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Aspherical photon and anti-photon surfaces

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ABSTRACT

In this note we identify photon surfaces and anti-photon surfaces in some physically interesting spacetimes, which are not spherically symmetric. All of our examples solve physically reasonable field equations, including for some cases the vacuum Einstein equations, albeit they are not asymptotically flat. Our examples include the vacuum C-metric, the Melvin solution of Einstein–Maxwell theory and generalisations including dilaton fields. The (anti-)photon surfaces are not round spheres, and the lapse function is not always constant.

1. Introduction

It is well known that the Schwarzschild solution contains circular photon orbits at $r = 3M$, where $M > 0$ is the ADM mass. These circular photon orbits are the projection onto the spatial manifold $t =$ constant of null geodesics in the spacetime. Moreover if the projection of the tangent vector of any null geodesic is tangent to the sphere at one time it remains tangent to the sphere at all future times. Because the Schwarzschild metric is static it is both possible and convenient to reformulate these properties using Fermat's principle in terms of the so-called optical geometry of the spatial sections. Any static spacetime metric may cast in the form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -N^2 dt^2 + g_{ij}dx^i dx^j \tag{1.1}$$

with $x^\mu = (t, x^i)$, $i = 1, 2, 3$ and the lapse function N and spatial metric g_{ij} independent of t. It is a straightforward exercise to show that the spatial projection of null geodesics are geodesics of the optical distance $ds_{\rm opt}$ defined by

$$ds^2_{\rm opt} = N^{-2} g_{ij}dx^i dx^j = f_{ij}dx^i dx^j \,. \tag{1.2}$$

For the Schwarzschild solution

$$ds^2_{\rm opt} = \frac{dr^2}{\left(1 - \frac{2M}{r}\right)^2} + \frac{r^2}{1 - \frac{2M}{r}}(d\theta^2 + \sin^2\theta d\phi^2)\,. \tag{1.3}$$

The circumference $C(r)$ of every great circle lying on the sphere $r =$ constant is given by

$$C(r) = \frac{2\pi r}{\sqrt{1 - \frac{2M}{r}}}. \tag{1.4}$$

The circumference $C(r)$ has a unique minimum at $r = 3M$. Thus every great circle lying on the sphere $r = 3M$ is a geodesic of the ambient three-dimensional optical manifold. Expressed differently: $r = 3M$ is a totally geodesic submanifold (in fact hypersurface) of the optical manifold.

Photon surfaces have attracted attention recently, in particular in the last two years there have been several results establishing the uniqueness of spacetimes admitting a photon surface under certain conditions [1–8]. These works typically assume that the spacetime is complete, asymptotically flat and with the exception of [8] assume that the lapse, N, is constant on the surface. In this paper we give some counter-examples to demonstrate that the conclusions of these theorems can be violated if one allows certain of the assumptions to be dropped. In particular, we shall show that there exist physically interesting metrics satisfying Einstein's equations (with or without matter) with non-spherically symmetric photon spheres such that the lapse is not constant on the photon sphere. Moreover these metrics are not of cohomogeneity one. The metrics contain relatively mild (conical) singularities, and are not asymptotically flat in the usual sense (although in the $\Lambda = 0$ case they contain regions in which the curvature approaches zero). These spacetimes we consider are all related to the C-metrics, first found by Levi-Civita [9], which are now understood to represent uniformly accelerated black holes.

