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Analytical Model for Fishnet Structures: A Systematic Circuit Approach

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Abstract – **Fishnet structures provide a popular procedure to obtain negative-refraction-index artificial media. They are basically stacked extraordinary-transmission structures that are closely spaced. Thus these structures can be built by combinig two or more metallic plates with a 2D periodic distribution of small apertures. Analytical models based on equivalent circuits are almost trivially obtained when the electric separation between metal plates is relatively large. However the more interesting situation arising from tightly coupled screens is more involved and only heuristic equivelent-circuit approaches have been reported in the literature. In this contribution a systematic approach to obtain a general circuit model for this kind of structures is developed.**

I. Introduction

Among the diverse artificial media that provide a negative index of refraction (NIM), fishnets have attracted a lot of attention due to their relative simplicity and the possibility of implementing NIM even at optical frequencies [1]. The most basic form of a fishnet consists of two closely spaced extraordinary transmission (ET) screens, although an arbitrary number of these ET perforated metallic plates can be stacked to obtain an electrically thick NIM sample [2]. In recent years the interest on NIM has experienced a regrowth because of their multiple applications [3, 4]. The theoretical study of this kind of structures mainly lies on the use of intensive computational methods, although some qualitative analytical models are also available in the literature to deal with the analysis of simplified versions of those structures [5]. More accurate analytical models can be developed using equivalent circuits [6, 7] where a number of approximations are required. However, in recent years, a systematic approach has been reported in [8, 9] to extract the appropriate network topology of equivalent circuits and their electrical parameters for several periodic structures of interest. The purpose of this contribution is to adapt that analytical methodology to the modelling of fishnet structures by means of the equivalent-circuit approach. In particular the interaction between adjacent ET screens through high-order modes (which is key to explain fishnet operation) is conveniently incorporated to the model.

II. Derivation of the circuit approach and numerical verification

Fig. 1(a) shows a basic fishnet structure formed by two closely spaced periodically perforated metal plates. Let us consider a uniform plane wave normally impinging on the structure from the left side. The periodic nature of the problem allows us to reduce the original scattering study to the waveguide problem highlighted in the figure, which is the unit cell of the periodic structure. This unit cell can be seen as a parallel-plate waveguide loaded with a couple of diaphragms separated by a dielectric layer. A simple heuristic and approximate equivalent circuit for this problem is the one depicted in Fig. 1(b), where the shunt admittances are split into internal [dielectric, index (1)] and external [air, index (0)] contributions. This separation between internal and external problems is key to obtain a general circuit model for a generalized fishnet made with an arbitrary number of dielectric slabs and perforated metallic layers. An example of application of a circuit model of this type can be seen in [10]. Unfortunately

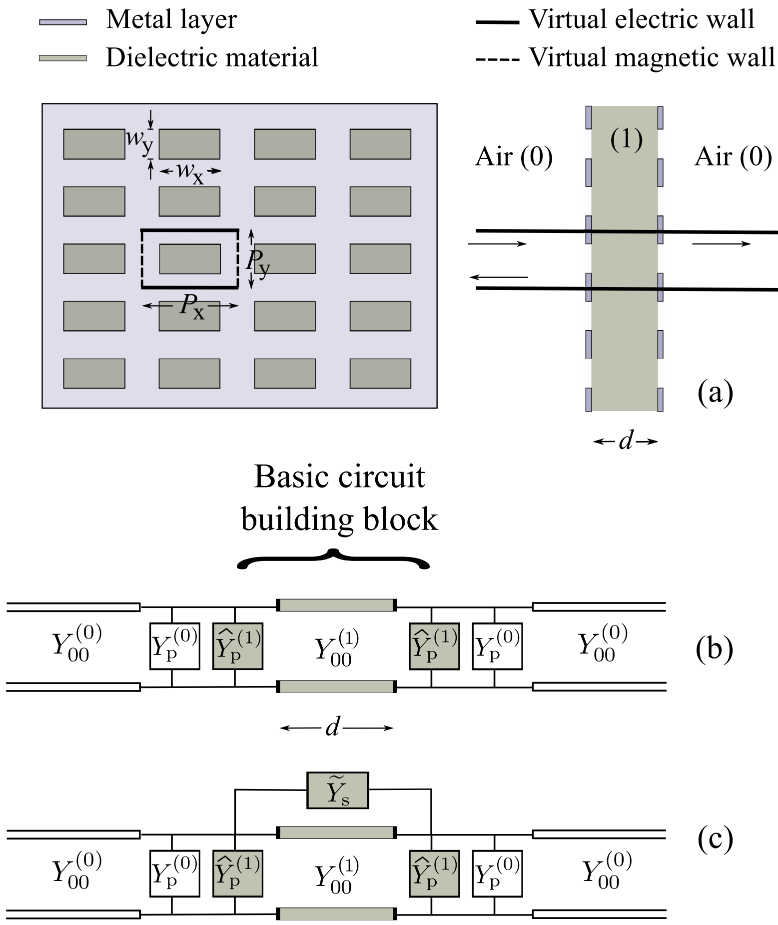

Fig. 1: (a) Sketch of the most basic implementation of a fishnet structure (front view and cross section). The waveguide–like unit cell is highlighted. (b) Topology of the conventional equivalent circuit used to calculate the scattering parameters (fundamental incident mode). (c) Proposed circuit accounting for high-order modes interactions.

