

1 **Flexural Behaviour of Headed Bar Connections between Precast Concrete Panels**

2 Jean Paul Vella^a, Robert L. Vollum^a, Andrew Jackson^b

3 ^aDepartment of Civil and Environmental Engineering, Imperial College London, London, UK

4 ^bEngineering Excellence Group, Laing O'Rourke, Dartford, UK

5

6 **Abstract**

7 The use of headed bars in joints between precast concrete elements allows continuity of reinforcement to
8 be achieved over very short splice lengths. The paper describes a series of flexural tests carried out on
9 specimens consisting of pairs of precast elements connected by overlapping headed bars of 25 mm
10 diameter. The headed bars overlapped by 100 mm within a 200 mm wide insitu concrete joint in which
11 transverse bars and vertical shear studs were installed to provide confinement. This type of joint
12 facilitates the construction of continuously reinforced slabs from precast elements thereby enabling
13 significant reductions in overall construction time and improvements in construction quality due to off-
14 site fabrication. The tests investigated the influence on joint strength, ductility and crack width of
15 concrete strength, out-of-plane offset of precast planks and confining shear studs. Ductile failure with
16 yield of 25 mm diameter high strength headed bars was achieved with joint concrete having a cylinder
17 compressive strength of 39 MPa. A nonlinear finite element model is presented, which gives good
18 predictions of joint strength as well as providing insight into joint behaviour.

19

20 **Keywords:** Precast concrete, Headed reinforcement, Lap length, Strut-and-tie, Nonlinear finite element
21 analysis.

22

23

24 Notation

25	ε_2	NLFEA reinforcement strain at end of yield/start of strain hardening
26	ε_3	NLFEA reinforcement ultimate strain
27	ε_c	Strain in the compression zone concrete
28	ε_{c1}	Strain at peak compressive stress
29	ε_s	Shear stud measured strain
30	η	Concrete strain ratio
31	σ_2	NLFEA reinforcement stress at end of yield/start of strain hardening
32	σ_3	NLFEA reinforcement ultimate stress
33	σ_c	Concrete compressive stress
34	σ_y	NLFEA steel yield stress
35	ν	NLFEA steel Poisson's ratio
36	\varnothing_b	Reinforcement diameter
37	\varnothing_{tr}	Transverse bar diameter
38	E_{cm}	Concrete elastic modulus
39	E_s	Reinforcement elastic modulus
40	H	NLFEA steel hardening modulus
41	L_{hb}	Headed bar lap length between bearing faces of heads
42	M	Maximum moment at joint-precast interface
43	M_{hb}	Bending moment at bar head
44	M_{fl}	Maximum applied bending moment at joint-precast interface
45	$M_{p,hb}$	Longitudinal headed bar plastic moment resistance
46	$M_{p,tr}$	Transverse bar plastic moment resistance
47	M_{test}	Maximum measured or equivalent calculated bending moment achieved in test
48	M_{tr}	Bending moment in transverse bar
49	N_{hb}	Measured longitudinal headed bar force
50	N_{2hb}	Longitudinal headed bar force on the two bar side
51	$N_{3hb,centre}$	Central longitudinal headed bar force on the three bar side
52	$N_{3hb,edge}$	Edge longitudinal headed bar force on the three bar side
53	$N_{y,hb}$	Longitudinal headed bar yield load

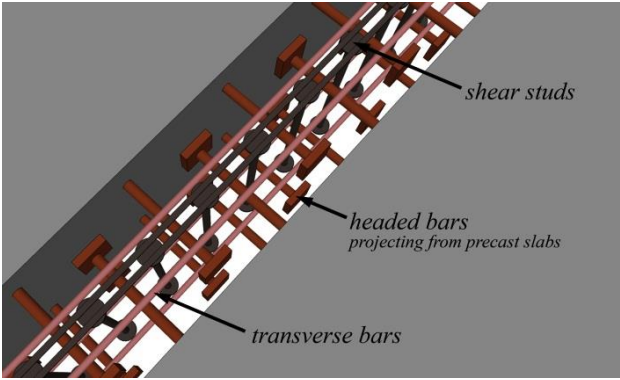
54	$N_{y,tr}$	Transverse bar yield load
55	N_{tr}	Force in transverse bar
56	N_s	Force in shear stud
57	P	Maximum flexural test load
58	P_{tens}	Maximum tensile test measured load
59	P_{fl}	Maximum flexural test measured load
60	S_{hb}	Spacing of headed bars with same orientation
61	S_F	Shear factor in NLFEA
62	d_g	Maximum aggregate size in NLFEA
63	f_c	NLFEA concrete cylinder compressive strength
64	f_{c0}	Onset of nonlinear behaviour in NLFEA
65	f_t	NLFEA concrete tensile strength
66	$f_{c,cyl,j}$	Measured joint concrete cylinder compressive strength
67	$f_{c,cyl,p}$	Measured precast concrete cylinder compressive strength
68	f_{cm}	Mean concrete cylinder compressive strength
69	$f_{ct,j}$	Measured joint concrete tensile strength
70	f_u	Reinforcement ultimate strength
71	f_y	Reinforcement yield strength
72	r_c	Compressive strength of cracked concrete factor in NLFEA
73	w_d	Plastic displacement in concrete softening law in NLFEA
74	x_2	Precast slab out-of-plane offset
75	x_t	Transverse bar offset from the centreline of the joint

76

77 **1. Introduction**

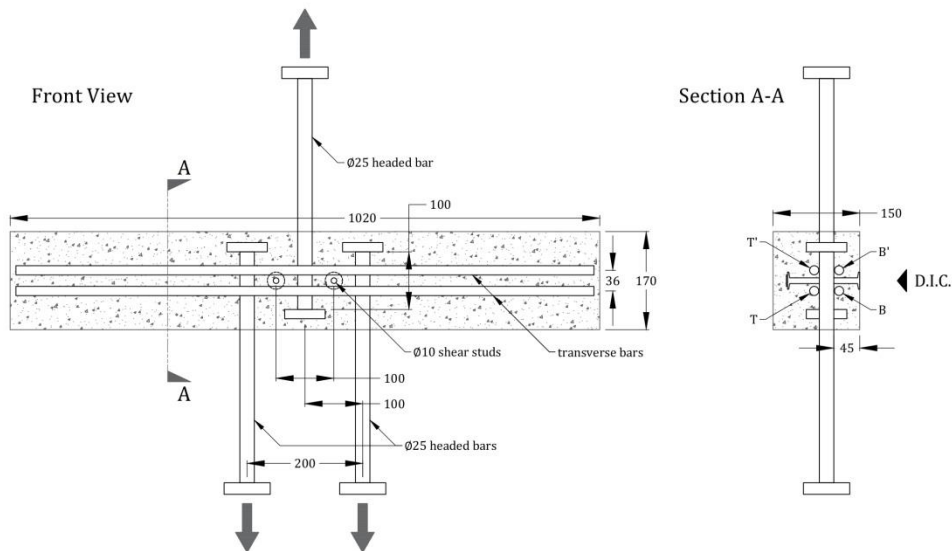
78 The paper investigates the performance of narrow cast in-situ joints between precast concrete elements
79 in which continuity of reinforcement is achieved through overlapping headed bars, as shown in Figure 1.
80 Using headed instead of straight bars, significantly reduces tension splice lengths, thereby facilitating
81 very efficient construction systems, like the 'E6 floor system' patented by Laing O'Rourke, in which
82 headed bar splices provide continuity between precast elements within the floor depth. The narrow joint

83 width adopted in the E6 system, made possible by the use of headed bars, allows adjacent precast units to
84 be supported off each other during construction with easily handled steel brackets. This significantly
85 reduces traditional propping, thereby enabling other follow-on trades to commence earlier. This in turn
86 reduces overall construction time and improves on-site health and safety as well as construction quality
87 due to trades being moved offsite into the factory. The system is ideal for regular slab layouts with
88 standardised components, but can accommodate bespoke floor arrangements.



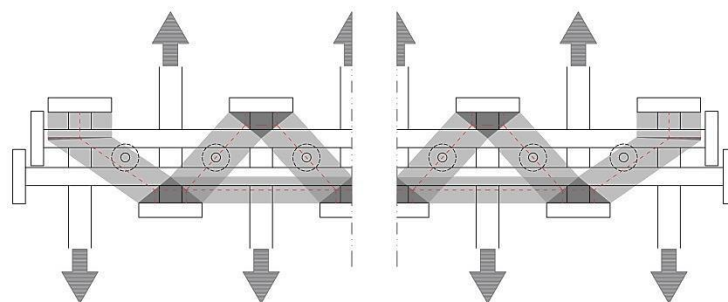
89
90 **Figure 1: Typical headed bar joint**

91 Similar connections using lapped headed bars, but with smaller diameter bars or longer laps, and U-bars
92 have been studied by other researchers with the main emphasis on bridge deck applications [1-13]. A
93 variety of design approaches have been proposed for these joints, including: models based on the ACI
94 318-02 [14] recommendations for side-blowout and bearing strength, strut-and-tie models [4, 9-11], and
95 an upper bound plasticity based model [12, 13]. The authors have previously tested a series of tension
96 specimens with the geometry shown in Figure 2 which is intended to simulate a headed bar splice within
97 the tension zone of a 300 mm thick slab loaded in flexure. The tension tests investigated the influence of
98 variables including concrete strength, transverse reinforcement area and arrangement and presence or
99 absence of confining shear studs [15].



100
101 **Figure 2: Typical tensile test specimen**

102 This paper describes a series of five flexural tests which were carried out to investigate the influence on
 103 joint strength of concrete strength, out-of-plane offset of precast slabs and confining shear studs. The bar
 104 heads used in the tension and flexural tests were sufficiently large to develop the full tensile strength of
 105 the bars without any contribution from bond [16]. Therefore, tension is mainly transferred between
 106 overlapping headed bars through a series of diagonal compressive struts as shown in Figure 3 in which
 107 the the transverse headed bars resist out of balance forces at ends of diagonal struts. The paper compares
 108 and contrasts the behaviour of the headed bar splice joints in the authors' tension and flexural tests.



109
110 **Figure 3: Tensile force transfer within headed bar joint (plan)**

111 **2. Laboratory Testing**

112 *2.1. Tension Test Specimen Details*

113 A full description of the direct tension tests is given elsewhere [15] so only pertinent points are
 114 summarised here. In total 27 tension specimens were tested to investigate the influence on joint strength
 115 of variables including concrete strength, transverse reinforcement and presence or absence of confining
 116 shear studs. The headed bars used in the tests were 25 mm in diameter with 70 mm square heads and

117 yield strength of 530 MPa. Only specimens G1-26-2H20:TT'-S-100-200, G1-40-2H20:TT'-S-100-200 and
118 G2-26-2H20:TT'-100-200 are discussed in this paper since they are directly comparable with flexural
119 tests B2-26-2H20-S-0, B2-39-2H20-S-0 and B2-24-2H20-/-0 respectively. The geometrical dimensions
120 and longitudinal reinforcement arrangement of these specimens (see Figure 2) are the same as for the
121 uncracked tension zone of the tested slabs. Where present, two 10 mm diameter 125 mm long shear studs
122 with 30 mm diameter heads were placed in the positions shown in Figure 2. The minimum and maximum
123 covers to the stud head were zero and 25 mm. The 36 mm spacing of the transverse bars shown in Figure
124 2 was chosen to allow sufficient space for concrete to be placed in contact with the bar heads and to allow
125 clearance for the friction weld flash. The tests focussed on concrete controlled failures with a view to
126 determining the critical concrete strength at which bar yield precedes concrete failure. Table 1 provides
127 details of the three tension specimens most pertinent to this study. The test ID describes the specimens as
128 follows:

129 For example, G1-26-2H20:TT'-S-100-200:

130 “ G1 ” – Test group

131 “ 26 ” – Measured concrete cylinder strength at time of testing

132 “ 2H20 ” – Number and diameter of transverse bars

133 “ TT' ” – Position of transverse bars as indicated in Figure 2

134 “ S ” – Shear studs included

135 “ 100 ” – Lap length of headed bars

136 “ 200 ” – Spacing of headed bars

137 In Table 1, $f_{c,cyl,j}$ and $f_{ct,j}$ are the measured concrete cylinder compressive strength and tensile splitting
138 strength respectively. ϕ_{tr} is the transverse bar size, S_{hb} is the spacing of the headed bars in the same
139 orientation, L_{hb} is the lap length between the bearing faces of the heads and x_t is the offset of the
140 transverse bars from the centreline of the joint. P_{tens} is the maximum load achieved in each test. For
141 reference, the headed bar yield load was 260 kN. The response of the three tension specimens in Table 1
142 is discussed later alongside that of the relevant slab tests.

