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Self-sustaining processes at all scales in wall-bounded turbulent shear flows

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We collect and discuss the results of our recent studies which show evidence of the existence of a whole family of self-sustaining motions in wall-bounded turbulent shear flows with scales ranging from those of buffer-layer streaks to those of large-scale and very-large-scale motions in the outer layer. The statistical and dynamical features of this family of self-sustaining motions, which are associated with streaks and quasi-streamwise vortices, are consistent with those of Townsend's attached eddies. Motions at each relevant scale are able to sustain themselves in the absence of forcing from larger- or smaller-scale motions by extracting energy from the mean flow via a coherent lift-up effect. The coherent self-sustaining process is embedded in a set of invariant solutions of the filtered Navier-Stokes equations which take into full account the Reynolds stresses associated with the residual smaller-scale motions.

1. Introduction

The flow visualizations of Kline *et al.* [1] revealed that the near-wall region of turbulent boundary layers is populated by very robust streaky motions. The average spanwise streak-spacing is $\lambda_z^+ \approx 100$, in wall units, in the buffer layer and it increases with the distance from the wall [1,2] as confirmed by early direct numerical simulations which also revealed the existence of quasi-streamwise vortices [3] associated with the streaks. Streaky motions also exist in the logarithmic and the outer regions which are populated by large-scale structures with dimensions of the order of the outer leng-

-th scale δ (e.g. the half channel width in Poiseulle flow, the full channel width in plane Couette flow, or the boundary-layer thickness). 'Large-scale motions' (LSM) have typical streamwise and spanwise sizes of $\lambda_x \approx 2\delta - 3\delta$ and $\lambda_z \approx \delta - 1.5\delta$ respectively [4,5]. Very large scale motions (VLSM), also known as 'superstructures', have been shown to exist with streamwise scales extending up to $\lambda_x \simeq O(10\delta)$ [6–8]. These large and very large scale motions are especially relevant at high Reynolds numbers where they account for an increasingly significant amount of the turbulent kinetic energy and Reynolds stress in the outer region [9] and they modulate the near-wall cycles [10].

Townsend hypothesized that the logarithmic region would be composed of self-similar energy-containing motions, the size of which are proportional to their distance from the wall [11]. By a suitable superposition of these hypothetical motions, termed as 'attached eddies', under the constraint of a constant Reynolds shear stress typical of the log-layer, he predicted that the wall-parallel velocity components of turbulence intensities in the logarithmic region would exhibit the logarithmic wall-normal dependence. Although Townsend speculated that the double-cone eddy is a possible statistical form of attached eddy, he did not commit to a specific type of eddy nor to any specific sustaining mechanism. Based on experimental observations Perry and Chong [12] extended Townsend's theory by introducing a more specific structural model based on the assumption that the attached eddies are, at least on average, in the form of Theodorsen's Λ-vortices with dimensions going from those of buffer-layer streaks to those of the outer layer structures. A series of theoretical studies have refined this model (e.g. [13,14]) and a number of experimental studies have supported these structural theories with the detection of intense vortical structures which where associated with Λ-vortices (see e.g. [15,16]).

In the cited structural models, it is assumed that the mechanism sustaining the whole turbulent motion is associated with the production of 'small' Λ-vortices in the buffer layer with the characteristic size of the buffer layer streaks observed by Kline et al. [1]. Those buffer-layer Λ-vortices then feed larger and larger Λ-vortices, up to large-scale motions, by merging or parent-offspring regeneration [17,18] where a sufficiently strong initial hairpin vortex generates a spatially growing packet of hairpins. Very-large-scale motions are attributed to the concatenation of large-scale motions [7,9]. Following this rationale, large-scale motions in the outer layer would not exist in the absence of supporting or co-supporting [19] motions in the buffer layer.

The relevance of this 'bottom-up' scenario based on Λ-vortices, especially in the high Reynolds number regime is, however, not universally accepted. Some recent studies have challenged the idea that hairpin vortices are a prominent feature of high Reynolds number turbulence and that the parent-offspring mechanism is active in turbulent environments with counter-evidence [20,21]. Even more importantly, and independent of the question of the existence or not of prominent Λ-vortices, a growing number of recent results suggest that buffer-layer motions are not ultimately necessary to sustain large-scale and very-large-scale motions. For instance, it has been shown that the strong modification of the buffer-layer energy-production processes does not lead to a significant change of the motions at larger scales [22,23]. These findings lead to at least two fundamental questions on the nature of high-Reynolds number turbulence: (a) If not the Λ-vortices, what is the nature of Townsend's attached eddies? (b) How do these eddies sustain?

