MASS-CONSISTENT WINDFIELDS FOR SIMULATING
RADIONUCLIDE DISPERSION OVER COMPLEX TERRAIN

KEN KITSON, BSc.

A thesis submitted for the degree of
Ph.D., London University and D.I.C., Imperial College

Environmental Safety Group,
Nuclear Power Section,
Department of Mechanical Engineering,
Imperial College,
London, SW7

February, 1988
ABSTRACT

This thesis is concerned with the development and evaluation of a computer model which produces three-dimensional windfields flowing over complex terrain. Similar models are used by the nuclear authorities in Italy and the USA to provide real-time estimates of airborne pollutant transport in the event of an accident occurring at a nuclear installation. The problem is characterised by a requirement to predict the often complicated flow field over complex terrain within about 100 km of the release site using the limited meteorological data typically available during an accident.

The work has involved a critical assessment of the mass-consistent models currently in use, and an examination of which physical conditions are treated inadequately. An improved mass-consistent model has been developed. The novel calculational technique used allows a good resolution of the ground surface whilst retaining a relatively simple mathematical approach. The model is able to represent a wider range of meteorological conditions; in particular the important effects of stable stratification and elevated temperature inversion have been demonstrated. The computational requirements have been minimised by incorporating new techniques for solving large sparse matrix problems.

The model has been tested by applying it to various theoretical flow patterns with many sensitivity tests to the assumptions made and the data used. A number of case studies in the British Channel region have been considered. Comparisons with simpler models have been made. The thesis concludes with an overall assessment of the mass-consistent approach.
ACKNOWLEDGEMENTS

I would like to express my gratitude to my supervisors, Professor A.J.H. Goddard and Dr. H. M. ApSimon for their useful discussions and support throughout the work. The project was sponsored by the Science and Engineering Research Council and the Central Electricity Generating Board under the CASE scheme; particular thanks to H. MacDonald and his staff at the CEGB's Berkeley Nuclear Laboratories for making my visits there enjoyable and instructive. I would like to acknowledge additional financial assistance provided by the Commission of the European Communities.

Thanks to Elizabeth Dean for typing this thesis.

Finally, thanks are also due to the many friends I've known over the past years who have encouraged me to complete this work.
CONTENTS

Abstract 3
Acknowledgements 5
Contents 7
List of Figures 13
List of Tables 19
Notation 21

CHAPTER 1 INTRODUCTION 27

CHAPTER 2 THE WAFT MODEL

2.1 Basic Outline of the Model
2.1.1 Introduction 33
2.1.2 The Calculational Mesh and Topography Specification 34
2.1.3 Generating the Initial Windfield 37
2.1.4 Generating the Final Mass Consistent Windfield 39

2.2 The WAFT Model Formulation
2.2.1 The Calculational Cell and its Associated Velocities 42
2.2.2 The Adjustment Process 44
2.2.3 Boundary Conditions 49

2.3 Solving the Set of Simultaneous Linear Equations
2.3.1 Characteristics of the WAFT Set of Equations 51
2.3.2 Solution Methods 54
2.3.3 The Successive-Over-Relaxation Method 57
2.3.4 The Conjugate Gradient Method 60
2.3.5 The Incomplete Cholesky Conjugate Gradient Method 63
CHAPTER 3  USING WAFT TO MODEL POTENTIAL FLOW

3.1 Introduction 75

3.2 Definition of the Cell Weighting Factors
   3.2.1 The Method used to Select a Valid Weighting Scheme 77
   3.2.2 The Original Model Formulation 79
   3.2.3 The Second Weighting Scheme 81
   3.2.4 The Third and Fourth Weighting Schemes 83

3.3 Sources of Error in the WAFT Model
   3.3.1 Introduction 86
   3.3.2 Comparison Methodology 87
   3.3.3 Numerical Experiments 89
   3.3.4 Errors from the Initial Windfield Specification 92
   3.3.5 Interpolation Schemes 94
   3.3.6 Comparison of the Interpolation Schemes 97
   3.3.7 Summary of Errors Investigated using 2D WAFT 99

3.4 Comparing WAFT with Three-Dimensional Potential Flow
   3.4.1 Introduction 100
   3.4.2 Tracking Streamlines in the Final Windfield 102
   3.4.3 Streamlines over a Hemisphere 104
   3.4.4 Comparison with the MATHEW Code 107
   3.4.5 Flow over a Hemisphere Stream Surface 108
   3.4.6 Summary of Three-Dimensional Potential Flow Comparisons 110

CHAPTER 4  MODELLING THERMAL STRATIFICATION IN WAFT

4.1 Introduction 133

4.2 Neutral Flow Overlain by Stable Air
   4.2.1 Introduction 133
   4.2.2 Modelling an Elevated 'Lid' in WAFT 135
4.2.3 Flow past a Low Slope Hill under an Elevated Inversion

4.3 Stably Stratified Flow around Hills
4.3.1 Introduction
4.3.2 Theoretical Description of Strongly Stratified Flow past a Three-Dimensional Obstacle
4.3.3 The Relationship between the Velocity Weighting Factors and the Vertical Velocities
4.3.4 Numerical Experiments relating Vertical Velocities to the Velocity Weighting Factors
4.3.5 The Failure of a Simple Approach to Model Stable Flow
4.3.6 Modelling Stable Flow using WAFT
4.3.7 Using WAFT to Model Stable Flow over Simple Hills
4.3.8 Modelling Stable Flow in Complex Terrain

4.4 Summary

CHAPTER 5 MESOSCALE FLOW IN THE ATMOSPHERE

5.1 Introduction

5.2 The Variation of Velocity with Height in the Atmosphere
5.2.1 Introduction
5.2.2 The Velocity Profile in the Boundary Layer
5.2.3 The Velocity Profile used in WAFT

5.3 Thermally Driven Mesoscale Features
5.3.1 Land/Sea Breeze Circulations
5.3.2 Mountain/Valley Winds
5.3.3 Urban Heat Islands
CHAPTER 6 USING WAFT IN COMPLEX TERRAIN

6.1 Introduction

6.2 The Bristol Channel Study Region

6.3 Using Uniform Initial Windfields
   6.3.1 Flow Under an Elevated Inversion
   6.3.2 Stable Flow

6.4 Meteorological Data in the Bristol Channel Study Area

6.5 Case Study One - 9.00 Hours January 29, 1976
   6.5.1 Meteorological Situation
   6.5.2 Using a Uniform Surface Windfield
   6.5.3 Using the Available Surface Station Data
   6.5.4 Sensitivity Studies

6.6 Case Study Two - 9.00 Hours February 6, 1976
   6.6.1 Meteorological Situation
   6.6.2 Using a Uniform Surface Windfield
   6.6.3 Local Winds in Sub-Grid Features
   6.6.4 Using the Available Surface Station Data

6.7 Case Study Three - 9.00 Hours February 7, 1976
   6.7.1 Meteorological Situation
   6.7.2 Using a Uniform Surface Windfield
   6.7.3 Using the Available Surface Station Data
   6.7.4 Using a Time Series of WAFT Windfields

6.8 Summary
| Figure 2.1 | The topography representation in WAFT | 68 |
| Figure 2.2 | The three types of calculational cell in the WAFT mesh | 68 |
| Figure 2.3 | A typical WAFT cell illustrating the notation used | 69 |
| Figure 2.4 | A WAFT cell and its associated Lagrangian Multipliers | 70 |
| Figure 2.5 | Boundary conditions at the ground surface | 70 |
| Figure 2.6 | Matrix of equation coefficients for a mesh of twenty-six cells | 71 |
| Figure 2.7 | Convergence of the different iterative techniques; 8x8x16 mesh | 72 |
| Figure 2.8 | Convergence of the different iterative techniques; 35x21x11 mesh | 73 |
| Figure 3.1 | Two-dimensional potential flow over a half-cylinder; velocities plotted at WAFT mesh points | 112 |
| Figure 3.2 | Uniform initial windfield used to assess different weighting schemes in flow over a half-cylinder | 112 |
| Figure 3.3 | Final windfield over a half-cylinder using the original formulation of WAFT ('first weighting scheme') | 113 |
| Figure 3.4 | Part of a two-dimensional WAFT mesh which illustrates the deficiencies of the first weighting scheme | 113 |
| Figure 3.5 | WAFT final windfield over a half-cylinder using the second weighting scheme | 114 |
| Figure 3.6 | WAFT final windfield over a half-cylinder using either the third or fourth weighting scheme | 114 |
| Figure 3.7 | Streamline of potential flow used to define ridge shape | 115 |
| Figure 3.8 | Potential flow over ridge defined in Figure 3.7 | 115 |
Figure 3.9 WAFT final windfield over a low slope hill illustrating the weakness of the third weighting scheme when elongated mesh cells are used

Figure 3.10 WAFT final windfield over a half-cylinder illustrating problems with the fourth weighting scheme when variable mesh spacing is employed

Figure 3.11 'Standard' mesh used to evaluate sources of error in the WAFT method

Figure 3.12 The effect of varying the size of the mesh cells and calculational domain on WAFT windfield accuracy

Figure 3.13 The effect of varying the mesh cell shape on the WAFT windfield accuracy

Figure 3.14 The effect of the initial specification on the final windfield - seven randomly placed data points and bicubic spline interpolation

Figure 3.15 The effect of the initial windfield specification on the final windfield - 12 symmetrically placed data points and inverse square interpolation

Figure 3.16 Two-dimensional flow around a cylinder used in the evaluation of different interpolation schemes

Figure 3.17 Initial windfield accuracies for three data point configurations using different interpolation techniques

Figure 3.18 Simple interpolation scheme used when computing streamlines

Figure 3.19 Streamlines illustrating WAFT final windfield over a hemisphere

Figure 3.20 Streamlines in WAFT windfield over a hemisphere compared to potential flow

Figure 3.21 Deformation of a 'stream-tube' as it passes over the hemisphere

Figure 3.22 Comparison of MATHEW and WAFT final windfields: uniform initial windfield

Figure 3.23 Comparison of WAFT initial and final windfields: non-uniform initial windfield
Figure 3.24  Three-dimensional stream surface used to define hill shape  

Figure 3.25  Comparison between streamlines in WAFT final windfield and potential flow streamlines in flow over hemisphere stream surface  

Figure 4.1  Flow past a simple hill with elevated inversions present at three different heights  

Figure 4.2  Low Froude number flow past a hill  

Figure 4.3  Flow past a simple hill with the vertical velocity weighting factor set to 1 and 10  

Figure 4.4  Final windfield generated by the stable flow WAFT model  

Figure 4.5  Magnitude of vertical velocities in WAFT windfield compared to corresponding theoretical stable flow values  

Figure 4.6  How the Bristol Channel region is divided up with two different values of the 'high-lying' factor  

Figure 5.1  Vertical velocity profiles used in WAFT  

Figure 5.2  Typical structure of a sea breeze  

Figure 5.3  Possible pollutant transport in a sea breeze  

Figure 6.1  The Bristol Channel Study region  

Figure 6.2  WAFT windfield over the Bristol Channel region under an elevated inversion at 2000m (uniform initial windfield)  

Figure 6.3  WAFT windfield over the Bristol Channel region under an elevated inversion at 800m (uniform initial windfield)  

Figure 6.4  WAFT windfield over the Bristol Channel region under an elevated inversion at 400m (uniform initial windfield)  

Figure 6.5  Stable WAFT windfield over Bristol Channel region using uniform initial windfield – Case 1; $h_{LF} = 0.33\%, \partial T/\partial z = 1.0^\circ C/100m$, $u_{10} = 1m/s$
Figure 6.6
Stable WAFT windfield over Bristol Channel region using uniform initial windfield - Case 2; $h_{LF} = 66\%, \frac{\partial \theta}{\partial z} = 1.0^\circ C/100m$, $u_{10} = 1m/s$

Figure 6.7
Stable WAFT windfield over Bristol Channel region using uniform initial windfield - Case 3; $h_{LF} = 33\%, \frac{\partial \theta}{\partial z} = 0.5^\circ C/100m$, $u_{10} = 2m/s$

Figure 6.8
Stable WAFT windfield over Bristol Channel region using a uniform initial windfield - Case 4; uniform vertical velocity suppression

Figure 6.9
29th January case study - measured surface winds in the Bristol Channel region at 9.00 hours

Figure 6.10
29th January case study - flow at 100m above S.L. in uniform initial windfield run

Figure 6.11
29th January case study - streamlines from nuclear power stations in uniform initial windfield run

Figure 6.12
29th January case study - initial windfield at 100m above S.L. using available surface data

Figure 6.13
29th January case study - final windfield at 100m above S.L. using available surface data

Figure 6.14
29th January case study - streamlines from the nuclear power stations in the initial windfield using available surface data

Figure 6.15
29th January case study - streamlines from the nuclear power stations in the corresponding final windfield

Figure 6.16
29th January case study - simulated plumes in the WAFT final windfield

Figure 6.17
29th January case study - simulated Gaussian plumes based on source meteorology

Figure 6.18
WAFT sensitivity studies - the effect of initial windfield interpolation scheme and data points used

Figure 6.19
WAFT sensitivity studies - the effect of the height of the elevated inversion
| Figure 6.20 | WAFT sensitivity studies - the effect of different assumed vertical profiles | 253 |
| Figure 6.21 | WAFT sensitivity studies - the effect of the high level wind specification | 254 |
| Figure 6.22 | 6th February case study - measured surface winds in the Bristol Channel region at 9.00 hours | 255 |
| Figure 6.23 | 6th February case study - final windfield at 160m above S.L. using a uniform initial windfield | 255 |
| Figure 6.24 | The topography around Aberdare meteorological station using mesh intervals of (a) 5.0 km and (b) 0.5 km | 256 |
| Figure 6.25 | WAFT windfields around Aberdare using uniform initial windfields and a high resolution mesh | 257 |
| Figure 6.26 | 6th February case study - initial and final windfields at 200m above S.L. using available surface data and inverse square interpolation | 258 |
| Figure 6.27 | 6th February case study - initial and final windfields at 200m above S.L. using available surface data and bicubic spline interpolation | 259 |
| Figure 6.28 | 6th February case study - trajectories in the final windfield produced using (a) inverse square interpolation and (b) bicubic spline interpolation | 260 |
| Figure 6.29 | 6th February case study - simulated plumes from the nuclear power stations | 261 |
| Figure 6.30 | 7th February case study - measured surface winds in the Bristol Channel region at 9.00 hours | 262 |
| Figure 6.31 | 7th February case study - final windfield at 200m above S.L. using uniform initial windfield | 262 |
| Figure 6.32 | 7th February case study - initial windfield at 10m above ground using available surface data and (a) inverse square or (b) bicubic spline interpolation | 263 |
| Figure 6.33 | 7th February case study - final windfield at 200m above S.L. using available surface data and inverse square interpolation | 264 |
Figure 6.34 7th February case study - trajectories in the final windfield produced using inverse square interpolation

Figure 6.35 7th February case study - measured surface winds in the Bristol Channel region at three-hourly intervals throughout the day

Figure 6.36 7th February case study - final windfields at 100m above S.L. for the time-series of runs

Figure 6.37 7th February case study - trajectories released from different heights in the time-series of windfields

Figure 6.38 7th February case study - trajectories released at different times in the time-series of windfields

Figure 6.39 7th February case study - simulated plume from Hinckley Point in the time-series of windfields

Figure 6.40 7th February case study - simulated plume from Oldbury in the time-series of windfields

Figure A.1 The WAFT code subroutine calling structure

Figure A.2 Hypothetical WAFT cell used to determine relationship between vertical velocity weighting factors and size of vertical velocities
<p>| Table 3.1 | 'Standard' reference run used in evaluating sources of error within WAFT | 128 |
| Table 3.2 | The effect of number of iteration cycles used on WAFT windfield accuracy | 128 |
| Table 3.3 | The effect of the value of over-relaxation parameter on the WAFT windfield accuracy | 129 |
| Table 3.4 | The effect of including a vertical density profile on the WAFT windfield | 129 |
| Table 3.5 | The importance of the initial windfield specification in determining the WAFT final windfield accuracy | 130 |
| Table 4.1 | The relationship between the size of the vertical velocity weighting factors used and the magnitude of the vertical velocities obtained in the final windfield | 167 |
| Table 4.2 | The ratio of vertical to horizontal velocities and the deflection of the crest-grazing stream-line in WAFT windfields | 167 |
| Table 4.3 | Comparison of the deflection of the crest-grazing streamline in windfields produced by the stable WAFT model with theoretical values | 168 |
| Table 6.1 | Computed Froude numbers for the sub-regions in the Bristol Channel in the stable flow runs | 273 |
| Table 6.2 | 7th February case study - input data for the time series runs | 274 |</p>
<table>
<thead>
<tr>
<th>NOTATION</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{mn}$</td>
<td>matrix of Lagrangian Multiplier equation coefficients</td>
</tr>
<tr>
<td>$AX_{ijk}$</td>
<td>general element of $A_{mn}$</td>
</tr>
<tr>
<td>$AY_{ijk}$</td>
<td>Lagrangian Multiplier equation coefficient associated with face (i,j,k)</td>
</tr>
<tr>
<td>$AZ_{ijk}$</td>
<td>normal to the x-axis (see Figure 2.4)</td>
</tr>
<tr>
<td>$A_o$</td>
<td>area of the open portion of a cell face</td>
</tr>
<tr>
<td>$A_t$</td>
<td>total area of a cell face</td>
</tr>
<tr>
<td>$b$</td>
<td>general right-hand side vector</td>
</tr>
<tr>
<td>$B_L$</td>
<td>lower triangular part of $A_{mn}$</td>
</tr>
<tr>
<td>$B_T$</td>
<td>transpose of $B_L$</td>
</tr>
<tr>
<td>$D$</td>
<td>diagonal part of $A_{mn}$</td>
</tr>
<tr>
<td>$E_{mn}$</td>
<td>some approximation to $A_{mn}$</td>
</tr>
<tr>
<td>$E^{-1}_{mn}$</td>
<td>the inverse of $E_{mn}$ and the pre-conditioning matrix of the Incomplete</td>
</tr>
<tr>
<td></td>
<td>Cholesky Conjugate Gradient method</td>
</tr>
<tr>
<td>$F$</td>
<td>Froude number</td>
</tr>
<tr>
<td>$f_{ijk}$</td>
<td>net flow [m^3s^-1] out of cell (i,j,k) in the initial windfield</td>
</tr>
<tr>
<td>$f$</td>
<td>right-hand side vector consisting of net outflow from cells in the initial</td>
</tr>
<tr>
<td></td>
<td>windfield</td>
</tr>
<tr>
<td>$f_{HL}$</td>
<td>'high-lying' factor used to divide a complex terrain region into separate</td>
</tr>
<tr>
<td></td>
<td>hills</td>
</tr>
<tr>
<td>$G_{ijk}$</td>
<td>mass-consistent constraint for cell (i,j,k)</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity [ms^-2]</td>
</tr>
<tr>
<td>$H_{BL}$</td>
<td>height of the planetary boundary layer [m]</td>
</tr>
<tr>
<td>$h$</td>
<td>hill height [m]</td>
</tr>
<tr>
<td>$h_c$</td>
<td>critical height in stable flow past a hill - flow below this height is</td>
</tr>
<tr>
<td></td>
<td>approximately horizontal [m]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$h_{SB}$</td>
<td>depth of sea-breeze inversion [m]</td>
</tr>
<tr>
<td>$I$</td>
<td>the identity matrix</td>
</tr>
<tr>
<td>$i,j,k$</td>
<td>position indices in the calculational mesh in the x,y and z axis direction respectively</td>
</tr>
<tr>
<td>$L_{ijk}$</td>
<td>Lagrangian Multiplier for cell (i,j,k)</td>
</tr>
<tr>
<td>$L$</td>
<td>Lagrangian Multiplier solution vector</td>
</tr>
<tr>
<td>$L$</td>
<td>horizontal length scale of terrain [m]</td>
</tr>
<tr>
<td>$m$</td>
<td>mean local slope of the topography</td>
</tr>
<tr>
<td>$N$</td>
<td>Brunt-Väisälä frequency [s$^{-1}$]</td>
</tr>
<tr>
<td>$n$</td>
<td>number of unsubmerged cells in the mesh</td>
</tr>
<tr>
<td>$NC_x$</td>
<td>number of mesh intervals in the x-axis direction</td>
</tr>
<tr>
<td>$NC_y$</td>
<td>number of mesh intervals in the y-axis direction</td>
</tr>
<tr>
<td>$NC_z$</td>
<td>number of mesh intervals in the z-axis direction</td>
</tr>
<tr>
<td>$O_{X_{ijk}}$</td>
<td>area of open portion of cell face (i,j,k) normal to x-axis [m$^2$]</td>
</tr>
<tr>
<td>$O_{Y_{ijk}}$</td>
<td>area of open portion of cell face (i,j,k) normal to y-axis [m$^2$]</td>
</tr>
<tr>
<td>$O_{Z_{ijk}}$</td>
<td>area of open portion of cell face (i,j,k) normal to z-axis [m$^2$]</td>
</tr>
<tr>
<td>$P$</td>
<td>population of a city</td>
</tr>
<tr>
<td>$q_n(r,r_n)$</td>
<td>weighting given to measurement made at station n at general position $\mathbf{r}$ in the initial windfield interpolation</td>
</tr>
<tr>
<td>$\mathbf{r} = (x,y,z)$</td>
<td>general position in WAFT calculational mesh [m]</td>
</tr>
<tr>
<td>$\mathbf{r}_n$</td>
<td>position vector of measurement station n [m]</td>
</tr>
<tr>
<td>$R_n$</td>
<td>scalar distance to measurement station n ($=</td>
</tr>
<tr>
<td>$R_c$</td>
<td>cutoff radius for radius of influence interpolation method [m]</td>
</tr>
<tr>
<td>$r_c$</td>
<td>cylinder radius (in two-dimensional flow over a half-cylinder) [m]</td>
</tr>
<tr>
<td>$T$</td>
<td>mean temperature [°C]</td>
</tr>
</tbody>
</table>
\( T_{ij} \) \( \text{topography height at mesh position (i,j) [m]} \)

\( TX_{ijk} \) \( \text{total area of cell face (i,j,k) normal to x-axis [m}^2] \)

\( TY_{ijk} \) \( \text{total area of cell face (i,j,k) normal to y-axis [m}^2] \)

\( TZ_{ijk} \) \( \text{total area of cell face (i,j,k) normal to z-axis [m}^2] \)

\( \Delta t \) \( \text{time increment used in generating streamlines ("streamline resolution" time) [s]} \)

\( U^\infty \) \( \text{upper triangular part of } A \)

\( U_F \) \( \text{potential flow free stream speed [m/s]} \)

\( \Delta u \) \( \% \text{ speed error in WAFT windfield} \)

\( u_n(\xi_n) \) \( \text{measured x-component of velocity at station n [m/s]} \)

\( \nu_{ijk} = (u_{ijk}, v_{ijk}, w_{ijk}) \) \( \text{final windfield velocity defined at mesh points [m/s]} \)

\( \nu_{o,ijk} = (u_{o,ijk}, v_{o,ijk}, w_{o,ijk}) \) \( \text{initial windfield velocity defined at mesh points [m/s]} \)

\( \nu_{ijk} = (\nu_{u,ijk}, \nu_{v,ijk}, \nu_{w,ijk}) \) \( \text{velocity adjustments at mesh point [m/s]} \)

\( \nu_{ijk} = (U_{ijk}, V_{ijk}, W_{ijk}) \) \( \text{final windfield defined as 'flow-average' velocities through cell faces [m/s]} \)

\( \nu_{o,ijk} = (U_{o,ijk}, V_{o,ijk}, W_{o,ijk}) \) \( \text{initial windfield defined as 'flow-average' velocities through cell faces [m/s]} \)

\( \nu_{ijk} = ( \nu_{u,ijk}, \nu_{v,ijk}, \nu_{w,ijk} ) \) \( \text{adjustments made to 'flow-average' velocities [m/s]} \)

WAFT \( \text{"Wind Adjusted For Topography" computer model} \)

\( X_i \) \( \text{position of mesh plane i normal to x-axis [m]} \)

\( Y_j \) \( \text{position of mesh plane j normal to y-axis [m]} \)

\( Z_k \) \( \text{position of mesh plane k normal to z-axis [m]} \)

\( XS_i \) \( \text{x-axis interval between mesh plane i and i+1 [m]} \)

\( YS_j \) \( \text{y-axis interval between mesh plane j and j+1 [m]} \)

\( ZS_k \) \( \text{z-axis interval between mesh plane k and k+1 [m]} \)
\( \mathbf{x} \) general solution vector

\( \alpha_{x_{ijk}} \) minimisation weighting factor applying to x-component adjustments

\( \alpha_{y_{ijk}} \) minimisation weighting factor applying to y-component adjustments

\( \beta_{ijk} \) minimisation weighting factor applying to z-component adjustments

\( \alpha^c_{x_{ijk}}, \alpha^c_{y_{ijk}}, \beta^c_{ijk} \) the part of \( \alpha_{x_{ijk}}, \alpha_{y_{ijk}}, \beta_{ijk} \) which account for the cell geometry

\( \alpha^*_{x_{ijk}}, \alpha^*_{y_{ijk}}, \beta^*_{ijk} \) the part of \( \alpha_{x_{ijk}}, \alpha_{y_{ijk}}, \beta_{ijk} \) which account for preferential weighting of velocity adjustment

\( \delta(z) \) turning of the wind from the free atmosphere direction [degrees]

\( \Theta \) direction of two-dimensional flow [degrees]

\( \Delta \Theta \) error in two-dimensional WAFT flow direction [degrees]

\( d\Theta/dz \) vertical potential temperature gradient [°C/m]

\( \rho(z) \) density of air [kg/m³]

\( \omega \) Successive-Over-Relaxation iteration parameter
CHAPTER 1

INTRODUCTION

In the event of a nuclear accident releasing radioactive material to the atmosphere the first consideration is to determine where the material is likely to travel and which areas may be contaminated. Dilution and dispersal of the material should also be assessed. Sampling of radiation levels in the air is obviously vital during a release, but effective numerical models also have an important role to play. They may be used for a variety of purposes if a release occurs:

- to estimate the doses received by the population as the accident proceeds
- to provide guidance on where sampling teams should be deployed
- as a quality control of the concentration measurements made
- to determine the rate at which pollutant is leaking from the source
- for post-accident analysis

A description of the use of numerical models during the Three Mile Island accident is given in Knox et al (1980).

These models should satisfy as many of the following requirements as possible:

- be as realistic as possible; ideally they should have been validated using field trials
- involve a minimum of subjective judgement
produce results within a reasonable time so that they are useful during the course of an accident

- use data which is readily available

- be predictive in order that future consequences can be foreseen and acted upon

The approach adopted when modelling pollutant transport depends upon the range being considered. At short distances, it is usually assumed that the meteorological conditions at the source still pertain. The pollutant plume is assumed to be embedded in a uniform flow over flat terrain. The flow velocity used is that measured at the source. Many simple models, such as the Gaussian Plume, are available for this short range modelling. The use of these models for pollutant modelling in the UK is described in Clarke (1979). However, topography and local thermal flows are likely to make these models increasingly unrealistic at greater distances (of the order of tens of kilometres). Similarly, as the distance of interest is increased still further (to several hundreds of kilometres), local flow distortions become less important, and diurnal and synoptic influences become dominant. Trajectory models such as MESOS, (ApSimon et al, 1985a,b) have been used successfully at these national scale distances.

This thesis is concerned with modelling mesoscale wind flow patterns, that is, at distances of the order of tens of kilometres from the source. This is the sort of distance at which it might be necessary to evacuate people in the event of a serious nuclear release, and therefore modelling pollutant dispersal at these scales is of particular interest. This is especially so in the UK, where several of the nuclear power stations are sited within fifty kilometres of large population centres. The meteorological data routinely available in such an area is likely to be rather limited. In the UK there would typically be ten or twenty meteorological stations in an area of this size. Moreover,
they will usually only measure wind flow at ten metres above the ground. Hence, the problem is to model what may be complex three-dimensional flow using the rather sparse data measured at the ground surface.

The current approach to mesoscale modelling in the UK is still to employ simple straight line models over these distances (for example, Clarke, 1979). At the other extreme, complicated fluid dynamics numerical models may be used to generate mesoscale flows. Anthes and Warner (1978), and Mahrer and Pielke (1975) describe two such sophisticated models. These generally consume large amounts of computer resources, and require boundary conditions to be specified which are usually unknown in practical situations. The model presented here falls in between these two approaches in terms of complexity and computing cost. It is particularly appropriate when topographic influences are dominant. The three-dimensional windfields produced may be used with a dispersion model to predict patterns of concentration and deposition for important nuclides such as the Noble gases and Iodine. A suitable Monte-Carlo dispersion model (ApSimon et al, 1984) has been developed at Imperial College.

The approach described in this thesis makes optimal use of the sparse data available. As well as the surface winds, some indication of the upper air flow (from radiosonde measurements or the geostrophic wind) is also required. As an initial guess, a wind field is interpolated (and extrapolated) from this available data. This initial wind field will not, in general, obey mass continuity; there will be artificial areas of convergence and divergence. Therefore, a final wind field is constructed which is as close as possible to the initial one, but which is mass consistent. Hence the final result is derived from the sparse input data available, but also models the effects of the underlying topography on the flow.

A similar mass-consistent model, MATHEW (Sherman, 1978),
forms the windfield generation part of the ARAC (Knox et al, 1980) system for accident analysis. Online systems based on MATHEW have been provided for use during accidental releases from American nuclear installations. Mass-consistent techniques have also been used in a variety of other recent mesoscale modelling works; for example, Anderson (1971) simulated flow in the Toronto region, and Bhumralkar et al (1980) examined potential wind energy sites. Thus the performance of mass-consistent models is of considerable practical importance.

A full description of the model and the way the final wind field is constructed is given in Chapter 2. The model is validated against potential flow theory over simple hill shapes in Chapter 3. Some of the sources of error in the method are also assessed there.

The physical content of the model is limited to the continuity equation. However, the thermal structure of the atmosphere has a large impact on how the land surface perturbs the air flow but is not included in the model formulation. In stable conditions air tends to flow around hills, whereas in neutral flow the air is more likely to rise over the top of the hill; this difference may have a large bearing on pollutant transport. Chapter 4 considers how the basic mass consistent method may be adapted to account for the atmosphere's vertical thermal structure. There are many other mesoscale phenomena which the simple model presented here cannot reproduce; some examples are discussed in Chapter 5.

A full validation of this model against tracer experiments is not possible as the data is not available. However, some case studies using real meteorological data are presented in Chapter 6 which illustrate some of the strengths and weaknesses of the mass consistent approach.
CHAPTER 2

THE WAFT MODEL

2.1 BASIC OUTLINE OF THE MODEL

2.1.1 Introduction

The WAFT (Windfield Adjusted For Topography) model is designed to produce a three-dimensional mesoscale windfield. Regions modelled using WAFT may have horizontal extents in the range several, to several hundreds, of kilometres. The vertical domain extents are typically between two hundred and two thousand metres. The windfield is defined over a three dimensional Cartesian mesh of points, with the ground surface as a lower impenetrable boundary. The three-dimensional flow field produced by WAFT may be used in conjunction with a dispersion model to predict how airborne pollutant may travel in the atmosphere. WAFT uses meteorological data normally available from weather stations in the region. This data is usually rather limited, and yet the flow patterns in mesoscale regions are often very complicated, being a complex interaction of large scale synoptic flow, and smaller scale flow features caused by hills, cities, water masses etc. It is envisaged that this type of model might be used 'on-line' at a nuclear site while an accidental release is occurring. It is therefore important that it does not require a great deal of computer resources.

The WAFT model is composed of two distinct stages as follows. Firstly, an initial estimate of the three-dimensional flow is obtained by interpolation from the available meteorological data. Typically, there are between ten and twenty weather stations in a mesoscale region which measure the wind velocity at ten metres above the surface. Generally, these stations will be unevenly distributed in the area. Additionally, some estimate of the high level (at

33
a height of say, one or two kilometres) flow will be available. These upper level winds may be obtained from radiosonde measurements or deduced from synoptic pressure charts. This interpolated windfield will be referred to as the initial windfield, and will be denoted by \((u_o, v_o, w_o)\). Note that, since only simple interpolation methods are used to create this initial windfield, it will not, in general, conserve mass. In particular, the flow near the ground surface will often have a considerable component normal to the land surface. This would not occur in the real atmosphere unless there was strong thermal convection present.

It is in the second phase that the final result, termed the final windfield, is computed. The final windfield is defined on the same mesh, and is denoted \((u, v, w)\). This final windfield is generated so as to be as close as possible to the initial interpolated windfield (in a least squares sense) subject to the constraint that it conserves mass. In the final windfield there is thus no velocity component normal to the land surface. In particular, whereas the initial windfield might flow into a hillside, the final windfield will be forced to flow either around or over the hill. The final windfield is therefore based on actual measurements made in the area, and yet models the effect of the underlying topography.

2.1.2 The Calculational Mesh and Topography Specification

The horizontal length scale will be no greater than several hundreds of kilometres, and so no account is made for the earth's curvature. The mesh points lie in horizontal planes which represent surfaces of equal height above Sea Level. The x and y-axes are parallel to the earth's surface with the z-axis vertically upwards. In the rest of this chapter calculational mesh cells will be referred to frequently. These cells are the smallest cuboidal volumes defined by the
mesh points. The concept of a cell is important since the mass consistency of the windfield is measured in terms of the net flow into each of these cells.

Other mass consistent models, for example Racher et al (1978) and Tuerpe et al (1978), have employed terrain following coordinates rather than the rectangular Cartesian coordinates used by WAFT. In that type of coordinate system the lowest layer of mesh points lie on the topographic surface. Cartesian coordinates were chosen for WAFT for the following reasons:

- the model formulation is much simpler if Cartesian geometry is employed. WAFT's main purpose is to evaluate mass consistent models rather than be used operationally. It is therefore desirable to keep the mathematical basis as simple as possible so that a clear understanding of the model's behaviour can be obtained.

- in order to take into account the atmosphere's thermal structure it is necessary to control the size of vertical velocities in the final windfield. This is much more convenient in Cartesian geometries.

- the cells all have the same simple cuboidal shape.

However, terrain following coordinates do have some important advantages, namely:

- many features of real atmospheric flows occur in relatively thin surface layers. These are more easily handled in terrain following coordinates. Examples of this type of phenomena are the large variation in windspeed which occur in the lowest 100 metres or so of the atmosphere, and the shallow drainage flows which occur in mountain/valley systems.
- terrain following coordinates allow a simple lower topographic boundary. The lowest layers of cells have an impenetrable boundary coincident with their bottom surfaces, and none of the other cells have any land in them.

- the upper boundary of the domain may be given an arbitrary shape if terrain following coordinates are adopted. This allows greater flexibility in the way atmospheric temperature inversions may be modelled.

The ground height at each (x,y) mesh position is specified in the input data for WAFT. When considering the flow around an idealised hill shape, these heights may be generated using a mathematical formula representing the hill shape. If a real terrain region is to be modelled these heights may be taken from maps, for example Ordnance Survey maps. For each column of cells the topography is defined by the four corner heights $T_{i,j}$, $T_{i+1,j}$, $T_{i+1,j+1}$, and $T_{i,j+1}$ as shown in Figure 2.1. It is not in general possible to construct a plane through four points, so an extra topography height, $T_{av}$, is defined over the middle of the base. This 'middle height' is computed as the mean of the four corner heights. The topography surface is defined by the four adjacent triangular planes, each having the 'middle height' of the cell and two corner heights as vertices. This surface representation is also depicted in Figure 2.1. A more accurate topography surface could be obtained if this 'middle height' was also supplied as a data value instead of being computed from the corner heights.

WAFT's topography representation gives a relatively smooth surface which is much more realistic than that used in the MATHEW model of Sherman (1978). In MATHEW the topography height at any (x,y) position may only be set equal to one of the vertical grid level heights. The land surface for each column of cells is a flat horizontal surface at one of the vertical grid levels; the terrain surface resulting from this is a 'building block' representation. Lewellen et al
(1982) in their review of MATHEW noted that this crude topography surface resulted in large velocity errors in a 'skin' (one or two grid lengths deep) over the land surface.

The mathematical formulation presented later on in this chapter, and the original WAFT code, allow the spacing between successive grid planes to be varied. For example, the vertical grid spacing may be made smaller near the ground surface to model thin surface layer flow. However, as work in Chapter 3 shows, it has proved impossible to produce reliable results using variable mesh spacing. Hence, all the runs presented in this thesis use a uniform mesh spacing in each axis (although the spacing used in each axis direction may be different).

2.1.3 Generating the Initial Windfield

In the experiments with WAFT that are presented in this thesis two different types of initial windfield, \((u_o, v_o, w_o)\), are used. In cases designed to validate the model against potential flow theory, an initial windfield with an identical velocity at every mesh point is used. The velocity is generally set to the potential flow free stream velocity. It is then possible to see how the hill deforms this uniform initial flow field.

When WAFT is used to model flows in the real atmosphere, measurements from meteorological stations in the area are used to create the initial windfield. The data consists of measured winds at ten metres above the ground and a specification of the wind flow at, or above, the top of the domain. This upper level wind vector is assumed to be uniform across the domain. Interpolation and extrapolation techniques are then used to generate the initial windfield \((u_o, v_o, w_o)\), defined at every mesh point from the limited amount of raw data available. This is done in stages as follows. Firstly, a velocity is calculated at every \((x, y)\) mesh position at ten metres above the ground surface using
the ten metre station measurements and a two dimensional interpolation/extrapolation scheme. This interpolation scheme may be a rather simple one whereby the velocity at each (x,y) position is a distance weighted average of the measurements, or a more sophisticated surface fitting technique such as bicubic splines may be employed. The various interpolation schemes used in WAFT, and their respective merits, are discussed in detail in Section 3.3. At the end of this stage each vertical mesh line has a velocity defined on it at ten metres above the ground and the high level velocity defined at, or above, the top of the domain.

In the second stage, some assumption about the way the wind speed and direction varies with height is used to interpolate between the surface and high level vectors. A description of the assumed vertical profile is given in Section 5.2. At this point, the initial windfield is defined at all mesh points lying above the ground surface.

It is important to emphasise that because the final windfield is produced by minimally adjusting the initial windfield, the form of the initial windfield has a large influence on the final result. Specific localised flows such as urban heat island circulations or mountain/valley thermal winds can, in principle, be incorporated into WAFT by using sub-models to generate these flows as part of the initial windfield specification.

In general, the initial windfields created by WAFT have zero vertical velocities, that is, $w_0 = 0$. In a practical situation there is usually little information about the scale of the vertical velocities which is one reason why they are set to zero initially. More importantly though, setting them to zero allows WAFT to model the vertical temperature structure of the atmosphere. To do this it is necessary to influence the size of the vertical velocities in the final windfield. In WAFT it is only possible to directly control the difference between the initial and the
final windfield velocities. Hence, setting the initial vertical velocities to zero provides a mechanism of controlling the size of the final vertical velocities. If sub-models are used to generate specific local flow features then it might be appropriate to have non-zero initial vertical velocities.

