Coherent structures shed by multiscale cut-in trailing edge serrations on lifting wings

S.L. Prigent, O.R.H. Buxton and P.J.K. Bruce
Imperial College London, London SW7 2AZ, United Kingdom

This experimental study presents the effect of multiscale cut-in trailing edge serrations on the coherent structures shed into the wake of a lifting wing. Two-probe span-wise hot-wire traverses are performed to study spectra, coherence and phase shift. In addition, planar Particle Image Velocimetry is used to study the spatio-temporal structure of the vortices shed by the airfoils. Compared to a single tone sinusoidal serration, the multiscale ones reduce the vortex shedding energy as well as the span-wise coherence. Results indicate that the vortex shedding is locked into an arch-shaped cell structure. This structure is weakened by the multiscale patterns which explains the reduction in both shedding energy and coherence.
I. INTRODUCTION

The study of trailing edge (TE) serrations goes back several decades, with analytical prediction of trailing edge noise scattering by sawtooth patterns on flat plates by Howe [14] [15]. For a straight trailing edge, the far field noise spectrum linearly depends on the span-wise correlation length of the surface pressure fluctuations [1], that can be directly induced by the turbulent boundary layer close to the surface, or generated by the periodic vortex shedding in the case of a blunt trailing edge [13]. The idea behind the sawtooth serrations was thus to reduce the coherence of the scattered fluctuations by introducing a span-wise variation of the scattering edge, placed at an appropriate angle relative to the flow direction. Numerical investigations have also been conducted on airfoils with flat extension plates. Conventional sawtooth serrations were seen to reduce trailing edge noise, with a particular interest on tonal noise due to the presence of a suction side separation bubble as reported by Sandberg and Sandham [30], and Sandberg and Jones [29]. Experimental investigations also reported noise reduction, such as that by Arce Léon et al. [3] on a lifting wing at incidence, who reported a reduction of the noise generated by the turbulent boundary layer’s interaction with the trailing edge. Additionally, Vathylakis et al. [37] focused on the effect of the serrations’ penetration angle in the flow and reported a strong dependence of the noise reduction on this parameter. Detailed studies of the flow over the trailing edge serrations for a plate by Chong et al. [9], Moreau et al. [20] [21] or a lifting wing with extension plate by Avallone et al. [4] [5], report that the flow itself is distorted by the serrations which may account for the discrepancy of the measured noise reduction with analytical models, which assume straight streamlines and focus on noise scattering only. The key role played by the flow distortion emphasizes the importance of the fluid mechanics when studying the impact of such serrations, and therefore the necessity of a better characterization of the flow properties.

Trailing edge serrations on extension plates have thus shown benefits in terms of noise reduction and are in fact now used in industrial applications. On-shore wind turbines now integrate such serrations in their design - cf. Mathew et al. [18] - for which noise reduction has been reported by Oerlemans et al. [25] [24] and Buck et al. [7]. Other applications involving arrays of blades could also be feasible. In fact, the noise reduction due to serrations was also investigated in blade cascades by Finez et al. [11] who found similar broadband noise reduction as that reported in previous studies on isolated blades.

Most studies have focused on simple patterns of the serrations, and more complex geometries, such as slitted sawtooth, were investigated by Gruber et al.[12] or Arce Léon et al. [2] [17] with the aim of combining a flow permeability at the trailing edge with a geometry that could better tackle high frequency noise and mitigate the flow distortion. Stronger noise reduction was reported than in the case of classical sawtooth, with the main weakness being a stronger dependence on flow conditions. Recent studies by van der Velden and Oerlemans [36] [35] suggest that stronger broadband noise reduction can also be achieved with combed serrations, mainly due to flow re-alignment.

A recent approach has been to directly cut the serrations into the wing [10][8][22][26][33], instead of using an extension plate, with the potential aim of gaining structural robustness. Cut-in serrated trailing edges are observed to lead to perceived broadband acoustic benefits in spite of their inherent periodic bluntness, as presented by Chong et al. [8], whilst at the same time being structurally strong in comparison to a serrated extension plate. Further, wings with blunt trailing edges are known to produce higher lift coefficient, and Nedić et al. [22] indeed observed a slight increase in lift to drag ratio compared to an unmodified wing when using fractal cut-in serrations. However, this inherent bluntness generates additional vortex shedding. The objective of the current research is thus to better understand the impact this shedding has on an airfoil’s wake and to devise ways to mitigate this effect.

Prigent et al. [26] have studied sinusoidal cut-in serrations with effective angles ($\theta$) greater than 45 deg - $\theta$ being defined as the angle between the stream-wise direction and the peak-to-trough straight line, as illustrated in fig. 1. Within this limit, they highlighted that vortex shedding intensity decreased - by up to 57% compared to a straight blunt case - when reducing the pattern’s wavelength. In addition, a reduction of span-wise coherence at the vortex shedding frequency was also reported by the authors; although a side effect was an increase of span-wise coherence at low frequencies. Given that span-wise coherence close to the trailing edge can be an indication of acoustic noise sources - or base drag in the case of vortex shedding by blunt wings, its reduction has potential benefits for industrial applications. However, Chong et al. [8] have shown that a shorter wavelength leads to an increased noise production associated with the vortex shedding; hence reduction of the serration wavelength itself is not a viable way to further reduce vortex shedding intensity and span-wise coherence. Recent studies by Nedić and Vassilicos [23], and Nedić et al. [22], have shown the possible use of multiscale/fractal geometries to reduce vortex shedding, in the context of cross-stream flat plates and trailing edge serrations, respectively. Melina et al. [19] have studied the flow behind a fractal grid for an investigation of heat-transfer, and found as a side effect that shedding phenomena downstream of the grid was reduced for the fractal cases. This multiscale approach could thus prove useful as a means to reduce vortex shedding intensity.

