NUMERICAL MODELLING OF THE

ROLL DAMPING OF SHIPS DUE TO

VOLOX SHEDDING

by

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Flow visualization of the vortex shedding from the bilges of a rolling rectangular cross-section barge with bilge keels
SUMMARY

The vortex shedding from the bilges of rolling ships has been simulated using three computational models.

The first model involves the application of the Discrete Vortex Method to the simulation of oscillatory flow around infinite wedges, in particular the square isolated edge with bilge keel and/or edge rounding. Computed drag forces were included, via a matching technique, in a ship motion response computer program, so as to yield a method for predicting the roll damping of transportation barges due to vortex shedding. Comparisons with experiment for rectangular cross-section barges with various bilge geometries revealed that whilst the method was successful at computing roll damping for barges with sharp bilges, it overestimated damping increasingly at higher bilge radii.

The second model is an extension of the basic Discrete Vortex Method to predict the vortex shedding from the bilges of rolling vessels of quite general cross-section subject to certain restrictions. Computations were performed for a ship representative of current research interest. A tendency to overestimate roll damping was explained in terms of weaknesses in the Discrete Vortex Method itself and in its application to shedding from three-dimensional bodies.

Finally, a more advanced computer model of the full two-dimensional unsteady Navier-Stokes equations of motion has been developed which has enabled the prediction of the roll damping of barges with rounded bilges. This model represents, through its novel solution approach, a considerable advance in the simulation of vortex shedding especially for configurations where the separation point is required to be calculated as part of the computational solution. Comparisons with experiment for a variety of cases clearly established the validity of the method.

Flow visualisation experiments were undertaken to study the vortex shedding processes from barges with bilge keels and/or bilge rounding. It is shown that both the Discrete Vortex Method and the full Navier-Stokes solver predict flow patterns for oscillatory flow which correspond closely to experiment.

The three methods described here are capable of quite general application to many types of vortex shedding problems. Suggestions for possible future areas of work are described.
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A, B coefficients used in the Discrete Vortex Method calculations
A_1, B_2 half-coefficients used in the Discrete Vortex Method calculations
B_1 radiation roll damping coefficient
B_C critical roll damping coefficient
B_jk element of matrix of damping coefficients
B_V vortex roll damping coefficient
C_D drag coefficient
C_FV vortex force coefficient
C_m inertia coefficient
C_P pressure coefficient
C_rdm1 roll damping moment coefficient (see Equation 5.5.1)
C_rdm2 total vortex roll damping moment coefficient, obtained by summing C_rdm2 over the length of the ship
C_Z complex number used in the determination of L_Z
C_ζ complex number used in the determination of L_ζ
F aerodynamic force
F_m inertia force
F_V vortex force
K_C Keulegan-Carpenter number \( ( = U_0 T / d ) \)
L_Z flow length scale in the physical plane
L_ζ flow length scale in the transform plane
M transformation constant
M_V vortex roll damping moment
Re Reynolds number
St Stokes number \( ( = Re / K_c ) \)
\[ T \] period of oscillation of flow
\[ U_r, V \] freestream flow velocities in the transform plane
\[ U_0 \] freestream velocity amplitude in the physical plane (used in oscillatory flow)
\[ U_\infty \] freestream velocity in the physical plane
\[ V_T \] transpiration velocity
\[ V_z \] flow velocity scale in the physical plane
\[ W \] complex potential

\[ a \] bilge keel span
\[ a_i \] \( i^{th} \) complex ordinate in the transform plane
\[ a_i^* \] \( i^{th} \) complex ordinate in the physical plane
\[ b_v \] sectional vortex roll damping coefficient (see Equation 3.9.10)
\[ c_{rdm2} \] sectional vortex roll damping moment coefficient (see Equation 5.5.2)
\[ d \] cylinder diameter
\[ i \] \( \sqrt{-1} \)
\[ l_a \] moment arm
\[ l_s \] length of ship section
\[ l_v \] measure of vortex size
\[ p \] pressure
\[ r \] bilge radius
\[ t \] time
\[ t' \] time since the beginning of a particular time cycle
\[ u, v \] local velocities in the real and imaginary directions
\[ x, y \] real and imaginary components of complex ordinate
$z$ complex physical plane coordinate

$z_R$ height of roll centre above ship base

$\alpha_i$ $i$th transformation angle in the physical plane

$\alpha$ first moment of the vorticity field

$\Gamma_m$ strength of $m$th vortex

$\Gamma_{i,j}$ circulation at mesh point $(i,j)$

$\delta$ interior edge angle (degrees)

$\zeta$ complex transform plane coordinate

$\hat{\eta}_k$ $k$th mode of barge motion ($\hat{\eta}_4$ is roll amplitude)

$\lambda$ representation of edge angle [$=(360-\delta)/180$]

$\mu$ coefficient of viscosity

$\nu$ kinematic viscosity

$\rho$ density

$\sigma$ source strength

$\tau$ shear stress

$\phi$ real part of complex potential

$\psi_{i,j}$ stream function at mesh point $(i,j)$

$\Omega_{i,j}$ vorticity at mesh point $(i,j)$

$\omega$ roll angular frequency
CHAPTER 1
INTRODUCTION

PREFACE

Accurate methods of assessing and predicting the roll damping of ships are becoming increasingly important for their effective design and efficient operation. Determination of roll damping has largely so far been conducted by experimentation. A useful by-product of such experiments has been the development of empirically or semi-empirically based models of roll damping. Now, however, with the advent of fast digital computers it is possible to develop wholly theoretical models for many types of flow processes. In particular, recent advances in computational vortex shedding methods have begun to enable the theoretical prediction of ship eddy-making roll damping. Solutions produced by such computational models would provide a cheap and effective alternative to model testing. The work presented in this thesis concerns firstly the development and use of one such vortex shedding computer model, the Discrete Vortex Method, for the prediction of the roll damping due to vortex shedding of flat-bottomed barges with various bilge geometries. Secondly, the model has been extended to predict the roll damping of vessels of quite general cross-section subject to certain restrictions. Finally, a more advanced computer model of the full two-dimensional unsteady Navier-Stokes equations of motion has been developed which has enabled the prediction of the roll damping of flat-bottomed barges with rounded bilges to be carried out. In addition to roll damping data, the methods have been used to provide
valuable information on other aspects of the vortex shedding processes.

Much of the work presented here concerning the Discrete Vortex Method is also presented in Cozens, Bearman and Graham (1986).

1.1 The Prediction of the Motions of a Vessel in a Seaway
1.1.1 The prediction of vessel response in waves using strip theory

The advent of large tankers and high-speed dry-cargo ships has brought a greater awareness of the importance of the correct prediction of the various vessel responses to wave loading. Since the pioneering work of Korvin-Kroukovsky and Jacobs (1957) there have been major advances in prediction methods. The basis of this early work is the utilisation of strip theory, where the three-dimensional problem is reduced to a series of two-dimensional problems by the division of the ship into a number of transverse strips. The response of each section to wave loading is calculated, taking into account the damping due to wave radiation. Notable improvements have been devised by Smith and Salvesen (1970) who used close-fit methods of geometry representation to predict head-seas heave and pitch motions quite accurately even for high-speed hulls with large bulbous bows. Salvesen, Tuck and Faltinsen (1970) employed Frank's close-fit method (Frank, 1967) in the development of a theory for the prediction of heave, pitch, sway, roll and yaw motions as well as wave-induced vertical and horizontal shear forces, bending moments and torsional moments for a ship advancing at constant speed with arbitrary heading in regular waves. Comparisons with experimental data, where available, showed satisfactory agreement.
Despite the major advances outlined above, the more lightly damped motion responses, and particularly roll near resonance, are often poorly described by conventional strip theory, mainly due to the neglect of the effects of viscous damping. Salvesen, Tuck and Faltinsen, for instance, reported that roll response could only be computed with reasonable accuracy in near-resonance conditions if a viscous roll damping term were included in addition to the radiation damping. The same also holds true for the prediction of roll by the strip theory of Ogilvie and Tuck (1969) which was based on slender body theory. Since the roll motion may typically have a natural frequency within the range of the wave spectrum, it is therefore sensitive to the magnitude of the roll damping.

Many versions of strip theory are capable of predicting vessel response at forward speed. Although there is great practical interest in the response of vessels at forward speed in waves, the work on roll damping in this thesis is restricted to zero forward speed. However, it would certainly be possible to make empirical corrections to the results presented here to take account of the effects of forward speed. This might require an appropriate correction to roll damping at every strip constituting the vessel.

1.1.2 The roll response of a vessel at zero forward speed

The roll response of vessels at zero forward speed may be affected by a number of factors. Potential strip theory predicts a damping due to wave
radiation which has a linear dependency upon roll amplitude. Such a theory, however, fails to take account of the non-linearities in the vessel response. The possible sources of non-linearity at zero speed are skin friction and eddy-making damping, which give rise to non-linear damping moments, and non-linearities in the roll restoring moment.

As far as the roll restoring moment is concerned, Denise (1983) suggests that for shallow-bottomed vessels such as transportation barges linear potential theory may be invalid because of "non-linearities in the external forces, other than damping, due to water-structure interaction in the splash zone". Robinson and Stoddart (1986) have confirmed that for transportation barges at moderate to high roll angles there are indeed non-linearities in the restoring moment which may have an important influence on the vessel response. For deeper draught vessels, however, such non-linearities are not believed to be generally significant.

Kato (1958) concludes that the effect of skin friction stress is negligible for most practical cases. One should expect it to be relatively more important at model scale than full scale. Patel and Brown (1985) have estimated that the skin friction damping for a model barge of 0.8m beam affects the roll response at resonance by only 1%, so that in the case of a barge even at model scale skin friction stress may be considered negligible.

Usually it is argued that the most significant source of non-linearity
in the damping is the force associated with the shedding of vortices from the vessel hull and/or its appendages. The importance of vortex shedding in the roll damping of vessels was first noted in connection with bilge keels, which are a strong source of vortex shedding. Froude (1874) hypothesised a non-linear damping term, based on experiments with a flat plate oscillating in water. White (1895) carried out experiments to determine the additional damping produced by fitting bilge keels. Gawn (1940) and Dalzell (1978) performed and analysed tests on battleships and found that there was a quadratic or cubic variation of damping with roll amplitude, indicating substantial non-linearities in the damping term.

Ensuing from earlier experimental work, Kato (1966) developed a semi-empirical formula for the determination of the roll damping of vessels due to the effect of bilge keels. Ikeda et al. (1978) have developed a semi-empirical prediction method for the roll damping terms due to friction, wave, eddy and lift components for ordinary ship forms. Agreement with experiment for the simple shapes tested was fair except for the wide beam/draft variation. The eddy-making prediction method was based on experiments with twelve two-dimensional sections representative of container and cargo ships. A near-quadratic damping was found, that is, damping was roughly proportional to the squares of both roll amplitude and frequency. However, unappended sections with sharply defined flow separation points were not tested. Since the method was empirically based, its validity must presumably be restricted to vessels of largely similar cross-section to those used in the development of the method.
1.2 The Discrete Vortex Method

In response to a need for more accurate and more generally applicable methods for the prediction of the eddy-making component of ship roll damping, researchers have turned to develop wholly theoretical models for its prediction. Research has been directed, until the later stages of the work reported here, towards the Discrete Vortex Method.

1.2.1 Early development of the Discrete Vortex Method

Many detailed reviews of the historical development of the Discrete Vortex Method (DVM) have been published - see, for example, Clements and Maull (1975), Leonard (1980) or Jaroch (1986).

Rosenhead (1931) was the first researcher to introduce the concept of the discrete vortex representation of vortex sheets. He modelled successfully the Kelvin-Helmholtz instability of a two-dimensional infinite perturbed vortex sheet by replacing one wavelength of the vortex sheet with a distribution of evenly spaced discrete potential vortex elements along the sheet, thereby ignoring the effects of diffusion at finite Reynolds number. However, subsequent work by Hama and Burke (1960) repeating Rosenhead's work with a larger number of vortices and smaller time steps showed that whilst the vortices did indeed form into clusters, as in Rosenhead's original calculations, their paths were extremely irregular and occasionally crossed, an occurrence which is physically impossible for vortex sheets. Hama and
Burke discovered an error in Rosenhead's calculations, which, when rectified, improved the stability of their calculations. Rosenhead's work was extended by Abernathy and Kronauer (1962) who examined the stability of two initially parallel shear layers using the discrete vortex model. It was found that the two layers interacted to form the familiar von Karman vortex street pattern.

The discrete vortex approximation was first applied to the sheet shed from an elliptically loaded wing by Westwater (1935). The rolling up process followed closely that predicted analytically by Kaden (1931). A re-examination of Westwater's work by Moore (1974) revealed difficulties which were similar to those discovered with Rosenhead's model especially once many point vortices and much smaller time steps were employed. Most reporters on the work of Westwater have not mentioned the somewhat irregular appearance of the vortex sheet even in his calculations. This irregularity points to the fact that even with large time steps and few vortices instabilities in the vortex sheet still can occur. Moore concluded that it was the method of discretising the vortex sheet into point vortices which caused the instabilities. At certain times during a typical calculation the discrete point vortices approached each other closely; a spuriously large interaction then sometimes occurred which disrupted the calculation. Moore achieved suppression of the instabilities by successively amalgamating the innermost vortices of the spiral. Chorin and Bernard (1973) chose an alternative approach for suppressing the instabilities. They gave each point vortex a cut-off radius, so that the velocity at the centre of the vortex was zero rather than infinite as was the case for the pure potential vortex. The cut-off
thus prevented spuriously large interaction between pairs of vortices.

Clements (1973) applied the Discrete Vortex Method to sharp-edged bluff bodies. His model of flow behind a square based block modelled separation by the introduction of potential vortices into the flow at the sharp shedding edges, their initial velocity and strength depending only on the flow velocity at the "outside edge of the boundary layer". Clements' model predicted the form of the shedding pattern and the Strouhal number to within reasonable agreement. The model was successfully extended in Clements and Maull (1975), where vortices were placed near the separation points, their strength now being determined by application of the Kutta Condition, which was used as the "inviscid" analogue of boundary layer separation.

Kuwahara (1973) and Sarpkaya (1975) performed calculations for an inclined flat plate, transforming the plate into a circle using a Joukowsky transformation. Sarpkaya estimated the strength of the nascent, newly introduced vortices from the equation:

$$\frac{d\Gamma}{dt} = iU^2_{sh}$$  \hspace{1cm} 1.2.1

where the velocity $U_{sh}$ was taken as the mean velocity of the latest four vortices introduced into the shear layer. But Kiya and Arie (1977) have argued convincingly that Sarpkaya should not have based his formula on the velocity $U_{sh}$ but rather $U_{max}$, the velocity at the outer edge of the shear layer. The position of the vortices was estimated from satisfaction of the Kutta Condition. The generalised unsteady Blasius theorem (Milne-Thomson, 1968) was employed for the derivation of the normal force, which was predicted 20 - 25% higher than in experiment.
Strouhal number was predicted satisfactorily.

Further computations of the separating flow past flat plates at different angles of attack and different freestream conditions have been carried out by Kiya and Arie (1977), with separation positions for the nascent discrete vortices fixed at a distance "a_s" away from the edge of the flat plate. They carried out a careful analysis of the optimum value of a_s. Kiya, Arie and Harigane (1979) calculated the flow past a nearly normal flat plate using vortex decay to obtain results in good accord with experiment. Kamemoto and Bearman (1978) carried out an extensive investigation into the parameter a_s and suggested a suitable range of values within which predicted quantities did not vary substantially.

Even bluff body flows where no geometrically fixed separation point exists have been modelled using the Discrete Vortex Method. In his pioneering work on the application of the DVM to bluff body flows, Gerrard (1967) modelled the shedding of vortices behind a circular cylinder in uniform flow. Rather than placing the nascent vortices near the flow separation points, Gerrard placed them roughly one cylinder radius out from the centre line of the flow and one radius downstream of the centre of the cylinder. Calculated lift and drag curves showed time dependent oscillations, which could in many cases be interpreted to yield a representative Strouhal number. Further discrete vortex representations of the flow past circular cylinders have been carried out by Sarpkaya (1968), who simulated the early roll up of the shear layers. Stansby (1977) modelled with some success the flow past a circular cylinder both by fixing the separation points at an average
position estimated from experiment and also by predicting the separation points by "using the condition that the separating streamlines are tangential to the surface". In reality the actual shedding positions are unsteady and Reynolds number dependent. Thus fixing the separation points at an average position estimated from experimental evidence leads to certain inaccuracies; no convincing argument can be found for determining them by the tangential streamline method. On the other hand, these simplifying assumptions did yield useful results. Stansby also calculated oscillatory flow past the cylinder by fixing the separation points.

In bluff body flows where no geometrically fixed separation point exists, discrete vortex models are most often used either, as described above, by fixing the separation points in advance from, for example, the results of experimental analysis, or else by interaction with some form of boundary layer analysis. Deffenbaugh and Marshall (1976) matched an outer discrete vortex model with an inner viscous flow region, calculating the separation points by a solution of the unsteady laminar boundary layer equations. Pleasing agreement with other work was obtained in the early stages of the flow, but at later times the separation points moved unrealistically far forward, adversely affecting lift. More successful for long times were the calculations of Sarpkaya and Schoaff (1979) who employed a heuristic model of vortex decay to reduce the levels of circulation in the vortex sheet. It was their observation, as it is of many workers with the Discrete Vortex Method (see, for example, Clements and Maull, 1975), that the concentrated vortices of the wake contained typically 30% more vorticity than would
be found in experiment. Vortex decay was employed as a means of reducing the circulation strength in the wake nearer to experimental values. Sarpkaya and Schoaff also utilised a method of rediscretisation of the vortex sheets which had been pioneered by Fink and Soh (1974). Fink and Soh demonstrated that a source of instability in the roll-up of vortex sheets which had been noticed, for example, by Hama and Burke (1960), could be removed by rediscretisation of the vorticity. Rediscretisation involves the placing of each vortex at each time step at the midpoint of its segment. One might view this as another method of smoothing out the instabilities referred to above caused by the discretisation inherent in the potential vortex model.

1.2.2 The application of the Cloud-in-Cell technique to the Discrete Vortex Method

One of the main limitations of the Discrete Vortex Method as described so far is that it depends on the calculation of the influence on each point vortex of the rest of the point vortices as determined by the Biot-Savart law. For a computation with N vortices this procedure requires $O(N^2)$ operations to compute the required velocities. This effectively limits the maximum number of vortices in a typical computation on a mainframe computer to hundreds rather than thousands. The Cloud-in-Cell (or Vortex-in-Cell) technique which is described in Chapter 6 retains the Lagrangian treatment of the vortex field with its attendant advantages in terms of its ability to represent without smearing (or without introducing the levels of artificial viscosity associated with conventional grid methods) small features of the flow
moving with the fluid through a large region, but solves the Poisson equation for the velocity field on a fixed Eulerian Mesh. The technique evolved out of methods such as the Particle-in-Cell method developed at Los Alamos in 1955. A comprehensive review of that method may be found in Harlow (1964). A development of the PIC method is the Fluid-in-Cell method of Gentry, Martin and Daly (1966). The advantage of the CIC hybrid Lagrangian/Eulerian approach is the greatly reduced computational cost for small loss in accuracy. For a mesh with M intersections the number of operations is $O(N) + O(M \log_2 M)$. Reasonably sized calculations might be speeded up by two orders of magnitude by use of the CIC method. Christiansen (1973) was the first to report the use of the CIC technique in studying the motion of a two-dimensional incompressible, inviscid and homogeneous fluid. He computed successfully such fundamental cases as the Kelvin-Helmholtz instability and the formation of the Karman vortex street from two parallel shear layers, echoing the work of Rosenhead (1931) and Abernathy and Kronauer (1962) respectively. Baker (1979) applied the method to the evolution of the vortex sheet from an elliptically loaded wing and a wing with a flap deployed. Naylor (1982) has employed the method for uniform flow past a normal flat plate. Basuki (1983) used it to calculate unsteady flow over aerofoils and cascades.

In the present work the Cloud-in-Cell method has not been used in conjunction with the Discrete Vortex Method. This is because alternative procedures for reducing the computational expense were found. The importance of the CIC method here lies in its use in the solution of the full Navier-Stokes equations. This will be discussed in the next section.
1.3 The Solution of the Full Two-Dimensional Unsteady Navier-Stokes Equations

There are two main limiting factors in the application of the Discrete Vortex Method to the computation of vortex shedding from ship hulls.

1) The Discrete Vortex Method, with its assumption of an infinitely thin vortex sheet, is incapable of representing the effects of either diffusion or turbulence in the vortex sheet.

2) The Discrete Vortex Method cannot predict the flow separation point. This is not a problem in cases where the separation point is fixed by geometry. Shedding off a flat plate or bilge keel are typical examples where it can be used with a high degree of success. On the other hand, rounding of the shedding surface, such as on a circular cylinder or barge with rounded bilges, poses many problems as to the location of the shedding point. One solution which is feasible for small amounts of rounding or more generally where the separation point is known not to move markedly is to fix the point either from the results of experiments or by a heuristic approach. An alternative would be to carry out a boundary layer calculation to determine the point of separation. An obvious weakness of this is that boundary layer methods fail at and very near the separation position so that it cannot be predicted with much certainty.

In many cases, then, it is clear that some model of the vortex diffusion process, not only in the wake but also near the separation point, is
required. Such models are described in the succeeding sections, and involve solution of the unsteady two-dimensional Navier-Stokes equations.

1.3.1 Eulerian solutions of the Navier-Stokes equations

A useful survey of solutions to the Navier-Stokes equations may be found in Roache (1972). When an Eulerian mesh method is employed the Navier-Stokes equations are often solved in their vorticity transport form (see Chapter 6), although solutions of the "primitive" Navier-Stokes equations (the so-called "u,v,p" solutions) are also found. The vorticity transport equation is parabolic in form. Since the computational expense of solving parabolic equations is often large much effort has been expended on producing fast and efficient solution schemes. As early as 1957, Richtmyer (1957) had presented more than ten numerical schemes for the solution of parabolic equations. Solution schemes for these equations can be divided into two categories, explicit and implicit. Explicit schemes advance to a new time step using only variable values determined at the old time step; implicit methods advance using variable values from the new time step as well. In general explicit methods are simpler to program but computationally more expensive since they are limited by stability arguments to short time steps; implicit methods are more complicated but generally computationally cheaper since time steps can often - at least in principle - be arbitrarily large. One important feature of all solutions of the Navier-Stokes equations of this type is the requirement of small
mesh size near the body side in order to enable adequate representation of the boundary layer. This has important implications on computational expense since the time step size will usually be governed for stability or accuracy reasons by the smallest mesh cell size.

A frequently referenced example of computations carried out with an explicit scheme is the work of Thoman and Szewczyk (1969) who used a first order explicit scheme with "upwind" differencing to improve the stability of the calculations. Their computations, for uniform flow past a circular cylinder, were remarkable in that a Reynolds number of nearly one million was achieved. Accuracy of the solution was poor, however, because the upwind differencing scheme introduced "artificial" viscosity into the computations, which had the effect of damping the flow and in many cases preventing vortices from detaching from the cylinder.

For many applications the Alternating Direction Implicit (ADI) second-order finite difference solution scheme for parabolic equations due to Peaceman and Rachford (1955) is considered to be the most suitable. Peaceman and Rachford themselves tested the ADI method by solving the heat flow equation. By comparison, a typical explicit procedure required 25 times as much computer time. Borthwick (1986) employed the ADI method in a comparison with Thoman and Szewczyk's work on the circular cylinder. The ADI method produced much more realistic results. Nevertheless, computing restrictions have, until recently at least, generally limited the ADI method to computations involving Reynolds numbers of hundreds rather than thousands.
1.3.2 Lagrangian solutions of the Navier-Stokes equations

In contrast with the Eulerian solutions to the vorticity transport equation a number of Lagrangian schemes have been developed which possess quite different properties. These have generally evolved from the inviscid Discrete Vortex Method which solves the convection portion of the vorticity transport equation, but they also have the effects of diffusion taken suitably into account. Three alternatives for the modelling of the diffusion term have been investigated. Generally, since a large number of vortices is required to represent adequately the effects of diffusion, the Cloud-in-Cell technique has been employed to reduce computation time.

1) Representation of the shear layer with discrete vortex "blobs" (see Leonard, 1980). Diffusion can be taken into account by allowing the blobs to increase in size with time. This approach has been little used.

2) Diffusion onto neighbouring point vortices (see Raviart, 1986). This requires a large number of particles to ensure that diffusion is adequately represented. Problems can be envisaged with the imposition of the wall vorticity boundary condition. No numerical tests or implementations of the algorithm are known.

3) Addition of a random walk to the discrete vortices at each time step to represent statistically the effects of viscosity. Chorin (1973) introduced this method for flow past a circular cylinder. Agreement with
experiment was good at Re = 1000, but worse at both higher and lower Reynolds numbers. At the end of the computations there were 300 vortices in the flow. Milinazzo and Saffman (1977) found that to achieve acceptably accurate results for the viscous decay of a uniform vortex using this scheme the number of vortex elements had to be large, and in direct proportion to Reynolds number. It is difficult to know how many vortices are required for a given degree of accuracy, but it seems likely that Chorin's computations did not contain enough vortices for adequate representation of either the initial separation of the vortices or the wake.

Stansby and Dixon (1983) employed the random walk to compute shedding from a circular cylinder in both uniform and oscillatory flow. In applying the random walk method, with certain modifications, to various bluff body flows, Lewis and Porthouse (1983) have produced results which seem qualitatively plausible, although comparisons with experiment were limited, and not particularly good, except for the Blasius boundary layer. Several problems have been encountered in the use of the random walk method. First, there is an upper Reynolds number limit in terms of computational cost imposed by the requirement that, for a given degree of accuracy, the number of vortices must increase as Reynolds number increases. Secondly, there is a lower Reynolds number limit which is due to the inadequacy of the random walk model at low Reynolds number. Thirdly, the random walk model is incapable with present computing power of realizing the transition to turbulence, which would require the representation of much smaller length and time scales. In fact, it appears for the above reasons that the random walk method is applicable
in quite a narrow Reynolds number range, although inadequate work has been carried out on the method to enable this statement to be quantified.

