Experimental characterization of intermittency of thermoacoustic instability in a swirl stabilized combustor.

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Abstract

In the context of this work we attempt to examine the intermittency characteristics of thermoacoustic instabilities in an experimental swirl stabilized combustor operating with lean premixed air methane mixture. The instabilities are examined employing nonlinear dynamics tools, which can reveal the structure of the oscillations and allow for the development of methods that can forecast the onset of oscillations.

Introduction

Concepts for combustion technologies in gas turbines are being developed to reduce NOx emissions [1]. These concepts rely on operating at lean-premixed combustion mode. However, lean premixed mode of operation causes the combustor to be susceptible to triggering of thermoacoustic oscillations, which can severely damage the structural integrity of the combustor. Combustion instabilities in gas turbines are characterized by large amplitude oscillations of pressure and heat release at the fundamental or other natural acoustic modes of the combustor [2].

These instabilities are spontaneously triggered and are amplified by a feedback loop between unstable combustion processes and pressure. After the initial triggering, the instabilities are self-sustainable, their amplitude initially increases in a linear manner until it saturates at maximum amplitude, whence nonlinear dynamics are involved. The Rayleigh [3] grants fundamental insight into the coupling processes causing the onset of the oscillations. Essentially, it is stated that the relative phase between the acoustic field (pressure) and the thermal field (heat release) can define whether a self-sustainable process is initiated. According to [4], oscillations are amplified if the rate that energy is provided to the acoustic field by the combustion process is greater than the rate that acoustic energy is dissipated within the combustor and through its boundaries.

A review on the coupling between the flow field and the flame [5] provides insight on the susceptibility of lean combustion to triggering of thermoacoustic instabilities. Flow induced time scales are related with the convection of modes contributing to the coupling such as pressure waves. The resonant interactions between the upstream and the downstream field in confined geometries are also emphasized as a dominant coupling mode.

In literature, the nonlinear nature of the thermoacoustic instability phenomenon is attributed to the saturation of the heat release rate. The necessity of implementing nonlinear analysis techniques is emphasized. An introduction to the nonlinear characteristics of the thermoacoustic instabilities is provided in [2].

The irregular emergence of high amplitude oscillations out of low amplitude unstructured fluctuations near the lean blow out limit has been the subject of numerous studies. In [6], the authors employ nonlinear analysis techniques to study the self-excited thermoacoustic oscillations in a cylindrical combustor by altering the flame anchoring point. In [7], the dynamic properties of combustion instabilities in an axial swirl stabilized combustor have been derived employing non-linear analysis techniques. In this work, the dynamic properties of the transition from stochastic pressure fluctuations to fully developed periodic oscillations are studied, while increasing the equivalence ratio in lean premixed operational regime. In [8], the authors characterize the transitional regime towards fully developed combustion instability as intermittency. They suggest that a typical intermittent pattern in combustion instability is the transition from quiescent unstructured fluctuations to irregular bursts and then reinjection back to a quiescent state again. In [9], an experimental investigation of the route from combustion noise towards fully developed oscillations through high amplitude bursts is studied by altering the Reynolds number of the flow. It is attempted to establish intermittency as a separate dynamic combustion state. The fact that the bursts appear in a random pattern, only if the underlying flow field is turbulent indicates that turbulence is the significant factor into establishing homoclinic orbits in the phase space of the attractor of the system. Homoclinic orbits are established when the stable and unstable parts of a hyperbolic saddle equilibrium solution meet each other. In this case there are intermittent excursions of the dynamics between the quiescent state and the oscillatory state. In [10] the establishment of homoclinic orbits before the establishment of fully developed flutter instability is emphasized. Hence, the role of turbulence in establishing homoclinic orbits in non-reacting dynamic systems seems to be important.

Specific Objectives

In this work we characterize the characteristics of the intermittency observed in self-excited instabilities of lean premixed combustion of methane-air mixtures in an experimental combustor consisting of an axial swirler with a transparent cylindrical combustion chamber. The bursts emerge in an irregular non-deterministic manner for experimental cases near the lean blow out limit and they become more frequent while increasing the equivalence ratio for a given Reynolds number. It is
observed that homoclinic orbits are established and the intermittency is characterized as type – II intermittency [11]. Moreover, we examine precursors that forewarn before the triggering of a burst. A precursor that accounts for the transition into structured oscillations and the loss of chaos is suggested as an indication of a pending instability.