To avoid the introduction of sharp angles that could hinder the reduction of vortex shedding intensity, for instance by the generation of stream-wise vorticity, smooth patterns are chosen. Furthermore, span-wise coherence close to the trailing edge can be an indication of acoustic noise sources - or base drag in the case of vortex shedding by blunt wings, therefore its reduction has potential benefits for industrial applications. Since not much is known about the effect of the serrations on span-wise coherence in the wake, this shall be one of the focuses of this study. Although linked to acoustics topics, its scope is thus on the relevant fluid mechanics.
II. EXPERIMENTAL APPROACH

The trailing edge patterns used in this study are illustrated in fig. 1. They correspond to the iterative addition of a sinusoidal curve normally to the previous one. The goal is to locally reduce the wavelength while adding multiscale features to limit vortex shedding intensity and further reduce span-wise coherence by breaking down the structures being shed at the trailing edge.

The patterns studied hereafter have wavelengths $W = 40$ and $W = 84$ mm with a peak-to-trough amplitude of 20 mm, corresponding to effective angles of $\theta = 45^\circ$ and $\theta = 64.5^\circ$ respectively, and following the work presented by Prigent et al. [26]. These relatively large serration angles minimize the ratio of experimental resolution (sensor size and minimum separation) to the wavelength of the patterns. The nomenclature for the trailing edge follows this example: $2iS20W40B$ has 2 iterations on top of the base Sinusoid, with a peak-to-trough amplitude of 20 mm, a wavelength $W = 40$ mm and exhibits Bluntness in the trough.

By definition $\bar{z}$ is the span-wise position relative to a fixed reference at the trough, such that $\bar{z}/W = 0$ at the trough and $\bar{z}/W = \pm 0.5$ at the peak. $x$ is the downstream location and $x = 0$ corresponds to the tip of the trailing edge.

In this experimental investigation, a NACA0012 wing with interchangeable trailing edges is used following Prigent et al. [26]. The wing has chord $c = 150$ mm and the boundary layer is tripped at 0.2 $c$ by a single strip of sand paper on both upper and lower surfaces. Interchangeable trailing edge sections cover 92% of the 455 mm span. A 5° angle of attack provides moderate lift while preventing stall occurrence. Measurements were conducted at Imperial College London in a 457 × 457 mm cross-section low speed wind tunnel at $Re_c = 150,000$.

A. Hot-wire apparatus

To measure span-wise coherence, vortex shedding intensity and de-phasing (the definition of which is given in section III C), two-point span-wise traverses were performed in the wake: one hot-wire being fixed downstream of a trailing edge’s trough, corresponding to the centreline, while a second was traversed in the span-wise direction.

Two gold plated Dantec hot-wires were used, connected to a Dantec StreamLine 90N10-Frame Constant Temperature Anemometer powered by an isolated power supply. The square wave responses of the hot wires were all above 40 kHz, while a low-pass filter was set at 30 kHz. The sampling frequency of 100 kHz ensures that Shannon’s principle is respected to avoid aliasing. Acquisition at each point lasted for 40 s to allow convergence of the statistics, and a periodogram method was used with 200 windows for the spectral analysis yielding a data point every 5 Hz. The over heat ratio was set at 1.8. Calibration was performed before each traverse, with a range of freestream velocities spanning from 8 m/s ($Re_c = 75,000$) to 23 m/s ($Re_c = 225,000$). The size of the sensing wire is $l_{hw} = 1.25$ mm with a length to diameter ratio of 250 ensuring that conductive heat fluxes are negligible. A way of discussing the inherent spatial filtering is to use the frozen turbulence hypothesis to define a cut-off frequency by $f_c = u_0/2 * l_{hw}$ to convert the spatial resolution into the frequency space, with $u_0$ the local bulk velocity. This approach can only give an estimate since the flow does not match the assumption of very low turbulence intensity, reaching maximum values of 10 − 12%, and $f_c = 5.6$ kHz was found, which is well above all phenomena discussed hereafter.

The probes were mounted with the sensing wire orientated vertically to allow for smaller separation between them. The minimum separation for the different traverses is 1.5 − 2 mm, and $\bar{U}$ and $\sqrt{\overline{u'^2}}$ were monitored when moving the probes together to ensure there was no aerodynamic probe interference.
B. PIV apparatus

Time resolved planar (2D-2C) Particle Image Velocimetry (PIV) was conducted in a span-wise/stream-wise plane, with a field of view encompassing the largest TE wavelength. Fig. 2 illustrates the apparatus.

The illumination was made using a Litron LDY300 (Nd:YLF) laser, and a Phantom M310 camera (25.6 x 16.0 mm/1280 x 800 px sensors) with a Nikon 135 mm f/2 DC lens was used for imaging. The illumination and imaging set-up was controlled by the commercial software Davis 8.2 from LaVision. Data were acquired at 1500 Hz (that corresponds to 2.5 times the vortex shedding frequency) with an inter-frame separation time of 12 µs. The horizontal laser sheet was placed 1 mm under the tip of the non-modified trailing edge, thus being under the wing at the maximum amplitude of the cut-in patterns, and in the middle of the wake right after the trailing edge.

