total radiated energy (3.42) is proportional to \( \gamma \). This property of transition radiation has made it a useful tool for particle detectors, as explained in the introduction.

The differential energy spectrum of the radiation, equation (3.41), shows a roll-off as \( \ln(\gamma \omega_p/\omega) \) for \( \omega \ll \gamma \omega_p \) and as \( (\gamma \omega_p/\omega)^4 \) for \( \omega \gg \gamma \omega_p \) (this is the result of the expansion \( \ln(1 + x) \approx x - x^2/2 + x^3/3 \)). Therefore, it is usually considered that transition radiation has a cut-off in the spectrum at \( \omega \sim \gamma \omega_p \) (Figure 3.1).

3.2.2. Angular distribution. For a single electron normally incident onto the interface, the angular distribution of the radiation shows a typical two-lobe distribution, symmetrical with respect to the direction of the particle (Figure 3.2). Along the direction of the particle, no radiation is emitted. Moreover, as previously mentioned, in the relativistic case the radiation is confined within a half angle of emission of \( 1/\gamma \), just as for the case of synchrotron radiation.

When the angle of incidence is non-zero, a non-symmetrical distribution of the radiation appears. However, for relativistic energies the distribution is always confined around the direction of the particle (Figure 3.3).

**Figure 3.2.** Angular distribution of the transition radiation radiation for a particle directed normal to the interface.
3.2. PROPERTIES OF THE TRANSITION RADIATION FROM A SINGLE CHARGE

![Graph](image)

**Figure 3.3.** Angular distribution of the transition radiation as a function of the observation angle, for a) an electron traversing normal to the interface and b) at an angle $\psi = 40^\circ$. The curves are obtained for two values of the electron energy, $\gamma = 2$ and $\gamma = 26$.

### 3.2.3. Polarisation.

Another interesting property of transition radiation is that it is polarised. The reason for this is that the dipole oscillations induced in the medium by the charged particle produce polarised radiation; the collective effect is that the overall radiation is still polarised. In order to understand this statement in a more formal way we can take advantage of the arguments used by Jackson [76]. In the radiation zone the elementary dipole radiation field can be written as

$$dE_{\text{rad}} = \frac{1}{4\pi\varepsilon_0} \frac{e^{iR}}{R} (\mathbf{k} \times \mathbf{P}) \times \mathbf{k} \, d^3x,$$

where $\mathbf{P}$ is the polarisation vector, $\mathbf{k}$ the wave number and $R$ is the distance between the dipole at $x'$ and the observation point $x$. This distance can be written, in the far-field, as $R = |x' - x| \simeq r - \hat{k} \cdot x'$ (Figure 3.4), in which $\hat{k}$ is a unit vector in the direction of $\mathbf{k}$ (i.e. is the direction of observation, $\hat{k} = x/|x|$). Assuming the medium to be linear, the polarisation has the same direction of the driving field of the particle $E_i(x, \omega)$,

$$\mathbf{P}(x, \omega) \simeq [\varepsilon(\omega) - \varepsilon_0] E_i(x, \omega).$$

The total dipole radiation field $E_{\text{rad}}(x, \omega)$ can then be evaluated summing over all the
elementary dipoles in the medium (note that \( r \) is a constant),

\[
E_{\text{rad}} \simeq \frac{\omega_p^2}{4\pi c^2} \frac{e^{ikr}}{r} \int_{z'<0} (\hat{k} \times \mathbf{E}_i) \times \hat{k} e^{-ikx' \sin \theta} d^3x',
\]

where we are assuming that the charge passes from a medium at \( z < 0 \) to the vacuum at \( z > 0 \) and we have used \( \varepsilon/\varepsilon_0 = 1 - \omega_p^2/\omega^2 \). Note that, in obtaining the latter formula, we have replaced \( e^{ikR}/R \) with \( e^{ik(r-\hat{k} \cdot x')} / r \).

The integral (3.43) shows that, for a particle normally incident to the interface, transition radiation is radially polarised. In other words, the electric field oscillates in the plane defined by the directions of the target normal and the direction of observation, which is often called the radiation plane (Figure 3.5a and b). This can be seen from the fact that the direction of oscillation of \( E_{\text{rad}} \) is defined by the integral

\[
\iint (\hat{k} \times \mathbf{E}_i) \times \hat{k} e^{-ikx \sin \theta} dx dy = \iint [(E_\rho \cos \theta \cos \phi - E_z \sin \theta) \hat{e}_\parallel + E_\rho \sin \phi \hat{e}_\perp] e^{-ikx \sin \theta} dx dy,
\]

where the integral over \( z \) is missing because it does not influence the direction of \( E_{\text{rad}} \).

In (3.44), we have written the incident field \( \mathbf{E}_i(x, \omega) \) in terms of cylindrical coordinates \( (E_\rho, E_z) \) and we are assuming that the direction of observation \( \hat{k} \) belongs to the plane \( xz \) (Figure 3.4c). Furthermore, we have indicated with \( \hat{e}_\parallel \) and \( \hat{e}_\perp \) the unitary vectors in the radiation plane (the plane \( xz \)) and normal to the radiation plane, respectively.
3.3. Coherent Transition Radiation

So far we have discussed transition radiation as emitted from a single electron. In a more realistic case we must deal with the electron bunches produced in laser-solid interactions. In this case, coherent radiation can be emitted. In particular, in the case of $N$

![Diagram of polarisation properties of transition radiation](image)

**Figure 3.5.** Polarisation properties of transition radiation. a) and b) For a charge crossing normal to the interface, the radiation is radially polarised. The magnetic field, not shown, would be azimuthal. c) In general, there is a component of the electric field parallel and normal to the radiation plane.

After integration over $y$, we are left only with the term parallel to $\hat{e}_\parallel$ (the term parallel to $\hat{e}_\perp$ is odd in $y$ and integrates to zero) which demonstrates that the electric field is radially polarised. The magnetic field is, instead, transverse to the electric field.

For a generic direction of the particle, the radiation is not radially polarised anymore as a component normal to the radiation plane appears (Figure 3.5c).

### 3.3. Coherent Transition Radiation

So far we have discussed transition radiation as emitted from a single electron. In a more realistic case we must deal with the electron bunches produced in laser-solid interactions. In this case, coherent radiation can be emitted. In particular, in the case of $N$
electrons the intensity of the radiation at the angular frequency $\omega$ is, at a given observation point, proportional to

$$\sum_{i=1}^{N} |E_i^2| + \sum_{i,j=1; i \neq j}^{N} E_i \cdot E_j,$$

where $E$ is the electric field of the transition radiation produced by each electron. The first term, arising from the summation of the single spectra, is the so-called incoherent transition radiation (ITR), while the contribution of the interference of all the radiation fields is called coherent transition radiation (CTR). Clearly, the CTR signal can be orders of magnitude higher than the ITR signal, as the former is made of $N(N-1) \simeq N^2$ terms and the latter of $N$ terms.

In general, for a train of electron bunches the coherent addition is the result of the interference of the electromagnetic fields radiated by the electrons in each bunch, considered independently, and of the different bunches. This last term reflects the periodicity of the bunches and gives the typical spectral peaks at harmonic frequencies. In fact, when the electrons are separated by a time delay $\Delta T$ in a succession of short bunches, the CTR is peaked at the harmonics of $n\omega_{\text{bunching}}$, for $n = 1, 2, 3,...$. This means that the CTR is peaked at $n\omega_L$ for the case of resonance absorption and vacuum heating and $2n\omega_L$ for the case of jxB absorption.

The properties of transition radiation relevant to laser-solid experiments will be discussed in more detail, in the following.

### 3.4. Transition radiation in laser-solid experiments: a model

The theory of transition radiation relevant to laser-plasma interactions is discussed in Zheng et al. [93, 94] and Schroeder et al. [90]. In Zheng et al. the emission of transition radiation in laser-solid interactions is presented, by first assuming electrons directed along the normal to the plasma-vacuum interface [93] and then for a generalized direction of the escaping particles [94]. However, in the general case both the fields in the radiation plane (defined by the target normal and the observation vector) and perpendicular to it should be considered, as we have seen. Instead, as it turns out the formulas for the total emitted radiation in [94] are those in the radiation plane, and the other component is missing. The theory presented in [90] is complete, however it is more relevant to laser-gas interactions.

The theory for a single electron is actually already treated by, for example, Ter-Mikaelian [95]. The new message in the papers relevant to plasma physics is that, in case of a population of electrons, coherent radiation can be emitted and the appropriate distribution function has to be chosen.

In order to study the problem of emission by a beam of particles, it is convenient to define a set of angular and spatial coordinates related to the target normal direction,
3.4. TRANSITION RADIATION IN LASER-SOLID EXPERIMENTS: A MODEL

The geometrical parameters involved in the calculations of transition radiation. a) The generation of electron bunches from a laser-solid interaction.

The direction of the particle and the direction of observation. As we consider the case of a plane interface, it is natural to take a cartesian coordinate system where $z$ is the axis normal to the interface and the $x$ and $y$ axis lay in the plane of the interface (Figure 3.6a).

Following Schroeder et al. [90], the particle momentum vector is represented by

$$\mathbf{p} = |\mathbf{p}|(\sin \psi \cos \varphi, \sin \psi \sin \varphi, \cos \psi)$$

and the observation vector by

$$\mathbf{k} = \omega/c(\sin \varphi \cos \alpha, \sin \varphi \sin \alpha, \cos \varphi) ,$$

where all the variables used are clarified in Figure 3.6a. The total emitted transition radiation per unit angular frequency and unit solid angle can be written as

$$\frac{d^2W}{d\omega d\Omega} = \frac{e^2N}{\pi^2c} \int d^3\mathbf{p} g(\mathbf{p}) (\mathcal{E}_\parallel^2 + \mathcal{E}_\perp^2)$$

$$+ (N-1) \left( \left| \int d^3\mathbf{p} g(\mathbf{p}) \mathcal{E}_\parallel F \right|^2 + \left| \int d^3\mathbf{p} g(\mathbf{p}) \mathcal{E}_\perp F \right|^2 \right). \quad (3.45)$$

Here $\mathcal{E}_\parallel$ and $\mathcal{E}_\perp$ are the Fourier-fields in the plane parallel and perpendicular to the radiation plane and $F$ is a coherence function, or form factor, that takes into account the exact time and position at which electrons reach the interface. The momentum distribution function $g(\mathbf{p})$ is defined as the spatial integral of the six-dimensional phase space distribution $h(\mathbf{r}, \mathbf{p}), g(\mathbf{p}) = \int h(\mathbf{r}, \mathbf{p}) d^3\mathbf{x}$.

It is useful to discriminate between the incoherent and coherent component of the emitted radiation. From (3.45), the incoherent radiation is given by

$$\frac{d^2W}{d\omega d\Omega}_{\text{ITR}} = \frac{e^2N}{\pi^2c} \int d^3\mathbf{p} g(\mathbf{p}) (\mathcal{E}_\parallel^2 + \mathcal{E}_\perp^2) ,$$
and the coherent component by

\[
\frac{d^2W}{d\omega d\Omega}_{\text{CTR}} = \frac{e^2N(N-1)}{\pi^2c} \left( \left| \int d^3p \, g(p) E_{||} F \right|^2 + \left| \int d^3p \, g(p) E_{\perp} F \right|^2 \right)
\] (3.46)

Expressions for \( E_{||} \) and \( E_{\perp} \) can be found in Ter-Mikaelian [95] for the case of two media with generic dielectric constants. However, for our purposes we can assume that the interface separates a perfect conductor with vacuum, in which case the Fourier-fields simplify becoming

\[
E_{||}(\theta, u, \phi, \psi) = \frac{u \cos \psi [u \sin \psi \cos \phi - (1 + u^2)^{1/2} \sin \theta]}{[(1 + u^2)^{1/2} - u \sin \psi \cos \phi \sin \theta]^2 - u^2 \cos^2 \psi \cos^2 \theta},
\] (3.47)

\[
E_{\perp}(\theta, u, \phi, \psi) = \frac{u^2 \cos \psi \sin \psi \sin \phi \cos \theta}{[(1 + u^2)^{1/2} - u \sin \psi \cos \phi \sin \theta]^2 - u^2 \cos^2 \psi \cos^2 \theta},
\] (3.48)

where \( u \) is the normalised momentum, \( u = \gamma \beta \). It is important to note that these expressions have no dependence on the wavelength, i.e. the spectrum appears flat. This is true as long as we can consider the medium to be a perfect conductor, otherwise the wavelength dependence of the dielectric constant should be retained.

The expression for the coherence function depends on the phase term from the single-electron theory. A convenient expression is

\[
F = \frac{1}{g(p)} \int d^2r_{\perp} \int d\tau \, e^{ik_{\perp} \cdot r_{\perp}} e^{i\omega \tau} h(r_{\perp}, t, p). \] (3.49)

The spatial integral in (3.49) is over the surface of the interface, while the integration over time runs from \(-\infty\) to \(+\infty\). Expression (3.49) is consistent with Zheng et al. [94]. An equivalent expression is given by Schroeder et al. [90],

\[
F = \frac{1}{g(p)} \int d^2r_{\perp} e^{-ik_{\perp} \cdot r_{\perp}} \int_{-\infty}^{+\infty} dz \, e^{-i\omega(z - k_{\perp} \cdot v_{\perp})/\gamma z} h(r, p). \] (3.50)

In the next section we will clarify the origin of the phase term that arises in equation (3.50), which is equivalent to the phase term in (3.49). We will show that, in the far field, the phase \( \Phi \) for a wave produced by an electron that at time \( t \) is in the position \( (r_{\perp}, z) \) is indeed

\[
\Phi = k_{\perp} \cdot r_{\perp} + \frac{z}{\gamma z} (\omega - k_{\perp} \cdot v_{\perp}),
\] (3.51)

where \( k_{\perp} \) and \( v_{\perp} \) are the projections of the observation vector (wave vector) and of the velocity vector in the plane of the interface.

### 3.4.1. Physical explanation for the phase factor

The phase factor, equation (3.51), can be easily explained with physical arguments. Referring to Figure 3.7, let us suppose that two charged particles are in position \( (r_{\perp1}, z_1) \) and \( (r_{\perp2}, z_2) \). Furthermore, we will
Figure 3.7. Parameters used for the evaluation of the phase term for a single electron.

denote the respective velocities \( (v_{\perp 1}, v_{z1}) \) and \( (v_{\perp 2}, v_{z2}) \). An observer is looking at a distance \( L \) from the origin and the observation vector forms an angle \( \theta \) with the \( z \) axis. Note that we are assuming that the observation vector belongs to the \( x-z \) plane, however the result is general.

The two particles reach the interface after the time \( \Delta \tau_{1,2} = -z_{1,2}/v_{z1,2} \). When they reach the interface, the distance from the origin is

\[
\overline{OA}_{1,2} = r_{\perp 1,2} - v_{\perp 1,2} \frac{z_{1,2}}{v_{z1,2}},
\]

where \( r_{\perp} = |r_{\perp}| \) and \( v_{\perp} = |v_{\perp}| \).

The plane waves emitted at positions \( A_1 \) and \( A_2 \) reach the observer after having travelled for a distance \( L_1 \) and \( L_2 \), respectively. It is readily seen that

\[
L_{1,2}^2 = \overline{OA}_{1,2}^2 + L^2 - 2\overline{OA}_{1,2}L \sin \theta,
\]

where \( \theta \) has to be taken with its sign. Taking the difference of the two relations,

\[
L_1 - L_2 = \frac{\overline{OA}_1^2 - \overline{OA}_2^2 - 2L \sin \theta (\overline{OA}_1 - \overline{OA}_2)}{L_1 + L_2}.
\]

Assuming the emission to be in the radiation zone \( (L \gg \overline{OA}_1 + \overline{OA}_2) \), we find

\[
L_1 - L_2 \simeq \sin \theta (\overline{OA}_2 - \overline{OA}_1).
\]

At this point we can write down the phase difference between the two waves, which is the sum of a spatial and a temporal component. Concerning the spatial component, from (3.52) and (3.54) it is
\[ \Delta \Phi_s = 2 \pi \frac{L_1 - L_2}{\lambda} \]
\[ = k_{\perp} \cdot (r_{\perp 2} - r_{\perp 1}) - k_{\perp} \cdot \left( \frac{z_2}{v_{\perp 2}} v_{\perp 2} - \frac{z_1}{v_{\perp 1}} v_{\perp 1} \right), \]
while the temporal component is due to the time delay between the two particles when crossing the interface,
\[ \Delta \Phi_t = \omega \left( \frac{z_2}{v_{\perp 2}} - \frac{z_1}{v_{\perp 1}} \right). \]

Therefore, from the total phase difference \( \Delta \Phi = \Delta \Phi_s + \Delta \Phi_t \) we can conclude that the phase associated to the waves emitted at the rear surface is given by (3.51).

### 3.4.2. Analytical expression for the coherence function.

The six-dimensional distribution function that enters in equation (3.49) is chosen so as to model a train of \( n_b \) bunches (Figure 3.6b) with a Gaussian transverse spatial distribution and a Boltzmann momentum distribution,
\[ h(r_{\perp}, t, p) = K f(\psi, \phi) \sum_{k=-n}^{k=n} \delta[t - k\Delta t - d_0/(\beta c \cos \psi)] \]
\[ \times \exp \left\{ -\frac{|r_{\perp} - v_{\perp} (t - k\Delta t)|^2}{2r^2} \right\} \exp \left( -\frac{u}{u_t} \right), \quad (3.56) \]

Here \( K \) is a normalisation constant and \( f(\psi, \phi) \) is a generic angular distribution of the fast electrons. The \( n_b = 2n + 1 \) bunches are modeled with Dirac generalized functions that arrive at the interface at the times \( t = k\Delta t + d_0/(\beta c \cos \psi) \), where \( \Delta t \) is the time delay between two successive bunches (emitted with frequencies of \( \omega_L \) or \( 2\omega_L \)), and \( d_0 \) is the target thickness. The term \( d_0/(\beta c \cos \psi) \) takes into account that, although the electrons are generated in phase, after propagating through the target they reach the interface at different times, due to either a difference in (normalised) velocity \( \beta \) or in direction of propagation \( \psi \).

In the following we will always assume a collimated beam of electrons directed along the direction defined by the angles \( \psi_0 \) and \( \phi_0 \), so that
\[ f(\psi, \phi) = \delta(\psi - \psi_0)\delta(\phi - \phi_0). \]

Regarding the normalization constant, this quantity is found by imposing that
\[ \int dt d^2 r_{\perp} d^3 p h(r_{\perp}, t, p) = 1, \]
and has the expression
\[ K = (4\pi r^2 n_b \sin \psi_0 u_t^3)^{-1}. \]
3.4. TRANSITION RADIATION IN LASER-SOLID EXPERIMENTS: A MODEL

After inserting (3.56), (3.57) and (3.58) into (3.49), the coherence function reads

\[ F = \frac{1}{2n_b \sin \psi_0 \omega_t} \frac{\sin (n_b \omega \Delta t/2)}{\sin (\omega \Delta t/2)} \exp \left(-\frac{k_{\perp}^2 r^2}{2}\right) \]
\[ \times \exp \left[i \left(\omega \frac{d_0}{\beta c \cos \psi_0} - \frac{d_0}{\beta c \cos \psi_0} k_{\perp} \cdot v_{\perp}\right) \right], \] (3.59)

where \( k_{\perp} = k \sin \theta = (\omega/c) \sin \theta \) and we have made use of the Dirichlet kernel

\[ \sum_{k=-n}^{n} e^{ikx} = \frac{\sin[(n+1/2)x]}{\sin(x/2)}. \]

The term \( \sin (n_b \omega \Delta t/2) / \sin (\omega \Delta t/2) \) gives rise to peaks in the spectrum of the transition radiation, at the harmonics of the bunching frequency \( 1/\Delta t \). The term \( \exp(-k_{\perp}^2 r^2/2) \) in (3.59) is the result of the transverse Fourier transform of the beam spatial profile. It can be understood by analogy with classical diffraction in the far field (Fraunhoferapproximation: the radiation emitted from the rear surface can be thought of as the diffraction of a plane wave by a circular aperture with a Gaussian transmission function (Figure 3.8).

The phase terms \( \exp[i\omega d_0/(\beta c \cos \psi)] \) and \( \exp[id_0/(\beta c \cos \psi)k_{\perp} \cdot v_{\perp}] \) also account for the temporal and spatial coherence of the emitted radiation. The first term accounts for the longitudinal de-bunching of the electrons as they travel through the target. The latter term accounts for de-phasing induced by a transverse temperature \( T_{\perp} \) which leads to an increase in the beam size, as was pointed out by Zheng et al. [94]. However, due to our assumption of a collimated beam (\( T_{\perp} = 0 \)), this term will not play a role in our case.

3.4.3. Useful formulas for the coherent radiation. Once the full expression for the coherence function is determined, it is possible to proceed with the integration in (3.46),

**Figure 3.8.** The coherent transition radiation emitted by an electron beam with a Gaussian transverse spatial profile is equivalent to the diffraction of a plane wave with an aperture with a Gaussian transmission function. In the far field, the spatial distribution of the radiation also exhibits a Gaussian profile, being maximum on the symmetry axis.
In (3.61) we have introduced the quantities \( \beta \), which describe the effects of these terms separately.

Furthermore, we introduce the parameter \( \tau \), which depends on the effective foil thickness and the number of bunches \( n \).

It is possible to re-arrange equation (3.60) in order to make it in a more familiar form of a Fourier transform. We do this with the transformation \( \tau = d_0/(c \cos \psi_0) \cdot \sqrt{1 + u^2}/u \).

Physically, \( \tau \) represents the time required by an electron with normalised velocity \( \beta = u/\sqrt{1 + u^2} \) to travel through the equivalent target thickness \( d_0/\cos \psi_0 \). Similarly, \( \tau_0 \) is the time required by an electron moving at \( c \) to travel through the equivalent thickness. With this transformation, equation (3.60) becomes

\[
\frac{d^2W}{d\omega d\Omega}_{\text{CTR}} = \frac{e^2N(N-1) \sin^2(\omega \Delta t/2)}{4\pi^2 c n_b^2 u_t^6 \sin^2(\omega \Delta t/2)} \exp(-k_{\perp}^2 r^2) \\
\times \left[ \int_0^{+\infty} du u^2 E_{||} \exp \left( -\frac{u}{u_t} \right) \exp \left( i\omega \frac{d_0}{c \cos \psi_0} \frac{\sqrt{1 + u^2}}{u} \right) \right]^2 \\
+ \left[ \int_0^{+\infty} du u^2 E_{\perp} \exp \left( -\frac{u}{u_t} \right) \exp \left( i\omega \frac{d_0}{c \cos \psi_0} \frac{\sqrt{1 + u^2}}{u} \right) \right]^2, \quad (3.60)
\]

where \( E_{||} \) and \( E_{\perp} \) have been defined in (3.47) and (3.48).

