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Highlight
• An advanced model is developed to predict the forming limits of an Al-Li alloy under hot stamping conditions
• The developed model concerns the combined effects of varying strain rate, temperature and loading path
• The onset of necking of the component during hot stamping did not necessarily occur at the maximum thinning region
• Incremental work per unit volume ratio is a significant parameter for forming limit prediction under hot stamping conditions
Forming limit prediction for hot stamping processes featuring non-isothermal and complex loading conditions

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Abstract

An intrinsic feature of the hot stamping process, in which a hot blank is quenched and formed between water cooled dies, is the severe thermo-mechanical deformation that the blank experiences under the combined influences of non-isothermal and non-proportional loadings. This results in challenges for conventional forming limit prediction models to accurately predict material behavior. In this paper, a novel viscoplastic-Hosford-MK model was developed to predict the forming limits of an Al-Li alloy under hot stamping conditions. The effectiveness of the developed model was verified by the demonstration of accurate responses to cold die quenching, strain rate and loading path changes, enabling the developed model to reveal a realistic critical material response under complex deformation conditions. Finally, by applying the developed model to the hot stamping of an AA2060 component, its accuracy was successfully validated. It was indicated that the onset of necking during hot stamping of the component did not necessarily occur at the maximum thinning region, and this was due to the comprehensive effects of varying loading path, strain rate and temperature. A detailed mathematical analysis of the developed M-K model was also conducted, and it was found that the incremental work per unit volume ratio ($\dot{\mathcal{W}}_b/\dot{\mathcal{W}}_a = \bar{\sigma}_b d\bar{\sigma}_b/\bar{\sigma}_a d\bar{\sigma}_a$) between Zone b (where a thickness inhomogeneity exists) and Zone a (the remainder of the material) was a significant parameter that determined the formability of AA2060 under hot stamping conditions.
Keywords: A. Forming limit; B. Aluminium lithium; C. Viscoplastic;

1. Introduction

The ever increasing demand for fuel economy has led the aerospace industry to utilise lightweight materials such as aluminium lithium alloys in structural applications such as fuselage panels of aircraft [1]. Significant research programs have been undertaken to develop novel manufacturing technologies to process these materials into complex-shaped components, with the aim of reducing the manufacturing costs and enhancing the productivity.

Solution Heat treatment, Forming and in-die Quenching (HFQ™) introduced by Lin et al. [2] is a hybrid forming technology that combines both forming and heat treatment processes into a single operation. This enables complex-shaped sheet components to be formed whilst maintaining the full mechanical strength of the as-received material. In the HFQ forming process, the workpiece material is heated and deformed plastically using a cold die and punch, and hence strong non-isothermal conditions and non-proportional loading conditions associated with loading path and strain rate variations occur. This results in difficulties in the forming limit prediction under such complex conditions.

To assess the onset of necking or tearing on the deformed sheet metal, a forming limit diagram (FLD) is used, which is defined in a strain or stress based coordinate system to quantify the values of strain or stress at which necking or tearing takes place and indicates the level of deformation the sheet metal can experience under various strain paths before fracture occurs. The FLDs are determined experimentally from formability tests on a series of standardised test-pieces [3,4]. By measuring the strain field in the formed test-pieces before visible cracking appears, the forming limit of the blank could be determined. However, the determination of FLDs using experimental methods is extremely time consuming, requiring multiple tests to be performed. Thus, a theoretical FLD prediction model is required.

Theoretical FLD prediction models have been reviewed previously by Banabic [5] and Schwindt et al. [6]. Classical FLD-prediction models are typically used to predict failure in sheet metal forming, such as the Swift model [7], Hill model [8], the bifurcation analysis based models [9], the ductile fracture models and the Marciniak-Kuczynski (M-K) model [10].
The first approach utilizes instability criteria by introducing diffuse necking and localized necking, as proposed by Swift [7] and Hill [8] with the assumption of homogenous sheet metals. The second approach was based on Hill’s pioneering work, where Storen and Rice [9] incorporated deformation plasticity theory into a classical bifurcation analysis in their approach, and indicated that the development of a vertex on the yield locus plays a dominant role on the onset of localized necking from a state of uniform deformation in thin sheets. The third approach concerns the process of failure under various loading conditions, caused by microcracks formed by the nucleation, growth and coalescence of defects [11]. Continuum damage mechanics (CDM) models [12–14] introduce damage variables to reflect the material structure degradation at the micromechanics scale, with the assumption that failure takes place once specified damage parameters approach a certain value, such as 0.7 [15]. In hot forming conditions, the damage development is mainly due to the nucleation of cavities at grain boundaries [16]. In the CDM models, the specified damage variable is expressed as an effective surface density of cavity or void intersections within a plane [17]. Eq. (1) shows the power-law viscoplastic constitutive equation as an example.

\[ \dot{\varepsilon}_p = \left( \frac{\sigma - R - k}{K} \right)^\alpha \]  

By introducing a damage specified variable into the power-law viscoplastic constitutive equation, the equation is expressed as:

\[ \dot{\varepsilon}_p = \left( \frac{\sigma/(1 - f_d) - R - k}{K} \right)^\alpha \]  

where \( f_d \) is the damage factor.

The final approach concerns the development of necking based on the hypothesis of an initial non-uniformity at local regions, as proposed by Marciniak and Kuczynski. The Marciniak-Kuczynski (M-K) model [10] is one of the most commonly used methods to predict sheet metal forming limits or instability [18–21]. In the model, it is hypothesized that there is a thickness variation at a local region of the specimen, and this initial geometrical non-homogeneity is represented by the variable \( f_0 \), known as the “imperfection factor”. The local region where the imperfection exists is defined as Zone b, and the rest of the material as Zone a, and the onset of necking is defined when the strain rate ratio in the through-thickness direction or the major strain direction between Zone b and Zone a reaches a critical value. Apart from the classical prediction models outlined above, other FLD prediction models have
also contributed significantly to the modelling of the formability of sheet metals such as the modified maximum force criterion model, the stress-based criterion model and the through-thickness shear instability criterion model [22–24].

Several studies in the literature have applied a range of models to predict the forming limit of aluminium alloys [25–32]. Khan and Baig [28] used the Khan-Huang-Liang (KHL) model to successfully predict the forming limit curves for AA5182-O under warm forming conditions by considering the effects of temperature, strain rate, strain rate sensitivity and anisotropic material behaviour. Abedrabbo et al. [30–32] demonstrated good agreement between their developed model and experimental tests by evaluating the effects of temperature and anisotropy on the failure location. However, the model only considered the effect of temperature; the strain rate and loading path effects required further study. Stoughton and Yoon [29] assessed the formability of an aluminium alloy under non-proportional loading and triaxial stress loading conditions, emphasizing the importance of a stress-based FLD, and the developed model was validated by real forming process. Additionally, a stress-based FLD prediction model for a two-stage forming technique was developed by Zhang and Wang [27] who utilised a strain-based FLD to indicate that the occurrence of localised necking is mainly due to the localised geometric softening at a particular level of deformation for an anisotropic material.

In addition to the consideration of temperature, strain rate and loading path on the FLD prediction, recent studies have emphasised the effects of sheet thickness [33], lubrication [34] and strain history [35]. Yoshida et al. [35] demonstrated the effect of the work hardening rate on the formability of an anisotropic material under a proportional loading path. In Yoshida et al. [36], the work hardening behaviour of a material under multiaxial stress paths was analysed using a crystal plasticity model, also indicating that the plastic work per unit volume was a significant parameter on the stress-dependent working behaviour of aluminium alloy. Similar analysis work by Khan et al. [37,38] also took into account the incremental work per unit volume parameter in the calculation of the forming limit. The effect of plastic work on their failure prediction was investigated in [39].

A novel constitutive model named the unified Viscoplastic-Hosford-MK (Marciniak-Kuczynski) model was developed in the present research to determine the forming limit of a high strength aluminium alloy, particularly for hot stamping processes featuring non-isothermal conditions and dramatic strain rate and loading path variations. The developed
model was validated using fundamental experiments (uniaxial tensile and formability tests) and HFQ forming tests of a real component. The effects of changes in the loading path (loading history), temperature (quenching rate) and strain rate on the FLD were predicted and numerically analysed. To assess the capability of the developed model, a detailed mathematical analysis of the effect of varying the loading path, strain rate and temperature conditions was carried out in terms of the major driving forces behind necking. The effectiveness and accuracy of the developed model was verified by applying it to the hot stamping of an AA2060 component to predict the regions where necking would occur.

