An MPEC approach for Analysing the Impact of Energy Storage in Imperfect Electricity Markets

Yujian Ye, Dimitrios Papadaskalopoulos and Goran Strbac
Department of Electrical and Electronic Engineering
Imperial College London
London, UK
yujian.ye11@imperial.ac.uk, d.papadaskalopoulos08@imperial.ac.uk, g.strbac@imperial.ac.uk

***Abstract*— Although recent studies have investigated the impacts of energy storage on various aspects of power system operation and planning, its role in imperfect electricity markets has not been explored yet. This paper provides for the first time theoretical and quantitative evidence of the beneficial impact of energy storage in limiting market power by generation companies. Quantitative analysis is supported by a bi-level optimization model of the imperfect electricity market setting, accounting for the time-coupling operational constraints of energy storage. This bi-level problem is solved after converting it to a Mathematical Program with Equilibrium Constraints (MPEC). Case studies are carried out on a test market with day-ahead horizon and hourly resolution.**

***Index Terms*—Bi-level optimization, energy storage, electricity markets, market power, mathematical program with equilibrium constraints.**

NOMENCLATURE

A. Indices and Sets

t	Index of time periods running from 1 to T
i	Index of generation companies running from 1 to I

B. Parameters

b_i^G	Linear cost coefficient of generation company i (£/MW)
c_i^G	Quadratic cost coefficient of generation company i (£/MW^2)
g_i^{max}	Maximum power output limit of generation company i (MW)
b_t^D	Linear benefit coefficient of demand at time period t (£/MW)
c_t^D	Quadratic benefit coefficient of demand at time period t (£/MW^2)
d_t^{max}	Maximum power limit of demand at time period t (MW)
s^{max}	Power limit of energy storage (MW)
E^{cap}	Capacity of energy storage (MWh)
E^{min}	Minimum energy limit of energy storage (MWh)
E^{max}	Maximum energy limit of energy storage (MWh)
E_0	Initial energy level in energy storage (MWh)
η^c	Charging efficiency of energy storage
η^d	Discharging efficiency of energy storage

C. Variables

$k_{i,t}$	Strategic offer variable of generation company i at time period t
$g_{i,t}$	Power output of generation company i at time period t (MW)
d_t	Power consumed by demand at time period t (MW)
s_t^c	Charging power of energy storage at time period t (MW)
s_t^d	Discharging power of energy storage at time period t (MW)
E_t	Energy level in energy storage at the end of time period t (MWh)
λ_t	Market clearing price at time period t (£/MWh)

I. INTRODUCTION

Electricity markets are better described in terms of imperfect instead of perfect competition. Generation companies owning a large share of the market or units at strategic locations of the transmission network are able to manipulate the electricity prices and increase their profits beyond the competitive equilibrium levels, through strategic bidding. This exercise of market power results in increased price levels and loss of social welfare [1].

At the same time and in line with the emerging *Smart Grid* paradigm, an increased penetration of demand response and energy storage technologies in power systems is observed [2]. Regarding the role of demand response in the imperfect market setting, previous work has demonstrated that demand reduction at high market prices limits generation companies' ability to exercise market power [3]-[5]. However, the single time period modeling framework of this work is not suitable for analyzing the impact of energy storage, due to its time-coupling characteristics. Although numerous studies have investigated the benefits of energy storage on various aspects of system operation and planning [6]-[11], its role in imperfect electricity markets has not been explored yet.

This paper aims to fill this knowledge gap by providing both theoretical and quantitative evidence of the beneficial impact of energy storage in limiting market power by generation companies. Quantitative analysis is supported by a bi-level optimization model of the imperfect electricity market setting, whose upper level represents the profit maximization objective of strategic generation companies and the lower level represents the energy market clearing including the time-coupling operational constraints of energy storage. This bi-level problem is solved after converting it to a *Mathematical Program with Equilibrium Constraints* (MPEC), by replacing the lower level problem by its equivalent *Karush-Kuhn-Tucker* (KKT) optimality conditions. Case studies with this MPEC model on a test market with day-ahead horizon and hourly resolution quantitatively demonstrate the benefits of energy storage in reducing the generation profit increase and demand utility reduction driven by the exercise of market power by generation companies.