1.3.3 Comparison of Eulerian and Lagrangian schemes

The main advantage of a Lagrangian scheme is that it is able to resolve small details of the flow moving with the fluid through a large region. It is economical for flows with a strong convection of vorticity because the vortex particles are the computational representation of vorticity. Areas away from the wake are not represented simply because there is no such requirement. Large time step size can be used because there are apparently no stability restrictions either in the flow or with the boundary conditions. One of the most serious drawbacks of a Lagrangian scheme is its inability to cope with large distortions or large amounts of diffusion in the fluid, whereas an Eulerian scheme proceeds without difficulty in these situations. The obvious disadvantage in particular with the random walk method is the limitation on Reynolds number and achievable accuracy imposed by computing restrictions on the number of vortices in the flow.

1.3.4 Hybrid Lagrangian/Eulerian solutions of the Navier-Stokes equations

It would appear from the foregoing discussion that Lagrangian vortex schemes are suited for solving the convection problem, but solution of the diffusion problem is not as accurate or efficient as with a typical
Eulerian scheme. A combination of Lagrangian convection and Eulerian diffusion schemes could therefore eliminate many of the problems of both methods. Such a combination was first described by Garder, Peaceman and Pozzi (1964) for the solution of high Peclet number miscible displacement equations such as might be encountered in oil recovery problems. The Peclet number is analogous in these equations to the Reynolds number in the vorticity transport equations. Price, Cavendish and Varga (1968) showed that such a Moving Point Method (MPM) avoided the problems of numerical viscosity and spurious oscillations of the solution so frequently encountered in pure finite difference solutions, and was more accurate than those solutions for sufficiently large Peclet number. They indicated, however, that the MPM was not convergent under mesh refinement. Farmer (1985a) has demonstrated the convergence of an improved MPM method and showed for a typical test case that it is much more accurate than either 1st or 2nd-order Eulerian upwind differencing schemes. In an excellent review of modern moving point techniques, Farmer (1985b) describes two other techniques, the Modified Method of Characteristics and the Hybrid Moving Point Method.

The methods described by Farmer for miscible displacement problems have obvious application to the solution of the Navier-Stokes vorticity transport equation. The Moving Point Method especially could be employed to extend the Discrete Vortex Method in its Cloud-in-Cell formulation to cope with the effects of diffusion. However, solution of the vorticity transport equation entails some different problems to the miscible displacement equations, especially in the treatment of the boundary conditions. The MPM as adapted to the solution of the transport of
vorticity (we shall call this the Hybrid Moving Vortex Diffusive Method, or HMVDM) has been adopted in this present study. No previous research in this area can be found. The HMVDM represents a new approach to the solution of the Navier-Stokes equations.

1.4 The Application of Vortex Methods to Vortex Shedding from Vessel Hulls

This thesis concerns the application of the Discrete Vortex and Hybrid Moving Vortex Diffusive Methods described above to the problem of vortex shedding from rolling vessel hulls. The research work took as its starting point the earlier work of Downie, Bearman and Graham (1984 and 1985) on the roll damping of barges including the effects of vortex shedding. Typically, such barges are large ocean-going flat-bottomed barges which are utilised in the offshore oil industry. These vessels are usually towed to their destination bearing bulky cargos such as sections of oil rig structures. During transportation the cargo structures are secured to the vessel deck with welded steel fastenings. The fastenings and the cargo itself have to be designed to withstand inertia forces due particularly to roll in a seaway. Both cargo modules and fastening designs are strongly dependent on accurate predictions of barge roll motions.

Downie et al. considered only barges of rectangular cross-section for the critical design case of a barge with zero forward speed in beam waves. Such a situation might arise due to engine failure or loss of tow lines causing the barge to turn beam to waves. The work was an
application of the earlier research using the Discrete Vortex Method on the drag due to vortex shedding for sinusoidally oscillating flow around a semi-infinite square edge (see Graham, 1980). A matching technique was derived to match the flow calculated by the Discrete Vortex Method to the flow round the barge cross-section. Use of such a matching technique is a powerful tool since it means that discrete vortex calculations for a particular edge geometry need be performed only once, for all time, regardless of the overall barge geometry. The success of this technique led to the development by the present author of a Discrete Vortex Method for more general barge bilge shapes, particularly the isolated square edge with rounding and/or bilge keel (see Cozens, Bearman and Graham, 1986). The work is also reported in this thesis. Subsequently the method was extended to cope with complete ship cross-sections, without the matching technique. Finally, for a barge with rounded bilges, where the Discrete Vortex Method was considered to be generally inappropriate, the Hybrid Moving Vortex Diffusive Method was developed.

Other work on the roll damping of rectangular cross-section barges using the Discrete Vortex Method has been carried out by Patel and Brown (1986) following the earlier research of Brown, Eatock Taylor and Patel (1983) on barge motion responses excluding the effects of vortex shedding. Patel and Brown were prevented by computational considerations from carrying out a discrete vortex computation with enough vortices or for a sufficient duration to enable an accurate answer to be obtained. Nevertheless, damping predictions were in many cases in fair agreement with experiment. They also presented damping results for a barge with rounded bilges, but, though this is not stated explicitly, it is
believed that these results are empirically based.

Robinson and Stoddart (1986) have presented results for a rounded bilge barge which are based on the rectangular cross-section calculations of Downie et al. (1984) coupled with an empirical correction for the effects of edge rounding based on the experiments of Ikeda, Himeno and Tanaka (1977).

Faltinsen and Sortland (1987) have presented results for a rectangular cross-section barge in pure sway. The effect of bilge radius and bilge keels was also tested. Pleasing comparisons with experiment were made, and calculations performed for a selection of the above parameters for barges of differing aspect ratios. This work is an interesting application of the Discrete Vortex Method at reasonable computational cost to shedding computations around complicated shapes.

Few workers have employed the Discrete Vortex Method for the modelling of vortex shedding from ships with more general cross-sections. Soh and Fink (1971) employed it for the impulsive start of the flow around a heaving ship with bilge keels. No damping results were presented. Ikeda and Himeno (1981) have modelled the sway of various Lewis-form cylinders representative of ship shapes. Shedding patterns appeared plausible but again no damping results were presented. They did not attempt to model roll motions.

Odabasi, Incerek and Edwards (1985) have developed a method which predicts the roll damping of ships due to vortex shedding via a process of local matching to the results of Discrete Vortex Method computations.
for simpler shapes. In order to extend the method to edges with rounding, calibrations against experiment were carried out to provide empirical constants which were used in the damping predictions.

Brook (1986) has carried out comparisons on selected vessels between experimental dampings, Ikeda's roll damping prediction method (Ikeda et al., 1978) based on experimental observations and the method due to Odabasi, Incecik and Edwards (1985). The latter method predicted dampings for naked hull configurations which were in reasonably good agreement with experiment and which were in some cases significantly better than the predictions using the Ikeda method. However, the method tended to overpredict the effect of bilge keels relative both to experiment and to Ikeda's method for comparisons carried out for the Fisheries Protection Vessel, the Sulisker.

No research involving the solution of the full Navier-Stokes equations for the rolling of vessels can be found.

The present work on the modelling of vortex shedding from vessel hulls represents two major innovative contributions. First, the Discrete Vortex Method has been developed to predict the vortex shedding from a variety of barge bilge shapes via a specially modified matching process, and also from complete general ship cross-sections in roll (or sway). Secondly, a new computational technique for solving the Navier-Stokes equations has been developed. Computations have been carried out for certain cases where bilge rounding would normally prevent Discrete Vortex Method calculations from being performed successfully.
It is convenient to describe the nature of vortex shedding in oscillatory flow in this Chapter so that reference can be made later in the thesis to the various flow patterns which are observed. The discussion here is related to observations of the vortex shedding from the bilges of a rolling model barge for which flow visualisation experiments have been carried out as part of the current research.

A programme of experiments was carried out about four years ago at Imperial College to visualize the flow round the bilges of a rolling rectangular cross-section barge. The most important results of these experiments are presented in Downie, Bearman and Graham (1984). A vortex shedding pattern was visualized, and this was in reasonable agreement with theoretical predictions as far as vortex paths and vortex sizes were concerned. In the current set of experiments the same basic barge model was roll tested with bilge keels of different spans and bilges of different radii in order that visualisations could be made of the fundamental shedding patterns involved and also of the effects of different combinations of edge rounding and bilge keels.
2.1 Experimental Method

2.1.1 Experimental apparatus

The basic barge model is that used in the previous experiments, with the addition of bilge keels and/or rounded bilges. Details of the model are shown in Figure 2.1. The model was tested in the wave tank at Imperial College. This tank has a working section width of 0.61m and depth of 1m, although for the present tests the depth of the water was only 0.6m. For these tests, the wave-maker was not used; instead, the barge was force rolled. A schematic representation of the experimental apparatus is shown in Figure 2.2.

2.1.2 Test procedure

Forced-roll tests were carried out at a variety of combinations of angular frequency, roll amplitude, bilge keel span and bilge radius. For the square edge with bilge keel most of the testing was performed at a roll angular frequency of 4 rad/sec and roll amplitude of 5.7°. Other tests were carried out at a higher frequency (6 rad/sec) and a higher amplitude (9.9°). For the rounded edge, the two different bilge radii were tested with no keels and keels of varying span at an angular frequency of 1.5 rad/sec and an amplitude of 11.4°. Details of the figures discussed in this chapter are shown in the table below.
Barge beam = 0.36 m, length = 0.61 m and draught = 0.07 m

<table>
<thead>
<tr>
<th>Fig. no.</th>
<th>Bilge rad. (mm)</th>
<th>Span (mm)</th>
<th>Ang. freq. (rad/s)</th>
<th>Amp. (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3</td>
<td>13</td>
<td>13</td>
<td>1.5</td>
<td>11.4</td>
</tr>
<tr>
<td>2.4</td>
<td>0</td>
<td>0</td>
<td>4.0</td>
<td>9.9</td>
</tr>
<tr>
<td>2.5</td>
<td>0</td>
<td>25</td>
<td>6.0</td>
<td>5.7</td>
</tr>
<tr>
<td>2.6</td>
<td>0</td>
<td>76</td>
<td>4.0</td>
<td>5.7</td>
</tr>
<tr>
<td>2.7</td>
<td>25</td>
<td>0</td>
<td>1.5</td>
<td>11.4</td>
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<tr>
<td>2.8</td>
<td>13</td>
<td>0</td>
<td>1.5</td>
<td>11.4</td>
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<td>2.9</td>
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<td>6</td>
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<td>2.10</td>
<td>13</td>
<td>13</td>
<td>1.5</td>
<td>11.4</td>
</tr>
<tr>
<td>2.11</td>
<td>0</td>
<td>76</td>
<td>4.0</td>
<td>5.7</td>
</tr>
</tbody>
</table>
In Chapter 7 a formulation for a characteristic value of the Stokes number is given. For the tests reported here Stokes number on that basis is of the order of 5000. A typical equivalent full scale Stokes number would be of the order of 5 million. It is not completely clear what differences in roll damping there would be due to Stokes number between model scale and full scale. If the main component of damping is skin friction then the differences would be large; otherwise the major unknown factor is the effect of Stokes number on the separation point.

2.1.3 Flow visualization techniques

The flow was visualized using two separate techniques - polystyrene beads and dye.

1) Polystyrene beads
Very small polystyrene beads with specific gravity near that of water were injected in solution into the water at low speed. A thin sheet of intense light in a plane parallel to that of the barge end-plate was shone towards the source of the beads. This had the effect of illuminating beads in the shedding plane. This particular method of visualization is best in situations where the vortex shedding is strong and where significant detail is not required. It is capable of yielding a good picture of overall flow patterns.
2) Dye

A concentrated solution of dye was injected into the flow through fine hypodermic tubing. Illumination was from the further side of the barge. This method of visualization is best for lower local flow speeds. In this case the flow processes near the shedding edge can be observed in great detail. Care was taken in the experimentation to avoid possible interference between the tubing admitting the dye into the flow and the flow patterns themselves. Dye was ejected into the flow from points near, but not at, the shedding edge. Concentrated dye was used which permitted very low ejection speeds.

2.2 Photographs of the Vortex Shedding Processes

Photographs were taken of the vortex shedding as part of the flow visualization process. The photographs were used for two main purposes - to gain a better qualitative understanding of the flow pattern, and to measure vortex sizes to enable an assessment of the quality of the theoretical vortex predictions. In this section a representative sample of the photographs is shown and their qualitative importance briefly analysed.

Figure 2.3 shows a close-up photo of a typical single vortex shed from a keel. Apart from the main vortex "spiral" and its central "core", it can be seen that there are a number of smaller, secondary vortices forming on the main vortex sheet.
Figure 2.4 shows a series of photographs representing one complete time cycle of flow round a square edge. Flow visualization was performed using polystyrene beads. At the beginning of the time cycle, with the barge at peak amplitude, a large vortex may be seen to the right of the barge. Its pair is just beginning to form at the edge on the underside of the barge. By photo 2 this nascent vortex has grown considerably, whilst the first vortex of the pair has been swept right round to a position below and to the right; by photo 3 it is below and to the left, indicating its speed of travel. At the same time in photo 3 (peak amplitude at the opposite extreme of roll) the second vortex of the pair has now ceased growing, and there is a new nascent vortex at the right of the shedding edge. It appears, in general, that vortex pairs are of roughly equal, but opposite, strength. Since for the square edge equalisation of vorticity does not necessarily occur at the end of the growth of the second vortex, the pairing process splits the second vortex sheet leaving a smaller amount of residual vorticity which may be seen below and to the right in photo 4 and to the right in photo 5 of the nascent vortex. This is soon engulfed by the nascent vortex. Photo 4 shows the nascent vortex having grown rapidly; the structure of the previous pair is now indistinct, although its effect is clear enough. Photos 5 and 6 show the further growth of the nascent vortex.

Figure 2.5 shows a set of photographs representing one time cycle. The figure refers to a bilge keel of 25mm span at the higher roll rate; the visualization was performed using dye. Photo 1 shows the situation at peak amplitude. One vortex of each pair has already been cast off, and
the second vortex can be seen just starting to grow. It grows through photo 2, until by photo 3 the elements of the pairs are almost evenly matched in strength. Notice how the structure of the right hand pair appears to be much better preserved than that of the left hand, although this effect could possibly be due to some asymmetry in the ejection of dye into the flow. In photo 4 (peak amplitude) both pairs have left the edge completely and there are signs of a new nascent vortex at each edge. These grow considerably through photos 5 and 6, until by photo 1 again peak amplitude has been reached (and the end of the time cycle) and the second vortex of each pair begins to be shed. Figure 2.6 is a bead visualization of flow round the 76mm keel for a complete time cycle. All the events described above for Figure 2.5 can be seen happening here in considerable detail, with the barge moving from one peak of amplitude (photo 1), through to the other (photo 6) and then back again. Shedding off each edge is 180 degrees out of phase with the other. Vortex pairs convect nearly straight downwards, at an angle of about 45 degrees to the plane of the bilge keels. Figures 2.5 and 2.6 show that the presence of a bilge keel makes the vortex structure more stable in comparison with the square edge (Figure 2.4). In the two cases shown, there is little or no residual vorticity at the end of the pairing process. The pairs convect away from the vessel surface, whereas for the square edge convection occurred along the barge surface. There is a bistability in the shedding process. Vortex pairs may either convect upwards, towards the free surface, or downwards. In general (see, for example, Figure 2.6) in the cases where the vortex pairs would have been strong enough to have been affected by the free surface, they have convected downwards away from it.
Figure 2.7 shows the flow pattern for an edge with rounding but no keel. A "double" shedding pattern is observed, in that two strong vortex pairs are shed per cycle. The two pairs can be seen about to convect away from the edge in photos 1 and 5. In the presence of rounding, the shedding appears weaker than for the square edge; the vortex pairs remain very near the barge surface as they convect away. The double shedding pattern is a result of the residual vorticity noted in Figure 2.4 being large enough to force a second strong pairing of vortices. In fact the residual vorticity has become as large as the "primary", thus setting up two shedding patterns apparently equal in strength. Figure 2.8 shows an intermediate stage where residual vorticity is greater than for the square edge, but not as great as for the larger bilge radius. A double shedding pattern per cycle is observed, with pairs forming at photos 1/2 and 5/6, but the first pairing, to the left, is stronger than the second.

In Figure 2.9, where the keel span is half the bilge radius, the "single" shedding pattern observed in earlier figures is regained, despite the relatively small size of the keel. However, small secondary disturbances can still be seen. Figure 2.10 shows the flow situation with a keel span twice as great. The general appearance of the flow is now much like that which would be expected from an edge without any rounding. The figure shows particularly clearly the various stages in the shedding process. In both Figures 2.9 and 2.10 a small amount of residual vorticity is apparent at the end of the main pairing process. This pairs up with a portion of the new nascent vortex to form a weaker secondary pair. This is clearly seen in photos 2-5 of Figure 2.10.
Figure 2.11 shows the same flow situation as that in Figure 2.6. Two separate dye sources have been used here, so that the degree of two-dimensionality of the shedding can be assessed. Slight differences can be observed with the far vortex appearing to have convected further from the edge than the nearer one. This example typifies the observations made for all the different bilge configurations. Only very occasionally could there be noted any significant difference in, say, the convection paths.
CHAPTER 3
THE DISCRETE VORTEX METHOD

The application of the Discrete Vortex Method to the flow around isolated edges has been described in detail by Graham (1980), who drew on the earlier work of Pullin (1978). This chapter describes its development in the first place for more complicated isolated edge geometries, particularly the edge with rounding and/or bilge keel. Vortex shedding from such shapes can be directly related to vortex shedding from the bilges of barges in roll or sway by a matching process. Section 3.9 deals with the extension of the method to cope with the vortex shedding in roll of general symmetric ship hull cross-sections with an arbitrary separation position and arbitrary roll centre.

3.1 Transformation

Two alternative methods of representing the influence of the body were considered. Firstly, it is possible to represent the body by a distribution either of sources and sinks or vortices. A good example of the use of the Discrete Vortex Method with such a singularity distribution may be found in Sakata, Adachi, Saito and Inamuro (1983). The second alternative is the use of a transformation technique. An important advantage of the transformation technique is usually higher accuracy especially for geometries with sharp edges such as will often be encountered in vortex shedding problems; one drawback is the fact
that transformations are more difficult to implement for complicated shapes.

Because of the possibility of greater accuracy of representation, for the current research a numerical Schwarz-Christoffel transformation was employed which enabled isolated edges of arbitrary geometry to be transformed to a straight line segment as shown in Figure 3.1. The general transformation can be written as:

\[
\frac{dz}{d\zeta} = M \prod_{i=1}^{n} (\zeta - a_i)^{\alpha_i/n}
\]

where the symbols are as defined in the figure. M is called the transformation constant and defines the scaling relationship between the physical and transform planes. The normal sign convention for angles is adopted, that is, angles are taken to be positive for an anticlockwise rotation when the body is circled in an anticlockwise sense. The full numerical integration of Equation 3.1.1 which is necessary when a large number of vertices is involved has been discussed in detail by Davis (1983), and will be treated in Section 3.1.3. It is noted first, however, that for simple edge geometries versions of Equation 3.1.1 can be derived which yield analytical solutions of the integral. Two such geometrical configurations are discussed below.

### 3.1.1 Plain isolated edge transformation

Such a geometry with its transformation is shown in Figure 3.2. For this problem, with only one vertex, this being at the origin, Equation 3.1.1
reduces to:

\[ \frac{dz}{d\zeta} = M_c^{\lambda-1} \quad 3.1.2 \]

where:

\[ \lambda = 2 - \frac{\delta}{180} \quad (= 1 + \alpha_1/\pi) \quad 3.1.3 \]

Integration of Equation 3.1.2 and setting the constant of integration to zero leads to a familiar simple form of the Schwarz-Christoffel transformation:

\[ z = \frac{M}{\lambda} \zeta^\lambda \quad 3.1.4 \]

The second derivative of the transformation, which is required for the determination of Routh's correction (Clements, 1973), is:

\[ \frac{d^2z}{d\zeta^2} = M (\lambda-1) \zeta^{\lambda-2} \quad 3.1.5 \]

### 3.1.2 Plain isolated edge with bilge keel transformation

Such a geometry with its transformation is shown in Figure 3.3. For this problem, with three vertices in total, Equation 3.1.1 reduces to:

\[ \frac{dz}{d\zeta} = M_c (\zeta^2 + b)^{\lambda-1} \quad 3.1.6 \]

Integration of Equation 3.1.6 and setting the constant of integration to zero leads to the following form of the Schwarz-Christoffel transformation:

\[ z = \frac{M}{\lambda} (\zeta^2 + b)^{\lambda/2} \quad 3.1.7 \]
The second derivative of the transformation is:

\[
\frac{d^2 z}{d\zeta^2} = M(\zeta^2 + b)^{-2} \left[ (\zeta^2 + b) + (\lambda - 2)\zeta^2 \right]
\]

3.1.3 Full numerical solution of the Schwarz-Christoffel transformation

In cases where many vertices are present, such as in the representation of rounding for the isolated edge case, analytical solutions are not available and instead a computational solution must be sought. The problem is centred on the determination of the \(a_i\)'s which are initially unknown.

Solution for the \(a_i\)'s proceeds as follows. Without loss of generality one of the vertices in the \(z\)-plane can be set at the origin, as can one of the vertices in the transform plane. Further, the first and last vertices can be set to +i and -i in the transform plane. In order to determine the remaining values of \(a_i\), Equation 3.1.1 must be integrated to give \(z\) in terms of \(\zeta\). In this work an extension of the midpoint integration rule has been used to determine \(z_{m+1}\) from \(z_m\), where \(\Delta z_m = z_{m+1} - z_m\) is small. This requires the division of each facet into sub-elements, the number of the sub-elements required depending on the facet size and the nature of the transformation in the region.

In order to obtain the unknown values of \(a_i\) and \(M\), the following iterative procedure was adopted.
1) Guess initial values for the unknown constants. M is complex and can be conveniently set to $(1 + 0i)$; the $a_i$'s are chosen such that $a_{i-1} < a_i < a_{i+1}$ and the interval between each $a_i$ is equal.

2) Integrate Equation 3.1.1 using the midpoint integration rule between appropriate limits as follows:

$$\Delta z_m = M \Delta \tau_m \prod_{i=1}^{n} \left[ \frac{(\tau - a_i) (\alpha_i / \pi) + 1}{\alpha_i / \pi + 1} \right] ^ {\frac{\tau_m + 1}{\Delta \tau_m}}$$

3.1.9

In the physical plane the vertices will then not be in their correct locations. Therefore adjust M so that the first and last vertices lie in their correct position.

3) Use the adjusted value of M and the calculated positions of the physical plane vertices to provide better estimates of the $a_i$'s.

4) Return to Stage (2) and perform a new iteration assuming the new values of M and $a_i$. Repeat the iteration procedure until convergence has been achieved.

More detail on the above process may be found in Davis (1983).

Once converged values of $a_i$ and M have been obtained, then the various properties of the transformation can be evaluated.
First derivative of the transformation. For geometries where symmetry is involved Equation 3.1.1 can be rewritten in a more efficient form:

\[
\frac{dz}{d\zeta} = M (\zeta-a_\alpha r^\alpha/\pi \prod_{i=1}^{r-1} (\zeta^2-a_i^2)^{\alpha_i/\pi}
\]

where \( r = (n+1)/2, \ n \) odd.

Second derivative of the transformation. Little computational simplification can be achieved by using symmetry arguments in the case of the second derivative of the transformation, which is accordingly derived as:

\[
\frac{d^2z}{d\zeta^2} = M \left[ \prod_{i=1}^{n} (\zeta-a_i) \right] \sum_{i=1}^{n} \frac{(\alpha_i/\pi)}{(\zeta-a_i)}
\]

Determination of \( z \) from \( \zeta \). Equation 3.1.9 should be employed to determine \( z \) for given \( \zeta \), using an appropriate number of integration sub-elements.

Determination of \( \zeta \) from \( z \). An iterative scheme was developed for the determination of \( \zeta \) for given \( z \). The scheme involved the calculation of successive approximations to \( \zeta \) using a "shooting" scheme. In the first part of the scheme, \( \zeta_{old} \), the first estimate of the required value of \( \zeta \), was set to unity. In the second stage, the corresponding value of the physical plane ordinate, \( z_{old} = r_{old} e^{i\theta_{old}} \), was computed by the method of the previous sub-section. In the third stage, an improved estimate of
\[ \zeta_{\text{new}} = \zeta_{\text{old}} \left[ (1-R) + R \frac{r}{r_{\text{old}}} \right] e^{iR(\theta-\theta_{\text{old}})} \]  

where \( z = r e^{i\theta} \) is the actual physical plane ordinate corresponding to \( \zeta \) and \( R \) is a relaxation factor. Optimum performance in most conditions was achieved by under-relaxing the solution by a factor of one half, i.e. putting \( R = \frac{1}{2} \). Iteration between stages two and three was then carried out, with \( \zeta_{\text{old}} \) successively being equated to \( \zeta_{\text{new}} \), until the difference between \( z \) and \( z_{\text{old}} \) was less than the required tolerance.