2. A unified Viscoplastic-Hosford-MK constitutive model for FLD prediction under hot stamping conditions

In metal forming processes conducted at elevated temperatures, materials deform viscoplastically, and their microstructure changes with time. Therefore, the use of time integration based constitutive equations to model the evolution of physical phenomena during plastic deformation is essential. The effects of strain hardening and recovery on the flow stress evolution were modelled via dislocation-based hardening laws, in which the generation of dislocations due to plastic strain and the annihilation of dislocations (recovery) under hot forming conditions were taken into account. These temperature dependent viscoplastic constitutive equations were combined with a Hosford yield function to model the anisotropic nature of plastic deformation in sheet metals, and the M-K model. The latter represented the inherent nature of the imperfection in the material, which physically exists due to any non-uniformity, such as a thickness non-uniformity or pre-existing micro-defect. Consequently, fracture under different stress and strain conditions could be predicted.

2.1 A viscoplastic material model for an aluminum-lithium alloy (AA2060)

In most of the previous research on FLD prediction, two major types of material models have been employed [28], namely physics-based models and phenomenological models. Under most circumstances, the stress-strain behaviours of the workpiece material were modelled phenomenologically as a function of strain and strain rate [6,28,40], for example $\sigma = K\varepsilon_p^n$ and $\sigma = K\varepsilon_p^n \dot{\varepsilon}_p^m$. The material constants in these equations are normally not temperature dependent. Therefore, each flow stress curve requires a unique set of fitting
parameters, which are determined based on experimental data. Most recently, this issue was addressed by Khan and Baig [28], who combined their sophisticated phenomenological KHL model with the M-K model to predict FLCs by considering the effects of strain rate and temperature. On the other hand, physically based viscoplastic constitutive models have not been used for FLD prediction, due to the difficulties in determining the material constants [28].

Such mechanism-based constitutive equations [41] have been developed by considering the generation and annihilation of dislocations due to plastic strain and annealing under hot forming conditions. At elevated temperatures, it is assumed that the flow stress ($\bar{\sigma}$) of the material obeys the power law $\sigma = K \varepsilon^{n} \dot{\varepsilon}^{m}$. The power law equation then can be modified as shown in Eq. (3) to obtain the plastic strain rate ($\dot{\varepsilon}_p$) as a function of the initial yield stress ($k$) and hardening of the material ($R$). Since the material in both the defect (Zone b) and non-defect (Zone a) zones are deformed under the same set of given constitutive equations (Eqs. 3 to 13), the subscripts a and b represent constitutive equations for the calculation of strain and stress in both Zones a and b respectively [42].

$$\dot{\varepsilon}_{pa} = \left( \frac{\sigma_a - R_a - k}{K} \right)^n$$

$$\dot{\varepsilon}_{pb} = \left( \frac{\sigma_b - R_b - k}{K} \right)^n$$

$$\bar{\sigma}_a = E(\varepsilon_{ta} - \varepsilon_{pa})$$

$$\bar{\sigma}_b = E(\varepsilon_{tb} - \varepsilon_{pb})$$

where $\varepsilon_t$ is the total equivalent strain and $\varepsilon_p$ is the equivalent plastic strain.

The hardening parameters $R_a$ and $R_b$ are directly related to the normalised dislocation density ($\bar{\rho}$)[43] shown in Eq. (5), which varies with plastic strain and recovery as shown in Eq. (6) [15].

$$R_a = B\bar{\rho}_a^{0.5}$$

$$R_b = B\bar{\rho}_b^{0.5}$$

$$\dot{\bar{\rho}}_a = A(1 - \bar{\rho}_a)\dot{\varepsilon}_{pa} - C\bar{\rho}_a^{n_2}$$

$$\dot{\bar{\rho}}_b = A(1 - \bar{\rho}_b)\dot{\varepsilon}_{pb} - C\bar{\rho}_b^{n_2}$$
where $\bar{\rho}$ is the normalised dislocation density [43], the parameters $K$, $B$, $C$, $A$, $n$ and $E$ are temperature-dependent material constants, while $n_2$ is a temperature-independent material constant.

Eqs. (7) to (13) represent the temperature-dependent parameters in the form of Arrhenius equations.

$$K = K_0 \exp\left(\frac{Q_K}{R_g T}\right)$$  \hspace{1cm} (7)

$$k = k_0 \exp\left(\frac{Q_k}{R_g T}\right)$$  \hspace{1cm} (8)

$$B = B_0 \exp\left(\frac{Q_B}{R_g T}\right)$$  \hspace{1cm} (9)

$$C = C_0 \exp\left(-\frac{Q_C}{R_g T}\right)$$  \hspace{1cm} (10)

$$E = E_0 \exp\left(\frac{Q_E}{R_g T}\right)$$  \hspace{1cm} (11)

$$A = A_0 \exp\left(\frac{Q_A}{R_g T}\right)$$  \hspace{1cm} (12)

$$n = n_0 \exp\left(\frac{Q_n}{R_g T}\right)$$  \hspace{1cm} (13)

The values of the temperature independent constants were determined by calibrating the above equations with the uniaxial tensile test results. From Eqs. (7) to (13), these values include $K_0$, $k_0$, $B_0$, $C_0$, $A_0$, $n_0$, $E_0$, and $Q_{K_0}$, $Q_k$, $Q_B$, $Q_C$, $Q_A$, $Q_n$, $Q_{E_0}$. The term $Q$ represents the activation energy, $R_g$ is universal gas constant and $T$ is the temperature.

### 2.2 Hosford anisotropic yield function

Over the years, great efforts have been made to develop yield functions representing the anisotropic behaviour of sheet metals [44–53]. The Hosford anisotropic yield function [50] is used in the present research because of its verified accuracy for the description of high temperature yield loci [40,54,55]. Eq. (14) shows the Hosford anisotropic yield criterion.

Zone a: $$R_2 \sigma_{11a} + R_1 \sigma_{22a} + R_1 R_2 (\sigma_{11a} - \sigma_{22a}) = R_2 (R_1 + 1) \overline{\sigma}_{a}$$  \hspace{1cm} (14)

Zone b: $$R_2 \sigma_{11b} + R_1 \sigma_{22b} + R_1 R_2 (\sigma_{11b} - \sigma_{22b}) = R_2 (R_1 + 1) \overline{\sigma}_{b}$$  \hspace{1cm} (14)

where $l$ is a material constant, and $\sigma_{11}$ and $\sigma_{22}$ are the major and minor stress, respectively. $R_1$ and $R_2$ are the strain ratios (r-values) measured in the longitudinal and transverse
directions respectively, i.e. \( R_1 = r_0 \) and \( R_2 = r_{90} \cdot r_0 \) and \( r_{90} \) were obtained from uniaxial tensile tests for aluminium alloys at evaluated temperature, and were determined to be 0.69 and 0.73 respectively.

### 2.3 M-K model for the prediction of the forming limit

During the loading process, strain increments are imposed on the material in Zone a at each time step, with stresses calculated according to the constitutive model. Zone a and Zone b are interlinked by compatibility, where the minor strain in zone a (\( \varepsilon_{2a} \)) is equal to the minor strain in zone b (\( \varepsilon_{2b} \)) at their interfaces (Eq. (15)), and force equilibrium, where the force in zone a is equal to that in zone b (Eq. (16)). A schematic diagram of the M-K model in Fig. 1.

![Schematic diagram of the M-K model](image)

**Fig. 1** Schematic diagram of the M-K model

\[
\varepsilon_{2a} = \varepsilon_{2b} \quad (15)
\]

\[
\sigma_{1a} = f \sigma_{1b} \quad (16)
\]

where \( f \) is the instantaneous imperfection factor (thickness ratio between Zone b and Zone a), which is updated as deformation progresses using Eq. (17).

\[
f = f_0 \exp(\varepsilon_{3b} - \varepsilon_{3a}) \quad (17)
\]

where \( \varepsilon_{3a} \) and \( \varepsilon_{3b} \) are the thickness strains in zones a and b. Experimental FLDS were used to calibrate the initial value of the imperfection factor \( f_0 \). The value of \( f \) decreases as deformation progresses and strain localization becomes more severe in Zone b. Necking occurs when the thickness strain increment ratio between Zone b and a, shown in Eq. (18), approaches a critical value.
When the critical value is reached, the final values of the major and minor strains become the forming limit strains. The equations presented above were developed into a new prediction model, the Viscoplastic-Hosford-MK model, and could be solved simultaneously using a time integration method to deduce the forming limit. This new model for the first time combines a viscoplastic material model, the Hosford anisotropic yield function and the M-K model together, to enable the prediction of the forming limit for a given temperature, forming speed, and ratio “\( \beta \)” between the minor and major strains (the loading path), under constant or varying conditions [42].

