Stochastic Idiosyncratic Cash Flow Risk and Real Options: 
Implications for Stock Returns

Harjoat S. Bhamra*  
Imperial College Business School

Kyung Hwan Shim†, ‡  
University of New South Wales

November 3, 2016

Abstract

Stocks with high idiosyncratic volatility perform poorly relative to low idiosyncratic volatility stocks. We offer a novel explanation of this anomaly based on real options, which is consistent with earlier findings on idiosyncratic volatility (the positive contemporaneous relation between firm-level stock returns and idiosyncratic volatility). Our approach is based on introducing stochastic idiosyncratic cash flow risk into an equity valuation model of firms with growth options. Within our model, a firm’s systematic risk depends on the delta of its growth option. The growth option’s delta is lower when idiosyncratic volatility rises, driving down the firm’s systematic risk and hence its expected return – firms with higher idiosyncratic volatility therefore have lower expected returns. Our model additionally offers the following novel empirical predictions: (i) returns correlate positively with idiosyncratic volatility during intervals between large changes in idiosyncratic volatility (the switch effect), and (ii) the anomalies and the switch effect are stronger for firms with more real options and which undergo larger changes in idiosyncratic volatility. Empirical results support the predictions of our model.

Keywords: Idiosyncratic return volatility, cross-section of stock returns, asset pricing, real options, growth options, stochastic volatility, regime switching, mixed jump-diffusion processes.

*Imperial College Business School, London, UK, SW7 2AZ, Email: bhamra.harjoat@gmail.com; Tel: +44 (0)20 7594 9077.
†School of Banking and Finance, UNSW Business School, University of New South Wales, Sydney NSW, Australia, 2052, Email: k.shim@unsw.edu.au; Tel: 61 (02) 9385 5852.
‡We would like to thank Efstatios Avdis, Tony Berrada, George Constantinides, Engelbert Dockner, Lorenzo Garlappi, Ralph Koijen, Chunhua Lan, Andrey Malenko, Erwan Morellec, Stijn Van Nieuwerburgh, Stavros Panageas, Konark Saxena, Paul Schneider, Mark Shackleton, Robert Tumarkin, Raman Uppal, Jin Yu, Tan Wang, seminar participants at the University of New South Wales, University of Southern California, Norwegian School of Economics (Bergen), McGill University, Nottingham University Business School, Carleton University, participants of the 2013 Adam Smith Asset Pricing Conference, the 2013 China International Conference in Finance, the 2013 Northern Finance Association Annual Meetings, the 2013 Tel Aviv University Finance Conference, the 2014 Frontiers of Finance, the 2014 European Finance Association Annual Meetings, the 2015 Society for Economic Dynamics Meetings, and the 2015 American Finance Association Annual Meetings for helpful comments and discussions. We would also like to thank the Canadian Institute of Chartered Business Valuators for the Best Paper Award at the 2013 NFA Annual Meetings. All errors are ours. Please forward comments and suggestions to k.shim@unsw.edu.au.
1 Introduction

Real option theory provides a useful way of understanding how the risk of firms evolves over time. This has proved important in many areas of finance. Initial applications of real option theory focused on corporate investments,\(^1\) while subsequent applications have ranged from corporate strategy to understanding empirical asset pricing anomalies.\(^2,3\) In this paper, we focus on how real option theory helps our understanding of asset pricing anomalies related to idiosyncratic volatility; empirical findings that contradict standard asset pricing theories.\(^4\)

Arguments rooted in real option theory can explain why increases in firm-level idiosyncratic volatility (\(IVol\)) lead to concurrent increases in equity returns (Grullon, Lyandres, and Zhdanov (2012)) – the positive \(IVol\)-return relation (Duffee (1995)). Ang, Hodrick, Xing, and Zhang (2006), however, document that future average returns relate inversely to idiosyncratic volatility (the negative \(IVol\)-return relation hereafter).\(^5\) Do these seemingly conflicting empirical findings force us to look beyond real options theory? Or is it possible to explain these puzzles within a single model of corporate investments?

Our paper’s main contribution is three-fold. First, we show that by introducing stochastic idiosyncratic cash flow volatility (Guo, Miao, and Morellec (2005)) in a firm valuation model with optimal investment decisions (Berk, Green, and Naik (1999)), it is possible to explain the negative \(IVol\)-return relation in addition to the positive \(IVol\)-return relation.\(^6\) Second, we show that our model provides a novel empirical prediction on the relation between idiosyncratic volatility and equity return (the ‘switch effect’) which we verify empirically. For our third contribution, and consistent with the predictions of the main features of the model, we show empirically that all

---

\(^{1}\)For real option literature related to corporate investments, see Brennan and Schwartz (1985), MacDonald and Siegel (1986), and Pindyck (1988). Dixit and Pindyck (1994) offers an excellent overview of this literature.

\(^{2}\)For how merger decisions impact equity returns, see Hackbarth and Morellec (2008). For real option signaling games in corporate finance, see Grenadier and Malenko (2011).

\(^{3}\)For real option theory to understand cross-sectional asset pricing anomalies related to size, book-to-market, and momentum returns, see Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), Cooper (2007), and Sagi and Seasholes (2007).

\(^{4}\)Standard asset pricing theories suggest that idiosyncratic factors should not be priced based on the principle that non-systematic risks are eliminated through portfolio diversification.

\(^{5}\)Ang, Hodrick, Xing, and Zhang (2006) document that portfolios composed of high \(IVol\) stocks have significantly lower returns than their low \(IVol\) counterparts. The negative \(IVol\)-return relation is not unique to US equity markets. See Ang, Hodrick, Xing, and Zhang (2009) for evidence from international equity markets.

\(^{6}\)For analytical tractability, we assume a 2-regime Markov switching process for idiosyncratic cash flow volatility. Other papers using a 2-regime Markov switching process for state dynamics include Guo, Miao, and Morellec (2005) and Hackbarth, Miao, and Morellec (2006), who investigate investment and capital structure policies, respectively.
three of the aforementioned IVol-return relations are stronger for firms which experience higher variability in idiosyncratic volatility and derive more value from real options.

We show that the positive IVol-return relation is satisfied if and only if a firm’s equity returns possesses one of two alternative key properties. The first of the two properties is that a firm’s equity returns and its idiosyncratic volatility are driven by a common idiosyncratic risk factor. The second alternative property is that a firm’s equity returns and its idiosyncratic volatility are driven by the systematic risk factors, which drive changes in the firm’s unexpected return. Our model satisfies the first property, in contrast with the models used in Kogan and Papanikolaou (2013) and Herskovic, Kelly, Lustig, and Nieuwerburgh (2015), which satisfy the the second property.

We also show that the negative IVol-return relation is satisfied if and only if a firm’s systematic volatility falls when idiosyncratic risk increases. This third property ensures firm-level expected returns, which load on systematic volatility, are higher when idiosyncratic risk is lower, giving rise to the negative IVol-return relation.

Any model which satisfies one of the the first two properties and the third property, regardless of whether it is purely classical, invokes market frictions, or features irrationality, will simultaneously generate the positive and negative IVol-return relations. We develop an investment-based model of firms that incorporates two classical building blocks: stochastic idiosyncratic risk combined with a model of a cross-section of firms possessing growth options. Bringing these two features together, the first and the third properties occur endogenously, making it possible to resolve the positive and negative IVol-return relations.\(^7\)

In the model, young firms can expand upon previously deployed assets and become mature by incurring an adjustment cost in response to cumulative shocks to cash flows. The shocks are driven by an idiosyncratic component that has stochastic volatility, driven by a 2-state Markov chain, and a systematic component. In contrast to mature firms, the value of young firms includes an investment opportunity, the growth option, which contributes to the systematic risk of the firms.\(^8\) We solve the model in closed-form, and investigate the role that the stochastic nature of

\(^7\)We depart from the explanations for the negative IVol-return relation based on limits to arbitrage (Pontiff (2006)) or investors’ cognitive biases and mispricing in financial markets (Daniel, Hirshleifer, and Subrahmanyan (1998)). In this sense, our model does not invoke any market frictions or irrationality.

\(^8\)The salient points related to stochastic idiosyncratic cash flow volatility are made through growth options, but as discussed below, the same features hold for real options in general.
idiosyncratic risk has on returns. The main feature of the model is the dependence of growth option values on the idiosyncratic volatility of cash flows. This occurs because option values benefit from a greater volatility in the value of the underlying assets due to a higher expectation of a larger option payoff, because of convexity. Since the value of options is increasing in volatility, returns exhibit jumps in the direction of the switches in idiosyncratic volatility, satisfying the first property of returns and thus producing the positive IVol-return relation. The third property arises in relation to idiosyncratic volatility. Growth options exhibit a lower cash flow elasticity in the high volatility regime, leading to a lower exposure to the systematic risk of the cash flows. Therefore, the model produces an inverse relation between IVol and future average returns thereby explaining the negative IVol anomaly.

Grullon, Lyandres, and Zhdanov (2012) provide extensive empirical evidence in support for the real option mechanism underlying the positive IVol-return relation. Their analysis is entirely motivated by standard option pricing theory with constant volatility. We extend their work on several fronts. We develop and solve in closed-form a model where idiosyncratic volatility is stochastic and firms endogenously choose when to exercise growth options. The exercise decision, therefore, is dependent on the idiosyncratic volatility regime. Formulating an investment model with stochastic idiosyncratic volatility also leads to a novel IVol-return result – the switch effect – which we substantiate empirically along with the other predictions of the model.

In addition to the positive IVol anomaly, we also offer an explanation for the negative IVol-return relation which is not studied by Grullon, Lyandres, and Zhdanov. To see this, consider the sensitivity of an option to the value of the underlying asset, i.e. the option delta. Holding the value of the underlying asset constant, a high volatility regime that leads to an increase in the value of an option also decreases the option delta. In the model, a lower option delta means growth options have a lower exposure to the systematic risk of the underlying cash flows when

---

9. The model solution is in closed-form up to two constants which are identified by a simple system of algebraic equations.
10. Standard option theory tells us that option values are increasing in the risk of the asset the option is written on, implying that the growth option of a firm is more valuable when its idiosyncratic risk increases. Therefore, the returns on a firm with growth options is exposed to the same underlying idiosyncratic risk factor as the level of idiosyncratic volatility itself.
11. Intuitively, ceteris paribus, a greater value stemming from a non-systematic risk factor makes options less sensitive to the assets they are written on. This translates to a lower exposure to the systematic risk of the underlying assets.
idiosyncratic risk is high.\textsuperscript{12,13} Absent a proper empirical factor to adjust for systematic risk, the lower risk premia related to high idiosyncratic volatility appear in the guise of an inverse empirical relation between $IVol$ and abnormal returns.

A novel prediction of the model is a third $IVol$-return relation, the switch effect, which we also substantiate empirically. In the model, abnormal returns correlate positively with the volatility regime during periods between changes in idiosyncratic volatility caused by the non-materialized jumps in return. This in turn leads to a continuation in returns which correlates with idiosyncratic volatility during periods of constant volatility, followed by reversals caused by the jumps which concur with switches in idiosyncratic volatility. The firm’s currently producing assets – the assets-in-place – have linear valuations in cash flow and are hence invariant with respect to idiosyncratic volatility. Therefore, all three $IVol$-return relations are attributed entirely to growth options, the values of which respond to variation in idiosyncratic cash flow risk.

We verify the theoretical predictions quantitatively via Monte Carlo simulations and recreate the posited $IVol$-return relations.\textsuperscript{14} The simulations, which are based on realistic calibrations of the model, reveal that all three $IVol$-return relations are driven entirely by young firms possessing growth options and become more pronounced if the spread in idiosyncratic volatility between regimes is larger. When we specify a single regime – the standard specification in real option models – the simulations produce no statistical relation between $IVol$ and return. Taken together, the results reassert that stochastic idiosyncratic cash flow volatility and real options are the drivers behind the $IVol$-return relations in the model.

Guided by the implications of the model, the empirical analysis is focused on verifying the predictions using the cross-section of US manufacturing firms. Does the data support the model’s falsifiable predictions? This is crucial for verifying the plausibility and the economics underlying our explanation for how the $IVol$-return relations can arise in a rational model.