![FIG. 2: Sketch of the PIV (left, 3D view) and Hot-Wire (right, top view) apparatus](image)

Images were pre-processed to subtract the background luminosity, and a sliding background average was applied to account for non-uniformity in the illumination. Recursive processing was done with one pass at an initial window size of 48 x 48 px, and two passes at a window size of 16 x 16 px and 50% overlap, giving a spatial resolution of 1.2 mm. The velocity vector fields were then post-processed to remove spurious vectors with three passes of a universal outlier method, with final processed/rejected vectors count being about 2% and the empty vectors were replaced by interpolating from the neighbouring vectors identified as being valid. A total of 4000 snapshots were used per data set. To quantify the convergence or statistical noise arising from this finite set, the cumulative mean was studied by iteratively increasing the range of data used to compute it from 0% to 100%. A 95% confidence interval fit of the form $\mu \pm \frac{\sigma}{\sqrt{N}}$ was applied to the range of data 50-100%, where $\mu$ is the local mean, $N$ the number of cumulative data points and $\sigma$ the parameter varied to obtain the tightest fit respecting the 95% confidence criteria. The final value of $\frac{\sigma}{\sqrt{N}}$ is then taken as the statistical noise, which amounts to 0.1% of the local convective velocity. The same approach was taken for the standard deviation of velocity fluctuations and gives a relative statistical noise of 0.4%.

III. RESULTS AND DISCUSSIONS

A. PSD

To follow the span-wise evolution of shedding associated energy and observe the effect of the different trailing edge patterns, the local value of energy associated with shedding, $E_{sh}$, is defined as:

$$E_{sh} = \int_{St_{1}}^{St_{2}} \frac{\Phi_{v_{1},v_{2}}}{U_{\infty} c} dSt,$$

(2)

where $\Phi_{v_{1},v_{2}}$ is the cross-power spectral density between two time series $v_{1}$ and $v_{2}$ ($\Phi_{v_{1},v_{1}}$ is thus the power spectral density of $v_{1}$). $St_{1} = St_{sh}(1 - \Delta St/2)$ and $St_{2} = St_{sh}(1 + \Delta St/2)$ with $\Delta St$ being the range of Strouhal number over which to...
integrate and $St_{sh}$ being the vortex shedding value of $St$, where $\Phi_{v1,v2}$ is maximum. The value $\Delta St = 0.2$ is chosen following a sensitivity analysis, so that small changes in $\Delta St$ would not affect the results. In this study, the Strouhal number $St = f h/U_\infty$ is defined using the maximum bluntness at the trough, $h$ as opposed to the bluntness as a function of $z$. This choice will be explained later in the discussion of the shedding cell structure in section III C. Fig. 3 shows the span-wise and downstream evolution of $E_{sh}$ for a single tone pattern ($S20W40B$), and for one ($1iS20W40B$) and two ($2iS20W40B$) additional iterations. The multiscale patterns clearly show a lower energy in the shedding signature in fig. 3b & 3c compared to the single tone pattern in fig. 3a. Far downstream, for $x/c > 1.6$, the energy involved is not significant as no shedding spike is discernible in the spectra any more, and the differences are minimal.

The shedding energy plotted in fig. 3a & 3b exhibits span-wise oscillations for both $S20W40B$ and $1iS20W40B$ at $x/c = 0.04$. Fig. 4 shows the span-wise traverses of shedding-associated energy at $x/c = 0.04$ for $S20W00B$, $S20W40B$ and $S20W84B$. The aforementioned oscillations are also observed in the case of the straight blunt trailing edge: $S20W00B$. The wavelength of the oscillations is 8 mm while $S20W40B$ and $S20W84B$ have wavelengths of 40 mm and 84 mm respectively.

Furthermore, these oscillations have not been observed in that wind tunnel in the study of turbulent boundary layers by Rodriguez-Lopez et al. [28]. It is therefore attributed to an imperfection of the apparatus that could have caused a slight inhomogeneity in the upstream boundary layer.

Although unfortunate for the results plotted in fig. 3, one can make two observations. Firstly, fig. 3 (a) and (b) show that these oscillations do not appear further downstream, which means they are cancelled out in the trailing edge’s wake. Secondly, fig. 4 shows that the serrations exhibit weaker oscillations than the straight blunt case, with the biggest difference, in comparison to the straight blunt TE, being observed for $S20W40B$. This would indicate the possible effect of the multiscale patterns on mitigating incoming disturbances which, although interesting, is not the primary focus of this study.

### B. Coherence

The strong variations observed in the span-wise evolution of shedding energy raise the question of the effect that the multiscale patterns have on coherent structures in the wake. Although the present measurements do not provide direct conclusions on noise effects, span-wise coherence close to the trailing edge can be an indication of acoustic noise sources, as well as base drag in the case of vortex shedding by blunt trailing edges. Further downstream, it gives an indication of the strength of coherent structures in the wake and thus needs to be investigated in both regions. In order to study the contribution of coherence at different frequencies in the wake, the following definition is taken:

$$\gamma^2 = \frac{|\Phi_{v1,v2}|^2}{\Phi_{v1,v1} \Phi_{v2,v2}}.$$  

Prigent et al. [26] reported a reduction of coherence at $St_{sh}$ ($St_{sh} \approx 0.19$) with the introduction of sinusoidal patterns, in comparison to a straight blunt trailing edge ($S20W00B$). However, as illustrated by fig. 5 (a) and (b), such sinusoidal patterns
introduce coherence over a broadband range of low frequencies, especially in the trough. Fig. 5 (c) and (d) show that by using multiscale patterns, this effect is mitigated to varying degrees. Indeed, when comparing the total low frequency content of coherence - obtained by integrating $\gamma^2$ over $-0.5 < \tilde{z} / W < 0$ and $St < 0.07$ - a reduction of 8\% is achieved between $S20W40B$ and $1iS20W40B$, and 57\% between $S20W40B$ and $2iS20W40B$. The corresponding PSD maps (not plotted here for brevity) show that the low frequency region of high coherence downstream of the trough corresponds to high values of energy; hence adverse effects such as acoustic noise emission would be non-negligible. Therefore, reduction of coherence in this region is of particular interest.