The rest of this paper is organised as follows. Section II outlines models of generation, demand and energy storage market participants. Section III provides a theoretical explanation of the beneficial impact of energy storage on market power. Section IV formulates the bi-level optimization problem and the corresponding MPEC problem expressing the decision making of strategic generation companies. Case studies and illustrative results are presented in Section V. Finally, Section VI discusses conclusions of this work.

II. Modelling Market Participants

A. Strategic Generation Companies

For presentation clarity reasons and without loss of generality, we assume that each generation company i owns a single generation unit, the quadratic cost function, linear marginal cost function and output limits of which are expressed by (1), (2) and (3) respectively:

$$C_{i,t}(g_{i,t}) = b_i^G g_{i,t} + c_i^G (g_{i,t})^2 \tag{1}$$

$$MC_{i,t}(g_{i,t}) = b_i^G + 2c_i^G g_{i,t} \tag{2}$$

$$0 \le g_{i,t} \le g_i^{max}, \forall t \tag{3}$$

Strategic generation companies can exercise market power through either submitting offers higher than their actual marginal costs (i.e. economic withholding) or offering less than their actual generation capacity to the market (i.e. physical withholding) [1]. In line with the majority of relevant works [12]-[16], the first approach of exercising market power is investigated in this paper. Following the model employed in [5], [13]-[14], the strategic marginal cost function is expressed by (4), where the value of the decision variable $k_{i,t} \ge 1$ represents the strategic behavior of generation company i at time period t.

$$SMC_{i,t}(g_{i,t}) = k_{i,t}(b_i^G + 2c_i^G g_{i,t}) \tag{4}$$

If $k_{i,t} = 1$, company i behaves competitively and reveals its actual marginal costs to the market at t. If $k_{i,t} > 1$, company i behaves strategically and reports higher than its actual marginal costs to the market at t. Company i should determine the value of $k_{i,t}$ by accounting for the trade-off between higher market clearing price and lower clearing quantity. More specifically, a higher $k_{i,t}$ will tend to increase the market price, but at the same time will tend to decrease the quantity sold by company i, since companies with lower offers may replace i in the merit order and / or the demand side and the energy storage may reduce the demand at t.

B. Demand Side

Following the model employed in [5], the benefit obtained by the demand side at each time period is expressed through a quadratic, non-decreasing and concave function (5). The marginal benefit or *willingness to pay* is thus expressed through a linear decreasing function (6) which captures the effect of demand's self-price elasticity. As the demand level increases the consumers are willing to pay a lower price; equivalently, as the market price increases the demand requested by the consumers is reduced. The limits in the requested demand level at each time period are expressed by (7). The coefficients of the marginal benefit function and the maximum demand limit are time-specific parameters, capturing the differentiated preferences of consumers across different time periods [17].

$$B_t(d_t) = b_t^D d_t - c_t^D (d_t)^2 \tag{5}$$

$$MB_t(d_t) = b_t^D - 2c_t^D d_t \tag{6}$$

$$0 \le d_t \le d_t^{max}, \forall t \tag{7}$$

C. Energy Storage

A single energy storage unit in the system is assumed, the operational characteristics of which are expressed by (8)-(12). Constraint (8) expresses the energy balance in the storage unit including charging and discharging losses. Constraint (9) corresponds to its maximum depth of discharge and state of charge ratings. Constraints (10)-(11) represent its power limits. For the sake of simplicity, the storage energy content at the start and the end of the examined temporal horizon are assumed equal (12).

$$E_t = E_{t-1} + \eta^c s_t^c - s_t^d / \eta^d = 0, \forall t \in T \tag{8}$$

$$E^{min} \le E_t \le E^{max}, \forall t \in T \tag{9}$$

$$0 \le s_t^c \le s^{max}, \forall t \in T \tag{10}$$

$$0 \le s_t^d \le s^{max}, \forall t \in T \tag{11}$$

$$E_0 = E_T \tag{12}$$

III. Impact of Energy Storage on Generation Market Power

As demonstrated in [6]-[11], the deployment of energy storage enables flattening of the demand profile by discharging during peak time periods and charging during off-peak time periods. Fig. 1 illustrates in a price-quantity graph the impact of this demand flattening effect on the extent of market power exercised by generation companies. The two curves represent the aggregate competitive and strategic marginal cost curves of the generation side; the price intercept and slope of each segment of the strategic curve are higher than the respective parameters of the competitive curve (Section II-A). Energy storage reduces the peak demand from Q_2 to Q_2' and increases the off-peak demand from Q_1 to Q_1'. The intersections of the marginal cost curves with the vertical demand lines determine the market clearing prices in the respective cases. The price increments $\Delta\lambda$ represent the increase of the market clearing prices driven by the exercise of market power in the respective cases.

