Optimal Sensor Placement for Maximum Area Coverage (MAC) for Damage Localization in Composite Structures

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Abstract

In this paper an optimal sensor placement algorithm for attaining the Maximum Area Coverage (MAC) within a sensor network is presented. The proposed novel approach takes into account physical properties of Lamb wave propagation (attenuation profile, direction dependant group velocity due to material anisotropy) and geometrical complexities (boundary reflections, presence of openings) of the structure. A feature of the proposed optimization approach lies in the fact that it is independent of characteristics of the damage detection algorithm (e.g. probability of detection) making it readily up-scalable to large complex composite structures such as aircraft stiffened composite panel. The proposed fitness function (MAC) is independent of damage parameters (type, severity, location). Statistical analysis carried out shows that the proposed optimum sensor network with MAC results in high probability of damage localization. Genetic algorithm is coupled with the fitness function to provide an efficient optimization strategy.

Introduction

The recent advances in the SHM techniques have been motivated by utilising composite materials to their full potential resulting in lighter structures which currently is not achievable due to high demands in their damage tolerance design [1-5]. Their safety can be maintained by progressing from Scheduled Based Maintenance (SBM) towards Condition Based Maintenance (CBM) leading to significant cost reductions. Depending on the SHM system and its application, different techniques can be adopted for monitoring the integrity of the structure. [6-9].

For aeronautical applications, the decision to have a permanently installed SHM system for structural prognosis will be driven by its reliability, cost and the added weight of the system. The technological developments of piezoelectric transducers, PZTs, gives the possibility to develop SHM systems for damage detection in composite panels without effecting the performance of the structure [10, 11]. An important feature of PZTs is their electro-mechanical coupling which makes them particularly suitable as sensors and actuators in both passive [12, 13] and active sensing[14, 15]. Lamb-waves have been extensively studied for damage detection in recent years due to their sensitivity to different types of damage and their superior propagation properties such as low attenuation and dispersion [2, 3]. The basic feature which is usually used for monitoring the state of a component is the damage scattered signal. Once the scattered signal is detected, different signal processing techniques can be applied for detecting damage in the structure. A review of different damage detection approaches is given in [16]. Among guided wave techniques, the delay and sum imaging approach has proven to be effective in detecting Barely Visible Impact Damage, BVID [17-21]. A challenge for guided wave based methods, in particular for composite structures, is in separating mode-superposition and reflection of waves from boundaries and those emanating from damage, see e.g. [22]. This highlights the need for determining the ideal actuation signal and optimal sensor placement, particularly in complex structures such as aircraft composite panels with stiffeners, frames, rivet holes and large man-holes.

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Optimal placement of sensors/actuators in order to detect, with high probability and reliability, any damage prior to it becoming critical is a key factor in uptake of any SHM system. The interference of the sensor system with the design of composite stiffened panel is required to be minimum. On the other hand the SHM system must be able to detect various probable damage scenarios with high reliability. Therefore, optimisation analysis needs to be carried out to find the best sensor layup (number and location). The position of sensors can have a significant influence on the values of the damage index (DI) [7]. The application of guided wave techniques for damage detection is well established for simple plates or pipes [18-21, 23]. For aeronautical applications, the focus is given to composite plate-like structures with complex geometries where the optimal sensor configuration may not be so trivial. A more rigorous study is required in order to determine the best sensor configuration for these applications.

Lee and Staszewski [24] studied the optimal sensor placement in an aluminium panel by maximizing the interaction of Lamb waves with a defect. This was done through wave interaction simulation to position sensors at the locations where the largest increase in amplitude is observed when the plate is damaged. Consequently, the optimal sensor positions are in the proximity of defect and do not cover all possible damage locations.

Worden and Burrows [25] compared different optimization approaches for fault detection in a rectangular plate. The objective function used in the optimization was selected for a fault detection procedure based on Artificial Neural Network (ANN). A similar approach based on genetic algorithm (GA), was proposed for passive sensing [26]. The proposed optimisation algorithms for passive sensing maximize a fitness function which is based on probability of detection (POD) of the proposed impact detection method [27]. The main difference between the optimisation procedure for passive and active SHM systems is the evaluation of the fitness function to be optimized. In a study by Croxford et al. [28] the effect of pattern of sensor layout (i.e. triangle, rectangle, trapezoid) on defect detection capabilities of guided waves in an aluminium plate were investigated through quantitative and qualitative study of the performance of multiple sensor pairs. They have concluded that there is no single optimum sensor layout and that is dependent on the type of defect to be detected. However, for most defect scenarios investigated in their study, a square or hexagon configuration provided a close to optimum performance. Similarly, Malinowski et al. have investigated the influence of transducer configuration pattern in two subsequent works [29, 30] where both triangular and concentrated configurations were examined. The triangular configuration coupled the two approaches of phased array and distributed sensors to successfully detect magnets on an aluminium plate when they were placed inside and outside of the network. In [30], various transducer configurations and spacing for phased array arrangement were investigated and the best results were obtained for circle, square and strip configurations.

In [31] an optimization approach for damage detection was proposed with the objective function derived using ANN. This approach would require a significant number of damage scenarios to be used in the training of the network. Guo et al. presented an improved GA for optimal sensors placement of a metallic truss structure [32]. Flynn and Todd [33, 34] proposed an optimization procedure which minimizes Bayes risk. The methodology was based on a statistical model of the active sensing process based on guided waves. In that study, the signal properties of the waves were derived taking into account the efficiency of the actuating-sensing system, the propagation path and scattering envelope due to a defect present in a metallic plate. The optimisation approach was demonstrated by adding magnets in various locations to simulate damage.