It is also clear from fig. 5 that the introduction of multiscale patterns reduces the coherence at the vortex shedding frequency ($St_{sh}$) and its second harmonic ($2St_{sh}$). For ease of comparison, $\gamma^2$ can be directly plotted at $St_{sh}$ with the following definition:

$$\gamma_{sh}^2 = \frac{1}{St_{sh} - St_1} \int_{St_1}^{St_{sh}} \gamma^2 dSt.$$  \hspace{1cm} (4)

Fig. 6 shows the span-wise evolution of $\gamma_{sh}^2$ at $x/c = 0.04$ behind the trailing edges discussed in this study. The values are plotted against $z/c$ to provide a comparison of the physical extent over which coherence is observed for the different TEs. All TEs with $W = 40$ mm show a significantly shorter extent than those with $W = 84$ mm. As reported by Prigent et al. [26] the single tone serrations already reduce the extent over which coherence is measured compared to a straight blunt geometry, despite some locally higher values. The introduction of multiscale geometry further reduces these values, the difference being the greatest between $S20W40B$ and $2iS20W40B$.

For both $S20W40B$ and $1iS20W40B$, a local peak of coherence at $St_{sh}$ is observed around $\tilde{z} / W = -0.35$ in fig. 5, as well as for both wavelengths in fig. 6 (note that $z/c$ is used in the latter, with $\tilde{z} / W_{10} = -0.35 \leftrightarrow \tilde{z} / c \approx 0.09$). This feature is likely to be due to the cross-flow generated by the serrations themselves as discussed by Prigent et al. [26]. It is interesting to note that $2iS20W40B$ does not exhibit this peak and is the TE that reduces coherence the most, which is most likely due to the higher waviness of its pattern following the introduction of the second iteration.

Fig. 7 shows the spanwise and downstream evolution of $\gamma_{sh}^2$ behind $S20W40B$, $1iS20W40B$ and $2iS20W40B$. The $x/c = 0.04$ position was shown in fig. 6, and it is interesting to see that the reduction of coherence is not only seen very close to the trailing edge, but also further downstream. For intermediate positions such as $x/c = 0.4$ or $x/c = 0.6$, the reduction is not obvious between $S20W40B$ and $1iS20W40B$ but is rather clear between $S20W40B$ and $2iS20W40B$.

Fig. 5 and fig. 7 show that coherence with reference to the trough is mainly contained within $|\tilde{z} / W| < 0.4$, and although the figures are truncated at the peak ($\tilde{z} / W = 0.5$), no coherence is found when moving further away towards the adjacent trough.

Using the time-resolved PIV data, it is possible to obtain correlation traverses with other locations of the reference probe.

**FIG. 4:** Span-wise traverses of shedding-associated energy (from hot-wire data) at $x/c = 0.04$ and comparison with $S20W00B$. 

**FIG. 5 and fig. 7:** Spanwise and downstream evolution of $\gamma_{sh}^2$ (with reference to the trough) for $S20W40B$, $1iS20W40B$ and $2iS20W40B$. The $x/c = 0.04$ position was shown in fig. 6, and it is interesting to see that the reduction of coherence is not only seen very close to the trailing edge, but also further downstream. For intermediate positions such as $x/c = 0.4$ or $x/c = 0.6$, the reduction is not obvious between $S20W40B$ and $1iS20W40B$ but is rather clear between $S20W40B$ and $2iS20W40B$. 

**FIG. 5 and fig. 7:** Spanwise and downstream evolution of $\gamma_{sh}^2$ (with reference to the trough) for $S20W40B$, $1iS20W40B$ and $2iS20W40B$. The $x/c = 0.04$ position was shown in fig. 6, and it is interesting to see that the reduction of coherence is not only seen very close to the trailing edge, but also further downstream. For intermediate positions such as $x/c = 0.4$ or $x/c = 0.6$, the reduction is not obvious between $S20W40B$ and $1iS20W40B$ but is rather clear between $S20W40B$ and $2iS20W40B$. 

Using the time-resolved PIV data, it is possible to obtain correlation traverses with other locations of the reference probe.
FIG. 5: \( \gamma^2 \) maps (from hot-wire data), \( x/c = 0.04 \) downstream of four tested trailing edges.

Fig. 8 shows span-wise traverses of the correlation coefficient, defined as

\[
\frac{u'(r, t) \cdot u'(r + \Delta r, t)}{\sqrt{u'(r, t)^2 \cdot u'(r + \Delta r, t)^2}}
\]

between two time series \( u' \) taken at locations \( r \) and \( r + \Delta r \). The traverses are performed at three downstream locations, with four span-wise positions of the reference probe for each, downstream of only \( S20W40B \) for brevity. These positions match the trough (\( r = \tilde{z}/W = 0 \)), mid-point (\( r = \tilde{z}/W = 0.25 \)) and peak (\( r = \tilde{z}/W = 0.5 \)), as well as an intermediary point between mid-point and peak (\( r = \tilde{z}/W = 0.375 \)). At \( x/c = 0.04 \), the correlation centred at the trough is symmetric, and drops to zero before going downstream of the adjacent serrations. When using \( r = \tilde{z}/W = 0.25 \) as a reference probe, the correlation traverse
becomes asymmetric, with non-zero values towards the negative separation, that is towards $\bar{z}/W = 0$. Putting the reference probe close to the peak does not show any correlation apart from the main peak, due to the absence of shedding in that part of the wake. Moving downstream, at $x/c = 0.12$, the overall values decay, but the same asymmetry is observed. However, when moving to $x/c = 0.2$, it is interesting to note that the traverse with reference probe at $r = \bar{z}/W = 0.375$ starts exhibiting a similar, but weaker, asymmetry. Overall, this shows that the velocity fluctuations in the wake of the serrations are only correlated behind their own serration, and only weakly with the part of the wake behind an adjacent one.