The scope of this article is to collect and discuss a series of recent investigations which provide some answers to these questions resulting in a description of high Reynolds number wall-bounded turbulence in terms of a continuum of self-sustaining motions which draw energy directly from the mean flow. In §2, we will discuss how linear models of coherent perturbations to the turbulent mean flow show that energy can be extracted *directly* from the mean flow at scales ranging from those of the buffer-layer streaks to those of large-scale and very-large-scale motions. In §3, we will discuss how the large-scale and very-large-scale motions can be isolated from smaller-scale motions, and show that motions in the logarithmic and outer regions do sustain themselves even when the energy production processes at smaller scales, and in particular in the buffer layer, are artificially switched-off. The coherent self-sustaining process identified by this analysis is crystallized in invariant solutions which take the averaged effect of smaller-scale
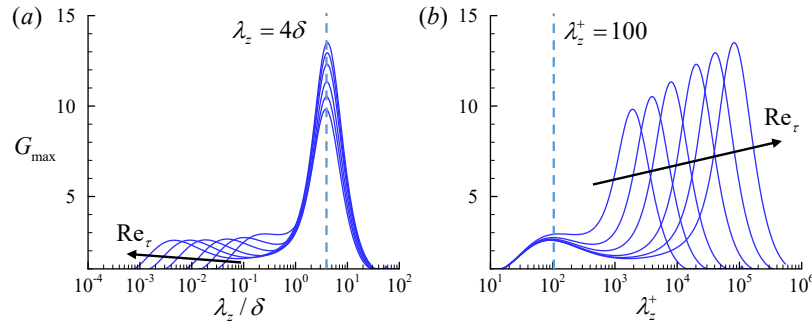
Figure 1. Maximum energy amplification G_{max} (for the definition of G_{max} see [24]) of streamwise uniform ($k_x = 0$) initial conditions versus the spanwise wavelength for the selected $Re_\tau = 500, 1000, 2000, 5000, 10000$, and 20000 (where $Re_\tau = \delta u_\tau / \nu$, with u_τ the friction velocity and ν the kinematic viscosity). The spanwise wavelength is scaled in outer (λ_z / h) and wall units ($\lambda_z^+ = \lambda_z u_\tau / \nu$), respectively, in panels a and b (replotted from channel flow data reported in [25,26]).

motions into account, as discussed in §4. In §5 it will be shown that the family of isolated self-sustaining motions has all the characteristics of Townsend's attached eddies. Some conclusions and perspectives will be drawn in §6.

2. The coherent lift-up effect

Contrary to canonical turbulent free shear flows, the turbulent mean flow profile of most wall-bounded shear flows is linearly stable [27–32] leaving unanswered the question of how turbulent motions can extract energy from the mean flow. The important progress realized in the understanding of non-normal energy amplifications in linearly stable laminar flows [24] has, however, motivated a re-examination of this question. Early studies along these lines have computed the optimal energy amplifications by using the turbulent mean flow profile as the base flow in the linearized Navier-Stokes equations (rearranged in the form of the usual Orr-Sommerfeld-Squire equations), neglecting Reynolds stresses associated with turbulent fluctuations [29]. It was found that perturbations leading to the (unconstrained) maximum energy growth have spanwise wavelength $\lambda_z \approx 3\,\delta$, where δ is the channel half-width, which is almost the same value found for laminar base flow profiles. The characteristic spanwise spacing of near-wall streaks ($\lambda_z^+ \approx 100$) was obtained under the constraint that the time scale of optimal solutions is of the order of the eddy turn-over time in the buffer layer [29]. Important progress was made by using generalized Orr-Sommerfeld and Squire operators which include a non-uniform eddy viscosity $\nu_T(y)$ associated with the turbulent mean flow used to model the turbulent Reynolds stresses.[1] An incorrect early expression of these operators [30] has been amended in later studies [25,31] to:

$$\mathcal{L}_{OS} = -ik_x(U\Delta - U'') + \nu_T\Delta^2 + 2\nu_T'\Delta\mathcal{D} + \nu_T''(\mathcal{D}^2 + k^2), \tag{2.1}$$

$$\mathcal{L}_{SQ} = -ik_x U + \nu_T\Delta + \nu_T'\mathcal{D}, \tag{2.2}$$

where $k_x = 2\pi/\lambda_x$, $k_z = 2\pi/\lambda_z$, $k^2 = k_x^2 + k_z^2$, λ_x and λ_z are the streamwise and spanwise wavelength, $\mathcal{D}$ and $'$ denote $\partial/\partial y$ and $\Delta = \mathcal{D}^2 - k^2$. Computing the optimal temporal energy amplification with these operators (and without any further restriction on the optimization times as in [29]) two peaks for the maximum energy amplification are found for long streamwise wavelengths $\lambda_x \gg \lambda_z$, as shown in figure 1. The main (outer) peak scales in outer units and roughly corresponds to large-scale streaky motions in the outer region. The associated maximum

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[1]The total eddy viscosity is educed from $\tau/\rho = \nu_T(y)(\partial U/\partial y)$ where τ is the total shear stress. Without $\nu_T(y)$ the turbulent mean flow U would not be a steady solution of the Reynolds-averaged Navier-Stokes equations.