For the positive $IVol$ anomaly, we go beyond Grullon, Lyandres, and Zhdanov (2012) by

\begin{itemize}
\item \textsuperscript{12}This is in contrast with the embodied technology shocks modelled in Garleanu, Panageas, and Yu (2012), which impact the level of output flow.
\item \textsuperscript{13}Interestingly, the result that an option’s delta is decreasing in risk has been used to resolve other anomalies: Johnson (2004) shows that the negative relationship between stock returns and the dispersion of analysts’ earnings forecasts can be explained in model where firms issues corporate debt, creating a default option. We pursue a different research question from Johnson. We focus on growth options where the exercise decision is endogenous and therefore depends directly on the idiosyncratic volatility regime. By contrast, Johnson models firms with default options as in Merton (1974), where the default decision is exogenous.
\item \textsuperscript{14}These findings, which confirm the model results numerically, are reported in a separate Online Appendix.
\end{itemize}
creating our own empirical variables for real option intensity and employing similar cross-sectional return regressions with additional specifications in which we include the difference between the 70th and 30th percentile values of $IVol$ to capture the variability in idiosyncratic volatility. For the negative $IVol$-return relation, we initially follow Ang, Hodrick, Xing, and Zhang (2006) by creating $IVol$-sorted portfolios, and then go further by additionally sorting on the basis of real option intensity and variability in idiosyncratic volatility for subsample analysis of extreme $IVol$ portfolios. We find evidence for $IVol$-return relations that strengthen in real option intensive and in the variability of idiosyncratic volatility in line with the predictions of the model.\(^{15}\)

Lastly, we examine empirically the switch effect; the model’s novel prediction that returns correlate with idiosyncratic volatility during intervals between large switches in idiosyncratic volatility. Using an event study methodology, we investigate the difference in 5-month average returns around the month in which stocks experience large changes in $IVol$. We find that post-switch returns are higher than pre-switch returns for stocks experiencing up switches in idiosyncratic volatility, whereas returns go in the opposite direction for stocks experiencing down switches. The switch effect is also stronger for more real option intensive firms and firms that undergo greater variability in idiosyncratic volatility in accordance with our model.

Several papers motivate our study. Berk, Green, and Naik (1999) were among the first to establish a correspondence between corporate investments and equity returns to explain anomalous regularities in the cross-section of stocks.\(^{16}\) Since then, the literature has been extended in many directions (Carlson, Fisher, and Gianmarino (2004), Cooper (2007), Zhang (2005), Hackbarth and Mauer (2012)). This literature demonstrates the value of firms evolve in response to optimal corporate investment decisions, giving rise to observable characteristics that proxy for time-varying risk premia.\(^{17}\) We expand the description of the operating environment of the firms by introducing stochastic idiosyncratic cash flow risk following the work in Guo, Miao, and Morellec (2005). Our work differs from Guo et al. (2005) however, since our focus is on the asset pricing implications of idiosyncratic volatility – instead of real investment – and we conduct extensive

\(^{15}\)For brevity, the empirical results for the positive $IVol$-return relations are discussed in a separate Online Appendix.

\(^{16}\)Fama and French (1993) provide evidence on the ability of a size factor and a book-to-market factor to explain returns. Fama and French (1992) and Fama and French (1996) provide a cross-sectional view of how average returns vary across stocks.

\(^{17}\)Carlson et al. (2006), Anderson and Garcia-Feijoo (2006) and Carlson et al. (2010) provide empirical evidence consistent with this literature.
empirical tests to verify the plausibility of our model.

To the best of our knowledge few inroads have been made to link idiosyncratic risk to asset pricing without appealing to market frictions or behavioral biases. Babenko, Boguth, and Tserlukevich (2013) view firms as portfolios of separate systematic and idiosyncratic divisions and rely on additivity of systematic and idiosyncratic cash flow shocks in the valuation of firms. Kogan and Papanikolaou (2013) model growth firms with greater sensitivity to technology shocks that are investment-specific to generate a lower risk premia. Yet a third paper, Herskovic, Kelly, Lustig, and Nieuwerburgh (2015), shows correlated shocks to idiosyncratic volatility across firms can be priced in incomplete markets with heterogeneous agents. The main driving force behind the results of our paper is time-varying stochastic idiosyncratic cash flow risk – not cash flow shocks – in the presence of real options. The underlying mechanism in our model is distinct from Babenko et al. (2013) and Kogan and Papanikolaou (2013), and permits a novel channel between the operating environment faced by the firms and equity returns leading to distinct asset pricing dynamics amenable to features observed empirically, such as heteroscedasticity, heavy-tails, return continuation, and return reversals. Our paper is also distinct from Herskovic, Kelly, Lustig, and Nieuwerburgh (2015). We assume independent shocks, as opposed to correlated shocks, to idiosyncratic volatility across firms which play a prominent role in returns through real options to explain a larger set of IVol-related anomalies. Importantly, we also find empirical support for the predictions of our model such as the switch effect, which are novel relative to the extant literature, further supporting the plausibility of our model.

The rest of the paper is organized as follows. Section 2 describes the key return properties necessary for the simultaneous resolution of the positive and negative IVol-return relations. Section 3 develops a model of firms with real options and stochastic idiosyncratic cash flow risk capturing

---

18 For limits to arbitrage related to short selling constraints and how they can lead to the negative IVol anomaly, see Pontiff (2006). For how investors’ cognitive biases can lead to mispricings in financial markets, see Daniel, Hirshleifer, and Subrahmanyam (1998).

19 Additivity implies that favorable idiosyncratic shocks decrease the importance of systematic cash flows leading to a lower risk premia.

20 In similar fashion to the results of our model, Das and Sundaram (1999) show asset returns must exhibit both heteroscedasticity and discontinuous jumps to empirically fit the heavy-tailed distributions in returns. For three-way relation between stock returns, idiosyncratic volatility and expected return skewness, see Boyer, Mitton, and Vorkink (2010). For continuation and reversals in stock returns, see Jegadeesh and Titman (1993) and Jegadeesh (1990), respectively.

21 For brevity, the focus of this paper is intentionally confined to the IVol-related puzzles. Further investigation of these other interesting extensions is left for future research.
the mentioned key return properties for the resolution of the IVol-return relations. Section 4 takes the predictions of the model and show empirical support in the data. Section 5 concludes. A separate Online Appendix contains all the proofs and derivations, other technical details, and additional empirical results omitted in the main body of the paper.

2 Key Properties of Returns: A Thought Experiment

In this section we describe properties of returns which are necessary and sufficient conditions for both the positive and negative IVol-return relations to hold simultaneously. By doing so, we can compare and contrast the modelling approach used in this paper with other approaches used in the literature.

First, assume the existence of a stochastic discount factor (SDF), $\pi$, such that

$$
\frac{d\pi_t}{\pi_t} = -r_t dt - \sum_{n=1}^{N} \Theta_{n,t} dB_{n,t}^{sys},
$$

(2.1)

The riskfree rate $r_t$ may be stochastic, and $B_{n,t}^{sys}$, $n \in \{1, \ldots, N\}$ are mutually orthogonal standard Brownian motions under the physical probability measure $\mathbb{P}$, each corresponding to some risk factor, where $\Theta_{n,t}$ is the associated price of risk, which can be stochastic. This way we make it clear that (2.1) nests models such as Bansal and Yaron (2004) and Kogan and Papanikolaou (2013), where the conditional CAPM does not hold. We can of course define a composite risk factor, $dB_t^{sys}$ and a composite price of risk, $\Theta_t$, via

$$
dB_t^{sys} = \sum_{n=1}^{N} \frac{\Theta_{n,t}}{\Theta_t} dB_{n,t}^{sys}, \quad \Theta_t = \sqrt{\sum_{n=1}^{N} \Theta_{n,t}^2}.
$$

(2.2)

Equity returns on a cross-section of firms, $k \in \{1, \ldots, K\}$ are exposed to aggregate risks and idiosyncratic risk. The equity return for Firm $k$ is given by

$$
dR_{k,t} = \mu_{k,t} dt + \sum_{n=1}^{N} \sigma_{n,k,t} dB_{n,t}^{sys} + \sigma_{id} dB_{n,t}^{sys},
$$

(2.3)

where the conditional expected return $\mu_{k,t}$ and the conditional volatilities $\sigma_{n,k,t}$ may be stochastic,
$dM_{id,k,t}^t$ is the increment in some idiosyncratic risk factor for Firm $k$, and $\sigma_{id,k,t}^t$ denotes the conditional idiosyncratic volatility. The idiosyncratic risk factors and conditional idiosyncratic volatility are independent across firms.\footnote{\textit{M}_{id,k,t}^t$ is a martingale under the physical probability measure $P$ and may be continuous, e.g. a standard Brownian motion or discontinuous such as a compensated Poisson process.}

Since $\pi$ is a SDF, the basic asset pricing equation holds, i.e.

$$E_t[dR_{k,t} - r_t dt] = -E_t \left[ dR_{k,t} \frac{d\pi_t}{\pi_t} \right], \quad (2.4)$$

which implies

$$\mu_{k,t} = r_t + \sum_{n=1}^{N} \sigma_{sys,n,k,t}^t \Theta_{n,t} = r_t + \sigma_{sys,k,t}^t \Theta_t, \quad (2.5)$$

where $\sigma_{sys,k,t}^t = \sqrt{\sum_{n=1}^{N} (\sigma_{sys,n,k,t}^t)^2}$.

So far, we have not considered any novel assumptions geared towards resolving the IVol anomalies. We now do so by considering the following properties of returns for firms $k \in \{1, \ldots, K\}$.

\textbf{Property 1} The idiosyncratic risk factor for Firm $k$’s returns is also a risk factor for the firm’s idiosyncratic volatility, i.e.

$$d\sigma_{id,k,t}^t = a_t dt + b_t dM_{id,k,t}^t, \quad (2.6)$$

where $b_t > 0$.

The above property states that unexpected changes in a firm’s idiosyncratic volatility are driven by the same factor as idiosyncratic changes in the firm’s unexpected return.

\textbf{Property 2} Systematic risk factors for Firm $k$’s returns are risk factors for the firm’s idiosyncratic volatility, i.e.

$$d\sigma_{id,k,t}^t = a_t dt + \sum_{n=1}^{N} b_{n,t} dB_{n,t}^{sys}, \quad (2.7)$$

where $\sum_{n=1}^{N} b_{n,t} \sigma_{sys,n,k,t}^t > 0$.

The above property states that unexpected changes in a firm’s idiosyncratic volatility are driven by the systematic factors, which drive changes in the firm’s unexpected return.
Property 3 Firm $k$’s systematic volatility is a decreasing function of its idiosyncratic volatility.

The above property states that firm’s with higher IVol will have lower systematic volatility.

Proposition 1 The positive IVol-return relation will be satisfied if and only if one or both of Properties 1 and 2 hold.

Provided that the composite price of risk is independent of firms’ idiosyncratic volatilities, the negative IVol-return relation will be satisfied if and only if Property 3 holds.

Observe that the dynamics of abnormal returns are given by

$$dR_{k,t}^a = \sigma_{k,t}^{sys} dB_{t}^{sys} + \sigma_{k,t}^{id} dM_{t}^{id}. \quad (2.8)$$

We can now see that Properties 1 and 2 are sufficient conditions for the correlation between changes in IVol and changes in abnormal returns to be positive, i.e. for the positive IVol anomaly to hold. Proposition 1 also shows that Properties 1 and 2 are necessary conditions.

The second part of Proposition 1 relates to the negative IVol-return relation. Idiosyncratic volatility is proxied by observed abnormal return volatility over a month. Firms are then sorted into portfolios based on this measure. How would such portfolios perform over the next month?

Consider a firm for which idiosyncratic volatility is currently high. The expected return for this firm is given by

$$E_t[dR_{k,t}] = (r_t + \sigma_{k,t}^{sys} \Theta_t) dt, \quad (2.9)$$

where, from Property 3, $\sigma_{k,t}^{sys}$ is a decreasing function of $\sigma_{k,t}^{id}$. Property 3 implies that firms with higher IVol will have lower expected returns, i.e. the negative IVol-return relation will hold. In portfolio sorts, Property 3 ensures that higher IVol portfolios perform poorly relative to lower IVol portfolios. Proposition 1 also shows that provided the composite price of risk, $\Theta_t$, is independent of firm-level idiosyncratic volatilities. Property 3 is also a necessary condition for the negative IVol-return relation to hold.

In light of Proposition 1, one would expect that theoretical models seeking to explain the positive and negative IVol-return relations will satisfy either Properties 1 and 3 or Properties 2 and 3. This is indeed the case.
Both Kogan and Papanikolaou (2013) and Herskovic, Kelly, Lustig, and Nieuwerburgh (2015) develop dynamic asset pricing models, which satisfy Property 2. By contrast, we do not use a model where idiosyncratic volatility is impacted by systematic shocks. Instead we rely on Properties 1 and 3. Babenko et al. (2013) develop a model, which satisfies Property 3, purely because firms’ values are not multiplicatively separable functions of systematic and idiosyncratic cash flows, and not because of the presence of real options. They then take the additional step of introducing real options to ensure the sensitivity of firm value to idiosyncratic cash flows rises when idiosyncratic cash flows rise, which implies that Property 1 is satisfied. Our model differs from Babenko et al. (2013) on two fronts. First, we need only one assumption to ensure Properties 1 and 3 are satisfied: firms experience stochastic idiosyncratic cash flow risk in an otherwise standard model of a cross-section of firms with growth options. Second, the presence of real options is crucial for understanding the negative IVol-return relation, whereas in Babenko et al. (2013) real options are irrelevant for the negative IVol-return relation. Importantly, we find strong empirical support for our model’s real option channel – when the intensity of real options in a firm is higher, the negative IVol-return relationship is significantly stronger.

