Spectral structure and linear mechanisms in a rapidly distorted boundary layer

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A B S T R A C T

The aim of the present work is to investigate the spectral structure of a rapidly distorted boundary layer that develops on a flat plate in presence of a localised patch of roughness or/and grid-generated freestream turbulence. We observe that, at a certain distance downstream of the roughness patch the boundary layer exhibits a bimodal shape in the energy spectrum of the streamwise velocity fluctuations, similar to that found in a fully-turbulent boundary layer at relatively high Reynolds numbers. The physical mechanism that gives rise to the low-wavenumber peak in the spectrum, which represents long streamwise motions or "superstructures", is identified to be the interaction of the broadband disturbances with the region of high shear near the wall in the boundary layer. We next show that the flat-plate boundary layer combined with surface roughness and grid turbulence can serve as building-block elements towards synthesising the wall-normal structure of a canonical turbulent boundary layer, in the context of large-scale streamwise motions. The rapidly distorted (or "synthetic") boundary layer presents a simpler environment in which the coherent motions can evolve and therefore can enable a better characterisation of these motions. To further illustrate the utility of the present approach we compare results from our measurements with the predictions of the Rapid Distortion Theory (RDT). We show that the streamwise turbulence energy in the near-wall region of the rapidly distorted boundary layer grows linearly with time consistent with the RDT results on the effect of pure shear on an initially isotropic turbulence. Moreover close to the edge of the boundary layer the large-scale fluctuations experience an enhancement in the streamwise turbulence energy in accordance with the linear blocking model in the RDT framework. The present work thus highlights the importance of linear processes in wall turbulence and can help us identify aspects of it to which the linear theories can be meaningfully applied.

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1. Introduction

One of the main themes of research on wall turbulence has been the structure of the wall-bounded flows, in both physical and spectral spaces. Since the work of Kim and Adrian (1999) it has been well recognised that a wall-bounded turbulent flow is primarily populated by two types of large coherent motion: the "Large Scale Motion" (LSM) and the "Very Large Scale Motion" (VLSM) in pipes and channels or "superstructures" in the boundary layer. The streamwise extent of the LSMs is of the order of 2 – 3δ whereas that of VLSMs/superstructures is about 6 – 10δ (Smits et al., 2011), where δ is the boundary-layer thickness, pipe radius or channel half-height. These motions appear in the pre-multiplied spectrum of the streamwise velocity fluctuations in the form of distinct humps in appropriate wavenumber ranges (see Kim and Adrian, 1999; Hutchins and Marusic, 2007). As the Reynolds number increases, the turbulence energy contained in VLSMs/superstructures is seen to progressively increase, and therefore these long streamwise structures are expected to play a prominent role at very high Reynolds numbers typical of those found in technological applications. As a result, a considerable effort has been invested in developing laboratory facilities that can achieve very high Reynolds numbers (see, for example, Hultmark et al., 2012; Mathis et al., 2009). Such efforts have provided important insights into the scaling and structure of wall turbulence and hold promising prospects for the future. However such facilities are highly specialised, and therefore alternative approaches for studying certain aspects of the coherent motion in high Reynolds number wall turbulence are worth pursuing. In this work we use such
an approach and present findings from an experiment on a boundary layer developing over a flat plate in presence of a localised patch of surface roughness and/or grid-generated freestream turbulence. We show that the streamwise structures generated in such a boundary layer mimic those found in the fully-turbulent boundary layers, and therefore the present experimental design can help us better understand spectral properties of the large coherent motions in wall turbulence.

1.1. Effect of surface roughness and grid turbulence

Surface roughness has been used in the past to study the effect of change in the surface condition on the near-wall structure in wall-rounded turbulent flows. Extensive work has been done towards understanding the structure of a turbulent boundary layer (TBL) developing over a rough surface - see, for example, Townsend (1976), Krogstad and Antonia (1994) and Jiménez (2004) among others. Experiments have also been carried out for the case of an abrupt transition in surface condition in the form of either rough-to-smooth (R-S) or smooth-to-rough (S-R) or smooth-rough-smooth (S-R-S) transition. Some of the early investigations on the response of a TBL subjected to such changes in surface condition have been summarised in Smiths and Wood (1985). It was realised that the effect of a surface perturbation on the boundary layer structure could be effectively described in terms of a new internal boundary layer which develops downstream of the perturbation. Furthermore, it was observed that the local-equilibrium hypothesis (Townsend, 1976) was no longer valid within the internal layer, wherein the effects of advection and diffusion of turbulence energy could not be neglected (Smiths and Wood, 1985). In this context, Morrison et al. (1992) investigated the non-equilibrium effects of an R-S surface transition on the bursting mechanisms associated with the near-wall streaks in a TBL.

In the relatively recent studies, surface roughness has been used to better understand the inner/outer interaction in wall turbulence. For example Birch and Morrison (2011) performed measurements on an extended region of distributed surface roughness in a channel and showed that the type of roughness used can have a significant influence on the self-similar nature of the mean velocity profile. With regard to the change in surface condition, Morrison (2007) showed that the flow downstream of an R-S transition experiences a strong inner/outer interaction between the internal boundary layer and the turbulence induced by the roughness. In this connection, Hanson and Ganapathisubramanyi (2016) have further investigated the effect of roughness-generated disturbances on the R-S transition layer in a TBL and highlighted the importance of the outer region in determining the near-wall scaling of the streamwise turbulence intensity.