Because of the cut-in approach, the exhibited bluntness varies along the span-wise direction, from maximum bluntness at the trough to sharpness at the peak. However, fig.5 has shown that the shedding signal measured in the wake has a single fundamental frequency along the span, which indicates that the shedding is locked into a cell-like mechanism across the trailing edge pattern, instead of a smoothly evolving shedding mechanism. This explains the choice of the maximum bluntness at the trough in the definition of $St$, rather than the local bluntness. This structure bears resemblance to the cell structure reported by Tombazis and Bearman [34] in the case of a thick plate at zero incidence with a sinusoidal trailing edge of constant bluntness, in a geometric sense. However, in that study, the cell structure exhibited regions shedding at different frequencies and their spatial arrangement would alternate, hence a single measurement point could capture different shedding frequencies over time. In the present case, only one fundamental shedding frequency is measured across the cell and the main physical difference between the two studies is the varying bluntness which appears to isolate consecutive cells as previously discussed. In summary, vortex shedding is locked into independent cells corresponding to the wavelength of the serrations.

FIG. 7: Span-wise and downstream evolution of shedding associated coherence, $\gamma_{sh}$ (from hot-wire data).

FIG. 8: Span-wise traverses of correlation coefficient with references at four span-wise locations, performed at three downstream locations for $S20W40B$ (from PIV data).
The previous observations indicate the presence of cell structures, however, so far there is no information as to how they develop downstream: either as straight lines or in more complex bent arches. The cross power spectral density provides information on the phase of coherent span-wise structures in the wake. Indeed, it may be written as:

$$\Phi_{v_1, v_2} = |\Phi_{v_1, v_2}| e^{i\phi_{v_1, v_2}},$$

and corresponds to the Fourier transform of the cross-correlation between the two time series $v_1$ and $v_2$. The phase $\phi_{v_1, v_2}$ thus gives an indication of how much lag there is between the two signals, and will henceforth be referred to as de-phasing. $\phi_{v_1, v_2}$ is a function of frequency and the local convection velocity, since a strong variation of the latter between the measurement points could modulate the lag/lead measured. The value of $\phi_{v_1, v_2}$ at the shedding frequency, denoted $\phi_{sh}$, is defined by averaging across a small range of frequencies, in an approach similar to eq. 4. Due to its definition, $\phi_{v_1, v_2} < 0$ corresponds to $v_2$ having a lead on $v_1$. Fig. 9a displays a simplified case of a vortical structure shed from the trailing edge shaped as an arch. In the type 1 configuration, $v_2$ has an increasing lead on $v_1$ when being traversed away from it. On the other hand, if the arch is upside down, i.e. type 2, $v_2$ then has a lag compared to $v_1$. The absence of de-phasing would correspond to a straight vortex, while a chevron-shaped one would exhibit a linear progression of de-phasing between the two time series when moving along the span-wise direction. Fig. 9c illustrates a weaker version of the arch shape structure, that has been rapidly broken down.

![Fig. 9: Sketch of three possible vortex shedding structures behind a serration.](image)

To focus on the values of de-phasing corresponding to significant coherence, $\Gamma_{sh}$ is introduced as follows:

$$\Gamma_{sh} = \gamma_{sh}^2 \cdot \phi_{sh}.$$  

It must be stated that weighting by $\gamma_{sh}^2$ does not affect the evolution of $\phi_{sh}$ discussed hereafter but filters out the fluctuations of de-phasing where coherence is negligible.

Fig. 10 shows the span-wise and downstream evolution of $\Gamma_{sh}$ between a reference point ($v_1$) downstream of the trough and a point traversed in the span-wise direction ($v_2$) (both at the same $x/c$), downstream of S20W40B, 1iS20W40B and 2iS20W40B, and S20W84B, 1iS20W84B and 2iS20W84B.

The value of $\Gamma_{sh}$ that has the greatest magnitude for each downstream location is plotted in fig. 10g & 10h for all serration patterns.

For S20W40B, fig. 10a shows a clear spike of de-phasing values for several downstream positions, and no significant de-phasing for $z/W < -0.5$ for any location. When moving downstream, the de-phasing initially decreases with negative values, indicating that the mid-point/peak parts of the wake have a lead time over the trough area, which increases from $z/W = 0$ to around $z/W = -0.3$. This shows that the coherent structures shed by the trailing edge initially have a curved shape. For $z/W < -0.4$, $\Gamma_{sh}$ quickly drops to zero as both coherence and de-phasing disappear. This corresponds to the edge of the vortex structure. These observations thus correspond to the idealised vortex structure of type 1 as illustrated in fig. 9a.

Fig. 10b & 10c, on the other hand, show that the multiscale patterns exhibit a stronger span-wise variation of the de-phasing in their wakes. Indeed, in the wake of 1iS20W40B, $\Gamma_{sh}$ reaches similar values as for S20W40B but its span-wise variations are stronger, which is particularly clear at $x/c = 0.4$ and $x/c = 0.6$. This observation is also valid for 2iS20W40B despite
the values being smaller. The downstream evolution of the $\Gamma_{sh,\text{max}}$ in fig. 10g also shows smaller values for $2iS20W40B$ and a tendency to decrease faster than $S20W40B$ when going downstream. This corresponds to the structure illustrated in fig. 9c.