energy growth increases proportionally to a Reynolds number based on outer units $Re_{out} = U_c \delta / \nu_{T,max}$ [25,31]. The secondary (inner) peak scales in inner (wall) unit and corresponds to $\lambda_z^+ \approx 90$ [25,30,31,33], i.e. the most probable spanwise wavelength of near wall streaks [2]. Optimal streaks associated with the secondary peak correspond well to the observed near-wall streaks. Structures with scales broadly lying between these two peaks correspond to log-layer streaks and are approximately self-similar [34]. This approach has then been extended by computing the optimal response to harmonic and stochastic forcing [32,34,35] confirming the existence of an inner and an outer amplification peak corresponding to the buffer-layer and large-scale streaky structures with intermediate (log-layer) scales also amplified. Experiments, where large-scale coherent streaks were steadily forced in the turbulent boundary layer, confirmed that coherent (temporally averaged) large-scale streaks can be transiently amplified in space [36]. The artificial forcing of large-scale coherent streaks was then used to reduce the turbulent drag [35,37].

These results obtained in most of the canonical wall-bounded turbulent shear flows (Couette flow, pressure-driven channel and pipe flows, zero pressure gradient boundary layer) provide sound evidence for the existence of the *coherent* lift-up effect[2] by which quasi-streamwise streaky motions with spanwise length scales ranging from those of the buffer-layer streaks to those of the large-scale and very-large scale motions are able to extract energy directly from the mean flow.

3. Self-sustaining process at large scale

In addition to the existence of the robust coherent lift-up effect discussed in §2 it was also shown that large-scale coherent streaks can undergo secondary instabilities at sufficiently large amplitudes [38,39], suggesting that a 'coherent' self-sustaining process similar to the buffer-layer one [40,41] might be at work also at larger scales in turbulent flows. To prove that such a type of process actually exists, however, it should be verified that motions of a given scale of interest are not sustained by larger- or smaller-scale motions.

The standard way to exclude larger-scale motions is to simulate the flow in domains of the desired streamwise and spanwise size and to enforce periodic boundary conditions on the boundaries. In this way, for instance, it has been possible to show that streaky motions in the buffer layer are self-sustaining [42]. Excluding smaller-scale motions without seriously damaging the solution at the considered scale has proven to be a challenging issue. Our preliminary tests, in which under-resolved Navier-Stokes simulations were used in order to exclude motions smaller than the grid spacing, were inconclusive because the considered large-scale structures were too poorly resolved and unphysical energy production peaks appeared corresponding to the grid size. This could have been expected because residual motions, unresolved by the grid, must be modelled as is well known by practitioners of large-eddy simulations (LES). The idea has therefore been to use large-eddy simulations and to model small-scale motions with a purely dissipative model which inhibits energy production by the (unresolved) small scales [43,44]. The equations used in LES are the usual ones [45,46] and can be obtained by applying a filter to the incompressible Navier-Stokes equations:

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0; \quad \frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{\partial \overline{q}}{\partial x_i} + \nu \frac{\partial^2 \overline{u}_i}{\partial x_j^2} - \frac{\partial \overline{\tau}_{ij}^r}{\partial x_j}, \tag{3.1}$$

where the overhead bar denotes the filtering action, $\overline{\tau}^r = \overline{\tau}^R - tr(\overline{\tau}^R)\, \mathbf{I}/3$, with $\overline{\tau}_{ij}^R = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$ and $\overline{q} = \overline{p} + tr(\overline{\tau}^R)/3$. A subgrid model in terms of eddy viscosity ν_t is used to model the anisotropic residual stress tensor $\overline{\tau}_{ij}^r = -2\nu_t \overline{S}_{ij}$, where $\overline{S}_{ij}$ is the rate of strain tensor associated with the filtered velocity field. The eddy viscosity is given by the *static* Smagorinsky model [47]: $\nu_t = D(C_s \overline{\Delta})^2 \overline{S}$, where $\overline{S} \equiv (2\overline{S}_{ij} \overline{S}_{ij})^{1/2}$, $\overline{\Delta} = (\overline{\Delta}_x \overline{\Delta}_y \overline{\Delta}_z)^{1/3}$ is the average length scale of the filter based on the mean grid spacing and C_s is the Smagorinsky constant and $D =$

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[2]The term 'coherent' lift-up here is used to emphasize the fact that the optimal energy amplification is computed on averaged streaks by explicitly accounting for the Reynolds stresses associated with turbulent fluctuations and to differentiate these results from standard lift-up computations, where Reynolds stresses are not taken into account and which give different results.