In the present work, we carry out measurements downstream of a localised patch of distributed grit roughness (akin to the S-R-S transition) with oncoming boundary layer in a laminar state. We observe that, at a certain distance from the roughness patch, the pre-multiplied power spectrum (or energy spectrum) of the streamwise velocity fluctuations exhibits a bimodal shape that resembles that found in a high-Reynolds number TBL. The new internal shear layer downstream of the roughness patch appears to be the key factor in generating the low-wavenumber spectral peak, which represents the long streamwise motions.

The interaction of a grid-generated freestream turbulence field with a TBL has been a topic of many investigations, especially to understand the effect of a modified outer layer on the structure of the near-wall turbulence (Hancock and Bradshaw, 1989; Sharp et al., 2009). Dogan et al. (2016) showed that the large outer-layer structures in such a boundary layer have a modulating effect on the small scales near the wall and that this effect increases with increase in the freestream turbulence intensity. “Bypass transition” has also been studied extensively, where the grid-generated turbulence interacts with a laminar boundary layer generating streamwise Klebanoff disturbances comprising streaks and vortices - see the review by Durbin and Wu (2007) and the references therein. The main thrust of the bypass-transition studies is to understand the penetration of the freestream disturbances into the boundary layer to form streamwise streaks, their subsequent amplification and final breakdown into turbulent spots (Zaki, 2013).

Here we study the interaction of grid-generated turbulence with a laminar boundary layer with or without the presence of a roughness patch. Over a certain distance close to the plate leading edge, the grid/roughness-generated turbulence is rapidly distorted (due to the effects of shear and blocking). We therefore term this boundary layer as a “rapidly distorted” boundary layer. Although the present arrangement is similar to that used in bypass-transition studies, our main focus here is to look at the spectral features of the rapidly distorted boundary layer as it evolves downstream, and which are relevant to a canonical TBL. We find that the most energetic streamwise wavenumber (scaled on δ) for the boundary layer subjected to grid turbulence alone is in close agreement with the low-wavenumber spectral peak seen in the boundary layer downstream of the roughness patch (in absence of grid turbulence). This suggests a possible physical mechanism, in the form of the interaction of broadband disturbances with the near-wall shear, for generating the long streamwise structures or superstructures observed in high Reynolds number turbulent boundary layers. We also investigate the wall-normal variation of the energy spectra in the rapidly distorted boundary layer and show that it compares favourably with that found in a canonical TBL. This suggests that it may be possible to synthesise the spectral structure of a canonical TBL using a combination of freestream turbulence and a localised patch of surface roughness along with a flat plate boundary layer.

1.2. Rapid Distortion Theory

The principal assumption of Rapid Distortion Theory (RDT) is that there should be sufficient separation between two timescales: one associated with the imposed mean-flow distortion (tD) and the other a time interval over which the turbulence field evolves through non-linear interactions (tN). When tD < tN the non-linear terms in the governing equation for turbulent fluctuations can be neglected, and therefore the “rapid” distortion of turbulent eddies can be described by a set of linearised equations (Hunt and Carruthers, 1990). One of the first investigations on the RDT was due to Batchelor and Proudman (1954) who considered the problem of turbulence subjected to large irrotational strain rates such as grid-generated turbulence passing through a rapid contraction. Since their work, the RDT has been used in a variety of flow situations such as a wake subjected to a rapid change in pressure gradient (Narasimha and Prabhu, 1972) or a wind flow past buildings (Bearman, 1972), among others. In the context of boundary layers developing under zero pressure gradient, the distortions primarily take place in the form of “shearing” due to the large velocity gradients present in the near-wall region and “blocking” due to the no-penetration condition imposed by the wall. In the present case of a rapidly distorted boundary layer it is reasonable to expect that the action of shear and blocking would be sufficiently rapid in a region close to the leading edge of the plate, and we show here that the predictions of the RDT are indeed applicable to this region.

There have been many studies in the literature that have considered the effect of homogeneous shear on an initial disturbance field. One of the main outcomes of such studies is the evolution of statistical quantities, such as turbulence intensity, two-point correlations and power spectra, with time (t) scaled on the local shear rate, S. Moffatt (1967) showed that, for an initial isotropic distur-
bance field, the total disturbance kinetic energy under the action of uniform shear (in the absence of blocking) increases linearly with time as \( t \to \infty \). He also found that from an initial field of random disturbances, the eddies which are cylindrical in shape with their axes aligned in the direction of the flow (akin to the near-wall quasi-streamwise vortices in a TBL) make a dominant contribution to the disturbance energy and Reynolds stresses. Townsend (1976) made extensive use of the results from the RDT towards understanding the structure of the near-wall turbulence; in this case the rapid time scale is provided by the wall shear and the large-eddy turnover time acts as a slow time scale. Townsend performed a detailed comparison of the two-point correlation functions measured in a TBL (and a wake) with those calculated from the RDT and showed a reasonably good agreement between the two. In an interesting numerical study Lee et al. (1990) subjected a homogeneous isotropic turbulence field to high rates of shear and showed, by solving the instantaneous equations of the RDT, that the resulting flow field consisted of long streaky structures similar to those found in the buffer layer of a TBL. More recently the RDT has been used to investigate the structure of a high-Reynolds-number TBL, such as the atmospheric surface layer, in which “top-down” interactions are believed to dominate the “bottom-up” interactions which prevail at low Reynolds numbers (Hunt and Morrison, 2000; Hunt and Carlotti, 2001).