In a more recent study, Fendzi et al [35] proposed an optimal sensor placement algorithm based on GA where the objective function is independent of the damage detection algorithm and is based on the geometry of the sensors and the POD function. The POD function was constructed from the energy scattered profile of Lamb waves based on ray tracing approach where only two parameters are required: Lamb wave group velocity and their spatial attenuation. Although the proposed approach significantly reduces the computational restrictions in calculating the POD function and is applicable to composite structures, it does not take into account
geometric complexities such as large openings or the presence of stiffeners or frames. Moreover, it results in the best sensor location for a pre-defined damage location.

All of the above mentioned methodologies require an objective function which merely relies on the performance of the damage detection methodology, i.e. POD. This means that a vast number of damage scenarios (various locations and severities) have to be analysed for each structure under investigation in order to determine the optimal sensor configuration. To generate this required data (numerically or experimentally) is too expensive, if not impossible. Moreover, for damage detection in metallic structure the development of POD function and reliability assessment are quite straightforward through receiver operating characteristic (ROC) curves [36]. This is not the case for composite structures and to this date there are no general established methods for evaluating the POD function for damage detection and characterization. Therefore a practical optimization technique is required which does not rely on POD function directly as input and hence will not vary with every change made to the damage detection algorithm.

The aim of this paper is to propose a sensor placement optimization approach for guided wave fault detection and localization techniques based on Maximum Area Coverage (MAC) within a sensor network. The advantage of this approach is that it is independent of the details of the damage detection algorithm and does not require determination of a POD function for a vast number of damage scenarios. Moreover, it can be applied to geometrically complex structures with pitch-catch sensor configuration and any active sensing procedure based on time of flight of damage reflected waves.

In order to demonstrate the suitability of the proposed objective function based on MAC, experimental verifications on a flat composite panel, in which BVID was introduced, are conducted. After demonstrating the validity of the proposed function, a statistical analysis, based on numerical simulation is performed. Once it is demonstrated that the sensor network with the minimum fitness function also provides the highest accuracy in damage detection, an optimization procedure for sensor placement is developed and demonstrated for a plate with an opening. The paper is organised as follows: first a brief overview of the damage detection method is given in section 1 followed by determination of the fitness function, which evaluates the network performance in section 2. An experimental verification is provided in section 3 and a statistical analysis is performed in section 4. Finally a genetic algorithm based optimization is introduced in section 5 and the optimal sensor placement for a plate with an opening is determined in section 5.

1. Active sensing procedure

Different damage identification techniques based on guided waves can be applied to structures [37-39]. Depending on the transducer configuration they can be used in pulse-echo or pitch catch [40] mode. The focus of this work is the optimal transducer configuration in composite structures for the pitch-catch approach, in which the sensors are placed in a network pattern [19, 39, 41] and the structure is interrogated using a round robin approach [42]. The basic assumption for damage detection with guided wave is that the presence of damage in the structure causes reflection and refraction of the wave. By comparing the energy transferred in a pristine structure and a damaged structure for all the transducer paths, the fault can be detected and characterized based on a damage index, with the highest value showing the most probable damage location. An imaging algorithm can then be used to plot the probability map of the DI for every point (represented by pixels) based on the Time of Flight (ToF) of the propagating wave [43]. Most of the detection techniques assume that the damage reflection is the first residual peak in any given sensor-actuator path which means that the damage should be present inside the transducer network for it to be correctly located (pitch catch configuration). Therefore, the first criterion for the proposed optimal sensor placement strategy is to maximize the area covered within the sensor network.

Consider the transducer path shown in Figure 1. The DI at pixel D (possible damage location) is evaluated from the ToF of the residual signal propagating from actuator A to sensor S. Knowing the group velocity of
the propagating wave and the distances between the pixel D, actuator A and sensor S, the possibility of a damage existing in that location can be evaluated by the DI calculated from the residual signal at time $t_{ADS}$. It needs to be pointed out here that the “probability” in this context merely refers to a measure of possibility of damage existing in a location based on the magnitude of DI and it is not measured in a probabilistic sense. The time of flight is calculated from

$$t_{ADS} = t_{off} + \frac{d_{A-D}}{v_{A-D}} + \frac{d_{D-S}}{v_{D-S}}$$

(1)

where $t_{off}$ represents the time lag of the interrogation system and $v_{A-D}$ and $v_{D-S}$ denote the corresponding wave velocities. Let us assume that the damage is located in pixel D. In this case, based on the calculated ToF of the damage scattered wave, the possible damage location is not a single point but rather all the points resting on an ellipse (sharing the same ToF) having the actuator and sensor at its foci, see Figure 2(a). Therefore each actuator-sensor path results in a probability distribution based on the measured DI. The imaging approach exploits every actuator-sensor pair to evaluate different damage indices at every pixel. All the images are subsequently fused together in order to improve the reliability of the investigation. This was shown for both isotropic [18] as well as quasi-isotropic material [44] by considering the directional velocity of the propagating wave.

The intrinsic assumption here is that when the point of interest (for example pixel D) coincides with the damage location, the damage index evaluated at that location is noticeably higher than the other pristine points in the structure (see [45]). Therefore, by defining a damage index based on selected features of the damage reflected wave, the location of the scatter source can be detected. The main challenge of the above approach is to distinguish between the damage scattered wave and the extra sources of reflection such as the boundary, in particular for higher frequencies where due to the high velocity of the propagating waves, superposition of different modes and wave reflections occur simultaneously. Several methods have been proposed to account for the negative effects of boundary reflections and mode superposition [22, 46, 47]. However, their applicability has been demonstrated on either aluminium or composite plates with simple geometry.

Figure 2 shows two different locations for the transducer path A-S with respect to the boundary. In both cases, the ellipse represents all the possible damage locations based on the ToF of the damage scattered wave from point D1. In the first scenario, Figure 2(a), the ellipse is fully included inside the boundaries of the structure. This means that the scattered signal at $t_{AD1S}$ or $t_{AD2S}$ is not affected by boundary reflections. In the second scenario, Figure 2(b), part of the ellipse is outside the plate boundaries which implies that at the time $t_{AD1S}$, the scattered signal has been already corrupted by boundary reflections. When the damage reflected and boundary reflected waves are superimposed, it is very difficult to distinguish between the two, in particular when mode conversion occurs.
Figure 2: Example of ellipses. a) Pixels completely included in the boundaries. b) Pixels partially included in the boundaries.