$1 - e^{-(y^+/A^+)^2}$ is the wall damping function proposed in [48] to drive (physically) to zero the eddy viscosity at the wall.
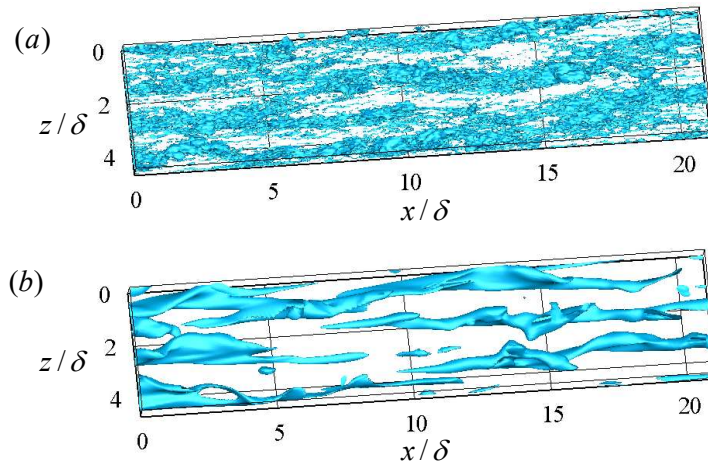
Figure 2. Iso-surface $u^+ = -2$ of instantaneous streamwise velocity fluctuations in the turbulent plane channel at $Re_\tau \simeq 950$ (data from simulations reported in [26,43,49]). At $C_s = 0.05$ (a), large-scale streaky motions are populated with small-scale structures. At $C_s = 0.3$ (b) small-scale structures are quenched and the self-sustaining large-scale motions are isolated.
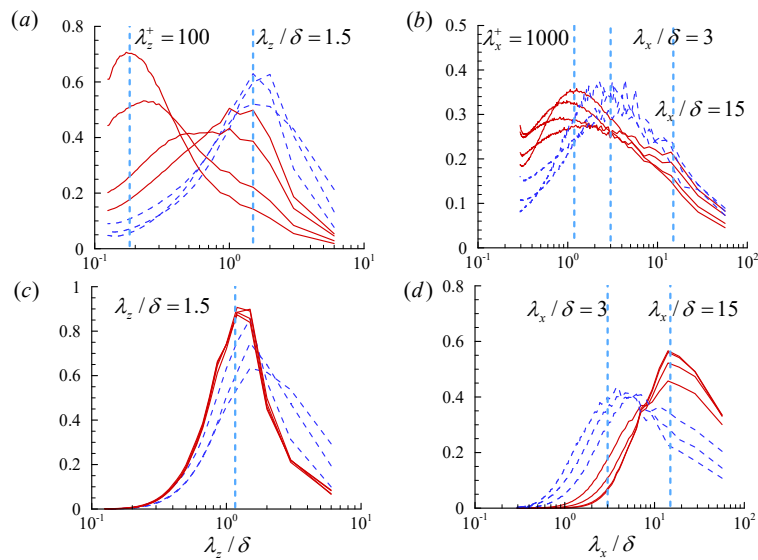
Figure 3. Spanwise premultiplied power spectrum $k_z E_{uu}(\lambda_z)$ [(a), (c)] and streamwise premultiplied power spectrum $k_x E_{uu}(\lambda_x)$ [(b), (d)] for respectively the reference simulation with $C_s = 0.05$ [(a) and (b)] and for $C_s = 0.2$ [(c) and (d)], where E_{uu} is the one-dimensional power spectrum of the streamwise velocity. The premultiplied spectra are extracted in the inner- ($y^+ = 16, 30, 70, 108$, solid red lines) and in the outer layer ($y/h = 0.38, 0.65, 1$, dashed blue lines) in the turbulent plane channel (replotted with data at $Re_\tau \simeq 550$ reported in [43]).

The use of the static Smagorinsky model, and not of better performing dynamic models, ensures that there is no energy transfer from residual motions to the (larger scale) filtered motions.

In this way it is therefore possible to simulate large-scale and very-large-scale motions excluding any energy input from motions at smaller scales. An additional problem, however, is that if motions on a single scale are to be isolated, *all* the other motions at smaller scales have to be filtered. The most immediate way to do this would be to greatly increase the grid spacing Δ. However, preliminary tests have shown that following this approach, only a few points in physical space would be available to represent well isolated large-scale structures, leading to inaccurate single-scale solutions. Recalling that the effective 'Smagorinsky mixing length' $l_0 = C_s\Delta$ is the product of the grid spacing by the Smagorinsky constant [50], we had the idea of increasing the filter width by increasing C_s instead of Δ, thereby preserving the (spatial) accuracy of the solutions [43,44]. The production term of an increasing range of small-scale motions is therefore quenched in overdamped simulations[3] by increasing C_s above its 'optimal' value $C_s = 0.05$ which provides the best *a posteriori* tests [52]. We have shown that in these overdamped simulations the friction Reynolds number is not greatly affected by the increase of C_s [43,44,51], indicating that the resulting flows maintain the high Reynolds numbers.