It is also of interest to investigate the linearised effect of blocking on the turbulence field. The term blocking refers to the attenuation of the wall-normal component of velocity associated with an eddy as it approaches a wall, due to the impermeability constraint at the wall. In an early work on the effect of blocking, Hunt and Graham (1978) developed a theory to study the effect of a plane boundary on freestream turbulence in absence of mean shear and compared the predictions with the experimental results on a “shearless” boundary layer obtained on a moving ground plane by Thomas and Hancock (1977), Hunt and Graham (1978) further argued that their theoretical results could also be relevant to the freestream turbulence in presence of a flat-plate boundary layer, especially for eddies having length scales sufficiently large compared to the boundary layer thickness (see Section 4.2). A numerical simulation of shear-free turbulent boundary layers was carried out by Perot and Moin (1995), which revealed the kind of turbulent structures (viz., “splits” and “anti-splits”) that are typically found in a turbulence field affected by blocking. In this connection Hunt and Carlotti (2001) showed that the effect of pure inviscid blocking at a given wall-normal location, \( y \), is to enhance the streamwise (and spanwise) component of the turbulence energy for wavenumbers given by \( k < 1/\gamma \) and to leave the high-wavenumber part (i.e. for \( k > 1/\gamma \) of the spectrum unaffected. The enhancement in the wall-parallel components of motion takes place at the expense of the wall-normal motion which is inhibited for \( k < 1/\gamma \) (Hunt and Carlotti, 2001). The principal mechanism for this inter-component transfer of energy is believed to be the pressure-strain correlation. However, the additional viscous effects such as dissipation and diffusion could also play an equally important role especially near the wall (Perot and Moin, 1995).

Here we examine the effects of shear and blocking on the turbulence generated by a grid and show that our results are broadly consistent with the RDT predictions.

The paper is organised as follows. Section 2 describes the experimental setup and measurement technique used. The results, primarily in the form of energy spectra, are presented in Section 3, along with the discussion of the physical mechanism that is responsible for their observed shapes. Section 4 deals with the relevance of the RDT predictions to the rapidly distorted boundary layer. The concluding remarks are presented in Section 5.

### 2. Experimental arrangement

The experiments were performed in a closed circuit wind tunnel, 457 mm \( \times \) 457 mm in cross section. An aluminium flat plate 454 mm wide, 10 mm thick and 760 mm long was installed in the wind tunnel test section. The plate has a sharp leading edge and a flap at its trailing edge which could be tilted upward to ensure that the front stagnation point was positioned on the measurement surface (Fig. 1). The flat plate itself was tilted at 2° with respect to the tunnel centreline (nose up) to achieve nominally zero pressure gradient, to within ±0.4% in terms of the freestream-velocity variation over the entire length of the plate.

Commercially available grit roughness sheets of two different grades, P120 and P80, were used as localised surface roughness elements. The two roughness strips were glued on to the measurement surface adjacent to each other, covering the entire span of the plate, with the upstream edge of the patch (P120) at 16 mm from the plate leading edge (Fig. 1). The total streamwise extent of the roughness patch was 40 mm (20 mm each for the two strips), and the total height of the patch was about 0.5 mm for the P120 strip and 0.7 mm for the P80 strip. The boundary layer thickness at the leading edge of the roughness (say \( \delta_\nu \)) can be estimated using the standard relation for the Blasius boundary layer, i.e. \( \delta_\nu / x = 4.9/ \sqrt{Re_x} \), where \( Re_x = U_\infty x / \nu \). Here \( U_\infty \) is the freestream velocity and \( \nu \) the kinematic viscosity. Using \( U_\infty = 17.6 \text{ m/s} \) (for the case of roughness alone) we find \( \delta_\nu \approx 0.6 \text{ mm} \). Thus the streamwise roughness length is about \( 67\delta_\nu \) while the roughness height is comparable to \( \delta_\nu \).

Freestream turbulence was generated by inserting a rectangular bi-planar grid into the test section at the end of the contraction section. The grid is made up of 3 mm square aluminium rods fastened together with the horizontal and vertical spacing between the rods, \( M = 16 \text{ mm} \), resulting in a solidity of 34%. The grid was positioned 625 mm upstream of the flat-plate leading edge, which is about 39M, ensuring that the freestream turbulence passing over the plate is nominally homogeneous in cross-stream directions and nearly isotropic (Corrsin, 1963).

Hot-wire measurements were carried out using the Streamline Pro Constant Temperature Anemometer from Dantec Dynamics Ltd. and using a single normal hot-wire probe for measuring the streamwise velocity. For measurements in the rapidly distorted boundary layers, a tungsten-wire probe with 5 \( \mu \text{m} \) diameter and 1.2 mm active length was used. Measurements were also taken in a fully-turbulent boundary layer for comparison, for which a platinum-Wollaston wire with 2.5 \( \mu \text{m} \) diameter and 0.5 mm active length was used to ensure that the length of the wire in wall units, \( l^+ < 20 \). The hot-wire voltages were corrected for ambient temperature variations using the standard square-root correction formula (Bruun, 1995). The hot-wire probe was calibrated against a Pitot-static tube mounted on the ceiling upstream of the plate. For the boundary layer measurements the acquisition rate for the hot-wire
signals was fixed at 40 kHz whereas the sampling duration was chosen to be either 45 s or 60 s depending on the flow condition. The sampling duration is of the order $10^3$ in terms of the eddy turnover time, estimated as $\delta/\nu_{\infty}$ (4.5 x $10^{-4}$ s), for the boundary layer at $x = 600$ mm developing in presence of the roughness patch alone.