2.1.4 Generating the Final Mass Consistent Windfield

Having generated the initial windfield, the next task is to construct the final windfield. This is as close as possible to the initial windfield whilst being mass consistent. Other models have relied upon the techniques of variational calculus to construct this final windfield, MATHEW (Sherman, 1978), MASCON (Dickerson, 1978), and the model of Bhumralkar et al (1980) provide examples of this type of approach. In these calculus treatments an integral of the form

\[ \iiint (u-u_o)^2 + (v-v_o)^2 + (w-w_o)^2 \, dV \]  \hspace{1cm} 2.1.4(1)

where

\((u,v,w)\) is the final windfield

\((u_o,v_o,w_o)\) is the initial windfield

\(\alpha, \beta\) are weighting coefficients

must be minimised subject to the constraint of mass conservation. This mass consistency constraint may be expressed mathematically as

\[ \text{div} \left( \sigma \mathbf{v} \right) = 0 \]

where \(\sigma\) is density [kg m\(^{-3}\)]  \hspace{1cm} 2.1.4(2)

The models are only used in the lowest kilometre or so of the atmosphere, and so constant density may be assumed. Hence 2.1.4(2) becomes

\[ \text{div} \left( \mathbf{v} \right) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]  \hspace{1cm} 2.1.4(3)
The minimisation of the integral 2.1.4(1) subject to this constraint may be undertaken in two ways. Firstly, there is the 'weak constraint' approach in which 2.1.4(2) is satisfied only approximately. This is realised by minimising a composite integral of the form

\[ \int \int \int \alpha (u-u_0)^2 + \alpha (v-v_0)^2 + \beta (w-w_0)^2 + \]
\[ I \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2 \, dV \] 2.1.4(4)

The parameter I controls how much weight is attached to reducing the divergence compared to keeping the final windfield as close as possible to the initial windfield. The other approach is the more widely used 'strong constraint' method. Here the constraint 2.1.4(3) is satisfied exactly. This is achieved by introducing a new function, \( L(x,y,z) \), which is known as the Lagrangian Multiplier. The following integral is then minimised

\[ \int \int \int \alpha (u-u_0)^2 + \alpha (v-v_0)^2 + \beta (w-w_0)^2 + \]
\[ L \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \, dV \] 2.1.4(5)

Details of this are given in Courant and Hilbert (1953). Minimising the integral 2.1.4(5) is equivalent to solving the following four differential equations (again, see Courant and Hilbert)

\[ u = u_0 + \frac{1}{2\alpha} \frac{\partial L}{\partial x} \] 2.1.4(6)
\[ v = v_0 + \frac{1}{2\alpha} \frac{\partial L}{\partial y} \] 2.1.4(7)
\[ w = w_0 + \frac{1}{2\beta} \frac{\partial L}{\partial z} \] 2.1.4(8)
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]  
2.1.4(9)

These four equations are known as the associated Euler-Lagrange equations. They may be reduced to a second order differential equation in \( L \), namely

\[ \frac{\partial^2 L}{\partial x^2} + \frac{\partial^2 L}{\partial y^2} + \frac{\partial^2 L}{\partial z^2} + \beta \frac{\partial^2 L}{\partial x \partial y} + \alpha \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = 0 \]  
2.1.4(10)

In the MATHEW model (Sherman, 1978) this differential equation is discretised using a finite difference scheme. The crude topography representation used in that model (described previously in Section 2.1.2) allows a simple differencing scheme to be used. The resulting set of simultaneous linear algebraic equations is solved using a Successive-Over-Relaxation numerical method. The finite difference formulations of equations 2.1.4(6), 2.1.4(7), and 2.1.4(8) are then used to generate the required final windfield. Terrain following coordinates allow a better representation of the topography but mean that the above equations become rather more complicated (see Racher et al, 1978). A further refinement is to use finite element rather than finite difference methods to solve the associated Euler-Lagrange equations; for example, see Caneill et al (1982), or Tuerpe et al (1978).

In contrast to the methods described above, WAFT does not employ variational calculus. Instead, the domain is divided up into a finite number of calculational volumes; these are the cells referred to previously. WAFT aims for zero net flow into each of these cells in the final windfield. In this way, a discrete condition of zero flow into each cell replaces the continuous spatial derivative condition given by 2.1.4(3). The mathematics of this technique are much simpler than those associated with the terrain following or finite element techniques mentioned above, and yet the terrain surface may be represented fairly accurately;
certainly, much more appropriately than the 'building block' surface used in MATHEW. The detailed mathematical basis of the model will be described in the next section.

2.2 THE WAFT MODEL FORMULATION

2.2.1 The Calculational Cell and its Associated Velocities

If a complex terrain surface is being modelled, it will not, in general, intersect the mesh in a regular fashion. In a given column of cells, there may be some cells which lie completely below the ground surface. These cells will be termed submerged cells and they play no role in the calculations. Those cells which are not submerged may be divided into two types. There are those which do not intersect the ground surface at all; these will be referred to as regular cells. Finally, there are those cells which are partially filled; these are the irregular cells. These three types of cell, submerged, regular unsubmerged, and irregular unsubmerged are illustrated in the column of cells depicted in Figure 2.2

Each mesh point within the grid is identified by three indices, \((i,j,k)\), \(i\) being the x-axis index, \(j\) the y-axis, and \(k\) the z-axis index. A typical cell showing the notation to be used in the rest of this section is shown in Figure 2.3. The cell shown is irregular since it contains some land. Figure 2.3(a) shows the three components of the initial windfield defined at each of the mesh points.

The WAFT procedure centres upon the concept of a calculational cell and the net flow into the cell. The WAFT equations therefore deal with average velocities flowing across cell walls rather than velocities defined at points. These flow-average velocities are depicted in Figure 2.3(b). The flow-average velocity across a cell face is defined as follows
\[ U_0 = \frac{A_o \cdot u_o}{A_t} \quad 2.2.1(1) \]

where

- \( A_o \) is the area of the open portion of the cell wall defined by the points \((i, j, k), (i, j, k+1), (i, k+1, k+1),\) and \((i, j+1, k)\)

- \( A_t \) is the total area of the cell wall defined by the points \((i, j, k), (i, j, k+1), (i, j+1, k+1),\) and \((i, j+1, k)\), so that \( A_t = YS_j \cdot ZS_k \)

- \( u_o \) is the mean x-component of initial velocity over the open portion of the cell wall defined by the points \((i, j, k), (i, j, k+1), (i, j+1, k+1),\) and \((i, j+1, k)\)

- \( U_0 \) is the initial flow-average velocity through the cell wall defined by the points \((i, j, k), (i, j, k+1), (i, j+1, k),\) and \((i, j+1, k+1)\)

Note that for cell walls which have no land obscuring any part of them (for example, the top cell face in Figure 2.3(b)) the flow-average velocity is the average velocity over the cell face, that is \( W_o = w_o \). Conversely, if the cell wall is completely below the ground surface then the flow-average velocity is zero. The net flow through a given cell face into a cell may be defined very simply using these flow-average velocities; it is simply the product of the total cell wall area and the flow-average velocity. For example, the initial net flow through the left hand face of the cell in Figure 2.3 is \( YS_j \cdot ZS_k \cdot U_{ijk} \). WAFT is formulated using these flow-average velocities because it is so easy to calculate net flows into cells using them. The following sections illustrate how this leads to a faster numerical solution method.
Simple interpolation techniques are used to calculate the average velocities over the open portion of the cell faces from the initial velocities defined at mesh points. Once this is done, the initial field flow-average velocities may be calculated. These initial flow-average velocities are denoted by \( U_0, V_0, \) and \( W_0 \). It is then necessary to create a corresponding final flow-average velocity field, denoted by \( U, V, \) and \( W \). The adjustments made to the initial field to generate the final field will be defined as follows

\[
\begin{align*}
U &= U_0 + \xi U \\
V &= V_0 + \xi V \\
W &= W_0 + \xi W
\end{align*}
\]

These velocity adjustments are chosen so that the final windfield is mass consistent; the calculation of these adjustments forms the central part of the WAFT code, and it is dealt with in the following subsection.

2.2.2 The Adjustment Process

The final flow field, \( U, V, W \), must be as close as possible to the initial flow field subject to the constraint that it conserves mass. The former requirement is expressed as the minimisation of the following sum

\[
S = \sum \alpha_{x_{ijk}} S_{ij}^2 + \alpha_{y_{ijk}} S_{jk}^2 + \beta_{ijk} S_{jk}^2
\]

The mass consistent constraint is expressed, for each computational cell as

\[
G_{ijk} = (U_{i+1jk} - U_{ijk}).YS_j.ZS_k + \\
(V_{ij+1k} - V_{ijk}).XS_i.ZS_k + \\
(W_{ijk+1} - W_{ijk}).XS_i.YS_j = 0
\]

The coefficients \( \alpha_{x_{ijk}}, \alpha_{y_{ijk}}, \) and \( \beta_{ijk} \) allow one to attach
different penalties to the adjustment of different velocities throughout the domain. For example, if it were felt that the initial windfield was a particularly good estimate of the actual windfield in a certain part of the domain, then the coefficients in that part of the domain could be set relatively large. WAFT would then tend to modify the initial windfield elsewhere to achieve a mass consistent flow. Alternatively, the coefficients corresponding to a particular coordinate axis direction might be increased, so that the windfield would be adjusted preferentially in the other directions. As later chapters will show, this may be used to model the thermal stratification of the atmosphere. Since WAFT is only used to model the lowest kilometre or so of the atmosphere the density is assumed to be constant; hence, no density terms appear in equation 2.2.2(2).

The equations in 2.2.1(2) may be used to rewrite 2.2.2(2) as

\[ G_{ijk} = (U^0_{ijk} + \Delta U_{ijk} - U^0_{ijk} - \Delta U_{ijk}) \cdot YS_j \cdot ZS_k + \]

\[ (V^0_{ijk} + \Delta V_{ijk} - V^0_{ijk} - \Delta V_{ijk}) \cdot XS_j \cdot ZS_k + 2.2.2(3) \]

\[ (W^0_{ijk} + \Delta W_{ijk} - W^0_{ijk} - \Delta W_{ijk}) \cdot XS_j \cdot YS_j = 0 \]

This is recast as

\[ G_{ijk} = (\Delta U_{ijk} - \Delta U_{ijk}) \cdot YS_j \cdot ZS_k + \]

\[ (\Delta V_{ijk} - \Delta V_{ijk}) \cdot XS_j \cdot ZS_k + 2.2.2(4) \]

\[ (\Delta W_{ijk} - \Delta W_{ijk}) \cdot XS_j \cdot YS_j + f_{ijk} = 0 \]

where \( f_{ijk} \) is the next flow out of cell \((i,j,k)\) in the initial windfield, that is

\[ f_{ijk} = (U^0_{ijk} - U^0_{ijk}) \cdot YS_j \cdot ZS_k + \]

45
If there are \( n \) unsubmerged cells in the domain, then \( 2.2.2(1) \) is summed over \( n \) cells subject to \( n \) constraints of the form \( 2.2.2(4) \). This is achieved using the method of Lagrangian Multipliers; see pp.164-167 of Courant and Hilbert (1953). For each of the \( n \) constraints, a new variable, \( L_{ijk} \), the Lagrangian Multiplier is introduced. A new sum \( \emptyset \), must then be minimised, where \( \emptyset \) is given by

\[
\emptyset = S + \sum L_{ijk} \cdot G_{ijk}
\]

Since there is one flow constraint for each cell, there is correspondingly one Lagrangian Multiplier defined for each cell.

The sum in \( 2.2.2(6) \) is minimised by differentiating with respect to \( S_{ijk} \) and equating to zero, to give

\[
2.\alpha \, x_{ijk} \cdot S_{ijk} - YS_j \cdot ZS_k \cdot (L_{ijk} - L_{i-1jk}) = 0
\]

Similarly, differentiating with respect to \( V_{ijk} \) and \( W_{ijk} \) yields

\[
2.\alpha \, y_{ijk} \cdot S_{ijk} - XS_i \cdot ZS_k \cdot (L_{ijk} - L_{ij-1k}) = 0
\]

and

\[
2.\beta \, z_{ijk} \cdot S_{ijk} - XS_i \cdot YS_j \cdot (L_{ijk} - L_{ijk-1}) = 0
\]

Rewriting \( 2.2.2(7) \) for two adjacent cells gives

\[
S_{ijk} = YS_j \cdot ZS_k \cdot (L_{ijk} - L_{i-1jk})
\]

and
\[ S_{i+1jk} = \frac{Y_{i}Z_{k}}{2\alpha_{x_{i}j}x_{i}k} \]  

Subtracting equation 2.2.2(10) from 2.2.2(11), and multiplying by \( Y_{i}Z_{k} \) leads to

\[ (S_{i} - S_{i}X_{x}Y_{i}Z_{k}) \]

2.2.2(12)

\[ \frac{(Y_{i}Z_{k})^{2}}{2} \left( \frac{L_{i}L_{i} - L_{i}L_{i}}{\alpha_{x_{i}}x_{i}k} - \frac{L_{i} - L_{i}}{\alpha_{x_{i}}x_{i}k} \right) \]

Similar expressions may be written for the other components

2.2.2(13)

\[ \frac{(X_{i}Z_{k})^{2}}{2} \left( \frac{L_{i} - L_{i} - L_{i}}{\alpha_{y_{i}j}y_{i}j} - \frac{L_{i} - L_{i}}{\alpha_{y_{i}j}y_{i}j} \right) \]

and

2.2.2(14)

\[ \frac{(X_{i}Y_{j})^{2}}{2} \left( \frac{L_{i} - L_{i} - L_{i} - L_{i}}{\beta_{i}i} - \frac{L_{i} - L_{i}}{\beta_{i}i} \right) \]

Substituting 2.2.2(12), 2.2.2(13), and 2.2.2(14) into 2.2.2(4) yields

2.2.2(15)

\[ \frac{(Y_{i}Z_{k})^{2}}{2} \left( \frac{L_{i} - L_{i} - L_{i} - L_{i}}{\alpha_{x_{i}j}x_{i}k} - \frac{L_{i} - L_{i}}{\alpha_{x_{i}j}x_{i}k} \right) \]

2.2.2(15) may be rewritten as

47
\[ \begin{align*}
A_{i,j,k} \cdot L_{i,j,k} & - A_{i+1,j,k} \cdot L_{i+1,j,k} - A_{i,j,k} \cdot L_{i-1,j,k} \\
A_{i,j+1,k} \cdot L_{i,j+1,k} & - A_{i,j,k} \cdot L_{i,j-1,k} \\
A_{i,j,k+1} \cdot L_{i,j,k+1} & - A_{i,j,k} \cdot L_{i,j,k-1} = f_{i,j,k}
\end{align*} \]

where

\[ \begin{align*}
A_{i+1,j,k} & = \frac{(Y_{S_{j}} \cdot Z_{S_{k}})^2}{2 \alpha_{x_{i+1,j,k}}} \\
A_{i,j,k} & = \frac{(Y_{S_{j}} \cdot Z_{S_{k}})^2}{2 \alpha_{x_{i,j,k}}} \\
A_{i,j+1,k} & = \frac{(X_{S_{i}} \cdot Z_{S_{k}})^2}{2 \alpha_{y_{i,j+1,k}}} \\
A_{i,j,k} & = \frac{(X_{S_{i}} \cdot Z_{S_{k}})^2}{2 \alpha_{y_{i,j,k}}} \\
A_{i,j,k+1} & = \frac{(X_{S_{i}} \cdot Y_{S_{j}})^2}{2 \beta_{i,j,k+1}} \\
A_{i,j,k} & = \frac{(X_{S_{i}} \cdot Y_{S_{j}})^2}{2 \beta_{i,j,k}}
\end{align*} \]

There is one equation of the form 2.2.2(16) for each unsubmerged cell in the mesh. The Lagrangian Multiplier, \( L_{i,j,k} \), associated with cell \((i,j,k)\) is related to the Lagrangian Multiplier in the six neighbouring cells; that is, four in the cells across each side cell face, one in the cell directly below, and one in the cell above. Each of the coefficients \( A_{i+1,j,k}, A_{i,j,k}, A_{i,j+1,k}, A_{i,j,k}, A_{i,j,k+1}, \) and \( A_{i,j,k} \) is associated with one of the cell faces. These relationships are depicted in Figure 2.4.

The problem is thus reduced to solving a set of simultaneous linear equations for the unknown Lagrangian Multiplier values. The solution of this set of equations is dealt with in Section 2.3. Note that since WAFT is formulated in terms of flow average velocities the coefficients of equation 2.2.2(16) given in 2.2.2(17) do not explicitly involve the areas of the open portions of the cell walls. This is
potentially a big advantage if an iterative method is used to solve the equations. This would generally require these coefficients to be recalculated during each iteration; for example, this is true for the Successive-Over-Relaxation method. It is much easier to compute the total area of the cell wall, rather than the portion of the wall that is open.

When the set of equations has been solved for the Lagrangian Multiplier, \( L_{ijk} \), equations such as 2.2.2(10) may be used to calculate the velocity adjustments \( \delta U \), \( \delta V \), and \( \delta W \). Hence, the final velocity field may be generated.

2.2.3 Boundary Conditions

There are two types of boundary enclosing the air mass under consideration. There is the lower terrain boundary through which air is not allowed to flow. Secondly, there are the 'open' boundaries of the domain above the topography through which air flows into and out of the domain. A cell which has one or more faces coincident with these outer open boundaries will have an associated Lagrangian Multiplier equation (equation 2.2.2(16)) involving the Lagrangian Multiplier of a nominal cell lying outside the calculational domain. The Lagrangian Multiplier for these nominal cells lying outside the open domain boundaries are arbitrarily set equal to zero. Note that in general this means there will be adjustments made to the velocities flowing through these open domain boundaries.

An alternative condition at these open domain boundaries would be to require that the Lagrangian Multiplier for cells lying outside the domain are held equal to the value of the Lagrangian Multiplier of their neighbour just inside the domain. This means that the velocity across the boundary face will not be adjusted. Hence, the flow through the open domain boundaries will be maintained in the adjustment process. However, this is of limited use since the flows into and out of the domain would not generally be known in a
practical situation. Moreover, even if it was required to fix the flow into the domain, the previous approach may also be used to do this. The weighting factors discussed in the previous subsection could be set to levy large penalties on the adjustment of velocities across the open domain boundaries.

The other type of boundary is that formed by the earth's surface at the bottom of the domain. At this lower impenetrable boundary there will be some irregular cells which have one or more of their faces completely under the ground surface. The boundary conditions here must ensure that the final flow-average velocity across such faces is zero. The initial flow-average velocity across such a face is set to zero so it is therefore necessary to have a zero velocity adjustment associated with these completely blocked faces. Referring to equation 2.2.2(10) (and analogous equations for the other adjustment components) it is clear that if the two Lagrangian Multiplier associated with the cells either side of the blocked face are equal then the velocity adjustment across this face will be zero. However there is a problem with adopting this kind of approach, which is illustrated by the cells in Figure 2.5. Three cell faces which are completely under the topography (shown blocked out) are depicted in this figure. If one was to adopt the above approach to guarantee zero velocity adjustments across these blocked faces then one would require that:

\[ L_{ijk} = L_{ijk-1} \]

\[ L_{i-1jk-1} = L_{ijk-1} \] 2.2.3(1)

\[ L_{i+1jk} = L_{ijk-1} \]

Generally there is one equation of the form 2.2.2(16) for each cell in the mesh, but for cell \((i,j,k-1)\) there would now effectively be three associated equations, that is, equations 2.2.3(1). Using this type of boundary condition, therefore, leads to a system of equations with more equations than unknowns.
This problem may be circumvented if one alters the coefficients in equation 2.2.2(16). For example, applying equation 2.2.2(16) to the cell (i,j,k) in Figure 2.5 one sees that setting AZ_{ijk} equal to zero is entirely equivalent to setting L_{ijk-1} equal to L_{ijk}; the Lagrangian Multiplier equation is exactly the same. Thus the coefficients in the Lagrangian Multiplier equation (eqn 2.2.2(16)) take the values given in equation 2.2.2(17) unless they are associated with a completely blocked cell face in which case they are set to zero. It is then not necessary to define Lagrangian Multiplier values for any submerged cells and the system of equations has the same number of equations as unknowns - one for each unsubmerged cell in the mesh. The boundary conditions used at the lower terrain boundary thus ensure zero velocity adjustments across completely blocked cell faces.

2.3 SOLVING THE SET OF SIMULTANEOUS LINEAR EQUATIONS

2.3.1 Characteristics of the WAFT Set of Equations

The system of n (n is the number of unsubmerged cells) simultaneous linear algebraic equations in n unknown Lagrangian Multipliers that must be solved in order to generate the final windfield may be expressed in matrix notation as

\[ A \mathbf{L} = \mathbf{f} \]  

2.3.1(1)

where

\[ A \] is the nxn matrix of equation coefficients described in equations 2.2.2(17)

\[ \mathbf{L} \] is the solution vector of Lagrangian Multipliers

\[ \mathbf{f} \] is the right-hand side vector consisting of the initial net flows out of each cell
Figure 2.6 shows the matrix $A$ written out in full for a WAFT mesh of twenty-six unsubmerged cells. The cell ordering system employed in WAFT is illustrated in the top part of the figure. The particular form of the equation 2.2.2(16) and the cell ordering system used means that the matrix $A$ has the following properties.

The matrix $A$ is very sparse; that is, most of the elements, $a_{mn}$, are zero. This is because the Lagrangian Multiplier equation for a given cell only involves the Lagrangian Multiplier in, at most, six adjacent cells. There are therefore no more than seven non-zero elements in any matrix row (or column). For a typical problem $n$ may be of the order of ten thousand, so that of the hundred million (that is, $n^2$) elements of $A$, only about seventy thousand are non-zero. Any solution method which kept all of the elements of $A$ in store would thus be extremely wasteful, as well as completely impractical for the large values of $n$ that will be used.

The matrix $A$ is symmetric, that is $a_{mn} = a_{nm}$. This follows because the coefficient relating Lagrangian Multipliers in adjacent cells is a function of the common cell face only. For example, consider two adjacent cells numbered $N_1$ and $N_2$. The Lagrangian Multiplier equation for cell $N_1$ has a coefficient $a_{N_1N_2}$ multiplying the Lagrangian Multiplier of the adjacent cell $N_2$. Similarly the Lagrangian Multiplier equation for cell $N_2$ has a coefficient $a_{N_2N_1}$ multiplying the Lagrangian Multiplier in cell $N_1$. These two coefficients are, from equations 2.2.2(17), both given by

$$a_{N_1N_2} = a_{N_2N_1} = \frac{(\text{area of common cell face})^2}{2.(\text{cell face weighting factor})}$$

This symmetry property means that only either the upper or lower triangular portion of the matrix $A$ need be considered.
Since flow-average velocities are used in the model formulation, the matrix elements, given by equations 2.2.2(17), can easily be calculated from a cell's dimensions and its associated weighting factors. This suggests it might not be necessary to store any of the non-zero elements of $A$ since they can be derived simply whenever they are required. However, (as will be seen in Chapter 3) it is, in fact, necessary to involve the open area of the cell walls (something which is not so readily calculated for an irregular cell) in the weighting factors. The best compromise between storage requirements and execution time is obtained by storing those matrix coefficients that are associated with irregular cells, and by calculating those for regular cells as they are needed.

The non-zero off-diagonal elements of $A$ lie in six distinct bands, three either side of the principal diagonal elements. This banded structure arises because of the Cartesian mesh that is employed and because of the fact that each Lagrangian Multiplier is only related to its six neighbouring values. The cell numbering system adopted means that the two outer bands correspond to adjacent cells in the x-direction, the two innermost bands relate to adjacent cells in the z-axis direction, whilst the other two bands are associated with the adjacent cells in the y-axis direction. The maximum bandwidth of a banded matrix is defined as the maximum number of elements between the two outer bands in any row. The matrix $A$ has a maximum bandwidth of $2 \cdot N_{C_y} \cdot N_{C_z}$ where $N_{C_y}$ is the number of mesh intervals along the y-axis of the mesh, and similarly $N_{C_z}$ is the number of mesh intervals along the z-axis of the mesh. The matrix $A$ has a variable bandwidth; that is, the bands are not straight (as can readily be seen from Figure 2.6). The bandwidth varies because different columns of cells have different numbers of
unsubmerged cells in them. The banded structure of matrix $\mathbf{A}$ means that not only are most of the matrix elements easily calculated but the positions of the non-zero elements of $\mathbf{A}$ are also easily obtained. In a sparse matrix with less structure it would generally be necessary to store the positions of the non-zero elements.

Examining the form of the Lagrangian Multiplier equations, 2.2.2(16), and its coefficients in 2.2.2(17), it is clear that the diagonal elements of $\mathbf{A}$ are all positive whilst the off-diagonal elements are either negative or zero. Additionally the modulii of the sum of all the off-diagonal elements in any row is less than or equal to the value of the diagonal element in that row.

From the foregoing it is apparent that matrix $\mathbf{A}$ possesses many special properties. It is obviously desirable to choose a solution method which takes advantage of these special features.

2.3.2 Solution Methods

There are essentially two basic approaches which may be used to solve a large set of simultaneous equations. One may either use a direct solution method which gives an exact answer if exact arithmetic is used, or one may use an iterative technique which only gives an approximate solution. An overview of the different methods available is given in Fox (1964), whilst some of the more recent developments are given in Duff (1980).

Direct solutions are formalised methods of eliminating variables between the equations in much the same way one would use for a small number of equations. The different direct solution methods are all basically similar. One of the simplest ones, Gaussian elimination, will be presented here. Consider $\mathbf{A} \mathbf{x} = \mathbf{b}$ which may be written in full as
The first step consists of subtracting \( a_{21}/a_{11} \) times row 1 from row 2, and \( a_{31}/a_{11} \) times row 1 from row 3, and \( a_{41}/a_{11} \) times row 1 from row 4. This gives the following new set of equations of the form

\[
\begin{pmatrix}
  a'_{11} & a'_{12} & a'_{13} & a'_{14} \\
  0 & a'_{22} & a'_{23} & a'_{24} \\
  0 & a'_{32} & a'_{33} & a'_{34} \\
  0 & a'_{42} & a'_{43} & a'_{44}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix}
= \begin{pmatrix}
b'_1 \\
b'_2 \\
b'_3 \\
b'_4
\end{pmatrix} \tag{2.3.2(2)}
\]

In this first step, multiples of row 1 are subtracted from the rows below so that column 1 is filled with zeros; row 1 is the pivotal row. In the second step, row 2 is the pivotal row, and the second column below row 2 is filled with zeros. This process is continued until the bottom row is reached when the system 2.3.2(1) will have become

\[
\begin{pmatrix}
a^{*11} & a^{*12} & a^{*13} & a^{*14} \\
0 & a^{*22} & a^{*23} & a^{*24} \\
0 & 0 & a^{*33} & a^{*34} \\
0 & 0 & 0 & a^{*44}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix}
= \begin{pmatrix}
b^{*1} \\
b^{*2} \\
b^{*3} \\
b^{*4}
\end{pmatrix} \tag{2.3.2(3)}
\]

\( x_4 \) may then be obtained directly from

\[
a^{*44}x_4 = b^{*4} \tag{2.3.2(4)}
\]

Knowing \( x_4 \), \( x_3 \) may be obtained from

\[
a^{*33}x_3 + a^{*34}x_4 = b^{*3} \tag{2.3.2(5)}
\]

This process of back substitution is continued until all the elements of the exact solution vector, \( x \), are calculated.

As mentioned above, only approximately \( 7n \) (n is the number
of unsubmerged cells) out of the $n^2$ elements of the matrix $A$ are non-zero in the WAFT system. However, as the Gaussian elimination proceeds, many of the zero elements between the bands become non-zero. This process is termed fill-in. For a symmetric banded matrix such as $A$ this means that the storage requirement for a direct method such as Gaussian elimination is roughly equal to half the maximum matrix bandwidth times the matrix dimension, $n$. For the situations that WAFT is required to model, $n$ may be 10,000 and the half bandwidth would be of the order of 300. This means that the memory requirement of a simple direct solution program would be prohibitively high.

In fact, there are some techniques available (for example Irons, 1970, and Hood, 1976) which only load sections of $A$ into central memory in sequence so that the central memory requirement is reduced (these are known as 'out-of-core solvers'). However, even these methods require relatively large amounts of central memory, and often involve time consuming transfers of data to and from peripheral storage. The large memory requirements for direct solution methods rule them out for WAFT.

In an iterative solution method, an initial guess at the solution is made, and then some algorithm which improves the solution is used repeatedly until a solution of the desired accuracy is achieved. Varga (1962) presents many of the available methods. An exact solution will not be obtained, but this is not a significant problem bearing in mind the modest accuracy expected in the WAFT final windfield.

The iterative process may be expressed generally as

$$ x_r^{r+1} = s_r + \tilde{O}_r (x_r^r) \quad 2.3.2(6) $$

where

- $x_r^r$ is the estimate after $r$ iterations
- $x_r^{r+1}$ is the (better) estimate after $r+1$ iterations
- $\tilde{O}_r$ is a function of $x_r^r$ and may also depend on $r$
If both $s^r$ and $\phi$ are independent of $r$ the iterative technique is said to be stationary. Different iterative methods arise from different choices of $s^r$ and $\phi_r$.

The simplest version of WAFT uses a stationary iterative technique known as Successive-Over-Relaxation. This runs reliably and economically in many of the cases presented in this thesis. However, there are some circumstances when this method converges impossibly slowly, and it is therefore necessary to adopt a more sophisticated non-stationary technique, Conjugate Gradients. This Conjugate Gradient method may be improved considerably with the addition of an accelerating technique known as Incomplete Cholesky Preconditioning. These different methods used in WAFT will be discussed in the following subsections.

2.3.3 The Successive-Over-Relaxation Method

The prime virtue of this method is its simplicity. It is also known as the extrapolated Gauss-Seidel scheme, and is the iterative technique adopted in MATHEW, Sherman (1978).

The matrix $A$ is decomposed into three parts as follows:

$$A = B + D + U$$  \hspace{1cm} 2.3.3(1)

where

- $B$ is the lower triangular part (with a zero diagonal) of $A$.
- $D$ is the diagonal part of $A$.
- $U$ is the upper triangular part (again with a zero diagonal) of $A$.

Since $A$ is symmetric then $U = B^T$, the transpose of $B$.

The stationary iterative scheme is then given by
\[(\omega^{-1} \cdot D + B) \cdot x^{r+1} = b - U \cdot x^r - (1 - \omega^{-1}) \cdot D x^r \]

where

- \(b\) is the right-hand side vector
- \(x^r\) is the estimate after \(r\) iterations
- \(x^{r+1}\) is the next estimate after \(r+1\) iterations
- \(\omega\) is the over-relaxation parameter

The over-relaxation parameter lies between 1.0 and 2.0, and within this range there will be some optimum value which produces the most rapid convergence. It is rather difficult to determine this optimum value for a general matrix and it is often selected by trial and error. Although equation 2.3.3(2) appears to be rather complicated it may be coded rather simply in WAFT. Equation 2.3.3(2) is implemented in WAFT by cycling through the cells in order, applying the following algorithm to update the Lagrangian Multipliers:

\[
L_{ij}^{r+1} = \frac{\omega}{A_{ijk}} \left\{ AX_{ijk} \cdot L_{ijk}^r + AX_{ijk} \cdot L_{ijk}^{r+1} + AY_{ijk} \cdot L_{ijk}^r + AY_{ijk} \cdot L_{ijk}^{r+1} + AZ_{ijk} \cdot L_{ijk}^r + AZ_{ijk} \cdot L_{ijk}^{r+1} + 2.3.3(3) \right\}
\]

where \(AX_{ijk}, AY_{ijk}, etc.\) have the meanings defined in equation 2.2.2(17). Note that as each cell's Lagrangian Multiplier is updated, new values in adjacent cells which have already been modified earlier in the same iteration sweep are used immediately to generate the new Lagrangian Multiplier. Hence, the right-hand side of equation 2.3.3(3) contains a mixture of values that have already been set earlier in the \((r+1)\)th iteration, and some that have yet to be set. This has the advantage that only one storage array
is required for the solution vector since new values may overwrite old values as soon as they are generated.

The Successive-Over-Relaxation (SOR) method is therefore very economical on central memory requirements. In fact, just three large arrays are needed, one to store the solution vector; another for the right-hand side vector (which represents the initial net flow from each cell), and an array which stores the cell weighting coefficients in equation 2.3.3(3) which are associated with irregular cells. The first two are both of length $n$. The length of the array holding cell weighting coefficients depends upon the complexity of the terrain; that is, what fraction of the unsubmerged cells are irregular. Typically though, it contains of the order of $n$ values. Overall, therefore, the storage requirement for SOR is proportional to $3n$. This should be compared to direct solution methods which have a central memory requirement proportional to $300n$.

An arbitrary estimate is used to initiate the iterative process. The null vector is used for this purpose in WAFT. The iterative process is terminated when the largest change in any Lagrangian Multiplier is below a prescribed value. Although there is no simple connection between this prescribed tolerance and the accuracy of the final wind-field, a tolerance of one percent generally gives sufficiently accurate final windfields. Some early trials with WAFT indicated that 1.60 was a reasonable value to use for the relaxation parameter.

The SOR method converges if the modulii of all the eigenvalues of $\frac{A}{\alpha}$ (the spectral radius of $\frac{A}{\alpha}$) are all less than unity. It is difficult to calculate the maximum modulus of any eigenvalue for a general matrix and it is therefore not possible to predict whether WAFT (using SOR) will converge before actually running it in a particular case. The SOR technique tends to work satisfactorily when there is equal suppression of vertical velocities throughout the vertical extent of the domain. In these cases, approximately fifty
iterations are required to produce sufficiently accurate final windfields over a mesh of 30 by 25 by 20. However, the SOR scheme converges extremely slowly in some cases which attempt to model flow under an elevated inversion. The worst possible type of case is when the inversion lies just above a hill peak. In this case, there are abrupt changes in the final windfield velocity field in the neighbourhood of the hill top. Correspondingly, there are large changes in the Lagrangian Multiplier solution vector near the hill top; SOR fails to converge in these circumstances. It is necessary to adopt the more sophisticated Conjugate Gradient technique in these difficult cases; this is described next.

2.3.4 The Conjugate Gradient Method

The Conjugate Gradient method is a non-stationary process which may be applied to symmetric matrix problems. However, it may also be regarded as a direct solution method since it has the property that, if exact arithmetic is used, the iterative process will arrive at the exact solution after a finite number of iterations. If the matrix is of size $n$ by $n$ then the exact solution is obtained after $n$ iterations. The method was originally developed by Hestenes and Stiefel (1952).

The general iterative formula, equation 2.3.2(6), becomes, for the Conjugate Gradient method

$$x^{r+1} = x^r + c_r^r \cdot \tilde{r}$$

2.3.4(1)

where $\tilde{r}^r$ is given by

$$\tilde{r}^r = \tilde{r}^r + c_{r-1}^r \cdot \tilde{r}^{r-1}$$

2.3.4(2)

and $r^r$ is the residual vector given by

$$r^r = b - \tilde{A} \cdot x^r$$

2.3.4(3)
The residual vector, \( \mathbf{r}^r \), tends to the null vector as the exact solution is approached. The scalars \( c_1^r \) and \( c_2^r \) are specified as follows

\[
c_1^r = \left( \mathbf{p}^r, \mathbf{z}^r \right) = \left( \mathbf{z}^r, \mathbf{z}^r \right) \quad (2.3.4(4))
\]

and

\[
c_2^r = \left( -\mathbf{r}^{r+1}, \mathbf{A}\mathbf{r}^r \right) = \left( \mathbf{z}^{r+1}, \mathbf{z}^{r+1} \right) \quad (2.3.4(5))
\]

These choices of \( c_1^r \) and \( c_2^r \) ensure that the following orthogonality relationships hold

\[
\left( \mathbf{r}^s, \mathbf{p}^t \right) = 0 \quad \text{for } s > t \quad (2.3.4(6))
\]

\[
\left( \mathbf{p}^s, \mathbf{A}\mathbf{p}^t \right) = 0 \quad \text{for } s \neq t \quad (2.3.4(7))
\]

\[
\left( \mathbf{z}^s, \mathbf{z}^t \right) = 0 \quad \text{for } s \neq t \quad (2.3.4(8))
\]

Details of the Conjugate Gradient method may be found in Reid (1971) and Fox (1964). The above orthogonality conditions mean that the nth residual vector, \( \mathbf{r}^n \), is zero; that is, the exact solution is obtained after \( n \) iterations.

The null vector is taken as the initial estimate for the solution vector, that is \( \mathbf{z}^0 = 0 \). The initial residual, \( \mathbf{r}^0 \), is calculated using

\[
\mathbf{r}^0 = \mathbf{b} - \mathbf{A}\mathbf{z}^0 = \mathbf{b} \quad (2.3.4(9))
\]

and \( \mathbf{c}^0 \) is given by

\[
\mathbf{c}^0 = \mathbf{r}^0 = \mathbf{b} \quad (2.3.4(10))
\]

The following set of recursive equations is then used for \( r=0,1,2,... \)

\[
c_i^r = \left( \mathbf{p}^r, \mathbf{A}\mathbf{c}^r \right) \quad (2.3.4(11))
\]
The residual vector $\mathbf{r}^r$ has a direct physical significance in WAFT. The elements in this vector are the net volume flows from each of the cells in the final velocity field corresponding to the approximate solution $\mathbf{X}^r$. It therefore provides a clear indication of the accuracy of the final velocity field as the iterative process is progressing. The iterative process is terminated when the maximum modulus value of any component of $\mathbf{r}^r$ is below a specified value. The net inflow or outflow of every cell in the final windfield will be less than this value. Sufficiently accurate velocity fields are obtained if the maximum net inflow in the final windfield is of the order of a thousand times smaller than that of the initial windfield.

The storage requirements of the Conjugate Gradient method are also quite modest. The matrix $\mathbf{A}$ only appears in a matrix by vector product and so full advantage may be made of its sparsity and the ease with which most of its elements may be calculated. Only those non-zero elements which relate to irregular cells are stored. In addition, four vectors of length $n$ are required: the residual vector, $\mathbf{r}$; the solution estimate, $\mathbf{X}$; the vector $\mathbf{p}$; and a work vector storing the result of $\mathbf{A}\mathbf{p}$. The total storage requirement is therefore approximately proportional to $5n$.

The rate of convergence of the Conjugate Gradient method is determined by the distribution of the eigenvalues of the matrix $\mathbf{A}$. If the eigenvalues are tightly clustered the method will converge quickly; in general, if there are $k$ tight clusters a good approximation will be obtained after $k$
iterations. In the cases when the SOR method performs adequately, the Conjugate Gradient method converges at roughly the same rate. However, in those when the SOR method performs poorly, the Conjugate Gradient technique produces satisfactory results after, typically \( n/10 \) repetitions. This represents a major improvement over the SOR scheme. Unfortunately, even this convergence rate means that typical WAFT runs (with \( n \) of the order of 10000) still take a considerable amount of machine time. It is desirable to accelerate the basic Conjugate Gradient method; this is dealt with in the following subsection.

2.3.5 The Incomplete Cholesky Conjugate Gradient Method

The Conjugate Gradient method may be accelerated using a technique known as preconditioning. A modified matrix equation

\[
E^{-1} A x = E^{-1} b \quad 2.3.5(1)
\]

is solved in place of the original system

\[
A x = b \quad 2.3.5(2)
\]

\( E^{-1} \) is termed the preconditioning matrix and \( E \) is chosen to be some approximation to \( A \). The matrix product \( E^{-1} A \) is therefore an approximation to the identity matrix, \( I \). The eigenvalue distribution of the product matrix \( E^{-1} A \) is more tightly clustered than the matrix \( A \); see, for example, Meijerink and Van der Vorst (1981). This means that the Conjugate Gradient scheme applied to system 2.3.5(1) will converge faster than the original system 2.3.5(2). Both \( A \) and \( E \) are symmetric and may therefore be expressed as the product of a lower triangular matrix and its transpose, so that

\[
E = B_E B_E^T \quad A = B_A B_A^T \quad 2.3.5(3)
\]
The construction of the matrix $E^{-1}$ will be considered now. The true lower decomposition of $A$, $B_A$, may be obtained using a direct solution method such as Cholesky Decomposition as described in Fox (1964), for example. The fill-in process associated with direct methods (as discussed in Section 2.3.2) means that the matrix $B_A$ will have many more non-zero elements than the matrix $A$. Thus storage restrictions preclude the generation of $B_A$. However, if one examines these fill-in elements of $B_A$ one finds that they are in general much smaller than the elements which lie in the same positions as the non-zero elements of $A$. It is therefore possible to generate $B_E$ (and hence $E^{-1}$) by performing a Cholesky decomposition on $A$ which neglects any elements that are created in positions occupied by zeros in $A$. This process is known as an Incomplete Cholesky decomposition of matrix $A$. The resulting approximation to $B_A$, $B_E$ has the same band structure as $A$ (that is, it is as sparse as $A$) and so storage requirements remain modest.