Using the overdamped-LES technique, it has been shown that large-scale and very-large-scale motions can sustain themselves in the turbulent plane channel and Couette flows even when small-scale structures (including buffer-layer and log-layer structures) are artificially quenched [43,51], as shown in figure 2. More quantitative measures such as the location of the peaks in the premultiplied energy spectra confirm that the surviving large-scale and very-large-scale motions have almost the same characteristic spanwise mean spacing as the natural ones (see figure 3). Intermediate-scale motions, which populate the logarithmic region, have been shown to be approximately self-similar and scale with their spanwise wavelength λ_z or, equivalently, with their distance y from the wall [44,49] which is compatible with the classical concept of attached eddies [11] (this issue will be discussed further in §5).

The detailed investigation of the temporal evolution of the isolated large-scale structures was performed in [53] and revealed that the coherent self-sustaining process is remarkably similar to that in the near-wall region. Figure 4 shows the temporal evolution of flow fields and the flow variables capturing streaks (E_u^+ in figure 4) and quasi-streamwise vortices (E_v^+ in figure 4). The streak is significantly amplified by the coherent lift-up effect from the quasi-streamwise vortical structures (fig. 4 c,d). The amplified streak subsequently undergoes 'rapid' streamwise meandering motions, reminiscent of streak instability or non-normal amplification of a sinuous mode on top of the streaks (fig. 4 e) [38,39,41,54]. This eventually results in the breakdown of the streaks and regeneration of new quasi-streamwise vortical structures[4] (fig. 4 f).

## 4. Large-eddy exact coherent structures

The specific mechanism by which the coherent large-scale structures discussed in §3 self-sustain can be further investigated by looking for the existence of invariant solutions of the filtered, possibly overdamped, Navier-Stokes equations similarly to what has been done to understand transitional structures (where the the unfiltered Navier-Stokes equations were considered at lower Reynolds numbers). Invariant solutions of the filtered equations take into full account small-scale motions but only their locally averaged effect (the local average is obtained through the filtering action). It is, in this way, possible to compute steady large-scale solutions even if small-scale motions are unsteady but their local average is statistically steady. To compute these solutions, the (possibly-overdamped) LES code used to show the existence of the self-sustaining process has been coupled to a Krylov-Newton algorithm capable of finding steady, travelling wave or relative periodic solutions where both the Reynolds number Re and the Smagorinsky constant C_s are used as continuation parameters. The computed large-scale solutions will be labelled 'large-eddy exact coherent solutions' (LECS) to differentiate them from the 'exact coherent solutions' (ECS, as

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[3]Most of the results described in this paper have been obtained using a Fourier spectral discretization in the horizontal plane and second-order accurate finite differences in the wall-normal direction. More details about the numerics can be found in the cited references like e.g. [43,44,51].

[4]The reader can refer to [53] for a detailed statistical quantification of this process.
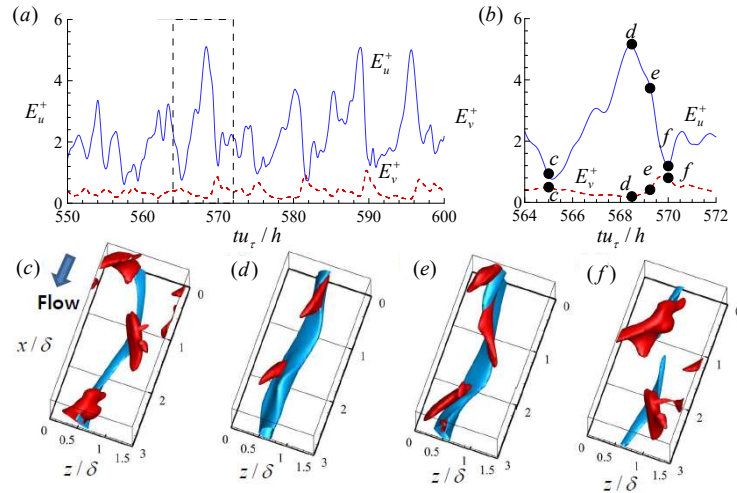
Figure 4. Time evolution of the self-sustaining large-scale motions in the minimal large-scale flow unit $L_x = 3\delta$ and $L_z = 1.5\delta$ [26,49]: (a) time trace of streamwise (E_u^+, solid blue line) and wall-normal (E_v^+, dashed red line) turbulent kinetic energy averaged over the computational domain; (b) magnification of (a) for $tu_\tau/h \in [564, 572]$; $(c - f)$ the corresponding flow visualisation at $tu_\tau/h = 565, 568.5, 569.3, 570$. In $(c - f)$, the red and blue iso surfaces indicate $u^+ = -4$ and $v^+ = 1.5$, respectively.
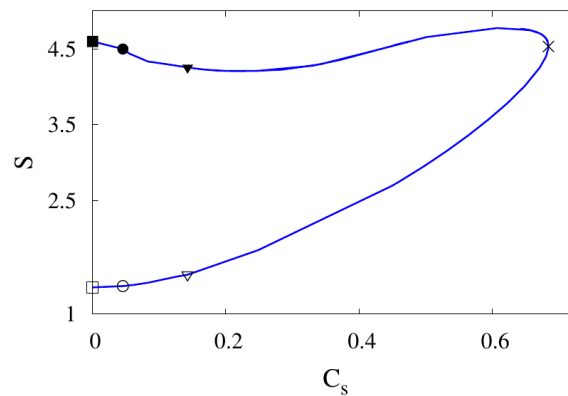
Figure 5. Continuation diagram of the steady LECS solutions in the Smagorinsky constant C_s at $Re = 750$ ($Re_\tau = 52$) in turbulent plane Couette flow. $S = \tau_w/(\mu U_w/h)$ is the wall shear rate of the solution normalized by its laminar value (replotted from data reported in [51,55]). The symbols denote the solutions found at $C_s = 0$ (Navier-Stokes, squares), $C_s = 0.05$ (reference LES, circles) and $C_s = 0.14$ (triangles), the value at which the lower-branch overdamped LECS solution was computed by edge-tracking in an overdamped-LES [51]. Empty symbols denote lower branch solutions, filled symbols denote upper branch solutions while the X symbol denotes the saddle-node C_s-bifurcation connecting the upper and lower C_s-branches.