Test ID	$f_{c,cyl,j}$ (MPa)	$f_{ct,j}$ (MPa)	ϕ_{tr} (mm)	Transverse Bar Positions	S_{hb} (mm)	L_{hb} (mm)	x_t (mm)	P_{tens} (kN)
G1-26-2H20:TT'-S-100-200	25.6	2.38	20	T T'	200	100	18	154
G1-40-2H20:TT'-S-100-200	40.4	3.60	20	T T'	200	100	18	242
G2-26-2H20:TT'-100-200	25.6	2.38	20	T T'	200	100	18	133

143

Table 1: Tension test specimen details

144

2.2. Flexural Test Specimen Details

145

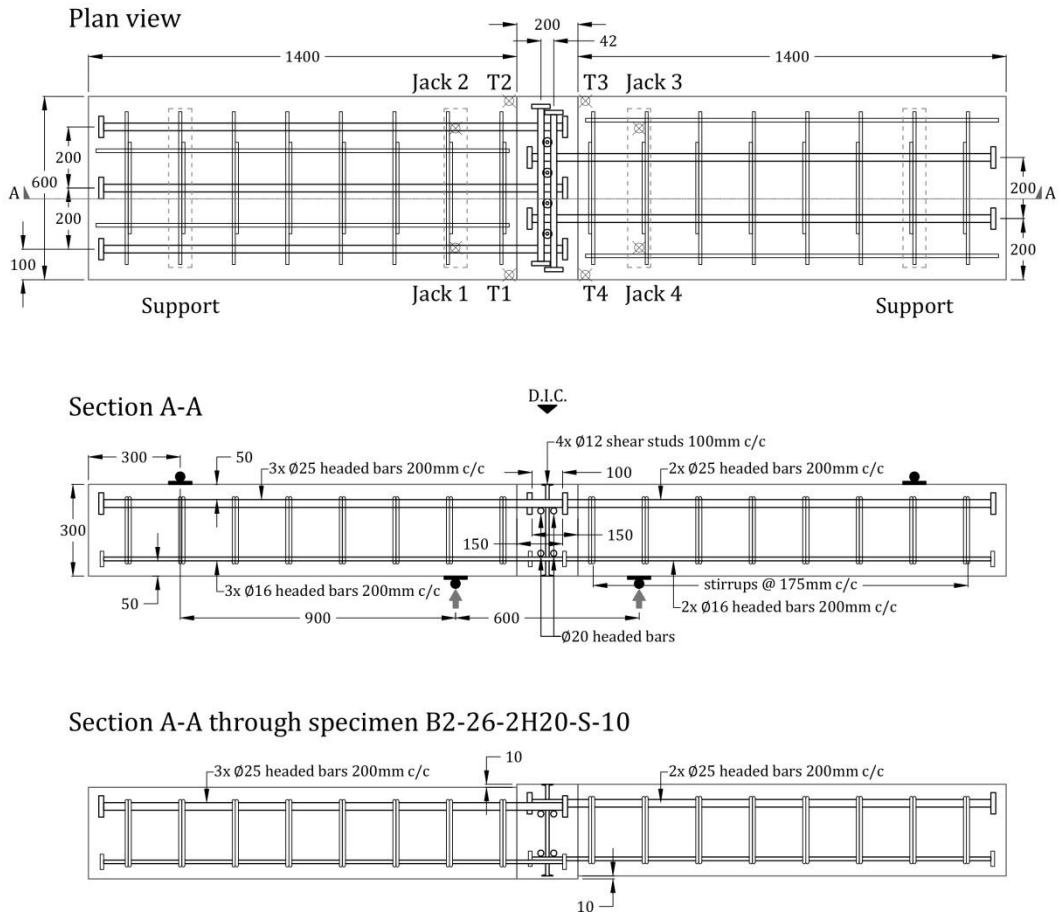
Five flexural specimens were tested of which one was a control specimen. A symmetrical five bar

146

reinforcement arrangement was chosen in preference to an unsymmetrical six bar arrangement to avoid

147

inducing secondary in-plane rotational stresses of the type observed by Gordon and May [7].



148

149

Figure 4: Typical flexural test specimen and section through specimen B2-26-2H20-S-10

150

The headed longitudinal bars were spaced at 200 mm centres with a 100 mm lap length between the

151

inside face of the heads as used in the standard arrangement of the Laing O'Rourke E6 system. 50 mm

152

cover was provided to the main bars in both layers. The tested variables were joint concrete strength, the

153

presence or absence of shear studs and out-of-plane offset of the precast planks. The headed bars in the

154

tensile zone were 25 mm in diameter with standard 70 x 70 x 16 mm friction welded square heads. The

155 diameter of the headed bars near the compression zone was 16 mm, with standard 50 x 50 x 12 mm
156 friction welded square heads. Two 20 mm transverse headed bars with 60 x 60 x 14 mm friction welded
157 square heads were provided on the inside face of each layer of longitudinal reinforcement as shown in
158 Figure 4. The spacing between the centrelines of the transverse bars of 42 mm is the minimum allowed by
159 the head size, including some tolerance. Where present, four 12 mm diameter shear studs were provided
160 in the positions shown in Figure 4. The shear studs were 300 mm long leaving no cover to the heads
161 which were 36 mm in diameter. Specimens were cast in timber moulds with the joint faces of the precast
162 sections left as cast and wetted lightly before the joint concrete was cast. The control specimen was
163 continuously reinforced as in the right hand precast unit of Figure 4. The end sections of the control
164 specimen were cast before the central 200 mm joint to isolate the influence of reinforcement detailing on
165 joint behaviour. In all cases, the joint was cast at least three weeks after the precast sections. Table 2
166 summarises the details of the tested specimens, with the test ID describing specimens as follows:

167 For example, B2-39-2H20-S-0:

168 “ B2 ” – Test group

169 “ 39 ” – Measured joint concrete cylinder strength at time of testing

170 “ 2H20 ” – Number and diameter of transverse bars

171 “ S ” – Shear studs included

172 “ 0 ” – Out-of-plane offset of precast planks

173 Specimens B2-26-2H20-S-0 and B2-39-2H20-S-0 investigated the influence on joint strength of concrete
174 strength, while specimens B2-24-2H20-/-0 and B2-26-2H20-S-10 respectively investigated the influence
175 of shear studs and out-of-plane offset of the precast planks. The offset was achieved by lowering the three
176 bar precast unit by 10 mm relative to the two bar unit and the joint infill was cast over the full depth of
177 310 mm as shown in Figure 4. In this case, zero cover was provided to the shear stud in the tensile zone.
178 The precast planks used in specimen B2-24-2H20-/-0 were reclaimed from specimen B2-26-2H20-S-0
179 after testing the joint to failure.

180 Concrete compressive and tensile strengths were measured from control specimens cured in the same
181 conditions and tested at the same time as the slabs. 100 mm diameter, 200 mm high cylinders were used
182 to determine compressive strength according to BS EN 12390-3:2009 [17] for both joint and precast

183 concrete. Joint concrete tensile strength was determined by means of splitting tests according to BS EN
 184 12390-6:2009 [18] on 300 mm high cylinders with a diameter of 150 mm. At least three specimens of
 185 each type were tested. The resulting concrete strengths are given in Table 2 which also gives geometrical
 186 details and failure loads of the slabs. In Table 2, $f_{c,cyl,p}$, $f_{c,cyl,j}$ and $f_{ct,j}$ are the measured precast slab concrete
 187 cylinder compressive strength, joint concrete cylinder compressive strength and joint concrete tensile
 188 splitting strength respectively, and x_2 is the out-of-plane offset of the precast planks.

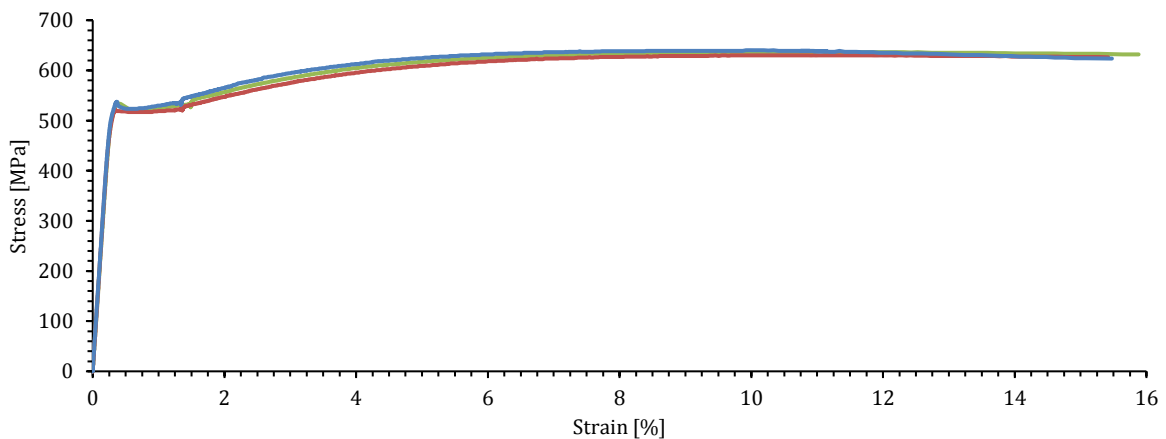
Test ID	$f_{c,cyl,p}$ (MPa)	$f_{c,cyl,j}$ (MPa)	$f_{ct,j}$ (MPa)	ϕ_{tr} (mm)	S_{hb} (mm)	L_{hb} (mm)	x_t (mm)	x_2 (mm)	P_{fl} (kN)	M_{fl} (kNm)
B1-39-/-/-	65.2	39.3	3.40	—	—	—	—	0	362	160
B2-26-2H20-S-0	60.4	25.7	2.46	20	200	100	21	0	253	111
B2-39-2H20-S-0	65.2	39.3	3.40	20	200	100	21	0	293	129
B2-24-2H20-/0	N/A	24.1	2.58	20	200	100	21	0	192	84
B2-26-2H20-S-10	60.4	25.7	2.46	20	200	100	21	10	225	99

189 **Table 2: Flexural test specimen details**

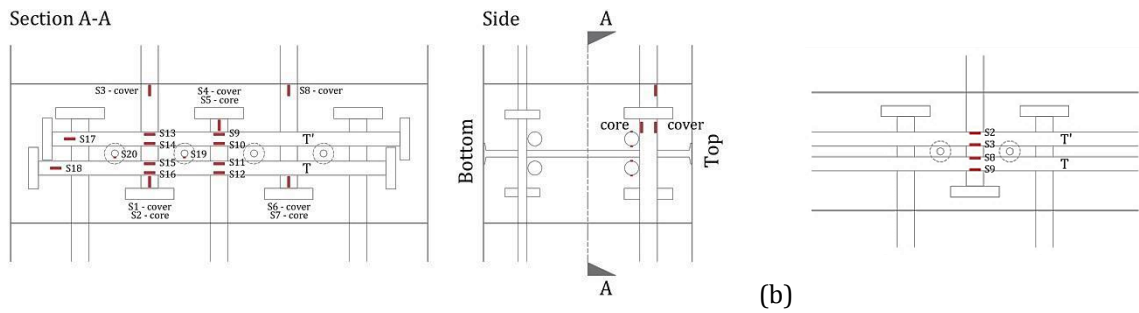
190 Reinforcement material properties were derived from coupon tension tests according to BS EN ISO 6892-
 191 1:2009 [19]. Three specimens of each bar type were tested with average results of the main
 192 reinforcement given in Table 3. Figure 5 shows stress-strain curves for the 25mm headed bar coupon
 193 tests wherein strains were measured with strain gauges and stresses calculated considering a cross-
 194 sectional bar area of 490.9 mm².