3 Model

In this section, we construct a growth option model similar in spirit to the models in Garlappi and Yan (2008) and Carlson, Fisher, and Giammarino (2004) while incorporating the key properties that help resolve the IVol-return relations.23

3.1 The Environment

Assume a finite number of two types of all-equity firms: young and mature. Mature firms produce at full capacity, while young firms produce at a lower operating scale but have the option to make an irreversible investment to increase production and become mature. For tractability, we assume that investments are financed with equity capital. Each firm starts out as young, but assume, without loss of generality, at moment $t$ there is an equal distribution of both. To ensure

\footnote{With no loss of generality, we rely specifically on growth options to incorporate convexity of firm valuations in the firms’ output price. Other forms of real options that incorporate convexities would accommodate similar results.}
that mature firms do not dominate the industry overtime, we assume mature firms face a non-

systematic risk of exiting the industry and being replaced by a new young entrant. This risk arrives

as an independent Poisson event which occurs with probability \( \lambda_M dt \) within the infinitesimally
time \( dt \).\(^{24}\)

Each Firm \( k, k \in \{1, \ldots, K\} \), produces a single commodity that can be sold at time-\( t \) in the

product market for price \( P_{k,t} \)

\[
P_{k,t} = X_{k,t}Z_t,
\]

(3.1)

where \( X_k \) and \( Z \) are the idiosyncratic and systematic components with dynamics

\[
\frac{dX_{k,t}}{X_{k,t}} = \sigma_{id,k,t}^i dB_{k,t}^id,
\]

\[
\frac{dZ_t}{Z_t} = \mu dt + \sigma_{sys}^t dB_{t}^sys,
\]

\[
(3.2)
\]

\( \mu \) denotes the constant growth rate, \( \sigma_{sys}^t \) the constant systematic volatility, \( \sigma_{id,k,t}^i \) the stochastic

idiosyncratic volatility, and \( dB_{k,t}^id \) and \( dB_{t}^sys \) are the increments of two independent Brownian

motions. \( dB_{k,t}^id \) is independent across firms.

The novel feature of the model is the presence of uncertainty shocks (see for example Bloom

(2009)) in the idiosyncratic volatility of the price process. In other words, idiosyncratic cash flow

risk is stochastic. The economic rationale for the assumption comes from allowing firms to have

random and time-varying potential to realize monopolistic rents.\(^{25}\)

We assume \( \sigma_{id,k,t}^i \) follows a 2-state Markov chain: \( \sigma_{id,k,t}^i \in \{\sigma_{L}^i, \sigma_{H}^i\} \), where \( 0 < \sigma_{L}^i < \sigma_{H}^i \) and

the probability of entering state \( s_{k,t} \in \{L, H\} \) within the infinitesimally small time \( dt \) is \( \lambda_{s_{k,t}} dt \).\(^{26}\)

Each firm has its own idiosyncratic volatility state and Markov chain, which is i.i.d. across firms

\(^{24}\)An earlier version of the paper that included endogenous entry and exit decisions by firms in a competitive

product market equilibrium model produced qualitatively identical asset pricing results. The generalization would

come at a cost of analytical tractability without adding to the underlying economics.

\(^{25}\)Dixit and Pindyck (1994) and Caballero and Pindyck (1996) show that idiosyncratic shocks translate to a

firm’s ability to retaining monopolistic rents – a firm that experiences a positive idiosyncratic technology shock

experiences an advantage that cannot be stolen by its competitors, while a positive aggregate shock is shared with

the firm’s competitors. Some micro-economic examples of changes in idiosyncratic cash flow risk are, among other

things, shifts in consumer needs and tastes, changes in production technology, and changes in the general operating

environment of the firm or industry.

\(^{26}\)Generalizing to an \( N \)-state Markov chain while preserving analytic solutions is possible, but would not add
to the underlying economics. It would also have been possible to assume that \( \sigma_{id,k,t}^i \) follows a continuous process,

but this generalization would come at the cost of analytical tractability, again without adding to the underlying

economics.
– hence the appearance of the subscript $k$ on $\sigma_{id}^{k,t}$ and $s_{k,t}$, but not on $\sigma_{id}^{L}$ and $\sigma_{id}^{H}$. We subscript quantities with $s_{k,t} \in \{H, L\}$ throughout to denote dependence on Firm $k$’s volatility regime.

Following several papers investigating the cross-section of equity returns (Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), Zhang (2005) and Livdan, Sapriza, and Zhang (2009)), we assume a pricing kernel with the following process:

\[
\frac{d\pi_t}{\pi_t} = -r dt - \Theta dB_{sys}^t
\]

(3.3)

where $\Theta = \frac{\mu_S - r}{\sigma_S}$ is the constant market price of risk, and $r$, $\mu_S$ and $\sigma_S$ denote respectively the risk-free rate, the market rate of return, and the market return volatility.\(^{27,28}\)

We can carry out the valuation of firms under the risk-neutral measure $Q$. Working under $Q$ changes the dynamics of the systematic component of the product market price to

\[
\frac{dZ_t}{Z_t} = \hat{\mu} dt + \sigma_{sys} dB_{sys}^t,
\]

(3.4)

where the risk-neutral drift, $\hat{\mu} = \mu - \sigma_{sys} \Theta$, is by assumption strictly less than the risk-free rate, $r$, and $d\hat{B}_{sys}^t = \Theta dt + dB_{sys}^t$ is the increment of a standard Brownian motion under $Q$.

### 3.2 The Value of a Mature Firm

We now derive the value of mature firms. For convenience, we omit firm subscripts throughout the rest of this section.

The cash flow of a mature firm stems solely from the output produced by the assets-in-place. Denote $A_M(P_t)$ the value of assets-in-place which produce a unit of output per unit time for a mature firm. The cost of producing a unit of output is $c$ per unit of time and so the profit per unit time is $P_t - c$. Therefore,

\[
A_{M,t} = A_M(P_t) = E_Q^t \left[ \int_t^\infty e^{-(r+\lambda_M)(u-t)}(P_u - c)du \right] = \frac{P_t}{r - \hat{\mu} + \lambda_M} - \frac{c}{r + \lambda_M}.
\]

(3.5)

\(^{27}\) The existence of the pricing kernel implicitly assumes investors in the market can trade a risk-free asset $B_t$ and a risky security $S_t$ whose price processes are given by $\frac{dB_t}{B_t} = r dt$ and $\frac{dS_t}{S_t} = \mu_S dt + \sigma_S dB_{sys}^t$ respectively.

\(^{28}\) A constant risk-free rate and market price of risk is purely for clarity. We could accommodate both a stochastic risk-free rate and market price of risk, but they are not central to the economics underlying this paper.
Since mature firms operate at scale $\xi_M$, their profit flow is $\xi_M(P_t - c)$ and the firm value is $\xi_M A_{M,t}$, which, as shown, is independent of the volatility state.

### 3.3 The Value of a Young Firm

Young firms derive value from the sum of assets-in-place and a growth option. Following the same steps as for mature firms, the value of the assets-in-place of a young firm is given by $Y_A_{Y,t}$, where

$$A_{Y,t} = A_Y(P_t) = E_t^Q \left[ \int_t^\infty e^{-r(u-t)}(P_u - c) du \right] = \frac{P_t}{r-\mu} - \frac{\xi}{r}.$$

Denoting $G(s_t) = G(P_t, s_t)$ the value of the growth option and summing together, a young firm’s total firm value is

$$V_{Y,s,t} = V_{Y,s,t} = \xi_Y A_{M,t} + G_{s,t}. \quad (3.6)$$

The growth option allows the firm to optimally increase production scale by $\xi = \xi_M - \xi_Y$. At the moment the option is exercised, $\tau$, the value of the young firm’s assets-in-place increases by $\xi_M A_{M,\tau} - \xi_Y A_{Y,\tau}$. After exercising the option, $t > \tau$, the young firm is mature. Therefore, the growth option value is merely the value of the incremental increase in assets-in-place. That is, $G_t = \xi_M A_{M,t} - \xi_Y A_{Y,t}$, and therefore $V_{Y,t} = \xi_Y A_{Y,t} + (\xi_M A_{M,t} - \xi_Y A_{Y,t}) = \xi_M A_{M,t}$.

Prior to exercise, $t < \tau$, the expected present-value of the payoff $\xi_M A_{M,\tau} - \xi_Y A_{Y,\tau} - I$ gives the value of the growth option

$$G_{s,t} = G_{s,t} = E_t^Q \left[ e^{-r(t-s_t)}(\xi_M A_{M,\tau} - \xi_Y A_{Y,\tau} - I) | s_t \right], t \leq \tau, \quad (3.7)$$

which depends on the volatility regime. This is indicated by $E_t^Q [\cdot | s_t]$, the expectation operator under Q conditional on date-$t$ when the current volatility regime is $s_t$. We prove the following in the Online Appendix:

**Proposition 2** In its low idiosyncratic volatility state, a young firm’s growth option value is given by

$$G_{L,t} = \frac{1}{l_1 - l_2} \left( \delta_L(l_2) \left( \frac{P_t}{P_L^*} \right)^{l_1} \right) \left( \frac{P_t}{P_L^*} \right)^{l_2}, P_t^{\max} < P_L^* \quad (3.8)$$

and in its high idiosyncratic volatility state, the growth option value is given by
where $P^*_t = \sup_{t \geq 0} \{ P_u : u \in [0, t) \}$ is the firm’s maximum output price, $l_2 > l_1 > 0$ are the positive roots of the quartic $q_L(l)q_H(l) - \lambda_L \lambda_H = 0$, $j_1 > j_2$ are the roots of the quadratic $q_H(j) = 0$, the optimal investment thresholds $P^*_i, i \in \{ L, H \}$, are the solution to the system of nonlinear algebraic equations (S-32) and (S-33), and $q_L(l), q_H(l), \delta_L(l), \epsilon(l)$ and $\delta_H(j)$ are algebraic expressions given in the Online Appendix.

The growth option is a convex function of $P_t$ since $l_1, l_2, j_1$ and $j_2 > 0$. The convexity of the option value with respect to $P_t$ ensures the optimal decision to expand, and hence young firm value, depend on the volatility regime $s_t$ (Guo, Miao, and Morellec (2005)). This is an important valuation property that young firms inherit due to Jensen’s inequality; a feature that contrast starkly from mature firms. In the model, Jensen’s inequality ensures that, ceteris paribus, the value of a growth option is increasing in the volatility of the output price. Hence, the option is worth less in the low idiosyncratic volatility state, making it optimal to exercise the option earlier, i.e. $P^*_L < P^*_H$, since the action entails forfeiting a lower-valued asset in exchange for the incremental increase in assets-in-place. In the low estate, exercise occurs as soon as the price process hits the threshold $P^*_L$ from below. In the high state, exercise occurs if the price process hits the threshold $P^*_H$ from below or, if $P_t \geq P^*_L$, as soon as the volatility state switches to low, whichever comes first. The latter scenario corresponds to the optimal decision to investment instantaneously independent of changes in the level of the output price, but purely from the fall in idiosyncratic volatility.

A stronger Jensen’s inequality associated with a higher idiosyncratic volatility also means that the value of a young firm’s growth option value jumps upward when idiosyncratic volatility shifts up. This contrasts starkly from assets-in-place because their value is independent of idiosyncratic volatility. Therefore, the returns of a young firm exhibit jumps which are attributed entirely to the dependence of the growth option on idiosyncratic volatility. Looking ahead to this, this
feature of the model, coupled with stochastic idiosyncratic risk, will prove to be essential for the simultaneous resolution of several IVol-return relations.

Insert Figure 1 here

Figure 1 depicts the ideas conveyed in Proposition 2 assuming different values of $\sigma^id_H$ and $\sigma^id_L$. Comparing the graphs across panels reveals that the growth option, as well as the investment threshold, has a higher value in the high volatility state than in the low state, and the difference across regimes is increasing in the spread between $\sigma^id_H$ and $\sigma^id_L$. The last panel shows the model leads to a single valuation profile if $\sigma^id_H = \sigma^id_L$, which is the usual specification in standard growth option models.

3.4 Implications for Returns

Having discussed the valuation, we turn our attention to the model-implied returns.

The return on a mature firm is independent of the production scale because the value of the firm exhibits constant returns with respect to production scale. A young firm’s assets-in-place exhibits the same property, therefore it makes more sense to refer to the return on assets-in-place, which we denote by $dR_{A,t}$ and given by

$$dR_{A,t} = \left[ r + (1 + L(P_t))\Theta \sigma^{sys} \right] dt + (1 + L(P_t))\sigma^{sys} dB^{sys}_t + (1 + L(P_t))\sigma^id_t dB^{id}_t,$$ (3.10)

where $L(P_t) = \frac{\zeta}{\zeta - \tau}$.

Equation (3.10) is intuitive. $\sigma^{sys}$ denotes the systematic risk of the product market, a constant by assumption, which multiplied by the market price of risk, $\Theta$, gives the implicit product market risk premium. The profits of the firm are net of fixed costs, therefore the expected return and volatility of assets-in-place are amplified by operating leverage, which is captured by $L(P_t)$.