The following sections present the fundamental experimental methods and results from uniaxial tensile tests and forming limit tests used to calibrate the viscoplastic-Hosford-MK model.
3. Experimental set-up and test program

Results from uniaxial tensile and isothermal forming limit tests were used to calibrate the developed viscoplastic-Hosford-MK model, which was subsequently verified by the HFQ™ forming of a wing stiffener component from AA2060 in the T83 temper, with a thickness of 2 mm. The chemical composition of the alloy is shown in Table 1.

**Table 1 AA2060 chemical composition**

<table>
<thead>
<tr>
<th>Composition</th>
<th>Li</th>
<th>Mg</th>
<th>Cu</th>
<th>Ag</th>
<th>Mn</th>
<th>Zn</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>wt%</td>
<td>0.6-0.9</td>
<td>0.6-1.1</td>
<td>3.4-3.5</td>
<td>0.05-0.5</td>
<td>0.1-0.5</td>
<td>0.3-0.5</td>
<td>Bal.</td>
</tr>
</tbody>
</table>

3.1 Uniaxial tensile test of AA2060-T83

The viscoplastic material response of the alloy over a range of strain rates and temperatures was evaluated by conducting uniaxial tensile tests. Dog bone shaped specimens were designed and laser cut with the gauge length parallel to the rolling direction. The dimensions of the test-piece are shown in Fig. 2(a). A Gleeble 3800 thermo-mechanical testing system was used to conduct the uniaxial tensile tests. The Gleeble system used direct resistance heating to heat up a test-piece, clamped between two continuously cooled jaws, up to a rate of 10,000°C/s. Thermocouple wires were spot welded and positioned at the central region of the specimen, which were used to provide temperature feedback to the system. The applied load, measured by the Gleeble load cell, and the strain, measured using a C-Gauge transducer positioned at the centre of the specimen, were recorded simultaneously. The tensile tests were conducted to failure at various temperatures and strain rates. To model the hot forming conditions for the HFQ formed AA2060, the test temperatures ranged between 400°C (above the recrystallization temperature) and 520°C (SHT temperature) [1]. The test matrices for the uniaxial tensile tests are shown in Table 2 and Table 3.
Fig. 2 (a) Geometry design (in mm) of specimen for uniaxial tensile test (b) Temperature profile against time for uniaxial tensile test; (c) strain rate profile in strain-rate variation test at constant temperature of 470 °C and (d) temperature profile in temperature variation test at constant strain rate of 0.02/s

Table 2 Test matrix for uniaxial tensile tests on AA2060

<table>
<thead>
<tr>
<th>Strain rate \ Temperature (°C)</th>
<th>400</th>
<th>450</th>
<th>470</th>
<th>500</th>
<th>520</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2/s</td>
<td></td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6/s</td>
<td></td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>11/s</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
</tbody>
</table>

Table 3 Test matrix for strain rate and temperature variation tensile test on AA2060

<table>
<thead>
<tr>
<th>Strain rate variation test shown in Fig. 2 (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
</tr>
<tr>
<td>Time (s)</td>
</tr>
<tr>
<td>t&lt;1.2</td>
</tr>
<tr>
<td>1.2s &lt; t &lt; 1.5</td>
</tr>
<tr>
<td>t &gt; 1.5</td>
</tr>
<tr>
<td>SR (s⁻¹)</td>
</tr>
<tr>
<td>SR₁=2</td>
</tr>
<tr>
<td>SR₁=2 to SR₂=0.2</td>
</tr>
<tr>
<td>SR₂=0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temperature variation test shown in Fig. 2 (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR (s⁻¹)</td>
</tr>
<tr>
<td>Time (s)</td>
</tr>
<tr>
<td>t &lt; 3.7</td>
</tr>
<tr>
<td>3.7 &lt; t &lt; 9.6</td>
</tr>
<tr>
<td>t &gt; 9.6</td>
</tr>
<tr>
<td>Temperature (°C)</td>
</tr>
<tr>
<td>420</td>
</tr>
<tr>
<td>370</td>
</tr>
</tbody>
</table>
The temperature profile [56] was defined using the control software of the Gleeble 3800 simulator. The heating rate was set to 50°C/s until the temperature reached 25°C below the target temperature, after which the heating rate was decreased to 5°C/s, to avoid overheating. The specimen was then held at the target temperature for 1 minute to ensure that homogenous distribution of temperature was achieved in the deformed region prior to HFQ forming. The temperature profile against time used for these tests is shown in Fig. 2(b).

A complex-shaped component may contain structural features, such as local recessed areas that enhance the stiffness, which inevitably cause step-changes in the strain rate and loading path. As an intrinsic feature of hot stamping processes, severe temperature reductions also take place as soon as the hot blank begins to contact the cold forming tools (typically water cooled). Therefore, in addition to the tensile tests outlined above, a test programme was prepared to reproduce these features, the results of which could be used to validate the developed material model. In the strain rate variation tests, a sharp change in the strain rate was applied between t=1.2s to t=1.5s. In the temperature variation tests, a sharp change in the temperature was applied at the times t=3.7s and t=9.6s to simulate the cold die quenching effect. As the temperature change occurred over such a short period of time using the Gleeble simulator, the strain induced during this period was negligible. The test profiles used for the strain rate and temperature variation tests are shown in Fig. 2(c) and (d).

3.2 Isothermal forming limit test of AA2060-T83

An isothermal dome test tool [57] was designed, manufactured and assembled for the high temperature forming limit tests of AA2060, to calibrate the proposed model. The rig was designed and integrated within an Instron furnace and mounted together on a 250 kN ESH press to perform isothermal forming limit tests, as shown in Fig. 3.
The geometry of the test-pieces, as shown in Fig. 4(a), was that of a circular blank with a central parallel shaft. Different widths of the parallel shaft (W) were used to obtain different loading paths, as shown by the points on the curves in the schematic FLC in Fig. 4(b). A regular grid pattern was electrolytically etched onto the surface of each specimen in order to measure the surface strains at the end of the forming process. Table 4 lists the widths of the parallel shafts of the blanks required to achieve the various loading paths. In Table 4, W is the width of the specimen (Fig. 4(a)), and D the diameter of specimen.

**Table 4** Widths of the test-pieces

<table>
<thead>
<tr>
<th>Geometry No.</th>
<th>Test Type</th>
<th>W (mm)</th>
<th>W/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Uniaxial tension</td>
<td>12</td>
<td>0.15</td>
</tr>
<tr>
<td>2</td>
<td>Plane strain</td>
<td>40</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>Biaxial strain</td>
<td>80</td>
<td>1</td>
</tr>
</tbody>
</table>
In order to conduct the forming limit tests, the furnace was first heated to the target temperature before the specimen was loaded in the test tool. The specimen temperature was then monitored independently before triggering the forming process. The test temperatures and forming speeds used are shown in Table 5.

**Table 5** Test parameters for the forming limit tests

<table>
<thead>
<tr>
<th>Forming speed (mm/s)</th>
<th>300°C</th>
<th>400°C</th>
<th>450°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td></td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td></td>
<td></td>
<td>√</td>
</tr>
</tbody>
</table>

The specimens were deformed until the point at which necking was observed. The surface strains were then measured using a strain visualization technique in the software ARGUS, and post-processed to determine the limit strains according to the procedure outlined in the standards [3].
3.3 HFQ™ forming of a wing stiffener

The developed viscoplastic-Hosford-MK model was validated using a HFQ stamped AA2060 wing stiffener component. The test-pieces, with dimensions of 200×65×2mm, were formed on a 250kN ESH hydraulic press; the forming test set up is shown in Fig. 5. In the forming process, the specimens were firstly heated up to a soaking temperature of 480°C in the furnace, which was below the solution heat treatment temperature of the Al-Cu based alloy (approximately 500°C - 540°C) [1] to avoid the melting of low melting point phases [58]. After that, the specimen was then transferred to the forming tool within 10 seconds, and formed at a temperature of approximately 450°C and a forming speed of 250 mm/s. A number of forming trials at increasing punch strokes and component depths were conducted, until necking started to occur in the central features of the wing stiffener. The strain distributions of the formed part were then analysed using ARGUS.

![Fig. 5 Forming test set up](image-url)
4. Results and discussion

The following section discusses two significant aspects associated with the development of the prediction model. The first aspect concerns the calibration of the viscoplastic-Hosford-MK model with the results obtained from the above mentioned uniaxial tensile and isothermal forming limit tests. The second aspect concerns the evaluation of the effectiveness of the unified viscoplastic-Hosford-MK model by taking into consideration intrinsic features of hot stamping, namely the strain rate, loading path and temperature variations.

