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11 *A gu* bound the top and ontime top quifieses $\frac{1}{12}$ Aspherical photon and anti-photon surfaces $\frac{1}{77}$

13 78 14 G.W. Gibbons ^{a,b}, C.M. Warnick ^c and contain the contact of the

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20 ARTICLE INFO ABSTRACT 85

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22 Article history: **Example 22 September 2016** In this note we identify photon surfaces and anti-photon surfaces in some physically interesting 87 23 Received / October 2016 24 Auctive 14 OCCOPET 2010
24 Austive 14 December 2010 *25 Figure State Courrier AAAA* flat. Our examples include the vacuum C-metric, the Melvin solution of Einstein–Maxwell theory and ₉₀ zand the second of the lapse separalisations including dilaton fields. The (anti-)photon surfaces are not round spheres, and the lapse $\frac{91}{20}$ function is not always constant.

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1. Introduction

 $\frac{34}{2}$ It is well known that the Schwarzschild solution contains circu-³⁵ 100

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³⁶ 101 ³⁶ circular photon orbits are the projection onto the spatial manifold $C(t) = \frac{C(t)}{\sqrt{1-2M}}$. $t =$ constant of null geodesics in the spacetime. Moreover if the $\sqrt{1-\frac{r}{r}}$ and $t =$ constant of null geodesics in the spacetime. Moreover if the ³⁸ 103
Interpretion of the tangent vector of any null geodesic is tangent to the circumference $C(r)$ has a unique minimum at $r = 3M$. Thus ⁴³
200 108 108
200 1401 Spatial sections. Any static spacetime metric may cast in the form a photon surfaces have attracted attention recently in particular mat's principle in terms of the so-called optical geometry of the

$$
ds^{2} = g_{\mu\nu}dx^{\mu}dx^{\nu} = -N^{2}dt^{2} + g_{ij}dx^{i}dx^{j}
$$
 (1.1)

optical distance *ds*opt defined by

$$
ds_{\rm opt}^2 = N^{-2} g_{ij} dx^i dx^j = f_{ij} dx^i dx^j.
$$
 (1.2)

For the Schwarzschild solution

$$
ds_{\text{opt}}^2 = \frac{dr^2}{\left(1 - \frac{2M}{r}\right)^2} + \frac{r^2}{1 - \frac{2M}{r}} \left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{1.3}
$$

c.warnick@imperial.ac.uk (C.M. Warnick).

32 **1. Introduction 1. Introduction 1. Introduction 1. Introduction 1.** Introduction **1.** Introduction **1.** Introduction **1.** Introduction **1.** Introduction **1.** Introduction **1.** Integral **1.** Introduction **1.** I $r = constant$ is given by

$$
C(r) = \frac{2\pi r}{\sqrt{1 - \frac{2M}{r}}}.\tag{1.4}
$$

 $\frac{39}{104}$ the sphere at one time it remains tangent to the sphere at all fu-
the sphere at one time it remains tangent to the sphere at all fu-
 $\frac{994}{104}$ every great circle lying on the sphere r = 3M is a geodesic ⁴⁰ ture times. Because the Schwarzschild metric is static it is both ambient three-dimensional ontical manifold Expressed differently ⁴¹ possible and convenient to reformulate these properties using Fer-
 $r = 3M$ is a totally geodesic submanifold (in fact hypersurface) of possible and convenient to reformulate these properties using $r = 3M$ is a totally geodesic submanifold (in fact hypersurface) of $\frac{107}{107}$ The circumference $C(r)$ has a unique minimum at $r = 3M$. Thus every great circle lying on the sphere $r = 3M$ is a geodesic of the ambient three-dimensional optical manifold. Expressed differently: the optical manifold.

 $_{44}$ spatial sections. Any static spacetime metric may cast in the form $_1$ photon surfaces have attracted attention recently, in particular $_{109}$ $\frac{45}{10}$ 10 $\frac{1}{10}$ 110 $\frac{1}{10}$ in the last two years there have been several results establishing 110 46 $dS = g_{\mu\nu}dx^{\mu}dx^{\nu} = -N^2dt^2 + g_{ij}dx^{\nu}dx^{\nu}$ (1.1) the uniqueness of spacetimes admitting a photon surface under 111 with $x^{\mu} = (t, x^{i})$, $i = 1, 2, 3$ and the lapse function *N* and spatial spacetime is complete, asymptotically flat and with the exception $\frac{1}{12}$ 48 148 113
A metric *g_{ij}* independent of *t*. It is a straightforward exercise to show 16 and 181 assume that the lanse N is constant on the surface. In ⁴⁹ that the spatial projection of null geodesics are geodesics of the $\frac{10}{100}$ assume that the tapse, it, is constant on the surface. In $\frac{114}{114}$ $\frac{50}{2}$ until the spanning of the state of the state of this paper we give some counter-examples to demonstrate that the $\frac{115}{2}$ $\frac{51}{16}$ optical distance display defined by $52 \t 12 \t N⁻²$ $N⁻²$ $N⁻¹$ $N<$ t_{53} $ds_{opt} = N$ $g_{ij}dx dx^j = J_{ij}dx dx^j$. s_{15} (s_{12}) there exist physically interesting metrics satisfying Einstein's equa-54 For the Schwarzschild solution **tions** (with or without matter) with non-spherically symmetric the 55 120 photon spheres such that the lapse is not constant on the pho- $\frac{1}{2}$ $\frac{1}{2}$ ton sphere. Moreover these metrics are not of cohomogeneity one. σ_{57} $ds_{\text{opt}} = \frac{ds_{\text{opt}}}{\sqrt{1-2M}} = \frac{2M}{1-2M}$ (ab + sin $\theta d\phi$). The metrics contain relatively mild (conical) singularities, and are θ 122 58 $\left(1-\frac{2m}{r}\right)$ r and the usual sense (although in the $\Lambda=0$ 123 59 124 case they contain regions in which the curvature approaches zero). 60 125 These spacetimes we consider are all related to the C-metrics, first 61 126 *E-mail addresses:* g.w.gibbons@damtp.cam.ac.uk (G.W. Gibbons), 62 c.warnick@imperial.ac.uk (C.M. Warnick). 127 **127 Server Containers and Server Containers and Server Container** certain conditions $[1-8]$. These works typically assume that the spacetime is complete, asymptotically flat and with the exception of [\[8\]](#page-4-0) assume that the lapse, *N*, is constant on the surface. In found by Levi-Civita $[9]$, which are now understood to represent