We now look at the returns of a young firm $dR_{Y,s,t}$, which is the weighted average of the return

\[29\] The choice of parameter values used in the calibration of the model is discussed separately in the Online Appendix.
on its assets-in-place and growth option, i.e.

\[ dR_{Y,s_t} = \left(1 - \frac{G_{s_t}}{V_{Y,s_t}}\right) dR_{A,t} + \frac{G_{s_t}}{V_{Y,s_t}} \frac{dG_{s_t}}{G_{s_t}}. \]  

(3.11)

where \( \frac{dG_{s_t}}{G_{s_t}} \) is the return on the firm’s growth option. Denote the idiosyncratic volatility state at date-\( t \) just before a change by \( s_{t-} \) and just after by \( s_t \). We prove the following in the Online Appendix:

**Proposition 3** The growth option return is given by

\[ \frac{dG_{s_t}}{G_{s_t}} = \mu_{G,s_t} dt + \Omega_{s_t} \left( \sigma_{s_t} dB^{sys}_t + \sigma_{s_t}^{id} dB^{id}_t \right) + \frac{G_{s_t} - G_{s_t-}}{G_{s_t-}} dM_{st- st,t}^{id}, \]  

(3.12)

where

\[ \mu_{G,s_t} = \Omega_{s_t} + \frac{G_{s_t} - G_{s_t-}}{G_{s_t-}} \lambda_{s_t} + \frac{1}{2} \frac{P_t^2}{G_{s_t-}} \left( \frac{\partial G_{s_t}}{\partial P_t} \right)^2 \left( \sigma_{s_t}^{sys} \right)^2 + \left( \sigma_{s_t}^{id} \right)^2, \]  

(3.13)

\[ \Omega_{s_t} = \frac{P_t}{G_{s_t-}^{2/3}} \frac{\partial G_{s_t}}{\partial P_t} \] is the elasticity of the growth option with respect to \( P_t \), and

\[ dM_{st- st,t}^{id} = dN_{st- st,t}^{id} - \lambda_{s_t} dt \]  

(3.14)

is a compensated Poisson process and hence a discontinuous martingale driven by changes in the young firm’s idiosyncratic volatility state, i.e. \( dN_{st- st,t}^{id} = 0 \) if \( s_t = s_{t-} \), or \( dN_{st- st,t}^{id} = 1 \) if \( s_t \neq s_{t-} \).

In the no action region, \( P_t^{max} < P_t^* \), the growth option’s elasticity with respect to the output price is lower when idiosyncratic volatility is high, i.e. \( \Omega_L > \Omega_H \), but the idiosyncratic volatility is higher when idiosyncratic volatility is high, i.e. \( \sigma_L^{id} \Omega_L < \sigma_H^{id} \Omega_H \), and the average return when there is no change in idiosyncratic volatility is higher when idiosyncratic volatility is high, i.e.

\[ \mu_{G,L} - \frac{G_L - G_H}{G_H} \lambda_L > \mu_{G,L} - \frac{G_H - G_L}{G_L} \lambda_H. \]

Proposition 3 shows that the growth option return is driven by three components. The first two terms of (3.12) correspond to the drift and diffusion terms common in standard continuous diffusion processes. The third term captures the jumps in returns which occurs whenever there is a switch in idiosyncratic volatility regime. Since the increment in the idiosyncratic risk factor \( dM_{st- st,t}^{id} \) also drives changes in idiosyncratic volatility, Property 1 is satisfied for the return of
growth options. The size of the jumps is proportionate to the change in the value of the option from the switch in volatility. The distinctiveness of the value of the option across volatility states establishes the concurrence between switches in idiosyncratic volatility regime and jumps in returns which happen in the same direction. This property of growth option returns is not present in assets-in-place or mature firms.

It is worth distinguishing the return dynamics (3.12) from other standard mixed jump-diffusion processes commonly found in option pricing (see Merton (1976), for example). Standard option pricing models assume a mixed jump-diffusion process where the jumps are drawn from a known distribution with a known jump amplitude for the underlying asset of the option. By contrast, the returns of a growth option in our model experiences jumps from the switches in idiosyncratic volatility of the underlying asset while the underlying asset does not exhibit jumps. This property of the model helps resolve the positive IVol puzzle.

Proposition 3 also reveals that as idiosyncratic volatility $\sigma^{id} \Omega$ rises, the growth option’s elasticity $\Omega$ falls and therefore the option’s systematic volatility $\Omega \sigma^{sys}$ decreases. This establishes a negative correspondence between idiosyncratic return volatility and systematic risk for growth options. The intuition follows from standard option pricing results – when volatility increases the value of an option increases but its sensitivity to changes in the underlying falls, and so does the exposure to the systematic risk of the underlying assets of the option. This drives down the expected return in the high idiosyncratic volatility state. Hence, Property 3 is satisfied for the return of growth options. The dependence of growth options on the idiosyncratic volatility state leads to differences in expected returns across idiosyncratic volatility regimes.

We also prove the following proposition in the Online Appendix:

**Proposition 4** A young firm’s conditional systematic return volatility is given by

$$\sigma_{R_Y, s_t}^{sys} = \left[ \left( 1 - \frac{G_{s_t}}{V_{Y, s_t}} \right) (1 + L(P_t)) + \frac{G_{s_t}}{V_{Y, s_t}} \Omega_{s_t} \right] \sigma^{sys},$$  

(3.15)

This result is similar to a result in Johnson (2004). Johnson shows that increasing uncertainty about the value of a firm’s assets while holding the risk premium constant lowers the expected returns of levered firms.
and is lower in the high idiosyncratic volatility regime, i.e.

\[
\sigma_{R_Y,H}^{sys} < \sigma_{R_Y,L}^{sys}
\]  

(3.16)

Similarly to a portfolio of two assets, the conditional systematic risk of a firm is a weighted average of the systematic risk of assets-in-place and the growth option. The expected return on assets-in-place is amplified by operating leverage, which is captured by \( L(P_t) \). \( L(P_t) \) relates inversely to \( P_t \) and becomes more prominent for firms that derive a greater proportion of firm value from assets-in-place as opposed to growth options. Differences in operating leverage can contribute to creating a value premium in the cross section of stock returns.

Growth options also play an amplifying role in the systematic risk of firms. As levered positions on underlying assets, options are riskier than revenues, i.e. \( \Omega_{st} > 1 \). Finite opportunities to expand adds to the importance of growth options for young firms. Young firms derive greater value from growth opportunities than mature firms, and hence they are more sensitive to the risk of growth options, separately contributing to the size effect in the cross section of returns.\(^{31}\)

The novel feature of the model is the dependence of young firms’ expected returns on idiosyncratic volatility. Proposition 4 generalizes the inverse relation between idiosyncratic risk and expected return to young firms by accounting for the effect of a change in the weight \( \frac{G_{st}}{V_{Y,s_t}} \) in the value of the firm. Intuitively, a higher valuation due to a stronger Jensen’s inequality coming from a higher idiosyncratic factor implies that the young firm has a lower exposure to the systematic risk of \( P_t \), and hence a lower expected return. Since idiosyncratic volatility has an effect on systematic risk through growth options only, mature firms do not have the same inverse relation between idiosyncratic volatility and systematic risk. To summarize, Propositions 3 and 4 together endogenously establish a positive contemporaneous correlation between a young firm’s idiosyncratic risk and returns, but a negative correlation between idiosyncratic risk and future expected returns.

For tractability, we assume that expansions are financed with equity capital. In the absence of debt financing, exercising growth options kills the option-induced leverage (and risk premia), and hence eliminates the dependence of the risk premium on stochastic idiosyncratic volatility.

\(^{31}\)See Carlson et al. (2004) for a thorough explanation of how the size and book-to-market effects arise in the context of the model.
This is how the model distinguishes young firms from mature firms. A question arises naturally: What would happen in our model if investments are partially or fully financed with debt capital? This is a sensible question since firms can choose to finance expansions with borrowed funds.

In the presence of debt financing, the equityholders of young firms would see a rise in financial leverage as firms exercise growth options, leading to a replacement of option-related leverage with financial leverage. Since the equity of an indebted firm is akin to a call option (Merton (1974) and Merton (1992)), allowing expansions to be financed with debt would reintroduce the dependence of the equity of the firm on stochastic idiosyncratic volatility. Therefore, equity values and equity returns would continue to exhibit dependence on stochastic idiosyncratic volatility after expansions. Since the extent of this dependence hinges on the amount of debt financing used to fund expansions, we conjecture that post-expansion equity returns should exhibit a stronger relation with idiosyncratic volatility the greater the amount of debt chosen to finance the expansion. Indeed, to study the effects of debt financing, in the empirical section of the paper we report the results for an empirical real option intensity variable (\textit{vega}) that captures the equity exposure of firms to financial leverage.

It is also worth distinguishing our model from the extant literature focusing on the negative $IVol$-return relation. By viewing firms as portfolios of separate systematic and idiosyncratic divisions, Babenko, Boguth, and Tserlukevich (2013) rely on additivity of systematic and idiosyncratic cash flow shocks in the valuation of the firms. Additivity implies that favorable idiosyncratic shocks decrease the relative importance of the systematic cash flow component leading to a lower risk premia for firms. Kogan and Papanikolaou (2013), by contrast, model high growth firms as having investment opportunities with higher sensitivity to investment-specific technology shocks to earn a lower risk premia. We view Babenko, Boguth, and Tserlukevich (2013) and Kogan and Papanikolaou (2013) as complementary since the inverse correspondence between risk premia – and hence expected returns – and idiosyncratic risk is their models are cash flow-based. The main driving force behind the results of our paper is \textit{time-varying stochastic} idiosyncratic cash flow risk – as opposed to idiosyncratic cash flow shocks – hence the underlying mechanism is distinct from Babenko et al. (2013) and Kogan and Papanikolaou (2013). Looking ahead, we discuss below that the distinct feature of our model permits a novel channel between the operating environment faced by the firms and equity returns leading to empirically observed asset pricing dynamics, such
as, among others, jumps, return skewness, heteroscedasticity, heavy-tails and return continuation.

Insert Figure 2 here

Figure 2 depicts the ideas conveyed in Propositions 3 and 4. Panel (a) depicts a negative difference in systematic volatility ($\Omega_H - \Omega_L$)$\sigma^{sys}$ between volatility states, while the difference in total return volatility $\Omega_H \sqrt{ (\sigma^{sys})^2 + (\sigma^{id})^2 } - \Omega_L \sqrt{ (\sigma^{sys})^2 + (\sigma^{id})^2 }$, and continuous drift terms $\mu_{G,H}^{C} - \mu_{G,L}^{C}$ are positive as shown in Panels (b) and (c). Panel (d) shows a negative difference in jump terms $\frac{G_L - G_H}{G_H} - \frac{G_H - G_L}{G_L}$ between volatility states. All the differences are increasing in the spread between $\sigma_{id}^{H}$ and $\sigma_{id}^{L}$, indicating that the relation between returns and idiosyncratic volatility is strengthened by a higher variation in idiosyncratic cash flow volatility. Lastly, the differences in all quantities are identically zero if $\sigma_{id}^{H} = \sigma_{id}^{L}$, which is the usual specification in standard growth option models.

3.5 Model Predictions

We summarize the model predictions for our empirical tests in this section.

Firstly, we show that the model can generate results consistent with the positive IVol-return relation (Duffee (1995)). To see this, from equations (3.10), (3.11) and (3.12) the abnormal return for a young firm can be written as

$$ dR_{Y,s}^{a} = dR_{Y,s} - E_t[dR_{Y,s} | s_{t-}] $$

$$ = \left[ \left( 1 - \frac{G_{s_{t-}}}{V_{Y,s_{t-}}} \right) (1 + L(P_t)) + \frac{G_{s_{t-}}}{V_{Y,s_{t-}}} \Omega_{s_{t-}} \right] (\sigma^{sys} dB_{s_{t-}}^{sys} + \sigma^{id}_{s_{t-}} dB_{s_{t-}}^{id}) + \frac{G_{s_{t-}}}{V_{Y,s_{t-}}} dM_{s_{t-} s_{t-},t}^{id} $$

$$ (3.17) $$

and the change in idiosyncratic volatility can be written as

$$ d\sigma_{s_{t-}}^{id} = \lambda_{s_{t-}} (\sigma_{s_{t-}}^{id} - \sigma_{s_{t-}}^{id}) dt + (\sigma_{s_{t-}}^{id} - \sigma_{s_{t-}}^{id}) dM_{s_{t-} s_{t-},t}^{id}. $$

$$ (3.18) $$

From the above expressions it is easy to see that a single common idiosyncratic factor $dM_{s_{t-} s_{t-},t}^{id}$

 drives changes in unexpected changes in return and idiosyncratic volatility in the same direction.