For $S20W84B$, fig. 10d shows that the first downstream position exhibits a similar trend as for $S20W40B$, with de-phasing initially forming a spike around $\bar{z}/W = -0.2$. The main difference from the shorter wavelength is the inversion of the de-phasing sign when moving downstream, with positive values first appearing around $\bar{z}/W = -0.3$ and subsequently becoming more widely spread. This means that when moving downstream, the mid-point area of the wake, which exhibited a lead over the trough area at the shedding frequency, starts to have a lag. The shape of the coherent structure would thus be inverted, corresponding to the type 2 structure in fig. 9b. This feature would be a logical outcome of the stronger velocity deficit observed at the peaks than at the trough, as reported by Prigent et al. [26]. Another possible explanation, that could add to the previous, is a phenomena similar to the axis switching in non-circular jets reported by Zaman [39] and Ramesh et al. [27], where the bent vortex would induce velocity fluctuations tending to straighten it and eventually invert its curvature. Given that vortex shedding was previously observed to be stronger for $S20W84B$ than $S20W40B$, it is logical to expect this phenomena to appear more so for the former.

FIG. 10: Span-wise and downstream evolution of weighted de-phasing, $\Gamma_{sh}$ (from hot-wire data), with reference point fixed at the trough ($\bar{z}/W = 0$).
is due to the fact that for this shorter wavelength, the field of view is not restricted to only one trough, and the same physical Ré2OMD spectra for (ii) and (iii) correspond to frequencies just above that of (i) but their Ré peak frequencies, one corresponding to the vortex shedding (i), at ∼ are the least damped modes, and therefore the most dynamically relevant. It is clear from fig. 11a that conjugate, only the positive half of the spectra is plotted. The numbers (i)-(iv) identify the modes being plotted in fig. 12. They studies applying the OMD to PIV data of strongly disturbed boundary layers and behind a multiscale array of bars, respectively. to avoid the dependence of the main peaks’ frequencies on the number of modes, and it was set at λMz that λMz is the last damped modes, and therefore the most dynamically relevant. It is clear from fig. 11a that S20W84B exhibits two peak frequencies, one corresponding to the vortex shedding (i), at ~ 588 Hz and a secondary one at ~ 340 Hz (iv). The modes (ii) and (iii) correspond to frequencies just above that of (i) but their Ré(λiOMD) are at similar level to (iv). Fig. 11b shows the OMD spectra for 2iS20W84B. The same two peaks are observed for modes (i) and (iv). However, it is clear that modes (ii) and (iii) have a lower Ré(λiOMD) value than for the single tone case. In addition, one notices an overall decrease of the modes’ Ré(λiOMD) values.

OMD is similar to Dynamical Mode Decomposition (DMD) (cf. Schmid [31] [32] and Jovanovic et al. [16]) in that it will approximate A by

\[ A = L M L^T \]  

(10)

However, DMD takes the Proper Orthogonal Decomposition (POD) modes of \( \Psi_0 \) for L. In our case, it is not conceivable to use as many modes as snapshots, and reducing the number of the latter yields unsatisfactory results. A low-rank \( r << N \) (i.e. a limited number of modes) is therefore imposed and we do not know a priori which basis is the most dynamically significant for L. Optimal Mode Decomposition (OMD) is thus used to take advantage of its dual optimisation scheme. Indeed, the OMD process finds an optimal lower order representation of A by performing the following optimisation:

\[ \min_{L,M} \| \Psi_0 - L M L^T \Psi_0 \|^2 \text{ such that } L \in \mathbb{C}^{k,r}, LL^T = I_r \text{ and } M \in \mathbb{C}^{r,r}. \]  

(11)

The matrix M is thus the low-order representation of the dynamics matrix A. It satisfies an eigenvalue problem written as \( M z_i = \lambda_i(M) z_i \), which gives the OMD modes as \( L z_i \), associated to a decay rate of \( \Re(\lambda_i^{OMD}) \) and a frequency \( \Im(\lambda_i^{OMD}) \), such that \( \lambda_i^{OMD} = \lambda_i(M)/\Delta t \). The pairs \( \Re(\lambda_i^{OMD}) \) and \( \Im(\lambda_i^{OMD}) \) form the OMD spectrum. In our case, r was chosen high enough to avoid the dependence of the main peaks’ frequencies on the number of modes, and it was set at \( r = 70 \).

The reader is referred to Wynn et al. [38] for further details, and to Rodríguez-López et al. [28] and Baj et al. [6] for other studies applying the OMD to PIV data of strongly disturbed boundary layers and behind a multiscale array of bars, respectively.

Fig. 11 shows the OMD spectra for S20W84B, 2iS20W84B, S20W40B and 2iS20W40B. The modes being complex conjugate, only the positive half of the spectra is plotted. The numbers (i)-(iv) identify the modes being plotted in fig. 12. They are the least damped modes, and therefore the most dynamically relevant. It is clear from fig. 11a that S20W84B exhibits two peak frequencies, one corresponding to the vortex shedding (i), at ~ 588 Hz and a secondary one at ~ 340 Hz (iv). The modes (ii) and (iii) correspond to frequencies just above that of (i) but their Ré(λiOMD) are at similar level to (iv). Fig. 11b shows the OMD spectra for 2iS20W84B. The same two peaks are observed for modes (i) and (iv). However, it is clear that modes (ii) and (iii) have a lower Ré(λiOMD) value than for the single tone case. In addition, one notices an overall decrease of the modes’ Ré(λiOMD) values.