**Experimental Configuration**

An overview of the rig that is used in this study is provided in Figure 1.

![Figure 1: Schematic description of the swirl stabilized combustor [12]. Dimensions in mm, not to scale. Details are provided in the text.](image)

Compressed air was provided at 6 barg from a compressor. The supply was reduced to 4 barg and it was measured through a thermal mass flow meter (M+W Instruments, Mass Stream D-6280) and controlled through a thermal mass flow controller (M+W Instruments, Mass Stream D-6383). The air flow was regulated through the following equipment. A pressure reducer was installed (Backpressure Regulator, Mackenberg UV 5.1) upstream of a critical Venturi (Cussons Technology Ltd, BS/ISO 9300:2005). The critical Venturi nozzle was used to choke the flow and provide a sonic boundary. The flow profile was conditioned before the nozzle through a tube bundle containing a wire mesh gauge and through a flow straightener. Upstream and downstream of the Venturi nozzle two absolute pressure gauges were installed (Omega, PXM319-3.5A10V) to estimate the extent of choking across the nozzle. The mass flow rate of fuel gas was metered through a thermal mass flow meter (M+W Instruments, Mass Stream D-6250) and controlled through a thermal mass flow controller (M+W Instruments, Mass Stream, D-6320). Fuel entered the injector at the rear of the centerbody and was forced out of ten equally spaced holes in the swirling annular air flow. The air flows from left to right in Figure 1. The majority of the air was delivered to the swirlir, while a small amount flowed through the centerbody for cooling purposes. The resulting swirl number was approximately 0.7 [13]. After the swirlir, the air-fuel mixture was expanded through a diffuser. At the end of the quartz silica section, a tapping allowed the spark plug igniter to access the ignitable mixture. The quartz silica length was 450mm and the duct diameter is 70 mm. A tapping 50 mm downstream of the igniter accommodated the installation of a semi-infinite tube (SIT) configuration [12]. It consisted of a coiled seamless tube of 2.93 mm internal diameter, whose length was 6m to avoid any interference to the acoustic field. It was then connected to a plastic hose for nitrogen flow for cooling purposes of the installed dynamic microphone (Kulite MIC-190M). The microphone was installed 130 mm over the bore of the exhaust and acoustic waves from the combustor were measured through a 1mm hole drilled on the SIT configuration. Chemiluminescence signals have been acquired through the use of the photomultiplier developed in [14]. Chemiluminescences emissions from the combustor have been detected by a set of UV plano-concave (Thorlabs, focal length -30 mm, ø 1 in) and UV plano-convex (Thorlabs, focal length 35mm, ø 1 in) lenses, focusing the light on a 1.3 mm diameter lens of an optical fibre (Thorlabs-FT1500UMT-0.39 NA, O 1500 µm, 300-1200 nm). The spectroscopic unit acquired the light transported through the optical fibre. The light is split into three spectral fractions using two dichroic mirrors. The reflected light was directed onto filters, to acquire the light from the spectra of interest (308.5 nm for OH*-bandwidth 18.0 nm, 430.5 nm for CH*-bandwidth 1.9nm, 516.0 nm for C2 – bandwidth 2.3 nm). The light signals were transformed into voltage through photomultipliers. In the context of this work we conduct experiments over a range of equivalence ratios (φ) from 0.55 to 0.65 with increments of 0.025 under constant Reynolds numbers (Re). The Reynolds numbers span from 15000 to 19000 with increments of 1000. Pressure fluctuations acquired through the mounted microphone describe the acoustic field, while OH* emissions indicate the heat release rate fluctuations.

**Results**

The effect of increasing the equivalence ratio, at constant Reynolds number, is the increase of the frequency of transition to the unstable state. To quantify this we measure the total number of local maxima (N) above a predefined threshold 0.5 kPa in the pressure time series in the first 20 seconds of the acquisition. We divide that over the total number of local maxima in the pressure fluctuations (N_{total}). The calculated normalized number of bursts (f), as defined in equation 1, is a probability measure for combustion to attain instability.

\[ f = \frac{N}{N_{total}} \] (1)

In Figure 2, the f number of Equation (1) is presented for various equivalence ratios. Near the lean blowout limit for equivalence ratio equal to 0.55 the transition from the quiescent to the oscillatory state takes place irregularly. The oscillating equivalence ratio between 0.57-0.60 there is an intermittent state of frequent transitions to the oscillatory regime and finally between values of 0.60 to 0.65 the instabilities are fully established and the transition back to the quiescent state is rare and for short periods of time. Figure 2 provides a qualitative description of the bifurcations of the dynamics of the system from quiescent conditions to fully unstable, while increasing the equivalence ratio, which is
considered the bifurcation parameter. In our case, the direction of the alteration of the parameter did not affect the qualitative nature of the instabilities. Furthermore, no hysteresis was observed. Hence, this may suggest that the global bifurcation of the transition to fully established instabilities is supercritical in nature [11].