**Flow length scale.** Flows around semi-infinite plain (i.e. single vertex) isolated edges possess no physical length from which a length scaling can be determined. Further, because of the semi-infinite nature of the body, there exists no freestream velocity except in certain special cases. Pullin (1978) derived a formulation of length scaling for plain isolated edges in uniform flow, this scaling being based on properties of the transformation, the "freestream" flow velocity in the transform plane, and the time scale of the flow (i.e. the time from the start of the flow). Self-similar solutions were thus provided for plain isolated edges of specified edge angle. Graham (1980) extended the concept to oscillatory flow, where the time scaling was now provided by the period, \( T \), of the oscillations. In the present research a generalised expression for the length scale \( (L_z) \) of the flow in the physical plane around isolated edges of arbitrary edge geometry has been derived from the velocity scale \( (V_z) \) in the physical plane as follows. A representative edge velocity in the physical plane will scale on \( dW/dz \). We choose the
constant of proportionality in the scaling to be $\lambda$, where $\lambda$ here represents the overall edge angle, so that:

$$V_z = \lambda \left| \frac{dW}{dz} \right|_{C_z} \quad 3.1.13$$

where $W$ is the complex potential in the absence of vortex shedding, and $dW/dz$ is taken as evaluated at $C_z$. $C_z$ is a complex ordinate of magnitude $L_z$ and orientation such that it lies on the edge bisector. We can further define a length scale in the transform plane, $L_\zeta$, where $L_\zeta = |C_\zeta|$, and $C_\zeta$ is the point in the transform plane corresponding to $C_z$ in the physical plane. Thus:

$$C_z = \lambda \frac{dW}{dz}_{C_z} \cdot T = \lambda \frac{dW}{dz}_{C_\zeta} \cdot \frac{dC_\zeta}{dz}_{C_\zeta} \cdot T \quad 3.1.14$$

Then we can write $L_z$ as:

$$L_z = |C_z| = \left| \frac{\lambda V T}{\prod_{i=1}^{n} (C_\zeta - a_i) / \pi} \right| \quad 3.1.15$$

In the above equation $V$ is treated as the transform plane velocity amplitude for oscillatory flow. This equation was solved by iteration between $C_z$ and $C_\zeta$. Since its solution is computationally expensive, for isolated edge cases where, for instance, keel span, $a$, or bilge radius, $r$, are small compared to the flow length scale an approximation to this equation can be solved instead as follows. Far away from the edge, i.e. where $|\zeta| >> a_i$ for all $a_i$, Equation 3.1.1 can be rewritten as:

$$\frac{dz}{d\zeta} = M \zeta \prod_{i=1}^{n} (a_i / \pi) = M \zeta^{\lambda - 1} \quad 3.1.16$$
Therefore, using Equations 3.1.14 and 3.1.15:

\[ L_z = \left( \frac{M}{\lambda} \right)^{-1/(2\lambda-1)} \frac{\lambda/(2\lambda-1)}{(VT)} \]  

Equation 3.1.17 is actually the expression used for the flow length scale for oscillatory flow round a plain isolated edge (see Graham, 1980).

3.2 Basic Concepts of the Discrete Vortex Method

The basis of the DVM, as applied here to vortex shedding off bluff bodies, consists in the representation of the shear layer from a bluff body by a series of point potential vortices (see Figure 3.4). For such a distribution of vortices and their images there exists a complex potential \( W \) which may be written for the transform plane as:

\[ W(\zeta) = iV\zeta + \frac{i}{2\pi} \sum_m \Gamma_m \log(\zeta - \zeta_m) - \frac{i}{2\pi} \sum_m \Gamma_m \log(\zeta + \zeta_m^*) \]  

which gives:

\[ \frac{\partial W(\zeta)}{\partial \zeta} = iV + \frac{i}{2\pi} \sum_m \Gamma_m \frac{1}{\zeta - \zeta_m} - \frac{i}{2\pi} \sum_m \Gamma_m \frac{1}{\zeta + \zeta_m^*} \]  

The concept of the complex potential enables the determination of the influence of each of the components of the flow field, namely the flow itself and the vortices and their images, at a given location. The equation of motion of the vortex sheet, which is force free and therefore moves with the fluid, is:
at each point $z_n$ of the sheet, where $(dW/dz)$ excludes the singularity due to the vortex element at $z_n$. From Equation 3.2.3 we derive that:

$$\frac{\partial z_n^*}{\partial t} = \frac{\partial W}{\partial z_n}$$  \hspace{1cm} 3.2.3

In the case of a core vortex, the vortex which represents the centre of a continuous spiral or cluster and whose strength grows with time according to the rate of amalgamation of vorticity into that core, a modified zero force equation is employed (Graham, 1980). Equation 3.2.4 is rewritten as:

$$\frac{\partial \zeta_n^*}{\partial t} = \left( \frac{\partial W}{\partial \zeta_n} \right) \frac{\partial \zeta_n}{\partial z_n} \left| \frac{\partial \zeta_n}{\partial z_n} \right|^2$$  \hspace{1cm} 3.2.4

$$\frac{\partial \zeta_n^*}{\partial t} = \left( \frac{\partial W}{\partial \zeta_n} \right) \frac{\partial \zeta_n}{\partial z_n} \left| \frac{\partial \zeta_n}{\partial z_n} \right|^2 - \frac{1}{r_n} \frac{\partial \Gamma_n}{\partial t} \left( z_n^* - z_s^* \right) \left( \frac{\partial \zeta_n}{\partial z_n} \right)^*$$  \hspace{1cm} 3.2.5

where $z_n - z_s$ is the cut joining the core vortex to the rest of the sheet. Equations 3.2.4 and 3.2.5 are used in conjunction with Equation 3.2.2 for the determination of the convection velocities of the vortices in the transform plane. With the inclusion of a core vortex and Routh’s correction (Clements, 1973), the transform plane convection equation is:

$$\frac{\partial \zeta_n^*}{\partial t} = \left[ i \nu + \frac{i}{2 \pi} \sum_{m \neq n} \Gamma_m \left( \frac{1}{\zeta_n - \zeta_m} - \frac{1}{\zeta_n + \zeta_m^*} \right) + \frac{i}{2 \pi} \Gamma_n \left( \frac{1}{\zeta_n + \zeta_n^*} \right) \frac{\partial \zeta_n^*}{\partial \zeta_n} \right]$$

$$x \left( \frac{\partial \zeta_n}{\partial z_n} \right) \left| \frac{\partial \zeta_n}{\partial z_n} \right|^2 - \frac{1}{r_n} \frac{\partial \Gamma_n}{\partial t} \left( z_n^* - z_s^* \right) \left( \frac{\partial \zeta_n}{\partial z_n} \right)^*$$  \hspace{1cm} 3.2.6
Equation 3.2.6 is employed in the Discrete Vortex Method via a straightforward one-step first order Euler integration to yield a means of convecting the discrete vortices through a time step.

3.3 Treatment of the Nascent Vortex

In discrete vortex representations where the vortex sheet is being fed with new vorticity from a shedding edge, a mechanism is required for determining the strength and complex coordinate of the new discrete vortex which represents this additional vorticity over a discrete period of time. Such a vortex is designated a "nascent" discrete vortex. The Kutta condition has most often in previous work been invoked to solve one of these three unknowns (vortex strength and complex coordinate); in this work it has been used to calculate the strength of the nascent vortex. There is then a requirement for the determination of the nascent vortex position; this is described in Section 3.3.2.

3.3.1 Satisfying the Kutta Condition

The Kutta Condition is satisfied when there is a finite flow velocity at the shedding edge in the physical plane. This implies because of the singularity of the transformation at the shedding edge that the velocity at this position in the transform plane is zero (i.e. \( \frac{dW}{d\zeta} = 0 \)). The general equation of motion of the fluid in the transform plane at any location other than at a vortex itself is Equation 3.2.2. Satisfying the Kutta Condition leads to an expression for the strength of the nascent
vortex, \( \Gamma_n \), placed at \( \zeta_n \):

\[
\Gamma_n = \left[ -2\pi V - \sum_{m \neq n} \Gamma_m \left( \frac{1}{\zeta_{\text{shed}} - \zeta_m} - \frac{1}{\zeta_{\text{shed}} + \zeta_m} \right) \right] \frac{1}{\zeta_{\text{shed}} - \zeta_n} - \frac{1}{\zeta_{\text{shed}} + \zeta_n} \quad 3.3.1
\]

where \( \zeta_{\text{shed}} \) is the position of the shedding edge. For the case where the shedding edge is at the origin this may be simplified to:

\[
\Gamma_n = \left[ -2\pi V - \sum_{m \neq n} \Gamma_m \frac{(\zeta_m^* + \zeta_m)}{|\zeta_m|^2} \right] \cdot \frac{1}{(\zeta_n^* + \zeta_n)} \quad 3.3.2
\]

### 3.3.2 Determination of the position of the nascent vortex

Various different methods of solution for the position in the flow of the nascent discrete vortex were tested, including a sheet vortex representation of the nascent vortex and a solution of the Brown and Michael equations for the nascent vortex (Brown and Michael, 1955). Eventually the simplest method, reported in detail by Kamemoto and Bearman (1978), was selected. The nascent vortex was placed at a specified point on the edge bisector, its position remaining invariant throughout the time cycle. In sinusoidal flow, a suitable position based on the peak flow velocity was found to be somewhat different from that for uniform flow, which was analysed by Kamemoto and Bearman. In fact, the position of the nascent vortex for satisfying the Kutta Condition in the physical plane \( z_{\text{kut}} \) was taken as:

\[
z_{\text{kut}} = c_3 C_z \quad 3.3.3
\]
where $c_3$ was determined from numerical experimentation to have a value of about 0.01, although the optimum value varied somewhat according to geometry. The principal factor in the determination of $c_3$ was the requirement to maintain an approximately even spacing between the vortices near the shedding edge. Equation 3.3.3 may then be used to determine $\zeta_n$ (i.e. $\zeta_{kut}$, the transform plane equivalent of $z_{kut}$) in Equations 3.3.1 or 3.3.2.

It has been noted by Naylor (1982) that instabilities quickly arise in computations if one nascent vortex is introduced into the calculation per shedding edge at each time step. Naylor reported that by placing a new nascent vortex in the flow less frequently, say once every two or three time steps, these instabilities were reduced. The drawback of this approach is that computation time is increased without any concomitant increase in definition of the vortex sheet. The instability problem appears to be related to the first order time integration scheme used for convecting the nascent vortex, in that because the nascent vortex is in a region of very high flow velocities it can be convected an unrepresentatively large distance. Having more than one convection step per nascent vortex increases the accuracy of convection. A less computationally expensive solution to the problem has been adopted in the present work. Recognising that in the formulation described in Section 3.3.1 the Kutta Condition is only satisfied at the beginning of a time step, and not at any other point in the time step while the nascent vortex is being convected away from the edge, it is clear that the Discrete Vortex Method must be somewhat insensitive to the precise location of the position of introduction of the nascent vortex. It is
advantageous to place the vortex at a position slightly further away from the shedding edge than would be required to satisfy the Kutta Condition at the beginning of the time step, because here flow velocities are lower and the vortex is actually generally convected a shorter and more representative distance than otherwise. Furthermore, introduction of the nascent vortex even slightly further away from the edge in the manner described above dramatically reduces instabilities. Location of the vortex in this manner is justifiable on the grounds of a degree of insensitivity to the position of introduction of the vortex; numerical experimentation confirms that indeed this location is not critical. This technique has been employed in all DVM calculations reported here. The position at which the nascent vortex is actually inserted into the flow in the transform plane \( \zeta_{\text{nascent}} \) is given in terms of the position in the transform plane where the Kutta Condition would be satisfied at the beginning of a time step by:

\[
\zeta_{\text{nascent}} = c_4 \zeta_{\text{kut}}
\]

\( c_4 \) was held between 1.25 and 1.4 depending upon the stability requirements.

3.4 A Scheme for Amalgamating Vortices

In the present model of vortex amalgamation each successive discrete vortex nearest the central core is amalgamated with it so as to create a core of increasing vortex strength (see Figure 3.4). Amalgamation has been employed for two reasons. First, in connection with the tendency to
randomisation of the vortex sheet which has been noted by a number of workers (see Chapter 1) amalgamation has been shown significantly to retard the randomisation process (Moore, 1974). Secondly, amalgamation saves computing time since the total number of vortices in the flow can be reduced. In fact, for oscillatory flow the computation will reach a "steady state" where the number of vortices in the calculation will remain roughly constant since the rate of production of new vortices will equal the rate of amalgamation. Typically, a new vortex from an initial cluster of, say, 30 vortices would be amalgamated once every 3 or 4 time steps. This would have the effect of limiting the total number of vortices in a flow with 64 time steps per cycle to about 60 or 70.

In the above model two discrete vortices are amalgamated so that their new combined position, \((x + iy)_{\text{new}}\), is given in terms of their old positions and circulations, designated by subscripts "old1" and "old2" as follows:

\[
(x + iy)_{\text{new}} = \frac{(x + iy)_{\text{old1}}|\Gamma_{\text{old1}}| + (x + iy)_{\text{old2}}|\Gamma_{\text{old2}}|}{|\Gamma_{\text{old1}}| + |\Gamma_{\text{old2}}|} \tag{3.4.1}
\]

Combined circulation is given by:

\[
\Gamma_{\text{new}} = \Gamma_{\text{old1}} + \Gamma_{\text{old2}} \tag{3.4.2}
\]

Deffenbaugh and Marshall (1976) have laid out the conditions for correct amalgamation. These are firstly that it yields an equivalent velocity field and secondly that it maintains the basic structure of the vortex distribution. The above method fulfils both of these conditions.
3.5 Starting the Flow Pattern

An initial superposition of velocity off the edge was used to start the vortex pairing pattern at the beginning of the computation. This velocity was decayed quickly with time so that it only significantly affected the first time cycle of flow and had no long term effect on the solution. The potential in the transform plane due to this velocity was:

\[ W(\zeta) = U_{\text{initial}} e^{-2(t/T)^2} \cdot \zeta^2 \]  

where \( U_{\text{initial}} \) was chosen so that vortices from the first half-cycle were convected well clear of the shedding edge. Thus by the end of two time cycles the velocity had decayed to only 0.03% of its initial value, \( U_{\text{initial}} \).

3.6 Decaying the Vortex Pairs

Many workers with the Discrete Vortex Method have employed a mechanism of removing vortices if they approach the body too closely - see, for example, Clements and Maull (1975). Otherwise, these vortices can convect at high speed close to the body surface due to the influence of their images and possibly interfere seriously with the shedding mechanism. A slightly different approach has been adopted with the present method - all vortex pairs are slowly decayed in strength once they have reached a certain degree of development, whether or not they near the body side. It was explained in Chapter 2 that in sinusoidal flow vortex pairs develop which have elements of nearly equal strength.
but opposite sign. Thus their decay only has a small effect on total circulation levels in the flow.

The decay mechanism chosen was programmed to come into effect as soon as the latter member of the vortex pair had ceased growing and had started to convect away from the edge. The decay was of the form:

\[
\Gamma = \Gamma_{\text{initial}} e^{-k_d(t/T)^2} \tag{3.6.1}
\]

The above method was entirely successful in ensuring that vortex pairs did not approach the edge too closely, or otherwise interfere too much with the shedding process in cases where they were slow to convect away. Since the circulation decay rates employed were low (\(k_d\) always less than 0.3), and decay was only employed for vortex pairs clear of the edge, there was a negligible effect on the computed vortex force, as was demonstrated by the results of computations with different values of \(k_d\).

Unlike the work of some other researchers, circulation decay was not used as an expedient for reducing the magnitude of the vortex force. For instance, Kiya, Arie and Harigane (1979) in their calculations of flow past a flat plate normal to the flow noted that the DVM without the use of vortex decay overestimated the amount of circulation in the wake and consequently the drag force. They employed vortex decay successfully as a means of reducing the circulation in the wake nearer to representative values. In the present work, however, it was accepted as part of the computations that forces would be overpredicted, it being thought wiser to take account of this at the final stages of analysis of the results.
3.7 Calculation of Pressure Coefficient

Bernoulli's equation for unsteady incompressible flow is:

\[ p + \frac{1}{2} \rho (u^2 + v^2) + \rho \frac{\partial \phi}{\partial t} = p_\infty + \frac{1}{2} \rho U_\infty^2 \]  

whence we obtain the following expression for the pressure coefficient:

\[ C_p = 1 - \frac{(u^2 + v^2)}{V_z^2} - \frac{2}{V_z^2} \frac{\partial \phi}{\partial t} \]  

with:

\[ (u^2 + v^2) = \left| V + \frac{1}{2\pi} \sum_m \Gamma_m \left( \frac{1}{\zeta - \zeta_m} - \frac{1}{\zeta + \zeta_m^*} \right) \frac{\partial \zeta}{\partial z} \right|^2 \]

and:

\[ \frac{\partial \phi}{\partial t} = - \text{Real} \left\{ \frac{i}{2\pi} \sum_m \Gamma_m \left[ \frac{1}{\zeta - \zeta_m} \frac{\partial \zeta_m}{\partial t} + \frac{1}{\zeta + \zeta_m^*} \frac{\partial \zeta_m^*}{\partial t} \right] \right\} \]

\[ + \frac{1}{2\pi} \frac{\partial \Gamma_n}{\partial t} \left[ \text{arg}(\zeta + \zeta_n^*) - \text{arg}(\zeta - \zeta_n) \right] \]

where the subscript "n" refers here to the nascent discrete vortex. Since there is in general no definable freestream velocity in the physical plane, \( V_z \) is employed as a reference velocity. The above method of determining the \( d\phi/dt \) term follows from the work of Jaroch (1986).
3.8 Calculation of Vortex Force

Following from the work of Graham (1980), the vortex force, $F_v$, may be obtained from an approximation of Blasius' equation for unsteady flow so that for isolated edge flows:

$$F_v = -i \rho \frac{3}{2} \sum_m \Gamma_m \left( \zeta_m + r_m^* \right)/2$$ \hspace{1cm} 3.8.1

A vortex force coefficient may be derived in terms of the length scales in the physical and transform planes as follows:

$$C_{Fv} = \frac{F_v}{(i \rho L_z^2 L^* \zeta^2)}$$ \hspace{1cm} 3.8.2

The values of force coefficient can be translated into drag and inertia coefficients for the isolated edge by taking the appropriate Fourier integral over one cycle of the flow to give:

$$C_D = A = \frac{3\pi}{4} \int_0^1 C_{Fv} \sin \left( \frac{2\pi t}{T} \right) d\left( \frac{t}{T} \right)$$ \hspace{1cm} 3.8.3

and:

$$C_m = C_{m0} + B, \text{ where } B = \frac{2}{\pi^2} \int_0^1 C_{Fv} \cos \left( \frac{2\pi t}{T} \right) d\left( \frac{t}{T} \right)$$ \hspace{1cm} 3.8.4

3.9 Extension of the Discrete Vortex Method to Model the Vortex Shedding from Rolling Ships

3.9.1 Vessel in pure sway

Extension of the basic DVM to cope with complete symmetric ship cross-sections in pure sway involved mainly modifications to the basic
numerical Schwarz-Christoffel transformation described in Section 3.1 above. An algorithm was developed to determine the exterior edge angles (α_i's) of a general hull cross-section composed of small segments as shown in Figure 3.5. A suitable total number of segments for adequate representation of the geometry without excessive computational expense was determined to be roughly 15, although the number varied somewhat according to the degree of complication of the geometry. The α_i's were obtained from raw sectional data using scalar products. Vector products were also calculated to assure the angles were attributed the correct sign.

The flow velocity in the transform plane (V) is found to be related to the freestream velocity in the physical plane (U_∞) by the following relationship:

\[ V = MU_\infty \]  

3.9.1

3.9.2 The representation of roll

The roll of a vessel can be represented by a transformation method as follows. Firstly, the ordinates themselves should be constantly changed to take account of the changing position of the vessel; secondly the motion of the vessel should be represented by a distribution of singularities (in this case sources and sinks) along the vessel surface. This second part of the representation may be seen as a modification of the surface boundary conditions to yield a normal surface velocity distribution which represents the motion due to roll. For small roll
amplitude the changing position of the vessel can be neglected, resulting in significant economies in programming and computation time. This simplification has been made here.

Flow issuing normal to the body segment can be represented by a source distribution of strength per unit length $\sigma(z)$ where the transpiration (outward normal) velocity is given as:

$$V_T = \frac{\sigma(z)}{2} \quad 3.9.2.$$  

A general complex velocity, $u - iv$ at $z$, is given by:

$$u - iv = \frac{1}{2\pi} \int_{a}^{b} \frac{\sigma(z')|dz'|}{z-z'} \quad 3.9.3$$

as shown in Figure 3.6. The source distribution may be transformed to the transform plane to yield a general equation for the velocity in the transform plane:

$$u - iv = \frac{1}{2\pi} \int_{c}^{d} \sigma(z') \frac{|dz'|}{dz'} \frac{dz'}{d\zeta} \frac{d\zeta}{\zeta - \zeta'} \quad 3.9.4$$

as shown in Figure 3.7. If segment $\overline{ab}$ in the physical plane is a body segment, and the transformation maps the body into a vertical line in the transform plane, the transpiration velocities in the transform plane will be purely real.
3.9.3 Method of solution for the velocity distribution due to a source distribution

For any but the simplest transformations or source distributions, Equation 3.9.4 has to be solved by numerical means. The process of numerical quadrature involved is complicated by the fact that Equation 3.9.4 has a singularity at $\zeta = \zeta'$. A sophisticated NAG library routine, D01AJF, was selected. This is an adaptive quadrature routine, using the Gauss 10-point and Kronrod 21-point rules. It was tested most successfully on certain cases for which analytical solutions were known.

3.9.4 Application of the theory to a rolling vessel

The various aspects of the implementation of the theory described in the sections above are presented below.

1) Roll motion was divided into two portions, pure roll about a fixed roll centre (which was actually fixed at the base of the vessel on the vessel centre line), and a sway component adjusted to yield the actual desired roll centre, as illustrated in Figure 3.8. Freestream velocity, $U_\infty$, is given by:

$$U_\infty = \omega z_R \hat{h}_u \sin \omega t$$

where $\omega$ is roll angular frequency and $\hat{h}_u$ is roll amplitude. $z_R$ is the height of the roll centre above the vessel base. Roll angle was defined
as positive clockwise. The advantages of fixing the roll centre in this way are twofold. First, it is convenient for calculating the roll velocities to use a roll centre at the origin of the coordinate system. Secondly, the computed roll velocities (i.e. due to the source distribution) at and near the shedding edge will often be nearly minimised by this arrangement. This means in practice that the order of accuracy of the computed roll velocities is not required to be as high as would have been the case otherwise.

2) The source strength on each segment is calculated using the value of surface (transpiration) velocity at the midpoint of the segment with the source strength assumed constant over the segment. This simplification thus requires that velocities only change slowly along the length of the segment or that segment size is small. The surface velocity ($V_T$) is taken as the normal component of the total velocity. Thus the slip velocity (velocity parallel to the surface) is not accounted for. This is quite correct for a potential flow model such as the DVM, but would be incorrect if a full solution of the Navier-Stokes equations were attempted.

3) The velocities due to roll were required at two separate stages within the computer program. In the first place, the velocity at the shedding edge due to roll was required to be calculated once and for all at the beginning of the program. The amplitude of this velocity does not change with time. It was required to be calculated with fair accuracy, since vortex strengths were partly dependent on it. For this velocity the NAG routine integration technique D01AJF was employed as described
above. The value of the transformation derivative \((dz/d\zeta)\) was required at each integration point. Secondly, velocities due to roll were required at every time step for each vortex. Since this would have been computationally very expensive using the NAG routine, an approximate analytical technique was developed. This technique made the assumption that the transformation derivative was constant over the segment, and equal to its value at the midpoint of the segment in the transform plane. It was possible to derive the following equation for a general velocity, \(u - iv\), in the transform plane at a point \(\zeta\) due to a segment \(cd\):

\[
u - iv = \frac{\sigma_{cd}}{2\pi} \left( \frac{dz}{d\zeta} \right)_{cd} \left( \frac{|dz|}{dz} \right)_{cd} \log \left[ \frac{(\zeta_c - \zeta)}{(\zeta_d - \zeta)} \right]
\]

3.9.6

The above equation was derived by recognising that in Equation 3.9.4 \(\sigma_{cd}\), \((dz/d\zeta)_{cd}\) etc. are assumed constant for the entire segment and represent the value of the appropriate variable at the midpoint of the segment. The velocity due to all the segments is simply the sum of the velocities due to each segment. However, the symmetry of the vessel can be used here with advantage. We know that if the source strength on segment \(cd^r\) on one side of the vessel is \(\sigma_{cd}^r\), the respective source strength on the other side \((\sigma_{cd}^l)\) is simply \(-\sigma_{cd}^r\). Thus a general velocity at a point \(\zeta\) due to a segment \(cd^r\) and its image segment \(cd^l\) is:

\[
u - iv = \frac{\sigma_{cd}^r}{2\pi} \left( \frac{dz}{d\zeta} \right)_{cd} \left( \frac{|dz|}{dz} \right)_{cd} \log \left[ \frac{(\zeta_c^r - \zeta)(\zeta_d^l - \zeta)}{(\zeta_d^r - \zeta)(\zeta_c^l - \zeta)} \right]
\]

3.9.7
4) Flow length scale, $L_z$, for the rolling ship case, is based on the total flow velocity at the shedding edge in the transform plane. It contains both the sway and the roll components.

3.9.5 Calculation of the vortex roll damping moment

The vortex roll damping moment (per unit length), $M_v$, is calculated using the method described in Downie, Bearman, and Graham (1984). This is essentially a Blasius momentum balance similar to that for the isolated edge flows (Equation 3.8.1). Assuming small amplitudes of motion:

$$M_v = -ip\frac{a}{b t} \sum_p \Gamma_m \frac{(\zeta_m + \zeta_m^*)}{2} \int_a^b \frac{\text{Real}((z-z_R)^* dz)}{\pi(z-\zeta_{\text{shed}})}$$  \[3.9.8\]

where $\zeta_{\text{shed}}$ is the position in the transform plane of the shedding edge, and the limits "a" and "b" denote the two junctions of the hull cross-section with the free surface. The moment arm, $l_a$, of the vortex damping force about the true roll centre is given as:

$$l_a = \left[ \frac{b}{a} \frac{\text{Real}((z-z_R)^* dz)}{(z-\zeta_{\text{shed}})} \right] / \left[ \frac{b}{a} \frac{dz}{(z-\zeta_{\text{shed}})} \right]$$  \[3.9.9\]
Sectional vortex damping coefficient, $b_v$, is defined as:

$$b_v = \frac{M_v \cdot l_s}{\omega h}$$  \hspace{1cm} 3.9.10

where $l_s$ is the section length. Total vortex roll damping coefficient, $B_v$, is obtained by summing the sectional damping coefficients over the length of the ship. Implicit in this analysis is the assumption of strip theory that for flow calculations a 3-dimensional body can be divided into a series of 2-dimensional transverse strips. In the case of vortex shedding, this is a fair assumption provided that changes in the sectional geometry occur gradually over the length of the ship and that end effects (i.e. at the bow and stern) can be ignored. Actually, in many practical cases, these conditions are not completely fulfilled. It is clear, for instance, that for a ship where the majority of the vortex damping is generated from a few sections near the bows (and possibly also the stern) end-effects could be important. In this case, strip theory might be expected to provide an over-estimate of roll damping.