Note that although Figure 2 represents an isotropic plate with constant group velocity, the concept is also applicable to anisotropic plates where the directional velocity of the wave can be used in constructing different ellipses for all possible damage locations [47].

Consequently, a constraint in the proposed optimisation algorithm is to reduce the amount of undesirable reflections to a minimum. A simplistic approach would consist of placing the transducers far from the boundaries. However, for the pitch-catch configuration, for a damage to be detected with high reliability it requires to be placed inside the sensor network [18, 48] and placing transducers too far from the boundaries will reduce this coverage area. Therefore it is necessary to determine a trade-off between the maximum area included within the transducer network and minimizing the boundary reflections.

In order to propose a strategy for optimal sensor configuration for damage localization, the first step is to assess the influence of the sensor positioning on the overall performance of the damage detection methodology. Therefore, a fitness function, capable of estimating the performance of an SHM system for different sensor configurations, must be defined.

2. Fitness function

The aim of the proposed optimisation approach in this work is to find optimal sensor placements for damage localization in composite structures. This strategy is built on the pitch-catch mode guided wave damage localization procedure presented in the previous section. The fitness function is introduced as an indicative measure of probability of detecting damage at any point in the structure. This value is based on maximizing the coverage area within the sensor network while reducing the negative effects of the boundary reflections. The proposed fitness function is geometrical and physical based. In this section the key factors used in construction of the fitness function are introduced. To introduce a coverage map of the structure, the geometry is divided into pixels where the fitness function is evaluated at the centre of each pixel. Increasing the number of pixels will increase the resolution of the fitness function. For every pixel, each transducer path resulting in constructing an ellipse passing through its centre will contribute to the fitness function measure at that point, i.e. the contribution equals to one. This will then be carried out for each pixel and summed for each transducer path. The higher the number of ellipses passing through a single point, the higher the probability of damage being detected at that given location. Given a total of $N$ transducers, the number of ellipses crossing a single pixel is given by $L = N(N - 1)$.

The fitness function to be maximized can then be built on the basis of the coverage area. However, without any physical and geometrical constraints the definition of this fitness function is not realistic as theoretically for any transducer path and any pixel in the structure, an ellipse can be constructed which passes through that point and the fitness function will be uniform over the whole structure.

2.1 Geometrical constraints

The first geometrical constraint influencing the fitness function is the direct path. Some damage detection algorithms (such as RAPID [49]) have a higher detectability of damage when it is located in the direct path of
any transducer pair. Therefore, a weighting function has been built into the fitness function to increase the coverage value for the points coinciding with the direct path of each transducer pair. This weighting function is a step function having the value 1 on the direct path and zero elsewhere. The choice of activating the weighting function for the direct path is up to the user, depending on the detection algorithm favouring the direct path.

The second geometrical constraint which was introduced is to minimize the boundary reflections. The boundary can either be the physical boundaries of the structure or parts such as stiffeners or frames. In order to determine if a transducer path should have a positive contribution to the overall fitness function or not, a threshold is defined for the length of the ellipse which falls outside the boundary. This threshold is representative of how close the transducers are to the boundary of the plate, openings or any complex geometrical features. If the length of the ellipse inside the boundaries, \( l_{ins} \), is above a certain threshold defined by a percentage of its total length \( \gamma * ell_{tot} \), the ellipse will be included in the fitness function evaluation \((\text{flag} = 1)\), otherwise it will be excluded \((\text{flag} = 0)\), i.e.

\[
\text{flag}(pix, path) = \begin{cases} 
1 & \text{if } ell_{ins} \geq \gamma * ell_{tot} \\
0 & \text{if } ell_{ins} < \gamma * ell_{tot} 
\end{cases}
\]

(2)

In the above approach, every actuator-sensor pair produces a binary image with one representing pixels at which the pair produces a good coverage (above the set threshold) and zero otherwise. Examples shown in Figure 3 illustrate a poor coverage (Figure 4a), in comparison to a better coverage (Figure 3b). The global coverage of the network is obtained by summing all the binary images produced by each transducer path, Equation (3).

\[
cov(pix) = \sum_{path=1}^{L} \text{flag}(pix, path)
\]

(3)

Figure 3: Examples of binary images. Black pixels represent acceptable ellipses

Figure 4 shows coverage areas of two different plates having 4 transducers. It can be observed that the maximum coverage is obtained inside the sensor network, while this coverage decreases as the pixels are placed further from the transducer network. In addition the choice of the boundary reflection coefficient, \( \gamma \), can change the coverage area. In Figure 4 (a) and (b) the coverage area of an L-plate is compared to a rectangular plate. To show the influence of the boundary reflection coefficient on the coverage area different values were tested. In this case the fitness function was defined to accept all the paths crossing a pixel where 75\% of its length falls inside the geometry of the structure. This factor reduced the boundary reflection and can be tuned by the user depending on how well the developed damage detection algorithm can deal with the effect of boundary reflections. Consequently, Figure 4(c) and (d) present the coverage area for the conservative case where the boundary reflections are fully eliminated \((\gamma * ell_{tot} = 1)\). It is clear that eliminating ellipses which fall outside of the boundary will reduce the coverage area. It should be pointed out that reducing the boundary reflections and its extent is an option in the fitness function evaluation and can be chosen by the user depending on the damage localization algorithm.
The detection method utilised is based upon ToF of the damage scattered wave. When an opening (for example a bolt hole) is located in the direct path of a pair of transducers, the wave propagation in the direct path is interrupted by reflections and refractions from the boundary of the opening which will increase the complexity of identifying the damage reflected wave in the close proximity of that path. Therefore the proposed fitness function is defined such that there will be no positive contribution from placing transducers at locations where the direct path goes through the opening. Consequently, pixels in the direct path going through an opening are assigned a value of $\text{flag}(\text{pix}, \text{path}) = 0$. Figure 5(a) shows an example of a square plate with a hole. There are three situations which can occur for any possible damage location $D$ in the structure. The ellipse, with foci $A$ and $S$, passing through $D$ has no intersections with the boundaries of the system. However the path between actuator and sensor is interrupted by the presence of a hole. In this case the reflections due to the hole will certainly affect the scattered signal at $t_{ADS}$.