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²⁷ Fig. 1. A section of the Penrose diagram of the maximally analytically extended uncharged C-metric without cosmological constant. The shaded region corresponds to a static ⁹² 28 93 patch.

30 Anti-photon surfaces are much less well known. They corre- Choosing the period of ϕ , one can eliminate the singularity on ei- 95 ³¹ spond in the static setting to totally geodesic submanifolds of the ther $x=1$ or $x=-1$. We can interpret the singularity as either ⁹⁶ ³² optical metric for which, however, the photon orbits lying in the representing a strut pushing the black hole or else a string pulling ⁹⁷ 33 98 surface are stable (as opposed to the unstable case characterising ³⁴ the photon surfaces). In the spherically symmetric case, in the ab-**The Hands and Fig. 1** we show the Penrose diagram of the maximally ex-³⁵ sence of naked singularities, it seems that these cannot occur if the tended C-metric. The shaded region in the figure corresponds to ¹⁰⁰ ³⁶ energy-momentum tensor satisfies reasonable energy conditions the region $-1/2$ ma $<$ y $<$ -1 , and the two Killing horizons are 101 ³⁷ [\[10\].](#page-4-0) However, in a class of cylindrically symmetric spacetimes of shown. Each point in the interior of the shaded region represents a ¹⁰² 38 Melvin type [11] we present anti-photon cylinders. topological sphere with coordinates x, ϕ . This sphere is not round, ¹⁰³ Anti-photon surfaces are much less well known. They corre-Melvin type [\[11\]](#page-4-0) we present anti-photon cylinders.

2. Some aspherical photon spheres

2.1. The vacuum C-metric

⁴⁴ While the existence of a photon surface surrounding a spheri- along causal curves. ⁴⁵ cally symmetric black hole is not surprising, the fact that it persists The optical metric is given by the same that the state of the state ⁴⁶ when the black hole undergoes a uniform acceleration and ceases **the example of the set of the example of the set of the example of the example of the set of the example of the example of the example of the example of** 49 114 ical significance was first elucidated by Kinnersly and Walker [\[12,](#page-4-0) see [\[15\].](#page-4-0)

The metric is given in Hong–Teo coordinates [\[16\]](#page-4-0) by

$$
ds^{2} = \frac{1}{a^{2}(x+y)^{2}} \left(F(y)dt^{2} - \frac{1}{F(y)}dy^{2} + \frac{1}{F(x)}dx^{2} + F(x)d\phi^{2} \right),
$$
\n(2.1)

where

$$
F(u) = (1 - u2)(1 + 2mau).
$$
 (2.2)

 126 *F (y)* is negative on the interval *(*−1*/*2*ma,*−1*)* and the metric is 127 static in this region, with Killing horizons at −1*/*2*ma*, −1 cor- responding to a black hole horizon and an acceleration horizon $\,$ We shall now show that the existence of a photon surface per- $\,$ 128 64 respectively. The coordinate *x* takes values in (−1, 1) and for $a \neq 0$, sists in the presence of cosmological constant and electric field, ¹²⁹

ther $x = 1$ or $x = -1$. We can interpret the singularity as either representing a strut pushing the black hole or else a string pulling it depending on which choice we make.

39 104 **104** but is axisymmetric and further has at least one conical singularity 104 40 **2. Some aspherical photon spheres** $\qquad \qquad$ on the axis (see [Fig. 2](#page-2-0) for an embedded example). The spacetime $\qquad \qquad$ 105 A 1 106 has an asymptotic region which is accessible from the static region 106 42 2.1. The vacuum C-metric **107** to the set of the set of the set of the acceleration horizon. This re-43 108 gion is asymptotically flat in the sense that the curvature decays In Fig. 1 we show the Penrose diagram of the maximally extended C-metric. The shaded region in the figure corresponds to the region $-1/2ma < y < -1$, and the two Killing horizons are shown. Each point in the interior of the shaded region represents a along causal curves.