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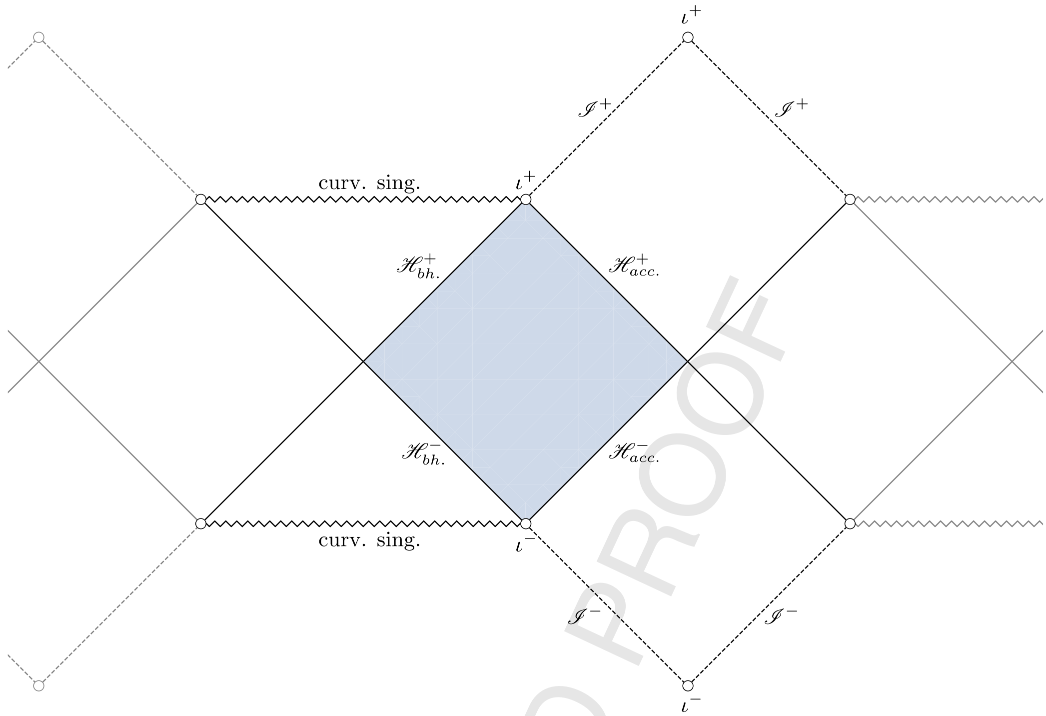

Fig. 1. A section of the Penrose diagram of the maximally analytically extended uncharged C-metric without cosmological constant. The shaded region corresponds to a static patch.

Anti-photon surfaces are much less well known. They correspond in the static setting to totally geodesic submanifolds of the optical metric for which, however, the photon orbits lying in the surface are stable (as opposed to the unstable case characterising the photon surfaces). In the spherically symmetric case, in the absence of naked singularities, it seems that these cannot occur if the energy-momentum tensor satisfies reasonable energy conditions [10]. However, in a class of cylindrically symmetric spacetimes of Melvin type [11] we present anti-photon cylinders.

2. Some aspherical photon spheres

2.1. The vacuum C-metric

While the existence of a photon surface surrounding a spherically symmetric black hole is not surprising, the fact that it persists when the black hole undergoes a uniform acceleration and ceases to be spherically symmetric is not at all obvious. This situation is described by the 'C-metric' first found by Levi-Civita [9]. Its physical significance was first elucidated by Kinnersly and Walker [12, 13]. For a subsequent review see [14]. For a uniqueness theorem see [15].

The metric is given in Hong–Teo coordinates [16] by

$$ds^2 = \frac{1}{a^2(x+y)^2}\left(F(y)dt^2 - \frac{1}{F(y)}dy^2 + \frac{1}{F(x)}dx^2 + F(x)d\phi^2\right), \tag{2.1}$$

where

$$F(u) = (1-u^2)(1+2mau). \tag{2.2}$$

$F(y)$ is negative on the interval $(-1/2ma, -1)$ and the metric is static in this region, with Killing horizons at $-1/2ma$, -1 corresponding to a black hole horizon and an acceleration horizon respectively. The coordinate x takes values in $(-1, 1)$ and for $a \neq 0$, there will in general be conical singularities on the axis $x = \pm 1$. Choosing the period of ϕ, one can eliminate the singularity on either $x = 1$ or $x = -1$. We can interpret the singularity as either representing a strut pushing the black hole or else a string pulling it depending on which choice we make.