\(^{32}\)The choice of parameter values used in the calibration of the model is discussed in a separate Online Appendix.
This is precisely Property 1 of Section 2. The other risk factors $dB^s_{t}^{sys}$ and $dB^i_{t}^{id}$ are independent of changes in idiosyncratic volatility. Therefore, according to Proposition 1, the model is consistent with the positive $IVol$-return relation. In empirical tests, if idiosyncratic volatility is estimated sufficiently accurately, the abnormal returns of firms with real options should be positively correlated with changes in their idiosyncratic volatility.

Secondly, we show that the model can simultaneously address the negative $IVol$-return relation (Ang et al. (2006)). To see this, consider the basic asset pricing equation

$$E_t[dR_{Y,t}|s_{t-}] = \left( r + \sigma_{R,s_{t-}}^{sys} \Theta \right) dt. \quad (3.19)$$

Proposition 4 shows that the systematic volatility of the firms with real options is higher when idiosyncratic volatility is lower, i.e. $\sigma_{R,Y,H}^{sys} < \sigma_{R,Y,L}^{sys}$. This is precisely the condition that satisfies Property 3 of Section 2. Therefore, according to Proposition 1, the model is also able to generate the negative $IVol$-return relation.

In empirical tests, portfolio sorts based on end-of-month realized $IVol$ is akin to sorting firms on the most recent idiosyncratic volatility state. Average returns of $IVol$-portfolios reflect expected returns from differences in idiosyncratic volatility. Hence, high $IVol$-portfolios subsequently have lower average returns than low $IVol$-portfolios. Absent proper empirical factors to adjust for systematic risk, differences in returns between $IVol$-portfolios appear in the puzzling guise of abnormal returns. This view of abnormal returns follows several recent papers that study seemingly anomalous return findings related to product market competition (Aguerrevere (2009)), corporate investments (Carlson, Fisher, and Giammarino (2004)), seasoned equity offerings (Carlson, Fisher, and Giammarino (2006)), mergers and acquisitions (Hackbarth and Morellec (2008)), and financial distress (Garlappi and Yan (2011) and Favara, Schroth, and Valta (2012)).

Thirdly, the model justifies a novel $IVol$-return prediction which we call the switch effect. Proposition 3 states that a growth option’s expected return contains a continuous component, i.e.

$$\mu_{G,s_{t-}}^c = \Omega_{s_{t-}} + \frac{1}{2} \frac{P_{t}^2}{G_{s_{t-}}} \frac{\partial^2 G_{s_{t-}}}{\partial P_{t}^2} \left( (\sigma_{s_{t-}}^{sys})^2 + (\sigma_{s_{t-}}^{id})^2 \right),$$
plus a probability-weighted jump term

\[
\frac{G_{s_t} - G_{s_{t-}}} {G_{s_{t-}}} \lambda_{s_t},
\]

where the jump term \( \frac{G_{s_t} - G_{s_{t-}}} {G_{s_{t-}}} \) is not materialized until a switch in \( \sigma_{s_t}^{id} \) occurs. The mean return when there is no change in \( \sigma_{s_{t-}}^{id} \) is positively correlated with idiosyncratic volatility \( \sigma_{s_{t-}}^{id} \), i.e. \( \mu_{G,H} - \frac{G_{L} - G_{H}} {G_{H}} \lambda_{L} > \mu_{G,L} - \frac{G_{H} - G_{L}} {G_{L}} \lambda_{H} \). Therefore, the model is consistent with realized returns correlating positively with idiosyncratic volatility during periods between large changes in \( \sigma_{s_{t-}}^{id} \). For a firm that experiences a positive change in idiosyncratic volatility, realized returns are relatively lower before changes in volatility than after changes. This property of returns is consistent with return momentum followed by strong reversals driven by the jumps which correlate with time-varying idiosyncratic volatility. These properties of asset returns are in accordance with several disparate empirical findings reported in the literature. Das and Sundaram (1999), for example, show asset returns must exhibit both heteroscedasticity and discontinuous jumps to empirically fit heavy-tailed distributions. Boyer, Mitton, and Vorkink (2010) reports the importance of return skewness, and Jegadeesh and Titman (1993) and Jegadeesh (1990) report strong evidence of continuation and reversals in stock returns, respectively.\(^{33}\)

Lastly, since the stochastic idiosyncratic risk channel takes effect through convexities in firm valuations, our model proposes that all three return-volatility predictions should be observed more strongly for firms that incorporate more real options and for firms that undergo greater changes in idiosyncratic risk. These prediction are confirmed quantitatively via numerical simulations and reported separately in the Online Appendix and further verified empirically in Section 4 below.

## 4 Empirical Analysis

The previous section discussed the model yields results in line with empirical relations between \( IVol \) and stock returns. In this section, we test the predictions of the model and show empirically support in the data.

\(^{33}\)For brevity, the focus of this paper is intentionally confined to the \( IVol \)-return relations. Further investigation of these other interesting properties of returns in the context of our model is left for future research.
4.1 Data Source, Variable Description and Summary Statistics

We collect stock returns from January, 1971 to December, 2010 from CRSP daily and monthly stock files.\textsuperscript{34} Data on factor returns and risk-free rates are from Ken French’s website, and accounting data is from COMPUSTAT annual files.\textsuperscript{35} We consider only ordinary shares traded on NYSE, AMEX and Nasdaq with primary link to companies in COMPUSTAT with US data source. Following most, we eliminate utility (SIC codes between 4900 and 4999) and financial companies (SIC codes between 6000 and 6999), companies with less than one year of accounting data, zero stock price, and negative book equity values. In order to rule out confounding effects from exchange delistings (see Shumway (1997)), we exclude returns within one year from the month of delisting if the stock has a delisting code with a first digit different from 1. This leads to a final sample of over 1 million observations of non-missing monthly stock returns and idiosyncratic return volatilities.

4.1.1 Idiosyncratic Return Volatility

We require a measure for idiosyncratic cash flow volatility for each firm in our sample.\textsuperscript{36} Following Ang, Hodrick, Xing, and Zhang (2006) we estimate idiosyncratic return volatility $IVol$ as the standard deviation of the residuals from regressing daily stock returns on the 3 factors of Fama and French (1993) for each firm $j$ and month $t$ in our sample as follows:

$$r_{j,t} = \alpha_i + \beta_{j,MKT}MKT_t + \beta_{j,SMB}SMB_t + \beta_{j,HML}HML_t + \varepsilon_{j,t}$$

where $IVol_{j,t} = \sqrt{var(log(1 + \varepsilon_{j,t}))}$ and $\varepsilon_{j,t}, \tau \in \{t-1, t\}$, is the residual.\textsuperscript{37}

We define $\Delta IVol_{j,t}$ as the month-on-month change in $IVol$, i.e., $IVol_{j,t} - IVol_{j,t-1}$. To capture the stochastic nature of idiosyncratic cash flow volatility, the thresholds for high and low volatility

\textsuperscript{34}Our sample begins in year 1971 because the number of firms with non-missing sales and net income observations is relatively low prior to the 70’s after we apply the reported filters discussed below.

\textsuperscript{35}The link to Ken French’s data library is http:// mba.tuck.dartmouth.edu/ pages/faculty/ken.french/data library.html

\textsuperscript{36}It is standard practice in empirical studies to use stock return volatility as a proxy for cash flow risk (Leahy and Whited (1996); Bulan (2005); Grullon, Lyandres, and Zhdanov (2012)). We provide further justification for using stock returns to work out cash flow volatility in the calibration of the model discussed in the Online Appendix.

\textsuperscript{37}Following Grullon, Lyandres, and Zhdanov (2012), we use the logarithm of the residuals in order to rule out the effects of return skewness on the relation between return and volatility (Duffee (1995); Chen, Hong, and Stein (2001); Kapadia (2007)).
regimes are defined by the 70th ($IVol_{70\text{pctl}}$) and the 30th percentile ($IVol_{30\text{pctl}}$) values of $IVol$ for each stock in the sample, and $\Delta IVol_j$ is defined as the difference to capture the variation in idiosyncratic volatility. In the context of our model, a higher $\Delta IVol$ captures a higher spread between $\sigma_{H}^{id}$ and $\sigma_{L}^{id}$.

4.1.2 Firm Characteristics

For our cross-sectional regressions, we require several controls known in the literature to affect stock returns. Guided by the literature, they are as follows: log market equity; log book-to-market; past stock returns; CAPM beta; and trading volume.\(^{38}\)

4.1.3 Real Option Intensity

We also require empirical measures for real option intensity ($RO$) for each firm in our sample.

The most common type of real options comes in the form of future growth opportunities (Brennan and Schwartz (1985); MacDonald and Siegel (1986); Majd and Pindyck (1987); Pindyck (1988)). Larger and older firms are more mature and have larger proportions of firm value from assets-in-place, while smaller and younger firms derive more value from future growth opportunities (Brown and Kapadia (2007); Carlson, Fisher, and Giammarino (2004); Lemmon and Zender (2010)). We consider market-to-book ratio as a measure of growth opportunities, and firm size and firm age as inverse measures. We capture firm size by book value of total assets, and firm age by the difference between the month of return observation and the month the stock of the firm

\(^{38}\)Following Fama and French (1993), market value of equity is defined as the share price at the end of June times the number of shares outstanding. Book equity is stockholders’ equity minus preferred stock plus balance sheet deferred taxes and investment tax credit if available, minus post-retirement benefit asset if available. If missing, stockholders’ equity is defined as common equity plus preferred stock par value. If these variables are missing, we use book assets less liabilities. Preferred stock, in order of availability, is preferred stock liquidating value, or preferred stock redemption value, or preferred stock par value. The denominator of the book-to-market ratio is the December closing stock price times the number of shares outstanding. We match returns from January to June of year $t$ with COMPUSTAT-based variables of year $t - 2$, while the returns from July until December are matched with COMPUSTAT variables of year $t - 1$. This matching scheme is conservative and ensures that the accounting information-based observables are contained in the information set prior to the realization of the market-based variables. We employ the same matching scheme in all our matches involving accounting related variables and CRSP-based variables. We define past returns as the buy-and-hold gross compound returns minus 1 during the six-month period starting from month $t - 7$ and ending in month $t - 2$. Following Karpoff (1987), trading volume is trading volume normalized by the number of shares outstanding during month $t$. Lastly, stock CAPM beta is the estimated coefficient from rolling regressions of monthly stock excess returns on the market factor’s excess returns. We use a 60-month window every month requiring at least 24 monthly return observations in a given window, and use the procedure suggested in Dimson (1979) with a lag of one month in order to remove biases from thin trading in the estimations.
first appeared in CRSP.

Growth opportunities are capitalized and revealed in the future in the form of increased sales and increased investments (Grullon, Lyandres, and Zhdanov (2012)). Therefore, we consider future sales growth and future investment growth as alternate measures of real option intensity. For each month and stock return observation of a firm in the sample, future growth is defined as the four-year sum of the corresponding growth rates starting 2 years after the month that the stock return is observed.\(^{39,40}\)

It is instructive to look to financial derivatives when searching for real option intensity. In option pricing theory, \textit{vega} captures the sensitivity of an option with respect to the volatility of the underlying stock. Since the equity of an indebted firm can be viewed as a call option on the assets of the firm with the value of debt as the strike price (Merton (1974); Merton (1992)), the \textit{vega} of the equity of an indebted firm should, in principle, capture the equity’s sensitivity to the idiosyncratic volatility of the assets of the firm. Therefore, for each firm \(j\) and year \(n\) in the sample, we rely on the Black and Scholes formula and the firm’s capital structure to compute the equity \textit{vega} as follows:

\[
\text{vega}_{j,n} = V_{j,n} N'(d_{j,n}) \sqrt{5}
\]

where \(d_{j,n} = \frac{\ln \left( \frac{V_{j,n}}{D_{j,n}} \right) + \left( r_{f,n} - \frac{\sigma_{j,n}^2}{2} \right) \times 5}{\sigma_{j,n} \sqrt{5}}\), \(N'(x) = \frac{\exp(-x^2/2)}{\sqrt{2\pi}}, \) \(r_{f,n}\) is the annualized risk free rate, \(\sigma_{j,n}\) is the annualized idiosyncratic return volatility based on the Fama French 3-factor model which is estimated using the previous six-months of return data, \(V_{j,n}\) is the sum of book value of debt \(D_{j,n}\) and market value of equity. We assume a 5-year time to expiration, and hence a 5-year debt maturity. Since \textit{vegas} are relatively invariant over a large range of values of the underlying asset, we classify firms as ‘high’ \textit{vega} if they have an equity \textit{vega} in the top tercile based on breakpoint values found among NYSE firms in our sample.\(^{41,42}\)

\(^{39}\)On the surface, these growth variables may raise concerns of look-ahead bias. The focus of this paper is on investigating the relation between return and volatility, and not on predicting future stock returns. Therefore, look-ahead bias is not a concern in this paper. See Grullon, Lyandres, and Zhdanov (2012) for a similar explanation.

\(^{40}\)Following Grullon, Lyandres, and Zhdanov (2012), we merge observed returns with the growth variables starting two years following the month of the return observation to rule out spurious correlations between growth surprises and returns.