In fact the Incomplete Cholesky decomposition has a particularly simple form for the WAFT matrix $A$. It is slightly more convenient to write $E$ as the product of three matrices, so that

$$E = B D^{-1} B^T$$

where

$B$ is a lower triangular matrix

$D$ is a diagonal matrix comprising the diagonal elements of $B$

The off-diagonal elements of $B$, $b_{mn}$, $m \neq n$, are simply equal to the corresponding elements in $A$, $a_{mn}$, $m \neq n$. The diagonal elements of $B$, $d_{nn}$ (and hence the elements of $D$ also) are calculated using the following recursive formula

$$d_{nn} = a_{nn} - a_{n1n1} - a_{n2n2} - a_{n3n3}$$
where

\( d_{mm} \) is the diagonal element to be calculated

\( a_{mm} \) is the corresponding element in \( A \)

\( a_{mn1}, a_{mn2}, a_{mn3} \) are the non-zero off-diagonal elements in the lower triangular part of \( A \) in row \( m \)

\( d_{n1n1}, d_{n2n2}, d_{n3n3} \) are previously calculated diagonal elements of \( B \)

Following Meijerink and Van der Vorst (1977) the iterative cycle for the preconditioned Conjugate Gradient method may be written

\[
\begin{align*}
\tilde{x}^0 &= 0 \\
\tilde{r}^0 &= b - \tilde{x}^0 \\
\tilde{p}^0 &= [B \tilde{D}^{-1} B^T]^{-1} \tilde{r}^0
\end{align*}
\]

and then for \( r=0,1,2... \)

\[
\begin{align*}
c_r &= \frac{\langle \tilde{x}^r, [B \tilde{D}^{-1} B^T]^{-1} \tilde{r}^r \rangle}{\langle \tilde{p}^r, \tilde{A} \tilde{p}^r \rangle} \\
\tilde{x}^{r+1} &= \tilde{x}^r + c_r \tilde{p}^r \\
\tilde{r}^{r+1} &= \tilde{r}^r - c_r \tilde{A} \tilde{p}^r \\
c_2 &= \frac{\langle \tilde{x}^{r+1}, [B \tilde{D}^{-1} B^T]^{-1} \tilde{r}^{r+1} \rangle}{\langle \tilde{x}^r, [B \tilde{D}^{-1} B^T]^{-1} \tilde{x}^r \rangle} \\
\tilde{p}^{r+1} &= [B \tilde{D}^{-1} B^T]^{-1} \tilde{r}^{r+1} + c_2 \tilde{p}^r
\end{align*}
\]

The Incomplete Cholesky Conjugate Gradient (ICCG) iterative cycle is similar to the basic Conjugate Gradient method.
(equations 2.3.4(11)), the only difference being the addition of matrix multiplications of the form

\[
[B \ D^{-1} \ B^T]^{-1} \ r
\]

2.3.5(10)

The operations in 2.3.5(10) may be rewritten as

\[
B^{-T} \ D \ B^{-1} \ r
\]

2.3.5(11)

The first multiplication in 2.3.5(11) is

\[
B^{-1} \ r = y, \ \text{say}
\]

2.3.5(12)

which may be reversed as

\[
r \ y = B
\]

2.3.5(13)

where \(y\) is the required vector. Writing 2.3.5(13) in full

\[
\begin{align*}
r_1 & = b_{11}y_1 \\
r_2 & = b_{21}y_1 + b_{22}y_2 \\
r_3 & = b_{31}y_1 + b_{32}y_2 + b_{33}y_3 \\
& \quad \text{etc.}
\end{align*}
\]

2.3.5(14)

one can see that the components of \(y\) (that is, \(B^{-1} \ r\)) may be calculated easily. The next multiplication in 2.3.5(11) is multiplication by the diagonal matrix \(D\) which is straightforward, and finally there is the multiplication by \(B^{-T}\) which is handled in the same way as the multiplication by \(B^{-1}\).

Since only the diagonal elements of \(B\) are different to those of \(A\), only one extra vector of length \(n\) is required to implement the ICCG method over and above the requirements of
the basic Conjugate Gradient method. The extra vector is used to store the diagonal elements of $B$ and overall this means that the total storage requirement for the ICCG technique is proportional to $6n$. In fact, WAFT uses magnetic disc files as backup stores, and thus the storage requirement of the WAFT ICCG code is only $4n$. The ICCG iterative process is terminated using the same criteria as the basic Conjugate Gradient method.

The convergence properties of the three alternative iterative methods available in WAFT are presented in Figures 2.7 and 2.8. Figure 2.7 relates to a small test mesh of 8 by 8 by 16, whilst Figure 2.8 corresponds to the sort of mesh used in real terrain simulations; the mesh size is 35 by 21 by 11. The dashed curves indicate cases which have a homogeneous vertical velocity suppression, whereas the solid curves illustrate cases where there is an abrupt change in the vertical velocity suppression in order to model a temperature inversion. The SOR method fails to converge satisfactorily when there is an inversion present on either mesh. The basic Conjugate Gradient method also converges rather slowly on the large mesh when there is an inversion present. The ICCG technique performs very well in all the cases. As a guide, sufficiently accurate velocity fields are obtained when the maximum flow into any one cell falls by a factor of a thousand or so.
Fig 2.1 - The topography representation in WAFT.

Fig 2.2 - The three types of calculational cell in the WAFT mesh.
(a) - velocities defined at mesh points

(b) - 'flow-average' velocities defined through cell faces

Fig 2.3 - A typical WAFT cell illustrating the notation used.
Fig 2.4 - A WAFT cell and its associated Lagrangian Multipliers.

Fig 2.5 - Boundary conditions at the ground surface.
(a) 3x3x3 WAFT mesh with one submerged cell

(b) matrix of coefficients for the set of Langrangian Multiplier equations corresponding to the above mesh

Fig 2.6 - Matrix of equation coefficients for a mesh of twenty six cells.
Log₁₀ \left( \text{max. flow into, or out of any cell } \frac{\text{m}^3}{\text{s}} \right)

- ICCG: Incomplete Cholesky Conjugate Gradient
- CG: Conjugate Gradient - no preconditioning
- SOR: Successive Over Relaxation

Fig 2.7 - Convergence of different iterative techniques; 8 x 8 x 16 mesh.
Fig 2.8 - Convergence of different iterative techniques; 35 x 21 x 11 mesh.
CHAPTER 3

USING WAFT TO MODEL POTENTIAL FLOW

3.1 INTRODUCTION

The work in this chapter checks whether the WAFT model produces sensible windfields. The windfields produced by WAFT are compared to potential theory flows. Whilst flow in the real atmosphere is often very different from potential flow, potential flow does provide a good model for many real atmospheric flows when buoyancy forces are not important, see for instance Hunt (1978). Flows over simple two and three-dimensional geometrical shapes are considered.

Section 3.2 begins by examining whether the model described in Chapter 2 will simulate potential flow. In particular, the assumption had been made that the weighting factors $\alpha_x$, $\alpha_y$, and $\beta$ in the minimisation sum 2.2.2(1) need not be related to either the dimensions of the cell faces or the fraction of the face that is above the ground surface; they are only used to preferentially weight adjustments to selected velocity components. For example, it was envisaged that this might be a way of modelling stable flow situations; the weighting factors for the vertical velocity components would be given relatively large values in order to suppress vertical motion.

There are no buoyancy forces present in potential flow which means one should not preferentially weight the velocity adjustments in any particular direction. Hence, if the original model formulation is correct, then WAFT should generate potential flow when the weighting factors are all set to unity. The method used to validate WAFT against potential flow is explained in subsection 3:2.1. Subsection 3:2.2 then goes on to examine whether the original model formulation does produce potential flow.
In fact, it turns out that the original formulation does not generate potential flow correctly. It is not necessary to abandon the formulation given in Chapter 2, but merely revise the function of the weighting factors. Instead of them being simply used to determine the preferential weighting given to certain velocity components, they must also take into account the dimensions of the appropriate cell face. These revised weighting factors can be considered as the product of a velocity weighting factor and a cell weighting factor. The velocity weighting factor performs the original role of weighting adjustments made to velocity components, whereas the cell weighting factors vary according to the cell face dimensions. These cell weighting factors are chosen so that potential flow is generated when the velocity weighting factors are all set to unity. Subsections 3.2.3 and 3.2.4 look at three possible definitions for the cell weighting factors.

These alternative weighting schemes are again validated against two-dimensional potential flow. No single cell weighting scheme is found which works satisfactorily on all the meshes tried. However, one scheme is identified which is valid providing the mesh intervals in any one axis direction are all the same (that is, variable mesh spacing is not allowed). This scheme is adopted for all subsequent WAFT runs presented in this thesis.

Having established the correct weighting scheme, section 3.3 goes on to examine sources of error within the WAFT method. Two classes of errors are distinguished: numerical errors arising from the fact that continuous fluid flow is modelled on a discrete mesh; and errors generated by the initial windfield generation based on sparse input data. The latter are found to be much more significant. Further work is undertaken to examine the effectiveness of various initial windfield interpolation schemes. This work is again based on the two-dimensional model.

Finally, three-dimensional WAFT windfields are compared with
potential flow. This work provides some indication of how hills deform neutral flow in the real atmosphere. A comparison with the MATHEW model (Sherman, 1978) is also made. WAFT is found to be significantly better at generating potential flow around an hemisphere. This is particularly true close to the hemisphere surface, showing the advantages of WAFT's more realistic surface representation.

3.2 DEFINITION OF THE CELL WEIGHTING FACTORS

3.2.1 The Method used to Select a Valid Weighting Scheme

Figure 3.1 shows the two-dimensional domain used to evaluate the candidate weighting schemes throughout this section. The hill is a 500 metre radius semicircle centred within a domain 2000 metres wide by 1500 metres high. The figure also shows the potential flow over such a hill. Potential flow velocity vectors are plotted at each of the mesh points used in the corresponding WAFT runs. These provide the yardstick whereby the accuracy of the WAFT field is judged. The potential flow velocity field is given by

\[ u(x,z) = U_F \left( \frac{1 + \frac{r_c^2}{x^2} (z^2 - x^2)}{(x^2 + z^2)^2} \right) \]

\[ w(x,z) = U_F \left( \frac{-2 \cdot \frac{x \cdot z \cdot r_c^2}{(x^2 + z^2)^2}} \right) \]

where

- \( x \) is the x-position relative to the cylinder axis
- \( z \) is the z-position relative to the cylinder axis
- \( u \) is the horizontal velocity component
- \( w \) is the vertical velocity component
- \( U_F \) is the free stream velocity - in this case 10 m/s
- \( r_c \) is the cylinder radius - set here to 500 m.
Each of the weighting schemes was assessed by comparing the final velocity field with the above analytic solution. Note, that as one might expect, the flow is accelerated over the crest of the semicircle, and there are relatively stagnant regions immediately upwind and downwind of the hill. This hill shape is rather extreme; one expects that if WAFT can produce acceptable results over this sort of shape, it will also perform reliably over the shallower hills found in real terrain.

Uniform initial windfields are used throughout this section. The initial velocity at each mesh point is set to the free stream velocity, \((10.0, 0.0)\). This initial windfield is illustrated in Figure 3.2. Note that the initial windfield flows into, and out of, the hill, not over it. The cells close to the hill surface have substantial net inflows (or outflows). Hence, the WAFT procedure is obliged to adjust the initial velocities in order to satisfy the constraint of no net flow into any cell in the final flow field.

Before going on to consider the results obtained using WAFT, it is appropriate to clarify some terms which will be used throughout this section. The model formulation given in Chapter 2 described the three-dimensional version of WAFT; it is this version which would be used in practical situations. The cells in the three-dimensional WAFT are cuboidal volumes and the weighting factors relate to the velocities flowing through the six cell faces. The work described here applies to both the two and three-dimensional WAFT models. For consistency with the model formulation in Chapter 2, reference will continue to be made to cell faces and their associated areas. Note, however, that the work in this section uses the two-dimensional version of WAFT where, strictly speaking, these terms should be replaced by cell boundary edge and its associated length.
3.2.2 The Original Model Formulation

The work here examines whether the model formulation presented in Chapter 2 may be used, without modification, to generate potential flow. The model was originally formulated using weighting factors that are not related to the cell face dimensions, or the relative size of the open portion, because this resulted in a set of equations which had very easily calculated coefficients. Hence, their solution would be particularly simple and efficient.

A two-dimensional version of WAFT based on the original model formulation with the weighting factors all set to unity produces the final windfield shown in Figure 3.3. In general, the flow field is similar to the potential flow shown in Figure 3.1. The flow has been deflected over the hill, and low speed regions have developed on either side of the hill. The flow has accelerated over the hill summit, and this acceleration has fallen off with height. These features are all present in the potential flow. However, there is obviously a problem with some of the mesh points lying close to the hill boundary. Huge velocities (of the order of thousands of metres per second) are generated here; these are indicated by the enormous arrows spiralling out from the semicircle in Figure 3.3. These abnormal velocities occur in cells where the faces are very nearly completely blocked. This situation is illustrated in Figure 3.4. The huge velocities are generated across cell faces such as BP and BQ.

These very large velocities may be explained as follows. In the original model formulation, adjustments are made to the initial velocity field with the following sum being minimised

\[ S = \sum \alpha_x \frac{\partial U}{\partial x} + \alpha_y \frac{\partial V}{\partial y} + \alpha_z \frac{\partial W}{\partial z} \]

As discussed in subsection 2.2.1, \( U \), \( V \) and \( W \) are flow-average velocities, so that, for instance,
\[ \delta u_{ijk} = \frac{A_o}{A_t} \delta u_{ijk} \]  
3.2.2(2)

where

- \(A_o\) is the open portion of the relevant cell wall
- \(A_t\) is the total area of the cell wall
- \(\delta u_{ijk}\) is the mean velocity adjustment over the open section of the cell wall
- \(\delta u_{ijk}\) is the flow-average velocity adjustment

In the original model formulation, and with all the weighting factors set to unity, the adjustments made to flow-average velocities are expected to be of equal magnitude across every cell face. For instance, in Figure 3.4

\[ \delta u_{AB} \approx \delta u_{BC} \]  
3.2.2(3)

Suppose that \(AB=BC\) and \(100\times BP=BC\) in Figure 3.4, then considering the true average velocities across \(AB\) and \(BP\) one can see that

\[ \delta u_{BP} = \frac{A_L}{A_o} \delta u_{BC} = 100 \cdot \delta u_{BC} \]  
3.2.2(4)

\[ \delta u_{AB} = \frac{A_L}{A_o} \delta u_{AB} = \delta u_{AB} \]  
3.2.2(4)

\[ \delta u_{BP} \approx 100 \cdot \delta u_{AB} \]

In this way unrealistically high velocities are generated across faces which are very nearly completely blocked. It is thus clear that the weighting factors must contain some part which is related to the cell faces. As was discussed in the introduction to this chapter, it is necessary to revise the functionality of the weighting factors: There will be a part, the cell weighting factor which is determined by the cell face's dimensions, in addition to the velocity weighting factor which weights the velocity adjust-
-ment through that face. The form of these cell weighting factors will be addressed in the following subsections. The scheme used in this subsection (i.e. when the cell weighting factors are effectively unity) will be referred to as the 'first weighting scheme'. The next two subsections discuss three alternative weighting schemes where the cell weighting factors do depend on cell face geometry.

3.2.3 The Second Weighting Scheme

One way of circumventing the problems inherent in the first weighting scheme (that is, the original model formulation) is to recast the minimisation sum in terms of true velocity averages rather than the flow-average velocities adopted initially. The new minimisation sum, \( S^* \), may then be written

\[
S^* = \sum \alpha_{x_{ijk}}^* S_{u_{ijk}}^2 + \alpha_{y_{ijk}}^* S_{v_{ijk}}^2 + \beta_{ijk}^* S_{w_{ijk}}^2
\]

Note, that \( S_u, S_v, \text{ and } S_w \) are true average velocities across the open portion of the face and not flow-average velocities. The weighting factors \( \alpha_{x_{ijk}}^*, \alpha_{y_{ijk}}^*, \text{ and } \beta_{ijk}^* \) are once again used to attach different penalties to the adjustment of particular velocity components. Again, setting these all to unity should model potential flow. In this case, one would expect that the adjustments made to all true velocity averages would be of the same order, and therefore this should prevent enormous velocities being created.

The true velocity averages, \( S_{u_{ijk}} \text{ etc.} \), are related to the flow average velocities, \( S_{U_{ijk}} \text{ etc.} \), by the relationship given above in 3.2.2(2). Hence, the new minimisation sum, \( S^* \), may be written using flow-average velocities as

\[
S^* = \sum \alpha_{x_{ijk}}^* \frac{T_{x_{ijk}}^2}{O_{x_{ijk}}^2} S_{u_{ijk}}^2 + \alpha_{y_{ijk}}^* \frac{T_{y_{ijk}}^2}{O_{y_{ijk}}^2} S_{v_{ijk}}^2 + \beta_{ijk}^* \frac{T_{z_{ijk}}^2}{O_{z_{ijk}}^2} S_{w_{ijk}}^2
\]

81
where,

\[ TX_{ijk} \] is the total area of cell face \((i,j,k)\) normal to the x-axis.
\[ TY_{ijk} \] is the total area of cell face \((i,j,k)\) normal to the y-axis.
\[ TZ_{ijk} \] is the total area of cell face \((i,j,k)\) normal to the z-axis.
\[ OX_{ijk} \] is the area of the open portion of cell face \((i,j,k)\) normal to the x-axis.
\[ OY_{ijk} \] is the area of the open portion of cell face \((i,j,k)\) normal to the y-axis.
\[ OZ_{ijk} \] is the area of the open portion of cell face \((i,j,k)\) normal to the z-axis.

Hence, in terms of the original model formulation, one now has weighting factors which consist of the product of the velocity weighting factors, for example \( \alpha^*_X \), and the cell weighting factors based on cell face geometry, which in this case are given by

\[
\alpha^C_{X_{ijk}} = \frac{TX^2_{ijk}}{OX^2_{ijk}}, \quad \alpha^C_{Y_{ijk}} = \frac{TY^2_{ijk}}{OY^2_{ijk}}, \quad \beta^C_{Z_{ijk}} = \frac{TZ^2_{ijk}}{OZ^2_{ijk}} \quad 3.2.3(3)
\]

It is not straightforward to calculate these cell weighting factors for faces which are partially obscured since they depend on the area of the open portion of the face. Since they are difficult to calculate, they are computed once at the beginning of WAFT and stored in an array for use during the iterative solution method.

The final windfield produced when this second weighting scheme is used is illustrated in Figure 3.5. This new weighting scheme has prevented abnormally high velocities from being generated. However, comparison with the potential flow in Figure 3.1 shows that the velocities near
the hill boundary are still incorrect. In this case, the velocity adjustments made across faces which have a small open portion are too small. This suggests that it might be appropriate to multiply the cell weighting factors used in this case by a factor proportional to the size of the open portion of the cell wall. In this way, less penalty would be attached to making adjustments across partially blocked faces, and hence the adjustments would tend to be relatively larger. Two different schemes for doing this are discussed in the next subsection.

3.2.4 The Third and Fourth Weighting Schemes

Two options will be considered for the multiplying factor discussed at the end of the above subsection. These are

- multiplying the cell weighting factors used previously by the area of the open portion of the cell face. The cell weighting factors then become

\[ \alpha_{X,ijk}^c = \frac{TX_{jk}^2 \cdot OX_{jk}^2}{OX_{jk}^2 \cdot TX_{jk}^2} \]  

3.2.4(1)

This will be referred to as the 'third weighting scheme'.

- multiplying the cell weighting factors by the ratio of the area of the open portion to the total area of the cell face. The cell weighting factors are then given by

\[ \alpha_{X,ijk}^c = \frac{TX_{jk}^2 \cdot OX_{jk}^2}{OX_{jk}^2 \cdot TX_{jk}^2} \]  

3.2.4(2)

This is the 'fourth weighting scheme'.

When these two weighting schemes are implemented and run on the mesh used previously they produce identical final
windfields. This common final windfield is depicted in Figure 3.6. The final windfield here is very close to the potential flow solution at all the mesh points including those close to the hill surface. The weighting factors in the two schemes differ by a factor equal to the total cell face area. In the grid used here, which uses identical square cells throughout, this factor is the same for all cell faces. The weighting factors control the relative importance attached to adjustments made across the cell faces, and hence this constant multiplying factor has no significance. This explains why the two weighting schemes produce identical results in this case. However, the two weighting schemes would give different relative weights to cell faces if the cells were not square, or if variable mesh spacing was employed. These cases will be considered now.

A low slope hill is used to examine whether these two weighting schemes produce valid windfields when the cells are not square, or irregular spacing is used. The hill shape is defined using one of the streamlines of potential flow over a semicircle. The particular streamline used is illustrated in Figure 3.7. Using a potential flow streamline to define the hill surface means that the flow over the hill may still be computed from equations 3.2.1(1) and 3.2.1(2) for potential flow over a semicircle. Other experiments have been performed using the semicircular ridge used before, and some simple wedge shaped hills. Most of these cases used the two-dimensional version of WAFT so as to conserve computer resources. However, some runs were conducted using the three dimensional WAFT code to check that this behaves in the same way.

These experiments show that both of the weighting schemes produce incorrect windfields over some of the grids used. In particular, the third weighting scheme does not produce acceptable results when the cell's height is very different from its length. This is apparent in the flows over the low slope streamline surface hill where it is appropriate to use mesh cells which are much longer than tall. Figure 3.8
illustrates the potential flow over this hill. Note that the vertical scale has been enlarged by a factor of two, and that the cells are 250 m long by 50 m high. In this case, the fourth weighting scheme produces results which are close to the analytic flow. However, as Figure 3.9 shows, the third weighting scheme produces vertical velocities which are much too small. This results in too great a flow acceleration over the hill crest. These difficulties are repeated whenever rectangular cells (or in the three dimensional case, non-cubic) are employed with the third weighting scheme. The problem intensifies as the cells are made more elongated.

An examination of the form of the cell weighting factors explains why this effect occurs. The weighting factors are proportional to the open area of the cell wall. If one considers, say, a three-dimensional mesh composed entirely of cubic cells one sees that the total area of cell faces normal to each axis is equal. If, on the other hand, the vertical dimension of the cell is ten times smaller than the other two dimensions, then the total area normal to the vertical axis is ten times bigger than the areas normal to the other two axes. Considering the grid as a whole, there would be a greater penalty attached to adjusting the vertical velocities. As was found experimentally, this effect becomes more pronounced as the cells are made more elongated. When one is modelling flows in the real atmosphere, one would tend to use domains which have horizontal dimensions which are much greater than their height. Hence, this problem with the third weighting scheme would be relevant in practical situations.

There are, however, some grids on which the third weighting scheme produces acceptable results but the fourth weighting scheme does not. These problems occur on grids which consist of mainly square cells but which have some layers of thin elongated cells (that is, grids which use variable mesh spacing). Figure 3.10 provides an example of a grid producing an incorrect windfield. There are two layers of
relatively thin cells at \( z = 290 \) m., and \( z = 590 \) m. The final windfield produced using the third weighting scheme compares well with potential flow; since most of the cells are square it does not suffer from the vertical velocity suppression problems discussed above. Figure 3.10 shows the results obtained using the fourth weighting scheme. It is apparent that there are abrupt, incorrect, changes of velocity at the layers of thin cells. This behaviour also occurs on three dimensional grids which have variable mesh spacing. The adjustments across the short faces are much too small; in other words, too large a penalty was being attached to these adjustments. These cases suggest that the third weighting scheme is the most appropriate one.

In summary then, there are some cases which support the use of the third weighting scheme, but others which suggest that the fourth weighting scheme should be used. A number of hybrid schemes have been tried which attempt to combine the advantages of both methods. However, it has not proved possible to find a scheme which works on all grids. The fourth weighting scheme is the most appropriate one to adopt for WAFT. Whilst the fourth weighting scheme does suffer from the disadvantage that it does not work with variable mesh spacing, the third weighting scheme has the much more serious drawback that it doesn't work on the sorts of grid one would wish to use over real terrain. Hence, all of the cases presented in the rest of this thesis are based on the fourth weighting scheme; that is, cell weighting factors specified as in equation 3.2.4(2). This does mean that it is not possible to use variable mesh spacing however.

3.3 SOURCES OF ERROR IN THE WAFT METHOD

3.3.1 Introduction

The experiments performed in Section 3.2 were used to select cell weighting factors which produce sensible adjustments
over a wide variety of calculational meshes. Having established the mathematical basis of the model, one can now investigate how accurate and reliable the method is. This section looks at possible sources of error using the two-dimensional version of WAFT. Two classes of errors are identified and studied. Firstly, there are those errors which result from the fact that a discrete calculational mesh is used to solve what is essentially a continuous fluid flow problem. For example, the sensitivity of the solution to the mesh spacing is examined. Secondly, there are those errors which result from the limited amount of source data which might be available during an accident. Evaluating these sources of error involves assessing how sensitive the final windfield is to the specification of the initial (arbitrarily generated) windfield.

The work here demonstrates that the second class of error is more significant. Hence, further work is undertaken to investigate the optimum method of generating the initial windfield from the sparse input data. A standard method is adopted to compare the various final windfields with known potential flow solutions. This allows a meaningful comparison of the various sources of error. The two-dimensional WAFT program provides the most economical route for these comparisons, and the results are generally easier to interpret than a three-dimensional flow field.

3.3.2 Comparison Methodology

The runs which assess the various sources of error are conducted using the streamline hill described previously in subsection 3.2.4. Once again, this hill is employed because of its smooth profile and easily calculated potential flow solution. The windfields produced by WAFT are always compared to this known potential flow solution. Different mesh spacings and domain areas are used throughout these experiments and this has to be borne in mind when specifying a standard comparison methodology. Each velocity field is
compared to the potential flow solution at forty points surrounding the hill; these points are illustrated in Figure 3.11. In some cases, these forty comparison points are not coincident with calculational mesh points; a simple interpolation scheme is used to generate a velocity from the nearest grid points. Two measures are made of the discrepancy between the WAFT generated velocity and the potential flow velocity. These are the percentage speed discrepancy, $\Delta u$, defined by

$$\Delta u = 100 \cdot \frac{u_{\text{WAFT}} - u_{\text{pot}}}{u_{\text{pot}}} \quad 3.3.2.(1)$$

where

- $u_{\text{WAFT}}$ is the speed in the WAFT windfield
- $u_{\text{pot}}$ is the speed in the potential flow

and the directional discrepancy, $\Delta \theta$, defined in degrees as

$$\Delta \theta = \theta_{\text{WAFT}} - \theta_{\text{pot}} \quad 3.3.2(2)$$

where

- $\theta_{\text{WAFT}}$ is the direction of the WAFT velocity at that point (in degrees)
- $\theta_{\text{pot}}$ is the direction of the potential flow at that point (in degrees)

These discrepancy measures are computed at each of the forty comparison points. The root mean square of the speed errors and direction errors are then computed to provide an overall indication of the differences between the WAFT and potential flow. Figure 3.11 also shows the WAFT calculational mesh which is used in all the initial windfield specification.
cases and those numerical experiments which are not concerned with the effects of changing the mesh specification.

3.3.3 Numerical Experiments

The initial windfield for all the examples in this subsection was (10.0,0.0) m/s at all mesh points; the runs in section 3.2 had already suggested that this choice would provide a reasonably accurate final windfield. A standard reference final windfield is generated using the standard mesh shown in Figure 3.11. The mesh and other details of this standard run are given in Table 3.1. The final windfield is much closer to the potential flow solution than is the initial windfield. In the following cases, either the mesh size or one of the other parameters listed in Table 3.1 are varied, and the effect on the accuracy of the final windfield noted.

The first series of experiments examines the effect of varying the size of the mesh cells and the total size of the calculational domain. The aspect ratio of the cells is fixed, with them being 2.5 times longer than high. The different meshes used are illustrated in Figure 3.12. The meshes are all drawn to the same scale, but note that in each case the vertical scale has been expanded by 2.5 times so that the cells are depicted as squares in the diagrams. The corresponding errors in the final windfields are given next to the meshes in Figure 3.12. Table 3.1 also gives the discrepancy between the initial windfield and the potential flow solution; this provides a reference with which to compare the error values given in Figure 3.12. Note that, for all the meshes considered, the final windfield is closer to potential flow than the initial flow. The poorest angular performance is found in meshes 1 and 5 which use large cells, and which therefore have a rather crudely defined hill shape. The grids which have the smallest domain area (meshes 4 and 6) give rise to the poorest speed performance. Potential flow theory says that the presence
of a hill of length $L$ disturbs the flow up to a height of order $L$, see, for example, Hunt (1978). The domain height in meshes 4 and 6 is smaller than $L$ and so it is not surprising that the final windfields are not very accurate.

The second set of cases looks at how changing the cell aspect ratio affects WAFT's behaviour. The standard domain size of 5000 metres by 1000 metres is used throughout. Figure 3.13 shows the cell shapes used and their corresponding final windfield errors. Case 1 produces rather poorer results than the rest, but examination of the net flows into cells suggests that this case converges slowly so that the values given here are pessimistic. The different cell shapes give acceptable results over an aspect ratio range of 0.1 to 2.0. This result confirms the work in section 3.2 validating the form of the cell weighting factors used.

The two-dimensional WAFT program uses the Successive-Over-Relaxation (SOR) method to solve the set of Lagrangian parameter equations. The importance of the number of iterations used in this process is discussed now. The standard mesh is employed and the results are presented in Table 3.2. Note that the final windfield is a considerable improvement on the initial windfield after only six iterations. At this stage the maximum flow into any cell has only been reduced by a factor of four. After twelve cycles, further iterations do little to improve the accuracy of the final windfield even though, at this point, the maximum flow into any one cell is only reduced by an order of magnitude. It would be quite futile to let the iterative process run for many iterations since this would not yield significantly more accurate windfields. This is especially true when one considers the other possible sources of error. Thus it is reasonable to terminate the iterative process after the maximum net flow into any cell has been reduced by two or three orders of magnitude. In contrast, the MATHEW model (Sherman, 1978) places great emphasis on reducing the windfield divergence (which is analogous to net flows into
cells in WAFT) by many orders of magnitude. Nothing is achieved by doing this.

WAFT's sensitivity to the value chosen for the over relaxation parameter is investigated in the next series of experiments. This parameter's role in the SOR process has been explained in subsection 2.3.3. The reference mesh is used and a standard number (50) of iterations is performed in each case. Table 3.3 illustrates the results obtained. The rate of convergence increases sharply as the optimum value for the over relaxation parameter is approached. For the hill and mesh used here the optimum value is approximately 1.7. A value of 2.0 produces divergent behaviour in the iterative process, and hence a very poor final windfield. Other experiments suggest that the optimum value for the over relaxation parameter depends on the hill shape and the particular mesh employed. In many cases the convergence rate is not as strongly dependent on the over relaxation parameter as is indicated in Table 3.3. Once again, since the net flow into cells need only be reduced by one order of magnitude for a reasonable windfield to be produced, there is not a strong relationship between the windfield accuracy and the over relaxation parameter.

Finally, the effect of incorporating a vertical density profile into the model is investigated. This is done so that some estimate of the importance of assuming a constant density may be obtained. A vertical density profile, \( \sigma(z) \), of the form

\[
\sigma(z) = \sigma_0 e^{-az}
\]

3.3.3(1)

is assumed, where

\( \sigma_0 \) is the density at Sea Level
\( z \) is the height above Sea Level
\( a \) is a constant controlling the rate at which density drops off with height
All densities are normalised so that $\bar{\rho}$ is unity. The net mass flow across each cell face is then computed as

$$u_{av} \times (\text{open area of cell face}) \times \bar{\rho}_{av}$$

3.3.2(2)

where

- $u_{av}$ is the mean velocity across the open part of the cell face
- $\bar{\rho}_{av}$ is the mean density across the cell face

The standard reference grid is employed again, and a range of values used for the constant $a$. Table 3.4 displays the results obtained. The ratio of the density at the top and bottom of the domain is also shown in the table. Note that even when the density varies by a factor of three throughout the domain height, the final windfield is very similar to the one produced assuming no density variation.

The numerical errors discussed in this subsection may be placed in context by comparing them to other sorts of error, in particular those arising from the initial windfield specification. These errors are considered in the following subsection.

3.3.4 Errors from the Initial Windfield Specification

The comparison method described previously is employed to assess the importance of the number and positions of the data points, and the interpolation scheme. The standard reference grid is used, and 50 iterations are performed with the over relaxation parameter set to 1.60. In these cases the initial windfield is not uniform, but interpolated from a few source data points. A different set of comparison points is also used in these cases. The comparison points used in the numerical experiments (illustrated in Figure 3.11) were grouped close to the hill. This was so all the points would still lie within the smallest domain areas used
in those experiments. However, in these experiments, the greatest discrepancies often occur near the edge of the domain away from the hill itself. In these cases, therefore, the comparison points are spread throughout the whole domain.

The non-uniform initial windfield is generated in the following way. Firstly, the data point's positions are chosen; they may, for example, be selected to give a good coverage of the whole domain, or more usually they are selected at random. The velocities defined by the potential flow solution at these points are then used as the input wind data for WAFT. Figures 3.14(a) and 3.15(a) illustrate two such data sets. Figure 3.14(a) shows seven input data points selected at random, and Figure 3.15(a) depicts twelve points selected in a symmetric pattern which gives a representative coverage of the potential flow.

Different two-dimensional interpolation schemes coded within the WAFT program are then used to generate the initial windfield from the source data. The initial windfield constructed from the seven data values in Figure 3.14(a) is illustrated in Figure 3.14(b). The bicubic spline method is used in this case. Similarly, the initial windfield created using the inverse square method on the data in Figure 3.15(a) is depicted in Figure 3.15(b). The corresponding final windfields appear in Figures 3.14(c) and 3.15(c). These two cases illustrate the importance of the initial windfield specification. The initial windfield interpolated from the twelve symmetrically placed points is itself quite close to the potential flow solution. Thus, the final windfield also compares well with the potential flow solution. In contrast, the initial windfield produced by the bicubic spline scheme from the seven randomly selected points is very different from the potential flow. The strong asymmetry of the data values is reflected in the initial windfield. The resultant final windfield is reasonably similar to the potential solution close to the hill where the land surface dominates. Elsewhere, however,
the final windfield follows the initial windfield pattern. In particular, there are large differences between the final windfield and the potential flow at the domain edges.

The results in Table 3.5 confirm the importance of the initial windfield creation. Case 1 in the table illustrates the speed and angular errors of the uniform initial windfield used in the previous subsection. Cases 2, 3 and 4 present the speed and angular errors when the same uniform initial windfield is used on a variety of different meshes. For comparison, cases 5 to 10, present the accuracy of final windfields produced from non-uniform initial windfields. The results show that the final windfield is much more sensitive to the data points and interpolation scheme used than to the calculational mesh employed. The final windfield is usually a better approximation to the potential flow than the initial flow, but they can both be seriously in error if the initial windfield is very different to the required solution. The directional accuracy of the final windfield is always reasonable close to the hill, but errors in excess of ninety degrees are found away from the hill surface.

It is clear that the initial windfield specification is more critical than other sources of error such as the mesh used, etc. The next subsection describes the different interpolation schemes that are available with WAFT, and following this, subsection 3.3.6 examines the behaviour of these different schemes.

### 3.3.5 Interpolation Schemes

The interpolation scheme generates a velocity component, \( u(r) \), at each mesh point, position vector \( \vec{r} \), from the \( N \) measured components, \( u_n(\vec{r}_n) \), at arbitrary positions, \( \vec{r}_n \). Two broad classes of schemes are available.
Firstly, there are those schemes which employ a weighted average of the measured components to generate the interpolated values, so that

\[ u(r) = \sum_{n} \frac{q_n(r, r_n) \cdot u_n(r_n)}{q_n(r, r_n)} \quad 3.3.5(1) \]

where \( q_n(r, r_n) \) is a weighting function dependent upon the position of the mesh point relative to the measurement point. All the weighting functions used in WAFT are relatively unsophisticated; they are all just simple functions of the scalar distance between the measurement and interpolation point. Denoting this scalar distance as \( R_n = |r - r_n| \), the three functions considered are

- The inverse square weighting method defined by

\[ q_n(r, r_n) = \frac{1}{R_n^2} \quad \text{for } R_n \neq 0 \quad 3.3.5(2) \]

Note that if \( R_n \) is zero, the measurement is coincident with the interpolation point, and thus the interpolated velocity is simply equal to the measured velocity. In this scheme, all the measurements contribute to each interpolated value (except when interpolation points lie on measurements).

- The 'radius of influence' scheme where

\[
\begin{align*}
q_n(r, r_n) &= \frac{R_c^2 - R_n^2}{R_c^2 + R_n^2} & R_n < R_c \\
q_n(r, r_n) &= 0 & R_n \geq R_c
\end{align*}
\]

\( R_c \) is the 'radius of influence'; measurements beyond this distance do not contribute to the interpolated value.

- The exponential weighting defined by
\[ q_n(r_1, r_n) = e^{-aR_n} \]  

3.3.5(4)

'a' is a constant controlling how quickly a measurement's influence falls off with distance. Once again, all the measurements contribute to each interpolated value.

The second type of interpolation scheme consists of fitting a surface of a particular mathematical form to the measured values. Three different types of surface fitting routines are considered.

- Polynomial least squares fit; here a polynomial of a specified power is chosen which gives the best least squares fit to the measured values. The number of data points imposes an upper limit on the order of the polynomial; there must be sufficient values to enable the coefficients in the general polynomial of the chosen order to be determined.

- Bicubic spline interpolation; the region is divided up into a number of rectangular panels, each of which has a bicubic polynomial fitted which gives the best least squares fit to the data points in that panel. In addition, it is arranged that the bicubic splines and their first derivatives are continuous across boundaries between panels. Panels are defined outside the perimeter of the region so that data points lying outside the domain may also be accounted for. These external panels also tend to prevent the large fluctuations that may arise at the edges of the domain. For a given number of panels an appropriate number of data points must be present in each panel for the problem to be completely determined. If WAFT detects any panels which do not have enough data points in it, then it automatically inserts extra 'pseudo' data points which are computed as inverse square averages of the true data points. This means that, for a given number of data points, the more
panels the domain is divided up into, the more the bicubic spline interpolation approaches the inverse square weighting method described above.

Triangular plane surfaces; in this scheme the region is triangulated using the measurement positions as vertices. In WAFT, the region is triangulated by hand. However, automatic means of doing this triangulation are available, see Lawson (1977). Extra 'pseudo' points are inserted around the region if they are required so that the whole of the domain is covered by triangles. Within each triangular area, interpolated values are computed using the equation of the plane that fits the measured data values at the vertices.

The above interpolation methods are used to generate two-dimensional velocity vector fields. This could be done in one of two ways: either the two velocity components may be interpolated separately, or the interpolation may be based on windspeed and wind direction. The former is adopted because of the difficulty of assigning an unambiguous direction when there are large variations in direction across the domain.

3.3.6 Comparison of the Interpolation Schemes

The comparison methodology used previously is employed here, but in this case the potential flow around a circular object provides the reference potential flow field. This velocity field is shown in Figure 3.16. The flow in the left-hand half is fairly uniform, but there are large changes in speed and direction in the right-hand half as the flow approaches the obstacle. The aim here is to assess the performance of the different interpolation schemes discussed previously. The assessment consists of a comparison between the interpolated initial windfield with the reference potential flow; corresponding final velocity windfields are not generated.
Note that there is no land included in this domain. As an analogy with three-dimensional flow in real terrain, one might imagine that the flow in Figure 3.16 represents a hypothetical uniform synoptic flow encountering a large mountain barrier.

Figure 3.17 depicts a small sample of the results using different interpolation schemes and various distributions of measurement points. Three configurations are illustrated, each consisting of ten randomly placed measurement points. The figure also gives the accuracies of the initial wind-fields using various interpolation methods. The results shown in the figure, together with many other cases, suggest the following conclusions. The methods which use weighted averages have the advantage that they do not generate velocities which are wildly different from the measured ones. On the other hand, they very rarely produce the best results since they don't take into account trends in the measured values. The 'radius of influence' and inverse square methods produce similar results, but the exponential scheme often produces rather poor velocity fields. In particular, this scheme is rather sensitive to the value of the parameter 'a' employed (see the previous subsection). If 'a' is given too high a value the influence of each measurement falls off very rapidly with distance, and the interpolated velocity at any point is basically equal to the velocity at the nearest measurement point. This implies a very disjointed velocity field. Conversely, if 'a' is set too low the interpolated field is excessively smoothed.

In contrast, the polynomial scheme does extrapolate trends which are present in the data, but because of this, it often produces 'wild' values in areas where there are few data points; particularly if these are close to the domain boundary. The very poor performance of the cubic polynomial fit for station array 'C' in Figure 3.17 provides an example of this. However, the cubic polynomial produces rather good interpolated fields for station arrays 'A' and 'B'. It is probable that this good performance is due to the simple
functional form of the reference velocity field. Other tests with the polynomial fitting scheme using reference velocity fields which don't have such a simple functional form give poorer results and tend to confirm this hypothesis.