In (3.61) we have introduced the quantities

\[
I_{1,2}(\tau) = F_{1,2}(\tau) \exp \left( -\frac{\tau_0}{u_t} \sqrt{\frac{\tau}{\tau_0}} \right), \quad (3.62)
\]

\[
F_1(\tau) = \frac{\tau}{(\tau^2 - \tau_0^2)^{5/2}} \left[ \frac{\cos \psi_0 \sin \psi_0 \cos(\phi - \alpha)}{\tau - \tau_0} \sin \psi_0 \sin(\phi - \alpha) \sin \theta \right]^2 - \tau_0^2 \cos^2 \psi_0 \cos^2 \theta, \quad (3.63)
\]

\[
F_2(\tau) = \frac{\tau}{(\tau^2 - \tau_0^2)^{5/2}} \left[ \frac{\cos \psi_0 \sin \psi_0 \sin(\phi - \alpha) \cos \theta}{\tau - \tau_0} \sin \psi_0 \sin(\phi - \alpha) \cos \theta \right]^2 - \tau_0^2 \cos^2 \psi_0 \cos^2 \theta. \quad (3.64)
\]

3.4.4. Spectrum of the CTR. The properties of the spectrum of the CTR depend on various parameters. In particular, the broadening of the spectral lines depends on the number of bunches \( n_b \) that constitute the electron beam, while the overall spectral decay depends on the effective foil thickness \( d_0/\cos \psi_0 \) and on the temperature \( u_t \). We will now describe the effects of these terms separately.
3.4. TRANSITION RADIATION IN LASER-SOLID EXPERIMENTS: A MODEL

3.4.4.1. Broadening of the spectral lines - effect of the number of bunches. Physically, we expect that the higher the number of the emitting bunches, the narrower the spectrum should appear at the harmonics of the bunch frequency. This is because, at non-resonant frequencies destructive interference should become more severe as we increase the number of interfering fields.

These effects are included, in eq. (3.61), in the term

\[ f(\xi) = \frac{\sin^2(n_b\xi)}{\sin^2(\xi)}, \]

where \( \xi = \omega\Delta t/2 \). The function \( f(\xi) \) has maxima at the harmonics of the bunch frequency \( \xi = m\pi \) or, in other words, \( \omega = m2\pi/\Delta t = m\omega_b \ (k = 0, 1, 2, \ldots) \). Due to the periodicity of this function, its general properties can be studied in the neighbourhood of \( \omega = 0 \). For \( \omega \to 0 \), \( f(\xi) \to n_b^2 \), therefore the FWHM width of the peaks is defined by twice the value \( \xi_{1/2} \) for which \( f(\xi_{1/2}) = n_b^2/2 \) (Fig. 3.9a).

Clearly, from the width of the harmonics it is thus possible to determine the number of bunches \( n_b \). A good numerical fit for \( n_b \) as a function of \( \xi_{1/2} \) is given by (Figure 3.9b)

\[ n_b = \frac{\pi}{2.25\xi_{1/2}}, \tag{3.65} \]

which, in dimensional units, translates into

\[ n_b = \frac{\pi}{1.12\Delta\omega_{\text{obs}}\Delta t} \approx 0.9 \frac{\lambda_{\text{obs}}}{n_h\Delta\lambda_{\text{obs}}}, \tag{3.66} \]

where \( \Delta\omega_{\text{obs}}, \Delta\lambda_{\text{obs}} \) are the experimental FWHM width of the peak (in terms of angular frequency and wavelength, respectively) and \( \lambda_{\text{obs}} \) is the mean wavelength of the peak. Furthermore, \( n_h \) is the ratio of the distance between two consecutive bunches and the wavelength of observation, \( n_h = c\Delta t/\lambda_{\text{obs}} \). For example, if the observed peak is at the 2nd harmonic of the laser frequency and one assumes that the bunches are emitted at the laser frequency, then \( n_h = 2 \). If, on the contrary, one assumes that the bunches are emitted at twice the laser frequency, then \( n_h = 1 \).

The expression on the right hand side of (3.66) is the value quoted in [96], except for the value 0.9 in front. The reason is that we find a better fit for \( n_b \) with eq. (3.65), instead of \( n_b = \pi/(2\xi_{1/2}) \) (Fig. 3.9b) that leads to the value found in the cited paper.

The total duration of the radiation \( \Delta t_{\text{CTR}} \) can also be estimated from the width of the harmonics as

\[ \Delta t_{\text{CTR}} = n_b\Delta t \approx 0.9 \frac{\lambda_{\text{obs}}^2}{c\Delta\lambda_{\text{obs}}}, \tag{3.67} \]

which is not dependent on the assumptions on the bunch frequency that are encoded in the parameter \( n_h \).
3. THEORY OF TRANSITION RADIATION

Figure 3.9. a) A plot of the function $f(\xi)$ for $n_b = 3$, showing the peaks at $\xi = m\pi$, $m = 0, 1, 2, \ldots$ b) Variation of the number of bunches $n_b$ as a function of the normalised HWHM of the spectral lines, $\xi_{1/2}$.

Figure 3.10. a) An example of the behaviour of the function $|F_1(\tau)|$, given in equation (3.63), for different temperatures $u_t = 2$ and $u_t = 20$. Parameters: $\psi = 0^\circ$, $\phi = 90^\circ$, $\alpha = 0^\circ$, $\theta = 20^\circ$. b) Decay of the spectrum (dashed lines) for the same parameters. The peaks given by the term $\sin^2(n_b\xi)/\sin^2(\xi)$ previously described are also given for completeness.

3.4.4.2. Spectral decay of the radiation - effect of target thickness and fast electron temperature. The integrals in (3.61) depend on the angular frequency of the radiation and give rise to a spectrum that decreases with $\omega$. This is a strictly coherent effect as the spectrum of the radiation from a single particle is flat, in our approximation of a perfect conductor. This was mentioned at the beginning of this section, on page 64.

The spectral decay of the radiation is ultimately related to the fact that we are dealing with a non-monoenergetic electron population. Even if the electrons are generated perfectly in phase (as we suppose in our model), a difference in velocity leads to a phase
3.4. TRANSITION RADIATION IN LASER-SOLID EXPERIMENTS: A MODEL

For a given wavelength of observation, the intensity of CTR decreases as the target thickness increases. This is, once more, a coherent effect. As the beam temperature increases, this dependence should be less prominent. In the limit of the fast electron temperature \( T_e \to \infty \), all the electrons travel at the speed of light and the radiation remains coherent, independently of target thickness.

This behaviour is clear from Figure 3.11, which demonstrates that the differential energy (defined as \( \frac{dW}{d\omega d\Omega} \)) decreases with target thickness. However, this effect becomes less evident as the fast electron temperature increases.

It should be noted that even for the case of incoherent transition radiation (ITR) it
is expected that the signal level should decrease with target thickness. This is due to scattering and losses of the fast electrons in the medium, which is an effect not included in the present analysis.

3.4.5. Measuring the polarisation of transition radiation. It is important to point out that the quantities $E_\parallel$ and $E_\perp$ that enter into (3.47) and (3.48) only refer to the components parallel and perpendicular to the radiation plane. In general, these quantities do not correspond to the directions that we refer to as “horizontal” and “vertical”, which are the quantities that we have actually measured in the experiments. In other words, we have that

$$
\frac{d^2W}{d\omega d\Omega} = \frac{d^2W}{d\omega d\Omega}^\parallel + \frac{d^2W}{d\omega d\Omega}^\perp = \frac{d^2W}{d\omega d\Omega}_{\text{hor}} + \frac{d^2W}{d\omega d\Omega}_{\text{ver}}. \tag{3.68}
$$

In this section we want to understand what are the relations between the terms in (1) and those in (2), in the previous formula. This is essential for interpretation of our experimental data. The quantities in (3.68) are all “local” quantities in the sense that they depend on the direction of observation. In a real experimental scenario, there is an infinite number

\[\text{Figure 3.12. A lens system is viewing the target rear side in the direction of target normal, while a second lens is viewing at an angle, in the horizontal plane. If an electron producing transition radiation is directed towards the target normal, for the first system there is no net polarisation; for the second system, the main net polarisation is horizontal. In figure, the angles defining the position of the center of the lens are also shown, that will be used for subsequent analysis. For example, for the lens viewing at target normal it is } \beta_1 = 0^\circ, \beta_2 = 0^\circ. \]
3.4. TRANSITION RADIATION IN LASER-SOLID EXPERIMENTS: A MODEL

Figure 3.13. A lens is viewing the target rear side along target normal ($\beta_1 = 0^\circ$, $\beta_2 = 0^\circ$). If we allow the electrons to be directed at different angles in the $yz$ plane ($\psi \in [-30^\circ, 30^\circ]$), the resulting polarisation ratio varies as in b).

of distinct directions of observations. They are defined, for example, by the aperture of a lens. Mathematically, this corresponds to integrating (3.68) over the solid angle.

The overall polarisation of the radiation is crucially dependent on the geometry of the collecting optics. As an example, consider the idealised case of an electron directed normal to the interface plasma-vacuum. A first lens is placed exactly on the symmetry axis, and focuses the light onto a detector. A second lens is placed in the horizontal plane, at a certain angle from the $z$ axis (Figure 3.12), and focuses the light on a second detector. Suppose the two detectors are equipped with two polarisers, enabling a measurement of the polarisation. We expect the first detector to collect an overall unpolarised radiation, while the second one will mainly detect horizontally polarised radiation.

The reason is that the first lens integrates radiation that is radially polarised. Due to the symmetry of the system, there is no preferential polarisation. The second system instead “breaks the symmetry” of the geometry, and mainly detects horizontally polarised radiation. We will prove these statements with some numerical experiments, later in this section.

In order to understand how to determine the horizontal and vertical polarisations from the knowledge of $E_\parallel$ and $E_\perp$, we start from the following relation, which represents the addition of the transition radiation fields for a population of $N$ electrons and ultimately gives rise to equation (3.45)

$$\frac{d^2W}{d\Omega d\omega} = \left| \frac{d^2W}{d\Omega d\omega} \right|_\parallel + \left| \frac{d^2W}{d\Omega d\omega} \right|_\perp = \frac{e}{\pi^2 c} \sum_{i=1}^{N} \sum_{j=1}^{N} (E_\parallel i^j E_\parallel j + E_\perp i^j E_\perp j) e^{i(\Phi_j - \Phi_i)}, \quad (3.69)$$

in which $\Phi$ represents the relative phase of the wave and is dependent on the time and position of arrival of each electron (cf. paragraph 3.4.1). At this point we note that the
"horizontal" and "vertical" components can be obtained by projecting the Fourier-fields with respect to a horizontal and vertical direction, defined by the unitary vectors $\hat{n}_{\text{hor}}$ and $\hat{n}_{\text{ver}}$. We start by writing the transition radiation field as

$$E = E_{\parallel} + E_{\perp} = E_{\text{hor}} \hat{n}_{\text{hor}} + E_{\text{ver}} \hat{n}_{\text{ver}},$$

from which

$$E_{\text{hor}} = (E_{\parallel} + E_{\perp}) \cdot \hat{n}_{\text{hor}}, \quad (3.70)$$

$$E_{\text{ver}} = (E_{\parallel} + E_{\perp}) \cdot \hat{n}_{\text{ver}}. \quad (3.71)$$

Equation (3.69) can be alternatively formulated in terms of the fields $E_{\text{hor}}$ and $E_{\text{ver}},$

$$\frac{d^2W}{d\Omega d\omega} = \frac{d^2W}{d\Omega d\omega}_{\text{hor}} + \frac{d^2W}{d\Omega d\omega}_{\text{ver}} = \frac{e}{\pi^2 c} \sum_{l=1}^{N} \sum_{j=1}^{N} (E_{\text{hor},l} E_{\text{hor},j} + E_{\text{ver},l} E_{\text{ver},j}) e^{i(\Phi_j - \Phi_l)}.$$
3.4. TRANSITION RADIATION IN LASER-SOLID EXPERIMENTS: A MODEL

Figure 3.15. Same as for Figure 3.14, but this time electrons do not move in the plane \(yz\) (\(\alpha = 100^\circ\) instead of \(\alpha = 90^\circ\), see Figure 3.6a for an explanation of the geometrical parameters used in this chapter). In this case, for electrons directed towards the imaging system (\(\psi \approx 45^\circ\)) the ratio of the polarisations \(r\) is <1.

from which, after noting that \(\hat{n}_{\text{hor}} \cdot \hat{n}_{\text{ver}} = 0\), we can separate the two contributions in the horizontal and vertical directions,

\[
\frac{d^2W}{d\omega d\Omega}|_{\text{hor}} = \frac{e}{\pi^2c} \sum_{l=1}^{N} \sum_{j=1}^{N} (E_{\|}^l \hat{e}_\| \cdot \hat{n}_{\text{hor}} + E_{\perp}^l \hat{e}_\perp \cdot \hat{n}_{\text{hor}}) (E_{\|}^j \hat{e}_\| \cdot \hat{n}_{\text{hor}} + E_{\perp}^j \hat{e}_\perp \cdot \hat{n}_{\text{hor}}) e^{i(\Phi_j - \Phi_l)},
\]

(3.72)

\[
\frac{d^2W}{d\omega d\Omega}|_{\text{ver}} = \frac{e}{\pi^2c} \sum_{l=1}^{N} \sum_{j=1}^{N} (E_{\|}^l \hat{e}_\| \cdot \hat{n}_{\text{ver}} + E_{\perp}^l \hat{e}_\perp \cdot \hat{n}_{\text{ver}}) (E_{\|}^j \hat{e}_\| \cdot \hat{n}_{\text{ver}} + E_{\perp}^j \hat{e}_\perp \cdot \hat{n}_{\text{ver}}) e^{i(\Phi_j - \Phi_l)},
\]

(3.73)

What is needed at this point is to find the different dot-products that appear in the previous two expressions. We will evaluate these quantities by expressing \(\hat{e}_\|, \hat{e}_\perp, \hat{n}_{\text{hor}}\) and \(\hat{n}_{\text{ver}}\) in the canonical system, which is the one pictured in Figure 3.6a. Furthermore, in this system we denote with \((\beta_1, \beta_2)\) the angles that define the position of the center of the lens (Figure
3. THEORY OF TRANSITION RADIATION

3.12). The coordinates of the relevant unit vectors are, in the canonical system,

\[ \hat{n}_{\text{hor}} = (0, \cos \beta_1, \sin \beta_1) \]
\[ \hat{n}_{\text{ver}} = (\cos \beta_2, -\sin \beta_1 \sin \beta_2, \cos \beta_1 \sin \beta_2) \]
\[ \hat{e}_\parallel = (\cos \alpha \cos \theta, \sin \alpha \cos \theta, -\sin \theta) \]
\[ \hat{e}_\perp = (\sin \alpha, -\cos \alpha, 0) \]

Thus, the terms appearing in (3.72) and (3.73) are

\[ \hat{e}_\parallel \cdot \hat{n}_{\text{hor}} = \cos \beta_1 \sin \alpha \cos \theta - \sin \beta_1 \sin \theta =: K_{11} \]
\[ \hat{e}_\perp \cdot \hat{n}_{\text{hor}} = -\cos \alpha \cos \beta_1 =: K_{12} \]
\[ \hat{e}_\parallel \cdot \hat{n}_{\text{ver}} = \cos \beta_2 \cos \alpha \cos \theta - \sin \beta_1 \sin \beta_2 \sin \alpha \cos \theta - \cos \beta_1 \sin \beta_2 \sin \theta =: K_{21} \]
\[ \hat{e}_\perp \cdot \hat{n}_{\text{ver}} = \cos \beta_2 \sin \alpha + \sin \beta_1 \sin \beta_2 \cos \alpha =: K_{22} \]

These quantities allow us to fully determine the horizontal and vertical components of the polarisation from the knowledge of \( E_\parallel \) and \( E_\perp \):

\[
\frac{d^2W}{d\omega d\Omega}_{\text{hor}} = \frac{e^2N}{\pi^2 c} \left[ \int d^3p g(p)(K_{11} E_\parallel + K_{12} E_\perp)^2 \right. \\
+ (N - 1) \left| \int d^3p g(p)(K_{11} E_\parallel + K_{12} E_\perp)F \right|^2 \right] , \tag{3.74}
\]

\[
\frac{d^2W}{d\omega d\Omega}_{\text{ver}} = \frac{e^2N}{\pi^2 c} \left[ \int d^3p g(p)(K_{21} E_\parallel + K_{22} E_\perp)^2 \right. \\
+ (N - 1) \left| \int d^3p g(p)(K_{21} E_\parallel + K_{22} E_\perp)F \right|^2 \right] . \tag{3.75}
\]

Using these results, we can perform the numerical experiments that we anticipated. As a first example, we study the case of a lens positioned on the z axis, \( \beta_1 = 0^\circ, \beta_2 = 0^\circ \) (Figure 3.13a). We vary the direction of an electron in the plane \( yz \) and compute the theoretical ratio \( r \) of horizontal/vertical polarisation. We do so by using equations (3.74) and (3.75) for the case of incoherent radiation, but we have verified that the coherent case gives very similar results. Figure 3.13b shows the results of this first analysis. We note that \( r = 1 \) for an electron directed towards target normal, as expected from the symmetry of the system. This is also a benchmark for the code that we have developed.

We have also studied the case for an imaging system viewing the target rear side at \( 45^\circ \) from the normal, \( \beta_1 = 45^\circ, \beta_2 = 0^\circ \) (Figure 3.14a). In this case, the radiation is almost completely horizontally polarised, unless the electrons are directed towards the imaging system, \( \psi \simeq 45^\circ \) (Figure 3.14b). In this eventuality, it is still \( r \sim 1 \). As it will be shown in chapters 4 and 5, this case is relevant for our experimental data and gives...
evidence that the electrons detected by our imaging systems were mainly directed towards the imaging systems themselves. This is a consequence of the fact that, in the relativistic case, transition radiation is mainly emitted in the direction of the particle momentum vector, as previously explained.

With respect to Figure 3.14, if we allow the electron to be directed not precisely in the plane $yz$ but at some small angle from it, the vertical polarisation can even become dominant ($r < 1$), still around the region where electrons are directed towards the imaging system.

### 3.4.6. Transition Radiation and PIC simulations

As transition radiation is a classical phenomenon, a solution of Maxwell’s equations, it is expected to be described in PIC simulations, although there is no evidence on this in the literature. To test whether this is the case, a number of PIC simulations have been performed with the code OSIRIS in 2D3V geometry.

The simulations turned out to be rather intensive: in order to describe the fields in the radiation zone, i.e. beyond the formation length, the simulation box has to be several tens of wavelengths wide. However, the number of particles per cell had to be kept reasonably high in order to describe the physics correctly.

A typical simulation used 32 nodes on the CX1 cluster at Imperial College. The box size was made of 7800 by 7800 cells, each cell containing 36 quasi-electrons and 36 quasi-ions. The initial density was $n = 100n_{cr}$, with a resulting resolution of 2 cells per skin depth in both directions (130 cells per laser wavelength). In order to be able to separate the laser field from the TR field, the laser electric field was set to oscillate along the direction $z$, i.e. out of the plane of the simulation. The geometry of the problem studied (2-dimensional) implies that the TR electric field should oscillate in the plane containing the target normal and the observation vector. In other words, the plane of the simulation ($xy$).

The target thickness was set to $31.42 \, c/\omega_0$ (i.e. $5 \, \mu m$ for $\lambda_L = 1 \, \mu m$) and the laser was initialised with $a_0 = 20$ and a flat-top temporal intensity. The Gaussian spot size was set to a FWHM of $15.7 \, c/\omega_0$ ($2.5 \, \mu m$ for $\lambda_L = 1 \, \mu m$). Figure 3.16a shows the method used to calculate the spectral and angular distribution: a series of rays were used to integrate the radiation at different angles. The origin of the rays was chosen where the radiation was clearly formed, beyond its formation length. Fourier-integration of the intensity modulations along the rays gives rise to the spectrum in 3.16b, which, despite the noise level intrinsic in PIC modelling, clearly shows the presence of peaks at harmonics of $2\omega$.

That only even harmonics are present is a clear indication that the electron bunches are produced at $2\omega$ rather than $\omega$. The reason is that, in these simulations, the angle of
incidence was normal to the target. In this case, the main absorption mechanism is $j \times B$. Bunches separated by half a laser wavelength can be observed from the plot of the phase space (see Figure 2.3 in chapter 2).

The angular distribution of the radiation is shown in Figure 3.17. This is made by integrating the radiation along the rays, at different angles. A zero in the emission can be seen along the direction of target normal. This is also consistent with emission from transition radiation.

In conclusion, we have demonstrated that transition radiation can be modeled in PIC simulations.

![Figure 3.16](image1.png)

**Figure 3.16.** a) Ray tracing used to determine the spectral and angular distribution of the radiation. b) Spectral distribution of the radiation.

![Figure 3.17](image2.png)

**Figure 3.17.** Angular distribution of the radiation emitted around the second harmonic of the laser frequency ($\lambda \in [488, 513] \text{ nm}$), showing the typical “two lobe” distribution of transition radiation.
CHAPTER 4

Experimental methods

In this chapter we describe some of the methods used for the detection and analysis of the experimental data. The first part details the method used for the polarisation measurements and some other diagnostics that will be of use throughout the rest of the thesis. The second part gives a brief introduction to the measurement of ultrashort laser pulses, with emphasis on the Frequency Resolved Optical Gating (FROG) method. This technique will be relevant to chapter 7.

4.1. Diagnostic techniques

4.1.1. Polarisation measurements of optical radiation. Polarisation measurements of optical radiation produced at the target rear side have been performed using either sheet polarisers or Wollaston prisms. Sheet polarisers are dichroic films that have a preferential absorption of one field component with respect to the other one. Typical sheet polarisers are made of nanoparticles embedded in sodium-silicate glass that are used to produce the polarising effect.

Wollaston prisms are made of two birefringent prisms (for example, calcite) with a right triangular section (Figure 4.1). A birefringent crystal has two indices of refraction, \( n_E \) and \( n_O \). A wave polarised parallel to the optical axis (extraordinary ray or E-ray) of the material moves at a velocity \( c/n_E \), while a wave polarized perpendicular to the optical axis (ordinary ray or O-ray) moves at a velocity \( c/n_O \).

In a Wollaston prism the optical axis of the two birefringent prisms are orthogonal to each other. Moreover, the two prisms are in contact, with no air spacing in between. An arbitrarily polarised beam incident on the first prism gives rise to an ordinary and extraordinary wave. However, at the interface between the two prisms, the O-ray in the first prism becomes the E-ray in the second one and vice versa. As a consequence, the first wave passes from a medium with index of refraction \( n_O \) to a medium with index of refraction \( n_E \), while the second one passes from \( n_E \) to \( n_O \). This produces two diverging, orthogonally polarised beams at the exit of the prism, according to Snell’s law of refraction.

Wollaston prisms are superior to sheet polarisers in terms of extinction ratio of the polarisations. For an incoming unpolarised beam, the extinction ratio is the ratio of the fluence of one polarisation to the orthogonal one, after passing through a polariser. Typical
extinction ratios for Wollaston prisms are about $10^{-5}$, while in the case of sheet polarisers it ranges between $10^{-3}$ and $10^{-4}$, depending on the material and wavelength of operation.

During the experimental campaigns presented in this thesis, Wollaston prisms have been usually preferred to sheet polarisers. Besides having a better extinction ratio, Wollaston prisms allow one to separate the beams into two orthogonal polarisations while still being detectable on the same CCD camera. For the case of sheet polarisers instead, a beam splitter (possibly unpolarising), two polarisers and two cameras are needed in order to detect both polarisations.

Calibration of the optical systems for the polarisation measurements has been achieved after sending a beam of known polarisation through the beamline. For this purpose, a helium-neon laser at 532 nm was sent through the beamline after passing through a sheet polariser. An example is shown in Figure 4.2.

Analysis of the polarisation data has required us to compare the signal for orthogonal polarisations. The natural and easiest way to compare two polarisations is by integrating the relevant signal on the CCD camera and then evaluating the ratio of the two signals. This has been done for both calibration and data analysis. Because small uncertainties in the optical paths of the two polarisations or a slight tilt of the CCD camera could affect the imaging conditions of the two polarisations on the CCD chip, we have chosen to integrate the signal at different cut-off intensities with respect to the maximum pixel value (after performing a background subtraction). An example is given in Figure 4.3. This method of analysis is restricted to the evaluation of the polarisation state of the radiation, and gives a way of estimating an error bar for these measurements.