4.1 Validation of unified Viscoplastic-Hosford-MK model

The viscoplastic-Hosford-MK model was validated using the experimental results from the uniaxial tensile tests and forming limit tests. The calibration process consisted of two steps. The first step was to calibrate the viscoplastic equations with the flow stress data obtained at different temperatures and strain rates. The second step was to calibrate the viscoplastic-Hosford-MK model with experimentally obtained FLDs for different temperatures and forming speeds.

4.1.1 Prediction of flow stress at different temperatures and strain rates

The calibration of the material model was performed by determining the material model constants from the uniaxial tensile test data. The flow stress curves of AA2060 at different strain rates and temperatures are shown in Fig. 6(a) and Fig. 6(b). The symbols in the figures represent the experimental flow stress at strain rates ranging from 0.2 to 13s\(^{-1}\) and temperatures ranging from 400°C to 520°C, whereas the solid curves show the predictions of flow stress using the viscoplastic material model with the derived constants shown in Table 6.

In Fig. 6 (b), a remarkable ductility drop on AA2060 was obtained when the temperature was above 470 °C due to the presence of low melting phases. A detailed explanation of this phenomena on AA2xxx is provided in the papers [58,59].

| Table 6: Material parameters for AA2060 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(K_0\) (MPa)   | \(Q_k\) (J/mol) | \(k_0\) (MPa)   | \(Q_k\) (J/mol) | \(B_0\) (MPa)   | \(Q_B\) (J/mol) | \(C_0\)         | \(n_2\)         | \(Q_C\) (J/mol) | \(E_0\) (MPa)   | \(Q_A\) (J/mol) |
| 0.510           | 29231.256       | 1.09E-10        | 98722.800       | 87.663          | 2892.790        | 31.171          | 3               | -2739.973       | 9917.930        | 9591.400        |
|                 |                 |                 |                 | 0.0304          | 1017.950        | 0.748           |                 | 11644.803       |                 |                 |
To assess the effectiveness of the viscoplastic material model, experimental results from the strain rate and temperature variation tests are presented as symbols in Fig. 6(c) and Fig. 6(d) respectively, and compared with the model predictions under such conditions. In the strain rate variation tests, a sudden drop of strain rate was applied during the uniaxial tensile test. At a constant temperature of 470°C, the strain rate was reduced from 2s\(^{-1}\) to 0.2s\(^{-1}\) within 0.3 seconds, and the flow stress suddenly dropped due to the reduction of strain rate hardening. In the temperature variation tests, a constant strain rate of 0.02s\(^{-1}\) was applied with three sharp reductions in temperature introduced during the uniaxial tensile test, which resulted in a corresponding sudden increases in the flow stress. This was mainly due to the reduction in the mobility of atoms and dislocations caused by the reduction of temperature. The solid curves of Fig. 6(c) and (d) show that the developed viscoplastic material model was able to accurately predict the flow stress under complex loading conditions of changing strain...
rate and temperature. A good agreement was achieved between the predicted and experimental results, with errors of less than 10%.

**Fig. 7** Comparison of the predicted (solid curves) and experimental forming limit curves (symbols) for AA2060 with different forming speeds at a constant temperature of 400°C.

**Fig. 8** Comparison of the predicted (solid curves) and experimental forming limit curves (symbols) for AA2060 at different temperatures with a constant forming speed of 250mm/s.

### 4.1.2 Prediction of forming limit diagrams

Fig. 7 and 8 show the forming limit diagrams (FLDs) of AA2060, as a function of forming temperature (ranging from 300 to 450°C) and speed (ranging from 75 to 400mm/s). The
symbols in both figures represent the experimental forming limit curves (FLCs) whereas the solid lines represent the FLD prediction results from the viscoplastic-Hosford-MK model. Based on the forming limit test, it was found that the formability of AA2060 increased with temperature, and decreased with forming speed. Small deviations between the experimental and prediction results can be found at 400°C and 450°C at 250mm/s under equi-biaxial loading. This was mainly due to the uncertainty in the level of localized necking detected during the strain measurement at these conditions. This uncertainty was the result of the degradation of the surface quality of the deformed specimen at high temperatures where larger strains were attained.

The viscoplastic-Hosford-MK model was calibrated using the experimental FLDs from Fig. 7 and 8. The predicted solid curves in the figures agreed well with the experimental FLDs. The dashed line in Fig. 7 represents the prediction of the FLD when the forming speed was increased from 250mm/s to 400mm/s at t=0.4s at a forming temperature of 400°C/s. The dashed line in Fig. 8 represents the prediction of the FLD with a quenching rate of 200°C/s at a forming speed of 250mm/s.
4.2 Validation of the model responses to strain rate changes, quenching rates and loading path changes

In this section, the responses of the developed model to complex loading histories are demonstrated and its intrinsic mechanisms revealed by analysing the evolution of the value \(\frac{d\varepsilon_{3b}}{d\varepsilon_{3a}}\) against time. In the viscoplastic-Hosford-MK model, for a given constant temperature, strain rate and loading path, the development of necking is indicated by a sharp increase in \(\frac{d\varepsilon_{3b}}{d\varepsilon_{3a}}\), as shown in Fig. 9, at \(t=0.55\)s. This occurs due to more plastic straining occurring in Zone b than in Zone a as deformation progresses [29]; as the heterogeneous plastic flow develops further and becomes more severe, this eventually causes the onset of necking.

![Graph showing the development of necking](image)

**Fig. 9** The development of a neck (\(d\varepsilon_{3b}/d\varepsilon_{3a}\)) against time at a constant temperature of 400°C, strain rate of 3s\(^{-1}\) and linear loading path of \(\beta=-0.5\).

To study the model’s responses to strain rate, quenching rate and loading path variations, a mathematical decomposition was conducted (as shown in Appendix A, Eqs. (26) to (50)) to distinguish between the effects of the individual terms in the model on the development of necking. Fig. 10 clearly shows that the increment work per volume ratio between Zones a and b (‘X’) plays a significant role on the development of necking. Such an assumption was similarly made by Yoshida and Khan [35–37], who analysed the effective stresses and effective strain increments during deformation. The principle of increment work per unit volume [37,38] or plastic work per unit volume [36] was emphasised by considering anisotropic material responses under non-proportional loading [36], various strain rates and
temperatures [37,38]. Based on their pioneering work, a detailed mathematical analysis was conducted, associated with the increment work per unit volume ratio and accounting for the effects of strain rate changes, quenching rates and loading path changes on formability in a hot stamping process.

From the numerical study on strain localisation, the development of necking \( \frac{d\varepsilon_{3b}}{d\varepsilon_{3a}} \) can be expressed by Eq. (19).

\[
\frac{d\varepsilon_{3b}}{d\varepsilon_{3a}} = \left( \frac{-R_2 - R_1 \alpha_i^{1-1}}{(-R_2 - R_1 \alpha_i^{1-1})} \right) \left( \frac{R_2 + \alpha_i^{1} R_1 + (1-\alpha_i) R \alpha \bar{R}_1}{R_2 + \alpha_i^{1} R_1 + (1-\alpha_i) R \alpha \bar{R}_1} \right) \cdot f_o \exp(\varepsilon_{3b} - \varepsilon_{3a}) \cdot \frac{\sigma_{ad} d\varepsilon_{3b}}{\sigma_{ab} d\varepsilon_{3a}}
\]

(19)

where \( \alpha \) is the ratio between the minor stress and major stress.

Let:

\[
X = \frac{\sigma_{ad} d\varepsilon_{3b}}{\sigma_{ab} d\varepsilon_{3a}}
\]

(20)

\[
Y = f_o \exp(\varepsilon_{3b} - \varepsilon_{3a})
\]

(21)

\[
Z = \left( \frac{-R_2 - R_1 \alpha_i^{1-1}}{(-R_2 - R_1 \alpha_i^{1-1})} \right) \left( \frac{R_2 + \alpha_i^{1} R_1 + (1-\alpha_i) R \alpha \bar{R}_1}{R_2 + \alpha_i^{1} R_1 + (1-\alpha_i) R \alpha \bar{R}_1} \right)
\]

(22)

where \( \sigma_a \) and \( \sigma_b \) are the equivalent stresses, \( d\varepsilon_{3a} \) and \( d\varepsilon_{3b} \) are the equivalent strain increments. These parameters all depend on the stress ratio (\( \alpha \)), temperature (\( T \)), strain ratio (\( \beta \)) and strain rate (\( \dot{\varepsilon} \)).
Fig. 10 The calculated expression terms “X”, “Y”, “Z” and \( d\varepsilon_{3b} \)/\( d\varepsilon_{3a} \) against time using the viscoplastic-Hosford-MK model at a strain rate of 3/s, a temperature of 400°C, and the loading path \( \beta = 0.5 \).