Fig. 11c shows that in the case of S20W40B, the main peak is not comprised of one mode, but three distinct ones. This is due to the fact that for this shorter wavelength, the field of view is not restricted to only one trough, and the same physical
phenomenon is observed for the two neighbouring troughs as well. However, it is interesting to note that these modes are indeed distinct, which corroborates the fact that successive serrations shed independently, as observed by the absence of coherence between them. For completeness, the OMD procedure was also run with a very limited number of modes, and these main modes were still separated for consecutive troughs, reinforcing the previous comment.

In fig. 11d, 2iS20W40B also shows an overall reduction of the modes’ $\Re(\lambda_{OMD}^i)$ values, which is stronger than in the case of 2iS20W84B. This is in line with the faster decay of span-wise coherence when moving downstream of 2iS20W40B, as observed in fig 7.

Fig. 12 shows the stream-wise ($u$) and span-wise ($w$) velocity components of the modes (i), (ii), (iii) and (iv) for S20W84B and 2iS20W84B. Note that the re-circulation region right behind the blunt trough is not visible as the field of view is situated slightly under the wing (cf section II B).

For mode (i), one notices the arched shapes predicted from the hot-wire data. The anti-symmetry of the span-wise component can be explained by the bending of the vortices: if they were straight and contained only span-wise vorticity, they would not induce span-wise velocity. It is thus the decomposition into stream-wise/span-wise components of the velocity fluctuations induced by the bent shed vortices that creates the antisymmetric structures. The symmetry of the stream-wise component structure follows from the same argument. Mode (i) hence corresponds to the idealised arch-shaped vortex shedding from fig.9a.

Prigent et al. [26] reported a span-wise component in the mean flow, from peak to trough in the lower part of the wake for sinusoidal trailing edges. The extent of the $w$ structures in mode (i) are similar to that of the mean flow. This modal decomposition is made on the velocity fluctuations, after subtracting the mean, so it is not a visualisation of the same effect. However, the link between the two is not hard to make. The mean flow will tend to bend the vortices shed in the trough; and the velocity fluctuations induced by these vortices will thus have a bent pattern with the symmetry discussed previously.

When using the multiscale 2iS20W84B, on the other hand, it is clear from fig. 12 that mode (i)’s structures are modified.
They are similar in the fact that they are also bent and the symmetry/antisymmetry is the same as for S20W84B, however, they appear to be flatter in the centre and to have weaker bent edges. This could be due to the stronger variations of the TE pattern, forcing the shed structures to develop into a smaller span-wise extent.

Modes (ii) and (iii), are antisymmetric for the stream-wise component, and therefore symmetric for the span-wise component. They are well structured for S20W84B, although not as neat as mode (i), but the spatial organisation is lesser for 2iS20W84B. This is particularly noticeable for mode (ii), as w displays patches in its span-wise evolution unlike the continuous structures of S20W84B. This is in fact expected, since S20W84B’s spectra showed a clear separation from the $\Re(\lambda_{OMD})$ values of modes ii and iii, and those from the rest of the spectra. Such a distinction is not visible for 2iS20W84B which means the modes are closer to the background motion, and therefore less meaningful.

Mode (iv) is similar to mode (i), however, the spatial period of the structure is smaller, as well as the associated frequency in the spectra, and there is no harmonic relation between the two. It is most likely a mode adding rippling and breaking down the spatial organisation of mode (i), to tend towards a weaker structure as depicted in fig. 9c.

Fig. 13 shows mode (i) for S20W40B and 2iS20W40B. The symmetric/anti-symmetric structures are also found. As noted before, the OMD spectra for those two cases showed three modes for the main peak. Mode (i)-b is a secondary mode out of the three forming the peak. This indeed corresponds to the three consecutive troughs visible in the field of view. The distinction between mode (i) and mode (i)-b shows the independence of the vortex shedding between adjacent troughs, as little structure is observable in them. Modes (ii)-(iv) are not discernible from the spectra and are therefore not plotted either.

Overall, both experimental approaches are complementary and it appears that the cell structures described by the hot-wire data analysis are qualitatively found with the first modes of the OMD decomposition. However, to link the two sets of data more quantitatively, the main OMD modes can be used to reconstruct a set of velocity fields. The $i^{th}$ reconstructed snapshot is taken as the sum of the OMD modes weighted by the projection of the original $i^{th}$ snapshot onto said modes. The modes being complex, the real part of the sum is taken. The set of reconstructed snapshots thus gives, for each position of the field of view, a time series extracted from the original data.

Fig. 14 shows the span-wise evolution of de-phasing $\phi_{sh}$ at $x/c = 0.04$ downstream of S20W84B obtained from hot-wire data, and OMD reconstructions with modes (i), (ii) and (iii), and with modes (i), (ii), (iii) and (iv). The time series obtained from this method are shorter than those obtained with the hot-wires, and the periodogram methods can only be used with a limited number of windows. The spectral analysis of reconstructed data thus yields higher noise than that of hot-wire data, and the comparison is hence made using the de-phasing without weighting by the coherence. Given that for $\tilde{z}/W < -0.4$, the data are more noisy and coherence drops down, this range is not included in the plot. Using the four main modes actually gives the best match to the hot-wire data, despite the fact that mode (iv) is actually not associated to the shedding frequency. This demonstrates its importance in the structure of the shedding cells. From $\tilde{z}/W = 0$ to $-0.35$, there is a reasonably good match of the hot-wire data with four modes reconstruction, noticing a decrease of $\phi_{sh}$ until around $\tilde{z}/W = -0.2$, and the extrema values are the same. After $\tilde{z}/W = -0.35$, the OMD data become more noisy, but coherence is very low in that area, so it is of lesser significance.