The origin of the co-ordinate system is located at centre of the plate leading edge. $x$, $y$, $z$ are the streamwise, wall-normal and spanwise directions respectively. $U$ and $u$ are the time-averaged and fluctuating streamwise velocities respectively. The hot-wire signals were filtered using a band-pass Butterworth filter between frequencies of 1 Hz and 30 kHz. The high-pass filter (at 1 Hz) was used to remove the low frequencies introduced by the wind tunnel fan. The high-pass cut-off frequency is much lower than those associated with the typical turbulence time scales. Furthermore, it was ensured that there was no significant aliasing effect in the frequency range of interest due to the low-pass cut-off frequency (30 kHz) being higher than the Nyquist frequency (20 kHz). Using the band-pass filtered signals, the streamwise turbulence intensity in the freestream at the plate leading edge was calculated to be about 0.2% without the grid and 2.75% with the grid inserted. The power-spectral density estimates for the streamwise velocity fluctuations are obtained using the Welch modified periodogram method with Hamming window and 50% overlap.

To check the standard conditions for grid turbulence, a streamwise traverse was carried out in the freestream above the plate. Fig. 2 shows the evolution in the streamwise turbulence kinetic energy with respect to the distance from the grid centre-plane, $X$ (= $x + 625$ mm). This variation is well represented by a power-law fit (shown as a solid line in Fig. 2) given by Eq. (1),

$$
\frac{U^2}{U_{rms}^2} = 16.57 \left( \frac{X}{M} - 4 \right)^{1.244}.
$$

The index and coefficient of the power-law fit (along with the offset to account for the virtual origin) fall well within the range given in Comte-Bellot and Corrsin (1966) indicating the presence of a nearly-isotropic regime.

### 3. Results

In this section we explore the spectral characteristics of the rapidly distorted boundary layer. We first present results for the cases of the roughness patch and grid turbulence applied separately, and then discuss the case where they are used together.

#### 3.1. Measurements in presence of roughness patch

Fig. 3 shows the mean velocity and streamwise intensity profiles for two representative measuring stations in the boundary layer downstream of the roughness patch. For comparison, the profiles for a canonical TBL at $Re_T = 861$ are shown. The TBL profiles were obtained in a separate experiment conducted in the same wind tunnel. Here $Re_T = Ut \delta/\nu$ is the friction Reynolds number, $u_T$ is the friction velocity ($u_T^2 = \nu (\partial U/\partial y)_{y=0}$) and $\delta$ the 99% boundary layer thickness. For the TBL, $u_T$ was determined using the Clauser chart method assuming a standard log-law region ($U^+ = (1/x) \ln y^+ + C$), and using $x = 0.41$, $C = 5$; here superscript + indicates normalisation with wall units. For the rapidly distorted boundary layers $u_T$ was estimated using the linearity of the mean velocity profile near the wall. For locations close to the plate leading edge, there is an extended region of linearity in the mean velocity profiles close to the wall and therefore such an estimate is presumed to be quite accurate (e.g. $x = 150$ mm; Fig. 3). As the boundary layer grows downstream, the wall-normal extent of the linearity shrinks and therefore the uncertainty in the $u_T$ estimates increases to about 3 – 4%. However this level of accuracy was considered to be sufficient for the present purposes.

It is evident from Fig. 3 that the rapidly distorted boundary layer is transitional, and even at the farthest measurement location ($x = 600$ mm) the boundary layer is not fully turbulent. Interestingly, the peak in the streamwise turbulence intensity scaled on $u_T$ matches reasonably well for all the three profiles. It must be noted that for the boundary layer at $x = 600$ mm the conditions of rapid distortion are not strictly valid. However we shall continue to refer to it as the rapidly distorted boundary layer for the sake of convenience.

Fig. 4a shows the energy spectra of streamwise velocity fluctuations ($k\Phi_{uu}$) at $x = 600$ mm. Here the streamwise wavenumber, $k$, is determined as $k = 2\pi f/\bar{U}$, where $f$ is frequency and $U$ is the local mean velocity. The spectra at $x = 600$ mm show a single broad hump centred around the wavenumber in the range of $k\delta = 2 – 4$. These spectra compare favourably with those found in a low- Reynolds number canonical TBL (Fig. 4b) both in terms of the shape and the energy content. The streamwise wavelengths for the most energetic motions for both the flows fall in the range (1.25 – 3)$\delta$, which is typical of the LSMs.

It is interesting to see how the shapes of the spectra evolve as we approach $x = 600$ mm starting from the roughness patch and this is shown in Fig. 5 at $y/\delta \approx 0.5$ (a) and $y/\delta \approx 0.8$ (b). The
Fig. 3. Variation of the streamwise mean velocity and intensity profiles for flow over a roughness patch, compared with a canonical TBL. Open and filled symbols represent the mean velocity and turbulence intensity respectively.

Fig. 4. Energy spectra of the streamwise fluctuating velocity for (a) boundary layer in presence of roughness patch, $x = 600$ mm, $Re_\tau = 457$. (b) canonical TBL, $Re_\tau = 861$. The arrow indicates the direction of increasing wall-normal distance, $y$.