The bicubic scheme offers a compromise since it does take account of trends in the data and yet does not tend to produce 'wild' interpolated values. It does not always produce the best results, but it very rarely generates the worst ones. If ten or more stations are employed then the bicubic spline method usually gives rise to the best wind-fields. For example, the inverse square method operating on twenty five randomly placed stations produces an RMS speed error of 22.3% and an RMS angular error of 10.3 degrees. Using the bicubic spline method on the same data produces corresponding figures of 5.1% and 2.9 degrees. Further experiments indicate that the bicubic spline method yields the best results when each panel has six to eight points in it.

3.3.7 Summary of Errors Investigated using 2D WAFT

The most important message from the work in this section is the importance of the initial windfield specification in determining the quality of the final windfield. The errors associated with the generation of the initial flow are much larger than the numerical errors. It is therefore not worthwhile to expend a lot of calculational effort solving the Lagrangian parameter equations to unrealistically high degrees of accuracy. The work also casts doubt on the wisdom of using very sophisticated methods, such as the Finite Element Method, to solve the minimisation problem when the initial windfield dominates the quality of the final result.

There are two factors which affect the initial windfield specification: firstly, the number and distribution of the
data points; secondly, the interpolation/extrapolation method used to construct the initial flow pattern. In a practical situation there is little one can do about the former. The work in subsections 3.3.5 and 3.3.6 studied some of the possible interpolation schemes. Work by Goodin et al (1979) indicated that a bicubic spline scheme produces the best results. The method they adopted was to triangulate the region with the data points as vertices, and then fit a bicubic spline to each triangular section. Whether their results are applicable to WAFT is open to some doubt however. Firstly, it should be noted that they evaluated the various interpolation schemes by reconstructing a smoothly varying polynomial function. It is possible that this biases their results in favour of interpolation methods which are based on polynomial surfaces. Secondly, fifty-six measurement points were used which is many more than would normally be available in a typical application of WAFT. Finally, a given station measurement may lie within a small scale local wind and may therefore be quite unrepresentative of the general wind patterns. It is difficult to envisage how any sort of interpolation scheme could deal with the latter problem, however.

The work in this section has indicated some interpolation schemes which are not suitable for various reasons, but in any given situation one cannot predict which interpolation scheme will produce the best results. Both the inverse square and bicubic spline method produced reasonable results, but one should be wary of using either blindly. It is always a sensible precaution to examine the interpolated fields by hand.

3.4 COMPARING WAFT WITH THREE-DIMENSIONAL POTENTIAL FLOW

3.4.1 Introduction

The aim of the work in this section is to ensure that the model produces a reasonable approximation to neutrally
stratified flow over simple three-dimensional obstacles. It is not expected that WAFT, with its rather limited atmospheric physics content, will produce the detailed flow agreement offered by, for example, the more sophisticated models of Walmsley et al (1982) or Mason and Sykes (1979). However, apart from a thin layer close to the ground surface, potential flow theory provides a reasonable approximation to neutrally stratified flow over a three-dimensional hill of low slope. The thickness of this layer is approximately $L_T/20$ where $L_T$ is half the streamwise extent of the hill. Neutrally stratified flow over low hills is discussed in Mason and Sykes (1979), and in Hunt (1978). It should be borne in mind that WAFT is not intended to model the detailed flow patterns around a hill, but rather, it should model the gross features of the flow through a mesoscale region. Hence, a comparison with three-dimensional potential flow provides a check that WAFT models neutrally stratified flow acceptably.

The initial work in this section uses a hemispherical hill, but subsequent work employs a more realistic low slope hill. It is convenient to use cells which have a small height compared to their horizontal dimensions when modelling the flow over shallow hills. The work therefore provides an additional check that the cell weighting functions selected in section 3.2 are indeed correct. The runs here examine whether WAFT can simulate potential flow given that the initial windfield is a fair approximation to the required solution. Hence, all the runs in this section employ a uniform initial windfield where the velocity at all points is set equal to the potential flow free stream velocity. Once again, since neutral flow is being simulated, the velocity weighting factors are all set to unity. These experiments also indicate by how much simple three-dimensional obstacles deform neutrally stratified flow. Hence, an insight may be gained into the likely effects of topography in neutral conditions.
3.4.2 Tracking Streamlines in the Final Windfield

WAFT is a tool for predicting pollutant transport over mesoscale distances. Thus, a relevant test of the final windfields is the deformation of streamlines by hills (rather than, for instance, a comparison of the velocity vectors). Throughout this section, WAFT's performance is assessed by comparing the streamlines in the final windfields with the corresponding potential flow streamlines. The streamline paths are computed from the final windfield flow-average velocities rather than the final velocities defined at mesh points. This allows a particularly simple interpolation scheme to be used to calculate the velocity at an arbitrary point, \((x,y,z)\). In fact, the velocities defined at mesh points are also interpolated from the flow average velocities and so it would involve an additional unnecessary interpolation step if these were used to compute streamline paths. It should also be noted that the flow-average form of the velocity field is also used as input to the Monte-Carlo dispersion program (described in ApSimon et al, 1984) used in conjunction with WAFT.

The simple interpolation scheme used is, referring to Figure 3.18, given by

\[
\begin{align*}
    u &= \left( \frac{x_{i+1} - x}{x_{i+1} - x_i} \right) U_i + \left( \frac{x - x_i}{x_{i+1} - x_i} \right) U_{i+1} \\
    v &= \left( \frac{y_{j+1} - y}{y_{j+1} - y_j} \right) V_j + \left( \frac{y - y_j}{y_{j+1} - y_j} \right) V_{j+1} \\
    w &= \left( \frac{z_{k+1} - z}{z_{k+1} - z_k} \right) W_k + \left( \frac{z - z_k}{z_{k+1} - z_k} \right) W_{k+1}
\end{align*}
\]

where

\((u,v,w)\) is the velocity at an arbitrary point \((x,y,z)\)
U, V, W are the flow-average velocities through cell faces.
X, Y, Z are the positions of mesh planes perpendicular to the x, y, and z-axes respectively.

The Monte-Carlo dispersion code must track many thousands of 'particles' as they are advected in the windfield which means that many interpolated velocities must be computed. The interpolation scheme must be simple and efficient; the above method fulfils these requirements. The scheme is, however, not very accurate if there are large variations in the velocities across a cell. For example, considering the equation for the x component of velocity, u, in 3.4.2(1), one sees that there is no dependence upon either the y or z position of the point. If there are large velocity gradients across the cells then this might lead to abrupt changes in the interpolated velocities as one crosses cell boundaries.

The streamlines are generated using the following algorithm. The streamline passing through (x, y, z) travels next to (x_l, y_l, z_l) where

\[
\begin{align*}
x_l &= x + St.u \\
y_l &= y + St.v \\
z_l &= z + St.w
\end{align*}
\]

3.4.2(2)

'St' is the 'streamline resolution' time; the smaller it is made, the smoother the generated streamlines. New velocity components are interpolated at (x_l, y_l, z_l) and then used to calculate the next position. The process is repeated until the streamline impacts the land surface or leaves the calculational domain. The method of computing the streamlines has been discussed in some detail since discrepancies between the potential and WAFT streamlines may be due to errors in the WAFT velocity field or the way the streamlines are generated. This point will be referred to again when the WAFT streamlines are discussed.
3.4.3 Streamlines over a Hemisphere

The calculational domain (in kilometres) for the first of these runs is defined as $-4<x<+4$, $-3<y<+3$ and $0<z<2.5$ with a hemispherical hill of radius 1 km placed centrally at $(0,0,0)$. The cells are all cubes of side 250m. The calculational domain and topography are illustrated in Figure 3.19. The triangular planes which define the land surface in WAFT give a good representation of a hemisphere. Care is taken that the calculational domain is large enough so that the velocity adjustments on the domain boundary are much smaller than those near the hill surface; the two-dimensional work earlier in this chapter showed how important this is.

The potential flow solution is axisymmetric about a line parallel to the $x$-axis running through the centre of the hemisphere. The streamlines lie in planes which are coaxial with this flow axis. Within any one of these planes, the equation of the potential flow streamlines may be written as

$$c = s^2 \left(1 - \frac{a^3}{(s^2 + x^2)^{3/2}}\right)$$

where

- $a$ is the radius of the hemisphere (metres)
- $s$ is the radial distance from a point on the streamline to the flow axis (metres)
- $x$ is the streamwise distance of the point from the hemisphere's centre (metres)
- $c$ is a constant for a given streamline. It determines the streamline's distance from the flow axis. If $s_\infty$ denotes the distance of the streamline from the flow axis at $x=\pm\infty$, then $c=s_\infty^2$.

Equation 3.4.3(1) may be used to compute a streamline's radial distance from the flow axis at an arbitrary downstream distance, $x$, given its distance from the flow axis at
$x=0$. This relationship provides a means of assessing the accuracy of the WAFT generated streamlines.

The solid curve in Figure 3.20 depicts the theoretical relationship between the distance from the streamline to the hemisphere's surface at $x=0$ and the radial distance at $x=4$ km. For comparison, streamlines are released at various distances from the hemisphere surface, at $x=0$, in the WAFT final windfield. These are tracked until they leave the domain at $x=4$ km, and their radial distance from the flow axis is noted. These values are compared with the theoretical relationship. Streamlines are released in three coaxial planes; one plane is vertical, and the other two are inclined at forty and eighty degrees to the vertical. Three of these WAFT streamlines in each of the three coaxial planes are shown in Figure 3.19. The circles in Figure 3.20 illustrate the results obtained with the WAFT windfield streamlines using the grid described above.

Generally, there is good agreement between the WAFT and theoretical streamlines. The points near the origin are not so good; for example, the streamline whose closest approach to the hemisphere is 21 m should be at a height of approximately 250 m at $x=4$ km, but is in fact at a height of about 400 m at that downwind distance. These deficiencies are not surprising, however. Consider, for instance, the streamline which is at 100 m above the ground at $x=4$ km; it should travel to within only 3.3 m of the hemisphere's summit. This would imply a quite unrealistic level of accuracy when plotting streamlines. The discrepancies near the origin reflect the coarseness of the grid (250 m) and the crude streamline generation algorithm described previously, rather than defects in the WAFT windfield. This explanation is supported by runs performed on different calculational meshes. These results are also depicted in Figure 3.20. The triangular symbols shows the results using a finer vertical grid spacing of 125 m, and the square symbols when a coarser spacing of 500 m are used. As one would expect, the fine grid produced better results, the coarse grid worse
ones. Again, the fact that grids employing non-cubic cells produces reasonable results is further evidence that the cell weighting factors are correctly formulated.

The streamline tracking algorithm may be improved to give better results. The improvement involves making an adjustment to the velocity component normal to the ground surface in the flow layers near the ground surface. This is done throughout a thin 'skin' of air close to the ground. At the top of this 'skin' no adjustment is made; that is, the velocity is as interpolated before. At the bottom of the 'skin' (the ground surface) the normal component is set to zero (the crude interpolation scheme described in equations 3.4.2(1) does not generally give a zero normal component at the ground). Throughout the 'skin' itself, there is a linear variation between zero adjustment at the top, and zero normal component at the bottom. The results obtained using this adjustment procedure with the 'skin' thickness set to 100m on the original mesh are depicted as the crosses in Figure 3.20. Much better results are obtained for the streamlines which pass close to the hemisphere surface.

Finally, a tube of streamlines is tracked through the domain to indicate how a pollutant plume might be deformed as it passes over a hemisphere. Streamlines are released from a circular array of sources on the upwind domain boundary. In purely uniform flow, they would generate a cylindrical tube as they moved through the domain. The presence of the hemisphere distorts the tube in several ways; this is shown in Figure 3.21. The streamlines have a much closer approach to the ground surface; at x=0km the bottom of the tube is 300m above the ground, but it comes to within 25m of the ground surface at the hill crest. The tube's cross-sectional shape is also affected. At x=0km it is a circle of diameter 400m, but at the hill crest its depth has been reduced to 145m, and its width increased to 860m. These two tube cross-sections are also shown in Figure 3.21. One other effect, which is not apparent in the figure, is the increase in velocity over the hill crest.
3.4.4 Comparison with the MATHEW Code

The approach adopted so far when evaluating three-dimensional WAFT windfields has been to examine the flow streamlines. This is because it is felt that this is the most relevant measure to pollutant dispersal. However, the velocities at mesh points are considered here so that a comparison with published results from MATHEW (Sherman, 1978) can be made. The work by Lewellen et al (1982) evaluating the MATHEW/ADPIC models gives details of the accuracy to which MATHEW predicts velocities around a hemisphere. A calculational mesh defined by \(-3 \leq x \leq 3, -3 \leq y \leq 3,\) and \(0 \leq z \leq 3\) is used, with a unit radius hemisphere centred at \((0,0,0)\) (all dimensions are in arbitrary units). The cells are cubes of side 0.3, and the initial windfield is uniform; the velocity at every point is \((1,0,0)\). An identical grid is used in the WAFT run and the results from the two models compared.

The results are presented in Figure 3.22 in terms of the maximum speed error in the final windfields outside various radial distances from the hemisphere's centre. The triangles represent the results from MATHEW, the circles those from WAFT. WAFT produces errors which are typically three to four times smaller than the corresponding MATHEW values. More strikingly however, WAFT is an order of magnitude more accurate at the smaller radii. This very much better performance close to the hill surface is probably a reflection of the more realistic topography representation employed in WAFT.

A run is also conducted which uses a non-uniform initial windfield. The input data consists of the potential flow velocities at eight surface points around the hemisphere base and one value from the top of the domain. A three-dimensional inverse square interpolation scheme constructs the initial windfield. The calculational mesh is as described previously. The speed errors for the WAFT initial and final windfields are depicted in Figure 3.23. Note that
the errors in this case in the final windfield are very much larger than when a uniform field is employed. The final windfield is no more accurate than the initial field. This case serves to underline the message from section 3.3 that the initial windfield specification is of great importance.

3.4.5 Flow over a Hemisphere Stream Surface

Subsection 3.4.3 showed that WAFT can produce a very good approximation to potential flow over a hemisphere given a reasonably good initial windfield as a starting point. The hemisphere has a rather abrupt shape and so it is desirable to perform some runs on more realistically shaped hills; this will give a clearer indication of how real hills deform neutral flow. The three-dimensional potential flow over an arbitrary obstacle is difficult to calculate; for instance, the potential flow over an ellipsoid shape requires very complicated integral functions as discussed in Milne-Thomson (1960), for example.

A convenient way of defining a low slope hill is to use a streamline surface of potential flow for the purpose. Once again, this provides an easily calculable potential flow solution. The potential flow around a sphere is axisymmetric, and hence the most obvious stream surfaces are also axisymmetric. However, these axisymmetric surfaces have several disadvantages. Firstly, whilst their downwind profiles are smooth, their crosswind profiles are semi-circles once again. Secondly, the land surface so generated does not tend to a plane away from the hemisphere itself.

The stream surfaces need not be axisymmetric, however; one may map out a more suitable stream surface by following streamlines released from a horizontal line source at $x=-\infty$. This type of surface is shown in Figure 3.24. The slope of the hill may be varied by choosing the height of the line source above the flow axis. Two hills are considered, one with a mean slope of around 0.2 (shown in Figure 3.24), and
one having a slope of approximately 0.05. Potential flow theory suggests that disturbances in the flow extend up to a height, L, above the hill, where L is the hill's half-length. Note, the depth of the atmosphere which is perturbed is not related to the hill height. The calculational domains used here have a vertical extent of the order of the hill's half-length, which is therefore much higher than the hill height.

The final windfields are again assessed in terms of streamline deflections. A typical case will be described. The 0.2 slope hill is used, it being about 370m high, and 3 km long. The hill is placed centrally in a domain defined by \(-6 \leq x \leq 6\), \(-4.8 \leq y \leq 4.8\) and \(0 \leq z \leq 2.55\) (all values in km). The mesh cells are 0.4 by 0.4 by 0.15 km high. Streamlines are tracked in a vertical plane passing through the centre of the hill which is aligned with the flow direction. Figure 3.25 illustrates the results obtained. The continuous lines depict the potential theory streamlines, and the circles indicate points on the WAFT streamlines. Good results are obtained, with the exception of those streamlines close to the land surface. Once again, as in subsection 3.4.3, this is probably a reflection of the algorithm used to generate streamlines. When a 'skin' correction is introduced the results are improved; the triangles in Figure 3.25 illustrate the corrected streamlines (the 'skin' thickness is 150m in this case). The horizontal deflections of WAFT streamlines also match potential theory closely. Note that horizontal deflections are quite modest even for a relatively steep sided hill such as this. For example, streamlines at half hill height (where the hill is 2 km wide) are deflected by, at most, 150m.

Similarly acceptable results are also obtained on the shallower slope hill. The cells here are very much longer and wider than high, so this again provides a validation of WAFT's cell weighting factors. If calculational domain heights are used which are much smaller than the hill length then the results are very poor, however. In these cases
WAFT produces too much adjustment at the top domain boundary leading to an overestimate of vertical velocities and an underestimate of the crosswind flows. Correspondingly, the vertical streamline deflections are too big, the horizontal ones too small. The failure to produce good streamlines in these domains is encouraging since it is in accord with potential flow theory in that perturbations extend to a depth similar to the hill's length.

Finally, the deformation of a stream tube over these low slope hills is considered. Similar effects to those obtained over the hemisphere are observed, but even with the 0.2 slope hill the distortions are much less pronounced. In the case of the 0.2 slope hill, a stream tube of diameter 200m at a height of 130m deforms into tube with an oval cross section of width 200m and height 150m at a position of 40m above the hill summit.

3.4.6 Summary of Three-Dimensional Potential Flow Comparisons

WAFT successfully predicts the potential flow around three-dimensional obstacles providing that a 'good' initial windfield is used and that the domain boundaries are sufficiently distant from the hill. The former constraint echoes the work in section 3.3. The work here has shown that the vertical domain extent must be of the same order as the hill length. This has a number of implications when using the model in real terrain. The typical hill length in real terrain might be 5 km, and so this implies a rather deep calculational domain. Most of this deep domain would not be of any interest in pollutant dispersal studies. The deep domain would also imply poor vertical mesh resolution or a great many vertical levels leading to increased computational effort. A more serious problem is that the constant density assumption made in the WAFT model becomes increasingly untenable in these deep domains.
In fact, however, the atmosphere's thermal stratification means that there is some height (generally in the lowest kilometre or so) at which vertical motion is suppressed. It is therefore not necessary to use such a deep calculational domain. However, one is no longer dealing with the homogenous neutrally stratified atmosphere assumed in this chapter. It is necessary to introduce thermal stratification into the WAFT model, and this is considered in the next chapter.
Fig 3.1 - Two-dimensional potential flow over a half-cylinder; velocities plotted at WAFT mesh points.

Fig 3.2 - Uniform initial windfield used to assess different weighting schemes in flow over a half-cylinder.
Fig 3.3 - Final windfield over a half-cylinder using the original formulation of WAFT ('first weighting scheme').

Fig 3.4 - Part of a two-dimensional WAFT mesh which illustrates the deficiencies of the first weighting scheme.
Fig 3.5 - WAFT final windfield over a half-cylinder using the second weighting scheme.

Fig 3.6 - WAFT final windfield over a half-cylinder using either the third or fourth weighting scheme.
Fig 3.7 - Streamline of potential flow used to define ridge shape.

Fig 3.8 - Potential flow over ridge defined in Figure 3.7.
**Fig 3.9** - WAFT final windfield over a low slope hill illustrating the weakness of the third weighting scheme when elongated mesh cells are used.

**Fig 3.10** - WAFT final windfield over a half-cylinder illustrating problems with the fourth weighting scheme when variable mesh spacing is employed.
Fig 3.11 - 'Standard' mesh used to evaluate sources of error in the WAFT method.
Fig 3.12 - The effect of varying the size of the mesh cells and calculational domain on WAFT windfield accuracy.
### Figure 3.13 - The effect of varying the mesh cell shape on the WAFT windfield accuracy.

<table>
<thead>
<tr>
<th>Cell shape</th>
<th>Cell dimensions (length x height)</th>
<th>R.M.S. speed, angular errors (% , degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>250 x 100</td>
<td>2.2% , 1.1°</td>
</tr>
<tr>
<td>I</td>
<td>250 x 25</td>
<td>6.6% , 2.2°</td>
</tr>
<tr>
<td>II</td>
<td>125 x 125</td>
<td>2.4% , 0.7°</td>
</tr>
<tr>
<td>III</td>
<td>250 x 250</td>
<td>2.7% , 2.0°</td>
</tr>
<tr>
<td>IV</td>
<td>125 x 250</td>
<td>2.8% , 1.4°</td>
</tr>
<tr>
<td>V</td>
<td>250 x 50</td>
<td>3.0% , 1.0°</td>
</tr>
</tbody>
</table>
Input data values

Initial wind field

Final wind field

Fig 3.14 - The effect of the initial windfield specification on the final windfield - seven randomly placed data points and bicubic spline interpolation.
Fig 3.15 The effect of the initial windfield specification on the final windfield - twelve symmetrically placed data points and inverse square interpolation.
Fig 3.16 Two-dimensional flow around a cylinder used in the evaluation of different interpolation schemes.
Station Array "A"

Station Array "B"

Station Array "C"

Initial Windfield Accuracy

<table>
<thead>
<tr>
<th>Station Array</th>
<th>Inverse Square</th>
<th>Inverse Square (radius of influence = 500 m)</th>
<th>Exponential (a = 250 m)</th>
<th>Polynomial (cubic)</th>
<th>Bicubic Spline (1 panel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>22.6% 23.1°</td>
<td>25.0% 22.3°</td>
<td>57.4% 27.7°</td>
<td>18.9% 18.5°</td>
<td>10.9% 4.5°</td>
</tr>
<tr>
<td>B</td>
<td>30.1% 13.1°</td>
<td>23.5% 12.0°</td>
<td>58.2% 25.6°</td>
<td>14.9% 7.3°</td>
<td>5.0% 4.0°</td>
</tr>
<tr>
<td>C</td>
<td>73.3% 23.6°</td>
<td>64.3% 18.1°</td>
<td>79.6% 28.9°</td>
<td>60.3% 21.7°</td>
<td>105.0% 47.8°</td>
</tr>
</tbody>
</table>

The accuracy of each field is specified by:

1. R.M.S. speed error, in percent
2. R.M.S. angular error, in degrees

Fig 3.17 - Initial windfield accuracies for three data point configurations using different interpolation techniques.
Fig 3.18 - Simple interpolation scheme used when computing streamlines.

Fig 3.19 - Streamlines illustrating WAFT final windfield over a hemisphere.
Fig 3.20 - Streamlines in WAFT windfield over a hemisphere compared to potential flow.

Fig 3.21 - Deformation of a "stream-tube" as it passes over the hemisphere.
Fig 3.22 - Comparison of MATHEW and WAFT final windfields; uniform initial windfield.

Fig 3.23 - Comparison of WAFT initial and final windfields; non-uniform initial windfield.
Fig 3.24 - Three-dimensional stream surface used to define hill shape.

--- potential flow streamline
○ points on WAFT streamline
△ points on WAFT streamline (normal component corrected)

Fig 3.25 - Comparison between streamlines in WAFT final windfield and potential flow streamlines in flow over hemisphere stream surface.
Number of mesh points: 21 by 11
X-grid line spacing: 250 m
Z-grid line spacing: 100 m
Domain size: 5000 by 1000 m
Successive-Over-Relaxation parameter: 1.6
Density profile: constant density assumed
Number of iterations: 50
Initial windfield specification: (10-0, 0-0) m/s at all mesh points

Table 3.1 - 'Standard' reference run used in evaluating sources of error within WAFT.

<table>
<thead>
<tr>
<th>Number of Iterations</th>
<th>Max. inflow/outflow of any cell (m/s)</th>
<th>R.M.S. speed error (%)</th>
<th>R.M.S. angular error (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.7x10^7</td>
<td>15.9</td>
<td>12.3</td>
</tr>
<tr>
<td>6</td>
<td>1.4x10^7</td>
<td>5.4</td>
<td>2.8</td>
</tr>
<tr>
<td>12</td>
<td>6.7x10^3</td>
<td>3.1</td>
<td>1.0</td>
</tr>
<tr>
<td>25</td>
<td>7.8x10^3</td>
<td>2.3</td>
<td>1.1</td>
</tr>
<tr>
<td>50</td>
<td>1.5x10^-1</td>
<td>2.2</td>
<td>1.2</td>
</tr>
<tr>
<td>100</td>
<td>8.3x10^-2</td>
<td>2.2</td>
<td>1.2</td>
</tr>
<tr>
<td>200</td>
<td>1.1x10^-3</td>
<td>2.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 3.2 - The effect of number of iteration cycles on WAFT windfield accuracy.
Table 3.3 - The effect of the value of the over-relaxation parameter on the WAFT windfield accuracy.

<table>
<thead>
<tr>
<th>Over-Relaxation parameter</th>
<th>Max. inflow/outflow of any cell (m/s)</th>
<th>R.M.S. speed error (%)</th>
<th>R.M.S. angular error (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>2.0x10^3</td>
<td>3.2</td>
<td>1.0</td>
</tr>
<tr>
<td>1.2</td>
<td>1.0x10^3</td>
<td>2.6</td>
<td>1.0</td>
</tr>
<tr>
<td>1.4</td>
<td>3.2x10^4</td>
<td>2.3</td>
<td>1.1</td>
</tr>
<tr>
<td>1.6</td>
<td>1.5x10^4</td>
<td>2.2</td>
<td>1.1</td>
</tr>
<tr>
<td>1.7</td>
<td>9.5x10^4</td>
<td>2.2</td>
<td>1.2</td>
</tr>
<tr>
<td>1.8</td>
<td>4.0x10^4</td>
<td>2.2</td>
<td>1.2</td>
</tr>
<tr>
<td>1.9</td>
<td>6.4x10^4</td>
<td>2.2</td>
<td>1.2</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0x10^5</td>
<td>10.0</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Table 3.4 - The effect of including a vertical density profile on the WAFT windfield.

<table>
<thead>
<tr>
<th>Density law coefficient, 'a' (m')</th>
<th>P(1km)</th>
<th>P(surface)</th>
<th>R.M.S. speed error (%)</th>
<th>R.M.S. angular error (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>uniform density</td>
<td>1.00</td>
<td></td>
<td>2.2</td>
<td>1.2</td>
</tr>
<tr>
<td>1x10^-3</td>
<td>1.00</td>
<td></td>
<td>2.2</td>
<td>1.2</td>
</tr>
<tr>
<td>1x10^-4</td>
<td>0.99</td>
<td></td>
<td>2.2</td>
<td>1.2</td>
</tr>
<tr>
<td>1x10^-5</td>
<td>0.91</td>
<td></td>
<td>1.9</td>
<td>1.3</td>
</tr>
<tr>
<td>1x10^-6</td>
<td>0.37</td>
<td></td>
<td>1.4</td>
<td>3.2</td>
</tr>
</tbody>
</table>
### Table 3.5 - The importance of the initial windfield specification in determining the WAFT final windfield accuracy

<table>
<thead>
<tr>
<th>Case</th>
<th>Data point positions</th>
<th>Interpolation scheme</th>
<th>Calculational mesh</th>
<th>Initial or final</th>
<th>R.M.S. speed error (%)</th>
<th>R.M.S. angular error (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>uniform initial windfield</td>
<td>standard</td>
<td>initial</td>
<td>9.7</td>
<td>6.8</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>uniform initial windfield</td>
<td>standard</td>
<td>final</td>
<td>3.4</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>uniform initial windfield</td>
<td>mesh 1</td>
<td>final</td>
<td>4.5</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>uniform initial windfield</td>
<td>mesh 3</td>
<td>final</td>
<td>1.1</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>as Fig. 3.14</td>
<td>quadratic</td>
<td>standard</td>
<td>initial</td>
<td>159.6</td>
<td>64.3</td>
</tr>
<tr>
<td>6</td>
<td>as Fig. 3.14</td>
<td>quadratic</td>
<td>standard</td>
<td>final</td>
<td>104.0</td>
<td>69.2</td>
</tr>
<tr>
<td>7</td>
<td>as Fig. 3.14</td>
<td>bicubic spline</td>
<td>standard</td>
<td>initial</td>
<td>33.7</td>
<td>24.3</td>
</tr>
<tr>
<td>8</td>
<td>as Fig. 3.14</td>
<td>bicubic spline</td>
<td>standard</td>
<td>final</td>
<td>30.2</td>
<td>8.9</td>
</tr>
<tr>
<td>9</td>
<td>as Fig. 3.15</td>
<td>inverse square</td>
<td>standard</td>
<td>initial</td>
<td>5.1</td>
<td>3.8</td>
</tr>
<tr>
<td>10</td>
<td>as Fig. 3.15</td>
<td>inverse square</td>
<td>standard</td>
<td>final</td>
<td>2.0</td>
<td>0.9</td>
</tr>
</tbody>
</table>

* see Figure 3.12
CHAPTER 4

MODELLING THERMAL STRATIFICATION IN WAFT

4.1 INTRODUCTION

The previous chapter compared WAFT windfields with potential flow theory. In the atmosphere, potential flow corresponds to unconstrained neutral flow extending through a depth of many kilometres. This is not a realistic situation. There are always stably stratified layers in the lowest couple of kilometres which influence flow in the boundary layer. Vertical motion is inhibited in these stable layers. It has already been mentioned that the vertical velocity weighting factors appear to provide a mechanism whereby the vertical velocities may be suppressed. Hence, the effects of stable stratification may be accounted for. This chapter examines how WAFT should be modified to incorporate the effects of stable stratification.

The chapter is divided into two main sections. The next section considers the common case where neutral air is overlaid by stably stratified layers. Following that, the more unusual case of stable flow extending throughout the boundary layer is discussed.

4.2 NEUTRAL FLOW OVERLAIN BY STABLE AIR

4.2.1 Introduction

Overall the atmosphere is generally slightly stably stratified. There is, however, usually a layer of neutrally (or less commonly, unstably) stratified air next to the ground. This neutral layer is typically between a few hundred metres and a couple of kilometres deep. In general, the neutral layer grows in depth during the daytime as heat input from the sun erodes the stable temperature gradient.
Hence, the daytime neutral layer tends to be deeper during the summer. High windspeeds also deepen the neutral layer since there is a greater amount of mechanical energy available to mix and extend the neutral flow. The depth of the neutral layer may be obtained from the vertical temperature soundings produced at several of the UK Meteorological stations. In neutral or unstable conditions, the lowest layer (the mixing layer) will have a temperature gradient close to the adiabatic lapse rate, while in stable air the temperature will decrease with height much less rapidly than this rate. In the absence of such a vertical temperature sounding, the nomogram given in Clarke (1979) based on the developing boundary layer model of Carson (1973) may be used to provide an estimate for the thickness of the neutral layer.

In certain meteorological conditions there may be a thin layer of very stable air overlying the neutral flow. This thin layer is usually referred to as an elevated inversion. They often appear on vertical temperature profiles as abrupt changes of temperature gradient. The temperature may rise by 5 to 10 degrees Centigrade in a few hundred metres. These elevated inversions are typical in developing anticyclones which are characterised by a subsidence of the upper air mass leading to the elevated inversion. These inversions may persist for several days and are quite strong. Elevated inversions are also associated with the passage of fronts. Typically, the elevated inversion lies at the cloud base.

For locations in the UK, the lowest layers of the atmosphere are neutrally stratified for between 70 and 90 percent of the time (Clarke, 1979). Coastal sites tend to have higher average windspeeds and thus a greater frequency of neutral conditions. The case of neutral flow overlaid by stable air is thus the dominant meteorological situation in the UK.
The overlying stable air will tend to suppress vertical motion. The boundary between the neutral and stable air may be regarded as something akin to a horizontal impenetrable 'lid' lying on the neutral flow below. In general the 'lid' will not be flat; how deformed it is will depend upon the nature of the topography, the neutral flow windspeed (which determines the amount of kinetic energy available to distort the boundary), and the inversion strength and altitude. Since WAFT is formulated in Cartesian coordinates it can only be used to model flat 'lids'. The ability to model inversions of a more general shape is one of the potential advantages of models which adopt terrain following coordinates (see subsection 2.1.2). However, a flat 'lid' should give some indication of the effects of overlying inversions. In any case, relatively little is known about the deformation of inversions in complex topography; there would not usually be any relevant information at the time of an accident.

The turbulence level is usually very low in the overlying stable air and the inversion 'lid' therefore provides a very effective barrier to the vertical transport of pollutant. The elevated inversion can be particularly important if it lies near, or below, the top of the topography. An inversion which occurs in a valley will often trap pollution. The previous chapter pointed out that it would be necessary to use deep calculational domains (of the order of tens of kilometres) to model potential flow correctly. However, the presence of an assumed flat 'lid' removes this problem. The domains which WAFT uses need only extend up to the height of the inversion. This usually occurs in the lowest two kilometres.

4.2.2 Modelling an Elevated 'Lid' in WAFT

The boundary between the neutral and overlying stable air may be modelled by suppressing the vertical velocities at the required height. The weighting factors $\alpha_{ij}^x$, $\alpha_{ij}^y$, $\alpha_{ij}^z$,
and $B_{ijk}$ (see subsection 2.2.2) provide a means of doing this. As discussed in subsection 3.2.2, these weighting factors consist of two separate parts, each performing a different role. One, the cell weighting factor, accounts for the particular geometry of each cell wall. The correct formulation of this part of the weighting factors was discussed in detail in section 3.2. The second part, the velocity weighting factor, may be used to preferentially weight adjustments made to velocities through particular cell faces. In particular, they may be used to encourage adjustment of horizontal velocities relative to vertical velocities. The initial vertical velocities are zero, and hence vertical motion would be suppressed in the final windfield.

The weighting factors associated with the horizontal cell faces at a particular vertical level control the size of the adjustments made to the vertical flows through those faces. Setting these velocity weighting factors to large values at a particular grid level will inhibit vertical motion there in the final windfield. The relationship between the size of the velocity weighting factors selected and the resulting vertical velocities is discussed in section 4.3. However, if they are set to sufficiently high values (say $10^6$) this will ensure that the final windfield vertical velocities are zero. In this way an impenetrable flat 'lid' is modelled. This vertical velocity suppression may be performed on any of the vertical grid levels so that both the thickness and the height of the horizontal flow layer can be varied in units of the vertical mesh spacing.

4.2.3 Flow past a Low Slope Hill under an Elevated Inversion

This section considers three WAFT runs which examine the effect of an elevated inversion on the final windfield. The hill is defined by the hemisphere stream surface described previously in section 3.4.5. The hill's mean slope is
approximately 0.2, and its height is 375 metres. A vertical mesh spacing of 75 metres is used. The three cases have inversions at 600, 450, and 300 metres (note that these heights are vertical mesh levels). The vertical velocity weighting factors below the inversion are all set to unity. At, and above, the height of the inversion the vertical velocity weighting factors are all set to $10^6$. Hence the flow at and above the inversion is horizontal and acts as a 'lid' on the neutral flow below.

Figures 4.1(a), 4.1(b), and 4.1(c) illustrate slices through the three-dimensional final velocity fields with the inversion at 600, 450 and 300 metres respectively. Each case is illustrated by a vertical section parallel to the flow which passes through the hill centre and a horizontal section at a height of 150 metres. One can clearly see the presence of the elevated inversion at the different heights in the vertical slices. The horizontal slices show how the horizontal deflections increase as the inversion is lowered toward the hill top. It should be noted that, even with this relatively steep hill, the horizontal deflections when the inversion is at 1.6 times hill height (that is at 600 metres) are very similar to those when there is no inversion present. Inversions only have a large effect on the cross-wind horizontal velocities when the inversion is close to, or below, the hill summit.

Referring to the vertical slice in Figure 4.1(b) where the inversion is at 450 metres, one sees that there is a substantial acceleration of the flow over the hill top. As the inversion is lowered this acceleration is increased. There are large adjustments of the horizontal velocities in the neighbourhood of the hill summit. These are reflected by large variations in the Lagrangian parameter solution used in the adjustment process. It is in these circumstances that the Successive-Over-Relaxation numerical solution method initially adopted for WAFT fails to converge satisfactorily. As related in Chapter 2, it is therefore
necessary to employ the more robust and efficient Conjugate Gradients method.

4.3 STABLY STRATIFIED FLOW AROUND HILLS

4.3.1 Introduction

The situation where the air at the ground surface is stably stratified is less common than the case described previously where neutrally stratified air is capped with stable layers. However, these conditions do occur quite often in the British Isles, particularly at night time or during cold weather.

This stable regime is often associated with winter time anticyclonic flow. Moreover, this regime is of particular interest, since the absence of vertical mixing and low windspeeds associated with this stable flow often represent worst case conditions for atmospheric pollution.

Once again, WAFT's velocity weighting factors offer a way of simulating these weather conditions. This section addresses the question of what values these weighting factors should take in order to successfully model stable flow. The section begins with a review of the current theoretical and experimental knowledge of stable flow over a simple isolated hill. This provides a model of the flow that WAFT is required to generate. The next two subsections examine how the ratio of vertical velocities to horizontal velocities in WAFT final windfields is related to the values of the velocity weighting factors chosen. Subsection 4.3.3 uses a theoretical argument based on a single computational cell to derive this relationship. The following subsection goes on to validate this relationship using a series of WAFT runs over a single hill. Thus, a connection is established between the values of the velocity weighting factors and the
magnitude of the vertical velocities in the WAFT final windfield.

The question which then arises is whether a simple direct relationship between the velocity weighting factors and some measure of atmospheric stability will suffice to model stable flow. A series of numerical experiments using WAFT is undertaken to address this question. An attempt is made to model stable flow over hills of differing height and slope, and using different incident windspeeds. These experiments demonstrate that a simple relationship between the velocity weighting factors and atmospheric stability cannot model stable flow correctly.

It becomes clear that the velocity weighting factors will also have to be a function of the local hill slope, hill height and incident windspeed, as well as the vertical temperature gradient. The basic WAFT formulation does not include enough atmospheric physics to model stable flow. The only viable solution is to work backwards; that is, the theoretical model presented in the first subsection is used to estimate the scale of the vertical velocities, and then the weighting factors are set accordingly.

The WAFT program is modified so that the local slope, hill height, windspeed, and thermal gradient are used to predict the scale of vertical velocities. Hence, the velocity weighting factors are then set so as to achieve vertical velocities of the required magnitude. The modified WAFT is then validated with a series of runs over simple hill shapes. Since the correct behaviour is being explicitly injected into the model it is expected that it will simulate stable flow correctly; however, this work provides a useful check that the additional coding is correct.

Having examined stable flow over a simple hill, stable flow in complex terrain is considered. Little is known about stable flow in realistic topographic regions, and hence a rather arbitrary approach is adopted. A scheme is devised
whereby the area is divided up into a number of subregions, each behaving as if it were an isolated simple hill. There is virtually no experimental evidence available to assess this approach. Some illustrative runs are performed in Chapter 6 to examine the effects which might occur in a real stable flow situation. Finally, WAFT's treatment of stable flow is compared with the schemes adopted in other similar models.

4.3.2 Theoretical Description of Strongly Stratified Flow past a Three-Dimensional Obstacle

The stratified flow past a three-dimensional hill is characterised by the Froude number, F. This is a dimensionless number which compares the potential energy required by the relatively dense air at the base of the hill to displace air at the top of the hill, with the kinetic energy in the approaching airstream available to drive this process. In this context, the Froude number is defined as

\[ F = \frac{\bar{u}}{Nh} \]  

where

- \(\bar{u}\) is the mean windspeed upstream of the hill (m/s)
- h is the height of the hill (m)
- N is the Brunt-Väisälä frequency (s\(^{-1}\))

The Brunt-Väisälä frequency is given by

\[ N = g \frac{d\Theta/dz}{T} \]  

where

- g is the acceleration due to gravity (ms\(^{-2}\))
- T is the mean temperature (°C)
- \(d\Theta/dz\) is the potential temperature gradient (°Cm\(^{-1}\))
For Froude numbers greater than about 1.5 the flow regime around the hill is broadly similar to neutrally stratified flow (neutral flow corresponds to $F=\infty$). Some separation effects may occur downwind of the obstacle.

As the Froude number is reduced to around unity (by increasing the stratification or decreasing the incident windspeed) strong lee waves are created by the hill. The ratio of the wavelength of these lee waves to the hill length characterises the nature of the flow. If the wavelength is of the same order as the hill length then separation around the hill is suppressed. If the wavelength is much smaller than the hill length, then separation occurs under the waves in the lee of the hill. If the wavelength is larger than the hill length then separation in the hill lee is similar to neutral flow. The flow at these moderate Froude numbers ($F\approx 1$) may be very complicated but there are generally significant mean vertical velocities present in the flow.