defined in [56]) which are solutions of the unfiltered Navier-Stokes equations and are therefore computed neglecting the Reynolds stresses associated with smaller-scale turbulent fluctuations.

The first LECS solutions have been computed in turbulent plane Couette flow and correspond to statistically steady large-scale motions [51,55]. Those solutions have been connected to previously known [57–59] solutions of the (unfiltered) Navier-Stokes equations by continuation to $C_s = 0$. It was also shown that the upper branch and the lower branch solutions at a given
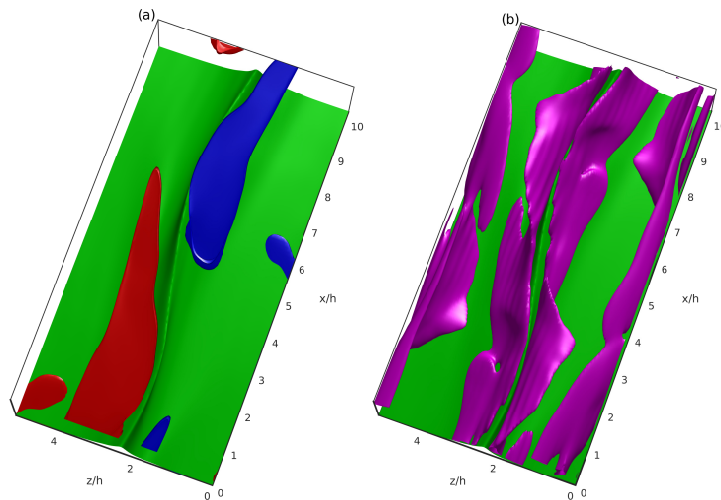
Figure 6. Visualisation of the upper branch large-scale coherent steady LECS solutions (figure courtesy of S. Rawat, replotted from data reported in [51,55]) for Re=2187 ($Re_\tau = 127$), with $C_s = 0.1$. Panel (*a*) represent the streaks and quasi-streamwise vortices: the green surface corresponds to a streamwise velocity value 50% of its maximum value while the blue and red surfaces correspond to streamwise vorticity values equal to $\pm 70\%$ of the maximum. Panel (*b*) represents the streaks and the relative eddy-viscosity associated with the filtered small-scale motions (the violet surface corresponds $\nu_t/\nu = 0.4$).

Reynolds number could be connected by an upper and a lower C_s-branch of solutions issued from a saddle-node bifurcation at high C_s (see figure 5). In a similar way, Rawat *et al.* [60] were also able to continue their travelling-wave (ECS) solution of the Navier-Stokes equations to a LECS with $C_s = 0.05$ in plane Poiseuille flow and Sasaki *et al.* [61] have been able to continue the 'gentle' relative periodic orbit ECS solution of Kawahara & Kida [62] to an LECS solution up to $Re = 400$ in plane Couette flow. More recently, it has been shown [63] that travelling wave solutions can be computed in the plane channel even at Reynolds numbers an order of magnitude higher than those previously considered in [51,61].

The computation of these large-eddy coherent solutions confirms the existence of a coherent self-sustaining process at large scales. This process does not rely on energy inputs from larger-scale nor from smaller-scale motions, including those in the buffer layer. These results also seem promising, if seen from the perspective of laminar-turbulent transition studies where subcritical transition to turbulence has been related to the appearance of invariant solutions of the Navier-Stokes equations. In this context upper branch (ECS) solutions of the Navier-Stokes equations display features consistent with the turbulent flow issued from the transition process [64,65]. As additional solutions appear when the Reynolds number is increased and additional small scales develop in the turbulent regime, it is, however, not clear that only a small number of ECS solutions could be used to describe the turbulent statistics even at moderate Reynolds numbers [66,67]. It remains to be investigated if turbulent statistics, at least at a given spatial scale, could be built using proper averaging of LECS (steady, travelling-wave or periodic) solutions which would naturally embed the effect of small-scale fluctuations motions without the need for additional small-scale solutions.