3.9.6 Range of validity of the method

Various simplifying assumptions are incorporated into the above method. These are:

1) The vessel is symmetric about its centre line.
2) The vessel undergoes a roll motion of small angular amplitude and the length scale of the vortex shedding is small in comparison with the length scale of the vessel. These two closely related assumptions are important not only in the implementation of roll itself but also in the use of the particular formulation of Blasius' theorem for the calculation of force which is employed here, which requires that the vortex shedding is localised to the shedding edge.

3) The segment lengths are small in comparison with the perimeter of the vessel.

4) The shedding edge is sufficiently sharp that the assumption of separation point fixed on the edge bisector is valid or approximately valid.

5) In the case of vortex shedding from more than one shedding edge, it is assumed that the interference between the vortex structures from each edge is negligible. This enables the method to compute separately the damping contributions from each source; it is unlikely to be a major restriction for such practical cases as shedding from bilge keels.

6) It is assumed that it is approximately valid to use a strip theory approach for the calculation of total vortex damping moment.
CHAPTER 4

RESULTS FROM THE DISCRETE VORTEX METHOD AND COMPARISON WITH EXPERIMENT FOR ISOLATED EDGE FLOWS

4.1 Assessment of the Sensitivity of the Method to Changes in the Run Parameters

Variations in the run parameters inherent in the method can in certain circumstances produce significant changes in force levels. An analysis of the effect of some of these parameters has therefore been carried out.

4.1.1 Effect of vortex decay rate

Excessive circulation decay rates applied to vortex pairs of slightly differing strength can affect the strength of the nascent vortex sufficiently to induce an instability into the calculation. They also have the effect, if applied to vortices too near the shedding edge, of reducing the magnitude of the vortex forces. Results have shown that drag values are practically unaffected for either of these two causes by the particular implementation of circulation decay in the current method (Section 3.6). In one particular case (square edge with bilge keel), the decay rate was reduced tenfold from its normal value with no change in drag in the third significant figure. Some calculations were performed
for the plain square edge with no circulation decay at all. Until the point when a vortex pair returned near to the edge, drag values were virtually identical to those obtained in calculations using decay.

4.1.2 Effect of the position of the nascent vortex

The Discrete Vortex Method is to some extent self-adjusting to the position ($z_{kut}$) of the nascent discrete vortex for satisfying the Kutta condition, through a feedback mechanism via the vortex strength in the shear layer. So long as the nascent vortex is introduced near the midpoint between the shedding edge and the previously shed vortex, changes in its position do not affect the drag excessively. For instance, for shedding off an edge of zero internal angle a doubling of $z_{kut}$ from its usual position made a 1% difference to the results. However, for a square edge, or edges of even greater edge angle, drag is only independent of $z_{kut}$ within narrower limits. It appears that an excessively large value of $z_{kut}$ has the same sort of action as a small bilge keel in increasing the drag and modifying the shedding behaviour. An excessively low value of the quantity leads to vorticity being convected along the body side under the influence of its image system rather than being convected into the flow. In a typical comparison for a square edge, a 40% reduction in $z_{kut}$ from a reasonable value to an excessively low value created a 15% reduction in drag and instabilities in the solution. Care was therefore taken to ensure for plain square edge calculations that the shedding position was as close to the edge as possible, but not too near as to create the above mentioned problems.
4.1.3 Effect of time step size and amalgamation rate

These two parameters are closely related - time step size affects the overall representation and behaviour of the vortex sheets, whereas amalgamation rate affects more specifically the behaviour of the vortex sheets once they begin to convect away from the edge. Amalgamation rate is defined as the number of vortices amalgamated per cluster per time step. A low amalgamation rate retains greater definition in the vortex sheets as they convect out into the field.

Two sets of numerical calculations have been performed to test the effect of refinement of both of the above parameters. All the calculations were performed with a value of $a/L_z$ of unity with force averaged over 8 time cycles. In the first set the number of time steps per cycle was kept constant at 64, the number used in most of the rest of the computations; amalgamation rate was decreased from 1 amalgamation every 2 time steps for a vortex cluster to 1 amalgamation every 4 time steps. The results of these computations are given in the table below:

<table>
<thead>
<tr>
<th>Amalgamation Rate</th>
<th>Coefficient A</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.500</td>
<td>6.453</td>
</tr>
<tr>
<td>0.333</td>
<td>6.552</td>
</tr>
<tr>
<td>0.250</td>
<td>6.443</td>
</tr>
</tbody>
</table>
In the second set amalgamation was held constant at 1 amalgamation every 3 time steps (the value used in most of the rest of the computations) whilst the number of time steps per cycle was varied from 40 to 88. The results of these computations are given in the following table:

<table>
<thead>
<tr>
<th>Number of time steps per cycle</th>
<th>Coefficient A</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>6.428</td>
</tr>
<tr>
<td>64</td>
<td>6.552</td>
</tr>
<tr>
<td>88</td>
<td>6.537</td>
</tr>
</tbody>
</table>

The results show that for shedding off a keel attached to a 90° edge changes in the amalgamation rate and number of time steps per cycle over the range tested (which spans the range used in all of the computations) have no significant effect on the value of the drag coefficient. Similarly, no significant effect was observed on the value of inertia coefficient.

For the plain square edge also, changes in the refinement of the vortex structure within the range described above did not produce a noticeable effect.
4.2 Results for Square Edge with Bilge Keel and Optional Rounding

4.2.1 Vortex positions and pressure distributions

A series of four pressure distributions, together with the appropriate plots of vortex position, is shown in Figures 4.1 - 4.4. These present distributions for a square edge with keel of span \( a/L_z = 2.2 \) but no rounding. The first figure shows the situation at the beginning of a time cycle \( (t'/T = 0) \); the second figure the situation 1/4 of the way through a time cycle \( (t'/T = 0.25) \), and so on. The qualitative agreement between the theoretical plots of vortex position and experiment may be assessed by a comparison with Figure 2.6. The shedding from the left hand keel is in phase with the computation. No experimental pressure distribution results are available, but the general form of the distributions seems correct. A suction peak is observed to grow on the surface adjacent to the nascent vortex cluster as the nascent vortex (cluster) grows. This is especially obvious in Figure 4.2. The pressures on the two surfaces should coincide at the shedding edge. In each of the distributions as the shedding edge is approached the two halves of the distribution start to come together before diverging again rapidly very near the edge. This is interpreted as a feature of the Discrete Vortex Method in that the discretisation of the vortex sheet means that the expected discontinuity in the velocity distribution at the shedding edge is "smeared" over a small distance in the theory, with consequent effects on the pressure distribution.
Figures 4.5 show four plots of velocity vectors for the case $\frac{r}{L_z} = \frac{a}{l_z} = 1.0$. The plots are for one complete time cycle, each plot being separated by a quarter of a time cycle. A velocity vector is drawn from the centre of each discrete vortex. Figure 4.5a shows a vortex cluster growing at the edge. Away from the edge a vortex pair is convecting towards the bottom right of the picture. In subsequent pictures the vortex growing at the edge pairs up with a new nascent vortex and this pairing also convects away from the edge, following the previous one.

Figure 4.6 shows two plots of vortex positions half a time cycle apart for the case of $\frac{r}{L_z} = \frac{a}{l_z} = 0.039$. Together with these figures are photos of the corresponding flows on the barge. In comparison to the keel span of the previous calculation this is a comparatively short keel. Nevertheless, even a short keel such as this has a profound effect on the vortex shedding pattern compared with the shedding off a plain square edge.

4.2.2 Comparison between theoretical and experimental vortex size

In Figure 4.7 experimental and theoretical vortex sizes are compared plotted against non-dimensional keel span for a square edge with keel. The apparent divergence between theory and experiment at the highest keel span is most likely due not to any deficiency in the Discrete Vortex Method but due to substantial uncertainties in the measurement of vortex size, $l_v$, in experiment. A method of matching the theoretical
isolated edge results to the experimental barge rolling results was employed.

4.2.3 Time dependent force coefficient

Plots of the variation of force coefficient, $C_{Fv}$, with time are presented in Figures 4.8. Figure 4.8a is for a square edge with bilge keel of span $a/L_z = 0.2$; Figure 4.8b is for a plain square edge. Both figures show traces taken over four time cycles, beginning six time cycles into the flow. The more structured vortex roll-up for the edge with the bilge keel, and more random shedding pattern for the plain square edge are reflected in the difference between the two force traces. Peak force in the first case is consistent over the four cycles, but varies quite dramatically in the second. A sudden trough in the third cycle of Figure 4.8b indicates the presence of a vortex very near the body side being convected at high velocity.

4.2.4 Drag and inertia coefficients

In the present formulation, drag coefficient ($C_D$) is equal to the coefficient $"A"$ whilst inertia coefficient ($C_m$) is related simply to, but not strongly dependent on, "$B$" (Equations 3.8.3 and 3.8.4). Values of the coefficients A and B for varying non-dimensional bilge keel span may be found plotted in Figures 4.9 and 4.10 respectively. Results are presented both for a square edge with keel and for a rounded square edge.
with "bounded" keel (i.e. where $a/r = \sqrt{2} - 1$ - see insets on these figures for a definition of a bounded keel). Also shown are the equivalent flat plate values (i.e. square edge with infinitely long bilge keel). Both $A$ and $B$ should be divided by a factor of 1.62 to provide comparison with the actual values of flat plate force coefficient (i.e. infinitely long flat plate) because of the matching technique involved. Keel span in these two figures is non-dimensionalised with respect to the equivalent length scale for the plain edge (Equation 3.1.17) for the sake of simplicity in its use with the matching process. Over the range of keel spans tested here, length scale was virtually invariant, and equal to $L_z(\text{plain edge})$. The form of the curves for both the coefficients is almost identical - a sharp initial rise at low keel span, followed by a slow asymptote to equivalent flat plate values. In the case of the rounded edge with bounded keel the drag results are, as would be expected, considerably lower than those for the keel with square edge. The edge rounding in the former case obviously reduces the flow velocities near the shedding edge.

Values of the coefficients $A$ and $B$ are 1.995 and 0.051 respectively for the square edge, and 5.38 and 0.27 for the flat plate. The relevant values calculated by Graham (1980) are 1.57 and -0.04, and 5.9 and 0.125. Unlike the coefficient $A$, coefficient $B$ is sensitive to the numerical parameters in the discrete vortex analysis. Also $B$ is relatively unimportant in comparison with $A$. The most meaningful comparison with Graham is therefore for $A$. Experimental results for $A$ for the square edge and flat plate are approximately 1.4 and 4.0 though there is a degree of uncertainty attached to them. The experiments - see
Bearman, Downie, Graham and Obasaju (1985) - were carried out for two-edged bodies (i.e. the diamond and the finite flat plate); the results have been matched to the isolated edge via a process described by Graham (1980). The matching process requires the use of experimental results from the very low Keulegan-Carpenter number range, Keulegan-Carpenter number ($K_C$) being defined as:

$$
K_C = \frac{U T}{d}
$$

where $d$ is a representative body length scale (the diameter in the case of a circular cylinder). Such a requirement implies that the scale of the shed vortices is much less than the body scale.

4.3 Results for Square Edge with Rounding

This case is more difficult because of the uncertainty over where the separation point(s) are. On the one hand in the limit of vanishing radius we can fix the separation point with some certainty. On the other hand, as radius is increased at fixed flow velocity, experiment has shown that the vortex shedding pattern observed for the plain edge (single pair per time cycle) gives way to a different pattern (double pair per cycle - see Figures 2.7 and 2.8), with a constantly varying separation point. It was not possible to determine from the experiments that were carried out in what radius range this change took place. Eventually vortex shedding ceases altogether. The correct simulation of the double shedding pattern is beyond the scope of the Discrete Vortex Method as it stands. However, when the radius is small it seems reasonable that a fair representation of the vortex shedding may be
obtained by fixing separation on the edge bisector, although this is not in general the true separation point. The results would be more representative the smaller the radius is. The approach of fixing separation thus has been taken here, the method being used for increasing radius until the computations became unstable. Alternative more sophisticated methods of determining the separation point could have been developed. However, at this stage of the research it was decided to proceed to full Navier-Stokes solutions of the flows around these shapes (Chapters 6 and 7) and so it was decided not to devote more development time to this method.

4.3.1 Vortex positions and pressure distributions

Results for three different edge radii are presented. Figures 4.11 to 4.14 refer to a non-dimensional edge radius \((r/L_z)\) of 0.039. The four figures are separated in time by a quarter of a time cycle. The vortex shedding is very similar to that off a plain square edge (see Figure 2.4 for the experimental results for a square edge) except perhaps that the nascent vortex cluster keeps closer to the edge. In this case the present method referred to above of fixing the separation point clearly works quite well inasmuch as shedding patterns are well-ordered and appear plausible. Figures 4.15 to 4.17 however show a situation where this approach does not function so well. Non-dimensional edge radius is 0.109; each figure is separated by a time interval of half a time cycle. The computation appears to be attempting to simulate the double pairing pattern referred to in the introduction to this section. The large
vortex cluster at the edge in Figure 4.16 has been split in two half a time cycle later in Figure 4.17, the smaller (residual) portion of the vorticity pairing up with the new vortex cluster near the edge. Despite some short periods (such as here) where the computation runs without major problems, as a whole it is highly unstable and seems to be on the point of complete breakdown.

Figures 4.18 to 4.21 are for a non-dimensional radius of 0.086. Here the radius is somewhat smaller and the calculation is more successful. Each figure is separated by a time interval of a quarter of a time cycle. The relevant pressure distribution is shown next to each plot of vortex positions. The general form of the pressure distributions is similar to those of the plain edge except that the magnitudes of the pressure values tend to be somewhat smaller. For instance a suction peak is still discernible in the case of a strong single nascent vortex cluster (Figure 4.21 - upper surface). There is also a suction peak at the junction between rounding and straight edge on the surface opposite that on which the nascent vortex is growing. This is followed by a strong recompression. This suction peak can be seen in all four cases, but is strongest when the nascent vortex is strongest (Figures 4.19 and 4.21). The feature is interpreted as a local acceleration of the flow due to the curvature discontinuity between the straight edge and the rounding.
4.3.2 Drag and inertia coefficients

Because of the incorrect position of the separation point it is thought that there will be an error in the drag magnitude which will increase with radius. Nevertheless drag values are presented here because it is felt that they give some qualitative guide as to its behaviour and also provide an indication of the limitations of the Discrete Vortex Method. The drag results presented in Chapter 7 and calculated using the Hybrid Moving Vortex Diffusive Method are considered much more reliable. Figure 4.22 shows a plot of the coefficient A against non-dimensional edge radius. A value of zero radius corresponds to a plain square edge. As radius increases, drag decreases as we should expect. No results are presented beyond a value of \( r/L_z \) of 0.064 since drag results for calculations at higher \( r/L_z \) are considered unreliable. In these cases, spuriously high values of drag were obtained when the vortex pairing switched direction, as it was prone to do frequently for the larger radii. This led to a sharp levelling off of the decreasing drag trend which was considered spurious. No plot of the coefficient B is presented since no discernible trend can be found in the distribution. The computed values are small and, except for the plain square edge, negative.
In the context of the current research, force data derived from the Discrete Vortex Method were required to be applied to the problem of the roll damping of vessels due to vortex shedding. Two different applications were made. Firstly, the isolated edge results for an edge with keel and/or rounding were applied via a matching process to the problem of barge roll damping. In this case the results were input into a specially modified version of the vessel motion response computer program named BMTIMP in order to obtain roll damping coefficients. This work will be described in the following sections. In the second place, the Discrete Vortex Method as developed for complete hull cross-sections was used directly in the computation of roll damping coefficients for the Fisheries Protection Vessel, the Sulisker. These results will be described in Section 5.5. In this section computed vortex shedding patterns are also presented and discussed.

5.1 Barge Motions and the BMTIMP Barge Rolling Computer Program

A schematic representation of barge motions is shown in Figure 5.1. The barge undergoes motion with six degrees of freedom. These can be divided into two coupled sets, one for sway, roll and yaw, the other for surge,
pitch and heave. Interest in the current work lies only with roll motion and the roll damping coefficient, $B_{44}$. Damping calculations were performed exclusively for a barge in forced-roll at zero forward speed. Response to waves was not considered.

The computer program now called "BMTIMP" was developed at Imperial College under joint support from BMT Ltd. and the SERC Marine Technology Directorate. It was formulated to enable the calculation of damping forces experienced by a rolling flat-bottom barge of rectangular cross-section due to the vortex shedding from its bilges. It was a full boundary integral method involving the application of linearised free surface boundary conditions, solved by strip theory. Since such a method is based on potential flow theory, it cannot itself model flow separation off the bilges and thus singularities arise there. The innovation of BMTIMP is in the treatment of these singularities. The flow is divided into an inner region close to the edge, consisting of the edge singularity and the consequent vortex shedding, and an outer inviscid region, consisting of the rest of the flow field. The inner region was modelled by use of the Discrete Vortex Method applied to an isolated edge, and was then matched to the outer region which was solved by the potential theory referred to above. BMTIMP has been well documented - see Downie, Bearman and Graham (1984). It could originally cope only with barges of rectangular cross-section, but has been modified by the present author to extend its scope to rectangular cross-section barges with bilge keels and/or bilge rounding. The modifications pertained mostly to the matching process and are described in Section 5.1.2 below.
5.1.1 Treatment of the roll damping contributions by BMTIMP

It is convenient at this point to outline the manner in which the various contributions to roll damping have been treated in BMTIMP. A much fuller description will be found in Downie, Bearman and Graham (1984) who originated BMTIMP. As was explained in Chapter 1, for the problem of a barge at zero Froude number rolling at small amplitude, there are only two significant contributions to roll damping, namely wave (radiation) and vortex damping. Thus the roll damping coefficient, $B_{44}$, can be written as:

$$B_{44} = B_1 + B_v$$

where $B_1$ and $B_v$ are the wave and vortex damping coefficient components. Damping coefficient is derived by dividing the damping by the roll velocity amplitude, defined as $\omega \hat{H}$. The former is analysed traditionally through the use of strip theory and is independent of roll amplitude. Generally in previous work the latter, as discussed in Chapter 1, has either been neglected altogether or been approximated by empirical or semi-empirical means. Here, we derive $B_v$ from the discrete vortex analysis, which provides a completely theoretically based representation of vortex shedding (see Section 3.9.5); the coefficient possesses linear or near-linear dependency on roll amplitude and frequency. This gives rise to the non-linear, near-quadratic vortex damping term.
5.1.2 Matching the isolated edge flow to flow round the rolling barge

The flow field about a rolling barge comprises an inner region in the vicinity of each keel edge, where the effects of flow separation and vortex shedding dominate, and an outer region associated with the motion of the barge and the effects of the free surface. The flow in the outer region may be found from potential flow theory. Provided the length scale of the flow round the shedding edge is small compared with a typical barge dimension (e.g. beam or draught) the flow in the inner region may be determined by a discrete vortex analysis of oscillatory flow about an infinite wedge with appropriate edge geometry, matched locally with the exterior flow field. The power of the matching technique lies in the fact that for one particular edge geometry and flow length scale only one discrete vortex calculation needs to be performed regardless of the overall barge geometry involved.

The principles of the matching process have been described by Graham (1980) and developed for the problem of a rectangular cross-section barge by Downie, Bearman and Graham (1984). In this section we describe the extensions necessary to this process to enable results for more general shedding geometries to be incorporated into the BMTIMP barge rolling program. In particular we are interested in square edges with keels and/or rounded bilges.

The matching technique involves matching of the length scale of the flow round the isolated edge to that of the flow round the rolling barge. For
the square isolated edge all flows are self-similar so that the vortex force for any barge of rectangular cross-section can be obtained by scaling from a unique value of isolated edge drag coefficient. With the inclusion of a bilge keel or bilge rounding we introduce a dimension into the isolated edge calculation. Now there will be a unique value of drag coefficient for every value of bilge dimension (e.g. keel span or edge radius) divided by length scale. This point is demonstrated in Figures 4.9 or 4.22. Therefore additionally in these cases we have to match the value of bilge dimension divided by length scale for the isolated edge to that for the rolling barge, and a number of discrete vortex calculations are required to cover the required range of bilge dimension to length scale ratios for each geometry.

In applying the matching technique two important simplifying assumptions were made.

1) It was assumed that, for identical external flow conditions, the length scale of the flow is independent of edge geometry for the rectangular barge with bilge keel and/or rounding. This assumption places an upper bound on the permissible values of \(\frac{a}{L_z}\) or \(\frac{r}{L_z}\); it is necessary for an efficient implementation of the matching process which matches the results for the isolated edge to those for the barge. The flow length and velocity scales, \(L_z\) and \(V_z\), vary by less than 10% in total over the range of bilge keel span, \(a\), or bilge radius, \(r\), for which drag results were presented in Chapter 4. This means that the drag results presented in this thesis for the edge with bilge keel or bilge rounding can be incorporated into the barge rolling problem by the
matching process as if they were results for the plain edge, provided the values of the coefficients A and B are selected via the correct values of \(a/L_z(\text{barge})\) or \(r/L_z(\text{barge})\) using the graphs in Figures 4.9, 4.10 and 4.22. In the case of the rounded bilge, B is taken as zero except for \(r/L_z(\text{barge}) = 0\). The above assumption is unlikely to be a restraint in any physically realistic situation except at very small roll amplitudes.

2) It is assumed that the bilge keel or rounding are details of the inner flow and do not affect the external flow around the rolling barge. This assumption is valid when not only the direct effect of the bilge appendage on the external flow but also the disturbance of the external flow by vortices shed from the appendage is negligible. These conditions are met when keel span or bilge radius and roll amplitude are small. This is essentially a restatement to include more arbitrary bilge geometries of the basic matching assumption already inherent in the method that the flow length scale round the shedding edge is small compared with a typical barge dimension. The assumption is particularly useful since it would mean that only that part of the barge rolling computer program concerned with vortex shedding need be modified. Modifications to the potential flow solution or the radiation damping, for instance, need not be made. The assumption is unlikely to be a serious further restraint on the validity of BMTIMP, since it is only a restatement of the basic matching assumption of BMTIMP.
5.2 Results for a Rectangular Cross-Section Barge with Bilge Keels

Calculations were carried out over a range of roll amplitudes for a force-rolled rectangular cross-section barge with and without bilge keels. The basic barge was the Noble Denton Consortium standard barge Case 3S with a length of 87.8m and height of roll centre above the water line of 7.17m; the sectional dimensions are shown in the insert in Figure 5.2. For all calculations the full scale roll period was 10s, which corresponded approximately to the resonant frequency. Results are actually presented at model scale, where the linear scaling relationship is 30:1 full scale to model scale. For more details see Noble Denton and Associates (1985). Figure 5.2 shows a comparison between experimental and theoretical roll damping coefficients in the absence of bilge keels. Although the experimental data has some level of uncertainty attached to it, it is clear that theory only slightly overpredicts experiment.

Since in the case of the plain 90 degree edge without bilge keel the vortex damping coefficient is linearly dependent on roll amplitude, it follows that vortex damping coefficient per unit of roll as a percentage of critical damping coefficient is independent of roll amplitude. For the particular case of the barge Case 3S above it takes the value of 56.2% (see Figure 5.3). When a bilge keel is added, the linear relationship above no longer holds. In fact, vortex damping per unit of roll is found to be a function of $a/L_z(\text{plain edge})$ since the drag and inertia coefficients $A$ and $B$ are functions of this quantity. It follows from this and the expression for length scale for the square edge
(Equation 3.1.17 with $\lambda = 1.5$) that vortex damping coefficient per unit of roll as a function of critical damping coefficient is therefore a function of $(\text{roll amplitude})/(\text{keel span})^{4/3}$. This function is shown graphically in Figure 5.3. This one graph contains, for this particular configuration, all the information required to calculate the roll damping for any combination of roll amplitude and keel span within the prescribed limits. The information is used in Figure 5.4, where vortex damping coefficient is shown plotted against roll velocity amplitude for six different full scale keel spans, 0, 0.1, 0.2, 0.5, 1.0 and 2.0 metres. The roll centre and basic barge geometry are as described for the barge Case 3S. No experimental data are available for comparison. The curve for a keel span of zero is, as explained above, a straight line. Increasing the keel span from zero raises the vortex damping as should be expected. Smaller keels are relatively more effective. For a given keel span the increase in damping over the plain edge case is proportionately greater the lower the roll amplitude. This point is shown in Figure 5.5, which shows the percentage gain in vortex damping over the plain edge case against roll velocity amplitude for a set of five keel spans.

The original computer program BMTIMP is quoted by Downie et al. (1984) as being valid for maximum roll amplitudes of over 5 degrees in the case of the barge tested here. The modifications to the program are also valid over this range of roll amplitudes, provided that the assumptions contained in Section 5.1.2 remain valid. These provide two restrictions. First, assumption (1) requires that non-dimensional keel span ($a/L_z$) should not be too great (i.e. should not exceed the highest value for
which isolated edge drag results have been presented). This would place a lower bound on the roll amplitude for which the method is applicable, but is not a practical restriction since even for the largest keel tested (2m) the minimum admissible roll amplitude was found only to be roughly 0.2°. The restriction arising from the second assumption is that the effect of the keel on the external flow should be negligible. This would provide an upper bound on keel span and roll amplitude. But it is not thought to be a practical restriction. The assumptions described in Section 5.1.2 are thought to permit a keel of at least one metre in span over the entire range of roll velocity amplitudes presented here, although with a lack of experimental data this figure is likely to be very imprecise.

5.3 Results for a Rectangular Cross-Section Barge with Rounded Bilges

Calculations were performed for the standard barge Cases 2 and 3 by matching round edge Discrete Vortex Method results. Both these barges are essentially the same as Case 3S. Case 3 has rounded bilges of radius 0.48m; in addition, Case 2 has a length of 65.1m as opposed to 87.8m and a slightly deeper draught. Graphs of predicted and experimental roll damping coefficients are shown in Figure 5.6 for Case 3 and Figure 5.7 for Case 2. Also shown for the sake of comparison are the BMTIMP predictions for a similar square-edged barge, and Robinson's empirical corrections to these predictions to cope with bilge rounding - see Noble Denton and Associates (1985). As would be expected, the effect of rounding in the theory is to reduce roll damping. The general trend of
the experiment is predicted. However, the reduction does not in general bring the prediction in line with experiment or with Robinson's correction. Probably a number of reasons contribute to this, including:

1) There is quite a high level of uncertainty in the experimental results.