The optical metric is given by

47 112 to be spherically symmetric is not at all obvious. This situation is 48 113 described by the 'C-metric' first found by Levi-Civita [\[9\].](#page-4-0) Its phys*ds*opt ⁼ ¹ *^F (y)*² *dy*² ⁺ 1 |*F (y)*| *dx*² *F (x)* ⁺ *^F (x)dφ*² *.* (2.3)

50 $\,$ [13\].](#page-4-0) For a subsequent review see [\[14\].](#page-4-0) For a uniqueness theorem Since $|F(y)|$ vanishes at the black hole horizon and the accelera- 115 51 see [15]. Sometimes that the interval of th 52 117 *(*−1*/*2*ma,*−1*)*. This corresponds to a photon surface, and further-53 118 more it is unstable, in the sense that geodesics which start close $\frac{54}{15}$ $\frac{1}{15}$ $\frac{1}{15}$ $\frac{1}{15}$ $\frac{1}{10}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ to the surface do not remain so. This surface will generically ha $a^2(x+y)^2$ $(x^2+y)^2$ $(x^3-x^2)^2$ $(x^2+y^2)^2$ $(x^2+y^2)^2$ $(x^3+y^2)^2$ a conical singularity corresponding to that of the full C-metric. In 56 121 [Fig. 2](#page-2-0) we show an isometric embedding of the C-metric photon 57 122 surface into Euclidean space. We identify *φ* so that the accelera-58 123 tion is induced by a string in this example (the other case does $\frac{59}{5}$ $\frac{F(t)}{24}$ $\frac{124}{12}$ $\frac{224}{124}$ $\frac{224}{124}$ $\frac{124}{124}$ $\$ Since $|F(y)|$ vanishes at the black hole horizon and the accelera-

60 (1.4) (1 (1.4)), (2.4) (2.4) Note that, in accordance with a remark in [\[19\]](#page-4-0) that the 125 Hamilton–Jacobi equation and the massless wave equation admit separation of variables for the metric (2.1) .

 65 there will in general be conical singularities on the axis $x = \pm 1$. and for other static generalizations of the C-metric [21–23]. These 130 We shall now show that the existence of a photon surface persists in the presence of cosmological constant and electric field, and for other static generalizations of the C-metric [\[21–23\].](#page-4-0) These

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geodesic.

²² examples show that the appearance of such surfaces is not re- The new scalar and electromagnetic field are given by 87 23 88 stricted to spacetimes of co-homogeneity one, even in the presence 24 of matter. $\lambda = 89$ of matter.

26 91 *2.2. C-metric with cosmological constant*

²⁸ The standard four dimensional "C-metric" with cosmological *magnetic charge, related to e and m* by ²⁹ constant and electric charge may be cast in the form $(2\pi \lambda)$

$$
\frac{3^4}{3^2} \quad ds^2 = \frac{1}{A^2(x+y)^2} \left(-F(y)dt^2 + \frac{1}{F(y)}dy^2 \right)
$$
\n(2.4)
\n
$$
+\frac{1}{G(x)}dx^2 + G(x)d\phi^2
$$
\n(2.4)
\n
$$
+\frac{1}{G(x)}dx^2 + G(x)d\phi^2
$$
\n(2.5)
\n
$$
+\frac{1}{G(x)}dx^2 + G(x)d\phi^2
$$
\n(2.6)
\n
$$
+\frac{1}{G(x)}dx^2 + G(x)d\phi^2
$$
\n(2.7)
\n
$$
= \frac{1}{2^8}
$$
\n(2.8)
\n
$$
=\frac{1}{2^8}
$$
\n(2.9)
\n
$$
=\frac{1}{2^8}
$$
\n(2.1) and (2.4). Thus we have
\nat least one photon surface and in addition the Hamilton-Jacobi
\nequation for null geodesics separates. Indeed, for sufficiently small

where

$$
F(y) = y^2 + 2mAy^3 + e^2A^2y^4 - 1 - \frac{\Lambda}{3A^2},
$$

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⁴² This solves the Einstein–Maxwell system with field strength $F =$
⁴² 1 $\int_{\Gamma(x)} \int_{\Gamma(x)} dy^2$ 107 ⁴⁸ maximal extension is considerably altered [\[17\].](#page-4-0) $\rho^{-2a\phi} = \frac{F(y)}{A_x - ay}$ $F(\xi) = (1 + r A\xi)^{\frac{2a^2}{(1+a^2)}}$ 113

$$
\frac{51}{52} \quad ds_{opt}^2 = \frac{1}{F(y)^2} dy^2 + \frac{1}{F(y)} \left(\frac{dx^2}{G(x)} + G(x) d\phi^2 \right). \tag{2.6} \qquad G(\xi) = \bar{G}(\xi)(1 + r - A\xi)^{\frac{1 - \omega^2}{(1 + \alpha^2)}}, \qquad \bar{G}(\xi) = \left[1 - \xi^2 (1 + r + A\xi) \right]. \tag{2.10}
$$