5. Relation to Townsend's attached eddies

The self-similar nature of the optimal perturbations [34] and the self-sustaining processes [44] discussed in §2 and §3 are clearly reminiscent of the concept of 'attached eddies' described by Townsend [11]. This issue was further investigated by a set of numerical experiments [49,53] based on LES introduced to isolate the statistical and dynamical behaviours of the self-sustaining

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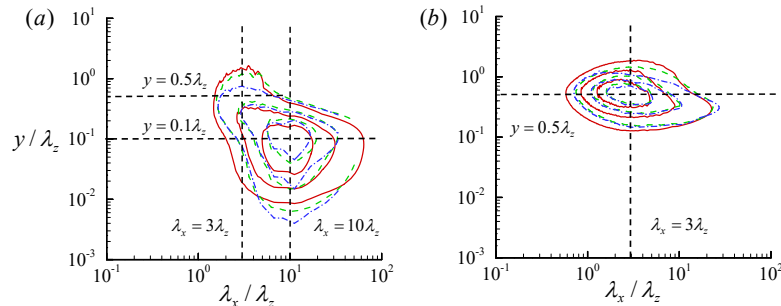
Figure 7. Premultiplied streamwise wavenumber spectra of (a) streamwise and (b) wall-normal velocities of the self-sustaining motions in a plane channel run at $Re_m = 2hU_m/\nu = 73333$, where U_m is the bulk velocity. (replotted from data reported in [49]). The solid (red), dashed (green) and dashed-dotted (blue) contours indicate $\lambda_z/\delta = 0.375, 0.5, 0.75$, respectively. The contour levels are chosen as 0.25, 0.5 and 0.75 times each of the maximum for comparison.

motions at each spanwise length scale λ_z between $\lambda_z^+ = 100$ and $\lambda_z = 1.5\delta$ by combining two techniques, one of which explicitly filters out the motions wider than λ_z [68] and the other damps out the motions smaller than λ_z by elevating C_s as described in §3. The statistical structures of the computed self-sustaining motions at each spanwise length scale were found to be approximately self-similar with respect to the given spanwise length λ_z, as shown in Fig. 7 by their spectra. Furthermore, the wall-parallel velocity components of the self-sustaining motions in the logarithmic region were large in the region relatively close to the wall, whereas their wall-normal velocity and Reynolds shear stress were small. It is important to highlight that the aforementioned features of the self-sustaining motions in the logarithmic region exactly fulfil all the theoretical requirements for predicting turbulence intensities in the logarithmic region in Townsend's original theory [11,12], suggesting that these motions are likely to be Townsend's attached eddies [44,49].

The numerical experiments by [49,53] revealed that each of the self-sustaining motions at a given spanwise length scale λ_z is composed of two structural elements: streaks and quasi-streamwise vortical structures. The former streaky structures, which predominantly carry the streamwise turbulent kinetic energy, are long, extending over $\lambda_x \simeq 10\lambda_z$ in the streamwise direction (fig. 7a), consistent with the linear analysis in §3. On the other hand, the latter quasi-streamwise vortical structures equally carry all the velocity components, and do not make any contribution to the region close to the wall (for example, compare the peak wall-normal location in the streamwise velocity spectra with the one in the wall-normal velocity spectra). Furthermore, the quasi-streamwise vortical structures are much shorter than the streaks and their streamwise extent was found to be only $\lambda_x \simeq 2 \sim 3\lambda_z$ (fig 7b). This size agrees well with that of the vortical structures either called hairpin vortex packets [5,16,18] or tall attached vortex clusters [69]. It should be mentioned that this length scale is well predicted by the most unstable wavelength of the streak instability [38,39], suggesting that they are presumably an outcome of this process. It should be stressed that the existence of the two structural elements, streaks and quasi-streamwise vortical structures, is essential for sustaining motions at a given λ_z, as their interaction lies in the heart of the self-sustaining process. It is thus appropriate to interpret streaks and quasi-streamwise vortices as that the single attached eddy two dynamically interconnected elements of a single attached eddy rather than two different classes of flow structures.

It is also important to highlight that the self-similar scaling of the streaky motions and quasi-streamwise vortical structures discussed retrieves most of the important features in one- and two-dimensional spectra of all the velocity components and the Reynolds stresses, including the emergence of k_x^{-1}. Each of the self-sustaining attached eddies also contain the Reynolds-stress lacking 'inactive' component, as Townsend originally hypothesized [11], and this was found

to correspond to each of the streaky motions below $y \simeq 0.5\lambda_z$ [49]. Further inspection of the dynamics of each of the attached eddies revealed that the streaks and quasi-streamwise vortical structures form a self-sustaining process [40,54,64] exactly the same as the one shown in Fig. 3 with a turn-over time scale of $Tu_\tau/\lambda_z \simeq 2$ [53]. The streaks are amplified from quasi-streamwise vortices via a coherent lift-up effect [25,29,34]; they then undergo rapid oscillation via secondary instability and/or transient growth on top of the streaks [38,39,41,54]; the quasi-streamwise vortical structures are nonlinearly regenerated [40,53,54]. The time scale of the self-sustaining process corresponds well to that of the 'bursting' in the logarithmic region, indicating that the bursting is naturally embedded in the self-sustaining process [53].