However, as the Froude number is reduced still further (say to below 0.4) the air flows past the hill in essentially horizontal layers. It is this strongly stratified flow regime which the rest of this section will concentrate upon. A more detailed description of the various flow regimes corresponding to different Froude numbers is given in Hunt and Snyder (1980) who use water flows which have a saline density gradient to model the effects of atmospheric stability. Work by Sykes (1978) using a sophisticated numerical model has also simulated stable flow.

The theory of strongly stratified stable flow was originally developed by Drazin (1961) and then extended by Brighton (1978). The low Froude number theory predicts, and experiments confirm, that the flow past a three-dimensional obstacle, of height $h$, may be split up into three regions as shown in Figure 4.2. The flow in region II is, to a first
approximation, horizontal; the flow in any horizontal plane being given by two-dimensional potential flow past the sectional shape of the hill at that height. There are two regions, at the base of the hill, and at its summit (region I, and region III in Figure 4.2) where the fluid is not constrained to move horizontally. Those layers have a thickness of approximately $Fh$. The height of the base of region I is known as the critical height, $h_c$ and is given by

$$h_c = (1 - F) h$$

Streamlines released upstream from heights greater than this critical height will go over the hill. Conversely, those released below the critical height will travel around the hill. The critical height is thus an important concept when considering pollutant transport in stable flows. The flow in region I may be treated as three-dimensional potential flow over that portion of the hill which extends higher than the critical height.

The flow in region II is not completely horizontal. As the flow passes around the obstacle the vertical pressure gradient is perturbed and this is balanced by perturbations in the density gradient caused by small vertical displacements of streamlines in region II (see for example Brighton). Equating these perturbations in the vertical density and pressure gradients gives an estimate of the vertical deflections of the streamlines in region II. Thus, the magnitude of the small vertical velocities in region II may be determined. It turns out that

$$w \sim u(h).F^2 \text{ local slope}$$

where

$u(h)$ is the upwind horizontal windspeed at that height.
The above theory assumes that the height of the hill is much less than the height over which the potential density decreases by a large fraction and that there are no abrupt changes of potential density as there might be, for example, at strong elevated inversions.

Experiments over an isolated hill in America, Cinder Cone Butte reported by Lavery et al (1982) confirmed the presence of a critical height in stably stratified atmospheric flow. Tracers were released upstream of the hill, above and below the calculated critical height, and as predicted, tracers released above the critical height went over the top of the hill, whilst those released below the critical height went around the hill. Some experiments were conducted when the critical height was predicted to be well below half the height of the hill (for example, the hill height was 180m, and critical heights as low as 32m were used) and yet the same behaviour of air moving over the hill above the critical height, and around the hill below it was observed. This suggests that the above theory may give reasonable results for Froude numbers somewhat higher than 0.5.

The theory presented above gives a prescription for the flow regions around the hill in stably stratified flow as well as an estimate of the scale of the vertical velocities encountered in the quasi-horizontal flow region. As was discussed in section 4.2.2, the vertical velocity adjustment weighting factors can be used to control the vertical velocities in the WAFT final windfields. It is hoped that, by adjusting the weighting factors suitably, the basic features of stably stratified flow may be modelled. The first step in doing this is to ascertain the relationship between the size of the vertical velocity adjustment weighting factors and the resulting vertical velocities in the final windfield. A short theoretical discussion of this relationship is presented in the next section.
4.3.3 The Relationship between the Velocity Weighting Factors and the Vertical Velocities

If one rewrites the minimisation sum given in Chapter 2 (equation 2.2.2(1)) as

\[ S = \sum \alpha_x^{c} U^2_{ijk} + \alpha_y^{c} V^2_{ijk} + \beta_{ijk}^{*} W^2_{ijk} \] 4.3.3(1)

where

- \( \alpha_x^{c} \) is the cell weighting factor for faces normal to the x-axis
- \( \alpha_y^{c} \) is the cell weighting factor for faces normal to the y-axis
- \( \beta_{ijk}^{*} \) is the cell weighting factor for faces normal to the z-axis
- \( \beta_{ijk}^{*} \) is the velocity weighting factor affecting the size of the vertical velocities

One wishes to know how the vertical velocities generated in the final windfield depend upon the value of \( \beta_{ijk}^{*} \). A cursory glance at equation 4.3.3(1) suggests that the following relationship may be appropriate

\[ w \propto \frac{1}{\sqrt{\beta_{ijk}^{*}}} \] 4.3.3(2)

In fact, this relationship is not valid. Whilst the minimisation sum, equation 4.3.3(1), generally refers to a complicated system of interrelated WAFT cells, an insight into the relationship between \( \beta_{ijk}^{*} \) and the vertical velocities is provided by considering one cell in isolation. The detailed analysis is given in Appendix 2, but the essential results are as follows

\[ w \sim \frac{m}{u} \text{ for } \beta_{ijk}^{*} m^2 \ll 1 \] 4.3.3(3)

and
where

\[ \frac{w}{u} = \frac{1}{m^2 \beta_{\text{ij}}^*} \quad \text{for} \quad \beta_{\text{ij}}^* m^2 \gg 1 \]

4.3.3(4)

\( w/u \) is the mean ratio between vertical and horizontal velocities in the WAFT final field in the neighbourhood of the hill

\( m \) is the local slope of the topography

\( \beta_{\text{ij}}^* \) is the velocity weighting factor controlling the magnitude of the vertical velocities as before

Note that in typical terrain the local slope, \( m \), would be of the order of 0.1 so that if \( \beta_{\text{ij}}^* \) were set equal to unity then one would be in the regime given in 4.3.3(3). This equation shows that \( w/u \) would be of the order of the local slope which is the relationship one would expect in neutrally stratified flow; this is consistent with the results obtained in Chapter 3 when \( \beta_{\text{ij}}^* \) was set to unity and WAFT generated a good approximation to potential flow over low hills. If, on the other hand, \( \beta_{\text{ij}}^* \) is set to a high value so that \( \beta_{\text{ij}}^* \gg 1/m^2 \) the analysis predicts that the velocity ratio, \( w/u \), is inversely proportional to \( \beta_{\text{ij}}^* \) (and not its square root) and perhaps more surprisingly it is also inversely proportional to the local slope.

These predictions are based upon the rather artificial case of an isolated cell. It is therefore desirable to conduct some numerical experiments with WAFT to see if they are indeed valid. These runs are considered next.

4.3.4 Numerical Experiments Relating Vertical Velocities to the Velocity Weighting Factors

The numerical experiments presented here employ three parabolic hills having mean slopes of approximately 1.0, 0.1 and 0.02. The hill height is 400 metres. In all cases, a uniform initial windfield is used; the velocity being
parallel to the x-axis. In each case, some power of ten is used for the vertical velocity weighting factor. This same velocity weighting factor is applied to every horizontal cell face. As before, the vertical velocity weighting factor is designated \( \beta^* \). Figures 4.3(a) and 4.3(b) illustrate the final windfields corresponding to vertical velocity weighting factors of 1.0 and 10.0 respectively. Each relates to the hill having unit slope and depicts two slices through the three dimensional windfield; a vertical slice through the centre of the hill parallel to the flow direction, and a horizontal slice at a height of 200 metres. As one would expect, as the size of the velocity weighting factor \( \beta^* \) is increased, the vertical velocities are suppressed, and hence the horizontal deflections become greater.

A mean value for the ratio of the vertical to horizontal windspeeds in the neighbourhood of the hill is computed for each case. Table 4.1 presents the results for the sixteen cases considered. The table gives the mean slope of the hill, the value used for \( \beta^* \), and the computed vertical to horizontal velocity ratio. Also shown are the values of \( \beta^* m^2 \) and \( 1/\beta^* m^2 \); these are relevant to the theoretical argument presented previously. Note that, in agreement with the previous section, when \( \beta^* m^2 >> 1 \) (that is \( \beta^* m^2 >> 100 \)) then the velocity ratio approximates to \( 1/\beta^* \). In some cases the agreement is rather approximate. For instance, in case 8, there is a factor of three difference between the predicted and actual ratio of vertical to horizontal velocities. This is not significant however, as the theoretical argument is only an order of magnitude estimate, and in any case, the measured ratio depends upon which mesh points one chooses to define as being 'in the neighbourhood' of the hill. Moreover, the results do show the expected behaviour as the mean slope or the velocity weighting factor is varied when the condition \( \beta^* m^2 >> 1 \) holds. For example, comparing case 3 with case 4, case 8 with case 9, or case 15 with case 16 demonstrates that the velocity ratio is inversely proportional to \( \beta^* \). Similarly, case 4 and case 8, or case 9 and case 14
illustrate the more surprising result that the velocity ratio is also inversely proportional to the mean slope. On the other hand, if the condition $\beta m^2 << 1$ applies then the velocity ratio is proportional to the mean slope, and does not depend on the value of $\beta^*$. Thus, these numerical experiments confirm the theoretical argument based upon a single cell given in the previous subsection. It is therefore possible to use the vertical velocity weighting factors to control the size of the vertical velocities in the final windfield in a predictable manner. This only applies if the condition $\beta m^2 >> 1$ is satisfied. Note that since when $\beta m^2 >> 1$, $\frac{1}{\beta} m \approx w/u$ this condition may be restated as $m >> w/u$. In other words, when $\beta^*$ is large enough to ensure that the ratio of the vertical to horizontal velocities, $w/u$, is much smaller than the mean slope, $m$, then the velocity ratio is inversely proportional to $\beta^*$.

4.3.5 The Failure of a Simple Approach to Model Stable Flow

The approach one might envisage using in order to model stable flow would be to obtain a simple direct relationship between the vertical velocity weighting factors used and some measure of atmospheric stability. This relationship would not involve other factors such as hill shape, or incident windspeed. One might, for instance, use the vertical temperature gradient to indicate the atmosphere's stability. Given the observed temperature profile one could derive a single value for the velocity weighting factor, $\beta^*$ and apply this over the whole calculational mesh. The work presented in this section demonstrates that such an approach will not work.

Recalling the theoretical results presented in subsection 4.3.2, the difference between the hill height and the critical height is
\[ h - h_c \sim h F = h \bar{u} = \bar{u} \]
\[ \frac{Nh}{N} \]

4.3.5(1)

and the ratio of vertical to horizontal velocities in the near-horizontal flow around the hill is

\[ \frac{w}{u} \sim F^2 m = \frac{\bar{u}^2}{N^2 h^2 L} = \frac{\bar{u}^2}{N^2 h L} \]

4.3.5(2)

For a particular atmospheric stability \( N \) is constant, and so

\[ (h - h_c) \propto \bar{u} \]

4.3.5(3)

and

\[ \frac{w}{u} \propto \frac{\bar{u}^2}{h L} \]

4.3.5(4)

If the simple approach postulated above is to be successful, then WAFT should obey 4.3.5(3) and 4.3.5(4) for any particular value of the vertical velocity weighting factor chosen (since one hopes that a single vertical velocity weighting factor corresponds to a particular value of \( N \)).

Five runs are considered using parabolic hills of various heights and lengths. The mean slope is either 0.5 or 1.0. The initial windfield is uniform in every case. The initial velocity at all points is \((2,0,0)\) m/s, except for case 3 when a velocity of \((1,0,0)\) m/s is employed. In each case, a value of 100 is used for the vertical velocity weighting factor, \( \beta \). This is applied uniformly throughout the calculational domain. Two quantities are measured in the final windfields. Firstly, the mean value of the ratio of horizontal to vertical windspeeds, \( w/u \), in the neighbourhood of the hill is computed. This should behave according to 4.3.5(4). Secondly, the vertical deflection of the centre-line streamline which just clears the hill summit. This corresponds to \( h - h_c \) in 4.3.5(3), and should therefore be proportional to \( \bar{u} \).
The results are given in Table 4.2. The relationships 4.3.5(3) and 4.3.5(4) are not obeyed. As just one example, consider cases 1 and 3. The runs are performed over an identical hill, but the initial windspeed in case 1 is half that of case 3. The theoretical relationships suggest that the value of $h-h_c$ in case 3 should be half that of case 1. In fact, they are the same. Similarly, the value of $w/u$ in case 3 should be one quarter of that for case 1. However, here again, the two cases give the same value. This result is not surprising bearing in mind WAFT's mathematical formulation; one would expect that the magnitude of the initial windspeed will merely be a scaling factor applying equally to all velocity components. If one examines a vertical crosswind section of the WAFT final windfield near the hill, one also sees no evidence of a 'critical' height as predicted theoretically.

The overall failure of WAFT to correctly predict stable flow phenomena should not be surprising. The theoretical discussion on stable flow presented above revolved around the Froude number. The Froude number relates the amount of energy required to overcome buoyancy forces to the amount of available kinetic energy. The WAFT model does not consider energy quantities and therefore cannot be expected to model stable flow.

4.3.6 Modelling Stable Flow using WAFT

It is clear from the work presented in the previous subsection that WAFT, in its basic form, cannot generate stable flow successfully. In fact, since the WAFT model does not consider energy balances, the only viable approach is to inject the theoretical results given above directly into the model. That is, the theoretical relations given in subsection 4.3.2 may be used to compute the expected ratio of vertical to horizontal windspeeds in a given stable flow situation. The vertical velocity weighting factors can then be set so as to achieve this ratio. The relationship
discussed theoretically in section 4.3.3, and confirmed experimentally in section 4.3.4, may be used to determine the weighting factors given the scale of the vertical velocities required. The steps WAFT goes through in order to produce stable flow over a single hill are described in the rest of this subsection.

The additional meteorological information required by WAFT if it is to model stable flow is some measure of the atmosphere's stability. This is presented to the program in the form of a value for $N^2$ (the Brunt-Väisälä frequency squared) at each vertical grid level. $N^2$ may be related to the potential temperature gradient using the relationship 4.3.2(2), and so values may be estimated from a vertical temperature profile. WAFT computes the mean value of $N$, denoted $N_{av}$, over the height of the hill. It should be noted that the theory presented in subsection 4.3.2 is only valid if there are no large variations in $d\Theta/dz$, so that $N_{av}$ should not be very different to $N$ at any grid level. Having generated the initial windfield in the normal fashion, the mean windspeed, $u_{av}$, is computed for the air between ground level, and the height of the hill, $h$. Hence, it is possible to define a mean Froude number, $F_{av}$, given by

$$F_{av} = \frac{u_{av}}{\frac{N_{av}h}{4.3.6(1)}}$$

If the mean Froude number is greater than 0.5, then WAFT rather arbitrarily decides that the flow over the hill may be approximated by neutral flow. Accordingly, all the vertical velocity weighting factors are set to unity. In fact, very complicated flow regimes (see Hunt and Snyder, 1980) may occur when the Froude number is of the order of unity, so that this may seem a rather crude assumption. However, one should bear in mind that WAFT is designed to predict the bulk flow through a mesoscale region, rather than detailed flow around a particular hill. It should be sufficient if the vertical velocities are of the correct magnitude. In any case, the flow effects may be so
complicated in these flow regimes that it is unlikely that anything other than the most sophisticated fluid dynamics model could cope. It should also be noted that the Cinder Cone Butte experiments (Lavery et al, 1982) suggested that, in some circumstances at least, the assumption of potential flow when the Froude number is greater than 0.5 might be reasonable. In the experiments at Cinder Cone Butte, a theoretical model which assumed potential flow above the critical height was validated when the Froude number was as high as 0.83. Such a high Froude number means that the bulk of the hill lies above the critical height, and therefore in potential flow.

If the mean Froude number, \( F_{av} \), is less than, or equal to, 0.5 then WAFT proceeds as follows. An effective critical height is computed for the hill,

\[
h_c = h (1 - F_{av}) \tag{4.3.6(2)}
\]

Above this critical height the velocity weighting factors are set to unity to generate potential flow. Below \( h_c \) the velocity weighting factors are set so as to achieve the vertical velocities predicted in subsection 4.3.2. Recalling the theoretical magnitude of the vertical velocities

\[
w \sim \frac{F^2 m}{u} \tag{4.3.6(3)}
\]

Previous work has shown that the vertical velocities are controlled by the weighting factors according to

\[
w \sim \frac{1}{u \beta^* m} \tag{4.3.6(4)}
\]

Hence the weighting factors should be set thus below the critical height

\[
\beta^* = \frac{1}{F^2 m^2} \tag{4.3.6(5)}
\]
Note the relationship given in 4.3.6(5) is only valid if

\[ \beta m^2 \gg 1 \quad 4.3.6(6) \]

Hence, 4.3.6(5) will only produce the correct vertical velocities if

\[ F^2 \ll 1 \quad 4.3.6(7) \]

WAFT computes the mean value for the slope of the land surface, \( m \), in each column of cells. The vertical velocity weighting factors are set according to 4.3.6(5) for every horizontal cell face below the critical height in each column of cells. Effectively, WAFT can only define the critical height in discrete intervals of the vertical grid spacing. Hence, it is often necessary to use a rather fine vertical mesh spacing when modelling stable flow. The condition 4.3.6(7) means that one should be wary of WAFT's results when the Froude number is in the range 0.3 to 0.5. Once again, however, it should be pointed out that WAFT is not expected to produce accurate flow patterns around single hills; it is a tool for predicting bulk flow through mesoscale regions. A different version of WAFT implements the above scheme; this is the 'stable flow version' of WAFT.

4.3.7 Using WAFT to Model Stable Flow over Simple Hills

This subsection describes some runs using the version of WAFT modified to generate stable flow as described in the previous subsection. The flow over parabolic hills of various slopes is considered. These experiments should not be considered as a validation of the basic WAFT formulation because, as described above, the theoretical stable flow behaviour is being explicitly injected into the model. Rather, these runs examine whether the method is viable, and check that the additional coding is correct. They also give
some insight into stable flow around a single hill and how this depends upon factors such as windspeed etc.

The first group of runs verifies that WAFT correctly generates the critical height and the neutral flow above it. The vertical deflection of the centreline crest grazing streamline is measured in each case as an indication of the location of the critical height. The hill height, approach windspeed, and the thermal stratification are varied to produce Froude numbers between 0.12 and 0.46. The two values of $N^2$ used (i.e. atmospheric stability) are $1.1 \times 10^{-3}$ s$^{-2}$ and $4.4 \times 10^{-3}$ s$^{-2}$ which correspond to vertical temperature gradients of 3.3 °C/100m and 6.6 °C/100m respectively. These values are somewhat higher than one would expect in the real atmosphere. Since the aim of the runs is to determine the position of the critical height, a relatively fine vertical mesh spacing is employed; the vertical grid spacing is approximately one tenth of the hill height.

The results are presented in Table 4.3. In general, there is good agreement between the theoretical and measured values of the centreline streamline deflection. However, case 3 is an exception to this. The poor result here is probably due to the difficulties of tracking streamlines close to the hill surface; this was discussed in subsection 3.4.2. This hypothesis is supported by the fact that case 8, which is identical to case 3 except that it has a gentler slope, produced good results. Experiments with neutral flow over that portion of the hill in case 3 which protruded above the critical height confirm that streamline tracking difficulties are indeed the cause of the problem. Figure 4.4 shows streamwise and crosswind vertical sections of the flow in case 1. The transition between the near horizontal flow layer and the neutral flow above can clearly be seen at a critical height of approximately 300 metres.

A different series of runs is used to check that the correct vertical velocities are being produced. Since the emphasis here is on producing correct vertical velocities which are
functions of the local slope in each column of cells a finer horizontal grid resolution is used at the expense of a coarser vertical mesh spacing. A range of parabolic hills with slopes between 0.02 and 1.0 is considered; the Froude number is in the range 0.07 to 0.46. The mean vertical speed close to the steepest part of the hill is compared to the theoretical prediction. The comparison between the theoretical, and measured vertical velocities is shown in the graph in Figure 4.5. There is general agreement, although in particular cases the theoretical and measured values may be different by a factor of up to six times. This variation is not too worrying bearing in mind that the theoretical relationship only gives an order of magnitude estimate, and that the measured value depends upon which points close to the hill surface one chooses to consider.

4.3.8 Modelling Stable Flow in Complex Terrain

The method which WAFT uses to model stable flow around a single hill was described in subsection 4.3.6, and the previous subsection confirmed that this approach is successful. It is now necessary to consider how stable flow in complex terrain should be treated. For instance, if one is considering stable flow around two adjacent hills of different height, are the results presented in subsection 4.3.2 applicable? If the results are valid in that situation, should the two hills be treated separately or as a conglomerate hill? There appears to be little theoretical or experimental evidence concerning stable flow in multiple hills besides the observation that vertical velocities tend to be suppressed.

In view of the sparcity of available information, the following, somewhat arbitrary, approach is adopted in WAFT. It is assumed that the complex terrain region may be considered as a number of subregions each behaving as if it were an isolated single hill. The approach given in subsection 4.3.6 is followed independently in each of these subregions. In each subregion, the maximum terrain height,
$h_{\text{max}}$, the mean windspeed, $u_{av}$, and the mean Brunt-Väisälä frequency, $N_{av}$ are computed. These are used to calculate a mean Froude number, $F_{av}$, for the subregion. If $F_{av}$ is greater than 0.5 then neutral flow is assumed, which corresponds to a critical height of zero metres. Otherwise, a critical height, $h_c$, is computed with neutral flow above, and near horizontal flow below. WAFT performs some smoothing of the critical heights over the whole domain so that abrupt changes do not occur across subregion boundaries.

The process which divides the whole domain into subregions is controlled by a parameter, $f_{HL}$, termed the 'high lying factor'. This is supplied to WAFT as a percentage and may take values between 1 and 99 percent. WAFT calculates the range of terrain heights in the domain. All topography heights which are in the lowest $f_{HL}$ percent of this total range are defined as 'low lying' points, whilst those above this are defined as 'high lying'. Each connected cluster of 'high lying' points represents the 'hill' in each of the subregions.

Figure 4.6 illustrates how the Bristol Channel area used in Chapter 6 may be divided up. Figure 4.6(a) shows what happens if the 'high lying factor' is set to 33 percent. The shaded areas represent 'high lying' land. Subregion 7 is the Welsh Mountains, subregion 3 is Exmoor, the Mendips are subregion 1, and so on. WAFT associates the 'low lying' points with the nearest 'high lying' points as shown by the subregion boundary lines. Hence, every (x,y) position lies in one of, in this case, seven subregions. Figure 4.6(b) shows the division when the 'high lying' factor is set to 66 percent. In this case, only two 'high lying' areas result; the Welsh Mountains and Exmoor. Thus the domain is modelled as if it consisted of just two hill masses.
4.4 SUMMARY

The work in this chapter has modelled thermal stratification in the atmosphere. Neutral flow beneath an elevated inversion has been simulated using the velocity weighting factors to create a flat overlying 'lid'. Experiments with a fairly steep sided hill (mean slope around 0.2) demonstrated that the inversion had to be quite close to, or below, the hill summit before there was any great effect on the crosswind velocities. Other mass consistent work seems to have paid little attention to the simulation of elevated inversions despite the fact that they can have a large effect on the horizontal flow patterns. For instance, whilst MATHEW (Sherman, 1978) can simulate elevated inversions, this capability has not been exercised; see the review by Lewellen et al (1982). The inversion in MATHEW has been implemented at a constant height above the topography, rather than a flat 'lid' at a fixed height above Sea Level as in WAFT. There is some doubt as to how realistic this is, and it will probably have a much smaller effect on the horizontal flow patterns than WAFT's flat inversion.

The case where stable flow extends upwards from the ground surface was also considered. Stable flow past a simple hill is characterised by a critical height. The flow below this height is very nearly horizontal. This situation cannot be generated with the basic WAFT formalism, and therefore the model has to be modified with the required behaviour being rather artificially injected. Complex topography is treated as a collection of isolated simple hills. The way stable flow is modelled in WAFT may be considered as somewhat arbitrary, but it does compare favourably with other mass consistent models.

Most of the other work with mass consistent models does mention that weighting functions may be used to suppress vertical motion, but little guidance is usually given about the values one should use. For instance, Bhumralkar et al (1980) mention that the ratio $W_H/W_V$ (which is equivalent to $f^*$ in WAFT) should be determined by numerical experiments in
their mass consistent model, COMPLEX. Sherman (1978) states that the ratio $\alpha_1/\alpha_2$ (again, equivalent to $\beta^*$) should be set equal to the expected ratio of the vertical to horizontal velocities in the MATHEW code. Again, no information is given as to how this value should be obtained for different meteorological conditions. In both of these models it appears that the weighting factors are to be applied uniformly over the whole grid. The work in subsection 4.3.5 suggested that this simple approach will not deal adequately with flows over differently shaped hills nor would it generate the two layered flow characteristic of stable conditions. Rather than relying on the fairly crude categorisation of stable flow offered by the Pasquill scheme, WAFT takes into account windspeed, potential temperature gradient and hill height when simulating stable flow.

A model which does take the hill shape into account when modelling stable flow is that developed at Electricite de France (EdF) by Caneill et al (1983). The approach adopted in that model is as follows. A theoretical relationship given by Buzzi et al (1977) is used which specifies the height to which a hill perturbs the stably stratified flow. This height is given by

$$H_o = L_T \frac{F}{N} (1 - R_o^2)$$  \hspace{1cm} 4.4(1)

where $R_o$ is the Rossby number given by

$$R_o = \frac{v}{C L_T}$$  \hspace{1cm} 4.4(2)

where

- $v$ is the typical horizontal velocity (ms$^{-1}$)
- $L_T$ is the terrain horizontal length scale (m)
- $N$ is the Brunt-Väisälä frequency (s$^{-1}$)
- $F$ is the Froude number
- $C$ is the Coriolis parameter
The model is then run with different weighting factors (K in their notation) and in each case the height to which the final windfield is perturbed is noted. It is thus possible to arrive at an empirical relationship between the weighting factors and the height to which the flow is perturbed. The theoretical relationship in 4.4(1) may then be used to generate a calibration curve describing the relationship between the weighting factors and atmospheric stability. It is important to note that this calibration curve is specific to a particular hill and calculational mesh. Whilst this method does take the hill's dimensions into account, and should therefore reproduce the correct vertical velocities, it does have several drawbacks compared to WAFT. Firstly, WAFT automatically selects appropriate weighting factors for any hill; it is not necessary to do several runs to create a calibration curve. The EdF model applies the weighting factors uniformly and therefore will not generate two layered flow. Finally, it is difficult to see how the EdF approach can be applied to complex terrain.

In contrast to the other models mentioned here, WAFT uses routinely measured atmospheric variables to model thermal stratification in a coherent way. This is particularly important because the thermal structure has a large effect on the interaction between the atmosphere and topography.
(a) 600 metre inversion height

(b) 450 metre inversion height

Fig 4.1 - Flow past a simple hill with elevated inversions present at three different heights (continued on next page)
Fig 4.1 - Flow past a simple hill with elevated inversions present at three different heights (continued from previous page)
Fig 4.2 - Low Froude number flow past a hill
Fig 4.3 (a) - Flow past a simple hill with the vertical velocity weighting factors set to 1.
Fig 4.3 (b) - Flow past a simple hill with the vertical velocity weighting factors set to 10.
Fig 4.4 - Final windfield generated by the stable flow WAFT model.
Magnitude of corresponding vertical velocities close to the hill in WAFT final windfields (m/s)

Fig 4.5 - Magnitude of vertical velocities in WAFT windfields compared to corresponding theoretical stable flow values.
(a) "High-lying" factor is 33 percent.

(b) "High-lying" factor is 66 percent.

Fig 4.6 - How the Bristol Channel is divided up with two different values of the "high-lying" factor.
### Table 4.1 - The relationship between the size of the vertical velocity weighting factors used and the magnitude of the vertical velocities obtained in the final windfield

<table>
<thead>
<tr>
<th>Case</th>
<th>Mean Slope, m</th>
<th>$B^1$</th>
<th>$B^m$</th>
<th>$\frac{1}{B^m}$</th>
<th>$\mathbf{w}_w$ in WAFT windfield</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>10</td>
<td>10</td>
<td>0.1</td>
<td>0.032</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>100</td>
<td>100</td>
<td>0.01</td>
<td>$4.1 \times 10^3$</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>1000</td>
<td>1000</td>
<td>0.001</td>
<td>$4.2 \times 10^4$</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>1</td>
<td>0.01</td>
<td>10</td>
<td>0.032</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
<td>10</td>
<td>0.1</td>
<td>1</td>
<td>0.026</td>
</tr>
<tr>
<td>7</td>
<td>0.1</td>
<td>100</td>
<td>1</td>
<td>0.1</td>
<td>0.013</td>
</tr>
<tr>
<td>8</td>
<td>0.1</td>
<td>1000</td>
<td>10</td>
<td>0.01</td>
<td>$3.2 \times 10^3$</td>
</tr>
<tr>
<td>9</td>
<td>0.1</td>
<td>$10^4$</td>
<td>100</td>
<td>0.001</td>
<td>$4.1 \times 10^4$</td>
</tr>
<tr>
<td>10</td>
<td>0.02</td>
<td>1</td>
<td>0.004</td>
<td>50</td>
<td>$5.9 \times 10^3$</td>
</tr>
<tr>
<td>11</td>
<td>0.02</td>
<td>10</td>
<td>0.04</td>
<td>5</td>
<td>$5.9 \times 10^3$</td>
</tr>
<tr>
<td>12</td>
<td>0.02</td>
<td>100</td>
<td>0.4</td>
<td>0.5</td>
<td>$5.4 \times 10^3$</td>
</tr>
<tr>
<td>13</td>
<td>0.02</td>
<td>1000</td>
<td>4</td>
<td>0.05</td>
<td>$3.4 \times 10^3$</td>
</tr>
<tr>
<td>14</td>
<td>0.02</td>
<td>$10^4$</td>
<td>40</td>
<td>0.005</td>
<td>$1.2 \times 10^3$</td>
</tr>
<tr>
<td>15</td>
<td>0.02</td>
<td>$10^5$</td>
<td>400</td>
<td>$5 \times 10^{-4}$</td>
<td>$1.8 \times 10^4$</td>
</tr>
<tr>
<td>16</td>
<td>0.02</td>
<td>$10^5$</td>
<td>4000</td>
<td>$5 \times 10^{-5}$</td>
<td>$1.9 \times 10^5$</td>
</tr>
</tbody>
</table>

### Table 4.2 - The ratio of vertical to horizontal velocities and the deflection of the crest-grazing streamline in WAFT windfields

<table>
<thead>
<tr>
<th>Case</th>
<th>Hill Height (m)</th>
<th>Hill Length (m)</th>
<th>Initial velocity $U_0$ (m/s)</th>
<th>$\frac{w}{u}$</th>
<th>$h-h_c$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
<td>400</td>
<td>2</td>
<td>$6.1 \times 10^{-3}$</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>400</td>
<td>2</td>
<td>$1.0 \times 10^{-2}$</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>400</td>
<td>1</td>
<td>$6.1 \times 10^{-3}$</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>800</td>
<td>2</td>
<td>$1.0 \times 10^{-2}$</td>
<td>17</td>
</tr>
<tr>
<td>5</td>
<td>800</td>
<td>800</td>
<td>2</td>
<td>$6.1 \times 10^{-3}$</td>
<td>8</td>
</tr>
<tr>
<td>Case</td>
<td>Windspeed (m/s)</td>
<td>Hill Height (m)</td>
<td>Hill Length (m)</td>
<td>Brunt-Vaisala Frequency Squared ($s^2$)</td>
<td>Froude Number</td>
</tr>
<tr>
<td>------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>---------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>400</td>
<td>400</td>
<td>$1.1 \times 10^{-3}$</td>
<td>0.23</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>200</td>
<td>400</td>
<td>$1.1 \times 10^{-3}$</td>
<td>0.46</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>400</td>
<td>400</td>
<td>$1.1 \times 10^{-3}$</td>
<td>0.46</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>400</td>
<td>800</td>
<td>$1.1 \times 10^{-3}$</td>
<td>0.23</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>800</td>
<td>800</td>
<td>$1.1 \times 10^{-3}$</td>
<td>0.12</td>
</tr>
<tr>
<td>6</td>
<td>1.5</td>
<td>400</td>
<td>400</td>
<td>$1.1 \times 10^{-3}$</td>
<td>0.12</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>400</td>
<td>400</td>
<td>$4.4 \times 10^{-3}$</td>
<td>0.12</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>400</td>
<td>2000</td>
<td>$1.1 \times 10^{-3}$</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Table 4.3 - Comparison of the deflection of the crest-grazing streamline in windfields produced by the stable WAFT model with theoretical values.
5.1 INTRODUCTION

The work in the previous two chapters has concentrated on the effects of topography on the bulk airflow. The thermal structure of the atmosphere has a large influence on this, and thus the previous chapter considered how stable flow and elevated inversions may be modelled in WAFT. In some situations, for example strong synoptic flow under an inversion, the mesoscale flow might be dominated by the effect of the underlying topography. On the other hand, the flow in a mesoscale region is often complicated by many other local scale effects which are not modelled in WAFT. This chapter looks at some of these local scale effects. A brief description is given, as well as the conditions when they are most likely to occur.

Section 5.2 considers how frictional forces at the land surface modify the synoptic flow aloft. The previous chapters have not considered frictional forces. However, the earth's surface is generally rough and frictional effects are usually very important in the lowest layers of the atmosphere. This is particularly so over land where buildings and vegetation produce a rather rough surface. This surface drag means that windspeeds at the bottom of the atmosphere are usually considerably smaller than those aloft. The variation of velocity with height is termed the velocity profile, and its typical form in the atmosphere is described.

The gross thermal structure of the atmosphere has already been considered, but local heating and cooling effects also affect the mesoscale flow. These are usually associated with relatively slack synoptic conditions. Land/sea breeze circulations are an example of this type of local phenomena.
They result from the temperature differences between neighbouring land and water masses. These thermally driven flows are discussed in section 5.3 where three common types are considered.

The effects which occur in the wake of hills provide further examples of complicated features of mesoscale flow. These effects include lee waves and wake rotors, and they are generally more significant over relatively steep sided hills. These phenomena are briefly touched on in section 5.4.

The WAFT model is based only on the continuity equation. Temperature and surface roughness, for example, are not included in the model formalism. This rather limited physical basis means that WAFT cannot intrinsically generate the phenomena mentioned above. They will only appear in the final windfield if they are present in the initial windfield specification. Thus, if the input wind measurements were sufficiently representative of the atmospheric flow, then these local flow phenomena would be included in the WAFT final windfield. Typically however, the input data is nowhere near comprehensive enough to resolve these local scale phenomena.

Normally only surface wind measurements are available; there is no information about the variation of velocity with height. Thus, WAFT has to use an assumed vertical profile. The form of this assumed profile is discussed in subsection 5.2.3. Similarly, the surface measurements themselves are not usually dense enough to resolve any local scale thermal flows. The only viable approach seems to be to include some simple sub-models in the initial windfield generation. These sub-models should use readily available meteorological information to generate the local scale flow pattern as part of the initial windfield. For instance, one might construct an urban heat island sub-model based on the urban temperature excess. Anderson (1971) employs this sub-model approach. No sub-models are as yet provided in WAFT, but
some indication of possible sub-models of thermally driven flows is given in section 5.3.

5.2 THE VARIATION OF VELOCITY WITH HEIGHT IN THE ATMOSPHERE

5.2.1 Introduction

Surface frictional forces cause a variation in (horizontal) velocity with height. The form of this velocity profile in the atmosphere will be discussed in this section.

The atmosphere may be considered as three layers as follows:

- a surface layer a few tens of metres thick. Atmospheric momentum is lost to the ground in this layer. The motion of a small 'parcel' of air is dependent on forces exerted on it by the surrounding turbulent motion. Pressure gradient and Coriolis forces are not important in this layer.

- the Planetary Boundary Layer. This is a region where momentum is transferred downwards from the flow aloft to the surface layer. Its depth and properties depend on the vertical temperature gradient. On clear hot days, with an unstable lapse rate, the turbulence is generated by thermal energy. The boundary layer is usually between a half and two kilometres deep in these circumstances. In neutral conditions, mechanical friction is the main agent causing turbulence. The boundary layer depth depends mainly on the surface roughness and windspeed, and typically ranges between several hundred metres and a kilometre. Turbulence is inhibited in stable conditions and the boundary layer may be very shallow, particularly if the winds are light. In this layer, the turbulent forces and the Coriolis and pressure gradient forces are of the same magnitude.
the free atmosphere aloft. Pressure gradient and Coriolis forces dominate here. The winds can generally be inferred from the pressure distribution, for example, using the geostrophic approximation.

In considering pollutant transport, one is most concerned with flow in the boundary layer. Indeed, the top of the boundary layer often acts as a ceiling on pollutant dispersion. Surface measurements are made at ten metres, and the high level flow may be obtained from either pressure charts, radiosonde data, or computer prognoses. The velocity variation in the boundary layer, that is, between the surface flow and high level flow, is discussed next.

5.2.2 The Velocity Profile in the Boundary Layer

The variation of velocity in the boundary layer depends, to a large degree, on the stability of the atmosphere. A comparison between the surface wind and the velocity at the top of the boundary layer gives an indication of the velocity variation throughout the boundary layer. Findlater et al (1966), for example, examined how the free atmosphere winds compared with surface winds for different stabilities.

On hot days, when thermal convection is vigorous, momentum transfer from the free atmosphere to the surface is very effective. The surface wind may be very similar to the free atmosphere velocity, and there is thus little velocity variation throughout the boundary layer. Typically, the surface windspeed might be 60 or 70 percent of the free atmosphere speed, and the surface wind might be backed by only 10 degrees or so.

In neutral conditions, downward momentum transfer is largely through mechanical mixing. Thus, the vertical profile depends upon the windspeed and the surface roughness. As the surface roughness becomes larger, the ratio of surface to upper windspeed becomes smaller. Thus the surface
windspeed over sea (which is a relatively smooth surface) is much closer to the free atmosphere velocity than corresponding surface measurements made at land stations. A typical value for the ratio of surface to upper windspeed over land in neutral conditions is 0.5 to 0.6. The backing of the surface wind may be around 30 degrees.

Thermal convection is inhibited in stable conditions, and the lights winds which are also characteristic of stable flow mean that mechanical mixing is also small. Hence, the surface wind is not coupled very closely to the free atmosphere velocity. As a consequence, the surface windspeed may be only 20 percent of the free windspeed, and may be backed by 60 degrees or more.

Measurement of the velocity profile in the boundary layer, using pilot balloons for example, indicates that $\frac{\Delta V}{\Delta z}$ falls off rapidly with height (see Pasquill, 1971). Thus, a common model of the variation of windspeed with height is a power law such as

$$V(z) = k z^P$$  \hspace{1cm} 5.2.2(1)

where

- $V(z)$ is the windspeed at height $z$
- $k$ is a constant
- $P$ depends upon the surface roughness and lapse rate

The turning of the wind from the free atmosphere wind direction, $\bar{\gamma}$, is often nearly linear, although Smith (1975) suggests that a sine law relationship may be more appropriate.

$$\sin \bar{\gamma} = s_o \left(1 - \frac{z}{H_{BL}}\right)$$  \hspace{1cm} 5.2.2(2)

where
\( \gamma(z) \) is the backing of the wind from the free atmosphere direction
\( H_{BL} \) is the height of the boundary layer
\( s_0 \) is a constant

Whilst the above relationships, 5.2.2(1) and 5.2.2(2) provide some indication of typical profiles, it is important to stress that observed profiles are often very different. Examples of the varied profiles measured in complex terrain are provided by Wooldridge and Fox (1981). For instance, they report some profiles which have several windspeed maxima and others which have deep layers of constant windspeed. Another feature which is often observed is a windspeed maximum occurring in conjunction with elevated inversions. These low level maxima are most often associated with nocturnal inversions in the lowest 300 metres or so. The vertical profile is often especially complicated in stable flow as a result of the weak coupling between the motion in different layers.

Nevertheless, simulations of observed tetroon trajectories by Nappo (1976) have indicated that vertical profiles such as those in 5.2.2(1) and 5.2.2(2) provide a reasonable description of the velocity variation in the boundary layer. This is particularly so in neutral conditions. The exact form of the velocity profiles used in WAFT is described in the next subsection.