4.1.2. Optical spectrometer. Typical optical spectrometers used in laser-plasma physics are grating spectrometers in the Czerny-Turner configuration (Figure 4.4). In this configuration, a beam enters the slit of the spectrometer that is placed at the focus of a spherical mirror. This mirror then collimates the light towards the grating, that diffracts the beam
4.1. DIAGNOSTIC TECHNIQUES

Figure 4.2. Calibration of the polarisation with a Wollaston prism. A He-Ne at 532 nm was sent through a sheet polariser and then through the optical beamline. In this configuration, the Wollaston prism allows both polarisations to be detected on the same CCD camera. a1) and a2) signal for sheet polariser with horizontal polarisation, for different integration times (as given in the figures). b1) and b2) signal for sheet polariser with vertical polarisation.

Figure 4.3. An example of a signal in a shot at the JETI laser in Jena. The signals of the two orthogonal polarisations were integrated at different cutoff intensities with respect to the maximum signal level: a) cutoff = 0.5, b) cutoff = 0.6; c) cutoff = 0.7, d) cutoff = 0.8.
according to the wavelength. The diffracted beam is then re-focused by another spherical
mirror onto the exit slit of the spectrometer where, usually, a CCD camera is placed.

The spectral region where this type of spectrometer is more efficient mostly depends
on the characteristics of the grating, since gratings are blazed for a particular spectral
region.

For some of the experimental investigations, a Bentham monochromator was used as
a spectrometer, after removing the exit slit. However, for the measurements of harmonics
from the rear side of solid targets, a spectrometer able to spectrally resolve the light from
the UV to the infrared would be ideal. For example, the second and third harmonics of
the fundamental of the Nd:YAG at 1054 nm are at 527 and 351 nm, respectively. In order
to achieve this, an Ocean Optics HR4000CG-UV-NIR Composite Grating Spectrometer
was used. This spectrometer has a grating and order-sorting filter that provides a 200-
1100 nm wavelength range with 0.75 nm FWHM optical resolution. The order-sorting
filter is necessary in order to avoid the presence of spurious multiple orders on the same
spectral line. The detector included on the spectrometer is a Toshiba TCD1304AP linear
CCD array, with 3648 pixels and a sensitivity of 100 photons per count at 800 nm. The
detector also has a spectral range that goes from 200 nm to 1100 nm.

For each experiment, the spectral response of the system including all the optics and
detector was evaluated using an absolutely calibrated Bentham CL6 white light source,
with a spectral range of 200-3000 nm. This source was positioned inside the chamber and
the light was then collected by the optical system up to the spectrometer. In this way, by
comparing the measured spectrum with the known spectrum of the source the effect of
the optics and detector could be accounted for.

4.1.3. Electron spectrometer. The energy spectrum of electrons escaping the target
was measured using a magnetic deflection spectrometer, where electrons are dispersed by
an externally imposed magnetic field. Deflection of the electrons depends on their energy,
according to the Lorentz formula \( \frac{dp}{dt} = -ev \times B \). Outside of the magnetic field region,
the trajectory of the electrons is rectilinear, and the position of the electrons at the detector
plane is simply related to the electron energy.

Commercially available Fuji imaging plates [97] are usually used for electron detection.
The sensitive part of the plate is made of a 100 \( \mu \)m layer of luminescent material,
BaFBr:Eu\(^{2+}\). This material, doped with Europium 2+ ions, can give rise to a process
called photo-stimulated luminescence (PSL). In this process, energy absorbed from the
ionising radiation excites the phosphor particles to a metastable state. This can be con-
sidered as information stored by the imaging plate of the amount of ionising radiation. In
order to access this information, after each shot the imaging plates are read by a Fujifilm
4.1. DIAGNOSTIC TECHNIQUES

Figure 4.4. Schematic of a Czerny-Turner spectrometer.

reader. During this scanning process, a He-Ne laser at 632 nm further excites the fluorescent particles to an unstable state, from which it relaxes to its initial stable state. As a result, a 3.2 eV photon is emitted. The amount of energy re-emitted is thus related to the dose absorbed by the imaging plate.

Imaging plates are sensitive to any ionising radiation, including x-rays, electrons and ions. The spurious signal from low energy photons ($E < 1$ keV) is, however, reduced by shielding the entrance of the electron spectrometer with a thin ($\sim 10 \mu$m) foil of aluminium.

4.1.4. Laser energy transmission. Measurements of laser energy transmission (for the case of interaction with gaseous targets) have been made by collecting the transmitted radiation onto a fast photo-diode. It was ensured that the solid angle covered for this measurement was larger than the solid angle of the laser in vacuum, so that virtually all of the transmitted radiation would be collected by the system even after interaction with the gaseous target.

The signal of the photo-diode was read on an oscilloscope. The total energy deposited by the photons is proportional to the integral of the oscilloscope trace. Calibration for this measurement was achieved by using vacuum shots as a reference. Transmission for the case of shots with plasma is then determined by comparison of the oscilloscope trace with the reference shot.
The sensitivity of the diode can be considered to be linear, and the spectral response can be considered to be flat over several tens of nm. This is especially true since, in our case, the central wavelength of the laser was at 0.8 µm which is far from the silicium band gap. For experiments at a central wavelength of ~ 1 µm, during the interaction photons are likely to be red-shifted close to the Si band gap at ~ 1.1 eV (i.e. λ ~ 1.1 µm). In this case, the efficiency in the photon detection is expected to rapidly drop and a flat response of the photo-diode cannot be assumed.

4.2. Pulse length measurements

Before going into the details of the device used for our pulse length measurements, we present here an account of the principles underlying the field of ultrashort pulse length measurements.

It is difficult to give a full account of the numerous experimental techniques used to measure the properties of ultrashort laser pulses. A review of some of these techniques can be found in reference [98], on which we base some of the following description.

The ultimate aim of all of these techniques is to find the temporal intensity \( I(t) \) and phase \( \phi(t) \) of the laser pulse. These two quantities fully define the properties of an electromagnetic pulse with electric field \( E(t) \) through the well known representation

\[
E(t) \propto \sqrt{I(t) \exp[i\phi(t)]} + c.c.,
\]

(4.76)

where with c.c. we denote the complex conjugate. Alternatively, one can express the field in the frequency domain rather than in the temporal domain. The temporal and frequency representation of the field are related through the Fourier transform

\[
E(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{E}(\omega) \exp(i\omega t) dt.
\]

Just as for (4.76), the field \( \tilde{E}(\omega) \) can be expressed in terms of a spectral intensity \( S(\omega) \) (or spectrum) and spectral phase \( \Phi(\omega) \)

\[
E(\omega) \propto \sqrt{S(\omega) \exp[i\omega\Phi(\omega)]} + c.c.
\]

It is important to notice that, while \( \tilde{E}(\omega) \) is the Fourier transform of \( E(t) \) (and vice versa), \( S(\omega) \), \( I(t) \) and \( \Phi(\omega) \), \( \phi(t) \) are not simply related by a Fourier transform. In fact, these quantities are all correlated. For example, the spectrum \( S(\omega) \) of a pulse depends, in general, on both the temporal intensity \( I(t) \) and temporal phase \( \phi(t) \).

In order to measure all the properties of a pulse, the ideal case would be to have a detector with a temporal resolution that is much shorter than the shortest feature of the pulse. Such a detector would measure, in practice, the intensity \( |E(t)|^2 \), which would be by itself very close to the final aim of measuring \( E(t) \). Unfortunately, for all the applications in
ultrafast optics there is no such a device and all the detectors can be considered to be, effectively, time integrating. This means that, rather than measuring $|\mathcal{E}(t)|^2$, a detector would measure $\int_{-\infty}^{+\infty} |\mathcal{E}(t)|^2 dt$ from which it would not be possible to uniquely determine $\mathcal{E}(t)$. However, even for the case of time integrating detectors some clever methods have been devised to recover all the information about the field $\mathcal{E}(t)$.

Two main ideas have been developed for this purpose: the first one is to use a replica of the pulse; the second one is to use this replica and nonlinear optical processes to gate the pulse with itself.

The nonlinear optical processes that are employed are mainly second (SHG) or third (THG) harmonic generation. What makes these processes attractive for the purpose of short pulse measurements is that they allow one to transform a sum of two fields into a product that can be uniquely discriminated (spatially and/or spectrally) from other signals. For example, in the case of SHG, considering two identical pulses $\mathcal{E}_{1,2}(\mathbf{r}, t) = E(\mathbf{r}, t) \exp[i(\omega t - \mathbf{k}_1 \cdot \mathbf{r})] + c.c.$ incident on a SHG crystal in non-collinear geometry (Figure 4.5), the SHG signal produced is proportional to

\begin{align*}
(\mathcal{E}_1 + \mathcal{E}_2)^2 &= 4|E|^2 + E^2 \exp[2i(\omega t - \mathbf{k}_1 \cdot \mathbf{r})] + E^*^2 \exp[-2i(\omega t - \mathbf{k}_1 \cdot \mathbf{r})] \\
&+ E^2 \exp[2i(\omega t - \mathbf{k}_2 \cdot \mathbf{r})] + E^*^2 \exp[-2i(\omega t - \mathbf{k}_2 \cdot \mathbf{r})] \\
&+ 2|E|^2 \exp[-i(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r})] + 2|E|^2 \exp[i(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r})] \\
&+ 2E^2 \exp[i(2\omega t - (\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r})] + 2E^*^2 \exp[-i(2\omega t - (\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r})].
\end{align*}

The last line is the most interesting for the purpose of short pulse measurements: it describes generation of second harmonic along the direction $\mathbf{k}_1 + \mathbf{k}_2$ (Figure 4.5).

These nonlinear processes are experimentally detectable for pulses intense enough that the polarisation $P$ induced in the crystal is no longer a linear function of the exciting field, instead (in the Fourier domain)

$$
\tilde{P} = \varepsilon_0 (\chi^{(1)} \tilde{\mathcal{E}} + \chi^{(2)} \tilde{\mathcal{E}}^2 + \ldots).
$$

(4.77)
The relevant physics is then described by the wave equation already discussed in chapter 2 equation (2.20), which was given in the time domain,

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2}.$$  \hspace{1cm} (4.78)

This equation is at the basis for understanding harmonic generation in nonlinear optics.

**4.2.1. Second harmonic generation.** Shortly after the discovery of the first laser in 1960, the generation of second harmonic was demonstrated by shining a ruby laser onto a quartz crystal [99], thus exploiting the nonlinear dependence of the polarisation of the dielectric with respect to the incident laser field, equation (4.77). In this section we review, with a simplified description, some of the physics of second harmonic generation in nonlinear media, which is of general importance in laser-plasma physics, not only for pulse length measurements but also for frequency doubling (and tripling) in optical probing and ICF research.

**4.2.1.1. Basic equations in collinear geometry.** In a 1D case, the wave equation (4.78) can be written as

$$\frac{\partial^2 \tilde{E}}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \tilde{E}}{\partial t^2} = \mu_0 \frac{\partial^2 P^L}{\partial t^2} + \mu_0 \frac{\partial^2 P^{NL}}{\partial t^2},$$

where the polarisation \( P \) has been split into a linear and nonlinear term. After Fourier-integrating the previous equation with respect to the time variable, one obtains

$$\frac{\partial^2 \tilde{E}(z, \omega)}{\partial z^2} + \frac{\varepsilon_r^{(1)}(\omega)}{c^2} \omega^2 \tilde{E}(z, \omega) = -\mu_0 \omega^2 \tilde{P}^{NL}(z, \omega),$$  \hspace{1cm} (4.79)

where \( \varepsilon_r^{(1)} = 1 + \chi^{(1)} \). The equation for second harmonic generation is usually written after writing the fields as

$$\tilde{E}_\omega = E_1 \exp[i(k_1 z - \omega t)],$$  \hspace{1cm} 
$$\tilde{E}_{2\omega} = E_2 \exp[i(k_2 z - 2\omega t)],$$

where \( E_1 \) and \( E_2 \) are the amplitude fields which are slowly varying in comparison to \( 1/\omega \).

The nonlinear polarisation has contributions from a term oscillating at \( \omega \) and one oscillating at \( 2\omega \),

$$\tilde{P}^{NL} = 2\varepsilon_0 \chi^{(2)} \tilde{E}_{2\omega} \tilde{E}_\omega^* + \varepsilon_0 \chi^{(2)} \tilde{E}_\omega \tilde{E}_\omega,$$

which, once substituted into equation (4.79), gives rise to two coupled equations: one for the pump wave and one for the second harmonic. For second harmonic generation, after substituting the latter expressions into the wave equation (4.79), the resulting equation is
\[
\left( \frac{d^2E_2}{dz^2} + 2ik_2 \frac{dE_2}{dz} - k_2^2 E_2 + \frac{\epsilon_r^{(1)}(\omega_2)}{c^2} \omega_2^2 E_2 \right) \exp[i(k_2z - \omega_2t)] = -\frac{\chi^{(2)}\omega_2^2}{c^2} E_1^2 \exp[i(2k_1z - 2\omega_1t)],
\]
where we have introduced \( \omega_2 = 2\omega \) and \( \omega_1 = \omega \). Since \( k_2 = \sqrt{\epsilon_r^{(1)}(\omega_2)/c} \), the third and fourth term on the left-hand side of this expression cancel. Moreover, at this point we apply the slowly varying envelope approximation (SVEA), in which we neglect the first term on the left-hand side with respect to the second one,

\[
\left| \frac{d^2E_2}{dz^2} \right| \ll \left| k_2 \frac{dE_2}{dz} \right|.
\]

In this way equation (4.80) simplifies into

\[
2ik_2 \frac{dE_2}{dz} = -\frac{\chi^{(2)}\omega_2^2}{c^2} E_1^2 e^{i\Delta kz},
\]
where \( \Delta k = 2k_1 - k_2 \). Equation (4.81) is a very important result, that allows to understand much of the physics of second harmonic generation such as the concept of phase matching and phase matching bandwidth.

4.2. Phase matching condition. In general, equation (4.81) should be coupled to a similar equation that describes the evolution of the pump amplitude \( E_1 \). However, for no pump depletion it can be easily solved to give the solution

\[
E_2(z, \omega_2) = -\frac{\chi^{(2)}\omega_2^2}{c^2} E_1^2(z, \omega_1) \left( \frac{e^{i\Delta kL} - 1}{i\Delta k} \right),
\]
from which, for the intensity,

\[
I_2(z, 2\omega) \propto I_1^2(z, \omega) \left[ \chi^{(2)} \right]^2 \frac{L^2 \sin^2(\Delta kL/2)}{(\Delta kL/2)^2} = I_1^2 \left[ \chi^{(2)} \right]^2 L^2 \text{sinc}^2(\Delta kL/2),
\]
where \( I_1 \) is the intensity of the pump wave.

The expression (4.82) clarifies that the intensity of the second harmonic signal is dependent on the parameter \( \Delta k = 2k_1 - k_2 \), on the length \( L \) of the crystal and, of course, on the strength of the generating field \( I_1 \). The case of perfect phase matching is the one for which \( \Delta k = 0 \), which maximises the efficiency of second harmonic generation. Microscopically, the phase matching condition ensures that all the individual dipoles in the medium radiate coherently [100]. As a consequence, the total radiation is proportional to the square of the number of dipoles in the medium. This is the origin of the dependence on \( L^2 \) in (4.82). We also note that there is a dependence on the square of the nonlinear sus-
ceptibility $\chi^{(2)}$, which is maximised in materials which are denoted as nonlinear. Typical nonlinear crystals used for SHG are beta barium borate (BBO) or potassium dihydrogen phosphate (KDP) crystals.

The experimental verification of equation (4.82) was first given in 1962 by Maker et al. [101]. In this experiment, a ruby laser was incident on a quartz crystal that was rotated at different angles with respect to the beam axis (Figure 4.6a). In this way the length $L$ of the crystal was effectively varied with respect to the beam direction ($L = d/\cos \theta$, where $d$ is the crystal thickness). According to (4.82), the resulting intensity of the second harmonic should vary in a way similar to Figure 4.6b, the exact curve depending on the thickness of the crystal $d$. The experimental curve, that can be appreciated in the original paper, is qualitatively similar to this curve although the effects of pump depletion for increasing $\theta$ (hence $L$) are also visible.

The phase matching condition $\Delta k = 2k_1 - k_2 = 0$ can be alternatively formulated in terms of the index of refraction. In these terms, the phase matching condition corresponds to $n(\omega) = n(2\omega)$. However, because almost all materials show normal dispersion in the optical range, that is $n(\omega) < n(2\omega)$, the radiation wave usually lags behind the polarisation wave and phase matching cannot be achieved: the efficiency in the generation of second harmonic is low.

The most common way of achieving phase matching is by using birefringent crystals. In this case, the second harmonic that is generated is always polarised in the direction that gives the lowest value of the two possible refractive indices. For a negative uniaxial crystal, this corresponds to an ordinary wave ($n_O < n_E$) and for a positive uniaxial crystal it corresponds to an extraordinary wave ($n_E < n_O$).

For what concerns the polarisation of the incoming pump waves (speaking of non-collinear SHG), at least one of the two pump waves must be polarised in the direction that gives the highest index of refraction ($E$ in a positive uniaxial crystal and $O$ in a negative one). If the other pump wave is polarised in the same direction, it is said that phase matching is of the type I. When the polarisations of the two pump waves are orthogonal, it is said that phase matching is of the type II.

4.2.1.3. Phase matching bandwidth. In general, perfect phase matching is only possible for a single frequency. However, a real pulse is made of many different frequencies. In order to take into account the effects of a range of frequencies, it is useful to define the phase matching bandwidth as the range of frequencies for which the $\text{sinc}^2$ term in (4.82) is between 0.5 and 1. This is found to be [98]

$$\Delta \lambda_{FWHM} = \frac{0.44 (\lambda_0/L)}{\left| \frac{dn}{d\lambda} \bigg|_{\lambda_0} - \frac{1}{2} \frac{dn}{d\lambda} \bigg|_{\lambda_0/2} \right|}. \quad (4.83)$$
4.2. PULSE LENGTH MEASUREMENTS

It is clear from (4.83) that the phase matching bandwidth decreases as the length of the crystal increases. It turns out that for pulse measurements, it is important to achieve phase matching over the entire bandwidth of the pulse. For ultrashort pulses, this means that the nonlinear crystal must be very thin. For example, a Gaussian transform-limited 10 fs pulse at 800 nm has a bandwidth \( \Delta \lambda = 0.44\lambda^2/(c\Delta t) = 94 \) nm. In order for the phase matching bandwidth to be greater than the pulse bandwidth on a single shot, it is necessary to use very thin crystals, of the order of tenths of \( \mu m \).

Unfortunately, according to equation (4.82), for thin crystals the efficiency in the generation of second harmonic is significantly reduced, due to the dependence on the square of the crystal length. Therefore there is a trade-off between increasing the crystal thickness because of efficiency arguments and decreasing the crystal thickness due to phase matching bandwidth considerations.

However, this is not always true. In a specific case, one in which we are very interested in as will be clear later, it is actually convenient to have a thick crystal. In fact, provided that the incoming beam is incident on the crystal at a range of angles, the crystal can then act as a spectrometer whilst still achieving phase matching over the entire bandwidth of the pulse [102, 103, 104]. This is due to the fact that, in this case, phase matching occurs at different angles, for different wavelengths.

4.2.1.4. Group velocity dispersion (GVD). As all materials present a degree of dispersion, \( n = n(\omega) \), a pulse that propagates in a medium is distorted, because the different frequency components of the pulse propagate at different group and phase velocities and
can also be attenuated at different rates. Here we want to consider the effects of linear propagation, i.e., we assume that no nonlinear processes occur while the pulse propagates through the medium.

If we denote the incoming field with $E_{\text{in}}(\omega)$ and the coefficient of attenuation with $\alpha(\omega)$, the outgoing field $E_{\text{out}}(\omega)$ after propagation through a medium for a distance $L$ is given by

$$E_{\text{out}}(\omega) = E_{\text{in}}(\omega) \exp\left[-\alpha(\omega)L/2\right] \exp\left[i n(\omega)\frac{\omega}{c} L\right].$$

The effect is to distort both the spectrum and the spectral phase of the original pulse:

$$S_{\text{out}}(\omega) = S_{\text{in}}(\omega) \exp[-\alpha(\omega)L]$$

$$\phi_{\text{out}}(\omega) = \phi_{\text{in}}(\omega) + i \cdot n(\omega)\frac{\omega}{c} L.$$

For methods that cannot retrieve the complete field (spectral intensity and spectral phase), such as SHG autocorrelation, it is very important to make sure that the pulse to be measured is not significantly distorted by passage through material. On the other hand, for methods that determine all the properties of the fields it is possible to take into account this effect by using the inverse function of (4.84). However, if possible, it is still preferable to avoid passing through material. In fact, a correct application of (4.84) requires the knowledge of the attenuation and refractive index of the material with good accuracy. Increasing the amount of material increases the effect of these small uncertainties, which may affect the retrieval of the original pulse.

### 4.3. SHG Autocorrelation

A common method used to get information on pulse properties is single shot second harmonic autocorrelation. For this measurement, the detector simply records the second harmonic signal produced when the two pulse replicas overlap in a nonlinear crystal, as shown in Figure 4.7. The signal detected is then proportional to the second order intensity autocorrelation,

$$A^{(2)}(\tau) = \int_{-\infty}^{+\infty} I(t)I(t-\tau)dt ,$$

where $I(t)$ is the temporal intensity of the pulse and $\tau$ represents the relative delay between the two pulse replicas that overlap in the nonlinear crystal.

The intensity autocorrelation is clearly dependent on the properties of the pulse. The question is how much information can be gained from the knowledge of this function.

As it turns out, the information that can be obtained is rather limited. First of all, no information on the temporal phase of the pulse is retained. Secondly, because the autocorrelation is symmetric in time, $A^{(2)}(\tau) = A^{(2)}(-\tau)$, this technique is unable to deal
with complex pulses. Finally, there is ambiguity in the pulse retrieval: different pulses could give rise to the same autocorrelation trace.

In order to deal with these problems, different techniques have been used. The simplest and most successfull one, which has been used for the investigations presented in chapter 7, is the FROG method, that will be described in the next section.

4.4. SHG Frequency Resolved Optical Gating (FROG)

In this method, the detector measures the following quantity, which is a function of both time delay and frequency,

$$I_{\text{FROG}}^{\omega}(\tau, \omega) = \left| \int_{-\infty}^{+\infty} E(t)E(t-\tau)\exp(-i\omega t)dt \right|^2,$$

where $E$ is the electric field of the pulse. The integral is the Fourier transform of a quantity that is related to the temporal delay of the two pulses. In fact, $E(t)E(t-\tau)$ is the output of the nonlinear crystal in Figure 4.7. The FROG signal is then the spectrum of this signal (i.e. the modulus square of its Fourier transform).

The advantage of this method is that it allows one to avoid the problems of autocorrelators that we have mentioned before. In fact, from (4.85) it is possible to retrieve all the properties of the pulse, including the phase. Moreover, this method is able to deal with complex pulses. Finally, the solution of the pulse retrieval problem can be considered to be, effectively, unique: for a given FROG trace there is only one complex electric field $E(t)$. In reality, it is not possible to absolutely determine the time direction, i.e. which part of the pulse corresponds to “early” and which one to “late”. Nevertheless, with some assumptions or some extra information it is possible to remove this uncertainty. The way we have achieved this will be detailed in chapter 7.