In addition to our own findings based on linear theory and on numerical experiments, more recently, there has been further evidence which directly supports the existence of the self-similar coherent structure. For instance, it has been recently experimentally shown [70] that POD modes extracted in the logarithmic region from Princeton's superpipe data are self-similar with respect to the spanwise length scale and/or the distance from the wall. Furthermore, it has recently been found that most turbulent skin friction at sufficiently high Reynolds numbers ($Re_\tau > 1000$) is generated by the self-similar self-sustaining motions in the logarithmic region [68,71]. In particular, the coherent lift-up effect was found to play a crucial role in transferring the streamwise momentum to the near-wall region (i.e. friction generation). Smart enforcing of this transfer at large scale was demonstrated to reduce a substantial amount of drag by modifying the self-sustaining processes at other scales [35], and its artificial inhibition also yielded a significant amount of drag reduction at high Reynolds numbers [53,68,71], suggesting the practical importance of the self-sustaining processes in skin-friction control.

6. Conclusions

Let us briefly summarize and then discuss the main results that we have collected and reviewed here. Probably the most important new finding is that large-scale [43,51] and moderate-scale [44] structures can self-sustain when smaller scale motions are artificially quenched in over-damped large-eddy simulations of the turbulent flow. A continuum of self-sustaining motions therefore exists which is, furthermore, statistically and dynamically consistent with the features of Townsend's attached eddies including an approximate self-similarity in the logarithmic region [34,44,49,63]. The existence of coherent self-sustained processes at large scale is also confirmed by the computation of invariant large-scale solutions of the filtered (fully nonlinear) Navier-Stokes equations [51,55,60,61,63]. The ability of the moderate- and large-scale structures to self-sustain is supported by generalized linear stability analyses revealing that the non-normal amplification of coherent quasi-streamwise streaks from coherent quasi-streamwise vortices provides a robust mechanism for the extraction of kinetic energy from the turbulent mean flow at scales ranging from those of buffer-layer streaks to those of the large-scale and very-large scale structures [25,30–35]. The mechanism by which these coherent motions self-sustain is similar to the SSP process which was proposed to model the buffer layer dynamics [40,41]. A notable difference, however, is that because the self-sustained 'coherent' structures exist in a suitably averaged or filtered sense, especially at large scales, it is essential to include in the picture the Reynolds stresses associated to the fluctuating or residual smaller-scale motions.

When put together, these results promote the idea that turbulence in wall-bounded flows is built upon a continuum of coherent *self-sustaining* processes with scales ranging from those of buffer-layer structures to those of large-scale and very-large scale motions. The whole picture is consistent with Townsend's 'attached eddies' paradigm, including the self-similarity of logarithmic layer structures [34,44,49] but it significantly differs from currently widespread *dynamical* interpretations based on bottom-up or top-down processes. Indeed, our results show that streamwise elongated streaky structures can dynamically self-sustain by drawing energy directly from the mean flow and therefore they do not necessarily need to be forced or generated by structures at smaller or larger scales as previously assumed.

The line of investigation we have presented is in no way closed. Many important questions need further investigation such as, for instance: (a) Can the continuum of self-sustained processes be educed from DNS or experimental data by a suitable combination of filtering and conditional averaging? (b) Is it possible to build a consistent low-dimensional dynamical system capturing the dynamics of each independent process at each single scale? (c) Can this understanding be used to build better predictive models e.g. of Reynolds shear stresses in wall-bounded turbulent flows? (d) Can this improved understanding be used to better control wall-bounded turbulence? These are important questions which are under active investigation.

Future work will also certainly aim at establishing a relation between the self-sustaining streaky motions and the specific shapes of attached eddies used in previous studies (e.g. [12,14]). Another sensitive issue will also be to establish the type and level of energy transfers between attached eddies of different sizes. These energy transfers were, through the Smagorinsky's model, reduced to a pure dissipative flux oriented from larger to smaller scale structures. Even if it has been shown that each single self-sustaining motion is, in principle, able to sustain itself by drawing energy uniquely from the mean flow, quantitative improvements can certainly be obtained by using more advanced subgrid-scale models better able to reproduce scale-to-scale energy transfers. Concerning the large-eddy invariant solutions (LECS), active work is under way to find more of these solutions, to understand if and how they are connected to the Navier-Stokes invariant solutions (ECS) and to understand their relevance to the dynamics of the attached eddies at each single scale. An important issue is to understand how many of these steady, travelling-wave or periodic LECS solutions are necessary to capture the first and second-order turbulent flow statistics at least at each single scale of motion.

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