In Fig. 1 we show the Penrose diagram of the maximally extended C-metric. The shaded region in the figure corresponds to the region $-1/2ma < y < -1$, and the two Killing horizons are shown. Each point in the interior of the shaded region represents a topological sphere with coordinates x, ϕ. This sphere is not round, but is axisymmetric and further has at least one conical singularity on the axis (see Fig. 2 for an embedded example). The spacetime has an asymptotic region which is accessible from the static region by causal curves falling through the acceleration horizon. This region is asymptotically flat in the sense that the curvature decays along causal curves.

The optical metric is given by

$$ds_{\text{opt}} = \frac{1}{F(y)^2}dy^2 + \frac{1}{|F(y)|}\left(\frac{dx^2}{F(x)} + F(x)d\phi^2\right). \tag{2.3}$$

Since $|F(y)|$ vanishes at the black hole horizon and the acceleration horizon, it must have at least one maximum on the interval $(-1/2ma, -1)$. This corresponds to a photon surface, and furthermore it is unstable, in the sense that geodesics which start close to the surface do not remain so. This surface will generically have a conical singularity corresponding to that of the full C-metric. In Fig. 2 we show an isometric embedding of the C-metric photon surface into Euclidean space. We identify ϕ so that the acceleration is induced by a string in this example (the other case does not allow an embedding into flat space).

Note that, in accordance with a remark in [19] that the Hamilton–Jacobi equation and the massless wave equation admit separation of variables for the metric (2.1).

We shall now show that the existence of a photon surface persists in the presence of cosmological constant and electric field, and for other static generalizations of the C-metric [21–23]. These

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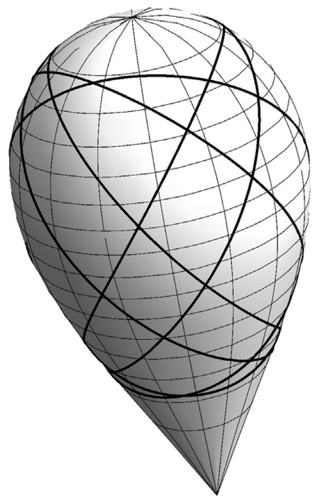

Fig. 2. The photon surface for the C-metric with $ma = 0.2$, showing a portion of a geodesic.

examples show that the appearance of such surfaces is not restricted to spacetimes of co-homogeneity one, even in the presence of matter.

2.2. C-metric with cosmological constant

The standard four dimensional "C-metric" with cosmological constant and electric charge may be cast in the form

$$ds^2 = \frac{1}{A^2(x+y)^2}\left(-F(y)dt^2 + \frac{1}{F(y)}dy^2 + \frac{1}{G(x)}dx^2 + G(x)d\phi^2\right) \tag{2.4}$$

where

$$F(y) = y^2 + 2mAy^3 + e^2A^2y^4 - 1 - \frac{\Lambda}{3A^2},$$
$$G(x) = 1 - x^2 - 2mAx^3 - e^2A^2x^4. \tag{2.5}$$

This solves the Einstein–Maxwell system with field strength $\mathcal{F} = edy \wedge dt$. The function F is positive on an interval (y_0, y_1) and the metric is static in this region, with Killing horizons at y_0, y_1 corresponding to a black hole horizon and an acceleration horizon. For sufficiently small e, Λ the geometry of the static region is essentially the same as for the uncharged C-metric, although the maximal extension is considerably altered [17].

The optical metric is given by

$$ds^2_{\text{opt}} = \frac{1}{F(y)^2}dy^2 + \frac{1}{F(y)}\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right). \tag{2.6}$$

After the transformation $y \to -1/r$, this is in precisely the form of equation (4.1) of [18] so we see immediately that the projective structure of the optical metric is invariant under changes of the cosmological constant. Since F vanishes at the black hole horizon and the acceleration horizon, it must have at least one maximum on the interval (y_0, y_1). For small values of e, Λ, this maximum will be unique. This corresponds to a photon surface, i.e. a totally geodesic submanifold of the optical metric. This surface will generically have a conical singularity corresponding to that of the full C-metric.