From the numerical study on Eq. (19), the effect of \( Y \) and \( Z \) on \( d\varepsilon_{3b} \)/\( d\varepsilon_{3a} \) can be considered to be negligible, and \( f_0 \) is the constant \((0.9967)\), thus

\[
\frac{d\varepsilon_{3b}}{d\varepsilon_{3a}} \propto \frac{\bar{\sigma}_b d\bar{e}_b}{\bar{\sigma}_a d\bar{e}_a}
\]

(23)

Utilising the expression for the incremental work per unit volume ratio:

Zone a: \( \dot{W}_a = \bar{\sigma}_a \cdot d\bar{e}_a \) (24)

Zone b: \( \dot{W}_b = \bar{\sigma}_b \cdot d\bar{e}_b \)

\[
\frac{d\varepsilon_{3b}}{d\varepsilon_{3a}} \propto \frac{\dot{W}_b}{\dot{W}_a} = \frac{\bar{\sigma}_b(\alpha, T, \varepsilon_b) d\bar{e}_b(\alpha, T, \varepsilon_b)}{\bar{\sigma}_a(\alpha, T, \varepsilon_a) d\bar{e}_a(\alpha, T, \varepsilon_a)}
\]

(25)

Thus, \( \dot{W}_b/\dot{W}_a \) can be regarded as a representative factor to evaluate the strain localisation under different strain rates, quenching rates and loading paths.

In the viscoplastic-Hosford-MK model, the failure criterion was defined as a critical value of the thickness strain increment ratio between Zone b and Zone a. This criterion is an overall response to the evolutions of stress and strain states in all directions in the material with changing temperature, strain rate and loading path. In Fig. 11, the equivalent stress and strain increments are plotted, and it is shown that initially \( d\bar{e}_b \) and \( \bar{\sigma}_b \) are almost equivalent to \( d\bar{e}_a \) and \( \bar{\sigma}_a \), respectively. Eventually, compared to Zone a, \( d\bar{e}_b \) and \( \bar{\sigma}_b \) reach such large values that localisation and failure occurs.

(a) \hspace{4cm} (b)
Calculations of (a) strain increment \(d\varepsilon\) and (b) effective stress \(\bar{\sigma}\) in Zone a and Zone b using the viscoplastic-Hosford-MK model under the same conditions as in Fig. 9.

4.2.1 Effect of strain rates on the development of a neck

For the following set of model constants: imperfection factor \(f_0 = 0.9967\), temperature \((400\, ^\circ C)\) and loading path \(\beta = -0.5\) applied to the viscoplastic-Hosford-MK model, the various strain rate histories shown in Fig. 12(a) were applied to the developed model. Fig. 12(b) shows the model response to various strain rates, and in general, it could be deduced that the development of a neck is strain-rate dependent: as the strain rate increases, necking development is accelerated. Namely, the formability of AA2060 would decrease when forming takes place at higher rates. In a forming process, higher forming rates would normally lead to higher overall strain rates, which makes the material less formable. The reduced formability at higher strain rates would compete with the increased drawability at higher forming speeds typically achieved in forming processes. As indicated by Eqs. (19) to (25), the development of necking represents the overall responses of a blank material to external loadings. In fact, the complex response of the blank material to strain rates might be more explicitly revealed by the incremental work per unit volume ratio, i.e. the joint effects of the strain increment (Fig. 12 c) and stress (Fig. 12d) evolution. Fig. 12(c) shows the evolution of strain increments with loading. The dashed lines show the strain increments in Zone a, loaded at constant strain rates of 3, 5 or 7/s. In Zone b, the strain increments were rather stable at the initial stages and their magnitudes were only slightly higher than those in Zone a, due to the thickness differences induced by the initial defects. The strain increments grow dramatically when necking starts to occur, due to the fact that a more severe stress concentration occurs when excessive plastic strain induced thinning takes place in Zone b. Therefore, the material in Zone b is deformed at a higher rate, as indicated by Fig. 12(c).
The equivalent stress values in Zone b (the dashed lines) are also fairly close to those found in Zone a at the initial stages of loading and increase gradually, which reflects the strain hardening behaviour of the blank material, as indicated by Fig. 12(d). When the blank material is continuously loaded with different strain rates, the dominant factor in the development of the neck is the strain rate hardening. The increase of strain rate not only leads to the acceleration of the plastic strain difference between Zone a and Zone b, but also results in a more pronounced difference in their strength. Consequently, a greater ratio in the
incremental work per unit volume (Eq. (25)) was obtained, resulting in premature fractures at lower strain levels, as shown in Fig. 12(b).

When the blank material is subject to an abrupt strain rate increase, e.g. from 3 to 5 $s^{-1}$, at 0.4s, as shown in Fig. 12(a), the model is able to immediately respond to the strain rate increase and the necking development is accelerated correspondingly, as indicated by Fig. 12(b). As a result of the abrupt strain rate increase applied in Zone a, the strain increment in Zone b showed a simultaneous steep increase (Fig. 12c). Since the strain rate change is introduced to Zone a at a late stage, when a neck has started to develop, an obvious deviation in the strain increment (Zone b) can be observed from Fig. 12(c), leading to the occurrence of fracture rather rapidly. The abrupt increase in strain rate brings step-changes in the equivalent stresses in both Zone a and Zone b, as shown in Fig. 12(d). The magnitude of the instant stress increase is determined by the strain rate hardening of the blank material: higher strain rate hardening leads to a greater stress increase as a response to the abrupt strain increase. During the necking development stage following the strain rate increase, Fig. 12(d) shows that there is a more rapid deviation of the equivalent stress in Zone b from that of Zone a, and subsequently necking occurs at a lower failure strain.

4.2.2 Effects of temperature and quenching rate changes on the development of necking

In a hot stamping process, the hot blank is formed and quenched simultaneously by water cooled forming tools. The effect of the quenching rate on the forming limit was therefore modelled. As shown in Fig. 13(a), the hot blank is formed and quenched from 400°C at a constant quenching rate of 30 or 150°C/s. The formability at isothermal conditions at contact temperatures of 300 and 400°C is also presented and analysed. For the following set of model constants: imperfection factor ($f_0=0.9967$), strain rate (3/s) and loading path ($\beta = -0.5$) applied to the viscoplastic-Hosford-MK model, as shown in Fig. 13(b), the development of necking is forming temperature and quenching-rate dependent; a temperature increase would enhance the formability of the hot blank, while a higher quenching rate would accelerate the development of necking. The predicted results suggest that, in light of formability, hot stamping performed at higher blank temperatures and lower quenching rates would be more favourable.

The material response to the quenching rate was revealed by analysing the incremental work per unit volume ratio, i.e. the corresponding joint effects of the strain increment (Fig.13c) and effective stress (Fig.13d). A constant strain rate (3s$^{-1}$) was assigned in Zone a,
while the strain increment in Zone b was fairly stable in the initial stages, until necking started to occur. The different increasing gradients of the strain increment were mainly induced from the corresponding changes in the flow stress, as shown in Fig. 13(c) and (d). In the application of a quenching rate, the material response is reflected by the flow stress of the material. Temperatures in both Zones a and b drop more quickly, and results in greater strain hardening. Thus, strain localisation occurred earlier due to the excessive stress concentration induced from strain hardening at Zone b. The higher the quenching rate in the quenching process, the more the strain localisation would be accelerated.

When an abrupt quenching rate increase from 30°C/s to 150°C/s was applied, e.g. at 0.4s, as shown in Fig. 13(a), at a constant strain rate (3s⁻¹), the model was able to immediately respond to the increase, and the necking development was accelerated correspondingly, as indicated by Fig. 13(b). The increase in the quenching rate brings step changes in the equivalent stresses in both Zone a and Zone b, as shown in Fig. 13(d). The magnitude of the sharp stress increase is determined by the instant increase in the strain increment (Fig. 13(c)) and subsequent strain hardening of the blank material: higher strain hardening leads to greater stress localisation. During the necking development stage following the quenching rate increase, the deviation of the equivalent stress in Zone b from that of Zone a is accelerated by the increase in strain hardening, causing an earlier onset of necking.
Fig. 13 (a) Temperature profiles against time with different quenching rates (b) Predictions of developments of necks ($d\varepsilon$, $d\varepsilon_n$) with different quenching rates using a linear loading path ($\beta = -0.5$) and strain rate of $3s^{-1}$. The developments of (c) $d\Sigma$ and (d) $\bar{\sigma}$ against time ($T_i$ is the initial temperature, $Q$ is the quenching rate)
4.2.3 Effect of changes in loading path on necking development

To form the wing stiffener component with its complex geometrical features required the forming process to be performed in two stages within a single pressing operation. Each region of the part was deformed in either a linear or non-linear loading path. The effect of one-stage straining (linear loading path) and two-stage straining (non-linear loading path) on the forming limit was therefore demonstrated by plotting their corresponding FLCs. For the following set of model constants: imperfection factor $f_o=0.9967$, a temperature of 400 °C and an average strain rate of 3s$^{-1}$, applied to the viscoplastic-Hosford-MK model, FLCs with changes in loading paths were predicted and shown in Fig. 14.