Figure 2: Normalized number of pressure peaks from Eq. (1) above a threshold of instability as a function of the equivalence ratio for different Re.

Dynamic Characteristics of Instabilities

To export the dynamic characteristics of the instability regimes at various equivalence ratios the time series were discretized into quiescent and unstable sections.

Figure 3: Time series and respective power spectra, of the oscillatory parts of the signals for three equivalence ratios (φ = 0.55, φ = 0.60, φ = 0.65) at Re = 17000.

We calculated the root mean square of the pressure over an interrogation window of 100 samples and defined a threshold below which operation was stable. The acquired oscillations were then used to calculate the amplitude spectra. In Figure 3, the pressure time series along with their respective amplitude spectra are presented for three equivalence ratios. An acoustic analysis of our combustor (assuming a displacement node at the closed end, a pressure node at the open end and speed of sound as a function of the adiabatic temperature) indicates the fundamental acoustic frequency of the first quarter wave mode is 164 Hz. The acquired spectra are broadband in nature, which is characteristic for signals where turbulence has a significant contribution. For the case near the lean blow out limit (φ = 0.55), distinct peaks are observed at the fundamental frequency and its harmonics. For φ = 0.60, where the intermittent regime has been established one can observe the phenomenon of period doubling. The half sub harmonic frequency of the fundamental frequency has emerged. Linear combinations of the harmonic and its sub harmonic can also be observed. In theoretical and numerical dynamic studies transition to chaos has been observed via sequences of period doubling bifurcations [15]. If noise is absent, an infinite number of period doubling bifurcations are possible. In the presence of a turbulent background, noise is present and this can suppress subsequent period doubling bifurcations [11]. Hence, for φ = 0.65 we see that the effect of the sub harmonics has been suppressed, there are distinct peaks at multiples of the fundamental frequency, while the spectra are more broadband.

Study of homoclinic orbits

Figure 4: State space representation of the attractors attained by the pressure signal. The attractors are representative for Re = 17000 and φ = 0.55, φ = 0.60, φ = 0.65. The dynamics move from the quiescent state at the centre of the disk to the quasi-periodic cycles in the outer parts of the state-space diagram before they become quiescent again (embedding dimension = 3, time delay=33 samples).

Typically, in experiments one can measure only a limited number of signals to describe the dynamic behaviour of the system. In topology, this is a problem of embedding. According to [16] it is possible to reconstruct the state space by scalar measurements, while preserving the invariants of the system (i.e., maximum Lyapunov exponent). In this
work, we adopt the embedding approach described in [17]. One can define delay-coordinate vectors by calculating the appropriate embedding dimension and an appropriate time delay. In Figure 4, the development of the reconstructed attractors for three equivalence ratios at a constant Reynolds number is depicted. In all three cases, one can deduce the homoclinicity of the trajectories by an inspection of the attractors. The dynamics transition from the low amplitude quiescent regime to the high amplitude oscillations and then following a spiral transitional orbit in the same plane defined by the oscillation disks returns at the quiescent state. In the transitional regime \((\phi=0.60)\), after the period doubling bifurcation, the trajectory attains intermittently two separate high amplitude oscillation states before it returns to the quiescent state. This is an effect of the trajectory oscillating at multiples of two frequencies before the return to the stable manifold. The separate oscillation states merge after the establishment of fully developed instabilities \((\phi=0.65)\), due to the suppression of the sub harmonic frequency and its multiples. The argument in favour of the existence of homoclinic orbits is supported by the observations in [18]. The authors suggest that systems whose characteristic is the occurrence of intermittent events attain typically homoclinic cycles and the passage time of the trajectory on the quiescent states follows a skewed probability distribution with a characteristic exponential tail.