\(^{41}\)The \textit{vega} of a call option is highest when the option is at-the-money, and relatively low and invariant when the the underlying stock is priced away in either direction from the strike price (see Hull (2011)).

\(^{42}\)The percentile breakpoint values of most financial variables tend to be more stable overtime for NYSE firms.
Once the empirical variables are measured as described above, we classify firms as small, growth, young, high sale growth and high investment growth if the corresponding variables have values in the top or bottom tercile, where the tercile breakpoints are determined by NYSE firms in the sample. Then we form real option measures by pairwise-combining the categorical variables, i.e. small growth, young high investment growth, etc. The motivation for combining the empirical proxies is to improve the ability to capture real option intensity.

Certain industries are perceived to have more growth opportunities than others. For our last set of variables, we follow Grullon, Lyandres, and Zhdanov (2012) and categorize firms as ‘natural resource’ if they belong to industries 27 (precious metals), 28 (mining), or 30 (oil and natural gas) based on the Fama and French (FF) industry classifications (Fama and French (1997)). Firms are assigned as ‘high-tech’ if they belong to any one of the FF industries 22 (electrical equipment), 32 (telecommunications), 35 (computers), 36 (computer software), 37 (electronic equipment), and 38 (measuring and control equipment). ‘Bio-tech’ firms are defined by the FF industries 12 (medical equipment) and 13 (pharmaceutical products). Lastly, firms in any one of the three aforementioned industries are assigned as ‘all-growth’.

4.1.4 Summary Statistics

Table 1 reports the summary statistics for the main variables in our sample. The mean (median) excess stock return is 0.9976% (-0.41%) per month or approximately 11.9712% (-4.92%) per annum, and the mean (median) daily idiosyncratic stock return volatility \( IVol \) is 2.9476% (2.2782%), or approximately 44.0171% (34.0208%) annualized. These statistics are similar to those reported in Ang, Hodrick, Xing, and Zhang (2006) and Grullon, Lyandres, and Zhdanov (2012). The sample mean (median) month-on-month change in \( IVol, \Delta IVol \), is -.0023% (-.011%), and the sample standard deviation is 2.1096%. These values are also similar to those reported in Grullon, Lyandres, and Zhdanov (2012).

\[ \text{Insert Table 1 here} \]

than for non-NYSE firms.
4.2 Real Option Intensity, $\Delta IVol$ and IVol-Return Puzzles

This section examines several empirical IVol puzzles motivated by the model and show they consistently go in the predicted direction.\textsuperscript{43}

4.2.1 Negative IVol-Return Relation

The model in section 3 offers a novel risk-based explanation for the negative IVol anomaly. Is the negative IVol relation stronger for more real option intensive firms and firms with a higher variation in idiosyncratic volatility in line with the predictions of the model? Answering this question helps address if the model is plausible.

To verify the predictions, we sort the firms in our sample into two groups, a high real option intensity (the top tercile) group and a ‘all the rest’ group, at the end of each June based on each one of the real option categorical variables, and separately, into three equally-sized groups by IVol at the end of each month.\textsuperscript{44} Then for each month and each one of the two-way classifications of option intensity and IVol, we form value-weighted portfolios and assess their performance over the following month. This approach corresponds to the 1/0/1 (formation period/waiting period/holding period) strategy of Ang, Hodrick, Xing, and Zhang which makes up most of their main analysis.

Portfolio performance is assessed on a risk-adjusted basis relative to the Fama and French 3-factor model:

$$r_t - r_{f,t} = \gamma_0 + \gamma_1 MKTRF_t + \gamma_2 SMB_t + \gamma_3 HML_t + \epsilon_t$$  \hspace{1cm} (4.3)

where $r_t$ is the portfolio return, $r_{f,t}$ is the riskless rate, and $MKTRF$, $SMB$, and $HML$ are the three factors of Fama and French (1993) that proxy for the market risk premium, size and book-to-market, respectively.\textsuperscript{45} Investigating risk-adjusted returns addresses whether differen-

\textsuperscript{43}In addition to the earlier work by Grullon et al. (2012) shown empirically on the positive IVol-return relation and growth options, the model in Section 3 of this paper leads to novel predictions for the correspondence between the positive IVol anomaly and variability in idiosyncratic volatility. For brevity, we do not provide a discussion of our empirical results supporting these predictions in the main text. These results are discussed in the Online Appendix.

\textsuperscript{44}The results based on the continuous real option variables yield qualitatively identical results and hence, for brevity, are not reported in the paper. These results are available from the authors upon request.

\textsuperscript{45}The results from raw returns, risk-adjusted returns relative to the CAPM, and risk-adjusted returns relative to the 4-factor model (Carhart (1997)) all yield qualitatively identical results. These results are available from the authors upon request.
tials in returns are not merely compensation for exposure to common risk factors, and hence addresses whether real option intensity and stochastic idiosyncratic volatility proxy for priced risk. The model predictions translate to a lower risk-adjusted return estimate, $\gamma_0$, for portfolios characterized by higher real option intensity.

**Insert Table 2 here**

Table 2 reports the results. As shown, the general pattern in risk-adjusted returns is decreasing from portfolios of low $IVol$ stocks to portfolios of high $IVol$ stocks, confirming the previous results of Ang, Hodrick, Xing, and Zhang. The table also reveals that the decreasing pattern in risk-adjusted returns is more pronounced (both economically and statistically) for high real option intensity firms. For instance, panel (b) of the table shows that ‘small growth’ stocks experience an annualized difference in risk-adjusted returns of -19.91% between high $IVol$ and low $IVol$ stocks. This contrasts starkly from ‘all the rest’ which has an annualized difference in risk-adjusted returns of -4.84% between extreme $IVol$ portfolios. This pattern in portfolio returns is present for virtually all the reported alternate variables for real option intensity.

Next, we investigate how the negative $IVol$-return relation relates to both real option intensity and variability in idiosyncratic volatility. In addition to the two-way sorts based on $IVol$ and each of the alternate variables for real option intensity, we independently sort the stocks at the end of each month into three equally-sized groups by $\bar{\Delta IVol}$. Then, we assess the performance of the zero-cost portfolio which invests in the highest $IVol$ portfolio from the funds of short selling the lowest $IVol$ portfolio. This strategy is repeated each month and for each of the two-way rank classifications of real option intensity and $\Delta IVol$. The model predictions translate to a lower risk-adjusted return estimate, $\gamma_0$, for portfolios characterized by higher real option intensity and higher $\Delta IVol$.

**Insert Table 3 here**

Table 3 reports the results. As shown in a consistent fashion with the previous results, the risk-adjusted returns of the zero-cost $IVol$ portfolios are generally lower for more real option intensive ($RO$) firms across all $\bar{\Delta IVol}$ ranks. Additionally, the zero-cost $IVol$ portfolios appear
to experience more sizeable negative returns (both economically and statistically) in the sample of higher real option intensity and higher $\Delta IVol$ firms. For instance, panel (b) of the table shows that ‘small growth’ stocks that belong to the top $\Delta IVol$ exhibit an annualized difference in risk-adjusted returns of -16.24%, which is substantially lower than the other $RO \times \Delta IVol$ groups. This pattern in risk-adjusted returns across $RO \times \Delta IVol$ stocks goes in the predicted direction and is present for most of the reported variables for real option intensity.

These results lend strong support for the predictions of our model and demonstrate that the stocks of firms that experience more extreme changes in $IVol$ and incorporate more real options experience a stronger inverse relation between $IVol$ and returns.

4.2.2 The Switch Effect on Returns

The model in section 3 offers another novel prediction on the relation between idiosyncratic risk and returns; that returns should correlate positively with idiosyncratic volatility during intervals between large changes in idiosyncratic volatility (the switch effect). Post-switch returns should be higher than pre-switch returns for stocks experiencing up switches in idiosyncratic volatility. By contrast, the difference between post and pre-switch returns should be negative for stocks that experience down switches.

We rely on event study methodology to verify this prediction. For each firm $j$ and month $t$, we define an up switch in $IVol$ if $IVol_{j,t-1} < IVol_{j,30\text{pctl}}$ and $IVol_{j,70\text{pctl}} < IVol_{j,t}$. Down switches are defined similarly, i.e. if $IVol_{j,t-1} > IVol_{j,70\text{pctl}}$ and $IVol_{j,t} < IVol_{j,30\text{pctl}}$. Once all the up and down switch events are recorded, we compute the difference between the 5-month mean return beginning from the month after the event month and the 5-month mean return ending in the month prior to the event month. Then we investigate how the differences in mean returns around the switch months relate to option intensity. More specifically, we risk-adjust monthly returns according to the Fama and French (1993) 3-factor model as follows:

$$r^*_j,t = r_{j,t} - r_{f,t} - \sum_{k=1}^{3} \beta_{j,k} F_{k,t}$$

where $r_{j,t}$ is the return on stock $j$ in month $t$, $r_{f,t}$ is the risk-free rate, $F_{k,t}$ for $k \in [1, 3]$, denote the

---

46See Chapter 4 of Campbell, Lo, and Mackinlay (1997) for an excellent review on event study analysis.
three Fama and French factors (market, size, and book-to-market), and $\hat{\beta}_{j,k}$ denote the estimated factor loading.\textsuperscript{47,48} Then, for each firm $j$ and event month $t$, the difference in average returns between post- and pre-switch episodes is computed as follows:

$$r_{j,t}^{\text{Diff}} = \frac{1}{5} \sum_{\tau=t+1}^{t+6} r_{j,\tau}^* - \frac{1}{5} \sum_{\tau=t-6}^{t-1} r_{j,\tau}^*.$$  \hfill (4.5)

To investigate how the events relate to stock returns around event months, we run separate cross-sectional return regressions based on the approach of Fama and MacBeth (1973) as follows:

$$r_t^{\text{Diff}} = \gamma_0 t + \gamma_1 RO_t + \eta_t$$  \hfill (4.6)

for each month $t$, each alternate real option intensity measure $RO$, and each of the up and down switch samples, where $r_t^{\text{Diff}}$ is the vector of differences in mean returns around the switch month $t$, $\nu$ is a vector of ones, and $RO_{t-1}$ is the vector of ones and zeros distinguishing high real option intensity firms from ‘all the rest’. The model predictions translate to tests of $\gamma_0 > 0$ and $\gamma_1 > 0$ for the up switch sample, and $\gamma_0 < 0$ and $\gamma_1 < 0$ for the down switch sample.

**Insert Table 4 here**

Table 4 reports the results. As shown, the estimates of $\gamma_0$ are positive for the up switch sample and negative for the down switch sample and highly statistically significant for each regression with an alternate $RO$ variable. The estimates of $\gamma_1$ are also consistently the predicted sign for most of the alternate $RO$ measures and both switch samples, highlighting the presence of a stronger switch effect for more real option intensity firms. These results lend strong support for the switch effect in line with the prediction of our model.

Next, we investigate how the switch effect relates to both real option intensity and the variability of idiosyncratic volatility. We consider $\Delta IVol$ and the interaction between $RO$ and $\Delta IVol$ as explanatory variables in separate regressions for each month $t$ and alternate $RO$ variable as

\textsuperscript{47}The results using unadjusted returns offer qualitatively identical results. These results are available from the authors upon request.

\textsuperscript{48}We estimate the factor loadings $\hat{\beta}_{j,k}$ for each stock and each month using monthly rolling regressions with a 60-month window requiring at least 24 non-missing return observations. We employ the approach in Dimson (1979) with a lag of one month in order to remove biases in the estimates due to thin trading.
follows:

$$r_t^{Diff} = \gamma_0 + \gamma_1 \Delta IVol + \gamma_2 \Delta IVol \times RO_{t-1} + \eta_t$$

(4.7)

where $r_t^{Diff}$, $\eta$ and $RO_{t-1}$ are as defined previously, and $\Delta IVol$ is a vector of $\Delta IVol_j$. The model predictions lead to tests of $\gamma_1 > 0$ and $\gamma_2 > 0$ for the up switch sample, and $\gamma_1 < 0$ and $\gamma_2 < 0$ for the down switch sample.

**Insert Table 5 here**

Table 5 reports the results. As shown, including $\Delta IVol$ renders the intercept estimates no longer significant for most of the regressions. The reason is the difference in mean returns around the switch months is subsumed by $\Delta IVol$, in line with the predictions of our model. $\Delta IVol$ has coefficient estimates that goes in the predicted direction for virtually all of the $RO$ variables. To demonstrate the economic significance, consider ‘small high investment’ as the real option intensity variable. Given a $\Delta IVol$ sample mean of 0.1061, the $\gamma_1$ estimates correspond to an additional annualized switch effect return of -11.83% from a down switch in idiosyncratic volatility for ‘small growth’ stocks in relation to ‘all the rest’, whereas an up switch leads to an incremental return of 15.37%. These incremental returns are highly economically and statistically significant.