It is striking that the projective symmetry of the optical metric first noticed by Islam for the Schwarzschild–de-Sitter metric [20] and recently seen to hold for a wide family of static spherically symmetric solutions of Einstein's equations [10] can persist under deformations away from spherical symmetry. Note also that the metric (2.4), is conformal to the metric product of two 2-manifolds each admitting an isometry. Thus it shares the property with the standard C-metric that the Hamilton–Jacobi equation for null geodesics separates. Since the Ricci scalar is constant, it also follows that the conformally invariant wave equation separates.

2.3. C-metric with conformally coupled scalar field

In [21] Charmousis et al. construct a generalisation of the C-metric to allow a magnetic charge and coupling to a conformally coupled scalar field. The metric takes the form (2.1) with the metric functions changed to

$$F(y) = y^2 + 2mAy^3 + m^2A^2y^4 - 1 - \frac{\Lambda}{3A^2},$$
$$G(x) = 1 - x^2 - 2mAx^3 - m^2A^2x^4. \tag{2.7}$$

The new scalar and electromagnetic field are given by

$$\sqrt{-\frac{\Lambda}{6\alpha}}\frac{Am(x-y)}{1+Am(x+y)}, \qquad \mathcal{F} = edy \wedge dt + gdx \wedge d\phi. \tag{2.8}$$

Here α is a coupling constant appearing in the action and g is the magnetic charge, related to e and m by

$$e^2 + g^2 = m^2\left(1 + \frac{2\pi\Lambda}{9\alpha}\right). \tag{2.9}$$

Clearly the modification of F and G does not change the conformal and product structure seen in (2.1) and (2.4). Thus we have at least one photon surface and in addition the Hamilton–Jacobi equation for null geodesics separates. Indeed, for sufficiently small m, Λ, the polynomial $F(y)$ has four distinct roots,[1] so in any static region there is at most one photon surface.

2.3.1. Dilaton C-metric

The dilaton C-metric of Dowker et al. [22] reads:

$$ds^2 = \frac{1}{A^2(x-y)^2}\left[F(x)\left(G(y)dt^2 - \frac{dy^2}{G(y)}\right) + F(y)\left(\frac{dx^2}{G(x)} + G(x)d\varphi^2\right)\right]$$
$$e^{-2a\phi} = \frac{F(y)}{F(x)}, \qquad A_\varphi = qx, \qquad F(\xi) = (1 + r_- A\xi)^{\frac{2a^2}{(1+a^2)}}$$
$$G(\xi) = \bar{G}(\xi)(1 + r_- A\xi)^{\frac{(1-a^2)}{(1+a^2)}}, \qquad \bar{G}(\xi) = \left[1 - \xi^2(1 + r_+ A\xi)\right]. \tag{2.10}$$

The region between the horizons satisfies $G(y) < 0$, $G(x) > 0$ so that the metric is static with respect to $\partial/\partial t$ and has optical metric

$$ds^2_{\text{opt}} = \frac{dy^2}{G(y)^2} - \frac{F(y)}{G(y)}\left(\frac{dx^2}{G(x)F(x)} + \frac{G(x)}{F(x)}d\varphi^2\right) \tag{2.11}$$

[1] Note that a degree p polynomial with p distinct roots must have at least one turning point between any two consecutive roots by the intermediate value theorem. Since there are $p - 1$ pairs of consecutive roots, and a degree p polynomial has at most $p - 1$ turning points, we conclude there is *exactly* one turning point between any two consecutive roots.

Between the black-hole and the acceleration horizons, $F(y)G(y)^{-1}$ has an extremum so that there is a photon surface whose geometry is given by the part of the metric in brackets in (2.11). Provided a is sufficiently small, this extremum is unique, so there is at most one photon surface in the static patch.

Note that (2.10) is conformal to the product metric

$$ds^2 = \frac{1}{F(y)}\left(G(y)dt^2 - \frac{dy^2}{G(y)}\right) + \frac{1}{F(x)}\left(\frac{dx^2}{G(x)} + G(x)d\varphi^2\right). \tag{2.12}$$

It again follows that the Hamilton–Jacobi equation for null geodesics separates.