In the case of an opening, there can situations where an ellipse is mostly included in the structure, but the following paths can be interrupted by the presence of a hole:

1. the direct path between actuator and sensor, Figure 5 (a);
2. the path between the actuator and pixel, Figure 6 (a);
3. the path between the pixel and the sensor, Figure 6 (b).

Figure 4: Coverage example for 2 different geometries and boundary reflection coefficient

(a) L- plate with coefficient $\gamma * ell_{tot} = 0.75$

(b) Rectangular plate with coefficient $\gamma * ell_{tot} = 0.75$

(c) L- plate with coefficient $\gamma * ell_{tot} = 1$

(d) Rectangular plate with coefficient $\gamma * ell_{tot} = 1$
In all three cases the propagation path is not a direct line anymore as it is interrupted by an opening. Hence the ToF of the incident wave cannot be evaluated based on the direct distance between the transducers and the pixel as it will result in a wrong damage location.

Therefore, if an opening is present in any of the three paths there will be no positive contribution from that transducer pair in the evaluation of the fitness function for that pixel.

Similar restrictions apply when dealing with complex geometries, for example an L-shaped plate where the transducers are placed in different flanges, Figure 5(b). Therefore in all these scenarios the fitness function is established to exclude the paths which are crossing boundaries of the structure or openings. This forces to position the transducers where there is a direct line of sight between them.

2.2 Physical constraints

So far, the fitness function has been based on geometrical constraints. However, the physical properties of the wave propagation in composite plates also influence the optimal sensor placement. One such factor is the group velocity. The triangulation method for damage detection is directly related to correct evaluation of ToA based on the group velocity of the reflected wave. In isotropic and quasi-isotropic composite plates the velocity of the wave is the same in all directions. However, in anisotropic composite structures, the directionality of the wave propagation in damage detection must be considered. Hence, the proposed fitness function also takes into account the directionality of the wave propagating for anisotropic structures when constructing ellipses for each pixel. The ellipses drawn in Figure 7 illustrate the possible damage locations for two different transducer pairs in an anisotropic plate [47] for a ToA measured for pixel X.

Another physical constraint which affects the wave propagation and therefore has an influence on the defined fitness function is the wave attenuation. The minimum detectable damage size is related to the actuation frequency through the wavelength of the guided wave. In addition, the relationship between the excitation frequency and attenuation of Lamb waves is well known [3], i.e. the energy of Lamb wave dissipates with distance. Different frequencies have different loss of power per unit distance therefore the coverage area of the propagating wave should also be related to the actuation frequency. An attenuation factors is introduced in the evaluation of the proposed fitness function. The first step is to obtain the attenuation profile of the structure either experimentally or with a validated numerical model. Each actuation frequency will have a different exponential trendline.
Clearly the optimal sensor placement cannot be different based on the actuation frequency. However, for any SHM system, based on several key factors such as minimum detectable damage size, the damage detection algorithm, type of damage to be detected (through thinness, surface), the resonance frequency of the sensorized structure and the hardware setup an optimum frequency range will be identified apriori. For thin composite plates this range is usually between 50 to 500 kHz. For this working range the worst attenuation profile can be chosen as an input to the optimization algorithm. In particular this is important for damage detection in large structures where the operating frequencies will be selected considering the attenuation of Lamb waves.

Once the attenuation profile is available, the influence of the actuation frequency can be added to the fitness function evaluation. For each pixel the distance between the actuator and the sensor is measured. By using this distance and the corresponding coefficients for the exponential trendlines a ‘frequency factor’ is assigned to each actuator-sensor pair for a particular pixel, enabling the integration of actuation frequency into the fitness function:

\[ \text{freq\_factor} = A \times \exp(B \times \text{distance}) \]  

where A and B are the two coefficients of the exponential trendline related to the selected actuation frequency. The resulting 0 or 1 from each actuator-sensor pair is multiplied by ‘freq\_factor’. The coverage level for each pixel is subsequently determined.

### 2.3 Definition of the global fitness function

The global fitness function introduced in this paper is based on maximizing the coverage area provided by a transducer network taking into account geometrical constrains such as cut-outs, stiffeners and boundaries as well as physical constraints such as material parameters (composite, metallic, material anisotropy) as explained in the previous sections. The two important physical parameters influencing the above constraints are the attenuation profile and the group velocity which can be determined experimentally or numerically.

To evaluate the fitness function representing the performance of a particular sensor network as a coverage plot, the intensity of the function at each pixel is obtained from equation (3), by applying the proposed geometrical and physical constraints as additional conditions in evaluation of \( \text{cov}(pix) \). Afterwards, the fitness function is determined by fusing the values from all the transducer paths and summed for all the pixels, to obtain a single coefficient as

\[ c = \sum_{pix=1}^{nr\_pixel} \text{cov}(pix) \times \text{freq\_factor} \]  

where \( nr\_pixel \) is the number of pixels.
The intensity of the coverage at each pixel is represented by the magnitude of the fitness function \( c \). For simple geometries where there is a direct line of sight between all transducer pairs and there are no discontinuities or stiffeners present in the structure, the maximum coverage level can be calculated as

\[
N(N - 1) + (N - 1)!
\]

where \( N \) is the number of transducers. For example, for the coverage plots shown in Figure 4 (b), the maximum intensity for 4 transducers is 18 which is indicated by 4x3=12 transducer paths crossing each pixel + 3! which is the coverage from direct paths. Recall that for each transducer pair crossing a pixel, the value of the fitness function equals to 1 (if coverage is above certain threshold). It is worth mentioning that the presented equations reflect the maximum intensity of the coverage and in case the boundary reflection constraints, \( \gamma \) defined in equation (2), are applied the intensity of the coverage will be reduced. The minimum value of the coverage is given by \( N(N - 1) \) while the maximum intensity is given by equation (6). For a rectangular plate with 4 transducers, although the maximum intensity is 18, depending on the coefficient \( \gamma \) the intensity can change significantly, see Figure 8 where the maximum intensity varies between 12 and 18 with different values of coefficient.