Type	ϕ_b (mm)	Head size (mm)	f_y (MPa)	f_u (MPa)	E_s (GPa)
Headed bar	25	70 x 70 x 16	530	636	197
Headed bar	20	60 x 60 x 14	516	631	201
Shear stud	12	$\phi 36$	564	656	223

195 **Table 3: Flexural test specimen reinforcement properties**



196 **Figure 5: Stress-strain curves for 25 mm headed bar coupon tests**
 197



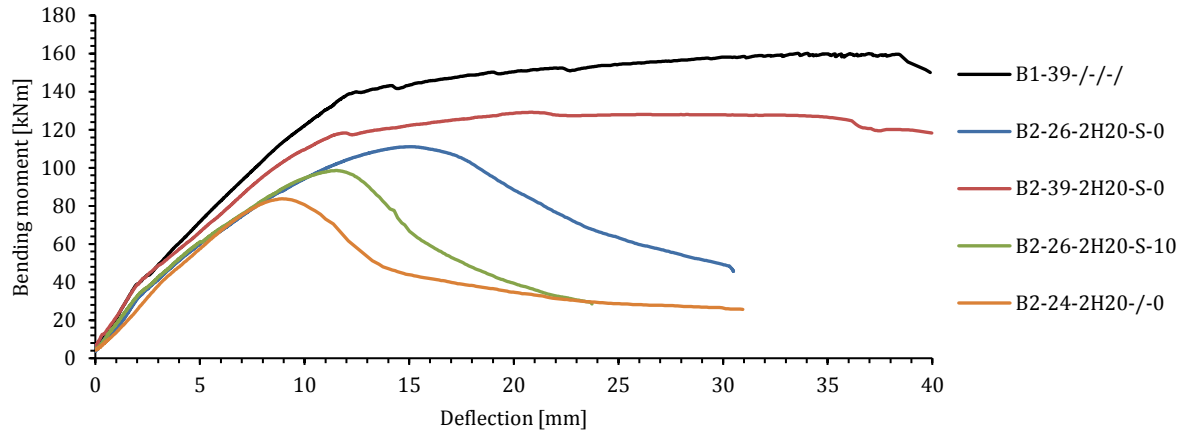
218 (a) **Figure 7: Strain gauge positions in flexural specimens (a) and tension specimens (b)**

219
 220 A random speckle pattern was sprayed onto the surface of the specimen over the constant moment region
 221 to enable the LaVision StrainMaster system [20] to track the movement of the pattern by comparing
 222 images captured by the DIC cameras. Two high resolution cameras captured images of the surface of the
 223 specimen in stereo mode every 3 seconds. Since two cameras were used, it was possible to capture 3D
 224 displacements, surface strains and crack propagation.

225
 226 **3. Test Results and Observations**

227 *3.1. Strength and load-deflection response*

228 For ease of reference, the slab response is henceforth related to the applied bending moment, including
 229 self-weight, at the precast to joint interface. The failure loads of the tested slabs are listed in Table 2 in
 230 which P_{fl} and M_{fl} respectively are the maximum applied load, including self-weight of around 13.5 kN, and
 231 corresponding bending moment. Load-deflection curves are presented in Figure 8. The load-deflection
 232 responses of the control specimen B1-39-/-/-/, with continuous reinforcement through the joint, and B2-
 233 39-2H20-S-0 are very similar up to near yield of the latter. The flexural reinforcement yielded on the two
 234 bar side of the joint-to-precast interface in B1-39-/-/-/and B2-39-2H20-S-0 at bending moments of 142
 235 kNm and 118 kNm respectively. The greater resistance of B1-39-/-/-/ was due to the additional two 10
 236 mm bars provided within the joint. Both specimens failed at large displacements, after extensive yielding
 237 of reinforcement, due to concrete crushing in the flexural compression zone within and adjacent to the
 238 joint. In the case of B2-39-2H20-S-0, concrete crushing also occurred in the joint around the headed bars
 239 loaded in tension.



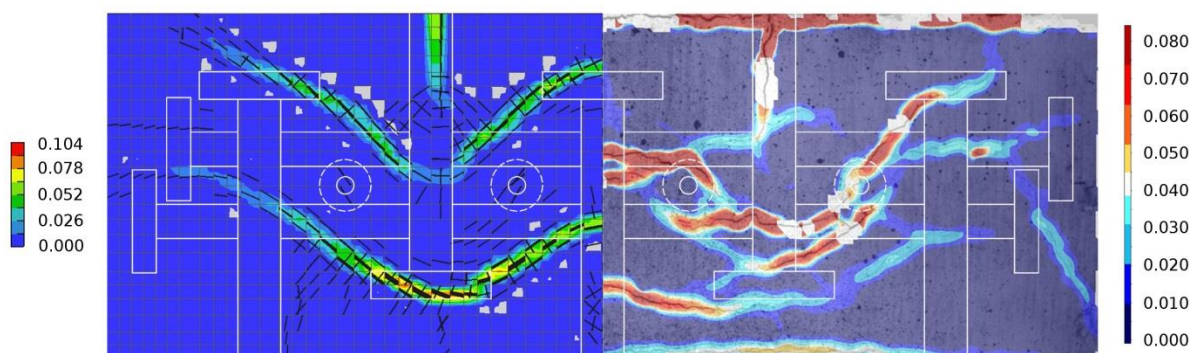
240
241 **Figure 8: Load-deflection curves**

242 The remaining specimens with lower joint concrete strengths of around 25 MPa failed within the joint
 243 prior to headed bar yield. Before failure, deflections of the lower concrete strength specimens were very
 244 similar and greater than that of B2-39-2H20-S-0 due to increased deformation within the joint. The post-
 245 peak load deflection curves of specimens with around 25 MPa concrete exhibit softening but also
 246 considerable ductility, unlike failure of straight bar splices which tends to be brittle. Offsetting the precast
 247 concrete planks in B2-26-2H20-S-10 out-of-plane by 10 mm reduced strength by 11%, but not stiffness,
 248 compared with B2-26-2H20-S-0. Specimen B2-24-2H20-/-/0, without shear studs, achieved a peak
 249 moment around 24% less than that its companion specimen with shear studs, possibly due to loss of
 250 restraint to prying action resulting from curvature of the beam, as described by Chun [1].

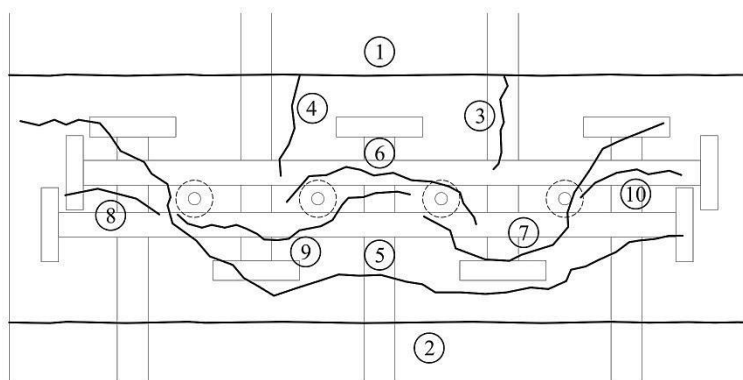
251 **3.2. Crack development and failure mechanism**

252 DIC was used to continuously monitor crack development and surface principal strains. Figure 9 shows
 253 surface principal tensile strains in specimen B2-26-2H20-S-0 alongside comparable strains from NLFEA
 254 which are discussed subsequently. Strains are shown at the measured and predicted failure loads which
 255 correspond to bending moments at the precast to joint interface of 111 kNm and 95 kNm respectively.
 256 The magnitude of strain calculated with DIC is dependent on the size in pixels of a user defined subset
 257 within which the speckle pattern is monitored and correlated between images. The subset size was
 258 chosen such that the DIC strain was calculated over a gauge length of approximately 5 mm as in the
 259 NLFEA. Regions of high strain in Figure 9 correspond to cracks. Figure 10 shows the crack pattern at
 260 failure for the same specimen, which is typical. The first cracks to appear were transverse flexural cracks
 261 at the precast-to-joint interfaces, starting with crack 1 at the two bar interface (right hand side interface
 262 in Figure 4) at an applied bending moment of around 11 kNm. The longitudinal cracks 3 and 4 along the

263 two headed bars propagated from the interface at a moment of 31 kNm. Crack 5 initiated near the two
 264 heads of the main bars at a moment of 45 kNm and subsequently extended towards the slab centreline
 265 and edges. Crack 6 near the head of the central headed bar appeared at a bending moment of 63 kNm,
 266 followed by cracks 7 and 8. Cracks 9 and 10 formed at bending moments of 70 kNm and 100 kNm
 267 respectively, followed by additional cracking close to the failure moment of 111 kNm.