There are other ambiguities in the pulse retrieval, which are not of importance for us (they are often referred to as “trivial” ambiguities): it is not possible to determine the absolute phase and the absolute time, as the trace remains unchanged for $E(t) \rightarrow \exp(i\phi_0)E(t)$ and for $E(t) \rightarrow E(t-t_0)$, respectively.
4.4.1. The GRENOUILLE. The instrument used for the measurements discussed in chapter 7 is a GRENOUILLE 8-20USB manufactured by Swamp Optics. In practice, a GRENOUILLE is a very simple FROG system [104]. In a GRENOUILLE, the two replicas of the pulse are obtained by means of a Fresnel biprism and overlap in the nonlinear crystal (III. in Figure 4.8) with varying delay, as it is the case for a single shot autocorrelator. However, at this point information on both the spectrum of the autocorrelation and the autocorrelation itself are mapped in a 2D image at the detector plane, giving rise to the signal expressed in (4.85).

The important simplifying elements of a GRENOUILLE are the Fresnel biprism, which allows one to obtain the two replicas of the pulse in a very simple way, and the crystal. The crystal thickness must be large enough that the crystal itself can also operate as a spectrometer. In order to achieve this, a first cylindrical lens (I. in Figure 4.8) focuses the beam onto the crystal. The range of angles provided by this focusing element must be wide enough that all the frequencies that compose the pulse can be phase matched at different angles of propagation in the crystal. Each angle corresponds to effective second harmonic generation within a certain spectral range, because in a birefringent medium the index of refraction is dependent on the angle of propagation of the wave with respect to the optical axis. As a result, the phase matching condition $2k_1(\omega) = k_2(2\omega)$ will be satisfied by different spectral components of the pulse for different angles of incidence [103].

The thicker is the crystal, the better is the spectral resolution that the crystal can provide, for a given angle of incidence. This is because, as we have discussed, the phase matching bandwidth decreases as the length of the crystal increases. A limit on the thickness of the crystal is, however, imposed by group velocity dispersion (GVD) that causes distortion of the pulse during the gating process.

Therefore, two conditions have to be fulfilled in a GRENOUILLE. First, the phase matching bandwidth must be much smaller than that of the pulse. This condition translates into

$$GVM \cdot L \gg \tau_p, \quad (4.86)$$

where $GVM$ is the group velocity mismatch and $L$ the length of the crystal. The GVM is nothing else than the counterpart of the phase matching bandwidth, expressed in the time domain rather than in the frequency domain. Physically, it expresses the difference in group velocity between the pulse and the second harmonic,

$$GVM = \frac{1}{v_g(\lambda_0/2)} - \frac{1}{v_g(\lambda_0)}.$$
4.4. SHG FREQUENCY RESOLVED OPTICAL GATING (FROG)

Condition (4.86) therefore ensures that the fundamental and the second harmonic cease to be overlapped before exiting from the target. In this way the crystal acts as a spectrometer.

The second condition that must be satisfied in a GRENOUILLE is that distortion of the pulse due to GVD is smaller than the coherence time $\tau_c$ of the pulse (which is roughly the duration of the shortest temporal structure within the pulse),

$$GVD \cdot L \ll \tau_c .$$

(4.87)

For a GRENOUILLE, the crystal length $L$ must therefore be chosen so as to fulfill conditions (4.86) and (4.87). These conditions can also be formulated at once, giving

$$GVM/GVD \gg \tau_\omega/\tau_c = TBP ,$$

where $TBP$ is usually referred to as the *time bandwidth product*. The latter equation has been referred to as the “fundamental equation of GRENOUILLE” by its developers [104].

4.4.2. Pulse retrieval. In general, it is not possible to analytically invert equation (4.85) to retrieve the complex electric field $E(t)$ from the knowledge of the FROG trace $I_{FROG}^{2\omega}(\tau, \omega)$. This is done, instead, using an iterative process. All the FROG algorithms start from an initial guess $E(t)$ for the complex electric field and then improve it in order to make the resulting FROG trace in equation (4.85) as close as possible to the experimental one. The process of minimising the “distance” between the theoretical and experimental
4. EXPERIMENTAL METHODS

FROG traces requires solving a multi-dimensional minimisation problem. In all FROG algorithms the error at the \( k \)-th iteration is defined as

\[
G^{(k)} = \sqrt{\frac{1}{N^2} \sum_{i,j=1}^{N} \left| I_{FROG}(\omega_i, \tau_j) - \mu I_{FROG}^{(k)}(\omega_i, \tau_j) \right|^2},
\]

where \( \mu \) is a real normalization constant that minimises the error \( G^{(k)} \). Also, \( I_{FROG} \) is the measured (experimental) FROG trace and \( I_{FROG}^{(k)} \) is the \( k \)-th iteration of the retrieved FROG trace. Finally, \( \omega_i \) and \( \tau_j \) are elements of the frequency and delay arrays that compose the \( N \times N \) matrix of the FROG trace.

The purpose of all FROG algorithms is to decrease the error in (4.88) and improve the solution iteration after iteration. In order to decrease the chance of stagnating around local minima, FROG algorithms usually apply different strategies when the error stops decreasing at some minimum rate.

The software that has been used for the data presented in this thesis is the commercial software FROG3, developed by Femtosoft Technologies [105]. This software takes, as an input, the experimental FROG trace. As an output, it gives the retrieved pulse shape, either in the temporal domain (temporal intensity and phase) or in the frequency domain.
(spectral intensity and phase).

An example of a retrieval for an experimental FROG is shown in Figure 4.9. The retrieved FROG trace appears to be very similar to the original one. This is a good indication that the algorithm run by FROG3 has led to a satisfactory solution, in that the retrieved pulse characteristics must be very close to the one producing the original trace. Quantitatively, FROG3 gives an error between the original and retrieved FROG traces. This is probably nothing else than the error evaluated according to the expression (4.88). Typical errors found for our experimental traces, in the case of a good retrieval, were below 2 percent.
CHAPTER 5

Angular distribution measurements of the optical rear emission

In this chapter we will present measurements of the optical rear emission in the interaction of a $\approx 4 \times 10^{19}$ W cm$^{-2}$ laser pulse onto Ti foils with varying thickness. These measurements were made at the JETI laser system at the Institut für Optik und Quantenelektronik in Jena, Germany. The pulse duration of this system is $\approx 80$ fs and the maximum energy on target is of $\approx 800$ mJ.

At the present status of the literature, most of the experiments on rear side optical emission have been performed with pulses that are longer and/or less intense than the JETI laser pulse. The earliest paper on the subject concerned the interaction of a 350 fs, 10 J laser pulse onto Al foils with thickness ranging from 17 to 400 $\mu$m [106]. The laser was incident along the target normal and focused to a peak intensity of $\sim 10^{19}$ W cm$^{-2}$. The subsequent investigations made by French groups were obtained with very similar parameters, as the laser system used was always the LULI 100 TW [91, 32]. More recently, Manclossi et al. [107] and Santos et al. [96] have presented measurements made at LOA, with a peak intensity on target of $6 \times 10^{19}$ W cm$^{-2}$ in 40 fs (FWHM). Similar investigations have also been made by Zheng et al.[108] from the group in Osaka, Japan, and from Teubner et al. [109] from MPQ, Germany, who explain the radiation as an effect of coherent wake emission (CWE).

The most recent papers on the optical rear emission have been published by Storm et al. from the Rochester group [110, 111]. In this case, the MTW laser at LLE was focused onto different targets to a peak intensity of $\sim 10^{19}$ W cm$^{-2}$.

With the exception of Teubner et al. [109], all these investigations have a main common characteristic: the radiation is collected in the direction of target normal. In this chapter we investigate, instead, the angular distribution of the radiation. This is done, experimentally, by using three imaging systems that view the target rear side at three different directions. Moreover, Wollaston prisms are used in order to investigate the polarisation of the radiation. The results of this investigation are that the radiation is, in fact, polarised and highly anisotropic. Moreover, the radiation is peaked at the first and second harmonic of the laser frequency. These observations suggest that the radiation is either coherent wake emission (CWE) or coherent transition radiation (CTR). However, using the results of chapter 6, we anticipate that the source is most likely CTR.
5. ANGULAR DISTRIBUTION MEASUREMENTS OF THE OPTICAL REAR EMISSION

5.1. The JETI laser system

At its current state, the JETI laser system is a 10 TW system that can deliver 800 mJ on target in 80 fs. The final optics consists of an f/2 parabolic mirror that can focus the beam to a spot size of $5 \mu m^2$ FWHM, which contains about 35% of the total laser energy. The peak intensity on target is thus $5 \times 10^{19}$ W cm$^{-2}$.

JETI is based on a chirped pulse amplification technique, as all current high-intensity laser systems. The oscillator is a Ti:sapphire cavity, that operates at a central wavelength $\lambda_0 = 795$ nm and is pumped by a 5W CW neodymium:YVO$_4$ laser. The laser is stretched by a double-pass grating stretcher and is first amplified by a regenerative amplifier. In a subsequent stage, an ultrafast Pockels cell is used to reduce the ASE and the prepulse level, thus improving the contrast ratio. The pulse then passes through a second and third stage of amplification before being recompressed. The transmission efficiency of the compressor is of about 65%. After the compressor, the pulse diameter is of about 7 cm before entering the target chamber.

Measurements of the laser contrast ratio have been made using a portion of the beam and sending it onto a THG autocorrelator. According to this diagnostic, the pulse presents a contrast $I_{ASE}/I_{main} < 10^{-8}$ between ASE and main pulse up to 40 ps before arrival of the main pulse, followed by a single pre-pulse of the order of $10^{-5}$ at 30 ps. After the pre-pulse, the ASE level decreases to $< 10^{-8}$ again before the pedestal of the main pulse. Further details on the JETI laser system can be found in the PhD thesis by S. Pfotenhauer [112].

5.2. Experimental parameters

5.2.1. Focal spot measurements. The last focusing optic (f/2 off-axis parabola) was optimised on a daily basis in order to achieve the highest intensity on target. For this purpose, the focal spot was imaged onto an 8 bit CCD camera and the parabola moved in all its 6 degrees of freedom in order to minimize spatial aberrations. The inevitable presence of wings around the main focal spot was evaluated by using different filters level (Figure 5.1).

5.2.2. Experimental set-up. The targets used for this experiment were extended foils of titanium of different thicknesses (2, 5, 10 and 20 $\mu m$). The use of extended foils allows hundreds of shots to be taken without the need to replace the target, thus improving the statistics. In fact, the target holder is attached to a xyz stage so that it is possible to move to a “fresh” position on the foil after every shot. However, although the repetition rate of the laser system allows, in principle, to shoot at 10 Hz, the actual frequency of operation was of one shot every 10-30 seconds. This was mainly limited by the operation of the diagnostics.
Alignment of the target was made by moving it towards or away from the off-axis parabola. An ionisation chamber was placed just outside the vacuum chamber, measuring the amount of energy deposited in the form of ionising radiation such as electrons or x-rays. The position of the target was then moved, so as to maximise the dose measured by the ionisation chamber. This is a standard procedure at JETI, used to maximise the intensity on target.

The experimental set-up is shown in Figure 5.2. The $p$–polarised laser pulse was incident at $45^\circ$ on target. As we have seen, this increases the absorption fraction of laser energy into hot electrons (cf. section 2.2). Moreover, it minimises unwanted back-reflections into the laser system and debris on the parabola.

Three optical systems collected the radiation emitted from the target rear side, at different viewing angles with respect to the target normal. In particular, an imaging line was looking towards the laser axis. We will call it the “$+45^\circ$” line. The magnification for this system was $\times 7$. In order to avoid the possibility of damaging the CCD camera in case of a null shot, a high-reflectivity mirror at 800 nm was used before the lens. A second imaging system was looking at the target normal (“$0^\circ$” line), with a magnification of $\times 11$ and a third one at $45^\circ$ from target normal and $90^\circ$ with respect to the laser axis (“$-45^\circ$” line), with a magnification of $\times 8$.

The lenses were all of a diameter of 2” and a focal length of 15-16 cm. The light was imaged at the second harmonic of the fundamental ($\lambda_0/2 \simeq 400$ nm) and then split by Wollaston prisms onto 12 bit Basler A102f CCD cameras, which allowed measurements of the polarisation. The theoretical resolution $\Delta x$ allowed by the collection optics was of about 2 $\mu$m for the three systems, evaluated, in first approximation, according to the Rayleigh criterion $\Delta x = 1.22 \lambda f\#$. 

**Figure 5.1.** Focal spot images taken with an 8 bit CCD camera for two different filtering levels.
5.3. Analysis of correlations

It is interesting to, first, make a comparison between the dose measured by the ionisation chamber and the intensity of the rear side optical emission. Both diagnostics should give a measure of the intensity on target. However, the ionisation chamber is more sensitive to the bulk heating of the target. For example, the fast electrons that travel through the target can give rise to bremsstrahlung photons, in the x-ray region. Electrons hitting the walls of the chamber could also, with the same process, produce x-ray photons detected by the ionisation chamber. On the other hand, mechanisms such as transition radiation should be more sensitive to the surface heating of the target rear side and, therefore, should be in principle more indicative of the energy of the fast electrons as they reach the rear surface.

In Figure 5.3 we plot the dose as a function of laser energy, for two different target thicknesses (2 µm and 10 µm). These plots suggest that the dose is increasing as the laser energy increases, which is not surprising. Moreover, to confirm that this diagnostic is mostly sensitive to the bulk heating of the target, we note that the dose was comparably higher for the thicker foils (with the exception of a couple of high-dose shots with the thinner foil). This is in contrast with the signal from the optical rear emission that, as we will see, is mostly due to coherent transition radiation and is more intense for the thinner targets.
5.3. ANALYSIS OF CORRELATIONS

The error in the dose measurements is given by the instrument scale, which is limited to the division of 0.1 µSv. Assuming a uniform distribution of probability for the measurement, this corresponds to a standard deviation \( \sigma = 0.1 \, \mu\text{Sv} / \sqrt{12} = 0.029 \, \mu\text{Sv} \). The amount of dose measured varied between 0.1 and 3.2 µSv per shot\(^1\), with some shot-to-shot variations that are also evident. The most likely cause for the fluctuations for comparable laser energies is to be ascribed to non-perfect alignment of the target front side, as the target foil was moved after every shot to a new position. Inhomogeneities of the target front side are less likely to be the cause because the Rayleigh range of the laser was \( \approx 20 \, \mu\text{m} \). The other possibility is, of course, that the laser conditions were also varying from shot to shot, in terms of pulse duration, prepulse level or focal spot intensity variations.

Given that the dose measurements appear to be correlated with the laser energy, it is interesting to compare if there is a correlation between the measurements of the ionisation chamber and those of the optical radiation. As we clarified, the latter should be mainly the result of a surface process rather than a volumic process. In Figure 5.4 we show the clearest data set, which was for a 10 µm Ti foil. On one axis is the energy emitted in the form of optical radiation, collected at 0° and +45°. For this analysis, the optical signal (imaged at the second harmonic of the laser frequency) was integrated for all the pixels in the CCD camera above a value of 1/e from the maximum pixel value, after background subtraction. On the other axis is the dose measured. The data set points to the expected result, which is that as one quantity increases the other one also tends to increase, but is also limited by the sensitivity of the ionisation chamber (0.1 µSv).

\(^1\)In order to have an idea for what a Sievert (Sv) is, we note that the International Commission for Radiation Protection (ICRP) recommends a limiting absorbed dose of 1 mSv per year for the “general public” [113].
5. ANGULAR DISTRIBUTION MEASUREMENTS OF THE OPTICAL REAR EMISSION

We have made a similar analysis by looking at the correlation between the three different optical channels. In Figure 5.5 we plot these results. On one axis is the optical energy collected at the second harmonic for one channel VS the optical energy for another channel, on the other axis. The data are rather scattered and there is only a weak correlation between the three channels.

This suggests that the source of the radiation is not only anisotropic but also that the radiation collected at a particular angle can give little information on the radiation collected at a different viewing angle. That the radiation is not isotropically emitted is also confirmed by the fact that the signal strength was generally higher for the imaging system viewing in the direction of the laser axis, and minimum for the system viewing at $-45^\circ$ from target normal.

To our knowledge, there are two possible mechanisms that could be responsible for this: coherent wake emission (CWE) and transition radiation. CWE has been proposed by Teubner et al. [109] in order to explain the presence of harmonics in the optical spectrum of the radiation. In their interpretation, the electron bunches that are produced at the target front side are able to excite plasma waves at locations where the local plasma frequency coincides with multiples of the laser frequency, $\omega_p = m\omega_L$ for $m = 1, 2, 3, \ldots$. Such plasma waves have associated electrical currents that can re-emit radiation. The authors of the paper argue that the radiation should be peaked along the laser direction and polarised as the laser pulse.

A rigorous theory of CWE is currently missing. Furthermore, the fluctuations of the polarisation that we have observed could be difficult to interpret in the framework of this theory. Results from chapters 6, that we anticipate, suggest that a more consistent ex-
5.4. Optical Spectra

In order to spectrally resolve the radiation we used different interference filters in front of the CCD camera at 0°. In this way it was possible to spatially resolve the radiation at different wavelengths. Figure 5.6 shows the results of this investigation. The interference filters were chosen at 375, 400, 450, 500, 600, 700, 750 and 800 nm. As the fundamental
second harmonic of the laser wavelength are at 800 nm and 400 nm, if coherent
transition radiation is present we would expect to observe an enhancement of the radiation
around these wavelengths. It should be noted, however, that the camera was not moved
during these shots. As a consequence, due to chromatic aberration the images are in focus
only close to the second harmonic, even though the lens used were achromatic doublets.
Despite this, we can surely infer from Figure 5.6 that the radiation of the central bright
spot is enhanced at the second and first harmonics (note that the color scale is adjusted for
each shot). This strongly suggests that these bright regions of emission are due to CTR.
Based on this observation, we argue that the emissions at $+45^\circ$ and $-45^\circ$ were also due to
CTR, although no analogous measurements were taken for these viewing angles. Besides
the bright central spots, the rest of the emission appears to be broadband emission from
the bulk plasma, in agreement with the polarisation measurements that show no polarised
radiation outside the CTR region (see next section).

Moreover, the presence of both the 1st and 2nd harmonic gives evidence that the
electron current produced at the target front side is modulated at the laser wavelength.
The possible absorption mechanisms that are responsible for this are, as we have seen,
resonance absorption and vacuum heating. This is not surprising, considering that the
laser was incident at $45^\circ$ from the normal to the target. This, of course, does not exclude
a contribution from $\mathbf{j} \times \mathbf{B}$ absorption as well.

5.5. Spatial imaging and polarisation measurements

5.5.1. General considerations. Spatial imaging of the rear side optical emission re-
veals the filamented structure of the fast electron beam. However, although some extra
care was taken in order to correctly image the target rear side, the resulting images appear
with some aberrations. This could be due to the small spatial structure of the filaments
at the target rear side, which was probably below the resolution of our imaging system.
In the next chapter, with a technique based on the idea of coherent diffraction, we will
demonstrate that such filamentary structure can belong to filaments as small as 1 $\mu$m,
although in that case the experimental parameters were considerably different from the
present ones.

The images appear to be more filamented for the case of the $0^\circ$ observation angle. In
the case of the $+45^\circ$ or $-45^\circ$ channels, usually one or two filaments appear.

Polarisation measurements have been made using the method discussed in chapter 4.
The signal was found to be polarised for all three channels (Figure 5.7), but the polarisa-
tion was also fluctuating from shot to shot.

From Figure 5.7d), it is apparent that the polarised radiation from the fast electrons
is surrounded by an unpolarised circular region of emission. This is most likely due to
Figure 5.6. Images at 0° for the case of a 10 µm Ti foil, for different interference filters placed in front of the CCD camera. The color scales are linear.
Figure 5.7. a), b) and c) Raw images for the three channels, for a shot with a 2 μm Ti target. a) laser direction (+45°), b) target normal (0°), c) −45° from target normal. d) An example of a result for 0° viewing angle, for a shot with a 10 μm Ti foil. We can appreciate the presence of an unpolarised background of radiation, on top of which is the radiation from the escaping fast electrons.
thermal emission, which is not expected to be polarised. For a single charged particle in motion, the radiation would be polarised. However, when summing the contribution of a statistical ensemble of particles with random motion, as in the case of thermal motion, the global radiation field is not polarised. It should also be noted that this region of emission is limited by rather circular boundaries, which are the result of the spherical hydrodynamic expansion that develops at the target rear side.

The signal from the fast electrons and from the thermal emission represent two phenomena that happen at completely different timescales, tens of femtoseconds compared to nanoseconds, but are combined in the same snapshot by the time-integrating CCD camera. This is probably one of the first of such observations made in laser-solid interactions. The only other observation of this kind that we are aware of is in the paper by Santos et al. [96]. In this case, a laser with an intensity of $\sim 9 \times 10^{19}$ W cm$^{-2}$ was directed onto Al targets. The radiation, just as for our case, was collected along target normal. With respect to our measurements, no polarisation measurements were taken and there were also no other angles of observation. The authors observe a small central bright spot surrounded by a halo of emission and conclude that the central spot is the coherent transition radiation emitted by relativistic fast electrons ($T_{CTR} \sim 10$ MeV), while the halo is due to thermal emission.

5.5.2. Polarisation measurements. For analysis of the polarisation data, we proceed as discussed in chapter 4. The two signals on the CCD camera, representing the two polarisations, are integrated in order to get an average signal and then their ratio is evaluated. This, of course, eliminates the “microscopic” information on the filamentary structure of the beam, however it simplifies the analysis considerably because it allows to get a single number out of each shot. For each shot, an estimate of the error for the polarisation ratio is made after integrating the signal of each polarisation at different cut-off values with respect to the maximum pixel value. For the present analysis, we have used cut-offs at 0.5, 0.6, 0.7, 0.8, 0.9 of the maximum signal level. A cut-off at 0.9 of the maximum signal level effectively corresponds to comparing the maximum signal level for the two polarisations.

For the statistical evaluation of different shots, we choose to evaluate the mean as a weighted mean, where the weights are represented by the “standard deviations” $\sigma_i$ for each shot,

$$\bar{x} = \frac{\sum_{i=1}^{n} (x_i/\sigma_i^2)}{\sum_{i=1}^{n} (1/\sigma_i^2)} .$$

(5.89)

Similarly, a weighted standard deviation is computed as
Figure 5.8. Plots of the polarisation ratios for 0° viewing angle, for different values of the laser energy. The laser was focused to a 2 µm Ti foil. a) and b) are the corresponding statistical averages evaluated according to expressions (5.89) and (5.90).

\[
\sigma = \frac{1}{\sum_{i=1}^{n}(1/\sigma_i^2)}. \tag{5.90}
\]

Variations in the laser conditions, target distance and local properties of the target surface (such as roughness [114]) may lead to the fluctuations observed in the experiment. Our statistical method may be considered to be valid only for “small” fluctuations of the polarisation data. In order to determine if this is the case, we first compute a mean \( r_1 \) and standard deviation \( \sigma_{r_1} \) for the ratio of horizontal to vertical polarisation, according to expressions (5.89) and (5.90). Then, with the same formulas we compute the mean and standard deviation of the vertical to horizontal polarisation, \( r_2 \) and \( \sigma_{r_2} \). If the data show little fluctuations, the result for \( r_2 \) will be, within the error bars, comparable to \( 1/r_1 \). If this is not true, we reject the analysis.