![Figure 8 influence of the boundary reflection coefficient on intensity of the coverage](image)

For more complex geometries such as an L-plate where there might not be a direct line of sight between all the transducer pairs the value of the maximum coverage is not that simple to evaluate. In this case the maximum coverage level is obtained by

\[
N(N - 1) + (M - 1)!
\]

where \( N \) is the number of transducers and \( M \) is the number of transducers pairs with direct line of sight.

The best performance of a transducer network is reached when \( c \) is maximum. An optimization algorithm typically searches for the global or local minimum of a problem. As it is known that for pitch catch configuration the area inside the network has higher probability of damage detection and it is desirable to maximize this area. Therefore the fitness function is determined as:

\[
F = \frac{1}{c * A_{\text{network}}}
\]

where \( A_{\text{network}} \) is the area included inside the network of sensors. Depending on the network configuration, different values of \( F \) can be obtained.

The philosophy of the optimisation strategy introduced is that the procedure will start with minimum number of transducers (input by the user) resulting in their optimal placement. This will show the extent of the coverage as well as the intensity of the coverage. Subsequently the user has the choice of increasing the number of transducers if a better coverage is required, see Figure 9 as the area and the intensity of the coverage increases with the number of transducers from 3 to 5. For each of the cases presented in Figure 9 the algorithm resulted in the optimal placement of sensors to allow the greatest possible coverage. Clearly increasing the sensors from 3 to 4, results in a complete coverage of the plate whereas 5 increases the intensity of the coverage. Therefore having the number of transducers as an input gives the user the flexibility to limit the coverage area to vulnerable zones.
To demonstrate the influence of geometrical factors on the fitness function, an L-plate with different sensor configuration has been investigated. Figure 10 illustrates different coverage area for this example by changing just one sensor location. The scaling on the contour plots represents the intensity of the coverage which is directly related to the number of transducers, the boundary reflection factor $\gamma$ and the direct path between each pair (without crossing boundaries or openings). As shown in Figure 10 (b), when there is no direct line of sight between an actuator and a sensor, the path has no contributions to the overall coverage map and leaves a large part of the structure with very low to no coverage. Comparing the values of the fitness function for both cases there is a significant increase for Network 2 resulting in much lower detection probability. The results of the two cases are summarized in Table 1 showing Network 1 (Figure 10 (a)) to yield a better performance. However, if the transducers are placed in such a way that there is a direct path between each pair without crossing the boundary, the coverage map improves significantly, see Figure 11.

**Figure 9** Coverage area and intensity for different transducer numbers

**Figure 10**: Comparison of two different network configurations. a) Network 1. b) Network 2

**Table 1**: Networks configurations and fitness functions. Positions are non-dimensional.

<table>
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<tr>
<th>Transducer</th>
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<th>Y</th>
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<td>2.157e-4</td>
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As it can be seen (Figure 12), six transducers cover the whole structure (fitness function = 6.9242e-05).

Before presenting the MAC optimization approach, it is important to demonstrate that the proposed fitness function is an appropriate representative of the performance of a network for damage detection using Lamb waves. In the next section experimental measurements are presented to demonstrate that a network with higher coverage area factor (fitness function) detects damage with higher precision in comparison to the networks with lower coverage area where unreliable result are obtained.

3. Experimental verification of the fitness function
In this section the suitability of the fitness function in assessing the performance of a sensor network for damage detection on composite CFRP plates is investigated through experimental tests for damage detection on composite CFRP plates. In all case the boundary reflection factor $\gamma = 0.75$ and the attenuation factor was obtained experimentally.

3.1 Damage detection in a composite plate
The first verification example is damage detection on a composite plate with different sensor networks. The properties of the composite plate are summarized in Table 2. The plate was instrumented with 9 transducers placed in a regular pattern listed in Table 3. The aim of the experiment is to demonstrate the relationship between the fitness functions of different sensor networks (networks of 4 transducer set) and their damage detection performance. It is demonstrated that the network providing the best coverage at the damage location is the one which can locate damage with the highest accuracy. As shown earlier, the minimum required number
of transducers to provide maximum coverage in a rectangular plate is 4, placed at each corner. Therefore, throughout this work, the optimal sensor placement is carried out to find the best four locations.

The results of two different sensor networks are compared here to verify the validity of the fitness function. Both networks consist of 4 transducers: Set 1 [1 2 3 9] and Set 2 [3 4 7 8]. The plate was impacted with a drop tower having a 2.41 kg mass, see [50]. The coverage of the plate obtained with the two sensor configurations is shown in Figure 14 where some preliminary observations can be made: Both networks reach a maximum level of detection coefficient equal to 12 in certain areas of the plate. Set 1 provides fewer blind spots (coverage coefficient equal to zero) than Set 2, as the top part of the plate is not covered with Set 2. The position of damage is indicated with a cross X in Figure 14. Set 1 provides an optimal coverage at the damage position, while Set 2 has lower coverage coefficient at the damage location. The next step is to investigate whether Set 1 detects damage with higher precision as expected in comparison to Set 2 where the coverage coefficient at the location of damage is lower.