Fig. 1 demonstrates that deployment of energy storage reduces the price increment at the peak period from $\Delta\lambda_2$ to $\Delta\lambda_2'$ while it increases it at the off-peak period from $\Delta\lambda_1$ to $\Delta\lambda_1'$. Although the peak demand reduction is slightly lower than the off-peak demand increase (due to the charging and discharging losses), i.e. $Q_2 - Q_2' < Q_1' - Q_1$, the price increment reduction at the peak period is higher than its increase at the off-peak period, i.e. $\Delta\lambda_2 - \Delta\lambda_2' > \Delta\lambda_1' - \Delta\lambda_1$, due to the larger slope of the strategic marginal cost curve. This effect also applies to the resulting generation profit increments and demand utility decrements (as quantitatively explored in Section V-B) and implies that deployment of energy storage results in an overall reduction of the extent of market power exercised by the generation side.

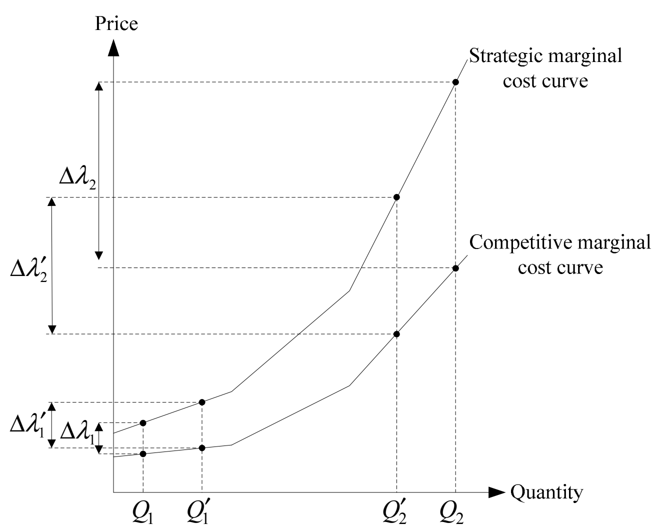

Figure 1. Impact of energy storage on the extent of market power exercised by the generation side.

IV. Modeling Imperfect Electricity Markets

A. Bi-level Optimization Model

Following the approach employed in [12]-[16], the decision making of strategic generation companies is modeled through the bi-level optimization problem (13)-(24). The upper level determines the optimal offering strategies maximizing the profit of the generation companies, and is subject to the lower level problem representing the market clearing process. These two problems are coupled, since the offering strategies determined by the upper level problem affect the objective function of the lower level problem, while the market clearing price and generation dispatch determined by the lower level problem affect the objective function of the upper level problem.

(Upper level)

$$\max_{k_{i,t},\forall i,\forall t} \sum_{i,t}\left[\lambda_t g_{i,t} - C_{i,t}\left(g_{i,t}\right)\right] \quad (13)$$

Subject to:

$$k_{i,t} \geq 1, \forall i, \forall t \quad (14)$$

(Lower level)

$$\min_{\boldsymbol{VL}}\left[\sum_{i\in I} k_{i,t} C_{i,t}\left(g_{i,t}\right) - B_t(d_t)\right] \quad (15)$$

Where:

$$\boldsymbol{VL} = \left[g_{i,t}, \forall i, \forall t\right] \cup \left[d_t, \forall t\right] \cup \left[s_t^c, s_t^d, E_t, \forall t\right] \quad (16)$$