In Figures 5.8 and 5.9 we show the result of this analysis for the 0° and −45° channels, for the 2 µm Ti foil. The left pane is a plot of \( r_1 \) versus the laser energy, the right pane shows \( r_2 \) and \( 1/r_1 \), for comparison. For the 0° channel, a qualitative agreement between the two curves is found. A good agreement is also present for the −45° channel, with the exception of the energy around 160 mJ.

According to this analysis, the 0° channel presents a smaller degree of polarisation than the −45° one. That the 0° channel presents a degree of polarisation is justified by the fact that the fast electrons are not necessarily directed along target normal. Still, the polarisation degree for the 0° channel is higher than expected, considering the results of
5.5. SPATIAL IMAGING AND POLARISATION MEASUREMENTS

**Figure 5.9.** Plots of the polarization ratios for $-45^\circ$ viewing angle, for different values of the laser energy. The laser was focused to a 2 $\mu$m Ti foil. a) and b) are the corresponding statistical averages evaluated according to equations (5.89) and (5.90).

**Figure 5.10.** Theoretical ratio of horizontal/vertical polarisation for the observation angles at $\pm45^\circ$ and different direction of electrons, for different electron temperatures (see chapter 3). In a), the range of angles defining the observation cone of the imaging system is highlighted. The behaviour of the curve around the observation cone is clarified in b).
chapter 3 (cf. Figure 3.13). It is not clear why this is the case. It is possible that the data are very sensitive to the actual position of the lens, especially for the imaging system that is viewing along target normal. For the $-45^\circ$ channel, a comparison of the experimental range of polarisations (between 1 and 4) with the theoretical one in Figure 5.10 suggests that the electrons responsible for the signal were directed along the imaging system.

A similar argument applies for the $+45^\circ$ channel, although in this case the fluctuations are more severe and we are unable to follow the same statistical approach as for the other two cases (Figure 5.11). This could be an indication that the pointing of the fast electron beam is unstable, although still within the cone of the collection optics.

5.5.3. Energy of the rear emission as a function of target thickness and laser energy. If the radiation collected at the target rear side is due to coherent radiation, as the spectral measurements suggest, we expect a rapid decrease of the signal strength with respect to target thickness. As we have discussed in chapter 3, this is the result of the dephasing of the electrons as they travel through the target.

In Figure 5.12, we can appreciate that the signal strength can indeed be considered a decreasing function of target thickness. The experimental curves refer to two different laser energies (325 and 650 mJ) and are fitted with theoretical curves resulting from the integration of equation (3.61) in chapter 3 over the solid angle of the imaging system. The clearest set of data, with respect to this fitting, is for the $+45^\circ$ channel.

However, as it was noted by Santos et al. [106], it is difficult to fit the data for the thinnest target thicknesses. This might be due to the effect of refluxing which should be larger for thinner targets.
Figure 5.12. Energy of the optical rear emission as a function of target thickness. The data correspond to statistical averages over, typically, 10 shots.

The fast electron temperature that we infer from our data ranges between 150/200 keV and 600 keV, and is higher for the 650 mJ case. For the same parameters, Beg’s scaling would give 440 keV for the 325 mJ case and 600 keV for the 650 mJ case. This scaling seems to work slightly better than Wilks’ scaling, that would predict 530 keV for the 325 mJ case and 870 keV for the 650 mJ case.

The effect of the laser intensity on the energy of the optical emission can be better appreciated in Figure 5.13. The figure includes, for reference, theoretical curves from Beg’s and Wilk’s scaling laws for the hot electron temperatures. For the parameters of the experiment, the data do not allow discrimination between the two scaling laws; both are in reasonable agreement with the experimental data.

These curves have been obtained by inverting the two scaling laws in order to find the irradiance $I \lambda^2$ as a function of the hot electron temperature,
Then, for each value of the fast electron temperature (corresponding to a particular value of $\Lambda^2$ according to the previous relations) the energy radiated as CTR is evaluated. In this way it is possible to construct an array of CTR energy with respect to irradiance.

### 5.6. Summary

In this chapter we have shown that the optical radiation emitted at the target rear side is highly anisotropic. For the conditions described here, for which the laser pulse was incident at $45^\circ$ from target normal, the radiation is mostly directed towards the laser direction. In our interpretation, this must be due to the fact that the majority of the fast electrons are produced in this direction. When they escape the target rear surface, they emit transition radiation which, due to the relativistic speeds of the electrons, is mostly confined along this direction.

That the emission is due to transition radiation is confirmed by polarisation measurements. We have further demonstrated that the brightest region of emission that appears in our data is peaked where the wavelength of emission is close to the harmonics of the laser frequency, implying that the radiation is coherent. This is due to a modulated current of fast electrons that is produced at the target front surface and propagates through the target, for the duration of the laser pulse. The coherence of the radiation is also confirmed by the dependence of the signal strength with target thickness, which shows a reasonable agreement with what is expected from the theory of coherent transition radiation developed in chapter 3. We have also observed an increase of the signal strength with laser energy, which is a consequence of the increase of the fast electron temperature with laser intensity.

Finally, we stress that only the radiation coming from the fast electrons is polarised. Unpolarised radiation is also, always, present and is indicative of thermal emission. This process happens at much longer time-scales than the signal from the fast electrons.
Figure 5.13. Energy of the optical rear emission as a function of $I\lambda^2$, for different parameters (observation angle, target thickness). The data correspond to statistical averages over, typically, 10 shots. The Beg’s and Wilks’ scaling laws are best fitted to the experimental data.
CHAPTER 6

Filamentation and recirculation measurements from the rear side optical emission

In this chapter we will present measurements of the optical rear emission for intensities ranging from $8 \times 10^{19} \text{ Wcm}^{-2}$ to largely beyond $10^{20} \text{ Wcm}^{-2}$, in experiments performed on the Vulcan laser system at the Rutherford Appleton Laboratory. With a technique that is related to the coherent diffraction of radiation, we will be able to quantitatively estimate the filament size at the target rear side, showing that filaments as small as 1 $\mu$m FWHM or below were produced in the interaction with Cu foils. In chapter 4 we have seen that spatially resolving the filamentary structure can prove to be difficult by optical imaging, because its scale can be below the optical resolution of the imaging system. Moreover, a correct imaging requires re-focusing the imaging diagnostic at the target rear side for every different target thickness, particularly when high-magnification systems are used. Our method avoids these complications. On the other hand, it is an indirect approach that relies on assumptions that model the emission of coherent transition radiation.

In the experiments described in this chapter, an extended halo of radiation that surrounds the brightest source of optical rear emission can also be observed, and is interpreted in terms of electrons refluxing in the target. The fact that this source of radiation is polarised can be explained by transition radiation emitted by electrons that are reflected at the target rear side by the sheath field produced during the interaction.

The study of fast electron transport is motivated by applications such as fast ignition or ion acceleration. This was clarified in chapter 2. Here we add that it is currently believed that the laser parameters required for the igniting beam in a fast ignition scenario will be an energy of $\sim 100 \text{ kJ}$ in $\sim 20 \text{ ps}$ and $\sim 20 \mu$m focal spot, resulting in a power and intensity of the laser driver of $\sim 5 \text{ PW}$ and $\sim 10^{21} \text{ Wcm}^{-2}$, respectively. This should be delivered at the second or third harmonic of the fundamental [26].

These intensity and power are marginally met by current laser systems such as those in the present chapter. This motivates much of the work carried on these facilities. However, the energy is about two orders of magnitude below what will be required and the pulse duration is shorter by about an order of magnitude. Moreover, such experiments usually involve the interaction with an initially un-ionised and un-compressed solid material. A notable exception is the experiment made by Kodama et al. in 2001 [56], where a 1.2 kJ
nanosecond laser at 0.53 \( \mu m \) wavelength was used to compress a deuterated-polystyrene (CD) shell with a gold cone attached. Then, a heater beam (100 TW, 60 J) was focused on the compressed material. An increase of the x-ray yield and evidence for increased D-D fusion yield were found when the short pulse laser was fired.

The techniques used for diagnosing fast electron transport are shadowgraphy \([115, 116, 117]\), \( K_\alpha \) \([118]\), XUV emission \([119]\) and optical emission \([106, 91]\). There is currently a general agreement that the bulk of the fast electrons generated in the interaction have a half-cone divergence of \(20^\circ \sim 60^\circ\) for intensities \(I > 10^{19} \text{ Wcm}^{-2}\) \([120]\). Recently Santos et al. \([96]\) have shown that the most energetic electrons are well collimated, although they carry only a small fraction of the overall fast electron energy. Previous measurements on the fast electron filamentation include the work of Manclossi et al. \([107]\) on the filamentation in Al targets and in insulators (CH). Within the optical resolution of their imaging system (\(\Delta x \lesssim 5 \mu m\)), the authors observe a homogenous signal for the case of aluminium, and filaments of \(\sim 13 \mu m\) for CH targets with a thickness of \(\sim 100 \mu m\). They conclude that the filamentation in CH is triggered by an instability of the ionisation front.

A very high-resolution optical imaging system (\(\Delta x \simeq 1.4 \mu m\)) was used by Storm et al. \([110, 111]\) to investigate filamentation in different metals, for an intensity of \(\sim 1 \times 10^{19} \text{ Wcm}^{-2}\). Filaments with a diameter as small as 2 \(\mu m\) were observed in the case of Al foils of thickness of 20 \(\mu m\).

Concerning previous work on recirculation, we note that this process was first hypothesised by Mackinnon et al.\([121]\) to account for the experimental increase in the proton yield for thin targets, when the thickness \(d\) was much smaller than half of the laser pulse length \(c \tau_p/2\). In fact, in this case electrons can be reflected by the sheath field at both the front and back of the target, and increase the hot electron density \(n_h\) at the target rear side. The resulting electric field at the target rear side is then increased, because \(E_{\text{sheath}} \propto n_h^{1/2}\) \([122]\) (we do not discuss the influence of the hot electron temperature here). According to the authors, this accounts for the increase in proton energy observed in the experiment.

Other indirect measurements have confirmed this original idea. These include the work by McKenna et al.\([123]\) and Nilson et al.\([124]\). In the former case, \([123]\), ion emission from the edges of flat targets was observed, millimiters away from the laser focus; this was interpreted by effects of lateral electron transport in the target, that is able to set up a strong sheath field at the target edges. In the latter case, \([124]\), strong target heating due to refluxing was inferred from \(K\)-shell measurements from mass-limited (volume as small as \(20 \times 20 \times 2 \mu m^3\)) copper targets.

However, so far a more direct proof on electron recirculation is missing. We believe our measurements fill this gap.
6.2. EXPERIMENTAL PARAMETERS

6.1. The Vulcan laser system

Vulcan is a Nd:glass laser with central wavelength at 1.053 μm. It can deliver beams into three separate target areas: Target Area East (TAE), Target Area West (TAW) and Target Area Petawatt (TAP). The target areas relevant to the experiments described in this chapter are TAW and TAP.

At the time of our experiment, TAW could use 6 long pulse beams and a 100 TW CPA beam that could deliver 60 J on target in ≈ 500 fs. In the interaction chamber, this beam was focused with an f/4 off-axis parabola to a peak intensity of almost $10^{20}$ Wcm$^{-2}$.

The CPA beam in TAP can deliver 500 J on target in ≈ 500 fs. Using an f/3 off-axis parabola, the resulting peak intensity is $\sim 10^{21}$ Wcm$^{-2}$. The first amplification stage makes use of the Optical Parametric Chirped Pulse Amplification (OPCPA) technique.

6.2. Experimental parameters

A series of experiments were performed with the intent of studying the properties of the optical radiation emitted at the rear side of solid targets, in terms of spatial imaging, optical spectrum and polarisation.

In Figure 6.1 we show the experimental set-ups for three experimental campaigns made on Vulcan. Experiments a) and b) were performed in TAP and experiment c) in TAW. The position of the optics were often dictated by the presence of other diagnostics and by the requirement that no optical diagnostic would be looking within the cone of the laser. This was to avoid the damaging of optics and cameras in the eventuality of a failed shot in which a significant fraction of the laser energy would be transmitted.

For experiments a) and b), the ≈ 300 J of energy on target were delivered in 500-600 fs duration. In experiment a), the $p$-polarized laser pulse was focused on to a range of flat Au foils of varying thickness at an angle of incidence of $\sim 40^\circ$. The imaging system, with two 16 bit CCD cameras equipped with 2ω interference filters and sheet polarisers, imaged the target rear side, $\simeq 55^\circ$ from target normal and $\simeq 15^\circ$ from laser axis in the horizontal plane. The magnification was $\times 5$ with a theoretical resolution of $\simeq 5 \mu m$.

In experiment b), two Czerny-Turner spectrometers were spectrally resolving the optical radiation, at angles of 50° and 30° with respect to the normal to the target.

Experiment c), as we have said previously, was conducted in TAW. In this case, the $p$-polarized laser pulse, with up to 60 J on target and a duration of 580 ± 114 fs, was focused to a spot containing about 35% of the total energy in a diameter of 6 μm, giving a peak intensity $I \simeq 8 \times 10^{19}$ Wcm$^{-2}$. The angle of incidence was fixed at 8° with respect to the normal of the target front surface. The radiation was collected using an f/3.5 imaging system, centered at $\simeq 33^\circ$ from the normal to the target front side and $\simeq 41^\circ$ from laser axis in the horizontal plane, with a magnification of $\times 9.4$ and a theoretical
resolution of 2.8 \( \mu \text{m} \). The collected radiation was split into two orthogonal polarisations with a Wollaston prism and then imaged onto the same chip of a 2\( \omega \) filtered 16 bit CCD camera. The radiation was also spectrally resolved from the NIR to the UV using an optical spectrometer with a 14-bit linear CCD detector. For this purpose, the radiation was collected with a silver parabolic mirror and then coupled onto a fiber with a 600 \( \mu \text{m} \) silica core. The fiber was connected to an Ocean Optics 4000CG spectrometer, that has been briefly discussed in chapter 4 (cf. section 4.1.2 on page 80).

Figure 6.1. Experimental configurations. Experiments a) and b) were performed in TAP and c) in TAW. In a), I is a sheet polariser. In a) and c), II is an interference filter. Finally, in c), III is a Wollaston prism.

The “effective” extinction ratio (including the effects of the polariser and of the optics) for experiment a) is estimated to be of the order of 10\(^{-2}\), mainly limited by a systematic error in the orientation of the two sheet polarisers used in front of the two cameras. For experiment c), this effective extinction ratio was measured using a green HeNe at 532 nm at the target chamber center. The laser was pointing towards the imaging system and a sheet polarizer was used in front of the laser in order to select a particular direction of polarisation (horizontal/vertical). After passing through the system, including the Wollaston
prism, the light was then detected by the CCD. The extinction ratio was then measured from the ratio of the two signals on the CCD, and amounted to about $10^{-2}$. This was once again mainly limited by a systematic error in the rotation of the polariser (Wollaston prism). Indeed, we were not just interested in selecting two generic orthogonal polarisations but in selecting the vertical and horizontal ones (with respect to the plane of incidence of the laser field).

As it will be noted, the measurements that will be described in this chapter are similar to those in chapter 5, albeit in that case we were mainly interested in the angular distribution of the radiation that we were simultaneously collecting at three different angles. In this case, besides the different intensity regime, we have carried out a more systematic evaluation of the optical spectra and of the polarisation as well.

Regarding the spectral measurements, in both experiments b) and c) the radiation was coupled into the spectrometers by focusing all of the radiation into the fiber optics, rather than imaging. In this way we could minimise the effects of chromatic aberration of the optical elements and be sure to couple the light correctly into the fiber, for a wide range of wavelengths. The disadvantage of this approach is, of course, that in this way we are not able to tell the source of the spectrally resolved radiation. However, spatial imaging of the radiation using different interference filters has revealed, in the previous chapter, that it is indeed the brightest source of emission that shows peaks at harmonics of the laser frequency.

6.3. Results and interpretation

6.3.1. Optical spectra. In Figure 6.2, typical spectra of the optical rear emission are shown. The left column refers to experiment b) and the right one to experiment c). For experiment b), the resolution was limited by the slit of the spectrometer to $\sim 1$ nm. A similar spectral resolution was achieved in experiment c).

As a first remark, we notice that all the spectra show characteristic peaks at the harmonics of the laser frequency, in agreement with the theory of coherent transition radiation for electron bunches separated by the laser wavelength. In Figure 6.2b1 we show a saturated spectrum for a shot with a 100 $\mu$m Au target which presents harmonics up to the 5th (211 nm). It is not clear why the fundamental is less bright than the second, third and fourth harmonic, a possible explanation being the different coupling of the light into the optical fiber.

As a second remark, we can evaluate the duration of the CTR signal from the expression (3.67) on page 69, $\Delta t_{CTR} = \frac{0.9 \lambda_{2obs}^2}{(c \Delta \lambda_{obs})}$. From the experimental width of the second and third harmonic, we estimate $\Delta t_{CTR} = 50 - 150$ fs or 10-30% of the total
FIGURE 6.2. Optical spectra for different shots. Left column: results from experiment b) in Figure 6.1 (red spectrum: viewing angle at 55°; blue spectrum: viewing angle at 30°). Right column: experiment c) in Figure 6.1.
duration of the laser pulse. This suggests a rapid decrease in the efficiency of transition radiation once the sheath is formed at the rear surface and the scale length becomes comparable to the formation length of the radiation.

As a final remark on Figure 6.2, we note that the peaks are red-shifted with respect to the values at $\lambda = 1054/n_h$ nm, where $n_h$ is the number of the harmonic. It is not clear why this is the case. At the end of this section we will discuss some possible explanations.

6.3.1.1. Effect of hole boring on laser absorption. The presence of odd harmonics in experiment c) might seem surprising. In fact, in this case the angle of incidence was close to $0^\circ$. As a result, it would be expected that electrons are mainly accelerated via the $j \times B$ mechanism which would imply electron bunches separated by half the laser wavelength and, therefore, that only the even harmonics should be present. This is also what is apparent from PIC simulations, at least when the laser interacts with a steep density gradient. In that case, only electron bunches separated by twice the laser frequency are visible and the corresponding optical radiation only presents even harmonics (Figure 3.16 in chapter 3).

Previous measurements have actually already shown the presence of odd harmonics even at $0^\circ$ incidence [32]. That this is the case is a clear indication, if it was necessary, that the process of laser-absorption is far more complicated than in an academic exercise. Clearly, laser energy absorption is a non-stationary process that can be significantly modified by the presence of a preplasma and by processes such as hole boring, that acts during the interaction itself. It is interesting to give a simple estimate of the effects of hole boring into laser absorption. For this purpose, we evaluate the motion of the critical surface using the formula for the hole boring (or shock) speed. Our method is validated by a calculation made by Henig et al. [125], which shows excellent agreement between the formula for the shock speed and PIC simulations. For total laser absorption, the non-relativistic hole boring speed can be written as [49, 126]

$$\beta_{hb} = \sqrt{\frac{I_L}{n_i m_i c^3}},$$

where $n_i$ and $m_i$ refer to the ion density and mass, respectively. For a Gaussian intensity profile, this means that the laser pushes the (relativistic) critical surface at different rates, depending on the transverse position. The maximum hole boring speed is achieved on the laser axis and a crater develops on the target surface.

After some time during the interaction, the curvature of the front surface is important enough that a significant portion of the laser cannot be considered to be incident at $0^\circ$ on target anymore. In this case absorption mechanisms such as vacuum heating or, in the presence of a prepluse, resonance absorption could become important. Introducing the notation as in Figure 6.3 and expressing the spatial dependence of the laser intensity as
6. FILAMENTATION AND RECIRCULATION MEASUREMENTS FROM THE REAR SIDE OPTICAL EMISSION

![Diagram of laser pulse and equiphase surfaces](image)

**Figure 6.3.** A laser pulse with a Gaussian spatial profile produces hole boring of the target front surface. Even assuming that the equi-phase surfaces are planes (i.e. the pulse is within its Rayleigh range) the deformation of the critical surface is such that, after some time, a fraction of the laser energy cannot be considered to be incident at normal anymore.

\[
I_L(x,t) = I_0 \exp\left(-2x^2/w_0^2\right),
\]
the position of the critical surface changes in time and space according to

\[
z(x,t) = c\beta_{hh}t = \sqrt{\frac{I_0}{n_i m_i c}} \exp\left(-x^2/w_0^2\right) t,
\]
so that the local angle of the surface is given by

\[
\frac{\partial z}{\partial x} = -\sqrt{\frac{I_0}{n_i m_i c}} \exp\left(-x^2/w_0^2\right) \frac{2x}{w_0^2} t.
\]

In order to determine the local angle of incidence of the wave we need to specify the equiphase surfaces, as the local wave vector is normal to them. We assume that they are represented by planes. In other words, we assume that the laser is within its Rayleigh range.

At this point we estimate the time required to achieve substantial deformation of the target that can trigger new absorption mechanisms. For this, we evaluate the time \(t_{20^\circ}\) for which the incidence angle is of 20° at the beam waist \(w_0\). At around this angle, as we have seen, resonance absorption is maximised. For times longer than \(t_{20^\circ}\), an increasing portion of the beam interacts with the critical surface at an angle \(\geq 20^\circ\).

By imposing that \(|\partial z/\partial x| = \tan(20^\circ)\) when \(x = w_0\), we find that this time is given by

\[
t_{20^\circ} = 111w_0[\mu m]\left(\frac{n_{23}}{I_{20}}\right)^{1/2} \text{ fs}, \tag{6.91}
\]
where \(n_{23}\) is the ion density in units of \(10^{23}\) cm\(^{-3}\) and \(I_{20}\) is the laser intensity in units of
10^{20} \text{Wcm}^{-2}. As an example, we use formula (6.91) for the case of experiment c). We assume the relativistic critical density to be \( n_e = 8 \times 10^{21} \text{cm}^{-3} \), that corresponds to \( \lambda_L = 1 \mu \text{m} \) and \( \gamma \simeq (1 + a_0^2/2)^{1/2} = 8 \). Assuming a fully ionised Cu plasma (\( Z=29 \)), this gives an ion density \( n_i \simeq 3 \times 10^{20} \text{cm}^{-3} \). As we had an intensity on target of \( 8 \times 10^{19} \text{Wcm}^{-2} \) and a focal spot of about \( 5 \mu \text{m FWHM} \), formula (6.91) gives \( t_{20} \simeq 34 \text{fs} \). Thus, at an early stage the geometry of the interaction has changed.

6.3.1.2. Discussion on the red-shift. The red-shift in the harmonic peaks that we have mentioned amounts to several nanometers. A possible explanation is that the electron bunches are actually produced at intervals that are not exactly coincident with \( \lambda_L \) or \( \lambda_L/2 \). However, there is no theoretical argument on which we can base these conclusions.

Alternatively, we could think that the red-shift is due to the motion of the rear surface, while the effect of the front surface is negligible for electron bunches that travel close to the speed of light. The movement of the rear surface has the effect of increasing the time delay between two successive bunches. We denote with \( \Delta T \) the time delay of two bunches for a stationary rear surface and with \( \beta_r \) the normalised velocity of the rear surface. After the first bunch has arrived at the interface, the second bunch arrives with a time delay \( \Delta T' = \Delta T \frac{1}{1 - \beta_r} \). (6.92)

Thus whenever the target rear side moves, a red-shift in the spectral peak should appear (\( \Delta T' > \Delta T \)). This is consistent with our measurements. As the experimental shift amounts to 1 – 3% of the central wavelength, according to formula (6.92) this should correspond to \( \beta_r = 0.01 – 0.03 \).