The coefficient estimates for the interaction term between $\Delta IVol$ and $RO$ also lend support for the predictions of the model. The sign of the estimates are as predicted for all of the reported $RO$ measures in the up switch sample, and as predicted for some of the measures in the down switch sample. To demonstrate the economic significance, consider again ‘small high investment’ as the $RO$ measure. Holding $\Delta IVol$ at the sample mean, the $\gamma_2$ estimates suggest that more option intensive firms, relative to ‘all the rest’, experience an incremental annualized switch effect return of 11.88% from an up switch in idiosyncratic volatility, whereas a down switch leads to an incremental return of -1.76%.

To summarize, the empirical results lend strong support for the predictions of our model. The strength of several $IVol$-return empirical relations simultaneously go in the direction predicted by the model.
5 Conclusion and Final Remarks

Portfolios of high idiosyncratic volatility stocks underperform their low idiosyncratic volatility counterparts while earlier studies find that firm-level returns correlate positively with contemporaneous changes in idiosyncratic volatility. These findings contradict standard asset pricing theory which tells us that idiosyncratic volatility bears no relation with returns.

We propose a novel explanation for these seemingly disparate and contradicting puzzles with testable hypotheses that link returns to idiosyncratic volatility. We do so by developing a model of firms with growth options and stochastic idiosyncratic cash flow risk (Guo, Miao, and Morellec (2005) and Hackbarth, Miao, and Morellec (2006)), and derive closed-form solutions for firm values and returns. Returns and idiosyncratic volatility are exposed to the same risk factor, and the systematic risk of returns falls when idiosyncratic risk rises. By distinguishing between mature firms, which do not possess growth option, and young firms, which do, the model offers a novel idiosyncratic volatility-return relation, the ‘switch effect’ – that returns correlate with the regime of idiosyncratic volatility – in addition to existing ones, and supposes them to be stronger for younger and smaller growth firms and firms with more variable idiosyncratic volatility. We verify these predictions empirically, confirming the distinctive features of the model are plausible.

Our work extends to other strands of the asset pricing literature. Das and Sundaram (1999) show asset returns must exhibit both heteroskedasticity and discontinuous jumps to empirically fit the heavy-tailed distributions in returns. The previous literature has often relied on behavioral biases to explain these statistical properties of returns (DeLong et al. (1990); Barberis et al. (1998); Hong and Stein (1999)). The model-implied return in this paper exhibits heteroskedasticity and discontinuous jumps that coincide with large changes in idiosyncratic volatility in predictable ways, additionally shedding new insights on the three-way relation between stock returns, idiosyncratic volatility and expected return skewness (Boyer, Mitton, and Vorkink (2010)). The rich features of our model also establish predictability amenable to return continuation as detected in Jegadeesh and Titman (1993), and return reversals encountered by Jegadeesh (1990). Our modeling framework is grounded on investment theory, hence it opens up potential for a rational explanation for several empirical asset pricing findings. We leave these other interesting extensions for future research.
References


Figure 1: Model Solution and Properties: Dependence of Growth Option Value on Idiosyncratic Volatility

The figure shows plots of growth option value $G_{s_t}$ as a function of $P_t$ and idiosyncratic volatility regime $s_t$. The solid 45 degree line corresponds to the intrinsic value of the growth option. Option values in the high and low volatility states are depicted by dashed and dashed dotted curves, respectively. The exercise thresholds are depicted by the vertical dotted lines where the lower threshold corresponds to $P_{L_t}$, and the higher threshold corresponds to $P_{H_t}$.

Panel (a) depicts option values based on baseline values $\sigma_{H_t}^{id} = 0.5473, \sigma_{L_t}^{id} = 0.3354$, panel (b) depicts option values based on $\sigma_{H_t}^{id} = 0.4837, \sigma_{L_t}^{id} = 0.3832$, and the plot in panel (c) is based on $\sigma_{H_t}^{id} = 0.4312, \sigma_{L_t}^{id} = 0.4312$.

(a) $\sigma_{H_t}^{id} = 0.5473, \sigma_{L_t}^{id} = 0.3354$  
(b) $\sigma_{H_t}^{id} = 0.4837, \sigma_{L_t}^{id} = 0.3832$  
(c) $\sigma_{H_t}^{id} = 0.4312, \sigma_{L_t}^{id} = 0.4312$
Figure 2: Model Solution and Properties: Dependence of Return on Idiosyncratic Volatility Regime

The figure shows differences in systematic volatility, drift term, jump term, and diffusion term of the option value process between the high and low volatility regimes for various values of $P$ based on the model developed in Section 3 of the paper. Panel (a) shows differences in systematic volatility $(\Omega_H - \Omega_L)\sigma^{sys}$. Panel (b) shows differences in total volatility $\Omega_H \sqrt{(\sigma_{sys}^H)^2 + (\sigma_{id}^H)^2} - \Omega_L \sqrt{(\sigma_{sys}^L)^2 + (\sigma_{id}^L)^2}$. Panel (c) shows differences in continuous drift term $\mu_{G,H} - \mu_{G,L}$, and Panel (d) shows differences in jump term $\frac{G_H - G_L}{e^{r_H} - e^{r_L}}$ between regimes. Separate plots are shown for each set of $\sigma_{sys}^H$ and $\sigma_{id}^H$ values.
Table 1: Empirical Results: Sample Summary Statistics

The table reports sample summary statistics for excess stock returns, idiosyncratic return volatilities $IVol$, month-to-month $IVol$ changes $\Delta IVol$, and select variables for real option intensity. The sample period is from January, 1971 to December, 2010 for all the market-based variables. Excess return is the difference between the end-of-month stock return and the risk-free rate. Stock return volatility $IVol$ refers to the end-of-month volatility of the log daily returns risk-adjusted based on the 3-factor model of Fama and French (1993). The mean of total asset value is in millions of dollars. The mean of firm age is expressed in months after computing the number of months since the stock’s first appearance on CRSP for each firm in our sample. Investment and sale growths are computed for each firm and each year as the sum of the $t + 2$ to $t + 5$ growth rates where $t$ is the fiscal year of the return observation. vegas are computed for each firm and each month based on equation (4.2).

<table>
<thead>
<tr>
<th>market variables</th>
<th>Mean</th>
<th>StdDev</th>
<th>P5</th>
<th>Median</th>
<th>P95</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>excess return</td>
<td>0.009976</td>
<td>0.180828</td>
<td>-0.22309</td>
<td>-0.0041</td>
<td>0.272627</td>
<td>1041266</td>
</tr>
<tr>
<td>$IVol$</td>
<td>0.029476</td>
<td>0.024979</td>
<td>0.0079</td>
<td>0.022782</td>
<td>0.072884</td>
<td>1038601</td>
</tr>
<tr>
<td>$\Delta IVol$</td>
<td>-2.3E-05</td>
<td>0.021096</td>
<td>-0.02552</td>
<td>-0.00011</td>
<td>0.026111</td>
<td>1035935</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Real Option variables</th>
<th>Mean</th>
<th>StdDev</th>
<th>P5</th>
<th>Median</th>
<th>P95</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(total assets)</td>
<td>4.804593</td>
<td>2.009753</td>
<td>1.789757</td>
<td>4.62188</td>
<td>8.352702</td>
<td>1041266</td>
</tr>
<tr>
<td>log(age)</td>
<td>3.953142</td>
<td>1.540425</td>
<td>0</td>
<td>4.290459</td>
<td>5.746203</td>
<td>1041266</td>
</tr>
<tr>
<td>investment growth</td>
<td>0.996235</td>
<td>18.22423</td>
<td>-0.64226</td>
<td>0.225036</td>
<td>2.237907</td>
<td>871778</td>
</tr>
<tr>
<td>sales growth</td>
<td>1.579677</td>
<td>79.57993</td>
<td>-0.46927</td>
<td>0.29381</td>
<td>1.83045</td>
<td>868519</td>
</tr>
<tr>
<td>vega</td>
<td>2.84E-69</td>
<td>1.49E-67</td>
<td>9.63E-110</td>
<td>9.89E-81</td>
<td>1.88E-70</td>
<td>1041104</td>
</tr>
</tbody>
</table>

38
Table 2: Empirical Results: $IVol$-Sorted Portfolio Returns and Real Option Intensity

The table reports mean risk-adjusted returns during month $t$ for the portfolios sorted on the basis of $IVol$ during month $t-1$ for each classification of real option intensity. Each panel corresponds to the results using an alternate measure for real option intensity. Columns labeled '3-1' correspond to the zero-cost portfolio that takes a long position in the highest $IVol$ portfolio funded from a short position in the lowest $IVol$ portfolio for each classification of real option intensity. Both $IVol$ and risk-adjusted returns are estimated relative to the 3-factor model of Fama and French (1993). All portfolios are value-weighted and rebalanced monthly. Reported risk-adjusted returns are annualized. Robust Newey and West (1987) t-statistics are reported in square brackets. **p < 0.01; * p < 0.05; p < 0.10.

<table>
<thead>
<tr>
<th>Real Option Intensity</th>
<th>Rank on $IVol$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>3-1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>3-1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>3-1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>small high vega</td>
<td>1.3118**</td>
<td>1.443</td>
<td>-4.1226**</td>
<td>-5.4345**</td>
<td>1.3399**</td>
<td>1.186</td>
<td>-3.5023</td>
<td>-4.8422**</td>
<td>1.1484*</td>
<td>0.8917</td>
<td>-4.9013**</td>
<td>-6.0497**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[2.2125]</td>
<td>[1.2015]</td>
<td>[-2.0369]</td>
<td>[-2.4447]</td>
<td>[2.3010]</td>
<td>[0.9753]</td>
<td>[-1.5847]</td>
<td>[-2.0239]</td>
<td>[1.8996]</td>
<td>[0.7600]</td>
<td>[-2.2452]</td>
<td>[-2.5556]</td>
</tr>
<tr>
<td></td>
<td>small high sale</td>
<td>1.3810**</td>
<td>1.2084</td>
<td>-3.6206**</td>
<td>-5.0016**</td>
<td>1.0777*</td>
<td>0.8581</td>
<td>-5.3113**</td>
<td>-6.3890**</td>
<td>1.2383**</td>
<td>1.8331</td>
<td>-3.2394</td>
<td>-4.4778*</td>
</tr>
<tr>
<td></td>
<td>small young</td>
<td>0.7477</td>
<td>0.4518</td>
<td>-3.7675*</td>
<td>-4.5152*</td>
<td>1.4748*</td>
<td>0.7158</td>
<td>-2.7805</td>
<td>-4.2553*</td>
<td>0.6634</td>
<td>1.2607</td>
<td>-5.4760**</td>
<td>-6.1394***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.9722]</td>
<td>[0.3995]</td>
<td>[-1.7082]</td>
<td>[-1.8697]</td>
<td>[1.9199]</td>
<td>[0.5762]</td>
<td>[-1.3525]</td>
<td>[-1.8781]</td>
<td>[1.0983]</td>
<td>[0.8937]</td>
<td>[-2.4337]</td>
<td>[-2.6350]</td>
</tr>
<tr>
<td></td>
<td>small high investment</td>
<td>3.2293</td>
<td>2.3808</td>
<td>-4.5156</td>
<td>-7.7450**</td>
<td>1.4316</td>
<td>1.2535</td>
<td>-6.0880</td>
<td>-7.5196**</td>
<td>5.4876***</td>
<td>5.0094</td>
<td>0.8018</td>
<td>-4.6858</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1.5656]</td>
<td>[0.6126]</td>
<td>[-0.9362]</td>
<td>[-2.0684]</td>
<td>[0.8085]</td>
<td>[0.4088]</td>
<td>[-1.7998]</td>
<td>[-2.4055]</td>
<td>[3.1616]</td>
<td>[1.5461]</td>
<td>[0.2237]</td>
<td>[-1.3247]</td>
</tr>
<tr>
<td></td>
<td>young high vega</td>
<td>0.7477</td>
<td>0.4518</td>
<td>-3.7675*</td>
<td>-4.5152*</td>
<td>1.4748*</td>
<td>0.7158</td>
<td>-2.7805</td>
<td>-4.2553*</td>
<td>0.6634</td>
<td>1.2607</td>
<td>-5.4760**</td>
<td>-6.1394***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.9722]</td>
<td>[0.3995]</td>
<td>[-1.7082]</td>
<td>[-1.8697]</td>
<td>[1.9199]</td>
<td>[0.5762]</td>
<td>[-1.3525]</td>
<td>[-1.8781]</td>
<td>[1.0983]</td>
<td>[0.8937]</td>
<td>[-2.4337]</td>
<td>[-2.6350]</td>
</tr>
<tr>
<td></td>
<td>natural resources</td>
<td>3.2293</td>
<td>2.3808</td>
<td>-4.5156</td>
<td>-7.7450**</td>
<td>1.4316</td>
<td>1.2535</td>
<td>-6.0880</td>
<td>-7.5196**</td>
<td>5.4876***</td>
<td>5.0094</td>
<td>0.8018</td>
<td>-4.6858</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1.5656]</td>
<td>[0.6126]</td>
<td>[-0.9362]</td>
<td>[-2.0684]</td>
<td>[0.8085]</td>
<td>[0.4088]</td>
<td>[-1.7998]</td>
<td>[-2.4055]</td>
<td>[3.1616]</td>
<td>[1.5461]</td>
<td>[0.2237]</td>
<td>[-1.3247]</td>
</tr>
<tr>
<td></td>
<td>high tech</td>
<td>3.2293</td>
<td>2.3808</td>
<td>-4.5156</td>
<td>-7.7450**</td>
<td>1.4316</td>
<td>1.2535</td>
<td>-6.0880</td>
<td>-7.5196**</td>
<td>5.4876***</td>
<td>5.0094</td>
<td>0.8018</td>
<td>-4.6858</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1.5656]</td>
<td>[0.6126]</td>
<td>[-0.9362]</td>
<td>[-2.0684]</td>
<td>[0.8085]</td>
<td>[0.4088]</td>
<td>[-1.7998]</td>
<td>[-2.4055]</td>
<td>[3.1616]</td>
<td>[1.5461]</td>
<td>[0.2237]</td>
<td>[-1.3247]</td>
</tr>
<tr>
<td></td>
<td>bio tech</td>
<td>3.2293</td>
<td>2.3808</td>
<td>-4.5156</td>
<td>-7.7450**</td>
<td>1.4316</td>
<td>1.2535</td>
<td>-6.0880</td>
<td>-7.5196**</td>
<td>5.4876***</td>
<td>5.0094</td>
<td>0.8018</td>
<td>-4.6858</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1.5656]</td>
<td>[0.6126]</td>
<td>[-0.9362]</td>
<td>[-2.0684]</td>
<td>[0.8085]</td>
<td>[0.4088]</td>
<td>[-1.7998]</td>
<td>[-2.4055]</td>
<td>[3.1616]</td>
<td>[1.5461]</td>
<td>[0.2237]</td>
<td>[-1.3247]</td>
</tr>
</tbody>
</table>
Table 3: Empirical Results: IVol-Sorted Portfolio Returns, Switch Size and Real Option Intensity