Subject to:

$$d_t + s_t^c - s_t^d - \sum_i g_{i,t} = 0 : \lambda_t, \forall t \quad (17)$$

$$0 \leq g_{i,t} \leq g_i^{max} : \mu_{i,t}^-, \mu_{i,t}^+, \forall i, \forall t \quad (18)$$

$$0 \leq d_t \leq d_t^{max} : \nu_t^-, \nu_t^+, \forall t \quad (19)$$

$$E_t = E_{t-1} + \eta^c s_t^c - s_t^d / \eta^d : \xi_t, \forall t \quad (20)$$

$$E^{min} \leq E_t \leq E^{max} : \pi_t^-, \pi_t^+, \forall t \quad (21)$$

$$0 \leq s_t^c \leq s^{max} : \rho_t^-, \rho_t^+, \forall t \quad (22)$$

$$0 \leq s_t^d \leq s^{max} : \sigma_t^-, \sigma_t^+, \forall t \quad (23)$$

$$E_0 = E_T : \varphi \quad (24)$$

The objective function (13) of the upper level problem constitutes the total profit of the generation companies. This problem is subject to the limits of the strategic offer variables (14) and the lower level problem (15)-(24). The latter represents the market clearing process at each time period, maximizing the perceived (since generation companies do not generally report their actual marginal costs) social welfare (15), subject to demand-supply balance constraints (17) (the Lagrangian multipliers of which constitute the market clearing prices), generation and demand limits (18)-(19) and the operational constraints of energy storage (20)-(24).

B. MPEC Formulation

In order to solve the above bi-level optimization problem, the lower level problem is replaced by its KKT optimality conditions, which is enabled by the continuity and convexity of the lower level problem. This converts the bi-level problem to an MPEC which is formulated as follows:

$$\max_{\boldsymbol{V}} \sum_{i,t}\left[\lambda_t g_{i,t} - C_{i,t}\left(g_{i,t}\right)\right] \quad (25)$$

Where:

$$\boldsymbol{V} = \left[k_{i,t}^G, \forall i, \forall t\right] \cup \left[g_{i,t}, \forall i, \forall t\right] \cup \left[d_t, \forall t\right] \cup \left[s_t^c, s_t^d, E_t, \forall t\right] \cup \left[\lambda_t, \forall t\right] \cup \left[\mu_{i,t}^-, \mu_{i,t}^+, \forall i \in I, \forall t\right] \cup \left[\nu_t^-, \nu_t^+, \forall t\right] \cup \left[\xi_t^-, \xi_t^+, \forall t\right] \cup \left[\pi_t^-, \pi_t^+, \forall t\right] \cup \left[\sigma_t^-, \sigma_t^+, \forall t\right] \cup \left[\rho_t, \forall t\right] \cup \left[\varphi\right] \quad (26)$$

Subject to:

$$k_{i,t} \geq 1, \forall i, \forall t \quad (27)$$

$$k_{i,t}\left(b_i^G + 2c_i^G g_{i,t}\right) - \lambda_t - \mu_{i,t}^- + \mu_{i,t}^+ = 0, \forall i, \forall t \quad (28)$$

$$-b_t^D + 2c_t^D d_t + \lambda_t - \nu_t^- + \nu_t^+ = 0, \forall t \quad (29)$$

$$\lambda_t - \rho_t^- + \rho_t^+ - \eta^c \xi_t = 0, \forall t \quad (30)$$

$$-\lambda_t - \sigma_t^- + \sigma_t^+ + \xi_t/\eta^d = 0, \forall t \quad (31)$$

$$-\pi_t^- + \pi_t^+ + \xi_t - \xi_{t+1} = 0, \forall t < T \quad (32)$$

$$-\pi_T^- + \pi_T^+ + \xi_T - \varphi = 0 \quad (33)$$

$$d_t + s_t^c - s_t^d - \sum_i g_{i,t} = 0, \forall t \quad (34)$$

$$E_t = E_{t-1} + \eta^c s_t^c - s_t^d/\eta^d, \forall t \quad (35)$$

$$E_0 = E_T \quad (36)$$

$$0 \leq \mu_{i,t}^- \perp g_{i,t} \geq 0, \forall i, \forall t \quad (37)$$

$$0 \leq \mu_{i,t}^+ \perp \left(g_i^{max} - g_{i,t}\right) \geq 0, \forall i, \forall t \quad (38)$$