Figure 5: Probability distribution of duration of visits in the quiescent regime. The distribution is skewed with an exponential fall-off (Re=17000, \(\phi=0.60\)).

The structure of the recurrence plot depicted in Figure 6 is representative of a burst near the lean blow off limit. We observe dense patches with perforated upper right edges. This is the characteristic transitional structure from and towards quiescent phases. We also observe the kite-like shape of the transitional edges. These observations indicate the existence of type-II intermittency mechanisms is associated with subcritical Hopf bifurcations, where a pseudoquasiperiodic quiescent regime transitions into a large amplitude periodic regime [23].

Qualitative characteristics of oscillations
Figure 7 depicts the probability distribution of the attained pressure maxima during the oscillatory regimes. Near the lean blow off limit ($\phi=0.55$) the pressure peaks are distributed around zero with few positive peaks. At the transitional regime ($\phi=0.60$) and the fully established oscillatory regime ($\phi=0.65$), there is a negative skewness, which increases as the equivalence ratio increases.

Figure 8 depicts the probability distribution of the phase difference between the pressure and chemiluminescent intensity (indicating heat release rate) fluctuations signals. The cross correlation between the two signals indicates the relative phase difference of their respective oscillations, based on the sample lead or lag where maximum correlation is observed.

The fundamental frequency of the oscillations indicates the number of samples in an oscillatory cycle. We relate the number of samples in each cycle to the calculated relative sample difference to estimate the phase difference between the two signals. The relative sample difference showcases a deviation between -250 samples and 250 samples (negative values indicate chemiluminescent intensity fluctuations signal lead, and positive values indicate pressure fluctuations signal lead). Each cycle consists of 120 samples. Sample delays in the range of [-30, 30], [-150, -120], [120, 150] signify [0, $\pi$/2] relative phase difference. In that case it can be deduced that the two signals are coupled.

We can estimate the probability of the signals attaining distinct coupling ranges by calculating the relevant area below the probability distribution. Near the lean blow off, limit stronger coupling is noticed at the sampling range of [120, 150]. Only 38% of the samples that are included in the oscillatory parts of the signals are in phase and this is reflected in the rapid transition to the quiescent states (decoupling) in this regime. In contrast, stronger coupling is noticed in the intermediate and fully established regime in the [-30, 30] sample range. In both regimes, 80% of the samples in both the signals are in phase. The remaining 20% of the samples correspond to the transitional phases away from and towards the quiescent state.

Precursors of pending instability

The dynamic characteristics of the transition to structured oscillations from pseudoquasiperiodic low amplitude fluctuations suggest it is profitable to employ the permutation entropy as a tool to detect the triggering of a high amplitude burst.
Figure 9, during the quiescent phases of the pressure signal permutation entropy rises, an indication of chaotic fluctuations, while during the oscillatory phases, the entropy decreases as the signal attains a periodic attractor. The above properties suggest that the online calculation of the permutation entropy has potential usage as a feedback indicator for active control of combustion instabilities [25].

Conclusions

In this work, we have presented the dynamic characteristics of intermittency of thermoacoustic oscillations, during the transition from the lean blow off limit to fully established thermoacoustic instabilities. We observe that by increasing the equivalence ratio while keeping the Reynolds number constant the quiescent phase duration decreases and the oscillation amplitude increases. Furthermore, the power spectrum of pressure fluctuations showcases distinct peaks at multiples of the fundamental frequency of the combustor. A period doubling bifurcation occurs in the transitional regime, with the emergence of the half sub harmonic frequency. We establish the presence of homoclinic orbits, which is typical for trajectories of signals that are affected by the dynamics of systems characterized by intermittency. The bifurcation between the laminar states and the oscillatory states is subcritical in nature and we suggest that a Hopf bifurcation occurs. Moreover, stronger coupling between pressure and chemiluminescent intensity, a proxy for heat released is noticed, while increasing the equivalence ratio. At the same time the duration of the oscillations increases. Finally, we employ the permutation entropy as a tool to detect the emergence of structured periodic oscillations out of chaotic small amplitude fluctuations. In the context of the same work, high speed imaging of natural luminosity and chemiluminescent emission from the flame during combustion instability has also been conducted. In conjunction with methods of image processing, such as proper orthogonal decomposition and dynamic mode decomposition insight can be granted into the coherent flow field structures that contribute into the onset of instabilities. The observations of this analysis is subject of future publication.

References


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