2) The error in the theoretical results due to inadequacies in the discrete vortex model used here for rounded edges is believed to increase with non-dimensional bilge radius (see Section 4.3.2). That section also explains that reliable drag results could not be obtained beyond a value of \((r/L_2)\) of roughly 0.06. The region beyond which reliable drag results could not be obtained is marked by a dotted line on each figure.

3) Even for the square edge theory overpredicts experiment; the larger the non-dimensional bilge radius the more the overprediction is expected to be.

It is believed that the modifications to the BMTIMP program when used for rounded bilges do not place any further restriction on its operation.

Chapter 7 presents damping results for barge Cases 3S, 3 and 2 calculated using the Hybrid Moving Vortex Diffusive Method which was developed because of the inadequacies of the present discrete vortex
model in computing flows round significantly rounded edges. The HMVDM results are considered to be substantially more reliable for the barges with rounded bilges (Cases 3 and 2). Discrete Vortex Method results for these cases are presented in this chapter for the sake of comparison only.

5.4 Results for a Rectangular Cross-Section Barge with Rounded Bilges and Bilge Keels

Comparisons have been made with two barge models tested by Ikeda, Komatsu, Himeno and Tanaka (1977). These are models B and C, both having a length of 0.8m, beam of 0.28m, draught of 0.112m and roll centre at the waterline. In each case keel span is half the bilge radius; this radius is 1cm for model B and 2cm for model C. Graphs of predicted and experimental vortex damping coefficients \( B_v \) plotted against roll angular frequency \( \omega \) at fixed roll amplitude are shown in Figures 5.8 and 5.9 for models B and C respectively. The ratio \( (a/L_z) \) is independent of frequency so that there is a predicted linear relationship between vortex damping coefficient and frequency. In both cases agreement with experiment is excellent and better than the comparisons for rounded bilges without keels. The Discrete Vortex Method predictions are in general more accurate the smaller the shedding edge interior angle. For a rounded edge this angle is essentially 180°, but for a bilge keel it is 0°.
Figure 5.10 shows the variation of vortex damping coefficient with roll amplitude for model C at two fixed angular frequencies, 6.29 and 4.49 rad/sec. The theoretical curves are non-linear since the ratio $a/L_z$ and hence the computed vortex drag coefficient varies with roll amplitude. Theory and experiment agree well at both frequencies, but slightly better at the lower frequency. An examination of the variation of vortex damping coefficient with frequency for this model (Figure 5.9) might suggest that this slight apparent inconsistency relates to the experimental, not the computational, results in that experiment is locally somewhat higher than theory around a frequency of 6 rad/sec.

5.5 Results for a Typical Ship in Roll Motion

Section 3.9 describes the further developments necessary to enable the Discrete Vortex Method to compute the vortex damping due to roll for complete ships. A set of calculations was carried out using this method for the Fisheries Protection Vessel, the Sulisker. These are described in Sections 5.5.2 to 5.5.4 below.

As a test of this version of the DVM computations were carried out for the standard barge Case 3S referred to previously in this chapter (see Noble Denton and Associates, 1985). Comparison was made between the direct calculations of damping for the complete vessel geometry and those obtained via matching to isolated edge calculations. Respectable agreement was obtained.
One further test case was carried out; this is described in the next section.

5.5.1 The roll damping of a circular cylinder with bilge keels

A roll damping comparison was carried out between experiment and Discrete Vortex Method predictions for the case of the circular cylinder with two symmetrically positioned bilge keels rolling in an oscillatory motion about its central axis.

The experimental results were obtained from model tests carried out by Mercier (1965). A schematic view of the model cylinder with one of its two identical keels is shown in Figure 5.11. The cylinder shown has an outside diameter of 15.2cm. A second set of tests was carried out on a larger cylinder with a diameter of 30.4cm, and equivalently proportioned bilge keels. Roll period for the smaller cylinder was 1.46 seconds and for the larger 2.04 seconds. The cylinders were suspended vertically on a torsion wire and were given an initial angular displacement. Roll damping moment was then deduced from the roll extinction (decrement) curves. Mercier defined a roll damping moment coefficient, \( C_{\text{rdm1}} \), as:

\[
C_{\text{rdm1}} = \frac{\text{damping moment}}{(\pi/2) A_k R_{ca}^3 (\omega_n^1)^2}
\]

where \( A_k \) is the frontal area of both keels, and \( R_{ca} \) is the radius from the axis of rotation to the centre of area of the keel.
Mercier calculated $C_{\text{rdm}}$ as 16 for the larger model and 15 for the smaller.

The computation was performed with the version of the DVM as developed for the prediction of the roll damping of vessels due to vortex shedding. It was carried out with 80 vortices per time cycle over six time cycles.

Roll damping moment coefficient results are as follows, based on Mercier's definition of $C_{\text{rdm}}$:

<table>
<thead>
<tr>
<th></th>
<th>Experiments</th>
<th>Discrete Vortex Method (vortex damping)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiments</td>
<td>16, 15</td>
<td>18.6</td>
</tr>
</tbody>
</table>

Assuming the vortex damping component to be the only significant contribution to the overall damping - and Mercier shows that this is so - the Discrete Vortex Method overestimates damping for this case by between 16 and 24%.

5.5.2 The Fisheries Protection Vessel "Sulisker"

A series of discrete vortex computations was carried out for the Fisheries Protection Vessel, the Sulisker. These are described in the following sections; this section outlines the main characteristics of the Sulisker.
The F.P.V. Sulisker has an overall length of 71.33 m, beam of 11.6 m, displacement of 1532 tonnes, and nominal draft of 4.6 m. A plan of the ship may be found in Figure 5.12; cross-sectional data are to be found in Figure 5.13. Detailed information concerning the roll testing of a 1/20th scale model of this vessel may be found in Spouge and Ireland (1986).

Digitisation of the sectional data was undertaken by BMT Ltd. The roll damping experiments on the model of the Sulisker were carried out with a bar keel of span 0.16 m (full-scale dimension) fitted over a significant portion of the ship (Sections 3 - 18). DVM calculations have been performed both with and without a bar keel. Comparisons with other roll damping prediction methods were only available without the keel. Experimental results have also been obtained for the Sulisker with added bilge keels. The bilge keels had a span of 0.35 m and length of 8.68 m. They were positioned 3.06 m below the waterline, 10 m apart at midships, and spanned Sections 12 to 15.

5.5.3 Vortex positions and velocity vectors

The Discrete Vortex Method was employed to compute vortex shedding patterns for the Sulisker on a sectional basis in the manner described in Section 3.9.5. The basic computation which was carried out was for a roll amplitude of 8° with the bar keel and a roll centre 4.65 m above the ship base. Other computations investigated the effect of neglecting the
bar keel, the effect of two additional bilge keels, and the effects of changing the height of the roll centre above the ship base ($z_R$) and roll amplitude ($\theta^A$). All calculations were performed at full scale. Precise details of these calculations are given in the table below:

<table>
<thead>
<tr>
<th>Run</th>
<th>$z_R$ (m)</th>
<th>$\theta^A$ ($^\circ$)</th>
<th>bar keel</th>
<th>bilge keel</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>4.65</td>
<td>8</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>(2)</td>
<td>4.48</td>
<td>8</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>(3)</td>
<td>4.65</td>
<td>4</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>(4)</td>
<td>4.48</td>
<td>4</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>(5)</td>
<td>4.65</td>
<td>8</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>(6)</td>
<td>4.65</td>
<td>8</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>(7)</td>
<td>4.48</td>
<td>4</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Because of the limitations of the method, it was not possible in Run 5 (no bar keel) to compute vortex shedding for all the sections since some possessed no sharp shedding edge or possessed a sharp shedding edge of internal angle greater than about $110^\circ$, and would have yielded unrepresentative dampings. By far the greatest proportion of damping is produced by the sharp sections, so that this is no significant limitation on the validity of the results. This is confirmed by a comparison with the sectional roll damping results of Brook (1987) which were obtained using the roll damping method of Odabasi et al. (1985). This method will be referred to as the "BMT" method and bases its prediction of vortex roll damping on a process of matching to simpler
body shapes for which vortex drag coefficients had previously been computed by Graham (1980) using the Discrete Vortex Method. In the current work, computations for different roll centres have been performed due to an uncertainty over the true position of the roll centre in experiment. The roll centre was in fact believed to vary somewhere between the waterline and the centre of gravity (i.e. \( z_R \) between 4.48 and 4.65) though even these limits may be too precise (Spouge, 1987).

Plots of vortex positions for twelve of the sections for which DVM calculations for Run 1 were carried out are presented in Figure 5.14. The plots are all of the flow situation half-way through the 4th time cycle. The size of the vortex cluster at the edge grows between Sections 1 and 3 due to the increased roll velocity at the edge. Between Sections 1 and 11 edge angle slowly increases. This is accompanied by a relative weakening of the vortex strengths. The vortex clusters at the edge grow nearer to the edge; the pairs are less structured as they convect away from the edge, and tend to convect away nearer the body side. By Section 9 the vortex structure is quite weak; the pairing direction has also switched, so that, at this time instant, instead of a single large vortex cluster at the edge, there is a less well-structured vortex pair convecting away from the edge. The least structured shedding patterns are observed for Sections 11 and 13, with a few point vortices actually having convected through the body side for Section 13 due to the proximity of the vortex structures to the surface combined with a convection scheme which is only first order accurate. For Sections 15 to 19 the shedding edge becomes sharper, with a consequent strengthening of
the vortex structure. The pairing direction preferred by the sharper edges is re-established at Section 17. Between Sections 19 and 22 the shedding edge again becomes blunter, whilst the draft of the sections reduces, with an attendant diminishing of the size of the vortex clusters.

Figures 5.15 to 5.18 show plots of velocity vectors for a variety of conditions. The plots in each figure are separated by a quarter of a time cycle each, starting at the beginning of the 4th time cycle. Figures 5.15 and 5.16 compare the shedding patterns at Section 17 for Runs 1 and 3 (roll amplitudes of 8° and 4° respectively). The size of the vortex clusters is substantially smaller for the lower roll amplitude as should be expected. Figure 5.17 shows velocity vectors for the same section and 8° roll amplitude in the absence of a bar keel (Run 5). There is considerable similarity in the shedding patterns between this run and Run 1, with a strong vortex pairing structure convecting out into the flow field in both cases, but the absence of a bar keel in Run 5 implies weaker vortex shedding, with vortex pairs staying closer to the body side. Figure 5.18 shows velocity vectors for Run 7, Section 14. Vortices are being shed from one of the two bilge keels present. The Discrete Vortex Method as developed here is only capable of calculating vortex shedding from one shedding edge. Thus in the case of two or more shedding edges on the same section, total vortex damping is obtained by performing calculations for each shedding edge separately and then summing the resulting individual vortex dampings. This assumes that the interference from the vortex structures emanating from each shedding edge is negligible (see Section 3.9.6).
5.5.4 Roll damping results

In this section damping results are presented in terms of the sectional vortex roll damping moment coefficient, \( c_{rdm2} \), defined as:

\[
  c_{rdm2} = \frac{b}{\omega H_u}
\]

Total vortex roll damping moment coefficient, \( C_{rdm2} \), is obtained by summing the sectional coefficients over the length of the ship.

Figure 5.19 provides a graph of sectional vortex roll damping moment coefficient, \( c_{rdm2} \), for Runs 1 - 3 and 5. Run 1 yields dampings which are consistently higher than those from Run 2, which has a lower roll centre and would thus be expected to predict lower edge velocities and therefore lower dampings. Use of the sectional vortex roll damping moment coefficient permits direct comparison between damping results of different amplitudes since damping has a near-quadratic dependence on roll amplitude and \( c_{rdm2} \) is a function of damping divided by the square of roll amplitude. A comparison between Runs 1 and 3 on this basis reveals small but interesting variations of \( c_{rdm2} \) with roll amplitude. It appears, in general, that a concave section geometry near the shedding edge is associated with a lower damping at higher roll amplitude, and that a convex geometry is associated with a higher damping. The reason for this may be that a convex geometry effectively becomes "sharper" the larger the vortex size (and hence the larger the roll amplitude). This would then imply an increase in damping. The
opposite would be true for a concave geometry.

Comparison is also made in Figure 5.19 between the DVM results and those due to Brook (1987) using the BMT method. Since the BMT method was employed for a geometry without a bar keel, the most meaningful comparison is with Run 5. Both runs were computed with the same roll centre and at the same roll amplitude. Significant differences in the distribution of damping over the ship may be seen, although both methods agree roughly on the sections where most damping is being generated and also on the magnitudes of the peak dampings for the fore and aft portions of the ship.

The sectional vortex roll damping moment coefficient distribution was summed over the whole hull to yield the total vortex roll damping moment coefficient \( C_{rdm2} \). The table below shows values of \( C_{rdm2} \) for the present DVM calculations (except the bilge keel calculations), for the method due to Ikeda et al. (1978) - the "Ikeda" semi-empirical method, and for the BMT method. Experimental results are given which have been derived from both roll decrement tests and forced-roll tests. A range of experimental values is shown, this representing the range of scatter in the experiment. All damping results presented apply to the full-scale situation.
<table>
<thead>
<tr>
<th>Method</th>
<th>$C_{rdm2}$ (tonnes m$^2$/ rad$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discrete Vortex Method:</td>
<td></td>
</tr>
<tr>
<td>Run 1 (bar keel)</td>
<td>22</td>
</tr>
<tr>
<td>Run 2 (bar keel)</td>
<td>18</td>
</tr>
<tr>
<td>Run 3 (bar keel)</td>
<td>21</td>
</tr>
<tr>
<td>Run 4 (bar keel)</td>
<td>17</td>
</tr>
<tr>
<td>Run 5 (no bar keel)</td>
<td>14</td>
</tr>
<tr>
<td>Ikeda method (no bar keel)</td>
<td>4</td>
</tr>
<tr>
<td>BMT method (no bar keel):</td>
<td></td>
</tr>
<tr>
<td>4° amplitude</td>
<td>13</td>
</tr>
<tr>
<td>8° amplitude</td>
<td>11</td>
</tr>
<tr>
<td>Experiment (bar keel):</td>
<td></td>
</tr>
<tr>
<td>Roll decrement tests</td>
<td>9 - 12</td>
</tr>
<tr>
<td>Forced-roll tests</td>
<td>10 - 13</td>
</tr>
</tbody>
</table>

The DVM computations show that both a lowering of the roll centre and a lowering of roll amplitude from the values used in Run 1 reduce the total vortex roll damping moment coefficient. A comparison between Runs 1 and 5 shows that the presence of the bar keel increases damping by 60%. Part of this increase can be accounted for by the fact that, in the absence of a bar keel, computations are restricted, as described in Section 5.5.3 above, to sections with sufficiently "sharp" shedding geometries. This will produce the effect of somewhat underestimating
Interpretation of the relative accuracy of the various results is difficult because of the uncertainty over the experimental damping and because neither the Ikeda nor the BMT method include the effects of the bar keel present in experiment.

Comparison between the Discrete Vortex Method and the other roll damping prediction methods should be made for the DVM calculations carried out in the absence of a bar keel (Run 5). Agreement is good with the BMT method, but extremely poor with the Ikeda method, there being a difference of a factor of nearly four between that method and the DVM. One should expect the Discrete Vortex Method to be the most reliable since it provides the most complete model of vortex shedding. There appear to be weaknesses in the Ikeda method when employed for geometries such as the naked hull Sulisker geometries which are substantially different from those against which it was originally validated.

There are uncertainties in the experimental vortex roll damping results for two basic reasons. In the first place, it is difficult to break total roll damping down into its linear (i.e. radiation damping) and its non-linear (i.e. vortex damping) portions. In the second place, there is a significant amount of scatter in the experimental total damping results. There are differences between the decrement and forced-roll tests (forced-roll tests being believed to be more reliable - Spouge, 1987); also, experimental vortex roll damping moment coefficient is averaged over a range of roll amplitudes, thus effectively ignoring any
variation with roll amplitude. If comparison between the DVM results is made only with the forced-roll experimental results, then the DVM overpredicts damping by between 30 and 120%. Of course, agreement might even be better (or indeed worse) since there could be a greater variation in roll centre than is allowed for in the DVM calculations here; also, it would appear that DVM calculations at still lower roll amplitudes might predict yet lower vortex roll damping moment coefficients. Thus comparison with overall experimental dampings produces no conclusive evidence as to precisely how accurate the DVM predictions are. One should expect the DVM to overpredict two-dimensional dampings by between 30 and 45% (see Chapter 4). However, another possible source of error (inherent, in fact, in all three prediction methods) is the use of a strip approach for calculating total vortex dampings (see Section 3.9.5). The vortex shedding processes will be affected by three-dimensionalities particularly near the bows and stern but the magnitude of this effect is unclear. It is likely that the use of strip theory will lead to an overprediction of damping.

A table of total vortex roll damping moment coefficient, $C_{rdm2}$, is presented below for the Sulisker with the addition of the two bilge keels.
<table>
<thead>
<tr>
<th>Method</th>
<th>$C_{\text{rdm}2}$ (tonnes m²/ rad²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discrete Vortex Method:</td>
<td></td>
</tr>
<tr>
<td>Run 6 (bar keel and two bilge keels)</td>
<td>27</td>
</tr>
<tr>
<td>Run 7 (bar keel and two bilge keels)</td>
<td>22</td>
</tr>
<tr>
<td>Ikeda method (no bar keel, two bilge keels)</td>
<td>7</td>
</tr>
<tr>
<td>BMT method (no bar keel, two bilge keels)</td>
<td>18</td>
</tr>
<tr>
<td>Experiment (bar keel and two bilge keels):</td>
<td></td>
</tr>
<tr>
<td>Roll decrement tests</td>
<td>11</td>
</tr>
<tr>
<td>Forced-roll tests</td>
<td>14</td>
</tr>
</tbody>
</table>

Run 6, which is otherwise identical to Run 1 except for the two bilge keels, shows that the DVM predicts an increment of 5 tonnes m²/ rad² due to the bilge keels. This increment does not vary significantly due to small changes in roll amplitude or roll centre, as can be seen from Run 7, which is otherwise identical to Run 4 except again for the two bilge keels.

It is not clear what the true experimental vortex damping increment due to the bilge keels is. One can only conclude that the DVM prediction appears feasible in the light of the vortex damping increments predicted by the Ikeda and BMT methods.
CHAPTER 6

DEVELOPMENT OF A HYBRID MESH / POINT VORTEX NAVIER-STOKES SOLVER
FOR THE PREDICTION OF THE FLOW AROUND ISOLATED EDGES

The limitations of the Discrete Vortex Method so far as the present work is concerned have been described in Section 1.3. These are primarily twofold - the inability of the method to model either turbulence or diffusion in the wake and the inability to predict the separation point, which is problematical where that point is not fixed by geometry. In the present research the problem arose in the calculation of vortex shedding from the isolated edge with rounding, drag results from which were then to be utilised via a matching process to predict the roll damping of rectangular barges with rounded bilges. Further problems can be envisaged in the prediction of the roll damping of more general ship cross-sections in cases where there is no sharp edge to fix the separation point. A method for these cases was clearly required which solved the full two-dimensional unsteady Navier-Stokes equations. The Introductory chapter, Chapter 1, explains the choice of the method, which will be described fully in this chapter, and results from which may be found in Chapter 7. This method will be designated here the Hybrid Moving Vortex Diffusive Method, or HMVDM for short.
6.1 Fundamental Equations

This section describes the fundamental fluid flow equations, together with their boundary conditions, which govern two-dimensional unsteady fluid flow. The HMVDM employs a system of point vortices superimposed onto a finite difference mesh to produce a numerical solution of the fluid flow governed by these equations.

6.1.1 Primitive equations

The fundamental equations for unsteady two-dimensional incompressible flow of a Newtonian fluid with no body forces and constant properties are the Navier-Stokes momentum equations in the x and y directions and the continuity equation (Roache, 1972).

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]  \hspace{1cm} 6.1.1

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \]  \hspace{1cm} 6.1.2

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \hspace{1cm} 6.1.3
6.1.2 Stream function and vorticity transport equations

By eliminating pressure from Equations 6.1.1 and 6.1.2 above, and defining vorticity \( \Omega \) as:

\[
\Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}
\]  

we obtain the parabolic vorticity transport equation:

\[
\frac{\partial \Omega}{\partial t} = -u \frac{\partial \Omega}{\partial x} - v \frac{\partial \Omega}{\partial y} + \nu \left( \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} \right)
\]  

We define the stream function \( \psi \) by:

\[
\frac{\partial \psi}{\partial y} = u; \quad \frac{\partial \psi}{\partial x} = -v
\]  

so as to rewrite Equation 6.1.4 as an elliptic Poisson equation,

\[
\nabla^2 \psi = -\Omega
\]  

It is in the form of the vorticity transport equation that the Navier-Stokes equations are solved in the present work. The vorticity transport equation consists of the unsteady term, \( \partial \Omega / \partial t \), the convective terms, \( u \partial \Omega / \partial x \) and \( v \partial \Omega / \partial y \), and the viscous diffusion term, \( \nu (\partial^2 \Omega / \partial x^2 + \partial^2 \Omega / \partial y^2) \). It is parabolic in time, and therefore poses a marching or initial-value problem, whereas the stream-function equation is elliptic and poses a "jury" or boundary-value problem.
6.1.3 Boundary conditions

The above equations are solved subject to certain boundary conditions, which will vary according to the problem concerned. The boundary conditions in the solution of the vorticity transport equation are conditions on vorticity. In the present problem the body side must be treated with a no-slip condition such that the wall vorticity ensures zero tangential velocity at the wall. At all the other computational boundaries slip conditions should be employed such that $\Omega = 0$.

The boundary conditions in the solution of the Poisson equation are conditions on the stream function. The two commonly used boundary conditions in the literature are either the Dirichlet or the Neumann (see Roache, 1970), with the Dirichlet typically setting the stream function constant on the upper (outer) and wall boundaries and to some inflow function at the upstream and downstream boundaries. Making the outer boundary a streamline produces the same restriction on the flow as a wind-tunnel wall. The Neumann boundary condition would be most commonly used at the outer boundary and involves setting the derivative of the stream function in the direction normal to the boundary to the value of freestream velocity there.

6.2 Finite Difference Forms

As a fundamental prerequisite of the computational solution of the above equations on a finite difference mesh, we require difference forms of each of the terms.
Forms of the first and second order difference formulae are required which apply to unequal mesh intervals. These can be derived using either a parabolic curve fit or a Taylor series expansion. The latter method is followed here since it enables the accuracy of the differencing scheme to be ascertained. The following analysis takes after the work of Salvadori and Baron (1961).

We define a function $f(x)$ whose differences we wish to establish at a point $m$, given $f_{m-1}$, $f_m$ and $f_{m+1}$. We define:

$$\Delta x_m = x_{m+1} - x_m; \quad \Delta x_{m-1} = x_m - x_{m-1}; \quad s = \Delta x_m / \Delta x_{m-1} \quad 6.2.1$$

(see Figure 6.1). Then we know that:

$$f_{m+1} = f_m + f'_m \Delta x_m + \frac{1}{2} f''_m \Delta x^2_m + \frac{1}{6} f'''_m \Delta x^3_m + \ldots \quad 6.2.2$$

and:

$$f_{m-1} = f_m - f'_m \Delta x_{m-1} + \frac{1}{2} f''_m \Delta x^2_{m-1} - \frac{1}{6} f'''_m \Delta x^3_{m-1} + \ldots \quad 6.2.3$$

Subtracting Equation 6.2.3 from Equation 6.2.2 we obtain:

$$f'_m = \frac{f_{m+1} - f_{m-1}}{\Delta x_m + \Delta x_{m-1}} + \frac{1}{2} f''_m \frac{\Delta x_m}{s} (1-s) + O(\Delta x_m)^2 \quad 6.2.4$$

Taking only the first term in the expansion, neglecting higher order terms, yields an expression for $f'_m$ which is at least first order accurate, and second order accurate if $s = 1$. Eliminating $f'_m$ between
the two Equations 6.2.2 and 6.2.3 we obtain an expression for $f''_m$:

$$f''_m = \frac{2s}{s+1} \frac{sf_{m-1} - f_{m}(1+s) + f_{m+1}}{\Delta x_m^2} + \frac{1}{3} f''_m \frac{\Delta x_m}{s} (1-s) + O(\Delta x_m)^2 \quad 6.2.5$$

Like the first derivative, the resultant expression for $f''_m$ obtained by taking the first term of the expansion is at least first order accurate.

For a regular mesh spacing, with $s = 1$, and taking only the first terms of the expansions, Equation 6.2.4 becomes:

$$f'_m = \frac{f_{m+1} - f_{m-1}}{2\Delta x} \quad 6.2.6$$

where $\Delta x = \Delta x_m = \Delta x_{m-1}$

and Equation 6.2.5 becomes:

$$f''_m = \frac{f_{m-1} - 2f_m + f_{m+1}}{\Delta x^2} \quad 6.2.7$$

The above two difference forms are found very commonly in the literature - see, for example, Roache (1972). They are both second order accurate.

6.3 Outline of the Basic Inviscid Cloud-In-Cell Method

The HMVDM computer program referred to above originated as an inviscid "Cloud-In-Cell" vortex shedding program written by Basuki at Imperial
College; it is convenient to summarise the operation of this method at this point. Although work on this particular program has never been reported in detail, the method follows closely an earlier Cloud-In-Cell method also due to Basuki (1983). The later method employs a rectangular mesh in the transform plane, whereas the earlier method transforms to a circle. Basuki's earlier computer program is also related closely to a program reported by Naylor (1982).

The Cloud-In-Cell codes represent modifications to the basic Discrete Vortex Method described in Chapter 3. These are as follows:

A) At every time step point vorticity is distributed onto the mesh, which has been set up at the beginning of the computation. The distribution function used in the above work was a reverse bilinear interpolation which distributed vorticity from a point vortex onto the four nearest grid points.

B) At every time step the Poisson's equation (Equation 6.1.7) is solved on the mesh using the known values of mesh vorticity to obtain mesh values of streamfunction. Equation 6.1.7 may be solved on a mesh by one of many fast computational methods; both Naylor (1982) and Basuki (1983) chose the Fast Fourier Transform method.

C) Mesh velocities are computed from the streamfunction using a finite difference approximation.

D) Convection velocities are computed at the point vortices from the
known mesh velocities using a bilinear interpolation scheme and the
vortices are convected according to their respective convection
velocities.