2.3.2. $U(1)^n$ charged C-metric

Another generalisation of the C-metric, due to Emparan [23] involves coupling extra $U(1)$ fields and scalars. The appropriate Lagrangian is

$$\mathcal{L} = R - \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\partial\sigma_i - \partial\sigma_j)^2 - \frac{1}{n}\sum_{i=1}^{n} e^{-\sigma_i}F^2_{(i)}, \tag{2.13}$$

where the scalars satisfy

$$\sum_{i=1}^{n}\sigma_i = 0. \tag{2.14}$$

The C-metric solution is then given by

$$ds^2 = \frac{1}{A^2(x-y)^2}\left[F(x)\left(\frac{G(y)}{F(y)}dt^2 - \frac{F(y)}{G(y)}dy^2\right) + F(y)\left(\frac{F(x)}{G(x)}dx^2 + \frac{G(x)}{F(x)}d\varphi^2\right)\right],$$

$$A_{(i)\,\varphi} = q_i x\frac{\sqrt{(1+r_0/q_i)(1-q_i^2A^2)}}{f_i(x)^{n/2}}.$$

where

$$F(\xi) = \prod_{i=1}^{n} f_i(\xi), \qquad f_i(\xi) = (1 - q_iA\xi)^{2/n}, \tag{2.15}$$

$$e^{-\sigma_i} = \frac{f_i(x)^nF(y)}{f_i(y)^nF(x)}, \qquad G(\xi) = (1-\xi^2)(1+r_0A\xi).$$

We take $q_i > 0$ and $r_0A > 1$. In the region $-1/r_0A < y < -1$, the metric is static with respect to $\partial/\partial t$. The Killing horizons at $y = -1/r_0A$ and $y = -1$ are the black hole horizon and the acceleration horizon respectively. The optical metric takes the form

$$ds^2_{\text{opt}} = \frac{F(y)^2}{G(y)^2}dy^2 - \frac{F(y)^2}{G(y)}\left(\frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\varphi^2\right). \tag{2.16}$$

Note that $G(y) < 0$ in this coordinate range. This has again at least one photon surface for a constant value of y located in the interval $(-1/r_0A, -1)$ where $F(y)^2/G(y)$ has an extremum. It appears that this photon surface is unique for sufficiently small q_i. The geometry of the photon surface is that of the metric in brackets in (2.16).

Note that Emparan's metric is conformal to one of the form (2.12) and hence the Hamilton–Jacobi equation for null geodesics separates.

2.4. The Melvin universe and anti-photon cylinders

The Melvin universe [11] is an electro-vac spacetime which is supported by a homogeneous magnetic field. A uniqueness property is established in [24], see also [25]. It has the spacetime metric

$$ds^2 = G^2(\rho)\left\{-dt^2 + d\rho^2 + dz^2\right\} + \frac{\rho^2}{G^2(\rho)}d\phi^2, \tag{2.17}$$

with

$$G(\rho) = 1 + \frac{B^2}{4}\rho^2 \tag{2.18}$$

and satisfies the Maxwell–Einstein equations with electromagnetic field

$$F = \frac{B\rho}{G^2(\rho)}d\rho\wedge d\phi, \tag{2.19}$$

corresponding to a homogeneous magnetic field aligned along the z-axis. The optical metric has line element

$$ds_{opt.} = d\rho^2 + dz^2 + \frac{\rho^2}{G^4(\rho)}d\phi^2. \tag{2.20}$$

The function $\rho^2G(\rho)^{-4}$ has a maximum at $\rho = \rho_0 := 2/(|B|\sqrt{3})$. Thus the cylindrical surface $\rho = \rho_0$ has vanishing second fundamental form and is therefore totally geodesic. In other words, geodesics initially satisfying $\rho = \rho_0$, $\dot\rho = 0$ remain tangent to $\rho = \rho_0$. Moreover, any null geodesic in the surface $\rho = \rho_0$ with $\dot\phi \neq 0$ is stable, in the sense that a small perturbation will remain close to $\rho = \rho_0$. Null geodesics in the surface with $\dot\phi = 0$ are marginally stable, since there are null geodesics with $\dot\phi = 0$, $\dot\rho = c \neq 0$. Thus $\rho = \rho_0$ is an anti-photon surface, with the conventions of [10]. Interestingly, this is in contrast to the spherically symmetric case of Reissner–Nordstrøm, metric with mass $M > 0$ and charge Q. In the sub-extreme case, $|Q| < M$, there is a unique photon sphere outside the horizon and for the super-extreme case, where $M < |Q| < \frac{3}{2\sqrt{2}}M$, there is both a photon and an anti-photon sphere [26].