5.2.3 The Velocity Profile used in WAFT

The previous subsection presented relationships for the variation of windspeed and wind direction with height in the boundary layer. These form the basis of the velocity profile used in WAFT. However, it should be remembered that WAFT uses the vertical profile to interpolate between a given surface velocity vector and a given wind at the top of the boundary layer. The surface wind velocity is computed using a two-dimensional interpolation of the ten metre winds
measured at meteorological stations, and the wind at the top of the boundary layer is provided in the input data. Hence, it is not always possible to use the relationships given previously. For instance, the speed power law is not appropriate if the surface windspeed is greater than the free atmosphere windspeed. Similarly, the sine turning law cannot be used when the surface wind direction differs from the free atmosphere flow by more than 90 degrees. Hence, it is necessary to provide a velocity profile which, whilst being based on the above relationships, copes with all the situations which may be encountered by WAFT.

The speed, \( v(z) \), at height \( z \), is computed using

\[
v(z) = v_s \left( \frac{z}{z_s} \right)^a \quad \text{if } v_f > v_s \text{ and } \frac{v_f}{v_s} < \frac{H_{BL}}{z_s} \tag{5.2.3(1)}
\]

or

\[
v(z) = b z^2 + c z \quad \text{for } z < z_s \\
v(z) = d z + e \quad \text{for } z > z_s \tag{5.2.3(2)}
\]

\[
v(z) = v_s \left( \frac{z}{z_s} \right)^a \quad \text{for } z < z_s \\
v(z) = v_f \left( \frac{z}{z_s} \right)^a \quad \text{for } z > z_s \tag{5.2.3(3)}
\]

if \( v_f < v_s \)

where

- \( v_f \) is the windspeed of the free atmosphere
- \( V_s \) is the surface windspeed
- \( H_{BL} \) is the height of the boundary layer
- \( z_s \) is the height at which surface windspeed measured
The constants $a$, $b$, $c$, $d$, and $e$ are chosen to fit the required velocities at $H_{BL}$ and $z_s$, and where appropriate, make $SV/Sz$ continuous at $z_s$. Profiles corresponding to the three formulae are shown in Figure 5.1. 5.2.3(1) is a power law profile. The other two relationships are used when a power law will not fit; they are chosen so as to give a smooth profile with the greatest change in the lower part of the boundary layer.

The deviation, $\gamma(z)$, from the free atmosphere wind direction at a height, $z$, is computed using

$$\gamma(z) = 180 \left( \frac{1}{c+z/H_{BL}} - \frac{1}{c+1} \right)$$

where

- $H_{BL}$ is the height of the boundary layer
- $c$ is a constant computed to give the required turning at the bottom of the boundary layer

This formula gives very similar results to the sine law presented in the previous subsection for small turning angles, but will also provide values when the surface turning is greater than 90 degrees. Again, it was selected on the criteria that it gives a smooth relationship with most variation in the lower part of the boundary layer.

Note that, theoretically, the surface wind should be backed from the free atmosphere wind direction. Once again though, this relationship is used to interpolate between a given surface wind direction and a given free atmosphere wind direction; it is used whether the surface wind is backed or veered from the upper flow. This leads to difficulties when the surface winds are around 180 degrees different to the free atmosphere flow. For example, it is quite possible that the surface two-dimensional interpolation will produce two neighbouring source vectors, one of which is backed by, say, 175 degrees, and the other which is veered by 175
degrees (of course, this means that the two surface wind directions only differ by 10 degrees). The turning law scheme adopted means that all the velocities above one point will be backed from the upper flow, and veered above the other flow. Hence, at a level somewhere near the middle of the boundary layer, the interpolation scheme will produce an unrealistic flow discontinuity with opposing flow directions at neighbouring points.

The relationships given in the previous subsection are not, in any case, applicable when the surface wind direction is very different to the upper layer flow. Also, it has been necessary to create ad hoc profiles for use when the surface windspeed is greater than the upper windspeed. Thus, the vertical profile scheme used in WAFT is somewhat arbitrary. Other models have used different schemes, for instance, Endlich et al (1982) use a logarithmic interpolation of each horizontal component so that

\[ u(z) = u_s + (u_f - u_s) \frac{\log(z) - \log(z_s)}{\log(H_{BL}) - \log(z_s)} \]  

5.2.3(4)

In view of the arbitrary nature of WAFT's standard velocity profile some alternative profiles are included in the model. One of these alternative profiles is the logarithmic interpolation of each horizontal component given in 5.2.3(4). A linear interpolation of each horizontal component is also provided. Finally, a linear windspeed and linear turning law are also available. The differences between these three alternative profiles and the 'standard' profile are examined as part of the sensitivity studies presented in subsection 6.5.4.

5.3 THERMALLY DRIVEN MESOSCALE FEATURES

5.3.1 Land/Sea Breeze Circulations

Nearly all the nuclear power stations in the United Kingdom are situated on the coast. Dispersion from sites close to
large water masses is therefore of particular interest. One feature of the water surface is that it is much smoother than the adjoining land. This difference in surface roughness affects the vertical velocity profile and mixing layer height. However, temperature differences between the land and water surfaces are usually more important. This thermal gradient arises as incoming solar radiation warms both the land and water. Heat is transported downwards much more effectively in the water (via turbulent mixing) and so relatively large masses of water are warmed. Hence, the rise in the surface water temperature is moderated. The land surface is therefore warmer during the day. Conversely, the temperature of this relatively large mass of water drops more slowly than the land surface at night. Thus, the water surface is warmer than the land surface during the night.

The variation in surface temperature may mean that the stability of the air over the land may be different to that over the water. This will affect dispersion from a coastal site. A more familiar effect of this temperature difference, however, is that a land/sea breeze circulation may be set up. In the daytime, air rising above the comparatively warm land surface leads to a high pressure zone developing above the land. Air thus moves from the high pressure zone above the land, to the lower pressure area above the sea. This influx of air aloft causes the pressure at the sea surface to rise, and a surface flow develops from sea to land. Thus a circulation develops, with a relatively shallow onshore flow at the surface, and a deeper, but slower, offshore flow aloft. Over the land there is ascending air which defines the sea breeze front. Correspondingly, there is subsiding air over the water.

Whether a sea breeze is formed or not depends largely upon the land/sea temperature difference. There must be unstable air over the land which leads to convection there. Conversely, the air over the sea should be stable with respect to the sea temperature. In the United Kingdom, this means
that sea breezes usually only occur between April and September. The frequency of sea breeze activity at Thorney Island, on the south coast, is put at 75 a year by Watts (1965). About a tenth of these penetrate as far as 45 km inland (Simpson et al, 1977). The synoptic scale winds also affect the sea breeze formation; they tend to occur when the prevailing winds are light. The offshore wind component is of particular interest; high offshore winds will prevent sea breezes occurring (Watts, 1965).

The sea breeze evolves throughout the day; in particular, the sea breeze front moves inland. Simpson et al (1977) found that the initial advance was typically 2.5 m/s, and that this increased by about 1 m/s during the day. The sea breeze itself has a higher onshore component so that the surface sea breeze flow is continually being 'fed' to the front where it rises as described above. Simpson et al measured vertical velocities in the range 0.7 to 3.0 m/s. Figure 5.2 illustrates the typical structure of a sea breeze. A 'head' forms at the sea breeze front. The front is a zone of mixing between the sea and land air; the mixed air flows back over the advancing sea air forming a stable layer. Ascending air at the front may cause cumulus or cumulonimbus clouds to form. The front is also indicated by changes in temperature, humidity, and wind velocity.

The prevailing synoptic wind affects the rate at which the sea breeze front advances. An offshore wind slows down the rate of advance but usually makes for a much more distinct front. If the coastline is convex, there will be horizontal convergence and the sea breeze front will again be more marked. The direction of the sea breeze also changes throughout the day. Initially, the sea breeze blows at rightangles to the coastline, but the Coriolis effect causes it to rotate, so that by the evening it may be flowing parallel to the coastline.

Experimental and theoretical work by, for example, Simpson and Britter (1980) has modelled sea breezes in terms of
density currents. The laboratory experiments have examined the advance of saline currents in a tank containing fresh water. It has thus been possible to arrive at some relationships which may be used to predict the sea breeze velocity, and the rate of advance of the front. These relationships have compared well with measurements made in the atmosphere (both in sea breezes and thunderstorm outflow). The rate of advance of the sea breeze front, $U_{SBF}$, is given by

$$U_{SBF} \sim \alpha_1 \sqrt{\frac{g \cdot S \cdot h_{SB}}{T}} + 0.6 \cdot U_A$$

where

- $\alpha_1$ is a constant, typically about 0.9 in UK conditions
- $g$ is the acceleration due to gravity
- $S$ is the land/sea temperature difference
- $T$ is the air temperature over the sea
- $h_{SB}$ is the depth of the sea breeze inversion
- $U_A$ is the ambient onshore wind

The sea breeze velocity relative to the sea breeze front, $U_{SB}$, is given as

$$U_{SB} \sim \alpha_2 \sqrt{\frac{g \cdot S \cdot h_{SB}}{T}}$$

where

- $\alpha_2$ is a constant, typically about 0.15 in UK conditions

The height of the sea breeze inversion, $h_{SB}$, is approximately

$$h_{SB} \sim \frac{H_{BL}}{4}$$

where

- $H_{BL}$ is the height of the mixing layer
A sea breeze is likely to have a large impact on pollutant dispersal from coastal sites. The sea breeze is a circulatory phenomenon which raises the possibility of material being swept back over the release point. There are large vertical velocities and large changes in horizontal velocity at the sea breeze front. Figure 5.3 illustrates how pollutant might be transported in the two cases of an ambient onshore and offshore wind. The circulatory motion and the low sea breeze inversion height can mean that pollutant concentrations may be very much higher than if there were no sea breeze.

Land breezes are the night-time counterpart of sea breezes. Their formation is favoured by clear nights and slack synoptic conditions. The land cools faster than the sea at night, and a circulation develops which is analogous to the daytime sea breeze. This time, the surface flow is seaward, air rises above the sea, the return flow aloft is towards the land, and there is subsidence over the land. In general, though, land breezes are much weaker features than sea breezes. They are often very shallow (about 50 metres). They may be more pronounced if they are reinforced by, for example, downslope winds flowing from coastal hills (see Moffit, 1956).

Land and sea breezes are local in scale and have a complicated evolutionary structure; the density of routinely measured surface winds would not, in general, be sufficient for WAFT to model these winds correctly. A single coastal station might, however, indicate whether a sea breeze existed.

An approach which could be used to include land/sea breezes in WAFT windfields is to incorporate a sea breeze sub-model in the initial windfield generation. The coastal winds could be modelled using relationships such as those presented above to estimate the likely strength and extent of the sea breeze. For instance, the surface sea breeze velocity may be obtained from 5.3.1(1) and 5.3.1(2); the
initial surface winds between the shore and the sea breeze front would be set to this value. Beyond the front, the usual method of computing surface initial winds would be employed.

The standard profile laws used to interpolate velocities between the ground and the top of the calculational domain would not be appropriate for sea breeze flow. Instead, one might use a constant onshore velocity up to the height of the sea breeze inversion, with a return offshore flow above that. The sea breeze front would be a line of horizontal convergence in the initial windfield. Thus, one would expect that WAFT would produce an appropriate updraft there as continuity is enforced. A series of WAFT runs, with the sea breeze front advancing inland, would be needed to model the dynamic nature of sea breezes.

Such a submodel would only provide a rough guide to the complexities of real sea breezes. In practice, other effects such as topography, stability and complex coastal geometries, will complicate the sea breeze structure. Nevertheless, such a crude model could at least indicate the type of local coastal flow patterns that might be present.

5.3.2 Mountain/Valley Winds

This subsection discusses the thermally driven winds which may occur in complex terrain. Katabatic winds are local down-slope flows which arise when air in contact with sloping ground is cooled. They tend to occur on nights with clear skies and weak synoptic flow. Katabatic winds may also form when mountain slopes are cooled quickly by precipitation, particularly snow.

These flows tend to be rather shallow, and are unlikely to exceed 50 metres in depth in the UK. A description of the vertical structure of observed katabatic winds is provided by Manins and Sawford (1979a). The major down-slope layer
was about 40 metres thick and was characterised by a strong thermal inversion and a windspeed maximum of about 3.5 m/s. Above this was a deeper layer of mixing (about 100 m. deep) between the surface down-slope flow and the ambient air aloft. The velocity profile in this layer was linear.

If the slopes drop down to the coast, then katabatic winds may reinforce land breezes to form 'nocturnal' winds. Moffitt (1956) has studied the nocturnal wind at Thorney Island, near Portsmouth; he related their occurrence to the ambient wind, and the difference between the maximum daytime temperature and the minimum temperature at night. Anabatic winds are the upslope equivalents of katabatic flow. They arise when the air next to a slope is heated. They occur less often than katabatic flow and tend to be a weaker phenomenon.

Recirculating flows often develop in valleys where these local slope winds exist. During the day, anabatic winds flow up the valley walls and air subsides over the valley floor. Southern facing slopes will tend to develop the strongest upslope flow. At night, the circulations are reversed, with katabatic winds flowing down the valley sides, and air rising above the valley floor.

The situation is further complicated if, for example, the valley opens out onto low lying plains. The air in the valley has a larger diurnal variation of temperature than the air in the plains. In the day, a local pressure gradient from the plains to the valley causes a valley wind to flow up the valley. Once again, the situation reverses at night-time, with a mountain wind flowing down the valley. Thus, the flow regime in a valley may be very complex, particularly because the mountain/valley winds are not generally in phase with the valley wall slope winds. Mountain/valley winds are a larger scale feature than anabatic/katabatic winds and may be more than 100 metres deep.
Some theoretical models of thermal slope winds are available (for example, Manins and Sawford, 1979b), but these usually apply to flows over broad homogenous slopes. Moreover, they are usually two-dimensional models, whereas field measurements (Manins and Sawford, 1979a) emphasise the three-dimensional nature of the flows. McNider and Pielke (1981) have used a sophisticated hydrostatic primitive equation model to study drainage flow in an idealised mountain valley. The simulated katabatic winds were very shallow with the velocity maximum occurring at 8 metres above the slopes. The down-slope flow increased in strength in the first hour after sunset, but thereafter weakened as pooling cold air in the valley floor reduced the horizontal temperature gradient. The katabatic winds were also weakened by the increasing stability of the valley air. The mountain wind filled the whole valley, and its development lagged that of the downslope winds by an hour.

The complex spatial and temporal development of drainage flow make it difficult to construct simple sub-models for use in WAFT. As noted in Chapter 2, this sort of shallow feature would be more conveniently treated in a model using terrain following mesh.

It may well be that anabatic and katabatic winds are relatively unimportant in the context of mesoscale pollutant transport because they are so shallow. Indeed, they might be more significant for the fact that they are reflected in surface measurements in complex terrain. Thus surface measurements taken in complex terrain might be very unrepresentative of the bulk mesoscale flow.

5.3.3 Urban Heat Islands

Heat islands provide a third example of local thermally driven flows. Whereas land/sea breezes are caused by temperature differences across an essentially linear feature (the coast), heat islands form above a zone which is
surrounded by an area at a different temperature. Heat islands often form above oceanic islands due to the temperature difference between the land and the surrounding sea. Oceanic heat islands are most marked during the early afternoon when the land/sea temperature contrast is greatest.

Urban heat islands are more relevant in the UK, however. The different surface roughness of cities can affect the local flow, but generally, heating effects are more significant. They arise because of the different thermal properties of the city as compared to the surrounding rural areas. Examples of the properties which differ include albedo, heat capacity, heat generation, and heating and cooling rates. The net result is that the city is often warmer than the surrounding countryside. Urban heat islands are usually much more complicated than their oceanic counterparts. They do not always show the clear diurnal variation found in oceanic heat islands although urban heat islands tend to be strongest at night. The isotherm pattern is often very complicated, being influenced by topography and small localised population concentrations; London's complex thermal structure is illustrated by Chandler (1965). Norwine (1972) has identified local temperature anomalies above individual shopping centres.

The urban temperature excess is a function of the city size. Oke (1972) has shown a logarithmic dependence between the maximum urban/rural temperature discontinuity and the urban population. The relationship given suggests that Bristol, for example, might have a maximum temperature anomaly of around 5°C. Although the air just above the city is warmer than the rural air, cooler air is often found at greater altitude (200-1000 metres) above the city. The reasons for this elevated cooler air are not clear but one possible cause is radiational cooling from polluted air. The top of the boundary layer and any elevated inversions are often deformed by the urban heat island; Spangler and Dirks
describe how elevated inversions are deformed upwards to form a domed shape over St. Louis.

The excess urban temperature gives rise to a local circulation. In calm, or near calm, ambient winds, radial flow inwards from the countryside to the city occurs at the surface. Typical velocities are 2-4 m/s; see, for example, Pooler (1963). Air rises above the city centre, and subsides over the surrounding suburbs and rural area. An outward radial flow completes the circulation at higher levels.

The presence of a pre-existing ambient wind modifies the above picture. If the ambient wind is above a certain critical speed, an urban heat island will not form. Oke and Hannell (1970) have used published data for various cities to suggest that the critical speed, \( U_{\text{crit}} \), is given by

\[
U_{\text{crit}} = 3.4 \log P - 11.6
\]

where \( P \) is the population of the city. A thermal plume may form when there is an ambient wind; the maximum temperature excess is shifted downwind of the city centre, and multiple elevated inversions may be present. The zones of updraught and downdraught are also shifted downwind so that air rises over the downwind suburbs, and subsides over the city centre itself.

The simple model proposed by Anderson (1971) might provide a suitable starting point for simulating urban heat islands in WAFT. He assumes that the vertical updraft velocity is proportional to the surface temperature excess. The constant of proportionality is determined from observations, such as those of Findlay and Hirt (1969) over Toronto. The surface temperature pattern is taken to be uniform over the city centre, falling over the suburbs to a lower constant temperature at the rural boundary. The inclusion of this sub-model would be one instance when the vertical velocities in WAFT's initial windfield might be non-zero. It might not be necessary to include the horizontal inflow associated
with urban heat islands in the initial windfield specification; WAFT's mass consistent adjustment process should generate horizontal convergence over the city to balance the updraft present in the initial windfield.

5.4 SUMMARY

Section 5.2 considered the way the velocity varies with height over real terrain. The effect of surface roughness is to create a velocity profile throughout the boundary layer. In practical situations, the WAFT input data consists only of surface wind measurements and the free atmosphere flow aloft. It is always therefore necessary to include some assumption about the velocity profile as part of the initial windfield generation. A simple profile based on sine law turning and a speed power law is adopted.

There will be many situations when the above assumed velocity profile is a very inadequate description of the complex profiles which occur in the atmosphere. Other models, for example MATHEW (Sherman, 1971), use observed profiles in the initial windfield generation. These presumably provide a better picture of the vertical wind structure, although questions must arise as to how representative they are of the whole mesoscale region. The situation is often better in field experiments when several vertical profiles may be available. For instance, five vertical profiles were used by Nappo (1976); this provided sufficient information to perform a horizontal interpolation from the profiles at each grid level and no vertical interpolation was necessary. The sparsity of available profile information in the UK precludes this approach in WAFT, at least when used operationally.

In slack synoptic conditions, the mesoscale windfield may be dominated by thermal flows. These occur where there is differential heating or cooling; examples at land/water boundaries, in mountain valleys, and over cities were
considered in the previous section. These flows are often circulatory and have complex spatial and temporal development. Interaction with the ambient synoptic flow further complicates the picture.

These thermal flows will only appear in WAFT windfields if they are adequately represented in the input data. A surface measurement which is substantially different from the general synoptic flow may indicate the presence of these thermal flows, but the existing meteorological station network is generally not comprehensive enough to describe these features adequately. There is no clear cut answer as to whether surface measurements lying in these thermal flows should be included in the initial windfield generation. If they are ignored, then what might be very important features in terms of pollutant transport will not appear in the final windfield. For instance, sea breeze circulations may have a profound effect on dispersion from coastal sites. On the other hand however, a very localised slope flow, for example, might be given an importance out of all proportion if it is included in the initial windfield data.

These questions echo the discussion in Chapter 3 concerning the representativeness of the input data and the choice of the surface interpolation scheme. What is clear is that, at the very least, one needs to be aware of the conditions favouring the development of these thermal flows. At the present stage of development of the initial windfield generation one may well need to examine the initial windfield by hand to filter out misleading surface observations. Certainly, one should be wary of using the generated initial windfield uncritically.

These thermal flows may be simulated by including simple sub-models in the WAFT initial windfield generation. Again, the paucity of the available input data (temperature, humidity, etc. as well as velocity fields) implies that these sub-models would inevitably have to have a rather simple physical basis. This in turn suggests that they will
not provide detailed agreement with the observed thermal flows. Nonetheless, they may be very valuable guides as to general nature of the mesoscale flow.

The simple inviscid flows discussed in Chapters 3 and 4 may not represent flow in the real atmosphere even in the absence of local thermal flows. This is particularly true of the flow downstream of the hill. A wake, a region of significantly increased turbulence, forms in the lee of the hill. The wake is also characterised by a mean horizontal velocity deficit and net downward motion. Increased turbulence in the wake enhances plume dispersion there. If the lee slope is sufficiently steep (typically, greater than 0.2 for a two-dimensional ridge, and somewhat higher for a three-dimensional hill) then separation may occur. It is difficult enough to predict whether separation will occur, let alone the size and the nature of the flow within the separation 'bubble'. In the case of a two-dimensional ridge, the separated region may extend up to 10h (h is the hill height) downstream and its depth may be anything from a small fraction of h to 2h (see Hunt, 1978). Wakes and separated zones behind three-dimensional objects are correspondingly more complex than those in two-dimensional flow; streamlines may be displaced both vertically and laterally in the separated flow. Recirculating eddies may develop in narrow steep-sided valleys.

As Chapter 4 discussed, lee waves form downstream of a hill in stable flow. Separated rotors may exist under the wave crest at varying distances downstream of the hill. In the three-dimensional case, if the stratification is sufficiently strong, the eddies which form behind the hill will themselves be largely horizontal features (see Brighton, 1978). Work by Anthes and Seamen (1979) using a dynamic forecast model illustrates the rotor which can develop both upstream and downstream of a ridge in two-dimensional stable flow.
If the stable stratification is in the form of an elevated inversion, then this too may cause complicated flow in the hill lee. The elevated inversion is deformed downwards over the hill summit. This deformation increases with windspeed or as the stratification of the elevated stable air decreases. If the inversion passes sufficiently close to the hill summit, then the whole nature of the flow changes; the whole of the mixed layer sweeps down the lee slope in a downwind rotor. In the atmosphere this feature is known as a Chinook, Fohn or Helm wind.

Trajectory experiments by Wooldridge and Fox (1981) for example, confirm that the lee slope features discussed above can be of relevance to pollutant dispersal. However, these effects are generally local to a hill, and therefore probably have most impact in short range modelling. Once again, as far as WAFT is concerned, their principal importance may be their effect on surface wind measurement made in the lee of the hill.

The previous two chapters have demonstrated that WAFT can model idealised stable and neutral flow over simple hills. This chapter has been a reminder of the complex nature of flow in the real atmosphere. This complexity will be underlined in the next chapter when case studies of real atmospheric flows are discussed.
Fig 5.1 - Vertical velocity profiles used in WAFT.
Fig 5.2 - Typical structure of a sea breeze.

<table>
<thead>
<tr>
<th>Onshore ambient wind</th>
<th>Offshore ambient wind</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>Before front reaches source</td>
<td>Before front reaches source</td>
</tr>
<tr>
<td>After front reaches source</td>
<td>After front reaches source</td>
</tr>
</tbody>
</table>

Fig 5.3 - Possible pollutant transport in a sea breeze.
CHAPTER 6

USING WAFT IN COMPLEX TERRAIN

6.1 INTRODUCTION

This chapter presents the windfields obtained when WAFT is used to model flow in a complex terrain region. The location selected is an area based on the Bristol Channel in South West Britain. The region, and the reasons for its selection, are described immediately after this introduction. The following subsection illustrates the final windfields that are generated when a uniform initial windfield is used. These experiments provide an indication of how the airflow might be deformed by the hills in the Bristol Channel region before the complexities of a spatially varying initial windfield are introduced. Both stable flow and neutral flow beneath an elevated inversion are considered.

WAFT is designed to use routinely available meteorological data. The sources used, and difficulties encountered, when collecting data for case studies in the Bristol Channel are discussed in section 6.4. Following this, three case study episodes in 1976 are described. The format of each of these studies is roughly the same. They begin with a run using a uniform initial windfield. The ten metre winds at weather station sites in the resultant final windfield are compared with those actually observed. If they are similar to those actually reported this gives some confidence that the topography was indeed exerting a major influence on the flow field on the day in question. A run using all the available surface data is also performed; this is how it is envisaged that WAFT would be used operationally. Streamline trajectories released at various heights from two of the nuclear power station sites in the area are used to illustrate the final flow pattern. A pollutant dispersal model, TOMCATS, is also employed to simulate plume behaviour.
in these final windfields. TOMCATS is described in ApSimon et al (1984). The plumes generated in the WAFT fields are compared with the corresponding Gaussian plumes based on source meteorology.

Each of the case studies is also used to illustrate some particular aspect of WAFT. The first case study is used as the basis for some sensitivity trials. In these, the differences between WAFT runs using alternative input data and/or arbitrary assumptions are compared to the differences between WAFT runs as a whole and the straight line Gaussian model. The reliability of measured ten metre surface winds is questioned in the second case study where the effects of sub-grid scale features near meteorological stations are examined. The last case employs a time sequence of WAFT fields to look at windfields that change with time. Finally, the last section contains a summary of the case studies, and the lessons learnt from them.

6.2 THE BRISTOL CHANNEL STUDY REGION

A complex terrain region, centred upon the Bristol Channel in South West Britain, is used in the rest of this chapter to investigate how WAFT might be used operationally in real topography. This case study region will now be described in a little more detail. The region's location with respect to the rest of Britain is shown in Figure 6.1. Note that the region's x and y axes are not parallel to the Grid Northings and Eastings. The size of the region is 150 by 120 kilometres. There is nothing particularly significant about the size of region used; WAFT's fairly limited physical basis means that it is not scale dependent. WAFT may, in principle, be used to study a region of a few kilometres up to many hundreds of kilometres in extent. The only restrictions are practical ones, in that a small region will not contain many weather stations, and, on the other hand, it will not be possible to resolve the topography very accurately in a large region.
There are several reasons why this particular region is suitable for the study of mesoscale pollution transport from nuclear installations using WAFT. Firstly, the region contains three nuclear power stations. The primary concern should an accidental release occur from one of these stations is the risk to the surrounding population. The study area is of particular interest because it is quite heavily populated with several large towns close to the nuclear power station sites; Figure 6.1 shows the towns and power station sites. Another requirement of the area is that it contains hills; there are major topographical features close to the stations in this area. This is not true of many of the other sites in the UK, for example Sizewell or Dungeness. The orography is illustrated in perspective view in Figure 6.1. The area is dominated by the mountains of South Wales which rise to a height of nearly 900 metres and contain many deep sided valleys. The second most important terrain feature is Exmoor in the bottom left of the figure. The Cotswolds, Mendips, and Malvern Hills constitute smaller features. The only major topography which lies just outside the region (and therefore might be expected to influence the flow) is the continuation of the Welsh Mountains northwards.

The x and y axis mesh intervals are both five kilometres. The vertical extent of the mesh varies from case to case, but typically it is of the order of a kilometre or so high, with a vertical mesh spacing of the order of one hundred metres. Hence the calculational mesh typically contains 31 by 25 by 11 points; that is around nine thousand points. This gives a fairly reasonable resolution of the topography without incurring large computer costs. The mesh location and orientation are used to compute the Ordnance survey Grid Reference of each (x,y) point in the mesh. Ordnance Survey maps of the area are then used to establish the height at each of these positions. In practice it is rather difficult to define a representative height at all of the points; the mesh point may happen to fall on a subgrid scale feature such as a narrow river valley. For example, there are a
number of deep river valleys in the Rhondda area which have valley floors which are only a few hundreds of metres wide and valley side slopes up to 0.3. Although some of the spot heights in the area are over 880 metres (in the Brecon Beacons), the topography used by WAFT only rises to 650 metres because of the way the mesh lies on the topography.

WAFT will produce windfields using the barest minimum of meteorological data; a single surface wind, a high level wind, and an assessment of the vertical thermal structure are sufficient. However, one would normally make use of all the available surface measurements. The Bristol Channel region has a good coverage of weather stations. The stations supplying data for each of the case studies are illustrated in the appropriate sections. These figures (for example Figure 6.9) name the weather stations inside the region, but note that WAFT also uses stations lying outside the study area to construct the initial field. These stations outside of the area perform a useful role in defining the initial windfield at the edges of the domain. Whilst there are no radiosonde weather stations actually in the area, there are three reasonably close by. These are used to provide an estimate of the upper level wind flow in the region, and the vertical temperature gradient. The locations of these three stations, Cambourne, Crawley and Hemsby are shown in Figure 6.1.

The reasons given above all illustrate why the Bristol Channel area is one of the most appropriate locations in the UK for testing WAFT. The major criticism of the area, perhaps, is that it is a little too complicated. As well as the complex topography deforming the synoptic flow, the Bristol Channel itself will probably produce complex land/sea breeze circulations in suitable conditions (see subsection 5.3.1 of the previous chapter). Similarly, there are a number of fairly steep parallel valleys in South Wales which may produce complex interacting eddy effects, and there also seems scope for drainage flow there too. These are all effects that the basic WAFT formulation cannot deal
with. They will only appear in the final windfield if the surface wind data accurately represents them.

6.3 USING UNIFORM* INITIAL WINDFIELDS

This section consists of runs using a uniform initial windfield. It is thus possible to examine how WAFT modifies the initial windfield without the complications of a non-uniform initial windfield obscuring matters.

6.3.1 Flow under an elevated inversion

The work here looks at the flow through the Bristol Channel region with an elevated inversion present. It has the general aim of illustrating the effect of an inversion in complex terrain, but more specifically, examines how the inversion height affects the flow deflection in the Bristol Channel.

Three cases are presented using inversions at differing heights. An identical initial windfield is used in each case. The initial windfield is defined so that the velocity at every grid point is parallel to the negative x-axis direction. The surface windspeed used is 10 m/s, whilst the high level wind is 20 m/s occurring at 2000m. Results for the three cases are depicted in Figures 6.2, 6.3 and 6.4. The inversion height in the three figures is 2000, 800 and 400m respectively. In each case, the paths of five streamlines released from the upstream boundary are plotted to illustrate how the orography deforms the flow. A perspective and plan view of the streamline paths is given in each figure. The top of the box shown in the perspective view represents the position of the elevated inversion. Each figure also shows an horizontal slice through the velocity field at 200m above Sea Level.

* Note that, from now on, a 'uniform initial windfield' will mean that the initial windfield is the same at all points 10m above the surface. A velocity profile will be used between 10m and the stated high level wind.
Figure 6.2 shows that, with the inversion at 2000m, there is virtually no horizontal deflection of the velocity field. The streamlines are straight; they rise over any hills they encounter. Even with the inversion lowered to 800m, which is only 150m above the highest hills, there is still relatively little deflection of the flow; though the Welsh Mountains are beginning to have some effect. However, reducing the inversion height to 400m (Figure 6.4) produces large horizontal deflections as the streamlines are forced to travel around the hills, particularly the Welsh Mountains.

These three runs suggest that, in a neutrally stratified atmosphere, the hills in the Bristol Channel region will not deflect the flow horizontally to any great extent unless there is an inversion present which is near, or below, the top of the orography (at about 650m). However, if there is a low inversion present, then the hills may be expected to exert a large influence on the flow patterns. The results here mirror those in subsection 4.2.3 which looked at flow over a single hill under an inversion.

6.3.2 Stable Flow

This section describes some runs using the stable flow version of WAFT over the Bristol Channel study region. These runs give some indication of how the stable version of WAFT behaves in a region of complex terrain. For comparison, a run (case 4) using a single value for the vertical velocity weighting factor throughout the whole domain is presented.

The initial velocity at every point in the domain is in the negative x-direction. Once again, a vertical velocity profile law is used so that the magnitude of the initial velocity at any point varies with height above ground. In all cases, a high level flow of 4 m/s is assumed to occur across the whole domain at a height of 1100m. In cases 1, 2
and 4, a ten metre wind of 1.0 m/s is used, whilst in case 3, the ten metre wind is set to 2.0 m/s. Subsection 4.3.8 described how the 'high lying factor', $f_{\text{HL}}$, controlled the way in which a complex terrain region was split into a number of hill masses behaving as single hills. In cases 1 and 3 $f_{\text{HL}}$ is 33 percent. This means that the Bristol Channel region is divided into seven subregions. It is 66 percent in case 2 which leads to a domain consisting of only two subregions. Cases 1 and 2 are identical except for the different values of $f_{\text{HL}}$ used. The potential temperature gradient for these two cases is set to 1.0°C/100m, which together with the light windspeed mean that these cases correspond to very stable flow - Pasquill Category F. In case 3 a less severe potential temperature gradient of 0.5°C/100m combined with a higher surface windspeed of 2 m/s leads to higher Froude numbers around the hills in the region. In case 4, the vertical velocity weighting factors are set to $2.5 \times 10^4$; this gives vertical velocities of a similar size to those of case 1.

The calculated mean Froude numbers and critical heights for each of the subregions in cases 1, 2 and 3 are given in Tables 6.1(a), 6.1(b) and 6.1(c). The relevant input parameters are also given. Note that in case 1 all of the seven subregions have mean Froude numbers below 0.5 so that the whole domain lies in stable flow. However, there are large variations in the critical height across the domain due to the large differences in the height of the land masses in the seven subregions. In case 3 only one subregion, the Welsh Mountains, is high enough to have a Froude number below 0.5; neutral flow is assumed to occur in the other six subregions. The two subregions in case 2 both have Froude numbers below 0.5, but since there are only two subregions there is less variation of the critical height across the whole domain.

Figures 6.5, 6.6, 6.7 and 6.8 depict streamlines released from above two points on the positive x-axis boundary of the domain as they move across the domain in the negative x
direction. The streamlines are released at 10, 100, 200, 300, 400 and 500m above the terrain at the two points. A plan and perspective view of the streamlines, together with a horizontal slice of the velocity field, are presented in each figure.

Several comments may be made concerning the effects of varying the various input parameters. For instance, since a large part of the domain in case 3 lies in neutral flow, the streamlines released near the front of the domain (as one looks at it in the perspective view) are not deflected horizontally by very much. In case 1, which represents more stable conditions, the lowest two streamlines travel around Exmoor, and only the upper four go over Exmoor. This represents a typical stable flow regime, whereby the lower air flows around a hill mass, but the upper air, above the critical height, rises over the hill. Similarly, the other group of streamlines, released near the rear of the domain, are deflected by the Welsh Mountains less in case 3 than in the more stable case 1. In case 3 the streamlines released from 300m and above lie above the critical height and therefore pass over the Welsh Mountains. In this particular set of runs the effect of the different 'high lying factor' used in case 1 and in case 2 is not very marked. The major difference has been a greater horizontal deflection of the flow around the hills (e.g., the Cotswolds) in the South Eastern corner of the domain in case 2. In case 1, with a 'high lying factor' of 33 percent, these small hills are assigned a subregion to themselves. The hills are not very high and hence the critical height in this part of the domain is also fairly low; therefore the bulk of the flow goes over the hills. On the other hand, in case 2 these hills are assigned to the subregion dominated by the much higher Welsh Mountains. Hence a much higher critical height is computed and more air is forced to travel around the Cotswolds.

In each of the first three cases there are examples of the reasonably abrupt change from near horizontal flow below the
critical height to neutral flow above this height. This may well have important consequences if a release occurred in this flow regime. It would very much depend upon the conditions whether pollutant would travel in a fairly straight line, or be forced to move around a given hill mass. Indeed, it is possible, if the plume already has a large vertical extent, that the pollutant may take two distinct routes as it encounters a hill mass in stable flow; some would go over, and some around the hill. Case 4 which uses a uniform value for the vertical velocity weighting factors does not produce this sort of behaviour. Instead, the flow at all levels tends to be near horizontal. Hence, a plume with a fairly large vertical extent encountering a hill mass would tend to be dispersed very quickly horizontally rather than two separate plumes developing.

6.4 METEOROLOGICAL DATA IN THE BRISTOL CHANNEL STUDY AREA

WAFT must use meteorological data that is routinely measured if it is to be a practical tool during accidents. For instance, its operational usefulness would be extremely limited if it required a detailed specification of the wind flow at several heights above the ground; such information is not generally available. This section describes how data that is routinely collected by the Meteorological Office is used to meet WAFT's requirements. In particular, it will describe the data used for the Bristol Channel case studies presented later in this chapter, but it will also serve to illustrate how WAFT may be used at other sites throughout the UK (and elsewhere). The meteorological data that WAFT requires may be divided into three elements:

- surface wind measurements
- high level flow (free atmosphere) specification
- vertical thermal structure

The sources for these three elements of data will now be discussed.
Imperial College already has access to a database of three hourly surface wind data covering the whole of Europe. The database only contains reports from the major synoptic stations, but it provides a basis for selecting days which are promising candidates for episodes which may be studied using WAFT. The existence of this database is the reason why all the case studies presented in this chapter are based on days in 1976. In this context 'promising candidates' are those days when the synoptic flow over the area is uniform, and the vertical temperature profile and surface winds give a fairly unambiguous picture of the atmosphere's thermal structure. Having selected suitable days for study, extra surface wind data may be obtained from the Meteorological Office. There are approximately 30 weather stations in, or just outside of, the study region. However, the number actually reporting data varies throughout the day. The major synoptic stations report every hour of the day, but others only report every three hours, and some only take measurements during the working day. Hence, the number of measurements available in the study region varies from approximately ten in the early hours of the morning, up to 30 or so at 9.00, 12.00, and 15.00 hours. The case studies in this chapter concentrate on windfields at 9.00 hours because of the large number of measurements available then.

Vertical wind profiles are obtained from the radiosonde stations at Cambourne, Crawley and Hemsby. These were given in the "Daily Weather Reports - Aerological Data" issued by the Meteorological Office. These soundings are only taken at 05.00 and 17.00 hours; one must interpolate values for other times. The wind velocity is not measured at consistent heights, and there are typically only three values reported in the lowest two kilometres. Hence, the single high level wind specified to WAFT is something of a crude approximation. However, the synoptic pressure charts may be used to confirm the high level flow suggested by the radiosonde data. The use of a uniform high level flow is something of a weak point, but the limited nature of the vertical wind data precludes anything more sophisticated.
The only other approach that might be adopted is to extract the high level flow pattern from either a large scale forecasting model, or from the synoptic pressure distribution using the geostrophic assumption.

The last data item required, the vertical thermal structure, is inferred from the vertical temperature profiles taken at Cambourne, Crawley and Hemsby. Unfortunately, the profiles are only taken at midnight and two of the sites are coastal and therefore might not reflect the inland profile at all well. Once again, the temperature profile, like the velocity profile, is only measured at a few points in the lowest kilometre or so. This crude measurement will not identify the thin stable layers which are often present in the atmosphere, and which may have a large effect on the atmosphere's behaviour. Some erosion of the stable gradient measured at midnight is assumed when estimating the profile at 9.00 hours, but the limitations mentioned above mean that the thermal structure used in the case studies might not be very accurate.

The episodes presented below have temperature profiles which give a consistent, well defined picture of the thermal structure of the atmosphere. However, in general, the profiles at Crawley, Cambourne, and Hemsby often tell very different stories, and it is therefore rather difficult to select an appropriate thermal structure for WAFT. This sparcity of upper air information might prove a significant difficulty when using WAFT operationally. If neutral stability is indicated, the vertical temperature profile is used to estimate the inversion height. The neutral flow version of WAFT is then used, with the vertical velocity weighting factors set appropriately to model a flat 'lid' at the required height. Otherwise, the stable version of WAFT is used, and the vertical temperature profile is used to estimate a potential temperature gradient. The Brunt-Vaisala frequency may be calculated from this using equation 4.3.2(2). It is possible to specify a different Brunt-Vaisälä frequency at each grid level, but the poor
temperature gradient definition usually means that it is only appropriate to define a single mean value for all levels.

WAFT first performs a 2-D interpolation of the supplied surface winds to define a 2-D flow pattern at ten metres above the ground. The inverse square method is used in most of the cases to be presented, but the bicubic spline technique is also employed. A vertical profile law is then used to create the windfield between the ten metre flow and the assumed high level flow. The profile law used is generally the 'standard' one described in subsection 5.2.3, but alternatives are tried as part of the sensitivity tests in subsection 6.5.4.