Thus the above steps replace the previously described influence function
method for the convection of vortices (see Equations 3.2.3 to 3.2.6).

6.4 Outline of the Hybrid Moving Vortex Diffusive Method

The essential features of the HMVDM are outlined here and then treated
in more detail in succeeding sections. The HMVDM represents a joint
development of the basic Cloud-In-Cell vorticity convection model by J.
M. R. Graham and the present author to take account of the diffusion
term within the Navier-Stokes equations, yielding a method which solves
a finite difference approximation to the Navier-Stokes equations.

The method solves, subject to the elliptic Poisson equation, the
Navier-Stokes vorticity transport equation. Performing the computation
in the transform plane gives rise to a term additional to those found in
the basic vorticity transport equation (Equation 6.1.5), this term being
the square of the transformation modulus (see Lecointe and Piquet, 1986):

\[
\frac{\partial \Omega}{\partial t} = \left[ \nu \left( \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} \right) - \frac{\partial \Omega}{\partial x} \cdot \frac{\partial \Omega}{\partial y} \right] \left| \frac{\partial z}{\partial x} \right|^2
\]

6.4.1
The solution is performed computationally in two distinct parts, each preceded by a solution of the Poisson Equation:

1) Diffusion

\[ \Omega^k = \Omega^n + \Delta t \left( \frac{\partial^2 \Omega^n}{\partial x^2} + \frac{\partial^2 \Omega^n}{\partial y^2} \right) \cdot | \frac{\partial \zeta}{\partial z} |^2 \]  

where the superscript "n" refers to conditions at time \( t \), and "k" to conditions intermediate between \( t \) and \( t+\Delta t \), with vorticity having been advanced one time step only so far as the diffusion term is concerned. Diffusion is carried out on the mesh using a conventional differencing scheme.

2) Convection

\[ \Omega^{n+1} = \Omega^k - \Delta t \left( u \frac{\partial \Omega^k}{\partial x} + v \frac{\partial \Omega^k}{\partial y} \right) \cdot | \frac{\partial \zeta}{\partial z} |^2 \]  

where the superscript "n+1" refers to conditions at time \( t+\Delta t \). Convection velocities are calculated on the mesh, but convection is actually carried out using the point vortices, not the mesh.

This gives rise to the following solution of the vorticity transport equation:
\[
\Omega^{n+1} = \Omega^n + \left\{ \Delta t \nu \left( \frac{\partial^2 \Omega^n}{\partial x^2} + \frac{\partial^2 \Omega^n}{\partial y^2} \right) - \Delta t \left( u \frac{\partial \Omega^n}{\partial x} + v \frac{\partial \Omega^n}{\partial y} \right) \right\} - \nu \Delta t^2 \left( u \frac{\partial}{\partial x} \left( \frac{\partial^2 \Omega^n}{\partial x^2} + \frac{\partial^2 \Omega^n}{\partial y^2} \right) + v \frac{\partial}{\partial y} \left( \frac{\partial^2 \Omega^n}{\partial x^2} + \frac{\partial^2 \Omega^n}{\partial y^2} \right) \right) \cdot \left| \frac{\partial z}{\partial z} \right|^2
\]

The solution is only first order in time, with a diffusion error being convected at each time step. This error is in addition to the error caused by forward differencing in time.

The HMVDM has the following stages:

1) Perform transformation.

2) (Stage A of the Cloud-in-Cell method described in the section above) At every time step distribute point vorticity onto the mesh, which has been set up at the beginning of the computation. The distribution function used is a reverse bilinear interpolation which distributes vorticity from a point vortex onto the four nearest grid points.

3) (Stage B of the CIC method described in the section above) At every time step solve the Poisson's equation (Equation 6.1.7) on the mesh using the known values of mesh vorticity to obtain mesh values of streamfunction. The Fast Fourier Transform method (Cooley and Tukey, 1965) is employed in the solution of the Poisson Equation.
4) Perform diffusion of the vortices on the mesh subject to the vorticity boundary conditions.

5) Solve the Poisson's equation (Equation 6.1.7) again as in Stage 3 above, using the values of mesh vorticity computed in Stage 4.

6) Recompute an improved estimate of the wall vorticity and diffuse the excess wall vorticity out into the field.

7) (Stage C of the CIC method described in the section above)
Compute mesh velocities from streamfunction using a finite difference approximation.

8) Transfer the extra mesh vorticity due to diffusion onto the point vortex system.

9) (Stage D of the CIC method described in the section above)
Compute convection velocities at the point vortices from the known mesh velocities using a bilinear interpolation scheme and convect the vortices according to their respective convection velocities.

A flow chart describing the operation of the HMVDM is shown in Figure 6.2.
6.5 Transformation

6.5.1 Utilisation of the general numerical Schwarz-Christoffel transformation

The transformation has been described fully in Section 3.1. For the present implementation the geometry input has been set up for an isolated edge with rounding; it would be simple, however, to change the input to any other reasonable configuration, and this is one of the possible areas of future work.

Two quantities were required as output from the transformation: $dz/d\xi$ in terms of $\xi$, and $z$ in terms of $\xi$. The derivative values on the body at the junction between two segments are zero if the interior angle between the two segments is less than 180°. To avoid an infinite reciprocal of the derivative its value was calculated at a position 1/10 of the distance to the next grid point out.

6.5.2 Setting up the mesh

The Schwarz-Christoffel transformation has a particular usefulness in the setting up of computational meshes. A mesh composed of equal rectangular elements in the transform plane transforms back to a mesh in the physical plane which is distorted in such a way as to concentrate mesh points in the regions where they are most required, i.e. in the boundary layer, especially near to body slope and curvature.
discontinuities. Actually, the mesh as described above in fact comprises a system of streamlines and equipotentials for the uniform flow situation in the absence of vortex shedding in both the transform and physical planes, which explains the fact that the mesh is finest near regions of significant flow acceleration.

For flows at high enough Reynolds number the experience of a number of workers (see, for example, Peace and Riley, 1983, or Bayliss, Gunzberger and Turkel, 1982) has shown that a simple mesh composed everywhere of equal rectangular elements is inadequate. There are two related reasons for this. Firstly, extra concentration of mesh points is required in and near the boundary layer. For laminar flow, the higher the Reynolds number, the thinner the boundary layer, all other aspects of the flow being identical, and therefore the mesh is required to be more concentrated near the surface. In fact, it is widely held that at least three points are required to represent the boundary layer - one at the surface, one at the edge of the boundary layer, and one actually in the boundary layer, see Roache (1972). Secondly, the mesh is required to extend out into the field a substantial distance compared with, say, some representative flow length scale.

Both of the above requirements could in theory be satisfied by a transform plane mesh composed of a sufficient number of equal rectangular elements. The problem would be that for high Reynolds number the total number of mesh points might be completely unaffordable computationally speaking. This problem is addressed in the present work by having a mesh spacing in the transform plane allowed to vary in the
direction normal to the surface \((y\text{-direction})\). It would also have been advantageous, though not so important, to have a variable mesh spacing in the streamwise \((x\text{-})\) direction. However, the requirements of the Fast Fourier Transform only permitted regular spacing in this direction.

The \(y\)-mesh spacing in the transform plane was chosen to have an exponential variation:

\[
y_j = b \left[ e^{a(j-1)} - 1 \right]
\]  

6.5.1

A typical example of the use of the Schwarz-Christoffel transformation for the generation of computational meshes is shown in Figure 6.3, with the \(y\)-mesh spacing in the transform plane expanding exponentially outwards.

An inverse is easily found for Equation 6.5.1:

\[
j = \text{INT} \left[ \log(y_j/b + 1) / a \right] + 1
\]  

6.5.2

An exponentially varying mesh has been used widely by other workers (see, for example, Peace and Riley, 1983). For values of \(a(j-1)\) sufficiently low the mesh distribution is linear. This fact enables comparison with uniform mesh methods. It would also be possible to use other mesh functions. If an analytical inverse did not exist a root finding process would be required which might be time consuming. An exponential mesh variation in the transform plane will lead to an even more significant concentration of points near the surface in the physical plane because of the properties of the transformation mentioned above.
6.6 Treatment of the Diffusion Term in the Solution of the Navier-Stokes Vorticity Transport Equation

This section deals with the treatment of the diffusion part of the Navier-Stokes vorticity transport equation, that is Equation 6.4.2 above. This mainly covers Stages 4 and 6 of the HMVDM.

6.6.1 Solution of the diffusion equation

This section refers to Stage 4 of the HMVDM

Equation 6.4.2 may be written in a finite difference form for variable y-mesh spacing as follows:

\[
\Omega_{i,j}^k = \Omega_{i,j}^n + \Delta t \left[ \frac{\Omega_{i-1,j}^n - 2\Omega_{i,j}^n + \Omega_{i+1,j}^n}{\Delta x_i^2} + \frac{2s}{s+1} \cdot \frac{s\Omega_{i,j-1}^n - \Omega_{i,j}^n (1+s) + \Omega_{i,j+1}^n}{\Delta y_j^2} \right] \frac{\partial \zeta}{\partial z}_{i,j}^2
\]

where \( \Delta y_j = y_{j+1} - y_j \), \( s = \Delta y_j / \Delta y_{j-1} \)

In fact, the method works mostly in terms of circulation values, where mesh circulation \( \Gamma_{i,j} \) is here defined as:

\[
\Gamma_{i,j} = \Omega_{i,j} \cdot \Delta x_i \cdot (\Delta y_j + \Delta y_{j-1})/2
\]
Then Equation 6.6.1 may be rewritten:

\[ \Gamma_{i,j}^k = \Gamma_{i,j}^n + \Delta t \nu \left[ \frac{\Gamma_{i-1,j}^n - 2\Gamma_{i,j}^n + \Gamma_{i+1,j}^n}{\Delta x_i^2} + \frac{2s}{s+1} \frac{\Delta y_{j-1} + \Delta y_{j-2}}{\Delta y_j^2} \right] \]

\[ \times \left( \frac{s\Gamma_{i,j-1}^n}{(\Delta y_j + \Delta y_{j-1})} - \frac{(1+s)\Gamma_{i,j}^n}{(\Delta y_j + \Delta y_{j-1})} + \frac{\Gamma_{i,j+1}^n}{(\Delta y_j + \Delta y_{j-1})} \right) \cdot \frac{\partial^2 \xi}{\partial z^2}_{i,j} \quad 6.6.3 \]

For an exponentially varying mesh,

\[ \frac{\Delta y_{j+1} + \Delta y_{j-1}}{\Delta y_{j-1} + \Delta y_{j-2}} = s \quad 6.6.4 \]

so that the y-direction diffusion term can be simplified to:

\[ \frac{\partial^2 \Omega}{\partial y^2}_{i,j} = \left[ \frac{2s}{s+1} \cdot \frac{s \Gamma_{i,j-1} + (1+s)\Gamma_{i,j} + \frac{1}{s} \Gamma_{i,j+1}}{\Delta y_j^2} \right] \quad 6.6.5 \]

and this is the form in which the diffusion equation is solved for every interior mesh point.
6.6.2 Vorticity boundary condition

This section refers to Stages 3 to 6 of the HMVDM

The diffusion equation is solved subject to the no-slip boundary condition at the wall (see Section 6.1.3 above). Boundary conditions on the other three boundaries are slip conditions with zero vorticity. A second order finite difference approximation for the wall vorticity boundary condition due to Woods (1954) is employed. This may be written in its circulation form, for variable y-mesh spacing, as:

\[
\tau^{n+1}_w = - \left[ \frac{3(\psi^{n+1}_w - \psi^n_w)}{\Delta y^2_w} + \frac{\tau^{n+1}_w}{\Delta x_w (\Delta y_w + \Delta y_{w+1})} \right] \Delta x_w \Delta y_w
\]

where the subscript "w" stands for conditions at the wall, "w+1" for conditions one mesh point out in the y-direction, and the superscript "n+1" stands for conditions at time t=n+1, that is one time step forward from the current conditions. Vorticity is positive anticlockwise. Figure 6.4 provides a sketch which illustrates the above points.

Roache (1972) outlines the problems encountered when applying the vorticity boundary condition. The most obvious one is that in the above equation neither streamfunction nor mesh circulation are known initially for the new time step (n+1). Mesh solutions of the vorticity transport equation which are second order in time will usually adopt some
iterative procedure between this equation and the Poisson Equation so that improved estimates of streamfunction and circulation may be successively obtained. Typical of such a method is the Alternating Direction Implicit Method used, for example, by Peace and Riley (1983). In the HMVDM this iteration process is not feasible on economic grounds and would be most difficult to implement computationally, and so an alternative approach was used.

1) **(Stage 3 of the HMVDM)** At each new time step Poisson's Equation is solved first, using values of $\Gamma_{i,j}^n$, i.e. circulation from the previous time step, but with the correct streamfunction boundary conditions for the new time $(n+1)$. Obtain a first estimate of streamfunction, $\psi_{i,j}^{k_l}$.

2) **(Stage 4 of the HMVDM)** Obtain a first estimate of $\Gamma_w$ by the method described below. Call this estimate $\Gamma_{w}^{kl}$. In the subsequent diffusion process an amount of circulation is diffused from the wall. It was noted during numerical experiments with the method that if this exceeds a critical amount instabilities develop. This is due to the fact that there is a feedback mechanism in operation with the method which means that an under- (or over-) specification of the boundary condition at one time step leads to an over- (or under-) specification at the next. Whether or not this feedback mechanism is stable or not depends largely on the size of the error in the wall circulation. In order to ensure that instabilities do not arise in this first estimate of wall circulation, an adjustment factor, $R$, is applied to the wall circulation formula (Equation 6.6.6) to yield the first estimate of the wall circulation, $\Gamma_{w}^{kl}$, using the value of $\Gamma_{w+1}$ from the old time step:
A suitable value of $R$ has been found by numerical experimentation to be 2. Diffusion is then carried out throughout the mesh as described in Section 6.6.1, using the first estimate of wall circulation, $\Gamma_{w}^{k1}$, to yield estimates of the mesh circulation elsewhere, $\Gamma_{i,j}^{k1}$.

3) (Stage 5 of the HMVDM) This is followed by a second solution of Poisson's Equation, with these revised values of mesh circulation, to yield $\psi_{i,j}^{k}$. 

4) (Stage 6 of the HMVDM) A second estimate of wall circulation is obtained as follows:

$$\Gamma_{w}^{k} = -\left[\frac{3(\psi_{w+1}^{k} - \psi_{w}^{k})}{\Delta y_{w}^{2}} + \frac{\Gamma_{w+1}^{k}}{\Delta x(\Delta y_{w} + \Delta y_{w+1})}\right]\Delta x\Delta y_{w} \tag{6.6.8}$$

that is, the correct boundary condition is applied on this second pass.

5) (Stage 6 of the HMVDM) The new wall circulation is employed to determine $\Gamma_{w+1}^{k}$ via the diffusion process, but further estimates elsewhere in the mesh are not carried out, so that $\Gamma_{i,j}^{k}$ is elsewhere set to $\Gamma_{i,j}^{k1}$.

Thus the above scheme makes two estimates of both streamfunction and
wall circulation per time cycle. Two estimates are also made of the circulation one mesh point out from the wall, which is crucially important since this is where new discrete vortices are introduced into the flow. Note that the above scheme only advances circulation and streamfunction to the intermediate stage (k), rather than the new time step (n+1). The new time step is achieved through convection of the circulation.

Roache (1972) mentions the existence of instabilities encountered in the use of the second order wall vorticity boundary condition. In fact numerical experimentation with the current method has established a clear link between these and time step size. It has also established that the same instabilities can exist even for the first order method. The strength of the current scheme lies in its exploitation of the feedback mechanism at the wall to enable a much more stable scheme than would normally be formulated and thus to enable for little extra computational cost larger time steps to be used.

6.7 Solution of the Poisson Equation

This section refers to Stages 3 and 5 of the HMVDM

Solution of the Poisson Equation (Equation 6.1.7) is carried out using a Fast Fourier Transform technique. Basuki (1983) has described the application of the technique to the Cloud-in-Cell method with a circular mesh; here we describe its use for rectangular meshes, with appropriate extensions for variable y-mesh spacing.
The Poisson Equation may be written in a discretised form as:

$$\nabla^2 \psi_{i,j} = \frac{-2T_{i,j}}{\Delta x(\Delta y_j + \Delta y_{j-1})}$$  \hspace{1cm} 6.7.1

where $\Delta y_j$ and $\Delta y_{j-1}$ are as defined in Section 6.2. Taking the Fourier Transform in the $x$-direction, we obtain:

$$-k^2 \psi_{k,j} + \frac{\partial^2 \psi}{\partial y^2}_{k,j} = \frac{-2G_{k,j}}{\Delta x(\Delta y_j + \Delta y_{j-1})}$$  \hspace{1cm} 6.7.2

where $\psi = \psi(k,j)$ is the Fourier Transform of $\psi(i,j)$

$$= \int \psi e^{-ikx} \, dx,$$

$k$ is the wave number, and $G$ is the Fourier Transform of circulation.

We can write a finite difference form of the left hand side of Equation 6.7.2 above using the finite difference approximation derived in Equation 6.2.5:

$$\frac{2s}{s+1} \psi_{k,j+1} + (-k^2 \Delta y_j^2 - 2s) \psi_{k,j} + \frac{2s^2}{s+1} \psi_{k,j-1}$$

$$= - \frac{2G_{k,j} \Delta y_j^2}{\Delta x(\Delta y_j + \Delta y_{j-1})}$$  \hspace{1cm} 6.7.3
Notice that as always $y_j$ is invariant of its $x$-location. This equation when applied successively to each interior mesh point forms a series of tridiagonal matrices which can be solved by a Gaussian elimination technique. In this work the elimination technique described by Varga (1962) is employed, with:

$$
v_{j} \psi_{k,j+1} + \lambda_{j} \psi_{k,j} + \mu_{j} \psi_{k,j-1} = \frac{-2G_{k,j} \Delta y_{j}^{2}}{\Delta x (\Delta y_{j}^{2} + \Delta y_{j-1}^{2})}
$$

6.7.4

where $v_{j} = \frac{2s}{s+1}$, $\mu_{j} = \frac{2s^{2}}{s+1}$, $\lambda_{j} = -(k^{2} \Delta y_{j}^{2} + 2s)$

As indicated in the previous section, solution of the Poisson Equation is performed twice per time step, once to provide values of stream function for the wall vorticity boundary condition, and once to provide values of stream function for the evaluation of mesh velocities.

### 6.7.1 Stream function boundary condition

The Dirichlet boundary condition with stream function set to zero at the inner boundary and the product of freestream velocity and $y$-distance on the outer boundary (see Figure 6.5), was adopted due to its ease of use with the Fast Fourier Transform method. Due to the periodic nature of the FFT, it is not possible to specify upstream (inlet) and downstream (outlet) boundary conditions. The FFT automatically sets streamfunction
values at the inlet equal to those at the outlet (i.e. a periodic boundary condition). This imposes the restriction on the HMVDM that any significant transport of vorticity should be performed well clear of the inlet and outlet boundaries. The use of the Dirichlet boundary condition on the upper boundary requires that this boundary should be sufficiently far from regions of significant vorticity that its effect on the vortex shedding processes is negligible. This can be verified by testing the sensitivity of results to outward mesh extent.

6.8 Determination of Mesh Velocities

This section refers to Stage 7 of the HMVDM

We can determine the mesh convection velocities from the values of streamfunction computed on the mesh from the Poisson Equation. If the x- and y- direction convection velocities in the transform plane are \( u \) and \( v \) respectively, then we can write from the definition of streamfunction (Equation 6.1.6):

\[
\begin{align*}
    u_{i,j} &= \left( \frac{\partial \psi}{\partial y} \left| \frac{\partial \zeta}{\partial z} \right|^2 \right)_{i,j} \quad 6.8.1 \\
    v_{i,j} &= \left( -\frac{\partial \psi}{\partial x} \left| \frac{\partial \zeta}{\partial z} \right|^2 \right)_{i,j} \quad 6.8.2
\end{align*}
\]

The \( \partial \zeta / \partial z \) term appears because of the transformation. It has been shown to be of the above form by Basuki (1983). Using Equations 6.2.4 and
6.2.6 we can write finite difference forms of the above equations as:

\[ u_{i,j} = \frac{\psi_{i,j+1} - \psi_{i,j-1}}{\Delta y_j + \Delta y_{j-1}} \cdot \left| \frac{\partial z}{\partial z} \right|_{i,j} \]  \hspace{1cm} 6.8.3

and

\[ v_{i,j} = \frac{\psi_{i-1,j} + \psi_{i+1,j}}{2\Delta x} \cdot \left| \frac{\partial z}{\partial z} \right|_{i,j} \]  \hspace{1cm} 6.8.4

6.9 Convection of Vortices

This section refers to Stage 9 of the HMVDM

The point vortices are convected at each time step in the transform plane using the known mesh velocities as calculated above. An interpolation scheme is required to determine the contribution of the mesh velocities at each point. Generally, such an interpolation scheme might pertain either to the nearest 1, 4 or 9 mesh points. Here we employ a bilinear interpolation scheme which employs the nearest four points to give an expression for the \( u \) and \( v \) components of the vortex convection velocity in terms of the \( u \) and \( v \) components of the mesh convection velocities:

\[ u_{\text{vortex}} = \sum_{L=1}^{4} u_L \cdot \frac{A_L}{A} \]  \hspace{1cm} 6.9.1

and
\[ v_{\text{vortex}} = \sum_{L=1}^{4} v_L \cdot \frac{A_L}{A} \quad \text{(6.9.2)} \]

For an explanation of the above equations see Figure 6.6.

6.10 Interpolation Between the Point Vortices and the Mesh

This section refers to Stage 2 of the HMVDM

At the beginning of each time step it is required to redistribute the circulation on the vortices onto the mesh so that the diffusion process can be carried out on the mesh. An inverse procedure to that of Equations 6.9.1 and 6.9.2 is employed:

\[ \Gamma_L = \frac{A_L}{A} \Gamma_{\text{vortex}} \quad \text{(6.10.1)} \]

for \( L = 1 \) to 4, where \( A_L \) are the areas as shown in Figure 6.7, \( A_\Gamma \) is the area of the relevant mesh box, and \( \Gamma_{\text{vortex}} \) is the strength of the point vortex.

6.11 Transfer of Mesh Circulation onto the Point Vortex System

This section refers to Stage 8 of the HMVDM

The diffusion process as described in Section 6.6 above amends the circulation values by an incremental amount at every mesh point at every time step. Some method of efficiently transferring this incremental mesh circulation onto the moving vortex system needs to be provided which minimises the number of new vortices created at each time step so that
computational costs do not become excessive. The most expensive scheme available would be simply to place into the flow at each time step vortices whose initial location is at the mesh vertices themselves. A more efficient method is to amalgamate this new circulation onto pre-existent point vortices; this method is employed in the current work. However, when there are no point vortices near enough to the relevant mesh vertices, no amalgamation takes place, in order that adequate definition of the vortex structure may be retained. In these circumstances vortices are released directly into the flow from the mesh vertices. The above scheme is described in more detail in the following section.

6.11.1 Amalgamation of vortices

A first order vortex amalgamation scheme has been devised as follows. Each mesh box is divided equally into four smaller boxes (see Figure 6.8a). A domain of four hyperbolic sections is constructed as shown in the figure. A sweep is carried out through all the current point vortices. During this sweep each point vortex is associated with its nearest grid point, and the mesh vortex there (i.e. the new circulation due to diffusion) is amalgamated with it if the point vortex lies inside the domain and if the two circulations share the same sign. The extent of the inclusion zone in this first sweep is indicated in the figure. In a second sweep through the vortices each mesh vortex is amalgamated with its nearest like-signed point vortex provided the point vortex lies within the region shown in Figure 6.8b. Amalgamation of like-signed vortices ensured that the total number of vortices in the flow was kept
within reasonable bounds. Vortices of opposite sign were not amalgamated, in order to avoid disturbing the small scale structure of the vortex sheet.

Vortices are amalgamated in such a way that the strength and position of the resultant vortex is as given in Section 3.4.

6.12 Calculation of Vortex Force

The force due to vortex shedding is determined by means of a momentum balance formed from Blasius' theorem in the way described in Chapter 3 for the Discrete Vortex Method.

It is necessary to show that this theorem is applicable to viscous as well as inviscid flows. Wu (1981) has shown, for two-dimensional viscous flows, that the aerodynamic force, $F$, exerted by a fluid on solid bodies immersed in and moving relative to the fluid is equal to the inertia force due to the mass of fluid displaced by the solid bodies ($F_m$) plus a term proportional to the rate of change of total first moment of the vorticity field in the solid bodies and the fluid (i.e. the "vortex" force, $F_v$, which, in viscous flows, is here defined as including the contribution of the boundary layer to the force summation). Thus an expression can be obtained for $F_v$:

$$F_v = -\rho \frac{d\alpha}{dt}$$  \hspace{1cm} 6.12.1
where \( \alpha \) is the first moment of the vorticity field, defined by:

\[
\alpha = \int_{R_L} r \times \Omega \, dR
\]

\[
6.12.2
\]

where \( r \) is a position vector, and \( R_L \) is a circular region \( r \ll L \). \( L \) is sufficiently large so that \( R_L \) contains all the solid regions, and that the total vorticity and total vorticity moment outside \( R_L \) are negligible. \( \Omega \) is the vorticity field in the infinite region \( R \).

As applied to the problem of vortex shedding off isolated edges, for a transform plane with the freestream acting in the \( y \)-direction (parallel to the body side) the above Equations 6.12.1 and 6.12.2 yield the following expression for vortex force:

\[
F_v = -i \rho \frac{\partial}{\partial t} \int_{R_L} x \Omega \, dx \, dy
\]

\[
6.12.3
\]

Converting vorticity in this equation into circulation and writing the equation in a discretised form, we obtain for a set of \( n \) point vortices:

\[
F_v = -i \rho \frac{\partial}{\partial t} \sum_{m=1}^{n} \Gamma_m x_m
\]

\[
6.12.4
\]

which is identical to Equation 3.8.1 except for the use here of the real part \( (x_m) \) of the transform plane point vortex complex ordinate. The real part of the position of its image is simply \( (-x_m) \). Equation 6.12.4 may be rewritten in its mesh form by recognising that the mesh ordinates do not change with time, but instead that the mesh circulation does:
Equations 6.12.4 and 6.12.5 not only furnish proof that Blasius' theorem in its present formulation can be used for viscous flows, but also confirm the validity of the particular implementation used by Graham (1980) and adopted for this present work. In the next chapter we examine more closely the use of the formula for one particular test case, namely the Stokes boundary layer (oscillatory boundary layer over a flat plate), and then examine the application of the HMVDM to the flow past isolated edges.
CHAPTER 7

RESULTS FROM THE HYBRID MOVING VORTEX DIFFUSIVE METHOD

7.1 Test Case of an Oscillatory Boundary Layer over a Flat Plate

A lengthy validation process was embarked upon to check the operation of the Hybrid Moving Vortex Diffusive Method. This included testing of the various subroutines in the computer method, especially those connected with the Fast Fourier Transform Poisson Solver. In the next stage of the validation, various practical flow cases of increasing complexity were computed. These culminated in computations on the oscillatory boundary layer over a flat plate, otherwise known as the Stokes boundary layer.