Further experimental investigations would be necessary in order to verify this model. If this model is applicable, measuring the red-shift of the transition radiation spectral peaks could become a simple technique for the determination of the speed of the rear surface.

6.3.2. Polarisation. Consistent with the observations discussed in chapter 5, the radiation was found to be highly polarised. Figure 6.4a shows an image of the rear-surface emission from a 5 \( \mu \text{m} \) Au target, for experiment a) (Figure 6.1). The first two panes are for horizontal and vertical polarisations, whilst the right pane shows the post-processed polarisation map resulting from their addition. The radiation is strongly (predominantly horizontally) polarized over the whole region. This is true for the extended halo of emission as well as for the brightest central area of emission. For our viewing angle (also in the horizontal plane) the polarisation of the emission is consistent with that expected for transition radiation. The overall size of the emission cannot be explained by the divergence of the electron beam, and is much larger than the dimensions of the laser focal spot
With increasing target thickness (Figures 6.4b, c), the signal strength decreases. However the spatial extent of the emission remains relatively unchanged (Figure 6.4d).

That the halo is also due to transition radiation indicates that the electrons must have been transported transversely by multiple reflections within the target, rather than traveling directly with large divergence angles. The size of the radiating region would otherwise imply that electrons are directed at $\approx 90^\circ$ from the normal to the target, which would inhibit the emission of transition radiation. In fact, equations (3.47) and (3.48) in chapter 3 (page 64) show that $E_\parallel, E_\perp \rightarrow 0$ for $\psi \rightarrow 90^\circ$ (electron at grazing incidence). Hence this observation is a direct evidence for refluxing of hot electrons which has previously only been deduced indirectly [121, 123, 124]. Refluxing occurs because of the inhibition of electrons exiting into vacuum by the strong sheath fields that they generate, and is most dominant for thin targets where the surface charge density is greatest.

However, the radiation of the halo is probably incoherent transition radiation, as it is unlikely that the modulation of the fast electron current would be maintained over multiple reflections at the target boundaries.

For completeness, we should stress that our measurements prove that the radiation was polarised. Strictly speaking, they do not allow to determine whether the polarisation was
linear or elliptical. In order to eliminate this uncertainty, it would be necessary to know the Stokes parameters, that define the polarisation state of an arbitrary wave (whether polarised or unpolarised) [127]. Our measurements only allow the determination of two of the four Stokes parameters (namely, the total incident irradiance and the difference between the irradiance in two orthogonal planes).

However, due to the fact that the theory of transition radiation predicts the generation of linearly polarised light (unless it is produced in anisotropic media), we believe our measurements reflect a polarisation state that is, essentially, linearly polarised. In principle, a magnetic field could produce elliptical polarisation, from an initially linearly polarised wave. However our argument is that, even if these magnetic fields are present at the target rear side, they do not significantly alter the polarisation state of the radiation because our measurements are consistent with the theory of transition radiation (in particular, that the radiation should be mainly horizontally polarised for our viewing angle).

Further investigations on the polarisation state of the radiation are discussed in the next paragraph.

6.3.2.1. Wedge targets. Within the theory of TR, it is predicted that the polarisation should depend on the angle of observation. To investigate whether this was the case, in experiment c) (Figure 6.1) a series of shots was taken on targets with varying rear-surface wedge angle.

Cu wedges with angles of $\alpha = 10^\circ$, $20^\circ$ and $35^\circ$ as well as a $50 \mu m$ flat ($\alpha = 0^\circ$) target were used. An example is given in Figure 6.5. The interaction point was chosen to ensure that the distance from front to rear surface was $50 \mu m$ for each target, thus keeping the effective foil thickness constant.

The use of wedge targets allowed the angle that the fast electron beam formed with respect to the rear surface to be varied without significantly changing the physics of the absorption mechanism at the front. Hence it can be assumed that the fast electrons produced in the interaction with the different targets had similar properties (temperature, direction, temporal envelope).
6. FILAMENTATION AND RECIRCULATION MEASUREMENTS FROM THE REAR SIDE OPTICAL EMISSION

It should be noted that, as the collecting optics were at a fixed position, this also enabled a variation of the angle of observation \( \theta \) with respect to the normal to the target rear side (Figure 6.6).

If the emission is mainly transition radiation then a variation in the degree of polarisation should be observed when changing the angle of the wedge. Indeed, the radiation exhibited a high degree of polarisation that was dependent on the wedge angle \( \alpha \), as shown in Figure 6.7 and analysed in Figure 6.8. For the flat targets it was found that the dominant polarisation is horizontal, as before. However increasing the angle of the wedge increases the respective contribution of the vertical polarisation (Figure 6.8, black circles).

As before, these images exhibit a halo surrounding the bright main region of CTR emission. This is seen most clearly in Figure 6.7c which demonstrates that this wide region is polarised. On the right side of this image, for both polarisations, a bright line of emission is seen from the target edge, the polarisation properties of which are difficult to interpret. Ion emission from the edge of solid targets has been observed previously [123], and has also been attributed to the field emission of hot electrons transported through the targets by recirculation. Both the extended halo and the emission from the target edge are at distances far from the initial interaction region, and can be explained by electron recirculation. Again this shows the importance of refluxing in transporting energy laterally in thin foils.

All these observations are, once more, a strong evidence that the radiation emitted is mainly due to transition radiation. Other sources of polarised radiation would be synchrotron radiation and coherent wake emission (CWE), that have both been considered in previous works [106, 109]. Synchrotron radiation is mainly polarized in the plane of motion of the electrons. In our case it would be produced by electrons being pulled back by the electrostatic field that builds up at the back of the target [96, 106], so that the polarization would vary across the emission region, depending on the direction in which electrons travel before restriking the surface. According to the present models, CWE should be polarized in the plane of polarization of the laser field and being mainly emitted in the direction of propagation of the laser field [109, 128]. Thus its polarization state should not be dependent on the angle of the rear surface.

**Figure 6.6.** Sketches of the wedge targets used in experiment c), showing our definition of angle of the wedge \( \alpha \) and angle of observation \( \theta \).
6.3. RESULTS AND INTERPRETATION

FIGURE 6.7. Polarisation analysed OTR images; LHS horizontal polarisation, RHS vertical polarisation for a) 50 µm foil, b) 35° wedge, c) 10° wedge (here top is horizontal and bottom vertical polarisation). In c), the center is over-exposed to enhance target visibility.

In order to use the information on the polarisation, and also for later use, here we rewrite the expression for the CTR emitted by a beam of electrons per unit angular frequency and unit solid angle [90] (see chapter 3, page 64),

\[
\frac{d^2W}{d\omega d\Omega}_{\text{CTR}} = \frac{e^2N(N - 1)}{\pi^2c} \left( \left| \int d^3p g(p) |E_{\parallel} F| \right|^2 + \left| \int d^3p g(p) |E_{\perp} F| \right|^2 \right),
\]

(6.93)

where \(N\) is the total number of electrons, \(g(p)\) the momentum distribution function, \(F\) a form factor and \(E_{\parallel}\) and \(E_{\perp}\) are the amplitudes of the Fourier-transformed electric field parallel and perpendicular to the radiation plane, respectively. It should be noted that, as it was pointed out in chapter 3, these two components do not correspond to our “horizontal” and “vertical” polarisations, although they are related by geometric arguments.

By integrating equation (6.93) over the solid angle of the collecting optics and using the information contained in the polarisation for the different wedge targets, we can estimate the direction of the electrons that were diagnosed by our imaging system, since \(\text{CTR is not an isotropic source of radiation}\). For this analysis we compute the theoretical ratios of horizontal versus vertical polarisation and compare these results with the experimental ones. For this purpose, in our model we vary the direction of identical, collimated electron beams in the horizontal plane and find the corresponding polarisation ratio. The results of this analysis are presented in Figure 6.8 and suggest that the electron filaments diagnosed were directed within the cone of our collection optics, i.e. \(\delta \in [25^\circ, 42^\circ]\), in agreement with the results in chapter 5.
Figures 6.8 and 6.9. Variation of polarisation with observation angle $\theta$ (or, equivalently, wedge angle $\alpha$). Experiment (black circles) and theoretical predictions for different directions of electron filaments $\delta$ (dashed lines). The shaded region is obtained assuming that the electrons are directed within the cone of the collection optics $\delta \in [25^\circ, 42^\circ]$.

Figures 6.8 and 6.9. Total normalised energy collected by the imaging system as a function of the angle of the electrons with respect to the front side target normal, for different electron temperatures (blue: 0.5 MeV, green: 2 MeV, red: 4.5 MeV, turquoise: 9.5 MeV). The two broken lines mark the range of angles included in the cone of the first lens. Each curve is re-scaled to its maximum value, in general more energy is radiated at higher temperatures.
This conclusion is also supported more directly by noting the distance of the main CTR from the laser axis in Figure 6.7c, which can be determined by measuring the distance from the target edge. The position of the signal from the CTR coincides with the position along the direction of the first lens.

This result is expected since, as we have discussed, for relativistic electrons the angular distribution of transition radiation is confined almost along the particle’s direction of propagation. In Figure 6.9 we show the (theoretical) amount of radiation collected by the imaging system as a function of the angle of the electron filament, for two different wedge targets \((\alpha = 0^\circ, 35^\circ)\). It is clear that electrons going towards the collection optics (denoted by the two vertical broken lines) are more easily detected by the imaging system. These plots are made assuming a Maxwellian distribution of electrons.

6.3.3. Coherent diffraction and filament size. A decrease of the signal strength with the angle of observation \(\theta\) (i.e., with decreasing wedge angle \(\alpha\)) was also observed (Figure 6.10, black circles). This observation, together with the polarisation measurements, would rule out the presence of CWE, which should not depend on \(\alpha\).

The strong dependence of the intensity of the radiation with the angle of observation (note that the scale in Figure 6.10 is logarithmic) is due to coherence effects and is related to the mean size of the electron filaments, as for CTR the divergence of the radiation is strongly dependent on source size. Hence we can use this information to estimate the mean size of the electron filaments producing the radiation.

For interpretation of the variation of the signal intensity with wedge angle, it is important to retain all the parameters in the expression (6.93). In particular, the role of the form factor \(F\) is essential in this case. Physically, this quantity accounts for the phase difference of the waves emitted at the back of the target. Therefore it depends on the time and position at which each electron reaches the back surface. In chapter 3, equation (3.59), we have given an expression for the form factor. It was assumed that a train of impulsive electron bunches is produced at the front of the target and that the transverse spatial profile of the electron beam is Gaussian. Here we rewrite this expression, in a slightly different way that highlights the important terms for our discussion,

\[
F = G \frac{\sin(n_b \omega_{\text{obs}} \delta T / 2)}{\sin(\omega_{\text{obs}} \delta T / 2)} \exp \left[ -\frac{1}{2} \left( \frac{2\pi r}{\lambda_{\text{obs}}} \right)^2 \sin^2 \theta \right].
\]

In the previous formula, \(\delta T\) is the bunching period, \(n_b\) the number of bunches and \(r\) the radius of the beam at \(1/\sqrt{\epsilon}\) maximum intensity. Moreover, \(\lambda_{\text{obs}}\) and \(\beta\) are, respectively, the wavelength and angle of observation (\(\lambda_{\text{obs}} = 527\) nm in our case). The quantity \(G\) is a function of several parameters, for example the target thickness. The main effect of this quantity is, in fact, that it models the decrease in signal strength with target thickness. The
second term in Eq. (6.94) gives rise to harmonics of the bunch frequency in the spectrum, as reported in the same chapter 3.

The third term is the result of the Fourier-transform of the transverse profile of each bunch [90, 93]. According to this expression, a variation of the signal intensity with the angle of observation is directly related to the size of the electron beam. This is because when the angle of observation $\theta$ differs from zero, this term can lead to a drastic decrease in the intensity of the signal, depending on the radius $r$ of the beam. This idea was also suggested by Zheng et al.[93] but, to our knowledge, in all previous experiments the observation angle was kept constant.

The data in Figure 6.10 have been fitted for different beam sizes. It was assumed that the electron temperature $T_e = 10$ MeV, but the dependence on $T_e$ is weak. A best fit over all the experimental data implies FWHM diameter $D = 1 \mu m$, however the fall-off in signal strength at large $\theta$ is too strong, suggesting the presence of smaller filaments. A better fit is found if only the data for $\alpha = 0^\circ$, $10^\circ$, and $20^\circ$ are considered. In this case, the best fit is for $D = 0.8 \mu m$. For $D = 0.5 \mu m$ the calculated fall-off in signal would be slower than measured.

It should be noted that, in the presence of multiple filaments, the filament size inferred from our indirect measurement is the mean size of the filaments. The term
Figure 6.11. Beam blooming (or better FWHM diameter of the electron beam) as a function of electron energy for an initially zero-radius beam that propagates through 50 µm of copper. The curves show Monte Carlo calculations, the analytical model as given by equation (6.95) and the same model where Λ_s is varied by a factor of 10. Image courtesy of J. R. Davies.

\[
\exp \left[-\frac{1}{2} \left(\frac{2\pi r}{\lambda_{\text{obs}}}\right)^2 \sin^2 \theta \right]
\]

in equation (6.94) shows that, when the angle of observation goes to zero, the effect of the filament size becomes less prominent. In the case of θ = 0, this term disappear and the signal is not dependent on the filament size anymore. As a result, more filaments, if they are present, should be visible when looking at target normal with respect to an angle θ > 0. This seems to be the case from Figures 6.7a and 6.7b. For the largest angle of observation (wedge angle α = 0°) only a single filament appears, with a size that is limited by the resolution of the imaging system. However, for the case of the smallest angle of observation (wedge angle α = 35°), a more filamented structure now appears.

6.3.4. Comparison with PIC simulations and Monte Carlo calculations. Numerous results from particle-in-cell codes have demonstrated the formation of µm-scale filaments directed over a wide range of angles [129, 130, 131, 132, 133]. However, for our conditions these filaments would have to propagate through ~ 50 µm of solid Cu, where angular scattering would be expected to cause a rapid expansion. An equation for “beam blooming” due to angular scattering, defined as the variance of the transverse beam size (see chapter 2, page 44), was given in [74]. Neglecting energy loss and assuming a small
6. FILAMENTATION AND RECIRCULATION MEASUREMENTS FROM THE REAR SIDE OPTICAL EMISSION

Figure 6.12. Electron spectra for different wedge targets, measured by an electron spectrometer positioned on the laser axis (experiment c). Image courtesy of S. R. Nagel.

The total angular deflection, beam blooming can be written as [134]

\[ B \approx \frac{e^2}{2\sqrt{3\pi\varepsilon_0}} Z n_a \ln\Lambda_s \frac{s^{3/2}}{p\nu}, \]

where \( s \) is the distance travelled, \( Z \) is the atomic number, \( n_a \) is the atom number density, \( \Lambda_s \) is a term that depends on the electron-atom scattering cross section and \( p, \nu \) are the momentum and velocity of the electron, respectively. This result gives excellent agreement with Monte Carlo modelling [135]. It indicates that, in order to obtain the features inferred of \( \sim 1.0 \mu m \) FWHM (\( \equiv 2\sqrt{2\ln2}B \)), electrons with energies \( > 40 - 50 \text{ MeV} \) are required for all realistic values of \( \Lambda_s \) (Figure 6.11). This is well above the ponderomotive potential of the laser (\( \sim 4 \text{ MeV} \)) and is in contradiction with the cut-off of 30 – 40 MeV measured with an electron spectrometer positioned on the laser axis in these experiments (Figure 6.12). This suggests that the small scale structure is the result of self-generated electric or magnetic fields inside the target or in proximity of the rear of the target.

This could be due to the resistive growth of the magnetic field inside the target [78]. Results from the code LSP have been published for parameters close to those of this experiment [136]. They show no sign of filaments at 0.5 ps for intensities of \( 10^{18} \text{ Wcm}^{-2} \) and \( 10^{19} \text{ Wcm}^{-2} \) and filaments 1-2 \( \mu m \) wide at \( 10^{20} \text{ Wcm}^{-2} \) (see Figure 12 in [136]), marginally higher than our peak intensity. In addition, rear surface magnetic focusing

\[ \text{Cu Wedge comparison} \]

\[ \text{Number of electrons/MeV/steradian} \]

\[ \text{Energy in MeV} \]

- flat target (50 \( \mu m \) Cu)
- 10° wedge
- 20° wedge
- 35° wedge
6.4. SUMMARY

Figure 6.13. Number of electrons per unit time (i.e. current) as seen at the target rear side, after propagation through 50 µm of copper. The electron bunches are initialised as impulses at the target front side. Even with collisional effects the current is clearly modulated after propagation. Image courtesy of J. R. Davies.

[137] and instabilities associated with the electron sheath at the back surface may contribute to the observed small emission size.

We have also used Monte Carlo calculations to test if it is possible to have a modulated current of fast electrons after propagation through 50 µm of copper. Arguments against this conclusion have often been based on the consideration that collisions would cause the beam to spread longitudinally (beam straggling), losing its modulated current and hindering the emission of coherent radiation. However, our Monte Carlo calculations suggest that, for our parameters, if at the front surface a train of impulsive electron bunches is injected into the target, at the target rear side the fast electron current still remains modulated. This is the first calculation that tests the concept of coherent transition radiation in the presence of collisions.

6.4. Summary

In this chapter we have confirmed some of the conclusions drawn in the previous chapter, particularly those concerning the polarisation measurements. We have shown that the optical spectrum of the radiation indeed shows peaks at the harmonics of the laser frequency and that this is a proof of the coherence of the emitted radiation. Furthermore, these peaks are red-shifted which could be an indication of the motion of the target rear surface.
For our thin (much smaller than the laser pulse length) targets, our measurements have shown the presence of an extended region of polarised radiation. Its spatial dimensions cannot be explained by ballistic transport of electrons, unless a half-cone divergence of $\approx 90^\circ$ is assumed. However, a much more likely explanation is that this is due to electrons refluxing in the target and emitting transition radiation when they traverse the rear surface. This radiation should be incoherent and we would not expect it to show peaks in the optical spectrum.

The presence of an extended source of polarised radiation represents the main difference with the results of the previous chapter, where an unpolarised halo of radiation was surrounding the source of CTR emission.

Finally, in this chapter we have made measurements of polarisation and intensity of the radiation for different wedge targets. Using the model developed in chapter 3, we have been able to interpret the strong dependence of the signal strength with the angle of the wedge. This has enabled us to infer diameters of $\sim 1.0\ \mu m$ FWHM for the electron filaments, which is consistent with published results from PIC and hybrid simulations.
CHAPTER 7

Nonlinear laser pulse evolution in an underdense plasma

The propagation of short ($c \tau_L \lesssim \lambda_p$), high-intensity ($I \gtrsim 10^{18}$ W cm$^{-2}$) laser pulses through plasmas has demonstrated its ability to produce plasma waves suitable for accelerating electrons in ultra-high fields ($E \sim 1$ GV/cm) [8, 13, 14, 87]. The properties of these plasma waves and, therefore, the quality of the accelerated electrons are inherently dependent not only on the initial parameters of the laser pulse, but also on its nonlinear evolution during propagation through the plasma. Understanding and controlling the laser pulse dynamics is therefore of upmost importance for optimising the laser-driven wakefield acceleration scheme.

In addition, the nonlinear pulse evolution is of interest on its own, as the complex interplay between the action of the ponderomotive force of the laser driver and the nonlinear response of the plasma can lead to efficient spectral broadening and self-compression of the laser pulse. This could lead to an increase of the power of currently existing high-intensity laser systems without increasing the energy of the pulse.

The advantage of a plasma as a nonlinear optical medium with respect to conventional optical media is that it does not suffer from limitations in terms of damage threshold, allowing for an arbitrary strength of the driver field. In the wakefield regime, $c \tau_L \lesssim \lambda_p$, simulations predict that the initial pulse evolution leads to the development of a positive chirp, red-shift at the front and blue-shift at the back of the pulse [86, 138]. Further theoretical work has also shown that this evolution could result in the production of an asymmetric pulse, steepened at either the front [139] or at the back [140].

The experimental demonstration of these theoretical predictions require the use of a technique that can uniquely retrieve the original pulse, including its temporal phase, and is able to deal with asymmetric pulses. Pulse compression from an initial 40 fs to $\simeq 13$ fs, with an efficiency of $\sim 20\%$, has been recently observed by Faure et al. [141]. However, in that experiment a second harmonic autocorrelation technique was used to measure the properties of the pulse. Second order autocorrelation has several disadvantages, as we have seen in chapter 4. In particular, this technique does not allow the retrieval of phase information about the original pulse and, as such, cannot uniquely retrieve the original pulse.

In this chapter we report on the fully temporal (amplitude and phase) characterisation of short ($\tau_L \simeq 45$ fs), relativistically intense ($I \gtrsim 1 \times 10^{19}$ W cm$^{-2}$) laser pulses after...
interaction with supersonic helium gas-jets. The experiment was conducted using the Astra-Gemini laser at the Rutherford Appleton Laboratory. Spectral broadening, pulse shortening and pulse profile steepening have been observed and the dependence of these nonlinear effects on plasma density and interaction length have been investigated. Furthermore, interferometric data and measurements of the spot size and transmitted energy of the laser pulse have allowed us to investigate the quality of the laser self-guiding and to determine the laser peak power as a function of plasma density and interaction length.

Before going into the experimental results, we will try to describe, from a theoretical point of view, the nonlinear evolution of a laser pulse in a plasma. This will be useful for the discussions on the experimental findings that will follow.

7.1. Pulse evolution: theory

In Chapter 1 we have described the equations that govern the production of the wakefield and the evolution of the laser pulse within the 1D quasi-static approximation, as in references [80, 81, 142, 143]. The starting equations for the nonlinear pulse evolution in a plasma are those already found in equations (2.33) and (2.34) in chapter 2, that we rewrite here in normalised form:

\[
\frac{2}{\xi} \frac{\partial^2 a}{\partial \xi \partial \tau} - \frac{\partial^2 a}{\partial \tau^2} = \frac{a}{1 + \phi} \]  
\[
\frac{\partial^2 \phi}{\partial \xi^2} = \frac{1}{2} \left[ \frac{1 + a^2}{(1 + \phi)^2} - 1 \right] \]  

(7.96)  
(7.97)

The normalised equations are obtained with the transformations \( \xi \rightarrow k_p \xi \) and \( \tau \rightarrow \omega_p \tau \). We stress again that equations (7.96) and (7.97) are written in the comoving frame of the pulse and are valid in the framework of the quasi-static approximation (QSA).

In the next few paragraphs we will write an analytic solution to a reduced set of equations, under the slowly varying envelope approximation. It will mainly serve to show that an initially un-chirped laser pulse propagating through a plasma develops a positive chirp.

Finally, we will present results of the numerical solution of the complete set of equations (7.96) and (7.97) and we will also show what particle-in-cell simulations predict.

7.1.1. Envelope approximation and Schrödinger equation. It is a common approximation in nonlinear optics to write the field as the product of a slowly varying envelope and a carrier frequency term:

\[
a(\xi, \tau) = \frac{1}{2} a_L(\xi, \tau) \exp(ik_0 \xi) + c.c. \]  

(7.98)
where $a_L$ is a complex quantity and $k_0 = \omega_L/\omega_p = \sqrt{n_{cr}/n_0}$ is a normalised wave number. This approximation is valid in most situations and allows one to separate out the high frequency components of the oscillating field. Inserting (7.98) in (7.96) and (7.97), we obtain

$$2 \frac{\partial^2 a_L}{\partial \xi \partial \tau} + 2ik_0 \frac{\partial a_L}{\partial \tau} \frac{\partial^2 a_L}{\partial \tau^2} = \frac{a_L}{1 + \phi}, \quad (7.99)$$

$$\frac{\partial^2 \phi}{\partial \xi^2} = \frac{1}{2} \left[ \frac{1 + |a_L|^2/2}{(1 + \phi)^2} - 1 \right], \quad (7.100)$$

where we are also considering an average field when writing the term $|a_L|^2/2$.