6.5 CASE STUDY ONE - 9.00 HOURS 29 JANUARY 1976

6.5.1 Meteorological Situation

A low pressure area over Ireland and a high over Denmark combined to give strong southerly flow over the whole of the British Isles. Tightly packed isobars indicated strong winds with 10 metre winds being between 10 and 20 m/s. The synoptic charts indicated that the southerly flow rotated toward south easterly flow as the day progressed. Surface temperatures were in the range three to six degrees Centigrade, and it was raining over the Bristol Channel study area.

The vertical wind profiles from Hemsley, Crawley and Cambourne Meteorological stations suggest that the wind at a height of 2000m was fairly uniform over the study region. The wind at that height was approximately 16 m/s blowing from compass direction 175 degrees. This 2000m wind defines the high level wind used by WAFT. Vertical temperature gradients from Crawley and Hemsley indicate a marked temperature inversion at a height of approximately 600m. The inversion's strength was about two to three degrees Centi-
grade. These temperature soundings were made at 0.00 hours on 29th January. Since this was a cold winter's morning, the inversion is assumed to persist at the same height until 9.00 hours. Hence an inversion at 600m is used for the WAFT simulation.

The surface winds reported by the Meteorological Stations are depicted as the solid arrows in Figure 6.9. The mean high level wind (at 2000m) is also shown. The winds reported by the coastal stations are, in general, much higher than the inland ones. This is often the case, since the relatively low surface roughness of the sea means that surface winds over water are much higher than those over land. The surface wind vectors shown in figure 6.9 suggest that the airflow may have been dividing as it encountered the Welsh Mountains.

6.5.2 Using a Uniform Surface Windfield

This first run employs a uniform surface windfield to indicate whether the topography might have been a significant factor in the flow pattern observed on the morning of 29th January. This run is similar to those presented in section 6.3, but in this case the initial surface wind direction is not the same as the high level wind direction. Hence there is directional as well as velocity shear in the initial windfield. The resulting final windfield is examined to see if the wind vectors at the locations of meteorological stations match those that were actually reported on the morning of 29th January. A mean surface wind of 10 m/s blowing from 155 degrees is used; it is an average of the reported surface winds shown in Figure 6.9. The high level wind is defined as 16 m/s, blowing from 175 degrees; both this high level wind, and the assumed mean surface wind are also depicted in Figure 6.9. The strength of the surface wind is approximately 60 percent of the high level wind, and it is backed by 20 degrees. These values are typical of neutral flow conditions, see for example Smith (1975).
Figure 6.10 illustrates a horizontal slice of the resulting final windfield at a height of 100m. The flow splits as it encounters the hills in South Wales. This is not unexpected, as the inversion height is below the top of the topography and therefore there is considerable horizontal deflection of the flow. Figure 6.11 displays notional streamlines leaving two of the nuclear power station sites in the region; Hinckley Point and Oldbury. At each of these locations the streamlines are released from 10, 100, and 200m above the ground surface. These cover the range of effective release heights one would expect in an accident. Hence, the importance of effective release height may be assessed. In addition, the differences between the routes taken by the streamlines released at different heights gives some indication of the amount of shear present in the final windfield. The 10m streamlines are deflected the most since they travel around the base of the hills in the area. The streamlines released at different heights above Hinckley Point take rather varied routes which indicates considerable vertical shear in this part of the flow. One would expect that this would lead to greatly enhanced horizontal plume dispersion. The arrowheads on the streamlines represent positions at ten minute intervals. The wider spacing of these arrowheads on the 200m streamline illustrates the higher speed flow to be found aloft.

The 'predicted' surface winds at the meteorological stations extracted from the final windfield are shown as the dashed arrows in Figure 6.9. There appears to be broad general agreement between these and the wind directions actually observed at the stations (shown as the solid arrows). The agreement is worst in the Eastern quarter of the area where, for example, the 'predicted' direction at Filton is wrong by approximately 30 degrees. It appears that WAFT predicts rather more deformation of the flow in this part of the region than actually occurs. Possible explanations for this discrepancy are that the inversion at this point has been deformed upwards or is rather weak, or possibly, the actual
high level wind direction here was different from that used over the rest of the area.

There is rather less general agreement between the 'predicted' and measured surface windspeeds. The 'predicted' surface windspeeds are lower than those actually measured suggesting that a higher assumed initial windspeed might have been appropriate. Moreover, the 'predicted' windspeeds do not follow the trends observed in the real data. For instance, the 'predicted' windspeed at Mumbles is relatively low, whilst the observed windspeed is much higher than average. One explanation for this difference is based on the fact that Mumbles is a coastal station. The WAFT formulation does not consider surface roughness. There is thus little reason to expect WAFT to generate the higher windspeeds found near the coast unless this feature is contained within the input surface data. Since in this case a single surface wind is used, it is little surprise that WAFT fails to generate higher windspeeds near the coast. In fact, this is one of many reasons why one would not expect WAFT to predict individual surface measurements with any degree of accuracy. This will be discussed in more detail in subsection 6.6.3.

Nonetheless, the general agreement between the 'predicted' and actual surface wind directions suggests that this episode might be a case where the topography in the Bristol Channel was exerting a significant effect on the flow field. Hence this is a case where one would expect WAFT to perform reasonably well.

6.5.3 Using the Available Surface Station Data

This section describes the results of a run using all the available surface station data measurements to generate the initial windfield. There are thirty surface measurements available for 9.00 on the morning of the 29th January 1976. The simple two dimensional inverse square interpolation
scheme is used to generate velocities at ten metres above the surface across the whole region. The calculational mesh, high level wind vector, and inversion height (at 600m) are the same as those in the uniform initial windfield run described in the previous subsection.

Figures 6.12 and 6.13 illustrate a horizontal section of the initial and final windfields respectively at a height of 100m. The flow patterns are rather similar, the major area of difference being around (30.0, 60.0) where there appears to be more channelling of the flow in the final windfield. This is a result of the adjustment procedure forcing air to go around the base of the Welsh Mountains. The fact that the initial and final windfields are so alike confirms earlier work in Chapter 3 which showed how important the specification of the initial windfield is. A comparison of Figures 6.10 and 6.13 illustrates that the final windfield produced using all the surface data is similar to the one generated when a uniform initial surface wind is assumed. The failure of WAFT to correctly generate the high surface winds found at coastal sites was discussed above. Note however, that Figures 6.10 and 6.13 demonstrate that there is little difference between the coastal windspeeds at 100m above the ground whether a uniform or interpolated initial surface windfield is employed. Hence the fact that WAFT may not predict surface coastal windspeeds very accurately is probably not very significant as far as pollutant transport through the region is concerned.

Streamlines leaving Hinckley Point and Oldbury at heights of 10, 100, and 200m above the ground in the initial and final windfields are depicted in Figures 6.14 and 6.15 respectively. The streamlines in the final windfield are calculated using velocity components interpolated from the final windfield vectors specified at each mesh point. Streamlines are computed in the initial windfield as well to provide an indication of how the WAFT adjustment process has modified pollutant trajectories. The computation of streamlines in the initial windfield is not quite as
straightforward as for the final windfield. This is because the initial windfield is not mass consistent and therefore streamlines released near the ground surface tend to hit the land surface close to their starting point. The following approach is adopted to compute meaningful streamlines in the initial windfields. The horizontal velocity components are obtained directly from the initial windfield as before, but a vertical velocity is introduced so that the streamlines remain at the same height above the ground surface. Hence, the generated streamlines behave as if the terrain is flat.

The streamlines in the initial and final windfields are rather similar; particularly those released from Oldbury. The streamlines that set off from Hinckley Point are deflected by the Welsh Mountains to a greater extent in the final windfield. Once again though, it appears that the initial windfield specification dominates the form of the final flow pattern. The divergent paths of the streamlines leaving Hinckley Point indicate that there is considerable vertical shear present which would be expected to lead to enhanced horizontal dispersion.

The TOMCATS Monte Carlo dispersion model developed at Imperial College (ApSimon et al, 1984) may be used to examine how pollutant would travel in the final windfield. Figure 6.16 illustrates plumes released at 50m above the ground at both Hinckley Point and Oldbury. The dispersion rate in the TOMCATS model is controlled by the value selected for the friction velocity. A value of 0.4 m/s is used which corresponds to neutral stability dispersion rates. To provide a comparison with this, Figure 6.17 shows the results obtained when TOMCATS is run in uniform windfields. Figure 6.17 actually depicts the results of two TOMCATS runs superimposed, each using a uniform velocity appropriate to one of the release sites. This uniform velocity is extracted from the WAFT final windfield produced using all the weather station data; the velocity at a height of 50m above the relevant site is extracted. The parameters used in this case are the same as those used in
Figure 6.16. Hence, Figure 6.17 simulates two uniform Gaussian plumes based on source meteorology.

Not unexpectedly, the plumes in the WAFT generated (that is, non-uniform) windfield are curved, although, in this particular case study, they do not differ very much from straight lines. The WAFT plumes are also considerably wider than those predicted by the Gaussian model. This enhancement of the horizontal dispersion is probably due to the vertical speed and directional shear present in the WAFT final windfield. This increased lateral spread means that a greater area is liable to be affected by the pollution, but on the other hand, centreline concentrations are lower. The straight line plume from Hinckley Point passes directly over the town of Barry, whilst the curved plume misses it, but does pass over the coastal towns of Bridgend, Margam, and Port Talbot. It is therefore not possible to say whether the Gaussian Plume model or WAFT would predict the more serious consequences. Detailed population data and dose calculations would be needed to assess which model gives rise to the worse case.

6.5.4 Sensitivity Studies

The work presented above showed how plumes released in WAFT generated windfields might differ from the more conventional straight line Gaussian Plume assumption. In the absence of actual experimental evidence (such as full scale atmospheric tracer releases) it is, however, difficult to judge how realistic these WAFT simulations are. Nevertheless, for reasons discussed previously, this episode might be a case where one would expect WAFT to perform comparatively well. It thus provides a good basis for some sensitivity studies. These sensitivity runs examine how changes to the input data, or the arbitrary sections of the WAFT model, affect the final windfield. These sensitivity studies are very relevant since some of the input data items required by WAFT may, in practice, be very difficult to measure; an example
is the inversion height. Similarly, previous work has already shown that, for instance, the arbitrarily chosen initial interpolation scheme may affect the final windfield significantly. It is of particular interest whether the differences between the various sensitivity runs are of the same magnitude as the differences between WAFT and other models such as the Gaussian Plume.

This sensitivity study consists of a further fifteen WAFT runs. These runs employ the data used above, but in each case one of the data values, or some aspect of the computational method is varied and the change in the final windfield noted. Unfortunately, it would consume too many resources to run the dispersion code, TOMCATS, in each of the cases. Instead, each final windfield is characterised by the streamlines released from 10, 100 and 200m above both Hinckley Point and Oldbury. The dotted lines shown in the Figures illustrate the corresponding Gaussian Plume straight line paths.

The first seven sensitivity cases look at the effects of choosing either a different initial surface windfield interpolation scheme or omitting some of the surface wind measurements. The streamlines in these seven cases, together with the 'base' run from the previous subsection, are presented in Figure 6.18. The odd numbered cases employ the simple inverse square interpolation method, whereas the even numbered ones use the more sophisticated bicubic spline scheme. Cases 1 and 2 make use of all the available surface station data; hence case 1 is the 'base' run referred to before. Each of the other pairs of cases have one surface data measurement missing; cases 3 and 4 have the Cardiff value omitted, cases 5 and 6 the one from Kymin, and cases 7 and 8 that from Filton.

Taken as a whole, the trajectories in the eight cases are broadly similar; that is, WAFT does not display any drastic instabilities. However, as previous work has indicated, significant differences appear when different initial
interpolation methods are used. Cases 5 and 6 provide just one example. The Oldbury trajectories are not very different from the straight line assumption when the inverse square interpolation scheme is employed; however, the bicubic spline case produces a much more curved path.

Omitting a data point only affected the final windfield locally. For example, comparing cases 1 and 3, it is apparent that leaving out the Cardiff data point affects the Hinckley Point trajectories (which pass relatively close to Cardiff) trajectories, but not those from Oldbury. In this particular series of runs the bicubic spline approach seems less sensitive to omitted data points. This suggests that there may be coherent trends within the surface data that the bicubic spline scheme recognises, and thus this lends weight to the argument that the surface data is representative of the mesoscale flow over the Bristol Channel occurring on the morning of the 29th.

The next two cases consider how the inversion height selected affects the final windfield. Subsection 6.3.1 indicated that the selection of this value is important, and yet it is difficult to ascertain in practical circumstances. Vertical soundings of the atmosphere are undertaken at relatively few stations in the UK, and the measurements are taken at widely spaced vertical intervals. It is thus often difficult to specify the inversion height over a mesoscale region, let alone assess how the inversion might be deformed by the orography. The streamlines in the two cases (cases 9 and 10) are plotted in Figure 6.19 as well as the 'base' case, case 1. The inversion ceiling in case 9 is at 400m, in case 10 at 800m, and at 600m in the 'base' case. As the inversion height is reduced, the streamlines are deflected around the Welsh Mountains more, and their paths diverge further away from the straight line assumption. The inversion starts having a large impact as it is lowered below the height of the hills in the region; case 1 has the inversion at the height of the highest hills in South Wales.
The consequences of altering the vertical velocity interpolation scheme are investigated next. Like inversion height, the real vertical profile is in general rather complex and difficult to define over a mesoscale area. As was discussed in subsection 5.2.3, WAFT treats this problem by assuming that the velocity varies with height according to some general profile law applicable over the whole region. Subsection 5.2.3 also discussed the four different profile laws available in WAFT. The choice of this profile law is arbitrary, and results corresponding to the four schemes coded are given here. Trajectories using each of the four laws are shown in Figure 6.20. The bicubic spline interpolation method is used throughout the four runs. The bicubic spline scheme is used since it tends to produce a less uniform surface windfield, and hence may highlight the differences between the various profile laws. Case 2, which was shown earlier as part of the surface interpolation sensitivity tests, uses the 'standard' vertical profile. This 'standard' profile consists of a sine law turning scheme with a power law speed profile. Case 11 uses a linear variation of both speed and turning angle. Cases 12 and 13 employ separate interpolations of the x and y velocity components, rather than expressing velocity as a combination of a speed and a direction. In case 12 the variation of the velocity components is linear, whereas a logarithmic relationship is adopted in case 13. The results in Figure 6.20 show that case 2 and 11 produce similar trajectories. The fact that the interpolation is done on the wind speed and direction, rather than the separate velocity components, seems to be the significant factor here.

The final three sensitivity runs deal with the effects of varying the high level wind specification. The WAFT model assumes a uniform wind velocity at some specified level, usually one to two kilometres, above the ground. This assumption is only justified in the simplest of meteorological conditions. Once again, the sparsity of upper air wind measurements in a mesoscale region means that it is not
straightforward to specify an appropriate mean high level wind vector. The 'base' case, case 1, has a high level wind of 16 m/s blowing from 170 degrees at a height of 2000m. This is a best estimate using the three nearest radiosonde readings at Cambourne, Crawley, and Hemsby. The three sensitivity comparisons assume a high level wind at 1000m; this lower value is used to accentuate differences in the final windfield due to different choices of the high level velocity. The high level windspeed is 15 m/s, and the wind directions are 170, 190, and 150 degrees in cases 14, 15, and 16 respectively. Figure 6.21 depicts the results of this final sensitivity experiment.

A comparison of cases 1 and 14 shows that the trajectories from Oldbury have not been affected very greatly by the change in the height at which the high level wind is specified. This is probably because the trajectories from Oldbury are for the most part aligned with the high level wind direction used in cases 1 and 14. However, when the near surface winds are not parallel to the high level flow, as is the case for the Hinckley Point trajectories, the high level wind height is more important. Here, the lower high level wind height used in case 14 leads to greater directional shear in the low level flow, and hence more divergent paths for trajectories released at different heights. The trajectories from Oldbury are not affected by very much as the high level wind direction is changed (ie cases 14, 15 and 16) either. Once again, though, because the streamlines leaving Hinckley Point are not aligned with the upper air flow, the high level flow direction takes on greater significance. As one might expect, the trajectories at different heights take more divergent routes as the upper level flow direction is rotated away from the surface flow direction.
6.6 CASE STUDY TWO - 9.00 HOURS 6 FEBRUARY 1976

6.6.1 Meteorological Situation

An anticyclone lying over Scandinavia gave south easterly flow over the Bristol Channel region. The isobars were not very tightly packed so that surface winds were moderate at about 5 m/s. Rain was reported in the area, and the situation was complicated by the passage of an occluded front during the morning. The three radiosonde velocity profiles at Hemsby, Crawley, and Cambourne taken at 5.00 hours and again at 17.00 hours are used to estimate a mean high level wind. There were marked differences between the profiles at the three radiosonde sites, and there is thus less justification for using a uniform high level wind than for the previous case study. However, the mean high level wind is taken as 12 m/s flowing from 150 degrees at a height of 1500m.

All three vertical temperature soundings indicate a very pronounced temperature inversion between 300 and 600m. The inversion at Crawley was very strong, being approximately 8°C. These soundings were taken at 0.00 hours, but with very little heat input between then and 9.00 hours, an inversion at 500m is assumed to have been present at 9.00 hours. The measured surface winds are shown in Figure 6.22. The surface windspeeds are of the order of 5 m/s in the main but they are backed by approximately 50 degrees from the high level wind (also shown) so that the surface winds are easterlies. However, the surface winds in the Welsh Mountains were all much lower; approximately 1-2 m/s. This is probably because these stations are in valley floors.

6.6.2 Using a Uniform Surface Windfield

As in the previous case study, this first run examines whether WAFT and a uniform initial surface windfield will generate ten metre winds in the final windfield which match
those actually observed. The uniform mean surface wind is set at 3 m/s blowing from compass direction 100 degrees. This represents an average of the measured winds shown in Figure 6.22. This assumed mean surface wind direction is backed 50 degrees from the high level flow, and it has a strength one quarter of the high level speed. This ratio of high level to ten metre windspeed is rather high, but it is influenced by the low measured windspeeds in the Welsh valleys. The reasonably fast general windspeed, and the measured vertical temperature gradient indicate that it is appropriate to model this case study as neutral flow under an elevated inversion at around 500m.

The final windfield generated from the uniform initial windfield is depicted in Figure 6.23. This shows the windfield at a height of 160m above Sea Level. At this height, there is a marked deflection around Exmoor and the Welsh Mountains. There is also a pronounced stagnant region just upwind of the Welsh Mountains. The majority of the topography in the Welsh Mountain area is above 160m, but the 'predicted' winds at those few points where the land is below 160m (that is, the Welsh valleys) are characterised by low windspeeds and variable directions. These variable light winds are also a feature of the observed winds in the valleys.

Detailed comparison of the observed and 'predicted' 10 metre winds at the station locations reveals marked differences, however. These 'predicted' station winds are illustrated as the dashed arrows in Figure 6.22. The 'predicted' winds are much more uniform than those actually observed. The agreement is particularly poor at the valley stations which actually measured light winds, but where WAFT has generated quite high surface windspeeds. Brynnamman provides an example where the 'predicted' direction is in error by 50 degrees, and the windspeed is eight times too high. In the previous case study, there was reasonable agreement between the actual and 'predicted' winds. The question arises whether the poor agreement in this case highlights a funda-
mental weakness in the WAFT method, or whether there are good reasons why one might not expect WAFT to always predict surface winds accurately. The next subsection examines this question.

6.6.3 Local Winds in Sub-Grid Features

A small area in the neighbourhood of one meteorological station, Aberdare, will be considered in order to examine WAFT's failure to predict ten metre winds correctly. Aberdare weather station is situated near the floor of a relatively steep sided NW-SE orientated valley in the Welsh Mountains. The local flow effects which might occur in this sort of location have already been discussed in Chapter 5. They might include mountain/valley winds in light wind conditions, or recirculating eddies in moderate or high wind flow. A ten metre wind measured in these circumstances might be quite unrepresentative of the bulk flow across a mesoscale region. However, even discounting these types of effects, there are good reasons why WAFT generated surface winds may be inaccurate. Figure 6.24 illustrates why this might be so. Figure 6.24(a) depicts the land surface around Aberdare defined at the horizontal mesh spacing of 5.0 km normally used in the runs presented in this chapter. Figure 6.24(b) shows the same area, but with the topography defined using horizontal mesh intervals of 0.5 km. With the normal grid resolution the land has no distinct form; it appears as a rather shallow dip. However, at the finer resolution a clearly defined valley appears. The valley floor is at 100m, with the surrounding hills rising to 500m. Aberdare weather station itself is situated at 174m above Sea Level.

This high resolution sub-grid area may be used to examine how local scale topography might give rise to correspondingly small scale flow effects which will not appear at the normal grid resolution. A domain of 20 by 20 by 17 cells based on the topography shown in Figure 6.24(b) is used. Separate computations use uniform initial surface winds in
each of the four cardinal compass directions. The uniform initial surface windspeed is 5 m/s in a direction backed by thirty degrees from the high level wind. An inversion is placed at 400m at the top of the valley sides. The results for two of the wind directions, westerly and northerly, are presented in Figure 6.25. They show velocity fields at 250m above Sea Level.

The flow in the valley floor has been channelled away from the main flow direction, and thus might be quite different from the bulk flow through the region. For example, when the bulk flow through the sub-grid area is from compass direction 270 degrees, as in Figure 6.25(a), the wind vector extracted from the final windfield which is 'predicted' at Aberdare is 4.3 m/s from 320 degrees. Similarly, when the bulk flow is from 180 degrees, the Aberdare ten metre wind is 'predicted' to be 2.2 m/s from 130 degrees. In both cases therefore, the wind direction hypothetically measured at Aberdare is 50 degrees away from the main direction. In addition, the windspeed at Aberdare differs by a factor of two in the two cases even though the mean flow windspeed is the same.

It might be objected that these cases do not constitute a very realistic experiment, particularly as a flat inversion was placed over the top of the valley. Whilst it is true that this might occur relatively infrequently, it is quite common for layers of stable air to be present in a valley; see, for example, Reid (1979).

These would constrain the flow in the valley to be in horizontal layers in much the same way that an overlying inversion would. In fact, these runs may have underestimated the effects of local scale terrain features. If, for example, an horizontal grid resolution of just 50m had been used it is possible that the 'predicted' winds at Aberdare might have been even more unrepresentative of the mesoscale flow.

To summarise, the work here suggests that the ten metre
winds measured by meteorological stations in complex topography might not be very representative of the mesoscale flow, but rather a feature of the local terrain. This is particularly so in stable conditions, or when there is a strong low lying inversion present. The fact that WAFT does not predict observed ten metre winds very successfully does not necessarily mean that WAFT is incorrectly predicting the general mesoscale flow. If the 'predicted' ten metre winds do agree with actual measurements then this is a good indication that the topography is playing a dominant role in the flow pattern and that WAFT might be expected to model the mesoscale flow well; the converse is not necessarily true however. The work here also suggests that it might not always be appropriate to use data from meteorological stations in complex orography when interpolating the initial windfield.

6.6.4 Using the Available Surface Station Data

Two runs are presented here which use all of the 31 surface wind measurements available on the morning of 6th February 1976 at 9.00 hours. The calculational mesh is identical to that used in subsection 6.6.2, and neutral flow under an inversion at 500m is again assumed. The only difference between the two runs is that one uses an inverse square interpolation scheme to generate the initial surface flow field, and the other a bicubic spline interpolation. Figures 6.26(a) and (b) respectively illustrate the initial and final windfields at a height of 200m when the inverse square method is employed. Similarly, Figures 6.27(a) and (b) depict the initial and final windfields when the bicubic spline scheme is used.

A comparison of the initial and final windfields in Figures 6.26 and 6.27 shows that the WAFT mass consistency adjustment process introduces a stagnant region where the flow meets the Welsh Mountains, and there appears to be some channelling of the flow along the Welsh valleys. The low
windspeeds present in the Welsh valleys in the initial windfield are strengthened in the final windfield. The bicubic spline interpolation method produces an initial windfield (see Figure 6.27(a)) which is rather different to the inverse square case. The differences are particularly marked in the bottom left and top right hand areas of the figures. The bicubic spline method tends to produce a less uniform flow pattern. For instance, in the bottom left of Figure 6.27(a) there is an area of horizontal convergence, and the flow in the top right hand corner is at right angles to the main flow through the study region. This increased variability is also reflected in the bicubic spline final windfield, although the WAFT adjustment process completely reverses the flow in the extreme bottom left of the area, so removing the area of horizontal convergence. The winds in the Welsh valleys have remained light and variable in the final windfield. This case study reinforces the importance of the initial windfield generation.

Figure 6.28 depicts hypothetical trajectories released in the two final windfields; 6.28(a) shows the inverse square trajectories, 6.28(b) the bicubic spline ones. The trajectories are released at three different heights, 10, 100 and 250m above the ground at Hinckley Point and Oldbury Nuclear Power station sites. The arrows on the trajectories represent half hourly intervals. The trajectories from Hinckley Point are rather similar in the two cases; they are moving in fast easterly flow. The trajectories released at 10m above the ground meander slightly more than the upper level ones, but they are all reasonably straight. The higher speeds aloft are illustrated by the wider spacing of the marker arrows. There are, however, more interesting differences between the trajectories released from Oldbury. The trajectories released at 10 and 100m are rather similar; those in Figure 6.28(b) take a slightly more northerly route, and the 10 and 100m trajectories take routes which are further apart indicating that there is more vertical shear in the bicubic spline windfield. It is, however, the trajectories released at 250m which are the most interest-
ing. In the inverse square field the 250m trajectory follows a fairly straight course westwards. The trajectory released at this height in the bicubic spline field heads off in a slightly more northerly direction, and then enters the complex flow in the Welsh valleys. It then turns northwards sharply to meander through the Welsh hills.

Some TOMCAT simulations illustrate how airborne pollutants might disperse in the different final windfields. Figure 6.29(a) depicts pollutant spread in the inverse square windfield, and Figure 6.29(b) in the bicubic spline flow. For comparison, Figure 6.29(c) illustrates Gaussian plumes based on source meteorology. The Gaussian plume assumption generates straight narrow plumes. In this particular case, the Gaussian plume from Oldbury passes right over Newbury, and would therefore give rise to relatively high doses there. In contrast, the plumes in the WAFT windfields have a much greater lateral spread; hence they would be less concentrated, but affect a much wider area. It is difficult to assess whether this would lead to more serious radiological consequences overall without doing detailed population dose studies. The plume from Oldbury in the inverse square windfield covers many of the towns of South Wales. When the bicubic spline windfield is used, the plume from Oldbury has a very complicated structure as it interacts with the Welsh hills. A plan view of the plume reveals 'holes' where the plume is forced around hills. Conversely, there are local areas of high concentrations in some of the valleys. If this type of pollutant distribution were to occur during an accident it would be rather difficult to interpret measured concentrations; there could be localised 'hotspots' a long way from the main plume axis.

It should be noted that in both this and the previous case study the trajectory and pollutant dispersal experiments have been conducted in a single WAFT flow field; that is to say the flow pattern has not changed with time. There has been an implicit assumption that the trajectories have left the study region before the flow pattern has changed signif-
significantly. The trajectories leaving Oldbury in this episode take approximately six hours to leave the area; there is thus some reason to doubt the validity of using a steady state flow field here. The final case study, discussed in the next section, is certainly one where it is necessary to use time varying flow fields.

6.7 CASE STUDY THREE - 9.00 HOURS, 7 FEBRUARY, 1976

6.7.1 Meteorological Situation

The high over Scandinavia remains from the day before, so that the British Isles lies in south to south easterly flow. However, the isobars are widely spaced and are no longer parallel indicating a much weaker synoptic flow than the previous day. The synoptic flow slowly rotates throughout the day so that the winds become south westerlies. Temperatures are fairly mild, in the range 5 to 7 degrees Centigrade, and mist and fog are reported.

The vertical wind profiles from Hemsby, Crawley and Cambourne indicate high level winds of the order of 8 m/s veering from 180 degrees at 0.00 to from 220 degrees at 12.00 hours. The surface winds at 9.00 hours are shown in Figure 6.30; the black squares represent stations reporting calms. The inland surface winds are very light (1-2 m/s) and there are large variations in speed and direction which are characteristic of stable flow. Many calms are reported at 6.00 hours, but the winds freshen during the morning and early afternoon.

The temperature profiles for 0.00 hours indicate surface inversions or low elevated inversions. The inland (and therefore most relevant) radiosonde station, Crawley, reports a strong surface inversion (of the order of five degrees) in the lowest 300 to 400m. This is overlaid by less stable (but nonetheless still stably stratified) air up to a height of approximately 1500m. The previous two case
studies have been modelled as neutral flow under an inversion lid, but the thermal profile described above indicates that it is appropriate to model this episode using the stable flow version of WAFT. At 0.00 hours the mean potential temperature gradient is estimated to be 1.2°C/100m.

6.7.2 Using a Uniform Surface Windfield

A uniform surface wind speed of 1 m/s blowing from 160 degrees is used for this part of the case study. The low windspeeds and erratic directions actually reported at 9.00 hours make the choice of an appropriate mean surface wind vector rather difficult. The low mean windspeed chosen reflects the many calms or near calms reported by inland stations, and the wind direction represents a realistic backing from the high level wind and the influence of the coastal stations near the eastern corner of the area which are probably more representative of the synoptic flow. The high level wind used by WAFT is estimated to be 8 m/s blowing from 210 degrees at an altitude of 1200m.

The stable flow version of WAFT is used for this uniform velocity case. The potential temperature gradient reported, 1.2°C/100m, translates to a Brunt-Väisälä frequency of 0.02 Hz. using equation 4.3.2(2). Therefore, in this case study, the Froude number over a given hill is given by

\[ F = \frac{50 \cdot u_{av}}{h} \]  

6.7.2(1)

where

- \( u_{av} \) is the mean windspeed in m/s
- \( h \) is the hill height in m

A 'hill defining factor', \( f_{HL} \), must be specified to control how WAFT splits up the Bristol Channel topography into
single hills (this is discussed in subsection 4.3.8). A value of 33 percent is used here which splits the region into seven subregions as shown previously in Figure 4.6(a). In this case, WAFT computes that two of the subregions, subregion three around Exmoor and subregion seven centred on the Welsh Mountains, have Froude numbers less than 0.5. The flow in these two subregions is therefore modelled according to the stable flow regime whereby there is near horizontal flow below the critical height and neutral flow above this height. A horizontal inversion 'lid' is placed at the top of the calculational domain at 1300m so that this neutral flow is constrained vertically. Whilst there is no evidence that such a 'lid' was actually present on the morning of 7th February, it is very likely that there were shallow stable layers which exerted a similar dampening effect on the neutral flow above the critical height. In any case, the presence of this elevated 'lid' has very little influence on the flow in the lower levels (where the pollutant and trajectory experiments are performed).

The resulting final windfield at 200m above Sea Level is illustrated in Figure 6.31. The first thing to note is that there is a wide variation in both windspeed and direction in the final field, even though a uniform initial field is used. WAFT generates light erratic winds in the Welsh hills and a stagnant region upwind of the Welsh hills over the Bristol Channel. These features are also apparent in the observed data. Similarly, WAFT produces a relatively fast flow area in the top left (eastern) corner of the study area which is also apparent in the reported surface winds.

Once again though, a station by station comparison of the reported and 'predicted' ten metre winds yields poor agreement. The dashed arrows in Figure 6.30 represent the 'predicted' winds extracted from the WAFT final windfield. Earlier work has already suggested that this disagreement is not too surprising. This is particularly true in this case; the stable conditions favour the development of many local scale flows. There are some stations where the agreement is
very good; Cardiff provides an example where both the observed and 'predicted' wind directions are at rightangles to the main synoptic flow direction. Even where the 'predicted' and reported winds are rather different, as at Filton for example, the windspeeds are very low and so the observed wind might well be indicative of a local flow effect. There are two stations at the left hand side of the figure, Brynamman and Exton, where WAFT generates fairly high surface winds, and yet calms were actually reported. This may be due to some local scale shielding effect. Brynamman is situated in an area where the WAFT final windfield was changing very rapidly, and it is quite likely that if a finer grid is used a very different 'predicted' wind would be produced.

6.7.3 Using the Available Surface Station Data

The measured ten metre winds from 28 surface stations provide the data for this example; of these, 12 reported calms. The calculational mesh and high level wind specification used for the uniform surface wind case are again used. However, some erosion of the stable vertical temperature gradient from the value at 5.00 hours is assumed here. A potential temperature gradient of 0.6°C/100m is employed. The 'hill defining factor', \( h_{FL} \) is again set at 33 percent.

The simple inverse square interpolation scheme is applied to the data to generate the initial flow field at ten metres above the surface. This initial ten metre flow pattern is shown in Figure 6.32(a). The 'holes' are areas close to meteorological stations which reported calms. The inverse square interpolation method tends to smooth out irregularities in the data, but note that in this case, the data is so variable that the initial windfield has large velocity gradients. For comparison, Figure 6.32(b) illustrates the ten metre flow when the bicubic spline method is employed. This flow field has extremely rapid changes of velocity; for example, in the extreme bottom right of the figure.
These abrupt changes might be realistic if they coincided with physical boundaries such as valley walls. However, the basic bicubic spline method used here does not take the region's topography into account and so these large velocity gradients are unrealistic. It is therefore probably safer to use the inverse square interpolation in this case.

The vertical potential temperature gradient employed here implies a Brunt-Väisälä frequency of approximately 0.014, so that in this case the Froude number is given by,

$$ F = \frac{70 \cdot u_{av}}{h} $$

6.7.3(1)

where the variables are the same as in 6.7.2(1). If the mean windspeeds were similar to those in the uniform initial windfield case, equation 6.7.3(1) would indicate higher Froude numbers over the hills of the region. However, the mean windspeeds are somewhat lower so that four of the seven subregions now have Froude numbers below 0.5.

The resultant final windfield at a height of 200m is presented in Figure 6.33. This windfield is quite similar to the one produced using the uniform initial windfield shown in Figure 6.31; this suggests that the observed data might reflect the mesoscale flow through the area quite well. There are two zones of relatively fast flow; in the top left hand corner, and from the bottom middle to top left. In between, there is a stagnant region trapped by the high ground of Exmoor and the Welsh mountains. There are also light winds upwind of Exmoor (extreme bottom left of figure) and in the Cotswolds (extreme bottom right). Once again, strong channelling with very variable wind directions occurs in the Welsh hills. WAFT calculates that virtually all of the study area at 200m is in stable flow below the critical height, and so the flow in the figure is basically two dimensional. WAFT reinforces the impression given by the observed surface winds that the mesoscale flow on the morning of 7th February is indeed very complex.
Trajectories from Hinckley Point and Oldbury are plotted in Figure 6.34. It is important to note that the arrows on the streamlines represent three hourly intervals. Thus, it takes over a day and a half for the 10m streamline from Hinckley Point to reach the edge of the study area. It is, of course, very unlikely that this flow pattern will persist for anything like that amount of time; unlike the trajectories presented in the two previous case studies (which had much higher surface windspeeds) these are very much hypothetical trajectories. However, they do illustrate the complexity of the final windfield. In particular, the 10m trajectory from Hinckley Point takes a completely different route from those released at 100 or 250m. There is also a marked variation in windspeed along the trajectories; pollutants travelling along these trajectories would alternately accelerate and decelerate. The speed also varies considerably with height; for instance, the speeds along the 250m trajectory are twice those of the 10m one. The streamlines from Oldbury actually converge as they travel away from the source.

It is clear that it will be necessary to use a temporally as well as spatially varying windfield to model this case study correctly. This is considered next.

6.7.4 Using a Time Series of WAFT Windfields

Figure 6.35 illustrates measured ten metre winds at three hourly intervals throughout 7th February 1976. The figure confirms the view that it is necessary to use a time varying windfield in order to model pollutant dispersal appropriately. Clearly, the surface winds are changing in a much shorter time scale than was needed for the trajectories to track across the study area in the static windfield used previously. Also, the available vertical velocity profiles indicate that the high level synoptic flow also changed
throughout the day. The WAFT model cannot produce time varying flows, and so the approach described below is adopted to model a changing flow pattern.

The available surface data at seven times throughout the morning and afternoon is used to create seven separate static windfields. These are 'snapshots' of the flow pattern at seven times throughout the day. The windfield at any intervening time is constructed using a linear interpolation of the two WAFT 'snapshot' windfields before and after the required time. The seven 'snapshot' windfields represent the flow pattern at 0.00, 3.00, 6.00, 9.00, 12.00, 15.00 and 18.00 hours. Hence, the wind vector at a particular mesh point, \((i,j,k)\) at 11.00 hours for example, is calculated as

\[
V_{11.00} = \frac{1}{3} V_{9.00} + \frac{2}{3} V_{12.00}
\]

Each of the 'snapshot' runs uses the same calculational mesh. Table 6.2 details the input data used in each case. The stable flow version of WAFT is used for the period up to and including 9.00 hours. The very stable potential temperature gradient actually measured at 5.00 hours, 1.2°C/100m, is employed for the 0.00, 3.00 and 6.00 hour cases. The 9.00 hours 'snapshot' windfield is the one presented in the previous subsection; the temperature gradient is 0.6°C/100m. In each of these stable flow cases, a 'hill defining factor' of 33 percent is used. The runs for 12.00 hours onwards employ the neutral flow version of WAFT. The developing boundary layer model of Carson (1973) is used to estimate the inversion height in these cases. The heights used are given in Table 6.2.

The available vertical wind profiles show that the high level wind initially blows from the south, but then rotates at a rate of about ten degrees per three hours in a clockwise sense. Again, Table 6.2 illustrates the high level wind specification used in each run. The number of surface
measurements available at each time is also shown, as well as the number of these that are calms. At 6.00 nearly three quarters of the stations reported calms; this is a very stagnant period.

Final windfields for 0.00, 6.00, 12.00 and 18.00 hours are illustrated in Figures 6.36(a), (b), (c) and (d) respectively. These all show the flow at 100m above Sea Level. The wind pattern at 0.00 hours is characterised by moderate easterly flow down the Bristol Channel, with stagnant regions upwind of the Welsh Mountains, Exmoor, and in the Cotswolds. By 6.00 hours winds are light and variable across the whole region; moderate flow only occurring in the extreme western corner of the area. The surface winds have increased by 12.00 hours but they are now predominantly southerlies. A pool of stagnant flow is trapped between Exmoor and the Welsh hills. The surface winds have freshened still further by 18.00 hours. The stagnant area has disappeared, but there is still significant deflection of the flow around the Welsh mountains.

Figures 6.37 and 6.38 illustrate plan and perspective views of trajectories plotted in the time series of WAFT windfields. Figure 6.37 shows trajectories released at 10, 100 and 250m above the Hinckley Point and Oldbury sites at 0.00 hours. Arrows denote hourly intervals, and every third one has the time of day marked. Initially, the trajectories leaving Hinckley move off in the same direction towards the west. The trajectories are moving at very different rates; it takes the 10m streamline nine hours to travel the same downwind distance that the 250m trajectory covers in just three hours. Interestingly, the trajectories all turn towards the north west at roughly the same position, although they actually do so at very different times. Presumably, this is where the trajectories encounter the south easterly flow near the western corner of the area.

The trajectories leaving Oldbury also highlight the complex nature of the flow. The 250m trajectory heads off toward
the north west, but as the day progresses and the winds rotate, it arcs around toward the north. Those released at 10 and 100m initially move towards the west. After about four hours or so, they encounter the pool of still air centred on the Bristol Channel. The trajectories are very nearly stationary at 6.00 hours, but as the winds pick up, they turn toward the east and very nearly go back the way they have come. Indeed, there is a point where the trajectories cross such that the 100m trajectory arrives at about 4.00 hours, but the 10m one gets there six hours later from the opposite direction.

The trajectories shown in Figure 6.38 are all released at 10m above the ground, but they leave at different times, namely, 0.00, 3.00, 6.00 and 9.00 hours. The numbers at the side of the trajectories indicate the time of day when the trajectory reaches a particular point. The direction in which the trajectories leave Hinckley Point rotates around clockwise during the day. As the winds freshen during the morning the speeds along the trajectories increase. By 12.00 hours the trajectory released at 9.00 has travelled the same distance as the one released three hours earlier.

The trajectory released from Oldbury at 3.00 hours performs a similar U-turn to the 0.00 hours trajectory, but in this case actually passes back over the release site. The later Oldbury trajectories don't follow such convoluted routes, but again those released later in the day catch up with those released earlier.