![FLC Diagram](image)

Fig. 14 shows the predicted FLCs with different changes in loading path.

In Fig. 14, the black line represents the predicted FLC under a linear loading path. NL-SP are the predicted FLCs under non-linear loading paths, where the material was pre-stretched under one type of loading path up to a specified equivalent strain level, and then stretched under a different type of loading path. As the loading path changes from linear to non-linear, the new FLCs shift towards different directions as indicated by the blue, green, red, yellow and green lines, thus revealing the loading path dependence of the developed model. To investigate the effect of non-linear loading paths on the formability of sheet metal, the development of the neck against time was analyzed. The loading paths corresponding to the forming limit strains in points A, B, C, D, and E in Fig. 14 are listed in Table 7.
Table 7 List of different loading path applied into developed model

<table>
<thead>
<tr>
<th>Loading path (LP)</th>
<th>Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP1 $\beta = 1$, equi-biaxial tension</td>
<td>A</td>
</tr>
<tr>
<td>LP2 $\beta = -0.5$, uniaxial tension</td>
<td>B</td>
</tr>
<tr>
<td>LP3 $\beta = -0.5$ to $\beta = 1$, uniaxial tension for $\varepsilon = 1.0$ followed by equi-biaxial stretching</td>
<td>C</td>
</tr>
<tr>
<td>LP4 $\beta = -0.5$ to $\beta = 1$, uniaxial tension for $\varepsilon = 1.5$ followed by equi-biaxial stretching</td>
<td>D</td>
</tr>
<tr>
<td>LP5 $\beta = -0.5$ to $\beta = -0.05$, uniaxial tension for $\varepsilon = 1.0$ followed by plane strain stretching</td>
<td>E</td>
</tr>
</tbody>
</table>

Fig. 15 Prediction of developments of necks ($\frac{d\varepsilon_{3y}}{d\varepsilon_{3a}}$) under different changes in loading path at an average strain rate of 3/s and constant temperature of 400°C

Fig. 15 shows the model responses to changes in loading path at the conditions listed in Table 7. In comparison to the failure strains under linear loading paths (LP1 and LP2), the failure strains under the non-linear loading paths LP3, LP4 and LP5, which each have different loading path conditions, were higher than LP1 (equi-biaxial tension), but lower than LP2 (uniaxial tension). In Fig. 15, it was predicted that the type of loading path and the amount of first-stage straining applied were two major factors that could have great effects on the forming limit strains.
Effects of different types of loading path

LP1: $\beta = 1$, biaxial tension;
LP2: $\beta = -0.5$, uniaxial tension;
LP3: $\beta = -0.5$ to $\beta = 1$, uniaxial stretching ($\varepsilon = 1.0$) and followed by biaxial tension until failure;
LP5: $\beta = -0.5$ to $\beta = -0.05$, biaxial stretching ($\varepsilon = 1.0$) and followed by plane strain stretching until failure;

![Diagram showing different loading paths and their effects](image)

Fig. 16 (a) Prediction of developments of necks ($d\varepsilon_{\text{eq}}/d\varepsilon_{\text{eq}}$) with the same amount of equivalent strain but different loading path sequences at an average strain rate of 3/s and constant temperature of 400°C and (b) detailed view of Area 1.

The HFQ forming process for the wing stiffener component occurred in two stages, the details of which are described in the paper [56]. From the FE model analysis, it was deduced that in the first stage of the process where the material was drawn into the die cavity, the loading was linear and uniaxial. In the second stage where the central geometrical features were formed, the material in the side wall was mostly stretched under plane strain conditions, whereas the material in the corner region was mostly stretched under biaxial conditions. Thus, capturing the effect of the loading path change and the amount of first-stage straining applied was critical.

Fig. 16(a) shows the model responses to different types of loading paths during second stage straining. The development of a neck under LP2 shows a higher failure strain than that under LP3 and LP5. The prediction results suggest that the formability under LP2 (a linear loading path) is higher than that under LP3 (a non-linear loading path) and LP5 (also a non-linear loading path). It was also predicted that the formability under LP3 is higher than that under LP5. At $\varepsilon = 1.0$, the evolutions of the development of the neck under LP3 and LP5...
show instant step increases in the value of $\frac{d\varepsilon}{d\varepsilon_{\mathrm{eq}}}$, the details of which are shown in Fig. 16(b). The instant increase with different magnitudes induced by different types of loading path (biaxial & plane strain) can be explained by the incremental work per unit volume ratio, i.e. the corresponding joint effects of the strain increment (Fig. 17(a)) and effective stress (Fig. 17(b)).

In Fig. 17(a), it is shown that the same amount of uniaxial straining was applied until $\varepsilon$ =1.0 for LP2, 3 and 5, and hence their strain increment evolutions up to this point were identical. When the loading path switched from the initial loading path ($\beta$=-0.5) to the new loading path ($\beta$=-0.05 or $\beta$=1), the strain increments under LP3 and LP5 begin to increase faster than that under LP2. This is due to the resulting instant changes in the equivalent strain rate, which would induce additional work hardening of the material, as shown by the subsequent increase in the effective stress evolutions. Although the initial increase in the strain increment and effective stress (Fig. 17(b)) is larger for LP3 at $\varepsilon$=1.0, failure occurs earlier in LP5. This is because the resulting difference in effective stress and strain increment between Zone a and Zone b for LP3 when the loading changes to biaxial conditions is much smaller than that for LP5 when the loading changes to plane strain conditions. Consequently, the incremental work per unit volume ratio for LP5 was larger than that in LP3. Strain localization under plan strain conditions would therefore occur more rapidly, leading to a faster evolution of the equivalent strain increment and effective stress, and hence failure.

**Effects of the amount of strain in the first-stage straining**

LP1: $\beta$=1, biaxial tension;
LP3: $\beta = -0.5$ to $\beta = 1$, uniaxial stretching ($\bar{\varepsilon} = 1.0$) and followed by biaxial tension until failure;
LP4: $\beta = -0.5$ to $\beta = 1$, uniaxial stretching ($\bar{\varepsilon} = 1.5$) and followed by biaxial tension until failure;

![Diagram](image1)

![Diagram](image2)

Fig. 18 (a) Prediction of developments of necks ($d\varepsilon_{\text{eq}}/d\varepsilon_{\text{eq}}$) with different amounts of equivalent strain in the first stage but the same loading path sequences at an average strain rate of 3/s and constant temperature of 400°C and (b) detailed view of Area 2

In Fig. 18(a), the development of a neck under LP4 shows a relatively higher failure strain than that under LP3. The prediction results suggest that the formability under LP4 is higher than that under LP3 (a non-linear loading path) and LP1 (a linear loading path, with $\beta = 1$). In the evolution of the development of the necks under LP1, LP3 and LP4, the onset of necking occurred during biaxial stretching. The differences in their corresponding formability were mainly due to the amount of first-stage uniaxial straining. The formability was enhanced when the amount of first-stage uniaxial straining was increased. Without any uniaxial straining, such as under LP1 (linear loading path), the formability was the lowest. Under the loading paths LP3 and LP4, the step increases in the values of $d\varepsilon_{\text{eq}}/d\varepsilon_{\text{eq}}$ were at $\bar{\varepsilon} = 1.0$ and $\bar{\varepsilon} = 1.5$, the details of which can be seen in Fig. 18(b). In Fig. 18(b), when the loading path changed from uniaxial to biaxial at $\bar{\varepsilon} = 1.0$ and at $\bar{\varepsilon} = 1.5$ for LP3 and LP4 respectively, the gradient of the curves under LP3 tended to be increasingly higher than that under LP4, which accelerates the occurrence of strain localization; hence the failure strain under LP3 was lower.
The material response to different amounts of first-stage straining was analysed in terms of the incremental work per unit volume ratio, i.e., the corresponding joint effects of strain increment (Fig. 19(a)) and effective stress (Fig. 19(b)). In Fig. 19(a), the strain increment evolution in Zone a and Zone b under LP1 was initially higher than that under LP3 and LP4 due to the relatively higher equivalent strain rate under LP1. Under the linear loading path LP1, the differences in strain increments between Zone a and Zone b increase at a higher rate than that of the non-linear paths LP3 and LP4. With the increasing level of uniaxial pre-straining from LP3 to LP4, the strain localisation was delayed. In Fig. 19(b), the stress evolution in Zone b under LP1, LP3 and LP4 shows different levels of strain hardening induced by the different strain levels. As the amount of first-stage uniaxial straining increases, the additional straining hardening induced from biaxial straining is delayed, resulting in the retardation of strain localisation.