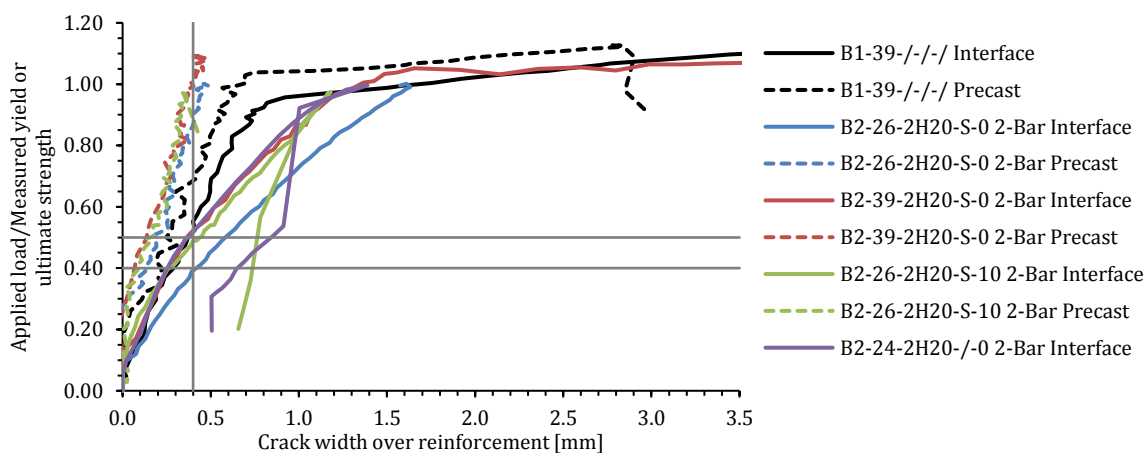
268
 269 **Figure 9: NLFEA (left) and test (right) crack pattern and maximum principal surface strain comparison for**
 270 **specimen B2-26-2H20-S-0**



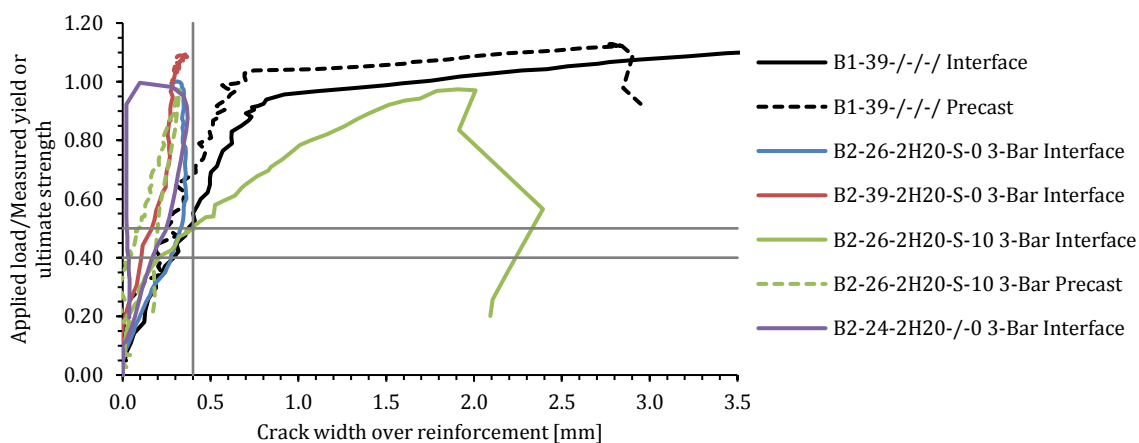
271
 272 **Figure 10: Specimen B2-26-2H20-S-0 schematic crack pattern**

273 Figure 11a and Figure 11b respectively show the development of crack widths over the H25 headed bars
 274 on the two and three bar side of the joint. The applied load is shown as a proportion of the least of the
 275 measured yield and failure loads. The yield loads of B1-39-/-/-/ and B2-39-2H20-S-0 are estimated from
 276 Figure 8 to be 142 kNm and 118 kNm. All other specimens failed before reinforcement yield. Crack widths
 277 are greatest on the two bar side (Figure 11a) as expected except for B2-26-2H20-S-10 with 10 mm offset.
 278 EC2 [21] calculates crack widths under the quasi permanent load which for slabs is typically around 50%
 279 of the measured yield load assuming a partial factor of 1.15 for reinforcement. For concrete controlled
 280 failures, the ratio of quasi permanent to measured failure load is at most 40% assuming a partial factor
 281 for concrete of 1.5. For these load ratios, interface crack widths in Figure 11a and Figure 11b are at most

282 0.40 mm. Consequently, design crack widths under quasi permanent load are considered acceptable at
 283 the joint-to-precast interfaces.



284 (a)



285 (b)

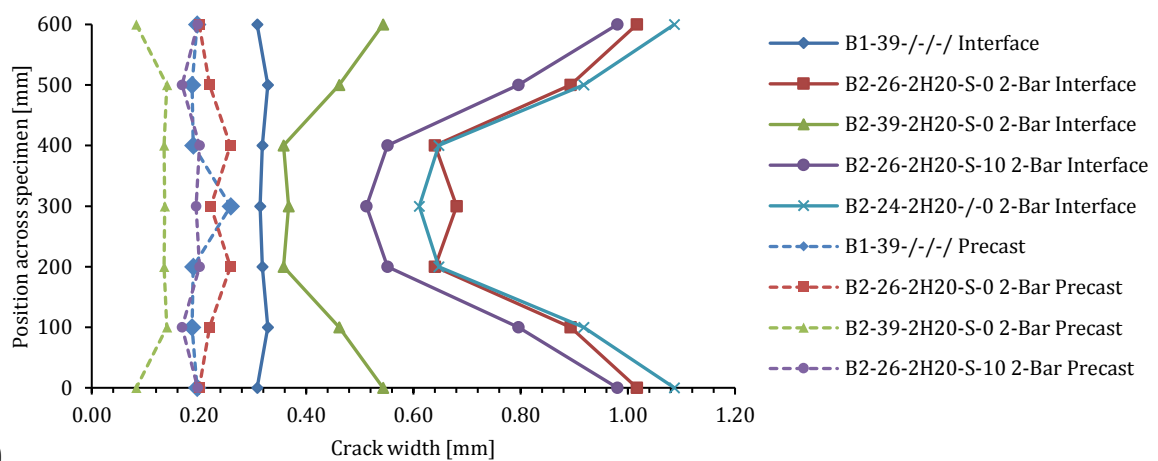
286 **Figure 11: Crack widths versus load on a) two bar side and b) over centre bar of three bar side**

287 Figure 12a and Figure 12b respectively show crack widths, at intervals of 100 mm, measured at the joint-to-
 288 to-precast interface crossed by two and three main bars respectively. Comparative crack widths in the
 289 precast units are also shown. The main longitudinal bars are located at ordinates of 200 mm and 400 mm
 290 in Figure 12a and at 100 mm, 300 mm and 500 mm in Figure 12b. Crack widths are shown at a moment of
 291 59 kNm which is 50% of the flexural yield load of B2-39-2H20-S-0. The only precast cracks captured by
 292 the DIC cameras were at the loading points which were around 200 mm from the joint-to-precast
 293 interfaces. Other flexural cracks were outside the field of view of the cameras.

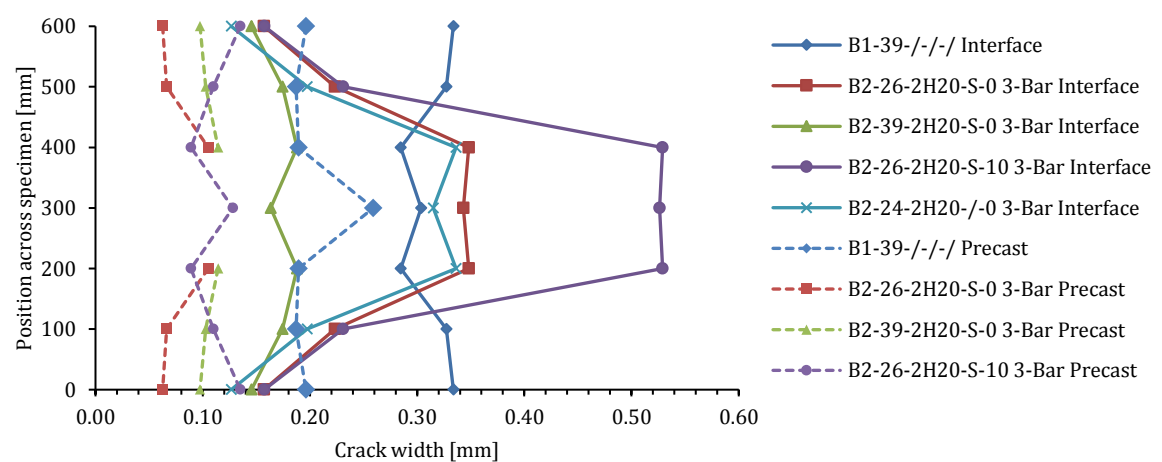
294 Crack widths were greatest at the joint-to-slab interface in both the control and headed bar specimens. In
 295 Figure 12a, crack widths are very wide at the edge of the joint due to the large distance of 200 mm to the
 296 nearest continuous longitudinal bar. Consequently, the crack widths between and over the two H25 bars
 297 are most relevant to practice. Crack widths at the interface were least in the control specimen B1-39-/-/-/

308 with a width of 0.31 mm at the centre of the specimen. At the same location, crack widths were 0.37 mm
 309 in specimen B2-39-2H20-S-0 and up to 0.68 mm in specimens with lower concrete joint strengths. The
 310 comparison between B1-39-/-/-/ and B2-39-2H20-S-0 is most pertinent since the concrete strength and
 311 failure loads of both slabs were comparable. The crack widths within the precast planks were similar for
 312 all specimens, with an average of 0.20 mm. Precast plank crack widths for specimen B2-26-2H20-/-/0 are
 313 not included as the planks were pre-cracked from previous testing.

314 Interface cracks in Figure 12b, on the three bar side, are generally lower than 0.40 mm, but up to 0.53 mm
 315 was observed for specimen B2-26-2H20-S-10 with the 10 mm vertical offset. Crack widths in the control
 316 specimen B1-39-/-/-/ are generally larger than in the headed bar specimens due to the smaller
 317 longitudinal bar area provided through the joint. Cracks in the precast planks for the headed bar
 318 specimens were typically around 0.10 mm wide on the three bar side.



309 (a)
 310



311 (b)
 312

Figure 12: Crack widths at 59 kNm on a) two-bar side and b) three bar side

313 Removal of loose concrete immediately after testing revealed a failure mechanism wherein the headed
 314 bars slipped over the transverse bars with diagonal failure planes only evident up to the depth of the

315 transverse bars. Figure 13 shows a photo of specimen B2-24-2H20-/-0 after two of the headed bars were
 316 cut at the interface and extracted with a wedge of concrete still attached to the head, and a corresponding
 317 sloping failure plane at the transverse bar. This mechanism is further facilitated by the offset joint since
 318 the transverse bars are not fully engaged within the overlapping head area of opposite headed bars. A
 319 similar failure mechanism was observed in the three bar tension tests [15].



320
 321 **Figure 13: Failure planes after removal of headed bars**

322 3.3. Longitudinal joint reinforcement forces

323 The maximum tensile forces that developed in the longitudinal headed bars of the flexural splice were
 324 compared with those developed in comparable direct tension tests. The tensile force in the slab tests was
 325 back calculated from sectional analysis assuming plane sections remain plane. The 16 mm bars near the
 326 compression zone were included in the analysis. The following compressive stress strain relationship
 327 from EC2 [21] was used for concrete:

$$328 \quad \sigma_c = f_{cm} \left(\frac{k\eta - \eta^2}{1 + (k-2)\eta} \right) \quad \text{for} \quad 0 \leq \varepsilon_c \leq 3.5 \quad (1)$$

329 where:

330 f_{cm} is the mean concrete cylinder strength, which is taken as the measured cylinder strength for the
 331 calculations in this paper,

$$332 \quad \eta = \frac{\varepsilon_c}{\varepsilon_{c1}} \quad (2)$$

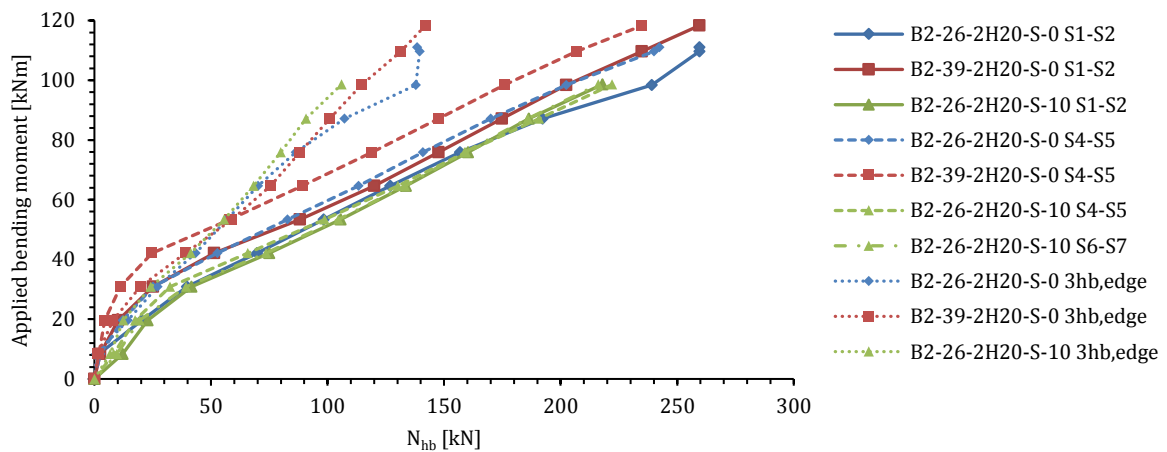
$$333 \quad \varepsilon_{c1} = 0.7f_{cm}^{0.31} \quad (3)$$

$$334 \quad k = 1.05E_{cm}\varepsilon_{c1}/f_{cm} \quad (4)$$

335
$$E_{cm} = 22 \left(\frac{f_{cm}}{10} \right)^{0.3} \quad (5)$$

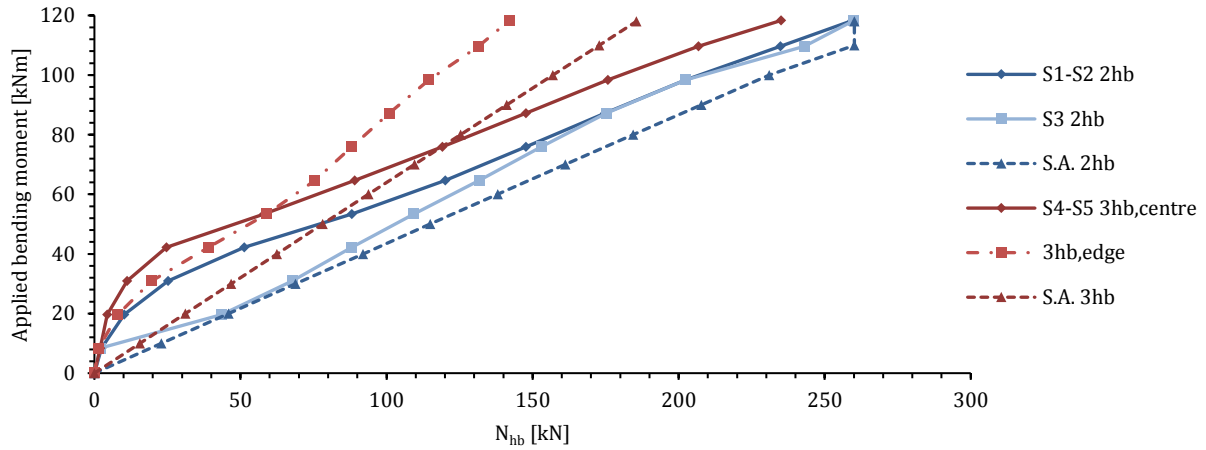
336 ϵ_c is the strain in the compression zone concrete, ϵ_{c1} is the strain at peak compressive stress, and E_{cm} is the
 337 concrete elastic modulus.

338 Figure 14 shows the relationship between applied bending moment and longitudinal bar forces N_{hb}
 339 derived from strains adjacent to bar heads. On the three bar side, the edge bar force $N_{3hb,edge}$ was
 340 calculated from equilibrium as: $N_{3hb,edge} = N_{2hb} - 0.5N_{3hb,centre}$, where N_{2hb} is the bar force on the two bar
 341 side (gauges S1-S2 and S6-S7), and $N_{3hb,centre}$ is the force in the central bar on the three bar side (gauges
 342 S4-S5). Figure 14 shows that $N_{3hb,centre}$ is around N_{2hb} making forces in the edge bars $N_{3hb,edge}$
 343 approximately $0.5N_{3hb,centre}$.



344
 345 **Figure 14: Longitudinal headed bar force comparisons**

346 Figure 15, which is typical, compares bar forces in specimen B2-39-2H20-S-0 derived from section
 347 analysis (S.A.) and measured strains (see Figure 7a). Measured bar forces on the two-bar side (S1-S2)
 348 were similar to those calculated with section analysis, but forces on the three bar side (S4-S5 and
 349 3hb,edge) are incorrectly calculated by section analysis to be equal.



350
351 **Figure 15: Specimen B2-39-2H20-S-0 longitudinal headed bar force comparisons to sectional analysis**

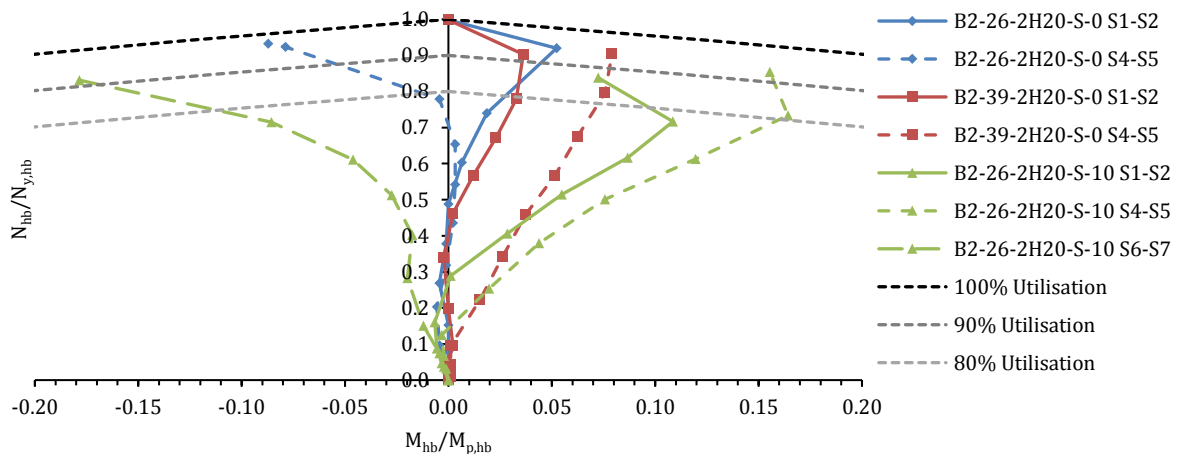
352 Figure 16 shows the interaction between axial load and bending moment at the heads of longitudinal bars
353 with the final point for each specimen corresponding to the least of the failure and measured yield loads.

354 Bending moments are in the same sense as the applied moment with positive moments corresponding to
355 maximum tensile strain within the cover zone. The forces are normalised by plastic capacities calculated

356 using an idealised elastic-plastic stress-strain curve since the reinforcement had a well-defined yield
357 plateau and measured strains did not approach the strain hardening region. Peak moments M_{hb} were

358 below 10% of the bar plastic moment capacity $M_{p,hb}$ except for the misaligned specimen B2-26-2H20-S-10

359 in which M_{hb} reached 18% of $M_{p,hb}$. The bending moment in the headed bars on the two bar side (solid
360 lines), which is critical, reduced to zero or near zero at peak N_{hb} .



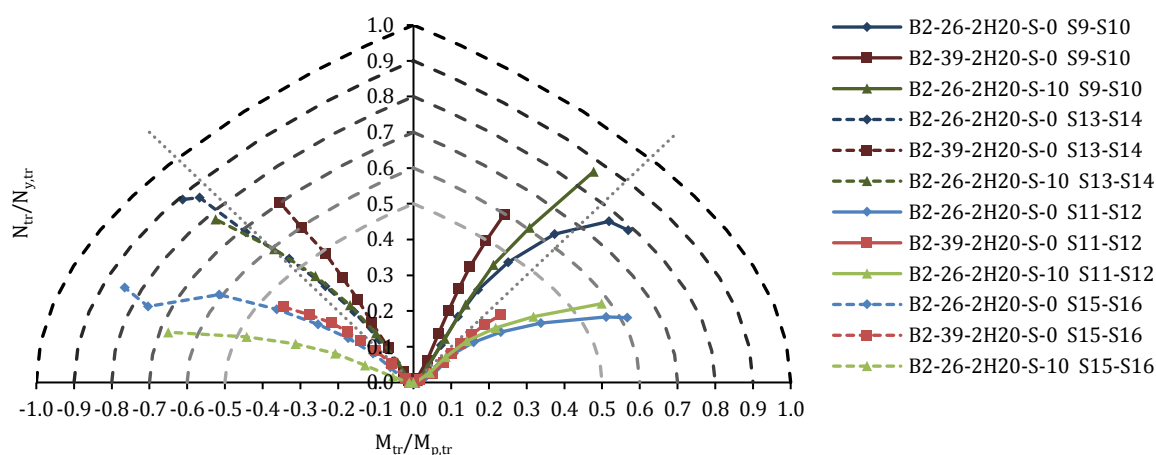
361
362 **Figure 16: Longitudinal headed bar force interaction**

363 3.4. Transverse Joint reinforcement forces

364 Figure 17 shows an axial load (N_{tr})-bending moment (M_{tr}) interaction diagram for the transverse bars
365 with final points for each specimen corresponding to the least of the failure and measured yield loads.

366 The plane of bending is horizontal with positive moment corresponding to maximum tension in gauges

367 S10, S12, S14 and S16 (see Figure 7a). Reversal of bending moments, due to continuous beam action,
 368 causes the change in sign visible in Figure 17 between gauges S9-S10 and S13-S14 as well as between
 369 gauges S11-S12 and S15-S16. At similar applied bending moments, forces in the transverse bars were
 370 lowest in specimen B2-39-2H20-S-0 with the highest joint concrete strength. Figure 17 shows that
 371 tension was greatest in the T' bar of Figure 7a (S9-S10, S13-S14) with the maximum force typically
 372 occurring at gauges S13-S14 midway between the edge and central bar on the three bar side. The tensile
 373 force in this bar was least at its end (S17) where the force was between 35-40% less than at gauges S13-
 374 S14. Bending moments were similar in the T and T' bars and proportionately greater in specimens with
 375 the least concrete strength. In no case was the full plastic capacity of the transverse bars achieved under
 376 combined axial and bending forces.



377
 378 **Figure 17: Transverse bar force interaction**

379 Figure 18 and Figure 19 respectively compare the development with N_{hb} of transverse bar axial force N_{tr}
 380 and bending moment M_{tr} in the slab and tension tests. In the slab tests, the longitudinal bar force N_{hb} is
 381 calculated at gauges S1-S2 adjacent to the head on the two bar side. Transverse bar axial tension N_{tr} and
 382 bending moment M_{tr} are shown at gauges where strains were greatest. N_{tr} and M_{tr} increase more rapidly
 383 with load in tension specimens (dashed line) than comparable flexural specimens (solid line) probably
 384 due to transverse bending in the former. Up to near failure, axial tension at corresponding gauge
 385 positions is almost independent of concrete strength for both slab and tension specimens with tension
 386 greatest in T' bars (see Figure 7). Conversely, bending moments were similar in T and T' bars of both slab
 387 and tension specimens with moments increasing as the concrete strength reduced.