54 After the transformation $y \rightarrow -1/r$, this is in precisely the form
The region between the borizons satisfies $C(y) < 0$, $C(y) > 0$ so 55 of equation (4.1) of [\[18\]](#page-4-0) so we see immediately that the projective the region between the inizons sausies $G(y) < 0$, $G(x) > 0$ so 120 56 structure of the optical metric is invariant under changes of the the limit include is static with respect to $\sigma/\sigma t$ and has optical metric σ 121 57 cosmological constant. Since *F* vanishes at the black hole horizon $\frac{1}{2}$, $\frac{1}{2$ ⁵⁸ and the acceleration horizon, it must have at least one maximum $ds_{\text{ext}}^2 = \frac{dy}{ds} - \frac{f(y)}{s} \left(\frac{dx}{ds} + \frac{g(x)}{g} d\omega^2 \right)$ (2.11) ¹²³ 59 on the interval (y_0, y_1) . For small values of *e*, Λ , this maximum (y_0, y_1) $G(y)$ $G(x)F(x)$ $F(x)$ $F(x)$ $F(x)$ $F(x)$ 60 125 will be unique. This corresponds to a photon surface, i.e. a totally 61 126 geodesic submanifold of the optical metric. This surface will gener-62 ically have a conical singularity corresponding to that of the full that a degree p polynomial with p distinct roots must have at least one the that a conical singularity corresponding to that of the full 63 128 turning point between any two consecutive roots by the intermediate value theo-C-metric.

⁶⁴ It is striking that the projective symmetry of the optical met-
bas at most $p-1$ turning points, we conclude there is exactly one turning points. ⁶⁵ ric first noticed by Islam for the Schwarzschild–de-Sitter metric between any two consecutive roots. The metric that the schwarzschild of the Schwarzschild–de-Sitter metric between any two consecutive roots. The metri

1 66 [\[20\]](#page-4-0) and recently seen to hold for a wide family of static spher-2 67 ically symmetric solutions of Einstein's equations [\[10\]](#page-4-0) can per-3 68 Sist under deformations away from spherical symmetry. Note also 68 4 **69 that the metric** (2.4), is conformal to the metric product of two 69 $\mathbb{Z} \rightarrow \mathbb{Z}$ 3-manifolds each admitting an isometry. Thus it shares the prop- \mathbb{Z}^2 $\begin{array}{ccc} 6 & \longrightarrow & \longrightarrow & \longrightarrow \longrightarrow \end{array}$ $\begin{array}{ccc} \longrightarrow & \longrightarrow & \longrightarrow \end{array}$ $\begin{array}{ccc} \longrightarrow & \longrightarrow & \longrightarrow \end{array}$ $\begin{array}{ccc} \longrightarrow & \longrightarrow & \longrightarrow & \end{array}$ erty with the standard C-metric that the Hamilton–Jacobi equation \end{array} 71 \mathcal{I} for null geodesics separates. Since the Ricci scalar is constant, it \mathcal{I}^z $\mathbb{R} \longrightarrow \mathbb{R}$ also follows that the conformally invariant wave equation sepa- \mathbb{R}^3 \sim 74 rates.

11 2.3. C-metric with conformally coupled scalar field **76** 2.3. C-metric with conformally coupled scalar field

 $\frac{13}{13}$ In [\[21\]](#page-4-0) Charmousis et al. construct a generalisation of the C-14 79 metric to allow a magnetic charge and coupling to a conformally ¹⁵ coupled scalar field. The metric takes the form (2.1) with the metand the contract of the contra

Fig. 2. The photon surface for the C-metric with
$$
ma = 0.2
$$
, showing a portion of a geodesic.
\n
$$
F(y) = y^2 + 2mAy^3 + m^2A^2y^4 - 1 - \frac{\Lambda}{3A^2},
$$
\n
$$
G(x) = 1 - x^2 - 2mAx^3 - m^2A^2x^4.
$$
\n(2.7)

The new scalar and electromagnetic field are given by

$$
\sqrt{-\frac{\Lambda}{6\alpha}} \frac{Am(x-y)}{1+Am(x+y)}, \qquad \mathcal{F} = edy \wedge dt + gdx \wedge d\phi. \tag{2.8}
$$

27 92 Here *α* is a coupling constant appearing in the action and *g* is the magnetic charge, related to *e* and *m* by

$$
e^{2} + g^{2} = m^{2} \left(1 + \frac{2\pi \Lambda}{9\alpha} \right).
$$
 (2.9)

 $\frac{36}{101}$ equation for null geodesics separates. Indeed, for sufficiently small $\frac{101}{101}$ where m, Λ , the polynomial $F(y)$ has four distinct roots,¹ so in any static $\frac{102}{102}$ $\frac{1}{38}$ region there is at most one photon surface.