The oscillatory boundary layer was thought to be a suitable test case for the method for two reasons:

1) It has direct relevance to the more general problem of vortex shedding from isolated edges with edge rounding in oscillatory flow. Indeed, it may be thought of as representing the case of infinite edge radius.

2) There is an analytical solution available so that comparison can be made with a known solution.

However, the Stokes boundary layer is not a good test of the convective part of the algorithm, although the HMVDM, originating as it does from
the DVM, was believed to have a good algorithm for representing convection.

**7.1.1 Analytical solution for an oscillatory boundary layer**

Schlichting (1960) states the analytical solution for the flow past an infinite oscillating flat plate. This solution can be adapted to that for a fixed wall and freestream of velocity:

\[ U_\infty = U_0 \sin(\omega t) \tag{7.1.1} \]

For the above freestream the analytical solution of the oscillatory boundary layer is:

\[ \frac{u}{U_0} = -[e^{-ky}\sin(\omega t-k\gamma) - \sin(\omega t)] \tag{7.1.2} \]

where:

\[ k = \sqrt{\frac{\omega}{2v}} \]

Now shear stress,

\[ \tau = \mu \frac{\partial u}{\partial y} \tag{7.1.3} \]

\[ = \mu U_0 k e^{-ky} [\sin(\omega t-k\gamma) + \cos(\omega t-k\gamma)] \]

\[ \tau = \sqrt{2} \mu U_0 k e^{-ky} \sin(\omega t-k\gamma + \pi/4) \tag{7.1.4} \]
Wall shear stress,

\[ \tau_w = \sqrt{2} \mu U_0 k \sin(\omega t + \pi/4) \]  

7.1.5

In other words, wall shear stress is \( \pi/4 \) out of phase with the freestream velocity. Force per unit width on the plate is given by:

\[ F_v = \tau_w \cdot (\text{domain length}) \]  

7.1.6

Vorticity can be simply derived from the above analysis by recognising that there will be no changes in the \( v \)-velocity in the \( x \)-direction (in fact, \( v = 0 \) throughout the flow). Then:

\[ \Omega = -\partial u/\partial y \]  

7.1.7

\[ \Omega = -\sqrt{2} U_0 k e^{-k y} \sin(\omega t - k y + \pi/4) \]  

7.1.8

It has been shown in the previous chapter that the Blasius momentum balance method of calculating drag is applicable to viscous as well as inviscid flows. In the case of viscous flows the momentum balance includes the boundary layer contribution to drag as well as the contribution due to vortex shedding. This can be demonstrated for the specific case of the oscillatory boundary layer, where the only component of drag is boundary layer (or skin friction). In terms of wall shear stress as opposed to force, Equation 6.12.4 gives us for a freestream parallel to the \( x \)-direction that:
\[ \tau_w = -\rho \frac{\partial \vec{v}}{\partial t} \sum_m \Omega_m \Delta y_m' y_m \tag{7.1.9} \]

Writing this in the continuum form,

\[ \tau_w = -\rho \frac{\partial \vec{v}}{\partial t} \int_0^\infty \left[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] y \, dy \]

But \( \frac{\partial v}{\partial x} = 0 \) so that:

\[ \tau_w = \rho \frac{\partial \vec{v}}{\partial t} \int_0^\infty \frac{\partial u}{\partial y} \, dy \]

Integrating by parts,

\[ \tau_w = -\rho \frac{\partial \vec{v}}{\partial t} \int_0^\infty \frac{\partial u}{\partial y} \, dy \tag{7.1.10} \]

The full Navier-Stokes equation of motion for the u-velocity is:

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{7.1.11} \]

with:

\[ \frac{\partial u}{\partial x} = \frac{\partial p}{\partial x} = v = 0 \]

So that:

\[ \tau_w = -\rho \nu \int_0^\infty \frac{\partial^2 u}{\partial y^2} \, dy \]
\[ \tau_w = \frac{\partial u}{\partial y} \]

which is the expression for the shear stress to be found in Equation 7.1.3. Thus the method used in this work for calculating vortex force (Equations 6.12.4 or 6.12.5) is shown to include and correctly predict, for this particular viscous flow case, the boundary layer (skin friction) force.

7.1.2 Comparison of the analytical solution with the computed results

Results of two test computations are presented here, together with the appropriate analytical results. The first was carried out on a 32 by 32 mesh with 80 time steps per cycle at a kinematic viscosity (ν) of 0.1, freestream velocity amplitude (U₀) of 1.0, and angular frequency (ω) of 0.2π. In the computation the y-mesh spacing was chosen to expand outwards with each mesh spacing being 1.051 times the previous. This represents quite a moderately expanding mesh. The x-mesh spacing was selected to be large enough so as to produce a computation where, far from the edges of the computational domain, changes in flow quantities in the x-direction were minimal. The mesh is shown in Figure 7.1a. The above aim was achieved, as Figure 7.1b shows. In Figure 7.1b velocity vectors have been plotted for every y-wise mesh location and every fourth x-wise location across the entire computational domain. The vectors represent the instantaneous situation at the end of two cycles of flow. The flow at the edges, as would be expected, is different from that nearer the middle of the domain, but one can see that there is a
substantial region where flow changes in the x-direction are minimal. Figure 7.1c shows an enlarged plot of the y-wise variation of velocity at a station near the middle of the domain at the end of two time cycles, that is, when freestream velocity is zero. The full line represents the analytical result; the symbols represent the calculated result. The analytical result has been calculated with a phase shift of -0.011 radians with respect to the computed, so as to compensate for a small error in computed freestream velocity by matching the two freestream velocities together. This phase shift represents 0.14 of one computational time step, which is considered low for a computer method which is only first order accurate in time. The level of agreement between the two velocity profiles is excellent.

Figure 7.2 shows a comparison between analytical and computed time-dependent wall shear stress for two cycles of flow beginning at the end of the first half-cycle. The computation was carried out with a somewhat more refined mesh and 320 time steps per cycle. The agreement between the two results is clearly very good. The main criticism of the computation is that it is up to about 2% of a time cycle out of phase with the analytical result.

On the evidence presented above it would appear that the HMVDM copes most effectively with a diffusion dominated flow such as the oscillatory boundary layer. This is encouraging, since the strength of the HMVDM is in its treatment of convection dominated problems, and any weakness that might exist in the method is likely to lie in its treatment of diffusion.
7.2 Vortex Shedding from an Isolated Square Edge in Uniform Flow

The case of uniform flow past an isolated square edge has previously been computed using the Discrete Vortex Method both by the present author and also by Pullin (1978). Because of the possibility of comparison with DVM results it therefore affords a good test of the HMVDM.

7.2.1 Definition of Reynolds number in uniform flow

One feature of the plain isolated edge flows being considered in this thesis is that they possess no physical length on which scaling may be carried out, though such a scaling is possible in the transform plane. Equation 3.1.15 gives an expression for the length scale, \( L_z \), of the flow in the physical plane based on the properties of the transformation. Such a length scale can be used in the definition of a flow Reynolds number, but it should be realised that such a Reynolds number is not based on the size of the vortex structure, but rather on a length scale which may be significantly larger than, although proportional to, the vortex size. Consequently, Reynolds numbers may appear to be quite high. A Reynolds number based on \( L_z \) would be:

\[
Re = \frac{L_z V_z}{\nu}
\]  

7.2.1
where $V_z$ and $L_z$ are defined as for oscillatory flow (Equations 3.1.13 and 3.1.15) save that here $V$ is the uniform flow velocity in the transform plane and $T$ is the time since the beginning of the computation.

7.2.2 Vortex positions and streamlines

The HMVDM was run for the isolated square edge in uniform flow using a 64 X 32 mesh with a mesh spacing which produced a high degree of refinement near the shedding edge, as Figure 7.3 shows. Figure 7.4 is a plot of the streamline pattern and vortex positions produced after 100 time steps, together with results calculated by Pullin (1978) using the Discrete Vortex Method (an essentially inviscid model). For the HMVDM run Reynolds number based on length scale was 3100. The boundary layer ahead of the vortex cluster was permitted to grow from the surface at a position five mesh points from the left hand (inlet) boundary. Flow velocity at the inlet and outlet boundaries was set to attached flow values. It was ensured that vortex extent was small in comparison with mesh size. In the plot showing vortex positions the cross symbols represent positive circulation, and the triangles negative circulation. Symbol size is representative of the value of circulation.

The comparison between the two results is extremely good, the slightly flatter appearance of the vortex in the Navier-Stokes solution arising, it is believed, from the action of viscosity in the HMVDM, although its effects are unlikely to be too pronounced in this case.
7.3 Results for Oscillatory Flow Past a Square Edge with Rounding

A series of computer runs were carried out using the HMVDM to predict the vortex shedding due to sinusoidally oscillating flow past an isolated square edge with various degrees of edge rounding. Results presented in this section thus echo the DVM computations of Section 4.3; the HMVDM is shown here to produce results of superior quality and reliability. The ultimate aim of the computations was to produce drag coefficient data which could be utilised via a matching process in the prediction of the roll damping of rectangular cross-section barges with rounded bilges. Section 7.4 treats this subject in detail; this section deals with the computations and the direct application of their results.

7.3.1 Definition of Stokes number in oscillatory flow

In oscillatory flow we employ the viscous parameter, Stokes number \( \text{St} \), otherwise known as the \( \beta \) parameter (Bearman et al., 1985), which is defined as:

\[
\text{St} = \frac{\text{Re}}{K_c}
\]

7.3.1

where both Reynolds number and Keulegan-Carpenter number are here defined in terms of flow length and velocity scales. Thus we obtain that:

\[
\text{St} = \frac{L_z v_z}{v}, \quad \frac{L_z}{V_{zT}} = \frac{L_z^2}{(\nu T)}
\]

7.3.2
One of the most important parameters in the computations was the Stokes number. Ideally, one would require results at Stokes numbers representative of both model and full scale flows. A typical model scale Stokes number based on flow length scale was of the order of five thousand for the rolling model barge described in Chapter 2, whilst a typical full scale Stokes number might be of the order of five million. It is not known in what Stokes number range transition to turbulence takes place, but it will depend on a combination of Stokes and Keulegan-Carpenter numbers; at higher $K_C$ transition will occur at lower St. It is presumed that at full scale the effects of turbulence are important. The higher Stokes number is therefore unachievable with the HMVDM in its present form because it lacks a turbulence model. The lower Stokes number is, however, achievable. In the computations which follow, all "high" Stokes number runs were carried out at Stokes numbers somewhat in excess of ten thousand. Two "low" Stokes number runs at a Stokes number of 4600 are reported, to demonstrate the effect of varying Stokes number.

7.3.2 Choice of computational mesh

Correct choice of mesh was most important and was complicated by two conflicting considerations. On the one hand adequate definition of mesh in and near the boundary layer was required. On the other hand the complete mesh was required to be as large as possible to maximise the possible path lengths of the vortex pairs and to ensure that the boundaries were far enough from the body and the vortex clusters for the
boundary conditions to be correctly imposed. Because of the computational expense it was not possible to employ meshes which were large enough to enable computations to be continued for more than about two cycles of flow without the outer boundaries imposing a restraint, since vortex pairs would tend, over a number of cycles, to convect towards these boundaries.

Four meshes were constructed; these are designated meshes A to D, and may be found plotted in Figures 7.5 to 7.8. Meshes A and B are plotted to nearly the same scale, whereas mesh C is about one quarter of the size of mesh A and mesh D one seventh of the size.

7.3.3 Choice of run parameters

Duration of computations

One critical parameter is the duration of the computations in terms of the number of time cycles. Ideally, one should want computations to be carried out over a large number of cycles, for example, ten as is the case in the Discrete Vortex Method calculations. Then it would be possible to average drag over many cycles, excluding at least the first cycle, when drag will invariably be lower. However, with the HMVDM it is not possible to average over several cycles, since the cost of computing more than two or so cycles of flow would be prohibitive. Further, the extent of the mesh is not sufficient to permit longer runs. A more extensive mesh would require more mesh points which would entail further computational expense.
Thus, a strategy alternative to obtaining drag coefficients averaged over several time cycles had to be adopted in the present work. It was possible to perform integrals of drag over half-cycle, instead of whole-cycle periods, by replacing Equation 3.8.3 by the following equation for the A "half-coefficient":

\[
A_{\frac{1}{2}} = \frac{3\pi}{2} \int_{0}^{\frac{1}{2}} C_{Fv} \sin \left(\frac{2\pi t}{T}\right) dt
\]

and similarly for the B half-coefficient.

This approach enabled a fuller assessment of the development of drag coefficient with time to be made (see Section 7.3.6). It was particularly important to exclude those coefficients from the average which appeared to have been affected by the starting conditions. Experience with the method showed that it was sufficient to exclude the coefficients calculated during the first cycle. It was therefore possible to limit the important computational runs to between 2 and 2\(\frac{1}{2}\) cycles, with drag coefficient usually being averaged over the second time cycle only.

**Non-dimensional edge radius**

Edge radius (r) itself is a redundant parameter in the set-up of the computations and is always set to unity. The important parameter in this context is non-dimensional radius \((r/L_{z})\), which determines the effective edge radius in terms of flow length scale. Computer runs were carried out at non-dimensional radii of 0, 0.0165, 0.0466 and 0.0924, in other
words practically the same range as the equivalent DVM runs (see Chapter 4).

X-wise mesh interval
Solution of the Poisson equation by means of the Fast Fourier Transform permitted variable mesh spacing only in the y-direction, and not in the x-direction. The x-wise mesh interval in the transform plane is determined by two factors: by how many surface mesh points are required in the rounding portion of the body, and by what the total mesh extent in the x-wise direction is required to be. These are conflicting requirements, in that as much definition in the rounding portion as possible is required, and this is detrimental to the total mesh extent, which is always required to be as great as is practicably possible. If such a conflict exists, the only way of resolving it is to increase the total number of mesh points in the x-direction. In the present work 64 points were used in the x-direction and 32 in the y-direction. It was found that provided the calculations were not continued past about 2 time cycles the mesh was sufficiently large that vortex pairs did not convect too near the inlet or outlet boundaries. The number of mesh points within the edge rounding itself varied between 5 and 15 depending on the flow length scale. For some test cases, several runs were performed at different mesh spacings to ensure that an optimum mesh spacing was being employed.

Y-wise mesh interval
Because of the facility in the program of varying the y-mesh spacing in the transform plane, it was possible to crowd points near the surface,
as described earlier. The mesh was chosen to vary so that in the transform plane the mesh interval increased at each station out by about 10%. Because of the nature of the transformation, this implies an increase of somewhat more than 10% in the physical plane. The aim in choosing the spacing itself (or, in other words, the value of "b" in Equation 6.5.1) was to place an adequate number of points in the boundary layer. The spacing was varied in order to find its optimum value for each case.

**Freestream velocity in the transform plane**

Freestream velocity amplitude in the transform plane was varied to yield the desired value of flow length scale and hence the desired value of non-dimensional radius \( r/L_z \).

**Time step size**

Time step size, \( \Delta t \), was optimised by a process of numerical experimentation. Too large a time step size leads to instabilities in the solution and inaccuracies in the vortex representation. Too small a time step leads to uneconomic computations. The same time step size was employed for all the higher Stokes number calculations, leading to 1200 time steps per cycle. For the lower Stokes number calculation with coarse mesh a much larger time step was possible, which led to 200 time steps per cycle. In all cases the period of the oscillation, \( T = 10 \).
7.3.4 Flow results

The table below provides a list of the major calculations (except test runs) which were carried out using the HMVDM, with corresponding figure numbers for plots of vortex position and, in some cases, plots of velocity vectors.

<table>
<thead>
<tr>
<th>Run</th>
<th>Mesh</th>
<th>( r / L_z )</th>
<th>( \Delta t )</th>
<th>( St )</th>
<th>Figure numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>A</td>
<td>0.0</td>
<td>0.00833</td>
<td>28300</td>
<td>7.9, 7.10</td>
</tr>
<tr>
<td>(2)</td>
<td>A</td>
<td>0.0</td>
<td>0.00833</td>
<td>18400</td>
<td>7.11, 7.12</td>
</tr>
<tr>
<td>(3)</td>
<td>B</td>
<td>0.0165</td>
<td>0.00833</td>
<td>36700</td>
<td>7.13, 7.14</td>
</tr>
<tr>
<td>(4)</td>
<td>C</td>
<td>0.0466</td>
<td>0.00833</td>
<td>36700</td>
<td>7.15, 7.16</td>
</tr>
<tr>
<td>(5)</td>
<td>D</td>
<td>0.0924</td>
<td>0.00833</td>
<td>36700</td>
<td>7.17, 7.18</td>
</tr>
<tr>
<td>(6)</td>
<td>C</td>
<td>0.0466</td>
<td>0.00833</td>
<td>4600</td>
<td>7.19</td>
</tr>
<tr>
<td>(7)</td>
<td>B</td>
<td>0.0466</td>
<td>0.05000</td>
<td>4600</td>
<td>7.20</td>
</tr>
</tbody>
</table>

In the plots of vortex positions it should be noted that the occurrence of large concentrations of symbols does not necessarily imply the presence of large concentrations of vorticity, since the process of diffusion may reduce the strength of the vortex cluster without substantially altering its plotted appearance. Also, the size of the vortex symbols plotted varies as the cube root of the vortex strength, to enhance the visibility of areas of weak vorticity. There will be occasions when many weak vortices in a plot will partially obscure the
presence of stronger areas of vorticity. For the above two reasons, plots of vortex positions can not be seen as analogous to experimental flow visualisations with either particles or dye. In this respect the plots of velocity vectors give a more complete picture of the flow situation. These are only presented here for cases where freestream velocity is zero, as is indicated by the fact that many regions of the velocity vector plots contain no plotted vector symbols.

RUN 1
Figures 7.9 and 7.10 show the first two time cycles of a square edge calculation (Run 1). Each position plot is separated by a quarter of a time cycle, and each vector plot by half a time cycle. Experimental flow patterns for the same type of configuration are shown in Figure 2.4. Experimental evidence would suggest that, after an initial period, usually only one strong pair of vortices would be shed per cycle, together with one weaker pair (the "residual vorticity"). In these plots two pairs of nearly equal strength are being shed per cycle (ignoring the first half cycle), a situation more reminiscent of shedding off rounded edges (see Chapter 2). However, this is quite feasible. Two pairs of nearly equal strength have been observed in experiment; also, in the present calculation it is possible that if the computation were extended a preferred shedding direction might eventually be established.

Both of the first two pairs which have been shed approach the body side due to the greater vortex strength in each case of the vortex nearer the body side. The first of the two pairs remains intact even by Figure 7.9h and produces substantial secondary separations (Figures 7.10c and d),
with the two clusters in the pair separating under the influence of their own images. On the other hand, the second vortex of the second of the two pairs is drawn back into the main vortex shedding pattern to form a part of the first vortex in the third pair. The rather intricate pairing behaviour is typical also of experimental observations.

RUN 2

Run 2 was carried out at a slightly lower Stokes number than Run 1 but with exactly the same mesh. The lower Stokes number is achieved through a lowering of the flow length scale. Thus we should expect smaller vortex structures to appear; the effect of the lower Stokes number itself should be minimal. Figures 7.11 and 7.12 contain plots of vortex position and velocity vectors for this case. Flow patterns are similar to those observed in Run 1, but differences appear noticeably at the end of $1\frac{1}{2}$ time cycles (Figures 7.11f and 7.12c). The second vortex to form in the first pair collides with the body so near the shedding edge that it immediately becomes drawn back into the shedding process and combines with a portion of the second part of the second pair to form a third weak pairing which convects away almost along the edge bisector. This pair is so weak that by the end of the second cycle (Figure 7.12d) its presence is scarcely noticeable, although from the position plot (Figure 7.11h) it might have appeared that its strength was quite considerable. However, the third pair is strong enough also to interfere with the next vortex pair to be shed by releasing some of its vorticity to be reformed into part of this pair (the pair to the left of the shedding edge in the above two figures). In Figure 7.11i yet another pair is seen forming, but the final figure (7.11j) might seem to suggest that both these last
two pairs have reformed into one larger pairing. Figure 7.12e confirms the presence of a strong vortex pair at the edge.

RUN 3
The flow results of Run 3 (Figures 7.13 and 7.14) are fairly similar to those of the previous runs despite the presence here of a small amount of edge rounding. The double shedding pattern is still observed; on this occasion the two vortex clusters in each pair are of more equal strength so that the pairs do not convect towards the body side but out into the field. Convection occurs at such a rate that by Figure 7.13g the first pair has reached the outward extent of the computational mesh. The general structure of the vortex clusters is very similar to those for the plain square edge - compare, for instance, Figures 7.13c and d with 7.9c and d.

RUN 4
The effect of a further increase in edge rounding may be seen in Figures 7.15 and 7.16 (Run 4). The nature of shedding of vorticity has begun to change. Due to the effects of edge rounding, vorticity is shed from the edge in small "packets" onto the main vortex core (Figures 7.15b and 7.15d, for example). This form of vortex structure was also noted in calculations performed with the HMVDM for uniform flow past rounded edges, and is also observed experimentally. Figure 2.8 shows flow visualisation pictures of the rolling barge at roughly the same values of \((r/L_z)\) and Stokes number. Photos 3, 7 and 8 show clearly the same types of shedding behaviour noted for Figures 7.15b and 7.15d. A rough correspondence between the computed flow results and the experimental
Considerable similarity between computation and experiment exists. The vortex pairing is much less well structured than for Run 3. The first pair to develop (Figures 7.15d and 7.16b) quickly collides with the wall, the right hand member being almost completely recombined into the next pair (Figure 7.15e), while the left hand member creates a secondary separation at the wall, vorticity from which is then convected away into the field (Figure 7.16c). Figures 7.15e and f, together with Figure 7.16c, show the second main pairing convecting away in two separate parts from the right of the shedding edge, whilst the last figures indicate another pairing convecting to the left and colliding with the wall. The seemingly confused state of Figure 7.15h is actually deceptive, as can be seen from the equivalent velocity vector plot.

**RUN 5**

Run 5 ($r/L_z = 0.0924$) has its approximate counterpart in the experimental flow visualisation pictures of Figure 2.7. A rough
correspondence between the figures might be as follows:

<table>
<thead>
<tr>
<th>Figure 7.17a</th>
<th>...</th>
<th>Photo 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot; 7.17b</td>
<td>...</td>
<td>&quot; 8</td>
</tr>
<tr>
<td>&quot; 7.17c</td>
<td>...</td>
<td>&quot; 2/3</td>
</tr>
<tr>
<td>&quot; 7.17d</td>
<td>...</td>
<td>&quot; 4</td>
</tr>
<tr>
<td>&quot; 7.17e</td>
<td>...</td>
<td>&quot; 5/6</td>
</tr>
<tr>
<td>&quot; 7.17f</td>
<td>...</td>
<td>&quot; 8/1</td>
</tr>
<tr>
<td>&quot; 7.17g</td>
<td>...</td>
<td>&quot; 2/3</td>
</tr>
<tr>
<td>&quot; 7.17h</td>
<td>...</td>
<td>&quot; 4</td>
</tr>
</tbody>
</table>

As is the case elsewhere, the location of the regions of concentration of vorticity should be ascertained with reference to plots of velocity vectors (Figures 7.18). Because of the large edge radius, vortex shedding is weak and less structured than at lower \((r/L_z)\). Two vortex pairs per cycle are shed, although the second vortex of a pair appears as little more than a thickening of the boundary layer - see for example Figure 7.17c.

RUNS 6 AND 7

Runs 6 and 7 (Figures 7.19 and 7.20) are both carried out at a lower Stokes number than the previous runs; the degree of edge rounding is the same as that for Run 4. Both figures present the first time cycle of flow. Run 6 is carried out on an identical mesh to that for Run 4 and has the obvious difference of thicker boundary layer. Run 7 is a test of sensitivity to mesh and time step size and is carried out on a
significantly larger, and therefore coarser, mesh than Run 6. Despite this, the forms of the shedding in both Run 6 and Run 7 are very similar.

### 7.3.5 Time dependent vortex force coefficient

Time dependent vortex force coefficient \( (C_{Fv}) \) is plotted against time in cycles \((t/T)\) in Figures 7.21 and 7.22.

Figure 7.21 shows a comparison between Runs 2 to 5, that is, between different values of \((r/L_z)\). The general form of all the force traces is similar. Increasing \((r/L_z)\) has the effect of reducing the force levels, but introducing into the traces "wiggles" of increasing relative size. The origin of the wiggles is thought to lie in the discretisation of the vortex sheet into point vortices and to be the unavoidable consequence of their occasional strong mutual interaction, especially near the surface. Such an event happens once every 100 time steps or so and has a negligible effect on force averages.

Force coefficient plots from Runs 4, 6 and 7 are plotted in Figure 7.22. A comparison between Runs 4 and 6 reveals the differences due to an almost tenfold reduction in Stokes number; a comparison between Runs 6 and 7 demonstrates the sensitivity of the HMVDM to changes in mesh and time step size. A more detailed discussion of these differences may be found in the next section. All the plots are of fairly similar appearance, the two fine mesh plots (Runs 4 and 6) showing much more detail. An instability in the computation can be seen starting to grow
at the end of the trace for Run 6. This prevented the run being extended further. Apart from this instability, the force trace for the higher Stokes number calculation shows a higher level of small-scale disturbances. It is not clear whether this is a genuine or purely numerical effect.