### Table 2: Plate properties

<table>
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<tr>
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<td>Length</td>
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<td>Height</td>
<td>300 mm</td>
<td>Thickness</td>
<td>2 mm</td>
</tr>
<tr>
<td>Layout</td>
<td>[0/+45/-45/90/0/+45/-45/90]s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E₁ [GPa]</td>
<td>136</td>
<td>E₂ [GPa]</td>
<td>8</td>
<td>ν₁₂</td>
<td>0.3</td>
</tr>
<tr>
<td>G₁₂ [GPa]</td>
<td>5</td>
<td>ρ [kg/m³]</td>
<td>1605</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: Sensors positions.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>X [mm]</th>
<th>Y [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-70</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>-70</td>
</tr>
<tr>
<td>3</td>
<td>-70</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>-70</td>
<td>-70</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>-70</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Results of damage detection with both sets, using the imaging based Lamb-wave approach described in [39], are shown in Figure 15. It is clear that damage is detected in both cases. However it can be correctly located using Set 1, which guarantees an optimal coverage at the corresponding position, while Set 2 does not locate damage with sufficient accuracy. This result reinforces the idea that the fitness function defined in Section 2 can provide a good estimation of the performance of a network for Lamb-wave investigation, without requiring a large number of damage simulations or experiments.
In order to further verify the influence of the attenuation factor on the fitness function a second CFRP plate [200 x 150 x 8.3 mm] representing thick composite structure, such as wing section of aircraft, was excited with various actuation frequencies ranging from 50-350 kHz. The first step was to obtain the attenuation profile for each frequency. Figure 16 shows the trendline and the frequency function for 100 kHz excitation frequency. Each trendline was normalized by the maximum amplitude (which occurred at 350 kHz actuation) to obtain the attenuation factor.
Figure 16. Attenuation Profile: Trendline for 100 kHz excitation frequency

The coverage area of the plate was obtained with and without the influence of frequency factor, see Figure 17. It can be seen that the attenuation coefficient has a significant influence on the coverage area and intensity depending on the distance of each pixel from transducer pairs. The next step is to investigate how different attenuation factors (related to different frequencies) will influence the fitness function.

![Coverage without attenuation coefficient](image1)

(a) Coverage without attenuation coefficient, fitness function = 5.4389e-05

![Coverage with attenuation coefficient](image2)

(b) Coverage with attenuation coefficient referring to 50 kHz, fitness function = 8.2178e-04

Figure 17. The influence of actuation frequency on the coverage map

To obtain the coverage plot for each actuation frequency, the fitness function introduced in the previous section was multiplied by the frequency factor defined by equation (4). Figure 18 illustrates the coverage map for 50, 100, 220 and 300 kHz actuation frequencies in the same scale. It is clearly seen that the intensity of the coverage map increases with the frequency, as expected. The inclusion of the attenuation factor is particularly important for large structures when optimizing the transducer locations to provide the maximum coverage.
The fitness function for each frequency is summarized in Table 4 with the lowest value referring to 350 kHz actuation frequency.

<table>
<thead>
<tr>
<th>Actuation frequency (kHz)</th>
<th>Fitness function</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>8.22E-04</td>
</tr>
<tr>
<td>100</td>
<td>5.01E-04</td>
</tr>
<tr>
<td>220</td>
<td>1.38E-04</td>
</tr>
<tr>
<td>350</td>
<td>1.04E-04</td>
</tr>
</tbody>
</table>

The reduction of fitness function with actuation frequency can be also represented by an exponential function as shown in Figure 19.

It is understood that depending on the detection algorithm and type of damage, one mode (A0, S0) could be more desirable over other modes and this can result in the user having a preferable actuation frequency. In that case the fitness function can be modified to consider different frequency factors.
3.3 Convergence

One parameter which could influence the value of fitness function is the pixel size. As the fitness function is evaluated at the centre of each pixel and the sum of all these values is indicative of the MAC for the plate, the question which remains is whether the size and number of pixels will have any influence on the fitness function evaluation. Therefore a parametric study was carried out to ensure the convergence of the coverage map for the same plate presented in the previous section. Figure 20 shows the coverage map for 4 different values of pixels. The variable Pixel_map indicates how many pixels are in each direction.

![Convergence of fitness function](image)
It was observed that at Pixel_map=50, Figure 20 (d), the fitness function reached its converged value. This means the composite plate 150 x 200 mm was divided into 2500 pixels (each 3x4 mm). There was no significant increase noticed in the computational time with increasing the number of pixels. The converged pixel size is then used in the evaluation of the fitness function in the optimization step.

4. Statistical numerical analysis

In order to demonstrate that the proposed methodology for optimal coverage is valid for different damage positions, a statistical analysis has been conducted. Finite Element Analysis of a plate with 8 different damage positions were carried out. Figure 21 depicts the plate configuration with transducers numbered 1 to 9 and damage locations indicated by crosses. The model is representative of the experimental case described in the previous section. Damage was introduced as local softening of the four central layers of the plate with an area of 200 mm². Different sets of sensors have been investigated to determine their coverage performance. A list of all the sets used is given in Table 5 and their corresponding fitness function is plotted in Figure 22(a). As it can be seen, Set 2 is the one which gives the highest value of the fitness function (maximum error in localizing the damage). Therefore it is expected to be the one with the worst overall performance. On the other hand, Set 3 is the one with the lowest value of the fitness function representing the best network. This is an expected result which has been confirmed experimentally.