$$0 \leq \nu_t^- \perp d_t \geq 0, \forall t \quad (39)$$

$$0 \leq \nu_t^+ \perp (d_t^{max} - d_t) \geq 0, \forall t \quad (40)$$

$$0 \leq \rho_t^- \perp s_t^c \geq 0, \forall t \quad (41)$$

$$0 \leq \rho_t^+ \perp (s^{max} - s_t^c) \geq 0, \forall t \quad (42)$$

$$0 \leq \sigma_t^- \perp s_t^d \geq 0, \forall t \quad (43)$$

$$0 \leq \sigma_t^+ \perp (s^{max} - s_t^d) \geq 0, \forall t \quad (44)$$

$$0 \leq \pi_t^- \perp (E_t - E^{min}) \geq 0, \forall t \quad (45)$$

$$0 \leq \pi_t^+ \perp (E^{max} - E_t) \geq 0, \forall t \quad (46)$$

The set of decision variables (26) includes the decision variables of the upper level and the lower level problem as well as the Lagrangian multipliers associated with the constraints of the lower level problem. The KKT optimality conditions of the latter correspond to equations (28)-(46).

V. Case Studies

A. Test Data and Implementation

The examined studies involve a test market with day-ahead horizon and hourly resolution. The market includes 7 generation companies, the cost coefficients and maximum output limits of which are given in Table I. Fig. 2 presents the assumed hourly values of the linear benefit coefficient b_t^D and the maximum demand d_t^{max}, which follow the daily pattern of consumers' activities. Based on the formulation of the marginal benefit function (6), the hourly values of the quadratic benefit coefficient are calculated as $c_t^D = b_t^D / 2d_t^{max}$. In order to analyze the impact of energy storage, different scenarios are examined regarding the size of energy storage, as expressed by its capacity E^{cap} as a percentage α of the daily energy demand. The assumed values of the rest of energy storage operational parameters are given in Table II.

The MPEC problem has been coded and solved using the optimization software FICO™ Xpress [18] on a computer with a 6-core 3.47 GHz Intel(R) Xeon(R) X5690 processor and 192 GB of RAM. The average computational time required for solving the MPEC problem across all the examined scenarios was around 10s.

TABLE I. Generation Companies Parameters

Generation company i	1	2	3	4	5	6	7
b_i^G (£/MW)	10	15	23	35	50	70	100
c_i^G (£/MW^2)	0.0001	0.0006	0.0014	0.0026	0.0042	0.0065	0.001
g_i^{max} (MW)	13,170	11,520	7,560	6,670	6,500	5,760	5,500

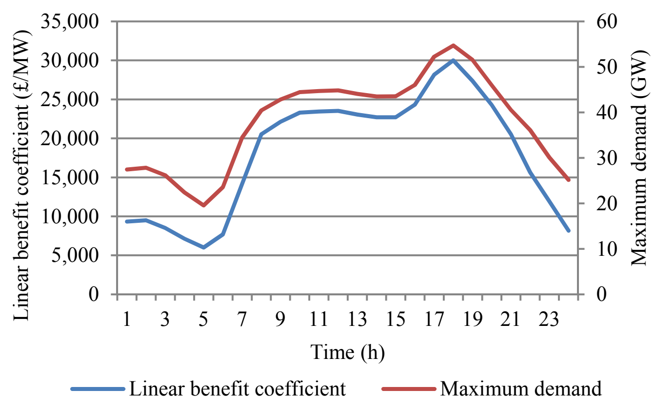

Figure 2. Hourly values of linear benefit coefficient and maximum demand.

TABLE II. Energy Storage Parameters

Parameter	E^{min}	E^{max}	E_0	s^{max}	η^c	η^d
Value	$0.2E^{cap}$	E^{cap}	$0.25E^{cap}$	$0.5E^{max}$/1h	0.9	0.9

B. Impact of Energy Storage on Generation Market Power

The exercise of market power by the generation side increases its profit while it decreases the utility of the demand side. Fig. 3 and 4 present the increment of the generation side's hourly profit and the decrement of the demand side's hourly utility driven by the exercise of market power, i.e. the difference between the profit / utility obtained under strategic generation behavior (as determined by the solution of the MPEC problem (25)-(46)) and the profit / utility obtained under competitive generation behavior (as determined by the solution of the market clearing problem (15)-(24) with $k_{i,t} = 1, \forall i, \forall t$), for different scenarios of energy storage capacity. Due to the reason comprehensively explained in Section III, energy storage reduces the hourly generation profit increment / demand utility decrement during peak hours and increases it during off-peak hours, with the former reduction being significantly higher than the latter increase. These effects are enhanced as the size of energy storage is increased.