The table reports mean risk-adjusted return during month $t$ for the zero-cost IVol portfolio for each of the 2-way classifications of real option intensity and rank on the mean difference between the 70th and 30th IVol percentile values $\Delta IVol$. Each panel corresponds to the results from using an alternate measure for real option intensity. The sorts are done on the basis of IVol and real option intensity observed during month $t$. Zero-cost portfolios take a long position in the highest IVol portfolio funded from a short position in the lowest IVol portfolio. Both IVol and risk-adjusted returns are estimated relative to the 3-factor model of Fama and French (1993). All portfolios are value-weighted and rebalanced monthly. Reported risk-adjusted returns are annualized. Robust Newey and West (1987) t-statistics are reported in square brackets. * * * $p < 0.01$; * * $p < 0.05$; * $p < 0.10$.

<table>
<thead>
<tr>
<th>Rank on $\Delta IVol$</th>
<th>Real Option Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>small high vega</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-2.4541</td>
</tr>
<tr>
<td></td>
<td>[-1.0159]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>growth</th>
<th>natural resources</th>
<th>high tech</th>
<th>bio tech</th>
<th>all growth ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.634</td>
<td>-0.0442</td>
<td>1.6231</td>
<td>6.8209</td>
<td>0.3569</td>
</tr>
<tr>
<td></td>
<td>[0.2129]</td>
<td>[-0.0145]</td>
<td>[-0.5745]</td>
<td>[1.3028]</td>
<td>[0.1429]</td>
</tr>
<tr>
<td>2</td>
<td>1.0354</td>
<td>-6.1101*</td>
<td>-1.7178</td>
<td>-2.7294</td>
<td>-0.8473</td>
</tr>
<tr>
<td></td>
<td>[0.3849]</td>
<td>[-1.8622]</td>
<td>[-0.6962]</td>
<td>[-0.6421]</td>
<td>[-0.3146]</td>
</tr>
</tbody>
</table>
Table 4: Empirical Results: Difference in Risk-Adjusted Returns Between Post and Pre-Switch Months Around Large Switches in Idiosyncratic Volatility and Its Dependence on Real Option Intensity

The table reports estimates from regressing difference in risk-adjusted returns between post and pre-switch months on real option intensity for stocks that experience switches in idiosyncratic volatility. The regression equation is $r_{t}^{Diff} = \gamma_0 + \gamma_1 RO_{t-1} + \eta_t$, where $r_{t}^{Diff}$ is a vector of differences in 5-month mean risk-adjusted returns between post and pre-switch months around the switch month $t$, and $RO_{t-1}$ is a vector of real option intensity values. An up switch in $IVol_{j,t}$ for stock $j$ is considered to have occurred during month $t$ if $IVol_{j,t-1} < IVol_{j,30\text{pctl}}$ and $IVol_{j,70\text{pctl}} < IVol_{j,t}$, where $IVol_{j,X\text{pctl}}$ denotes the $X$th percentile $IVol$ value for stock $j$. A down switch is considered to have occurred during month $t$ if $IVol_{j,t-1} > IVol_{j,70\text{pctl}}$ and $IVol_{j,t} < IVol_{j,30\text{pctl}}$. Both $IVol_{j,t}$ and risk-adjusted returns are estimated relative to the 3-factor model of Fama and French (1993). Each column corresponds to a separate set of estimates using an alternate variable for real option intensity. The top panel of the table reports the estimates for the down switch sample, and the bottom panel reports the results for the up switch sample. The reported estimates are the time-series means of the monthly coefficient estimates following the approach of Fama and MacBeth (1973). Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R-squareds. **p<0.01; * *p<0.05; * p<0.10.

<table>
<thead>
<tr>
<th>switch sample</th>
<th>Coefficient</th>
<th>Real Option Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>small</td>
<td>small</td>
</tr>
<tr>
<td></td>
<td>high vega</td>
<td>growth</td>
</tr>
<tr>
<td></td>
<td>RO</td>
<td>-0.0116** -0.0071** -0.0045 -0.0108*** -0.0036 0.001 -0.0015 0.0013 -0.001</td>
</tr>
<tr>
<td></td>
<td>RSQR</td>
<td>0.0482 0.0376 0.0388 0.0384 0.0439 0.0357 0.0333 0.0273 0.0288</td>
</tr>
<tr>
<td>up</td>
<td>Intercept</td>
<td>0.0048*** 0.0047*** 0.0043*** 0.0051*** 0.0036*** 0.0054*** 0.0056*** 0.0057*** 0.0056***</td>
</tr>
<tr>
<td></td>
<td>RO</td>
<td>0.0082** 0.0095*** 0.0107*** 0.0077*** 0.0164*** 0.0079*** 0.002 0.0025 0.0028</td>
</tr>
<tr>
<td></td>
<td>RSQR</td>
<td>0.0408 0.0349 0.0368 0.0324 0.0397 0.0325 0.0299 0.0237 0.0265</td>
</tr>
</tbody>
</table>
Table 5: Empirical Results: Difference in Risk-Adjusted Returns Between Post and Pre-Switch Months Around Large Switches in Idiosyncratic Volatility and Its Dependence on Real Option Intensity and Switch Size.

The table reports estimates from regressing difference in risk-adjusted returns between post and pre-switch months on real option intensity and the mean difference between the 70th and 30th IVol percentile values for stocks that experience switches in idiosyncratic volatility. The regression equation is $r_{diff}^t = \gamma_0 + \gamma_1 \Delta IVol + \gamma_2 \Delta IVol \times RO_{t-1} + \eta_t$, where $r_{diff}^t$ is a vector of differences in 5-month mean risk-adjusted returns between post and pre-switch months around the switch month $t$, $RO_{t-1}$ is a vector of real option intensity values, and $\Delta IVol$ is a vector of mean differences between $IVol_{j,70\text{pctl}}$ and $IVol_{j,30\text{pctl}}$, where $IVol_{j,X\text{pctl}}$ denotes the $X$th percentile IVol value for stock $j$. An up switch in $IVol_{j,t}$ for stock $j$ is considered to have occurred during month $t$ if $IVol_{j,t-1} < IVol_{j,30\text{pctl}}$ and $IVol_{j,70\text{pctl}} < IVol_{j,t}$. A down switch is considered to have occurred during month $t$ if $IVol_{j,t-1} > IVol_{j,70\text{pctl}}$ and $IVol_{j,30\text{pctl}} < IVol_{j,t}$. Both $IVol_{j,t}$ and risk-adjusted returns are estimated relative to the 3-factor model of Fama and French (1993). Each column corresponds to a separate set of estimates using an alternate variable for real option intensity. The top panel of the table reports the estimates for the down switch sample, and the bottom panel reports the results for the up switch sample. The reported estimates are the time-series means of the monthly coefficient estimates following the approach of Fama and MacBeth (1973). Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R-squareds. * * * $p < 0.01$; * * $p < 0.05$; * $p < 0.10$.

<table>
<thead>
<tr>
<th>Real Option Intensity</th>
<th>small</th>
<th>small</th>
<th>small</th>
<th>small</th>
<th>small</th>
<th>young</th>
<th>Natural</th>
<th>High</th>
<th>Bio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>high vega</td>
<td>growth</td>
<td>high sale</td>
<td>young</td>
<td>high inv</td>
<td>high vega</td>
<td>Resources</td>
<td>Tech</td>
<td>Tech</td>
</tr>
<tr>
<td>down</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>0.0014</td>
<td>0.0011</td>
<td>0.0001</td>
<td>0</td>
<td>-0.0002</td>
<td>0.0015</td>
<td>0.0009</td>
<td>0.0003</td>
<td>0.0006</td>
</tr>
<tr>
<td></td>
<td>[0.6783]</td>
<td>[0.5057]</td>
<td>[0.0229]</td>
<td>[0.0216]</td>
<td>[-0.0799]</td>
<td>[0.6820]</td>
<td>[0.4145]</td>
<td>[0.1555]</td>
<td>[0.2746]</td>
</tr>
<tr>
<td>$\Delta IVol$</td>
<td>-0.8262***</td>
<td>-0.8361***</td>
<td>-0.6459***</td>
<td>-0.7449***</td>
<td>-0.6130***</td>
<td>-0.8933***</td>
<td>-0.7969***</td>
<td>-0.7817***</td>
<td>-0.7619***</td>
</tr>
<tr>
<td>$\Delta IVol \times RO$</td>
<td>-0.2349</td>
<td>0.0228</td>
<td>-0.0493</td>
<td>-0.1334</td>
<td>-0.0914</td>
<td>0.3063</td>
<td>-0.0335</td>
<td>0.0778</td>
<td>0.0886</td>
</tr>
<tr>
<td></td>
<td>[-1.0029]</td>
<td>[0.1089]</td>
<td>[-0.2414]</td>
<td>[-0.7354]</td>
<td>[-0.3688]</td>
<td>[1.3479]</td>
<td>[-0.0943]</td>
<td>[0.4351]</td>
<td>[0.3732]</td>
</tr>
<tr>
<td>RSQR</td>
<td>0.0959</td>
<td>0.0962</td>
<td>0.1007</td>
<td>0.0930</td>
<td>0.1045</td>
<td>0.0941</td>
<td>0.0937</td>
<td>0.0913</td>
<td>0.0888</td>
</tr>
<tr>
<td>up</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.0038*</td>
<td>-0.0024</td>
<td>-0.0054**</td>
<td>-0.0032</td>
<td>-0.0048**</td>
<td>-0.0031</td>
<td>-0.0037*</td>
<td>-0.0042*</td>
<td>-0.0037</td>
</tr>
<tr>
<td>$\Delta IVol$</td>
<td>0.7808***</td>
<td>0.6350***</td>
<td>0.8880***</td>
<td>0.7471***</td>
<td>0.7969***</td>
<td>0.7031***</td>
<td>0.7761***</td>
<td>0.8545***</td>
<td>0.7441***</td>
</tr>
<tr>
<td></td>
<td>[3.9437]</td>
<td>[3.2559]</td>
<td>[4.2814]</td>
<td>[3.7131]</td>
<td>[4.1374]</td>
<td>[3.4753]</td>
<td>[4.0162]</td>
<td>[3.7309]</td>
<td>[3.6893]</td>
</tr>
<tr>
<td>$\Delta IVol \times RO$</td>
<td>0.0379</td>
<td>0.2846</td>
<td>0.3161*</td>
<td>0.0841</td>
<td>0.6159***</td>
<td>0.0601</td>
<td>0.339</td>
<td>0.0486</td>
<td>0.0432</td>
</tr>
<tr>
<td></td>
<td>[0.1882]</td>
<td>[1.5988]</td>
<td>[1.7296]</td>
<td>[0.5751]</td>
<td>[3.2721]</td>
<td>[0.3100]</td>
<td>[1.0305]</td>
<td>[0.3155]</td>
<td>[0.1791]</td>
</tr>
<tr>
<td>RSQR</td>
<td>0.0809</td>
<td>0.0784</td>
<td>0.0841</td>
<td>0.0701</td>
<td>0.0842</td>
<td>0.0777</td>
<td>0.0753</td>
<td>0.0723</td>
<td>0.0711</td>
</tr>
</tbody>
</table>