Overall, this comparison shows that the approach proposed for the study of de-phasing not only shows qualitative information on the shape of the modes, but also corresponds to a quantitative match of the cell structures.

### E. Reynolds stress

In this section, we consider the Reynolds stresses in the wake of the wing to explore the role that such stresses play in shaping the structures shed by the trailing edge. The link between $w'$ and $u'$ had been highlighted with the different structures of the OMD modes, however, this further analysis can be done by looking directly into the time series.

Fig. 15 shows the normalized Reynolds stress term $-\bar{u'}w'/U_2^2$ in the wake of four trailing edges, obtained from PIV data. For both S20W40B and S20W84B, the patterns can be divided in two areas, one close to the trailing edge and one further downstream. However, while S20W40B exhibits high stress close to the trailing edge and little downstream in fig.15a, S20W84B shows the opposite in fig.15c. In fig.15d 2iS20W84B shows a reduction of stress downstream and an increase close to the trailing edge, with regards to S20W84B. However, the maximum values remain low compared to the high stress of S20W40B. 2iS20W40B has very similar patterns to that of 2iS20W84B. The symmetry observed in all cases is due to the curvature of the shapes which, as explained for the OMD modes, inverts the sign of the $w'$-component of the fluctuations generated by the shed structures when going from one half of the serration to the other one.

Mode (i) for S20W40B is compact and the structure rather straight; and the edges of the structures correspond to the areas of high stress. This high stress, mainly due to $u'$, could thus be constraining the shed structures within a specific area, preventing them from fully developing into a bent shape. On the other hand, S20W84B does not exhibit this region of high stress close to the trailing edge, and its mode (i) is indeed a smoothly bent arch shape. 2iS20W84B has a higher stress than S20W84B close to the trailing edge, and its mode (i) displays a structure that is compact and rather straight in the centre, with bent tails, which is in-between those of S20W84B’s mode (i) and S20W40B’s mode (i).
From hot-wire data, one can estimate the maximum value of $\partial U/\partial y$ and compare it to the values found for $\partial U/\partial z$ and $\partial W/\partial x$ from the PIV data. The two latter are found to be dominated by the first by over an order of magnitude. Indeed, looking into all production terms available from the PIV data does not provide significant information and does not highlight differences between the tested trailing edges. Overall, the slight reduction of stress and associated production term downstream of the multiscale trailing edge cannot explain the more rapid downstream decay of coherence and energy previously observed at the shedding frequency. Hence, with these components, one can only say that the stress plays a role in shaping the structures shed by the trailing edges by limiting their span-wise development.

IV. CONCLUSIONS

Two experiments were conducted in the wake of a NACA0012 with multiscale cut-in serrations, at moderate angle of incidence, to study how the vortex shedding is affected by the patterns of such serrations. Hot-wires were used to span-wisely traverse the wake with a reference probe behind a trough, and to focus on spectral information such as energy, coherence and de-phasing. 2D-2C Particle Image Velocimetry (PIV) was used to obtain further information on the spatio-temporal structure of the wake.

Analysis of the hot-wire data revealed that multiscale serrations reduce the energy associated with vortex shedding, as well as the span-wise coherence, both at vortex shedding and at low frequencies. The reduction of span-wise coherence, both in physical extent and magnitude, is clear very close to the trailing edge and persists downstream.

Vortex shedding is found to be locked into cell structures behind the cut-in serrations. The study of de-phasing between the trough position and other span-wise locations shows that these cell structures are bent and can be reversed (for the large wavelength) by the span-wise inhomogeneity of the stream-wise velocity. The reduction of coherence by the introduction of multiscale patterns is thus explained by the strengthening of these cell structures.

Analysis of the PIV data by Optimal Mode Decomposition offers an insight into the spatio-temporal organisation of these structures. The bent shape is indeed clearly observed for modes corresponding to the vortex shedding frequency. The modes obtained from the wake of the multiscale trailing edges show less spatial organisation than those of the single tone pattern. This corroborates the results from the hot-wires on the weakening of the cell structures. Moreover, Reynolds stress is found to play a role in limiting the span-wise development of the cell structures, and therefore in altering their shapes.

In conclusion, the introduction of cut-in serrations exhibits bluntness that in turn generates vortex shedding that locks into a bent cell structure. This shedding is however weaker than that of a straight trailing edge of the same maximal bluntness. The use of multiscale patterns weakens the cell structures which translates into a reduction of vortex shedding intensity as well as a reduction of span-wise correlation at both vortex shedding and low frequencies.

ACKNOWLEDGMENTS

This work is supported by the European Commission, under a Marie Curie action (EU FP7 project “MULTISOLVE”, grant agreement No. 317269).


FIG. 12: Normalized OMD modes (i)-(iv) for $S20W84B$ (top two rows) and $2iS20W84B$ (bottom two rows), from PIV data.
FIG. 13: Normalized OMD mode (i) for $S20W40B$ (left) and $S20W40B$ (right), from PIV data. Mode (i)-b is a secondary mode of the main peak.

FIG. 14: Comparison between de-phasing obtained for $S20W84B$ at $x/c = 0.04$, by HW and OMD reconstructions with modes (i) (ii) (iii), and (i) (ii) (iii) (iv).
FIG. 15: Maps of partial Reynolds stress downstream of four tested trailing edges, from PIV data.