7.3.6 Drag and inertia coefficients

Figures 7.23, 7.25, and 7.27 present plots of drag coefficient for the isolated edge (\(C_D\) - referred to here and in the work of Graham, 1980, as the coefficient "A") against time for various computer runs. In fact it is the half-coefficient, \(A_\frac{1}{2}\), which is actually shown plotted at every half time cycle (Equation 7.3.3) and represents the integration of vortex force coefficient over the previous half-cycle multiplied by two to yield a coefficient compatible with a full-cycle coefficient. Figure 7.24 shows a plot of the half-coefficient, \(B_\frac{1}{2}\) (related to the inertia coefficient, \(C_m\)), which has been similarly derived.

The method of calculation of "vortex" force takes account not only of the component due to vortex shedding, but also of that due to skin friction. However, there are two pieces of evidence to suggest that the skin friction component is negligible. Firstly, a rough assessment of the skin friction drag, in the absence of flow separation, was made using oscillatory boundary layer theory (Section 7.1), and it was found to be less than 5% of the vortex drag in all cases. Second, comparison of force and drag coefficient data for runs at different Stokes number
reveals no significant trend due to lowering the Stokes number and hence increasing the relative skin friction contribution, although, since a lower Stokes number would also affect the pressure drag via changes in the separation point and amount of diffusion in the wake, it is possible that an actual significant increase in skin friction contribution has been partially obscured by a corresponding decrease in pressure drag. Also, in cases where the flow has developed significantly to cover much of the body, it becomes difficult to distinguish between drag due to vortex shedding and that due to skin friction.

**SQUARE EDGE**

Figure 7.23 shows the half-coefficient $A_{\frac{1}{2}}$ results from two sets of square edge computer runs (Runs 1 and 2). Run 2 is at a slightly lower Stokes number and was carried out to determine whether Run 1, which could not be extended to longer times because of mesh restrictions, had been continued long enough to provide an adequate assessment of the coefficient $A$ for fully developed flow, that is, away from starting conditions. Assuming the effect of changing Stokes number to be very slight indeed, it can be seen from Run 2 that drag appears to have settled down by the third half-cycle. A similar plot for the half-coefficient $B_{\frac{1}{2}}$ is presented in Figure 7.24. This coefficient is a most sensitive quantity and can be seen to vary quite markedly with time.

Average values of the coefficients $A$ and $B$, calculated as described in Section 7.3.3, are 1.398 and 0.017 for Run 2. The same results as
computed by the Discrete Vortex Method are 1.995 and 0.051. Most interest lies in $A$, which, if we are to take the HMVDM results as reliable, the DVM has overestimated by some 45%. This overestimation is considered quite normal. Experiment (see Section 4.2.4) gives $A$ as approximately 1.4, which would make the HMVDM result seem most respectable. The result for $B$ is bracketed by the DVM results of on the one hand the present work and on the other hand Graham (1980) who computed a value of -0.04.

SQUARE EDGE WITH ROUNding

Figure 7.25 shows the variation of half-coefficient $A_{1/2}$ with edge rounding over two time cycles. At higher edge non-dimensional radii drag appears to settle down more quickly. This is especially clear at $r/L_z = 0.0924$. Average values of $A$ were derived from the above figure using the method of Section 7.3.3, save that at $r/L_z = 0.0165$ only one data point was used, that at $t/T = 1.5$. At later times in the computation the drag predictions for this case were considered unreliable due to the proximity of the vortex pairs to the mesh boundary. Figure 7.26 shows the variation of these averaged values of coefficient $A$ with non-dimensional radius. This figure is the HMVDM counterpart of the DVM plot, Figure 4.22. A comparison of these two figures, again assuming that the HMVDM is reliable, shows that the DVM overestimates $A$ by an increasing percentage as $r/L_z$ is increased. This is in accordance with the problems encountered with the DVM at the higher edge radii and described in Section 4.3.2.
No graph of the variation of coefficient B with non-dimensional edge radius is presented since, except for the square edge, no discernible trend may be found in the computed values. In all cases they are very small and positive.

### 7.3.7 Effect of variation of Stokes number

The effect of a nearly ten-fold reduction in Stokes number, with all other computational variables held identical, can be seen in Figure 7.27a by a comparison of the half-coefficient $A_{\frac{1}{2}}$ for Runs 4 and 6. The onset of computational instabilities prevented Run 6 from being taken any further than 1 time cycle, yet on the two data points available for comparison the effect of this change in Stokes number would appear to be insignificant. Of course, one should expect larger differences for flows at very different Stokes number, especially when one enters the turbulent regime. It is beyond the scope of this work to consider the effects of turbulence; in laminar flow, a change in Stokes number might be expected to affect both skin friction and pressure drag, in the latter case via changes in the separation point and in the amount of diffusion and mixing in the wake.

### 7.3.8 Sensitivity to mesh density and time step size

It is necessary to establish the sensitivity of the HMVDM to changes in mesh density and time step size ($\Delta t$). As was reported earlier,
considerable testing of the HMVDM was carried out to verify that it was insensitive to such parameters. In this work the results of only one such test are presented. Runs 6 and 7 (Figure 7.27b) differ in that the mesh for Run 7 (Mesh B) is four times as coarse as that for Run 6 (Mesh C) and also in that time step for the former is six times as large. Considering the vast difference in the parameters, and the fact that the vortex shedding processes presented here are not in any case deterministic, the HMVDM seems fairly insensitive to mesh and time step refinement. Also, comparison may legitimately be made with the higher Stokes number run (4) for the reasons described in the previous section; such a comparison demonstrates that though especially at time $t/T = 1$ the coarse mesh predicts a higher drag level than the finer mesh this discrepancy has virtually disappeared at $t/T = 2$.

7.4 Application of the Hybrid Moving Vortex Diffusive Method Results to the Roll Damping of a Rectangular Cross-Section Barge with Rounded Bilges

The above described drag coefficient results for the isolated edge with edge rounding may be applied to the case of the rolling rectangular barge described in Chapter 5 by the matching technique incorporated into the computer program BMTIMP also described in that chapter. Graphs of roll damping coefficient calculated using the Discrete Vortex Method drag predictions may be found in Figures 5.2, 5.6 and 5.7 for the standard barge Cases 3S, 3 and 2 respectively. Exactly equivalent graphs using the HMVDM predictions are given in Figures 7.28, 7.29 and 7.30. In
all three cases the HMVDM damping predictions seem to perform well at low roll velocity amplitude, although there is very considerable scatter in the experimental data. At higher roll velocity amplitudes, where comparison is made with experimental forced roll results, the HMVDM predictions underestimate damping by up to 30% or so, whereas the DVM predictions significantly overestimate damping throughout the roll velocity amplitude range.

The possible sources of error in the DVM damping predictions and the experimental results have been outlined in Section 5.3. Any discrepancy between theory and experiment here will be due to inaccuracies either in the HMVDM, the matching process or the experiments themselves. The HMVDM isolated square edge drag result apparently compares so well with the square edge experiment (see Section 7.3.6) that the discrepancy between the theoretical and experimental roll dampings for the rectangular barge with square bilges (Case 3S) cannot be attributed except in small measure to inaccuracies in the HMVDM. The underestimation of experiment by theory in this case at the larger roll velocity amplitudes might point to a systematic error in the forced-roll experimental results which could explain the underestimation also present for Cases 3 and 2.
CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

8.1 Conclusions Concerning the Discrete Vortex Method

8.1.1 Isolated edge results

Conclusions concerning the Discrete Vortex Method may be drawn from the work at two different levels. First the predictions, especially the drag predictions, of the Discrete Vortex Method can be assessed directly by comparison with experimental results. Second the predictions of the BMTIMP computer program, which requires the DVM drag results as input, can also be assessed by comparison with experiment. This will in turn provide some insight into the reliability of the Discrete Vortex Method, but will also test other aspects of the roll damping prediction program such as the applicability of the matching technique. Conclusions concerning the BMTIMP computer program may be found in Section 8.3; this section discusses the conclusions which may be drawn directly relating to the Discrete Vortex Method results.

Plots of theoretical vortex positions in general follow well the trends shown in the experimental flow visualisation studies, especially for edges with keels, where all the essential features are present in both the shedding and convection phases. In the case of small amounts of bilge rounding, theoretical flow patterns correspond well to the type of flow visualised for a square edge, as we should expect. At higher radii,
however, the program, in its present form at least, is unable fully to simulate the double shedding pattern observed experimentally. Theoretical vortex sizes for flow round a square edge with varying keel span compare favourably with experimentally measured sizes given the uncertainty in the experimental results.

Calculated drag coefficients for the square edge and the flat plate are near Graham's calculated results (Graham, 1980). A well known feature of the Discrete Vortex Method is that it overestimates drag - see for example Sarpkaya (1975). This overestimation in the present case is about 30% for the flat plate and 45% for the square edge. The magnitude of these figures is in accordance with other workers' findings. The fact that drag results for the square edge with bilge keel are bounded on both sides, in the limit of vanishing keel span by the square edge and in the limit of long keel by the flat plate, by values which have been compared with experiment and those of other workers must give confidence that the error bound for the bilge keel results lies within these limits.

8.1.2 General hull cross-section results

The test computations performed using the DVM as developed for general hull sections indicated that despite the complexities of the geometry and the difficulties of implementing the representation of roll the method was performing well. Drag predictions for a rolling cylinder with bilge keels were within 16 to 24% of experiment.
The results of the computations for the FPV Sulisker were less easy to validate against experiment because of the fairly high level of uncertainty in the experimental dampings and the position of the roll centre. The DVM overestimates experimental total damping by between roughly 30 and 120%. These two figures represent approximate lower and upper bounds on the error, with all comparisons falling within these bounds. Given the fact that the DVM itself would be expected to overpredict damping, and also that the assumptions of strip theory are expected to lead to total dampings which are too high, the DVM predictions appear reasonable. Comparison with the semi-empirical method of Ikeda et al. (1978) was very poor, with the Ikeda method yielding an overall damping a factor of four lower than the DVM. The Ikeda method underpredicts experiment by roughly a factor of three. There appear to be weaknesses in Ikeda's method when employed for geometries substantially different from those against which it was originally validated. Comparison with the method of Odabasi et al. (1985) for the same case is good. One should expect a correctly implemented Discrete Vortex Method to yield more accurate answers than either of the other two methods because it would provide a better model of the physical flow processes. In fact, it is believed that this is the most complete theoretical model of vortex roll damping for ships yet to be constructed.

Discrete Vortex Method calculations were performed for the Sulisker with added bilge keels. Since the damping increment due to the keels was relatively small, comparisons either with the other damping prediction methods or with experiment for this case were inconclusive, although the
DVM predictions appeared at least feasible.

Damping computations with the DVM for the Sulisker have shown that there is a substantial damping increment due to the addition of a small bar keel. The method has also demonstrated the sensitivity of the damping to changes in the roll centre. It is anticipated that it will be of great value in determining the effect of even small changes in geometry or flow parameters.

Computations using the Discrete Vortex Method for other ships would provide further means of assessing the accuracy of its predictions. One particularly useful set of tests would involve on the one hand comparisons between computed and experimental overall vessel dampings, and on the other hand comparisons between computed sectional dampings and dampings derived from experiments involving equivalent two-dimensional sections. This would not only test the accuracy of the DVM itself, but also the implications of the use of strip theory.

One possible future development of the DVM is its extension to three dimensions. Although this would require a major research effort, it would be expected to yield more accurate roll damping data for three-dimensional vessels than the present strip approach. It would also enable the effects of forward speed to be investigated.
8.1.3 General conclusions

The above work has highlighted many interesting features of the Discrete Vortex Method as formulated here for both isolated edges and general ship hull cross-sections.

- The method is very economical even for many cycles of sinusoidal flow around intricate geometries.

- Sinusoidal flow, in that there is no mean convective velocity, and therefore any mean convection of the vortices is produced through the shedding and pairing of the vortices themselves, is among the most difficult types of flow for which the Discrete Vortex Method is used, and yet feasible and consistent results can still be achieved for a wide variety of complex geometries especially when the edge is sharp.

- A variety of different quantities can be deduced from the calculations without too much effort.

- As the edge becomes less "sharp", either by an increase in angle or by the application of edge rounding, the method becomes less able to cope for two distinct reasons:

  1) The separation point becomes more difficult to define and hence the Kutta condition becomes more difficult to satisfy in a meaningful way.
2) The subsequent convection of vortices becomes more dominated by viscous effects. Experiments indicate that for the less "sharp" edges the vortex pairs do not convect out into the flow but convect along the edge. Also diffusion takes place rapidly so that the convection velocities reduce quickly. These effects cannot be modelled adequately by the Discrete Vortex Method.

Because of the above deficiencies in the method adequate results cannot be provided by the Discrete Vortex Method employed in oscillatory flow around significantly rounded edges. In such circumstances a full Navier-Stokes solution of the flow provides the most realistic alternative.

8.2 Conclusions Concerning the Hybrid Moving Vortex Diffusive Method

The HMVDM provides a model of the unsteady two-dimensional Navier-Stokes equations, excluding the effects of turbulence. How good a model of vortex shedding this then makes depends partly on what Reynolds number is being considered, partly on what the effects of three-dimensionalities in the real flows are likely to be and partly on the adequacy of the finite difference approximations inherent in the method.

The quality of results from test computations including the oscillatory boundary layer case was encouragingly good. The HMVDM was then applied
to vortex shedding from an isolated square edge with rounding. Plots of vortex positions and velocity vectors demonstrated the ability of the method to predict fundamental flow processes correctly. Even small details of the shedding structure which were noted in experimental flow visualisations were also apparent in the theoretical predictions. Drag results displayed virtual insensitivity to mesh refinement, time step size or Stokes number within reasonable limits. The one direct comparison with an experimental drag result which was possible - that for the plain square edge - was excellent, the two results differing by just 0.14%, although there is a degree of uncertainty in experiment. Comparison with the DVM drag result for the plain square edge showed that the DVM result exceeded the HMVDM by about 45%, which would be expected considering the limitations of the DVM. There was worsening agreement at higher edge radii, due presumably to inadequacies in the DVM model. In the limit of infinite edge radius, we retrieve effectively the problem of the oscillatory boundary layer, which was predicted very well by the HMVDM, the maximum value of skin friction force being overestimated by 4% compared with the analytical.

The HMVDM drag results have applicability, via matching processes, not only to the prediction of the roll damping due to vortex shedding of barges with rounded bilges, but also to other related problems such as the prediction of damping from a square cylinder with corner rounding.

The hybrid nature of the HMVDM appears to encompass the best features of both mesh and particle methods. On the one hand, the plots of velocity vectors formed on the mesh indicate an entirely satisfactory mesh
representation, whilst on the other hand the plots of vortex positions provide an extremely detailed picture of the vortex shedding processes. This method would provide an ideal means of analysing in a detailed way the processes of vortex shedding and making more detailed comparisons with experiment than was feasible here. Analysis would not need to be confined to oscillatory flow around isolated edges. It is considered that the method is capable of extension to other geometries and flow configurations.

A particular area where more work would be valuable is in extending the HMVDM to cope with complete ship cross-sections in the same way that the DVM was extended. This would be a major exercise, but would provide a useful tool for the assessment of the damping contribution from sections of ships where no geometrically definable shedding point existed.

One limitation with the present HMVDM calculations was that it was not possible to continue them for much more than two cycles of oscillation because of the excessive computational expense involved. Although the accuracy of the results was not believed to have been seriously impaired by this, an important area of future work would be to devise ways of continuing the calculations much longer, in order to assess long term computational trends.

8.3 Conclusions Concerning the BMTIMP Computer Program

Comparisons with experiment were made for a rectangular cross-section
barge with square bilges using both the DVM and the HMVDM drag results as input to BMTIMP. In general, experimental roll damping values were bounded by on the one hand the DVM prediction and on the other the HMVDM. Since we can lay considerable confidence in the HMVDM drag prediction, this might suggest either that there is a small systematic error in experiment or that the matching process in BMTIMP has caused damping to be slightly underpredicted.

The predictions using the HMVDM for barges with rounded bilges underestimate experiment by up to 30% at the higher roll amplitudes. Predictions at lower roll amplitudes are often well within the scatter of experimental points. This again might suggest some doubt about the accuracy either of BMTIMP or the experimental results themselves.

There are no applicable experimental data available for the rectangular barge with bilge keels, but the experiments by Ikeda, Komatsu, Himeno and Tanaka (1977) for barges with bilge rounding and bilge keels compare extremely well with the BMTIMP theoretical predictions using drag values predicted by the DVM.

Use of the modified BMTIMP program in forced-roll tests in conjunction with Discrete Vortex Method and Hybrid Moving Vortex Diffusive Method isolated edge drag predictions has yielded the following important findings.

- The modified version of BMTIMP is capable of the prediction, to useful accuracy, of the roll damping of barges of rectangular cross-section
with a variety of bilge geometries.

- Even very small bilge keels produce a significant damping increment over the square edge or rounded bilge. Particularly noteworthy is the increase in drag if a "bounded" keel (i.e. of span 0.414 r) is added to a rounded bilge.

- The trend of the damping versus roll amplitude curve for both the edge with keel and the rounded bilge is that the damping at low roll amplitude diverges from that for the square bilge. At the higher amplitudes, however, the two curves run parallel.

Further experimental evidence would be useful in establishing the validity of the current modifications in matching to the BMTIMP barge rolling program and to fix the domain within which the assumptions underlying the modifications may be considered to be valid. This experimentation could include forced-roll tests on a rectangular cross-section barge with different sizes of bilge keel and different degrees of bilge rounding. The experiments could also include tests on a freely floating barge in regular beam waves, since the program has also been modified to cope with this condition.
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Bilge keel spans tested (mm)

0
1 (chamfered)
2
6
13
25
76

Bilge radii tested (mm)

square edge

13
25

Figure 2.1 Details of the model barge.
Figure 2.2 Schematic representation of experimental apparatus.
Figure 2.3 Flow visualization of vortex shedding from a rolling barge with rounded edges and bilge keels. Bilge radius, $r = 13 \text{ mm}$; keel span, $a = 13 \text{ mm}$; roll angular frequency, $\omega = 1.5 \text{ rad/sec}$; and roll amplitude, $\bar{\theta}_\infty = 11.4 \text{ degrees}$. 
Figure 2.4  Visualization of flow round a rolling barge at successive stages through one cycle of vortex shedding, using tracer particles. Barge with square edges; roll angular frequency, $\omega = 4.0$ rad/sec; and roll amplitude, $\hat{\theta} = 9.9$ degrees.
Figure 2.5 Visualization of flow round a rolling barge at successive stages through one cycle of vortex shedding, using dye. Barge with square edges and bilge keels; keel span, $a = 25$ mm; roll angular frequency, $\omega = 6.0$ rad/sec; and roll amplitude, $\hat{n} = 5.7$ degrees.
Figure 2.6 Visualization of flow round a rolling barge at successive stages through one cycle of vortex shedding, using tracer particles. Barge with square edges and bilge keels; keel span, $a = 76$ mm; roll angular frequency, $\omega = 4.0$ rad/sec; and roll amplitude, $\tilde{\phi} = 5.7$ degrees.
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Figure 2.8 Visualization of flow round a rolling barge at successive stages through one cycle of vortex shedding, using dye.
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Beginning of time cycle

Half way through time cycle

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O experiment; --- theory
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- ■ square edge with keel;
- □ rounded edge with bounded keel ($a/r = \sqrt{2} - 1$).
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- O square edge with keel;
- □ rounded edge with bounded keel ($a/r = \sqrt{2} - 1$).
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(VORTEX LENGTH SCALE) × 0.5
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\( \eta_1 = \text{surge}, \quad \eta_2 = \text{sway}, \quad \eta_3 = \text{heave} \)

\( \eta_4 = \text{roll}, \quad \eta_5 = \text{pitch}, \quad \eta_6 = \text{yaw} \)

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Full scale keel spans:  
- $\circ$ 2m;  
- $\triangle$ 1m;  
- $\Delta$ 0.5m;  
- $\diamond$ 0.2m;  
- $\triangledown$ 0.1m;  
- $\times$ no keel - equivalent to Case 35.
Figure 5.5 Percentage gain in vortex damping over square edge case for keels of different spans, as a function of roll velocity amplitude (model scale).

Full scale keel spans: ○ 2m; □ 1m; △ 0.5m; ▽ 0.2m, ▽ 0.1m.
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(beam 0.28m, draft 0.112m, keel span $a = 0.5$ cm, bilge radius $r = 1$ cm).
VORTEX DAMPING COEFFICIENT \( B_v \) (Nm/Rad/Sec)

\[ \eta_{14} = 0.227 \text{ Rad} \]

Figure 5.9 Variation of \( B_v \) with roll angular frequency \( \omega \) for Ikeda's barge model C; (beam 0.28m, draft 0.112m, keel span \( a = 1 \text{ cm} \), bilge radius \( r = 2 \text{ cm} \)).
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Typical velocity profile near the wall.

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Figure 6.5 Boundary conditions on stream function.
Figure 6.6 The velocity interpolation scheme.

\[ A_T = A_1 + A_2 + A_3 + A_4 \]

Figure 6.7 Redistribution of point vorticity onto the mesh system.

\[ A_T = A_1 + A_2 + A_3 + A_4 \]
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Oscillatory boundary layer over a flat plate: detail of velocity distribution at one x-station at the end of the second time cycle.

Figure 7.1c
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Figure 7.10a
\( t'/T = 0.5 \)

Figure 7.10b
\( t'/T = 1.0 \)
Figure 7.10c
$t'/T = 1.5$

Figure 7.10d
$t'/T = 2.0$

Figures 7.10 (cont'd)
Figures 7.11 Vortex positions at successive stages through two and a half time cycles. Run 2; Square edge.
Figures 7.11 (cont’d)

Figure 7.11e
$t'/T = 1.25$

Figure 7.11f
$t'/T = 1.50$

Figure 7.11g
$t'/T = 1.75$

Figure 7.11h
$t'/T = 2.00$
Figures 7.11 (cont'd)
Figures 7.12 Velocity vectors at successive stages through two and a half time cycles. Run 2; Square edge.
Figures 7.12 (cont'd)

Figure 7.12c
$t'/T = 1.5$

Figure 7.12d
$t'/T = 2.0$

Figures 7.12 (cont'd)
Figure 7.12e
$t'/T = 2.5$

Figures 7.12 (cont'd)
Figures 7.13 Vortex positions at successive stages through one and three quarter time cycles. Run 3; non-dimensional edge radius, $r/l_z = 0.0165$. 

- Figure 7.13a
  $t'/T = 0.25$

- Figure 7.13b
  $t'/T = 0.50$

- Figure 7.13c
  $t'/T = 0.75$

- Figure 7.13d
  $t'/T = 1.00$
Figure 7.13e
$t'/T = 1.25$

Figure 7.13f
$t'/T = 1.50$

Figure 7.13g
$t'/T = 1.75$

Figures 7.13 (cont'd)
Figures 7.14 Velocity vectors at successive stages through one and three quarter time cycles. Run 3; non-dimensional edge radius, $r/L_z = 0.0165$. 
Figure 7.14c
$t''/T = 1.5$

Figures 7.14 (cont'd)
Figures 7.15 Vortex positions at successive stages through two time cycles. Run 4; non-dimensional edge radius, $r/L_z = 0.0466$. 
Figures 7.15 (cont'd)

Figure 7.15e
$t'/T = 1.25$

Figure 7.15f
$t'/T = 1.50$

Figure 7.15g
$t'/T = 1.75$

Figure 7.15h
$t'/T = 2.00$
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Figure 7.16a
$t'/T = 0.5$

Figure 7.16b
$t'/T = 1.0$
Figures 7.16 (cont'd)

Figure 7.16c
$t'/T = 1.5$

Figure 7.16d
$t'/T = 2.0$
Figures 7.17a  
$t'/T = 0.25$

Figures 7.17b  
$t'/T = 0.50$

Figures 7.17c  
$t'/T = 0.75$

Figures 7.17d  
$t'/T = 1.00$

Figures 7.17  Vortex positions at successive stages through two time cycles. 
Run 5; non-dimensional edge radius, $r/L_z = 0.0924$. 
Figure 7.17e
$t'/T = 1.25$

Figure 7.17f
$t'/T = 1.50$

Figure 7.17g
$t'/T = 1.75$

Figure 7.17h
$t'/T = 2.00$

Figures 7.17 (cont'd)
Figures 7.18  Velocity vectors at successive stages through two time cycles. Run 5; non-dimensional edge radius, $r/L_z = 0.0924$. 

Figure 7.18a  
$t'/T = 0.5$

Figure 7.18b  
$t'/T = 1.0$
Figure 7.18c
$t' / T = 1.5$

Figure 7.18d
$t' / T = 2.0$

Figures 7.18 (cont'd)
Figures 7.19 Vortex positions at successive stages through one time cycle. Run 6; non-dimensional edge radius, $r/L_z = 0.0466$. 

Figure 7.19a  
$t'/T = 0.25$

Figure 7.19b  
$t'/T = 0.50$

Figure 7.19c  
$t'/T = 0.75$

Figure 7.19d  
$t'/T = 1.00$
Figures 7.20 Vortex positions at successive stages through one time cycle. Run 7; non-dimensional edge radius, $r/L_z = 0.0466$. 

Figure 7.20a $t'/T = 0.25$

Figure 7.20b $t'/T = 0.50$

Figure 7.20c $t'/T = 0.75$

Figure 7.20d $t'/T = 1.00$
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Figure 7.22 Traces of time dependent vortex force coefficient \( (C_{FV}) \) against time in cycles \((t/T)\) for different Stokes numbers, meshes and time step sizes.

Run 4, \( St = 36700 \), mesh C, \( \Delta t = 0.00833 \)

Run 6, \( St = 4600 \), mesh C, \( \Delta t = 0.00833 \)

Run 7, \( St = 4600 \), mesh B, \( \Delta t = 0.05000 \)
Figure 7.23  Half-coefficient, $A_\frac{1}{2}$, versus time in cycles (t/T). Comparison between Runs 1 and 2.

Figure 7.24  Half-coefficient, $B_\frac{1}{2}$, versus time in cycles (t/T). Comparison between Runs 1 and 2.
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Figure 7.30 Variation of the roll damping coefficient $B_{44}$ with roll velocity amplitude $\omega_{14}^\wedge$ for barge Case 2 (model scale values). Theory calculated using the Hybrid Moving Vortex Diffusive Method.