Preprocessing Algorithm for Source Localisation in a Multipath Environment

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Abstract—Several methods have been developed which allow the estimation of the location of an existing source with considerable accuracy in the absence of multipaths. However, if, in addition to the Line-of-Sight (LOS) path, non-LOS (NLOS) paths are also present, then all existing localisation algorithms dramatically fail to estimate the location of the source. In this paper, a passive array processing algorithm is proposed, which, if used prior to a localisation approach, suppresses all the multipath contributions in the received signal except for that of the LOS path. The performance of the proposed algorithm is evaluated through computer simulation studies.

Index Terms—Array signal processing, localisation, multipath, spherical wave propagation.

NOTATION

$\alpha$, $A$ Scalar
$\alpha^a$, $A^a$ Column vector
$\mathbf{A}$ Matrix
$\mathbb{I}_N$ $(N \times N)$ identity matrix
$\mathbf{0}_N$ Column vector of $N$ zeros
$\mathbf{1}_N$ Column vector of $N$ ones
$(\cdot)^T$, $(\cdot)^H$ Transpose, Hermitian transpose
$|\cdot|$ Magnitude of a complex number
$\lfloor a \rfloor$ Largest integer smaller than or equal to $a$
trace($\mathbf{A}$) Sum of the diagonal elements of matrix $\mathbf{A}$
diag($\mathbf{g}$) Matrix with vector $\mathbf{g}$ as main diagonal
$\odot$, $\otimes$ Hadamard and Kronecker product
$\mathcal{E}\{\cdot\}$ Expectation operator
$\mathbb{R}$, $\mathbb{C}$ Set of real and complex numbers

I. INTRODUCTION

The aim of a localisation algorithm is to process the signals transmitted by a target/source in order to accurately estimate its location. Localisation algorithms can be described as Range-based and as Direction-based, such as Direction-Of-Arrival (DOA). Range-based methods are in turn divided into "Received Signal Strength" (RSS) [1][2] and "Time" based methods, such as Time-Of-Arrival (TOA) [3][4] and Time-Difference-Of-Arrival (TDOA) [5] approaches. Furthermore, hybrid methods, such as TOA/DOA [6] or jointly range and DOA [7], have been developed. However, all of these methods fail dramatically in the presence of multipath.

Several approaches have been developed to suppress the Non-Line-Of-Sight (NLOS) paths. Many of them employ a statistical model of the transmission channel [8][9], however they often treat NLOS paths as noise [10], which limits their ability to completely remove NLOS contributions from the received signal. A localisation approach for NLOS environments is presented in [11], however it requires more than one receiver array and poses further requirements as to the array geometry of the source. Similarly, the approach in [12] requires several "agent" sensors in addition to the array elements. Another solution is thresholding as in [13] and [14], but the threshold’s value is arbitrarily set and thus the LOS signal might be rejected. The TOA approach in [15] can locate the source using only NLOS paths, but only if the multipath has pure reflectors. In [16] and [17], a TOA/DOA method is presented but it is only shown to work for a circular array and the location estimate error can exceed one meter even for high SNR. The hybrid DOA/TDOA in [18] is not very accurate either. A ray-tracing approach is presented in [19] but it requires a database for fingerprinting. A ray-tracing method that uses TDOA is proposed in [20], which requires a "Multipath Characteristic Database". The approach in [21] also uses a predefined database of template reflected signals. In [22] a multi-frequency array decorrelates the signals by averaging the signal covariance matrices obtained from different frequencies, but this requires several downconverted versions of the received signal to be stored.

The preprocessing algorithm that is proposed in this paper estimates the signal subspace of the LOS path by using only temporal information extracted from the received signal. By applying the projection operator of the estimated subspace to the received signal, the contributions of all NLOS paths are completely removed.

This paper is organised as follows: in Section II, the problem which the proposed algorithm is aiming to solve is mathematically formulated. In Section III, the theoretical framework of the proposed preprocessing algorithm is presented and then described in a compact form as a series of steps. In Section IV, computer simulation studies demonstrate the successful operation of the algorithm as well as the influence of factors such as the number of multipaths and the power of noise when the algorithm is applied prior to two different localisation methods. Finally, the paper is concluded in Section V.