2.3.1. Dilaton C-metric

This solves the Einstein–Maxwell system with field strength
$$
\mathcal{F} =
$$

\n⁴³ $edy \wedge dt$. The function *F* is positive on an interval (y_0, y_1) and
\nthe metric is static in this region, with Killing horizons at y_0 , y_1
\n⁴⁴ corresponding to a black hole horizon and an acceleration hori-
\n⁴⁵ corresponding to a black hole horizon and an acceleration hori-
\n⁴⁶ essentially the same as for the uncharged C-metric, although the
\n⁴⁷ essentially the same as for the uncharged C-metric, although the
\n⁴⁸

$$
e^{-2a\phi} = \frac{F(y)}{F(x)}
$$
, $A_{\phi} = qx$, $F(\xi) = (1 + r - A\xi)^{\frac{2a^2}{(1+a^2)}}$

$$
\frac{51}{52} \quad ds_{opt}^2 = \frac{1}{F(y)^2} dy^2 + \frac{1}{F(y)} \left(\frac{dx^2}{G(x)} + G(x) d\phi^2 \right).
$$
\n(2.6)
$$
G(\xi) = \bar{G}(\xi)(1 + r - A\xi)^{\frac{(1 - a^2)}{(1 + a^2)}}, \quad \bar{G}(\xi) = \left[1 - \xi^2 (1 + r + A\xi) \right].
$$
\n(2.10)

The region between the horizons satisfies $G(y) < 0$, $G(x) > 0$ so that the metric is static with respect to *∂/∂t* and has optical metric

$$
ds_{\rm opt}^2 = \frac{dy^2}{G(y)^2} - \frac{F(y)}{G(y)} \left(\frac{dx^2}{G(x)F(x)} + \frac{G(x)}{F(x)} d\varphi^2 \right)
$$
 (2.11)

Note that a degree *p* polynomial with *p* distinct roots must have at least one rem. Since there are *p* − 1 pairs of consecutive roots, and a degree *p* polynomial has at most *p* − 1 turning points, we conclude there is *exactly* one turning point between any two consecutive roots.

ARTICLE IN PRES JID:PLB AID:32354 /SCO Doctopic: Theory [m5Gv1.3; v1.190; Prn:19/10/2016; 14:19] P.4 (1-5) 4 *G.W. Gibbons, C.M. Warnick / Physics Letters B* ••• *(*••••*)* •••*–*•••

1 Between the black-hole and the acceleration horizons, $F(y)G(y)^{-1}$ 2.4. The Melvin universe and anti-photon cylinders 66 ² has an extremum so that there is a photon surface whose geomeone photon surface in the static patch.

$$
\frac{8}{9} \quad ds^2 = \frac{1}{F(y)} \left(G(y) dt^2 - \frac{dy^2}{G(y)} \right)
$$
\n
$$
= \frac{1}{F(x)} \left(\frac{dx^2}{G(x)} + G(x) d\varphi^2 \right).
$$
\n
$$
\frac{11}{F(x)} \left(\frac{dx^2}{G(x)} + G(x) d\varphi^2 \right).
$$
\n
$$
(2.12)
$$
\n
$$
\frac{11}{G(\rho)} = 1 + \frac{B^2}{\rho^2} \rho^2
$$
\n
$$
(2.13)
$$
\n
$$
\frac{73}{74} \tag{2.14}
$$
\n
$$
\frac{73}{74} \tag{2.15}
$$

14 It again follows that the Hammon Jacobi equation for han geo-
and satisfies the Maxwell–Einstein equations with electromagnetic contractions. It again follows that the Hamilton–Jacobi equation for null geodesics separates.

18 Another generalisation of the C-metric, due to Emparan [\[23\]](#page-4-0) $1 - G^2(\rho)^{\alpha \rho}$ 19 involves coupling extra *U*(1) fields and scalars. The appropriate Lagrangian is

$$
\mathcal{L} = R - \frac{1}{2n^2} \sum_{i=1}^{n} \sum_{j=i+1}^{n} (\partial \sigma_i - \partial \sigma_j)^2 - \frac{1}{n} \sum_{i=1}^{n} e^{-\sigma_i} F_{(i)}^2, \qquad (2.13) \qquad ds_{opt.} = d\rho^2 + dz^2 + \frac{\rho^2}{G^4(\rho)} d\phi^2.
$$
 (2.20)

where the scalars satisfy

$$
\sum_{i=1}^{n} \sigma_i = 0. \tag{2.14}
$$

The C-metric solution is then given by

$$
ds^{2} = \frac{1}{A^{2}(x - y)^{2}} \left[F(x) \left(\frac{G(y)}{F(y)} dt^{2} - \frac{F(y)}{G(y)} dy^{2} \right) + F(y) \left(\frac{F(x)}{G(x)} dx^{2} + \frac{G(x)}{F(x)} d\varphi^{2} \right) \right],
$$

$$
A_{(i)} \varphi = q_{i} x \frac{\sqrt{(1 + r_{0}/q_{i})(1 - q_{i}^{2} A^{2})}}{f_{i}(x)^{n/2}}.
$$

where

$$
F(\xi) = \prod_{i=1}^{n} f_i(\xi), \qquad f_i(\xi) = (1 - q_i A \xi)^{2/n}, \qquad (2.15) \qquad ds^2 = G^2(\rho) \left\{ -dt^2 + dz^2 + \frac{d\rho^2}{H(\rho)} \right\} + \frac{H(\rho)}{G^2(\rho)} \frac{\rho^2 d\phi^2}{1 - \frac{\Lambda}{B^2}}, \qquad (2.21) \qquad \text{100} \qquad \text{with } G \text{ as previously defined and} \qquad (2.12) \qquad (2.22) \qquad (2.23)
$$

$$
e^{-\sigma_i} = \frac{f_i(x)^n F(y)}{f_i(y)^n F(x)}, \qquad G(\xi) = (1 - \xi^2)(1 + r_0 A \xi).
$$
\nWith G as previously defined and