In [27] a generalisation of the Melvin universe to include a cosmological constant is constructed. The metric is modified to:

$$ds^2 = G^2(\rho)\left\{-dt^2 + dz^2 + \frac{d\rho^2}{H(\rho)}\right\} + \frac{H(\rho)}{G^2(\rho)}\frac{\rho^2d\phi^2}{1-\frac{\Lambda}{B^2}}, \tag{2.21}$$

with G as previously defined and

$$H(\rho) = 1 - \frac{\Lambda}{3}\left(\frac{3}{B^2} + \frac{3\rho^2}{2} + \frac{B^2\rho^4}{4} + \frac{B^4\rho^6}{64}\right). \tag{2.22}$$

The electromagnetic field strength becomes:

$$F = \frac{B^2}{\sqrt{B^2-\Lambda}}\frac{\rho}{G^2(\rho)}d\rho\wedge d\phi. \tag{2.23}$$

The $\Lambda \to 0$ case reduces to the Melvin universe above. The $B \to 0$ limit is singular, however after a coordinate transformation the metric can be shown to be equivalent in the $\Lambda < 0$ case to a vacuum anti-de Sitter solution found by Bonnor [28]. Both metrics are (up to a coordinate transformation) equivalent to a Horowitz–Myers AdS Soliton [29].

One can verify that, provided $-3B^2 < \Lambda < B^2$, the spacetime (2.21) contains an anti-photon surface located at:

$$\rho = \rho_0 := \frac{2}{B}\sqrt{\frac{B^2-\Lambda}{3B^2+\Lambda}}, \tag{2.24}$$

in the $\Lambda \to 0$ limit, we recover the anti-photon cylinder of the Melvin universe.

Finally, there are also exist anti-photon cylinders in the dilaton-Melvin [22,30] metrics. Using (3.2) of [22], the optical metric is:

$$ds^2_{\text{opt.}} = dz^2 + d\rho^2 + \frac{\rho^2 d\phi^2}{\left(1+\frac{(1+a^2)B^2}{4}\rho^2\right)^{\frac{4}{1+a^2}}} . \tag{2.25}$$

If the dilaton–photon coupling constant a satisfies $a^2 < 3$ there is a unique value of ρ at which

$$\frac{\rho^2}{\left(1+\frac{(1+a^2)B^2}{4}\rho^2\right)^{\frac{4}{1+a^2}}} \tag{2.26}$$

has a maximum, and hence the situation is the same as for the Melvin universe.

3. Comments

The examples given above may be compared with various uses in the literature of the term "photon sphere". Firstly the word "sphere" seems inappropriate since it could be construed to mean a 2-surface which has the intrinsic geometry of a round or canonical sphere. A less misleading term is "photon surface". In the case of a static metric, the most natural definition would be a totally geodesic submanifold of the optical manifold. As such, it need not be a level set of the lapse function N. Indeed in the case of the vacuum C-metric

$$N = \frac{af(y)}{x+y} , \tag{3.1}$$

which depends upon *both* x and y, while the photon surface is at a fixed value of y. For the Melvin universe, the lapse is constant on the anti-photon surface.

The definition given above is much less restrictive than that used in several recent uniqueness results [1–7] where it is insisted that a photon sphere be a level set of g_{tt} and any electrostatic potentials. A recent attempt has been made to remove that restriction [8] and we suggest therefore, at least in the static situation, that the term photon surface be limited to that used in the present paper.

Another distinction to be borne in mind is that from what Teo [31] calls "Spherical photon orbits around a Kerr black hole". He finds a family of orbits which lie in a surface of constant r in a certain coordinate system but the surface is not geometrically a sphere and moreover not every photon orbit whose initial tangent lies in the sphere remains in the sphere.

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