II. PROBLEM FORMULATION

Consider a calibrated array of $N$ omnidirectional sensors with known geometry operating in the presence of a single source, whose location is unknown. The source transmits a sequence of message symbols $\{a[n], \forall n\}$ as a baseband signal.
The shifting matrix \( J \) denoting the shifting matrix, which is defined as follows

\[
J \triangleq \begin{bmatrix}
\Omega_{N_c-1}^T & 0 \\
\Omega_{N_c} & \Omega_{N_c-1}
\end{bmatrix}
\] (10)

The shifting matrix \( J \) (or \( J^T \)), when raised to the power of \( \ell \), downshifts (or upshifts) a vector by \( \ell \) positions. This provides a convenient and compact way to model multipaths.
with $\delta_k$ being a $2N_c \times 1$ vector of zeros whose $k^\text{th}$ element is equal to 1.

It should be mentioned here that the magnitude of the elements of the manifold vector $\mathbf{g}_k$, which under planewave propagation is unity for all elements, under spherical wave propagation is non-unity. As a result, in the case of spherical wave propagation, the magnitude of the received snapshots must be properly equalised using the matrix $\mathbf{G}$, which is defined in Equation (20).

Using vector $\mathbf{f}$, the matrix $\mathbf{C}_{\text{int}}$, whose columns are the basis of the signal subspace comprising the NLOS and ISI signal contributions to $\mathbf{x}[n]$, is constructed as follows

$$\mathbf{C}_{\text{int}} \triangleq [\mathbf{C}_{\text{NLOS}}, \mathbf{C}_{\text{ISI}}] \in \mathbb{R}^{2N_c \times (3K-1)}$$

where

$$\mathbf{C}_{\text{NLOS}} \triangleq \begin{bmatrix} \mathcal{g}^T \mathbf{L}_1, \mathcal{g}^T \mathbf{L}_2, \ldots, \mathcal{g}^T \mathbf{L}_K \end{bmatrix}$$

$$\mathbf{C}_{\text{ISI}} \triangleq \begin{bmatrix} \mathcal{g}^T \mathbf{L}_1, \mathcal{g}^T \mathbf{L}_2, \ldots, \mathcal{g}^T \mathbf{L}_K \end{bmatrix}$$

with

$$\mathbf{C}_{\text{prev}} \triangleq \begin{bmatrix} (\mathcal{g}^T \mathbf{L}_1)^N \mathcal{g}_1, (\mathcal{g}^T \mathbf{L}_2)^N \mathcal{g}_2, \ldots, (\mathcal{g}^T \mathbf{L}_K)^N \mathcal{g}_K \end{bmatrix}$$

$$\mathbf{C}_{\text{next}} \triangleq \begin{bmatrix} (\mathcal{g}^T \mathbf{L}_1)^N \mathcal{g}_1, (\mathcal{g}^T \mathbf{L}_2)^N \mathcal{g}_2, \ldots, (\mathcal{g}^T \mathbf{L}_K)^N \mathcal{g}_K \end{bmatrix}$$

Using the matrix $\mathbf{C}_{\text{int}}$, the projection operator $\mathbf{P}_{\text{LOS}}$ is constructed, which projects to the complementary subspace of the NLOS and ISI signals

$$\mathbf{P}_{\text{LOS}} \triangleq \mathbf{I}_N \otimes \left( \mathbf{I}_N - \mathbf{C}_{\text{int}} \left( \mathbf{C}_{\text{int}}^H \mathbf{C}_{\text{int}} \right)^{-1} \mathbf{C}_{\text{int}}^H \right)$$

By applying this projection operator to $\mathbf{x}[n]$ given by Equation (5), the transformed discretised signal is obtained

$$\mathbf{z}_{\text{LOS}}[n] = \mathbf{P}_{\text{LOS}} \mathbf{x}[n] = \mathbf{P}_{\text{LOS}} \mathbf{h}_1 \beta_1 n[\mathbf{a}] + \mathbf{P}_{\text{LOS}} \mathbf{n}[\mathbf{a}] + \mathbf{P}_{\text{LOS}} \mathbf{h}_2 \beta_2 n[\mathbf{a}][\mathbf{b}] + \mathbf{P}_{\text{LOS}} \mathbf{n}[\mathbf{a}][\mathbf{b}]$$

The transformed noise vector $\mathbf{n}[\mathbf{a}][\mathbf{b}]$ from Equation (29) has the following theoretical covariance matrix

$$\mathbf{R}_{\text{nc}} = \mathbf{E} \left\{ \mathbf{n}[\mathbf{a}][\mathbf{b}] \mathbf{n}[\mathbf{a}][\mathbf{b}]^H \right\} = \mathbf{P}_{\text{LOS}} \mathbf{E} \left\{ \mathbf{n}[\mathbf{a}][\mathbf{b}] \mathbf{n}[\mathbf{a}][\mathbf{b}]^H \right\} \mathbf{P}_{\text{LOS}}^H = \sigma_n^2 \mathbf{P}_{\text{LOS}} \mathbf{P}_{\text{LOS}}^H$$