Fig. 5. Energy spectra at three different streamwise locations downstream of the roughness patch at (a) $y/\delta \approx 0.5$ (b) $y/\delta \approx 0.8$. 
spectrum at $x = 150$ mm (and $y/\delta \approx 0.5$) has a unimodal shape similar to that at $x = 600$ mm, except that the former peaks at wavenumbers that are an order-of-magnitude lower as compared to the latter. At an intermediate location of $x = 300$ mm the spectrum shows a bimodal shape with peaks of comparable magnitude. A similar trend is also observed at $y/\delta \approx 0.8$ (Fig. 5b) except that in this case there is a plateau around $k\delta \approx 1$ for $x = 150$ mm. Now the bimodal shapes of the spectra as seen in Fig. 5 are typical of those found in the moderate to high Reynolds number TBLs (Hutchins and Marusic, 2007). The streamwise wavenumber corresponding to the low-wavenumber peak at $x = 300$ mm is $k\delta \approx 0.3$, which gives the streamwise length of the associated structures to be $20\delta$ approximately. This is somewhat higher than the values of $6 – 10\delta$ for the length of the superstructures found in a canonical TBL, although the shapes of the spectra are quite similar. A better agreement between the two cases can be obtained (as shown in Section 3.3) if we use the local wall-normal distance, $y$, instead of $\delta$ for scaling the wavenumber.

At first sight, one might find it surprising that a patch of grit roughness can give rise to the bimodal shape of the spectra downstream of it, as a randomly distributed roughness can be expected to excite broadband disturbances without introducing any preferred length scales in its close vicinity. This behaviour can be understood if we take into consideration the thickness of the oncoming boundary layer in relation to the height of the roughness patch. As estimated in Section 2 the boundary layer thickness at the leading edge of the roughness ($\approx 0.6$ mm) is of the same order as the height of the roughness patch (0.5 – 0.7 mm), and therefore the structure of the boundary layer can be expected to be significantly altered as it passes over the patch. This is shown schematically in Fig. 6.

A new internal boundary layer (shown by a thick dashed line in Fig. 6) develops downstream of the roughness patch and grows outward into the external boundary layer as it evolves downstream. The internal layer provides a region of enhanced shear, highlighted by single dashed hatching (Fig. 6), over a certain distance downstream of the roughness patch. The broadband disturbances introduced by the roughness can be expected to interact with (and be sheared by) the internal boundary layer to produce long streamwise streaky structures (Moffatt, 1967) and this would explain the low-wavenumber hump that is observed at $x = 150$ mm ($k\delta = 0.1 – 0.2$) in Fig. 5. A second and rather weak hump around $k\delta = 1$ seen at $y/\delta = 0.8$ at this streamwise location (Fig. 5b) can be attributed to the direct effect of the roughness-induced disturbances in exciting comparatively shorter streamwise structures. Now the effect of the broadband disturbance field can be felt throughout the boundary layer except for a certain distance near the wall (outlined by a thick dash-dot line in Fig. 6) where only long-wavelength disturbances can be present due to the effect of shear sheltering (Durbin and Wu, 2007). This could be the reason for the absence of the high-wavenumber hump at $y/\delta = 0.5$ for $x = 150$ mm (Fig. 5a). As we move downstream from $x = 150$ mm the energy content of the high-wavenumber hump increases as the action of the roughness-induced disturbances becomes more vigorous, and over a certain region, shown cross-hatched in Fig. 6, the magnitude of the high-wavenumber peak can become comparable to that of the low-wavenumber peak generated by shear interaction. This would correspond to the spectral shapes observed at $x = 300$ mm (Fig. 5). Further downstream, the intensity of the shear slowly diminishes and therefore the low-wavenumber hump can be expected to become progressively weaker. Beyond a certain distance, presumably after the new internal layer has grown completely into the external boundary layer, the low-wavenumber hump disappears due to weak shear and the spectrum assumes a unimodal shape centred around $k\delta \approx 2$ as seen at $x = 600$ mm (Fig. 5).

3.2. Measurements in presence of grid turbulence

The discussion in the preceding section suggests that the low-wavenumber hump seen in the energy spectra could be attributed to the interaction of the roughness-generated disturbances with the high rates of shear present in the internal boundary layer. To investigate this aspect further we carry out experiments on a boundary layer subjected to grid-generated freestream turbulence in the absence of surface roughness. The energy spectra for such a boundary layer at $x = 150$ mm are shown in Fig. 7. The spectra show a unimodal shape with a broad hump centred around $k\delta = 0.1 – 0.2$. This range of wavenumbers matches well with that observed for the case of the boundary layer in presence of a roughness patch discussed above (e.g. at $x = 150$ mm in Fig. 5).
servation reinforces our conclusion that the primary mechanism for the generation of the low-wavenumber peak in the bimodal shape of the spectra seen in Fig. 5 is the shearing of the broadband turbulent eddies by the near-wall region. An immediate implication is that it may be possible to understand the present results in the framework of RDT outlined in Section 1. This aspect is dealt with in some detail in Section 4.

3.3. Measurements in presence of both roughness patch and grid turbulence

In this section we explore the wall-normal structure of the energy spectra in a rapidly distorted boundary layer in presence of the roughness patch and grid turbulence. The streamwise evolution of the spectra for this flow (not shown here) in the region close to the roughness patch is similar to that in the boundary layer subjected to roughness alone (Fig. 5). The motivation for introducing grid turbulence is that it enables us to study the blocking of large-scale freestream disturbances near the boundary-layer edge (where shear is weak/absent) due to the presence of the wall.