\n
$$
H(\rho) = 1 - \frac{\Lambda}{2} \left(\frac{3}{R^2} + \frac{3\rho^2}{R^2} + \frac{B^2 \rho^4}{R^2} + \frac{B^4 \rho^6}{R^4} \right).
$$
\n(2.22)

50 115 the metric is static with respect to *∂/∂t*. The Killing horizons at $y = -1/r_0 A$ and $y = -1$ are the black hole horizon and the accel-
 $z = 2$ 52 117 eration horizon respectively. The optical metric takes the form

$$
\frac{54}{55} \quad ds_{opt}^2 = \frac{F(y)^2}{G(y)^2} dy^2 - \frac{F(y)^2}{G(y)} \left(\frac{dx^2}{G(x)} + \frac{G(x)}{F(x)} d\varphi^2\right).
$$
\n(2.16) The $\Lambda \to 0$ case reduces to the Melvin universe above. The $B \to 0$ are linearly independent, however after a coordinate transformation the

58 ene photon surface for a constant value of y located in the inter- uum anti-de Sitter solution found by Bonnor [\[28\].](#page-4-0) Both metrics 123 59 124 val *(*−1*/r*⁰ *^A,*−1*)* where *^F (y)*²*/G(y)* has an extremum. It appears 60 that this photon surface is unique for sufficiently small q_i . The ge-
Myers AdS Soliton [29]. 61 ometry of the photon surface is that of the metric in brackets in The Can verify that, provided $-3B^2 < \Lambda < B^2$, the spacetime 126 (2.16) . (2.16) Note that $G(y) < 0$ in this coordinate range. This has again at least one photon surface for a constant value of *y* located in the inter-(2.16).

63 128 Note that Emparan's metric is conformal to one of the form ⁶⁴ (2.12) and hence the Hamilton–Jacobi equation for null geodesics $2 \int B^2 - \Lambda$ (2.2.4) ¹²⁹ 65 separates. $\begin{array}{ccc}\n\sqrt{2.47} & 130 \\
130 & 130\n\end{array}$ separates.

2.4. The Melvin universe and anti-photon cylinders

³ try is given by the part of the metric in brackets in [\(2.11\).](#page-2-0) Provided The Melvin universe [11] is an electro-vac spacetime which is 68 ⁴ a is sufficiently small, this extremum is unique, so there is at most supported by a homogeneous magnetic field. A uniqueness prop- ⁶⁹ 5 one photon surface in the static patch. \sim \sim \sim erty is established in [\[24\],](#page-4-0) see also [\[25\].](#page-4-0) It has the spacetime met- \sim 70 ⁶ Note that [\(2.10\)](#page-2-0) is conformal to the product metric ric ric and the state of the state of the product metric state of the state The Melvin universe $[11]$ is an electro-vac spacetime which is supported by a homogeneous magnetic field. A uniqueness propric

$$
ds^{2} = \frac{1}{F(y)} \left(G(y)dt^{2} - \frac{dy^{2}}{G(y)} \right)
$$
\n
$$
ds^{2} = G^{2}(\rho) \left\{ -dt^{2} + d\rho^{2} + dz^{2} \right\} + \frac{\rho^{2}}{G^{2}(\rho)} d\phi^{2},
$$
\n(2.17)

with

$$
+\frac{\overline{F(x)}}{F(x)}\left(\frac{\overline{G(x)}}{G(x)} + G(x)d\varphi^2\right). \tag{2.12}
$$
\n
$$
G(\rho) = 1 + \frac{B^2}{4}\rho^2 \tag{2.13}
$$
\n
$$
F(x) = \frac{B^2}{4\pi\sigma^2} \tag{2.14}
$$

 15 desits separates. 80 field

¹⁶ 2.3.2.
$$
U(1)^n
$$
 charged C-metric
\n¹⁷ Another generalisation of the C-metric, due to Emparan [23] $F = \frac{B\rho}{G^2(\rho)}d\rho \wedge d\phi,$ (2.19)

20 grangian is
2² *z*-axis. The optical metric has line element 21 and the contract of the con corresponding to a homogeneous magnetic field aligned along the

$$
\mathcal{L} = R - \frac{1}{2n^2} \sum_{i=1}^{n} \sum_{j=i+1}^{n} (\partial \sigma_i - \partial \sigma_j)^2 - \frac{1}{n} \sum_{i=1}^{n} e^{-\sigma_i} F_{(i)}^2, \qquad (2.13) \qquad ds_{opt.} = d\rho^2 + dz^2 + \frac{\rho^2}{G^4(\rho)} d\phi^2.
$$
 (2.20)