$$= \sigma_n^2 \mathbf{P}_{\text{LOS}}^2 = \sigma_{n,\text{LOS}}^2 \mathbf{P}_{\text{LOS}}$$

where, above, the following properties of the projector operator have been used: $\mathbf{P} = \mathbf{P}^H$ and $\mathbf{P} = \mathbf{P}$. The theoretical covariance matrix of $\mathbf{z}_{\text{LOS}}[n]$ given by Equation (29) is then given as follows

$$\mathbf{R}_{\text{LOS}} = \mathbf{E} \left\{ \mathbf{z}_{\text{LOS}}[n] \mathbf{z}_{\text{LOS}}[n]^H \right\}$$

$$= \beta_1 |\beta_1|^2 \mathbf{P}_{\text{LOS}} \mathbf{h}_1 \left( \frac{\mathcal{g}_1[n][\mathbf{a}][\mathbf{b}]}{L} \right) \mathbf{h}_1^H \mathbf{P}_{\text{LOS}} + \sigma_{n,\text{LOS}}^2 \mathbf{P}_{\text{LOS}}$$

$$= \mathbf{P}_{\text{LOS}} \left[ \beta_1 |\beta_1|^2 \mathbf{h}_1 \mathbf{h}_1^H + \sigma_n^2 \right] \mathbf{P}_{\text{LOS}}$$

It becomes clear that the $1^\text{st}$ term in Equation (31) is a matrix whose rank equals 1, containing only the contribution of the desired LOS path. The covariance matrix $\mathbf{R}_{\text{LOS}}$ or the data vector $\mathbf{z}_{\text{LOS}}[n]$ given by Equation (29) can be used consequently as input to any localisation algorithm that is based on LOS data.

Based on the above discussion, using an array of $N$ sensors, the proposed preprocessing algorithm can be summarised in a step-by-step format as follows:

**STEP-1** Collect data at the output of the Tapped-Delay Line and form the matrix $\mathbf{X}$ described in Equation (14).

**STEP-2** Estimate the path delay vector $\mathbf{f}$ described in Equation (17) by performing one-dimensional search of the cost function $\xi_{\text{delay}}(\ell)$ described in Equation (18).

**STEP-3** Using the delay path vector $\mathbf{f}$ construct the matrix $\mathbf{C}_{\text{int}}$ described in Equation (23) and calculate the projection operator $\mathbf{P}_{\text{LOS}}$ described in Equation (28).

**STEP-4** Construct the transformed signal vector $\mathbf{z}_{\text{LOS}}[n]$ and its covariance matrix $\mathbf{R}_{\text{LOS}}$ as described in Equations (29) and (31) respectively.

**STEP-5** Use any localisation algorithm based on the LOS data of Step 4.

**IV. COMPUTER SIMULATION STUDIES**

Consider a calibrated planar antenna array of 4 elements distributed over a large area, thus having large inter-antenna spacing and operating at 2.4GHz.

The Cartesian coordinates of the antenna locations are given by the columns of the following matrix in meters,

$$\begin{bmatrix} 0, -90, -44.95, -40.55 \\ 0, 0, 30, -20 \\ 0, 0, 0, 0 \end{bmatrix}$$

Consider also that a single source is transmitting from a location with polar coordinates

$$(\rho_m, \theta_m) = (43.5074 \text{m}, 170.646^\circ)$$

There is no Multiple Access Interference (MAI) present, only the multipaths of a frequency-selective channel. Without loss of generality, an $m$-sequence of length $N_c = 31$ and a path loss exponent $\alpha = 1$ are considered.

Figure 1 shows the performance of the RSS localisation algorithm from [2] and of the Large Aperture Sparse Array (LAA) localisation algorithm proposed in [7] as the average input SNR increases and for increasing number of paths $K$. The average input SNR is the average of the SNRs at the array elements over all realisations. It can be seen in Figure 1 that both algorithms fail when more paths other than the LOS path are present.

However, the performance of both algorithms for the same frequency-selective channel can be immensely improved if the proposed preprocessor is employed prior to localisation. Indeed this is shown in Figure 2 for the exact same environment and data as used in Figure 1. The LAA algorithm
exhibits better performance, mainly because the RSS algorithm depends too much on the accuracy of power measurements at the array sensors to locate the source.

V. CONCLUSIONS

In this paper, a novel algorithm has been proposed which suppresses NLOS multipaths in order to enable source localisation. The algorithm first obtains estimates of the multipaths’ delays using a novel cost function and then produces a weight matrix which completely removes the contributions of the NLOS paths from the received signal.

REFERENCES