this model neglects the interactions between the metal screens through high-order modes, which are essential for closely spaced screens and the fundamental feature of fishnets. This drawback is overcome by using the simple modification introduced in Fig. 1(c); i.e., the inclusion of $\widetilde{Y}_s$. It is worth mentioning that simple expressions (in terms of numerical series that can be efficiently summed up) have been obtained for all the lumped impedances appearing in the model. The obtained expressions are valid for any set of geometrical dimensions and they, as expected, trivially reduce to the model in Fig. 1(b) when high-order mode interactions are not relevant.

Four examples of validation are included in Fig. 2. In that figure, circuit model predictions and full-wave CST calculations are compared for a couple of metal grids separated by air (top left) or a dielectric slab (top right). A more complex structure involving four metal screens separated by three dielectric slabs (bottom right) or air gaps (bottom left) is also studied in the same figure. The circuit model for this latter case is trivially obtained by cascading the internal part of the circuit model represented in Fig. 1(c). In all the curves the non-physical predictions of the conventional approximate heuristic model in Fig. 1(b) are included as blue lines. These figures make apparent that the role of the first higher order modes is critical in this kind of structures.

III. Conclusion

Circuit models por fishnet–like structures are reported in this contribution. All the circuit components are analytically computed. The circuit model results agree very well with full-wave data obtained making use of important computational resources.

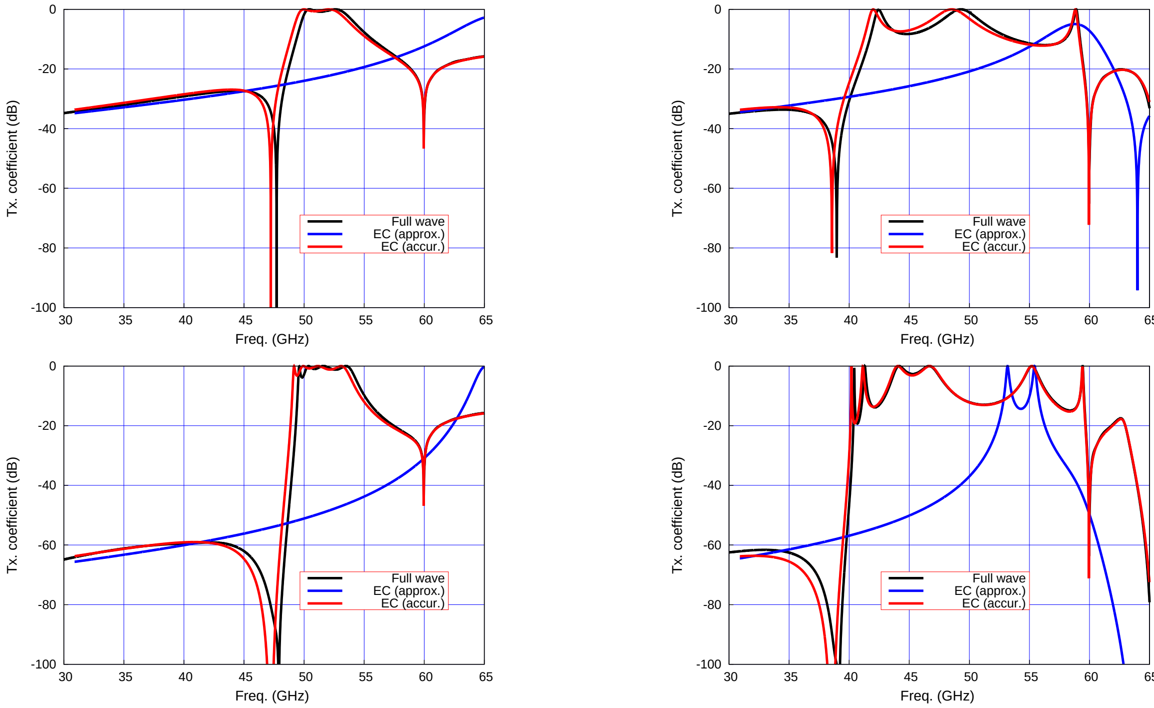

Fig. 2: Transmission for four different fishnets. Top panels: two coupled metal screens having air between them (left) or a dielectric material (right). Bottom panels: stacked four screens / three slabs structure with air (left) or dielectric (right). Dimensions: $P_x = P_y = 5.0\,\text{mm}$; $w_x = 3w_y = 2.4\,\text{mm}$; $d = 1.0\,\text{mm}$; $\varepsilon_r = 1.5$.

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