Due to the fact that the positive impact of energy storage during peak hours is more significant than its negative impact during off-peak hours, the total (daily) generation profit increment and demand utility decrement driven by the exercise of market power are significantly reduced as the size of energy storage is increased, as illustrated in Fig. 5. This result means that deployment of energy storage reduces the generation profit made by the exercise of market power, and allows consumers to more efficiently preserve their economic surplus against generation companies' strategic behavior.

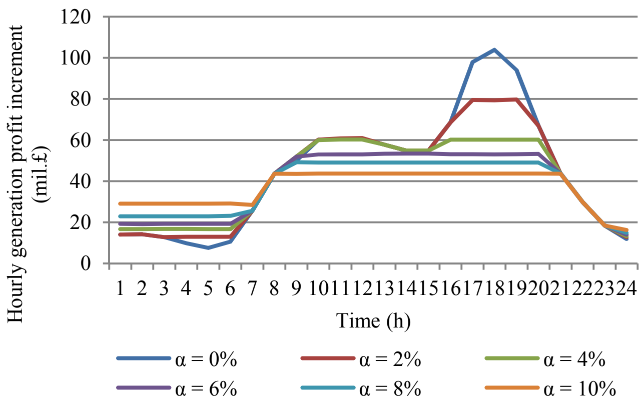

Figure 3. Hourly generation profit increment driven by the exercise of market power for different energy storage scenarios.

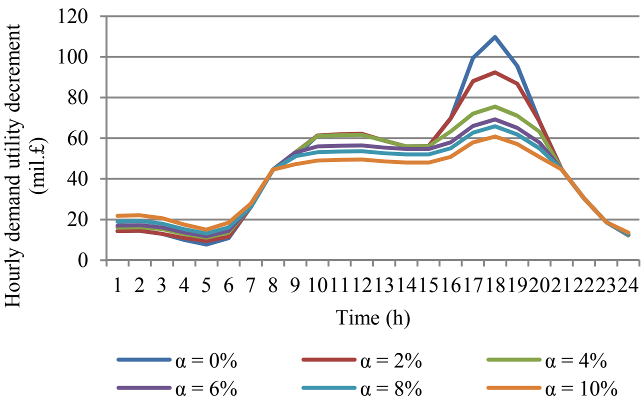

Figure 4. Hourly demand utility decrement driven by the exercise of market power for different energy storage scenarios.

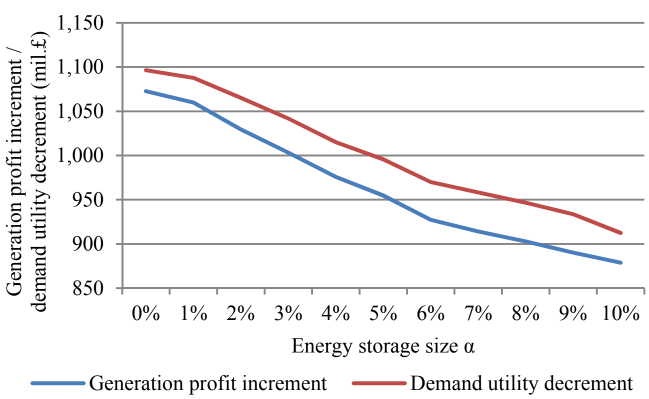

Figure 5. Generation profit increment and demand utility decrement for different energy storage scenarios.

VI. Conclusions

This paper has provided for the first time theoretical and quantitative evidence of the beneficial impact of energy storage in limiting market power by strategic generation companies. Theoretical explanation of this impact has been presented through a simple price-quantity graph, demonstrating that storage reduces the extent of exercised market power at peak periods and increases it at off-peak periods, with the former reduction dominating the latter increase and resulting in an overall positive impact. Quantitative analysis has been supported by a bi-level optimization model of imperfect electricity markets, accounting for the time-coupling operational constraints of energy storage and solved by converting it to an MPEC. Case studies with this MPEC model on a test market with day-ahead horizon and hourly resolution have quantitatively demonstrated the benefits of storage in limiting market power. An increasing storage capacity has been shown to reduce the generation profit made by the exercise of market power, and allow consumers to more efficiently preserve their economic surplus against generation companies' strategic behavior.

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