Fig. 8 shows a comparison of the spectra, for three sets of wall-normal locations, between the rapidly distorted boundary layer under consideration and a canonical TBL at a moderately-high Reynolds number \(Re = 2363\) measured in a different facility. Note that the wavenumber axis in Fig. 8 is now scaled with respect to the local wall-normal distance, \(y\), instead of the boundary layer thickness, \(\delta\). The right-hand panels in the figure show the changes that the spectral shape of the canonical TBL undergo as we move away from the wall. In the buffer layer (Fig. 8-a2) there is a single spectral hump with a second hump (at the low-wavenumber end) seen emerging close to the edge of the layer. The bimodal shape of the spectra is clearly evident in the logarithmic layer (Fig. 8-b2) wherein both the humps are of comparable magnitudes. This is consistent with the spectra reported in the literature at similar Reynolds numbers (see for example Hutchins and Marusic, 2007). As we move into the wake region the spectral shape again becomes unimodal (Fig. 8-c2), peaking at higher wavenumbers due to the scaling by the local wall-normal distance.

A similar variation of the spectral shape is also observed for the rapidly distorted boundary layer and we can identify regions in this flow (shown in the left-hand panels in Fig. 8) wherein the shapes of the spectra are qualitatively similar to those in the buffer, log and wake regions of the canonical TBL. More significantly, the streamwise wavenumbers (scaled on \(y\)) and magnitudes (scaled on \(u^2\)) of the spectral peaks for the two flows compare favourably, especially in the buffer and log regions of the TBL (Fig. 8-a1,a2 and b1,b2). In this sense almost the entire thickness of the rapidly distorted boundary layer corresponds to the wall layer of the TBL, and the region in the freestream adjacent to its edge appears to mimic the wake region of the TBL (although not as convincingly as the wall layer). This suggests that there is a close correspondence between the two flows in terms of the structure of the streamwise energy spectra, and this is depicted schematically in Fig. 9.

The similarity of the spectral modes between the two flows implies that the rapidly distorted boundary layer is populated by coherent motions which resemble the LSMs and superstructures found in the wall region of a canonical TBL. It is worth noting that the rapidly distorted boundary layer is far from an equilibrium turbulence state (such as involving local production-dissipation balance or the equilibrium energy cascade), and therefore the complex interactions of eddies present in a canonical TBL are not expected to be present in this flow. Thus the rapidly distorted boundary layer presents a much simpler environment in which the coherent motions can evolve.

The preceding discussion leads us to a conclusion that the rapidly distorted boundary layer has the essential primary elements that give rise to the spectral shapes and associated motions typical of a canonical TBL. These building-block elements could be identified as the near-wall region of the boundary layer as a source of shear, the grid-generated turbulence as a source of large-scale external (or ‘outer-region’) disturbances, and the localised roughness patch which introduces broadband disturbances near the wall and enhances the relatively high-wavenumber part of the spectrum. We have already seen that the broadband disturbances are sheared in the near-wall region to generate long streamwise structures. At the same time the disturbances are also blocked due to the impermeability constraint at the wall, which is experienced at distances from the wall smaller than the length scale of the disturbance (Section 1.2). The effect of blocking can be expected to be prominent close to the edge of a boundary layer where the effect of mean shear is negligible. This would be the case particularly in a region in the freestream immediately outside the edge of the boundary layer and for disturbances whose length scales are sufficiently large compared to the boundary-layer thickness. Now this part of the rapidly distorted boundary layer has been shown to be analogous to the wake region of a canonical TBL (Fig. 8-c1,c2) wherein the shear, although not absent, is weak. This suggests that large-scale eddies that are primarily blocked by the wall but are only marginally affected by shear can be expected to be present in the wake region of a TBL (presumably along with other types of eddies). Table 1 summarises the aspects of a rapidly distorted boundary layer that are qualitatively similar to a canonical TBL.

The implication of the present findings is that it may be possible to synthesise the spectral structure of the streamwise velocity fluctuations in a canonical TBL using the building blocks listed in Table 1. Therefore the rapidly distorted boundary layer studied here could be termed a “synthetic” boundary layer. The advantage of such a flow configuration is that the principal elements can be independently manipulated (such as the roughness height, grid solidity etc.) to generate coherent motions of required scale and strength, and this might help us better understand the origin of such motions in a canonical TBL. The present experimental design, which identifies essential building blocks and incorporates them in a simple experimental setup, thus represents an alternative approach for studying the structure of wall-bounded turbulent flows.

4. Considerations from Rapid Distortion Theory

It is clear from the discussion in the preceding sections that the results from the rapidly distorted boundary layer could be compared with the predictions of RDT. In this section we examine the effect of shear and blocking on turbulent fluctuations in the RDT framework. We choose the case of the boundary layer subjected to grid-generated turbulence in absence of surface roughness (Section 3.2), partly because of the simplicity of the flow configuration and partly because more extensive measurements on the freestream turbulence field are available for this case.

4.1. Effect of shear

The effect of shear can be expected to be important over a certain distance close to the leading edge of the plate where the boundary layer is relatively thin. To check if the conditions for the RDT are met by the boundary layer subjected to grid turbulence, we calculate the shear \(\left(\frac{\partial u}{\partial y}\right)_w\) and nonlinear \(\left(\frac{\partial u}{\partial y}\right)_s\) interaction time scales for a streamwise location sufficiently close to the plate leading edge, \(x = 150\) mm. The shear interaction time scale given by \(\tau_s = 1/(\partial u/\partial y)_w = 1/u'\delta u'/\delta y\) is \(\tau_s = 1.5 \times 10^{-6}\) s (subscript ‘w’ indicating wall). The nonlinear time scale is taken as the integral time scale \(\tau_l\) of the freestream turbulence sufficiently away from
Fig. 8. Comparison of energy spectra between the rapidly distorted boundary layer at \( x = 300 \) mm in presence of roughness and grid turbulence (\( a_1, b_1, c_1 \)), and a canonical TBL (\( a_2, b_2, c_2 \)) at \( Re_\tau = 2363 \). The arrow indicates the direction of increasing \( y \).