The envelope $a_L(\xi, \tau)$, as written in (7.98), is a complex number. It is convenient to write it as

$$a_L(\xi, \tau) = m(\xi, \tau) \exp[i\psi(\xi, \tau)].$$

After inserting this expression into (7.99) and equating the real and imaginary parts of the left and right hand side, we are left with two equivalent expressions to be solved,

$$2 \frac{\partial^2 m}{\partial \xi \partial \tau} - 2\frac{\partial m}{\partial \tau} \frac{\partial \psi}{\partial \xi} - 2k_0m \frac{\partial \psi}{\partial \tau} - \frac{\partial^2 m}{\partial \tau^2} + m \left( \frac{\partial \psi}{\partial \tau} \right)^2 = \frac{m}{1 + \phi}, \quad (7.101)$$

$$2 \frac{\partial m}{\partial \xi} + 2 \frac{\partial m}{\partial \tau} \frac{\partial \psi}{\partial \xi} + 2m \frac{\partial^2 \psi}{\partial \xi \partial \tau} + 2k_0 \frac{\partial m}{\partial \tau} - 2 \frac{\partial m}{\partial \tau} \frac{\partial \psi}{\partial \xi} - m \frac{\partial^2 \psi}{\partial \tau^2} = 0. \quad (7.102)$$

Only two terms in these two equations are multiplied by the factor $k_0 = \sqrt{n_{cr}/n_0}$. As a result, for $k_0 \gg 1$ (very diluted plasmas) they will represent the two leading terms in the equations. With these simplifications, that are in principle only valid at the beginning of the pulse evolution, (7.101) and (7.102) become

$$-2k_0m \frac{\partial \psi}{\partial \tau} = \frac{m}{1 + \phi}, \quad (7.103)$$

$$2k_0 \frac{\partial m}{\partial \tau} = 0. \quad (7.104)$$

Combining these two equations in one, we can see that, instead of the full expression (7.99), these are equivalent to solving the equation

$$2ik_0 \frac{\partial a_L}{\partial \tau} = \frac{a_L}{1 + \phi}. \quad (7.105)$$

In this way we have justified the applicability of the Slowly Varying Envelope Approximation (SVEA) to the case of the wave equation (7.99). Equation (7.105) can be interpreted as a Schrödinger equation where the laplacian term $\nabla^2 a_L$ vanishes and the potential is
represented by $1/(1 + \phi)$. We can learn from it that the characteristic time for the pulse evolution is of the order of $k_0$: $\Delta \tau = O(k_0)$. This makes sense, as it implies that the evolution is slower in more dilute plasmas. In the vacuum limit ($n_e = 0$), $\Delta \tau \to \infty$ and there is no pulse evolution, as expected.

For completeness, we also note that equation (7.105) has been already used by Bendib et al. [144]. However, in that case this equation was coupled to another equation describing the ionisation of the medium instead of Poisson’s equation (7.100).

The second equation, (7.104), clearly says that $m(\xi, \tau) = m(\xi, \tau_0)$ or, in other words, that the pulse envelope does not evolve in time. As a result, the wave equation (7.105) and Poisson’s equation (7.100) are decoupled. From (7.100), we discover that the potential is also not evolving in time, $\phi = \phi(\xi)$, since we have just learnt that the driving term $|a_L| = m$ is constant over time.

As a result, it is possible to solve (7.103) exactly, finding that

$$\psi(\xi, \tau) = -\frac{1}{2k_0} \frac{\tau}{1 + \phi(\xi)} + \frac{1}{k_0} K(\xi), \quad (7.106)$$

where we have applied the initial condition $\psi(\xi, 0) = K(\xi)/k_0$. This initial condition $K(\xi)$ will be specified later. The phase $\psi(\xi, \tau)$ determines the chirp of the pulse. In order to show this, we first need to obtain an expression for the normalised shift in the instantaneous frequency, $\Delta \omega_L = \omega_L/\omega_0 - 1$,

$$\Delta \omega_L = -\frac{1}{\omega_0} \frac{\partial \psi(x, t)}{\partial t} = \frac{1}{\omega_0} \left( -\omega_p \frac{\partial}{\partial \tau} + k_pc \frac{\partial}{\partial \xi} \right) \psi(\xi, \tau)$$

$$= \frac{1}{k_0} \left( \frac{\partial \psi}{\partial \xi} - \frac{\partial \psi}{\partial \tau} \right), \quad (7.107)$$

where we have passed from the stationary coordinates $(x, t)$ to the normalised coordinates in the comoving frame $(\xi, \tau)$.

In order to understand the chirp of the pulse as it evolves through the plasma, we have to evaluate the sign of the instantaneous frequency that appears in (7.107): a positive (negative) sign for $\Delta \omega_L$ implies a higher (lower) local frequency with respect to the initial frequency of the pulse, i.e. a blue (red) shift. For this purpose we can express both the derivatives that appear in (7.107) as a function of the potential $\phi(\xi)$:

$$\frac{\partial \psi}{\partial \xi} = \frac{\tau}{2k_0 (1 + \phi)^2} \frac{d\phi}{d\xi} + \frac{1}{k_0} \frac{dK}{d\xi}, \quad (7.108)$$

$$\frac{\partial \psi}{\partial \tau} = -\frac{1}{2k_0} \frac{1}{1 + \phi}, \quad (7.109)$$

so that the instantaneous frequency $\Delta \omega_L$ can be rewritten as
\[ \Delta \omega_L(\xi, \tau) = \frac{1}{2k_0^2} \frac{1}{1 + \phi(\xi)} \left[ \frac{\tau}{1 + \phi(\xi)} \frac{d\phi}{d\xi} + 1 \right] + \frac{dK}{d\xi}. \] (7.110)

At this point we specify the quantity \( K(\xi) \) so that the pulse is initially un-chirped, \( \Delta \omega_L(\xi, 0) = 0 \). If we do so, the instantaneous frequency then evolves as

\[ \Delta \omega_L(\xi, \tau) = \frac{1}{2k_0^2} \frac{\tau}{[1 + \phi(\xi)]^2} \frac{d\phi}{d\xi}. \] (7.111)

This is the final result of our discussion. We can see that the local frequency of the pulse is dependent on the details of the potential or, in other words, on the density profile in the wake. Numerical solution of Poisson’s equation (7.100) for a Gaussian pulse and in the wakefield condition \( c \tau_L \lesssim \lambda_p \) reveals the behaviour of \( \phi(\xi) \) across the pulse: \( d\phi/d\xi < 0 \) at the front of the pulse and \( d\phi/d\xi > 0 \) at the back of the pulse. Therefore, the solution (7.111) describes a pulse that is blue-shifted \( (\Delta \omega_L > 0) \) at the back of the pulse for any time \( \tau > 0 \) and is red-shifted \( (\Delta \omega_L < 0) \) at the front.

However, we should point out that the result in (7.111) ceases to have a physical meaning where and when \( \Delta \omega_L < -1 \), in which case \( \omega_L < 0 \). The time required to reach this unphysical scenario is

\[ \tau > -\frac{(1 + \phi(\xi))^2}{d\phi/d\xi} \frac{2k_0^2}{1 + \phi(\xi)}. \]

This occurs in regions where \( d\phi/d\xi < 0 \), for times \( \tau = O(k_0^2) \). This is reasonable, because at this point the approximations that we made to arrive at (7.105) are no longer valid. For example, according to our solutions (7.107) and (7.109), the term \( m(\partial\psi/\partial\xi)(\partial\psi/\partial\tau) \) grows linearly in time

\[ m \frac{\partial\psi}{\partial\xi} \frac{\partial\psi}{\partial\tau} = m \frac{\tau}{4k_0^2(1 + \phi)^3} \frac{d\phi}{d\xi} + \frac{1}{2k_0(1 + \phi)} \frac{dK}{d\xi} \]

and thus becomes comparable to the leading terms after a finite time. This time is dependent on \( k_0 \) and \( d\phi/d\xi \).

In Figure 7.1 we show the pulse properties for a laser pulse propagating through a plasma with \( k_0 = 100 \), in the SVEA approximation. This corresponds to solving equations (7.105) (this can be solved analytically, as discussed) and (7.100). The shape of the potential leads to the formation of a positive chirp, as anticipated.

### 7.1.2. Numerical solution of the full system of equations

We now discuss the numerical solution of the full system of equations (7.99) and (7.100).

#### 7.1.2.1. Choice of the numerical scheme

The equations to be numerically solved consist of a coupled set of equations. Poisson’s equation, (7.100), being a second order
Figure 7.1. Pulse evolution in the SVEA approximation, equations (7.105) and (7.100), for the case of $a_0 = 3$ and $k_0 = 100$. a1)-a4) Initial conditions ($\tau = 0$). b1)-b4) Pulse properties when $\tau = 80$. Note that figures a1) and a4) are identical to b1) and b4), which is due to the approximations made, while the instantaneous frequency of the pulse evolves from an un-chirped pulse a3) to a positively chirped pulse b3).
ODE, does not present any numerical difficulty. The scheme chosen for this integration is a Runge-Kutta of the 4th order.

For the solution of equation (7.99), we transform this second order partial differential equation into two first order ones. The advection term is then treated using a Sweby’s flux-limited method. The boundary condition at the edge of the box (ξ = 0) is set according to the zeroth-order analytical solution that has been discussed in the previous section. Further details on the numerical scheme can be found in the appendix.

7.1.2.2. Discussion of the results. The main difference between the analytical solution of the reduced problem and the numerical solution of the complete set of equations is that, in the latter case, the pulse shape \( |a_L(\xi, \tau)| \) is changing over time. Regarding the instantaneous frequency, this shows a qualitatively similar behaviour to the analytical model with the pulse being red-shifted at the front and blue-shifted at the back. Moreover, the pulse self-steepens from the back, becoming shorter and with a broader spectrum.

There is some controversy on whether self-steepening should happen at the front or at the back of the pulse. Decker et al. [145] predict steepening from the front, due to pulse erosion. Esarey et al. [139] also predict steepening at the front of the laser pulse. However, according to Gordon et al. [140], this paper neglected an important term in the wave equation. Results of Gordon et al. [140], Sprangle et al. [146] and Jha et al. [147] show self-steepening at the back of the pulse, however in these works only weakly relativistic pulses (\( a_0 \leq 0.5 \)) are considered.

As we mentioned here, the numerical solution of (7.99), (7.100) is mainly in favour of compression from the back, particularly for weakly relativistic pulses. Figure 7.2 shows the pulse shape and instantaneous frequency after propagation through 1 cm of plasma, for pulses with increasing \( a_0 \). The pulses are all initiated with a FWHM pulse length equal to the non-relativistic plasma wavelength, \( \lambda_p = \frac{2\pi c}{\omega_p} \). As \( a_0 \) increases, the plasma wavelength relativistically increases as \( \sqrt{\gamma} \lambda_p \) and the pulse produces a region more and more depleted of electrons. As a consequence, when \( a_0 \) increases the laser pulse sits in a region more and more void of electrons. For the case of complete blow-out, (7.99) predicts that the pulse propagates as in vacuum, i.e. without losing its shape (RHS of (7.99)=0), so that the pulse evolves maintaining its pulse shape and no compression from the back occurs. On the front side, for increasing \( a_0 \) the pulse is able to push the electrons more and more effectively as it can be noted by the increase in electron density \( \Delta n/n_0 = n/n_0 - 1 \) at the pulse front.

The 1D approach probably underestimates the amount of self-compression that happens at the front. In particular, in a 2D or 3D case self-focusing effects may lead to a more efficient push of the electrons at the front. Also, the presence of Raman back-scattering could result in the erosion of the pulse at the leading front. These effects are all neglected
Regarding the instantaneous frequency, the pulse is always red-shifted at the front and the amount of red-shift increases with $a_0$. At the back of the pulse, blue shift appears for weakly relativistic pulses, however the frequency flattens towards zero as the intensity increases. This is another consequence of the fact, that for $a_0 \gg 1$, the back of the pulse is in a region depleted of electrons.

7.1.3. Results from particle-in-cell simulations. We now turn our attention on the solution of the nonlinear pulse evolution problem as given by particle-in-cell codes. This numerical model predicts the formation of a positively chirped pulse, consistently with what we have seen already. The front of the pulse continuously loses energy and is red-shifted while the laser pulse compresses. Tsung et al. [138] predict the possibility of compressing the pulse up to a few laser cycle pulses. This is accompanied by an increase in the peak vector potential $a_L$ of the pulse.

In Figure 7.3 we show an example from a 2D3V simulation made with the PIC code OSIRIS, for the design parameters of the Gemini laser and a plasma density of $2 \times 10^{18}$ cm$^{-3}$ [148]. This simulation confirms the broadening of the pulse spectrum, with the front of the pulse being red-shifted. For completeness, we also show the simulated electron spectrum after 9.6 mm of propagation, which goes beyond 2 GeV with an energy spread of 2%.

7.2. The Astra Gemini laser system

The Astra Gemini laser system is an upgrade of the previous Astra laser system, that is based on a Ti:sapphire oscillator that operates at a central wavelength of 800 nm. The project formally started in July 2004 [149] and the first experiment, described in this chapter, was conducted by the Imperial College group on April 2008. At the time of this experiment, Astra Gemini could deliver up to 10 J in 45 fs, for a peak power of 200 TW. However, it will eventually deliver two beams at a peak power of up to 500 TW.

7.3. Experimental set-up

The experimental set-up is shown in Figure 7.4. The 200TW laser pulses were focused by an f/20 off axis parabolic mirror to a spot size of $(22.0 \pm 0.6) \mu$m FWHM diameter, corresponding to a confocal parameter of 1 mm. The interaction with a supersonic helium gas jet with electron densities of $2.3 - 6.6 \times 10^{18}$ cm$^{-3}$ and various lengths between 4 and 15 mm were investigated. A transverse probe beam was used for interferometric measurements of the electron density and to investigate the length and shape of the formed channel for every shot. The transmitted laser pulses were collimated by a large aperture (f/10) spherical mirror and imaged onto a 12-bit CCD camera which allowed
Figure 7.2. Laser pulse and plasma properties for $k_0 = \sqrt{n_{cr}/n_e} = 25$ and after propagation through 1 cm of plasma, for a) $a_0 = 0.5$, b) $a_0 = 1.0$, c) $a_0 = 3.0$, d) $a_0 = 6.0$, e) $a_0 = 10.0$, f) $a_0 = 20.0$. The dashed line represents the original shape of the pulse envelope.
7. NONLINEAR LASER PULSE EVOLUTION IN AN UNDERDENSE PLASMA

characterisation of the spot size at the end of the channel, both to confirm the quality of self-guiding and the length of the channel. Furthermore, the complete beam was focused onto the slit of an optical spectrometer with sensitivity in a wavelength region of 300 – 1000 nm which was calibrated with an absolutely calibrated white light source. An absolutely calibrated photodiode also measured the transmitted laser energy. The temporal dependence of intensity and phase were measured using a second order frequency resolved optical gating (FROG) technique by guiding a small part of the transmitted laser close to the center into a commercial SHG-FROG device, i.e. a GRENOUILLE [104] by Swamp optics.

In a GRENOUILLE, the images can be asymmetric with respect to the time axis due to spatial chirp and pulse front tilt. Although we did not observe noticeable asymmetries, the FROG traces were symmetrized prior to the retrieval in order to improve the contrast. The result of the retrieval was not affected, but a larger data set became accessible for analysis. The removal of the ambiguity in the time direction, after the FROG retrieval, is explained in the next section.

7.4. FROG - Removing the time direction ambiguity

As briefly discussed in chapter 4, the SHG FROG retrieval suffers from an ambiguity in the time direction. In fact, as it can be verified, the pulse complex amplitude $E(t)$ and its complex-conjugated time-reversed replica $E^*(-t)$ yield the same FROG trace

$$I_{FROG}^{\omega}(\tau, \omega) = \left| \int_{-\infty}^{+\infty} E(t) E(t - \tau) \exp(-i\omega t) \, dt \right|^2.$$
Figure 7.4. Set-up of the experiment. The relevant diagnostics for our pulse measurements are the GRENOUILLE, optical spectrometer, probe (for the interferometric analysis of the plasma density) and the diode.
This can be seen by writing
\[
\left| \int_{-\infty}^{+\infty} E^*(-t)E^*(-t+\tau) \exp(-i\omega \tau) d\tau \right|^2 = \left| \int_{-\infty}^{+\infty} E(-t)E(-t+\tau) \exp(i\omega \tau) d\tau \right|^2 = \left| \int_{-\infty}^{+\infty} E(t'-\tau)E(t') \exp(-i\omega' \tau) d\tau \right|^2 ,
\]
where, in the second line, we made the change of variable \( t' \rightarrow -t + \tau \). Note that the amplitude \( E^*(-t) \) corresponds to a pulse with intensity \( I(-t) \) and phase \( -\phi(-t) \).

The way we have removed the time direction ambiguity will be explained in this section. For this purpose, it proved useful to show the Wigner transform for each shot. This will be explained in the “method” paragraph. But first, we will briefly introduce the Wigner transform.

**7.4.1. The Wigner transform.** The Wigner transform was first introduced by Paul Wigner in the context of quantum mechanics [150]. It is not just a mathematical tool but it is also widely used in signal analysis. Given a complex electric field \( E(t) \), the Wigner transform is defined as
\[
W(t, \omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} E^*(t+\tau)E(t-\tau) \exp(2i\omega \tau) d\tau. \tag{7.112}
\]
The Wigner transform is related to the probability of finding a photon in the point \((t, \omega)\) in the photon phase space (within the boundaries of the uncertainty principle). The marginals of this function have a very interesting meaning. If we integrate over the frequency variable, what we find is the delay marginal,
\[
M_{Wig}(t) = \int_{-\infty}^{+\infty} W(t, \omega) d\omega = \int_{-\infty}^{+\infty} E^*(t+\tau)E(t-\tau) \delta(\tau) d\tau = |E(t)|^2 ,
\]
so that this marginal gives the *temporal intensity* of the wave (in the context of quantum mechanics, as in the original paper, we would instead talk about the quantum-mechanical wave probability \( |\psi(x)|^2 \)). If (7.112) is, instead, integrated over the time variable, what we find is the frequency marginal
\[
M_{Wig}(\omega) = \int_{-\infty}^{+\infty} W(t, \omega) dt = \int_{-\infty}^{+\infty} E^*(u) \exp(i\omega u) du \int_{-\infty}^{+\infty} E(v) \exp(-i\omega v) dv = \left| \int_{-\infty}^{+\infty} E(v) \exp(-i\omega v) dv \right|^2 = |E(\omega)|^2 ,
\]
where we have made the substitutions \( t - \tau \rightarrow v \) and \( t + \tau \rightarrow u \) and used the fact that the Jacobian of this transformation is \( J = 1 \). As we have just proved, integration over time gives the *spectral intensity* (spectrum) of the wave packet. Integration over time *and* frequency gives the total energy contained in the wave packet.
Another interesting property of the Wigner transform regards its relation to the instantaneous frequency. What we want to show here is that the normalised mean value of the angular frequency (i.e. the normalised first moment of the Wigner distribution) is nothing else than the instantaneous frequency. From the definition (7.112), the mean value of the angular frequency is

\[
\langle \omega \rangle = \int_{-\infty}^{+\infty} \omega W(t, \omega) d\omega = i \int_{-\infty}^{+\infty} E^*(t + \tau) E(t - \tau) \delta'(-\tau) d\tau
\]

\[
= \frac{i}{2} \frac{d[E^*(t + \tau)E(t - \tau)]}{d\tau} \bigg|_{\tau=0} = |E(t)| \frac{d\phi}{dt},
\]

(7.113)

where we have used \( E(t) = |E(t)| \exp[i\phi(t)] \). To conclude this proof we finally write that, normalising,

\[
\langle \omega \rangle(t) = \frac{\int \omega W(t, \omega) d\omega}{\int W(t, \omega) d\omega} = \frac{d\phi}{dr},
\]

(7.114)

Note that in the integrand of (7.113) the first derivative of the Dirac distribution function appears. This is obtained from the integral

\[
\int_{-\infty}^{+\infty} \omega \exp(2i\omega\tau) d\omega = \pi i \delta'(-\tau)/2,
\]

where the prime ’ sign denotes a derivative. In order to obtain the subsequent line we have then made use of the fact that

\[
\int_{-\infty}^{+\infty} f(\omega) \delta'(\omega - \omega_0) d\omega = f'(\omega_0).
\]

The properties just discussed represent, essentially, the power of the Wigner transform: in a single 2D image the Wigner transform contains all the information on the pulse. It may be argued that this is the same for the FROG trace. Actually, a FROG trace is not as straightforward to interpret. For a SHG FROG, for example, the delay marginal \( \int_{-\infty}^{+\infty} I_{\text{FROG}}(\tau, \omega) d\omega \) corresponds to the second order autocorrelation. The frequency marginal \( \int_{-\infty}^{+\infty} I_{\text{FROG}}(\tau, \omega) d\tau \) corresponds, instead, to the autoconvolution of the spectrum, \( M_{\text{FROG}}(\omega) = 2I(\omega) * I(\omega) \). These quantities are not immediately related to the pulse itself, so the interpretation of the FROG trace is not as straightforward as in the case of the Wigner transform.

7.4.2. Method. A simple way of removing the time direction ambiguity is, for example, to use a thin glass plate with known thickness, capable of producing two or more pulses onto the FROG device. If the relative intensity of these pulses is known, say that the pulse reflected at the front surface of the glass plate is the brightest, it is therefore possible to determine which part of the pulse comes early and which later, for every shot.

This method is, however, not suitable in the presence of complex pulses, which is our case. It would add even more complexity to the pulse to be retrieved. Nevertheless, it is also true that this ambiguity can be removed if the chirp of the pulse is known\(^1\). In this

\(^1\)Strictly speaking, only if the instantaneous frequency is not symmetric .
Figure 7.5. Schematic of the method used to determine the time direction of the reference pulse (left: early in time; right: late in time). From the FROG trace a) we retrieve the pulse b) and we choose the time direction so as to give the correct chirp (positive). We then construct the Wigner transform c), which will be used as a reference for the other shots. In the Wigner transform we have also overlayed the trace of the instantaneous frequency (red curve).

In our case, the time direction can be chosen so as to ensure that the pulse has the correct chirp.

Our method for determining the time direction is based on the knowledge of the chirp of the pulse (i.e. of the time direction) for a particular shot. Then, the determination of the time direction for other shots is made on the following physical assumption: that the variation of the pulse properties are a smooth function of the experimental parameters. That is to say, that for small changes in the relevant conditions (plasma density/laser energy) we expect to see small changes in the pulse characteristics.

This requires knowledge of the chirp of the pulse for a particular shot. This shot is, in our case, a vacuum shot. In a vacuum shot, the gas-jet is not fired. Therefore, the only distortion that can occur for the pulse is due to the linear propagation through glass before entering the FROG device. This effect is very well known. For a thick enough amount of glass in the beam path, the resulting pulse must be positively chirped, because in a normally dispersive material such as glass the longer wavelengths travel faster. Therefore, if we know the pulse to be positively chirped, the time direction can be chosen so as to ensure that this is the case.