![Damage positions](image)

**Figure 21:** Scheme of the plate with transducers and damage locations highlighted.

**Table 5:** Sets of sensors used in the numerical analysis.

<table>
<thead>
<tr>
<th>Set</th>
<th>Sensors</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[1 2 3 9]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>[3 4 7 8]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>[1 2 3 4]</td>
<td>[1 2 3 4]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>[1 2 5 7]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>[5 6 7 8]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Eight damage cases are not sufficient to develop a statistical model. In order to increase the number of possible scenarios, different levels of noise are introduced in the post-processing step, before performing the damage detection algorithm to create new scenarios. In total, 5 different levels of noise are investigated (SNR=Inf., SNR=90, SNR=70, SNR=50, SNR=30). As the level of noise may be different between pristine and damaged case, a total of 200 evaluations can be performed for each sensor configuration. The distance between the simulated damage and the predicted one is considered as the localization error of the chosen sensor configuration. Once all the 200 investigations for each sensor configuration have been performed, the average value of the errors is evaluated. The results obtained for all the different sets are shown in Figure 22(b). From Figure 22(a) and (b) it can be seen that the errors resemble the values of the fitness functions. This means that the lower the fitness function, the better the results of damage localization. This confirms that the proposed fitness function based on MAC for damage localization is suitable for the determination of the optimal sensor placement without the need to perform numerous damage investigations, which is a long and expensive procedure in particular from an experimental point of view. Results shown in Figure 22(b) are further confirmed by the evaluation of the cumulative distribution function (CDF) of the errors presented in Figure 23. It is clear that Set 3 can evaluate the highest probability of detection with the smallest error, whereas Set 2 shows the opposite behaviour. This is in line with the results of both the coverage and the average error assessment.
5. Optimization using Genetic Algorithms

The problem of finding the optimal placement for sensors requires the evaluation of the fitness function described in Section 2 for all possible combinations of the sensors. Once the maximum number of sensors, $N$, to be placed in a structure is chosen, their optimal positions have to be determined. In theory any sensor is free to be placed at any location within the structure. However, practical engineering constraints can be embedded into the algorithm to ensure realistic positioning of sensors. For example factors such as distance from edges, minimum distance between 2 transducers, ease of access in the structure and providing network connections needs to be taken into account for application to real structures. Based on these factors it is more feasible to have a vector of all possible sensor positions which will be searched during the optimisation process, rather than having no constraint on the transducer location. Once all the possible sensor locations, $POS$, have been selected, the total number of possible networks, $NET_{tot}$, with $N$ transducers is given by:

$$NET_{tot} = \frac{POS!}{N! \times (POS - N)!}$$  \hspace{1cm} (8)

For example a square plate of 1 m length, $POS = 361$ possible sensor locations can be identified, having 50 mm distance between two consecutive positions. Limiting the maximum number of sensors to $N = 4$, the total number of possible configurations is $NET_{tot} = 695,946,630$. An exhaustive search is therefore not possible when large structures are involved. Therefore, in this paper the optimal sensor configuration is obtained using the Genetic Algorithm. GA can reduce the number of computation, yet giving an optimal solution. The algorithm consists of evaluating the fitness function over a set of possible network configurations, called population. The size of the population is much smaller than $NET_{tot}$. The networks included in the population are then ranked according to their values of fitness function. After that the population is updated keeping the best networks and generating a new population of sets via crossover and mutation; it is important to define these functions in order to produce sets which are likely to have low values of fitness function in order to reach convergence of the algorithm, but at the same time including sets which have not been previously tested to investigate a wide range of possible combinations. After this new population is formed, the process starts again until a certain tolerance in the variation of the optimal solution or the maximum number of generations is reached. Details of the crossover and mutation are given in the following sections. For a more detailed description of GAs, the reader can refer to [45]. The parameters of the GA has been chosen after a thorough parametric study following [26] to ensure convergence.

Crossover function

After every generation, a set of best sensor configurations is selected. This choice is made based on the values of the fitness function associated to those networks. This set, called elite, is then brought to the next generation, while the remaining part of the total population is updated via crossover and mutation to maintain generality. The crossover operation aims at producing a new child from two parents. In order to reach convergence the parents are chosen from the elite. The developed crossover function aims at avoiding repeated sensors in a single gene. When two parents are chosen, first of all their common sensors are identified and transferred to the child while the remaining sensors are chosen randomly from the other chromosomes of the parents. If the parents don’t have any chromosome in common, half of each parent’s genetic information is passed to the child.

Mutation function

The mutation function ensures that a mutated version of the parents is included in the next generation. This step is to include new possible combinations of sensors into the analysis. The mutation function proposed in this paper is a modification of what was proposed in [26] where a probability of mutation $p_m$ was introduced and the mutated parent was generated based on a random variation of its genes according to the given probability. This approach can result in a high mutation of parents, but also in mutated parents which correspond exactly to the parents itself. This is due to the random selection of the genes to be mutated.
At initial generations it is desirable that multiple possible sensor configurations are analysed, so that the mutation rate is high. For this reason, the mutation function in this paper has been modified so that at the initial stages of the optimization the mutation is enhanced, by keeping only part of a parent and randomly picking the other part from the rest of the possible sensors. The length of the part to be kept is fixed rather than depending on a random distribution. The function ensures that the same sensor is not picked twice in the same network. After a certain amount of generations, the amount of genes to be kept during the mutation process is increased, in order to avoid the evaluation of solutions which would produce high values of the fitness function when the optimal solution is close to be reached.

5  Benchmark example for optimum sensor placement

In this section the proposed optimisation algorithm is applied to a structure with a circular opening. Initially, the influence of the hole on the MAC is investigated. The number of sensors in this example is fixed (4 transducers) and only their optimal location is sought.

5.1 Assessment of the fitness function for a plate with opening

The optimized network configuration obtained for a square flat plate of 200 x 200 mm, Figure 9 (b) (fitness function=6.4466e-05), is applied to the same plate but with an openings positioned at different locations. In this example the option of increasing the coverage value in the direct path of transducers has been applied. Figure 24 illustrated the coverage map of the same plate but with an opening. The coverage of two pixels A and B and some of the corresponding transducer paths that passes through them is highlighted. It can be observed that pixel B has a high coverage since none of the transducer paths go through the opening, therefore no reduction of the coverage value is applied. In addition, pixel B is located on the direct path of one transducer pair thus it will have higher coverage than its adjacent pixels. In contrast at least one of the transducer paths crosses the opening and therefore does not make any positive contributions to the coverage value.