Finally, TOMCATs is used to model plume behaviour in the sequence of WAFT windfields. Figures 6.39(a)-(e) illustrate how a notional plume emanating from Hinckley Point might behave during the morning and afternoon of 7th February. A constant rate continuous release beginning at 0.00 hours is assumed. Plan and perspective views of the plume at five times are shown; Figure 6.39(a)-(e) correspond to 03.30, 07.00, 11.00, 14.30 and 18.00 hours respectively. The plume is released at 50 metres. Figure 6.40(a)-(e) illustrates
the corresponding plume from Oldbury at the same series of times.

The plume from Hinckley Point heads off in a straight line toward the west. In the first 3-4 hours of its life it is rather similar to a Gaussian plume. As 6.00 hours is approached, however, the winds drop and begin to rotate toward the North. This happens relatively quickly so that the westward progress of the plume is halted, and the straight plume begins to act as a line source as material is swept North West over the Bristol Channel. These light southerlies continue throughout the morning, so that at 11.00 (Figure 6.39(c)) the plume has been 'smeared' across the Bristol Channel and is moving into South Wales. As the winds pick up from midday onwards, the main, distinct, portion of the plume is advected in the relatively fast flow around the Eastern side of the Welsh Mountains. Material released near the start of the day is still being swept westward over the Welsh hills. The irregular plume pattern indicates considerable channelling along the Welsh valleys. In the period 0.00 to 18.00 hours, the plume axis has rotated by about 120 degrees, and for the major part of the time it has borne little resemblance to a Gaussian plume.

The plume from Oldbury also begins by moving westward. There is a large amount of vertical shear present with the result that the plume is very wide. Around 7.00 hours, high concentrations develop around the site as very weak winds carry material back over the site. Most of the pollutant released earlier is now heading towards the North, but some, caught up in the complex flow pattern over the Bristol Channel, continues to move westwards. As midday comes around, the plume continues to be swept around towards the North, whilst the light winds mean a comparatively dense plume near Oldbury. During the afternoon, the strengthening southerlies/south westerlies lead to a well developed plume heading north eastwards up the Severn Valley. With no topographical mass deforming the flow in this part of the area, the plume assumes a Gaussian character in the latter part of the afternoon.
6.8 SUMMARY

The work in this chapter does not constitute an exhaustive validation of WAFT's behaviour in real terrain. Rather, it uses three case studies to get some idea of how useful WAFT might be in practice. The work at the beginning of the chapter using uniform initial fields confirmed earlier work with single hills which indicated that topography only exerts a significant effect in stable or low inversion conditions. The first lesson learnt from the case studies concerned the quality of the input data one might expect when using WAFT operationally. Two items in particular, the vertical velocity profile and the vertical temperature gradient, have a large impact on the final result, and yet available data is poorly defined and often conflicting. It should be noted that the work here considered days when there was a reasonably distinct thermal structure, and when the vertical velocity profiles across the area were consistent.

The first case study had fast neutral flow beneath an inversion at 600m. Experiments with a uniform initial windfield indicated that the topography was dominating the flow pattern, and therefore that WAFT should work well; the episode thus provided a good basis for some sensitivity studies. These showed that variations between the WAFT runs using different input data or interpolation schemes were as great as the differences between the WAFT trajectories and the straight line assumption used by the Gaussian Plume model.

The WAFT trajectories in the second episode considered were quite different from the straight line Gaussian assumption. However, the light variable surface winds also meant that the choice of initial windfield interpolation scheme was also more important. Thus, the variations between WAFT runs using different interpolation schemes were also correspondingly greater. The accuracy of WAFT windfields is to a very large extent determined by the quality of the input data; it must be representative of the mesoscale flow. Work using
a small subgrid scale domain demonstrated that in many cases the reported ten metre winds might be quite unrepresentative of the general flow patterns. Chapter 5 described local scale thermal flows which might be quite different from the mesoscale flow, but this case study also showed that local terrain features could also mean that ten metre winds are poor indicators of the bulk flow.

The final case involved a very complicated stable flow episode. The very low windspeeds and changing synoptic flow meant that it was necessary to use a sequence of WAFT windfields. It is possible to employ time varying meteorology with the Gaussian model (for example, Pendergast, 1979), but this episode illustrated that there are some situations in which a more sophisticated model, such as WAFT, is required. For example, an abrupt change of surface wind direction meant a plume from Hinckley Point started acting as a line source as the plume was 'smeared' across the region.

The plumes generated using WAFT were usually much wider than Gaussian plumes using corresponding stability assumptions. This was because of the large vertical shear often present in the WAFT windfields. Thus, WAFT tends to predict lower concentrations but spread over a wider area. WAFT may predict localised 'hot spots' as pollutant is trapped in valleys, however. WAFT plumes were not homogenous, sometimes they could split into two distinct plumes, or they might have a very complicated structure as the flow was channelled into many small valleys. The most obvious difference between WAFT and the Gaussian model is that the former often produces curved plumes; however, it is usually changes in wind direction, rather than deflections produced by topography, which give rise to the most pronounced curves.

The work in this chapter is not comprehensive enough to say whether the WAFT model is generally better than the simpler Gaussian model. The sensitivity studies suggest that WAFT
is no more reliable. On the other hand, there are certainly situations which do occur where the Gaussian model is completely unrealistic. In these cases, WAFT offers better results providing the input data is sufficiently representative.
Fig 6.1 - The Bristol Channel study region
Fig 6.2 - WAFT windfield over the Bristol Channel region under an elevated inversion at 2000 metres (uniform initial windfield).
Fig 6.3 - WAFT windfield over the Bristol Channel region under an elevated inversion at 800 metres (uniform initial windfield).
Fig 6.4 - WAFT windfield over the Bristol Channel region under an elevated inversion at 400 metres (uniform initial windfield).
Fig 6.5 - Stable WAFT windfield over the Bristol Channel region using a uniform initial windfield -
Case 1: $h_{LF} = 33\%$, $d\theta/dz = 1.0^\circ C/100m$, $u_{10} = 1$ m/s
Fig 6.6 - Stable WAFT windfield over the Bristol Channel region using a uniform initial windfield -
Case 2: \( h_{LF} = 66\% \), \( \frac{\partial \theta}{\partial z} = 1.6 \, ^\circ C/100m \),
\( u_{10} = 1 \, m/s \)
Fig 6.7 - Stable WAFT windfield over the Bristol Channel region using a uniform initial windfield -
Case 3: $h_{LF} = 33\%$, $d\theta/dz = 0.5^\circ\text{C}/100\text{m}$,
$u_{10} = 2 \text{ m/s}$
Fig 6.8 - Stable WAFT windfield over the Bristol Channel region using a uniform initial windfield - Case 4: uniform vertical velocity suppression
Fig 6.9 - January 29th case study: measured surface winds in the Bristol Channel region at 09:00 hours.
Fig 6.10 - 29th January case study - flow at 100m above S.L. in uniform initial windfield run

Fig 6.11 - 29th January case study - streamlines from the nuclear power stations in uniform initial windfield run
Fig 6.12 - 29th January case study - initial windfield at 100m above S.L. using available surface data

Fig 6.13 - 29th January case study - final windfield at 100m above S.L. using available surface data
Fig 6.14 - 29th January case study - streamlines from the nuclear power stations in the initial windfield using available surface data.

Fig 6.15 - 29th January case study - streamlines from the nuclear power stations in the corresponding final windfield.
Fig 6.16 - 29th January case study - simulated plumes in the WAFT final windfield

Fig 6.17 - 29th January case study - simulated Gaussian plumes based on source meteorology
Fig 6.18 - WAFT sensitivity studies - the effect of initial windfield interpolation scheme and data points used.
**Case 1** - Inversion at 600 m

**Case 9** - Inversion at 400 m  
**Case 10** - Inversion at 800 m

Fig 6.19 - WAFT sensitivity studies - the effect of the height of the elevated inversion

**Case 2** - standard profile  
**Case 11** - linear speed and direction

**Case 12** - linear velocity components  
**Case 13** - logarithmic velocity components

Fig 6.20 - WAFT sensitivity studies - the effect of different assumed vertical profiles
Case 1 - 16 m/s from 170° at 2000 m

Case 14 - 15 m/s from 170° at 1000 m

Case 15 - 15 m/s from 190° at 1000 m

Case 16 - 15 m/s from 150° at 1000 m

indicates high level wind direction

Fig 6.21 - WAFT sensitivity studies - the effect of the high level wind specification
**Fig 6.22** - 6th February case study - measured surface winds in the Bristol Channel region at 09.00 hours

**Fig 6.23** - 6th February case study - final windfield at 160m above S.L. using a uniform initial windfield
Fig 6.24 - The topography around the Aberdare meteorological station with mesh intervals of (a) 5.0 km and (b) 0.5 km
Fig 6.25 - WAFT windfields around Aberdare using uniform initial windfields and a high resolution mesh.
Fig 6.26 - 6th February case study - initial and final windfields at 200m above S.L. using available surface data and inverse square interpolation
Fig 6.27 - 6th February case study - initial and final windfields at 200m above S.L. using available surface data and bicubic spline interpolation.
Fig 6.28 - 6th February case study - trajectories in the final windfield produced using (a) inverse square interpolation and (b) bicubic spline interpolation.
Fig 6.29 - 6th February case study - simulated plumes from the nuclear power stations
Fig 6.30 - 7th February case study - measured surface winds in the Bristol Channel region at 09.00 hours

Fig 6.31 - 7th February case study - final windfield at 200 m above S.L. using uniform initial windfield
Fig 6.32 - 7th February case study - initial windfield at 10m above the ground using available surface data and (a) inverse square or (b) bicubic spline interpolation
Fig 6.33 - 7th February case study - final windfield at 200m above S.L. using available surface data and inverse square interpolation

Fig 6.34 - 7th February case study - trajectories in the final windfield produced using inverse square interpolation
Fig 6.35 - 7th February case study - measured surface winds in the Bristol Channel region at three-hourly intervals throughout the day
Fig 6.36 - 7th February case study - final windfields at 100m above S.L. for the time-series of runs
Fig 6.37 - 7th February case study - trajectories released from different heights in the time-series of windfields.
Fig 6.38 - 7th February case study - trajectories released at different times in the time-series of windfields
Fig 6.39 - 7th February case study - simulated plume from Hinckley Point in the time-series of windfields (continued on next page)
(d) 14.30 hours

(e) 18.00 hours

Fig 6.39 - 7th February case study - simulated plume from Hinckley Point in the time-series of windfields (continued from previous page)
Fig 6.40 - 7th February case study - simulated plume from Oldbury in the time-series of windfields (continued on next page)
Fig 6.40 - 7th February case study - simulated plume from Oldbury in the time-series of windfields (continued from previous page)
(a) Case 1

Hill defining factor = 33%
Temperature gradient = 1.0°C / 100 m
Surface windspeed = 1.0 m/s

<table>
<thead>
<tr>
<th>Subregion</th>
<th>Mean Froude number</th>
<th>Critical Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.41</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>0.36</td>
<td>160</td>
</tr>
<tr>
<td>3</td>
<td>0.27</td>
<td>230</td>
</tr>
<tr>
<td>4</td>
<td>0.41</td>
<td>130</td>
</tr>
<tr>
<td>5</td>
<td>0.41</td>
<td>130</td>
</tr>
<tr>
<td>6</td>
<td>0.37</td>
<td>170</td>
</tr>
<tr>
<td>7</td>
<td>0.21</td>
<td>510</td>
</tr>
</tbody>
</table>

(b) Case 2

Hill defining factor = 66%
Temperature gradient = 1.0°C / 100 m
Surface windspeed = 1.0 m/s

<table>
<thead>
<tr>
<th>Subregion</th>
<th>Mean Froude number</th>
<th>Critical Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.28</td>
<td>330</td>
</tr>
<tr>
<td>2</td>
<td>0.21</td>
<td>510</td>
</tr>
</tbody>
</table>

(c) Case 3

Hill defining factor = 33%
Temperature gradient = 0.5°C / 100 m
Surface windspeed = 2.0 m/s

<table>
<thead>
<tr>
<th>Subregion</th>
<th>Mean Froude number</th>
<th>Critical Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.83</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.79</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.51</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.90</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.88</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.78</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.37</td>
<td>410</td>
</tr>
</tbody>
</table>

Table 6.1 - Computed Froude numbers for the subregions in the Bristol Channel in the stable flow runs.
<table>
<thead>
<tr>
<th>Time of day</th>
<th>Number of stations</th>
<th>Number of reported calms</th>
<th>Assumed thermal profile</th>
<th>Assumed high level wind at 1500 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>00.00 hrs</td>
<td>16</td>
<td>6</td>
<td>Stable, thermal gradient = 1.2°C / 100 m</td>
<td>9 m/s from 180°</td>
</tr>
<tr>
<td>03.00 hrs</td>
<td>16</td>
<td>6</td>
<td>Stable, thermal gradient = 1.2°C / 100 m</td>
<td>9 m/s from 190°</td>
</tr>
<tr>
<td>06.00 hrs</td>
<td>19</td>
<td>14</td>
<td>Stable, thermal gradient = 1.2°C / 100 m</td>
<td>8 m/s from 200°</td>
</tr>
<tr>
<td>09.00 hrs</td>
<td>28</td>
<td>12</td>
<td>Stable, thermal gradient = 0.8°C / 100 m</td>
<td>8 m/s from 210°</td>
</tr>
<tr>
<td>12.00 hrs</td>
<td>24</td>
<td>4</td>
<td>Neutral, Inversion at 600 m</td>
<td>8 m/s from 220°</td>
</tr>
<tr>
<td>15.00 hrs</td>
<td>26</td>
<td>2</td>
<td>Neutral, Inversion at 700 m</td>
<td>8 m/s from 230°</td>
</tr>
<tr>
<td>18.00 hrs</td>
<td>21</td>
<td>3</td>
<td>Neutral, Inversion at 900 m</td>
<td>8 m/s from 240°</td>
</tr>
</tbody>
</table>

Table 6.2 - 7th February case study - input data for the time series runs.
This thesis has considered the generation of three-dimensional windfields which can be used to model the movement of airborne pollutants. In particular, it has been concerned with pollutant transport from nuclear sites at distances of the order of tens of kilometres (mesoscale modelling). A program (WAFT) which generates a three-dimensional flow field from routinely measured meteorological data has been developed and evaluated. The windfield is interpolated from measured winds but is then minimally adjusted so as to be mass-consistent. This type of approach has been adopted by a number of countries in on-line systems (notably ARAC in the USA) which are to be used in the event of a nuclear accident. It is therefore of considerable practical importance to examine how these models behave.

Mass-consistent models have been used in a variety of applications such as land/sea breeze simulation (Anderson, 1971), wind energy site evaluation (Bhumralkar et al, 1980) as well as pollutant transport. They have all tended to follow the approach adopted in Sherman's model MATHEW (Sherman, 1978). However, WAFT uses a different method which allows a much more realistic topography representation than MATHEW, and thus improves the simulation of flow close to the land surface. Moreover this improved surface representation has been achieved without needing the complicated mathematics employed in finite-element mass-consistent models (see Tuerpe et al, 1978). WAFT has also exploited recent developments in numerical analysis (such as the Incomplete Cholesky Conjugate Gradient method) to provide a more efficient and robust solution method than MATHEW. WAFT is capable of producing windfields from the barest minimum of input data; for instance, unlike MATHEW it does not require a measured vertical wind profile.
Care has been taken to ensure that WAFT models flow over simple geometrical shapes realistically. WAFT has been validated against known potential flow solutions. This is necessary given that detailed three-dimensional atmospheric data is not available for comparison. Experiments have confirmed that the WAFT windfields are not unduly dependent on the cell geometry; this is important since meshes used in practical applications may be only a kilometre or so in height but have horizontal dimensions of hundreds of kilometres. It is not clear whether similar validation work has been undertaken for many of the other mass-consistent models. It has also been established that the calculational mesh must extend up to a height equal to the hill's length if potential flow is to be simulated accurately; again, this seems to have been neglected in most of the other mass-consistent work.

Comparative assessment of the sources of error within the mass-consistent approach emphasised the sensitivity of the final windfield to the initial windfield specification. This was particularly marked in strongly varying flow fields or where few data points were available. The number or position of data points or using alternative initial windfield interpolation schemes were all far more significant than changing the mesh geometry or the accuracy to which the mass-consistent equations were solved. Sherman (1978) placed great emphasis on ensuring that the divergence of the final windfield is very low (that is the equations were solved to a high degree of accuracy). The work here showed that this has no material effect on the accuracy of the final windfield. Similar comments apply to work which employs sophisticated Finite-Element techniques - the effort is wasted unless the initial windfield is representative of the atmospheric flow. It is more appropriate to think of mass-consistent models as interpolation tools rather than complete fluid dynamics models; they will not generate flow features which are not represented in the input data.
The importance of the initial windfield specification was reinforced by the case studies presented in Chapter 6. Topographic effects were thought to be dominant in the first of these episodes but the final windfield was sensitive to data point omission and interpolation method even in this case. Sensitivity studies indicated that differences in WAFT trajectories caused by uncertainties in meteorological data could be as great as the differences between WAFT and straight line trajectories. This suggests that WAFT is no more reliable than the simpler Gaussian Plume model. The other two case studies featured rather more complicated meteorological conditions. Surface winds were light and very variable in direction. In one case the air was judged to be stably stratified down to ground level and in the other the elevated inversion was very low. Such conditions are of particular interest as they represent 'worse case' scenarios for pollutant transport following a nuclear accident. One would not expect a simple straight line model such as the Gaussian Plume to perform well in these conditions. However, whilst WAFT is potentially better able to model these situations the variability of the surface flow emphasises WAFT's sensitivity to the initial windfield specification.

WAFT, MATHEW and most of the other mass-consistent models employ measured surface winds in the generation of the initial windfield. The quality of the final windfield depends to what extent the surface winds represent the overall mesoscale flow. The work presented here has shown that the surface winds are often unreliable indicators of the overall flow. Experiments in Chapter 6 using a high resolution mesh in the vicinity of a meteorological station situated on a valley floor illustrated this. The measured ten metre wind can be influenced by very local scale topography. This criticism applies even if one assumes that atmospheric flow approximates potential flow. However, as the survey of local scale phenomena in Chapter 5 illustrated, local slope winds or lee rotors, for example, may also affect measured ten metre winds. Furthermore the
air velocity varies with height particularly in stable conditions. This variation is usually most marked in the lowest hundred metres or so. Thus, for all the reasons given above one cannot assume that measured ten metre winds are indicative of the bulk mesoscale flow across the region. This uncertainty casts doubt on the validity of the mass-consistent approach. Certainly, the inclusion of unrepresentative surface measurements might make the final windfield much poorer.

The behaviour of the atmosphere in complex terrain is strongly influenced by thermal stratification. It is thus necessary to account for the atmosphere's thermal structure in mass-consistent models. The minimisation coefficients ('weighting factors' in this thesis) provide a mechanism of preferentially weighting adjustments in either different directions or in different parts of the grid. In particular, they can be used to suppress or enhance velocity adjustment in the vertical direction and hence control the size of the vertical velocities. Thus, they may be used to model thermal stratification. This has been recognised in other mass-consistent work but little guidance has been given as to how these weighting factors should be determined. The relationship between the weighting factors and the vertical temperature gradient has been examined in some detail here. A consistent approach to modelling thermal stratification has been developed here which uses readily available meteorological data.

Two types of thermal stratification have been treated. The first is where neutrally stratified air is overlain by more stable air; this constitutes the commonest situation in the UK. The boundary between the neutral air and the overlying stable air is treated as a flat impenetrable 'lid' at a constant height above Sea Level. It should be noted that the inclusion of an elevated 'lid' removes the requirement (mentioned above) that the mesh height must be of the order of the hill length. Flow around single hills described in Chapter 4 and the case studies in Chapter 6 demonstrated the
importance of specifying the height of the 'lid' correctly. There is only significant horizontal deflection around hills as the 'lid' is lowered close to the hill summit or below. The case studies illustrated the difficulty of obtaining reliable information about the inversion height across the UK. Vertical temperature soundings are taken at only a few locations in the UK, and then only twice a day. The case study days were selected to have relatively unambiguous thermal profiles but even then the inversion height was difficult to pinpoint. This could prove a significant problem when using WAFT operationally. The above approach may be contrasted with that adopted in MATHEW (and similar models) where thermal stratification is modelled by suppressing vertical motion uniformly at all mesh levels. No guidance is given as to how these weighting factors should be determined from available meteorological data even though they may have a profound effect on the predicted flow.

The other type of thermal stratification considered was when the stably stratified flow extended right down to the ground surface. This is rarer than the elevated inversion case but is of interest because the constrained vertical motion and low windspeeds often produce the most severe pollution episodes. Current experimental and theoretical knowledge of stable flow around a single hill suggests that the flow is largely horizontal below a certain 'critical height' and close to potential flow above this. The uniform vertical velocity suppression available in MATHEW is not sufficient to model this two-layered flow. A new approach was adopted in WAFT whereby the weighting factors are related to the hill's dimension and the vertical thermal gradient. Less is known about the behaviour of stable air in complex terrain. Hence, a rather arbitrary solution of treating complex orography as a group of independent single hills had to be adopted.

WAFT's treatment of thermal stratification has many weaknesses but it does represent a first attempt to include a
very important feature of the atmosphere into the mass-consistent approach. It is not suggested that it models the detailed features of atmospheric flow but it does provide a systematic way of obtaining vertical velocities of a realistic magnitude. The need to parameterise thermal stratification in the weighting factors is just one reflection of the limited physical basis of the model. WAFT models flow distortion due to topography but other physical effects must be included in the initial windfield generation. For instance, the frictional drag of the earth's surface is incorporated as the vertical velocity profile used to interpolate the initial windfield. It is possible that other physical effects such as thermally driven flow could also be treated in this way. The mass-consistent approach gives a static view of the atmosphere; no time variation or predictive ability is included. As the final case study illustrated, the temporal evolution of the flow can be the major determinant of pollutant trajectories. A time varying flow may be simulated using a series of WAFT windfield. Any predictive ability must rely on the assumption of persistence.

This work has discussed the weaknesses of the mass-consistent approach in some detail but these should be viewed in the context of the deficiencies of other methods of mesoscale modelling. The Gaussian Plume model is the most widely used alternative and yet relies on the assumption that the flow is uniform. Two of the three case studies in Chapter 6 show situations when this is not the case. At the other end of the scale, sophisticated fluid dynamics codes may consume prohibitive amounts of computing power. More fundamentally, they may require meteorological data for boundary conditions or initialisation which is not generally available. In the end the different approaches must be assessed with reference to atmospheric tracer experiments. The work here has indicated the type of data that should be collected in order to run and verify a model such as WAFT. It would be desirable to have comprehensive velocity data throughout the boundary layer. Selections
from this data could then be used to drive WAFT in an attempt to gauge the minimum amount of data required for WAFT to produce accurate results. A vertical temperature sounding would also be required. A variety of meteorological situations would be desirable to identify which conditions the mass-consistent approach dealt with best.

Any further work with WAFT should concentrate on the initial windfield specification. In particular, alternative schemes which are not based on surface measurement might be tried. For instance, the initial windfield could be derived from the geostrophic wind or from a national scale weather prediction program. Since the majority of UK nuclear sites are coastal a land/sea breeze sub-model would be a valuable addition to WAFT.

In conclusion, whilst this work has improved the mass-consistent model developed by Sherman, too many uncertainties exist for it to be considered a viable operational tool. These centre around its limited physical basis and the inadequacy of routinely measured meteorological data. A final judgement must await comparisons with tracer experiments in the atmosphere.
REFERENCES


APPENDIX 1

THE WAFT CODE

A.1.1 INTRODUCTION

This appendix gives a brief description of the input and output files used by the WAFT code, as well as an overview of the code structure. The three-dimensional WAFT code used throughout this thesis consists of about four thousand lines of Fortran IV code written to run under NOS1.4 on CDC machines. Structured programming techniques are followed to improve readability and reduce programming errors.

Two different versions of WAFT exist; one which is used to model neutral flow, and the other which simulates stable flow. As described in subsection 4.3.8, the stable flow version contains a lot of extra code which splits the domain into subregions and allots a local Froude number to each. Within both the neutral and stable versions of WAFT, there are numerous variants which implement different surface interpolation routines, velocity profiles and numerical solution methods.

The WAFT code is kept on the CDC machine in the form of 'Update' files. 'Update' is a CDC utility which stores programs in a compressed form and aids the construction of different versions of a particular program. It provides simple instructions to insert, delete or merge lines. Different variants of the WAFT code are constructed from a base listing amended by the appropriate 'Update' files. An interactive program is provided to simplify assembly of the different WAFT variants. It asks the user whether the neutral or stable flow version is required, and which initialisation and numerical solution schemes are to be used. It then automatically generates the appropriate instruction stream for the 'Update' utility.
NOS1.4 (under which WAFT was developed) is a fixed memory operating system. This type of system imposes much stricter limits on the amount of central memory a program may use, in comparison with the virtual operating systems available on most modern computers. As a result, considerable care is taken to minimise the size of arrays used in the WAFT code. For example, intermediate results in the Conjugate Gradient solution method are written to disc files rather than being stored in an array.

The sizes of arrays in Fortran code must be specified before compilation; it is not possible to set their sizes dynamically at runtime, as it is, for example, in Algol programs. Another function of the interactive assembly program is therefore to edit the array declarations in the code. It determines the minimum acceptable size of these arrays by reading in the WAFT data file (described below in section A.1.3) for a particular calculational mesh and topography. On virtual memory operating systems, where program storage requirements are not so critical, one could set the array dimensions to values large enough to cope with all conceivable meshes.

Many other programs have been written besides the WAFT code itself. These include programs to calculate trajectories, generate analytic flows, and compare WAFT windfields with analytic flows, for example. The pictorial representations of WAFT windfields included in this thesis are generated using supporting graphics programs developed for use with WAFT.

A.1.2 DISC FILES PRODUCED BY WAFT

WAFT creates as many as five disc files which are then processed by, for example, graphics programs etc. A naming convention is adopted for these files on the CDC. It is thus easy to identify what type of information is contained in a particular file, and associate each output file with
the input data used to create it. The input data file (see next section) contains an alphanumeric string which forms part of the name of all disc files produced by WAFT from that data. This string is termed the 'run identifier' and may be up to four characters long. The first character must be alphabetic. The other part of the file name indicates the type of information held and is supplied by WAFT itself.

Assuming a run identifier of 'RED1', for example, the names and contents of the five disc files would be as follows:

**CONRED1** contains information about the mesh structure and the topography heights. This file is required by all programs which process WAFT output disc files.

**INVRED1** holds the initial windfield vectors defined at each mesh point. It is usually processed to provide vector plots of two-dimensional sections of the initial windfield. These are included throughout this thesis, for example Figure 6.12.

**FIVRED1** is the same as INVRED1 but pertaining to the final windfield.

**IAVRED1** is the initial windfield defined as average velocities flowing across each cell face. This format is more suitable for subsequent processing by the trajectory plotting and Monte-Carlo dispersion programs.

**FAVRED1** as IAVRED1, but for the final windfield.

Disc space is quite limited on the CDC installation at Imperial College. It is thus desirable to take advantage of the large word length of the CDC (60 bits) and pack the three velocity components at a particular mesh point into one computer word before transfer to disc files. This packing imposes limits on the size and accuracy of velocity components stored on the disc files. All velocity
components must lie between -50 and 50 m/s; this should be sufficient for all windfields of practical interest. The modest accuracy of the velocities is also not a problem considering all the other uncertainties associated with WAFT windfields.

A.1.3 INPUT DATA FOR WAFT

All the input data required to run WAFT is specified in one data file. This file contains details of the calculational mesh, orography and meteorological measurements. The user would normally create this file using the system editor.

The input file must specify the following data in the order and formats indicated below. The formats of individual lines are described using the Fortran IV format statement syntax; please refer to a Fortran IV manual for an explanation of this system.

1. The first line contains the 'run identifier' described above. The 'run identifier' may be between one and four alphanumeric characters long and begin in column one. The first character must be alphabetic. A period must follow immediately after the final character.

2. The second line contains an integer between one and six which controls the amount of detail provided in the line printer output from WAFT. This single digit must be in column one. The information listed for the different values of this parameter is as follows:

\[1\] error messages, run identifier, number of irregular cells, number of unsubmerged points

\[2\] calculational mesh, topography heights, meteorological data, flow statistics
3. final velocity field

4. initial velocity field

5. Lagrangian Multiplier solution vector

6. initial and final net flow into each cell

One would normally set this value to either two or three; bigger values produce copious amounts of output.

3. The third line controls which of the disc files discussed in the previous section is created during the WAFT run. The first value controls the production of the disc files containing the velocities defined at mesh points (that is INVxxxx and FIVxxxx). This single digit is placed in column one. The second value determines which of the average velocity disc files (IAVxxx and FAVxxx) is created. This single digit is entered in column three. Acceptable values and their effects are:

0    neither initial nor final field
1    final field only
2    initial field only
3    both final and initial fields

The file containing the mesh description (CONxxxx) is produced unless both values are zero.

4. This line marks the start of the data dealing with the mesh specification and topography surface. The number of mesh planes normal to each of the coordinate axes is described first on a single line.
The order they are specified is: x-axis, y-axis, and finally z-axis. The format of this line is:

\[ I2,8X,I2,8X,I2 \]

The minimum value for each axis is 02 while the maximum is 50 for the x and y-axes, and 30 for the z-axis.

5. The next set of lines contains the positions of the mesh planes normal to the x-axis. The positions are specified in kilometres. Each line except the last one in the set must contain ten values. The number of positions entered must match the number of mesh planes normal to the x-axis specified on the fourth line. Each line has the format:

\[ 10(F5.1,3X) \]

The values must be in strictly ascending order. The work in Chapter 3 established that WAFT only works correctly if the interval between successive mesh planes in any given axis direction is identical. One should therefore make sure that the difference between successive values is the same. The format of the data file was decided upon before this restriction was established; thus the only reason why all the mesh positions have to be specified is a historical one.

6. The next group of lines defines the positions of the mesh planes normal to the y-axis. These are specified in the same way as those normal to the x-axis.

7. The positions of the planes normal to the z-axis is entered next. The format is similar to that described for the other axes. However, the z-axis positions are specified in metres and negative
values are not allowed. The format of each line is slightly different being:

10(F6.1,2X)

8. The topography height in metres at each (x,y) location follows. The heights at increasing x-coordinates along each plane normal to the y-axis are given in turn. The values for each new y-coordinate position start on a new line. The format is as for the positions of the z-axis mesh planes described above.

9. A single digit in column one on the next line determines whether a slip or no-slip condition is assumed at the topography surface. A value of zero indicates no-slip whereas one means slip is allowed. Two other values must be supplied on this line if the stable flow version of WAFT is being used (they are ignored by the neutral flow version). The first specifies the 'high lying factor', $f_{HL}$, used to split the domain into subregions. Permissible values are 0.01 through 0.99. The second value determines how the top domain boundary is treated. It may either be zero which indicates no-flow at the top boundary, or unity which means flow is allowed through the top boundary. The format for the line is:

$\text{I1}, \text{9X,F4.2,6X,I1}$

10. The number of surface measurements to be used is specified on the next line. This is the beginning of the meteorological data. Values between 01 and 50 are allowed. The two digit number starts in column one.

11. The surface wind data is then described, using a new line for each measurement. The following
information is supplied for each surface measurement:

a. station name; up to ten alphanumeric characters

b. x-position in kilometres

c. y-position in kilometres

d. height above Sea Level in metres

e. x-component of velocity in m/s

f. y-component of velocity in m/s

g. station weighting (not used currently)

The format for each line is:

A10,F5.1,5X,F5.1,5X,F6.1,4X,F6.2,4X,F6.2,4X,E7.1

12. The next set of lines define the vertical thermal structure. There are ten values per line (except for the last line in the set); one value for each plane normal to the z-axis. The format is

10(E7.1,1X)

In the neutral flow version of WAFT the actual values of the vertical velocity weighting factor should be supplied. Normally, the value would either be unity for neutral flow, or a very high value such as $10^6$ to suppress vertical motion completely. These very high values model an inversion at the grid levels where they are applied. In the stable flow version of WAFT the values supplied should be the squares of the Brunt-Väisälä frequency (see subsection 4.3.2) for each horizontal mesh.
plane. However, if a no-flow condition has been selected for the top of the domain (see point 9 above) then the value supplied for the top horizontal mesh plane should be a vertical velocity weighting factor set to $10^6$ to suppress vertical motion there.

13. The high level wind is described in this line. Three values are supplied: the windspeed in m/s; the wind direction in degrees; and the altitude in metres at which the high level wind occurs. The wind direction is defined as the clockwise angle between the positive y-axis and the high level wind vector. The format of this line is:

\[ F4.1,6X,F5.1,5X,F7.1 \]

14. The next line marks the start of data controlling the Successive-Over-Relaxation (SOR) numerical solution method. These values are ignored if the Conjugate Gradient solution technique is used. The first value supplied is the over-relaxation parameter itself. This is a real number (format F4.2 starting in column one) between 1.00 and 1.99; see subsection 2.3.3.

15. A limit on the permissible size of any Lagrangian Multiplier during the iterative process is defined on the next line. This had improved the convergence of the two-dimensional model but has not proved useful in the three-dimensional code. It is therefore set to an extremely high value ($10^{40}$, say) so that it plays no role in the iterative process. The required format is E8.2 beginning in column one.

16. The next line determines when the iterative process will be terminated. The iterative process is deemed complete when the percentage change in all the Lagrangian Multipliers in a particular iteration
step is less than this value. The value is entered in E8.2 format starting in column one. A value of 0.10E+01 usually produces acceptably accurate final windfields (see subsection 2.3.3).

17. The final data value places a limit on the number of iterative steps which may be performed. This prevents huge amounts of computer resources being used if the SOR method is converging particularly slowly. The three digit integer should start in column one.

A.1.4 THE WAFT CODE STRUCTURE

This section presents a brief overview of the code structure. The subroutine calling structure is shown in Figure A.1. The main program calls six major subroutines; their names and functionality are as follows:

GRIDIN reads the input data describing the calculational mesh and topography surface.

CELTYP characterises each cell as either submerged, regular or irregular (these terms are explained in subsection 2.2.1). This information is stored in coded form so that subsequent code can quickly identify a cell's type. In the stable flow version of WAFT this routine is also responsible for dividing the topography up into the various subregions according to the 'high lying factor' (as described in subsection 4.3.8).

METDAT reads in the meteorological data; that is, the surface wind measurements, vertical thermal structure, and high level wind specification.

INITV generates the initial windfield. It does this in two stages. Firstly a surface two-dimensional
interpolation scheme and an assumed vertical profile are used to define an initial velocity vector at each unsubmerged mesh point. Secondly, simple averaging schemes are then used to define this initial windfield in terms of the flow average velocity travelling across each cell face. This requires the computation of the areas of the open portion of the cell faces. These areas are difficult to calculate for irregular cells and are also required during the iterative process; hence, their values are stored in an array at this stage. Finally, the net initial flow from each cell is computed. This is the right hand side vector in equation 2.3.1(1).

LAGRAN performs the Successive-Over-Relaxation iterative solution method. It first reads in the parameters controlling the iterative process from the data file. The Lagrangian Multiplier for every cell is then set to zero; this represents the initial solution estimate. The iterative process then proceeds until the Lagrangian Multipliers reach a prescribed accuracy. Most of the coefficients in matrix $A$ (see subsection 2.3.1) are easily calculable; the difficult ones associated with irregular cells have already been generated in INITV. Hence, the solution loop is reasonably efficient.

FINALV generates the final windfield. Again, this is done in two stages. Firstly, the final windfield formulated in terms of flow average velocities is constructed from the initial windfield and the Lagrangian Multiplier solution vector. This is done using the equations such as those given in 2.2.1(2) and 2.2.2(10). The final net flow into each cell is also computed here to check that the program is internally consistent, and that therefore, the final windfield is indeed mass consistent. Finally, simple interpolation methods are used to construct
the final velocity vectors defined at the mesh points.

The above structure pertains to early WAFT versions. They use the SOR iterative technique. Theoretically, the later versions which use Conjugate Gradient methods could just have a different version of the subroutine LAGRAN. However, the extra arrays required to implement this more sophisticated solution technique mean that the whole program would then exceed the very limited central memory available on the CDC system. It is therefore necessary to break WAFT into three, smaller, programs. The first one just calls GRIDIN, METDAT and INITV; it then writes values stored in arrays to a disc file. A new program which implements the Conjugate Gradient technique is then used. This replaces the subroutine LAGRAN in the earlier WAFT versions. This new program first reads in the disc file containing array values produced in the first part of WAFT. It then performs the iterative solution and ends by again writing the array values (which now include the Lagrangian Multiplier solution vector) to another intermediate disc file. Finally, the second part of WAFT (which just calls FINALV) reads this second intermediate disc file, and then generates the final windfield as before.
Fig A.1 - The WAFT code subroutine calling structure
This appendix uses an argument based upon a single cell to indicate the possible relationship between the size of the vertical velocity weighting factors selected and the magnitude of the vertical velocities obtained. The argument is not intended to be particularly rigorous; some guidance as to the nature of the relationship is all that is required. Indeed, since WAFT models the flow in a large assembly of cells, one would not expect an analysis based upon an isolated cell to provide a complete picture. Nevertheless, this analysis does provide an accurate insight into the effects of the weighting factors. The relationships obtained here are verified by the experiments with WAFT described in subsection 4.3.4.

Consider the single cell shown in Figure A.2. The topography surface intersects this cell in such a way that the bottom cell face and two of the side faces are completely blocked. The topography intersects the other three faces at their diagonals. Hence, these faces each have an open portion which is half their total area. The cell has unit length and width (in some arbitrary units) and a height 'm'. Hence, the slope of the topography is of order m too.

The situation in the initial windfield is shown in Figure A.2(a). The initial windspeed is $u_0$ in the positive x-direction. One could envisage such a cell lying off the axis centre on the upwind side of a simple hill, for example. The situation in the final windfield is depicted in Figure A.2(b). Note that the assumption is made that no adjustment has been made to the velocity in the x-axis direction. However, in order to obey the mass consistency constraint, the adjustment process has created new velocities through the top face and the side face normal to the y-axis.
The cell weighting factor for each face is equal to 0.5 and so this constant factor may be neglected. There is no adjustment made to the x-axis velocity, the adjustment to the y-axis velocity is \( v \), and \( w \) for the vertical velocity. Thus the general minimisation sum given in 2.2.2(1) written for this isolated cell is

\[
v^2 + \beta^* w^2 \quad \text{A.2(1)}
\]

The condition of no net flow into the cell in the final windfield is expressed by

\[
\frac{m_i u_i}{2} = \frac{m_i v}{2} + \frac{1}{2} \frac{m_i w}{2} \quad \text{A.2(2)}
\]

which may be rearranged to give a relationship for \( v \)

\[
v = \frac{1}{m} (m_i u_i - w) \quad \text{A.2(3)}
\]

Substituting this relationship for \( v \) into A.2(1), the minimisation sum becomes

\[
\frac{1}{m^2} (m_i u_i - w)^2 + \frac{\beta^*}{m^2} w^2 \quad \text{A.2(4)}
\]

Differentiating with respect to \( w \) gives, for minimisation:

\[
\frac{-2}{m^2} (m_i u_i - w) + 2 \frac{\beta^*}{m^2} w = 0 \quad \text{A.2(5)}
\]

or

\[
w \cdot (\beta^* + \frac{1}{m^2}) = \frac{1}{m} u_i \quad \text{A.2(6)}
\]

which gives the following for the ratio \( w/u_i \).
\[ w = \frac{m}{u_o \left(1 + \beta^* m^2\right)} \]  
\hspace{1cm} \text{A.2(7)}

Hence, there are two regimes depending upon the size of \( \beta^* m^2 \):

\[ w \sim m \quad \text{if} \quad \beta^* m^2 \ll 1 \]  
\hspace{1cm} \text{A.2(8)}

and

\[ w \sim \frac{1}{u_o \beta^*} \quad \text{if} \quad \beta^* m^2 \gg 1 \]  
\hspace{1cm} \text{A.2(9)}

The significance of these two regimes is discussed in subsection 4.3.3.
Fig A.2 - Hypothetical WAFT cell used to determine relationship between vertical velocity weighting factors and size of vertical velocities.