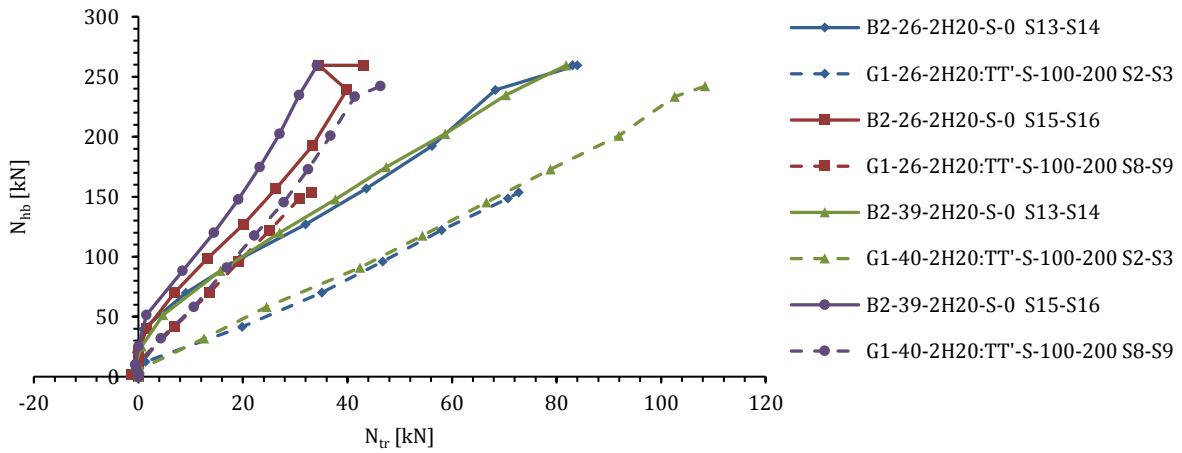


Figure 18: Transverse bar axial forces

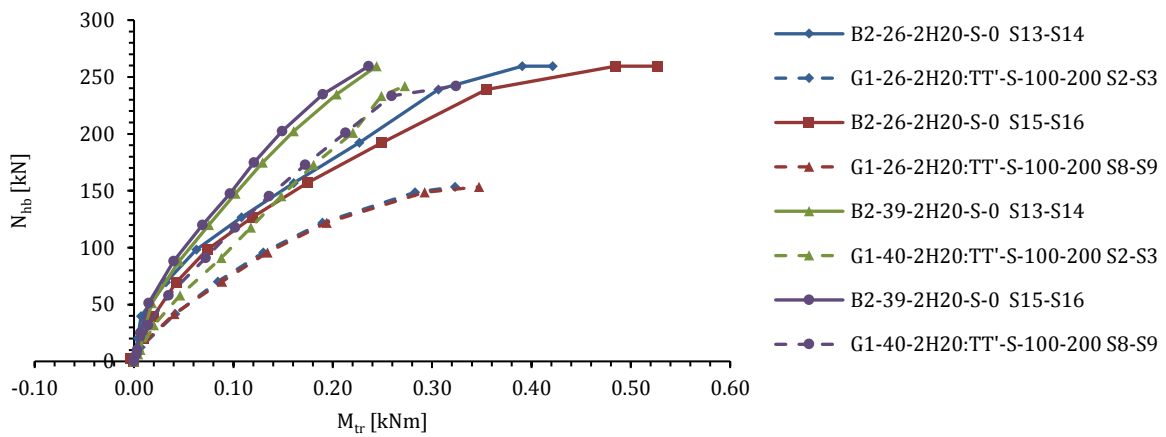


Figure 19: Transverse bar bending moments

3.5. Shear stud strains

Figure 20 compares the development with N_{hb} of strains in the shear studs of the slab and tension tests in which the stud diameters were 12 mm and 10 mm respectively. In the slab tests, N_{hb} is calculated at gauges S1-S2 adjacent to the head on the two bar side. Strains are always greater at gauge S19 than S20, suggesting out-of-plane confinement is greatest between highly stressed longitudinal bars. Up to near failure, strains were similar in aligned slab and tension specimens and fairly independent of concrete strength. Maximum strains did not exceed 65% of yield at ultimate strength except for the misaligned specimen in which yielding occurred at gauge S19. This suggests that the shear studs not only provide confinement to the joint concrete, but also balance out-of-plane forces arising from the transfer of force between the misaligned bars.

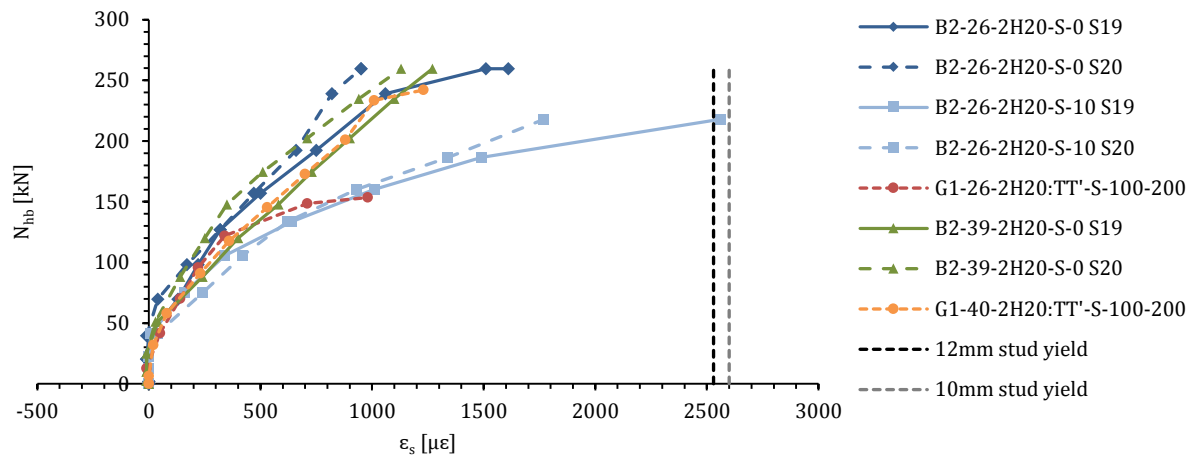


Figure 20: Shear stud axial strain

4. Numerical Modelling

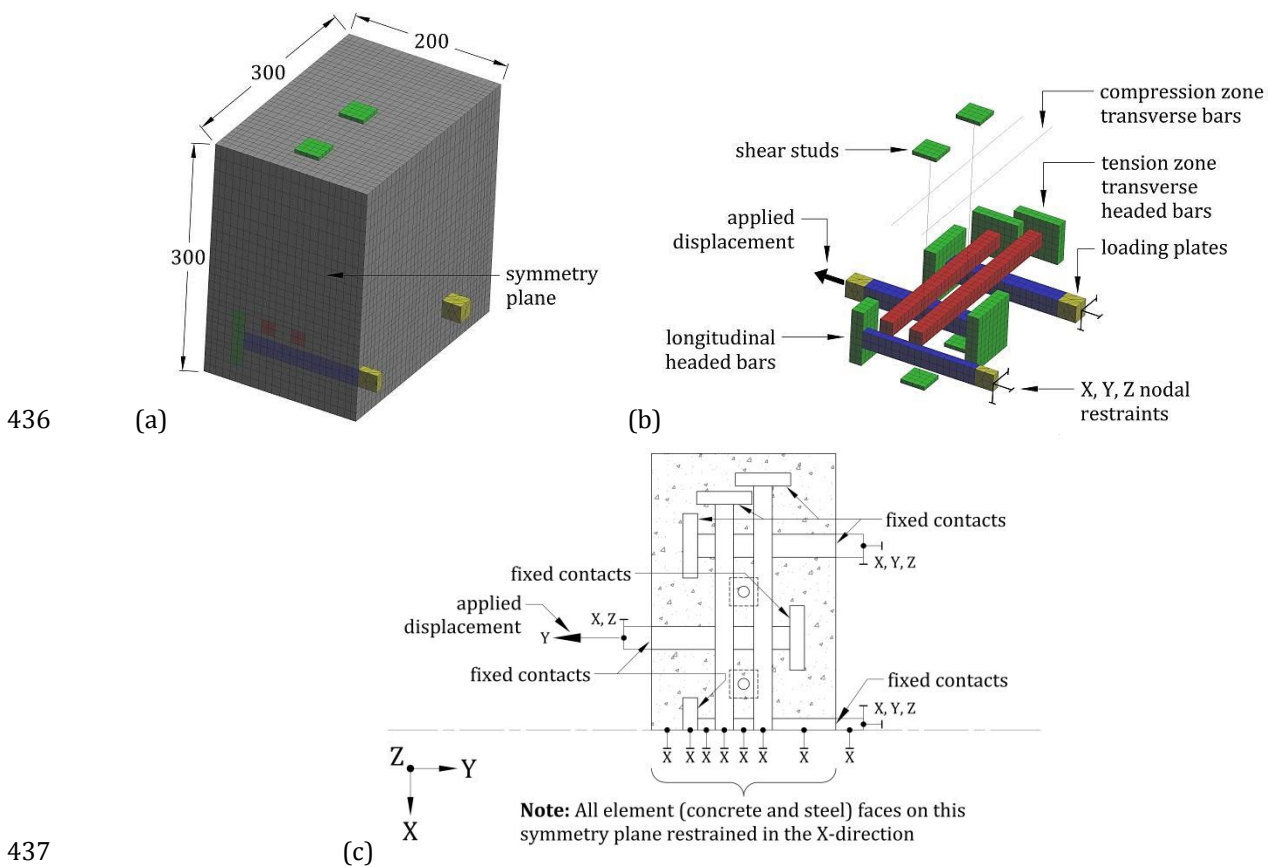
As shown in this section, the flexural resistance of the joints was greater than calculated from the strength of comparable direct tension specimens. Furthermore, as discussed in Section 3.3, the central headed bar resisted twice the load of the edge bars contrary to the predictions of section analysis assuming plane sections remain plane. Therefore, NLFEA was carried out to determine whether it could explain these observations. The adopted NLFEA procedure has previously been shown to be capable of simulating the response of the three bar direct tension specimens [22]. The nonlinear finite element model (NLFEM) was developed with the package ATENA-GiD [23]. GiD was used for pre-processing, while analysis and post-processing were carried out in ATENA Studio [24].

4.1. Geometry and finite element mesh

To reduce computational requirements and exploit symmetry, only half of the joint region was modelled, omitting the precast sections. Furthermore, only the steel reinforcement in the tensile zone was modelled in full detail. Adopted model parameters were validated by NLFEA of the authors' tensile tests [22]. Quadratic 20-noded "CCIsoBrick" isoparametric brick elements with 27 gauss points were used for all the concrete and steel components except for shear studs and transverse bars near the compression zone where linear embedded "CCIsoTruss" truss elements were used. Bars in the tensile zone were modelled with brick elements with square cross-sections having the same second moment of area as the provided circular bars. Brick elements were needed to capture bending behaviour since 1D truss elements in ATENA only have axial stiffness [25]. The maximum element size was limited to 9 mm in the tensile zone of the specimen, producing nodes roughly every 4.5 mm along element edges. The ends of the headed bars were modelled with linear elastic 4-noded "CCIsoTetra" isoparametric tetrahedral elements,

425 depicted loading plates, to avoid issues from stress concentrations. For simplicity, the heads of the shear
 426 studs were modelled entirely outside of the concrete and square (Figure 21a, b) with a similar surface
 427 area to the actual heads.