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Principal features of a rapidly distorted boundary layer compared to those of a canonical TBL</th>
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<td>Rapidly distorted boundary layer</td>
<td>Canonical TBL</td>
</tr>
<tr>
<td>A thin boundary layer (that rapidly shears large-scale disturbances to generate long structures away from the wall)</td>
<td>Buffer and log regions of the TBL, with the latter populated by superstructure-type motions</td>
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<tr>
<td>Grid (and roughness) generated turbulence blocked by the presence of wall</td>
<td>A class of large eddies in the Wake region near the TBL edge</td>
</tr>
<tr>
<td>Localised patch of roughness to excite shorter structures in the boundary layer</td>
<td>Relatively high-wavenumber part of the turbulence spectrum (typical of LSMS)</td>
</tr>
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</table>
the plate \(y = \text{90 mm}, y/\delta = \text{34.4}\) and is calculated to be \(t_n = t_s = 1.8 \times 10^{-3}\) s. Thus \(t_n/t_s = 1200\), which is large enough to meet the conditions of the RDT.

Here our aim is to test the predicted linearity of the evolution of the disturbance kinetic energy with time in the long-time limit (Moffatt, 1967), and towards this we make following assumptions/observations.

1. We assume that the effect of blocking is negligible compared to shear inside the boundary layer and we limit ourselves to a region sufficiently remote from the wall such that the direct viscous effects are unimportant (they are not included in the RDT analysis).

2. In the near-wall region the mean streamlines are assumed to be approximately parallel to the wall. It is well known that the wall-normal velocity decays much more rapidly than the wall-parallel velocities near the wall in a boundary layer and therefore the streamline inclination can be expected to be small. Furthermore we assume that turbulent eddies which enter the boundary layer are carried along the mean streamlines consistent with Taylor’s hypothesis.

3. The RDT analysis (Moffatt, 1967) assumes a uniform shear acting on a homogeneous initial field. In the present flow the shear rates change as we move downstream due to growth of the boundary layer. For regions close to the plate leading edge, the change in the shear rate can be expected to be small. Therefore for the present purposes we assume that the turbulent eddies evolve as if they are subjected to ‘locally-uniform’ regions of pure shear.

4. We use the streamwise disturbance energy as a surrogate for the total disturbance energy used in the RDT analysis, since the streamwise energy is seen to dominate the other components in the long-time limit (Moffatt, 1967). This is also consistent with the transient-growth studies which show that the disturbance energy in the streamwise velocity component increases at the expense of the transverse components as the time increases (del Álamo and Jiménez, 2006).

With these assumptions, we proceed to find out the evolution of turbulent eddies under the action of shear in the present measurements. For this we choose three stations that are fairly close to the plate leading edge, namely \(x = \text{80 mm}, \text{150 mm} \) and \(300 \text{ mm} \) (with \(\delta_{99}\) at these locations being 1.8 mm, 2.6 mm and 4.4 mm respectively). The wall-normal distances chosen, consistent with assumption (1) above, are \(y = 0.4 \text{ mm}, 0.45 \text{ mm} \) and 0.55 mm. It is observed that the mean velocity profiles for these stations follow an approximately linear variation up to \(y = 0.6 \text{ mm}\) and therefore we have chosen 0.55 mm as the uppermost location. The initial condition at \(x = 0\) is taken to be the mean velocity and turbulence intensity as measured in the freestream at that location. The time of flight for a turbulent eddy at a given \(y\) inside the boundary layer is given by

\[
t_{\text{flight}} = \int_0^x \frac{1}{U} dx. \tag{2}
\]

Here \(U\) is the local mean streamwise velocity. To calculate \(t_{\text{flight}}\) in Eq. (2) we interpolate \(x\) and \(U\) over the streamwise extent of interest so as to have sufficient number of points to evaluate the integral.

**Fig. 9.** A schematic depicting the corresponding mean velocity profile for a canonical TBL and a rapidly distorted boundary layer, wherein the spectral structure is nearly analogous.

**Fig. 10.** The evolution of the streamwise turbulence energy \(\left(u'^2/2\right)\) with respect to time of flight scaled on the local shear rate \(S_{\text{flight}}\). Here \(u'^2\) is the turbulence energy at \(x = 0\). It is evident from the figure that the initial evolution of the disturbance energy follows an approximately linear trend for all the \(y\) locations, with the degree of linearity decreasing with increasing \(y\). The linearity persists up to \(S_{\text{flight}} \approx 200 - 250\) (corresponding to \(x \approx 150 \text{ mm}\)) which can be considered to be sufficiently long time in terms of the shear rate, \(S\). This is consistent with the long-time behaviour predicted by the RDT (Moffatt, 1967) and thus the early disturbance evolution in the rapidly distorted boundary layer studied here is well described by the linearised shearing model. Note that beyond \(S_{\text{flight}} = 250\), the trend departs from linearity (Fig. 10) presumably due to the increasing influence of the nonlinear processes as the boundary layer thickness increases with distance downstream.