25 where the scalars satisfy The function $\rho^2 G(\rho)^{-4}$ has a maximum at $\rho = \rho_0 = 2/(|B|\sqrt{3})$. 26 26 Thus the cylindrical surface $\rho = \rho_0$ has vanishing second fun- $\frac{n}{27}$ $\frac{n}{27}$ amental form and is therefore totally geodesic. In other words, $\frac{92}{27}$ $\sum \sigma_i = 0.$ (2.14) geodesics initially satisfying $\rho = \rho_0$, $\dot{\rho} = 0$ remain tangent to ⁹³ $\rho = \rho_0$. Moreover, any null geodesic in the surface $\rho = \rho_0$ with ⁹⁴ 30 **6 7 is stable, in the sense that a small perturbation will re-** $\phi \neq 0$ is stable, in the sense that a small perturbation will re-31 96 main close to *ρ* = *ρ*0. Null geodesics in the surface with *φ*˙ = ⁰ 32 are marginally stable, since there are null geodesics with $\phi = 0$, ⁹⁷
are marginally stable, since there are null geodesics with $\phi = 0$, ⁹⁷ $ds^2 = \frac{1}{4^2(\gamma - \nu)^2} \left[F(x) \left(\frac{\sigma(y)}{F(y)} dt^2 - \frac{F(y)}{F(y)} dy^2 \right) \right]$ $\dot{\rho} = c \neq 0$. Thus $\rho = \rho_0$ is an anti-photon surface, with the con- $A^2(X - Y)^2$ $\bigcup_{i=1}^n Y_i(Y_i, Y_i)$ $\bigcup_{i=1}^n Y_i(Y_i, Y_i)$ ventions of [\[10\].](#page-4-0) Interestingly, this is in contrast to the spherically ³⁵ 100 symmetric case of Reissner–Nordstrøm, metric with mass $M > 0$ ¹⁰⁰ 36 $+ F(y) \left(\frac{G(x)}{G(x)} dx^- + \frac{H(y)}{F(x)} d\varphi^- \right)$. 37 102 102 102 102 102 photon sphere outside the horizon and for the super-extreme case, 102 38 where $M < |Q| < \frac{3}{2\sqrt{2}}M$, there is both a photon and an anti-
20 $A_{(i),\alpha} = 0 \cdot x^{\frac{1}{2}}$ 104 photon sphere [\[26\].](#page-4-0)

 $J_i(x)^{1/2}$ 105 $J_i(x)^{1/2}$ 105 $\ln [27]$ $\ln [27]$ a generalisation of the Melvin universe to include a cos-And the mological constant is constructed. The metric is modified to: the metric is modified to: the metric is modified to:

$$
F(\xi) = \prod_{i=1}^{n} f_i(\xi), \qquad f_i(\xi) = (1 - q_i A \xi)^{2/n}, \qquad (2.15) \qquad ds^2 = G^2(\rho) \left\{ -dt^2 + dz^2 + \frac{d\rho^2}{H(\rho)} \right\} + \frac{H(\rho)}{G^2(\rho)} \frac{\rho^2 d\phi^2}{1 - \frac{\Lambda}{B^2}}, \qquad (2.21) \qquad \frac{107}{100} \left(\frac{d\rho^2}{1 - \frac{\Lambda}{B^2}} \right)
$$

$$
H(\rho) = 1 - \frac{\Lambda}{3} \left(\frac{3}{B^2} + \frac{3\rho^2}{2} + \frac{B^2 \rho^4}{4} + \frac{B^4 \rho^6}{64} \right).
$$
 (2.22) 113
We take $q_i > 0$ and $r_0 A > 1$. In the region $-1/r_0 A < y < -1$,

The electromagnetic field strength becomes:

$$
F = \frac{B^2}{\sqrt{B^2 - \Lambda}} \frac{\rho}{G^2(\rho)} d\rho \wedge d\phi.
$$
 (2.23)

56 **121 12** 57 Note that $G(y) < 0$ in this coordinate range. This has again at least metric can be shown to be equivalent in the $\Lambda < 0$ case to a vac-
122 are (up to a coordinate transformation) equivalent to a Horowitz– Myers AdS Soliton [\[29\].](#page-4-0)

One can verify that, provided $-3B^2 < \Lambda < B^2$, the spacetime

$$
\rho = \rho_0 := \frac{2}{B} \sqrt{\frac{B^2 - \Lambda}{3B^2 + \Lambda}},\tag{2.24}
$$

G.W. Gibbons, C.M. Warnick / Physics Letters B ••• *(*••••*)* •••*–*••• 5

1 in the $\Lambda \rightarrow 0$ limit, we recover the anti-photon cylinder of the [2] C. Cederbaum, G.J. Galloway, Uniqueness of photon spheres via positive mass 66 2 67 rigidity, arXiv:1504.05804 [math.DG]. Melvin universe.

$$
ds_{\text{opt.}}^2 = dz^2 + d\rho^2 + \frac{\rho^2 d\phi^2}{\left(1 + \frac{(1 + a^2)B^2}{4}\rho^2\right)^{\frac{4}{1 + a^2}}}.
$$
\n(2.25) [9] S. Yazadjiev, B. Lazov, Classification of the static and asymptotically flat Einstein-Maxwell-dilaton spacetimes with a photon sphere, arXiv:1510.04022\n
\n[10] S. Yazadjiev, B. Lazov, Classification of the static and asymptotically flat
\n[11] S. Yazadjiev, B. Lazov, Classification of the static and asymptotically flat
\n*g*-*q*-*q*}.