428 Perfect bond was assumed between reinforcement and concrete. Pyl and Cervenka [25] suggest this is
 429 realistic for ribbed bars provided the mesh size is comparable with the bar diameter as in the current
 430 analysis. Physical justification is provided by the reduction in stiffness of the concrete surrounding the
 431 reinforcement bar that arises due to localised cracking. Bearing faces of bar heads were connected to the
 432 concrete using “fixed contacts” consisting of master-slave constraints as shown in Figure 21c. None of the
 433 other bar head faces were connected to the concrete. Fixed contacts were also required at interfaces
 434 between loading plates and longitudinal bars, due to differences in element type, as well as between steel
 435 and concrete at the bearing faces of shear stud heads.



437
 438 **Figure 21: Full NLFEM (a) NLFEM steel components only (b) and Boundary conditions (c)**

439 **4.2. Boundary conditions**

440 The boundary conditions applied to the model are shown in Figure 21c. The central node of the end faces
 441 of the loading plates of the supporting bars were constrained in all directions. A prescribed displacement
 442 of 0.0125 mm per load step was applied in the negative y-direction at the central node of the end plate of

443 the loaded bar, which was also restrained vertically in the z-direction. End displacements were monitored
 444 at the end of this bar, on the inside face of the loading plate. All element faces on the plane of symmetry
 445 were restrained in the x-direction.

446 4.3. Material constitutive models

447 Concrete was modelled in ATENA with “CC3DNonLinCementitious2” which is a smeared crack fracture-
 448 plastic model that combines constitutive models for tensile and compressive behaviour. The fixed crack
 449 option of ATENA with variable shear retention factor was chosen on the basis of sensitivity studies for the
 450 three bar tension specimens [22]. Concrete material parameters were automatically generated in ATENA-
 451 GiD in terms of the mean measured concrete cylinder compressive strength and then modified as shown
 452 in Table 4 which summarises key material parameters used in the analysis. The parameters w_d and r_c
 453 were derived from sensitivity studies on three bar tension specimens [22]. Transverse and longitudinal
 454 bars and heads in the tensile zone were modelled using the “CC3DBiLinearSteelVonMises” material model
 455 which has a bilinear elastic-plastic stress-strain law. The perfectly elastic material “CC3DElastIsotropic”
 456 was used for the loading plates at the ends of the longitudinal headed bars. The shear studs and
 457 transverse bars near the compression zone were modelled as “CCReinforcement”. A trilinear elastic-
 458 plastic stress strain relationship was used for the shear studs, while a bilinear law was used for the
 459 transverse bars for consistency with those in the tension zone. Adopted steel material properties are
 460 given in Table 5 and Table 6. More information on the material constitutive models can be found in
 461 reference [26].

Parameter	Function	
Cylinder strength	$f'_c = \text{Measured}$ (given in Table 2)	
Tensile strength	$f'_t = \text{Measured}$ (given in Table 2)	
Compression softening	$w_d = -1.8 \text{ mm}$	
Compressive strength in cracked concrete	$r_c = 0.3$	
Onset of nonlinear behaviour	$f'_{c0} = 2f'_t$	(Default function)
Fracture energy	$G_f = 73f_{cm}^{0.18}$	(Default value)
Shear factor	$S_F = 20$	(Default value)
Maximum aggregate size	$d_g = 10 \text{ mm}$	

462 **Table 4: ATENA concrete constitutive model parameters**

Component	E_s (GPa)	σ_y (MPa)	H (MPa)	ν
Bar heads	200	355	1.0e+4	0.3
25 mm headed bars	197	530	1.0e+4	0.3
20 mm headed bars	201	516	1.0e+4	0.3
Loading Plate	2.0e+4	—	—	0.05

463

Table 5: ATENA 3D steel material properties

Component	E_s (GPa)	σ_y (MPa)	ϵ_2	σ_2 (MPa)	ϵ_3	σ_3 (MPa)
20 mm compression headed bars	201	516	0.040	890	—	—
12 mm shear studs	223	564	0.026	564	0.060	637

464

Table 6: ATENA 1D reinforcement material properties

465 5. Numerical Modelling Results and Discussion

466 The predicted and observed crack patterns and surface principal tensile strains compare well as shown in
 467 Figure 9 for specimen B2-26-2H20-S-0. As developed, the NLFEM does not allow comparison of measured
 468 and predicted interface crack widths.

469 Table 7 shows measured and predicted failure loads P for the tested slabs and corresponding joint-to-
 470 precast interface bending moments M . Equivalent results are also given for the corresponding tension
 471 specimens for which the measured failure loads N_{2hb} are similar to bar forces at the bar head [15] derived
 472 from strains. The values of M and P for tension specimens were derived from N_{2hb} using section analysis
 473 and equilibrium respectively. Table 7 also shows corresponding “measured” individual bar forces on the
 474 two and three bar sides of the joint denoted N_{2hb} , $N_{3hb,centre}$ and $N_{3hb,edge}$. The measured slab test bar forces
 475 N_{2hb} and $N_{3hb,centre}$ were derived from strains measured adjacent to the bar head while $N_{3hb,edge}$ was
 476 calculated from equilibrium as $N_{3hb,edge} = N_{2hb} - 0.5N_{3hb,centre}$.

Test ID	Specimen type	P (kN)	M (kNm)	N _{2hb} (kN)	N _{3hb,centre} (kN)	N _{3hb,edge} (kN)
B2-26-2H20-S-0 (test)	Slab	253	111	260	242	139
B2-26-2H20-S-0NLFEA	Slab	211	95	211	206	108
G1-26-2H20:TT-S-100-200 (test)	Tension	171	74	154	-	-
G1-26-2H20:TT-S-100-200NLFEA	Tension	197	86	187	-	-
B2-39-2H20-S-0 (test)	Slab	293	129	260	235	143
B2-39-2H20-S-0NLFEA	Slab	260	121	260	253	134
G1-40-2H20:TT-S-100-200 (test)	Tension	260	114	242	-	-
G1-40-2H20:TT-S-100-200NLFEA	Tension	251	110	232	-	-
B2-24-2H20-/-0 (test)	Slab	192	84	-	-	-
B2-24-2H20-/-0NLFEA	Slab	179	82	179	170	94
G2-26-2H20:TT-100-200 (test)	Tension	153	66	133	-	-
G2-26-2H20:TT-100-200NLFEA	Tension	175	76	159	-	-
B2-26-2H20-S-10 (test)	Slab	225	99	218	222	107
B2-26-2H20-S-10NLFEA	Slab	187	81	187	173	101

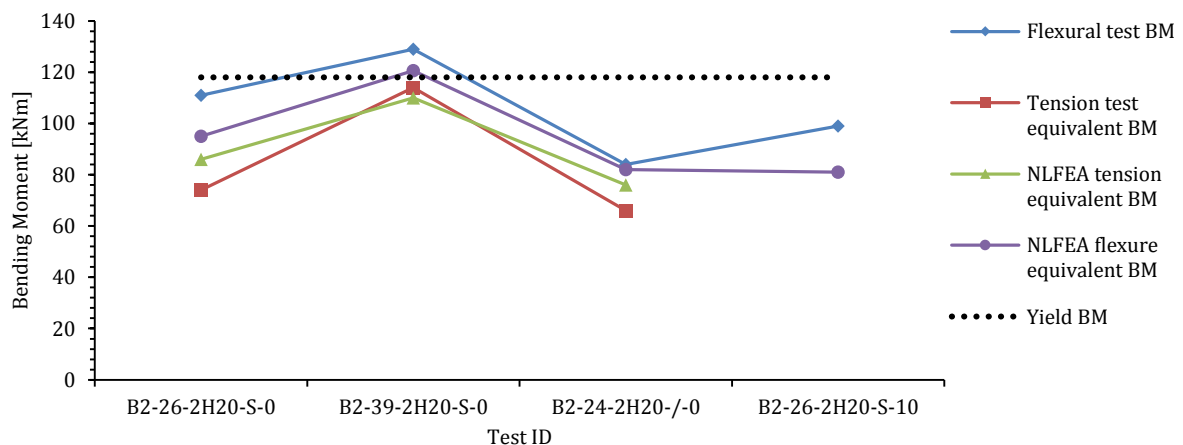
Table 7: Test specimen capacities and longitudinal bar axial forces

477

478 The slab NLFEA longitudinal headed bar forces in Table 7 are reactions at the end of the bars (X, Y, Z
479 nodal restraints shown in Figure 21b), rather than internal forces at the head as measured in the tests.

480 The NLFEA closely predicts the observed distribution of load between the centre and edge bars, with edge
481 bars resisting approximately half the force in the central bar unlike section analysis which predicts equal
482 forces in all three bars. As observed, the NLFEA also predicts lower strengths N_{hb} for tension than flexural
483 tests though prying action is not modelled in the latter. The capacity of the tension specimens is thought
484 to be reduced by bending in the plane of loading being more severe than in the longer slab joints.

485 Figure 22 shows that the flexural resistances derived from the tension tests broadly follow the trend of
486 the measured slab flexural resistances giving added confidence that it is safe to base the flexural design
487 strength of headed bar splices, of the tested geometry, on three bar tension tests of the type undertaken.

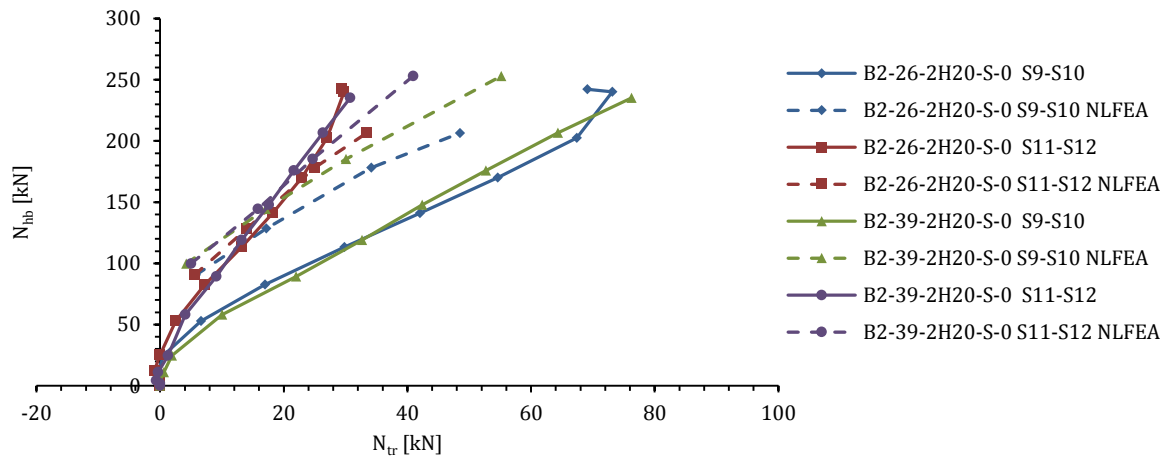


488

489

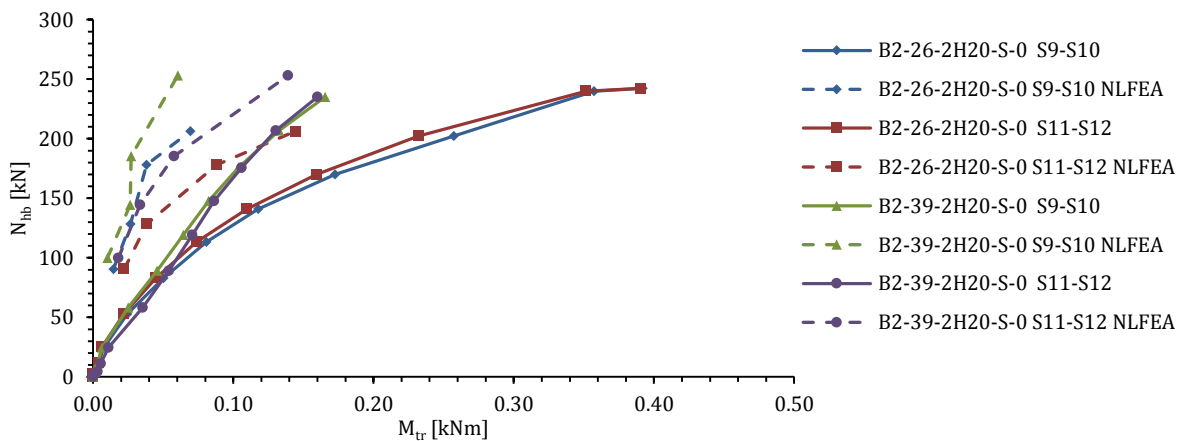
Figure 22: Comparison between test and NLFEA results

490 Figure 23 and Figure 24 compare transverse bar forces derived from strain gauge readings and NLFEA at
 491 the cross-over with the central longitudinal headed bar. Transverse bar forces are plotted against the
 492 force in the central longitudinal headed bar. The numerical results follow the trends of the test data
 493 relatively well, but the measured forces are generally underestimated.