![Figure 24: Examples of coverage map in presence of a hole](image)

To determine how the position of an opening affects the coverage map, the fitness function introduced in section 2 is evaluated for the same plate and sensor locations as in Figure 24 but with different position of openings. The results are presented in Figure 25 and the corresponding values of the fitness function is plotted in Figure 26. The positions of the sensors and of the hole are summarized in Table 6. It is clear from Figure 25 that the pixels with their path passing through the opening (either direct or indirect) have reduced value of the coverage map. As the hole is located away from the centre of the plate, the global coverage increases, i.e. the Fitness Function value decreases. This is attributed to the reduced
number of paths crossing the opening. Once the opening is very close to a sensor, most of the paths will cross the opening and that sensor has very small contribution to the overall coverage map, see Figure 25 (b) which has the highest value of the fitness function resulting in the least overall coverage level. By placing the sensor in front of the circular hole, direct paths with other three sensors are established while the area behind the hole would be partially hidden to the other three sensors, see Figure 27(a) and (b). By placing the sensor in front of the opening there is a noticeable increase in the coverage area and hence decrease in the fitness function, see the values of the fitness function in Figure 28. Simply by looking at the coverage map it is not easy to determine which of the two configurations provides the best coverage: Set 1 has a higher coverage value (Figure 27 (a)) in comparison to Set 2 which has a wider coverage area (Figure 27 (b)). The values of the corresponding Fitness Functions can be seen in Figure 28 where Set 2 provides a small improvement in the coverage. Both Set 1 and Set 2 are preferable to the previous configuration with 4 corner transducers, indicated as Set 0 (Figure 25 (b)).

(a) Hole at (0.5, 0.5), fitness= 2.6536e-04
(b) Hole at (0.75, 0.75), fitness= 3.2813e-04
(c) Hole at (0.7, 0.5), fitness= 1.8012e-04
(d) Hole at (0.5, 0.8), fitness= 2.7673e-04

Figure 25: Examples of coverage for a square plate with a hole in different positions
The main objective for optimum placement of a four sensors network is which locations would result in a maximum coverage and whether our fitness function can correctly evaluate the better network. It can be concluded from the tests carried out that the proposed fitness function provides a good indication of the coverage area of a sensor network with a view of maximising it while limiting the influence of boundary reflection of Lamb waves and incorporating the effect of direct paths. Next step, the proposed optimisation algorithm is applied to find the optimum sensor locations.

Table 6: Normalized sensors and holes positions

<table>
<thead>
<tr>
<th>Coordinates</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
<th>Hole 1</th>
<th>Hole 2</th>
<th>Hole 3</th>
<th>Hole 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0.95</td>
<td>0.9</td>
<td>0.05</td>
<td>0.05</td>
<td>0.5</td>
<td>0.75</td>
<td>0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>Y</td>
<td>0.9</td>
<td>0.05</td>
<td>0.85</td>
<td>0.15</td>
<td>0.5</td>
<td>0.75</td>
<td>0.5</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Figure 26: Values of the Fitness Functions for different location of hole.

Figure 27: Examples of coverage when sensors positions are changed.
Table 7: Coordinates of sensors for the two new configurations.

<table>
<thead>
<tr>
<th>Coordinates</th>
<th>Set 1</th>
<th>Set 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transducer</td>
<td>Sensor 1</td>
<td>Sensor 2</td>
</tr>
<tr>
<td>X</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Y</td>
<td>0.05</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Figure 28: Fitness Function values for the three network configurations, hole 4.

5.2 Optimal sensor placement for a plate with opening
The optimal sensor placement is now determined with the optimization procedure described in Sections 2 and 5. The optimization is performed using the following parameters:

- a population of 30 networks for every generation;
- max generation of 100;
- mutation rate of 0.02 and
- crossover fraction of 0.7.

The optimal sensor network resulted in the sensor locations given in Figure 29 and the corresponding value of the fitness function resulted to be:

$$FF = 2.5221 \times 10^{-4}$$

Figure 29: Optimal sensor position for a plate with a hole.
The value of the obtained fitness function is lower than the values obtained before, Figure 28. This confirms that the optimization approach is able to determine an optimal configuration for the sensors, based on the fitness, crossover and mutation functions. Analysing Figure 29 it is clear that part of the plate is blind to damage detection, due to the presence of the hole. However the sensor configuration guarantees a good coverage on the rest of the plate, meaning that damage can be detected in this area. To have a better coverage, the number of sensors should be increased.

Conclusions
A novel methodology for optimal sensor placement based on Maximum Area Covered (MAC) for damage detection using Lamb waves was presented. The proposed optimisation algorithm utilizes GA to minimize a fitness function resulting in optimal sensor locations given a fixed number of transducers. The fitness function is a physical based function which takes into account the properties of Lamb waves, boundary reflection, attenuation profile, actuation amplitude and geometrical complexities such as presence of an opening or stiffeners. The validity of the fitness function was first determined through an experimental verification followed by a statistical analysis. A parametric study was carried out using experimental signals to integrate the influence of the actuation frequency based on the attenuation factor together with a convergence study of the fitness function. After demonstrating the proposed fitness function is an appropriate indicator of the damage detection performance of a network, the optimal sensor placement for a plate with an opening was demonstrated. The coverage areas of the optimal sensor network was compared to several sub-optimal one and it showed lower value of the fitness function resulting in higher coverage area. One of the most important advantages of the proposed optimization strategy, MAC, is that it is a physical based approach and does not rely on POD values for each possible sensor combination which is extremely expensive. Therefore the proposed method is easily extendable to larger and more complex structures.

References