From this “initial condition”, we then choose the time direction for all the other shots. In order to do this analysis, it proved useful to show the Wigner transform of the laser pulse. This is because, as we have clarified, the Wigner transform gives, in a single 2D map, all the relevant properties of the pulse (temporal intensity, spectrum, instantaneous frequency).

Therefore the Wigner transform for different shots was compared to shots with similar parameters. The time direction was then chosen so that the Wigner transform would change smoothly from shot to shot. In Figure 7.5 we show the procedure used for the construction of the Wigner transform for a reference shot. In Figure 7.6 we then show
Figure 7.6. FROG traces and Wigner transforms for three different laser pulses, starting (upper row) from the same reference shot in Figure 7.5. For the other shots, of the two possible Wigner transforms the one that presents the smallest change with respect to the previous shot is chosen (left: early in time; right: late in time).
7.5. Experimental results

In this section we describe the most important results of the chapter, which concern experimental results on the pulse evolution for varying plasma density and interaction length. It is important to stress that, for this analysis, we restricted our data set to shots for which the total spectrum measured in the spectrometer was in good agreement with the retrieved spectrum from the FROG trace. This gives confidence that the FROG results are representative of the complete pulse even though we were collecting only a small portion of the pulse (while the optical spectrum was representative of the full aperture of the beam).

7.5.1. Pulse evolution for varying plasma density and interaction length. First, we present the results for the pulse propagation with varying plasma density. The inter-
action length was fixed at 4 mm (gas-jet nozzle with diameter of 5 mm). The density
was measured from interferometric data and was varied between $2.3 \times 10^{18}$ cm$^{-3}$ and
$6.6 \times 10^{18}$ cm$^{-3}$.

Figure 7.7 shows the results of this investigation. In the upper row are the raw data of
the FROG images. The left column, a1, belongs to the vacuum shot and corresponds to a
pulse with a FWHM duration of 45 fs. The other traces are for increasing plasma density.
As it is apparent, the FROG traces are varying with plasma density and the pulse is the
shortest for the highest density, d1 (it should be remembered that the delay marginal of a
SHG FROG is the intensity autocorrelation).

In the lower row of Figure 7.7 are the respective Wigner transforms of the FROG
traces. In particular, from the retrieval of the FROG traces a1, b1, c1 and d1 we first
back-propagate the pulse to take into account the presence of glass in the beam path (due
to the presence of neutral density filters in front of the FROG device and of a collimating
lens). In this way we can determine the properties of the laser pulse just after propagation
through the plasma. At this point we construct the Wigner transforms and we choose
the time direction with the method previously described. Note that we choose the time
direction so that “left” (more negative values) corresponds to “early” in time and “right”
(more positive values) to “late” in time.

The pulse shapes and spectra resulting from this analysis are shown in Figure 7.8.
As we anticipated, the laser pulse is shortened for increasing plasma density. Moreover,
interestingly the pulse in Figure 7.8b1 is clearly steepened from the back and red-shifted
at the front and blue-shifted at the back. This is the scenario predicted by the numerical
solution of the 1D quasistatic equations, as we have seen.

The evolution of the pulse for varying interaction length shows a behaviour very sim-
ilar to the one for the density scan. For this scan, the density was set at $2.3 \times 10^{18}$ cm$^{-3}$, as
for the lowest density in the density scan. Concerning the pulse duration, in Figure 7.9 the
pulse shapes for different interaction lengths are shown. The a) and b) panes correspond
to the same shots used for the density scan (vacuum shot and 4 mm interaction length).
As it can be seen, the pulse shortens as the interaction length increases.

A plot of the FWHM duration of the laser pulse for both the density and interaction
length scans is presented in Figure 7.10. For the highest densities, the pulse duration was
probably limited by the resolution of the GRENOUILLE which is of $\approx 20$ fs. This is
due to the fact that the nonlinear crystal present in this device could not phase-match the
whole spectrum for pulses shorter than this limit. However, in principle for these high
densities the broadening of the spectrum could support pulses as short as $\approx 5$ fs.

So far we have not discussed the energy losses of the laser pulse through the plasma.
This will be done in the next paragraph.
Figure 7.8. Temporal intensity (upper row) and spectrum (lower row) for the same shots in Figure 7.7. In the spectral data, the black curve refers to the spectrum retrieved from the FROG trace. The red broken curve refers to the spectrum measured from the independent optical spectrometer.
7.5. EXPERIMENTAL RESULTS

**Figure 7.9.** Pulse shapes for different interaction lengths: a) vacuum, b) 4 mm, c) 6 mm and d) 8.5 mm.

**Figure 7.10.** Dependence of the FWHM pulse duration with electron density a) and interaction length b). The broken line denotes the resolution limit of the GRENOUILLE device.
7.5.2. Energy transmission. The LWFA scheme for electron acceleration is ultimately related to the transformation of energy from the driver, the laser pulse, to the electrons injected in the plasma waves. Energy is, however, also converted into energy of the plasma wakefield itself which is eventually transformed into thermal energy. As a result, the laser pulse loses energy as it travels through the plasma.

The rate at which this happens has been written in terms of pulse front erosion by Decker et al. [145]. According to this model, the leading edge of the pulse should gradually be etched backwards, with a velocity \( v_{\text{etch}} \simeq c \omega_p^2 / \omega_0^2 \). This is due to the fact that the pulse front interacts with the plasma electrons and accelerates them via the ponderomotive force. This, in turn, results in the leading edge of the pulse losing energy. This model predicts that the pulse should be self-steepened at the front. As we have seen, this is not what we have observed in the experiment. However, results from Ralph et al. [151] agree with this model for what concerns the evaluation of the pump depletion length, which is defined as the distance travelled by the laser pulse before losing its energy. These experimental findings suggest that Decker’s model gives the correct dependencies. It is in this sense that we apply it to our results.

The energy transmission of the laser pulse after propagating for a distance \( l \) can be written as

\[
T(l) = 1 - k \sqrt{\frac{2}{\pi \Delta \tau_0}} \int_{\Delta \tau_0/2 - v_{\text{etch}}l/c}^{\Delta \tau_0/2} \exp(-2\tau^2/\Delta \tau_0^2) d\tau
\]

where we consider a Gaussian laser pulse (in time) \( a = a_0 \exp(-2\tau^2/\Delta \tau_0^2) \) and we have used \( \int_{-\infty}^{+\infty} \exp(-2\tau^2/\Delta \tau_0^2) d\tau = \sqrt{\pi/2} \Delta \tau_0 \). Also, we are assuming that the front (or back) of the pulse is etching at a speed \( v_{\text{etch}} \) and we use \( k \) as a free parameter. This free parameter originates from the fact that the position at which the pulse starts to be etched is not clearly defined in the literature; roughly, it should correspond to the half width at half maximum [145], in which case \( k = 1 \).

It should be noted that equation (7.115) can be used also for the case of the density scan. In fact, even though \( l \) is constant for this set of data, the plasma density is not and therefore neither is the etching velocity. Using the expression for \( v_{\text{etch}} \) that was previously given (\( v_{\text{etch}} \simeq c \omega_p^2 / \omega_0^2 \)), it is found that the transmission data can be fitted reasonably well for a factor \( k \approx 1 \) (Figure 7.11), for both data sets. However, the initial energy loss is higher than what this model would predict.
7.5. EXPERIMENTAL RESULTS

7.5.3. Pulse power. With the knowledge of pulse shape and energy transmission it is possible to evaluate the peak power for every shot. To evaluate this quantity, we denote the temporal pulse profile normalized to a maximum of 1 with $f(l, n_e, t)$ where $l$ is again the propagation length and $n_e$ is the electron density. The reference pulse is characterized by $l = n_e = 0$. The transmitted laser energy is given by $E_L(l, n_e) = \int P_P(l, n_e) f(l, n_e, t) dt$ which can be used to calculate the relative peak power,

$$P_P(l, n_e)P_P(0,0) = T(l, n_e) \cdot \frac{\int f(l, n_e, t) dt}{\int f(0,0,t) dt},$$

where $T(l, n_e) = E_L(l, n_e)/E_L(0,0)$ is the energy transmission.

The dependence of the normalised peak power with respect to density and interaction length is presented in Figure 7.12. These plots clarify that the pulse power decreases at first. This is because for these parameters pulse compression does not compensate for the energy loss. However, for higher densities (or longer propagation lengths) the peak power stays fairly constant: the pulse compresses more efficiently and the energy losses are comparatively less.

To conclude, we point out that, even though the power decreases from the vacuum case, it is likely that the pulses observed to compress down to 20 fs or below are, at least, among the most powerful ever produced for such short duration. In other words, it would be difficult to obtain a 20 fs pulse with a few Joules of energy, with conventional optical media.
**Figure 7.12.** Dependence of the pulse peak power with electron density a) and interaction length b).
CHAPTER 8

Conclusions

In this thesis we have presented experimental results on the transport of relativistic electrons produced in laser-solid interactions and on the nonlinear propagation of high intensity laser pulses in underdense plasmas. This chapter is aimed at summarising the results and giving future prospects.

8.1. Electron transport in solid targets

In chapters 5 and 6 we have investigated electron transport by looking at the optical radiation emitted at the rear side of laser-irradiated solid targets. Several conclusions can be drawn from these results. Specifically, we have given

- the first full characterisation of the polarisation properties of the optical rear surface emission generated by a laser-generated hot electron current. These measurements allow us to unequivocally determine for the first time that the emission is primarily due to optical transition radiation. This rules out other possible sources of emission, such as synchrotron radiation and coherent wake emission, whose effect has not been precluded by previous work;
- the first direct demonstration of the recirculation of electrons transported by multiple reflections from front and back surface. Up to now, this effect has only been inferred indirectly;
- the first demonstration of the presence of \( \mu \text{m}-\text{size} \) electron filaments at distances at which scattering effects ("beam blooming") would be expected to diffuse the filaments to larger dimensions (several microns). Thus, these small size features hint to the presence of self-generated electric and magnetic fields within the targets strongly affecting the transport.

We now give a more detailed account of the results of chapters 5 and 6.

8.1.1. Chapter 5. In chapter 5 we have presented measurements from an experiment conducted on JETI (Jena), with \( I_L \lesssim 4 \times 10^{19} \text{ Wcm}^{-2} \) and a pulse duration \( \tau_L \simeq 80 \text{ fs} \). The optical radiation was collected simultaneously at three different angles, at the second harmonic of the laser frequency (\( \lambda_{2\omega_L} = 400 \text{ nm} \)). We have found that most of the radiation is emitted in the laser direction (the laser was incident at 45\(^\circ\) on target). A high degree of polarisation was also observed, which is consistent with the properties of transition
8. CONCLUSIONS

FIGURE 8.1. Particle tracking in the 2D3V Osiris simulation. The laser, with an $a_0 = 20$ and a spot size of 5 µm FWHM, is incident (normal incidence) from the left of the simulation box to a 5 µm target with $n_e = n_i = 100 n_{cr}$. Resolution: 1.3 cells/plasma wavelength, 72 particles per cell (electrons+ions). The simulation was initialised with immobile ions and an electron temperature of 0.75 keV.

radiation. However, we have observed shot-to-shot fluctuations of the degree of polarisation. This is especially true for the imaging system viewing along the laser direction. The simplest explanation is that this is due to a fluctuation of the electron beam pointing. We further stress that the absolute degree of polarisation leads to the conclusion that the imaging system is mainly sensitive to electrons going towards the collecting optics. This is, once more, a characteristic which is consistent with emission of transition radiation from relativistic electrons, as for these energetic particles transition radiation is mainly emitted in the direction of propagation.

On some occasions, the signal from the fast electrons was clearly distinguishable from a background of unpolarised radiation. This unpolarised radiation is believed to be the result of thermal emission from the plasma background expanding at the target rear side. This should happen on a longer time-scale than the signal from the fast electrons.

A variation of the signal intensity with target thickness was also observed. This can be easily interpreted in the framework of coherent transition radiation (CTR) theory, as an effect of dephasing of the electron bunches as they reach the target rear surface. An increase of the signal of the CTR was also measured for increasing laser intensity, which is consistent with the increase in electron energy with $I \lambda^2$.

8.1.2. Chapter 6. In chapter 6, we have presented results from a series of experiments carried out on VULCAN (RAL), with $I_L = 8 \times 10^{19} - 10^{21}$ Wcm$^{-2}$ and $\tau_L \simeq 500$ fs.

It was shown that the spectrum of the radiation is peaked at the harmonics of the laser frequency, implying that, as in the previous experiment on JETI, the main source
of emission is CTR. At these laser intensities, a halo surrounding the main source of CTR emission was also observed, with dimensions much larger than the laser focal spot (> 200 µm). This spatially extented emission region is also polarised (this represents a major difference with the JETI results) and is attributed to recirculating electrons, since its dimensions cannot be explained by the initial divergence of the electrons produced at the target front side; hence, it is considered to be the result of multiple reflections of the fast electrons at the rear and front of the target.

Electron recirculation is predicted by particle-in-cell simulations. In Figure 8.1 we show the result of a PIC simulation, in which we have added particle tracking in order to visualize the trajectory of individual quasi-particles. Most of the electrons are bound to the target by the sheath fields produced both at the front and rear surface of the target, and only a small fraction of the overall fast electron population can escape the target sheath potential. As a result, electrons can be transported transversely, far from the interaction region. As they traverse the target rear surface, polarised radiation is emitted which can be detected.

By using different Cu wedge targets with a thickness of \( \approx 50 \, \mu m \), we have also been able to determine the angular distribution properties of the CTR. This angular distribution should be analogous to the diffraction of radiation by a circular aperture, in the far-field approximation. Such analogy was clarified in chapter 3, and has allowed us to estimate the diameter of the fast electron filaments reaching the rear surface. The result is that filaments of \( \sim 1 \, \mu m \) FWHM were measured in these interactions.

Our technique has allowed us to go beyond the resolution limit of our imaging system, which was well above 1 \( \mu m \). The drawback is that we had to make some assumptions, for example that the transverse spatial profile of the electron beam is Gaussian. Also, our approach only considers the presence of a single filament or of filaments all of the same size. In fact, it seems reasonable to fit the data to filaments of different diameter, around 1 \( \mu m \) FWHM; however, the smallest feature size is of sub-\( \mu m \) in order to fully describe the observed angular distribution for large angles of observation (cf. Figure 6.10 on page 130).

The small filamentary structure inferred in our experiment is consistent with several simulation results that can be found in the literature. In particular, in Figure 8.2 we show results published by Evans in 2006 [136], using the hybrid code LSP which has an implicit PIC description of the hot electrons and a fluid description of the cold background. These simulations show that, for an intensity of \( 10^{20} \, \text{Wcm}^{-2} \) and a target thickness of 50 \( \mu m \), filaments 1-2 \( \mu m \) wide are present 500 fs after the beginning of the interaction.

This small scale structure is the result of self-generated electric and magnetic fields. Understanding the formation and behavior of these small scale structure will be of vital
Figure 8.2. Electron density (top) and temperature (bottom) from LSP simulations, for a target with thickness of 50 µm, Z=30 and ρ = 10 g cm⁻³ and after 500 fs from the beginning of the interaction. The fast electron transport is investigated for different laser intensities at 10¹⁸ W cm⁻², 10¹⁹ W cm⁻² and 10²⁰ W cm⁻² (left-right). Image courtesy of R. G. Evans, from reference [136].

importance for applications of high intensity laser physics, such as fast ignition and ion acceleration.

8.1.3. Future work. The simple technique presented in chapter 6, which relies on the idea of coherent diffraction of the radiation, can be used in future investigations on electron filamentation, for different experimental conditions. For example, it would be of interest to vary the target material, target thickness and the scale-length at the target front side. Such measurements could improve our understanding of filamentation instabilities in laser-solid interactions, which is one outstanding problem in electron-driven fast ignition.

An interesting and simple study could also be made by looking at how the optical spectrum varies with respect to the laser irradiance. This is because, as we have seen in chapter 2, the absorption mechanism is dependent on laser intensity and produces bunches that are separated by either λ_L/2 or λ_L. This would be reflected in the CTR spectrum. We would expect the spectrum to be more peaked at even harmonics for the highest intensities, as in this case the j × B mechanism should become dominant. For lower intensities or for circular polarisation, this mechanism of absorption should become less efficient and the odd harmonics should then appear, with the fundamental being the most intense.
8.2. Nonlinear pulse evolution in a laser-driven wakefield

In chapter 7 we investigated the mechanism of nonlinear laser pulse propagation in a supersonic helium gas-jet, with plasma densities ranging from $2.3 \times 10^{18}$ cm$^{-3}$ to $6.6 \times 10^{18}$ cm$^{-3}$ and for different interaction lengths (4, 6, 8.5, 15 mm). We have found that

- the pulse is subject to substantial compression, from 45 fs to below 20 fs for the highest densities or interaction lengths;
- pulse compression is accompanied by spectral broadening;
- the pulse develops a positive chirp (red photons at the front, blue photons at the back), which is in agreement with an analytic estimate and with results from particle-in-cell and kinetic simulations (see, for example, [86]);
- during propagation, an asymmetric pulse develops that appears to be steepened from the back for the lowest density ($n_e \approx 2.3 \times 10^{18}$ cm$^{-3}$) and shortest interaction length (4 mm).

Furthermore, we add that while the laser pulse propagates through the plasma it continuously loses energy due to ionisation of the gas, scattering and since it drives a wakefield. Electromagnetic energy is thus converted into internal energy of the plasma background and into energy of the accelerated electrons. For our parameters, the result is that the peak laser power decreases with respect to the vacuum case, even though the pulse shortens during the interaction.

8.2.1. Future work. Future work could be aimed at optimising the mechanism of pulse compression, which could be achieved by changing pulse energy and electron density so that the pulse can efficiently self-compress. This would allow to increase the power of existing laser systems without the need of increasing the energy of the pulse, which would otherwise require increasing the size and cost of the laser system.

For the purpose of controlling this pulse shortening, it would also be necessary to address some inconsistencies that we have found between PIC simulations and experimental data. In particular, for the conditions of our experiment, simulations predict an amount of broadening which is not in agreement with that observed in the experiment.

This is evident in Figure 8.3, which shows the Wigner transform from a 2D3V OSIRIS simulation, for different interaction lengths. The simulated Wigner transforms show the same trend as that observed in the experiment; namely, that the photon distribution rotates in the $t - \omega$ phase space, the pulse envelope compresses and the spectrum broadens. However, this is accompanied by substantial shift of the mean wavelength and broadening towards the infrared region: after 8 mm of propagation the mean frequency of the simulated spectrum shifts to 920 nm and the spectrum extends to over 1 $\mu$m. This should be compared with the initial central wavelength at 800 nm with a spectral width of 40 nm.
and to the experimental data after 8.5 mm of propagation, which show no significant shift of the mean wavelength and a spectrum that extends to 900 nm or less, in the infrared region.

Our measurements are not believed to be the result of an instrument sensitivity effect as our FROG results and spectrum results have different spectral responses but show good agreement. The sensitivity of the spectrometer (including the reflectivity of all mirrors, the grating and CCD detector) falls off from 60% to 40% over the range 800 - 900 nm, but this is taken into account in the analysis.

Understanding this disagreement between simulations and experiments will prove useful for the purpose of optimising and controlling pulse compression. PIC simulations that can be found in the literature predict that an efficient compression can lead to the production of few-cycle (few femtoseconds) laser pulses and an increase of the pulse peak power [152, 138]. This optimal region in the parameter space is likely not to be found in the ultra-relativistic regime ($a_0 \gg 1$), which would produce highly nonlinear plasma waves and, eventually, wavebraking with a corresponding high rate of absorption of laser energy in the plasma.

In order to measure the short pulses predicted by these simulations, a device different from the one used for our experiment would be required. In particular, a normal FROG device could be used (rather than a GRENOUILLE) because this would permit the use of a nonlinear crystal as thin as 5 µm. This is important to ensure that the full bandwidth of the pulse is phase-matched in the crystal, so that the FROG trace would represent a reliable measure of the pulse properties.
APPENDIX A

Numerical solution of the quasistatic equations

We rewrite here the equations that describe the nonlinear laser pulse evolution in an underdense plasma, in the quasistatic approximation,

\[ 2 \frac{\partial^2 a_L}{\partial \xi \partial \tau} + 2ik_0 \frac{\partial a_L}{\partial \tau} - \frac{\partial^2 a_L}{\partial \tau^2} = \frac{a_L}{1 + \phi}, \quad (A.116) \]

\[ \frac{\partial^2 \phi}{\partial \xi^2} = \frac{1}{2} \left[ \frac{1 + |a_L|^2}{(1 + \phi)^2} - 1 \right]. \quad (A.117) \]

Here we explain the numerical method used to solve this system of equations. Poisson’s equation (A.117) is a second order ODE and does not present any numerical complication. We have chosen a Runge-Kutta method of the 4th-order for its numerical integration.

For the solution of equation (A.116), we transform this second order partial differential equation into two first order ones,

\[ \frac{\partial a_L}{\partial \tau} = F, \]

\[ \frac{\partial F}{\partial \tau} - 2 \frac{\partial F}{\partial \xi} - 2ik_0 F = -\frac{a_L}{1 + \phi}. \quad (A.118) \]

In our work we were mainly concerned in understanding the pulse evolution over long propagation lengths (about a centimeter), for pulse lengths of the order of a plasma period. For these conditions the effects of self-compression are evident. In fact, a high gradient in the pulse envelope forms. We have found that the use of a leap-frog method produces spurious oscillations that destroy the solution over the course of the simulation. Therefore, we have eventually opted for a Sweby’s flux-limited method that ensures the presence of a smooth solution.

Flux-limited methods have been especially developed to deal with gasdynamics problems where the presence of discontinuities such as shock waves has to be treated carefully. Essentially, they are adaptive linear combinations of two first-generation methods [153]. In this sense, they are considered to be second-generation methods because the numerical approach is solution-sensitive (adaptive), marking a difference with first-generation methods. In particular, Sweby’s algorithm for the advection term is a combination of a FTFS (or FTBS, depending on the sign of the wave speed) and a Lax-Wendroff finite dif-
ference schemes. The weight of each of these schemes is regulated by the presence of a
flux limiter, the value of which depends on the gradients in the solution.

For our case, the application of this scheme to equation (A.118) leads to

\[
F_{j}^{n+1} = F_{j}^{n} + 2\lambda \left( F_{j+1}^{n} - F_{j}^{n} \right) + \lambda (1 - 2\lambda) \left[ \phi_{j+1}^{n} (F_{j+1}^{n} - F_{j}^{n}) - \phi_{j}^{n} (F_{j}^{n} - F_{j-1}^{n}) \right] + 2i k_{0} \Delta \tau F_{j}^{n} - \Delta \tau \frac{d_{j}^{n}}{1 + \phi_{j}^{n}},
\]

where \( \lambda = \Delta \tau / \Delta \xi \) and \( \phi_{j}^{n} \) is the flux limiter. We have chosen to use a “minmod” flux
limiter, which is defined by

\[
\phi(r) = \begin{cases} 1 & r \geq 1, \\ r & 0 \leq r < 1, \\ 0 & r < 0. \end{cases}
\]

The parameter \( r \) in the previous definition is dependent on the gradient of the solution,

\[
r_{j}^{n} = \frac{F_{j+1}^{n} - F_{j}^{n}}{F_{j}^{n} - F_{j-1}^{n}}.
\]

It can be shown that this flux limiter switches between FTFS (FTBS), the Beam-Warming
second-order upwind method and the Lax-Wendroff method [153].

Different choices for the flux limiter are possible, however we found no appreciable
differences when using different flux limiters such as the “superbee” or the “Van Leer”.
Bibliography


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176 Bibliography