494
 495

Figure 23: Comparison between measured and NLFEA transverse bar axial forces



496
 497

Figure 24: Comparison between measured and NLFEA transverse bar bending moments

498 Figure 25 compares measured and predicted forces in the shear studs. As with transverse bar forces, the
 499 NLFEA captures the general trend of behaviour but underestimates measured forces. The underestimate
 500 of stud force may result from the absence of prying action in the NLFEA since only tension is applied in
 501 the model.

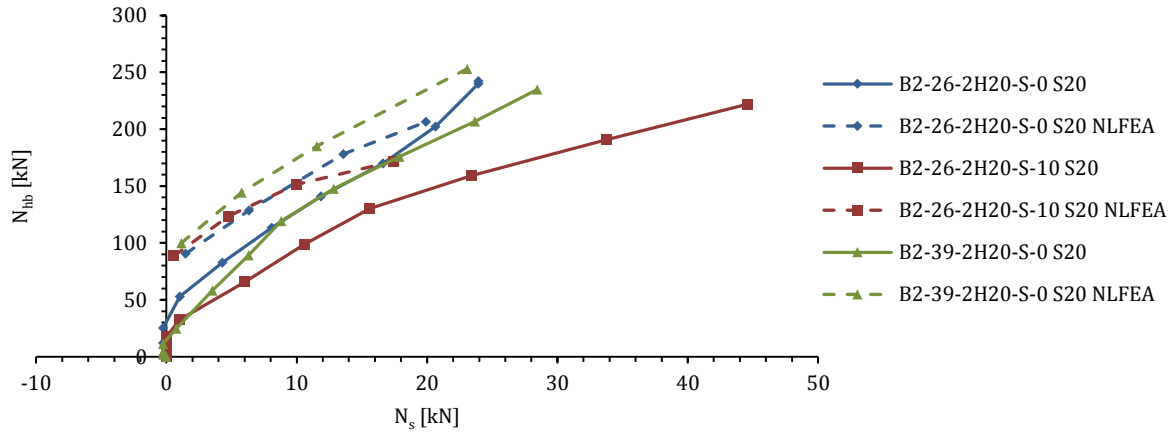


Figure 25: Comparison between measured and NLFEA shear stud axial forces

502
503

504 6. Conclusions

505 The paper describes a series of flexural tests on precast concrete slabs connected by headed bar splices.
 506 This type of connection facilitates precast concrete construction thereby increasing construction
 507 efficiency due to significantly reduced on-site work. The tests investigated the influence on slab strength
 508 and stiffness of joint concrete strength, shear studs, and out-of-plane offset of precast planks. Lap
 509 strengths are compared with strengths of comparable direct tension splices previously tested by the
 510 authors as part of the same project. NLFEA of these joints with ATENA-GiD captures the overall joint
 511 behaviour reasonably well and gives good estimates of flexural strength. The NLFEM is considered
 512 suitable for the design of standard joint configurations provided a suitable safety format is adopted.
 513 Alternatively, the splice strength can be calculated using STM [15] or the upper bound plasticity model of
 514 Joergensen and Hoang [13] as presented in [15].

515 Key conclusions from the tests are:

- 516 a) A lap length of 100 mm in 39 MPa joint concrete was found sufficient to develop the full yield
 517 strength of H25 headed bars when detailed with confining reinforcement as shown in Figure 4.
 518 Ductility was comparable to that of the control specimen with continuous reinforcement through
 519 the joint.
- 520 b) Small scale tension tests give a good indication of joint behaviour and conservative predictions of
 521 flexural strength.
- 522 c) Flexural strength and stiffness is mainly affected by joint concrete strength.
- 523 d) Vertical out-of-plane offset within the joint does not significantly reduce flexural strength.

- 524 e) Shear studs increase joint strength by providing confinement to the joint concrete and
525 restraining prying action of the headed bars in flexural specimens. Shear studs also balance any
526 out-of-plane stresses arising due to construction tolerances.
- 527 f) Precast-to-joint interface crack widths reduce with increasing joint concrete strength but appear
528 to be within acceptable limits at the serviceability limit state as defined in EC2.

529 The understanding of joint behaviour at both ultimate and serviceability limit state gained in this
530 research gives better confidence for a more widespread use of headed bar splices in precast concrete
531 construction.

532 7. Acknowledgements

533 This is a research project funded by Laing O'Rourke. The authors would like to thank Arup for their
534 collaboration in this project. Thanks also to the technicians at the Structures Laboratory at Imperial
535 College London, in particular Mr. Leslie Clark and Mr. Stefan Algar for their availability and assistance
536 throughout the testing programme.

537

538 References

- 539 1. Chun, S.C., *Lap Splice Tests Using High-Strength Headed Bars of 550 MPa (80 ksi) Yield Strength*.
540 ACI Structural Journal, 2015. **112**(6): p. 679 - 688.
- 541 2. Li, L., Ma, Z., Griffey, M.E., and Oesterle, R.G., *Improved Longitudinal Joint Details in Decked Bulb*
542 *Tees for Accelerated Bridge Construction: Concept Development*. Journal of Bridge Engineering,
543 2010. **15**(3): p. 327-336.
- 544 3. Li, L., Ma, Z., and Oesterle, R.G., *Improved Longitudinal Joint Details in Decked Bulb Tees for*
545 *Accelerated Bridge Construction: Fatigue Evaluation*. Journal of Bridge Engineering, 2010. **15**(5):
546 p. 511-522.
- 547 4. Li, L. and Jiang, Z., *Flexural Behavior and Strut-and-tie Model of Joints with headed bar details*
548 *Connecting Precast Members*. Perspectives in Science, 2016. **7**: p. 253-260.
- 549 5. Thompson, M.K., Jirsa, J.O., and Breen, J.E., *Behaviour and Capacity of Headed Reinforcement*. ACI
550 Structural Journal, 2006. **103**(4): p. 522-530.

- 551 6. Thompson, M.K., Ledesma, A., Jirsa, J.O., and Breen, J.E., *Lap Splices Anchored by Headed Bars*. ACI
552 Structural Journal, 2006. **103**(2): p. 271-279.
- 553 7. Gordon, S.R. and May, I.M., *Development of In-situ Joints for Pre-cast Bridge Deck Units*.
554 Proceedings of the Institution of Civil Engineers: Bridge Engineering, 2006. **159**(BE1): p. 17-30.
- 555 8. Gordon, S.R. and May, I.M., *Precast Deck Systems for Steel-Concrete Composite Bridges*.
556 Proceedings of the Institution of Civil Engineers: Bridge Engineering, 2007. **160**(BE1): p. 25-35.
- 557 9. He, Z., Ma, Z.J., Chapman, C.E., and Liu, Z., *Longitudinal Joints with Accelerated Construction*
558 *Features in Decked Bulb-Tee Girder Bridges: Strut-and-Tie Model and Design Guidelines*. Journal of
559 Bridge Engineering, 2013. **18**(5): p. 372-379.
- 560 10. Ong, K.C.G., Hao, J.B., and Paramasivam, P., *A Strut-and-Tie Model for Ultimate Loads of Precast*
561 *Concrete Joints with Loop Connections in Tension*. Construction and Building Materials, 2005.
562 **20**(3): p. 169-176.
- 563 11. Ma, Z.J., Lewis, S., Cao, Q., He, Z., Burdette, E.G., and French, C.E.W., *Transverse Joint Details with*
564 *Tight Bend Diameter U-Bars for Accelerated Bridge Construction*. Journal of Structural
565 Engineering, 2012. **138**(6): p. 697-707.
- 566 12. Joergensen, H.B. and Hoang, L.C., *Tests and Limit Analysis of Loop Connections between Precast*
567 *Concrete Elements Loaded in Tension*. Engineering Structures, 2013. **52**: p. 558-569.
- 568 13. Joergensen, H.B. and Hoang, L.C., *Strength of Loop Connections between Precast Bridge Decks*
569 *Loaded in Combined Tension and Bending*. Structural Engineering International, 2015. **25**(1): p.
570 71-80.
- 571 14. ACI, *Building Code Requirements for Structural Concrete (ACI 318-02) and Commentary (ACI 318R-*
572 *02)*. 2002, American Concrete Institute Committee 318. p. 443.
- 573 15. Vella, J.P., Vollum, R.L., and Jackson, A., *Investigation of Headed Bar Joints between Precast*
574 *Concrete Panels*. Engineering Structures, 2017. **138C**: p. 351-366.
- 575 16. Brooker, O., *Use of Headed Bars as Anchorage to Reinforcement*. The Structural Engineer, 2013: p.
576 49-57.
- 577 17. BSI, *Testing hardened concrete Part 3: Compressive strength of test specimens*. 2009, British
578 Standards Institution. p. 22.
- 579 18. BSI, *Testing hardened concrete Part 6: Tensile splitting strength of test specimens*. 2009, British
580 Standards Institution. p. 14.

- 581 19. BSI, *Metallic materials - Tensile testing - Part 1: Method of test at room temperature*. 2009, British
582 Standards Institution. p. 76.
- 583 20. LaVision, *Product Manual for DaVis 8.3*. 2016, LaVision GmbH: Gottingen, Germany. p. 356.
- 584 21. BSI, *Eurocode 2: Design of concrete Structures - Part 1-1: General Rules and Rules for Buildings*.
585 2004, British Standards Institution. p. 225.
- 586 22. Vella, J.P., Vollum, R.L., and Jackson, A., *Numerical Modelling of Headed Bar Joints subjected to*
587 *Tension*. Magazine of Concrete Research, 2017. DOI: 10.1680/jmacr.17.00011.
- 588 23. Cervenka, V., Cervenka, J., Janda, Z., and Pryl, D., *ATENA Program Documentation, Part 8: User's*
589 *Manual for ATENA-GiD Interface*. 2014, Cervenka Consulting: Prague. p. 112.
- 590 24. Benes, S., Mikolaskova, J., and Altman, T., *ATENA Program Documentation, Part 12: User's Manual*
591 *for ATENA Studio*. 2015, Cervenka Consulting: Prague. p. 58.
- 592 25. Pryl, D. and Cervenka, J., *ATENA Program Documentation, Part 11: Troubleshooting Manual*. 2014,
593 Cervenka Consulting: Prague. p. 51.
- 594 26. Cervenka, V., Jendele, L., and Cervenka, J., *ATENA Program Documentation, Part 1: Theory*. 2013,
595 Cervenka Consulting: Prague. p. 268.