We have seen in Section 3.2 that the long streamwise motions in a rapidly distorted boundary layer, which mimic the superstructures in a canonical TBL, could be attributed to the rapid shearing of large-scale disturbances in the near-wall region. This observation combined with the success of the RDT seen in Fig. 10 suggests that the origin of the superstructure-type motions could be understood in the framework of linearised models. This is an important result as the relevance of linear mechanisms towards understanding the coherent motions is currently one of the major themes.
of research in wall turbulence (Sharma et al., 2011; McKeon and Sharma, 2010; Jiménez, 2013).

It must be noted that in the classical RDT analysis the disturbance amplitudes do not change in the cross-shear direction as the flow field is taken to be homogeneous. On the other hand, in a boundary layer flow the disturbance amplitude is a strong function of y and therefore the initial disturbance energies $u^2_0$ for the three y locations are likely to be different. This explains the different slopes of the linear trends for different y locations seen in Fig. 10. It is relevant to refer to the work of Landahl (1990) who considered a more realistic initial disturbance field and found that the resulting disturbance amplitudes, calculated using a linear model, had a similar wall-normal dependence.

4.2. Effect of blocking

Now we examine the effect of linear inviscid blocking (Hunt and Carlotti, 2001), which results in an enhancement in the streamwise turbulence energy for wavenumbers $k < 1/f$. For this, we look at the changes in the shape of the spectrum in a rapidly distorted boundary layer as we move from the freestream towards the wall, at $x = 150$ mm (Fig. 11). It is evident from the figure that close to the edge of the boundary layer ($y \approx \delta$) there is an enhancement in energy for $k \lesssim 1/\delta$ in accordance with the RDT. In this region the boundary layer shear is negligible and hence the primary mechanism for the increase in the streamwise turbulence energy is the blocking of the large scale fluctuations by the presence of wall. It is also seen that for $k > 1/\delta$ there is an attenuation of energy and this can be attributed to the influence of nonlinearity which becomes important at smaller scales (see Fig. 8 in Hunt and Carlotti, 2001). Closer to the wall, the behaviour of the spectrum starts to depart from the linearised blocking model due to the increased effect of shear, as can be seen at $y/\delta = 0.35$ (Fig. 11).

It is now interesting to consider how the integral length scale of the freestream turbulence ($\ell_{t}$) is related to the boundary layer thickness. $\ell_{t}$ can be estimated from the integral time scale as $\ell_{t} = t_{f}U_{0}$. At $x = 150$ mm, $\ell_{t} = 16.2$ mm ($U_{0} = 9$ m/s, $t_{f} = 0.0018$ s). With $\delta = 2.6$ mm, we get $\ell_{t}/\delta = 6.2$ which is O(10). Thus close to the leading edge of the plate we can expect $\ell_{t}/\delta$ to be O(10). We can obtain a similar estimate for a canonical TBL by considering the thickness of the wall layer in a TBL to be equivalent to the entire thickness ($\delta$) of a rapidly distorted boundary layer (Fig. 9). The thickness of the wall layer in a high-Reynolds-number TBL is typically observed to be $\delta_{w} = (0.1 - 0.15)\delta$ (Vassilicos et al., 2015), which is the upper edge of the logarithmic region. If we take the integral length scale of the turbulent eddies in the wake region of a TBL to be O(10) (Vassilicos et al., 2015), we get $\ell_{t}/\delta_{w} = O(10)$ which is again of the same order as that obtained above for the
rapidly distorted flow. These considerations suggest that the linear blocking model could be relevant to a class of eddies in the wake region of a canonical TBL that are about to impinge on the wall. A pictorial depiction of such a scenario can be found in Hunt and Carlotti (2001).

5. Concluding remarks

In this work we use an alternative approach for studying the spectral structure of a canonical TBL by carrying out experiments on a rapidly distorted boundary layer evolving towards a fully-turbulent state. The results on the boundary layer subjected to a roughness patch (in absence of grid turbulence) show that over a certain distance downstream of the patch, the energy spectrum of the streamwise velocity fluctuations exhibits a bimodal shape typical of that found in a TBL at moderately high Reynolds numbers. We find that the interaction of the roughness-induced broadband disturbances with the new internal boundary layer that develops downstream of the roughness patch is responsible for the generation of the low-wavenumber peak in the spectrum. Such a spectral peak represents long streamwise motions which are analogous to the superstructures present in a TBL. Using both grid turbulence and surface roughness we show that the rapidly distorted boundary layer synthesises the spectral structure of a canonical TBL in the wall-normal direction. We also identify regions in the synthetic boundary layer which display spectral shapes that are similar to those found in the buffer, logarithmic and wake regions of a TBL. Such a correspondence allows us to identify the basic building blocks which give rise to the observed shapes of the spectra and streamwise scales of the associated coherent motions.

Next we use the results on the rapidly distorted boundary layer to test the predictions of RDT. In a region close to the wall, the streamwise turbulence energy advected along the mean streamlines is seen to increase linearly with time, in accordance with the RDT analyses that consider the effect of pure shear on an initial isotropic turbulence field. Furthermore, near the edge of the rapidly distorted boundary layer there is an enhancement in the streamwise energy content of the large-scale disturbances in a way that is consistent with the effect of linear inviscid blocking. Thus the present results are in good agreement with the effect of shear and blocking as predicted by the RDT. In view of the structural similarity between the rapidly distorted boundary layer and a canonical TBL seen above, it should now be possible to identify aspects of a canonical TBL to which linear analyses like the RDT could be meaningfully applied. Similar attempts have been made in the past (for example Hunt and Morrison, 2000: Högström et al., 2002), and the present work provides a further support for using such linearised tools in the study of wall turbulence.

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References