Fig. 19 Evolution curves of (a) $d\varepsilon$ and (b) $\bar{\sigma}$ in Zone a and Zone b under the same conditions as in Fig. 18.
4.3 Application of the forming limit prediction model in the forming of the AA2060 complex-shaped component

Having verified the capabilities of the model for the range of loading conditions discussed previously, the model was experimentally validated using the results obtained from the forming of a practical component. The following section discusses the results obtained by utilizing the viscoplastic-Hosford-MK with a numerical simulation of the process and the comparisons made with a formed wing stiffener component.

4.3.1 Finite Element model verification

The finite element (FE) software PAM-STAMP was utilised to simulate the stamping of the sheet metal AA2060 wing stiffener component. The detailed FE model set up was fully described in the paper [56,60]. The model was verified by comparing the material thinning results from the FE simulation with the forming tests. In the experiments, the part was formed at a temperature of 450°C and a forming speed of 250mm/s. In order to study the forming limit of AA2060, the forming stroke was set to 22.3mm such that localised necking occurred in the formed part. The thinning distributions of the formed part from the FE simulation and the experiments are shown in Fig. 20.

![Thinning distributions](image)

*Fig. 20 Thinning distributions (a) measured from the experimental forming test using ARGUS software and (b) predicted from the PAM-STAMP FE simulation*
In Fig. 20, the numerical and the experimental results for the thinning distribution show that the largest deformation occurred at the corner region of the central features of the part (as shown in Appendix B), with a maximum thinning level of 0.68. Along section A-B and section C-D, the thinning values of the part were compared as can be seen in Fig. 21; a good agreement between the simulated and experimental thinning results was achieved, thus verifying the FE model.

![Comparison of experimental and prediction results in thinning](image)

**Fig. 21** Comparison of experimental (solid symbols) and prediction results in thinning (solid curve) through (a) section AB and (b) section CD

### 4.3.2 Forming limit prediction for the AA2060 wing stiffener component

The forming limit of the AA2060 sheet wing stiffener component was predicted using the developed forming limit prediction model (viscoplastic-Hosford-MK model) and the PAM-STAMP FE simulation, the full operation procedures were demonstrated in the paper [61]. In Fig. 22, the final formed part with localized necking and the corresponding prediction of the forming limit is shown. In the experiment, the localized necking occurred at the side wall region rather than the corner region of the central feature of the part. The same failure location was observed when forming the wing stiffener component from AA5754 and AA6082, as shown in Appendix C. Thus, the localized necking did not occur at the corner region as expected. However, the location of necking could be determined correctly by the developed model, which predicted that the forming limit was exceeded in the region as expected. Based on the simulation results in Fig. 23, the FE simulation results in the corner region (Region 2) and side wall region (Region 1) were analyzed by considering the thinning values and $d\varepsilon_{33}/d\varepsilon_{33}$ values.
Fig. 22 Formed component: (a) Observation of localised necking located at side wall and (b) Prediction of localised necking using the developed model

In Fig. 23, the elements selected for analysis in Regions 1 and 2 are highlighted (10 elements and 5 elements, respectively), and the thinning and corresponding $\frac{\Delta e_{ba}}{\Delta e_{3u}}$ values plotted for each as shown in Fig. 24.

Fig. 23 Analysis of selected elements from region 1 and region 2

Based on the results from Fig. 24, in the forming of the AA2060 wing stiffener component, the localised necking did not occur at the region of maximum thinning (element B3). The onset of localised necking is strongly affected by many factors, such as the quenching rates and forming speed and loading path variations that occur in hot stamping processes; using the
developed forming limit prediction model, the localised necking was predicted based on the combined effects of these factors, as shown in Fig. 25.

![Graph showing thinning and thinning ratio across elements A1-A10 and B1-B5]

**Fig. 24** Development of a neck and thinning for element: (a) A1-A10 and (b) B1-B5 of the FE simulation
In the predicted localised necking region and maximum thinning region, two elements (A6 and B3) were selected. The evolutions of the development of the neck, temperature, strain rate and loading path were all plotted in Fig. 25. In Fig. 25(a), the development of a neck curve ($d\varepsilon_{30}/d\varepsilon_{33}$) against time is shown. Along this curve for element A6, numerous spikes are observed at different times, which were the combined result of the simultaneous changes in the strain rate, temperature and loading path. For example, at $t=0.72$ in the hot stamping process for the wing stiffener component, the central parts of the component were formed, which involved changes in the strain rate and loading path. Meanwhile, the central part was quenched, causing a change in temperature. Regarding the loading path change at $t=0.72$ shown in Fig. 25(d), it was found that the loading on the side wall (A6) changes from a
uniaxial tensile mode to a plane strain mode when the central regions were started to be formed, whereas the loading on the corner region (B3) changes from a uniaxial tensile mode to a biaxial mode. In Fig. 25(b), at t=0.72s, the temperature changes from approximately 451 °C to 423 °C for both elements, corresponding to quenching rates of approximately 100 to 200 °C/s, because the hot central region of the blank makes contact with the cold central punches. Fig. 25(c) also shows that the strain rate also increased dramatically when the central region was formed for both elements. Based on the detailed analysis of the viscoplastic-Hosford-MK model, the poor predicted formability of element A6, where the failure point was observed, was due to a high strain rate, a high quenching rate to a low temperature, and a change in the loading path from a uniaxial mode to a plane strain mode.

Conclusion

The forming limit prediction for the hot stamping of an AA2060 wing stiffener component was successfully conducted using the developed viscoplastic-Hosford-MK model. The location of the failure regions in the wing stiffener component was accurately predicted, which agrees well with results obtained from experimental forming tests. It was concluded that the position of the onset of the neck might not coincide with the maximum thinning region. The onset of necking under complex loading conditions is mainly due to the combined influences of non-isothermal loading, strain rate and loading path changes, which are typical intrinsic features of hot stamping processes. Two significant features of the developed model were found:

(1) It was shown that accurate model responses could be obtained when forming parameters were varied, such as the forming temperature, strain rate and loading path. Particularly, the instantaneous change of strain rate, quenching rate and loading path could be captured.

(2) It was shown that the incremental work per unit volume ratio plays a dominant role in the prediction of necking in the developed model, and the temperature, strain rate and loading path all have great effects on the hardening behavior of the materials.
Acknowledgements

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Reference

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Appendix

A. Development of the viscoplastic-Hosford-MK model

The Hosford anisotropic yield function can be expressed as:

\[ R_2 \sigma_i^l + R_2 \sigma_2^l + R_2 (\sigma_1 - \sigma_2) = R_2 (R_1 + 1) \bar{\sigma}^l \]

According to the Hosford anisotropic yield function, the equivalent stress in Zones a and b were calculated as follows:

Zone a: \( \bar{\sigma}_a = \left\{ \frac{1}{R_2 (R_1 + 1)} \left[ R_2 \sigma_{1a}^l + R_1 \sigma_{2a}^l + R_1 \sigma_{1a} - \sigma_{2a} \right] \right\}^{\frac{1}{2}} \) (26)

Zone b: \( \bar{\sigma}_b = \left\{ \frac{1}{R_2 (R_1 + 1)} \left[ R_2 \sigma_{1b}^l + R_1 \sigma_{2b}^l + R_1 \sigma_{1b} - \sigma_{2b} \right] \right\}^{\frac{1}{2}} \)

In thin sheet metals, it is assumed that \( \sigma_3 = 0 \). Let \( \sigma = \alpha \sigma_1 \).