10 **a unique value of** ρ **at which** the static Latitude of ρ at which a unique value of *ρ* at which

$$
\frac{\rho^2}{(1+\frac{(1+a^2)B^2}{4}\rho^2)^{\frac{4}{1+a^2}}}
$$
\n
$$
= \frac{(2.26)}{(1+\frac{(1+a^2)B^2}{4}\rho^2)^{\frac{4}{1+a^2}}}
$$
\n
$$
= \frac{(2.26)}{(1+\frac{(1+a^2)B^2}{4}\rho^2)^{\frac{4}{1+a^2}}}
$$
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$$
= \frac{(2.26)}{(1+a^2)(1+a^2)} = 12
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16 Mehrin universe and the context of the came as the care and the dividend of the Mehrin universe 81 and 16 Mehrin universe 81 has a maximum, and hence the situation is the same as for the Melvin universe.

3. Comments

21 86 B225, [http://dx.doi.org/10.1103/PhysRev.139.B225.](http://dx.doi.org/10.1103/PhysRev.139.B225) 22 III LIC ILCLACULE OF LIC LETTI PROCOT SPIELE . THISLY LIC WOLD [12] W. Kinnersley, M. Walker, Uniformly accelerating charged mass in general rel- 87 $_{23}$ \sim Sphere seems inappropriate since it could be construed to mean ativity, Phys. Rev. D 2 (1970) 1359, [http://dx.doi.org/10.1103/PhysRevD.2.1359.](http://dx.doi.org/10.1103/PhysRevD.2.1359) 28 be a level set of the lapse function *N*. Indeed in the case of the *vol.* 14, 1972, pp. 48–85. The examples given above may be compared with various uses in the literature of the term "photon sphere". Firstly the word "sphere" seems inappropriate since it could be construed to mean a 2-surface which has the intrinsic geometry of a round or canonical sphere. A less misleading term is "photon surface". In the case of a static metric, the most natural definition would be a totally geodesic submanifold of the optical manifold. As such, it need not vacuum C-metric

$$
N = \frac{af(y)}{x + y},
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33 98 which depends upon *both x* and *y*, while the photon surface is at 34 a fixed value of y, for the Melvin universe, the lapse is constant $\frac{1}{1}$ cal constant, Phys. Rev. D 91 (6) (2015) 064014, [http://dx.doi.org/10.1103/](http://dx.doi.org/10.1103/PhysRevD.91.064014) 99 35 100 [PhysRevD.91.064014,](http://dx.doi.org/10.1103/PhysRevD.91.064014) arXiv:1501.01355 [gr-qc]. a fixed value of *y*. For the Melvin universe, the lapse is constant on the anti-photon surface.

36 The definition given above is much less restrictive than that [18] G.W. Gibbons, C.M. Warnick, M.C. Werner, Light-bending in Schwarzschild-de- 101 37 used in several recent uniqueness results $[1-7]$ where it is insisted sitter: projective geometry of the optical metric, class. Quantum Gravity 25 $_{102}$ 38 that a photon sphere be a level set of g_{tt} and any electrostatic $\frac{1387 \times 10000 \times 100000 \times 100000 \times 1000000 \times$ 39 potentials. A recent attempt has been made to remove that restric- kerr–Taub-NUT–de Sitter metrics in higher dimensions, Phys. Lett. B 609 (2005) 104 40 $\,$ tion $\,$ [8] and we suggest therefore, at least in the static situation, $\,$ 124, http://dx.doi.org/10.1016/j.physletb.2004.07.066, arXiv:hep-th/0405061. $\,$ 105 41 that the term photon surface be limited to that used in the present [20] J.N. Islam, The cosmological constant and classical tests of general relativity, 106 potentials. A recent attempt has been made to remove that restricpaper.

43 Another distinction to be borne in mind is that from what Teo **the contract the contract of the calar** field Class Quantum Gravity 26 (2009) 175012 108 44 [31] calls "Spherical photon orbits around a Kerr black hole". He arxiv:0906.5568 [gr-qc]. 2013 109 45 finds a family of orbits which lie in a surface of constant *r* in a [22] F. Dowker, J.P. Gauntlett, D.A. Kastor, et al., Pair creation of dilaton black holes, 110 46 certain coordinate system but the surface is not geometrically a $_{201}$ Phys. KeV D 49 (1994) 2909-2917, arxiv:nep-th(9309075, 111 47 sphere and moreover not every photon orbit whose initial tangent $\frac{125}{265-300}$ results then this finitial tangular to the set of the set lies in the sphere remains in the sphere.

Acknowledgements

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 $\frac{1}{2}$ contained and the distribution of the distribution of the distribution of the light of 4 Melvin $[22,30]$ metrics. Using (3.2) of $[22]$, the optical metric is:
 $\frac{101088/0264-9381/33/7/075006}{101088/0264-9381/33/7/075006}$ arxiv:1508 00355 [math DG] [3] C. Cederbaum, G.J. Galloway, Uniqueness of photon spheres in electro-vacuum spacetimes, Class. Quantum Gravity 33 (2016) 075006, [http://dx.doi.org/](http://dx.doi.org/10.1088/0264-9381/33/7/075006) [10.1088/0264-9381/33/7/075006](http://dx.doi.org/10.1088/0264-9381/33/7/075006), arXiv:1508.00355 [math.DG].
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