Zone a: \( \bar{\sigma}_a = \sigma_{1a} \left\{ \frac{1}{R_2 (R_1 + 1)} \left[ R_2 + \alpha_1' R_1 (1 - \alpha_a) \right] \right\}^{\frac{1}{2}} \) (27)

Zone b: \( \bar{\sigma}_b = \sigma_{1b} \left\{ \frac{1}{R_2 (R_1 + 1)} \left[ R_2 + \alpha_2' R_1 (1 - \alpha_b) \right] \right\}^{\frac{1}{2}} \)

Let,

Zone a: \( \sigma_{1a} = \varphi_a \bar{\sigma}_a \)

Zone b: \( \sigma_{1b} = \varphi_b \bar{\sigma}_b \) (28)

Zone a: \( \varphi_a = \left\{ \frac{1}{R_2 (R_1 + 1)} \left[ R_2 + \alpha_a' R_1 (1 - \alpha_a) \right] \right\}^{\frac{1}{2}} \) (29)

Zone b: \( \varphi_b = \left\{ \frac{1}{R_2 (R_1 + 1)} \left[ R_2 + \alpha_b' R_1 (1 - \alpha_b) \right] \right\}^{\frac{1}{2}} \)

where \( \varphi \) is the ratio of the major stress to the equivalent stress [62].
Let $d\varepsilon_2 = \beta d\varepsilon_1$,

$$d\varepsilon_3 = -(1 + \beta)d\varepsilon_1$$ \hspace{1cm} (30)

According to the associated flow rule,

$$d\varepsilon_{ij} = d\lambda \frac{\partial f}{\partial \sigma_{ij}} = d\lambda \frac{\partial \bar{\sigma}}{\partial \sigma_{ij}} \rightarrow d\lambda = \frac{d\varepsilon_{ij}}{\partial \sigma_{ij}}$$ \hspace{1cm} (31)

$$\text{Zone a:} \quad \frac{d\varepsilon_{aA}}{R_2 \sigma_{a1}^{-1} + R_2 R_1 (\sigma_{a1} - \sigma_{a2})^{-1} + R_2 (\sigma_{a1}^{-1} - \sigma_{a2})} = \frac{d\varepsilon_{aB}}{R_2 \sigma_{a2}^{-1} - R_2 R_1 (\sigma_{a1} - \sigma_{a2})^{-1} + R_2 (\sigma_{a1}^{-1} - \sigma_{a2})^{-1}} = \frac{d\bar{\varepsilon}_a}{R_2 (R_1 + \alpha) \bar{\sigma}_a^{-1}}$$ \hspace{1cm} (32)

$$\text{Zone b:} \quad \frac{d\varepsilon_{bA}}{R_2 \sigma_{b1}^{-1} + R_2 R_1 (\sigma_{b1} - \sigma_{b2})^{-1} + R_2 (\sigma_{b1}^{-1} - \sigma_{b2})} = \frac{d\varepsilon_{bB}}{R_2 \sigma_{b2}^{-1} - R_2 R_1 (\sigma_{b1} - \sigma_{b2})^{-1} + R_2 (\sigma_{b1}^{-1} - \sigma_{b2})^{-1}} = \frac{d\bar{\varepsilon}_b}{R_2 (R_1 + \alpha) \bar{\sigma}_b^{-1}}$$ \hspace{1cm} (33)

Thus,

$$d\varepsilon_3 = \frac{d\bar{\varepsilon}}{-R_2 - R_2 \alpha^{-1}} = \frac{1}{R_2 (R_1 + 1)} \left[ R_2 + \alpha R_1 + (1 - \alpha)^\dagger R_1 R_2 \right]$$ \hspace{1cm} (34)

$$\text{Zone a:} \quad d\varepsilon_{aA} = \frac{d\bar{\varepsilon}_a}{-R_2 - R_2 \alpha_a^{-1}}$$ \hspace{1cm} (35)

$$\text{Zone b:} \quad d\varepsilon_{bA} = \frac{d\bar{\varepsilon}_b}{-R_2 - R_2 \alpha_b^{-1}}$$ \hspace{1cm} (35)
According to Eq. (35), $d\varepsilon_{3a}$ and $d\varepsilon_{3b}$ can be expressed as:

$$d\varepsilon_{3a} = \frac{-R_z - R_i \alpha_a^{i-1}}{R_z(R_i + 1)^{i-1}} d\bar{\varepsilon}_a$$

(36)

$$d\varepsilon_{3b} = \frac{-R_z - R_i \alpha_b^{i-1}}{R_z(R_i + 1)^{i-1}} d\bar{\varepsilon}_b$$

(37)

Eq. (37) divided by Eq. (36) gives

$$\frac{d\varepsilon_{3b}}{d\varepsilon_{3a}} = \frac{(-R_z - R_i \alpha_a^{i-1})R_x(R_i + 1)^{i-1}}{(-R_z - R_i \alpha_b^{i-1})R_y(R_i + 1)^{i-1}} \frac{\left[ \frac{1}{R_z(R_i + 1)} \left[ R_z + \alpha_a^i R_i + (1-\alpha_a)^i R_i R_z \right] \right]^{\frac{1}{i-1}}}{\left[ \frac{1}{R_z(R_i + 1)} \left[ R_z + \alpha_b^i R_i + (1-\alpha_b)^i R_i R_z \right] \right]^{\frac{1}{i-1}}} d\bar{\varepsilon}_b$$

(38)

(39)

Substitute $\varphi_a = \left[ \frac{1}{R_z(R_i + 1)} \left[ R_z + \alpha_a^i R_i + (1-\alpha_a)^i R_i R_z \right] \right]^{\frac{1}{i-1}}$ and $\varphi_b = \left[ \frac{1}{R_z(R_i + 1)} \left[ R_z + \alpha_b^i R_i + (1-\alpha_b)^i R_i R_z \right] \right]^{\frac{1}{i-1}}$ into Eq. (39),

$$\frac{d\varepsilon_{3b}}{d\varepsilon_{3a}} = \frac{(-R_z - R_i \alpha_a^{i-1})}{(-R_z - R_i \alpha_b^{i-1})} \left[ R_z + \alpha_a^i R_i + (1-\alpha_a)^i R_i R_z \right] \varphi_a d\bar{\varepsilon}_b$$

(40)
In the M-K model,

The initial imperfect factor is: \( f_0 = \frac{t_b}{t_a} \)

The imperfect factor can be expressed as:

\[
f = \frac{t_b}{t_a}
\] (41)

where \( t_a \) and \( t_b \) is the thickness in Zone a and Zone b.

\[
e_{3b} = \ln \left( \frac{t_b}{t_a} \right) > t_b = t_0 \exp(e_{3b})
\] (42)

\[
e_{3a} = \ln \left( \frac{t_a}{t_b} \right) > t_a = t_0 \exp(e_{3b})
\] (43)

\[
f = \frac{t_b}{t_a} = f_0 \exp(e_{3b} - e_{3a})
\] (44)

The assumption of the M-K model is that:

\[
e_{2b} = e_{2a}
\] (45)

\[
\sigma_{la} = f \sigma_{lb}
\] (46)

Thus, based on Eq. (28) and Eq. (46),

\[
\sigma_a \varphi_a = f \sigma_b \varphi_b
\] (47)

\[
\varphi_a \sigma_a = f_0 \exp(e_{3b} - e_{3a}) \frac{\sigma_b}{\sigma_a} \varphi_b
\] (48)

\[
\frac{\varphi_a}{\varphi_b} = f_0 \exp(e_{3b} - e_{3a}) \frac{\sigma_b}{\sigma_a}
\] (49)

Finally, substitute Eq. (49) into Eq. (39),

\[
\frac{d e_{3b}}{d e_{3a}} = \frac{-R_2 - R_3 \alpha_a^{-1}}{-R_2 - R_3 \alpha_b^{-1}} \left[ \frac{R_a + \alpha_a^{-1} R_1 + (1 - \alpha_a)^{1/2} R_2}{R_a + \alpha_a^{-1} R_1 + (1 - \alpha_a)^{1/2} R_2} \right] \cdot f_0 \exp(e_{3b} - e_{3a}) \cdot \frac{\sigma_a d \sigma_b}{\sigma_a d e_a}
\] (50)
B. Photo of formed wing stiffener component

Fig. 26 shows the demonstration of central features in formed wing stiffener component
C. Forming test on AA2060, AA6082 & AA5754

The same failure locations (failure in the side wall) were observed on different aluminium alloys when forming the wing stiffener component under hot stamping conditions.

![AA5754, AA2060, AA6082 components](image)

Fig. 25 shows the failure positions of the formed wing stiffener component on AA5754 (Left), AA2060 (middle) and AA6082 (Right) under hot stamping conditions with different forming strokes.

**Table 8** Forming conditions for forming wing stiffener component on AA5754, AA2060 & AA6082

<table>
<thead>
<tr>
<th>Material</th>
<th>Forming Temperature</th>
<th>Forming Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA5754</td>
<td>450 °C</td>
<td>250mm/s</td>
</tr>
<tr>
<td>AA2060</td>
<td>450 °C</td>
<td>250mm/s</td>
</tr>
<tr>
<td>AA6082</td>
<td>450 °C</td>
<td>250mm/s</td>
</tr>
</tbody>
</table>
Graphical abstract

An advanced model concerns the forming limit of material under the combined effects of changes in strain rate, temperature and loading path. Incremental work per unit volume ratio \( \left( \frac{\partial W}{\partial vol} \right) \) play an important role for the development of necking. Numerical results in good agreements with experimental results. The onset of necking did not necessarily occur at the maximum thinning region.